Fuzzy preference relation rough sets

Qinghua Hu, Daren Yu, Congxin Wu
Harbin Institute of Technology, Harbin 150001, P. R. China

Abstract

Preference analysis is a class of important tasks in multi-criteria decision making. The classical rough set theory was generalized to deal with preference analysis by replacing equivalence relations with dominance relation. However, crisp preference relations can not reflect the fuzziness in criteria. In this paper, we introduce the logsig function to extract fuzzy preference relations from samples characterized with numerical attributes. Then we integrate fuzzy preference relations with an improved fuzzy rough set model and develop a fuzzy preference rough set model. We generalize the dependency used in classical rough sets and fuzzy rough sets to compute the relevance between the criteria and decision. The proposed model is used to analyze a fuzzy preference data. It shows the effectiveness of the proposed model.
Keywords: *preference analysis, multi-criteria decision making, fuzzy sets, rough sets, dependency*

1. Introduction

There are a lot of applications associated with an assignment of objects evaluated by a set of criteria to pre-defined preference-ordered decision classes, such as credit approval, stock risk estimation, and teaching evaluation [1]. A lot of models, methods and algorithms were proposed for extracting and aggregating fuzzy preference relations from distinct criteria [2, 3].

Rough set theory is proven to be an effective tool for dealing with inconsistent information [4]. This theory was widely discussed and applied in dependency analysis, feature evaluation, attribute reduction and rule extraction. However, Pawlak rough set model is constructed on equivalence relations and partition, which is considered as one of the main problems in employing the model to complex decision analysis and classification tasks. As to the problem of decision making with multiple criteria, there are pre-defined preference structures in condition and decision. The classical model is not able to receive and extract the information of ordinal structure. Therefore, this model can not be used to analyze the information with preference relations. Pawlak first discussed this problem in [5], and Greco, Matarazzo, Slowinski proposed a novel rough set model for preference analysis, where dominance relations were constructed based on the decision preference [6, 7, 8]. However, there are still some differences between preference relations and fuzzy preference relations. Fuzzy preference relations can reflect the degree that an object is less than or greater than another, while crisp preference relation can not survey this information. In this work, we will introduce a fuzzy preference rough set model for fuzzy preference analysis [9].

As claimed in many literatures, fuzzy sets and rough sets are complementary, rather than competitive in dealing with uncertainty. It is useful to integrate rough sets with fuzzy sets in a uniform framework. The fuzzy rough model was firstly constructed by Dubois and Prade in 1990 [10], where fuzzy similarity relations satisfying the properties of reflectivity, symmetry and max-min transitivity were used. Moreover, the t-norm *min* and t-conorm *max* were introduced in defining fuzzy lower approximations and fuzzy upper approximations. In fact, t-norm operator *min* and its dual operator *max* are one pair of special triangular norms. There are a number of other fuzzy operators used in fuzzy reasoning [11]. Radzikowska and Kerre gave a more general definition of fuzzy rough sets in [12]. They defined a broad family of fuzzy rough sets with respect to a fuzzy similarity relation; the fuzzy lower approximation and upper approximation are determined by a border implicator and a t-norm, respectively. In [11] t-norms and T-residuated implication were introduced in defining fuzzy rough sets. Moreover, Mi and Zhang proposed a new definition of fuzzy rough sets based on the residual implication θ and it's dual σ, where fuzzy similarity relations were also employed to generate fuzzy granulated spaces [14].

Obviously, the above theories and models are constructed based on fuzzy equivalence relations. They are not able to analyze data with preference relations. In 2003, Wu, Mi and Zhang first showed a generalized fuzzy rough set model based on general fuzzy relations and max-min fuzzy operators [13]. In 2004, Mi and

Zhang presented a novel fuzzy rough set model based on fuzzy residual implication θ and its dual σ [14]. In [15] Yeung, Chen, et al reviewed the previous work and showed two classes of models of defining fuzzy rough sets based on arbitrary fuzzy relations. It can be imagined that fuzzy preference relations, as a special class of fuzzy relation to model fuzzy preference in multi-criteria decision making, can be integrated with these general fuzzy rough set models. Thus a fuzzy preference rough set model is constructed for fuzzy preference analysis. However, to our best knowledge, there has little work reporting fuzzy preference rough set model so far. In this work, we will discuss the model and its properties of fuzzy preference relation based rough sets for analyzing the dependency of criteria.

2. Fuzzy preference relations and fuzzy preference granules

In employing a fuzzy rough set model to real world applications, there are three key issues to be addressed: how to build a fuzzy relation on samples with respect to each attribute or criterion, how to aggregate multiple attributes or criteria and how to define the lower and upper approximations of arbitrary fuzzy subsets. In this section, we will introduce algorithms to compute the fuzzy preference relations between samples.

There are two kinds of preference relations widely used in many important decision models [16].

Multiplicative preference relations: a multiplicative preference $R \in U \times U$ is represented by a relation matrix $(r_{ij})_{n\times n}$, where r_{ij} is interpreted as the ratio of the preference degree of x_i to that of x_j, i.e. x_i is r_{ij} times as good as x_j. It was suggested that $r_{ij} \in \{1,2,\cdots,9\}$. In this case, the fuzzy preference relation R is usually assumed multiplicative reciprocal, i.e., $r_{ij} \cdot r_{ji} = 1, \forall i, j \in \{1,2,\cdots,n\}$.

Fuzzy preference relations: a fuzzy preference relation R is a fuzzy set on the product set $U \times U$, which is characterized by a membership function $\mu_R : U \times U \to [0, 1]$. If the cardinality of U is finite, the fuzzy preference relation can also be represented by a $n \times n$ matrix $(r_{ij})_{n\times n}$, where r_{ij} is interpreted as the preference degree of x_i over x_j: $r_{ij} = 1/2$ indicates there is no difference between x_i and x_j; $r_{ij} > 1/2$ shows x_i is preferred to x_j and $r_{ij} = 1$ means x_i is absolutely preferred to x_j. On the other hand, $r_{ij} < 1/2$ indicates x_j is preferred to x_i. In this case, the preference matrix is usually assumed additive reciprocal, i.e., $r_{ij} + r_{ji} = 1, \forall i, j \in \{1,2,\cdots,n\}$.

In real-world applications, dominance structures or preference structures are usually represented with criteria characterized by a set of ordinal discrete values or numerical values.

Given a universe of finite objects $U = \{x_1,\ldots,x_m\}$, a is a numerical feature to describe the objects. Then the upward and downward fuzzy preference relations over U are computed by

$$r_{ij}^{\uparrow} = \frac{1}{1+e^{-k(x_i-y_j)}} \quad \text{and} \quad r_{ij}^{\downarrow} = \frac{1}{1+e^{k(x_i-y_j)}},$$

where k is positive constant to control the gradient.

In fact, $f(x) = \dfrac{1}{1+e^{-x}}$ is the well known Logsig transfer function used in BP neural networks. The curve of the function in [-5 5] is shown in figure 1.

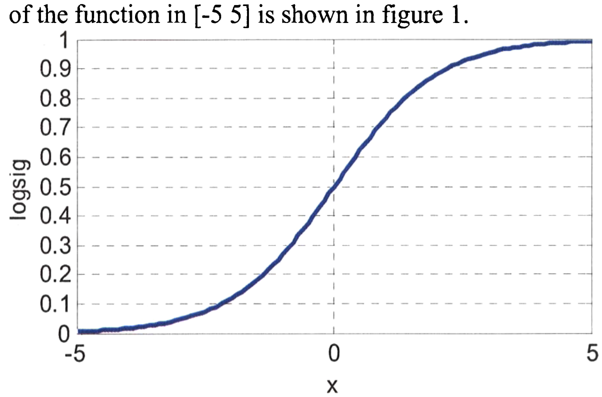

Fig. 1. Curve of function logsig in interval [-5, 5]

Obviously, fuzzy preference relations not only reflect the fact that an object x_i is greater (less) than another x_j, but also measure how much x_i is greater (less) than x_j. This shows fuzzy preference relations are more powerful in extracting information from fuzzy data than dominance relations.

We have the properties of $r_{ii}^{\uparrow} = r_{ii}^{\downarrow} = 0.5$, $\dfrac{1}{1+e^{-x}} = 1$ if $x = +\infty$ and $\dfrac{1}{1+e^{-x}} = 0$ if $x = -\infty$. These results are consistent with our intuition that $r_{ij} = 0.5$ indicates there is no difference between x_i and x_j if they have the same value on criterion a, while $r_{ij}^{\uparrow} = 1$ if x_i is much greater than x_j in terms of criterion a. Moreover, It is easy to know that $\forall i, j \in \{1,2,\cdots,n\}$, $r_{ij}^{\uparrow} = r_{ji}^{\downarrow}$; $r_{ij} + r_{ji} = 1$ if the fuzzy preference relation is computed with the logsig function. We also see that these fuzzy preference relations are neither reflexive nor symmetric. But they are transitive, i.e., $\min_y \left(R(x,y), R(y,z)\right) \le R(x,z)$.

Fuzzy preference relations produce nested granularity structures over the universe, the fuzzy subsets of objects which are greater or less than x_i are defined as

$$[x_i]_a^{\uparrow} = \frac{r_{1i}}{x_1} + \frac{r_{2i}}{x_2} + \cdots + \frac{r_{ni}}{x_n}, \quad [x_i]_a^{\downarrow} == \frac{r_{i1}}{x_1} + \frac{r_{i2}}{x_2} + \cdots + \frac{r_{in}}{x_n},$$

where $r_{ij} = \dfrac{1}{1+e^{-k(x_i - y_j)}}$.

Generally speaking, there is more than one criterion used in decision analysis. One can generate a fuzzy preference relation from each criterion. So we require a technique to aggregate multiple criteria. There are some methods developed to deal with this problem, such as t-norm and weighted aggregation operators [22, 35, 44]. In this work, we are not going to discuss the effect of aggregation operators on multi-criteria decision analysis and we introduce the fuzzy intersection operator.

Given a task of multi-criteria decision making, x_i and x_j are two arbitrary objects in the universe. If r_{ij} and s_{ij} are the fuzzy preference degrees that x_i is preferred over x_j derived from two criteria a and b, then the aggregated preference of a and b is defined as $\min(r_{ij}, s_{ij})$. Certainly, we can also use other t-norms, such as algebraic product and bold intersection operators.

Obviously, we have the following conclusion.

1) $x \geq_a y \Leftrightarrow [x]_a^{\uparrow} \subseteq [y]_a^{\uparrow} \Leftrightarrow [y]_a^{\downarrow} \subseteq [x]_a^{\downarrow}$;
2) $x \geq_B y \Leftrightarrow [x]_B^{\uparrow} \subseteq [y]_B^{\uparrow} \Leftrightarrow [y]_B^{\downarrow} \subseteq [x]_B^{\downarrow}$.

3. Approximating preference decision with fuzzy preference granules

As fuzzy preference granules are defined, now we approximate preference decisions with these fuzzy granules. In this section, we introduce the fuzzy rough set model to construct fuzzy preference approximations.

we select the following fuzzy approximation operators:

- $S-$ lower approximation operator: $\underline{R_S} A(x) = \inf_{u \in U} S(N(R(x,u)), A(u))$;
- $T-$ upper approximation operator: $\overline{R_T} A(x) = \sup_{u \in U} T(R(x,u), A(u))$,

where $N(a) = 1 - a$, $S = \max$ and $T = \min$. In this case, fuzzy approximation operators can be written as

$\underline{R} A(x) = \inf_{u \in U} \max(1 - R(x,u)), A(u))$,

$\overline{R} A(x) = \sup_{u \in U} \min(R(x,u), A(u))$.

In fact, we can get the similar results for other fuzzy approximation operators, s-norms and t-norms. We are not going to show the details about them in this work.

In multi-criteria decision analysis, the decision is usually crisp. Just like example 1, the decision class is $\{1, 2, 3\}$. They maybe stand for poor, good and excellent, and so on. Therefore, the preference decision forms a crisp nested structure over the universe. Given a preference decision table $< U, C \bigcup D >$ and assumed the value domain of D is $\{d_1, d_2, \cdots, d_N\}$, $d_1 \leq d_2 \leq \cdots \leq d_N$. The nested preference decision structure is given by $d_i^{\geq} = \bigcup_{j=i}^{N} d_j$ or $d_i^{\leq} = \bigcup_{j=1}^{i} d_j$.

Here $d_i^{\geq}$ and $d_i^{\leq}$ are both crisp subsets of *U*. obviously, we have $d_i^{\geq} \supseteq d_j^{\geq}$ and $d_i^{\leq} \supseteq d_j^{\leq}$ if $i \leq j$ (d_i denotes the set of samples with decision d_i).

Definition 1. Given fuzzy preference relations $R^{\uparrow}$ and $R^{\downarrow}$, preference decision classes $d_i^{\geq}$ and $d_i^{\leq}$, we define the membership of sample x to the lower and upper approximations of $d_i^{\geq}$ and $d_i^{\leq}$ as

- upward fuzzy lower approximation:
  $\underline{R^{\uparrow}} d_i^{\geq}(x) = \inf_{u \in U} \max(1 - R^{\uparrow}(u,x),\ d_i^{\geq}(u))$,
- upward fuzzy upper approximation:
  $\overline{R^{\uparrow}} d_i^{\geq}(x) = \sup_{u \in U} \min(R^{\uparrow}(x,u),\ d_i^{\geq}(u))$,
- downward fuzzy lower approximation:
  $\underline{R^{\downarrow}} d_i^{\leq}(x) = \inf_{u \in U} \max(1 - R^{\downarrow}(u,x),\ d_i^{\leq}(u))$,
- downward fuzzy upper approximation:
  $\overline{R^{\downarrow}} d_i^{\leq}(x) = \sup_{u \in U} \min(R^{\downarrow}(x,u),\ d_i^{\leq}(u))$,

where $X(x)$ means the membership of x to the fuzzy subset *X*. Moreover, we specify that $\underline{R^{\uparrow}} d_1^{\leq}(x) = 1$ and $\underline{R^{\downarrow}} d_N^{\leq}(x) = 1$, where d_1 and d_N are the worst and the best decision classes, respectively.

It is remarkable that there is little difference between these definitions and those proposed in [13, 14]. In this work, we replace $R^{\uparrow}(x,u)$ and $R^{\downarrow}(x,u)$ in [14] with $R^{\uparrow}(u,x)$ and $R^{\downarrow}(u,x)$ in the definitions of $\underline{R^{\uparrow}} d_i^{\geq}(x)$ and $\underline{R^{\downarrow}} d_i^{\leq}(x)$. These definitions are identical if R is symmetric. However, fuzzy preference relations do not satisfy the property of symmetry, therefore these definitions are different. We will show the rationality of the revision in the below.

Now, let us first show how to compute the lower and upper approximations of preference decision classes.

- As to $\underline{R^{\uparrow}} d_i^{\geq}(x)$, we assume that $u \in d_i^{\geq}$, then $d_i^{\geq}(u) = 1$, $\max(1 - R^{\uparrow}(u,x),\ d_i^{\geq}(u)) = 1$. On the other hand, assumed that $u \notin d_i^{\geq}$, i.e., $u \in d_{i-1}^{\leq}$, then $d_i^{\geq}(u) = 0$, $\max(1 - R^{\uparrow}(u,x),\ d_i^{\geq}(u)) = 1 - R^{\uparrow}(u,x)$. As

$1-R^{\uparrow}(u,x)\le 1$, $\underline{R^{\uparrow}}d_i^{\ge}(x)=\inf_{u\notin d_i^{\ge}}1-R^{\uparrow}(u,x)$. As $1-R^{\uparrow}(u,x)=R^{\downarrow}(u,x)$, thus $\underline{R^{\uparrow}}d_i^{\ge}(x)=\inf_{u\notin d_i^{\ge}}R^{\downarrow}(u,x)$. If $i=1$, $d_1^{<}=\varnothing$. In this case, we specify $\underline{R^{\uparrow}}d_1^{\ge}(x)=1$.

- Assumed that $u\notin d_i^{\ge}$, then $d_i^{\ge}(u)=0$, $\min(R^{\uparrow}(x,u),\ d_i^{\ge}(u))=0$; otherwise $u\in d_i^{\ge}$, then $d_i^{\ge}(u)=1$, $\min(R^{\uparrow}(x,u),\ d_i^{\ge}(u))=R^{\uparrow}(x,u)$. So $\overline{R^{\uparrow}}d_i^{\ge}(x)=\sup_{u\in d_i^{\ge}}R^{\uparrow}(x,u)$.

Analogically, $\underline{R^{\downarrow}}d_i^{\le}(x)=\inf_{u\notin d_i^{\le}}R^{\uparrow}(u,x)$ and $\overline{R^{\downarrow}}d_i^{\le}(x)=\sup_{u\in d_i^{\le}}R^{\downarrow}(x,u)$.

Formula $\underline{R^{\uparrow}}d_i^{\ge}(x)=\inf_{u\notin d_i^{\ge}}1-R^{\uparrow}(u,x)$ indicates that the membership of x to the lower approximation of $d_i^{\ge}$ depends on the sample which does not belong to $d_i^{\ge}$ and has the greatest preference degree over *x*, while $\overline{R^{\uparrow}}d_i^{\ge}(x)=\sup_{u\in d_i^{\ge}}R^{\uparrow}(x,u)$ shows that that the membership of x to the upper approximation of $d_i^{\ge}$ depends on the sample which belongs to $d_i^{\ge}$ and has the least preference degree over *x*. If there is only one criterion to derive the fuzzy preference relation, we can say that $\underline{R^{\uparrow}}d_i^{\ge}(x)$ is determined by the greatest sample with a preference decision class $d_{i-1}^{\le}$, while $\overline{R^{\uparrow}}d_i^{\ge}(x)$ depends on the smallest sample with the preference decision class $d_i^{\ge}$. The similar interpretation can also be attained with $\underline{R^{\downarrow}}d_i^{\le}(x)=\inf_{u\in d_i^{\le}}1-R^{\downarrow}(u,x)$ and $\overline{R^{\downarrow}}d_i^{\le}(x)=\sup_{u\in d_i^{\le}}R^{\downarrow}(x,u)$.

Proposition 1. $R^{\uparrow}$ and $R^{\downarrow}$ are fuzzy preference relations generated with criteria B and functions $\frac{1}{1+e^{-k(x_i-x_j)}}$ and $\frac{1}{1+e^{k(x_i-x_j)}}$. $R^{\ge}$ and $R^{\le}$ are crisp preference relations generated with criteria B. If $k=+\infty$, then $\forall d_i\in D$, we have $\underline{R^{\uparrow}}d_i^{\ge}=\underline{R^{\ge}}d_i^{\ge}$; $\underline{R^{\downarrow}}d_i^{\le}=\underline{R^{\le}}d_i^{\le}$.

Proposition 1 shows that fuzzy preference rough sets will degrade to the classical preference rough sets if gradient coefficient $k=+\infty$. In fact, fuzzy preference relations are degenerated to crisp preference relations if $k=+\infty$. In this context, $\underline{R^{\uparrow}}d_i^{\ge}(x)=1$ if all the samples better than x get the preferential decision class $d_i^{\ge}$, and vice versa. According to the classical preference relation rough set $x\in\underline{R^{\ge}}d_i^{\ge}$ if $[x]_B^{\uparrow}\subseteq d_i^{\ge}$.

Proposition 2 Given fuzzy preference decision table $<U,C,D>$, $x\in U$, $B_1,B_2\subseteq C$, $R_1^{\uparrow},R_1^{\downarrow}$ are fuzzy preference relations generated with B_1; $R_2^{\uparrow},R_2^{\downarrow}$ are fuzzy preference relations generated with B_2.Then the upwards and downwards fuzzy preference relations generated with $B_1\cup B_2$ are $R^{\uparrow}=\min(R_1^{\uparrow},R_2^{\uparrow})$ and $R^{\downarrow}=\min(R_1^{\downarrow},R_2^{\downarrow})$, respectively. for $\forall x\in d_i^{\ge},y\in d_i^{\le}$, we have $\underline{R^{\uparrow}}d_i^{\ge}(x)\ge\underline{R_1^{\uparrow}}d_i^{\ge}(x)$, $\underline{R^{\uparrow}}d_i^{\ge}(x)\ge\underline{R_2^{\uparrow}}d_i^{\ge}(x)$ and $\underline{R^{\uparrow}}d_i^{\le}(y)\ge\underline{R_1^{\uparrow}}d_i^{\le}(y)$, $\underline{R^{\uparrow}}d_i^{\le}(y)\ge\underline{R_2^{\uparrow}}d_i^{\le}(y)$.

There are multiple criteria in a preference decision table. The more criteria we can use, the more the decision is consistent. Proposition 2 indicates that adding new criteria into the current subset, the certainty of decision improves.

Definition 2. Given $<U,C,D>$, $R^{\uparrow}$ and $R^{\downarrow}$ are two fuzzy preference relations generated by $B\subseteq C$. The value domain of D is $\{d_i,i=1,\cdots N\}$. Furthermore, we have $d_1\le d_2\le\cdots\le d_N$. The fuzzy preference approximation qualities of D with respect to B are defined as:

- upward FPAQ: $\gamma_B^{\uparrow}(D^{\ge})=\dfrac{\sum_i\sum_{x\in d_i^{\ge}}\underline{R^{\uparrow}}d_i^{\ge}(x)}{\sum_i|d_i^{\ge}|}$,
- downward FPAQ: $\gamma_B^{\downarrow}(D^{\le})=\dfrac{\sum_i\sum_{x\in d_i^{\le}}\underline{R^{\downarrow}}d_i^{\le}(x)}{\sum_i|d_i^{\le}|}$,
- global FPAQ:

$$\gamma_B(D)=\frac{\sum_i\left(\sum_{x\in d_i^{\le}}\underline{R^{\downarrow}}d_i^{\le}(x)+\sum_{x\in d_i^{\ge}}\underline{R^{\uparrow}}d_i^{\ge}(x)\right)}{\sum_i\left(|d_i^{\ge}|+|d_i^{\le}|\right)},$$

Obviously, $0\le\gamma_B^{\uparrow}(D^{\ge})\le 1$, $0\le\gamma_B^{\downarrow}(D^{\le})\le 1$ and $0\le\gamma_B(D)\le 1$. We say that D is upward, downward or global consistent if $\gamma_B^{\uparrow}(D^{\ge})=1$, $\gamma_B^{\downarrow}(D^{\le})=1$ or $\gamma_B(D)=1$, respectively.

Proposition 3. Given a fuzzy preference decision table $<U,C,D>$, $a\in B\subseteq C$. We have $\gamma_B^{\uparrow}(D^{\ge})\ge\gamma_{B-a}^{\uparrow}(D^{\ge})$, $\gamma_B^{\downarrow}(D^{\le})\ge\gamma_{B-a}^{\downarrow}(D^{\le})$ and $\gamma_B(D)\ge\gamma_{B-a}(D)$.

Proof. we can easily derive the conclusion according to proposition 2 and definition 2.

Definition 3. $R^{\uparrow}$ and $R^{\downarrow}$ are two fuzzy preference relations generated by $B\subseteq C$. $a\in B$, $S^{\uparrow}$ and $S^{\downarrow}$ are two fuzzy preference relations generated by $B-a$. a

is upward redundant if $\gamma_B^{\uparrow}(D^{\geq}) = \gamma_{B-a}^{\uparrow}(D^{\geq})$; otherwise a is upward indispensable and $\gamma_B^{\uparrow}(D^{\geq}) > \gamma_{B-a}^{\uparrow}(D^{\geq})$. a is downward redundant, globally redundant, downward indispensable or globally indispensable if $\gamma_B^{\downarrow}(D^{\leq}) = \gamma_{B-a}^{\downarrow}(D^{\leq})$, $\gamma_B(D) = \gamma_{B-a}(D)$, $\gamma_B^{\downarrow}(D^{\leq}) > \gamma_{B-a}^{\downarrow}(D^{\leq})$, $\gamma_B(D) > \gamma_{B-a}(D)$.

Definition 4. Given a fuzzy preference decision table $<U,C,D>$, $B \subseteq C$ we say B is an upward reduct if

1) $\gamma_B^{\uparrow}(D^{\geq}) = \gamma_C^{\uparrow}(D^{\geq})$;

2) for $\forall a \in B$ $\gamma_B^{\uparrow}(D^{\geq}) > \gamma_{B-a}^{\uparrow}(D^{\geq})$.

Correspondingly, we can also define the downward reducts and global reducts.

Definition 5. Given a fuzzy preference decision table $<U,C,D>$, $B \subseteq C$ we say B is an downward reduct if

1) $\gamma_B^{\downarrow}(D^{\leq}) = \gamma_C^{\downarrow}(D^{\leq})$;

2) for $\forall a \in B$ $\gamma_B^{\downarrow}(D^{\leq}) > \gamma_{B-a}^{\downarrow}(D^{\leq})$.

Definition 6. Given a fuzzy preference decision table $<U,C,D>$, $B \subseteq C$ we say B is an global reduct if

1) $\gamma_B(D) = \gamma_C(D)$;

2) for $\forall a \in B$ $\gamma_B(D) > \gamma_{B-a}(D)$.

4. Experiments

Assume that we evaluate 10 candidates with two criteria A1, A2, as shown in table 1. D is the score set of candidates. The task we confront is to construct a fuzzy preference relation from the data and compute the dependency between each criterion and decision.

Table 1 Samples of multi-criteria decision making

	X1	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**	**X8**	**X9**	**X10**
A1	0.28	0.25	0.74	0.48	0.42	0.55	0.78	0.75	0.83	0.85
A2	0.28	0.31	0.42	0.47	0.51	0.58	0.71	0.78	0.80	0.91
D	1	1	1	2	2	2	2	3	3	3

The fuzzy preference relations between 10 samples in example 1 are presented in tables 2 and 3, where table 2 shows the preference matrix computed with attribute A1 and function logsig, and K is specified as 10.

Table 2. Fuzzy preference relations computed with A1

	X1	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**	**X8**	**X9**	**X10**
X1	0.5	0.39	0.99	0.95	0.89	0.98	1	1	1	1
X2	0.61	0.5	0.99	0.97	0.93	0.99	1	1	1	1
X3	0.01	0.01	0.5	0.14	0.06	0.32	0.96	0.9	0.97	0.98
X4	0.05	0.03	0.86	0.5	0.29	0.74	0.99	0.98	0.99	1
X5	0.11	0.07	0.94	0.71	0.5	0.88	1	0.99	1	1
X6	0.02	0.01	0.68	0.26	0.12	0.5	0.98	0.95	0.99	0.99
X7	0	0	0.04	0.01	0	0.02	0.5	0.26	0.54	0.61
X8	0	0	0.1	0.02	0.01	0.05	0.74	0.5	0.77	0.82
X9	0	0	0.03	0.01	0	0.01	0.46	0.23	0.5	0.57
X10	0	0	0.02	0	0	0.01	0.39	0.18	0.43	0.5

Table 3. Fuzzy preference relations computed with A2

	X1	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**	**X8**	**X9**	**X10**
X1	0.5	0.61	0.89	0.95	0.97	0.99	1	1	1	1
X2	0.39	0.5	0.84	0.92	0.95	0.98	1	1	1	1
X3	0.11	0.16	0.5	0.68	0.79	0.92	0.99	1	1	1
X4	0.05	0.08	0.32	0.5	0.65	0.84	0.97	0.99	0.99	1
X5	0.03	0.05	0.21	0.35	0.5	0.74	0.95	0.98	0.99	1
X6	0.01	0.02	0.08	0.16	0.26	0.5	0.88	0.95	0.96	0.99
X7	0	0	0.01	0.03	0.05	0.12	0.5	0.74	0.79	0.95
X8	0	0	0	0.01	0.02	0.05	0.26	0.5	0.57	0.88
X9	0	0	0	0.01	0.01	0.04	0.21	0.43	0.5	0.84
X10	0	0	0	0	0	0.01	0.05	0.12	0.16	0.5

as to object X1 and criterion A1, the fuzzy granule of objects which are greater than X1 in terms of A1 can be written as

$$[XI]_{A1}^{\uparrow} = \frac{0.5}{x_1} + \frac{0.39}{x_2} + \frac{0.99}{x_3} + \frac{0.95}{x_4} + \frac{0.89}{x_5} + \frac{0.98}{x_6} + \frac{1}{x_7} + \frac{1}{x_8} + \frac{1}{x_9} + \frac{1}{x_{10}}.$$

First, Let us compute the dependency of decision D on

A1 and A2 in table 1. We try crisp the preference rough set model and the proposed fuzzy rough set model. The derived results are shown table 4. Moreover, we change the attribute A1 value of sample X3 from 0.6 to 0.74. Clearly, the inconsistency of decision based on A1 becomes greater in this case. We compute the dependency of A1 and A2 based on preference rough sets and fuzzy preference rough sets again, shown in table 5. Although the inconsistency gets greater, there is not difference between the dependencies computed with crisp rough sets. As to fuzzy preference rough sets based dependency, the values of dependency get smaller after the revision. This experiment shows that preference rough sets can not reflect the degree of preference, while fuzzy preference rough sets are able to measure this kind of information.

Table 4. Dependency computed with example 1

	Crisp		Fuzzy	
	A1	A2	A1	A2
Upward	0.6000	1.0000	0.7988	0.9159
down	0.8000	1.0000	0.8900	0.9362
global	0.7000	1.0000	0.8444	0.9261

Table 5. Dependency after a sample is revised.

	Crisp		Fuzzy	
	A1	A2	A1	A2
Upward	0.6000	1.0000	0.7270	0.9159
downward	0.8000	1.0000	0.8702	0.9362
global	0.7000	1.0000	0.7986	0.9261

5. Conclusion

Preference analysis is a class of important tasks in intelligent data analysis and machine learning. In this paper, we introduce the logsig function, which is widely used in BP neural networks, to extract fuzzy preference relations from samples. Then we integrate fuzzy preference relations with an improved fuzzy rough set model and thus develop a fuzzy preference rough set model. The relations between the proposed model and the crisp preference rough set model are discussed. Moreover, we generalize the dependency used in classical rough sets and fuzzy rough sets to compute the relevance between the criteria and decision. We propose the definitions of upward dependency, downward dependency and global dependency.

Acknowledgement

This work is partly supported by National Natural Science Foundation of China under Grant 60703013 and Development Program for Outstanding Young Teachers in Harbin Institute of Technology under Grant HITQNJS.2007.017

Reference

[1] H. M. Moshkovich, A. I. Mechitov, D. L. Olson. Rule induction in data mining: effect of ordinal scales. Expert Systems with Applications, 22(2002) 303-311

[2] Z. S. Xu. Multiple-Attribute Group Decision Making With Different Formats of Preference Information on Attributes. IEEE Transactions on systems, man, and cybernetics—Part B: Cybernetics, 37 (2007) 1500-1511

[3] R. Slowinski, S. Greco, B. Matarazzo, Axiomatization of utility, outranking and decision-rule preference models for multiple-criteria classification problems under partial inconsistency with the dominance principle, Control & Cybernetics 31 (2002) 1005–1035.

[4] Y. Y. Yao. A comparative study of fuzzy sets and rough sets. Information Sciences, 109 (1998) 227-242

[5] Z. Pawlak. Rough set approach to multi-attribute decision analysis. European Journal of Operational Research 72 (1994) 443-459

[6] S. Greco, B. Matarazzo, R. Slowinski. Rough approximation of a preference relation by dominance relations. European Journal of Operational Research 117 (1999) 63-83

[7] S. Greco, B. Matarazzo, R. Slowinski. Rough approximation by dominance relations. International journal of intelligent systems, 17 (2002) 153–171

[8] S. Greco, B. Matarazzo, R. Slowinski. Rough sets theory for multi-criteria decision analysis. European Journal of operational research 129 (2001) 1-47

[9] Y.-M. Wang, C. Parkan. Optimal aggregation of fuzzy preference relations with an application to broadband internet service selection. European Journal of Operational Research 187 (2008) 1476–1486

[10] D. Dubois, H. Prade. rough fuzzy sets and fuzzy rough sets. International Journal of General Systems, 17 (1990) 191-209

[11] N. N. Morsi, M. M. Yakout, Axiomatics for fuzzy rough set, Fuzzy Sets Syst., vol. 100, pp. 327–342, 1998.

[12] A. M. Radzikowska, E. E. Kerre. A comparative study of fuzzy rough sets. Fuzzy Sets and Systems 126 (2002) 137–155

[13] W. -Z. Wu, J.-S. Mi, W.-X. Zhang. Generalized fuzzy rough sets. Information Sciences 151 (2003) 263–282

[14] J.-S. Mi, W.-X. Zhang. An axiomatic characterization of a fuzzy generalization of rough sets. Information Sciences 160 (2004) 235–249

[15] D. S. Yeung, D.-G. Chen, E. C. C. Tsang, J. W. T. Lee, X.-Z Wang. On the generalization of fuzzy rough sets. IEEE Transactions on fuzzy systems, 13(2005)343-361

[16] E. Herrera-Viedma, F. Herrera, F. Chiclana, M. Luque. Some issues on consistency of fuzzy preference relations. European Journal of operational research 154 (2004) 98-109