Research on Edge Detection of Gray-scale Image based on Mathematical Morphology Algorithm and Rough Sets

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***Abstract*—A novel mathematical morphology algorithm based on rough sets is given for edge detection of gray-scale image. According to geometrical feature of image, the structure element is selected automatically to image details matching. The experimental results show that the method is better than the classical edge detection. It is also better than traditional mathematical morphological used single structure element.**

Keywords- mathematical morphology; rough sets; gray-scale image; edge detection

I. INTRODUCTION

Mathematical morphology is a new mathematical theory which can be used to process and analyze the images. It provides an alternative approach to image processing based on shape concept stemmed from set theory, not on traditional mathematical modeling and analysis. Image processing problem such as edge detection, noise restrain, feature extraction and image partition are usually adopt the Mathematical morphology method. For general morphological edge detection, some simply and symmetrical shape structure elements such as crisscross, diamond and disk are adopted. But there exist a lot of complex image in true life. Single structure element selection are only sensitive to image edge which has the same direction of structure elements, and are not so effective to the edge which has the direction other than the structure elements. Therefore, they are difficult to detect complex edge feature.

In this paper, a novel mathematic morphology algorithm based on and rough set is proposed and apply it to edge detection. Rough set has been widely applied in many fields since it was put forward by Poland mathematician Z.pawlak in 1982. The method is effective in artificial intelligence (AI) and knowledge process field. Especially in machine learning, pattern recognition, decision support and data mining. Lower approximation and upper approximation are two basic concepts in rough set theory. All objects is divided into three sets, that is negative, positive and uncertainty. These three sets is denote by lower approximation, negative space and boundary. This novel algorithm proposed in paper can select appropriate structure element automatically according image detail using mathematic morphology based on rough sets concept. Traditional mathematic morphology adopt single structure element in edge detection, there exist many disadvantages of coarseness and inaccurateness. The novel algorithm proposed in paper can overcome these disadvantages and make the image edge more precious than traditional mathematic morphology algorithm.

II. BASIC CONCEPTS OF ROUGH SETS AND MATHEMATICAL MORPHOLOGICAL THEORIES

A. Basic Concepts of Rough Sets

Rough sets have been introduced by pawlak to serve as approximate description of sets that are unknown, incompletely specified, or whose exact specification is complex. Human knowledge about a domain is expressed by classification. Rough set theory[1]-[3] treats knowledge as an ability to classify perceived objects into categories. Objects belonging to the same category are considered to be indistinguishable to each other.

Definition 1: Let U denote a finite and non-empty set of objects called the universe, and let $R \subseteq U \times U$ denote an equivalence relation on U. The pair (U, R) is called an approximation space. The equivalence classes partitioned by R are called elementary sets of (U, R).

The equivalence relation and the induced equivalence classes may by regarded as the available information or knowledge about the objects under consideration. Given an arbitrary set $X \subseteq U$, it may be impossible to describe X precisely using the equivalence classes of R. That is, the available information is not sufficient to give a precise representation of X. In this case, one may characterize X by a pair of lower and upper approximations:

$$\underline{RX} = \{x \in U : [x]_R \subseteq X\} \quad (1)$$

$$\overline{RX} = \{x \in U : [x]_R \bigcap X \neq \Phi\} \quad (2)$$

Where $[x]_R$ is the equivalence class containing x. The lower approximation $\underline{RX}$ is the union of all the elementary sets which are subset of X. It is the largest composed set contained in X. The upper approximation $\overline{RX}$ is the union of all the elementary sets which have a non-empty intersection with X. It is the smallest composed set containing X. The Fig.1 is figural description of rough sets.

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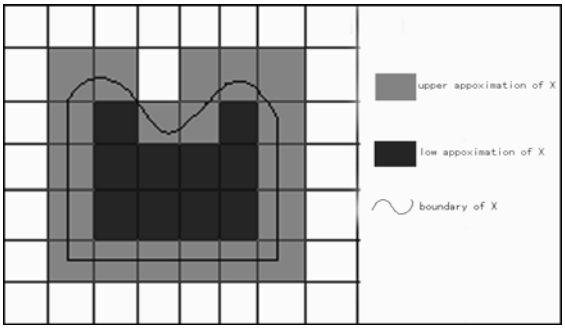


Figure 1. Figural description of rough sets

Definition 2: Let $K=(U,R)$ denote approximation sets. The roughness measures of rough set $X \subseteq U$ is presented as

$$\gamma_X = \frac{|\underline{RX}|}{|X|} \tag{3}$$

Roughness measures describe the measurement of sets. The bigger is the roughness measures, the more accurate is the sets. The smaller is the roughness measures, the more rough is the sets.

B. Basic Mathematical Morphology Theories

Mathematical morphology[4]-[6] is a powerful tool for dealing with various problems in image processing and computer vision. It is developed from set theory. The image which will be processed by mathematical morphology theory must been changed into set and mathematical morphology operation must be defined by set arithmetic. The basic morphological operations are erosion, dilation, opening, closing, and they are used for detection, modifying, manipulation the features present in the image based on their shapes. The shape and the size of structure elements (SE) play crucial roles in such type of processing and are therefore chosen according to the need and purpose of the associated application.

Let X denote image sets. B denote a SE. It is also a sets. B_x denote the centre of B locate at x. Through Moving B in X, B_x have three state:

① $B \subseteq X$; ② $B_x \subseteq X^c$; ③ $B_x \bigcap X \neq \phi, B_x \bigcap X^c \neq \phi$.

The first relation denote SE and X have the most correlativity. The second relation denote SE and X have no correlativity. The third relation denote SE and X have partly correlativity. Mathematical morphological define the erosion and dilation operator as follow

$$F \Theta B = \{x \in X : B_x \subseteq X\} \tag{4}$$

$$F \oplus B = \{x \in X : B_x \bigcap X \neq \phi\} \tag{5}$$

From above concept of rough sets and concept of mathematical morphological, there exist approximation between them. Low approximation approximate to erosion operator. Upper approximation approximate to dilation operator. Morphology process image depending on same structure B. Namely It use same granularity to process image from rough sets angle. This is not available in edge detection.

III. THE NOVEL MATHEMATIC MORPHOLOGY EDGE DETECTION ALGORITHM BASED ON ROUGH SETS

Firstly, we introduce some basic mathematical morphological operators of gray-scale images.

In the two-dimensional Euclidean space z^2 ,Let $F(x,y)$ denote a gray-scale two dimensional image, B denote SE. Dilation of a gray-scale image $F(x,y)$ by a gray-scale SE $B(s,t)$ is denoted by

$$(F \oplus B)(x,y) = \max\{F(x-s, y-t) + B(s,t)\} \tag{6}$$

Erosion of gray-scale image $F(x,y)$ by gray-scale SE $B(s,t)$ are denoted by

$$(F \Theta B)(x,y) = \min\{F(x+s,\ y+t) - B(s,t)\} \tag{7}$$

Erosion is a transformation of shrinking, which decreases the gray-scale value of the image, while dilation is a transformation of expanding, which increases the gray-scale value of the image. But both of them are sensitive to the image edge whose gray-scale value changes obviously. Erosion filters the inner image while dilation filters the outer image.

The edge of image F, which is denoted by $E_d(F)$, is defined as the difference set of dilation domain of F and the domain of F. This is also known as dilation residue edge detector:

$$E_d(F) = (F \oplus B) - F \tag{8}$$

Accordingly, the edge of image F, which is denoted by $E_e(F)$, can also be defined as the difference set of the domain of F and the erosion domain of F. This is also known as erosion residue edge detector:

$$E_e(F) = F - (F \Theta B) \tag{9}$$

The dilation and erosion often are used to compute the morphological gradient of image F, denoted by $E(F)$:

$$E(F) = (F \oplus B) - (F \Theta B) \tag{10}$$

The morphological gradient highlights sharp gray-level transition in the input image, and therefore, it is often used as edge detector.

Usually, people use single and symmetrical structure elements morphology to detect image edge. But they are

difficult to detect complex edge feature, because they are only sensitive to image edge which has the same direction of structure elements and are not so effective to the edge which has the direction other than the structure elements. So we proposed a noval algorithm based on rough sets. Firstly Look following definition 3.

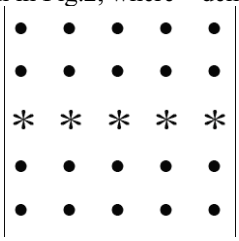
Definition 3. structure element $B_1,B_2, \ldots B_n$, Target image X, $\forall x \in X$, the best fit structure element B' to x is [7]

$$B' \in \{B^i \mid r_{B^i} = \max(r_{B_x^1}, r_{B_x^2}, \ldots, r_{B_x^n}), r_{B^i} = \frac{|RB_x^i|}{|B_x^i|}\} \tag{11}$$

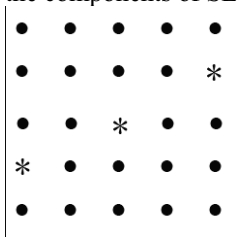
We select structure element according to the biggest roughness measure. That means the best structure element most felicitously match the pixel in image. The most difficult problem is selecting SE. The different SE selected turn out different result. According to define 3, we can select SE which have different granularity and different direction.

The mathematical morphological edge detection algorithm base on rough sets is follow.

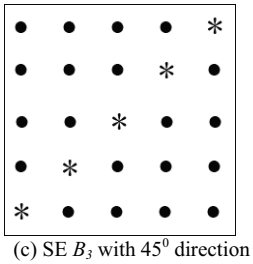
Step1 construct structure elements B_i of different directions. In this paper we construct eight SE which have different directions angle. The angles are 0^0,22.5^0, 45^0, 67.5^0, 90^0, 112.5^0, 135^0 and 157.5^0. These structure elements are shown in Fig.2, where * denotes the components of SE.



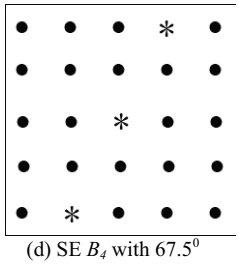
(a) SE *B1* with 0^0 direction angle



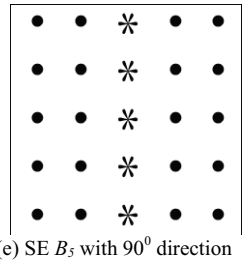
(b) SE B_2 with 22.5^0 direction angle



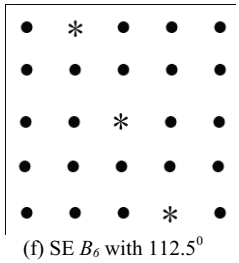
(c) SE B_3 with 45^0 direction angle



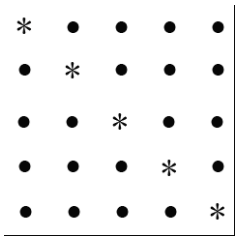
(d) SE B_4 with 67.5^0 direction angle



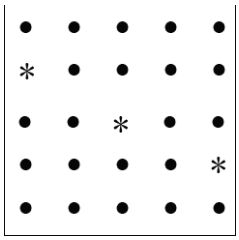
(e) SE B_5 with 90^0 direction angle



(f) SE B_6 with 112.5^0 direction angle



(g) SE B_7 with 135^0 direction angle



(k) SE B_8 with 157.5^0 direction angle

Figure 2. Eight different directional structure elements in 5×5 square window

Step 2. Choose the SE according following equation at each pixel.

$$B' = \{B^i \mid r_{B^i} = \max(r_{B_x^1}, r_{B_x^2}, \ldots, r_{B_x^8}), r_{B^i} = \frac{|RB_x^i|}{|B_x^i|}\}$$

Step3. Calculate the edge according (10) . SE is B' get by step 2.

The following Figural is experiment results.

(a) original lena image

(b) Sobel operator

(c) Prewitt operator

(d) Roberts operator

(e) Traditional morphology algorithm

(f) The algorithm proposed by paper

Figure 3. experiment results

the proposed mathematical morphological edge detection algorithm is compared with a variety of existing edge detection methods. Fig.3(a) is the original lena gray-scale image with 256 gray-scale levels. Fig.3(b), Fig.3(c) and Fig.3(d) are the results of processed original lena image after respectively applying sobel edge detector, Prewitt edge detector and roberts edge detector. Fig.3(e) are the result of result of processed original lena image by traditional mathematical morphology used by single SE. Fig.3(f) is the result of processed original lena image by the novel mathematical morphological edge detection algorithm based on rough sets.

According to the experiment results shown in Fig.3(c) and Fig.3(d), Prewiit operator and Roberts operator can't detect the edge integrally and continually. The Fig.3(b) resulted by Sobel operator can detect integrally and continually, but the edge information is more rough. Single SE morphological edge detect better than Sobel operator, Prewiit operator and Roberts operator. But comparing the novel algorithm proposed by paper, It is thicker than it. We can look hair, the edge detail information Fig.3(f) resulted by the novel algorithm is more than the Fig.3(e) resulted by the traditional mathematic morphology algorithm.

IV. CONCLUSIONS

Recently someone study multi-structure morphology on edge detection[8]. The edge detection use synthetic weighed method after result processed by each SE. although using multi-structure, they can not find the essence. In this paper, a novel mathematic morphological edge detection algorithm based on rough sets conception is proposed to detect image edge. Choose SE dynamically in processing image. The experimental results show that the edge more pinpointed, integral and continual than Canny operator, Sobel operator and Prewitt operator. And the result is thinner than the usually used single SE morphological edge detection operator. Meanwhile, the algorithm avoid problem of choosing the SE.

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