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Hybrydowy klasyfikator wykorzystujący zbiory
rozmyte i przybliżone-badania symulacyjne

Hybrid classifier using fuzzy logic and rough
sets- simulation investigations

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OCENA PRACY:

Streszczenie

Niniejsza praca porusza problem klasycznego rozpoznawania obiektów (ang. pattern recognition). Istnieje wiele metod, a także dziedzin, w których można zastosować tę wiedzę: od rozpoznawania obrazów, chorób medycznych, czy też obiektów opisanych za pomocą różnych miar. Rozwój komputerów spowodował znaczący rozwój sztucznej inteligencji. Proponowane rozwiązania są bardzo zaawansowane i uwzględniają wiele czynników związanych z określonym problemem. Naukowcy prześcigają się w swoich osiągnięciach, przez wiele lat dopracowali algorytmy rozpoznawania w najdrobniejszym szczególe jednak jeszcze nikt nie zdołał stworzyć algorytmu, który nigdy nie popełnia błędu. W ostatnich czasach znacznie większym zainteresowaniem cieszą się hybrydowe rozwiązania, w których pojedyncze klasyfikatory zostają połączone w jeden. Głównym założeniem jest wykorzystanie różnorodnych klasyfikatorów, które popełniają błędy w różnych obszarach przestrzeni cech, lub też stosują różne wnioskowanie na podstawie dostępnych danych. W takich sytuacjach istnieje możliwość, że zbiór klasyfikatorów będzie w stanie się wzajemnie uzupełniać, dzięki czemu wydajność hybrydowego klasyfikatora będzie znacznie lepsza. W literaturze istnieje wiele przykładów hybrydowego łączenia klasyfikatorów, natomiast w tej pracy zaproponowano wieloetapowy klasyfikator wykorzystujący zbiory przybliżone (ang. Rough Sets) i zbiory rozmyte (ang. Fuzzy logic).

Praca została podzielona na dwie części. W pierwszej z nich zaprezentowany został przegląd literatury, a także przykłady dostępnych rozwiązań w dziedzinie rozpoznawania i klasyfikacji obiektów. Rozdział 2 stanowi wprowadzenie do tej tematyki, szczególnej uwadze należy poświęcić rozdziały 2.3, 2.4, w których dokładnie opisano algorytmy Rough Sets i Fuzzy Logic, ich główne zalety, wady, a także możliwe zastosowania. W rozdziale 3 opisano sposób implementacji algorytmów wykorzystanych w tej pracy magisterskiej. W drugiej części przedstawiono wyniki przeprowadzonych badań. Rozdział 4 zawiera informacje o danych testowych, a także sposobie oceny efektywności algorytmów. Sposób przeprowadzenia badań, środowisko symulacyjne, a także wyniki testów wraz z krótkim omówieniem zostały zamieszczone w rozdziale 5. Całość pracy zakończona jest podsumowaniem, a także propycją przyszłych badań stanowiących rozwój tego projektu (rozdziały 6, 7).

Rough sets, czyli teoria zbiorów przybliżonych została zaproponowana w 1982 r. przez prof. Zdzisława Pawlaka jako rowinięcie klasycznej teorii zbiorów, w której element albo należy do określonego zbioru lub nie (jednoznaczna decyzja true albo false). Zbiór przybliżony opisywany jest za pomocą dwóch zbiorów:

- przybliżenie dolne- zbiór obiektów, które na pewno należą do klasy decyzyjnej X ,
- przybliżenie górne- zbiór obiektów, które być może należą do klasy decyzyjnej X ,

Zbiory przybliżone zostały utworzone w celu aproksymacji pojęć (zbiorów), ze względu na niedostateczny stan wiedzy wynikający z braku informacji o obiekcie czy też niewystarczających pomiarów. Do głównych zalet zbiorów przybliżonych w systemach decyzyjnych można zaliczyć:

- redukcję danych, zdolność do selekcji kluczowych atrybutów w procesie klasyfikacji,
- generowanie reguł decyzyjnych $IF - THEN$,
- odkrywanie wzorców z danych,
- odkrywanie zależności pomiędzy atrybutami opisującymi dany obiekt,

W teorii zbiorów rozmytych przyjęło się, że reguły decyzyjne są przedstawione w postaci tabeli decyzyjnej, w której wiersze odpowiadają obiektom, natomiast kolumny przedstawiają wartości poszczególnych atrybutów. Spośród wszystkich atrybutów można wydzielić atrybuty decyzyjne, które mówią o klasie danego obiektu, a także atrybuty warunkowe opisujące cechy obiektu.

Fuzzy logic, czyli logika rozmyta została zaproponowana przez Lofti Zadeha, który zauważył, że pomiędzy stanem 0 (fałsz), a stanem 1 (prawda) istnieje szereg wartości pośrednich, które można opisać za pomocą stopnia przynależności danego elementu do zbioru. W tym celu została wprowadzona funkcja przynależności μ , która przybiera wartości z przedziału $[0, 1]$. Istnieje wiele typów tych funkcji, jednak najczęściej stosowane to funkcja trapezowa, trójkątna i funkcja Gaussa. Zbiory rozmyte zostały stworzone w celu opisanie zjawisk wieloznacznych i nieprecyzyjnych, używanych w języku naturalnym np. średni wzrost. Bardzo ważną rzeczą w przypadku logiki rozmytej są zmienne lingwistyczne, które w języku naturalnym określają pewne wielkości wejściowe lub wyjściowe. Wnioskowanie odbywa się na podstawie reguł decyzyjnych, które składają z kilku warunków, a także jednego wniosku.

Jednym z najważniejszych elementów tej pracy magisterskiej są rezultaty przeprowadzonych za pomocą własnego oprogramowania utworzonego w języku *Python*. Aby umożliwić porównanie wyników i powtórzalność środowiska symulacyjnego wykorzystane zostały ogólnie dostępne zbiory testowe z repozytorium UCI:

- Haberman,

- Iris,
- Wine,
- Thyroid,
- Bupa,
- Pima,
- Wdbc,

Zostały wybrane różnorodne zagadnienia, szczególnie pod względem złożoności problemu (liczby obiektów opisujących obiekt), a także tematyki, od diagnostyki cukrzycy, raka piersi, wątroby do rozpoznawania typu wina. Każdy z testów został powtórzony 10 razy, natomiast aby wydzielić zbiór testujący i uczący zastosowano technikę 4-fold cross-validation, która polega na tym, że w każdej turze dane wejściowe podzielone są na cztery części. Jedna z nich traktowana jest jako zbiór testujący, natomiast pozostałe jako zbiór uczący. Cały proces powtarzany jest cztery razy.

Głównym celem tej pracy magisterskiej była zaproponowanie hybrydowego klasyfikatora, wykorzystującego zbiory rozmyte i przybliżone. W tym celu został przeprowadzony szereg badań mających początkowo na celu zbadanie podstawowych właściwości zaimplementowanych algorytmów, a na końcu zbiorcze podsumowanie efektywności hybrydowego rozwiązania. Na potrzeby badań autor zaimplementował następujące algorytmy:

- Podstawowy algorytm Rough Sets z ustalonym krokiem granulacji
 - Rekursywny algorytm Rough Sets z modyfikacją reguł decyzyjnych wynikających ze zmiany kroku granulacji
 - Hybrydowy klasyfikator wykorzystujący zbiory rozmyte i przybliżone. Był to dwuetapowy klasyfikator, w którym jeżeli w pierwszym kroku nie znaleziono reguły pewnej lub możliwej to uruchamiany był drugi etap, a w nim klasyfikator oparty na fuzzy logic. Całość wymagała zaimplementowania dwóch dodatkowych algorytmów:
 - Algorytm Rough Sets połączony z algorytmem genetycznym wykorzystanym do znalezienia optymalnego kroku granulacji, a także selekcji atrybutów,
 - Algorytm Fuzzy logic połączony z algorytmem genetycznym użytym w celu znalezienia odpowiednich reguł decyzyjnych,
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WROCLAW UNIVERSITY OF TECHNOLOGY

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MASTER THESIS

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Abstract

This paper concerns the problem of pattern recognition. Advancements in computers technology has spawned the development of artificial intelligence science and many robust algorithms have been invented. In the recent years many scientists have proposed many solutions for solving complex and high dimensional problems. Some algorithms give good results, but very often they are devoted to specific problems. Until now no one has managed to invent classifier with 100% classification accuracy. Properties of single classifiers have been deeply exploited and it is hard to do something more in this field. Nowadays, it is common to take advantage of the diversity of simple classifiers and build hybrid classifier from the basic ones.

This master thesis focuses on two algorithms for pattern recognition: rough sets and fuzzy logic. Firstly, the basic algorithm properties are presented and later tests on real datasets are performed to check the classification accuracy of each classifier. The final result is the recommendation of multistage hybrid classifier.

All tests were performed on Linux platform using the simulator written in *Python* language. As the input to the system real datasets from *UCI* repository were taken. To ensure diversity in simulations, datasets with different number of attributes were chosen. To divide dataset into training and testing patterns 4-fold cross validation was used, which means that in each run data were splitted into 4 even parts and one subset was treated as a testing and the remaining ones as training and it was done four times. To ensure more reliable results each algorithm configuration was repeated ten times.

This paper is divided into two parts. In the first, the review of literature and the basic notation used in the whole thesis is presented. Section 2 is the source of the basic knowledge about pattern recognition and algorithms. Sections 2.3, 2.4 present the description of algorithms used in this thesis. The second part, which is the main point of this thesis, presents experiments analysis. Testing environment setup and program used in simulations are described in section 4, while the results with comments are placed in section 5. Proposal of future work as the continuation of this topic and the final summary can be found in sections 6, 7.

The first tests have shown that the basic rough sets classifier, even with the modification of decision rules can deal with simple datasets, but when the dimensionality of the problem increases then the classification accuracy is significantly deteriorated. An arbitrary chosen step of granulation strongly affects algorithm performance. For this reason, multistage hybrid classifier is proposed which uses rough sets and fuzzy logic. Classifier were connected in a sequence when the input to the system was directed to rough sets classifier. Where no proper rule was

found either *certain* or *possible* at the first stage then the pattern was rejected and directed to fuzzy logic classifier, otherwise class label was assigned by the first classifier as the final result.

Hybrid classifier was compared with other classifiers commonly used in the literature: 3-KNN, SVM, Linear Discriminant classifier, Decision Tree. It turned out that proposed solution can compete with other algorithms and in four out of seven tests this classifier was the best, in other cases results were comparable.

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1 Introduction

1.1 Preface

With the improvement of computers we are able to tackle with high dimensional problems in many areas of human life. At first glance everything looks so easy, but when we go deeper into a problem more and more obstacles become evident and unavoidable. For example, reconsider pattern recognition tasks in which measured objects (patterns) coming from the environment can be described by many attributes. Some of them are very meaningful while others brings only noise and distortion. This is the role of the algorithm to pick valuable attributes, but finding a minimal attribute reduct is \mathcal{NP} -hard problem. Another important issue is connected with overlaps in the attribute space. When features are easily separable even a simple classifier works perfectly, but for more tricky cases very sophisticated approaches have to be applied.

For the past years, many scientist in the world tried to invent new algorithms to simplify and improve the accuracy of classification. There are many solutions, but ones of the most eminent in the literature are: neural networks, fuzzy logic or evolutionary algorithms such as genetic algorithm or tabu search. Because simple approaches failed in more complicated problem, scientists tried to apply algorithms for dimensionality reduction and merge abilities of single classifier into combined one. This improved the quality of classification significantly, but until now no one has managed to invent such a classifier that will never make no mistakes.

This thesis touches the broad topic of pattern recognition task which is very difficult and demanding problem in every aspect of science. Generally, pattern classification is about assigning label to an unknown object based on the available knowledge coming from training dataset or expert knowledge. This process be compared to the capability of human brain which is able to put certain scenario into context and identify distinguishable object components. The whole process of classification can be divided into into few parts and each phase has a significant impact on the final results (more detailed description about classification task will be presented in section 2).

1.2 Main goals of the thesis

Examples of pattern recognition application can be found in many areas of science. As it was previously noticed, in the literature one can find many algorithms used for object classification, while here the rough sets, fuzzy logic and genetic algorithms are used and investigated. The general purpose of this thesis is to propose a hybrid classifier which merges the power of fuzzy logic and rough sets.

First of all the mathematical background and basic properties of these algorithms will be presented and later investigations will be carried out to prove the usefulness of proposed fusion. It should be noted that for the simulation purposes author implemented basic fuzzy logic, rough sets algorithms and created a hybrid classifier.

This thesis is a continuation of author's work in the field of pattern recognition, especially connected with rough sets theory. The results of previous experiments can be found in [36], [37]. The aim of this paper is to collect all experiments scenarios in one place and present the most valuable conclusions for further usage. It is meant to show the steps of how to construct an advanced hybrid classifier from the basic algorithms. For this reason, at the beginning of this thesis the basic properties of rough sets algorithm are checked in simulations, later the algorithm for modification decision rules is introduced and at the end as the final point the hybridization of algorithms is proposed using fuzzy logic, rough sets and genetic algorithm.

At the end of this section the experiment environment should be characterized in few words. It is done by defining input/output of the system.

- **given:**

- dataset divided into training and testing objects. in this thesis datasets from *UCI* Repository are used (described in section 4). Those datasets are broadly available and everyone in the future would be able to repeat tests and compare the results with those presented in this paper,
- algorithm for classification: basic rough sets, rough sets with modification of decision rules, hybrid classifier,

- **to find:**

- classification accuracy on the given dataset by the particular algorithm,

Figure 1.1 shows the general schema of the simulation environment. The term *Simulator* in box means software (Python program) created by author to perform all simulations.

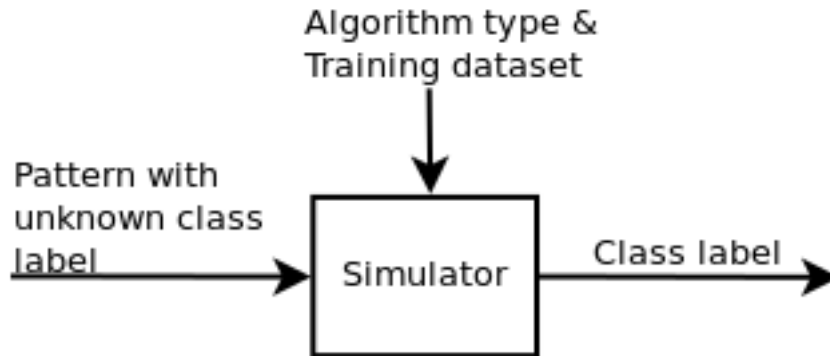


Figure 1.1: General schema of simulation environment. Input/output of the system

1.3 Scope of this project

This paper comprises of two parts. In the first, the review of literature and the basic notation used in the whole thesis is presented. Section 2 is the source of the basic knowledge about pattern recognition. Sections 2.3, 2.4 present the description of algorithms used in this thesis. Additionally, this part shows algorithm construction steps using pseudo-code.

The second part, which is the main point of this thesis, presents experiments analysis. Paragraph 4 describes the experiment environment settings and program written in *Python* to simulate all implemented algorithms. Results with comments are placed in section 5. General conclusions and plans for the future work are placed in sections 6, 7, respectively.

2 Pattern recognition algorithms

2.1 Introduction

This section is devoted to presenting information about pattern recognition algorithms. The main purpose is to describe algorithms that are used in this thesis.

Pattern recognition is a wide area of science in which we are interested in assigning label from a given set of classes to every unknown pattern. The whole process of classification can be divided into few phases, where each has a significant impact on the final classification accuracy:

1. data collection,
2. feature selection,

3. model selection,
4. classifier selection,
5. training,
6. testing.

At first, few basic information and categories will be given about pattern classification task. Generally, the whole process can be divided into two main categories:

- unsupervised- objects (patterns) incoming to the system are not previously labeled and this is the system task to find an appropriate structure of the data, to establish the organization of the classes basing only on the available data; there is no statistical or expert knowledge at a hand,
- supervised- in this approach incoming patterns have labels and can be treated as a training set. A supervised learning algorithm analyzes the training data and produces an inferred function, which is called a *classifier*.

In this thesis supervised learning is considered because available datasets are labeled with class number. Another division is connected with the type and structure of data:

- statistical classification- attempts to classify objects based on a set of extracted features and available statistical model for the generation of these patterns. Each pattern is represented in terms of d features (measurements) and is viewed as a point in d -dimensional feature space;
- syntactic or structural pattern recognition- assumes that pattern structure is quantifiable and extractable so that structural similarity of patterns can be assessed. It formulates hierarchical descriptions of complex patterns built up from simpler primitive elements. EKG waveforms, textured images and shape analysis of contours are the examples of syntactic approach;

2.2 Problem statement for pattern recognition task

In this section the problem statement for the pattern recognition algorithm will be presented. A supervised learning is assumed here which means that each pattern is labelled by the label $j \in M$, where M is an m -element set of possible states numbered with the successive natural numbers. The state j is unknown and does not undergo the direct investigation. What can only be measured are attributes or features by which a state manifests itself. For this reason each object (pattern) is described by a d -dimensional measured feature vector $x \in X$. In order to classify

an unknown pattern classification algorithm uses knowledge stored in the training set consisting of N training patterns:

$$S = (x_1, j_1), (x_2, j_2), \dots, (x_N, j_N) \quad (2.1)$$

In practice the decision algorithm with the learning phase should use knowledge included in the training set S so as the consequence the classifier has the following form:

$$i = \Psi(S, x), i \in M \quad (2.2)$$

In decision theory, to ensure that Ψ approximates the problem as closely as possible an additional loss function is introduced that assigns a specific value to the loss resulting from producing an incorrect label. The particular loss function depends on the type of label being predicted. In case of classification problem it is usually zero-one loss function. This corresponds simply to assigning a loss value of 1 to any incorrect labeling and is equivalent to computing the accuracy of classification procedure over the set of training data.

2.3 Rough sets

2.3.1 Introduction

Rough sets theory represents the mathematical approach to deal with imperfect knowledge. In the standard pattern recognition task we need precise information about pattern to recognize it, while rough sets can deal with vague or incomplete data. The problem of imperfect information has been tackled for a long time and it has become a crucial issue for many scientist. One of the most prominent approaches in the recent years are fuzzy logic and rough sets. In this section the latter approach is presented in greater details.

Comparing with other methods, rough sets have many advantages, but one of the most important one is its ability to works only on the raw data, with no additional assumption such as density probability in Bayesian algorithm [40], [15]. The main facts about rough sets algorithm can be summarized in few point presented below:

1. simplifies data by the granulation pre-processing,
2. is able to reduce attributes,
3. generates set of easy to understand and readable decision *IF-THEN* rules,
4. evaluates significance of data.

When talking about rough set theory one has to understand the concept of a set and how a rough set is related to the classical set represented in mathematic. From the mathematical point of view the crisp (precise) set is a collection of objects of interest and is uniquely determined by its elements. In other words, it means that every element must be uniquely classified as belonging to the set or not (true or false). For example, the set of odd numbers is crisp because every number is either odd or even and can not be partially in both.

This is the pure theory, but generally problems from the nature are much more complicated and require more complex decisions than whether an objects belong to the set or not, because for some phenomenon we can not precisely describe element membership. Consider the group of people and division into the set of small and high people. The height is not a precise but a vague concept and data vagueness can be met in many problems from the nature. Here is the spot for rough sets theory where the vagueness is expressed by a boundary region of a set.

2.3.2 Basic notation

Rough sets theory can be described in many ways and notations. In this paper the approach presented in [51] is applied.

In rough sets theory to represent datasets (information) we introduce a notion called an *information system* [39], [41]. It can be described by 4-tuple

$$IS = \langle U, Q, V, f \rangle \quad (2.3)$$

where the notation used in eq. (2.3) is as follows:

- U is the universe of discourse which is a finite set of objects (patterns),
- Q is a finite set of attribute by which each patterns manifests itself (is described),
- $V = \bigcup V_q$, V_q represents a domain of attribute q ,
- $f : U \times Q \rightarrow V$ is a total information function, such that $\bigvee_{q \in Q, x \in U} f(x, q) \in U$,

The information system can be represented as a finite table in which columns are labeled by attributes and each rows stands for an object from IS . Over the information table we can define decision table T where the set of attributes Q is disjointed into two subset C and D . The set C is a subset of condition attributes, and the set D contains decision attributes by which we can partition set U into decision classes.

From the granular nature of rough sets it may happen that some objects in the U are indistinguishable due to the limited information caused by granulation process. Taking this into account let define an indiscernibility relation $R \rightarrow U \times U$,

representing the lack of knowledge about patterns in the set U . The indiscernibility relation on U can be extended and associated with every non-empty subset of attributes $P \subseteq Q$ and is defined by eq. (2.4)

$$I_P = \{(x, y) \in U \times U : f(x, q) = f(y, q), \bigvee_{q \in P}\} \quad (2.4)$$

Now having I_P we can say that objects x and y are P -indiscernible by a set of attributes P if $y I_P x$. Relation I_P divides the set U into blocks (concepts) of P -indiscernible objects. The P -elementary set containing objects P -indiscernible with $x \in U$ is referred as $I_P(x)$ and defined as follows:

$$I_P = \{y \in U : y I_P x\} \quad (2.5)$$

By representing a target concept X as a subset of U we would like to describe it with respect to R . Additionally let assume that P is a non-empty subset of attributes from Q . In rough sets reasoning an object membership to a set can be represented in two ways:

1. an object $x \in U$ certainly belongs to X if all objects from the P -elementary set defined by $I_P(x)$ also belong to X . A set of all objects certainly belonging to X creates the P -lower approximation of X and can be represented as follows:

$$\underline{I}_P = \{x \in U : I_P(x) \subseteq X\} \quad (2.6)$$

2. an object $x \in U$ can possibly belong to X if at least one object from P -elementary set $I_P(x)$ can possibly belong to X . All the objects that could possibly belong to X are denoted as P -upper approximation of X , defined as:

$$\overline{I}_P = \{x \in U : I_P(x) \cap X \neq \emptyset\} \quad (2.7)$$

Therefore the set $U - \overline{I}_P$ represents the negative region containing the set of objects that can be definitely ruled out as members of the target set X .

The tuple $\langle \underline{I}_P, \overline{I}_P \rangle$ representing a lower boundary of the target X and the upper boundary of the target X creates a rough set [36]. Using above notion we can define P -boundary region which is a difference between upper and lower approximation [15].

$$BN_P(X) = \overline{I}_P - \underline{I}_P \quad (2.8)$$

The $BN_P(X)$ is a set of elements which can not be certainly classified neither as X nor as not- X with respect to the set of attributes P . If the boundary region of X is empty then the set represented by tuple $\langle \underline{I}_P, \overline{I}_P \rangle$ is crisp, otherwise we deal with inexact set which is called rough set. Until this moment we can see that rough sets

concept can be defined quite generally by means of topological operations: interior and closure, called approximations. They express the knowledge about pattern in terms of granules, not by a precise measure [42].

The illustrative example of rough sets reasoning is presented in fig. 2.1

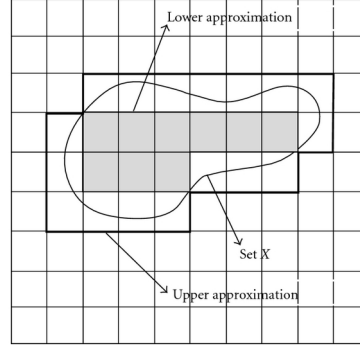


Figure 2.1: Rough sets example presenting lower and upper approximations

2.3.3 Rough sets indicators

In rough sets theory we can define few indicators to measure the quality of approximation. First of all with every subset of $X \subseteq U$ described by P subset of attributes we can associate an indicator called an accuracy of approximation defined as:

$$\alpha_P(X) = \frac{I_P(X)}{\overline{I_P(X)}} \quad (2.9)$$

The greater $\alpha_P(X)$ is the better approximation we have which means that many objects can be certainly classified as belonging to the target set X .

Another important indicator is a quality of approximation of $X \subseteq U$ by attributes from subset P . It represents the percentage of correctly classified objects using attributes P from subset X :

$$\gamma_P(X) = \frac{\overline{I_P(X)}}{|X|} \quad (2.10)$$

Assuming that we can partition U into n decision classes using P non-empty subset of attributes from C , the quality of classification can be defined as the ration of all correctly classified objects into classes:

$$\gamma_P(CLASS) = \frac{\sum_{i=1}^n \overline{I_P(CL_i)}}{|U|} \quad (2.11)$$

These indicators can be used in determining the quality of rough set algorithm or for finding the optimal reduct. Generally, the main goal in constructing rough sets algorithm is to obtain rule set with possible the greatest number of certain decisions.

2.3.4 Properties of rough sets

Similarly to classical sets, rough sets can be described by the following properties:

1. $\overline{I_P} \subseteq X \subseteq \underline{I_P}$
2. $\overline{I_P}(\emptyset) = \underline{I_P}(\emptyset) = \emptyset; \overline{I_P}(U) = \underline{I_P}(U) = U$
3. $\overline{I_P}(X \cup Y) = \overline{I_P}(X) \cup \overline{I_P}(Y)$
4. $\underline{I_P}(X \cap Y) = \underline{I_P}(X) \cap \underline{I_P}(Y)$
5. $\overline{I_P}(U - X) = -\underline{I_P}(X)$
6. $\underline{I_P}(U - X) = -\overline{I_P}(X)$

It is easily seen that the lower and the upper approximations of a set are interior and closure operations in a topology generated by the indiscernibility relation $I_P(X)$. Additionally, in rough sets theory we can define four types of data vagueness [42]:

- $\overline{I_P}(X) \neq \emptyset, \underline{I_P}(X) \neq \emptyset$ IF X is roughly I_P -definable. It means that using I_P for some elements from U we can decide whether they belong to X or $U - X$;
- $\overline{I_P}(X) \neq \emptyset, \underline{I_P}(X) \neq U$ IF X is internally I_P -indefinable. It means that using I_P we are able to decide which elements from U belong to $U - X$, but we don't know if they belong to X ;
- $\overline{I_P}(X) \neq \emptyset, \overline{I_P}(X) = U$ IF X is externally I_P -definable. It means that using I_P we are able to decide which elements from U belong to X , but we don't know if they belong to $U - X$;
- $\overline{I_P}(X) \neq \emptyset, \overline{I_P}(X) = U$ IF X is totally I_P -indefinable. It means that using I_P we are unable to decide for any element from U if it belongs to X or $-X$;

2.3.5 Rough sets reasoning from data

The category description can be done in two ways:

1. extensional,
2. intentional.

Each of these approaches define how pattern from dataset is described and assigned to the proper class. To correctly represent a target concept (class we try to recognize from all objects) from dataset an algorithm should be able to identify all objects belonging to this category. With the first method presented above (extensional), we have no insight into decision engine so we do not know how to assign new objects to the category. In the second approach the particular category is described by the set of rules. The same method is applied in rough sets algorithm where elementary granules (concepts) of knowledge build blocks consisting of indiscernible patterns from the universe of discourse U .

As stated in section 2.3.2 considered dataset or problem can be represented as a table. When decision attributes from D are added to this table it becomes a decision table T where each row is associated with decision rule. Below, the practical example is presented to clear all the things out.

Let consider the well-known problem of patients suffering from flu [41], [42]. The task is to decide if a patient described by 3 attributes is ill or not. Table 2.1 represents an information system IS about healthy patient and those suffering from flu. Attributes: *Headache*, *Muscle-pain*, *Temperature* are called condition attributes and are denoted as $(c_i, i \in (1, \dots, q))$, while the attribute *Flu* (last column in table 2.1) is chosen as a decision attribute c_d .

Table 2.1: Example dataset showing healthy patients and suffering from flu

Patient	Headache c_1	Muscle pain c_2	Temperature c_3	Flu c_d
p1	no	yes	high	yes
p2	yes	no	high	yes
p3	yes	yes	very high	yes
p4	no	yes	normal	no
p5	yes	no	high	no
p6	no	yes	very high	yes

Each row of a decision table determines a decision rule.

Rule 1: IF c_1 IS 'NO' AND c_2 IS 'YES' AND c_3 IS 'HIGH' THEN c_d IS 'YES'

Rule 2: IF c_1 IS 'YES' AND c_2 IS 'NO' AND c_3 IS 'HIGH' THEN c_d IS 'YES'

Rule 3: IF c_1 IS 'YES' AND c_2 IS 'YES' AND c_3 IS 'VERY HIGH' THEN c_d IS 'YES'

Rule 4: IF c_1 IS 'NO' AND c_2 IS 'YES' AND c_3 IS 'NORMAL' THEN c_d IS 'NO'

Rule 5: IF c_1 IS 'YES' AND c_2 IS 'NO' AND c_3 IS 'HIGH' THEN c_d IS 'NO'

Rule 6: IF c_1 IS 'NO' AND c_2 YES 'NO' AND c_3 IS 'VERY HIGH' THEN c_d IS 'YES'

Analyzing table 2.1, it is noticeable that some patients can not be distinguished between each other. It is possible to generate different indiscernible relations based on the chosen attributes. In case of *Headache* attribute patients: p_2, p_3, p_5 are indiscernible; patients p_2, p_5 are indiscernible with respect to attributes *Headache*, *Muscle-pain* and *Temperature*, or if we use *Headache* and *Muscle-Pain* the set U can be divided into three sets: $\{p_1, p_4, p_6\}, \{p_2, p_5\}, \{p_3\}$.

Now it is time for defining key features of rough sets. Over the table 2.1 we can choose two concepts (target sets): *Flu* and *Not Flu*. For the first concept the lower approximation set of patient certainly having flu is $\{p_1, p_3, p_6\}$, while the upper approximation of patients possibly suffering from flu is $\{p_1, p_2, p_3, p_5, p_6\}$. The boundary region for concept *Flu* is a set of $\{p_2, p_5\}$ patients. For the concept *Not Flu* the lower approximation is the set $\{p_4\}$, the upper approximation is the set $\{p_2, p_4, p_5\}$, whereas the boundary region is again the set $\{p_2, p_5\}$.

Additionally, we can measure the accuracy of approximation $\alpha_P(x)$ for each concept. This can indicate if the set of attributes used for describing the concepts is correctly chosen. For the *Flu* concept where $X=\{p_1, p_2, p_3, p_6\}$ (those patterns where the decision attribute is Yes) is described by set of attributes $P=\{\text{Headache, Muscle-pain, Temperature}\}$ the accuracy of approximation is:

$$\alpha_P(Flu) = \frac{|\{p_1, p_3, p_6\}|}{|\{p_1, p_2, p_3, p_5, p_6\}|} = \frac{3}{5}$$

On the other hand, when we take only one attribute $P=\{\text{Temperature}\}$, then for the concept *Flu* we get the lower approximation of as a set of $\{p_3, p_6\}$ and the upper approximation of $\{p_1, p_2, p_3, p_5, p_6\}$. In this case the accuracy of approximation is as follows:

$$\alpha_P(Flu) = \frac{2}{5}$$

To sum up, $\alpha_P(x)$ is a very important indicator in rough sets theory and tells which attributes better characterize target concept. This example has shown that because of the granule representation of knowledge in rough sets approach some object can not be discerned. It is very important to choose proper condition attributes because they determine how upper and lower approximations are represented by objects from U .

2.4 Fuzzy logic

2.4.1 Introduction

In fuzzy logic an element membership to a set is described by a membership function which assigns value from interval $[0, 1]$:

$$\mu_A(x) : U \rightarrow [0, 1], A \subseteq U, x \in A \quad (2.12)$$

The properties of membership function μ are as follows:

- $\mu_{\emptyset}(x) = 0, x \in U$
- $\mu_U(x) = 1, x \in U$
- $\mu_X(x) \leq \mu_Y(x), IF X \subseteq Y$
- $\mu_{\bar{A}}(x) = 1 - \mu_A(x), x \in U$

where \emptyset is an empty set and \bar{A} is the complement of the set A .

Fuzzy logic is a superset of Boolean logic that has been extended to handle the concept of partial truth - values between “completely true” and “completely false” [7], [13]. This theory was introduced by Dr. Lotfi Zadeh in the 1960's as a tool for modeling the uncertainty of natural language. This approach can be applied to many problems, here analyze only few examples:

- days of a week- there is a question which day is mostly associated with a weekend. Generally, we would assume that the membership value g associated with each day will be as follows: {Monday=0, Tuesday=0.2, Wednesday=0.4, Thursday=0.5, Friday=0.7, Saturday=1.0, Sunday=0.9};
- the seasons- there is not evident boundary when one season stops and another starts. This example can be illustrated quite precisely by fig. 2.2;

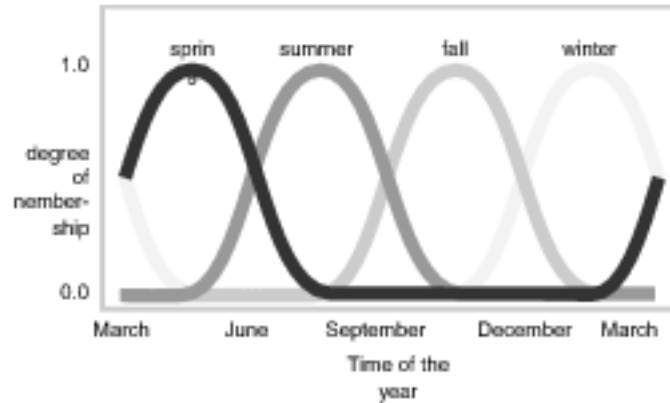


Figure 2.2: Example of how fuzzy logic can be used to describe the seasons of the year²

- the height (small and tall definitions)- from the group of people it is hard to define who is tall and who is small interchangeably. To solve this problem we can define the concept *Tall* and assign value to each person describing its height. This process is presented in fig. 2.3;

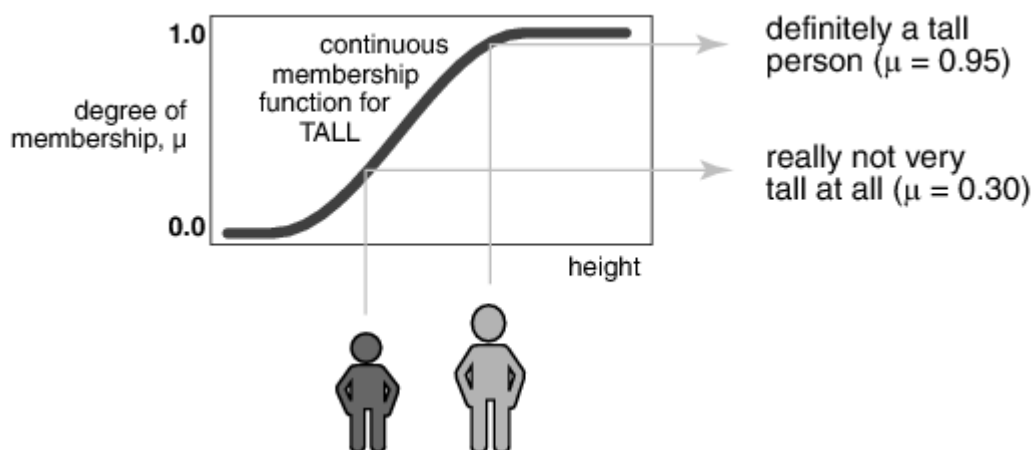


Figure 2.3: Fuzzy approach for defining person height³

²source: http://www.mathworks.com/help/toolbox/fuzzy/bp7816_1.html

Similarly to the section 2.3.1, let consider more deeply the problem of defining if person is small or tall in the group of people. First of all, we have to define a fuzzy set (concept) *Tall* which will answer the question "to what degree is person x tall?". Zadeh describes *Tall* as a linguistic variable, which represents our cognitive category of "tallness" [26]. The procedure is as follows: to each person in the group assign a degree of membership (value from $[0, 1]$ range) in the fuzzy subset *Tall* based on the membership function (for example the same as presented in fig. 2.3). Table 2.2 shows the degree of membership how person is tall in the group of six people.

Table 2.2: Table describing how person is tall by the fuzzy logic linguistic variable

Person	Height [m]	degree of tallness
p1	1.5	0.00
p2	1.6	0.21
p3	1.7	0.44
p4	1.8	0.52
p5	1.9	0.69
p6	2.0	1.00

Fuzzy numbers are fuzzy subsets generated over the attribute domain. They have a peak or plateau with membership grade 1 over which the members of the universe are completely in the set. The membership function is increasing towards the peak and decreasing away from it. There are different types of membership functions and their usage strongly depends on the type of considered problem. One of the most commonly used in the literature are: triangular, trapezoidal, Gaussian shapes (see example in fig. 2.5).

2.4.2 Fuzzy reasoning from data

Reasoning in fuzzy logic is based on decision rules, the same as in rough sets approach [12]. Rules are expressed in the form of IF *CONDITIONS* THEN *DECISION* where *CONDITIONS* can be divided into antecedent set ($A_{ri}, i \in (1, \dots, q)$) and *DECISION* is the consequent determining the output of the rule. In the pattern recognition task when we deal with patterns described by d features then decision rules have the form presented by eq. (2.13)

$$IF x_1 = A_{r1} AND x_2 = A_{r2} AND \dots AND x_d = A_{rd} THEN class C_r with CF_r \quad (2.13)$$

³source: http://www.mathworks.com/help/toolbox/fuzzy/bp78l6_1.html

The *AND*, *OR*, and *NOT* operators of Boolean logic exist in fuzzy logic and are usually defined in two ways:

- the minimum, maximum, and complement operator from the set. When they are defined this way, they are called the Zadeh operators [3];
- multiplication, addition and subtraction from one. In this case normalization procedure is needed because after these operations the final result can be greater than one;

Fuzzy logic classification is based on three main steps:

1. fuzzyfication- in the fuzzyfication process we convert continuous quantity into fuzzy number based on the appropriate membership function value. It requires defining membership grade of crisp input x in the fuzzy set;
2. rule induction- there are different types of fuzzy inference systems, but one of the most commonly used (the same as in this paper) is Mamdani inference system [48], another method is Sugeno-Type Fuzzy Inference;
3. defuzzification- the process of producing a quantifiable (crisp) result from available fuzzy sets and corresponding membership degrees;

The whole process of fuzzy reasoning is presented in fig. 2.4.2



Figure 2.4: Steps applied in fuzzy reasoning.⁴

Let consider a simple example, which should clear all ambiguities (the following example is based on [1]). The problem is connected with estimating the level of the risk involved in software engineering project. There are two inputs to the system (funding, staffing) and one output (risk). Membership functions are represented as of triangular shapes (fig. 2.5)

⁴source: http://www.mathworks.com/help/toolbox/fuzzy/bp7816_1.html

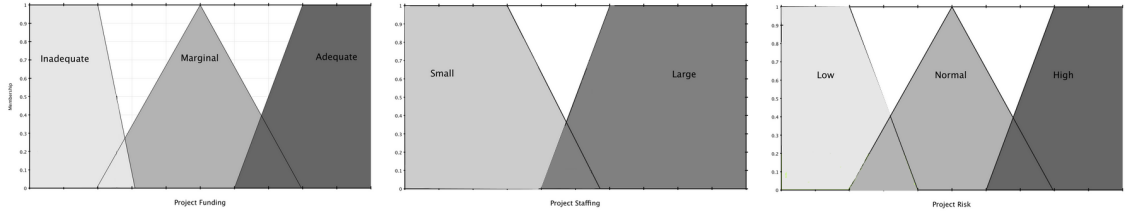


Figure 2.5: Input and output fuzzy linguistic variables for the described system.

The next important thing in fuzzy logic beside fuzzy values are fuzzy rules. In this problem they are provide a priori and are as follows:

- Rule 1: IF funding IS adequate OR funding IS small THEN risk IS low
 Rule 2: IF funding IS marginal AND staffing IS large THEN risk IS normal
 Rule 3: IF funding IS inadequate THEN risk IS high

Let say that we want to calculate project risk for inputs: funding = 35 and staffing = 60.

1. The first step is to convert crisp values into fuzzy representation. After activating each input membership functions the fuzzy values are as follows:

$$\begin{array}{l} \text{input 1} \\ \hline \mu_{inadequate}(35) = 0.5 \\ \mu_{marginal}(35) = 0.2 \\ \mu_{adequate}(35) = 0.0 \end{array}$$

$$\begin{array}{l} \text{input 2} \\ \hline \mu_{small}(60) = 0.1 \\ \mu_{large}(60) = 0.7 \end{array}$$

2. Now it is time for rule induction where OR is treated as a summation, AND as multiplication operator.

$$\begin{array}{l} \text{Rule 1 : } \mu_{low} = 0.0 + 0.1 = 0.1 \\ \text{Rule 2 : } \mu_{normal} = 0.2 \cdot 0.7 = 0.14 \\ \text{Rule 3 : } \mu_{high} = 0.5 \end{array}$$

3. After performing clipping of consequent membership functions for each rule (example given in fig. 2.6), the final crisp output can be calculated. It is

done by defuzzification method, where one of the most commonly used is a centroid formula given by eq. (2.14) [1], [2].

$$CO = \frac{\sum_{x=a}^b \mu_A(x) \cdot x}{\sum_{x=a}^b \mu_A(x)} \quad (2.14)$$

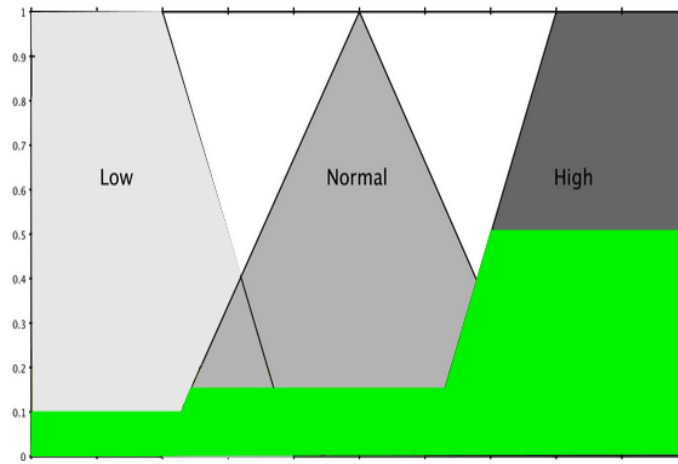


Figure 2.6: Example of clipping the consequent membership function as the result of rule induction

Using eq. (2.14) the output of the system is as follows:

$$CO = \frac{(0 + 10 + 20) \cdot 0.1 + (30 + 40 + 50 + 60) \cdot 0.14 + (70 + 80 + 90 + 100) \cdot 0.5}{0.1 \cdot 3 + 0.14 \cdot 4 + 0.5 \cdot 4} = 69.3$$

It means that for input values funding = 35 and staffing = 60 the project risk is about 69%.

2.4.3 Genetic-based machine learning approaches

In the literature one can find many examples of fuzzy logic applications for pattern recognition task [4], [10]. Generally, there are two main approaches:

1. some expert knowledge is available and fuzzy rules are created beforehand,
2. no knowledge about data set is present so some techniques of data mining must be applied to extract rules from the training set.

At this point it must be strongly emphasized that in this thesis the main focus is based on fuzzy algorithm for pattern recognition task where there is no prior

knowledge about fuzzy system and decision rules. There are different approaches for construction fuzzy logic algorithm from raw dataset, for example optimization techniques such as gradient descent or heuristic algorithms [9], [17], [30]. In this paper the genetic algorithm is proposed for creating an optimal rule set.

One can wonder what is the purpose of applying genetic algorithm into rule generation. For the simplicity let consider such a case where N training patterns are available and nothing more. Now the following question arises: how to properly divide the feature space into fuzzy set, and how to generate decision rules when there is no expert knowledge. The simplest and straightforward solution is to create let say 10 triangular membership functions for each attribute, and check each possible combination. Is this approach optimal? The efficiency of this solution strongly depends on the complexity of the problem. For d -dimensional feature space and k fuzzy membership functions per each attribute the number of possible combination for generating one rule is equal to k^d . For greater d (for example wine dataset from *UCI* repository has 13 attributes) it is impossible to find the proper rule combination in a reasonable time. Here is an open spot for genetic algorithm to show its searching abilities.

There are two main methods of genetic-based machine learning approaches [32], [33]:

1. Michigan template- it is a population of fuzzy rules and a single fuzzy rule is handled as an individual (see fig. 2.7). The evaluation of each fuzzy rule is performed by classifying all the given training patterns by the available rule set N_{rule} . At the end of each iteration new individuals are created through genetic operators and merged to the current population. For the next generation N_{rule} best individuals are taken. The whole procedure can be summarized as follows:
 - (a) generate N_{rule} fuzzy rules,
 - (b) evaluate the fitness of each fuzzy rule in the current population,
 - (c) generate $N_{replace}$ fuzzy rules using genetic operators,
 - (d) merge $N_{replace}$ fuzzy rules with current population and choose the best N_{rule} individuals for the next generation,
 - (e) return to point (b) if stopping condition is not fulfilled (number of generations).

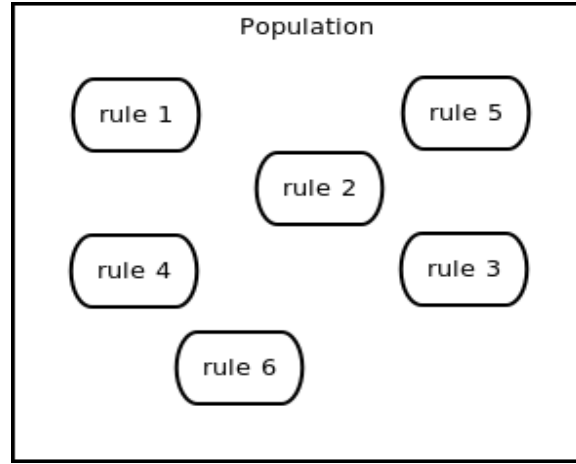


Figure 2.7: Example of Michigan approach for constructing Genetic Fuzzy logic algorithm

2. Pittsburgh template- in this approach a set of fuzzy rules is handled as an individual (see fig. 2.8). In this case the length of a single individual is equal to $n \cdot N_{rule}$, where n is the length of single fuzzy rule. Algorithm starts with N_{pop} randomly generated rule sets. The fitness value of a single individual is the number of correctly classified patterns in the training set by a given rule set. The procedure for this algorithm is as follows:
 - (a) generate N_{pop} individuals consisting of N_{rule} fuzzy rules each,
 - (b) calculate the fitness value of each rule set (individual) using training dataset,
 - (c) generate $N_{replace}$ new rule set using genetic operators,
 - (d) merge $N_{replace}$ fuzzy rule sets with current population and choose the best N_{pop} individuals for the next generation,
 - (e) return to point (b) is stopping condition is not fulfilled (number of generations).

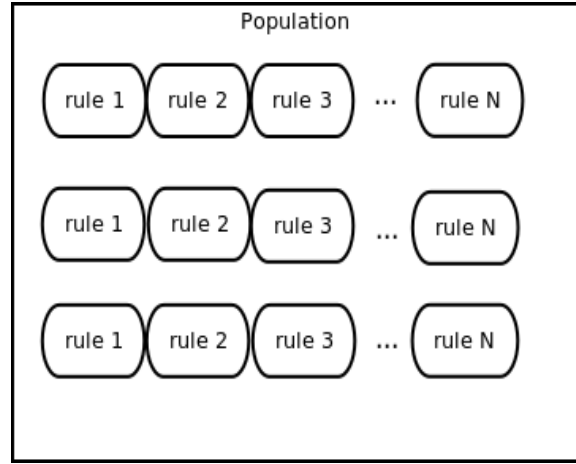


Figure 2.8: Example of Pittsburgh approach for constructing Genetic Fuzzy logic algorithm

In this thesis the first approach is implemented in simulations which means that individual is modeled as a single rule.

2.5 Attribute selection

Many problem are complex and multidimensional. For example Sonar dataset from *UCI* Repository has 60 attributes describing a single pattern. Usually we hope to recognize pattern in a relatively lower dimension to reduce the cost in measuring and processing information and enhance the interpretability of learned models. Feature selection or reduction is done for classifiers to remove the noise and superfluous data. Generally, this is not an easy task and requires a lot of computation [2], [6].

A reduct is a set of attributes that ensures the same classification of elements from U as the rudimentary set of attributes. More than one reduct can exist for one information system, but there is only one core reduct in Q . The core reduct is the set of attributes from Q that all the attributes are indispensable. It means that we cannot remove those features from the set because we will loose information about patterns.

An attribute is dispensable if the following criterion is fulfilled:

$$I(P) = I(P - a), \text{ for } \{a\} \in P \subseteq Q$$

In the literature there are many examples of how to apply attribute selection:

- information measures,

- distance measures,
- dependence measures,
- consistency measures.

Different approaches provide various results and their application strictly depends on the type of data. One of the best known methods are: Principal Component Analysis, Factor analysis or optimization and heuristic techniques. In the recent years heuristic methods are very often used because of their speed and searching abilities. A lot of work has been devoted to feature extraction and this aspect is not the main goal of this research. Basing on the results and conclusions presented in the literature, in this thesis a genetic algorithm is proposed for extracting valuable features from the set of all attributes [29], [50].

2.6 Genetic algorithm

Genetic Algorithm is an element of evolutionary computation, which is a rapidly growing area of soft computing [44], [45]. GA is based on the principles of natural selection and genetic modification. As optimization methods, GA operates on a population of points, designated as individuals. Each individual of the population represents a possible solution of the optimization problem. Individuals are evaluated depending upon their fitness which indicates how well an individual of the population solves the optimization problem. To sum up, GA has the following general features:

1. GA operates with a population of possible solutions (individuals) instead of a single individual. Thus, the searching process can be carried out in a parallel form or sequentially;
2. GA is able to find the optimal or sub-optimal solutions in complex and large search spaces. Moreover, it can be applied to nonlinear optimization problems with constraints defined in discrete or continuous search spaces;
3. GA examines many possible solutions at the same time, so there is a higher probability that the search process can converge to an optimal solution;

There are four main parts in each GA process to consider (graphically presented as a flow chart 2.9):

1. the problem representation or encoding,
 2. fitness or objective function definition,
 3. fitness-based selection,
-

4. evolutionary reproduction of candidate solutions (individuals or chromosomes).

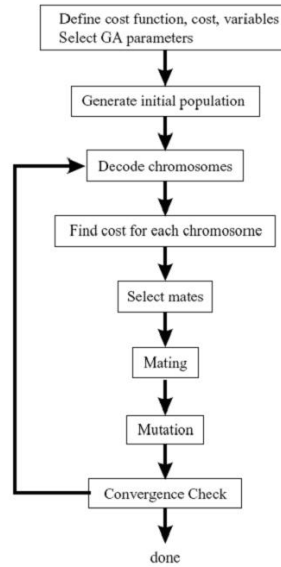


Figure 2.9: Diagram representing phases in genetic algorithm evaluation

Genetic algorithms are widely used as a search techniques in the various fields. In this thesis it will be used for finding an optimal cuts in the attribute space for rough sets algorithm and for obtaining optimal decision rule set in fuzzy logic algorithm. The success of a genetic algorithm can be quantified by estimating the cost and the quality of final obtained solution.

In the literature there can be found many examples of how GA is useful in solving hard optimizations problems, but beside unquestionable advantages there also exist downsides. A traditional GA without any diversity maintenance mechanism often suffers from getting stuck on the suboptimal peaks, because almost the entire GA population would have converged to a single peak, as a result of the rapid loss of population diversity [46], [47].

There is a great deal of work showing how to set the optimal parameters in an evolutionary algorithm to obtain required speedup and solution accuracy, but this is not the main issue in this thesis. For more exact information see [44], [45].

When designing genetic algorithm one of the most important thing is to ensure proper crossover and mutation operations. More precise information about these operators will be presented in section 3.

2.7 Hybrid classifiers

In the recent years there is an increasing interest in methods of combining multiple learning systems into hybrid one [6], [7]. The main advantage of such approach is its ability to find different relations and explanation for the dataset for each classifier. If classifiers make errors on different parts of the feature space or apply different reasoning it is possible that the ensemble of classifiers will complement each other and the final classification will be better.

Generally, there are two types of hybrid classifiers (example presented in fig. 2.10):

1. multiexpert systems- classifiers work in parallel, each of them is trained and tested on the same data and independent decisions are combined to compute the final result. The most common example is a majority voting;
2. multistage systems- classifiers are connected in a sequence where the next classifier is trained and used for classification only if the previous classifier rejected the pattern;

It is hard to decide which approach is better. Each system has its pros and cons and the choice depends of the type of dataset and available classifiers. In this thesis the second approach is implemented where the first segment is built as rough sets classifier and the second one is constructed using fuzzy logic.

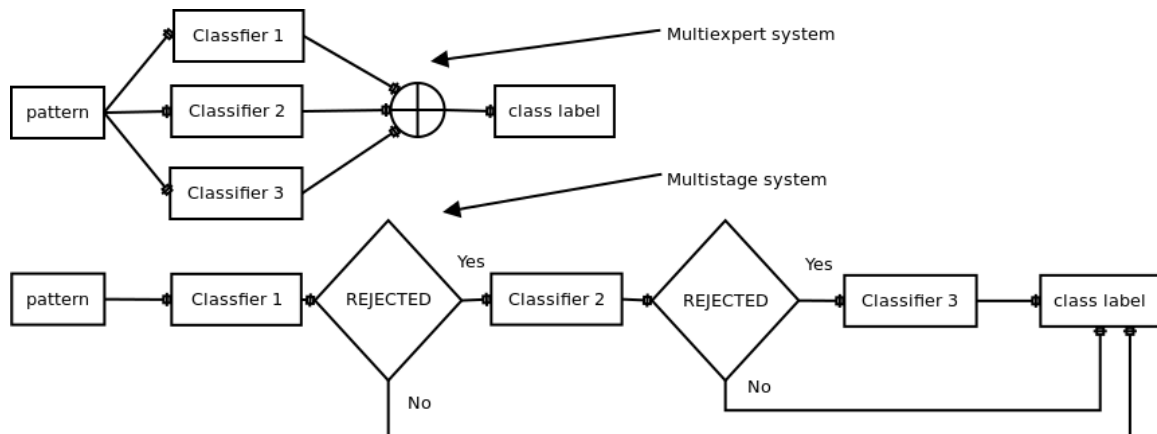


Figure 2.10: Example of two approaches for constructing hybrid classifiers

3 Algorithm construction

In this section basic information about algorithm construction will be presented. For the classification task three algorithms were implemented and com-

pared. At the beginning, we start with the basic algorithm of rough sets, later present rough sets algorithm with modification of decision rules and at last the multistage hybrid algorithm consisting of genetic algorithm, rough sets and fuzzy logic is shown in greater details.

3.1 Rough sets algorithm construction

The basic rough sets algorithm with constant step of granulation can be summarized in six steps [36], [37]:

1. Algorithm starts with an arbitrary chosen step of granulation which requires dividing every domain of feature into G intervals. If the attributes are the real numbers then the granulation preprocessing is needed first. To calculate the proper interval eq. (3.1) is used:

$$v_{pi}^i = \lfloor \frac{f_i}{G} \cdot (f_{i,max} - f_{i,min}) \rfloor \quad (3.1)$$

where v_{pi}^i is the interval label for i -th attribute; f_i is the i -th real attribute value for pattern x , $f_{i,min}$ and $f_{i,max}$ denote the extend of the i -th feature calculated in the training phase and $\lfloor \cdot \rfloor$ is the operator extracting integer value from real number. If for any attribute from x in the testing phase $f_i > f_{i,max}$ then $v_{pi}^i = G - 1$ or $f_i < f_{i,min}$ then $v_{pi}^i = 0$, in other cases eq. (3.1) is applied. If the attributes are discrete then no preprocessing is required and interval label is the same as feature discrete value.

After this step, the value of each attribute is represented by the number of interval in which this attribute is included. For each attribute from $l = (1, \dots, q)$ we choose the same numbers of intervals K_l called step of granulation G . For the l -th attribute denoted by v_{pl}^l it is defined its p_l interval from $p_l = (1, \dots, K_l)$

2. Using training dataset, construct the decision table T where each row represents a pattern with additional class label. Over the table T define the set $FOR(C)$ of decision rules of the following form:

$$IF (x_1 = v_{p_1}^1) AND (x_2 = v_{p_2}^2) AND \dots (x_q = v_{p_q}^q) THEN \Psi(S, x) = j$$

Each generated rule is evaluated and the strength factor is assigned to it determining the accuracy of approximation (see section 2.3.3)

3. For the created set of formulas $FOR(C)$ for each $j = 1, \dots, m$ an algorithm calculates lower \underline{I}_P , upper \overline{I}_P approximations and the boundary region BN_P which are determined by the set of conditional attributes $P \subseteq Q$

4. In order to classify new pattern x algorithm looks for matching rules in the set $FOR(C)$ (rule is activated if the left condition is fulfilled by attributes describing x).
5. If there is only one matching rule r , then the pattern x is classified to the class which is indicated by r -th decision attribute j , because for sure such a rule is belonging to the lower approximation of all rules indicating j . Rule r is denoted as a *certain* rule.
6. If there is more than one matching rule in the set $For(C)$, it means that the recognized pattern should be classified by the rules from the boundary regions and in this case as a decision algorithm takes the index of boundary region for which the strength of corresponding rule is maximal. All activated rules are denoted as *possible* rules.
7. In other cases: no appropriate rule was found or few rules have the same strength factor then the unknown pattern x is rejected.

3.2 Rough sets algorithm construction with modification of decision rules

It can happen that for particular number of intervals basic rough sets algorithm can not find patterns in the training set, so as the consequence *dummy* rules are generated, useless in the classification process (the strength of the rule is 0). The main drawback of algorithm presented in section 3.1 is the fact that it starts with an arbitrary chosen step of granulation and its accuracy strongly depends on it. In this section the recursive modification of the previous algorithm is presented for automatically changing the step of granulation if the pattern x is rejected. The modification is as follows [38]:

1. Algorithm starts with an arbitrary chosen step of granulation G , generally it is a high value to ensure that recursion can be invoked by decreasing G . At first algorithm repeats the whole procedure 1-6 described in the previous section.
2. If for the pattern x algorithm cannot find neither *certain* nor *possible* decision rule, it means that there is no proper representation in the learning set. In such a situation algorithm tries to find matching rule by decreasing recursively the current interval G by factor $\epsilon = 1$ for every condition attribute $l = 1, \dots, q$ until the proper rule is found. If *certain* rule or *possible* rules with maximum strength are induced then the algorithm returns decision attribute j , otherwise if for every attribute $G = 1$, then the pattern x is rejected.

The recursion is time consuming so to enhance the process of finding the proper decision formulas for different G the decision set $FOR(C)$ is stored in the memory for faster retrieval.

3.3 Multistage hybrid algorithm construction

3.3.1 Motivations

When we deal with complex data it can happen that a single classifier is not sufficient. There arises a question if connection of different classifier will improve the classification? In this thesis the hybridization of rough sets and fuzzy logic is proposed and investigated. The next subsections show algorithm construction.

3.3.2 Fuzzy logic algorithm construction

3.3.2.1 Problem formulation

In the fuzzy logic algorithm one of the most important key is creation of rules which will ensure correct classification. When there is no expert knowledge about dataset it is not an easy task to generate them from scratch. In the literature one can find many practical examples of how to generate *IF-THEN* rules, from the statistical tools to heuristic algorithms. In the recent years the most appealing and effective approach is genetic algorithm [5], [14], [24]. For the algorithm construction we have to make few assumptions:

- for the learning procedure N training patterns are available,
- a set F of linguistic values and their membership functions are given for describing each attribute.

From points enumerated above, the second one is the most important and the choice of the proper membership functions affects the classification accuracy [18], [7]. It requires in-depth explanation of how set F is generated in this thesis, how to partition each attribute into linguistic values and how to describe each membership function. Genetic algorithm is used here for finding an optimal parameter settings.

Fig. 3.1 shows an example of how to generate fuzzy set F of possible membership functions. In this example 14 membership functions are used. Each function has a subscript defining its linguistic value and a proper location in the feature space. To emphasize the complexity of the problem let take into account that each attribute is divided in the same way as depicted in fig. 3.1. Having d -dimensional feature space, we would like to generate decision rules consisting of d antecedents. For each position we can choose 1 out of 14 membership functions which gives 14^d

possibilities. It is impossible to evaluate each potential solution in a reasonable time, especially for greater d [29], [35].

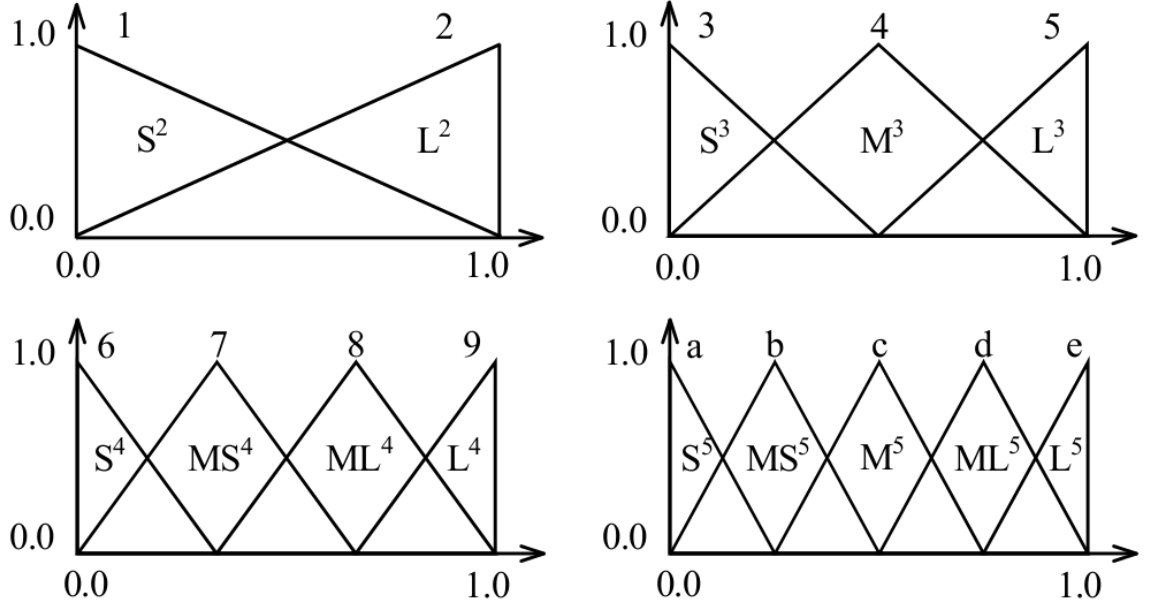


Figure 3.1: Example fuzzy partition set for an attribute

For classification task single fuzzy rule is of the following form [32], [21]:

$R_r : IF x_1 = A_{r1} AND x_2 = A_{r2} AND \dots AND x_d = A_{rd} THEN class C_r with CF_r$

where $x = (x_1, x_2, \dots, x_d)$ is a d -dimensional pattern vector; A_{ri} is an antecedent fuzzy set with a linguistic label (taking into account the example from 3.1 A_{ri} can have one label from the set $\{1, 2, \dots, 9, a, b, \dots, e\}$); C_r is a consequent fuzzy set determining the class label $\{1, \dots, m\}$, CF_r is a rule strength; r denotes the number of the rule from the set of all possible rules $\{1, \dots, N_{rule}\}$.

In simulations presented in this paper N_{rule} was set to ten. In this place it should be explained why this particular number was chosen. In the literature it is recommended to use N_{rule} rather small because for greater N_{rule} algorithm's search abilities are deteriorated significantly [5], [12]. Additionally, special tests were performed to determine the optimal number of rules in author's software. From researches where N_{rule} was changed from 5 to 50 the best results were obtained for $N_{rule} = 10$.

3.3.2.2 Rule generation

The process of rule generation for fuzzy logic without the expert knowledge is complex and can be done in couple steps. At first, using available training set,

generate randomly N_{rule} rules. For each training pattern x_p calculate the compatibility grade of a single rule connected with antecedent part $A_r = (A_{r1}, A_{r2}, \dots, A_{rd})$ using the product operator of each membership function $\mu_{A_{ri}}$ determined for A_{ri} :

$$\mu_{A_r}(x_p) = \mu_{A_{r1}}(x_p) \cdot \mu_{A_{r2}}(x_p) \cdot \dots \cdot \mu_{A_{rd}}(x_p) \quad (3.2)$$

If we know how to calculate the compatibility grade of each training pattern now we can determine C_r and CF_r for each rule. The fuzzy probability $P(class\ j|A_r)$ of class j , $j = (1, \dots, m)$ indicating how pattern x can be associated with class j is given by eq. (3.3) [32]:

$$Pr(class\ j|A_r) = \frac{\sum_{x_p \in class\ j} \mu_{A_r}(x_p)}{\sum_{p=1}^m \mu_{A_r}(x_p)} \quad (3.3)$$

For the r -th rule R_r the label of class is assigned according to the winning rule, which means that the label with maximal probability is chosen:

$$R_r : C_r = \max_{j=\{1, \dots, m\}} \{Pr(class\ j|A_r)\} \quad (3.4)$$

In the learning phase it can happen that rule R_r can be activated by patterns coming from different classes. To ensure the proper classification, each rule has a strength factor which tells how precisely rule R_r predicts the consequent class j .

$$R_r : CF_r = Pr(class\ j|A_r) - \sum_{j=1, j \neq C_r}^M Pr(class\ j|A_r) \quad (3.5)$$

If CF_r in eq. (3.5) is negative then rule R_r is denoted as *dummy* and is not taken for further reasoning, otherwise it is used in defuzzification process to determine the final class label [19], [27].

3.3.2.3 Fuzzy reasoning

Let assume that N_{rule} fuzzy rules are generated with indicators C_r , CF_r determined by eq. (3.4), (3.5). Then the process of classification is done as follows:

$$\Psi(S, x_p) = C_q \leftarrow \max_{j=\{1, \dots, M\}} \{\mu_{A_q}(x_p) \cdot CF_r\} \quad (3.6)$$

The label of the class for unknown pattern is determined by a winner rule R_w that has the maximum compatibility grade and the rule strength CF_r .

If multiple fuzzy rules have the same maximum product μ_{A_r} but different consequent classes then the classification is rejected. The same action is taken if no fuzzy rule is compatible with the incoming pattern x_p .

3.3.2.4 Genetic algorithm for fuzzy algorithm construction

In this section genetic algorithm will be described in greater details. This algorithm was used to generate an initial number N_{rule} set of fuzzy rules for classification. Basic assumptions:

- fuzzy rule encoding is the same as presented in section 3.3.2.1,
- training data set is given with N patterns,
- triangular membership functions are used and are described by 2-tuple (a, b) , where a is the center value (where $\mu(x) = 1$), and b determines left and right extend of membership function, respectively,

$$\mu(x) = \begin{cases} \frac{-1}{b} \cdot x + \frac{a+b}{b} & x \geq a \text{ and } x \leq (a + b) \\ \frac{1}{b} \cdot x - \frac{a-b}{b} & x \geq (a - b) \text{ and } x < a \\ 0 & \text{otherwise} \end{cases} \quad (3.7)$$

- possible partitions of the feature space are determined in the same way as in the example presented in fig. 3.1,
- genetic algorithm uses standard operations such as cross-over, mutation, population generation, fitness evaluation,
- as the template for genetic fuzzy algorithm Michigan approach is used which means that we have N_{rule} number of individuals in the population.

Next few step will present the whole structure of genetic algorithm used in fuzzy logic:

- chromosome representation and encoding:
 - each individual represents a single fuzzy rule R_r from the set of containing N_{rule} rules. The length of the chromosome is the same as the number of attributes describing the pattern x . Each allele has value determining which linguistic variable is used in the current rule. Reconsider Iris dataset which is a 4-dimensional classification problem where for each attribute 14 membership functions plus one variable telling to omit the attribute (called *DON'T USE*) are generated. An exemplary individual can be as follows:

$$1|c|DON'T USE|4||1|0.85$$

Above individual can be decoded into rule presented in fig. 3.2

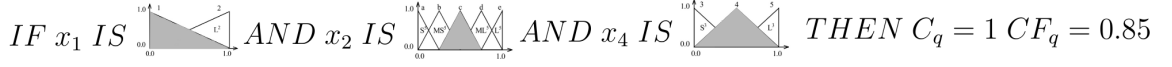


Figure 3.2: Example rule decoded from the individual chromosome (attribute x_3 was omitted in this rule)

This rule assigns patterns to class $j = 1$ with the strength equal to 0.85.

- individual evaluation
 - to ensure proper genetic algorithm process an appropriate fitness function must be defined [11], [22]. Firstly the nature of pattern recognition task must be taken into account and secondly the structure of the fuzzy algorithm. Generally, the main goal is to generate rules with the highest CF_r grade, the smallest number of attributes and the highest classification rate. Fitness function is given by eq. (3.8)

$$F_{fg} = w_1 \cdot NC + w_2 \cdot NNC + \left(\frac{1}{NOF}\right)^2 + w_3 \cdot CF \quad (3.8)$$

where w_1, w_2 are weights for a reward and punishment to the rule based on the classification result (in simulations $w_1 = 5, w_2 = 10$); NC and NNC are the numbers of correctly recognized and misclassified patterns by a particular rule, respectively; NOF is the number of attributes used by the rule (in the above example $NOF = 3$); CF is the strength factor of the rule and w_3 is the weight (in the simulations $w_3 = 10$). The best individuals are those which maximize function F_{fg} . It is worth noting that function F_{fg} was proposed by author as the extension for most commonly met functions in the literature where only the number of correctly recognized patterns is taken into account in individuals evaluations. In this thesis the complexity of the rule and its strength had to be reconsidered additionally.

- cross-over
 - from the whole population two individuals are chosen randomly to constitute parents for further reproduction. With a probability of 0.5 each allele is picked either from mother or father chromosome. In this way two new individuals are generated (see example in fig. 3.3),

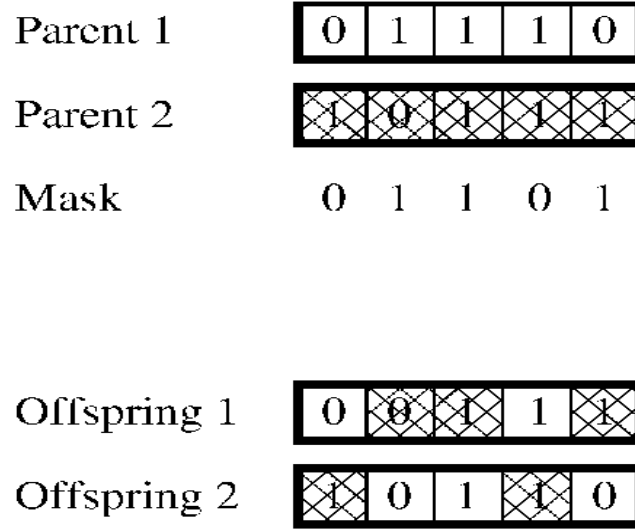


Figure 3.3: Cross-over operation used in genetic algorithm

- mutation
 - in the particular generation one chromosome is chosen randomly and later in each allele new membership function is taken from other possible functions. For example if in the first allele the first membership function is chosen the set of candidates is given by $\{2, \dots, 9, a, \dots, e, DON'T\ USE\}$,
- selection
 - in genetic algorithm one of the most important issue is how to generate next population [23]. Here, after the end of one generation individuals from the population are merged with those created through cross-over and mutation operations. Later, the average fitness value F_{avg} is calculated in the whole set. To the next generation those individuals are passed which their fitness indicator F_{fg} is greater than the average $F_{fg} \geq F_{avg}$,

Of course it can happen that for cross-over and mutation operator newly-generated individual will be invalid (the whole chromosome contains only *DON'T USE* linguistic variables). In such a situation rule is rejected and the whole generation process is repeated again.

Table 3.1 presents basic genetic algorithm parameters. Optimal values were determined during simulations by trial and error method.

Table 3.1: Parameter settings for genetic algorithm used in fuzzy logic

Parameter	value
N_{rule}	10
$N_{replace}$	$N_{rule}/2$
Crossover probability	0.9
Mutation probability	0.3
Generations	500

3.3.3 Rough sets and genetic algorithm

Rough sets algorithm presented in section 3.1 uses an arbitrary chosen step of granulation and each attribute has the same granulation intervals. In some cases this approach gives good results, but in more complex problems algorithm efficiency is low [25], [31]. Additionally, the basic rough set algorithm uses all attributes for rule construction.

Finding the optimal attribute reduct and rough set partition is NP problem [20], [2]. To overcome this obstacle a genetic algorithm is used in the similar way as in section 3.3.2.4. Now, a single individual describes the partition for each attribute independently. Reconsider individual encoding for 4-dimensional Iris dataset. The number of granulation intervals for each attribute is chosen from the set $\{1, 2, \dots, K_{max}\}$, where K_{max} is the maximum value of discretization. Additionally, *DON'T USE* variable is used to determine that a given attribute is excluded from the rule. The example of individual is given below:

$$|2|DON'T USE|K_{max}|3||120$$

It means that the first feature is divided into two intervals, the second is not used and the third and fourth are discretized into K_{max} and 3 intervals, respectively. The fitness indicator of this individual is 120. To evaluate individual the following fitness function given by eq. (3.9) is proposed:

$$F_{rg} = w_1 \cdot NC + w_2 \cdot NNC + \left(\frac{1}{NOF}\right) + w_3 \cdot \left(\frac{1}{NOCR}\right)^2 \quad (3.9)$$

,where w_1, w_2 are weights for a reward and punishment to the individual on the classification result ($w_1 = 5, w_2 = 10$); NC and NNC are the numbers of correctly recognized and misclassified patterns; NOF is the number of attributes used by the rule (in the example above $NOF = 3$); $NOCR$ is the number of *certain* rules which are derived from partition given by the particular individual; w_3 is the weight (in the simulations $w_3 = 10$).

The whole procedure of constructing genetic rough sets algorithm can be summarized in few steps:

1. Determine the maximum partition value for each attribute K_{max} . In this thesis K_{max} is the same for all features and is equal to 7,
2. Generate N_{pop} individuals by randomly assigning value from the set $\{1, 2, \dots, K_{max}, DON'T\ USE\}$ to each allele in the chromosome,
3. Treat each individual as a rough set partition and calculate lower, upper approximations and the boundary region. Evaluate individual using fitness function F_{rg} ,
4. Generate $N_{replace}$ individuals using genetic operators and merge with the current population,
5. Choose N_{pop} individual for the next generation
6. If stopping criteria is not fulfilled go to 2,

Cross-over and mutation operations are done in the same way as presented in section 3.3.2.4. Parameters for genetic algorithm used in this section are presented in table 3.2:

Table 3.2: Parameter settings for genetic algorithm used in rough sets

Parameter	value
N_{pop}	10
$N_{replace}$	$N_{pop}/2$
Crossover probability	0.9
Mutation probability	0.2
Generations	100

In case of genetic algorithm for rough set 100 generations were sufficient to obtain reliable results.

3.3.4 Hybrid rough sets and fuzzy logic algorithm

Multistage classifier created in this thesis can be divided into three phases:

1. rough sets classifier construction using genetic algorithm presented in section 3.3.3,
2. fuzzy logic classifier construction with rule generation by heuristic approach described in 3.3.2.4,
3. pattern recognition using sequential hybrid classifier,

Each step plays an important role in the whole process and affects the final classification accuracy [13], [29], [34]. Proper parameters of genetic algorithm are especially important. The whole hybrid algorithm can be summarized in the following steps:

1. Divide available dataset into three separated subsets: the first for genetic algorithm operation, the second and third as training and testing.
2. Train rough sets, fuzzy logic classifiers
3. Classify pattern using rough sets algorithm:
 - If pattern is classified by a *certain* or *possible* rule then it is a final label
 - If no proper rule is found or more than one rule have the same strength but different label then pattern is rejected and processed by fuzzy logic classifier.

An illustrative scheme of hybrid classifier is presented in fig. 3.4.

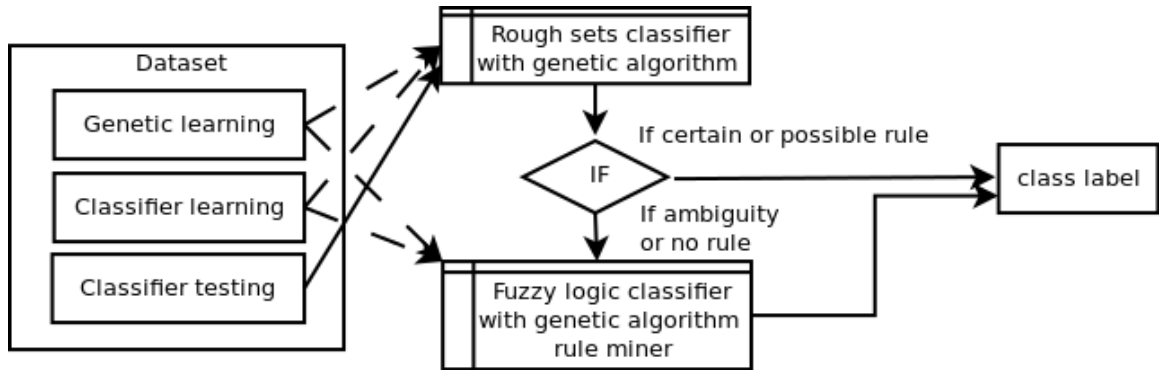


Figure 3.4: Diagram presenting how hybrid classifier works

3.3.5 Difference between rough sets and fuzzy logic reasoning

At the end of this section the difference between rough sets and fuzzy logic should be explained more deeply. In rough sets algorithm we provide discretization for the whole feature and the number of generated decision rules is equal the number of objects in the set. In previous section it was stated that rough sets can be associated with table. Opposite to rough sets algorithm is fuzzy logic in which the number of rules is much smaller and in this thesis it is 10. We do not want to partition the whole feature space into fuzzy sets, instead we pick only those membership functions which ensure the greatest compatibility grade with training patterns.

4 Experimentation system

4.1 Assumptions

This section starts the second part of this thesis. In the first, algorithms were presented in case of construction and their properties. For the simulation purposes the following algorithms have been implemented:

1. Basic Rough sets algorithm
2. Rough sets algorithm with modification of decision rules basing on the granulation step G
3. Hybrid algorithm consisting of:
 - Rough sets algorithm with genetic approach for finding the optimal partition in feature space and attribute reduction
 - Fuzzy logic algorithm with genetic approach for constructing decision rules

4.2 Datasets

To perform all the experiments, testing datasets from *UCI* repository were used [49]. This approach is commonly used in the literature because ensures that someone in the future would be able to retake the tests and compare the results. The main motivation in choosing those datasets was to ensure diversity and test algorithms with complicated and complex problems. Below each dataset is shortly described:

- Dataset name: Haberman
 - #attributes: 3
 - #instances: 306
 - #classes: 2
 - Description: This dataset contains cases from a study that was conducted between 1958 and 1970 at the University of Chicago's Billings Hospital on the survival of patients who had undergone surgery for breast cancer.
 - Dataset name: Iris
 - #attributes: 4
-

-
- #instances: 150
 - #classes: 3
 - Description: This dataset is one the most commonly used in pattern recognition task. Attribute information:
 - sepal length in cm
 - sepal width in cm
 - petal length in cm
 - petal width in cm
 - Dataset name: Wine
 - #attributes: 13
 - #instances: 178
 - #classes: 3
 - Description: These data are the results of a chemical analysis of wines grown in the same region in Italy but derived from three different cultivars. The analysis determined the quantities of 13 constituents found in each of the three types of wines.
 - Dataset name: Thyroid
 - #attributes: 5
 - #instances: 215
 - #classes: 3
 - Description: This dataset was created at the University of California at Irvine by Ross Quinlan during his visit in 1987 for the 1987 Machine Learning Workshop. It contains 5 features describing thyroid symptoms.
 - Dataset name: Bupa
 - #attributes: 6
 - #instances: 345
 - #classes: 2
 - Description: The first 5 variables are all blood tests which are thought to be sensitive to liver disorders that might arise from excessive alcohol consumption. Each line in the bupa.data file constitutes the record of a single male individual.
-

- Dataset name: Pima
 - #attributes: 8
 - #instances: 768
 - #classes: 2
 - Description: This dataset comes from National Institute of Diabetes and Digestive and Kidney Diseases. The binary-valued decision indicates whether the patient shows signs of diabetes according to World Health Organization criteria. All patients are females at least 21 years old of Pima Indian heritage.
- Dataset name: Wdbc
 - #attributes: 32
 - #instances: 569
 - #classes: 2
 - Description: Features are computed from a digitized image of a fine needle aspirate (FNA) of a breast mass. Each real-valued features are computed for each cell nucleus.

4.3 Efficiency indicators

To evaluate effectiveness of algorithm it has to be consistent approach used in all experiments and additionally a priori knowledge about each dataset must be know. By a-priori knowledge one should understand the label of class for each pattern. Below, there are listed methods of algorithm fitness scoring:

- Classification accuracy CA - the number of correctly classified patterns out of O objects.

- The best value from n probes

$$B = \max\{CA_1, \dots, CA_n\} \quad (4.1)$$

- The worst value from n probes

$$W = \min\{CA_1, \dots, CA_n\} \quad (4.2)$$

- The average value from n probes

$$A = \frac{1}{n} \cdot \sum_{i=1}^n (CA_i) \quad (4.3)$$

- Error variance from n simulations

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (AC_i - A)^2} \quad (4.4)$$

4.4 Program description

To perform all simulations in this project a program was written in *PYTHON* (more information can be found in A). It allows simulating all algorithms with chosen dataset and obtain algorithm efficiency indicators.

Program was tested on Linux platform with Intel Pentium Dual Core 2.4 GHz, 2GB memory. To run the program one has to install *Python* environment at least in version 2.6. Because implemented algorithms have many setting parameters, they were written to the file so that easily change their value in testing procedure. Output results (efficiency indicators) are written to CSV for further processing. The most preferable environment for running this project is Eclipse, free to download from the Internet. Each classifier is written in the form of *Python* class so that further extension would be very easy. An example of classifiers usage is presented in appendix A.

5 Results of experiments

This section presents environment setup and later the results of simulation investigations. It is important to describe simulation setup so that in the future someone could repeat test or maybe extend the application.

5.1 Testing environment

When it comes to classification task there is always a problem of how to divide available dataset into training and testing sets. One of the most common approach to ensure proper classifier evaluation is cross validation (more information about cross validation methods can be found in [43]). There are different types:

- **holdout cross validation**- data set is separated into two sets, called the training set and the testing set. Classifier is trained using the training set only. Then the classifier is asked to predict the output values for the data in the testing set. The advantage of this method is that it is usually preferable to the residual method and takes no longer to compute. However, its evaluation can have a high variance, because it depends heavily on which data points end up in the training set and which end up in the test set,

- **Leave-one-out cross validation**- the classifier is trained on all the available data except for one point and a prediction is made for that point and stored to compute the average error from all points,
- **K-fold cross validation**- the data set is divided into k subsets, and the hold-out method is repeated k times. Each time, one of the k subsets is used as the test set and the other $k - 1$ subsets are merged together to form a training set,

In this thesis 4-fold cross validation was used (see fig. 5.1). To ensure that presented results are reliable each test was repeated 10 times.

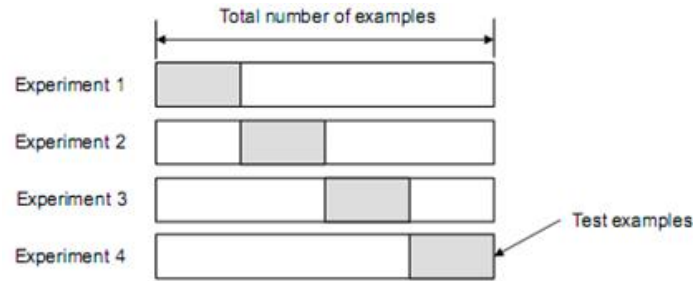


Figure 5.1: Example of 4-fold cross validation.

5.2 Testing results

The main goal of performed tests was to check classification accuracy for different algorithms. As stated before three algorithms were used, but in simulations were carried out. To sum up, the main motivations are as follows:

- check the accuracy of classification for basic rough sets algorithm and show that in more complex problem it is useless,
- show how the modification of decision rules improves the efficiency comparing with the basic rough sets algorithm,
- check how genetic algorithm can improve the classification accuracy for both rough sets and fuzzy logic algorithms,
- prove that proposed hybrid classifier can compete with other classifiers,

5.2.1 Impact of granulation step G on rough sets algorithm efficiency

The aim of this test was to find out how the discretization of feature space affects the classification accuracy. The number of intervals for each attribute was the same and denoted as K_l , where $l \in (1, \dots, q)$. Habernam dataset was taken as the input to the system. Dataset was divided into training and testing dataset using 4-fold cross-validation method. The number of objects to recognize was equal to 153. Results of the simulation are presented in table 5.1 where the notation is as follows:

- G - granulation step
- O - total number of correctly recognized objects
- C/CD - number of patterns for which a *certain* decision rules were used/number of correctly recognized patterns using these rules,
- P/PD - number of patterns for which a *possible* decision rules were activated/number of correctly classified objects using these rules,
- V number of patterns rejected from classification. There was no suitable rule or more rules than one have the same strength but different class label.,
- C^*, P^*, V^* - total number of *certain* , *possible* or *dummy* (strength is equal to zero) decision rules, respectively

In the experiment, for every feature the initial step of granulation was changed from 4 to 18 while the factor of its increasing was equal to one.

Analyzing the results of simulation presented in table 5.1 one can see that the quality of the algorithm depends heavily on the step of granulation G and better results are obtained rather for small G . It can be concluded that increasing G results in growing the number of rules with the strength equal to zero (parameter V^*), because in the learning phase algorithm is unable to find class representatives. In this case classification is impossible and pattern is rejected. The bigger granulation step G is, more *certain* or *possible* rules are obtained, but on the other hand the number of cells without any representatives is increasing. Generally, it is better to keep V^* parameter rather small and correlate its value with C and P factors. Let notice that for $G = 18$ the classification accuracy is very poor even though the number of *certain* rules is the biggest comparing with other cases. The reason for low classification is connected with V^* where 5794 rules are *dummy*.

Table 5.1: Result of simulation for finding the dependency between granulation step and classification accuracy

G	O	C/CD	P/PD	V	C^*	P^*	V^*
4	95	0/0	125/95	28	3	7	54
5	97	0/0	121/97	32	2	4	119
6	52	3/1	65/51	85	5	8	203
7	74	0/0	91/74	62	3	6	334
8	18	2/1	24/17	127	8	9	495
9	52	4/4	59/48	90	6	7	716
10	57	36/25	38/32	79	12	13	975
11	25	5/2	30/23	118	7	9	1315
12	45	49/41	9/4	95	19	12	1697
13	9	0/0	13/9	140	17	12	2168
14	30	9/6	28/24	116	24	14	2706
15	29	30/25	17/4	106	27	13	3335
16	18	2/1	20/17	131	22	12	4062
17	25	3/1	38/24	112	24	17	4872
18	9	2/1	11/8	140	25	13	5794

5.2.2 Impact of recursive modification of granulation step G on rough sets algorithm efficiency

In the previous section (5.2.2) it was shown that the granulation step strongly affects the classification accuracy. Greater G implies that we have more *certain* or *possible* rules, but on the other the hand number of patterns without rule covering is increasing. A lot of patterns are rejected because no proper rule is found. To improve that situation an algorithm with modification of decision rules is proposed. More details are presented in section 3.2, but generally when an object is rejected from classification, current G is decreased by $\epsilon = 1$ until *certain* for *possible* rule is found. The same dataset was used as in the previous test with the same algorithm settings to show that this approach is more efficient comparing with the previous algorithm. Results of classification are presented in fig. 5.2 for the basic rough sets (blue line-BRS) and algorithm with modification of decision rules (orange line-MRS).

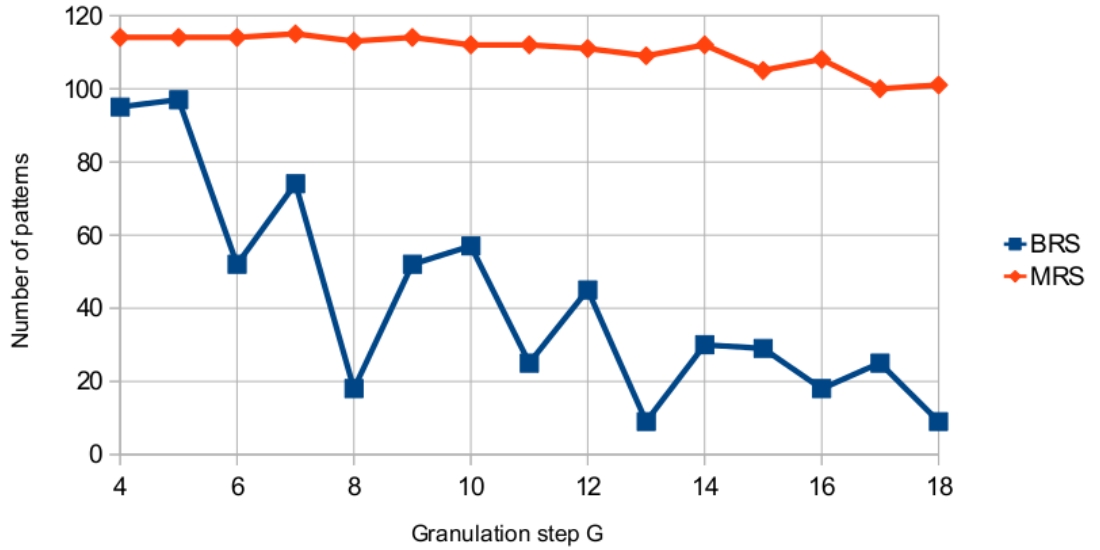


Figure 5.2: Comparison of Basic Rough sets algorithm with algorithm where modification of decision rules is introduced.

Looking at fig. 5.2 one can observe that the modification of granulation step G during algorithm execution increases the classification accuracy and even if algorithm is started with different G the classification stays almost at the same level while in the basic approach the greater G implies worse results. Additionally, it is visible that increasing granulation step is not the right solution. Even if the number of *certain* or *possible* rules is greater, the final result is worse. What is more important, computational time is longer for greater G . In this case the optimal G would be 5, but for each problem G should be chosen independently because it must reflect how patterns are located in the feature space.

The main two disadvantages of proposed rough sets algorithm are as follows:

- it uses an arbitrary chosen step of granulation. Modification of decision rule improves classifier quality, but for the prize of computational time,
- it uses all attributes for creation of decision rules. When the problem is complex then the decision rules are long and tangled. Additionally, some features are useless in classification, instead of valuable information they bring noise to the system and deteriorate final results,

5.2.3 Impact of number of membership functions on genetic fuzzy logic algorithm efficiency

5.2.3.1 Example of rule generation

In this section the results of fuzzy logic classifier simulation are presented. The goal is to show that proposed algorithm construction is correct and gives satisfactory results. Fuzzy logic algorithm is used in hybrid classifier so it was required to check its properties and find the best parameter settings, especially in case of the number of membership functions.

At first, let remind what are the requirements for fuzzy logic classifier in this thesis. For the input it is provided a dataset without no expert knowledge of how to appropriately divide the feature space into fuzzy sets, and how many membership functions are need. The goal is to find minimal rule set with the possible highest classification accuracy.

As the first step, let present how fuzzy logic classifier deals with pattern recognition and what is the minimal rule set for exemplary Iris dataset. At the beginning each feature is divided into 14 membership functions in the same way as presented in fig. 3.1 plus one linguistic variable *DON'T USE*. The final rule set which was able to classify 32 out of 34 testing patterns is presented in fig. 5.3. From 10 test the following rule construction occurred 7 times. In this example each attribute was normalized and after this process feature values were from $< 0, 1 >$ range.

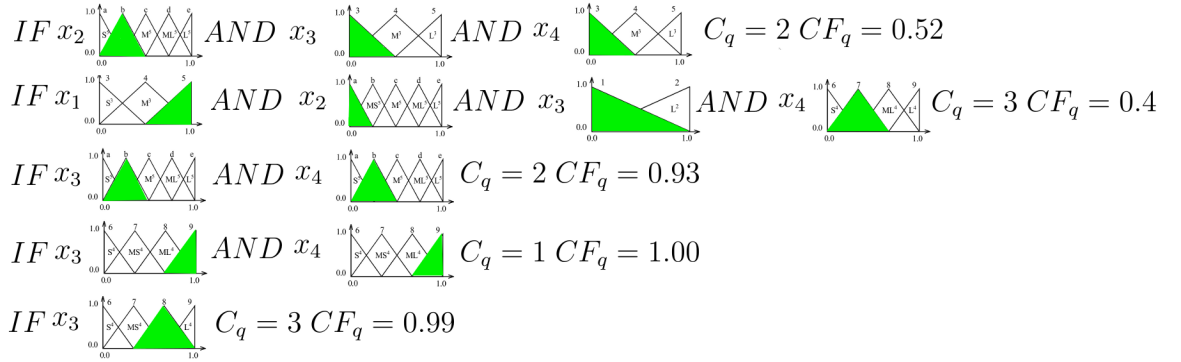


Figure 5.3: Example of rule set generated for Iris dataset

Analyzing figure 5.3 it is visible that some attributes were omitted from classification, but the results of classification are quite satisfactory. Additionally, only five rules were needed for correct pattern recognition.

5.2.3.2 Classification accuracy and the number of membership functions in genetic fuzzy logic algorithm

The goal of the second part of this test was to check how the number of initial membership functions affects the final result of classification. There is a question if it is better to use many small membership functions (for example 14 functions such as presented in fig. 3.1) or only few functions with greater area coverage. In the first case the solution space is much greater than in the second approach so many rules must be created to obtain reliable results. Additionally, in most recognition problems we do not need so precise feature partitions because it can happen that for many created regions we can not find proper representatives in the training set.

Parameters for genetic algorithms are the same as presented in table 3.1. In each simulation the level of partitions k was changed from 7 to 2. Few words of explanation should be written about how k determines the number of membership functions MF for each attribute. This number is described by eq. (5.1)

$$MF = \sum_{i=1}^k (n_i + 1) + 1 \quad (5.1)$$

Wine dataset was used as the input to the system. After dividing the set into three sets (the first one for genetic algorithm, the second and the third for fuzzy logic training and testing) 45 patterns were used for classification.

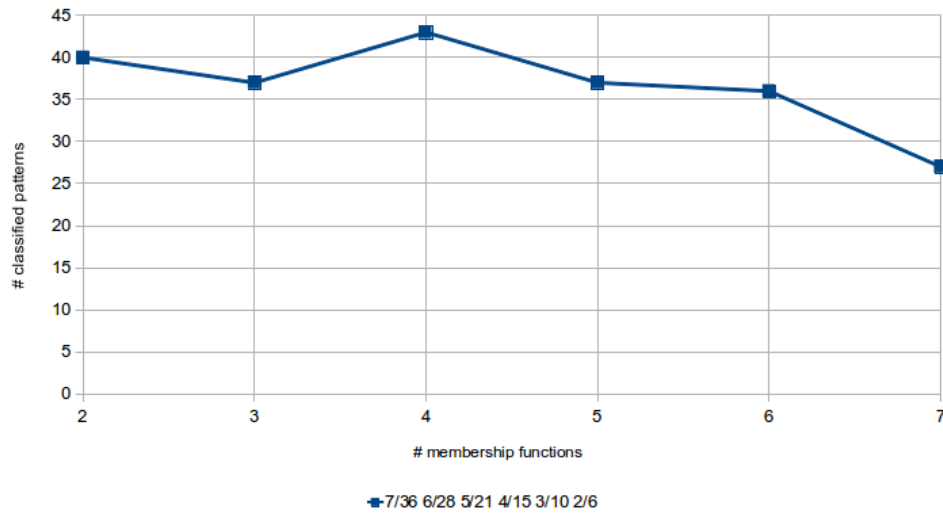


Figure 5.4: Impact of initial number of membership functions MF per attribute on the final fuzzy classification rate

Looking at the fig. 5.4 one can conclude that it is not worth constantly increasing the number of membership functions (MF) per attribute because for greater MF algorithm obtains worse results. From conducted simulations it was noticed that the optimal k value is 4 which leads to 14 different membership functions per attribute. If not implicitly stated, $k = 4$ will be always used in next tests.

5.2.3.3 Classifiers performance comparison

Here the genetic fuzzy logic algorithm is compared with rough sets algorithms. For basic rough sets algorithm granulation step G was set to 4 while for rough sets algorithm with modification of decision rules this factor was equal to 7. Parameters for genetic fuzzy logic classifier were the same as presented in table 3.1, parameter k was set to 4. Basic rough sets algorithm and algorithm with modification of decision rules used all available features for classification. In case of genetic fuzzy logic classifier the average number of attributes used in 10 rules was calculated.

The main purpose of this simulation was to check the efficiency of proposed genetic algorithm, especially how many attributes and how many rules are needed for the proper classification. Table 5.2 presents the results of simulation. Notation used in the table is as follows:

- F - number of attributes in the dataset
- O - number of objects used for classification
- W, B, A, σ -performance indicators described in section 4.3
- RSR - number of objects correctly recognized by basic rough sets algorithm
- $RSMRS$ - number of patterns correctly recognized by rough sets algorithm with modification of decision rules
- GFL - number of objects correctly recognized by genetic fuzzy logic classifier
- FU - an average number of features used by genetic rough fuzzy logic classifier in the best rule set

Table 5.2: Comparison of accuracy of classification for genetic fuzzy logic classifier with rough sets algorithms for different datasets

Dataset	F	O	W	B	A	σ	FU
RSR							
haberman	3	77,0	50,0	55,0	53,0	1,9	3,0
iris	4	39,0	27,0	31,0	29,3	1,5	4,0
thyroid	5	56,0	41,0	46,0	44,0	1,9	5,0
bupa	6	87,0	25,0	37,0	30,3	4,3	6,0
pima	8	192,0	85,0	98,0	91,5	4,9	8,0
wine	13	45,0	0,0	2,0	0,8	0,8	13,0
wdbc	30	142,0	10,0	15,0	11,8	2,0	30,0
RSMR							
haberman	3	77,0	53,0	58,0	55,5	2,1	3,0
iris	4	39,0	32,0	38,0	34,8	2,4	4,0
thyroid	5	56,0	48,0	50,0	49,3	0,8	5,0
bupa	6	87,0	23,0	35,0	28,8	4,3	6,0
pima	8	192,0	62,0	69,0	65,5	2,5	8,0
wine	13	44,5	15,0	23,0	19,3	3,0	13,0
wdbc	30	142,3	10,0	15,0	11,8	2,0	30,0
GFL							
haberman	3	77,0	54,0	58,0	55,9	1,2	8,5
iris	4	39,0	29,0	39,0	34,3	3,1	8,2
thyroid	5	56,0	41,0	45,0	43,4	1,2	8,4
bupa	6	87,0	49,0	58,0	53,4	2,7	8,3
pima	8	192,0	124,0	140,0	134,1	4,8	7,7
wine	13	45,0	35,0	42,0	38,0	1,9	8,3
wdbc	30	142,0	107,0	127,0	119,2	5,6	7,9

Results from table 5.2 indicate that genetic fuzzy logic classifier can compete with other classifier. The main advantage is connected with the number of attributes building decision rules. In each case some features were removed from the set. Let notice that for more complex problems, for d greater than 4 fuzzy logic algorithm obtained the best results, much better than basic rough sets and rough sets with modification of decision rules.

5.2.4 Impact of granulation step G on genetic rough sets algorithm efficiency

The goal of this test is to check which approach is better:

1. use the same granulation step G for each attribute and additionally take all

feature into classification,

2. use different partition for each attribute independently and try to remove some features treating them as a noise,

The first approach is simulated by algorithm with modification of decision rules (see section 3.2), while the second is genetic rough sets algorithm (described in section 3.3). Results of simulations are placed in table 5.3. ,where the notation is as follows:

- F - number of attributes in the dataset
- O - number of objects used for classification
- W, B, A, σ -performance indicators described in section 4.3
- RSR number of objects correctly recognized by basic rough sets algorithm
- $RSMR$ number of patterns correctly recognized by rough sets algorithm with modification of decision rules
- GRR number of objects correctly recognized by genetic rough sets algorithm
- FU number of features used by genetic rough sets algorithm for classification.

Parameters for genetic rough sets algorithm were the same as presented in table 3.2 and for the first rough sets algorithm granulation step G was equal to 4 and for algorithm with modification of decision rules starting granulation value was 7. These parameters were selected from the previous simulations because in such configuration the best results were obtained. For genetic rough sets classifier K_{max} was set to 7.

Table 5.3: Accuracy of classification for genetic rough sets and basic rough sets algorithms for different datasets

Dataset	F	O	W	B	A	σ	FU
RSR							
haberman	3	77,0	50,0	55,0	53,0	1,9	3,0
iris	4	39,0	27,0	31,0	29,3	1,5	4,0
thyroid	5	56,0	41,0	46,0	44,0	1,9	5,0
bupa	6	87,0	25,0	37,0	30,3	4,3	6,0
pima	8	192,0	85,0	98,0	91,5	4,9	8,0
wine	13	45,0	0,0	2,0	0,8	0,8	13,0
wdbc	30	142,0	10,0	15,0	11,8	2,0	30,0
RSMR							
haberman	3	77,0	53,0	58,0	55,5	2,1	3,0
iris	4	39,0	32,0	38,0	34,8	2,4	4,0
thyroid	5	56,0	48,0	50,0	49,3	0,8	5,0
bupa	6	87,0	23,0	35,0	28,8	4,3	6,0
pima	8	192,0	62,0	69,0	65,5	2,5	8,0
wine	13	44,5	15,0	23,0	19,3	3,0	13,0
wdbc	30	142,3	10,0	15,0	11,8	2,0	30,0
GRR							
haberman	3	77,0	58,00	61,00	59,50	1,50	2,00
iris	4	39,0	35,00	39,00	37,00	1,58	2,42
thyroid	5	56,0	49,00	54,00	51,17	1,91	2,58
bupa	6	87,0	55,00	61,00	57,67	2,06	2,67
pima	8	192,0	145,00	155,00	149,42	3,15	3,00
wine	13	45,0	40,00	44,00	42,67	1,43	2,92
wdbc	30	142,0	128,00	135,00	132,67	1,65	5,08

From table 5.3 one can conclude that genetic rough sets algorithm obtains better results than other algorithms, especially it is visible for more complex problems such as wine or pima datasets. Additionally, let analyze the last column FU . It determines how many attributes are used in genetic rough sets algorithm for classification. It is noticeable that some features are useless and genetic rough sets algorithm is able to find valuable attributes. Another thing to reconsider is how the complexity of the problem affects algorithm classification accuracy. Basic rough sets algorithm tackles quite well with simple problems, for example iris or haberman datasets, but when the number of attributes is greater than 4 then the algorithm efficiency decreases, while genetic rough sets is not affected by this problem and can deal with complex datasets.

When you compare the performance of genetic rough sets and fuzzy logic classifier it is visible that genetic rough sets classifier obtains better results. Taking this into account, hybrid classifier is constructed in such a way that genetic rough sets algorithm is used as the first stage. Only if pattern is rejected fuzzy logic classifier is invoked.

5.2.5 Comparison of hybrid multistage classifier with other classifiers

In this section results of simulation for hybrid classifier are presented. The accuracy of classification is compared with other classifiers trained and tested with the same datasets. The main goal of this simulation was to check if proposed hybridization (rough set and fuzzy logic) can compete with other classifiers. As the source of reference different types of classifiers were chosen to ensure the greatest diversity:

- LDAC classifier (Linear Discriminant classifier)- The linear discriminant analysis method consists of searching some linear combinations of selected variables, which provide the best separation between the considered classes. These different combinations are called discriminant functions (see example in fig. 5.5)

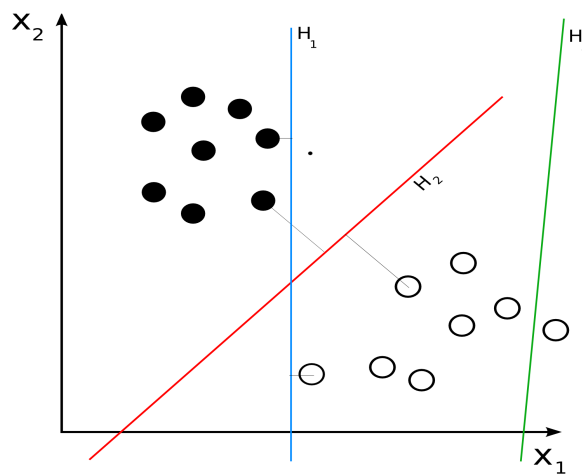


Figure 5.5: Example of linear discriminant classifier for 2-dimensional problem

- 3-KNN Classifier- it is one of the simplest approach in the pattern recognition, it classifies objects based on closest training examples in the feature space.
- Gini index classifier- it is an example of decision tree algorithm where the decision is represented in case of decision rules. Decision node specifies a

test on a single attribute, leaf node indicates the value of the target attribute, arc/edge splits of one attribute and indicate the disjunction of test to make the final decision. Decision trees classify instances or examples by starting at the root of the tree and moving through it until a leaf node is reached.

- Maximum likelihood classifier- this classifier is commonly used in image recognition tasks. It assigns a pixel to a class on the basis of its probability of belonging to a class whose mean and covariance are modelled as forming a normal distribution in multidimensional feature space.
- Svm classifier- it is non-probabilistic linear classifier which deals with finding an optimal linear hyperplanes for class separation.

Results of simulation are presented in table 5.4, where notation is as follows:

- O - number of patterns to be recognized
- W, B, A, σ -performance indicators described in section 4.3
- MACL- Maximum Likelihood Classifier
- Hybrid- multistage hybrid rough sets fuzzy logic classifier described in section 3.3.4

Numbers in bold font indicates this classifier which obtained the best result for a particular dataset. In cases when the same result was obtained for more classifier then the number of attributes is taken into account.

Table 5.4: Comparison of hybrid rough fuzzy classifier with other common classifiers

Classifier	O	W	B	A	σ
iris					
LDAC	39,0	36,0	38,0	36,8	0,8
KNN	39,0	34,0	38,0	36,2	1,8
GINI	39,0	31,0	38,0	35,5	2,9
MACL	39,0	34,0	38,0	36,3	1,5
SVM	39,0	24,0	26,0	25,0	1,0
Hybrid	39,0	35,0	39,0	37,0	1,6
Bupa					
LDAC	87,0	56,0	62,0	59,3	2,8
KNN	87,0	55,0	65,0	59,3	4,0
Continued on next page					

Table 5.4 – continued from previous page

Classifier	O	W	B	A	σ
GINI	87,0	54,0	56,0	55,0	0,7
MACL	87,0	48,0	56,0	52,5	3,2
SVM	87,0	53,0	63,0	59,5	3,8
Hybrid	87,0	55,0	62,0	58,8	2,6
Pima					
LDAC	192,0	146,0	149,0	147,5	1,5
KNN	192,0	131,0	136,0	132,3	2,2
GINI	192,0	123,0	136,0	130,5	5,3
MACL	192,0	142,0	145,0	143,3	1,1
SVM	192,0	128,0	141,0	134,8	4,7
Hybrid	192,0	146,0	152,0	149,0	2,1
haberman					
LDAC	77,0	57,0	58,0	57,5	0,5
KNN	77,0	54,0	57,0	55,8	1,1
GINI	77,0	45,0	58,0	52,3	4,7
MACL	77,0	57,0	58,0	57,8	0,4
SVM	77,0	55,0	58,0	56,5	1,1
Hybrid	77,0	58,0	63,0	60,0	2,1
wdbc					
LDAC	142,0	133,0	138,0	135,5	1,8
KNN	142,0	127,0	134,0	131,3	2,7
GINI	142,0	128,0	131,0	128,8	1,3
MACL	142,0	133,0	139,0	135,8	2,2
SVM	142,0	127,0	135,0	131,8	2,9
Hybrid	142,0	127,0	138,0	134,3	4,3
thyroid					
LDAC	56,0	47,0	51,0	49,3	1,8
KNN	56,0	48,0	52,0	50,8	1,6
GINI	56,0	48,0	53,0	50,5	1,8
MACL	56,0	50,0	54,0	51,0	2,1
SVM	56,0	50,0	53,0	51,0	1,2
Hybrid	56,0	50,0	55,0	52,0	2,1
wine					
LDAC	45,0	41,0	43,0	43,0	0,7
KNN	45,0	27,0	35,0	39,9	3,0
Continued on next page					

Table 5.4 – continued from previous page

Classifier	<i>O</i>	<i>W</i>	<i>B</i>	<i>A</i>	σ
GINI	45,0	37,0	41,0	39,0	2,0
MACL	45,0	43,0	45,0	44,3	0,8
SVM	45,0	38,0	44,0	41,3	2,4
Hybrid	45,0	41,0	43,0	43,0	0,7

From table 5.4 one can conclude that hybrid classifier obtains quite good results comparing with other classifiers. From seven datasets four times it was the best. In other cases results are comparable. What is more important hybrid classifier is able to classify pattern with reduced number of attributes. In this case created decision rules are simpler and more readable for user. This is especially important in medicine where physician is provided with decision rules and basing on them makes the final diagnosis. Another thing to reconsider is the stability of proposed classifier. It uses genetic algorithm for rule construction, so taking into account its random nature hybrid classifier can be unstable, but simulations show that this is not a problem. Appropriate number of generations for genetic algorithm assures the proper convergence and as the consequence hybrid classifier is stable.

To prove that proposed hybridization is effective let compare results from tables 5.4, 5.3. In every case the results of classification were better than for single genetic rough sets algorithm. It means that if pattern was rejected from classification in the first stage in some cases it was possible to classify pattern by fuzzy logic classifier.

6 Summary and conclusions

6.1 Conclusions from tests

In this paper the results of simulation investigations were presented. The main purpose of five test scenarios was to evaluate prepared classifiers in case of classification accuracy. Implemented and tested algorithms in this paper are as follows:

1. Basic Rough sets algorithm
2. Rough sets algorithm with modification of decision rules
3. Genetic Based Fuzzy Logic algorithm
4. Multistage hybrid classifier using Rough sets and Fuzzy logic

Author intention was to present the whole process of constructing complex classifier from simpler ones. Researches started with the basic rough sets algorithm. Because the results of classification were not satisfactory new algorithm with modification of decision rules was introduced. Significant improvement was visible for simple problems, but for multidimensional datasets the classification accuracy remained poor. The next proposal consisted in constructing multistage hybrid classifier where the power of fuzzy logic and rough sets reasoning were connected. Original algorithm presented in this thesis used genetic algorithm for finding the reduct of attributes and the optimal granulation step G for each attribute in case of rough sets and for fuzzy logic genetic algorithm determined the best decision rule set. Conducted test confirmed that proposed algorithm can compete with other classifiers and obtains reliable results. Next paragraphs will shortly summarize each research.

The main goal of the first simulation was to check how the granulation step G affects the classification accuracy. The efficiency of rough sets algorithm is determined by the number of *certain* and *possible* rules. Ideally, it would be great that all rules are *certain* and we have 100% coverage in the rule set. Conveyed simulations confirmed the thesis that granulation step has the greatest impact on the classification results. This parameter must be chosen very carefully and few factors must be taken into account:

- the complexity of the problem, how many attributes are used to describe the pattern,
- how the granulation intervals are generated, equally for all attributes or independently,

Another important conclusion from the first test concerns the optimal value for G . It is not worth increasing G , good results of classification are obtained rather for small numbers such as five or six. As the evidence analyze table 5.1 where for $G = 18$ the classification accuracy was very poor. Increasing G ensures that the number of *certain* decision rules is greater, but on the other hand there are more dummy rules (rule with strength equal to 0).

Taking into account the fact that the classification accuracy of algorithm presented in section was not satisfactory, the recursive rough sets algorithm was proposed to tackle with situation where for the particular granulation G there is no appropriate rule in the rule sets. From simulations it can be concluded that this approach improved the classification significantly and what is more important, thanks to the modification of decision rules algorithm was not affected by the granulation step G . The stability of classifier is the biggest advantage, but on the other hand the main drawback is connected with algorithm execution. In situation when the pattern is rejected from classification algorithm is recursively invoked until a proper rule is found. This requires much more time than the basic approach without rule modification. For simple classification tasks this is not a problem, but for more complex datasets algorithm computation is much longer.

This thesis concerns the problem of classifying unknown patterns using rough sets approach and fuzzy logic reasoning. To successfully accomplish this task new algorithm of fuzzy logic had to be implemented. In the literature there are many examples of fuzzy logic controllers or classifiers. In this paper genetic based fuzzy logic classifier was implemented. The main advantages of proposed solution are as follows:

- it manages to construct decision rules without any expert knowledge. It means that the input to the system constitutes only raw data from dataset without additional information about the number of membership functions or their location in the feature space. As the output we obtain rule set with the highest classification rate,
- it is able to apply attribute reduction,

In section it was shown that proposed fuzzy classifier was able to classify 32 out of 34 testing patterns from iris dataset with five rules (presented in fig. 5.3).

As fuzzy logic was finished and genetic algorithm turned out to be a good solution it was decided to connect genetic algorithm with rough sets and check how this fusion works. The main goal of the heuristic approach was to find an optimal granulation for each feature independently and try to remove those attributes that are useless in classification. Conducted tests proved that this direction of researches is good and gives promising results. Especially it was visible for high

dimensional datasets where the classification rate for basic rough sets and rough sets with modification of decision rules was very low.

The last part in this master thesis consisted in constructing a multistage hybrid classifier connecting implemented by the author genetic rough sets algorithm and genetic based fuzzy logic algorithm. There are many types and methods of classifier fusion, but in this paper two-phased sequential classification was proposed. Comparison with different well-known classifiers gave optimistic results, but of course more profound test are required to fully prove its usefulness.

6.2 General conclusions

To sum up, this paper presents the result of work in the field of pattern recognition and classification. It reviewed some of the most representative publications connected with rough sets, fuzzy logic and genetic algorithms. Here an original construction of hybrid classifier has been presented. Attached results are very promising and encourage for further more complex investigations. Generally, pattern recognition is not an easy task and many factors and parameters must be taken into account. Until now, no-one has managed to invent the classifier with 100% accuracy. Even if for one dataset results are satisfactory, but for others classification will be worse.

7 Future work

In this thesis the basic parameters and settings were checked for rough sets algorithm and fuzzy logic algorithm. In the future, it is strongly recommended to carry out more profound simulations to fully understand behaviour of algorithm in different environment and settings. In this paper to evaluate each classifier well-known datasets from *UCI* Repository were used. This offers reliable environment for testing, but the next step in the future work is to apply proposed algorithm into real life problem, such as optimal control or image pattern recognition. Using rough sets properties it would be advisable to detect tumor tissue on CT, MRI images or bone structures for further 3D reconstruction.

Another thing to reconsider in the next researches is how to generate partition of feature. Here, the genetic algorithm was used to find the reduct of attributes and the number of intervals for each attribute independently. Results of simulations confirmed the usefulness of this approach, but here arises the question if there is another solution for finding an optimal feature granulation. Two possible future tests:

1. Testing the classification accuracy of rough sets algorithm when granulation is based on the frequency of patterns in the training dataset. This approach assumes that for clusters with many patterns the granulation will be more precise while in other places it would be sparse.
2. Testing the classification accuracy of rough sets algorithm when granulation is determined by fuzzy logic and triangular membership functions. A concept of fuzzy discretization of feature space for a rough sets theoretic classifier is presented in [2]

The last, but not least aspect of the future work is to check different types of classifier hybridization. In this thesis rough sets algorithm was the most important and only in cases when pattern was rejected fuzzy logic classifier was used. It would be required to simulate different scenarios of classifier ensemble, for example majority voting with fuzzy logic, rough sets and neural network or 3-KNN classifiers. Additionally, it would be great to compare different algorithms for feature reduction with genetic approach used in this paper.

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Appendix A Program description

A.1 Installation requirements

For the test purposes the simulator in *Python* language was written. To successfully run this program few requirements must be met. In this master thesis this software was executed on Linux platform and here this approach will be described. Of course it is possible to run the program on Windows, but proper preparation must be undertaken. Requirement for the software:

- *Python* in version at least 2.6
- NumPy
- SciPy
- mlp library with Gsl. Steps for the proper installation:
 1. `sudo apt-get install python2.6-dev`
 2. go to: <http://www.gnu.org/prep/ftp.html>
 3. click on an ftp link close to your location
 4. find the gsl/ directory and click on it
 5. find the gsl-VERSION.tar.gz file, where version is 1.14 or greater. Click on that file to download it.
 6. In a terminal window extract the tar.gz file using `tar -xzf gsl-VERSION.tar.gz` and then `cd` to the `./gsl-VERSION` directory
 7. Look at the INSTALL file. It will probably tell you to run `./configure`, then `make`, and then `make install`
 8. download mlp from <http://sourceforge.net/projects/mlpy/files/>
 9. unzip file and inside directory run from command line `python setup.py install`

The whole project is divided into modules and each classifier is implemented as python class:

- BasicClassifiers- this class implements basic classifiers such as: LDAC, 3-KNN, MaximumLikelyHood Classifier, Gini Index Classifier, svm Classifier
- RoughSetsClassifier- this class implements basic rough sets classifier. Depending on the chosen module it is an algorithm with modification of decision rules or not.

- **GeneticFuzzyLogicClassifier**- this class simulate genetic fuzzy logic classifier. In the beginning genetic algorithm is run to obtain the best decision rule set and later classification is done
- **GeneticRoughSetsClassifier**- this class implements genetic rough sets classifier. It comprises of two parts:
 - genetic algorithm for obtaining an optimal partition for each feature
 - classification procedure which uses partition from the previous step for pattern recognition
- **HybridClassifier**- this class implements hybrid classifier. This is a multistage classifier in which rough sets algorithm is treated as the first classifier and fuzzy logic as the second.

A.2 Example usage

To run each classifier few basic steps must be done. First of all proper parameters with cross-validation and dataset type must be chosen. Below, a simple example is presented showing how to run genetic rough sets classifier fir iris dataset

```

1: #!/usr/bin/python
2: # -*- coding: utf-8 -*-
3:
4: from genetic_rough_sets_classifier import GeneticRoughSetsClassifier
5:
6: if __name__ == '__main__':
7:     K_FOLD_NUMBER = 4
8:     K = 1
9:     GENERATIONS = 500
10:    MUTATION = 0.3
11:    CROSS_OVER = 0.9
12:    G = 7
13:    POPULATION = 10
14:    grs = GeneticRoughSetsClassifier(debug=DEBUG)
15:    filename = 'datasets/iris.data.txt'
16:    grs.read_data(filepath=filename, label_is_last=False)
17:    grs.prepare_data(k_fold_number=K_FOLD_NUMBER)
18:    grs.k_fold_cross_validation(k=K)
19:    grs.initialize_genetic(generations=GENERATIONS, mutation_prop=MUTATION, ↵
        crossover_prop=CROSS_OVER)
20:    grs.create_population(population_size=POPULATION, division=G)
21:    # run genetic algorithm
22:    grs.run()
23:    # genetic algorithm found granulation
24:    # now classify patterns
25:    classified = 0
26:    for i in range(len(patterns)):
27:        if labels[i] == grs.classify(patterns[i]):
28:            classified += 1
29:    grs.print_summary()

```

A.3 Results

In each simulation results of classification for each classifier are saved in csv file. Names of the files are as follows:

- *results/rough_sets_classifier.csv* for RoughSetsClassifier
- *results/genetic_fuzzy_logic_classifier.csv* for GeneticFuzzyLogicClassifier
- *results/genetic_rough_sets_classifier.csv* for GeneticRoughSetsClassifier
- *results/hybrid_classifier.csv* for hybrid classifier

After execution of the program each file is created from scratch so be sure that the previous version of results is save in another location or another name.
