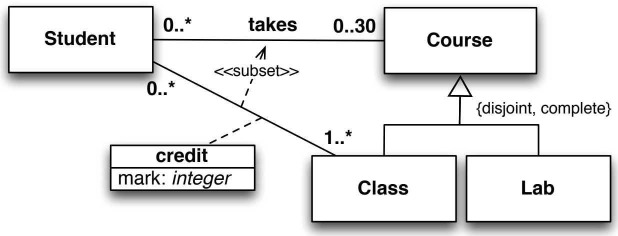
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

Formal Methods – Feb. 13, 2017

(Time to complete the test: 2 hours)

Exercise 1. Express the following UML class diagram in FOL.



Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

Student

peter
paul
mary
jane

Class

calculus
AI
FM
algorithms

Lab

IoT lab
db lab
hacking lab

credit/mark

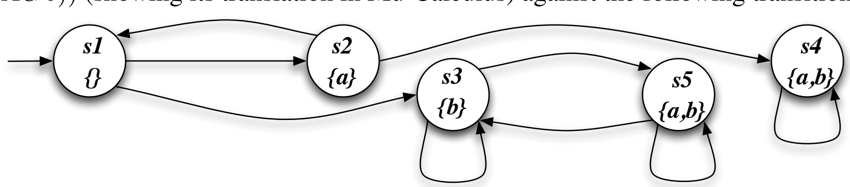
peter	algorithm	30
paul	calculus	27
mary	algorithms	28
mary	AI	30
jane	FM	30
jane	algorithms	30

takes

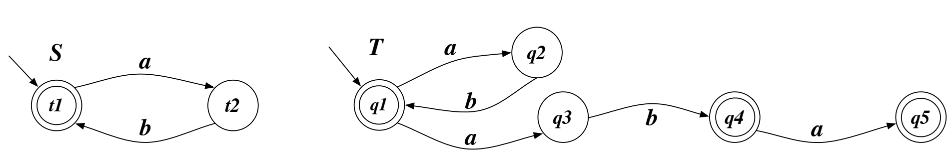
peter	IoT lab
paul	IoT lab
mary	FM
jane	db lab
jane	hacking lab
jane	IoT lab

1. Check whether the instantiation (once completed) is correct (and explain why it is or it is not).
2. Express in FOL and evaluate the following queries:
   (a) Return students that have taken at least 3 courses.
   (b) Return students that have taken only classes.
   (c) Check if there exists a student that has taken all labs.
   (d) Check if there is a student that has taken all classes, but not for credit.

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge [next]X) \vee [next]Y)$ and the CTL formula $EF(\neg a \supset (EX\, a \wedge EX AG\, b))$ (showing its translation in Mu-Calculus) against the following transition system:



Exercise 4. Consider the following two transition systems:



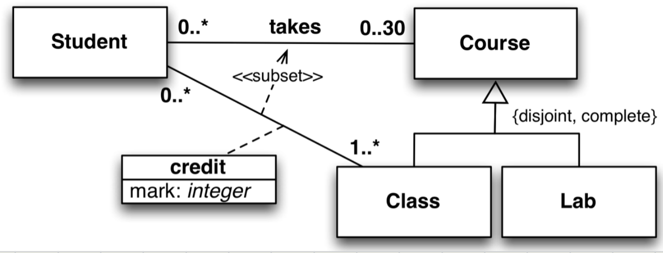
Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

Exercise 5. Given the following conjunctive queries:

```
q1(x) :- edge(x,y), edge(y,z), edge(z,x).
q2(x) :- edge(x,y), edge(x,w), edge(y,z), edge(z,x), edge(z,v), edge(v,y), edge(v,w), edge(w,z).
```

check whether $q1$ is contained into $q2$, explaining the technique used and, in case of containment, showing the homomorphism between the canonical databases.

Exercise 1. Express the following UML class diagram in FOL.



$S(x)$, $C(x)$, $CL(x)$, $L(x)$
$TAKES(x,y)$
$CREDIT(x,y)$
$MARK(x,y,z)$

$\forall x,y.\ TAKES(x,y) \supset S(x) \land C(y)$
$\forall x.\ S(x) \supset 0 \leq \#\{y \mid TAKES(x,y)\} \leq 30$
$\forall y.\ C(y) \supset 0 \leq \#\{x \mid TAKES(x,y)\}$

$\forall x,y.\ CREDIT(x,y) \supset S(x) \land CL(y)$
$\forall x.\ S(x) \supset \exists y.\ CREDIT(x,y)$
$\forall y.\ CL(y) \supset 0 \leq \#\{x \mid CREDIT(x,y)\}$
$\forall x,y,z.\ MARK(x,y,z) \supset CREDIT(x,y) \land INT(z)$

$\forall x,y.\ CREDIT(x,y) \supset TAKES(x,y)$

$\forall x.\ CL(x) \supset C(x)$
$\forall x.\ L(x) \supset C(x)$
$\forall x.\ CL(x) \supset \lnot L(x)$
$\forall x.\ C(x) \supset CL(x) \lor L(x)$

Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

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1) $C = \{\text{CALCULUS}, \text{AI}, \text{FM}, \text{ALG}, \text{IOT}, \text{DB}, \text{HACK}\}$

$\forall x,y.\ \text{TAKES}(x,y) \supset S(x) \land C(y)$

THE RELATION CONTAINS STUDENT AND COURSES, AND CARDINALS ARE OK ✓

$\forall x,y.\ \text{CREDIT}(x,y) \supset S(x) \land CL(y)$

THE RELATION CONTAINS STUDENT AND CLASSES, AND CARDINALS ARE OK ✓

2) a. $\exists c,c',c''.\ S(x) \land \text{TAKES}(x,c) \land \text{TAKES}(x,c') \land \text{TAKES}(x,c'') \land c \neq c' \land c \neq c'' \land c' \neq c''$

$\{\text{JANE}\}$

b. $\forall c.\ S(x) \land (\text{TAKES}(x,c) \supset \text{CLASS}(c))$

$\{\text{MARY}\}$

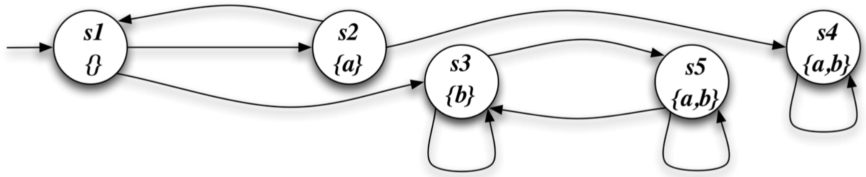
c. $\forall \ell.\ \exists s.\ S(s) \land (L(\ell) \supset \text{TAKES}(s,\ell))$

$\{\text{TRUE}\}$

d. $\exists s.\ S(s) \land (\forall c.\ \text{CLASS}(c) \supset \text{TAKES}(s,c) \land \neg\exists m.\ \text{CREDIT}(s,c,m))$

$\{\text{FALSE}\}$

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge [next]X) \vee [next]Y)$ and the CTL formula $EF(\neg a \supset (EX\, a \wedge EX\, AG\, b))$ (showing its translation in Mu-Calculus) against the following transition system:



1) $\nu X.\mu Y.((a \wedge [NEXT]X) \vee [NEXT]Y)$

$[X_0] = \{1,2,3,4,5\}$

$[X_1] = [\mu Y.((a \wedge [NEXT]X_0) \vee [NEXT]Y)$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREA(NEXT, X_0) \cup PREA(NEXT, Y_0) =$

$= (\{2,4,5\} \cap \{1,2,3,4,5\}) \cup \phi = \{2,4,5\}$

$[Y_2] = ([a] \cap PREA(NEXT, X_0) \cup PREA(NEXT, Y_1) =$

$= (\{2,4,5\} \cap \{1,2,3,4,5\}) \cup \{4\} = \{2,4,5\}$

$[Y_1] = [Y_2] = [X_1] = \{2,4,5\}$

$[X_2] = [\mu Y.((a \wedge [NEXT]X_1) \vee [NEXT]Y)$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREA(NEXT, X_1) \cup PREA(NEXT, Y_0) =$

$= (\{2,4,5\} \cap \{4\}) \cup \phi = \{4\}$

$[Y_2] = ([a] \cap PREA(NEXT, X_1) \cup PREA(NEXT, Y_1) =$

$= (\{2,4,5\} \cap \{4\}) \cup \{4\} = \{4\}$

$[Y_1] = [Y_2] = [X_2] = \{4\}$

$[X_3] = [\mu Y.((a \wedge [NEXT]X_2) \vee [NEXT]Y)$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREA(NEXT, X_2) \cup PREA(NEXT, Y_0) =$

$= (\{2,4,5\} \cap \{4\}) \cup \phi = \{4\}$

$[Y_2] = ([a] \cap PREA(NEXT, X_2) \cup PREA(NEXT, Y_1) =$

$= (\{2,4,5\} \cap \{4\}) \cup \{4\} = \{4\}$

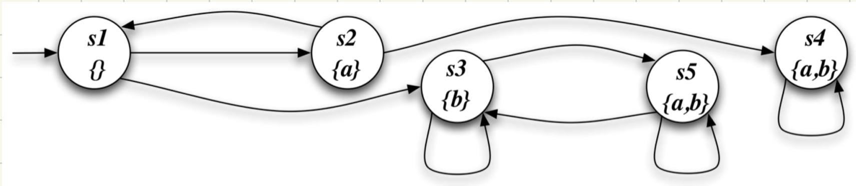
$[Y_1] = [Y_2] = [X_3] = \{4\}$

$[X_2] = [X_3] = \{4\}$

$s_1 \in [\nu X. \mu Y. ((a \wedge [NEXT]X) \vee [NEXT]Y)] = \{4\}$? NO!

2) $EF(\neg a \supset (\underbrace{EX\, a}_{\gamma} \wedge EX \underbrace{AG\, b}_{\alpha}))$

β, δ, ε, η



$[\alpha] = [AG\, b] = [\nu Z. b \wedge [NEXT]Z]$

$[Z_0] = \{1,2,3,4,5\}$

$[Z_1] = [b] \cap PREA(NEXT, Z_0) =$

$= \{3,4,5\} \cap \{1,2,3,4,5\} = \{3,4,5\}$

$[Z_2] = [b] \cap PREA(NEXT, Z_1) =$

$= \{3,4,5\} \cap \{3,4,5\} = \{3,4,5\}$

$[Z_1] = [Z_2] = [\alpha] = \{3,4,5\}$

$[\beta] = [EX\, \alpha] = [\langle NEXT \rangle \alpha] = PREE(NEXT, \alpha) = \{1,2,3,4,5\} = [\beta]$

$[\gamma] = [EX\, a] = [\langle NEXT \rangle a] = PREE(NEXT, a) = \{1,2,3,4,5\} = [\gamma]$

$[\delta] = [\gamma \wedge \beta] = [\gamma] \cap [B] = \{1,2,3,4,5\} \cap \{1,2,3,4,5\} = \{1,2,3,4,5\} = [\delta]$

$[\varepsilon] = [\neg a \supset \delta] = [a] \cup [\delta] = \{2,4,5\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5\} = [\varepsilon]$

$[\eta] = [EF\, \varepsilon] = [\mu Z\, \varepsilon \vee \langle NEXT \rangle Z]$

$[Z_0] = \phi$

$[Z_1] = [\varepsilon] \cup PREE(NEXT, Z_0) =$

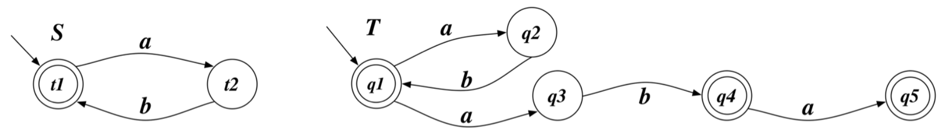
$= \{1,2,3,4,5\} \cup \phi = \{1,2,3,4,5\}$

$[Z_2] = [\varepsilon] \cup PREE(NEXT, Z_1) =$

$= \{1,2,3,4,5\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5\}$

$[Z_1] = [Z_2] = [\eta] = \{1,2,3,4,5\}$ $\quad T_{s_1} \models \varepsilon$? $\rightarrow s_1 \in [\varepsilon]$? YES!

Exercise 4. Consider the following two transition systems:



Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

TWO TS ARE BISIMILIAR IF THEY HAVE THE SAME BEHAVIOR:

- LOCALLY THEY LOOK INDISTINGUISHABLE
- EVERY ACTION THAT CAN BE DONE ON ONE, CAN ALSO BE DONE ON THE OTHER, AND THEY REACH THE SAME RESULT

$R_0 = T \times S = \{(q_1,t_1),(q_1,t_2),(q_2,t_1),(q_2,t_2),(q_3,t_1),(q_3,t_2),$
$(q_4,t_1),(q_4,t_2),(q_5,t_1),(q_5,t_2)\}$

$R_1 = \{(q_1,t_1),(q_2,t_2),(q_3,t_2),\cancel{(q_4,t_1)},\cancel{(q_5,t_1)}\}$ FINALI O NON FINALI

$R_2 = \{(q_1,t_1),(q_2,t_2),\cancel{(q_3,t_2)}\}$

$R_3 = \{\cancel{(q_1,t_1)},(q_2,t_2)\}$

$R_4 = \{\cancel{(q_2,t_2)}\}$

$R_5 = \{\ \}$

$R_6 = \{\ \}$

$R_5 = R_6$ GFP FOUND

$(q_1,t_1) \notin$ GFP SO T AND S ARE NOT BISIMILIAR

Exercise 5. Given the following conjunctive queries:

```
q1(x) :- edge(x,y), edge(y,z), edge(z,x).
q2(x) :- edge(x,y), edge(x,w), edge(y,z), edge(z,x), edge(z,v), edge(v,y), edge(v,w), edge(w,z).
```

check whether $q1$ is contained into $q2$, explaining the technique used and, in case of containment, showing the homomorphism between the canonical databases.

CHECK WHETHER $q_1(x) \subseteq q_2(x)$

FREEZE

$q_1(c) \subseteq q_2(c)$

$$\begin{cases} q_1(c): \text{EDGE}(c,y), \text{EDGE}(y,z), \text{EDGE}(z,c) \\ q_2(c): \text{EDGE}(c,y), \text{EDGE}(c,w), \text{EDGE}(y,z), \text{EDGE}(z,c), \text{EDGE}(z,v), \text{EDGE}(v,y), \text{EDGE}(v,w), \text{EDGE}(w,z) \end{cases}$$

BUILD CANONICAL INTERPRETATION

$I_{q_1(c)}$: $\Delta^{q_1(c)}: \{c, y, z\}$

$\text{EDGE}^{q_1(c)}: \{\langle c,y\rangle, \langle y,z\rangle, \langle z,c\rangle\}$

$I_{q_2(c)}$: $\Delta^{q_2(c)}: \{c, y, z, v, w\}$

$\text{EDGE}^{q_2(c)}: \{\langle c,y\rangle, \langle c,w\rangle, \langle y,z\rangle, \langle z,c\rangle, \langle z,v\rangle, \langle v,y\rangle, \langle v,w\rangle, \langle w,z\rangle\}$

QUERY ANSWERING

$I_{q_1(c)} \models q_2(c)$?

$\alpha(y) = ? \rightarrow \alpha(y) = y$
$\alpha(v) = ? \rightarrow \alpha(v) = c$
$\alpha(w) = ? \rightarrow \alpha(w) = y$
$\alpha(z) = ? \rightarrow \alpha(z) = z$

$I_{q_1(c)}, \alpha \models q_2(c)$? YES

HOMOMORPHISM

$h(c) = c$
$h(y) = \alpha(y) = y$
$h(v) = \alpha(v) = c$
$h(w) = \alpha(w) = y$
$h(z) = z$

$(c,y) \in \text{EDGE}^{q_2(c)} \Rightarrow (h(c), h(y)) \in \text{EDGE}^{q_1(c)}$

$(c,w) \in \text{EDGE}^{q_2(c)} \Rightarrow (h(c), h(w)) \in \text{EDGE}^{q_1(c)}$

$(y,z) \in \text{EDGE}^{q_2(c)} \Rightarrow (h(y), h(z)) \in \text{EDGE}^{q_1(c)}$

$(z,c) \in \text{EDGE}^{q_2(c)} \Rightarrow (h(z), h(c)) \in \text{EDGE}^{q_1(c)}$

$(z,v) \in \text{EDGE}^{q_2(c)} \Rightarrow (h(z), h(v)) \in \text{EDGE}^{q_1(c)}$

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