Centrality_1

March 17, 2020

In network analysis we often seek to distill the large network into a few attributes so that we can better understand its structure. A good place to start is finding which subgraphs of the network contain the most information. Thinking of nodes as the most elementary subgraph we arrive at our first step in understanding our network: *Which nodes are most important?*

This is what data scientists refer to as Node Centrality and is a Local-Scale Feature.

0.0.1 Measuring node centrality

Degree Centrality What if we simply measured a node's important by how many nodes it is connected to? Also known as Degree Centrality, it takes a myopic approach to a node's importance. Let's say that you are friends with a hundred people, but your friend Jake is only friends with 10. Degree centrality would say that you are more important in the social network than Jake, but what it failed to account for is that one of Jake's 10 friends was the President of the United States!

We need a measure that accounts for the importance of its neighboring nodes (i.e. the importance of your friends).

Eigenvector centrality If you have a computer science background you can probably sense recursion is near, but don't worry, eigenvectors will save the day!

We define the importance of node i as

$$\phi(k) = k\sum_{j=1}^{n} A(i,j)\phi(j)$$

where A is the adjacency matrix.

All this means is that the importance of node i is the sum of the importance of its neighboring nodes. In the social network example, your importance is the sum of the importance of your friends. This recursive approach does not seem very attractive, but if we rearrange things an important pattern emerges:

First, notice that $\sum_{j=1}^{n} A(i,j)\phi(j)$ is the element-wise view of matrix-vector multiplication: $\sum_{j=1}^{n} A_{i,j}x_j = Ax$

$$\phi(k) = k\sum_{j=1}^{n} A(i,j)\phi(j)$$

$$\phi = k(A\phi)$$

$$A\phi = \frac{1}{k}\phi$$

This last expression is simply an eigenvector problem [^](note this and that) with eigenpair: $(\lambda = \frac{1}{k}, v = \phi)$. *Recall canonical form $Av = \lambda v$

Therefore, although the recursive definition is easier to understand, we can solve for each node's centrality score by simply solving the eigenvector problem (i.e. find ϕ) once.

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