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Section 002

Project 03 – Network Routing – Write Up

1. **Code:**

```
        //-------------------------------------------------------------------------------//
        //------------------------- Dijkstra's Algorithm --------------------------------//
        //-------------------------------------------------------------------------------//


        /**
         * This Dijkstra's is implemented based off of the pseudocode found in the text.
         * Time complexity for the Binary Heap Priority queue is O(|V| * |logV|)
         * because the implementation requires all the nodes to be visited but the
         * update methods require |log V| time.  The array priority queue is O(|V|^2)
         * because it has to visit all the nodes on implementation and has to itterate
         * through the nodes in order to delete and update the queue.  The space
complexity
         * for both is O(|V|) because each queue stores all of the nodes generated
         **/
        private List<int> dijkstrasAlgorithm(PriorityQueue queue, bool isArray)
        {
            queue.makeQueue(points.Count);

            List<int> previous = new List<int>();
            List<double> distances = new List<double>();

            // Sets all distance values to "infinity"
            // sets previous as 'undefined'
            for(int i = 0; i < points.Count; i++)
            {
                previous.Add(-1);
                distances.Add(double.MaxValue);
            }

            // distance to start node
            distances[startNodeIndex] = 0;

            // checks which priority queue will be used
            if(isArray)
            {
                queue.insert(startNodeIndex, 0);
            }
            else
            {
                queue.insert(ref distances, startNodeIndex);
            }

            // main loop
            // loops until there arent any nodes with a permanent distance to the end
node
            while(!queue.isEmpty())
            {
                int minIndex;

                if(isArray)
```

```
                {
                    minIndex = queue.deleteMin();
                }
                else
                {
                    minIndex = queue.deleteMin(ref distances);
                }

                PointF u = points[minIndex];

                // finds the best path amongst its neighbors, if it exists
                foreach(int index in adjacencyList[minIndex])
                {
                    PointF alt = points[index];
                    double altDistance = distances[minIndex] + distanceBetween(u, alt);

                    if(altDistance < distances[index])
                    {
                        previous[index] = minIndex;
                        distances[index] = altDistance;

                        if(isArray)
                        {
                            queue.decreaseKey(index, altDistance);
                        }
                        else
                        {
                            queue.decreaseKey(ref distances, index);
                        }
                    }
                }
            }

            //queue.printQueue();

            // gives the path of the nodes
            return previous;
        }

//--------------------------- Helper Method ----------------------------------//

        private double distanceBetween(PointF u, PointF v)
        {
            double deltaXSqr = Math.Pow(v.X - u.X, 2);
            double deltaYSqr = Math.Pow(v.Y - u.Y, 2);
            double distance = Math.Sqrt(deltaXSqr + deltaYSqr);
            return distance;
        }

        //------------------------------------------------------------------------//
        //--------------------------- Draw Methods -------------------------------//
        //------------------------------------------------------------------------//

        private PointF findMidPointOfPoints(int firstIndex, int secondIndex)
        {
            PointF midpoint = new PointF();

            midpoint.X = (points[firstIndex].X + points[secondIndex].X) / 2;
```

```
            midpoint.Y = (points[firstIndex].Y + points[secondIndex].Y) / 2;

            return midpoint;
        }

        // draws the shortest path
        private void drawPath(ref List<int> path)
        {
            int cur = stopNodeIndex;
            int previousIndex = cur;
            double totalPathCost = 0;

            Console.Write("path nodes are: ");
            foreach(int num in path)
            {
                Console.Write(num + ", ");
            }
            Console.WriteLine();

            while(true)
            {
                cur = path[cur];
                if (cur == -1) break;

                Pen pen = new Pen(Color.Black, 1);
                graphics.DrawLine(pen, points[cur], points[previousIndex]);

                double distance = distanceBetween(points[cur], points[previousIndex]);
                totalPathCost += distance;

                PointF midPoint = findMidPointOfPoints(previousIndex, cur);

                int distanceTruncated = (int)distance;

                graphics.DrawString(distanceTruncated.ToString(),
SystemFonts.DefaultFont, Brushes.Black, midPoint);
                previousIndex = cur;
            }

            pathCostBox.Text = totalPathCost.ToString();
        }
```

2. Priority Queue Classes

```
    /**
     * Abstract class that is implemented by HeapPriorityQueue and ArrayPriorityQueue
     **/

    abstract class PriorityQueue
    {
        public PriorityQueue() { }
        public abstract void makeQueue(int nodeNum);
        public virtual int deleteMin() { return 0; }
        public virtual int deleteMin(ref List<double> distances) { return 0; }
        public virtual void decreaseKey(int index, double key) { }
        public virtual void decreaseKey(ref List<double> distances, int index) { }
        public virtual void insert(int index, double value) { }
        public virtual void insert(ref List<double> distances, int index) { }
```

```
        public abstract void printQueue();
        public abstract bool isEmpty();
    }

class HeapPriorityQueue : PriorityQueue
    {
        private int capacity;
        private int count;
        private int lastElement;
        private int[] distancesHeap;
        private int[] pointers;

        public HeapPriorityQueue() { }

        /**
         * Is empty is time complexity of O(1) because it is a comparison.  There is no
space complexity.
         **/
        public override bool isEmpty()
        {
            return count == 0;
        }

        /**
         * This is for debugging purposes and is of time complexity O(|V|) because it
itterates through all
         * the nodes in the distanceHeap array. There is no space complexity.
         **/
        public override void printQueue()
        {
            Console.Write("Contents of Heap are: ");

            for (int i = 1; i < capacity; i++)
            {
                if (distancesHeap[i] != -1) Console.Write(distancesHeap[i] + ", ");
            }
            Console.WriteLine();
        }

        /**
         * Here both time and space complexity is O(|V|). Time is such because it
itterates through the
         * nodes to assign values in increasing number.  Space is of the same order
because there are
         * two arrays that are allocated to the size V.
         **/
        public override void makeQueue(int nodeNum)
        {
            distancesHeap = new int[nodeNum + 1];
            pointers = new int[nodeNum];

            for (int i = 1; i < nodeNum + 1; i++)
            {
                distancesHeap[i] = i - 1;
                pointers[i - 1] = i;
            }

            // distancesHeap[0] = -1;
```

```
            capacity = nodeNum;
            count = 0;
            lastElement = capacity;
        }

        /**
         * Time complexity here is O(|logV|) because we  know the minimum node and then
we just have to
         * bubble up and sort the tree to accomadate for deleting the root. The worse
case scenario
         * would be the height of the tree which is |logV|.  There is no space complexity
because there
         * are no new space allocations
         **/
        public override int deleteMin(ref List<double> distancesArray)
        {
            int minValue = distancesHeap[1];
            count--;

            if (lastElement == -1) return minValue;

            distancesHeap[1] = distancesHeap[lastElement];
            pointers[distancesHeap[1]] = 1;
            lastElement--;

            int whileIndex = 1;
            while (whileIndex <= lastElement)
            {
                int leftChildIndex = whileIndex * 2;
                if (leftChildIndex > lastElement) break;

                if (leftChildIndex + 1 <= lastElement &&
distancesArray[distancesHeap[leftChildIndex + 1]] <
distancesArray[distancesHeap[leftChildIndex]])
                {
                    leftChildIndex++;
                }

                if (distancesArray[distancesHeap[whileIndex]] >
distancesArray[distancesHeap[leftChildIndex]])
                {
                    int temp = distancesHeap[leftChildIndex];
                    distancesHeap[leftChildIndex] = distancesHeap[whileIndex];
                    distancesHeap[whileIndex] = temp;

                    pointers[distancesHeap[leftChildIndex]] = leftChildIndex;
                    pointers[distancesHeap[whileIndex]] = whileIndex;
                }

                whileIndex = leftChildIndex;
            }

            return minValue;
        }

        /**
         * Time complexity is O(|logV|) because it finds the new distance to update,
switches it
```

```
         * and then updates the tree like in delete min.  This gives the worse case to be
the height
         * of the tree, |logV|. There is no space complexity because there is no new
space allocations.
         **/
        public override void decreaseKey(ref List<double> distancesArray, int index)
        {
            int heapIndex = pointers[index];
            count++;

            int whileIndex = heapIndex;
            while (whileIndex > 1 && distancesArray[distancesHeap[whileIndex / 2]] >
distancesArray[distancesHeap[whileIndex]])
            {
                int temp = distancesHeap[whileIndex / 2];
                distancesHeap[whileIndex / 2] = distancesHeap[whileIndex];
                distancesHeap[whileIndex] = temp;


                pointers[distancesHeap[whileIndex / 2]] = whileIndex / 2;
                pointers[distancesHeap[whileIndex]] = whileIndex;

                whileIndex = whileIndex / 2;
            }
        }


        /**
         * Insert has a time complexity of O(|logV|) because it inserts the given node
         * and updates the tree by bubbling up.  |log V| is the height of the tree so
         * that would be the worse case scenario.  There is no space complexity because
         * there are no new space alloations.
         **/
        public override void insert(ref List<double> distances, int pointerIndex)
        {
            count++;

            int whileIndex = pointers[pointerIndex];
            while (whileIndex > 1 && distances[distancesHeap[whileIndex / 2]] >
distances[distancesHeap[whileIndex]])
            {
                int temp = distancesHeap[whileIndex / 2];
                distancesHeap[whileIndex /2] = distancesHeap[whileIndex];
                distancesHeap[whileIndex] = temp;

                pointers[distancesHeap[whileIndex / 2]] = whileIndex / 2;
                pointers[distancesHeap[whileIndex]] = whileIndex;

                whileIndex = whileIndex / 2;
            }
        }
    }

class ArrayPriorityQueue : PriorityQueue
    {
        private double[] queue;
        private int count;
```

```
        public ArrayPriorityQueue() { }

        /**
         * Time Complexity is O(1). There is no space complexity.
         * */
        public override bool isEmpty()
        {
            return count == 0;
        }

        /**
         * makeQueue time and space complexity is O(|V|) because it itterates through all
the nodes.
         * it also makes a new array the same size of the number of nodes.
         * @Param the number of nodes generated
         * */
        public override void makeQueue(int nodeNum)
        {
            queue = new double[nodeNum];
            count = nodeNum;

            for(int i = 0; i < nodeNum; i++)
            {
                queue[i] = double.MaxValue;
            }
        }

        /**
         * Print queue method for debugging
         * If it needs to be included because the instructions say so it is of time
complexity O(N)
         * where N is the length of the queue
         **/
        public override void printQueue()
        {
            Console.WriteLine("Queue: ");
            for(int i = 0; i < queue.Length; i++)
            {
                Console.Write(i + ": " + queue[i] + ", ");
            }
            Console.WriteLine();
        }

        /**
         * Time Complexity is O(1) because it just puts the key in the queue at the given
index.
         * There is no space complexity.
         **/
        public override void decreaseKey(int index, double key)
        {
            queue[index] = key;
        }

        /**
         * The time complexity is O(|V|) because it itterates through all the nodes in
the system.
         * There is no space complexity because there is no new arrays made.
         **/
```

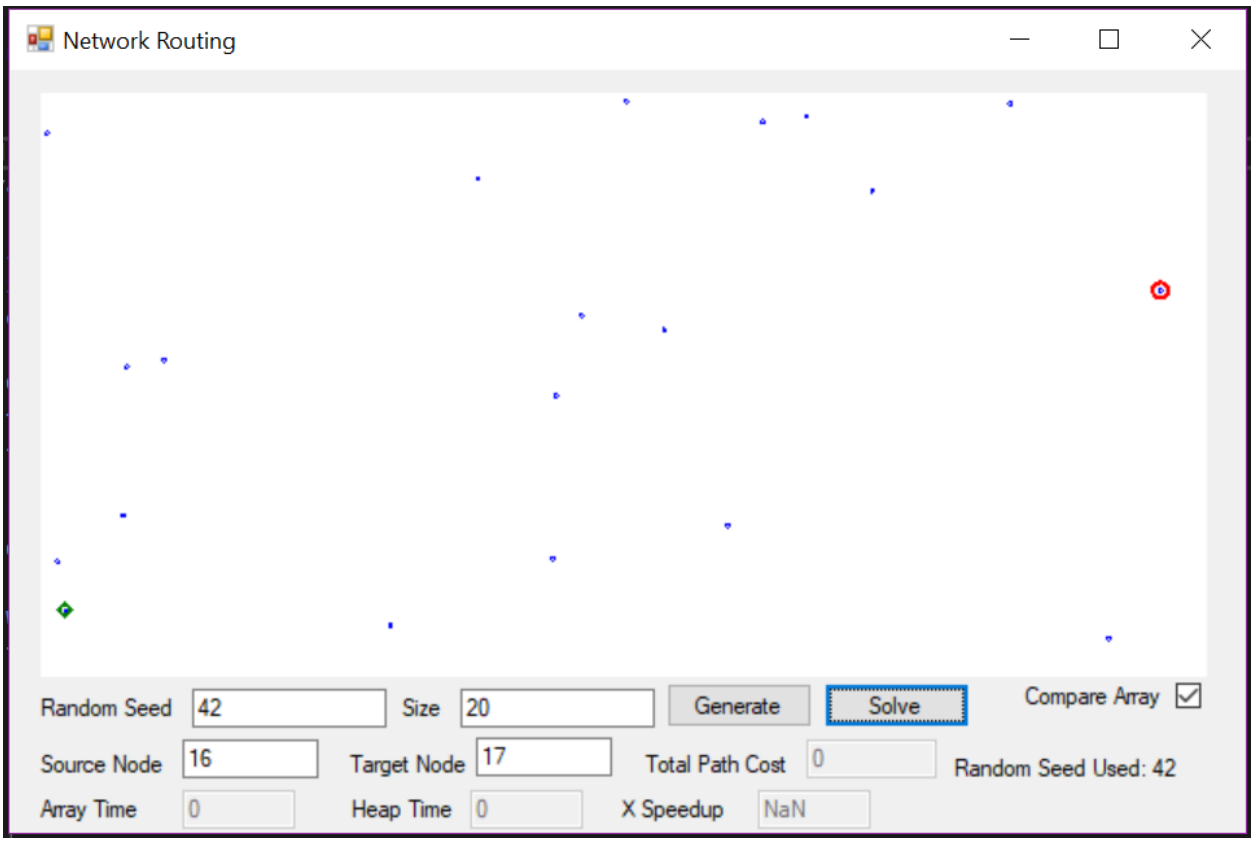
```
        public override int deleteMin()
        {
            double minValue = double.MaxValue;
            int minIndex = 0;

            for(int i = 0; i < queue.Count(); i++)
            {
                if(queue[i] < minValue)
                {
                    minValue = queue[i];
                    minIndex = i;
                }
            }
            count--;
            queue[minIndex] = double.MaxValue;

            return minIndex;
        }

        /**
         * The time complexity of the insert method is O(1) because it directly inserts
into the
         * array at the given index.  There is no space complexity because there are no
new
         * allocations.
         **/
        public override void insert(int index, double value)
        {
            queue[index] = value;
            count++;
        }
    }
```

3. Time and Space Complexity

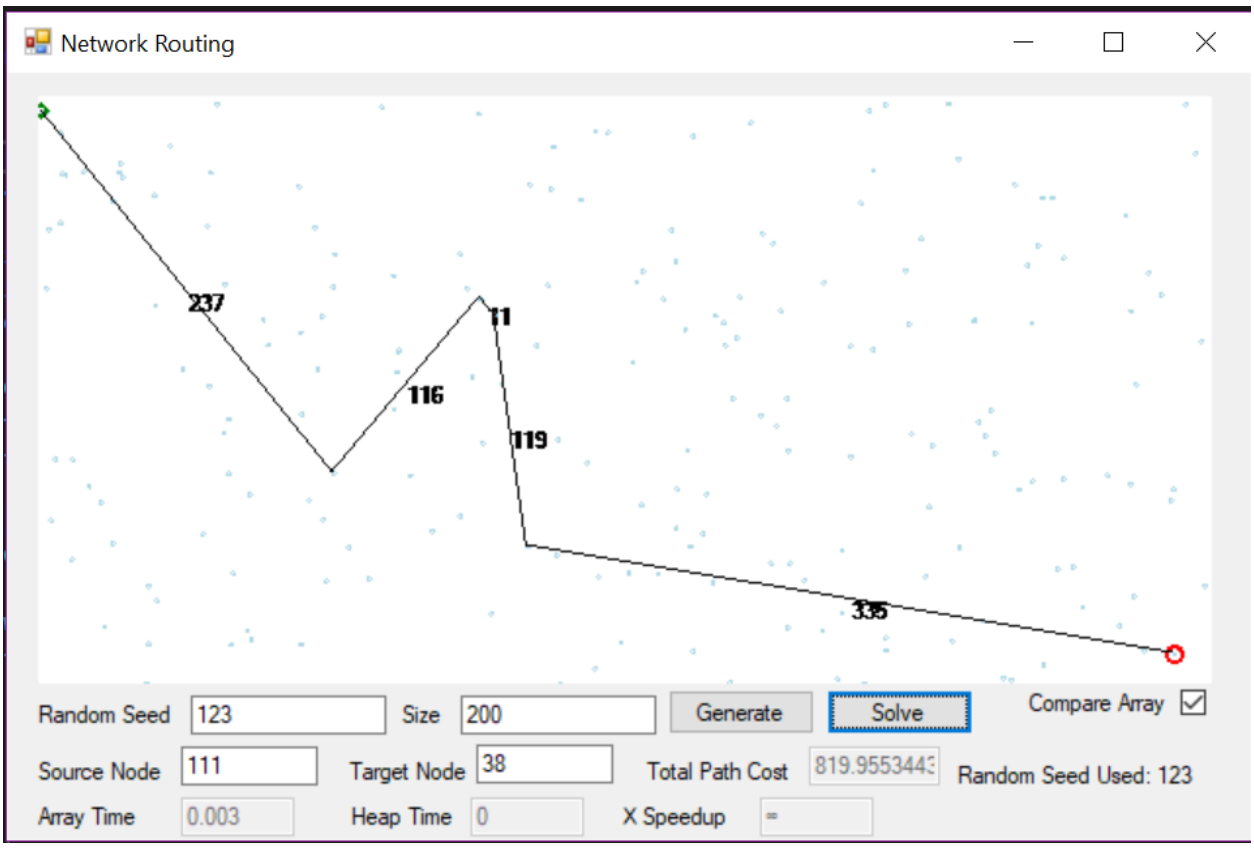
```
/**
         * This Dijkstra's is implemented based off of the pseudocode found in the text.
         * Time complexity for the Binary Heap Priority queue is O(|V| * |logV|)
         * because the implementation requires all the nodes to be visited but the
         * update methods require |log V| time.  The array priority queue is O(|V|^2)
         * because it has to visit all the nodes on implementation and has to itterate
         * through the nodes in order to delete and update the queue.  The space
complexity
         * for both is O(|V|) because each queue stores all of the nodes generated
         **/
```

4. Screenshots

Random seed 42 – Size 20



Random seed 123 – Size 200

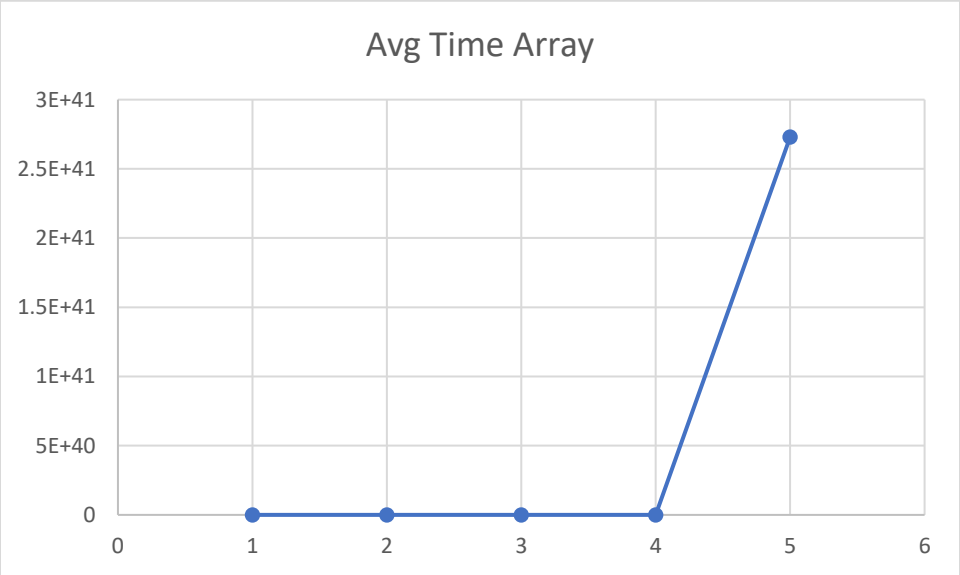


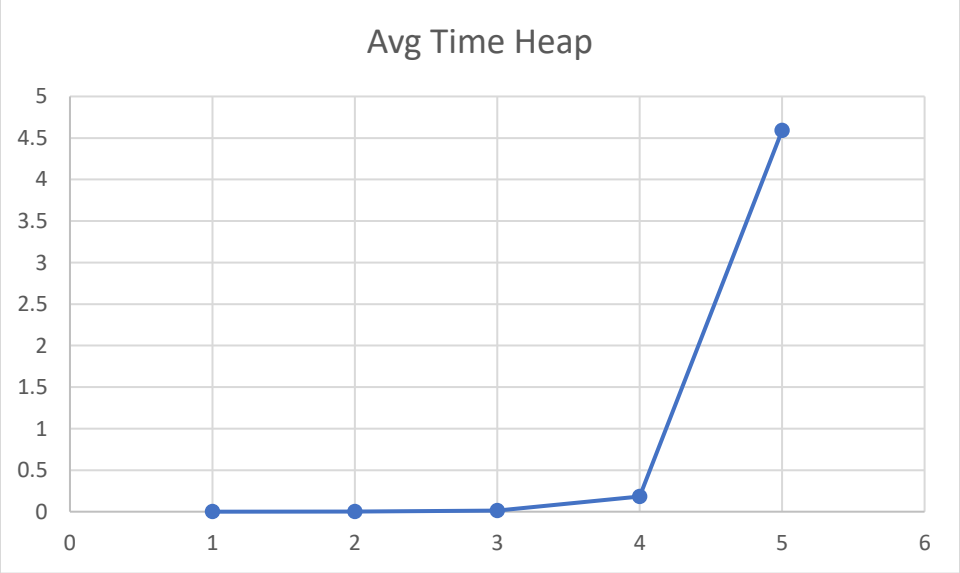
5. Empirical Analysis

Array						
Nodes	1	2	3	4	5	Avg
100	0.001	0	0	0	0	0.0002
1000	0.056	0.058	0.054	0.055	0.064	0.0574
10000	4.367	4.348	4.328	4.521	4.478	4.4084
100000	99707.85	103278.3	127755.8	174552.8	161706.2	133400.19
1000000	9.77E+39	1.29E+40	1.29E+40	8.62E+41	4.68E+41	2.73E+41

Heap						
Nodes	1	2	3	4	5	Avg
100	0.001	0	0	0	0	0.0002
1000	0.001	0.001	0.001	0.001	0.001	0.001
10000	0.015	0.012	0.013	0.015	0.015	0.014
100000	0.179	0.184	0.177	0.174	0.192	0.1812
1000000	4.606	4.487	4.58	4.553	4.73	4.5912

Number Nodes	Avg Time Array	Avg Time Heap
100	0.0002	0.0002
1000	0.0574	0.001
10000	4.4084	0.014
100000	133400.19	0.1812
1000000	2.73E+41	4.5912





The results look like they are both square functions, but that's not true looking at the time on the y axis. The array method would have taken about a lifetime to finish where as the heap implementation finished it a lot quicker. I was surprised at how dramatic the difference was between the two. It is really eye opening because the array queue is really simple to implement but not practical for scaling.