1 Syntax

$i \in \text{Integer} ::= \mathbf{0} \mid \mathbf{1} \mid \mathbf{\text{-}1} \mid \mathbf{2} \mid \mathbf{\text{-}2} \mid \ldots$

$id \in \text{Identifier} ::= \ldots$

$str \in \text{String} ::= \ldots$

$p \in \text{PythonCode} ::= \ldots$

$sP \in \text{SubProgram} ::= \mathbf{sp}_{id} \mid \ldots$

$rsP \in \text{RawSubProgram} ::= \mathbf{`}str\mathbf{`}$

$s \in \text{State} ::= \mathbf{s}_{id}$

$\Gamma \in \text{Environment} ::= \mathbf{\Gamma}_{id}$ (in code: G)

$c \in \text{Construction} ::= rsP \mid s \mid \Gamma \mid sP$

$cl \in \text{ConstructionList} ::= cl_1 \ \mathbf{,}\ cl_2 \mid c \mid \varepsilon$

$ap \in \text{ApplyPredicate} ::= id\mathbf{(}\ cl_1 \mid cl_2\ \mathbf{)}$

$d \in \text{DefinePredicate} ::= id\mathbf{(}\ cl_1 \mid cl_2\ \mathbf{)\{}\ p\ \mathbf{\}}$

$tr \in \text{Transition} ::= \mathbf{<}c_1\ \mathbf{,}\ s_1\mathbf{>} \rightarrow \mathbf{<}c_2\ \mathbf{,}\ s_2\mathbf{>} \mid \mathbf{<}c\ \mathbf{,}\ s_1\mathbf{>} \rightarrow s_2$ (in code: $=>$)

$uo \in \text{UpOpSemRule} ::= uo_1\ uo_2 \mid \text{ap} \mid \text{tr} \mid \varepsilon$

$rel \in \text{Relation} ::= \mathbf{:} \mid \mathbf{<:}$

$ty \in \text{Typing} ::= \Gamma \vdash c_1\ rel\ c_2$ (in code: $|-$)

$ut \in \text{UpTypingRule} ::= ut_1\ ut_2 \mid ap \mid ty \mid \varepsilon$

$P \in \text{Program} ::= P_1P_2 \mid R \mid D \mid \mathbf{\{}\ P\ \mathbf{\}} \mid \mathbf{!popEnv} \mid code$

$r \in \text{Rule} ::= S \mid O \mid T$

$rs \in \text{SyntaxRule} ::= \mathbf{syntax}(id)\mathbf{\{}\ p\ \mathbf{\}}$

$ro \in \text{OperationalSemanticsRule} ::= \mathbf{semantics}(id)\mathbf{\{}\ \dfrac{uo}{tr}\ \mathbf{\}}$ (in code: $---$)

$rt \in \text{TypingRule} ::= \mathbf{typing}(id)\mathbf{\{}\ \dfrac{ut}{ty}\ \mathbf{\}}$

$code \in \text{Code} ::= \mathbf{code}\ rsP$

2 Semantics

State: $\hat{s} = \{[E_n, E_{n-1}, \ldots, E_0], s\}$ ({environments, program state : $s =< st_s, \Gamma_s >$})
Starting configuration (<construction, state>): $< P, \{[\emptyset], < \emptyset, \emptyset >\} >$ we expect state as pair of store, gamma
Environment: $E =< S_E, O_E, T_E, D_E >$- elements of environment E
Syntax Environment: S_E : Identifier $\rightharpoonup$ PythonCode
Operational Semantics Environment: O_E : Identifier $\rightharpoonup$ $\frac{\text{UpOpSemRule}}{\text{Transition}}$ (formally: UpOpSemRule × Transition)
Typing Environment: T_E : Identifier $\rightharpoonup$ $\frac{\text{UpTypingRule}}{\text{Typing}}$
Predicate Environment: D_E : Identifier $\rightharpoonup$ (ConstructionList | ConstructionList){ PythonCode }

Predefined system predicates: `PARSE(`S`, `rsP`, `sP`)`, `UNIFY-AND-RUN(`$(\ cl_1 \mid cl_2\ )$`, `$(\ cl_3 \mid cl_4\ )\{\ p\ \}$`)`

$$\frac{S \vdash \texttt{PARSE}(uo,\ uo') \qquad S \vdash \texttt{PARSE}(tr,\ tr')}{S \vdash \texttt{PARSE}(\ \frac{uo}{tr}\ ,\ \frac{uo'}{tr'}\ )}\ \text{P-RO} \qquad \frac{S \vdash \texttt{PARSE}(ut,\ ut') \qquad S \vdash \texttt{PARSE}(ty,\ ty')}{S \vdash \texttt{PARSE}(\ \frac{ut}{ty}\ ,\ \frac{ut'}{ty'}\ )}\ \text{P-RT}$$

$$\frac{S \vdash \texttt{PARSE}(uo_1,\ uo_1') \qquad S \vdash \texttt{PARSE}(uo_2,\ uo_2')}{S \vdash \texttt{PARSE}(uo_1\ uo_2,\ uo_1'\ uo_2')}\ \text{P-uo12} \qquad \frac{S \vdash \texttt{PARSE}(ut_1,\ ut_1') \qquad S \vdash \texttt{PARSE}(ut_2,\ ut_2')}{S \vdash \texttt{PARSE}(ut_1\ ut_2,\ ut_1'\ ut_2')}\ \text{P-ut12}$$

$$\frac{\texttt{PARSE}(S,\ c_1,\ c_1') \qquad \texttt{PARSE}(S,\ c_2,\ c_2')}{S \vdash \texttt{PARSE}(<c_1\ ,\ s_1> \rightarrow <c_2\ ,\ s_2>,\ <c_1'\ ,\ s_1> \rightarrow <c_2'\ ,\ s_2>)}\ \text{P-tr} \qquad \frac{\texttt{PARSE}(S,\ c,\ c')}{S \vdash \texttt{PARSE}(<c\ ,\ s_1> \rightarrow s_2,\ <c'\ ,\ s_1> \rightarrow c'\ ,\ s_2)}\ \text{P-trend}$$

$$\frac{\texttt{PARSE}(S,\ c_1,\ c_1') \qquad \texttt{PARSE}(S,\ c_2,\ c_2')}{S \vdash \texttt{PARSE}(\Gamma \vdash c_1\ rel\ c_2,\ \Gamma \vdash c_1'\ rel\ c_2')}\ \text{P-ty} \qquad \frac{S \vdash \texttt{PARSE}(cl_1,\ cl_1') \qquad S \vdash \texttt{PARSE}(cl_2,\ cl_2')}{S \vdash \texttt{PARSE}(cl_1\ ,\ cl_2,\ cl_1'\ ,\ cl_2')}\ \text{P-cl} \qquad \frac{\texttt{PARSE(S,}\ c,\ c')}{S \vdash \texttt{PARSE}(c,\ c')}\ \text{P-c}$$

$$\frac{\texttt{PARSE}(S,\ cl_1,\ cl_1') \qquad \texttt{PARSE}(S,\ cl_2,\ cl_2')}{S \vdash \texttt{PARSE}(id(\ cl_1 \mid cl_2\ ),\ id(\ cl_1' \mid cl_2'\ ))}\ \text{P-ap} \qquad \frac{\texttt{PARSE}(S,\ cl_1,\ cl_1') \qquad \texttt{PARSE}(S,\ cl_2,\ cl_2')}{S \vdash \texttt{PARSE}(id(\ cl_1 \mid cl_2\ )\{\ p\ \},\ id(\ cl_1' \mid cl_2'\ )\{\ p\ \})}\ \text{P-d}$$

$$\frac{< P_1, \{[E, \ldots], s\} > \rightarrow < P_1', \{[E', \ldots], s'\} >}{< P_1 P_2, \{[E, \ldots], s\} > \rightarrow < P_1' P_2, \{[E', \ldots], s'\} >}\ \text{SR-P1P2} \qquad \frac{< P_1, \{[E, \ldots], s\} > \rightarrow \{[E', \ldots], s'\}}{< P_1 P_2, \{[E, \ldots], s\} > \rightarrow < P_2, \{[E', \ldots], s'\} >}\ \text{SR-P1P2end}$$

$$\frac{E' =< S_E[id \mapsto p], O_E, T_E, D_E >}{< \textbf{syntax}(id)\{\ p\ \}, \{[E, \ldots], s\} > \rightarrow \{[E', \ldots], s\}}\ \text{SR-RS}$$

$$\frac{S \vdash \texttt{PARSE}(\ \frac{uo}{tr}\ ,\ \frac{uo'}{tr'}\ ) \qquad E' =< S_E, O_E[id \mapsto \frac{uo'}{tr'}\ ], T_E, D_E >}{< \textbf{semantics}(id)\{\ \frac{uo}{tr}\ \}, \{[E, \ldots], s\} > \rightarrow \{[E', \ldots], s\}}\ \text{SR-RO}$$

$$\frac{S \vdash \texttt{PARSE}(\ \frac{ut}{ty}\ ,\ \frac{ut'}{ty'}\ ) \qquad E' =< S_E, O_E, T_E[id \mapsto \frac{ut'}{ty'}\ ], D_E >}{< \textbf{typing}(id)\{\ \frac{ut}{ty}\ \}, \{[E, \ldots], s\} > \rightarrow \{[E', \ldots], s\}}\ \text{SR-RT}$$

$$\frac{S \vdash \texttt{PARSE}(id(\ cl_1 \mid cl_2\ )\{\ p\ \},\ id(\ cl_1' \mid cl_2'\ )\{\ p\ \}) \qquad E' =< S_E, O_E, T_E, D_E[id \mapsto (\ cl_1' \mid cl_2'\ )\{\ p\ \}] >}{< id(\ cl_1 \mid cl_2\ )\{\ p\ \}, \{[E, \ldots], s\} > \rightarrow \{[E', \ldots], s\}}\ \text{SR-D}$$

$$\frac{}{< \{\ P\ \}, \{[E, \ldots], s\} > \rightarrow < P\ \textbf{!popEnv}, \{[E, E, \ldots], s\} >}\ \text{SR-\{P\}}$$

$$\frac{}{< \textbf{!popEnv}, \{[E_n, E_{n-1}, \ldots], s\} > \rightarrow \{[E_{n-1}, \ldots], s\}}\ \text{SR-popEnv}$$

$$\frac{\texttt{PARSE}(S_E,\ rsP,\ sP) \qquad T_E, D_E \vdash \Gamma_s \vdash sP : \textbf{unit}}{< \textbf{code}\ rsP\ , \{[E, \ldots], s\} > \rightarrow < sP, \{[E, \ldots], s\} >}\ \text{SR-rsP}$$

$$\frac{D[id] = (\ cl_3 \mid cl_4\ )\{\ p\ \} \qquad \texttt{UNIFY-AND-RUN}(\ (\ cl_1 \mid cl_2\ ),\ (\ cl_3 \mid cl_4\ )\{\ p\ \})}{D \vdash id(\ cl_1 \mid cl_2\ )}\ \text{OT-sP-ap}$$

$$\frac{T_E, O_E, D_E \vdash <c_1\ ,\ s_1> \rightarrow <c_2\ ,\ s_2>}{< c_1, \{[E, \ldots], s_1\} > \rightarrow < c_2, \{[E, \ldots], s_2\} >}\ \text{SR-sP} \qquad \frac{T_E, O_E, D_E \vdash <c\ ,\ s_1> \rightarrow s_2}{< c, \{[E, \ldots], s_1\} > \rightarrow < \{[E, \ldots], s_2\} >}\ \text{SR-sPend}$$

$$\frac{\dfrac{uo_1\ uo_2}{<c\ ,\ s_1> \rightarrow s_2} \in Img(O) \qquad T, O, D \vdash uo_1, uo_2}{T, O, D \vdash <c\ ,\ s_1> \rightarrow s_2}\ \text{O-sP-uo12end}$$

$$\frac{\dfrac{uo_1\ uo_2}{<c_1\ ,\ s_1> \rightarrow <c_2\ ,\ s_2>} \in Img(O) \qquad T, O, D \vdash uo_1, uo_2 \qquad T, D \vdash \Gamma_{s_2} \vdash c_2 : \textbf{unit}}{T, O, D \vdash <c_1\ ,\ s_1> \rightarrow <c_2\ ,\ s_2>}\ \text{O-sP-uo12}$$

$$\frac{\dfrac{uo}{<c_1\ ,\ s_1> \rightarrow s_2} \in Img(O) \qquad T, O, D \vdash uo}{T, O, D \vdash <c_1\ ,\ s_1> \rightarrow s_2}\ \text{O-sP-uoend}$$

$$\frac{\dfrac{uo}{<c_1\ ,\ s_1> \rightarrow <c_2\ ,\ s_2>} \in Img(O) \qquad T, O, D \vdash uo \qquad T, D \vdash \Gamma_{s_2} \vdash c_2 : \textbf{unit}}{T, O, D \vdash <c_1\ ,\ s_1> \rightarrow <c_2\ ,\ s_2\ >}\ \text{O-sP-uo}$$

$$\frac{\dfrac{}{<c_1\ ,\ s_1> \rightarrow s_2} \in Img(O)}{T, O, D \vdash <c_1\ ,\ s_1> \rightarrow s_2}\ \text{O-sPend} \qquad \frac{\dfrac{}{<c_1\ ,\ s_1> \rightarrow <c_2\ ,\ s_2>} \in Img(O) \qquad T, D \vdash \Gamma_{s_2} \vdash c_2 : \textbf{unit}}{T, O, D \vdash <c_1\ ,\ s_1> \rightarrow <c_2\ ,\ s_2>}\ \text{O-sP}$$

$$\frac{\dfrac{ut_1\ ut_2}{ty} \in Img(T) \qquad T, D \vdash ut_1, ut_2}{T, D \vdash ty}\ \text{T-sP-ut12} \qquad \frac{\dfrac{ut}{ty} \in Img(T) \qquad T, D \vdash ut}{T, D \vdash ty}\ \text{T-sP-ut} \qquad \frac{\dfrac{}{ty} \in Img(T)}{T, D \vdash ty}\ \text{T-sP}$$