SAPIENZA Università di Roma – MSc. in Engineering in Computer Science
Artificial Intelligence & Machine Learning – Jan 24, 2025

Part 1 - Artificial Intelligence

(Time to complete the test: 2:00 hours)

An autonomous vacuum cleaner can move among some rooms, connected by doors. The vacuum can move in 4 directions: up, right, down, left. Rooms can be dirty or clean. When in a room, the vacuum can clean it. Consider the following formalization of this scenario.

Non-Fluents:

- $Room(r)$, denoting that r is a room;
- $Direction(d)$, denoting that d is a direction;
- $Connected(x, d, y)$, denoting that room x has a door in direction d, connecting x to y. E.g., $Connected(r_1, right, r_2)$ denotes that room r_2 is on the right of room r_1 and that the two rooms are connected through a door. Observe that connections are bidirectional, e.g., $Connected(r_1, right, r_2)$ implies $Connected(r_2, left, r_1)$, and viceversa.

Fluents:

- $In(r)$, denoting that the vacuum is in room r.
- $Dirty(r)$, denoting that room r is dirty.
- $Clean(r)$, denoting that room r is clean.
- $Closed(x, d, y)$, denoting that the door connecting y to x in direction d is closed.

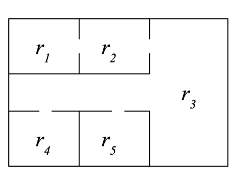
Actions:

- $move(x, d, y)$, which allows the vacuum to move along direction d from room x to room y. The action can be done only if there is a door in direction d connecting x to y, and that door is open (i.e., not closed). The effect is that the vacuum is in room y (and not in x anymore).
- $open(x, d, y)$, which allows the vacuum to open the door connecting x to y along d. The action can be done only if the vacuum is in x and x is connected to y along d. The effect is that the door is open (i.e., not closed).
- $close(x, d, y)$, which allows the vacuum to close the door connecting x to y along d. The action can be done only if the vacuum is in x and x is connected to y along d. The effect is that the door is closed.
- $clean(r)$, which allows the vacuum to clean room r. The action can be done only if the vacuum is in r and r is dirty. The effect is that r is clean (and not dirty).

Initial situation:
There are four directions, u(p), r(ight), d(own), l(eft), and five rooms, $r1, \ldots, r_5$, connected as follows (recall that connections are bidirectional):

- r_1 is connected to r_2 through a door on the right (thus r_2 is connected to r_1 through a door on the left),
- r_2 is connected to r_3 through a door on the right (and r_3 to r_2 through a door on the left),
- r_3 is connected to r_4 through a door down (and r_4 to r_3 through a door up),
- r_3 is connected to r_5 through a door down (and r_5 to r_3 through a door up).

The corresponding map is depicted below.



All rooms are dirty, all doors are closed, and the vacuum starts in r_1.

Exercise 1.

1. Formalize the above scenario as a Basic Action Theory.
2. Using regression, check whether the action sequence

$$\varrho = open(r_1, r, r_2); move(r_1, r, r_2); open(r_2, r, r_3); move(r_2, r, r_3); clean(r_3);$$

   leads to a situation where room r_3 is clean.

Exercise 2.

Considering goal $\gamma = In(r_3) \land Clean(r_1) \land Clean(r_4)$,

1. Formalize the above scenario as a PDDL domain file and a PDDL problem file.
2. Draw a fragment of the corresponding transition system, containing at least 15 (arbitrarily selected) connected states, including one goal state, and such that for every state in the fragment all outgoing transitions are present (even if the target state of the transition is not included in the fragment);
3. Solve planning for achieving γ, using forward depth-first search (uninformed), reporting the various steps of the forward search computation, and returning the resulting plan (you can pick the most convenient choice at every step).

Exercise 3.

1. Consider the following CNF formula:

$$\phi_1 = (p \lor q \lor r \lor s) \land (\neg p \lor q \lor \neg r) \land (\neg q \lor \neg r \lor s) \land (p \lor \neg q \lor r \lor s) \land (q \lor \neg r \lor \neg s) \land (\neg p \lor \neg r \lor s) \land (\neg p \lor \neg s) \land (p \lor \neg q)$$

   Using the DPLL Procedure, check whether ϕ_1 is satisfiable.

Exercise 1.

1. Formalize the above scenario as a Basic Action Theory.
2. Using regression, check whether the action sequence

$$\varrho = open(r_1, r, r_2); move(r_1, r, r_2); open(r_2, r, r_3); move(r_2, r, r_3); clean(r_3);$$

leads to a situation where room r_3 is clean.

1) PRECONDITION AXIOMS

$Poss(MOVE(x,d,y),s) \equiv CONNECTED(x,d,y) \land \neg CLOSED(x,d,y,s)$
$Poss(OPEN(x,d,y),s) \equiv IN(x,s) \land CONNECTED(x,d,y)$
$Poss(CLOSE(x,d,y),s) \equiv IN(x,s) \land CONNECTED(x,d,y)$
$Poss(CLEAN(r),s) \equiv IN(r,s) \land DIRTY(r,s)$

SUCCESSOR STATE AXIOMS

EFFECT AXIOMS

$a = MOVE(x,d,y) \supset \neg IN(x, DO(a,s)) \land IN(y, DO(a,s))$

$a = OPEN(x,d,y) \supset \neg CLOSED(x,d,y,DO(a,s))$

$a = CLOSE(x,d,y) \supset CLOSED(x,d,y,DO(a,s))$

$a = CLEAN(R) \supset \neg DIRTY(R, DO(a,s)) \land CLEAN(R, DO(a,s))$

NORMALIZE

$\exists y,d.\ a = MOVE(x,d,y) \supset \neg IN(x, DO(a,s))$

$\exists x,d.\ a = MOVE(x,d,y) \supset IN(y, DO(a,s))$

$a = OPEN(x,d,y) \supset \neg CLOSED(x,d,y,DO(a,s))$

$a = CLOSE(x,d,y) \supset CLOSED(x,d,y,DO(a,s))$

$a = CLEAN(R) \supset \neg DIRTY(R, DO(a,s))$

$a = CLEAN(R) \supset CLEAN(R, DO(a,s))$

EXPLANATION CLOSURE

$IN(R, DO(a,s)) \equiv (\exists x,d.\ a = MOVE(x,d,y)) \lor$
$(IN(R,S) \land \neg(\exists y,d.\ a = MOVE(x,d,y)))$

$CLOSED(x,d,y,DO(a,s)) \equiv a = CLOSE(x,d,y) \lor$
$(CLOSED(x,d,y,s) \land \neg a = OPEN(x,d,y))$

$DIRTY(R, DO(a,s)) \equiv DIRTY(R,S) \wedge \neg a = CLEAN(R)$

$CLEAN(R, DO(a,S)) \equiv a = CLEAN(R) \vee CLEAN(R,S)$

INITIAL SITUATION

$DIRECTION(d) \equiv (d = UP) \vee (d = RIGHT) \vee (d = LEFT) \vee (d = DOWN)$
$ROOM(R) \equiv (R = R_1) \vee (R = R_2) \vee (R = R_3) \vee (R = R_4) \vee (R = R_5)$
$CONNECTED(x,d,y) \equiv (x = R_1 \wedge y = R_2 \wedge d = RIGHT) \vee (x = R_2 \wedge y = R_1 \wedge d = LEFT) \ldots$
$DIRTY(R, S_0) \equiv (R = R_1) \vee (R = R_2) \vee (R = R_3) \vee (R = R_4) \vee (R = R_5)$
$IN(R, S_0) \equiv (R = R_1)$
$CLOSED(x,d,y,S_0) \equiv (x = R_1 \wedge y = R_2 \wedge d = RIGHT) \vee (x = R_2 \wedge y = R_1 \wedge d = LEFT) \ldots.$

2) $\varrho = open(r_1, r, r_2); move(r_1, r, r_2); open(r_2, r, r_3); move(r_2, r, r_3); clean(r_3);$ R_3 CLEAN

$S_1 = DO(OPEN(R_1, R, R_2), S_0)$
$S_2 = DO(MOVE(R_1, R, R_2), S_1)$
$S_3 = DO(OPEN(R_2, R, R_3), S_2)$
$S_4 = DO(MOVE(R_2, R, R_3), S_3)$
$S_5 = DO(CLEAN(R_3), S_4)$

$D_0 \cup D_{UNA} \models \mathcal{R}[CLEAN(R_3, S_5)]$

$\mathcal{R}[CLEAN(R_3, S_5)] = \mathcal{R}[CLEAN(R_3, DO(CLEAN(R_3), S_4))] =$
$= \mathcal{R}[CLEAN(R_3) = CLEAN(R_3) \vee CLEAN(R_3, S_4)] =$
$= CLEAN(R_3) = CLEAN(R_3) \vee \mathcal{R}[CLEAN(R_3, S_4)] =$ TRUE

ρ LEADS TO A SITUATION WHERE R_3 IS CLEAN ✓

Exercise 2.

Considering goal $\gamma = In(r_3) \wedge Clean(r_1) \wedge Clean(r_4)$,

1. Formalize the above scenario as a PDDL domain file and a PDDL problem file.
2. Draw a fragment of the corresponding transition system, containing at least 15 (arbitrarily selected) connected states, including one goal state, and such that for every state in the fragment all outgoing transitions are present (even if the target state of the transition is not included in the fragment);
3. Solve planning for achieving γ, using forward depth-first search (uninformed), reporting the various steps of the forward search computation, and returning the resulting plan (you can pick the most convenient choice at every step).

1)
```
(DEFINE (DOMAIN VAC.DOM)
  (:REQUIREMENTS :ADL)
  (: TYPES ROOM DIR)
  (: PREDICATES
      (ROOM ?R - ROOM)
      (DIRECTION ?d - DIR)
      (CONNECTED ?x ?y - ROOM ?d - DIR)
      (IN ?R - ROOM)
      (DIRTY ?R - ROOM)
      (CLEAN ?R - ROOM)
      (CLOSED ?x ?y - ROOM ?d - DIR)
  )
  (: ACTION MOVE
     : PARAMETERS (?x ?y - ROOM ?d - DIR)
     : PRECONDITIONS (AND (CONNECTED ?x ?d ?y)(NOT (CLOSED ?x ?d ?y)))
     : EFFECT (AND (NOT (IN ?x)) (IN ?y))
  ); END OF MOVE
  (: ACTION OPEN
     : PARAMETERS (?x ?y - ROOM ?d - DIR)
     : PRECONDITIONS (AND (IN ?x)(CONNECTED ?x ?d ?y))
     : EFFECT (NOT (CLOSED ?x ?d ?y))
  ); END OF OPEN
  (: ACTION CLOSE
     : PARAMETERS (?x ?y - ROOM ?d - DIR)
     : PRECONDITIONS (AND (IN ?x)(CONNECTED ?x ?d ?y))
     : EFFECT (CLOSED ?x ?d ?y)
  ); END OF CLOSE
  (: ACTION CLEAN
     : PARAMETERS (?r - ROOM)
     : PRECONDITIONS (AND (IN ?r)( DIRTY ?r))
     : EFFECT (AND (NOT (DIRTY ?r)) (CLEAN ?r))
  ); END OF CLEAN
); END OF DEFINE DOMAIN
```

```
(DEFINE (PROBLEM VAC. PROB)(: DOMAIN VAC. DOM)
  (: OBJECTS    R1 R2 R3 R4 R5 - ROOM
                UP RIGHT DOWN LEFT - DIRECTION
  )
  (: INIT   (IN R1) (DIRTY R1)(DIRTY R2)(DIRTY R3)(DIRTY R4)(DIRTY R5)
            (CONNECTED R1 RIGHT R2)(CONNECTED R2 LEFT R1)...
            (CLOSED R1 RIGHT R2)(CLOSED R2 LEFT R1) ...
  )
  (: GOAL (AND (IN R3) (CLEAN R1)(CLEAN R4))
  )
); END OF DEFINE PROBLEM
```

Exercise 3.

1. Consider the following CNF formula:

$$\phi_1 = (p \vee q \vee r \vee s) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg q \vee \neg r \vee s) \wedge (p \vee \neg q \vee r \vee s) \wedge (q \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg r \vee s) \wedge (\neg p \vee \neg s) \wedge (p \vee \neg q)$$

Using the DPLL Procedure, check whether ϕ_1 is satisfiable.

$\phi : \{\{p,q,r,s\},\{\neg p,q,\neg r\},\{\neg q,\neg r,s\},\{p,\neg q,r,s\},\{q,\neg r,\neg s\},\{\neg p,\neg r,s\},\{\neg p,\neg s\},\{p,\neg q\}\}$

SR) $p = T \rightarrow \phi : \{\{q,\neg r\},\{\neg q,\neg r,s\},\{q,\neg r,\neg s\},\{\neg r,s\},\{\neg s\}\}$

UP) $s = F \rightarrow \phi : \{\{q,\neg r\},\{\neg q,\neg r\},\{\neg r\}\}$

UP) $r = F \rightarrow \phi : \{\ \}$

$I = \{\neg r, \neg s, p\}$