The close-degree of covering rough sets

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***Abstract*—Close-degree is a measurement to describe the closeness level of two fuzzy sets, which is an important means of pattern recognition with close optional Principle. The concept of the close-degree of covering rough sets based on neighborhood to describe the closeness of two rough sets is proposed, and the basic properties of the close-degree of covering rough sets are given in this paper. A more perfect definition of lattice close-degree in covering rough sets based on neighborhood is given which satisfies all axioms for degree of approaching. We also discuss the property of lattice close-degree in covering rough sets.**

Keywords- *fuzzy sets ; covering rough sets ; close-degree; lattice close-degree*

I. INTRODUCTION

Rough set theory(RST), proposed by Pawlak [1], [2], is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. It provides a systematic approach for classification of objects through an indiscernability relation. Many examples of applications of the rough set method to process control, economics, medical diagnosis, biochemistry, environmental science, biology, chemistry psychology, conflict analysis and other fields can be found in[3] [4], [5].

An equivalence relation is the simplest formulization of the indiscernability. However, it cannot deal with some granularity problems we face in real information systems, thus many interesting and meaningful extensions have been made to tolerance relations [6], [7], similarity relations [8], coverings [9], [10], etc. This paper studies covering based rough sets.

Close-degree is a measurement to describe the closeness level of two fuzzy sets, which is an important means of pattern recognition with close optional Principle. Many scholars have been studied it. Zhan shaofeng [11] give a more perfect definition and property of the neartude in fuzzy sets. Jiang hao[12] discussed the defects of various lattice degree of approaching are discussed and a new lattice degree of approaching is introduced which satisfies all axioms for degree of approaching in fuzzy sets. LI Xiuhong[13] proposed the concept of the close-degree of rough sets to describe the closeness of two rough sets , the basic properties of the close-degree of rough sets are given. And the lattice close-degree of rough sets are defined. But it has defect that dissatisfies all axioms for degree of approaching. In this paper, we discuss the close-degree imitate to [13] in covering rough set and the concept are defined based on neighborhood.

II. PRELIMINARIES

Definition1[14] $N: F(U)\times F(U)\rightarrow[0,1]$, $(A,B)\rightarrow N(A,B)$, $N(A,B)$ is called the close-degree of fuzzy sets A and B ,if $N(A,B)$ meets the conditions the following four conditions.

(1) if $\mu_A(x)\in\{0,1\}$, then $N(A,A^c)=0$;
(2) $N(A,B)=N(B,A)$;
(3) $N(A,A)=1$, $N(U,\varnothing)=0$;
(4) if $A\subseteq B\subseteq C$, then
$N(A,C)\le N(A,B)\wedge N(B,C)$.

Definition2 [10] Let U be a universe of discourse, C a family of subsets of U. If none subsets in C is empty, and $\cup C = U$, C is called a covering of U. The pair (U,C) is called a covering approximation space.

In the following discussion, the universe of discourse U is considered to be finite.

Definition3[15] Let (U,C) be a covering approximation space. For each $X\subseteq U$,

$$n(x)=\cap\{K\in C; x\in K\}$$

is called the neighborhood of x.

Definition4[15]Let (U,C) be a covering approximation space. For each $X\subseteq U$,

$$\text{Set } \underline{C}X=\{x\in U \mid n(x)\subseteq X\}$$

is called the covering lower approximation of X.

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Set $\bar{C}X=\{x\in U \mid n(x)\bigcap X\neq\varnothing\}$

is called the covering upper approximation of X .

Set $bn_C X=\bar{C}X-\underline{C}X$ is called the covering boundary approximation of X .

$CX=(\underline{C}X,\bar{C}X)$ is called the covering rough set of X .

Definition5[16]Let (U,C) be a covering approximation space.

$u_X^C:U\to[0,1],u_X^C(x)=\frac{|n(x)\bigcap X|}{|n(x)|}$ is called the covering Membership function.

It can be regarded as a special fuzzy set F_X^C on the domain , That is,

$$F_X^C=\left\{(x,\mu_{F_X^C}(x))\middle| x\in U,\mu_{F_X^C}(x)=\frac{|n(x)\cap X|}{|n(x)|}\right\}.$$

Where $|X|$ expresses the cardinal number of X .

III. THE CLOSE-DEGREE OF COVERING ROUGH SETS

Definition6 Let (U,C) be a covering approximation space. $X,Y\subseteq U$, CX,CY are covering rough set and F_X^C,F_Y^C are fuzzy sets. The close-degree $N(F_X^C,F_Y^C)$ of F_X^C and F_Y^C is called the close-degree of CX and CY .Note as $N(CX,CY)$ and

$$N(CX,CY)=N(F_X^C,F_Y^C).$$

Definition7 Let (U,C) be a covering approximation space. $CX=(\underline{C}X,\bar{C}X)$ is covering rough sets, $(CX)^c=\left((\bar{C}X)^c,(\underline{C}X)^c\right)$ is called the complement set of $CX=(\underline{C}X,\bar{C}X)$.

Theorem 1 Let (U,C) be a covering approximation space, $X\subseteq U$, then $CX^c=(CX)^c$.

Proof. We only need proof $\underline{C}X^c=(\bar{C}X)^c$ and $\bar{C}X^c=(\underline{C}X)^c$,because of $CX^c=(\underline{C}X^c,\bar{C}X^c)$ .For any $x\in\underline{C}X \Leftrightarrow n(x)\subseteq X \Leftrightarrow n(x)\cap X=\varnothing \Leftrightarrow x\notin\bar{C}X^c \Leftrightarrow x\in(\bar{C}X^c)^c$, hence $\underline{C}X=(\bar{C}X^c)^c$.

That means $\bar{C}X^c=(\underline{C}X)^c$. Then using X^c replace the X in formula $\underline{C}X=(\bar{C}X^c)^c$,we have $\underline{C}X^c=(\bar{C}X)^c$.

Therefore, $CX^c=(CX)^c$.

Theorem2 Let (U,C) be a covering approximation space, then $F_{X^c}^C=(F_X^C)^c$.

Proof. For any $x\in U$,

$$\mu_{F_X^C}(x)+\mu_{F_{X^c}^C}(x)=\frac{|n(x)\cap X|+|n(x)\cap X^c|}{|n(x)|}$$

$$=\frac{|n(x)|}{|n(x)|}=1$$

So we have $F_{X^c}^C=(F_X^C)^c$.

Theorem3 Let (U,C) be a covering approximation space, then $N\left(CX,(CX)^c\right)=0$,when $\underline{C}X=\bar{C}X$.

Proof. For any $x\in X$,$\underline{C}X\subseteq X\subseteq\bar{C}X$ [2] and $\underline{C}X=\bar{C}X$, then $\underline{C}X=X=\bar{C}X$. hence $n(x)\cap X=n(x),\ n(x)\subseteq X$, that $\mu_{F_X^C}(x)=\frac{|n(x)\cap X|}{|n(x)|}=\frac{|n(x)|}{|n(x)|}=1$;

For any $x\in X^c,n(x)\cap X=\varnothing,$ which yields $\mu_{F_X^C}(x)=0$.

Therefore, $\mu_{F_X^C}(x)\in\{0,1\}$. From (1) of Definition1 , $N(F_X^C,(F_X^C)^c)=0$. Therefore

$$N(CX,CX^c)=N(F_X^C,F_{X^c}^C)=N(F_X^C,(F_X^C)^c)=0.$$

According to definition 1 and definition 6, we can easily get the theorem4 and theorem5.

Theorem4 Let (U,C) be a covering approximation space, then $N(CX,CY)=N(CY,CX)$.

Theorem5 Let (U,C) be a covering approximation space, then $N(CX,CX)=1$.

Theorem6 Let (U,C) be a covering approximation space, $X\subseteq Y\subseteq Z$, then

$$N(CX,CZ)\le N(CX,CY)\wedge N(CY,CZ)$$

Proof. For any $x \in U$, $X \subseteq Y$,we have $|n(x) \cap X|$ $\leq |n(x) \cap Y|$. Hence $\mu_{F_X^C}(x) = \frac{|n(x) \cap X|}{|n(x)|}$ $\leq \frac{|n(x) \cap Y|}{|n(x)|} = \mu_{F_Y^C}(x)$, which yields $F_X^C \subseteq F_Y^C$.

Similarly, we have $F_Y^C \subseteq F_Z^C$. That we have $F_X^C \subseteq F_Y^C \subseteq F_Z^C$. So we can get $N(F_X^C, F_Z^C) \leq$ $N(F_X^C, F_Y^C) \wedge N(F_Y^C, F_Z^C)$ from the (4) of definition1.

Therefore, $N(CX, CZ) \leq N(CX, CY) \wedge N(CY, CZ)$.

IV. 4THE LATTICE CLOSE-DEGREE OF COVERING ROUGH SETS

Definition8 Let (U, C) be a covering approximation space, CX, CY are covering rough sets,

$N(CX, CY)_L$ is called the lattice close-degree of covering rough sets CX and CY ,which is defined by

$$N(CX, CY)_L = \left|1 - \left[(\overline{x} \vee \overline{y}) - (\underline{x} \vee \underline{y}) + F_X^C \otimes F_Y^C - F_X^C \oplus F_Y^C\right]\right|$$

Where $F_X^C, F_Y^C \in F(U)$,

$$\overline{x} = \underset{x \in U}{\vee} \mu_{F_X^C}(x), \underline{x} = \underset{x \in U}{\wedge} \mu_{F_X^C}(x),$$

$$F_X^C \otimes F_Y^C = \underset{x \in U}{\vee} \left(\mu_{F_X^C}(x) \wedge \mu_{F_Y^C}(x)\right),$$

$$F_X^C \oplus F_Y^C = \underset{x \in U}{\wedge} \left(\mu_{F_X^C}(x) \vee \mu_{F_Y^C}(x)\right).$$

We will discuss the proprieties of the lattice close-degree of covering rough sets

Theorem7 Let (U, C) be a covering approximation space, $F_X^C, F_Y^C \in F(U)$, CX, CY are covering rough sets, then $0 \leq N(CX, CY)_L \leq 1$.

Proof. We have $\overline{x} \geq F_X^C \otimes F_Y^C$, $\overline{x} \vee \overline{y} \geq F_X^C \otimes F_Y^C$ from $\overline{x} = \underset{x \in U}{\vee} \mu_{F_X^C}(x) \geq \mu_{F_X^C}(x) \geq \mu_{F_X^C}(x) \wedge \mu_{F_Y^C}(x)$ and $\underset{x \in U}{\vee} \mu_{F_X^C}(x) \geq \underset{x \in U}{\vee} \left(\mu_{F_X^C}(x) \wedge \mu_{F_Y^C}(x)\right)$. And $\mu_{F_X^C}(x) \geq \underline{x}$ $\mu_{F_Y^C}(x) \geq \underline{y}$, hence $\mu_{F_X^C}(x) \vee \mu_{F_Y^C}(x)$ $\geq \underline{x} \vee \underline{y}$. Then $\underset{x \in U}{\wedge} \left(\mu_{F_X^C}(x) \vee \mu_{F_Y^C}(x)\right) \geq \underline{x} \vee \underline{y}$ which yields $F_X^C \oplus F_Y^C \geq \underline{x} \vee \underline{y}$. it is clear that $0 \leq \overline{x}, \underline{x}, \underline{y}, \overline{y}, F_X^C \oplus F_Y^C, F_X^C \otimes F_Y^C \leq 1$, we have $0 \leq \overline{x} \vee \overline{y} - \underline{x} \vee \underline{y} + F_X^C \oplus F_Y^C - F_X^C \otimes F_Y^C \leq 1$, i.e. $1 - \left[\overline{x} \vee \overline{y} - \underline{x} \vee \underline{y} + F_X^C \oplus F_Y^C - F_X^C \otimes F_Y^C\right] \leq 1$, therefore,

$$0 \leq \left|1 - \left[\overline{x} \vee \overline{y} - \underline{x} \vee \underline{y} + F_X^C \oplus F_Y^C - F_X^C \otimes F_Y^C\right]\right| \leq 1$$

so we have $0 \leq N(CX, CY)_L \leq 1$ from the definition 8.

Theorem8 Let (U, C) be a covering approximation space, $F_X^C, F_Y^C \in F(U)$, CX, CY are covering rough sets, then $N(CX, CY)_L = N(CY, CX)_L$.

Proof. Since $N(CX, CY)_L$

$$= \left|1 - \left[\overline{x} \vee \overline{y} - \underline{x} \vee \underline{y} + F_X^C \oplus F_Y^C - F_X^C \otimes F_Y^C\right]\right|$$

$$= \left|1 - \left[\overline{y} \vee \overline{x} - \underline{y} \vee \underline{x} + F_X^C \oplus F_Y^C - F_X^C \otimes F_Y^C\right]\right|$$

$$= N(CY, CX)_L,$$

Now we have proved that

$$N(CX, CY)_L = N(CY, CX)_L.$$

Theorem9 Let (U, C) be a covering approximation space, $F_X^C, F_Y^C, F_Z^C \in F(U)$ and $F_X^C \geq F_Y^C \geq F_Z^C$, CX, CY, CZ are covering rough sets, then

$$N(CX, CY)_L \geq N(CX, CZ)_L.$$

Proof. For $F_X^C \geq F_Y^C \geq F_Z^C$, we have $F_X^C \geq F_Y^C$, $F_X^C \geq F_Z^C$, Therefore, $\mu_{F_X^C}(x) \geq \mu_{F_Y^C}(x)$ and $\mu_{F_X^C}(x) \geq \mu_{F_Z^C}(x)$ from Definition8.so we have $\overline{x} \geq \overline{y}$ and $\underline{x} \geq \underline{y}$, which yields $\overline{x} \vee \overline{y} = \overline{x}$ and $\underline{x} \vee \underline{y} = \underline{x}$. we also have $F_X^C \oplus F_Y^C = \underline{x}$, $F_X^C \otimes F_Y^C$ $= \overline{y}$ As a result, $N(CX, CY)_L = |1 - \overline{x} + \underline{y}| = 1 - \overline{x}$ $+ \overline{y}$. Similarly, we have $N(CY, CZ)_L = 1 - \overline{y} + \overline{z}$ and $N(CX, CZ)_L = 1 - \overline{x} + \overline{z}$. Hence, $N(CX, CY)_L + N(CY, CZ)_L = 2 - \overline{x} + \overline{z}$ .so we have $N(CX, CY)_L + N(CY, CZ)_L = 1 + N(CX, CZ)_L$, which yields

$$N(CX, CY)_L \geq N(CX, CZ)_L.$$

Now we have proved that

$$N(CX, CY)_L \geq N(CX, CZ)_L.$$

Theorem10 Let (U,C) be a covering approximation space, $F_X^C \in F(U)$, then

$$N(CX,CX)_L = 1, N(U,\varnothing)_L = 0.$$

Proof. For any $F_X^C \in F(U)$, $F_X^C \geq F_X^C$, so according to the proof process of Theorem9, we have $N(CX,CX)_L = 1-\overline{x}+\overline{x} = 1$. Similarly, $U \geq \varnothing$, so $N(U,\varnothing)_L = 1-\overline{x} = 1-1 = 0$. Now we have proved that $N(CX,CX)_L = 1, N(U,\varnothing)_L = 0$.

Theorem11 Let (U,C) be a covering approximation space, $F_X^C, F_Y^C \in F(U)$ and $F_X^C \cap F_y^C = \varnothing$, CX, CY are covering rough sets, then

$$N(CX,CY)_L \leq 1-\overline{x} \vee \overline{y} + \underline{x} \vee \underline{y}.$$

Proof. For $F_X^C \cap F_y^C = \varnothing$, we assumption that $F_y^C = \varnothing$ or $F_y^C \neq \varnothing$.

When $F_y^C = \varnothing$, which yields $F_X^C \geq F_y^C$. So we have $N(CX,CY)_L = 1-\overline{x} \vee \overline{y} + \underline{x} \vee \underline{y}$;

When $F_y^C \neq \varnothing$, then $\exists x_0 \in U, \mu_{F_X^C}(x_0) = 0$, $\mu_{F_Y^C}(x_0) \neq 0$. Hence $F_X^C \otimes F_Y^C(x) = \underset{x\in U}{\vee} \mu_{F_X^C \cap F_y^C}(x) = 0$ and $F_X^C \oplus F_Y^C(x) = \underset{x\in U}{\wedge} \mu_{F_X^C \vee F_y^C}(x) \geq 0$. Therefore,

$$\begin{aligned} &N(CX,CY)_L \\ &\leq \left|1-\left[\left(\overline{x} \vee \overline{y}\right)+\left(\underline{x} \vee \underline{y}\right)-F_X^C \square\ F_Y^C - F_X^C \otimes F_Y^C\right]\right| \\ &\leq \left|1-\overline{x} \vee \overline{y} + \underline{x} \vee \underline{y}\right| \\ &= 1-\overline{x} \vee \overline{y} + \underline{x} \vee \underline{y}. \end{aligned}$$

Thus, $N(CX,CY)_L \leq 1-\overline{x} \vee \overline{y} + \underline{x} \vee \underline{y}$.

Corollary Let (U,C) be a covering approximation space, $F_X^C, F_Y^C \in F(U), F_X^C \cap F_y^C = \varnothing$, and $\overline{x} = 1, \underline{x} = \underline{y} = 0$, then

$$N(CX,CY)_L = 0.$$

V. CONCLUSIONS

In this paper, The concept of the close-degree of covering rough sets based on neighborhood to describe the closeness of two rough sets is proposed, and the basic properties of the close-degree of covering rough sets are given. A more perfect definition of lattice close-degree in covering rough sets based on neighborhood is given which satisfies all axioms for degree of approaching. We also discuss the property of lattice close-degree in covering rough sets..

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