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Evolution of Unplanned Coordination in a Market Selection Game

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Short Abstract (ACE-related contribution)

This paper examines the evolution of unplanned coordination among independent agents in a market selection game, which is a non-cooperative repeated game with many agents (e.g., 100 agents) and several markets (e.g., five markets). Every agent is supposed to simultaneously choose a single market for maximizing its own payoff obtained by selling its product at the selected market. It is assumed that the market price is determined by a linear decreasing function of the total supply at each market. For example, if many agents choose a particular market, the market price at that market is low. On the contrary, the market price is high if only a small number of agents choose that market. In this manner, the market prices are determined by the actions of all agents. The point of our market selection game is to choose a market with a high market price, i.e., a market that is not chosen by many other agents. In this paper, the evolution of game strategies is performed by localized selection and mutation. A new strategy of an agent is probabilistically selected from its neighbors' strategies by the selection operation or randomly updated by the mutation operation. It is shown that the maximization of each agent's payoff through the genetic operations leads to the unplanned coordination of the market selection where the undesired concentration of agents is avoided. Through computer simulations, the unplanned coordination is compared with the planned global coordination realized by the maximization of the total payoff over all agents. Simulation results show that almost the same average payoff is obtained by these two schemes. That is, the selfish maximization of each agent's payoff through the genetic operations leads to a near-optimal result with respect to the total payoff over all agents. The unplanned coordination of the market selection realized by the evolution is also compared with some simple market selection methods such as a minimum transportation cost strategy, a random selection strategy, and an optimal strategy for the previous actions. Simulation results show that much higher payoff is obtained by the unplanned coordination than those methods.

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***Abstract* -** This paper examines the evolution of unplanned coordination among independent agents in a market selection game, which is a non-cooperative repeated game with many agents and several markets. Every agent is supposed to simultaneously choose a single market for maximizing its own payoff obtained by selling its product at the selected market. It is assumed that the market price is determined by the total supply of products. For example, if many agents choose a particular market, the market price at that market is low. The point of the market selection is to choose a market that is not chosen by many other agents. In this paper, game strategies are genetically updated by localized selection and mutation. A new strategy of an agent is probabilistically selected from its neighbors' strategies by the selection operation or randomly updated by the mutation operation. It is shown that the maximization of each agent's payoff leads to the unplanned coordination of the market selection where the undesired concentration of agents is avoided. The unplanned coordination is compared with the planned global coordination obtained by the maximization of the total payoff over all agents.

***Index Terms* -** Non-cooperative repeated game, evolution of game strategies, genetic algorithms, unplanned coordination, agent-based computational economics.

I. INTRODUCTION

Evolutionary computation has been successfully applied to various research fields from optimization to machine learning [1]-[5]. One interesting application area of evolutionary computation is game theory. Evolution of game strategies has been mainly studied for the Iterated Prisoner's Dilemma (IPD) game [6]-[9]. In those studies, a population of game strategies was evolved by genetic operations such as selection, crossover, and mutation. Evolution of game strategies was guided by the fitness value of each strategy evaluated through the execution of the IPD game. In this paper, we examine the evolution of game strategies for a market selection game, which was formulated in our former study [10] as a non-cooperative repeated game with many agents and several markets. An example of our market selection game is shown in Fig. 1 where 100 agents and five markets are located in a two-dimensional world $[0,100]\times[0,100]$. In every round of our game, each agent is supposed to simultaneously choose a single market for maximizing its own payoff obtained by selling its product at the selected market. Competition among various strategies such as a minimum transportation cost strategy and an optimal strategy for the previous actions was examined in [11]. Our market selection game was handled as a supervised learning problem in [12].

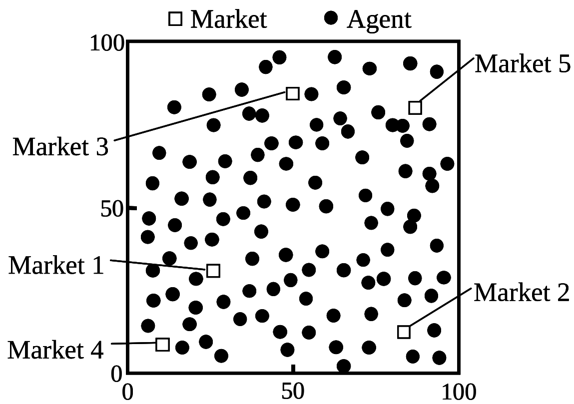


Fig. 1. Example of our market selection game.

In our market selection game, it is assumed that the market price at each market is determined by a linear decreasing function of the number of agents who choose that market. For example, if many agents choose a particular market, the market price at that market is low. On the contrary, the market price is high if the market is chosen by only a small number of agents. It should be noted that markets in our market selection game are not places where agent want to meet other agents. In such places, the more agents they can meet, the higher payoff they will obtain. On the other hand, makers in our market selection game are places where agents (i.e., producers) meet consumers. Thus the concentration of agents to a particular market causes the decrease in the market price at that market. In this manner, the market prices are determined by the actions of all agents. The payoff of each agent is defined by the market price at the selected market and the transportation cost from the agent's location to the selected market. If an agent chooses a market that is also chosen by many other agents, the payoff of that agent is low because the market price of the selected market is low. On the contrary, high payoff may be obtained from markets that are not chosen by many other agents. Thus the point of the market selection is to choose a market that is not chosen by many other agents. Since each agent is supposed to simultaneously choose a single market in every round of our market selection game, it is impossible to know the actions of the other agents before the market selection. Thus no agent knows its optimal choice in advance. That is, the market selection is performed based on the estimation of the actions of the other agents or the estimation of the expected payoff from each market.

The main characteristic feature of our market selection game is that many heterogeneous agents (e.g., 100 agents) with different locations are simultaneously involved. It contrasts with the IPD game where only two agents usually play the dilemma game. While the IPD game was extended to its *N*-player version [13], all the *N* players were still homogeneous. Each agent in our market selection game has its own location (i.e., different transportation costs to the markets). This means that a good strategy for an agent is not always appropriate for another agent. The payoff of each agent depends on the actions of the other agents. The interaction among many heterogeneous agents is the main characteristic feature of our market selection game.

In this paper, we show how the unplanned coordination of the market selection among independent agents can be evolved by simple genetic operations. We mean by "coordination" that the undesired concentration of agents to a particular market is avoided. The coordination also means that all the agents can enjoy high payoff. On the other hand, the term "unplanned" means that such coordination is realized by independent agents, each of which tries to maximize its own payoff (not the total payoff over all agents). In [14], "coordination" is defined as "*managing dependencies between activities*". Since there is no central planner (i.e., manager at the system level) in our market selection game, the coordination is not planned at the system level. Of course, each agent plans to maximize its own payoff. In this sense, the market selection is planned at the agent level while it is not planned at the system level. We use the term "unplanned" at the system level. We also use the term "undesired" at the system level. Since there is no central planner, one may think that the term "undesired" is meaningless. The concentration of many agents is not always undesirable for all agents. The concentration to a particular market is undesirable only for those agents who choose that market because the other agents may enjoy high market prices at the other markets that are not chosen by many agents. While the term "undesired" has a semantic problem, we use this term because the situation of the undesired concentration is easily and intuitively understood.

A strategy of each agent is denoted by a market selected by that agent. In the case of the market selection game in Fig. 1, a strategy of an agent is denoted by one of the five indexes {1, 2, 3, 4, 5}. In the evolution of game strategies in this paper, first one of the five markets is randomly assigned to each agent for executing the first round of our market selection game. After the first round, the strategy of each agent is updated based on the obtained payoff. The strategy update is performed by probabilistically selecting a new strategy of each agent from its neighbors' strategies. The selection probability of each strategy is specified based on the obtained payoff from that strategy. That is, the fitness value of each strategy is the payoff obtained by the agent with that strategy. Similar localized selection operations have been used in the spatial IPD game [15]-[18] and cellular (or fine-grained) genetic algorithms [19]-[21]. The strategy of each agent is also randomly replaced with another strategy with a prespecified mutation probability. The second round of the game is executed using the updated strategies. The strategy update and the game execution are iterated until a prespecified stopping condition is satisfied. Since the payoff of each agent is used as the fitness function, we can see that the evolution of game strategies corresponds to the maximization of the individual payoff of each independent agent.

Simulation results clearly show that the unplanned coordination of the market selection is realized by the evolution of game strategies for maximizing the individual payoff of each independent agent. The unplanned coordination is compared with the planned global coordination realized by the maximization of the total payoff over all agents. Simulation results show that almost the same average payoff is obtained from these two schemes: the unplanned coordination and the planned global coordination. That is, the maximization of the individual payoff by each independent agent leads to a near-optimal result with respect to the maximization of the total payoff over all agents. The obtained unplanned coordination is also compared with some non-evolutionary market selection methods such as a minimum transportation cost strategy, a random selection strategy, and an optimal strategy for the previous actions. Simulation results show that much higher payoff is obtained by the unplanned coordination than those non-evolutionary methods.

This paper is organized as follows. Section II briefly describes the formulation of our market selection game. Section III describes our algorithm for the evolution of game strategies. This algorithm can be viewed as a kind of a cellular genetic algorithm. Section IV shows through computer simulations that the unplanned coordination of the market selection is rapidly evolved among independent agents. The effect of parameter specifications such as the size of the neighborhood structure and the mutation probability is also examined. Section V compares the realized unplanned coordination of the market selection with some other market selection methods. The unplanned coordination is also compared with the planned global coordination obtained by a genetic algorithm for maximizing the total payoff over all agents. Finally Section VI concludes this paper.

II. PROBLEM FORMULATION

In this section, we briefly explain our market selection game formulated in [10]. Our market selection game is a non-cooperative repeated game with n agents and m markets (e.g., $n = 100$ and $m = 5$ in Fig. 1). Agents and markets are indexed by i and j, respectively ($i = 1,2,...,n$ and $j = 1,2,...,m$). Let us denote the total number of rounds of our game by T ($T = 1000$ in our computer simulations of this paper). The number of rounds is indexed by t where $t = 1,2,...,T$. We assume that every agent has a single product to be sold in each round of our game. The action of each agent in each round is to choose a single market where its product is sold. Let us denote the action of the i-th agent in the t-th round as

$$x_{ij}^{t} = \begin{cases} 1, & \text{if the } i\text{-th agent chooses the } j\text{-th market at the } t\text{-th round,} \\ 0, & \text{otherwise,} \end{cases} \qquad \text{for } j = 1,2,\ldots,m . \tag{1}$$

Because every agent is supposed to choose a single market from the given m markets for selling its product, the following relation holds:

$$\sum_{j=1}^{m} x_{ij}^{t} = 1 . \tag{2}$$

The market selection is simultaneously performed by all agents in every round of our market selection game. No agent knows the current actions of the other agents before the current round is completed. This means that no agent knows the optimal market selection in advance. In Fig. 2, we show an example of the market selection where the line from each agent indicates the selected market (i.e., the line corresponds to the flow of the product). The payoff of each agent is determined by the market price at the selected market and the transportation cost of the product from the agent's location to the selected market.

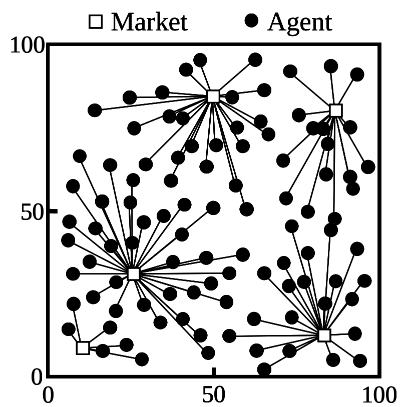


Fig. 2. Example of the market selection. Every agent chooses its nearest market.

It is assumed in our market selection game that the market price at each market is determined by a linear decreasing function of the total amount of products brought to that market. The total amount of products brought to the *j*-th market in the *t*-th round is calculated from the actions of all agents as

$$X_j^t = \sum_{i=1}^{n} x_{ij}^{t} . \tag{3}$$

The market price at the *j*-th market in the *t*-th round is determined by the following linear decreasing function:

$$p_j^t = a_j - b_j \cdot X_j^t , \tag{4}$$

where a_j and b_j are positive constants that specify the market price in the *j*-th market. In our computer simulations of this paper, we used the same market price determination mechanism for all the five markets in Fig. 1:

$$p_j^t = 200 - 3 \cdot X_j^t \quad \text{for} \quad j = 1,2,...,5 . \tag{5}$$

The transportation cost of the product is dependent on the distance between the agent's location and the selected market. Let d_{ij} be the distance from the i-th agent's location to the j-th market. The transportation cost to the j-th market is defined for the i-th agent as follows:

$$c_{ij} = c \cdot d_{ij}, \tag{6}$$

where c is the transportation cost for the unit distance. In our computer simulations of this paper, we specified the value of c as $c = 1$ (i.e., $c_{ij} = d_{ij}$). As shown by (6), the location of each agent is related to the transportation cost to each market. The location of each agent is also related to the available information for that agent. In the evolution of the unplanned coordination of the market selection, we assume that each agent knows the previous strategies and payoff of its neighboring agents. A fixed number of agents are defined for each agent as its neighbors using the distance between agents.

The payoff of each agent is determined by the market price at the selected market and the transportation cost to that market. When the i-th agent chooses the j-th market at the t-th round, its payoff is defined as follows:

$$r_i^t = p_j^t - c_{ij}. \tag{7}$$

From (1)-(2), this formulation can be rewritten as

$$r_i^t = \sum_{j=1}^{m} x_{ij}^t \cdot (p_j^t - c_{ij}). \tag{8}$$

It should be noted that the payoff of each agent depends on the actions of the other agents through the market price (see (3) and (4)). The aim of each agent in our market selection game is to maximize its own payoff over T rounds.

III. EVOLUTION OF GAME STRATEGIES

In this section, we propose an algorithm for the evolution of game strategies. The proposed algorithm is based on the strategy update and the game execution. The strategy update is performed by localized selection and mutation.

A. Initial Strategy of Each Agent

As shown in Fig. 2, the strategy of each agent corresponds to the choice of a single market. Let s_i^t be the market selected by the i-th agent in the t-th round. Thus s_i^t can be viewed as the strategy of the i-th agent in the t-th round. The strategy of each agent in the initial round is randomly specified as one of the given m markets with the probability $1/m$.

B. Fitness Evaluation

Since the strategy of each agent is deterministic, its fitness value can be calculated by the execution of a single round of our market selection game. All agents simultaneously perform the market selection according to their current strategies. Then the payoff of each agent is calculated by (8). The calculated payoff is used as the fitness value of the strategy of that agent. Let $r_i^t(s_i^t)$ be the payoff of the i-th agent with the strategy s_i^t in the t-th round. $r_i^t(s_i^t)$ is used as the fitness value of s_i^t for updating the current strategies after the t-th round.

C. Localized Selection

With a prespecified replacement probability P_r , the current strategy of each agent is replaced with one of its neighbors' strategies. Let $N(i)$ be the neighbors of the i-th agent. We define $N(i)$ by the N nearest neighbors from the i-th agent. It should be noted that the i-th agent itself is included in $N(i)$ as the nearest neighbor. Thus $N(i)$ consists of the i-th agent and the other $(N-1)$ nearest neighbors. In Fig. 3, we illustrate the neighborhood structure $N(i)$ for the case of $N=4$. $N(i)$ in Fig. 3 consists of the i-th agent and the other three nearest neighbors.

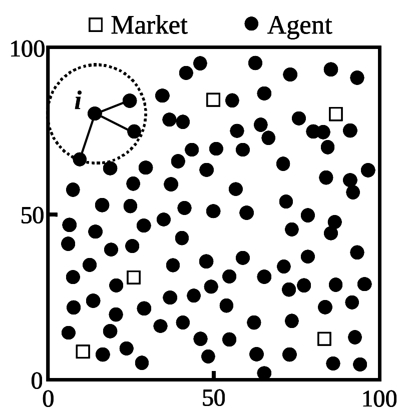


Fig. 3. Illustration of the neighborhood structure $N(i)$ with four neighbors (i.e., $N=4$).

With the replacement probability P_r , s_i^t of the i-th agent is replaced with one of its neighbors' strategies in $N(i)$. When s_i^t of the i-th agent is replaced, the strategy s_k^t of its neighbor k ($k \in N(i)$) is selected with the following selection probability:

$$P(s_k^t) = \frac{r_k^t(s_k^t) - r_{\min}^t(N(i))}{\sum_{k \in N(i)} \{r_k^t(s_k^t) - r_{\min}^t(N(i))\}}, \tag{9}$$

where $r_{\min}^t(N(i))$ is the minimum payoff among the neighbors in $N(i)$:

$$r_{\min}^t(N(i)) = \min\{r_k^t(s_k^t) : k \in N(i)\}. \tag{10}$$

The replacement procedure is applied to each agent with the replacement probability P_r. Thus strategies of some agents are not updated.

D. Mutation

A mutation operation is applied with a prespecified mutation probability P_m to each of the current strategies after the replacement procedure. The mutation operation randomly changes the current strategy of an agent into a different strategy. The application of the mutation operation is independent of the application of the replacement procedure with the localized selection. Thus both operations may be applied to some agents while neither operation may be applied to other agents. The updated strategies after these two operations are used in the next round of our market selection game as s_i^{t+1}, $i = 1,2,...,n$.

E. Algorithm

We have already explained each procedure in our algorithm for the evolution of game strategies. Our algorithm is summarized as follows:

Step 1: Randomly specify the initial strategy of each agent.

Step 2: Execute a single round of our market selection game using the current strategies of the agents. Then calculate the payoff of each agent.

Step 3: With the replacement probability P_r, replace the strategy of each agent with one of its neighbors' strategies.

Step 4: With the mutation probability P_m, randomly replace the strategy of each agent with a different strategy.

Step 5: If a prespecified stopping condition is not satisfied, return to Step 2.

IV. COMPUTER SIMULATIONS

We performed computer simulations by applying the proposed algorithm to the market selection game in Fig. 1. Our algorithm was iterated until the 1000th generation (i.e., the 1000th round of our market selection game). In this section, we first demonstrate that the unplanned coordination of the market selection can be rapidly evolved by our algorithm with the best parameter specifications. Then we examine the effect of parameter specifications on the evolution of the unplanned coordination.

A. Parameter Specifications

In our computer simulations, we examined $18 \times 10 \times 4$ combinations of the following parameter specifications for the size *N* of the neighborhood structure $N(i)$, the replacement probability P_r, and the mutation probability P_m:

$N = 2, 3, 4, ..., 10, 20, 30, ..., 100,$

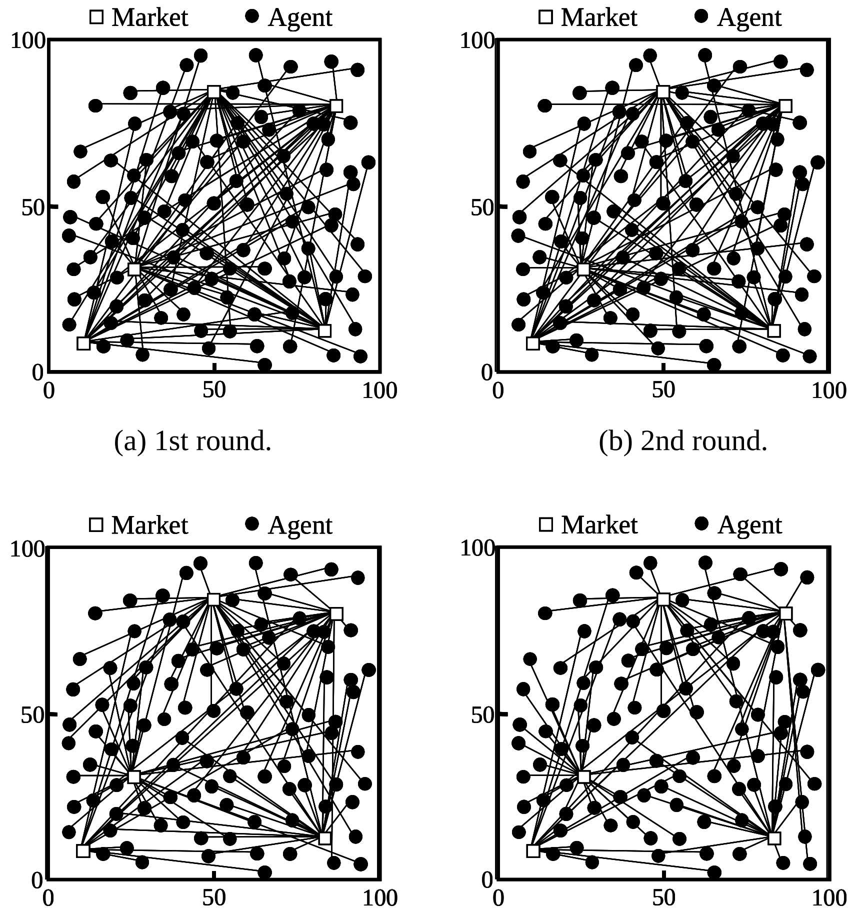
$P_r = 0.1, 0.2, 0.3, ..., 1.0,$

$P_m = 0, 0.1, 0.01, 0.001.$

Good results were obtained from wide ranges of these parameter values: $N = 3 \sim 10$, $P_r = 0.1 \sim 1.0$ and $P_m = 0$, 0.01, 0.001. The best result was obtained from $N = 4$, $P_r = 0.5$, and $P_m = 0$. In the next subsection, we show simulation results by our algorithm with these best parameter values for demonstrating that the unplanned coordination of the market selection among independent agents can be rapidly evolved. We discuss the effect of parameter specifications on the evolution of the unplanned coordination in later subsections.

B. Simulation Results with the Best Parameter Specifications

In this subsection, we show that the unplanned coordination of the market selection among independent agents can be rapidly evolved by the proposed algorithm. We applied our algorithm with $N = 4$, $P_r = 0.5$, and $P_m = 0$ to the market selection game in Fig. 1. In the initial round, the strategy of each agent was randomly specified. Thus the market selection in the first round was in great disorder as shown in Fig. 4 (a). After the first round, the localized selection operation was applied to each agent with the replacement probability P_r. When this operation was applied to an agent, the current strategy of that agent was replaced with one of its neighbors' strategies. Such a simple replacement procedure gradually organized the market selection as shown in Fig. 4.



(a) 1st round. (b) 2nd round.

(c) 3rd round. (d) 4th round.

Fig. 4. Market selection in the first four rounds.

This computer simulation was continued until the 1000th round. The unplanned coordination of the market selection was almost fully evolved until the 20th round as shown in Fig. 5. In this figure, each market was selected by almost the same number of agents. That is, the undesired concentration of agents was avoided. Furthermore each agent selected one of its nearest markets for avoiding unnecessarily high transportation cost.

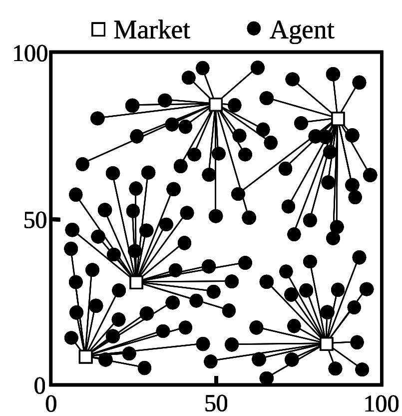


Fig. 5. Market selection in the 20th round.

Table 1 summarizes simulation results over the 1000 rounds in terms of the payoff of each agent. This table shows the highest, average, and lowest payoff among the 100 agents in each round. From this table, we can see that the average payoff was rapidly improved in the first 10 rounds. After the 20th round, the average payoff was almost the same. This means that the unplanned coordination of the market selection was almost fully evolved until the 20th round. From Table 1, we can also see that the lowest payoff was also rapidly improved in the first 10 rounds. This means that all the agents enjoyed the benefit of the unplanned coordination.

Table 1. Highest, average, and lowest payoff among the 100 agents in each round.

Round	1	2	3	4	5	10	20	30	40	50	100	500	1000
Highest	145.9	139.9	154.9	151.9	142.4	145.9	142.9	148.9	142.9	139.4	136.9	136.9	139.9
Average	83.9	90.3	96.4	102.8	106.1	115.7	118.7	117.7	118.1	118.6	118.5	118.4	118.6
Lowest	38.7	41.8	44.7	48.4	49.9	86.4	92.8	85.9	89.8	93.6	93.6	93.3	99.6

Fig. 4, Fig. 5 and Table 1 are simulation results obtained from a single trial. Since our algorithm is based on the random assignment of initial strategies and the probabilistic strategy update, one may think that totally different simulation results can be obtained from multiple independent trials. For examining this issue, we independently performed the same computer simulation ten times from different initial strategies. Fig. 6 shows the average payoff obtained by each of the ten independent trials. That is, Fig. 6 simultaneously shows ten results in a single figure while they are not clearly distinguishable. From this figure, we can see that the average payoff was improved in a very similar way in all the ten independent trials. That is, the rapid evolution of the unplanned coordination was observed in all the ten independent trials. High average payoff was obtained until the 50th round in all the ten independent trials.

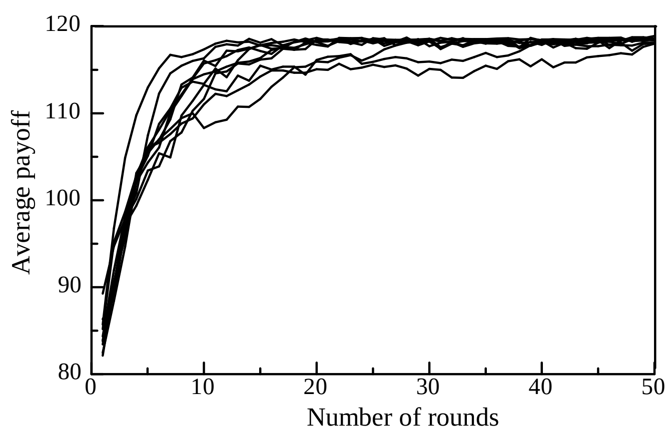


Fig. 6. Simulation results of ten independent trials.

C. Effect of Neighborhood Structure

The localization of the selection operation has a significant effect on the evolution of the unplanned coordination of the market selection among independent agents. This is because the coordination of the market selection requires the speciation of strategies as shown in Fig. 5. That is, spatially close agents should have the same strategy (i.e., should choose the same market) for avoiding unnecessarily high transportation cost. At the same time, each strategy (i.e., the choice of each market) should not be shared by too many agents for avoiding the undesired concentration of agents. The speciation of strategies is disturbed when the neighborhood structure is too large. On the other hand, when the neighborhood structure is too small, the propagation of a good strategy may be restricted in a small number of agents. For example, if two agents are closely located as shown in Fig. 7, their strategies cannot be propagated by the localized selection operation to the other agents in the case of $N = 2$. Thus too small neighborhood structures as well as too large neighborhood structures have bad effects on the evolution of the unplanned coordination of the market selection.

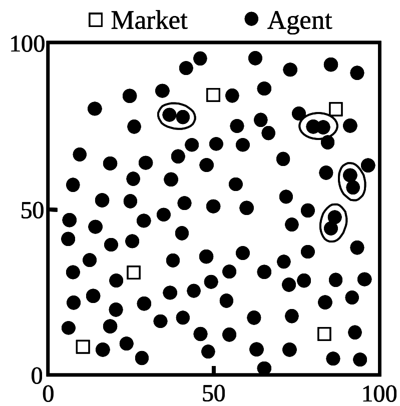


Fig. 7. Examples of pairs of closely located agents in our market selection game.

For clearly illustrating the above discussion, we performed computer simulations using various specifications of the size of the neighborhood structure. The other conditions such as the replacement probability and the mutation probability were specified in the same manner as in the previous subsection. For each specification of the neighborhood structure, we performed ten independent trials. Then we calculated the average payoff obtained by the 100 agents among the 1000 rounds in the ten trials. Simulation results are summarized in Table 2. From Table 2, we can see that high average payoff was obtained when N was specified as $N = 4 \sim 10$. We can also see that too small and too large neighborhood structures led to poor average payoff. It should be noted that the average payoff 84.4 in the case of $N = 100$ is almost the same as the result by the randomly assigned initial strategies in Table 1 (i.e., average payoff 83.9). Since our market selection game involved 100 agents, the selection was not localized in the case of $N = 100$. That is, a new strategy of each agent was selected from all the current strategies. As a result, very poor payoff was obtained in the case of $N = 100$.

Since large neighborhood structures prevent the speciation of strategies (e.g., Fig. 5), high average payoff could not be obtained in Table 2 when *N* was large.

Table 2. Average payoff obtained by each specification of the neighborhood structure.

# of neighbors (*N*)	2	3	4	5	6	7	8	9	10	20	50	100
Average payoff	97.7	110.1	118.0	117.9	117.7	117.5	117.3	116.9	116.6	111.7	93.3	84.4

The value of *N* in Table 2 can be viewed as an index of the amount of the available information for each agent. Each agent knows the previous strategies and payoff of *N* agents (i.e., that agent and its $(N-1)$ nearest neighbors). For example, each agent only knows the previous strategies and payoff of that agent and its single nearest neighbor in the case of $N = 2$. The low average payoff in this case (i.e., 97.7 in Table 2) is due to such limitation of the available information. The best average result was obtained in Table 2 when each agent had the information about its three neighbors (i.e., when $N = 4$). It is interesting to note that good results were not obtained when each agent knew the previous strategies and payoff of all agents (i.e., when $N = 100$). That is, too much available information hindered the evolution of the unplanned coordination.

D. Effect of Replacement Probability

The replacement probability P_r controls the number of replaced strategies after each round of our market selection game. We examined ten specifications of P_r : P_r = 0.1, 0.2, ..., 1.0. The other parameters were specified in the same manner as in the previous computer simulations (i.e., their best specifications were used). While small values of P_r slowed down the evolution of the unplanned coordination in early rounds (see Fig. 8), the specification of P_r does not have a large effect in the long run. The average payoff obtained from ten independent trials for each value of P_r is shown in Table 3. From this table, we can see that very good results were obtained for all values of P_r .

Table 3. Average payoff obtained by each specification of the replacement probability.

Replacement probability	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Average payoff	117.4	117.9	118.0	118.0	118.0	118.0	118.0	117.9	117.8	117.7

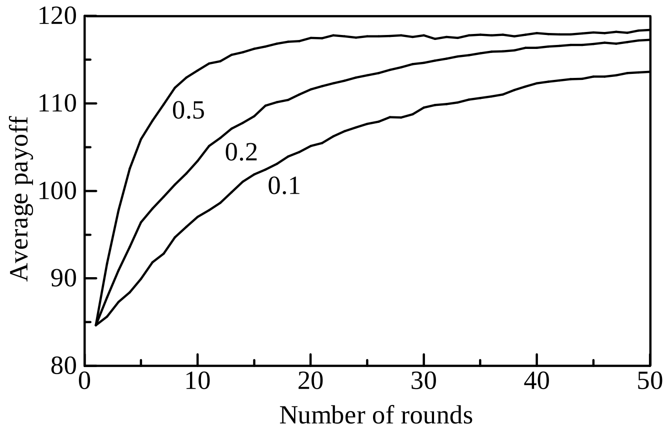


Fig. 8. Effect of the specification of the replacement probability on the evolution in early generations. Ten independent trials were performed for each specification. The average result over those ten trials for each specification is shown in this figure.

E. Effect of Mutation Probability

The role of the mutation operation is to randomly introduce new strategies to agents. If a good strategy cannot propagate to some agents through the iterative execution of the localized selection operation, the mutation operation is necessary for the evolution of the unplanned coordination of the market selection. At the same time, the mutation operation may disturb the coordination. For examining such positive and negative effects of the mutation operation, we examined 4×3 combinations of four mutation probabilities (P_m = 0.1, 0.01, 0.001, 0) and three neighborhood structures ($N = 2, 3, 4$). The other parameter values were specified in the same manner as in the previous computer simulations. Simulation results are summarized in Table 4. Table 4 shows the average payoff obtained from ten independent trials for each combination of P_m and N . In this table, the negative effect of the mutation operation is clear. That is, good results were not obtained in the case of $P_m = 0.1$. In this case, ten agents among the 100 agents randomly changed their strategies at each generation on the average. Such mutation disturbed the coordination of the market selection. The best result was obtained in the case of no mutation (i.e., $P_m = 0$) when the size of the neighborhood structure was specified as $N = 4$. We can also observe the positive effect of the mutation operation in Table 4. That is, the average payoff was improved by increasing the mutation probability from 0 to 0.001 and 0.01 when the neighborhood structure was too small (i.e., when $N = 2, 3$). In these cases, the available information for each agent was limited. Each agent only knew the previous strategies and payoff of only a few agents. Occasional random change of the market selection complements the limitation of the available information when N is too small.

Table 4. Effect of the specification of the mutation probability.

Neighborhood structure	Mutation probability			
	0.1	0.01	0.001	0
$N = 2$	105.3	112.3	102.2	97.7
$N = 3$	109.9	116.2	112.1	110.1
$N = 4$	110.4	117.3	118.0	118.0

F. Effect of Probabilistic Selection

In the previous computer simulations, we used the probabilistic selection defined by the roulette wheel in (9). For examining the necessity of the probabilistic selection, let us consider a deterministic selection of the best strategy among the neighboring agents. This deterministic selection scheme always selects the best strategy with the highest payoff in the neighborhood structure $N(i)$ for the *i*-th agent. Of course, the replacement using the deterministic selection is applied to each agent with the replacement probability P_r as in the case of the probabilistic selection.

In the same manner as in the previous computer simulations, we examined the evolution of the unplanned coordination using the deterministic selection. Simulation results are summarized in Table 5. For comparison, we also show simulation results with the probabilistic selection in Table 5. From this table, we can see that the probabilistic selection was not necessary when the size of the neighborhood structure was small (i.e., when *N* was small). On the other hand, the average payoff from the deterministic selection was much smaller than that from the probabilistic selection when the size of the neighborhood structure was larger (i.e., when *N* was large). This inferiority of the deterministic selection can be easily understood if we consider the extreme case with $N = 100$. In this case, all agents are included in the neighborhood structure of any agent. This means that the current strategy of an agent is replaced with the best strategy in the current round among all agents with the replacement probability. As a result, many agents choose the same market in the next round. In this manner, the undesired concentration of agents to some markets happens in each round.

Table 5. Average payoff obtained from the two selection schemes.

# of neighbors (*N*)	2	3	4	5	6	7	8	9	10	20	50	100
Probabilistic	97.7	110.1	118.0	117.9	117.7	117.5	117.3	116.9	116.6	111.7	93.3	84.4
Deterministic	97.9	110.9	118.3	118.2	117.0	115.6	114.3	112.5	111.7	105.3	83.4	19.0

V. COMPARISON OF DIFFERENT METHODS

In the previous section, we have demonstrated that the unplanned coordination of the market selection could be rapidly evolved by our algorithm designed for maximizing the individual payoff of each independent agent. In this section, we examine our market selection game and the realized unplanned coordination through computer simulations using some non-evolutionary market selection methods in our former studies [10]-[12].

A. Random Selection Strategy

One important issue in our market selection game is to avoid the undesired concentration of agents to particular markets. The simplest way for avoiding such concentration is to randomly select a market. We iterated our market selection game for 1000 rounds by assigning this random selection strategy to all the 100 agents. This computer simulation was performed ten times. The average payoff over the ten independent trials was 84.8. This average payoff is much worse than the average result 118.0 by our algorithm with the best parameter specifications.

B. Mimic Strategy

Another simple strategy of each agent is to mimic the previous choice of its nearest agent (excluding the agent itself). In the first round, every agent randomly chooses a market because there is no available information to be mimicked. After the first round, this strategy mimics the previous choice of the nearest agent independent of its performance (i.e., independent of the payoff of the nearest agent in the previous round). While the mimic strategy looks similar to the case of $N = 2$ in our evolutionary model in the previous section, they are totally different from each other. As shown in the selection probability in (9), the replacement of the agent's strategy with its neighbor's strategy depends on the payoff of each agent in the previous round in our evolutionary model. Each agent with the mimic strategy, however, always mimics the previous choice of its nearest neighbor even when the agent's previous payoff was much higher than its nearest neighbor's previous payoff. In other words, each agent only knows the previous choice of its nearest neighbor agent and does not know the nearest neighbor's previous payoff in the mimic strategy.

In the same manner as in the case with the random selection strategy, we performed computer simulations using this mimic strategy. The average payoff over ten independent trials was 82.4. This average payoff is almost the same as the result by the random selection strategy (i.e., average payoff 84.8). That is, no coordination was evolved by the mimic strategy. This is because any information about the payoff of each agent was not utilized in the mimic strategy.

C. Minimum Transportation Cost Strategy

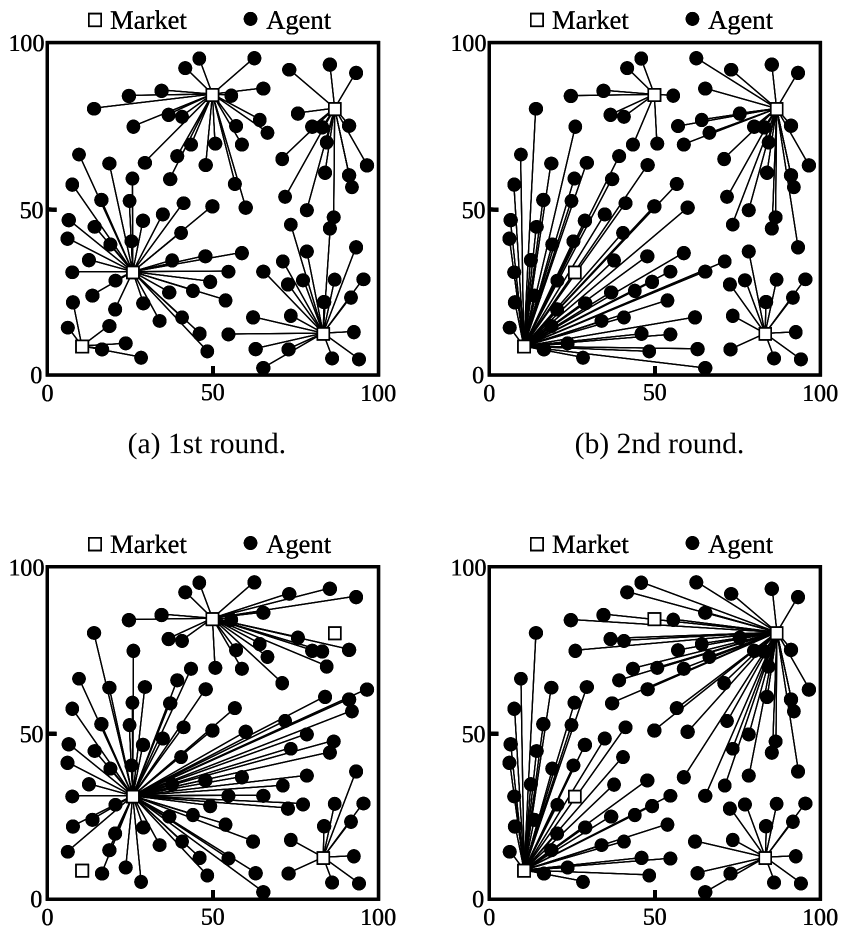
In our market selection game, the transportation cost can be minimized by choosing the nearest market. Fig. 2 shows the market selection when all agents use this minimum transportation cost strategy. The average payoff in this situation is 108.0. This is the average payoff by the minimum transportation cost strategy for the same market selection game as in the previous computer simulations. While the transportation cost is minimized, the undesired concentration of agents is observed in Fig. 2. That is, some markets are chosen by too many agents. As a result, the average payoff 108.0 by the minimum transportation cost strategy is not as high as the result obtained from the unplanned coordination (i.e., average payoff 118.0).

D. Optimal Strategy for the Previous Actions

As we have already explained, no agent knows the optimal market selection for the current round because every agent is supposed to simultaneously select a market. Every agent, however, knows the best market for the previous round of our market selection game. Of course, the best market was not always actually selected in the previous round. When the previous round is completed, every agent knows its best market in the previous round. In general, the best market for the previous round is not the best for the current round. When the other agents do not change their choices, the best market for the previous round is also the best for the current round. The optimal strategy for the previous actions chooses a market that was the best choice in the previous round. In the first round, this strategy chooses the nearest market. This is because the nearest market is the optimal choice in the situation where no market is chosen by any other agents.

We iterated our market selection game for 1000 rounds by assigning the optimal strategy for the previous actions to all agents. The obtained average payoff from ten independent trials was 47.2. This surprisingly low payoff was due to the synchronized oscillation of the market selection. In this computer simulation, no coordination of the market selection was evolved. The market selection was gradually organized in the form of the synchronized oscillation. In the first round, the market selection was the same as Fig. 2 (also shown in Fig. 9 (a)) because the nearest market was selected by each agent. For the market selection in the second round, each agent calculated its best market for the given (i.e., actually chosen) actions of the other agents in the first round. Then each agent chose the calculated best market in the second round. In this manner, the selection of the best market for the previous actions was iterated until the 1000th round. Fig. 9 shows how the synchronized oscillation was evolved.

In Table 6, we summarize the average simulation result and the available information in each market selection method. As shown in this table, the average payoff by the optimal strategy for the previous actions is very small while it needs a lot of information. This shows that a large amount of available information does not always lead to good results. Similar results were observed in Table 2 of Subsection IV.C where the relation between the average payoff and the specification of the neighborhood structure $N(i)$ was examined (also see the case of $N = 100$ in Table 6). Table 6 also shows that good results were not obtained when the amount of available information was too small (e.g., when the random strategy was used). Similar results were observed in Table 2 where good results were not obtained from too small neighborhood structures (also see the case of $N = 2$ in Table 6).



(a) 1st round. (b) 2nd round.

(c) 3rd round. (d) 4th round.

Fig. 9. Evolution of the synchronized oscillation of the market selection.

Table 6. Average payoff obtained by each strategy and the available information.

Market selection method	Average payoff	Available information
Unplanned coordination	97.7	Previous actions and payoff of N agents ($N = 2$)
	118.0	Previous actions and payoff of N agents ($N = 4$)
	84.4	Previous actions and payoff of N agents ($N = 100$)
Random selection	84.8	No information
Mimic strategy	82.4	Previous actions of the nearest neighbor agent
Minimum cost strategy	108.0	Transportation cost to each market
Optimal strategy for previous actions	47.2	Previous actions of all the other agents Transportation cost to each market Price mechanism at each market

E. Global Coordination by Genetic Algorithms

All the previous computer simulations were performed in the framework of non-cooperative games. That is, each agent tried to maximize its own payoff. In this subsection, we handle our market selection game as a cooperative game for comparing the unplanned coordination of the market selection with the planned coordination that maximizes the total payoff over all agents. Of course, the handling as a cooperative game is just for analyzing our market selection game. In other words, we assume only in this subsection that there is a central planner (i.e., manager) who can observe and control the actions of all agents. The objective of the central planner is to maximize the total payoff over all agents.

In the framework of cooperative games, the total payoff over all agents is maximized. That is, our objective function is defined as

$$r^t = \sum_{i=1}^{n} r_i^t \, , \tag{11}$$

where r_i^t is the payoff of the i-th agent in the t-th round of our market selection game, which is defined by (8). For maximizing the total payoff in (11), we use a genetic algorithm.

In this subsection, the market selection by n agents is coded as a string of length n as $s = s_1 s_2 \cdots s_n$ where s_i denotes the market selected by the i-th agent. The fitness value of the string s is defined by the total payoff in (11). In our genetic algorithm in this subsection, first a number of strings of length n are generated by randomly assigning a market to each agent. Next the fitness value of each string is calculated as the total payoff obtained from our market selection game. Then new strings are generated by selection, crossover, and mutation. The generated new strings form the next population. The best string in the current population is inherited to the next population with no modification. Such generation update is iterated until a prespecified stopping condition is satisfied.

We applied our genetic algorithm to our market selection game for maximizing the total payoff over all agents. We used the roulette wheel selection with the linear scaling as in (9), the standard uniform crossover, and the mutation operation in the previous sections. Parameters in our genetic algorithm were specified as follows:

Population size: 100,

Crossover probability: 0.8,

Mutation probability: 0.001,

Stopping condition: 10,000 generations.

We performed computer simulations with these parameter specifications 20 times. The best average payoff obtained from the 20 trials was 119.2. This best result was obtained from 19 trials among the 20 trials. Fig. 10 shows the market selection corresponding to this result. From the comparison between Fig. 10 and Fig. 5, we can see that the unplanned coordination in Fig. 5 is very similar to the planned global coordination in Fig. 10. Actually the average payoff 118.7 in Fig. 5 is almost the same as 119.2 in Fig. 10. From these observations, we can conclude that the evolved unplanned coordination is near-optimal with respect to the maximization of the total payoff over all agents.

In Fig. 10, the highest payoff and the lowest payoff over the 100 agents are 139.9 and 97.9, respectively. The lowest payoff 97.9 in Fig. 10 is slightly worse than the case of the finally obtained unplanned coordination in Table 1 (i.e., the lowest payoff 99.6). This is because the global coordination in Fig. 10 was obtained for maximizing the total payoff over all agents while the unplanned coordination was obtained for maximizing the individual payoff of each independent agent.

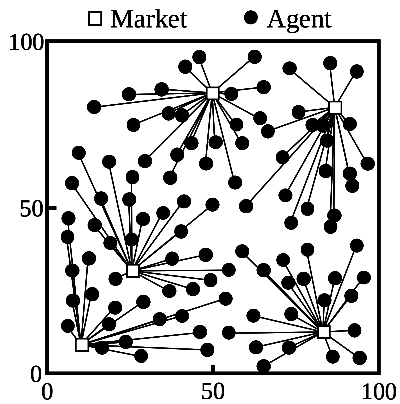


Fig. 10. Best result with respect to the total payoff over all agents.

In Fig. 11, we show how the average payoff was improved in each of the 20 independent trials of the genetic algorithm. That is, 20 results are shown in this figure while they are not distinguishable. From Fig. 11, we can see that the average payoff was quickly improved in the first 100 rounds (i.e., 100 generations) and gradually improved until the 500th generation. From the comparison of Fig. 11 (i.e., evolution of the planned coordination by the central planner) with Fig. 6 (i.e., evolution of the unplanned coordination with no central planner), we can see that the pattern of the increase in the average payoff is very similar to each other. The important point is that the genetic algorithm in Fig. 11 required much more generations than that in Fig. 6 (note the difference in the scale of the horizontal axis between Fig. 6 and Fig. 11). Moreover, it should be noted that each population includes 100 strings (i.e., 100 solutions of the market selection) in Fig. 11 while each population in Fig. 6 corresponds to a single solution. This means that the evolution of the planned coordination in Fig. 11 required much more computation load than the unplanned coordination in the previous section.

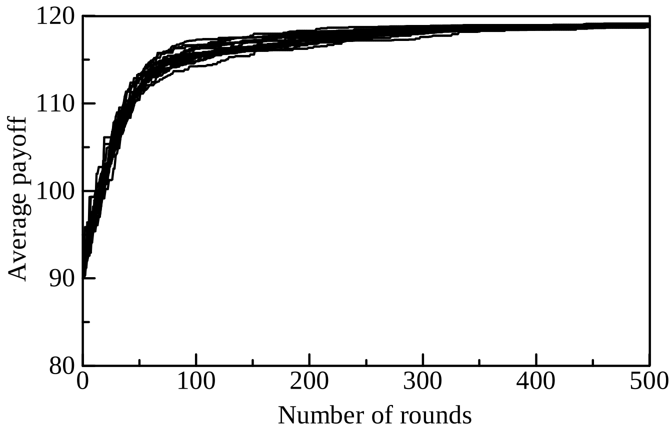


Fig. 11. Simulation results of 20 independent trials by the genetic algorithm with the central planner. The average payoff was calculated for the elite string in each round (i.e., generation) of each trial.

VI. CONCLUSIONS

In this paper, we demonstrated that the unplanned coordination of the market selection could be rapidly evolved by simple genetic operations designed for maximizing the individual payoff of each independent agent. That is, the evolution was driven by each independent agent without any central planner (i.e., manager) who can observe and control the actions of all agents. The undesired concentration of agents was avoided by the evolution of the unplanned coordination where all agents could enjoy high payoff. It was also shown that the evolved unplanned coordination with no central planner was very similar to the planned global coordination with the central planner whose objective was the maximization of the total payoff over all agents. This means that the evolved unplanned coordination was near-optimal with respect to the maximization of the total payoff over all agents. These results show that the selfish maximization of the individual payoff by each independent agent leads to the unplanned coordination where all agents enjoy high payoff. From the viewpoint of computation load required for the evolution, the market selection by each independent agent was much more efficient than that by the central planner. Roughly speaking, the computation load required for the evolution of the unplanned coordination was about 1/1000 of that for the planned global coordination.

Through computer simulations, we demonstrated that the appropriate localization of the selection operation was necessary for the evolution of the unplanned coordination. When we did not localize the selection, simulation results were almost the same as the random selection. On the other hand, when the neighborhood structure for the localization was too small, the unplanned coordination was not well evolved. Good results were obtained from the neighborhood structure with four to ten agents. The localization of the selection can be viewed as the restriction on the amount of the available information for each agent. When the neighborhood structure was large (or when there was no neighborhood structure), each agent knew the previous actions and payoff of many neighboring agents (or all agents). On the other hand, when the neighborhood structure was small, each agent knew the previous actions and payoff of only a few neighboring agents. As expected, when the amount of the available information was too small, good results were not obtained. When the amount of available information was too large, good results were not obtained either. Only when each agent knew the previous actions and payoff of several agents (i.e., four to ten neighbors), very good results were obtained.

Our market selection model can be extended in various manners. All those extensions are left for future research. Some of them are:

(1) Modification of the assumptions on the market selection and the strategy update: Our model in this paper is based on the simultaneous market selection and the synchronized strategy update by all agents. Different results may be obtained if we modify these assumptions.

(2) Studies on more complicated models: Our market selection model is very simple. For example, each agent has a single unit of the product, each agent should choose a single market, the market price at each market is determined by the fixed decreasing function, there is no alliance between agents, etc. We can easily generate more complicated models by modifying some of these settings of our model.

(3) Modelling for other situations different from market selection: The basic characteristic feature of the payoff mechanism in our model is that the concentration of agents leads to poor payoff. This characteristic feature is shared by various everyday situations. For example, a highway chosen by too many drivers dose not give them high payoff due to traffic jams, the entrance examination becomes very tough when many students choose the same department, etc. Our model can be applied to such situations after minor modifications.

Some computer programs and data used in computer simulations of this paper are available from our web site: http://www.ie.osakafu-u.ac.jp/~hisaoi/ci_lab_e/index.html.

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