Rough Data and Rough Data Law Recognition*

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Abstract

S-rough sets (singular rough sets) theory was proposed by improving Z.Pawlak rough sets theory. S-rough set has dynamic characteristics, since it is defined by R -element equivalence class $[x]$ which has dynamic characteristic. The concept of rough data was presented by employing S-rough sets, and rough data has dynamic characteristic, furthermore, the rough data law generation is given. Based on the concepts above, the generation theorem and the reversion theorem of rough data law, the recognition principle of rough data law, and the discernible theorem of rough data law were given, finally the applications of rough data law in risk investment system are presented.

Keywords: *one direction S-rough sets; dual of one direction S-rough sets; rough data; rough data law; law recognition.*

1. Introduction

In the year of 2002, ref.[1] improved Z.Pawlak rough sets[2], and proposed S-rough sets(singular rough sets), besides, refs.[3-7] gave several new discussions and applications about S-rough sets. S-rough set has dynamic characteristics (one direction dynamic characteristics, two direction dynamic characteristics), since it is defined by R -element equivalence class $[x]$ which has dynamic characteristic. S-rough sets have three forms[3-5,8,9]: one direction S-rough sets (one direction singular rough sets), dual of one direction S-rough sets(dual of one direction singular rough sets) and two direction S-rough sets(two direction singular rough sets). The results presented in this paper are achieved on the basis of one direction S-rough sets and dual of one direction S-rough sets. In one direction S-rough sets, R-element equivalence class $[x]$ possesses the characteristics as follows: if y_i is the characteristic value of $x_i \in [x]$, $y_i \in \mathrm{R}$, and R is real number set, then $[x]$ has a data set $y = \{y_1, y_2, \cdots, y_n\}$ correspondingly, where $[x] = \{x_1, x_2, \cdots, x_n\}$; data set y can generates polynomial law[10] $w(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$; if adding some element to $[x]$, then $[x]$ will generate $[x]^f = \{x_1, x_2, \cdots, x_m\}$, moreover $[x]^f \supseteq [x]$, and $[x]^f$ has data set $y^f = \{y_1, y_2, \cdots, y_m\}$ which can generate polynomial law[10] $w(x)^f = b_{m-1}x^{m-1} + b_{m-2}x^{m-2} + \cdots + b_1 x + b_0$, $m \geq n$; obviously, because some elements are added to $[x]$, the generated law $w(x)$ of $[x]$ becomes $w(x)^f$, and the curve shape of $w(x)$ is changed. In dual of one direction S-rough sets, R -element equivalence class $[x]$ possesses the characteristics as follows: if some elements are deleted from $[x]$, then $[x] = \{x_1, x_2, \cdots, x_n\}$ becomes $[x]^{\bar{f}} = \{x_1, x_2, \cdots, x_k\}$, $n \geq k$, moreover $[x] \supseteq [x]^{\bar{f}}$, $[x]^{\bar{f}}$ has data set $y^{\bar{f}} = \{y_1, y_2, \cdots, y_k\}$ which can generate polynomial law[10] $w(x)^{\bar{f}} = c_{k-1}x^{k-1} + c_{k-2}x^{k-2} + \cdots + c_1 x + c_0$, $n \geq k$; obviously, because some elements are deleted from $[x]$, the generated law $w(x)$ of $[x]$ becomes $w(x)^{\bar{f}}$, and the curve shape of $w(x)$ is changed. The characteristics which the R -element equivalence class $[x]$ of one direction S-rough sets or dual of one direction S-rough sets possesses are equal to the characteristics which the dynamic information system possesses. Given information system law $\mu(x) = \rho_{n-1}x^{n-1} + \rho_{n-2}x^{n-2} + \cdots + \rho_1 x + \rho_0$ of dynamic information system, the curve shape of system law $\mu(x)$ will change following the interference information elements intrude into or are deleted from information system. This paper transplants one direction S-rough sets and dual of one direction S-

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rough sets to the analysis and recognition of disturbances in information system, some important results and applications are presented, which indicate that the research of this paper is valuable.

One direction S-rough sets and dual of one direction S-rough sets are new tools for the research of dynamic information system. The intersection, penetration and communion between one direction S-rough sets or dual of one direction S-rough sets and dynamic information system can generates a new research direction for the dynamic information recognition.

For the convenience of discussion and accepting the results of this paper easily while keeping the contents integral, one direction S-rough sets and dual of one direction S-rough sets are simply introduced to section 2 as the theory basis of the discussion of this paper.

2. S-rough sets [1, 3-9]

Assumptions: U is the finite element universe of discourse; $X \subset U$ is a finite element set, R is the element equivalence relation on U; $[x]$ is a R-element equivalence class; $f \in F$ is an element transfer, and $F=\{f_1, f_2, \cdots, f_m\}$ is element transfer family, where the characteristic of $f \in F$ is that: $u \in U$, $u \overline{\in} X$, $f(u)=x \in X$; $\overline{f} \in \overline{F}$ is an element transfer, and $\overline{F}=\{\overline{f}_1, \overline{f}_2, \cdots, \overline{f}_n\}$ is element transfer family, where the characteristic of $\overline{f} \in \overline{F}$ is that: $x \in X$, $\overline{f}(x)=u \overline{\in} X$.

One direction S-rough sets

Given element set $X \subset U$, X° is called one direction S-set (one direction singular sets) of X, if

$$X^{\circ}=X \bigcup \{u \mid u \in U, u \overline{\in} X, f(u)=x \in X\}. \tag{1}$$

X^{f} is called f-extension of X, if

$$X^{f}=\{u \mid u \in U, u \overline{\in} X, f(u)=x \in X\}. \tag{2}$$

$(R,F)_{\circ}(X^{\circ})$ and $(R,F)^{\circ}(X^{\circ})$ are called lower approximation and upper approximation of $X^{\circ} \subset U$ respectively, if

$$(R,F)_{\circ}(X^{\circ})=\bigcup[x]=\{x \mid x \in U, [x] \subseteq X^{\circ}\}, \tag{3}$$

$$(R,F)^{\circ}(X^{\circ})=\bigcup[x]=\{x \mid x \in U, [x] \bigcap X^{\circ} \neq \phi\}. \tag{4}$$

The set pair composed of $(R,F)_{\circ}(X^{\circ})$ and $(R,F)^{\circ}(X^{\circ})$ is called one direction S-rough sets of $X^{\circ} \subset U$, moreover

$$((R,F)_{\circ}(X^{\circ}), (R,F)^{\circ}(X^{\circ})). \tag{5}$$

$B_{nR}(X^{\circ})$ is called R-boundary of $X^{\circ} \subset U$, if

$$B_{nR}(X^{\circ})=(R,F)^{\circ}(X^{\circ})-(R,F)_{\circ}(X^{\circ}). \tag{6}$$

$A_s(X^{\circ})$ is called assistant set of one direction S-rough sets, if

$$A_s(X^{\circ})=\{x \mid u \in U, u \overline{\in} X, f(u)=x \tilde{\in} X\}. \tag{7}$$

Where "$\tilde{\in}$" is a special symbol, its meaning can be seen in [3-5,8,9].

Dual of one direction S-rough sets

Given one direction S-set $X^{\circ} \subset U$, X' is called dual of one direction S-set of X°, if

$$X'=X-\{x \mid x \in X, \overline{f}(x)=u \overline{\in} X\}. \tag{8}$$

$X^{\overline{f}}$ is called $\overline{f}$-contraction of X, if

$$X^{\overline{f}}=\{x \mid x \in X, \overline{f}(x)=u \overline{\in} X\}. \tag{9}$$

$(R,\overline{F})_{\circ}(X')$ and $(R,\overline{F})^{\circ}(X')$ are lower approximation and upper approximation of $X' \subset U$ respectively, if

$$(R,\overline{F})_{\circ}(X')=\bigcup[x]=\{x \mid x \in U, [x] \subseteq X'\}, \tag{10}$$

$$(R,\overline{F})^{\circ}(X')=\bigcup[x]=\{x \mid x \in U, [x] \bigcap X' \neq \phi\}. \tag{11}$$

The set pair composed of $(R,\overline{F})_{\circ}(X')$ and $(R,\overline{F})^{\circ}(X')$ is called dual of one direction S-rough sets of $X' \subset U$, moreover

$$((R,\overline{F})_{\circ}(X'), (R,\overline{F})^{\circ}(X')). \tag{12}$$

$B_{nR}(X')$ is called R-boundary of $X' \subset U$, if

$$B_{nR}(X')=(R,\overline{F})^{\circ}(X')-(R,\overline{F})_{\circ}(X'). \tag{13}$$

$A_s(X')$ is called assistant sets of dual of one direction S-rough sets, if

$$A_s(X')=\{x \mid x \in X, \overline{f}(x)=u \underset{\sim}{\in} X\}. \tag{14}$$

Where "$\underset{\sim}{\in}$" is a special symbol, its meaning can be seen in [3-5, 8, 9]

More concepts about one direction S-rough sets, dual of one direction S-rough sets and two direction S-rough sets can be seen in [3-5, 8, 9].

Based on the expressions (1)-(14) and ref.[3-5, 8, 9], following theorems can be obtained.

Theorem 1 (Relation theorem between one direction S-rough sets and Z.Pawlak rough sets) If $F=\phi$, then one direction S-rough sets and Z.Pawlak rough sets fufil

$$((R,F)_{\circ}(X^{\circ}), (R,F)^{\circ}(X^{\circ}))_{F=\phi}=(R_{-}(X), R^{-}(X)). \tag{15}$$

Where $(R_{-}(X), R^{-}(X))$ is Z.Pawlak rough set [2], $X \subset U$; $R_{-}(X)$ is lower approximation of X, $R^{-}(X)$ is upper approximation of X, and R is equivalence relation.

In fact, if $F=\phi$, then $\{u \mid u \in U, u \overline{\in} X, f(u)=x \in X\}=\phi$ in expression (1), and $X^{\circ}=X$, furthermore $(R,F)_{\circ}(X^{\circ})=\bigcup[x]=\{x \mid x \in U, [x] \subseteq X^{\circ}\}=\{x \in U, [x] \subseteq X\}=\bigcup[x]=R_{-}(X)$ in expression (3), while $(R,F)^{\circ}(X^{\circ})=\bigcup[x]=\{x \mid x \in U, [x] \bigcap X^{\circ} \neq \phi\}=\{x \mid x \in U, [x] \bigcap X \neq \phi\}=\bigcup[x]=R^{-}(X)$ in expression (4).

So there is $((R,F)_{\circ}(X^{\circ}),(R,F)^{\circ}(X^{\circ}))_{F=\phi}=(R_{-}(X), R^{-}(X))$.

Theorem 2 (Relation theorem between dual of one direction S-rough sets and Z.Pawlak rough sets) If $\overline{F}=\phi$, then dual of one direction S-rough sets and Z.Pawlak rough sets fulfil

$$((R,F)_{\circ}(X'),(R,F)^{\circ}(X'))_{\overline{F}=\phi}=(R_{-}(X),R^{-}(X)) . \quad (16)$$

On the basis of theorems 1-2, propositions 1-4 can be proved.

Proposition 1 On the dynamic-static condition, one direction S-rough sets is the general form of Z.Pawlak rough sets, while Z.Pawlak rough sets is the special case of one direction S-rough sets.

Proposition 2 On the dynamic- static condition, dual of one direction S-rough sets is the general form of Z.Pawlak rough sets, while Z.Pawlak is the special case of dual of one direction S-rough sets.

Proposition 3 Assistant set $A_s(X^{\circ})$ in Z.Pawlak rough sets fulfils $A_s(X^{\circ})=\phi$.

Proposition 4 Assistant set $A_s(X')$ in Z.Pawlak rough sets fulfils $A_s(X')=\phi$.

Proposition 5 The attribute set $\{(\alpha^{\circ})_{-},(\alpha^{\circ})^{-}\}$ of one direction S-rough set and the attribute set $\{\alpha_{-},\alpha^{-}\}$ of Z.Pawlak rough set fulfil

$$\{(\alpha^{\circ})_{-},(\alpha^{\circ})^{-}\}\subseteq\{\alpha_{-},\alpha^{-}\} . \quad (17)$$

Where $(\alpha^{\circ})_{-}$ and $(\alpha^{\circ})^{-}$ are the attribute sets of $(R,F)_{\circ}(X^{\circ})$ and $(R,F)^{\circ}(X^{\circ})$ respectively, α_{-} and α^{-} are the attribute sets of $R_{-}(X)$ and $R^{-}(X)$ respectively. Expression (17) denotes that $(\alpha^{\circ})_{-}\subseteq\alpha_{-}$ and $(\alpha^{\circ})^{-}\subseteq\alpha^{-}$.

Proposition 6 The attribute set $\{(\alpha')_{-},(\alpha')^{-}\}$ of dual of one direction S-rough set and the attribute set $\{\alpha_{-},\alpha^{-}\}$ of Z.Pawlak rough set fulfil

$$\{\alpha_{-},\alpha^{-}\}\subseteq\{(\alpha')_{-},(\alpha')^{-}\} . \quad (18)$$

Where $(\alpha')_{-}$ and $(\alpha')^{-}$ are the attribute sets of $(R,\overline{F})_{\circ}(X')$ and $(R,\overline{F})^{\circ}(X')$ respectively. Expression (18) denotes that $\alpha_{-}\subseteq(\alpha')_{-}$ and $\alpha^{-}\subseteq(\alpha')^{-}$.

On the basis of Expressions (1)-(14), theorems 1-2 and propositions 1-6, rough data and rough data law generation are presented in section **3.**

3. Rough data and rough data law generation

Assumptions: $[x^{\circ}]_{-}=(R,F)_{\circ}(X^{\circ})=\bigcup[x]=\{x_1,x_2,\cdots,x_m\}$, $[x^{\circ}]^{-}=(R,F)^{\circ}(X^{\circ})=\bigcup[x]=\{x_1,x_2,\cdots,x_n\}$, and $[x^{\circ}]_{-}\subseteq[x^{\circ}]^{-}$, $m\le n$; $[x']_{-}=(R,\overline{F})_{\circ}(X')=\bigcup[x]=\{x_1,x_2,\cdots,x_t\}$, $[x']^{-}=(R,\overline{F})^{\circ}(X')=\bigcup[x]=\{x_1,x_2,\cdots,x_{\lambda}\}$, $t\le\lambda$. In the discussions of sections **3** and **4,** elements $x_i\in[x^{\circ}]_{-}$ and $x_j\in[x^{\circ}]^{-}$ (or $x_i\in[x']_{-}$ and $x_j\in[x']^{-}$, or $x_k\in[x]_{-}$ and $x_{\lambda}\in[x]^{-}$) are the elements with single characteristic value, or x_i has a single characteristic value $y_i\in\mathrm{R}$, and R is the real number set.

- Given R-element equivalence class $[x]=\{x_1,x_2,\cdots,x_r\}$, $y_i\in\mathrm{R}$ is the characteristic value of $x_i\in[x]$, $i=1,2,\cdots,r$, and R is real number set; y is data set generated by $[x]$, or the data set of $[x]$ for short, moreover

$$y=\{y_1,y_2,\cdots,y_r\} . \quad (19)$$

The data points $(x_1,y_1),(x_2,y_2),\cdots,(x_r,y_r)$ structured by expression (19) can generate polynomial $p(x)$, and $p(x)$ is called the data law generated by $[x]$, moreover

$$p(x)=\sum_{j=1}^{r}y_j\prod_{\substack{i=1\\ i\ne j}}^{r}\frac{x-x_i}{x_j-x_i} = a_{r-1}x^{r-1}+a_{r-2}x^{r-2}+\cdots+a_1x+a_0 . \quad (20)$$

- Given $[x^{\circ}]_{-}=\{x_1,x_2,\cdots,x_m\}$ and $[x^{\circ}]^{-}=\{x_1,x_2,\cdots,x_n\}$. $(y^{\circ})_{-}$ and $(y^{\circ})^{-}$ are the data sets of $[x^{\circ}]_{-}$ and $[x^{\circ}]^{-}$ respectively, moreover

$$(y^{\circ})_{-}=\{y_1,y_2,\cdots,y_m\} , \quad (21)$$

$$(y^{\circ})^{-}=\{y_1,y_2,\cdots,y_n\} . \quad (22)$$

The set pair composed of $(y^{\circ})_{-}$ and $(y^{\circ})^{-}$ is called the rough data generated by one direction S-rough sets, moreover

$$((y^{\circ})_{-},(y^{\circ})^{-}) . \quad (23)$$

Similar to expressions (19) and (20), expressions (21) and (22) can generate the polynomials $p(x)_{-}$ and $p(x)^{-}$ respectively, moreover

$$p(x)_{-}=b_{m-1}x^{m-1}+b_{m-2}x^{m-2}+\cdots+b_1x+b_0 , \quad (24)$$

$$p(x)^{-}=c_{n-1}x^{n-1}+c_{n-2}x^{n-2}+\cdots+c_1x+c_0 . \quad (25)$$

$p(x)_{-}$ and $p(x)^{-}$ are called the data laws generated by lower approximation $(R,F)_{\circ}(X^{\circ})$ and upper approximation $(R,F)^{\circ}(X^{\circ})$ respectively.

- The data law pair composed of $p(x)_{-}$ and $p(x)^{-}$ is called the rough data law generated by one direction S-rough sets, moreover

$$(p(x)_{-},p(x)^{-}) , \quad (26)$$

$p(x)_{-}$ and $p(x)^{-}$ are called lower law and upper law of $(p(x)_{-},p(x)^{-})$ respectively.

- Given $[x']_{-}=\{x_1,x_2,\cdots,x_t\}$ and $[x']^{-}=\{x_1,x_2,\cdots,x_{\lambda}\}$. $(y')_{-}$ and $(y')^{-}$ are the data sets of $[x']_{-}$ and $[x']^{-}$ respectively, moreover

$$(y')_{-}=\{y_1,y_2,\cdots,y_t\} , \quad (27)$$

$$(y')^{-}=\{y_1,y_2,\cdots,y_{\lambda}\} . \quad (28)$$

The data pair composed of $(y')_{-}$ and $(y')^{-}$ is called the rough data generated by dual of one direction S-rough sets, moreover

$$((y')_-, (y')^-)\,. \quad (29)$$

Similar to expressions (19) and (20), expressions (27) and (28) can generate the polynomials $q(x)_-$ and $q(x)^-$ respectively, moreover

$$q(x)_- = d_{t-1}x^{t-1} + d_{t-2}x^{t-2} + \cdots + d_1 x + d_0\,, \quad (30)$$

$$q(x)^- = e_{\lambda-1}x^{\lambda-1} + e_{\lambda-2}x^{\lambda-2} + \cdots + e_1 x + e_0\,. \quad (31)$$

$q(x)_-$ and $q(x)^-$ are the data laws generated by lower approximation $(R,\overline{F})_\circ(X')$ and upper approximation $(R,\overline{F})^\circ(X')$ respectively.

● The data law pair composed of $q(x)_-$ and $q(x)^-$ is called the rough data law generated by dual of one direction S-rough sets, moreover

$$(q(x)_-, q(x)^-)\,, \quad (32)$$

$q(x)_-$ and $q(x)^-$ are called lower law and upper law of $(q(x)_-, q(x)^-)$ respectively.

● $\pi(x)_-$ and $\pi(x)^-$ are called the data laws generated by lower approximation $R_-(X)$ and upper approximation $R^-(X)$ respectively, moreover

$$\pi(x)_- = \varepsilon_{p-1}x^{p-1} + \varepsilon_{p-2}x^{p-2} + \cdots + \varepsilon_1 x + \varepsilon_0\,, \quad (33)$$

$$\pi(x)^- = \mu_{q-1}x^{q-1} + \mu_{q-2}x^{q-2} + \cdots + \mu_1 x + \mu_0\,. \quad (34)$$

● The data law pair composed of $\pi(x)_-$ and $\pi(x)^-$ is called the rough data law generated by Z.Pawlak rough sets, moreover

$$(\pi(x)_-, \pi(x)^-)\,, \quad (35)$$

$\pi(x)_-$ and $\pi(x)^-$ are called lower law and upper law of $(\pi(x)_-, \pi(x)^-)$ respectively.

Where $[x]_- = R_-(X) = \bigcup[x] = \{x_1, x_2, \cdots, x_p\}$, and $y_- = \{y_1, y_2, \cdots, y_p\}$ is the data set of $[x]_-$; $[x]^- = R^-(X) = \bigcup[x] = \{x_1, x_2, \cdots, x_q\}$, and $y^- = \{y_1, y_2, \cdots, y_q\}$ is the data set of $[x]^-$; (y_-, y^-) is the rough data generated by Z.Pawlak rough sets.

Based on expressions (19)-(35), we can get:

Proposition 7 There must be an unique rough data law $(p(x)_-, p(x)^-)$ generated by one direction S-rough set.

Proposition 8 There is must be an unique rough data law $(q(x)_-, q(x)^-)$ generated by dual of one direction S-rough set.

Proposition 9 There must be an unique rough data law $(\pi(x)_-, \pi(x)^-)$ generated by Z.Pawlak rough set.

The proofs of propositions 7-9 can be obtained by the uniqueness of Lagrange interpolation polynomial, and the proofs are omitted.

Based on expressions (19)-(35), propositions 7-9 can be obtained.

Theorem 3 (F-reversion theorem of rough data law $(p(x)_-, p(x)^-)$) If $\{(\alpha^\circ)_-, (\alpha^\circ)^-\}$ is the attribute set of $(p(x)_-, p(x)^-)$, and $\{\alpha_-, \alpha^-\}$ is the attribute set of $(\pi(x)_-, \pi(x)^-)$, then the necessary and sufficient condition for $(p(x)_-, p(x)^-)$ is reverted to $(\pi(x)_-, \pi(x)^-)$ or

$$(p(x)_-, p(x)^-) = (\pi(x)_-, \pi(x)^-) \quad (36)$$

is that

$$(\alpha^\circ)_- \bigcup_{\beta_i \in V, \beta_i \overline{\in} (\alpha^\circ)_-} \{f(\beta_i) = \alpha_i\} = \alpha_-\,, \quad (37)$$

$$(\alpha^\circ)^- \bigcup_{\beta_j \in V, \beta_j \overline{\in} (\alpha^\circ)^-} \{f(\beta_j) = \alpha_j\} = \alpha^-\,. \quad (38)$$

Theorem 4 ($\overline{F}$-reversion theorem of rough data law $(q(x)_-, q(x)^-)$) If $\{(\alpha')_-, (\alpha')^-\}$ is the attribute set of $(q(x)_-, q(x)^-)$, and $\{\alpha_-, \alpha^-\}$ is the attribute set of $(\pi(x)_-, \pi(x)^-)$, then the necessary and sufficient condition for $(q(x)_-, q(x)^-)$ is reverted to $(\pi(x)_-, \pi(x)^-)$ or

$$(q(x)_-, q(x)^-) = (\pi(x)_-, \pi(x)^-) \quad (39)$$

is that

$$(\alpha')_- - \{\overline{f}(\alpha_i) = \beta_i\}_{\alpha_i \overline{\in} (\alpha')_-} = \alpha_-\,, \quad (40)$$

$$(\alpha')^- - \{\overline{f}(\alpha_j) = \beta_j\}_{\alpha_j \overline{\in} (\alpha')^-} = \alpha^-\,. \quad (41)$$

The proof of theorems 3 and 4 is direct and here are omitted.

By employing the concepts in expressions (19)-(41), section 4 gives the applications of rough data law in dynamic information system.

4. The applications of rough data law in risk investment law recognition

● σ_- and σ^- are called the discernibility metrics of $p(x)_-$ with respect to $\pi(x)_-$ and $p(x)^-$ with respect to $\pi(x)^-$ respectively, if

$$\sigma_- = \|(y^\circ)_-\| / \|y_-\|\,, \quad (42)$$

$$\sigma^- = \|(y^\circ)^-\| / \|y^-\|\,. \quad (43)$$

The number pair composed of σ_- and σ^- is called the discernibility metric of $(p(x)_-, p(x)^-)$ with respect to $(\pi(x)_-, \pi(x)^-)$, moreover

$$(\sigma_-, \sigma^-)\,. \quad (44)$$

Where $\|(y^\circ)_-\|$ and $\|(y^\circ)^-\|$ are the 2-norms of vectors $(y^\circ)_- = (y_1, y_2, \cdots, y_m)^T$ and $(y^\circ)^- = (y_1, y_2, \cdots, y_n)^T$ generated by $(y^\circ)_- = \{y_1, y_2, \cdots, y_m\}$ and $(y^\circ)^- = \{y_1, y_2, \cdots, y_n\}$ respectively, where $\|(y^\circ)_-\| = (y_1^2 + y_2^2 + \cdots + y_m^2)^{1/2}$ and $\|(y^\circ)^-\| = (y_1^2 + y_2^2 + \cdots + y_n^2)^{1/2}$. $\|y_-\|$ and $\|y^-\|$ are the 2-norms of vectors $y_- = (y_1, y_2, \cdots, y_p)^T$ and $y^- = (y_1, y_2, \cdots, y_q)^T$ generated by $y_- = \{y_1, y_2, \cdots, y_p\}$ and $y^- = \{y_1, y_2, \cdots, y_q\}$ respectively, where $\| y_- \| = (y_1{}^2 + y_2{}^2 + \cdots + y_p{}^2)^{1/2}$ and $\| y^- \| = (y_1{}^2 + y_2{}^2 + \cdots + y_q{}^2)^{1/2}$.

● η_- and η^- are called the discernibility metrics of $q(x)_-$ with respect to $\pi(x)_-$ and $q(x)^-$ with respect to $\pi(x)^-$ respectively, if

$$\eta_- = \| (y')_- \| / \| y_- \| , \tag{45}$$

$$\eta^- = \| (y')^- \| / \| y^- \| . \tag{46}$$

The number set composed of η_- and η^- is called the discernibility metric of $(q(x)_-, q(x)^-)$ with respect to $(\pi(x)_-, \pi(x)^-)$, moreover

$$(\eta_-, \eta^-) . \tag{47}$$

Where $\| (y')_- \|$ and $\| (y')^- \|$ are the 2-norms of vectors $(y')_- = (y_1, y_2, \cdots, y_t)^T$ and $(y')^- = (y_1, y_2, \cdots, y_\lambda)^T$ generated by $(y')_- = \{y_1, y_2, \cdots, y_t\}$ and $(y')^- = \{y_1, y_2, \cdots, y_\lambda\}$ respectively, where $\| (y')_- \| = (y_1{}^2 + y_2{}^2 + \cdots + y_t{}^2)^{1/2}$ and $\| (y')^- \| = (y_1 + y_2 + \cdots + y_\lambda)^{1/2}$.

By employing expressions (42)-(47), we can obtained that:

The F-recognition principle of rough data law

If the discernibility metrics of $(p(x)_{-,k}, p(x)^-_k)$ with respect to $(\pi(x)_-, \pi(x)^-)$ fulfils

$$\sigma_{-,k} > 1 ,\ \sigma^-_k > 1 , \tag{48}$$

then there must be

$$(p(x)_{-,k}, p(x)^-_k) \neq (\pi(x)_-, \pi(x)^-) . \tag{49}$$

Where expression (49) denotes that $p(x)_{-,k} \neq \pi(x)_-$ and $p(x)^-_k \neq \pi(x)^-$, $k \in (1, 2, \cdots, t)$.

The $\bar{F}$-recognition principle of rough data law

If the discernibility metrics of $(q(x)_{-,k}, q(x)^-_k)$ with respect to $(\pi(x)_-, \pi(x)^-)$ fulfils

$$\eta_{-,k} < 1 ,\ \eta^-_k < 1 , \tag{50}$$

then there must be

$$(q(x)_{-,k}, q(x)^-_k) \neq (\pi(x)_-, \pi(x)^-) . \tag{51}$$

Where expression (51) denotes that $q(x)_{-,k} \neq \pi(x)_-$ and $q(x)^-_k \neq \pi(x)^-$, $k \in (1, 2, \cdots, t)$.

By employing expressions (42)-(51), the following theorems can be got directly.

Theorem 5 (the discernible theorem of $(p(x)_-, p(x)^-)$ and $(\pi(x)_-, \pi(x)^-)$) Rough data law $(p(x)_-, p(x)^-)$ with respect to rough data law $(\pi(x)_-, \pi(x)^-)$ fulfil

$$\underset{\sigma_-, \sigma^- > 1}{\mathrm{DIS}} ((p(x)_-, p(x)^-), (\pi(x)_-, \pi(x)^-)) . \tag{52}$$

Theorem 6 (the discernible theorem of $(q(x)_-, q(x)^-)$ and $(\pi(x)_-, \pi(x)^-)$) Rough data law $(q(x)_-, q(x)^-)$ with respect to rough data law $(\pi(x)_-, \pi(x)^-)$ fulfil

$$\underset{\eta_-, \eta^- < 1}{\mathrm{DIS}} ((q(x)_-, q(x)^-), (\pi(x)_-, \pi(x)^-)) . \tag{53}$$

Where DIS=discernibilty.

Corollary 1 If $(p(x)_{-,*}, p(x)^-_*)$ is the F-reversion of $(\pi(x)_-, \pi(x)^-)$, then

$$\mathrm{IND}((p(x)_{-,*}, p(x)^-_*), (\pi(x)_-, \pi(x)^-)) . \tag{54}$$

Corollary 2 If $(q(x)_{-,*}, q(x)^-_*)$ is the $\bar{F}$-reversion of $(\pi(x)_-, \pi(x)^-)$, then

$$\mathrm{IND}((q(x)_{-,*}, q(x)^-_*), (\pi(x)_-, \pi(x)^-)) . \tag{55}$$

Where IND=indiscernibility.

By employing the expressions (42)-(55) in sections **3** and **4**, the applications of rough data law in risk investment law recognition is given as follows. For briefness and without losing universality, the example just gives the data law $p(x)_-$, and the discernibility-recognition of $q(x)_-$ with respect to data law $\pi(x)_-$.

The example in this section comes from the investment profit data of China T • T company during January to July of the years 1997,1998 and 1999. T • T company has 5 subsidiary companies u_1, u_2, u_3, u_4, u_5, and $\forall u_i$ has profit sequence $y_i = (y_{i,1}, y_{i,2}, y_{i,3}, y_{i,4}, y_{i,5}, y_{i,6}, y_{i,7})$, $\forall y_{i,k} \in$ R, $i = 1, 2, \cdots, 5$; R is the real number set. T • T company has profit sequence y, moreover

$$y = (y_1, y_2, y_3, y_4, y_5, y_6, y_7) . \tag{56}$$

Where $y_k = \sum_{i=1}^{5} y_{i,k}, y_k \in y, k = 1, 2, \cdots, 7; y_{i,k} \in y_i, i = 1, 2, \cdots, 5$.

Fig.1 gives the investment profit law distribution of T • T company during January to July of the years 1997, 1998 and 1999.

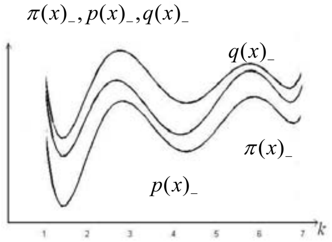

Fig.1. $\pi(x)_-$ is the profit distribution during January to July of the years 1997; $p(x)_-$ is the profit distribution during January to July of the years 1998, the investment risk is great (some risk factors intruded into the investment system); $q(x)_-$ is the profit distribution during January to July of the years 1999, the investment risk is small in this year (many risk factors disappeared from the investment system).

The risk investment law recognition of T·T company

To make the profit laws $p(x)_-$, $q(x)_-$ and $\pi(x)_-$ in Fig.1 discrete respectively, Tab.1 can be obtained.

Tab.1. The discrete distributions of $p(x)_-$, $q(x)_-$ and $\pi(x)_-$.

	1	2	3	4	5	6	7
$\pi(x)_-$	1.5207	1.2938	1.4703	1.1842	1.3768	1.5576	1.4938
$p(x)_-$	1.2679	1.0362	1.3714	1.0944	1.2074	1.4083	1.3092
$q(x)_-$	1.5996	1.4321	1.6795	1.3992	1.4836	1.6019	1.5893

The data in Tab.1 has been standardized, and which won't affect the analysis of the result.

Fig.1 and Tab.1 indicate the fact that the investment profit law $\pi(x)_-$ changes following the investment environment changes. If the investment risk factors (attribute) increase, then the profit law $\pi(x)_-$ will drop, and $\pi(x)_-$ becomes $p(x)_-$. If some investment risk factors (attribute) disappear, then the profit law $\pi(x)_-$ will rise, and $\pi(x)_-$ becomes $q(x)_-$. Every investor may encounter this fact.

By using the $\overline{F}$-recognition principle of rough data law, the data of $p(x)_-$ and $\pi(x)_-$ in Tab.1, and expression (45), there is

$$\sigma_- = \|(y')_-\| / \|y_-\| = 11.5782/14.1036 = 0.8209 .$$

so the discernibility metric of $p(x)_-$ with respect to $\pi(x)_-$ is 0.8209, which tells the investors that the reason for the profit law $\pi(x)_-$ dropping is that the investment risk factors (attribute) increase, and $p(x)_-$ with respect to $\pi(x)_-$ fulfils $\mathrm{DIS}(p(x)_-, \pi(x)_-)$.

5. Discussion

S-rough sets (singular rough sets) have more advantages than Z.Pawlak rough sets, and S-rough sets can be applied to much more application research fields for S-rough sets has dynamic characteristics. By employing the dynamic characteristics of S-rough sets, the applications of risk investment law recognition are given. Because risk investment system and its profit law has dynamic characteristic, we can say that S-rough sets theory is a new mathematical tool for dynamic information recognition researches.

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