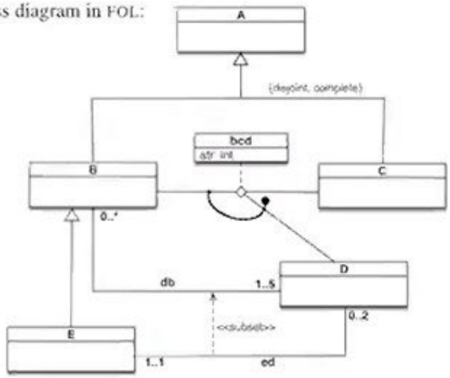
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

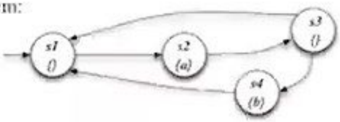
Formal Methods – February 21, 2025

(Time to complete the test online: 2:00 hours)

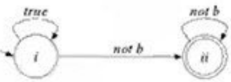
Exercise 1 Express the following UML class diagram in FOL:



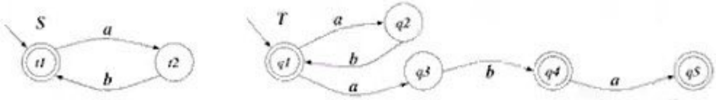
Exercise 2 Consider the following transition system:



- **Exercise 2.1:** Model check the Mu-Calculus formula: $\nu X.\mu Y.((a \wedge \langle next \rangle X) \vee (\neg b \wedge \langle next \rangle Y)$
- **Exercise 2.2:** Model check the CTL formula $AG(AFa \wedge EFb \wedge EG\neg b)$, by translating it in Mu-Calculus.
- **Exercise 2.3:** Model check the LTL formula $\Box\Diamond b$, by considering that the Büchi automaton for $\neg\Box\Diamond b$ (i.e., $\Diamond\Box\neg b$) is:



Exercise 4. Consider the following two transition systems:



Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

Exercise 5. Compute the certain answers to the CQs $q(x) \leftarrow contains(x, y), genre(y, z)$ and $q(x, z) \leftarrow contains(x, y), genre(y, z)$ over the following incomplete database (naive tables), and discuss how you obtained the result:

contains

cd	*song*
cd1	$null_1$
$null_2$	sg1
cd2	sg2
cd3	sg1
cd4	$null_3$
$null_4$	$null_3$

genre

object	*color*
$null_1$	rock
sg1	rock
sg2	blues
$null_3$	$null_5$

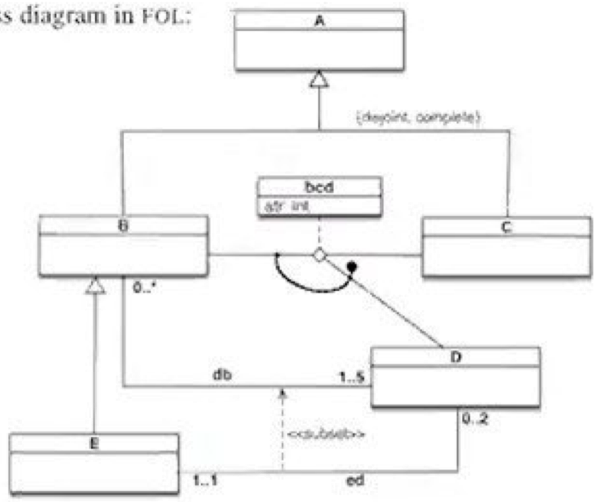
t c check

$D \models q(c)$

$q_D = q(c)$

$T \models q(c)$

Exercise 1 Express the following UML class diagram in FOL:



$A(x),\ B(x),\ C(x),\ D(x),\ E(x)$
$BCD(x,y,z)$
$ATR(x,y,z,w)$
$DB(x,y)$
$ED(x,y)$

$\forall x,y,z.\ \ BCD(x,y,z) \supset B(x) \wedge C(y) \wedge D(y)$
$\forall x,y,y',z.\ \ BCD(x,y,z) \wedge BCD(x,y',z) \supset y = y'$
$\forall x,y,z,w.\ \ ATR(x,y,z,w) \supset BCD(x,y,z) \wedge INT(w)$

$\forall x,y.\ \ DB(x,y) \supset D(x) \wedge B(y)$
$\forall y.\ B(y) \supset 1 \leq \#\{x \mid DB(x,y)\} \leq 5$

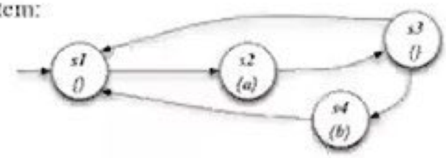
$\forall x,y.\ \ ED(x,y) \supset E(x) \wedge D(y)$
$\forall x.\ E(x) \supset 0 \leq \#\{y \mid ED(x,y)\} \leq 2$
$\forall y.\ D(y) \supset 1 \leq \#\{x \mid ED(x,y)\} \leq 1$

$\forall x,y.\ \ ED(x,y) \supset DB(x,y)$

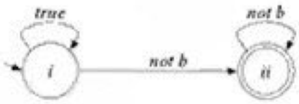
$\forall x.\ E(x) \supset B(x)$

$\forall x.\ B(x) \supset A(x) \wedge \neg C(x)$
$\forall x.\ C(x) \supset A(x)$
$\forall x.\ A(x) \supset B(x) \vee C(x)$

Exercise 2 Consider the following transition system:



- **Exercise 2.1:** Model check the Mu-Calculus formula: $\nu X.\mu Y.((a \wedge \langle next \rangle X) \vee (\neg b \wedge \langle next \rangle Y)$
- **Exercise 2.2:** Model check the CTL formula $AG(AFa \wedge EFb \wedge EG\neg b)$, by translating it in Mu-Calculus.
- **Exercise 2.3:** Model check the LTL formula $\Box\Diamond b$, by considering that the Büchi automaton for $\neg\Box\Diamond b$ (i.e., $\Diamond\Box\neg b$) is:



1) $\varphi = \nu X.\mu Y.((a \wedge \langle NEXT \rangle X) \vee (\neg b \wedge \langle NEXT \rangle Y))$

$[X_0] = \{1,2,3,4\}$

$[X_1] = [\mu Y.((a \wedge \langle NEXT \rangle X_0) \vee (\neg b \wedge \langle NEXT \rangle Y))]$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREE(NEXT, X_0)) \cup ([\neg b] \cap PREE(NEXT, Y_0)) =$

$= (\{2\} \cap \{1,2,3,4\}) \cup (\{1,2,3\} \cap \phi) = \{2\}$

$[Y_2] = ([a] \cap PREE(NEXT, X_0)) \cup ([\neg b] \cap PREE(NEXT, Y_1)) =$

$= (\{2\} \cap \{1,2,3,4\}) \cup (\{1,2,3\} \cap \{1\}) = \{1,2\}$

$[Y_3] = ([a] \cap PREE(NEXT, X_0)) \cup ([\neg b] \cap PREE(NEXT, Y_2)) =$

$= (\{2\} \cap \{1,2,3,4\}) \cup (\{1,2,3\} \cap \{1,3,4\}) = \{1,2,3\}$

$[Y_4] = ([a] \cap PREE(NEXT, X_0)) \cup ([\neg b] \cap PREE(NEXT, Y_3)) =$

$= (\{2\} \cap \{1,2,3,4\}) \cup (\{1,2,3\} \cap \{1,2,3,4\}) = \{1,2,3\}$

$[Y_3] = [Y_4] = [X_1] = \{1,2,3\}$

$[X_2] = [\mu Y.((a \wedge \langle NEXT \rangle X_1) \vee (\neg b \wedge \langle NEXT \rangle Y))]$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREE(NEXT, X_1)) \cup ([\neg b] \cap PREE(NEXT, Y_0)) =$

$= (\{2\} \cap \{1,2,3,4\}) \cup (\{1,2,3\} \cap \phi) = \{2\}$

$[Y_2] = ([a] \cap PREE(NEXT, X_1)) \cup ([\neg b] \cap PREE(NEXT, Y_0)) =$

$= (\{2\} \cap \{1,2,3,4\}) \cup (\{1,2,3\} \cap \{1\}) = \{1,2\}$

$[Y_3] = ([a] \cap \text{PREE}(\text{NEXT}, X_1)) \cup ([\neg b] \cap \text{PREE}(\text{NEXT}, Y_2)) =$

$= (\{2\} \cap \{1,2,3,4\}) \cup (\{1,2,3\} \cap \{1,3,4\}) = \{1,2,3\}$

$[Y_4] = ([a] \cap \text{PREE}(\text{NEXT}, X_1)) \cup ([\neg b] \cap \text{PREE}(\text{NEXT}, Y_3)) =$

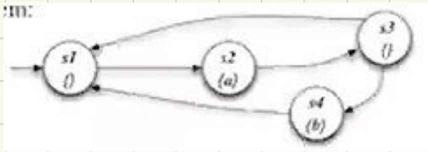
$= (\{2\} \cap \{1,2,3,4\}) \cup (\{1,2,3\} \cap \{1,2,3,4\}) = \{1,2,3\}$

$[Y_3] = [Y_4] = [X_2] = \{1,2,3\}$

$[X_1] = [X_2] = \{1,2,3\}$

$s_1 \in \varphi$? YES!

2) $$\underbrace{AG(\underbrace{\underbrace{AF\,a}_{\gamma} \wedge \underbrace{EF\,b}_{\beta} \wedge \underbrace{EG\,\neg b}_{\alpha})}_{\delta}}_{\varepsilon}$$



$$[\alpha] = [EG\ \neg b] = [\nu Z.\ \neg b \wedge \langle NEXT \rangle Z]$$

$$[Z_0] = \{1,2,3,4\}$$

$$[Z_1] = [\neg b] \cap PREE(NEXT, Z_0) =$$

$$= \{1,2,3\} \cap \{1,2,3,4\} = \{1,2,3\}$$

$$[Z_2] = [\neg b] \cap PREE(NEXT, Z_1) =$$

$$= \{1,2,3\} \cap \{1,2,3,4\} = \{1,2,3\}$$

$$[Z_1] = [Z_2] = [\alpha] = \{1,2,3\}$$

$$[\beta] = [EF\ b] = [\mu Z.\ b \vee \langle NEXT \rangle Z]$$

$$[Z_0] = \phi$$

$$[Z_1] = [b] \cup PREE(NEXT, Z_0) =$$

$$= \{4\} \cup \phi = \{4\}$$

$$[Z_2] = [b] \cup PREE(NEXT, Z_1) =$$

$$= \{4\} \cup \{3\} = \{3,4\}$$

$$[Z_3] = [b] \cup PREE(NEXT, Z_2) =$$

$$= \{4\} \cup \{2,3\} = \{2,3,4\}$$

$$[Z_4] = [b] \cup PREE(NEXT, Z_3) =$$

$$= \{4\} \cup \{1,2,3\} = \{1,2,3,4\}$$

$$[Z_5] = [b] \cup PREE(NEXT, Z_4) =$$

$$= \{4\} \cup \{1,2,3,4\} = \{1,2,3,4\}$$

$$[Z_4] = [Z_5] = [\beta] = \{1,2,3,4\}$$

$[\gamma] = [AF\ \alpha] = [\mu Z.\ \alpha \vee [NEXT]\, Z]$

$[Z_0] = \emptyset$

$[Z_1] = [\alpha] \cup PREA(NEXT, Z_0) =$

$= \{2\} \cup \emptyset = \{2\}$

$[Z_2] = [\alpha] \cup PREA(NEXT, Z_1) =$

$= \{2\} \cup \{1\} = \{1,2\}$

$[Z_3] = [\alpha] \cup PREA(NEXT, Z_2) =$

$= \{2\} \cup \{1,4\} = \{1,2,4\}$

$[Z_4] = [\alpha] \cup PREA(NEXT, Z_3) =$

$= \{2\} \cup \{1,3,4\} = \{1,2,3,4\}$

$[Z_5] = [\alpha] \cup PREA(NEXT, Z_3) =$

$= \{2\} \cup \{1,2,3,4\} = \{1,2,3,4\}$ $\quad [Z_4] = [Z_5] = [\gamma] = \{1,2,3,4\}$

$[\delta] = [\alpha \wedge \beta \wedge \gamma] = [\alpha] \wedge [\beta] \wedge [\gamma] = \{1,2,3\} \cap \{1,2,3,4\} \cap \{1,2,3,4\} = \{1,2,3\} = [\delta]$

$[\varepsilon] = [AG\ \delta] = [\nu Z.\ \delta \wedge [NEXT]\, Z] =$

$[Z_0] = \{1,2,3,4\}$

$[Z_1] = [\delta] \cap PREA(NEXT, Z_0) =$

$= \{1,2,3\} \cap \{1,2,3,4\} = \{1,2,3\}$

$[Z_2] = [\delta] \cap PREA(NEXT, Z_1) =$

$= \{1,2,3\} \cap \{1,2,4\} = \{1,2\}$

$[Z_3] = [\delta] \cap PREA(NEXT, Z_2) =$

$= \{1,2,3\} \cap \{1,4\} = \{1\}$

$[Z_4] = [\delta] \cap PREA(NEXT, Z_3) =$

$= \{1,2,3\} \cap \{4\} = \emptyset$

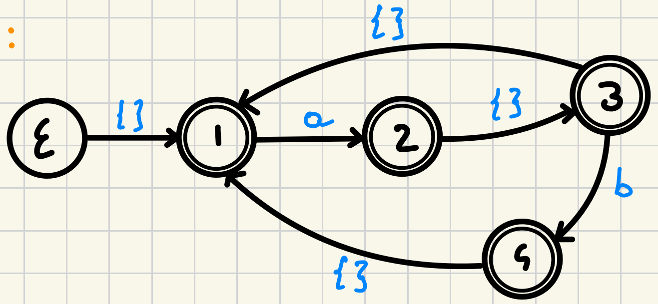
$[Z_5] = [\delta] \cap PREA(NEXT, Z_4) =$

$= \{1,2,3\} \cap \emptyset = \emptyset$

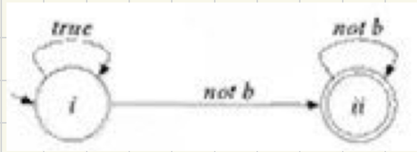
$[Z_4] = [Z_5] = [\varepsilon] = \emptyset$ $\quad s_1 \in [\varepsilon] = \emptyset$? NO!

3)

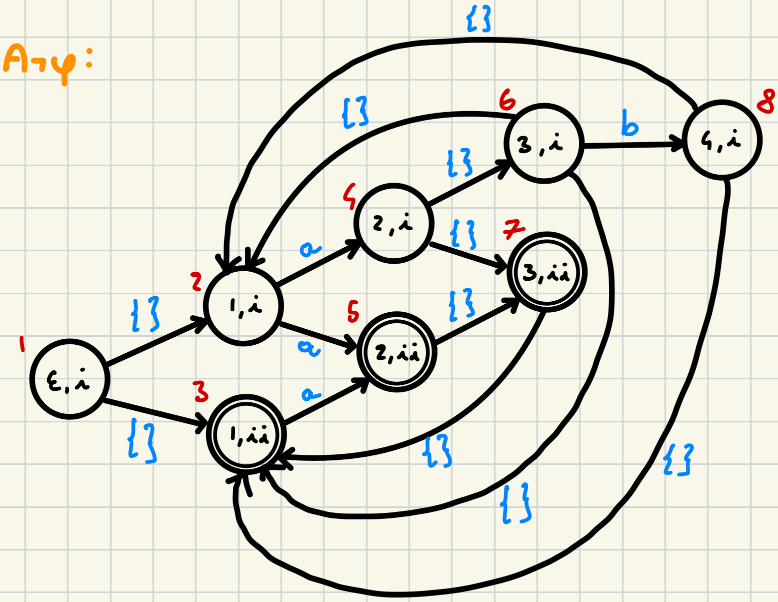
A_γ:



$A_{\neg\varphi}$:



$A_\gamma \cap A_{\neg\varphi}$:



$\varphi = \nu X\, \mu Y. \left(F \wedge \langle NEXT \rangle X \vee \langle NEXT \rangle Y\right)$

$[X_0] = \{1,2,3,4,5,6,7,8\}$

$[X_1] = [\mu Y. (F \wedge \langle NEXT \rangle X_0 \vee \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_0) =$

$= \{3,5,7\} \cap \{1,2,3,4,5,6,7,8\} \cup \phi = \{3,5,7\}$

$[Y_2] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_1) =$

$= \{3,5,7\} \cap \{1,2,3,4,5,6,7,8\} \cup \{1,2,3,4,5,6,7,8\} = \{1,2,3,4,5,6,7,8\}$

$[Y_3] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_2) =$

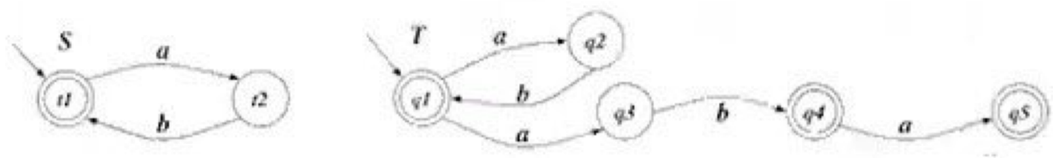
$= \{3,5,7\} \cap \{1,2,3,4,5,6,7,8\} \cup \{1,2,3,4,5,6,7,8\} = \{1,2,3,4,5,6,7,8\}$

$[Y_2] = [Y_3] = [X_1] = \{1,2,3,4,5,6,7,8\}$

$[X_0] = [X_1] = \{1,2,3,4,5,6,7,8\}$

$s_1 \in [\varphi] = ?$ YES!

Exercise 4. Consider the following two transition systems:



Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

TWO TS ARE BISIMILIAR IF THEY HAVE THE SAME BEHAVIOR:

- LOCALLY THEY LOOK INDISTINGUISHABLE
- EVERY ACTION THAT CAN BE DONE ON ONE, CAN ALSO BE DONE ON THE OTHER, AND THEY REACH THE SAME RESULT

$$R_0 = S \times T = \{(t_1, q_1), (t_1, q_2), (t_1, q_3), (t_1, q_4), (t_1, q_5), (t_2, q_1), (t_2, q_2), (t_2, q_3), (t_2, q_4), (t_2, q_5)\}$$

$$R_1 = \{(t_1, q_1), \cancel{(t_1, q_4)}, \cancel{(t_1, q_5)}, (t_2, q_2), (t_2, q_3)\}$$

$$R_2 = \{(t_1, q_1), (t_2, q_2), \cancel{(t_2, q_3)}\}$$

$$R_3 = \{\cancel{(t_1, q_1)}, (t_2, q_2)\}$$

$$R_4 = \{\cancel{(t_2, q_2)}\}$$

$$R_5 = \{\}$$

$$R_6 = \{\}$$

$R_5 = R_6 = \{\}$ $(t_1, q_1) \notin R_6$, SO S AND T ARE NOT BISIMILIAR

Exercise 5. Compute the certain answers to the CQs $q(x) \leftarrow contains(x, y), genre(y, z)$ and $q(x, z) \leftarrow contains(x, y), genre(y, z)$ over the following incomplete database (naive tables), and discuss how you obtained the result:

contains

cd	*song*
cd1	$null_1$
$null_2$	sg1
cd2	sg2
cd3	sg1
cd4	$null_3$
$null_4$	$null_3$

genre

object	*color*
$null_1$	rock
sg1	rock
sg2	blues
$null_3$	$null_5$

$\forall c$ check

$D \models q(c)$

$q_D \subseteq q(c)$

$I_D \models q(c)$

COMPUTE THE QUERY q_{DB}

$$\begin{aligned} q_{DB} \rightarrow\ & CONTAINS(CD_1, x_1) \wedge CONTAINS(x_2, SG_1) \wedge \\ & \wedge CONTAINS(CD_2, SG_2) \wedge CONTAINS(CD_3, SG_1) \wedge \\ & \wedge CONTAINS(CD_4, x_3) \wedge CONTAINS(x_4, x_3) \wedge \\ & \wedge GENRE(x_1, ROCK) \wedge GENRE(SG_1, ROCK) \wedge \\ & \wedge GENRE(SG_2, BLUES) \wedge GENRE(x_3, x_5) \end{aligned}$$

$q(x) \rightarrow CONTAINS(x, y) \wedge GENRE(y, z)$ NOT BOOLEAN!

LET SOSTITUTE x WITH CD_1 (FOR EXAMPLE)

$q(CD_1) \rightarrow CONTAINS(CD_1, y) \wedge GENRE(y, z)$

⋮

$q(SG_1) \rightarrow CONTAINS(SG_1, y) \wedge GENRE(y, z)$

⋮

$q_{DB} \subseteq q(CD_1)$

$D_{q_{DB}} \models q(CD_1)$ YES

$D_{q_{DB}} \models q(CD_2)$ YES