NEW RESEARCH ON FUZZY ROUGH SETS

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Abstract:

For probing into further the relations of the general algebraic theories and fuzzy rough sets theories, this paper presents several new theories and definitions on fuzzy rough sets including half-group, group, subgroup, ring, subring, ideal, field. These new algebraic theories studied on fuzzy rough sets will break through the boundary of deterministic attribute space in general algebraic theories so that we may discuss and solve some fuzzy or uncertain problems on approximate space of fuzzy rough sets. Thus, the new theories development in this paper leads to advancements in general algebra and in turn can form the theoretic base for further applications of fuzzy rough sets. Advancements in general algebra through the study of algebraic theories on fuzzy rough sets are not only applicable to fuzzy rough sets fields, but also are of important theoretical value to modern algebra.

Keywords:

Fuzzy rough sets; pseudo-complement; group; ring; field

1. Introduction

In Pawlak's [1] rough sets (RS) model and subsequently the algebraic approach to rough sets was given by Iwinski [2], any classical set A in universe U cannot be accurately described with the knowledge of knowledge base (U,R), which is in turn defined by the upper approximation and lower approximation of set A in (U,R). In real life, however, knowledge and concept of which people often are involved with are fuzzy and ambiguous, i.e., the set A in U is a fuzzy set (FS) [3][6]. Pawlak provided the comparison between FS and RS and showed the difference between the two [4]. The present question is: How is the set A described with the knowledge of the pair (U,R) that is hit? The fuzzy rough sets (FRS) model was introduced to answer such question.

In previous works of [5][7-17] concepts of which inclusion, be equal to, operation and operation law of intersection, union and complement, some properties of upper approximation and lower approximation were introduced. However, the algebraic theories of FRS were not introduced all along. In this paper, we will discuss some algebraic theories of FRS. Based on operation laws and theories of group and ring in modern algebra, and theories on FS and RS again, we introduce new algebraic systems on FRS. The algebraic systems include half-group, group, subgroup, ring, subring, ideal, field on FRS are provided and discussed mainly. Thus, by studying the algebraic theories on FRS, some new theories and new achievements in general algebraic theories can be not only adapted to FRS fields, but also these researches will impulse further development of general algebraic theories and FRS theories and enrich content of modern algebra and FRS. It will be a theoretic base for further applications of FRS, such as, which it enables FRS by its algebraic systems to complete complex pattern recognition tasks in real-time. Finally, some problems on FRS to be solved and development trends are discussed.

2. Fuzzy Rough Sets (FRS)

[5] gives one definition of FRS. Here we give a new definition of FRS.

Definition 2.1 Assume $L=[0,1]$ and let $\{U,R\}$ be a given approximate space. Let $A=\langle A_L, A_U\rangle$ with $A_L \subseteq A_U$ be a rough set on $\{U,R\}$. If $A=\langle A_L, A_U\rangle$ is also a fuzzy set in U, i.e., $A=\langle A_L, A_U\rangle$ in U is characterized by the mappings $A:U\to L$, A_L : $U\to L$ and $A_U:U\to L$, the set A is called fuzzy rough set in U .

According to $A_L \subseteq A \subseteq A_U$, it implies $A_L(x)\le A(x)\le A_U(x)$ for all $x\in U$.

All FS in U are represented as $F(U)$, i.e.,

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$F(U)=\{A|A:U\to L\}$. The same, all FRS in U are represented as $FR(U)$.

Definitions of inclusion, intersection, union and complement of FRS have been provided in [5]. See [5].

Definition 2.2 Let $\{U,R\}$ be a given approximate space. For any two FRS $A=\langle A_L,A_U\rangle$ and $B=\langle B_L,B_U\rangle$ in U, we define $A_L=B_L$ and $A_U=B_U$ iff $A_L(x)=B_L(x)$ and $A_U(x)=B_U(x)$ for each $x\in U$. Then A and B are called fuzzy roughly equal and denoted by $A\approx B$.

Obviously, $\approx$ is an equivalence relation in U and equivalence class of $\approx$ are called FRS. The equivalence class $\approx$ of FRS A is labeled as $[A]_\approx$. All kinds of FRS $[A]_\approx$ are represent as R^F, i.e.,

$$R^F=\left\{[A]_\approx \middle| A=\langle A_L,A_U\rangle, A_L:U\to L, A_U:U\to L\right\}$$

Similarly, according to the definitions of inclusion in [5], define $A_L\subseteq B_L$ and $A_U\subseteq B_U$ iff $A_L(x)\le B_L(x)$ and $A_U(x)\le B_U(x)$ for each $x\in U$. Then A and B are called fuzzy rough inclusion and label $A\subseteq B$.

3. The New Algebraic Systems on FRS

Since FS do not satisfy the complementary law, i.e., $(F(U),\bigcup,\bigcap,c)$ is a soft algebra instead of the Boolean algebra, FRS do not satisfied either complementary law, i.e., every one have not complementary one in $(R^F,\bigcup,\bigcap,c)$, but we can define a pseudo-complement $*$ for every one in $(R^F,\bigcup,\bigcap,*)$ so that make it satisfy a pseudo complementary law.

Definition 3.1 Let L be a given set. The pseudo–complement of $x\in L$ is an element $x^*\in L$ defined by: $x^*=\sup\{a\in L:a\wedge x=0\}$.

In particular, the pseudo–complement of $A\in FR(U)$ is an element $A^*\in FR(U)$ defined by: for $B\in FR(U)$, there is $B\subseteq A^*$ while $A^*\bigcap A=\phi$ and $B\bigcap A=\phi$.

Since the fuzzy membership degree is between 0 and 1 according to [3], here we assume $L=[0,1]$. Where $\wedge$ is a 'and' operator that includes the min operator and infimum, etc. The standard minimum operation $\wedge$ is the greatest t–norm.

Definition 3.2 Let $\{U,R\}$ be a given approximate space. Assume $FR(U)$ be all FRS in U. For any two FRS $A=\langle A_L,A_U\rangle$ and $B=\langle B_L,B_U\rangle$ in $FR(U)$, define a pair of single maps $\langle \underline{S},\overline{S}\rangle$ on $FR(U)$, i.e., $\underline{S},\overline{S}$: $FR(U)\to FR(U)$, let them respectively satisfy $\bigcap_{t\in T}A^t$ is an FRS equivalence class of $\bigcap_{t\in T}A_L^t\bigcup\underline{S}\left(\bigcap_{t\in T}A_U^t\right)$, and $\bigcup_{t\in T}A^t$ is an FRS equivalence class of $\overline{S}\left(\bigcup_{t\in T}A_L^t\right)\bigcap\left(\bigcup_{t\in T}A_U^t\right)$. For instance, $\underline{S}(A_U\cap B_U)=A\cap B$ and $\overline{S}(A_L\cup B_L)=A\cup B$. Then, there are:

$$\bigcap_{t\in T}A^t=\left[\bigcap_{t\in T}A_L^t\bigcup\underline{S}\left(\bigcap_{t\in T}A_U^t\right)\right]_\approx \overset{\Delta}{=}\left\langle\bigcap_{t\in T}A_L^t,\bigcap_{t\in T}A_U^t\right\rangle$$

$$\Leftrightarrow \bigwedge_{t\in T}A^t(x)=\left\langle\bigwedge_{t\in T}A_L^t(x),\bigwedge_{t\in T}A_U^t(x)\right\rangle$$

$$\bigcup_{t\in T}A^t=\left[\overline{S}\left(\bigcup_{t\in T}A_L^t\right)\bigcap\left(\bigcup_{t\in T}A_U^t\right)\right]_\approx \overset{\Delta}{=}\left\langle\bigcup_{t\in T}A_L^t,\bigcup_{t\in T}A_U^t\right\rangle$$

$$\Leftrightarrow \bigvee_{t\in T}A^t(x)=\left\langle\bigvee_{t\in T}A_L^t(x),\bigvee_{t\in T}A_U^t(x)\right\rangle$$

Where T is an index set, and $\cup$, $\cap$ stand for the standard fuzzy union and intersection. In particular, $[A_L\bigcup\underline{S}(A_U)]_\approx=A$ and $\left[\overline{S}(A_L)\bigcap A_U\right]_\approx=A$ are true for any $A\in FR(U)$.

So $[A_L\bigcup\underline{S}(A_U)]_\approx=\left[\overline{S}(A_L)\bigcap A_U\right]_\approx$.

Theorem 3.1 The $\underline{S}(\phi)=\phi$ and $\overline{S}(U)=U$ are true and unique respectively, where U denotes a universe set, and ϕ is an empty set.

In particular, there are $\underline{S}([\phi]_\approx)=[\phi]_\approx$ and

$\overline{S}([U]_{\approx}) = [U]_{\approx}$.

Proof: According to the definition 3.2, because $\underline{S}(\phi)$ signifies $\bigcap_{t\in T} A_U^t = \phi$, and then $\bigcap_{t\in T} A^t = \phi$ must be true by the definition 2.1. However, according to the above definition 3.2, $\bigcap_{t\in T} A^t = \phi$ is satisfied if only $\underline{S}(\phi) = \phi$ holds.

Similarly, $\overline{S}(U) = U$ is also true. Again, $\underline{S}$ and $\overline{S}$ are all single maps, so $\underline{S}(\phi) = \phi$ and $\overline{S}(U) = U$ are unique.

Theorem 3.2 Let $\{U, R\}$ be a given approximate space. For $A = \langle A_L, A_U \rangle \in R^F$, define $A^* = \left[A_U^* \right]_{\approx}$, where the $A_U^* \in F(U)$ is a pseudo–complement of $A_U \in F(U)$, then, $A^* \in R^F$ must be a pseudo- complement of A.

Proof: By $\left[\overline{S}(A_L) \cap A_U \right]_{\approx} = A$, then,

$$A_L \cap (A^*)_L = \left[\overline{S}(A_L) \cap A_U \cap A_U^* \right]_{\approx} = [\phi]_{\approx}$$

and $A_U \cap (A^*)_U = \left[\overline{S}(A_L) \cap A_U \cap A_U^* \right]_{\approx} = [\phi]_{\approx}$

So

$$A \cap A^* = \left[\left(A_L \cap (A^*)_L \right) \cup \underline{S}\left(A_U \cap (A^*)_U \right) \right]_{\approx}$$

$$= \left[[\phi]_{\approx} \cup \underline{S}\left([\phi]_{\approx} \right) \right]_{\approx} = \left[[\phi]_{\approx} \cup [\phi]_{\approx} \right]_{\approx} = [\phi]_{\approx}$$

Assume for any $Y \in R^F$, $A \cap Y = [\phi]_{\approx}$, i.e., $\left[\left(A_L \cap Y_L \right) \cup \underline{S}\left(A_U \cap Y_U \right) \right]_{\approx} = [\phi]_{\approx}$, therefore, there are $A_L \cap Y_L = [\phi]_{\approx}$ and $\underline{S}(A_U \cap Y_U) = [\phi]_{\approx}$. Since $\underline{S}([\phi]_{\approx}) = [\phi]_{\approx}$ and is unique, then $A_U \cap Y_U = [\phi]_{\approx}$. Again, the A_U^* is a pseudo–complement of A_U, so $Y_U \subseteq A_U^*$.

Since $Y_L \subseteq Y_U$, then $Y_L \subseteq A_U^*$, i.e., $Y_L \subseteq (A^*)_L$, $Y_U \subseteq (A^*)_U$.

Again, $Y_L \subseteq Y \subseteq Y_U$, $(A^*)_L \subseteq A^* \subseteq (A^*)_U$, according to the fuzzy rough inclusion of definition 2.2, so $Y \subseteq A^*$ holds true.

Simultaneously, there are $A(x) \wedge A^*(x) = 0$ by $A \cap A^* = [\phi]_{\approx}$ and $A(x) \wedge Y(x) = 0$ by $A \cap Y = [\phi]_{\approx}$, and then there is $Y(x) \leq A^*(x)$ for all $x \in U$. Therefore, that A^* is a pseudo-complement of A holds by the definition 3.1.

Q.E.D.

Thus, the algebraic system $\left(R^F, \cup, \cap, *\right)$ satisfies the pseudo complementary law.

Let $(G, \cup)$ and $(G, \cap)$ be two given algebraic systems. Then

(a) For any A, $B \in G$, there are $A \cup B \in G$ and $A \cap B \in G$;

(b) The intersection and union operation all satisfy the associative law;

(c) The intersection and union operation all satisfy the commutative law;

(d) The intersection and union operation all satisfy the distributivity law between the two;

(e)To $\cup$ or $\cap$ operation, there is a unit;

(f)To $\cup$ or $\cap$ operation, there is an inverse one – pseudo complement A^* for every one A on G.

Definition 3.3 Let S that consists of a FRS with one operation be an algebraic system, if the algebraic system S satisfies the above condition (a)(b), then it is called FR half-group; if the algebraic system S satisfies the above condition (a)(b)(e)(f), then it is called FR group; if the algebraic system S satisfies the above condition (a)(b)(e)(f)(c), then it is called FR Abelian group. In addition, let S' that consists of a FRS with two operations be also an algebraic system, for the one operation on S', if the algebraic system S' satisfies the above condition (a)(b)(e)(f)(c), however, for the other operation on S', S' satisfies (a)(b)(d), then S' is called FR ring; if S' satisfies (a)(b)(d)(c) for the other operation on S', then S' is called FR commutative ring; if S' satisfies (a)(b)(d)(c)(e)(f) for the other operation on S', then S' is called FR field.

Based on the above definitions of intersection and union of FRS, and the operation laws of FS in [3], here we give the operation laws of intersection and union on R^F as follows: the following ①②③ are easy to be proved. Here it is omitted.

Let $\left(R^F, \cup\right)$ and $\left(R^F, \cap\right)$ be two given algebraic systems. Then

① For any A，$B \in R^F$，there are $A \bigcup B \in R^F$ and $A \bigcap B \in R^F$;

② The intersection and union operations all satisfy the associative law;

③ The intersection and union operations all satisfy the commutative law;

④ The intersection and union operations all satisfy the distributivity law between the two;

For any $A, B, C \in R^F$, according to the definition 3.2, there are:

$$\left(A \bigcup B\right) \bigcap C = \left\{\left[\left(A \bigcup B\right)_L \bigcap C_L\right] \bigcup \underline{S}\left[\left(A \bigcup B\right)_U \bigcap C_U\right]\right\}_{\approx}$$

$$= \{[(A_L \bigcap C_L) \bigcup (B_L \bigcap C_L)] \bigcup \underline{S}[(A_U \bigcap C_U) \bigcup (B_U \bigcap C_U)]\}_{\approx}.$$

$$(A \bigcap C) \bigcup (B \bigcap C) = \left\{\overline{S}\left[(A_L \bigcap C_L) \bigcup (B_L \bigcap C_L)\right] \bigcap \left[(A_U \bigcap C_U) \bigcup (B_U \bigcap C_U)\right]\right\}_{\approx}.$$

Since $\left[A_L \bigcup \underline{S}(A_U)\right]_{\approx} = \left[\overline{S}(A_L) \bigcap A_U\right]_{\approx}$, then $\left(A \bigcup B\right) \bigcap C = (A \bigcap C) \bigcup (B \bigcap C)$.

Similarly, there is $\left(A \bigcap B\right) \bigcup C = (A \bigcup C) \bigcap (B \bigcup C)$.

⑤ To $\bigcup$ operation，there is a unit $\left[\phi\right]_{\approx}$, where ϕ denotes an empty set.

For $A \in R^F$, there are

$$A \bigcup \left[\phi\right]_{\approx} = \left[\overline{S}\left(A_L \bigcup \phi_L\right) \bigcap \left(A_U \bigcup \phi_U\right)\right]_{\approx} = \left[\overline{S}(A_L) \bigcap A_U\right]_{\approx} = A;$$

Similarly, according to [3], there are

$A_L\left(x\right) \vee \phi_L\left(x\right) = A_L\left(x\right)$ and

$A_U\left(x\right) \vee \phi_U\left(x\right) = A_U\left(x\right)$ for all $x \in U$.

Where $\vee$ is an 'or' operator, it includes the max operator, supremum operator, etc.

The same, to $\bigcap$ operation, there is a unit $\left[U\right]_{\approx}$, where U denotes a universe set.

⑥ To $\bigcup$ operation，by the theorem3.2 again, there is an inverse one – pseudo complement A^* for every one A on R^F and

$$A \bigcup A^* = \left[\overline{S}\left(A_L \bigcup (A^*)_L\right) \bigcap \left(A_U \bigcup (A^*)_U\right)\right]_{\approx} = \left[\overline{S}(\tilde{U}_L) \bigcap \left(\tilde{U}_U\right)\right]_{\approx} = \left[\tilde{U}\right]_{\approx}.$$

Where $\tilde{U}$ denotes an approximate universe set.

Define $\left[\tilde{U}\right]_{\approx} = [\tilde{\phi}]_{\approx}^*$, where $\tilde{\phi}$ denotes an approximate empty set, and $[\tilde{\phi}]_{\approx}^*$ is the pseudo-complement of $[\tilde{\phi}]_{\approx}$ and is called a pseudo-unit, i.e., $A \bigcup A^* = [\tilde{\phi}]_{\approx}^*$.

At the same time, there are $A_L\left(x\right) \vee (A^*)_L\left(x\right)$ and $A_U\left(x\right) \vee (A^*)_U\left(x\right)$ for all $x \in U$.

The same, to $\bigcap$ operation, there is also an inverse one – pseudo complement A^* for every one A on R^F and $A \bigcap A^* = \left[\phi\right]_{\approx}$, define $\left[\phi\right]_{\approx} = [U]_{\approx}^*$, where $[U]_{\approx}^*$ is the pseudo complement of $[U]_{\approx}$ and is called a pseudo-unit, i.e., $A \bigcap A^* = [U]_{\approx}^*$.

Theorem 3.3 The algebraic systems $\left(R^F, \bigcup\right)$ and $\left(R^F, \bigcap\right)$ are all FR half-group; and they are also all FR group and are all FR Abelian group; The algebraic system $\left(R^F, \bigcup, \bigcap\right)$ is a FR ring and is a FR commutative ring and FR field.

Proof: From the above ①②③④⑤⑥, it proves that the theorem holds true.

If the unit $\left\langle \bigcup, \bigcap \right\rangle$ is regarded as an operation, there is the following definition.

Definition 3.4 Let $\left\{U, R\right\}$ be a given approximate space. All sets that consist of FRS in U are represented as $FR\left(U\right)$. In the algebraic system $\left(FR\left(U\right), \left\langle \bigcup, \bigcap \right\rangle\right)$, there is always $B \in FR\left(U\right)$ for any $A \in FR\left(U\right)$ so as to make $A \bigcup B = \left[U\right]_{\approx}$ or $A \bigcup B = \left[\tilde{U}\right]_{\approx}$, $A \bigcap B = \left[\phi\right]_{\approx}$, then, the system $\left(FR\left(U\right), \left\langle \bigcup, \bigcap \right\rangle\right)$ is claimed to have pseudo complement-one, and B is called the pseudo complement-one of A, label as A^*.

From the above ⑥ and the theorem 3.2, we can know, there are $A \bigcup A^* = \left[\tilde{U}\right]_{\approx}$ and $A \bigcap A^* = \left[\phi\right]_{\approx}$, therefore, the algebraic system $\left(R^F, \langle \bigcup, \bigcap \rangle\right)$ has the pseudo complement-one A^* for any $A \in R^F$.

And then, from the above ①②③④⑤⑥, there are:

(i) For any A, $B \in R^F$, there are $A \bigcup B \in R^F$ and $A \bigcap B \in R^F$;

(ii) To $\bigcup$ and $\bigcap$ operation, $\langle \bigcup, \bigcap \rangle$ satisfies the associative law, commutative law respectively and the distributivity law between the two;

(iii) There is a unit $\left\langle [\phi]_{\approx}, \left[\tilde{U}\right]_{\approx} \right\rangle$ for every one on $\left(R^F, \langle \bigcup, \bigcap \rangle\right)$;

(iv) There is an inverse one A^* for every one A on R^F and $A \bigcup A^* = \left[\tilde{U}\right]_{\approx}$, $A \bigcap A^* = [\phi]_{\approx}$.

Definition 3.5 The algebraic system $\left(R^F, \langle \bigcup, \bigcap \rangle\right)$ is called a FR pseudo-group and called FR Abelian pseudo-group.

Now, the definitions of a subring, an ideal and a subgroup are introduced as follows:

Definition 3.6 Let $R_r^F = \left(R^F, \bigcup, \bigcap\right)$ be a FR ring. The nonempty subset X of ring R_r^F is called the subring of R_r^F if there exists $XX \subseteq X$, where XX is a synthesis operation; X is called the left ideal of R_r^F if there exists $R_r^F X \subseteq X$; X is called the right ideal of R_r^F if there exists $XR_r^F \subseteq X$; X is called the (two sides) ideal of R_r^F if X is both the left ideal and the right ideal or there exists $XR_r^F X \subseteq X$.

Especially, for subgroup, there is also a similar definition 3.6: let $R_{g_1}^F = \left(R^F, \bigcup\right)$ and $R_{g_2}^F = \left(R^F, \bigcap\right)$ be two FR group, the nonempty subset X of group $R_{g_1}^F$ and nonempty subset Y of group $R_{g_2}^F$ are respectively called the subgroup of $R_{g_1}^F$ and $R_{g_2}^F$ if there exist $XX \subseteq X$ and $YY \subseteq Y$ respectively.

4. Conclusions

This paper discusses mainly the algebraic systems theories on FRS. By studying the algebraic theories on FRS, some new achievements in general algebraic theories can impulse further development of modern algebraic theories and FRS theories.

At present, although some theories and few applications of FRS have obtained remarkable achievements, it also has a series of problems to be solved, such as how functions of FRS will be selected for neural network trained by the algebraic theories on FRS, how the properties of the field on FRS will be discussed on the basis of some operations and rules of FRS, and how the algebraic theories on FRS will be applied to image processing and other pattern recognition etc. Again, we only discuss the half-group, group, subgroup, ring, subring, ideal, field, the homomorphism and isomorphism between rings on FRS are not introduced. The analysis theorems on FRS are not also presented. It will be worthwhile further studying these problems. These problems to be solved will directly impulse the development of FRS.

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