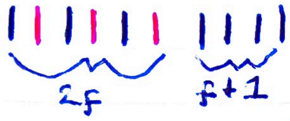
*if $n=1$ theorem 1 holds trivially

THEOREM 1 View sychronisation - There exists infinite views with honest leaders, where all honest replicas will simaltaneously be in that view for long enough to make progress.

LEMMA 1 There exists infinite consecutive assignments of two honest leaders to views. That is, we can always find future views V_1 and V_2 with honest leaders l_1 and l_2.



We have a round-robin system for choosing leaders. If we attempt to alternate honest and byzantine leaders, there will always be $f+1$ consecutive honest leaders at the end. Even if $f=0$ there will always be at least 2 consecutive honest leaders*.

LEMMA 2 Honest leader l_1 will eventually enter V_1 (as defined in lemma 1).

For any $V < V_1$, leader l_1 will eventually transition out of it. This can happen in these ways

- l_1 receives a proposal - line 23 or line 31
- l_1 receives a quarum of votes - line 40
- The honest replicas timeout and complain to the next leader. If the next leader does not progress they timeout and complain again. Eventually they will complain to an honest leader that sends a NEXTVIEW (line 28), advancing l_1.

Each advancement of view requires a quorum, so a byzantine node cannot `skip' l_1 past V_1. Hence l_1 will eventually enter V_1.

LEMMA 3 Once l_1 enters V_1 (lemma 2), there will be some view with an honest leader and all honest replicas in that view simaltaneously.

We consider each ~~case for two~~ line where l_1 could have entered V_1 in turn.

+ where δ is the bound on latency once GST is reached.

A1 LINE 23: This can only occur if l_1 receives a proposal from the leader of V_0. Since l_1 is the leader of V_1 it could not have entered V_1 this way.

A1 LINE 31: This can only occur if l_1 is not the next leader (that is, the leader of V_1). Since it is, this cannot occur.

A1 LINE 40: l_1 receives a quorum of votes from $V_1 - 1$.
l_1 will then broadcast this QC in its proposal, which all honest replicas will receive by δ^+. All honest replicas except l_2 will transition to V_2 and send a vote to l_2, and l_2 itself will transition to V_2 once it receives a quorum. N.B. the honest replicas must vote for the proposal since it is safe - it has been proposed by an honest leader.
Hence all honest replicas will simultaneously be in V_2.

A2 LINE 31: l_1 receives a quorum of COMPLAINs from itself.
l_1 must have sent this to all replicas (line 28), so all honest replicas will receive it after δ has elapsed and transition to all be in V_1 simultaneously.

In any case, all honest replicas will simultaneously be in V_1 or V_2.

LEMMA 4 Once all honest replicas enter a view V with an honest leader they will have sufficient time to progress.
To exit V a replica must either:
→ A1 L23: Receive a higher proposal with a QC, such a ~~proposal~~ QC cannot exist as all honest replicas are currently in V.
→ A1 L31: Receive a proposal from the honest leader, this means they have had time to make progress
→ A1L40: Be the next leader and receive a quorum of votes. Again progress has been made.
→ A2 L31: Receive a quorum of ~~complains~~ COMPLAINs. This should not happen is the timeout is sufficiently long.
In any case progress is made.

Theorem 1 follows from lemmas ~~[illegible]~~ 1 through 4 ~~etc~~. □

~~THEOREM 2 Synchronisation [illegible] validity – A view will only be entered if some honest node wants* it to.~~

~~On line 31 (algorithm 1) a replica may advance itself once it has made progress, so an honest replica *~~
~~In all other cases of a view advancing a quorum is required, so at least one honest replica wishes the new view to be entered.~~

~~* advances itself only if it wishes to.~~

~~* Our algorithm doesn't quite match the formalism in the cogsworth paper~~

THEOREM 2 Synchronisation validity – The pacemaker will only advance the state* if at least one honest ~~replica~~ consensus machine wishes it to be advanced.

~~*of one honest [illegible] consensus machine~~

This holds trivially for the 3 calls to ONNEXTSYNCVIEW in algorithm 1. The consensus machine commands the pacemaker to advance its own state, so if it is honest then there is one honest state machine that wishes the state to be advanced.

The only other way the view can be advanced is on line 31 of algorithm 2. This requires a quorum of COMPLAIN messages, so at least one honest state machine wishes the view to be advanced. □