FUZZY CONTROL AND ROUGH SETS

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1. Introduction

Zdzislaw Pawlak introduced the notion of rough sets [1]. It was a result of fundamental research on logical properties of information systems that are instances (snapshots) of relational databases. The theory of rough sets is concerned with the classificatory analysis of imprecise, uncertain or incomplete information or knowledge expressed in terms of data acquired from experience. Over the past one and half decade the theory has evolve into a new technology. It has proven to be a viable technique in data mining, especially extracting rules from databases. In this paper, we explore, however, another direction, intelligent control. As in [2], [3], [4], rough set theory, fuzzy logic control, and modern differential geometric view of non-linear dynamic systems are integrated into one mathematical formalism, called rough logic government. Under this formalism, *fuzzy logic can be viewed as a methodology of constructing solutions of system equations*, without explicitly constructed system models.

Equivalently, fuzzy logic chooses to model the control subsystem directly. Among others, it allows us to investigate simple kind of Lyapunov stability problem [5] that is a relatively unexplored area of fuzzy logic control. A ship-mounted satellite tracking antenna is used to illustrate the notion.

2. Modeling of Dynamic Systems

Given a set of differential equations, one can model *either* the solution directly -- the theory of differentiable manifolds, *or* the set of differential equations as an embedded "surface" in high dimensional Euclidean space. In control theory, we can model either the systems (plant model) or the solutions (control functions). In classical control (CC), one models the whole systems in which control functions are solutions of system equations, while in fuzzy logic control (FLC), one models the solutions directly. FLC approach avoids the complex system modeling; it focuses on systems that have no analytic models. It pays, however, its price in the step of verification and validation.

In CC, verification and validation is easy, because system models are often derived from the laws of nature and principles of engineering [4].

For our exposition, we quote from a common text [10] the following: "A mathematical model is a mathematical characterization of a phenomena or a process." "So mathematical model has three essential parts:

- a process or phenomena that is to be modeled,
- a mathematical structure capable of expressing the important properties of the object to be modeled,
- an explicit correspondence between the two."

We would like to add fourth point:

- verification and validation; proving the correspondence or the model is correct by experiments

3. Rough Set Theory

In this section, we will recall some notions about rough sets [1], [6]. To avoid monotonous, we use space, collection, and family as synonym for set.

3.1. Rough Sets

Let U be an non-empty set, called the universe. Let R be an equivalence relation on U; R induces a partition. We will use R to denote either equivalence or partition. The pair (U, R) is called an approximation space. Equivalence classes are called *R-elementary sets* or simply elementary sets. The empty set ϕ is assumed to be elementary. A definable set is any union of elementary sets. The set of all elementary sets is the quotient set Q(U). For each subset X in U, let [X, R], or [X] (when R is understood) denote the equivalence class of R containing X. Then, the lower approximation of X is defined as

$$R\_L[X] = \cup\{ [u, R]: [u, R] \subseteq X \},$$

and upper (higher) approximation is defined as

$$R\_H[X] = \cup\{ [u, R]: [u, R] \cap X \neq \phi \}$$

3.2. Rough Logic and Model Theory

Syntactically, a rough logic is a S_5 modal logic [7], [8], i.e., the language structure is the same. So their model theory is also similar to, but different. More specifically, the model theory of rough logic has a similar possible-world-model structure as S_5 but different in its semantics.

A *relational structure* on E is a 4-tuple
$$E = (E, N, R, F)$$
where (1) E is a set of entities $\{e, e_1, ...\}$, the domain of objects; (2) N is a set of distinguished entities, the domain of constants; (3) R is a set of relations; (4) F is a set of partial functions.
We have used the same notation for the set of entities and the relational structure. The relational structure will be referred to as the **ideal universe or ideal world.** We assume that there is an equivalence relation R on E, and hence two rough operators, the upper (denoted by H) and lower approximation (denoted by L) [1]. Let $P=\{H_i, i= 1,2,... \}$ be the partition.

Let W^h , h> 0, be a representative set that consists of one representative from each H_i,

$$W^h=\{e \mid \text{a unique } e \in H_i, \ i=1, 2, ... \}$$

Each W^h inherit a relational structure from E
$$W^h=(W^h, N^h, R^h, F^h)$$
(1) $N^h = N \cap W^h = N$; all possible should have the same content, (otherwise some sentences will lost its meaning) (2) R^h are set of relations restricted to W^h, (3) F^h are set partial functions restricted to W^h (if any function values are outside of W^h, they undefined). W^h is called a **possible world** (called observable world in [7])

The pair $W^0=(E, P)$ is an approximation space of E. Its relational structure induced from E.
$$W^0=(W^0, N, R, G)$$
W^0 is called **Approximation World**.
(W^0, as a set, is the approximation space)

Accessibility Relations

For any given two observable worlds W^h, W^k, we can define a binary relation as follows: For each element x in W^h, x belongs to, at least, one of R-elementary set, say H_i, In this H_i, there is a

unique y in W^k. The set of such pairs (x, y)'s forms a binary relation that relates two possible worlds. It is an equivalence relation.

Rough Models

A Rough Model is a 6-tuple

    RM =(E, N, R, F, RO, W)

where (1) E is a set (of entities) with a partition; (2) N is a set of distinguished entities; (3) R is a set of relations; (4) F is a set of partial functions; (5) RO is a set of rough operators induced from the partition, i.e., RO={H}; (H= Upper approximation); (6) W is a set of possible worlds $W^h = (W^h, N^h, R^h, F^h)$;

Interpretations

In order to express the validity of formulas, we define an interpretation from the language to rough model. We assign each n--ary predicate to an n--ary relation in R^h; each partial function symbol to each n--ary partial function in F^h ; to each constant to a distinguished entity in N^h. The function γ which defines these assignments is called an interpretation to W^h .

4. Fuzzy Logic Control

First let us formulate the classical FLC in terms of our new mathematical formalism.

(1) A *theory:* The set of linguistic rules in classical FLC is expressed by a set of predicates of rough logic. Such a set is called a theory in formal logic; it consist of a formal system and a set of proper axioms [4]. The set of linguistic rules is a set of proper axioms for a rough logic system. The advantage is that any qualitative information that even includes those differential geometric property of the non-linear dynamic systems can be expressed by a set of predicates.

(2) *Interpretations:* It was called *f*uzzification in classical FLC. Typically this step was handled by linguistic variables; linguistic constants are replaced by associated membership functions. In our formulation, it is *an interpretation of a theory of rough logic. We will consider all possible interpretation; each interpretation is called possible fuzzy world.*

(3) *Defuzzification*: Using TVFI or Mandani's inference method, each possible world (a set of

fuzzy rules) determines a candidate (potential) control function [9]. We view this control function as a candidate (potential) "solution" of *unconstructed* system equations.

(4) *Verification and Validation*. In the first two steps, domain experts suggested the set of linguistic rules and the membership functions for each linguistic constant, to be precise, a *theory* and interpretations. These suggestions are based on their experience and intuitions, so the control function produced by step (3) has to be verified and validated by experiments. This is a requirement of mathematical modeling.

(5) If verification and validation of all candidate control functions fail, then the symbolic model must be wrong, so we need to restart from step (1) again. Step (4) and (5) is called tuning in classical FLC design.

5. Tublar Neighborhoods-Towards Lyapunov Stability

We will review some differential geometry of non-linear dynamic systems. If dynamic systems satisfy certain mild conditions, such as Lipschitz condition, solution trajectories are continuously and uniquely dependent on initial conditions [5, pp. 137]. In terms of geometry, there is a tubular neighborhood of integral submanifolds [11]. Tubular neighborhoods can be intuitively visualized as curved tubes of integral submanifolds. A curved tube is a subspace of E × T with tube pointing to the T direction, where E is a Euclidean Space and T is time axis;. Let S be a "solid circle" (disc) in E, then S × T is a cylinder in E × T.

Proposition A dynamic system is stable in Lyapunov sense, if there exists a cylinder that contains the tublar neighborhood [5].

Now we return to our problem. Let us assume ideally, we have found all control functions that are verified and validated. Such a family of all solutions of unconstructed system equations should form a tubular neighborhood.

In practices, how could we know we have found a tubular neighborhood of solutions? From real world phenomena or geometry, we should focus on verifying and validating solutions in the boundary (this can cut down the experiments).

Since tubular neighborhood should satisfy curve linear convexity properties. From these boundary set we will examine the following curvel inearity: Let us assume that the candidate control functions are F, F' are verified and validated. The we should try to verify and validate the curve linear sum, aF $\oplus$ bF', where a+b=1. If we can accomplish this, then we have a tubular neighborhood of "real" control functions.

First, we would like to remind readers that we model control phenomena directly. So we will assume experts are skillful enough to encode all these physical phenomena in *the qualitative model - a theory* of logic. Such information may even come from differential geometric analysis of the real world as practiced by hard computing camp.

5. Ship-Mounted Satellite Tracking Antenna Control

A robust satellite tracking antenna is designed to cope with the sensor imprecision and the highly noisy sea environment. The problem can be encoded by (1) a set of linguistic rules on two variables, longitude and altitude angles. They are represented by THETA and PHI. We also concluded that (2) two angles can varies only in closed and bounded ranges. Both information are expressible by rough logic. So the symbolic model is a *theory* of rough logic.

It is relatively easy to find that the fuzzy logic controller works for certain pairs of THETA and PHI. So we found those points which are in the boundary. Moreover, we found that the fuzzy logic controller work for the curve linear sum, AF $\oplus$ b.f.', where a+b=1, if F and F' work. So we establish that there is a tubular neighborhood of solutions. Moreover by the properties of (2). we know that the whole tubular neighborhood is contained in a cylinder S $\times$ T. By the proposition above. we no only show that fuzzy controller work. We also show that it is stable in Lyapunov sense.

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