Learning brain connectivity using tensor-response regression

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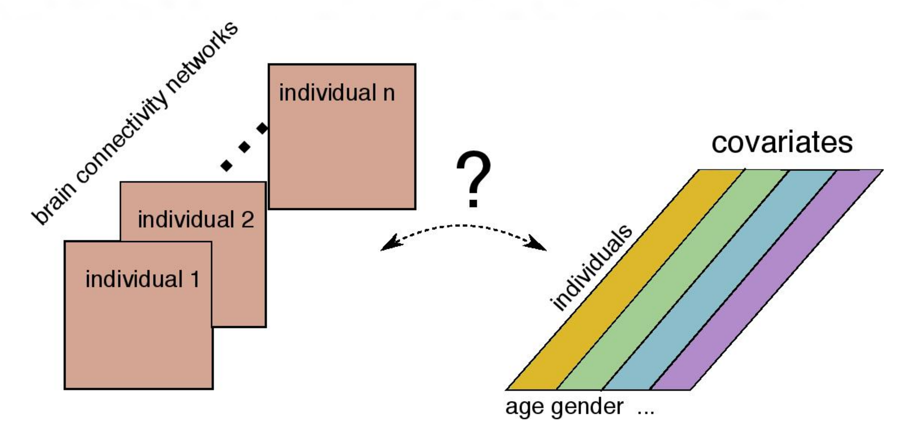
Motivation

Network response model is recently developed in the context of neuroimaging analysis. Data in form of multidimensional arrays, a.e. tensors has been gaining increasing attentions in recent years. Tensor is a effective way to represent the underlying structure in multidimension data. Several usage:

- Network population model. Study the relationship between the network-valued response with the individual covariates.

$$\operatorname{logit}\left(\mathbb{E}\left(\boldsymbol{Y}_i|\boldsymbol{x}_i\right)\right)=\mathcal{B}\times_3\boldsymbol{x}_i,\quad \text{for } i=1,\ldots,n$$



- Link model with node attributes. Study the relationship between two certain individuals based on their side information.

$$\operatorname{logit}\left(\mathbb{P}((i,j)\in E)\right)=\boldsymbol{x}_i^T\boldsymbol{B}\boldsymbol{x}_j=\left\langle\boldsymbol{B},\boldsymbol{x}_i^T\boldsymbol{x}_j\right\rangle$$

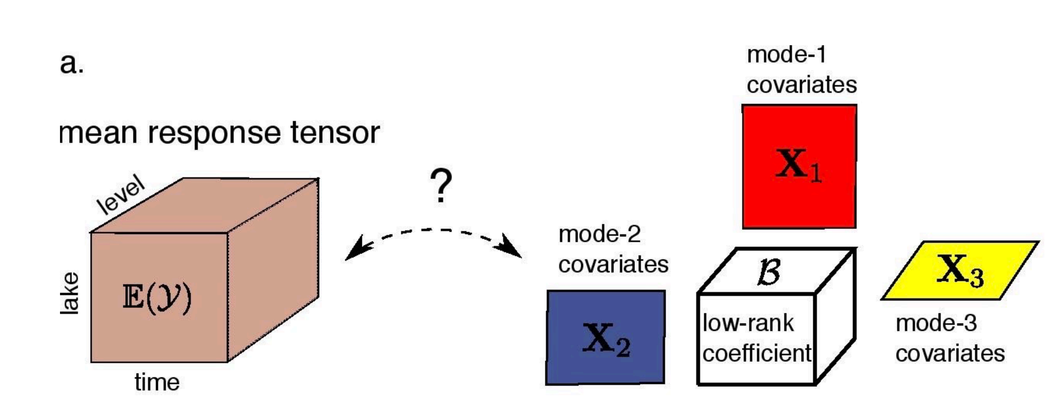
Model

Low dimension on tensor response is our key assumption. Inspired by normal form regression, model of our tensor-response generalized regression is:

$$\operatorname{logit}(\mathbb{E}\mathcal{Y})=\mathcal{U}=\mathcal{B}\times_1 X_1\times_2 X_2\times_3 X_3$$

$$\mathcal{B}=\mathcal{G}\times_1 W_1\times_2 W_2\times_3 W_3$$

Where X_1, X_2, X_3 is the covariate on three modes. The $\mathcal{B}$ is the coefficient tensor. $\mathcal{U}$ is the ground truth indicate the probability of binary data tensor.



Theorem Statement

$$\operatorname{Loss}\left(\mathcal{B}_{true},\hat{\mathcal{B}}\right)\leq\frac{2}{c_1^{2K}}\min\left\{\frac{C(\boldsymbol{r},\alpha)\sum_k p_k}{\prod_k d_k},2\alpha^2\right\}$$

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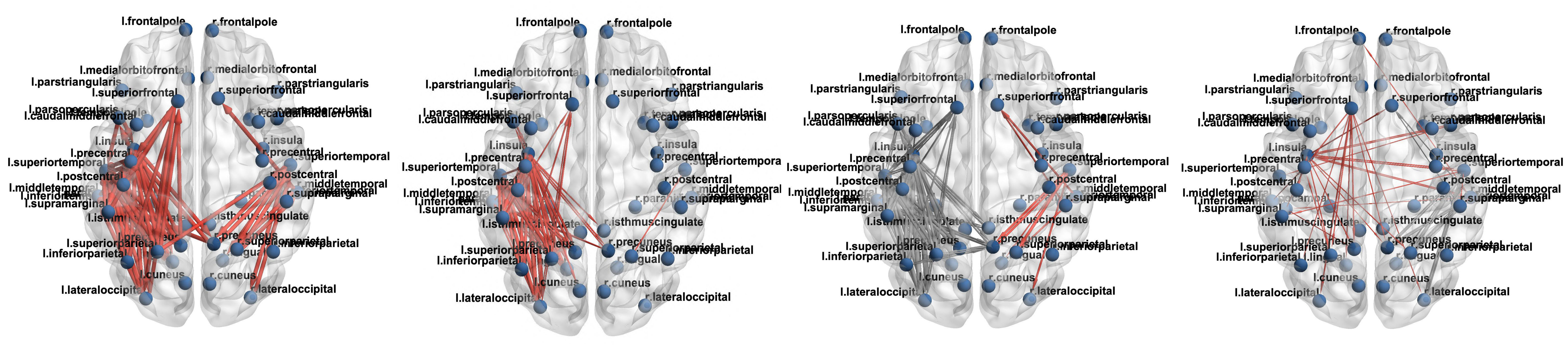
Global physical brain connectivity network pattern and local brain regions affected by age/gender can be identified by tensor regression model.

Brain Net for Global Connectivity and Significant Areas affected age/gender



(a) Global Connectivity (b) Age 26-30 (c) Age 31+ (d) Gender(Male)

Figure 1 : (a) Global Brain Connectivity; (b) Significant regions activated/non-activated when age is 26-30; (c) Significant regions activated/non-activated when age is 31+; (d) Significant regions activated/non-activated in males compared with female.

Numerical Result for representative active regions in Global pattern and affected by age/gender

Table 1: Representative active regions in Global Connectivity

Active Area		Index*		Coefficient	rank
Left. Superior Temporal	*Left. Lateral Occipital*	11	24	101.26	1
		10	34		2
Left. Insula	*Left. Supra-marginal*	3	21	93.00	7
		3	32		8
Left. Superior Temporal	*Left. Supra-marginal*	11	21	91.75	12
		32	22		15
Right. Superior Parietal	*Right. Superior Temporal*	53	44	65.38	46
		53	45		46

Generally, connections within each Hemisphere are more significant than connections cross two Hemispheres.

Table 2: Representative active regions affected by age 26-30

Active Area		Index*		Coefficient	rank
Left. Lateral Occipital	*Left. Insula*	24	3	2.75	1
		34	3		2
Left. Superior Temporal	*Left. Lateral Occipital*	11	24	2.37	3
		10	34		5
Left. Superior Temporal	*Left. Lateral Occipital*	10	24	2.27	11
		11	34		14

More connections among Left Hemisphere appear in people of Age 26-30.

Table 3: Representative active regions affected by Age 31+

Active Area		Index*		Coefficient	rank
Right. Superior Parietal	*Right. Superior Temporal*	53	44	1.80	1
		44	53	0.75	20
Right. Precuneus	*Right. Superior Temporal*	62	44	1.32	2
		62	45	1.32	2
Right. Superior Frontal	*Right. Middle Temporal*	38	46	1.28	3
		60	46	1.28	4

Some Right Hemisphere connections are strengthen in people of Age 31+, while well connected regions in Age 26-30 turn to non-activated.

Table 4: Representative active regions affected by gender

Active Area		Index*		Coefficient	rank
Left. Lateral Occipital	*Left. Superior Frontal*	24	4	0.89	1
		24	26	0.89	2
Left. Precuneus	*Left. Superior Frontal*	28	4	0.83	2
		26	4	0.83	3
Right. Superior Temporal	*Left. Insula*	45	3	0.69	10
		67	3	0.69	10

Several Left Hemisphere connections are more connected in males than females.