Untitled

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```
logitGD <- function(y, x, optim.method = "GDI", eps = 10e-4,</pre>
                     max.iter = 100, alpha = function(t)\{1/t\}, beta_0 = c(0,0))\{
  stopifnot(length(y) == length(x) & optim.method %in% c("GDI", "GDII", "SGDI")
            & is.numeric(c(max.iter, eps, x)) & all(c(eps, max.iter) > 0) &
               is.function(alpha))
  iter <- 0
  err <- list()
  err[[iter+1]] <- eps+1
  w_old <- beta_0
  res <-list()
  while(iter < max.iter && (abs(err[[ifelse(iter==0,1,iter)]]) > eps)){
    iter <- iter + 1
    if (optim.method == "GDI"){
      w_new <- w_old + alpha(iter)*updateWeightsGDI(y, x, w_old)</pre>
    if (optim.method == "GDII"){
      w_new <- w_old + as.vector(inverseHessianGDII(x, w_old)%*%
                                     updateWeightsGDI(y, x, w_old))
    if (optim.method == "SGDI"){
      w_new <- w_old + alpha(iter)*updateWeightsSGDI(y[iter], x[iter], w_old)</pre>
    res[[iter]] <- w_new
    err[[iter]] <- sqrt(sum((w_new - w_old)^2))
    w_old <- w_new
  return(list(steps = c(list(beta_0),res), errors = c(list(c(0,0)),err)))
}
updateWeightsGDI <- function(y, x, w_old){</pre>
  (1/length(y))*c(sum(y-p(w_old, x)), sum(x*(y-p(w_old, x))))
  \#c(sum(y-p(w_old, x)), sum(x*(y-p(w_old, x))))
}
updateWeightsSGDI <- function(y_i, x_i, w_old){</pre>
  c(y_i-p(w_old, x_i), x_i*(y_i-p(w_old, x_i)))
p <- function(w_old, x_i){</pre>
  1/(1+\exp(-w_{old}[1]-w_{old}[2]*x_{i}))
inverseHessianGDII <- function(x, w_old){</pre>
```

```
solve(
    matrix(c(
        sum(p(w_old, x)*(1-p(w_old, x))),
        sum(x*p(w_old, x)*(1-p(w_old, x))),
        sum(x*p(w_old, x)*(1-p(w_old, x))),
        sum(x*x*p(w_old, x)*(1-p(w_old, x)))
    ),
    nrow =2 )
)
```

```
# wstępna inicjalizacja parametrów
eps = 1e-5
                                          # warunek stopu.
                                          # data jest listą ramek danych.
n = length(data)
diff = eps + 1
                                          # różnice w oszacowaniach parametrów
                                          # między kolejnymi krokami.
learningRates = function(x) 1/x
                                         # długości kroku algorytmu.
beta_old = numeric(0, length = k)
                                         # punkt startowy dlugosci k,
                                          # qdzie k to liczba zmiennych
                                          # objaśniających w modelu.
                              # estymacja
i = 1
                                          # iterator kroku algorytmu
while(i <= n | diff < eps) do</pre>
                                          # do zbieżności lub wyczerpania zbiorów
 batch = data[[i]]
 beta_new = beta_old - learningRates(i) * U_Batch(batch)
                                          # U_Batch to częściowa funkcja
                                          # log-wiarogdności dla zaobserwowanego
                                          # zbioru `batch`
 diff = euclidean_dist(beta_new, beta_old) # odległość euklidesowa
 beta_old = beta_new
  i = i + 1
end while
return beta_new
```

```
# be tracked in the future
  # estimate
  while(i <= n & diff > epsilon) {
    #tryCatch({
    beta_new[[i]] <- coxphSGD_batch(formula = formula, data = data[[i]],</pre>
                                      learningRate = learningRates(i),
                                      beta = beta_old)
    diff <- sqrt(sum((beta_new[[i]] - beta_old)^2))</pre>
    beta_old <- beta_new[[i]]</pre>
    i < -i + 1
    #}, error = function(cond) \{i <<-n+1\})
  # return results
  fit <- list()</pre>
  fit$Call <- match.call()</pre>
  fit$coefficients <- beta new
  fit$epsilon <- epsilon
  fit$learningRates <- learningRates</pre>
  fit$steps <- i
  class(fit) <- "coxphSGD"</pre>
  fit
}
coxphSGD_batch <- function(formula, data, learningRate, beta){</pre>
  # Parameter identification as in `survival::coxph()`.
  Call <- match.call()</pre>
  indx <- match(c("formula", "data"),</pre>
                names(Call), nomatch = 0)
  if (indx[1] == 0)
      stop("A formula argument is required")
  temp <- Call[c(1, indx)]</pre>
  temp[[1]] <- as.name("model.frame")</pre>
  mf <- eval(temp, parent.frame())</pre>
  Y <- model.extract(mf, "response")</pre>
  if (!inherits(Y, "Surv"))
      stop("Response must be a survival object")
  type <- attr(Y, "type")</pre>
  if (type != "right" && type != "counting")
      stop(paste("Cox model doesn't support \"", type, "\" survival data",
          sep = ""))
  # collect times, status, variables and reorder samples
  # to make the algorithm more clear to read and track
  cbind(not_censored = 1 - unclass(Y)[,2],
        times = unclass(Y)[,1],
        mf[, -1]) %>%
    arrange(times) -> batchData
```

```
# calculate the log-likelihood for this batch sample
  partial_sum <- list()</pre>
  for(k in 1:nrow(batchData)) {
    # risk set for current time/observation
    risk_set <- batchData %>%
      filter(times <= batchData$times[k])</pre>
    nominator <- apply(risk_set[, -c(1,2)], MARGIN = 1, function(element){</pre>
      element * exp(element * beta)
    }) %>%
      t %>%
      colSums()
    denominator <- apply(risk_set[, -c(1,2)], MARGIN = 1, function(element){</pre>
      exp(element * beta)
    }) %>%
      t %>%
      colSums()
    partial_sum[[k]] <-</pre>
      batchData[k, "not_censored"] * (batchData[k, -c(1,2)] - nominator/denominator)
  do.call(rbind, partial_sum) %>%
    colSums() -> U_batch
  beta_out <- beta + learningRate * U_batch</pre>
 return(beta_out)
checkArguments <- function(formula, data, learningRates,</pre>
                              beta_0, epsilon) {
  assert_that(is.list(data) & length(data) > 0)
  assert_that(length(unique(unlist(lapply(data, ncol)))) == 1)
  # + check names and types for every variables
  assert that(is.function(learningRates))
  assert_that(is.numeric(epsilon))
  assert_that(is.numeric(beta_0))
    # check length of the start parameter
  if (length(beta_0) == 1) {
    beta_0 <- rep(beta_0, as.character(formula)[3] %>%
                     strsplit("\\+") %>%
                     unlist %>%
                     length)
 }
  return(beta_0)
}
```

```
x <- runif(1000)
z < -2 + 3*x
pr <- 1/(1+exp(-z))
y <- rbinom(1000,1,pr)
logitGD(y, x, optim.method = "GDI", eps = 10e-5, max.iter = 500)$steps -> GDI
logitGD(y, x, optim.method = "GDII", eps = 10e-5, max.iter = 500)$steps -> GDII
ind <- sample(length(y))</pre>
logitGD(y[ind], x[ind], optim.method = "SGDI",
        max.iter = 500, eps = 10e-5)$steps -> SGDI.1
ind2 <- sample(length(y))</pre>
logitGD(y[ind2], x[ind2], optim.method = "SGDI",
        max.iter = 500, eps = 10e-5)$steps -> SGDI.2
ind3 <- sample(length(y))</pre>
logitGD(y[ind3], x[ind3], optim.method = "SGDI",
        max.iter = 500, eps = 10e-5)$steps -> SGDI.3
ind4 <- sample(length(y))</pre>
logitGD(y[ind4], x[ind4], optim.method = "SGDI",
        max.iter = 500, eps = 10e-5)$steps -> SGDI.4
ind5 <- sample(length(y))</pre>
logitGD(y[ind5], x[ind5], optim.method = "SGDI",
        max.iter = 500, eps = 10e-5)$steps -> SGDI.5
do.call(rbind, c(GDI, GDII, SGDI.1, SGDI.2, SGDI.3, SGDI.4, SGDI.5)) -> coeffs
unlist(lapply(list(GDI, GDII, SGDI.1, SGDI.2, SGDI.3, SGDI.4, SGDI.5), length)) -> algorithm
data2viz <- cbind(as.data.frame(coeffs),</pre>
      algorithm = unlist(mapply(rep, c("GDI", "GDII", "SGDI.1", "SGDI.2", "SGDI.3", "SGDI.4", "SGDI.5")
names(data2viz)[1:2] <- c("Intercept", "X")</pre>
library(ggplot2); library(ggthemes)
ggplot(data2viz) +
  geom_point(aes(x = X, y = Intercept, col = algorithm)) +
  geom_line(aes(x = X, y = Intercept, col = algorithm,
                group = algorithm)) +
 theme_tufte(base_size = 20)
logitGD() asda graphSGD()
graphSGD(c(0,0), y, x)
graphSGD(c(3.1,2.1), y, x)
graphSGD(c(4,3), y, x)
graphSGD(c(1,2), y, x)
dataCox <- function(N, lambda, rho, x, beta, censRate){</pre>
  # real Weibull times
  u <- runif(N)
  Treal <- (-\log(u) / (lambda * exp(x %*% beta)))^(1 / rho)
  # censoring times
  Censoring <- rexp(N, censRate)</pre>
```

```
# follow-up times and event indicators
 time <- pmin(Treal, Censoring)</pre>
 status <- as.numeric(Treal <= Censoring)</pre>
 # data set
 data.frame(id=1:N, time=time, status=status, x=x)
x <- matrix(sample(0:1, size = 40, replace = TRUE), ncol = 2)
head(dataCox(20, 3, 2, x, beta = c(2,3), 5))
           time status x.1 x.2
 id
1 1 0.021530741
                  0 0
2 2 0.022200468
                     0
                        0
                             1
                        0
3 3 0.009355016
                    1
                            1
4 4 0.156673908
                   0 1 0
5 5 0.151352826
                    1 0
6 6 0.132864938
                            1
                     0
                         0
graphSGD(c(0,0), y, x, 4561);graphSGD(c(0,0), y, x, 456)
graphSGD(c(2,1), y, x, 4561); graphSGD(c(2,1), y, x, 456);
graphSGD(c(1,0), y, x, 4561);graphSGD(c(1,0), y, x, 456);
graphSGD(c(2.1,3.1), y, x, 4561)graphSGD(c(2.1,3.1), y, x, 456)
x <- matrix(sample(0:1, size = 20000, replace = TRUE), ncol = 2)
dCox \leftarrow dataCox(10^4, lambda = 3, rho = 2, x, beta = c(1,3), censRate = 5)
vizCoxSGD(dCox)
```

```
coxphSGD <- function(formula, data, learningRates = function(x){1/x},</pre>
                     beta_0 = 0, epsilon = 1e-5) {
  checkArguments(formula, data, learningRates,
                   beta_0, epsilon) -> beta_old # check arguments
  n <- length(data)</pre>
  diff \leftarrow epsilon + 1
  i <- 1
  beta_new <- list() # steps are saved in a list so that they can
                      # be tracked in the future
  # estimate
  while(i <= n & diff > epsilon) {
    beta_new[[i]] <- coxphSGD_batch(formula = formula, data = data[[i]],</pre>
                       learningRate = learningRates(i), beta = beta_old)
    diff <- sqrt(sum((beta_new[[i]] - beta_old)^2))</pre>
    beta_old <- beta_new[[i]]</pre>
    i <- i + 1
 }
  # return results
 list(Call = match.call(), coefficients = beta_new, epsilon = epsilon,
       learningRates = learningRates, steps = i)
}
coxphSGD_batch <- function(formula, data, learningRate, beta){</pre>
  # collect times, status, variables and reorder samples
  # to make the algorithm more clear to read and track
  batchData <- prepareBatch(formula = formula, data = data)</pre>
  # calculate the log-likelihood for this batch sample
  partial_sum <- list()</pre>
  for(k in 1:nrow(batchData)) {
    # risk set for current time/observation
    risk_set <- batchData %>% filter(times <= batchData$times[k])</pre>
    nominator <- apply(risk_set[, -c(1,2)], MARGIN = 1, function(element){</pre>
      element * exp(element * beta)
    }) %>% rowSums()
    denominator <- apply(risk_set[, -c(1,2)], MARGIN = 1, function(element){</pre>
      exp(element * beta)
    }) %>% rowSums()
    partial_sum[[k]] <-</pre>
      batchData[k, "event"] * (batchData[k, -c(1,2)] - nominator/denominator)
  do.call(rbind, partial_sum) %>%
    colSums() -> U_batch
 return(beta + learningRate * U_batch)
}
```