

Rough Set Theory – Fundamental Concepts, Principals, Data Extraction, and Applications

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1. Introduction

Rough Set Theory, proposed in 1982 by Zdzislaw Pawlak, is in a state of constant development. Its methodology is concerned with the classification and analysis of imprecise, uncertain or incomplete information and knowledge, and of is considered one of the first non-statistical approaches in data analysis (Pawlak, 1982).

The fundamental concept behind Rough Set Theory is the approximation of lower and upper spaces of a set, the approximation of spaces being the formal classification of knowledge regarding the interest domain.

The subset generated by lower approximations is characterized by objects that will definitely form part of an interest subset, whereas the upper approximation is characterized by objects that will possibly form part of an interest subset. Every subset defined through upper and lower approximation is known as Rough Set.

Over the years Rough Set Theory has become a valuable tool in the resolution of various problems, such as: representation of uncertain or imprecise knowledge; knowledge analysis; evaluation of quality and availability of information with respect to consistency and presence a not of date patterns; identification and evaluation of date dependency; reasoning based an uncertain and reduct of information data.

The extent of rough set applications used today is much wider than in the past, principally in the areas of medicine, analysis of database attributes and process control. The subject of this chapter is to present the Rough Set Theory, important concepts, and Rough Set Theory used with tools for data mining, special applications in analysis of data in dengue diagnosis. The chapter is divided into the four following topics:

- Fundamental concepts
- Rough set with tools for data mining
- Applications of rough set theory;
- Case – Rough set with tools in dengue diagnosis.

2. Fundamental concepts

Rough Sets Theory has been under continuous development for over years, and a growing number of researchers have become its interested in methodology. It is a formal theory derived from fundamental research on logical properties of information systems. From the

outset, rough set theory has been a methodology of database mining or knowledge discovery in relational databases. This section presents the concepts of Rough Set Theory; which coincide partly with the concepts of other theories that treat uncertain and vagueness information. Among the existent, most traditional approaches for the modeling and treatment of uncertainties, they are the Theories of the Uncertainty of Dempster-Shafer and Fuzzy Set (Pawlak et al., 1995). The main concepts related to Rough Set Theory are presented as the following:

2.1 Set

A set of objects that possesses similar characteristics it is a fundamental part of mathematics. All the mathematical objects, such as relations, functions and numbers can be considered as a set. However, the concept of the classical set within mathematics is contradictory; since a set is considered to be "grouping" without all elements are absent and is know as an empty set (Stoll, 1979). The various components of a set are known as elements, and relationship between an element and a set is called of a pertinence relation. Cardinality is the way of measuring the number of elements of a set. Examples of specific sets that treat vague and imprecise date are described below:

a. Fuzzy Set

Proposed by mathematician Loft Zadeh in the second half of the sixties, it has as its objective the treatment of the mathematical concept of vague and approximate, for subsequent programming and storage on computers.

In order for Zadeh to obtain the mathematical formalism for fuzzy set, it was necessary to use the classic set theory, where any set can be characterized by a function. In the case of the fuzzy set, the characteristic function can be generalized so that the values are designated as elements of the Universe Set U belong to the interval of real numbers $[0,1]$.

The characteristic Function Fuzzy is $\mu_A: U \rightarrow [0,1]$, where the values indicate the degree of pertinence of the elements of set U in relation to the set A , which indicated as it is possible for an element of x of U to belong to A , this function is known as Function of Pertinence and the set A is the Fuzzy Set (Zadeh, 1965).

b. Rough Set

An approach first forwarded by mathematician Zdzislaw Pawlak at the beginning of the eighties; it is used as a mathematical tool to treat the vague and the imprecise. Rough Set Theory is similar to Fuzzy Set Theory, however the uncertain and imprecision in this approach is expressed by a boundary region of a set, and not by a partial membership as in Fuzzy Set Theory. Rough Set concept can be defined quite generally by means of interior and closure topological operations know approximations (Pawlak, 1982).

Observation:

It is interesting to compare definitions of classical sets, fuzzy sets and rough sets. Classical set is a primitive notion and is defined intuitively or axiomatically. Fuzzy set is defined by employing the fuzzy membership function, which involves advanced mathematical structures, numbers and functions. Rough set is defined by topological operations called approximations, thus this definition also requires advanced mathematical concepts.

2.2 Information system or information table

An information system or information table can be viewed as a table, consisting of objects (rows) and attributes (columns). It is used in the representation of data that will be utilized by Rough Set, where each object has a given amount of attributes (Lin, 1997).

These objects are described in accordance with the format of the data table, in which rows are considered objects for analysis and columns as attributes (Wu et al., 2004). Below is shown an example of an information Table 1.

| Patient | Attributes | | | |
|---------|------------|----------|-------------|---------------|
| | Headache | Vomiting | Temperature | Viral illness |
| #1 | No | Yes | High | Yes |
| #2 | Yes | No | High | Yes |
| #3 | Yes | Yes | Very high | Yes |
| #4 | No | Yes | Normal | No |
| #5 | Yes | No | High | No |
| #6 | No | Yes | Very high | Yes |

Table 1. Example of information table

2.3 Indiscernibility relation

Indiscernibility Relation is a central concept in Rough Set Theory, and is considered as a relation between two objects or more, where all the values are identical in relation to a subset of considered attributes. Indiscernibility relation is an equivalence relation, where all identical objects of set are considered as elementary (Pawlak, 1998)

In Table 1, presented in section 2.2, it can be observed that the set is composed of attributes that are directly related to the patients' symptoms whether they be headache, vomiting and temperature. When Table 1 is broken down it can be seen that the set regarding {patient2, patient3, patient5} is indiscernible in terms of headache attribute. The set concerning {patient1, patient3, patient4} is indiscernible in terms of vomiting attribute. Patient2 has a viral illness, whereas patient5 does not, however they are indiscernible with respect to the attributes headache, vomiting and temperature. Therefore, patient2 and patient5 are the elements of patients' set with unconcluded symptoms.

2.4 Approximations

The starting point of rough set theory is the indiscernibility relation, generated by information concerning objects of interest. The indiscernibility relation is intended to express the fact that due to the lack of knowledge it is unable to discern some objects employing the available information. Approximations is also other an important concept in Rough Sets Theory, being associated with the meaning of the approximations topological operations (Wu et al., 2004). The lower and the upper approximations of a set are interior and closure operations in a topology generated by the indiscernibility relation. Below is presented and described the types of approximations that are used in Rough Sets Theory.

a. Lower Approximation (B'')

Lower Approximation is a description of the domain objects that are known with certainty to belong to the subset of interest.

The Lower Approximation Set of a set X , with regard to R is the set of all of objects, which certainly can be classified with X regarding R , that is, set B'' .

b. Upper Approximation (B^*)

Upper Approximation is a description of the objects that possibly belong to the subset of interest. The Upper Approximation Set of a set X regarding R is the set of all of objects which can be possibly classified with X regarding R , that is, set B^* .

c. Boundary Region (BR)

Boundary Region is description of the objects that of a set X regarding R is the set of all the objects, which cannot be classified neither as X nor $-X$ regarding R . If the boundary region is a set $X = \emptyset$ (Empty), then the set is considered "Crisp", that is, exact in relation to R ; otherwise, if the boundary region is a set $X \neq \emptyset$ (empty) the set X "Rough" is considered. In that the boundary region is $BR = B^* - B''$.

Mathematically speaking, let a set $X \subseteq U$, B be an equivalence relation and a knowledge base $K = (U, B)$. Two subsets can be associated:

1. B-lower: $B'' = \bigcup \{Y \in U/B : Y \subseteq X\}$
2. B-upper: $B^* = \bigcup \{Y \in U/B : Y \cap X \neq \emptyset\}$

In the same way, $POS(B)$, $BN(B)$ and $NEG(B)$ are defined below (Pawlak, 1991).

3. $POS(B) = B'' \Rightarrow$ certainly member of X
4. $NEG(B) = U - B^* \Rightarrow$ certainly non-member of X
5. $BR(B) = B^* - B'' \Rightarrow$ possibly member of X

Figure 1 presents a graphic representation of these regions (Lambert-Torres et al., 1999).

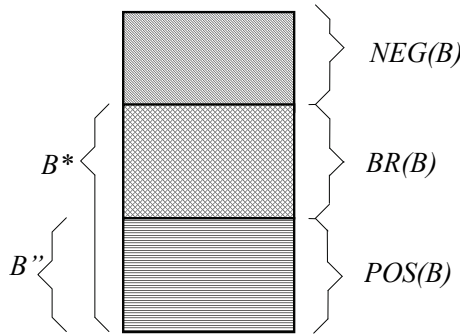


Fig. 1. Definition of B-approximation sets and B-regions

d. Quality Approximation

It is obtained numerically using its own elements, specifically those of lower and upper approximation. The coefficient used in measuring the quality is represented by $\alpha_B(X)$, where X is a set of objects or registrations regarding B . The quality of approximation uses two coefficients that are presented below:

- Imprecision coefficient $\alpha_B(X)$

Where α_B is the quality of approximation of X , being denoted by:

$$\alpha_B(X) = |B''(X)| / |B^*(X)| \quad (1)$$

Where $|B''(X)|$ and $|B^*(X)|$ it represents the cardinality of approximation lower and upper, and the approximation are set $\neq \emptyset$. Therefore, $0 \leq \alpha_B \leq 1$, if $\alpha_B(X)=1$, X it is a definable set regarding the attributes B , that is, X is crisp set. If $\alpha_B(X) < 1$, X is rough set regarding the attributes B .

Applying it formulates for Table 1, it has $\alpha_B(X) = 3/5$ for the patients with possibility of they are with viral illness.

- Quality Coefficient of upper and lower approximation

- Quality Coefficient of upper approximation $\alpha B(B^*(X))$

It is the percent of all the elements that are classified as belonging to X , being denoted for:

$$\alpha B(B^*(X)) = |B^*(X)| / |A| \quad (2)$$

In the Table 1, $\alpha B(B^*(X)) = 5/6$, for the patients that have the possibility of they be with viral illness.

- Quality Coefficient of lower approximation $\alpha B(B''(X))$

It is the percentage of all the elements that possibility are classified as belonging to X , and is denoted as:

$$\alpha B(B''(X)) = |B''(X)| / |A| \quad (3)$$

In the Table 1, $\alpha B(B^*(X)) = 3/6 = 1/2$, for the patients that have viral illness.

Observation: In Quality coefficient upper and lower, presented in number 2, of this section, $|A|$ represents the cardinality of any given set of objects.

2.5 Decision tables and decision algorithms

A decision table contains two types of attributes designated as the condition attribute and decision attribute. In Table 1, shown in section 2.2, the attributes of headache, vomiting and temperature can all be considered as condition attributes, whereas the viral illness attribute is considered a decision attribute.

Each row of a decision table determines a decision rule, which specifies the decisions (actions) that must be taken when conditions are indicated by condition attributes are satisfied, e.g. in Table 1 the condition (Headache, no), (vomiting, yes), (Temperature, high) determines the decision (Viral illness, yes).

Table 1 shows that both patient2 and patient5 suffer from the same symptoms since the condition attributes of headache, vomiting and temperature possess identical values; however, the values of decision attribute differ. These set of rules are known as either inconsistency, non-determinant or conflicting. These rules are known as consistency, determinant or non conflicting or simply, a rule.

The number of consistency rules, contained in the decision table are known as a factor of consistence, which can be denoted by $\gamma(C, D)$, where C is the condition and D the decision. If $\gamma(C, D) = 1$, the decision table is consistent, but if $\gamma(C, D) \neq 1$ the table of decision is inconsistent.

Given that Table 1, $\gamma(C, D) = 4/6$, that is, the Table 1 possesses two inconsistent rules (patient2, patient5) and four consistent rules (patient1, patient3, patient4, patient6), inside of universe of six rules for all the Table 1 (Ziarko & Shan, 1995). The decision rules are frequently shown as implications in the form of "if... then... ". To proceed is shown one rule for the implication viral illness:

| | | | |
|------|---------------|--------|-----|
| If | Headache | = no | and |
| | Vomiting | = yes | and |
| | Temperature | = high | |
| Then | Viral Illness | = yes | |

A set of decision rules is designated as decision algorithms, because for each decision table it can be associated with the decision algorithm, consisting of all the decision rules that it occur in the respective decision table. A may be made distinction between decision algorithm and decision table. A decision table is a data set, whereas a decision algorithm is a collection of implications, that is, a logical expressions (Pawlak, 1991).

2.6 Dependency of attributes

In the analysis of data, it is important discover the dependence between attributes. Intuitively, a set of attributes D depends totally on a set of attributes C , denoted as $C \Rightarrow D$, if all values of attributes from D are uniquely determined by values of attributes from C , then D depends totally on C , if there exists a functional dependency between values of D and C . For example, in Table 1 there are no total dependencies whatsoever, if in Table 1, the value of the attribute Temperature for patient p5 was "no" instead of "high", there would be a total dependency $\{\text{Temperature}\} \Rightarrow \{\text{viral illness}\}$, because to each value of the attribute Temperature there would correspond a unique value of the attribute viral illness.

It would also necessitate a more global concept of dependency of attributes, designated as partial dependency of attributes, in Table 1, the temperature attribute determines some uniquely values of the attribute viral illness. That is, (temperature, very high) implies (viral illness, yes), similarly (temperature, normal) implies (viral illness, no), but (temperature, high) does not imply always (viral illness, yes). Thus the partial dependency means that only some values of D are determined by values of C . Formally the dependence among the attributes can be defined in the following way: If D and C are subsets of A , can be affirmed that D depends on C in degree K ($0 \leq k \leq 1$), denoted $\Rightarrow_k D$ and if $k = \gamma(C, D)$. If $K=1$, D depends totally on C , if $K < 1$, it is said that D depends partially (in a degree K) on C .

The concept of dependent attributes is strongly coupled to the concept of consistency of decision table (Pawlak, 1991). For example, for dependency attributes $\{\text{Headache, Vomiting, Temperature}\} \Rightarrow \{\text{viral illness}\}$, it get $k=4/6= 2/3$, because four out of six patients can be uniquely classified as having viral illness or not, employing attributes Headache, Vomiting and Temperature.

It can be interesting to note exactly how patients can be diagnosed using only the attribute Temperature, that is, the degree of the dependence $\{\text{Temperature}\} \Rightarrow \{\text{viral illness}\}$, diagnose is obtained $k = 3/6 = 1/2$, in this case there are only three patients {patient1, patient3, patient6} out of six, only these three, can be classified exclusively as having viral illness. In contrast, with the case of patient4, who cannot be classified as having viral illness, since the value of the attribute temperature, in this case, is normal. Hence the single attribute Temperature offers worse classification than the whole set of attributes Headache, Vomiting and Temperature. It is interesting to observe that neither Headache nor Vomiting can be used to recognize viral illness, because for both dependencies $\{\text{Headache}\} \Rightarrow \{\text{viral illness}\}$ and $\{\text{Vomiting}\} \Rightarrow \{\text{Viral illness}\}$ it has $k=0$.

It can be easily seen that if D depends totally on C then $I(C) \subseteq I(D)$. That means that the partition generated by C is finer than the partition generated by D , and that the concept of dependency presented in the section corresponds to that considered in relational databases.

2.7 Reduction attributes in information system

For many application problems, it is often necessary to maintain a concise form of the information system, but there exist data that can be removed, without altering the basic

properties and more importantly the consistency of the system (Cerchiarì et al, 2006). If is subtract relative data from the headache and vomiting, the resultant data set is equivalent to original data in relation to approximation and dependency, as it has the same the approximation precision and the same dependency degree using the original set of attributes, however with one fundamental difference, the set of attributes to be considered will be fewer.

The process of reducing an information system such that the set of attributes of the reduced information system is independent and no attribute can be eliminated further without losing some information from the system, the result is known as reduct. If an attribute from the subset $B \subseteq A$ preserves the indiscernibility relation RA , then the attributes $A - B$ are dispensable. Reducts are such subsets minimal, i.e., that do not contain any dispensable attributes. Therefore, the reduction should have the capacity to classify objects, without altering the form of representing the knowledge (Geng & Zhu, 2006). When the definition above is applied, the information system presented in the Table 1, B is a subset of A and belongs to B :

- a is dispensable in B if $I(B) = I(B - \{a\})$; otherwise a is indispensable in B ;
- Set B is independent if all its attributes are indispensable;
- Subset B' of B is a reduct of B if B' is independent and $I(B') = I(B)$; and

A reduct is a set of attributes that preserve the basic characteristics of the original data set; therefore, the attributes that do not belong to a reduct are superfluous with regard to classification of elements of the Universe.

3. Rough set with tools for data mining

The great advances in information technology have made it possible to store a great quantity of data. In the late nineties, the capabilities of both generating and collecting data were increased rapidly. Millions of databases have been used in business management, government administration, scientific and engineering data management, as well as many other applications. It can be noted that the number of such databases keeps growing rapidly because of the availability of powerful database systems. This explosive growth in data and databases has generated an urgent need for new techniques and tools that can intelligently and automatically transform the processed data into useful information and knowledge. One of the processes used to transform data into knowledge is Knowledge Discovery Database (KDD), which is divided in three stages (preprocessing, data mining and post processing) that are shown in the section 3.1.

3.1 Knowledge Discovery in Database - KDD

Knowledge Discovery in Database - KDD is a process, with several stages, no trivial, interactive and iterative, for the identification of comprehensible patterns, valid, new and potentially useful starting from great groups of data (Fayyad et al., 1996a). KDD is characterized as a process composed of several operational stages: preprocessing, data mining and the post processing. Figure 2 presents the sequence of the stages executed during the process of KDD.

a. Preprocessing Stage

The preprocessing stage understands the functions related to the reception, the organization and to the treatment of data, this stage has as its objective the preparation of the data for the following stage of the data mining.

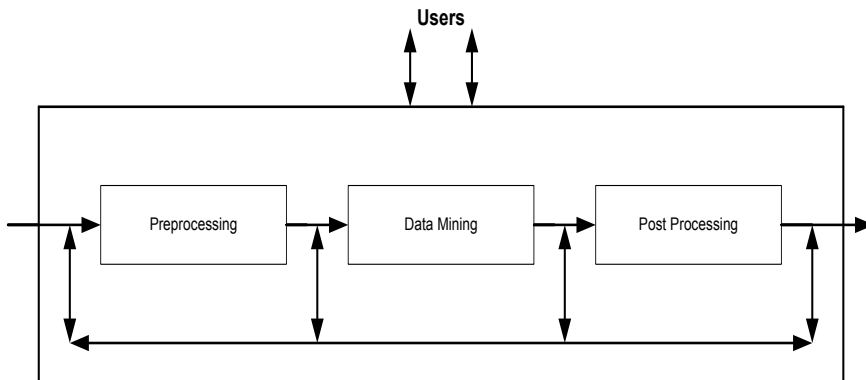


Fig. 2. The KDD process

b. Data Mining Stage

The data mining stage defines the techniques and the algorithms to be used by the problem in question, as are examples of techniques that can be used in this stage such as neural network, rough set, genetic algorithms, statistical models and probabilistic. The choice of technique depends, in many cases, on task type to be developed. In Table 2 is shown a summary of tasks to be accomplished and some alternative methods that can be useful. It is important to observe that Table 2 does not drain the universe of methods of data mining that can be used in each task of KDD and is purely a summary (Fayyad et al., 1996b).

| Tasks of KDD | Methods of Data Mining |
|--------------------------------------|--|
| Discovery Associations | Basic, Apriori, DHP, Partition, DIC, ASCX-2P |
| Discovery Generalize of Associations | Basic, Apriori, DHP, Partition, DIC, ASCX-2P |
| Discovery of Sequences | GSP, MSDD, SPADE |
| Discovery Generalize of Sequences | GSP, MSD, SPADE |
| Classification | Neural Network, C4.5, Rough Sets, Genetic Algorithms, CART, K-NN, Bayes Classifier |
| Regression | Neural Network, Fuzzy Set |
| Summarization | C4.5, Genetics Algorithms |
| Clustering | K-Means, K-Modes, K-prototypes, Fuzzy K-Means, genetics Algorithms, Neural Network |
| Forecast of Temporal Series | Neural Network, Fuzzy Set |

Table 2. Methods of Data Mining that can be applied in the tasks of KDD

During the data mining stage much useful knowledge is gained in respect of the application. Many authors consider data mining synonymous with KDD, in the context this stage, the

KDD process is often known as Data Mining; in the research it will Data Mining be indented as KDD (Piatetsky-Shapiro & Matheus, 1995; Mitchell, 1999; Wei, 2003).

Data mining has become an area of research increasing importance, and is also referred to as knowledge discovery in databases (KDD), consequently this has resulted in a process of non trivial extraction of implicit, previously unknown and potentially useful information, such as knowledge rules, constraints, regularities from data in databases

c. Post Processing Stage

In the post processing stage the treatment of knowledge obtained during the data mining stage. This stage is not always necessary; however, it allows the possibility of validation of the usefulness of the discovered knowledge.

3.2 Rough set in data mining

Rough set theory constitutes a consistency base for data mining; it offers useful tools for discovering patterns hidden in data in many aspects. Although in theory rough set deals with discreet data, rough set is commonly used in conjunction with other techniques connected to discretization on the dataset. The main feature of rough set data analysis is both non-invasive and notable ability to handle qualitative data. This fits into most real life applications nicely.

Rough Set can be used in different phases of the knowledge discovery process, as attribute selection, attribute extraction, data reduction, decision rule generation and pattern extraction (Komorowski et al., 1999). Furthermore, recent extensions of rough set theory with rough mereology have brought new methods of decomposition of large data sets, data mining in distributed and multi-agent based environments and granular computing (Polkowski, 2002). It includes mechanisms for defining partial memberships of sets, but does not introduce additional measures of probabilities or degrees of membership.

The basic idea is that there is some information or data associated with each object in the universe of discourse. Based on this information, it is possible to tell some of the objects apart, while others are impossible to distinguish. The latter objects are indiscernible from each other, and form a set. Each set of indiscernible objects is a knowledge granule, and they form the building blocks for knowledge about the universe. The rough set community has been a very active research community since its inception in the eighties, and a large number of rough set methods for knowledge discovery and data mining have been developed. The entire knowledge discovery process has been subject to research, and a wide range of contributions has been made.

Data mining technology provides a new thought for organizing and managing tremendous data. Rough set theory is one of the important methods for knowledge discovery. This method can analyze intact data, obtain uncertain knowledge and offer an effective tool by reasoning.

Rough set has shed light on many research areas, but seldom found its way into real world application. Data mining with rough set is a multi-phase process consisted of mainly: discretization; reducts and rules generation on training set; classification on test set.

Rough Set Theory, since it was put forward, has been widely used in Data Mining, and has important functions in the expression, study and conclusion of uncertain knowledge, it is a powerful tool, which sets up the intelligent decision system. The main focus is to show how

rough set techniques can be employed as an approach to the problem of data mining and knowledge extraction.

4. Applications of rough set theory

Rough set theory offers effective methods that are applicable in many branches of Artificial Intelligence, one of the advantages of rough set theory is that programs implementing its methods may easily run on parallel computers, but several problems remain to be solved.

Recently, much research has been carried out in Rough Set together with other artificial Intelligence methods such as Fuzzy Logic, Neural Networks, and Expert Systems and some significant results have been found. Rough set theory allows characterization of a set of objects in terms of attribute values; finding dependencies total or partial between attributes; reduction of superfluous attributes; finding significance attributes and decision rule generation.

The applications of Rough Set have resolved complex problems; and therefore have been attractive to researchers in recent years, already it has been applied successfully in a number of challenging fields such the a soft computing method.

This section provides a brief overview of some of the many applications of rough set. There are several properties of rough sets that make the theory an obvious choice for use in dealing with real problems:

a. Pattern Recognition

Pattern Recognition using Rough Set is one such successful application field, but in 2001 A. Mrozek and K. Cyran (2001) proposed a hybrid method of automatic diffraction pattern recognition based on Rough Set Theory and Neural Network. In this new method, the rough set is used to define the objective function and stochastic evolutionary algorithm for space search of a feature extractor, and neural networks are employed to model the uncertain systems. The features obtain end by optimized sampling of diffraction patterns are input to a semantic classifier and the pattern recognition algorithm is performed with optimized and standard computer-generated holograms.

b. Emergency room diagnostic medical

A common and diagnostically challenging problem facing emergency department personnel in hospitals is that of acute abdominal pain in children. There are many potential causes for this pain, most usually non-serious. However, the pain may be an indicator that a patient has a serious is illness, requiring immediate treatment and possibly surgery. Experienced doctors will use a variety of relevant historical information and physical observations to assess children. Such attributes occur frequently in recognizable patterns, allowing a quick and efficient diagnosis. Inexperienced doctors, on the other hand, may lack the knowledge and information to be able to recognize these patterns. The rough set based clinical decision model is used to assist such inexperienced doctors. In this research, rough sets are used to support diagnosis by distinguishing between three disposition categories: discharge, observation/further investigation, and consultation. Preliminary results show that the system gives accuracy comparable to doctors, though it is dependent on a suitably high data quality (Rubin et al., 1996)

c. Acoustical analysis

Rough Set was applied for the assessment of concert hall acoustics. Rough set algorithms are applied to the decision table containing subjectively quantified parameters and the results of

overall subjective preference of acoustical objects described by the parameters. Fuzzy membership functions map the test results to approximate the tested parameter distribution, which is determined on the basis of the separate subjective test of individual parameter underlying overall preference. A prototype system based on the rough set theory is used to induce generalized rules that describe the relationship between acoustical parameters of concert halls and sound processing algorithms (Kotek, 1999)

d. Power system security analysis

Rough Set is a systematic approach used to help knowledge engineers during the extraction process of facts and rules of a set of examples for power system operation problems. This approach describes the reduction the number of examples, offering a more compact set of examples to the user (Lambert-Torres et al., 1999).

e. Spatial and meteorological pattern classification

Some categories of sunspot groups are associated with solar flares. Observatories around the world track all visible sunspots in an effort to detect flares early, the sunspot recognition and classification are currently manual and labor intensive processes which could be automated if successfully learned by a machine. The approach employs a hierarchical rough set based learning method for sunspot classification. It attempts to learn the modified Zurich classification scheme through rough set-based decision tree induction. The resulting system has been evaluated on sunspots extracted from satellite images, with promising results (Nguyen et al., 2005).

A new application of rough set theory for classifying meteorological radar data has been introduced. Volumetric radar data is used to detect storm events responsible for severe weather. Classifying storm cells is a difficult problem as they exhibit a complex evolution throughout their lifespan. Also, the high dimensionality and imprecision of the data can be prohibitive. Rough set approach is employed to classify a number of meteorological storm events (Shen & Jensen, 2007).

f. Intelligent control systems

An important application field of rough set theory is that of intelligent control systems especially when incorporated with fuzzy theory (Xie et al., 2004).

g. Measure the quality of a single subset

Ant Colony System algorithm and Rough Set Theory proposed a hybrid approach to feature selection, in Rough Set Theory offers a heuristic function in order to measure the quality of a single subset. It has studied the influence of the setting of the parameters for this problem, in particular for finding a reduct. Experimental results show this hybrid approach is a potential method for features selection (He et al., 2007).

There are infinite possibilities in the development of methods based on Rough Set Theory such as nonstandard analysis, nonparametric statistics and qualitative.

5. Case – rough set with tools in dengue diagnosis

In this section, several patients data set is shown with possible dengue symptoms. Through data are analysis is accomplished, using a Rough Set approach for the elimination of redundant data and the development of a set of rules that it can aid the doctor in the elaboration of the diagnosis. Below the Table 3 is shown with the patients data set and respective symptoms, and the data are of the discreet type.

5.1 Information system or information table

| Patient | Conditional Attributes | | | Decision Attribute |
|---------|------------------------|-----------------------------|-------------|--------------------|
| | blotched_red_skin | muscular_pain_articulations | Temperature | Dengue |
| P1 | No | No | Normal | No |
| P2 | No | No | High | No |
| P3 | No | No | Very High | Yes |
| P4 | No | Yes | High | Yes |
| P5 | No | Yes | Very High | Yes |
| P6 | Yes | Yes | High | Yes |
| P7 | Yes | Yes | Very High | Yes |
| P8 | No | No | High | No |
| P9 | Yes | No | Very High | Yes |
| P10 | Yes | No | High | No |
| P11 | Yes | No | Very High | No |
| P12 | No | Yes | Normal | No |
| P13 | No | Yes | High | Yes |
| P14 | No | Yes | Normal | No |
| P15 | Yes | Yes | Normal | No |
| P16 | Yes | No | Normal | No |
| P17 | Yes | No | High | No |
| P18 | Yes | Yes | Very High | Yes |
| P19 | Yes | No | Normal | No |
| P20 | No | Yes | Normal | No |

Table 3. Patients with respective symptoms

Where, B are all of the objects or registrations of the system, given set $B=\{P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, P12, P13, P14, P15, P16, P17, P18, P19, P20\}$ the set conditional attributes is represented by $C=\{\text{blotched_red_skin, muscular_pain_articulations, Temperature}\}$ and the set D represented the decision attribute, where $D=\{\text{dengue}\}$. The set A or Table 3, can be shown in relation to the function of nominal values of considered attributes, in the Table 4:

| Conditional Attributes | Attributes | Nominal Values |
|------------------------|-----------------------------|-------------------------|
| | blotched_red_skin, | Yes, No |
| | muscular_pain_articulations | Yes, No |
| Decision Attributes | Temperature | Normal, High, Very High |
| | Dengue | Yes, No |

Table 4. Nominal Values of Attributes

5.2 Indiscernibility relation

Indiscernibility Relation is the relation between two objects or more, where all the values are identical in relation to a subset of considered attributes. In Table 3, presented in section 5.1, it can be observed that the set is composed of attributes that are directly related to patients'

symptoms, where $C = \{\text{blotched_red_skin}, \text{muscular_pain_articulations}, \text{temperature}\}$, the indiscernibility relation is given to $\text{INDA}(C)$. When Table 3 is broken down it can be seen that indiscernibility relation is given in relationship to conditional attributes:

- The blotched_red_skin attribute generates two indiscernibility elementary sets: $\text{INDA}(\{\text{blotched_red_skin}\}) = \{\{P1, P3, P4, P5, P8, P12, P13, P14, P20\}, \{P6, P7, P9, P10, P11, P15, P16, P17, P18, P19\}\}$.

| Patient | Conditional Attributes | | | Decision Attribute |
|---------|------------------------|-----------------------------|-------------|--------------------|
| | blotched_red_skin | muscular_pain_articulations | Temperature | Dengue |
| P1 | No | No | Normal | No |
| P12 | No | Yes | Normal | No |
| P13 | No | Yes | High | Yes |
| P14 | No | Yes | Normal | No |
| P2 | No | Yes | High | No |
| P20 | No | Yes | Normal | No |
| P3 | No | No | Very High | Yes |
| P4 | No | Yes | High | Yes |
| P5 | No | Yes | Very High | Yes |
| P8 | No | No | High | No |
| P10 | Yes | No | High | No |
| P11 | Yes | No | Very High | No |
| P15 | Yes | Yes | Normal | No |
| P16 | Yes | No | Normal | No |
| P17 | Yes | No | High | No |
| P18 | Yes | Yes | Very High | Yes |
| P19 | Yes | No | Normal | No |
| P6 | Yes | Yes | High | Yes |
| P7 | Yes | Yes | Very High | Yes |
| P9 | Yes | No | Very High | Yes |

Table 5. Table 3 organize in relations blotched_red_skin attribute

- The muscular_pain_articulations attribute generates two indiscernibility elementary sets: $\text{INDA}(\{\text{muscular_pain_articulations}\}) = \{\{P1, P2, P3, P8, P9, P10, P11, P16, P17, P19\}, \{P4, P5, P6, P7, P12, P13, P14, P15, P18, P20\}\}$.

| Patient | Conditional Attributes | | | Decision Attribute |
|---------|------------------------|-----------------------------|-------------|--------------------|
| | blotched_red_skin | muscular_pain_articulations | Temperature | Dengue |
| P1 | No | No | Normal | No |
| P2 | No | No | High | No |
| P3 | No | No | Very High | Yes |
| P8 | No | No | High | No |
| P9 | Yes | No | Very High | Yes |
| P10 | Yes | No | High | No |

| | | | | |
|-----|-----|-----|-----------|-----|
| P11 | Yes | No | Very High | No |
| P16 | Yes | No | Normal | No |
| P17 | Yes | No | High | no |
| P19 | Yes | No | Normal | No |
| P4 | No | Yes | High | Yes |
| P5 | No | Yes | Very High | Yes |
| P6 | Yes | Yes | High | Yes |
| P7 | Yes | Yes | Very High | Yes |
| P12 | No | Yes | Normal | No |
| P13 | No | Yes | High | Yes |
| P14 | No | Yes | Normal | No |
| P15 | Yes | Yes | Normal | No |
| P18 | Yes | Yes | Very High | Yes |
| P20 | No | Yes | Normal | No |

Table 6. Table 3 organize in relation muscular_pain_articulations attribute

- The temperature attribute generates three indiscernibility elementary sets: $INDA(\{temperature\}) = \{ \{P2, P4, P6, P8, P10, P13, P17\}, \{P3, P5, P7, P9, P11, P18\}, \{P1, P12, P14, P15, P16, P19, P20\} \}$.

| Patient | Conditional Attributes | | | Decision Attribute |
|---------|------------------------|-----------------------------|-------------|--------------------|
| | blotched_red_skin | muscular_pain_articulations | Temperature | Dengue |
| P13 | No | Yes | High | Yes |
| P2 | No | Yes | High | No |
| P4 | No | Yes | High | Yes |
| P8 | No | No | High | No |
| P10 | Yes | No | High | No |
| P17 | Yes | No | High | No |
| P6 | Yes | Yes | High | Yes |
| P1 | No | No | Normal | No |
| P12 | No | Yes | Normal | No |
| P14 | No | Yes | Normal | No |
| P20 | No | Yes | Normal | No |
| P15 | Yes | Yes | Normal | No |
| P16 | Yes | No | Normal | No |
| P19 | Yes | No | Normal | No |
| P3 | No | No | Very High | Yes |
| P5 | No | Yes | Very High | Yes |
| P11 | Yes | No | Very High | No |
| P18 | Yes | Yes | Very High | Yes |
| P7 | Yes | Yes | Very High | Yes |
| P9 | Yes | No | Very High | Yes |

Table 7. Table 3 organized in relation temperature attribute

5.3 Approximation

The lower and the upper approximations of a set are interior and closure operations in a topology generated by a indiscernibility relation. Below is presented and described the types of approximations are followed using in Rough Set Theory; the approximations concepts are applied in the Table 3, shown to proceed:

| Patient | Conditional Attributes | | | Decision Attribute |
|---------|------------------------|-----------------------------|-------------|--------------------|
| | blotched_red_skin | muscular_pain_articulations | Temperature | Dengue |
| P1 | No | No | Normal | No |
| P2 | No | No | High | No |
| P8 | No | No | High | No |
| P10 | Yes | No | High | No |
| P11 | Yes | No | Very High | No |
| P12 | No | Yes | Normal | No |
| P14 | No | Yes | Normal | No |
| P15 | Yes | Yes | Normal | No |
| P16 | Yes | No | Normal | No |
| P17 | Yes | No | High | No |
| P19 | Yes | No | Normal | No |
| P20 | No | Yes | Normal | No |
| P3 | No | No | Very High | Yes |
| P4 | No | Yes | High | Yes |
| P5 | No | Yes | Very High | Yes |
| P6 | Yes | Yes | High | Yes |
| P7 | Yes | Yes | Very High | Yes |
| P9 | Yes | No | Very High | Yes |
| P13 | No | Yes | High | Yes |
| P18 | Yes | Yes | Very High | Yes |

Table 8. Table 3 organized in relation decision attribute

- a. Lower Approximation set B''
 - Lower Approximation set (B'') of the patients that are definitely have dengue are identified as $B'' = \{P3, P4, P5, P6, P7, P13, P18\}$
 - Lower Approximation set (B'') of patients that certain have not dengue are identified as $B'' = \{P1, P2, P8, P10, P12, P14, P15, P16, P17, P19, P20\}$
- b. Upper Approximation set B^*
 - Upper Approximation set (B^*) of the patients that possibly have dengue are identified as $B^* = \{P3, P4, P5, P6, P7, P9, P13, P18\}$
 - Upper Approximation set (B^*) of the patients that possibly have not dengue are identified as $B^* = \{P1, P2, P8, P10, P11, P12, P14, P15, P16, P17, P19, P20\}$
- c. Boundary Region (BR)
 - Boundary Region (B^*) of the patients that not have dengue are identified as: $BR = \{P1, P2, P8, P10, P11, P12, P14, P15, P16, P17, P19, P20\} - \{P1, P2, P8, P10, P12, P14, P15, P16, P17, P19, P20\} = \{P11\}$;

- Boundary Region (B^*), the set of the patients that have dengue are identified as: $BR = \{P3, P4, P5, P6, P7, P9, P13, P18\} - \{P3, P4, P5, P6, P7, P13, P18\} = \{P9\}$

Observation: Boundary Region (BR), the set constituted by elements P9 and P11, which cannot be classified, since they possess the same characteristics, but with differing conclusions differ in the decision attribute.

5.4 Quality of approximations

The two coefficients of quality of approximation are:

- Imprecision coefficient, using Eq. (1)):
 - for the patients with possibility of they are with dengue $\alpha B(X) = 7/8$;
 - for the patients with possibility of they are not with dengue $\alpha B(X) = 8/12$.
- Quality Coefficient of upper and lower approximation, using Eq. 2 and 3:
 - $\alpha B(B^*(X)) = 8/20$, for the patients that have the possibility of they be with dengue;
 - $\alpha B(B^*(X)) = 11/20$, for the patients that not have the possibility of they be with dengue;
 - $\alpha B(B''(X)) = 7/20$, for the patients that have dengue;
 - $\alpha B(B''(X)) = 8/20$, for the patients that not have dengue.

Observations:

1. Patient with dengue: $\alpha B(B''(X)) = 7/20$, that is, 35% of patients certainly with dengue.
2. Patient that don't have dengue: $\alpha B(B''(X)) = 11/20$, that is, approximately 55% of patients certainly don't have dengue.
3. 10% of patients (P9 and P11) cannot be classified neither with dengue nor without dengue, since the characteristics of all attributes are the same, with only the decision attribute (dengue) not being identical and generates an inconclusive diagnosis for dengue.

6. Data reduction in information system

The form in which data is presented within an information system must guarantee that the redundancy is avoided as it implicates the minimization of the complexly computational in relation to the creation of rules to aid the extraction knowledge. However, when the information system possesses redundancy situations, it is necessary to treat it One of the ways of accomplishing this is to use the concept of reduct, without altering the indiscernibility relations.

A reduct is a set of necessary minimum data, since the original proprieties of the system or information table are maintained. Therefore, the reduct must have the capacity to classify objects, without altering the form of representing the knowledge.

The process of reduction of information is presented below in Table 3, it can be observed that the data is of a discreet type.

a. Verification inconclusive data

Step 1 – Analysis of data contained in Table 3 shows that possess information inconclusive, being that the values of conditional attributes same and the value of decision attribute is different.

Conclusion of Step 1: The symptoms of patient P9 and patient P11 are both inconclusive, since they possess equal values of conditions attributes together with a value of decision

attribute that is different. Therefore, the data of patient P9 and patient P11 will be excluded from Table 3.

b. Verification of equivalent information

Step 2 – Analysis of data contained in Table 3 shows that it possesses equivalent information.

| | | | | |
|-----|-----|-----|-----------|-----|
| P2 | No | No | High | No |
| P8 | No | No | High | No |
| P4 | No | Yes | High | Yes |
| P13 | No | Yes | High | Yes |
| P7 | Yes | Yes | Very High | Yes |
| P18 | Yes | Yes | Very High | Yes |
| P10 | Yes | No | High | No |
| P17 | Yes | No | High | No |
| P12 | No | Yes | Normal | No |
| P14 | No | Yes | Normal | No |
| P20 | No | Yes | Normal | No |
| P16 | Yes | No | Normal | No |
| P19 | Yes | No | Normal | No |

Conclusion of Step 2 – The Table 3 has it reduced data presented in a revised version in Table 9 shown below:

| Patient | Conditional Attributes | | | Decision Attribute |
|---------|------------------------|-----------------------------|-------------|--------------------|
| | blotched_red_skin | muscular_pain_articulations | Temperature | Dengue |
| P1 | No | No | Normal | No |
| P2 | No | No | High | No |
| P3 | No | No | Very High | Yes |
| P4 | No | Yes | High | Yes |
| P5 | No | Yes | Very High | Yes |
| P6 | Yes | Yes | High | Yes |
| P7 | Yes | Yes | Very High | Yes |
| P8 | No | No | High | No |
| P10 | Yes | No | High | No |
| P12 | No | Yes | Normal | No |
| P15 | Yes | Yes | Normal | No |
| P16 | Yes | No | Normal | No |
| P19 | Yes | No | Normal | No |

Table 9. Reduct of Information of Table 3

Step 3 – Analysis of each condition attributes with the attributes set.

| Patient | Conditional Attributes | Decision Attribute |
|---------|------------------------|--------------------|
| | blotched_red_skin | Dengue |
| P1 | No | No |
| P2 | No | No |
| P3 | No | Yes |
| P4 | No | Yes |
| P5 | No | Yes |
| P6 | Yes | Yes |
| P7 | Yes | Yes |
| P8 | No | No |
| P10 | Yes | No |
| P12 | No | No |
| P15 | Yes | No |
| P16 | Yes | No |
| P19 | Yes | No |

Table 10. Analysis of Attribute blotched_red_skin in Table 9

| Patient | Conditional Attributes | Decision Attribute |
|---------|-----------------------------|--------------------|
| | muscular_pain_articulations | Dengue |
| P1 | No | No |
| P2 | No | No |
| P3 | No | Yes |
| P4 | Yes | Yes |
| P5 | Yes | Yes |
| P6 | Yes | Yes |
| P7 | Yes | Yes |
| P8 | No | No |
| P10 | No | No |
| P12 | Yes | No |
| P15 | Yes | No |
| P16 | No | No |
| P19 | No | No |

Table 11. Analysis of Attribute muscular_pain_articulations in Table 9

| Patient | Conditional Attribute | Decision Attribute |
|---------|-----------------------|--------------------|
| | Temperature | Dengue |
| P1 | Normal | No |
| P2 | High | No |
| P3 | Very High | Yes |
| P4 | High | Yes |
| P5 | Very High | Yes |
| P6 | High | Yes |

| | | |
|-----|-----------|-----|
| P7 | Very High | Yes |
| P8 | High | No |
| P10 | High | No |
| P12 | Normal | No |
| P15 | Normal | No |
| P16 | Normal | No |
| P19 | Normal | No |

Table 12. Analysis of Attribute Temperature in Table 9

Conclusion of analysis: In this analysis, no data was excluded.

- c. Given analysis of condition attributes in Table 9, it can be observed that the same data exists in proceeding tables.
- Analysis of attributes blotched_red_skin and muscular_pain_articulations in Table 9.

| Patient | Conditional Attributes | | Decision Attribute |
|---------|------------------------|-----------------------------|--------------------|
| | blotched_red_skin | muscular_pain_articulations | Dengue |
| P1 | No | No | No |
| P2 | No | No | No |
| P3 | No | No | Yes |
| P4 | No | Yes | Yes |
| P5 | No | Yes | Yes |
| P12 | No | Yes | No |
| P6 | Yes | Yes | Yes |
| P7 | Yes | Yes | Yes |
| P8 | No | No | No |
| P16 | Yes | No | No |
| P19 | Yes | No | No |
| P10 | Yes | No | No |
| P15 | Yes | Yes | No |

Table 13. Analysis of Attributes blotched_red_skin and muscular_pain_articulations in Table 9

Result of analysis

| Patient | Conditional Attributes | | Decision Attribute |
|---------|------------------------|-----------------------------|--------------------|
| | blotched_red_skin | muscular_pain_articulations | Dengue |
| P1 | No | No | No |
| P3 | No | No | Yes |
| P4 | No | Yes | Yes |
| P6 | Yes | Yes | Yes |
| P10 | Yes | No | No |
| P15 | Yes | Yes | No |

Table 14. Result of Analysis of Attributes blotched_red_skin and muscular_pain_articulations in Table 9

- Analysis of Attributes attributes blotched_red_skin and temperature.

| Patient | Conditional Attributes | | Decision Attribute |
|---------|------------------------|-------------|--------------------|
| | blotched_red_skin | Temperature | Dengue |
| P1 | No | Normal | No |
| P12 | No | Normal | No |
| P2 | No | High | No |
| P8 | No | High | No |
| P3 | No | Very High | Yes |
| P5 | No | Very High | Yes |
| P7 | Yes | Very High | Yes |
| P4 | No | High | Yes |
| P6 | Yes | High | Yes |
| P10 | Yes | High | No |
| P15 | Yes | Normal | No |
| P16 | Yes | Normal | No |
| P19 | Yes | Normal | No |

Table 15. Analysis of Attributes blotched_red_skin and temperature in Table 9

Result of analysis

| Patient | Conditional Attributes | | Decision Attribute |
|---------|------------------------|-------------|--------------------|
| | blotched_red_skin | Temperature | Dengue |
| P1 | No | Normal | No |
| P2 | No | High | No |
| P3 | No | Very High | Yes |
| P4 | No | High | Yes |
| P6 | Yes | High | Yes |
| P10 | Yes | High | No |
| P15 | Yes | Normal | No |

Table 16. Result of it Analysis of Attributes blotched_red_skin and temperature in Table 9

- Analysis of attributes muscular_pain_articulations and temperature in Table 9.

| Patient | Conditional Attributes | | Decision Attribute |
|---------|-----------------------------|-------------|--------------------|
| | muscular_pain_articulations | Temperature | Dengue |
| P1 | No | Normal | No |
| P16 | No | Normal | No |
| P19 | No | Normal | No |
| P2 | No | High | No |
| P8 | No | High | No |
| P10 | No | High | No |

| | | | |
|-----|-----|-----------|-----|
| P3 | No | Very High | Yes |
| P4 | Yes | High | Yes |
| P6 | Yes | High | Yes |
| P5 | Yes | Very High | Yes |
| P7 | Yes | Very High | Yes |
| P12 | Yes | Normal | No |
| P15 | Yes | Normal | No |

Table 17. Analysis of Attributes muscular_pain_articulations and temperature in Table 9

Result of analysis

| Patient | Conditional Attributes | | Decision Attribute |
|---------|-----------------------------|-------------|--------------------|
| | muscular_pain_articulations | Temperature | Dengue |
| P1 | No | Normal | No |
| P2 | No | High | No |
| P3 | No | Very High | Yes |
| P4 | Yes | High | Yes |
| P5 | Yes | Very High | Yes |
| P12 | Yes | Normal | No |

Table 18. Result of it analysis of Attributes muscular_pain_articulations and temperature in Table 9

Step 4 – Verification of equivalent (intersection) data in the Tables 14, 16 and 18 correspond where data is the element of reduct information in relation to Table 9.

| Patient | Conditional Attributes | | | Decision Attribute |
|---------|------------------------|-----------------------------|-------------|--------------------|
| | blotched_red_skin | muscular_pain_articulations | Temperature | Dengue |
| P1 | No | No | Normal | No |
| P3 | No | No | Very High | Yes |
| P4 | No | Yes | High | Yes |

Table 19. Table with result of information reduct of Table 9

6. Decision rules

With the information reduct shown above, it can be generated the necessary decision rules for aid to the dengue diagnosis. The rules are presented to proceed:

Rule-1

R1: If patient

blotched_red_skin = No and

muscular_pain_articulations = No and

temperature = Normal

Then dengue = No.

Rule-2

R2: If patient

 blotched_red_skin = No and
 muscular_pain_articulations = No and
 temperature = Very High

Then dengue = Yes.

Rule-3

R3: If patient

 blotched_red_skin = No and
 muscular_pain_articulations = Yes and
 temperature = High

Then dengue = Yes.

7. Conclusion

This study, it has discussed the Rough set theory, was proposed in 1982 by Z. Pawlak, as an approach to knowledge discovery from incomplete, vagueness and uncertain data. The rough set approach to processing of incomplete data is based on the lower and the upper approximation, and the theory is defined as a pair of two crisp sets corresponding to approximations.

The main advantage of rough set theory in data analysis is that it does not need any preliminary or additional information concerning data, such as basic probability assignment in Dempster-Shafer theory, grade of membership or the value of possibility in fuzzy set theory. The Rough Set approach to analysis has many important advantages such as (Pawlak, 1997): Finding hidden patterns in data; Finds minimal sets of data (data reduction); Evaluates significance of data; Generates sets of decision rules from data; Facilitates the interpretation of obtained result

Different problems can be addressed though Rough Set Theory, however during the last few years this formalism has been approached as a tool used with different areas of research. There has been research concerning be relationship between Rough Set Theory and the Dempster-Shafer Theory and between rough sets and fuzzy sets. Rough set theory has also provided the necessary formalism and ideas for the development of some propositional machine learning systems.

Rough set has also been used for knowledge representation; data mining; dealing with imperfect data; reducing knowledge representation and for analyzing attribute dependencies.

Rough set Theory has found many applications such as power system security analysis, medical data, finance, voice recognition and image processing; and one of the research areas that has successfully used Rough Set is the knowledge discovery or Data Mining in database.

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