

## COMPARISON OF MAMDANI AND SUGENO FUZZY INFERENCE SYSTEM MODELS FOR RESONANT FREQUENCY CALCULATION OF RECTANGULAR MICROSTRIP ANTENNAS

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**Abstract**—Models based on fuzzy inference systems (FISs) for calculating the resonant frequency of rectangular microstrip antennas (MSAs) with thin and thick substrates are presented. Two types of FIS models, Mamdani FIS model and Sugeno FIS model, are used to compute the resonant frequency. The parameters of FIS models are determined by using various optimization algorithms. The resonant frequency results predicted by FIS models are in very good agreement with the experimental results available in the literature. When the performances of FIS models are compared with each other, the best result is obtained from the Sugeno FIS model trained by the least-squares algorithm.

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## 1. INTRODUCTION

MSAs have many desirable features such as low profile, light weight, conformal structure, low production cost, and ease of integration with microwave integrated circuit or monolithic microwave integrated circuit components [1–5]. MSAs are therefore used extensively in a broad range of commercial and military applications.

In MSA designs, it is important to determine the resonant frequencies of the antenna accurately because MSAs have narrow bandwidths and can only operate effectively in the vicinity of the resonant frequency. So, a technique to compute accurately the resonant frequency is helpful in antenna designs. Several techniques [1–39] are available in the literature to calculate the resonant frequency of rectangular MSA, as this is one of the most popular and convenient shapes. These techniques can be broadly classified into two categories: analytical and numerical techniques. The analytical techniques offer both simplicity and physical insight, but depend on several assumptions and approximations that are valid only for thin substrates. The numerical techniques provide accurate results but usually require considerable computational time and costs.

The neural models trained by various algorithms were used in calculating the resonant frequency of rectangular MSAs [26, 28, 29, 31]. A neuro-fuzzy network was presented in [30] to compute the resonant frequencies of MSAs. In [30], the number of rules and the premise parameters of Sugeno FIS were determined by the fuzzy subtractive clustering method and then the consequent parameters of each output rule were determined by using linear least squares estimation method. The training data sets were obtained by numerical simulations using a moment-method code based on electric field integral equation approach. To validate the performances of the neuro-fuzzy network, a set of further moment-method simulations was realized and presented to the neuro-fuzzy network.

The concurrent neuro-fuzzy system models were proposed in [34, 35] to calculate simultaneously the resonant frequencies of the rectangular, circular, and triangular MSAs. The concurrent neuro-fuzzy system comprises an artificial neural network and a fuzzy system. In a concurrent neuro-fuzzy system, neural network assists the fuzzy system continuously (or vice versa) to compute the resonant frequency.

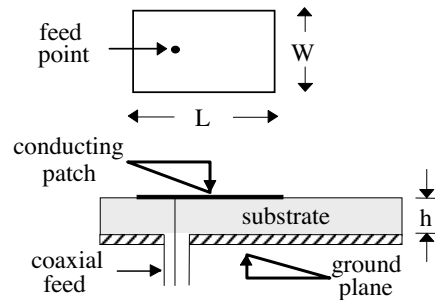
In this paper, the models based on Mamdani FIS [40, 41] and Sugeno FIS [41, 42] are presented for computing the resonant frequency of rectangular MSAs with thin and thick substrates. The FIS is a popular computing framework based on the concepts of fuzzy set theory, fuzzy if-then rules, and fuzzy reasoning [40–43]. FISs are

nonlinear systems capable of inferring complex nonlinear relationships between input and output data. The high-speed real-time computation feature of the FIS recommends its use in antenna computer aided design (CAD) programs.

In previous works [27, 33–35, 44–50], we successfully used Sugeno FISs for computing accurately the various parameters of the rectangular, triangular, and circular MSAs. In [27, 33], the tabu search algorithms and the hybrid learning algorithm were used to train the Sugeno FISs. However, in this paper, six different optimization algorithms, least-squares (LSQ) algorithm [51–53], nelder-mead (NM) algorithm [54, 55], genetic algorithm (GA) [56, 57], differential evolution algorithm (DEA) [58–60], particle swarm optimization (PSO) [61, 62], and simulated annealing (SA) algorithm [63–65], are used to train the Sugeno FISs. Furthermore, in this paper, Mamdani FIS models trained by LSQ, NM, GA, DEA, PSO, and SA are proposed to compute the resonant frequency of rectangular MSAs.

## 2. FIS MODELS FOR RESONANT FREQUENCY COMPUTATION

Figure 1 shows a rectangular patch of width  $W$  and length  $L$  over a ground plane with a substrate of thickness  $h$  and a relative dielectric constant  $\epsilon_r$ . A survey of the literature [1–39] clearly shows that only four parameters,  $W$ ,  $L$ ,  $h$ , and  $\epsilon_r$ , are needed to describe the resonant frequency. In this paper, the resonant frequency of the rectangular MSA is computed by using FIS models. The FIS is a very powerful approach for building complex and nonlinear relationship between a set of input and output data [40–43].



**Figure 1.** Geometry of rectangular MSA.

Three types of FISs, Mamdani FIS, Sugeno FIS, and Tsukamoto FIS, have been widely used in various applications [41]. The

**Table 1.** Measured resonant frequency results and dimensions for electrically thin and thick rectangular MSAs.

Patch No	W (cm)	L (cm)	h (cm)	$\epsilon_r$	$(h/\lambda_d) \times 100$	Measured ( $f_{me}$ ) (MHz)
1	5.700	3.800	0.3175	2.33	3.7317	2310 <sup>+</sup>
2	4.550	3.050	0.3175	2.33	4.6687	2890 <sup>+</sup>
3	2.950	1.950	0.3175	2.33	6.8496	4240 <sup>+</sup>
4	1.950	1.300	0.3175	2.33	9.4344	5840 <sup>+</sup>
5*	1.700	1.100	0.3175	2.33	10.9852	6800 <sup>+</sup>
6	1.400	0.900	0.3175	2.33	12.4392	7700 <sup>+</sup>
7	1.200	0.800	0.3175	2.33	13.3600	8270 <sup>+</sup>
8	1.050	0.700	0.3175	2.33	14.7655	9140 <sup>+</sup>
9	1.700	1.100	0.9525	2.33	22.9236	4730 <sup>+</sup>
10	1.700	1.100	0.1524	2.33	6.1026	7870 <sup>+</sup>
11	4.100	4.140	0.1524	2.50	1.7896	2228 <sup>A</sup>
12*	6.858	4.140	0.1524	2.50	1.7671	2200 <sup>A</sup>
13	10.800	4.140	0.1524	2.50	1.7518	2181 <sup>A</sup>
14	0.850	1.290	0.0170	2.22	0.6535	7740
15*	0.790	1.185	0.0170	2.22	0.7134	8450
16	2.000	2.500	0.0790	2.22	1.5577	3970
17	1.063	1.183	0.0790	2.55	3.2505	7730
18	0.910	1.000	0.1270	10.20	6.2193	4600
19	1.720	1.860	0.1570	2.33	4.0421	5060
20*	1.810	1.960	0.1570	2.33	3.8384	4805
21	1.270	1.350	0.1630	2.55	5.6917	6560
22	1.500	1.621	0.1630	2.55	4.8587	5600
23*	1.337	1.412	0.2000	2.55	6.6004	6200
24	1.120	1.200	0.2420	2.55	9.0814	7050
25	1.403	1.485	0.2520	2.55	7.7800	5800
26	1.530	1.630	0.3000	2.50	8.3326	5270
27	0.905	1.018	0.3000	2.50	12.6333	7990
28	1.170	1.280	0.3000	2.50	10.3881	6570
29*	1.375	1.580	0.4760	2.55	12.9219	5100
30	0.776	1.080	0.3300	2.55	14.0525	8000
31	0.790	1.255	0.4000	2.55	15.1895	7134
32	0.987	1.450	0.4500	2.55	14.5395	6070
33*	1.000	1.520	0.4760	2.55	14.7462	5820
34	0.814	1.440	0.4760	2.55	16.1650	6380
35	0.790	1.620	0.5500	2.55	17.5363	5990
36	1.200	1.970	0.6260	2.55	15.5278	4660
37	0.783	2.300	0.8540	2.55	20.9105	4600
38*	1.256	2.756	0.9520	2.55	18.1413	3580
39	0.974	2.620	0.9520	2.55	20.1683	3980
40	1.020	2.640	0.9520	2.55	19.7629	3900
41	0.883	2.676	1.0000	2.55	21.1852	3980
42	0.777	2.835	1.1000	2.55	22.8353	3900
43	0.920	3.130	1.2000	2.55	22.1646	3470
44*	1.030	3.380	1.2810	2.55	21.8197	3200
45	1.265	3.500	1.2810	2.55	20.3196	2980
46	1.080	3.400	1.2810	2.55	21.4787	3150

<sup>+</sup> These frequencies measured by Chang et al. [14]. <sup>A</sup> These frequencies measured by Carver [8]. The remainder measured by Kara [22, 23].

\* Test data.  $\lambda_d$  is the wavelength in the dielectric substrate.

differences between these three FISs lie in the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedures differ accordingly. In this paper, Mamdani and Sugeno FIS models are used to calculate accurately the resonant frequency of rectangular MSAs. The data sets used in training and testing Mamdani and Sugeno FIS models have been obtained from the previous experimental works [8, 14, 22, 23], and are given in Table 1. The 37 data sets in Table 1 are used to train the FISs. The 9 data sets, marked with an asterisk in Table 1, are used for testing. For the FIS models, the inputs are  $W$ ,  $L$ ,  $h$ , and  $\varepsilon_r$  and the output is the measured resonant frequency  $f_{me}$ .

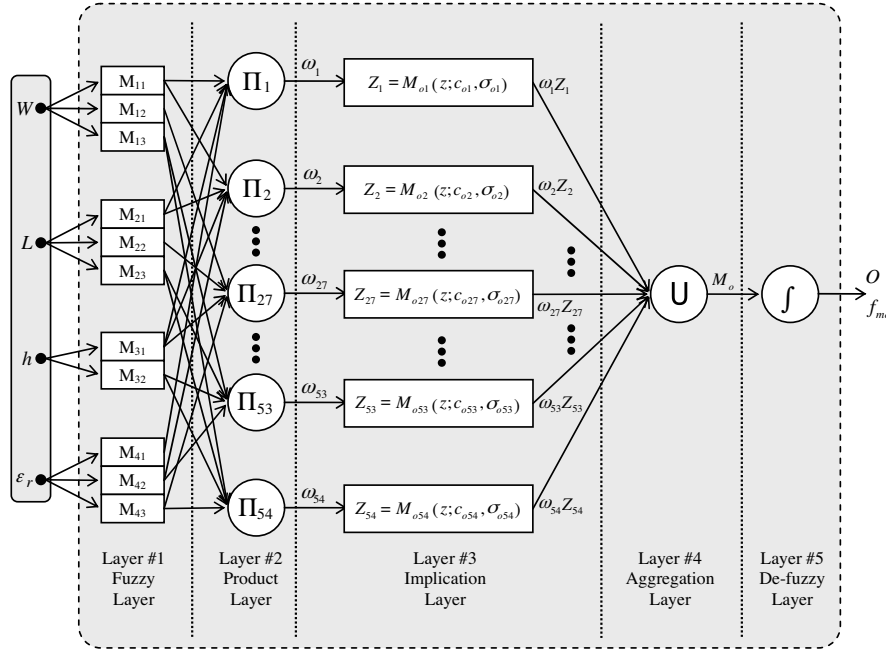
In this paper, the grid partitioning method [41] is used for the fuzzy rule extraction. In the grid partitioning method, the domain of each antecedent variable is partitioned into equidistant and identically shaped membership functions (MFs). A major advantage of the grid partitioning method is that the fuzzy rules obtained from the fixed linguistic fuzzy grids are always linguistically interpretable. Using the available input-output data, the parameters of the MFs can be optimized.

In this paper, the number of MFs for the input variables  $W$ ,  $L$ ,  $h$ , and  $\varepsilon_r$  are determined as 3, 3, 2, and 3, respectively. Each possible combination of inputs and their associated MFs is represented by a rule in the rule base of the Mamdani and Sugeno FIS models. So, the number of rules for FIS models is 54 ( $3 \times 3 \times 2 \times 3 = 54$ ).

The application of the Mamdani and Sugeno FIS models to the resonant frequency calculation is given in the following sections.

### 2.1. Mamdani FIS Models for Resonant Frequency Computation

Mamdani FIS [40, 41] was proposed as the very first attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. The Mamdani FIS architecture used in this paper for the resonant frequency computation of rectangular MSAs is illustrated in Figure 2, in which a circle indicates a fixed node, whereas a rectangular indicates



**Figure 2.** Architecture of Mamdani FIS for resonant frequency computation of rectangular MSA.

an adaptive node. The rule base for Mamdani FIS can be written as

1. if ( $W$  is  $M_{11}$ ) and ( $L$  is  $M_{21}$ ) and ( $h$  is  $M_{31}$ ) and ( $\varepsilon_r$  is  $M_{41}$ )  
then  $Z_1 = M_{o1}(z; c_{o1}, \sigma_{o1})$
2. if ( $W$  is  $M_{11}$ ) and ( $L$  is  $M_{21}$ ) and ( $h$  is  $M_{31}$ ) and ( $\varepsilon_r$  is  $M_{42}$ )  
then  $Z_2 = M_{o2}(z; c_{o2}, \sigma_{o2})$
3. if ( $W$  is  $M_{11}$ ) and ( $L$  is  $M_{21}$ ) and ( $h$  is  $M_{31}$ ) and ( $\varepsilon_r$  is  $M_{43}$ )  
then  $Z_3 = M_{o3}(z; c_{o3}, \sigma_{o3})$
4. if ( $W$  is  $M_{11}$ ) and ( $L$  is  $M_{21}$ ) and ( $h$  is  $M_{32}$ ) and ( $\varepsilon_r$  is  $M_{41}$ ) (1)  
then  $Z_4 = M_{o4}(z; c_{o4}, \sigma_{o4})$
- $\vdots$
53. if ( $W$  is  $M_{13}$ ) and ( $L$  is  $M_{23}$ ) and ( $h$  is  $M_{32}$ ) and ( $\varepsilon_r$  is  $M_{42}$ )  
then  $Z_{53} = M_{o53}(z; c_{o53}, \sigma_{o53})$
54. if ( $W$  is  $M_{13}$ ) and ( $L$  is  $M_{23}$ ) and ( $h$  is  $M_{32}$ ) and ( $\varepsilon_r$  is  $M_{43}$ )  
then  $Z_{54} = M_{o54}(z; c_{o54}, \sigma_{o54})$

with

$$Z_k = M_{ok}(z; c_{ok}, \sigma_{ok}) \quad k = 1, \dots, 54 \quad (2)$$

where  $M_{ij}$ ,  $Z_k$ , and  $M_{ok}$  represent the  $j$ th MF of the  $i$ th input, the output of the  $k$ th rule, and the  $k$ th output MF, respectively. In Eq. (1),  $c_{ok}$  and  $\sigma_{ok}$  are the consequent parameters that characterize the shapes of the output MFs.

As shown in Figure 2, the Mamdani FIS architecture consists of five layers: fuzzy layer, product layer, implication layer, aggregation layer, and de-fuzzy layer. Layered operating mechanism of the Mamdani FIS can be described as follows:

**Layer 1:** In this layer, the crisp input values are converted to the fuzzy values by the input MFs. In this paper, the following generalized bell, trapezoidal, and gaussian MFs for the inputs are used:

i) Generalized bell MFs for  $(i = 1 \text{ or } i = 4), (j = 1, 2, 3), (x = W \text{ or } x = \varepsilon_r)$ :

$$M_{ij}(x) = Gbell(x; a_{ij}, b_{ij}, c_{ij}) = \frac{1}{1 + \left| \frac{x - c_{ij}}{a_{ij}} \right|^{2b_{ij}}} \quad (3a)$$

ii) Trapezoidal MFs for  $i = (2), j = (1, 2, 3), (x = L)$ :

$$M_{ij}(x) = Trap(x; a_{ij}, b_{ij}, c_{ij}, d_{ij}) = \begin{cases} 0, & x \leq a_{ij} \\ \frac{x - a_{ij}}{b_{ij} - a_{ij}}, & a_{ij} \leq x \leq b_{ij} \\ 1, & b_{ij} \leq x \leq c_{ij} \\ \frac{d_{ij} - x}{d_{ij} - c_{ij}}, & c_{ij} \leq x \leq d_{ij} \\ 0, & d_{ij} \leq x \end{cases} \quad (3b)$$

iii) Gaussian MFs for  $(i = 3), (j = 1, 2), (x = h)$ :

$$M_{ij}(x) = Gauss(x; c_{ij}, \sigma_{ij}) = e^{-\frac{1}{2} \left( \frac{x - c_{ij}}{\sigma_{ij}} \right)^2} \quad (3c)$$

where  $a_{ij}$ ,  $b_{ij}$ ,  $c_{ij}$ ,  $d_{ij}$ , and  $\sigma_{ij}$  are the premise parameters that characterize the shapes of the input MFs.

**Layer 2:** In this layer, the weighting factor (firing strength) of each rule is computed. The weighting factor of each rule, which is expressed as  $\omega_k$ , is determined by evaluating the membership expressions in the antecedent of the rule. This is accomplished by first converting the input values to fuzzy membership values by using

the input MFs in the layer 1 and then applying the “*and*” operator to these membership values. The “*and*” operator corresponds to the multiplication of input membership values. Hence, the weighting factors of the rules are computed as follows:

$$\begin{aligned}
\omega_1 &= M_{11}(W) M_{21}(L) M_{31}(h) M_{41}(\varepsilon_r) \\
\omega_2 &= M_{11}(W) M_{21}(L) M_{31}(h) M_{42}(\varepsilon_r) \\
\omega_3 &= M_{11}(W) M_{21}(L) M_{31}(h) M_{43}(\varepsilon_r) \\
\omega_4 &= M_{11}(W) M_{21}(L) M_{32}(h) M_{41}(\varepsilon_r) \\
&\vdots \\
\omega_{53} &= M_{13}(W) M_{23}(L) M_{32}(h) M_{42}(\varepsilon_r) \\
\omega_{54} &= M_{13}(W) M_{23}(L) M_{32}(h) M_{43}(\varepsilon_r)
\end{aligned} \tag{4}$$

**Layer 3:** In this layer, the implication of each output MF is computed by

$$M_{imp,k} = \omega_k Z_k \quad k = 1, \dots, 54 \tag{5}$$

where  $M_{imp,k}$  represents the implicated output MFs.

**Layer 4:** In this layer, the aggregation is performed to produce an overall output MF,  $M_o(z)$ , by using the union operator:

$$\begin{aligned}
M_o(z) &= \bigcup_{k=1}^{54} M_{imp,k} = \bigcup_{k=1}^{54} \omega_k Z_k \\
&= \bigcup_{k=1}^{54} \omega_k M_{ok}(z; c_{ok}, \sigma_{ok}) = \bigcup_{k=1}^{54} \omega_k e^{-\frac{1}{2} \left( \frac{z - c_{ok}}{\sigma_{ok}} \right)^2}
\end{aligned} \tag{6}$$

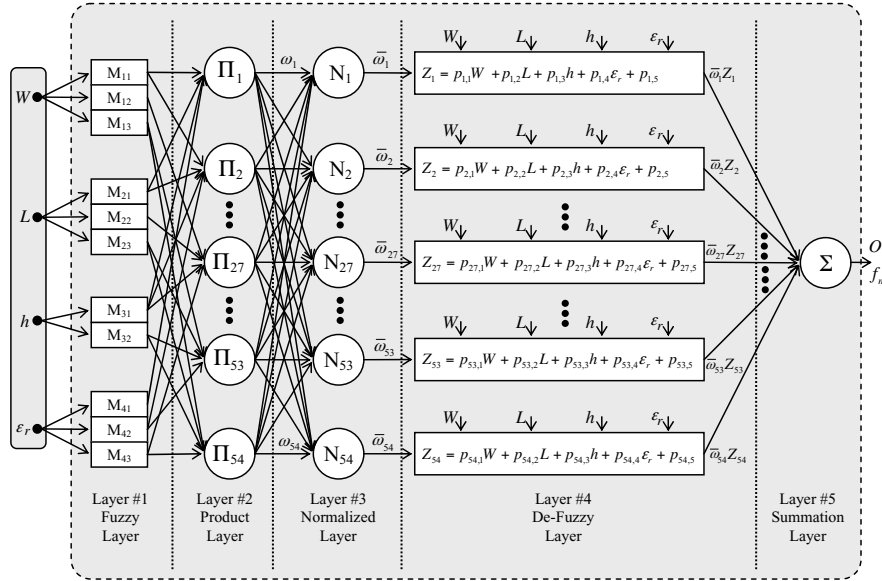
The types of the output MFs ( $M_{ok}$ ) are Gaussian. Here, the “*union*” operation is performed using the maximum operator.

**Layer 5:** In this layer, the defuzzification is performed by using the centroid of area method:

$$O = \frac{\int_Z M_o(z) z dz}{\int_Z M_o(z) dz} \tag{7}$$

It is clear from Eq. (3) that the generalized bell, trapezoidal, and gaussian MFs are specified by three, four, and two parameters, respectively. Therefore, the Mamdani FIS used in this paper contains a total of 142 fitting parameters, of which 34 ( $3 \times 3 + 3 \times 4 + 2 \times 2 + 3 \times 3 = 34$ ) are the premise parameters and 108 ( $2 \times 54 = 108$ ) are the consequent parameters.





**Figure 3.** Architecture of Sugeno FIS for resonant frequency computation of rectangular MSA.

## 2.2. Sugeno FIS Models for Resonant Frequency Computation

Sugeno FIS [41, 42] was proposed to develop a systematic approach to generate fuzzy rules from a given input-output data. The Sugeno FIS architecture used in this paper for the resonant frequency calculation is shown in Figure 3. The rule base for Sugeno FIS is given by

1. if ( $W$  is  $M_{11}$ ) and ( $L$  is  $M_{21}$ ) and ( $h$  is  $M_{31}$ ) and ( $\varepsilon_r$  is  $M_{41}$ )  
then  $Z_1 = r_1(W, L, h, \varepsilon_r)$
2. if ( $W$  is  $M_{11}$ ) and ( $L$  is  $M_{21}$ ) and ( $h$  is  $M_{31}$ ) and ( $\varepsilon_r$  is  $M_{42}$ )  
then  $Z_2 = r_2(W, L, h, \varepsilon_r)$
3. if ( $W$  is  $M_{11}$ ) and ( $L$  is  $M_{21}$ ) and ( $h$  is  $M_{31}$ ) and ( $\varepsilon_r$  is  $M_{43}$ )  
then  $Z_3 = r_3(W, L, h, \varepsilon_r)$
4. if ( $W$  is  $M_{11}$ ) and ( $L$  is  $M_{21}$ ) and ( $h$  is  $M_{32}$ ) and ( $\varepsilon_r$  is  $M_{41}$ ) (8)  
then  $Z_4 = r_4(W, L, h, \varepsilon_r)$
- $\vdots$   $\vdots$

53. if ( $W$  is  $M_{13}$ ) and ( $L$  is  $M_{23}$ ) and ( $h$  is  $M_{32}$ ) and ( $\varepsilon_r$  is  $M_{42}$ )  
 then  $Z_{53} = r_{53}(W, L, h, \varepsilon_r)$
54. if ( $W$  is  $M_{13}$ ) and ( $L$  is  $M_{23}$ ) and ( $h$  is  $M_{32}$ ) and ( $\varepsilon_r$  is  $M_{43}$ )  
 then  $Z_{54} = r_{54}(W, L, h, \varepsilon_r)$

with

$$Z_k = r_k(W, L, h, \varepsilon_r) \quad k = 1, \dots, 54 \quad (9)$$

where  $M_{ij}$ ,  $Z_k$ , and  $r_k$  represent the  $j$ th MF of the  $i$ th input, the output of the  $k$ th rule, and the  $k$ th output MF, respectively.

As shown in Figure 3, the Sugeno FIS structure consists of five layers: fuzzy layer, product layer, normalized layer, de-fuzzy layer, and summation layer. It is clear that **Layer 1** and **Layer 2** of Sugeno FIS are the same as those of Mamdani FIS. The operating mechanism of the other layers for Sugeno FIS can be described as follows:

**Layer 3:** The normalized weighting factor of each rule,  $\bar{\omega}_k$ , is computed by using

$$\bar{\omega}_k = \frac{\omega_k}{\sum_{i=1}^{54} \omega_i} \quad k = 1, \dots, 54 \quad (10)$$

**Layer 4:** In this layer, the output rules can be written as:

$$\begin{aligned} \bar{\omega}_k Z_k &= \bar{\omega}_k r_k(W, L, h, \varepsilon_r) \\ &= \bar{\omega}_k (p_{k1}W + p_{k2}L + p_{k3}h + p_{k4}\varepsilon_r + p_{k5}) \quad k = 1, \dots, 54 \end{aligned} \quad (11)$$

where  $p_k$  are the consequent parameters that characterize the shapes of the output MFs. Here, the types of the output MFs ( $r_k$ ) are linear.

**Layer 5:** Each rule is weighted by own normalized weighting factor and the output of the FIS is calculated by summing of all rule outputs:

$$O = \sum_{k=1}^{54} \bar{\omega}_k Z_k = \frac{\sum_{k=1}^{54} \omega_k Z_k}{\sum_{k=1}^{54} \omega_k} \quad (12)$$

The Sugeno FIS used in this paper contains a total of 304 fitting parameters, of which 34 ( $3 \times 3 + 3 \times 4 + 2 \times 2 + 3 \times 3 = 34$ ) are the premise parameters and 270 ( $5 \times 54 = 270$ ) are the consequent parameters.

**Table 2.** Comparison of measured and calculated resonant frequencies obtained by using Mamdani FIS models presented in this paper for electrically thin and thick rectangular MSAs.

Patch No	Measured ( $f_{me}$ ) (MHz)	Present Mamdani FIS Models					
		LSQ	NM	GA	DEA	PSO	SA
1	2310	2309.9	2309.9	2309.8	2310.0	2309.8	2309.5
2	2890	2889.8	2889.8	2889.7	2889.7	2889.5	2889.1
3	4240	4239.7	4239.6	4239.5	4239.4	4239.2	4238.4
4	5840	5839.5	5839.4	5839.3	5839.1	5838.9	5837.8
5	6800	6799.4	6799.3	6799.2	6798.9	6798.7	6797.4
6	7700	7699.4	7699.2	7699.1	7698.8	7698.5	7697.1
7	8270	8269.3	8269.2	8269.0	8268.7	8268.4	8266.9
8	9140	9139.2	9139.1	9138.9	9138.5	9138.3	9136.6
9	4730	4729.6	4729.5	4729.4	4729.2	4729.1	4728.2
10	7870	7869.3	7869.2	7869.0	7868.7	7868.5	7867.0
11	2228	2227.9	2227.8	2227.8	2227.9	2227.7	2227.4
12	2200	2200.0	2199.9	2199.9	2200.2	2199.9	2199.8
13	2181	2181.2	2181.2	2181.3	2182.1	2181.4	2181.8
14	7740	7739.4	7739.2	7739.1	7738.7	7738.5	7737.1
15	8450	8449.3	8449.1	8449.0	8448.6	8448.4	8446.8
16	3970	3969.7	3969.6	3969.5	3969.4	3969.3	3968.6
17	7730	7729.4	7729.2	7729.1	7728.8	7728.5	7727.1
18	4600	4599.8	4599.8	4599.7	4600.0	4599.5	4599.1
19	5060	5059.6	5059.5	5059.4	5059.2	5059.0	5058.1
20	4805	4804.6	4804.5	4804.4	4804.2	4804.1	4803.2
21	6560	6559.5	6559.3	6559.2	6558.9	6558.8	6557.5
22	5600	5599.5	5599.4	5599.3	5599.1	5598.9	5597.9
23	6200	6199.5	6199.4	6199.3	6199.0	6198.8	6197.7
24	7050	7049.4	7049.3	7049.1	7048.9	7048.7	7047.4
25	5800	5799.5	5799.4	5799.3	5799.1	5798.9	5797.8
26	5270	5269.6	5269.5	5269.4	5269.2	5269.0	5268.0
27	7990	7989.3	7989.2	7989.0	7988.7	7988.5	7987.0
28	6570	6569.5	6569.3	6569.2	6568.9	6568.8	6567.5
29	5100	5099.6	5099.5	5099.4	5099.2	5099.0	5098.1
30	8000	7999.3	7999.2	7999.0	7998.7	7998.5	7997.0
31	7134	7133.4	7133.3	7133.1	7132.9	7132.6	7131.3
32	6070	6069.5	6069.4	6069.3	6069.0	6068.9	6067.7
33	5820	5819.5	5819.4	5819.3	5819.1	5818.9	5817.8
34	6380	6379.5	6379.4	6379.2	6379.0	6378.8	6377.6
35	5990	5989.5	5989.4	5989.3	5989.0	5988.9	5987.8
36	4660	4659.6	4659.5	4659.4	4659.3	4659.1	4658.3
37	4600	4599.6	4599.5	4599.5	4599.3	4599.1	4598.3
38	3580	3579.7	3579.6	3579.6	3579.5	3579.3	3578.7
39	3980	3979.7	3979.6	3979.5	3979.4	3979.3	3978.5
40	3900	3899.7	3899.6	3899.5	3899.4	3899.3	3898.6
41	3980	3979.7	3979.6	3979.5	3979.4	3979.3	3978.5
42	3900	3899.7	3899.6	3899.5	3899.4	3899.3	3898.6
43	3470	3469.7	3469.7	3469.6	3469.5	3469.4	3468.7
44	3200	3199.7	3199.7	3199.6	3199.5	3199.4	3198.9
45	2980	2979.8	2979.7	2979.7	2979.6	2979.5	2978.9
46	3150	3149.7	3149.7	3149.6	3149.5	3149.4	3148.9

**Table 3.** Comparison of measured and calculated resonant frequencies obtained by using Sugeno FIS models presented in this paper for electrically thin and thick rectangular MSAs.

Patch No	Measured ( $f_{me}$ ) (MHz)	Present Sugeno FIS Models					
		LSQ	NM	GA	DEA	PSO	SA
1	2310	2310.0	2310.2	2310.1	2310.1	2310.1	2309.5
2	2890	2890.0	2890.3	2890.1	2890.1	2890.0	2890.2
3	4240	4240.0	4240.0	4240.1	4240.1	4240.1	4239.4
4	5840	5840.0	5839.8	5839.8	5839.4	5839.7	5839.4
5	6800	6800.0	6800.1	6800.1	6799.9	6800.3	6800.9
6	7700	7700.0	7700.0	7699.7	7700.1	7699.6	7701.4
7	8270	8270.0	8270.0	8269.7	8270.1	8269.7	8270.5
8	9140	9140.0	9140.0	9139.7	9140.1	9139.8	9140.9
9	4730	4730.0	4730.0	4730.0	4730.0	4729.9	4728.6
10	7870	7870.0	7869.8	7869.8	7869.6	7869.9	7869.1
11	2228	2228.0	2228.0	2228.1	2228.0	2228.0	2228.1
12	2200	2200.0	2200.0	2200.1	2200.0	2200.0	2200.0
13	2181	2181.0	2181.0	2181.1	2181.0	2181.0	2181.0
14	7740	7740.0	7739.7	7739.8	7739.4	7739.7	7738.5
15	8450	8450.0	8449.9	8450.0	8449.8	8450.5	8449.3
16	3970	3970.0	3970.1	3970.0	3970.1	3970.0	3970.1
17	7730	7730.0	7729.3	7729.2	7728.6	7728.3	7725.4
18	4600	4600.0	4600.0	4600.0	4600.0	4600.0	4600.5
19	5060	5060.0	5060.1	5060.3	5059.9	5061.3	5061.3
20	4805	4805.0	4805.5	4805.2	4804.9	4804.4	4803.9
21	6560	6560.0	6559.0	6559.5	6558.5	6560.5	6561.7
22	5600	5600.0	5599.3	5598.9	5598.1	5597.5	5596.3
23	6200	6200.0	6203.0	6203.5	6202.6	6204.7	6203.6
24	7050	7050.0	7050.1	7050.7	7050.1	7052.9	7054.6
25	5800	5800.0	5798.6	5798.8	5797.9	5799.7	5799.9
26	5270	5270.0	5270.5	5270.3	5269.7	5270.2	5270.6
27	7990	7990.0	7990.3	7991.0	7991.3	7992.1	7989.9
28	6570	6570.0	6568.8	6568.1	6567.4	6564.6	6560.1
29	5100	5100.0	5100.2	5100.1	5099.8	5100.1	5099.7
30	8000	8000.0	7999.5	7999.2	7999.3	7998.5	7998.6
31	7134	7134.0	7134.5	7134.7	7134.6	7135.6	7138.1
32	6070	6070.0	6069.6	6069.6	6069.4	6069.1	6065.0
33	5820	5820.0	5820.5	5820.4	5820.4	5821.2	5828.1
34	6380	6380.0	6379.5	6379.0	6379.1	6377.7	6379.1
35	5990	5990.0	5989.9	5989.8	5990.0	5989.0	5988.3
36	4660	4660.0	4660.0	4659.9	4660.1	4659.6	4658.7
37	4600	4600.0	4600.1	4600.2	4600.2	4600.1	4599.6
38	3580	3580.0	3580.0	3580.0	3580.0	3579.9	3579.3
39	3980	3980.0	3979.7	3979.9	3979.9	3979.4	3978.5
40	3900	3900.0	3900.2	3900.3	3900.4	3900.8	3899.5
41	3980	3980.0	3979.9	3979.7	3979.7	3978.8	3977.1
42	3900	3900.0	3900.0	3900.3	3900.3	3900.1	3899.5
43	3470	3470.0	3470.0	3470.2	3470.2	3470.2	3469.4
44	3200	3200.0	3200.1	3199.8	3199.8	3198.9	3199.4
45	2980	2980.0	2980.0	2980.1	2979.9	2979.9	2980.0
46	3150	3150.0	3149.9	3150.5	3150.5	3151.0	3150.3

**Table 4.** Resonant frequencies obtained from the Sugeno FIS models available in the literature for electrically thin and thick rectangular MSAs.

Patch No	Measured ( $f_{me}$ ) (MHz)	Sugeno FIS Models in the Literature [27]		
		ITSA	MTSA	CTSA
1	2310	2310.0	2310.0	2310.0
2	2890	2890.0	2890.0	2890.0
3	4240	4240.1	4240.1	4240.2
4	5840	5839.5	5839.7	5840.0
5	6800	6800.7	6800.7	6799.8
6	7700	7698.7	7698.4	7699.1
7	8270	8271.7	8272.1	8271.3
8	9140	9139.2	9139.1	9139.3
9	4730	4730.0	4730.0	4730.0
10	7870	7870.0	7869.9	7869.7
11	2228	2228.0	2228.0	2228.0
12	2200	2200.0	2200.0	2200.0
13	2181	2181.0	2181.0	2181.0
14	7740	7740.1	7740.1	7740.0
15	8450	8449.7	8449.6	8449.6
16	3970	3970.1	3970.1	3970.2
17	7730	7730.3	7730.4	7730.7
18	4600	4600.0	4600.0	4600.0
19	5060	5060.0	5060.4	5061.9
20	4805	4804.9	4804.4	4802.9
21	6560	6559.8	6560.0	6560.5
22	5600	5600.1	5600.2	5600.3
23	6200	6200.0	6199.2	6197.0
24	7050	7050.1	7050.6	7050.5
25	5800	5800.5	5801.1	5801.9
26	5270	5269.6	5269.0	5268.5
27	7990	7990.2	7990.6	7991.8
28	6570	6569.6	6569.0	6570.0
29	5100	5100.0	5098.3	5095.1
30	8000	7999.5	7998.7	7997.0
31	7134	7134.9	7136.1	7137.9
32	6070	6069.5	6066.6	6068.8
33	5820	5821.6	5829.2	5838.2
34	6380	6378.9	6377.0	6369.3
35	5990	5989.7	5987.9	5986.5
36	4660	4660.1	4660.4	4660.8
37	4600	4599.9	4600.0	4600.2
38	3580	3579.5	3579.4	3579.5
39	3980	3978.0	3975.5	3974.5
40	3900	3901.7	3903.8	3904.7
41	3980	3981.2	3981.7	3982.0
42	3900	3899.0	3899.0	3898.6
43	3470	3471.2	3470.8	3471.0
44	3200	3200.7	3200.3	3200.3
45	2980	2980.6	2980.4	2980.2
46	3150	3148.1	3149.0	3149.0

**Table 5.** Resonant frequencies obtained from the conventional methods available in the literature for electrically thin and thick rectangular MSAs.

Patch No	Measured ( $f_{me}$ ) (MHz)	Conventional Methods in the Literature											
		[6]	[7]	[8]	[1]	[2]	[11]	[15]	[16]	[19]	[23]	[23]	[32]
1	2310	2586	2381	2373	2452	2296	2458	2389	2377	2323	628	786	2450
2	2890	3222	2911	2893	3013	2795	3042	2915	2908	2831	963	1219	3016
3	4240	5039	4327	4239	4529	4108	4681	4296	4331	4191	2294	2983	4539
4	5840	7559	6085	5928	6448	5700	6918	5965	6089	5919	5032	6712	6449
5	6800	8933	6958	6806	7405	6467	8109	6761	6909	6798	6955	9375	7398
6	7700	10919	8137	7956	8711	7482	9836	7820	7889	7528	10261	14005	8701
7	8270	12284	8905	8986	9579	8132	11054	8508	8402	8254	12894	17725	9605
8	9140	14038	9831	11330	10621	8894	12588	9306	8709	9141	16706	23151	10753
9	4730	8933	5101	1094597	5614	4320	7958	4525	5109	4922	19581	28126	7391
10	7870	8933	7829	7694	8154	7463	8341	7795	7825	7585	3494	4500	8171
11	2228	2292	2209	2232	2259	2175	2248	2245	2222	2197	281	266	2252
12	2200	2292	2208	2204	2241	2158	2228	2221	2206	2180	281	266	2221
13	2181	2292	2208	2184	2230	2148	2216	2204	2210	2168	281	266	2192
14	7740	7804	7697	7750	7791	7635	7737	7763	7720	7717	295	412	7765
15	8450	8496	8369	8431	8478	8298	8417	8446	8396	8389	349	488	8451
16	3970	4027	3898	3949	3983	3838	3951	3950	3917	3887	358	510	3977
17	7730	7940	7442	7605	7733	7322	7763	7639	7551	7376	1775	1610	7730
18	4600	4697	4254	4407	4641	4455	4979	4729	4614	4430	14548	113	4618
19	5060	5283	4865	4989	5070	4741	5101	4958	4924	4797	1294	1621	5077
20	4805	5014	4635	4749	4824	4520	4846	4724	4688	4573	1169	1460	4830
21	6560	6958	6220	6421	6566	6067	6729	6382	6357	6114	2719	2550	6563
22	5600	5795	5270	5424	5535	5158	5625	5414	5374	5194	1907	1769	5535
23	6200	6653	5845	6053	6201	5682	6413	5987	5988	5735	3019	2860	6193
24	7050	7828	6566	6867	7052	6320	7504	6682	6769	6433	4942	4792	7030
25	5800	6325	5435	5653	5801	5259	6078	5552	5586	5326	3399	3259	5787
26	5270	5820	4943	5155	5287	4762	5572	5030	5081	4842	3281	3383	5273
27	7990	9319	7334	7813	7981	6917	8885	7339	7570	6822	8153	8674	8101
28	6570	7412	6070	6390	6550	5794	7076	6135	6264	5951	5236	5486	6543
29	5100	5945	4667	4993	5092	4407	5693	4678	4830	4338	5457	5437	5193
30	8000	8698	6845	7546	7519	6464	8447	6889	7160	6367	8089	8067	7948
31	7134	7485	5870	6601	6484	5525	7342	5904	6179	5452	7241	7242	7169
32	6070	6478	5092	5660	5606	4803	6317	5125	5341	4735	6113	6103	6026
33	5820	6180	4855	5423	5352	4576	6042	4886	5100	4513	5881	5875	5817
34	6380	6523	5101	5823	5660	4784	6453	5122	5396	4729	6529	6546	6515
35	5990	5798	4539	5264	5063	4239	5804	4550	4830	4196	5950	5976	6064
36	4660	4768	3746	4227	4141	3526	4689	3770	3949	3479	4600	4600	4613
37	4600	4084	3201	3824	3615	2938	4209	3168	3446	2921	4556	4603	4550
38	3580	3408	2668	3115	2983	2485	3430	2670	2845	2461	3554	3574	3628
39	3980	3585	2808	3335	3162	2590	3668	2790	3015	2572	3920	3955	3956
40	3900	3558	2785	3299	3133	2573	3629	2771	2987	2555	3863	3895	3907
41	3980	3510	2753	3294	3112	2522	3626	2721	2966	2509	3940	3982	3922
42	3900	3313	2608	3147	2964	2364	3473	2554	2823	2356	3852	3903	3747
43	3470	3001	2358	2838	2675	2146	3129	2317	2549	2137	3450	3493	3381
44	3200	2779	2183	2623	2474	1992	2889	2151	2357	1983	3160	3197	3123
45	2980	2684	2102	2502	2370	1936	2752	2086	2259	1924	2954	2982	2972
46	3150	2763	2168	2600	2453	1982	2863	2139	2338	1972	3125	3160	3096

**Table 6.** Train, test, and total absolute errors between the measured and calculated resonant frequencies for FIS models.

Models	Algorithms	Train Absolute Errors (MHz)	Test Absolute Errors (MHz)	Total Absolute Errors (MHz)
Present Mamdani FIS Models	LSQ	16	4	20
	NM	20	5	25
	GA	24	5	29
	DEA	31	7	38
	PSO	37	9	46
	SA	73	17	90
Present Sugeno FIS Models	<b>LSQ</b>	<b>0</b>	<b>0</b>	<b>0</b>
	NM	10	5	15
	GA	15	5	20
	DEA	19	4	23
	PSO	31	9	40
	SA	57	16	73
Sugeno FIS Models in the Literature [27]	ITSA	20	4	24
	MTSA	36	14	50
	CTSA	52	30	82

**Table 7.** Total absolute errors between the measured and calculated resonant frequencies for the conventional methods.

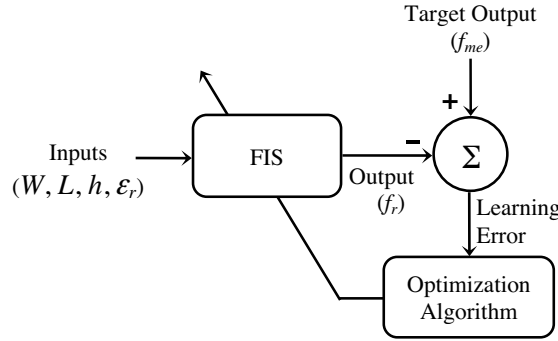
Conventional Methods in the Literature	[6]	[7]	[8]	[1]	[2]	[11]
Errors (MHz)	36059	26908	1104916	19179	32930	23746
Conventional Methods in the Literature	[15]	[16]	[19]	[23]	[23]	[32]
Errors (MHz)	23761	19899	31436	108707	126945	10132

### 2.3. Determination of Design Parameters of Mamdani and Sugeno FIS Models

The determination of the design parameters of the FIS is of vital importance. An optimization algorithm is used to determine these design parameters. The parameter optimization is done in a way such that the error measure between the target and the actual output is minimized. During the learning process of the FIS, the premise parameters and the consequent parameters are tuned until the desired response of the FIS is achieved. In this paper, six different optimization

algorithms, LSQ, NM, GA, DEA, PSO, and SA, are used to determine the optimum values of the design parameters and adapt the FISs. Basic optimization framework steps of these algorithms for the calculation of resonant frequency by using Mamdani and Sugeno FIS models are summarized below:

**Step 1:** Choose the initial FIS structure. In this step, the number and the types of MFs for the input variables  $W$ ,  $L$ ,  $h$ , and  $\varepsilon_r$  are found. Thus, the numbers of the premise parameters ( $a_{ij}$ ,  $b_{ij}$ ,  $c_{ij}$ ,  $d_{ij}$ , and  $\sigma_{ij}$ ) and the consequent parameters ( $c_{ok}$  and  $\sigma_{ok}$  or  $p_k$ ) are determined.



**Figure 4.** FIS model for calculating the resonant frequency.

**Step 2:** Initialize the values of the premise and consequent parameters. Initial values of the premise and consequent parameters are produced. Hence, the input MFs are equally spaced and made to cover the universe of discourse.

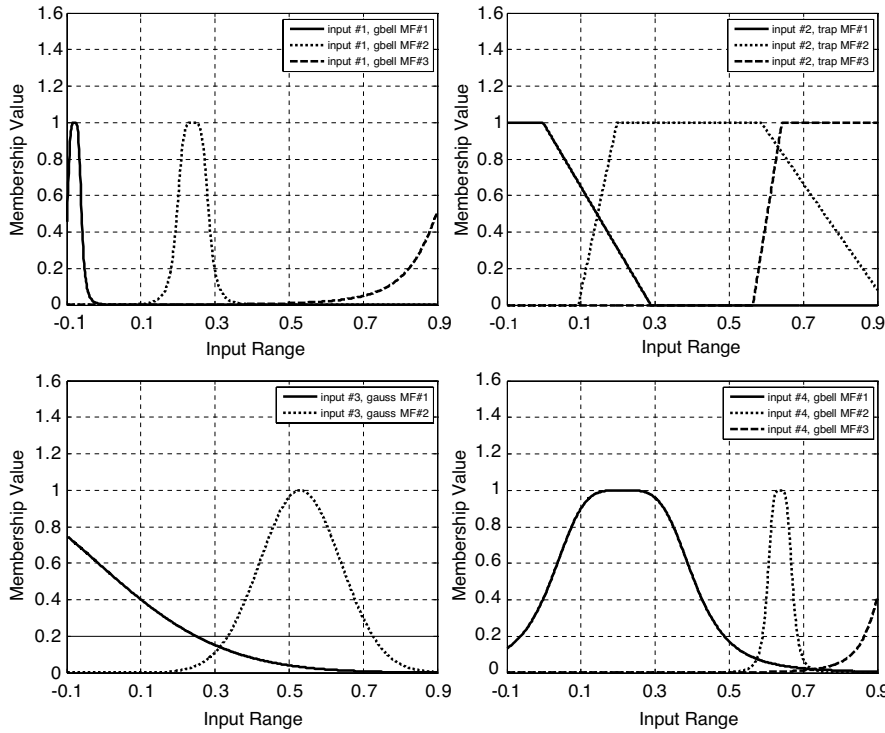
**Step 3:** Compute the output (resonant frequency  $f_r$ ). FIS produces the output ( $f_r$ ) for a given input data ( $W$ ,  $L$ ,  $h$ , and  $\varepsilon_r$ ) using the premise and the consequent parameters in the inference mechanism.

**Step 4:** Evaluate the performance of FIS. The learning error between the target (measured resonant frequency  $f_{me}$ ) and the actual output is computed. In this paper, mean square error (MSE) criterion is used for this purpose.

**Step 5:** Continue or terminate the process. In this step, termination criterion of the optimization algorithm is checked. If the learning error is equal to/small from the predetermined error value or iterations reach to a final value, the process is stopped. Otherwise, the process continues to Step 6.

**Step 6:** Update the parameter values using the optimization algorithm. In this step, an optimization algorithm (LSQ, NM, GA,





**Figure 5.** Shapes of the MFs of input variables (Input #1 =  $W$ , Input #2 =  $L$ , Input #3 =  $h$ , and Input #4 =  $\varepsilon_r$ ) for the Sugeno FIS model trained by the LSQ.

DEA, PSO, and SA) is used for updating the values of the premise and the consequent parameters so as to decrease the learning error. Then, the process continues from Step 3.

The process explained above is simply given in Figure 4.

### 3. RESULTS AND CONCLUSIONS

The resonant frequencies calculated by using Mamdani and Sugeno FIS models presented in this paper for electrically thin and thick rectangular MSAs are given in Tables 2 and 3, respectively. LSQ, NM, GA, DEA, PSO, and SA in Tables 2 and 3 represent, respectively, the resonant frequencies calculated by the FIS models trained by LSQ, NM, GA, DEA, PSO, and SA. For comparison, the resonant frequency results obtained by using the Sugeno FIS models [27] and the conventional methods [1, 2, 6–8, 11, 15, 16, 19, 23, 32] are listed in

Tables 4 and 5, respectively. ITSA, MTSA, and CTSA in Table 4 represent, respectively, the resonant frequencies calculated by the Sugeno FIS models trained by the improved tabu search algorithm (ITSA), the modified tabu search algorithm (MTSA), and the classical tabu search algorithm (CTSA). The resonant frequency results in twelfth and thirteenth columns of Table 5 are obtained by using the curve-fitting formula [23] and by using the modified cavity model [23], respectively. The sum of the absolute errors between the theoretical and experimental results for FIS models and conventional methods is listed in Tables 6 and 7.

When the performances of Mamdani and Sugeno FIS models are compared with each other, the best result is obtained from the Sugeno FIS model trained by the LSQ algorithm, as shown in Tables 2, 3, 4 and 6. The final shapes of the input MFs are illustrated in Figure 5 for the Sugeno FIS model trained by the LSQ algorithm. For brevity, the final shapes of the MFs of other FIS models are not given.

It is clear from Tables 5 and 7 that the conventional methods [1, 2, 6–8, 11, 15, 16, 19, 23, 32] give comparable results. Some cases are in very good agreement with measurements, and others are far off. When the results of FIS models are compared with the results of the conventional methods, the results of all FIS models are better than those predicted by the conventional methods. The very good agreement between the measured resonant frequency values and the computed resonant frequency values of FIS models supports the validity of the FIS models and also illustrates the superiority of FIS models over the conventional methods.

In this paper, the FIS models are trained and tested with the experimental data taken from the previous experimental works [8, 14, 22, 23]. It is clear from Tables 5 and 7 that the theoretical resonant frequency results of the conventional methods are not in very good agreement with the experimental results. For this reason, the theoretical data sets obtained from the conventional methods are not used in this work. Only the measured data set is used for training and testing the FIS models.

As a consequence, Mamdani and Sugeno FIS models are used to accurately calculate the resonant frequency of electrically thin and thick rectangular MSAs. Six optimization algorithms, LSQ, NM, GA, DEA, PSO, and SA, are used to determine the design parameters of the FIS models. In order to validate the performances of the FIS models, comprehensive comparisons have been made. The results of FIS models are in very good agreement with the measurements. The best result is obtained from the Sugeno FIS trained by LSQ algorithm.

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