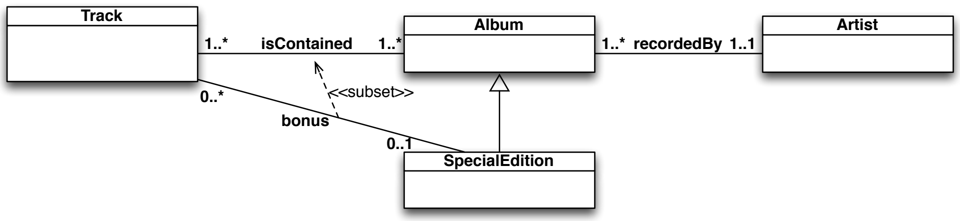
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

Formal Methods - Final Test A – December 16, 2016

(Time to complete the test: 2 hours)

Exercise 1. Express the following UML class diagram in FOL.



Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

$Track$

t1
t2
t3
t4
t5
t6

$Album$

a1
a2
a3

$SpEd$

s1
s2

$Artist$

bt
rs

$isContained$

t1	a1
t2	a1
t3	a1
t1	a2
t4	a2
t5	a2
t5	a3

$bonus$

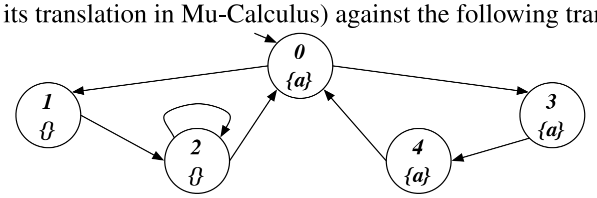
t5	s1
t6	s2

$recordedBy$

a1	bt
a2	bt
a3	rs
s1	rs
s2	bt

1. Check whether the above instantiation, once completed, is correct, and explain why it is or it is not.
2. Express in FOL the following queries and evaluate them over the completed instantiation:
   (a) Return the artist that recorded an album and a special edition containing the same track.
   (b) Return those artist that have recorded only special editions.
   (c) Check if there is a track appearing in all albums that are not special editions.

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge \langle next \rangle X) \vee ([next]Y))$ and the CTL formula $AF(EG(a \supset AXEX\neg a))$(showing its translation in Mu-Calculus) against the following transition system:



Exercise 4. Consider the following two transition systems:

Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

Exercise 5. Compute the certain answers to the following CQs over the following incomplete database (naive tables), and discuss how you obtained the result:

$$q() \leftarrow contains(x, y), genre(y, z) \qquad q(x, z) \leftarrow contains(x, y), genre(y, z)$$

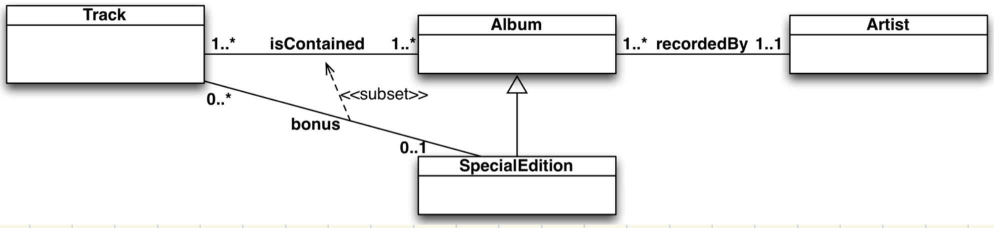
$contains$

$album$	$song$
wywh	$null_1$
$null_2$	$null_3$
$null_4$	$null_5$
$null_6$	$null_3$

$genre$

$song$	$type$
$null_1$	progressive
$null_3$	blues
$null_5$	$null_7$

Exercise 1. Express the following UML class diagram in FOL.



$T(x)$, $A(x)$, $SE(x)$, $ART(x)$
$ISCON(x,y)$
$BONUS(x,y)$
$RECBY(x,y)$

$\forall x,y.\ ISCON(x,y) \supset T(x) \wedge A(y)$
$\forall x.\ T(x) \supset \exists y.\ ISCON(x,y)$
$\forall y.\ A(y) \supset \exists x.\ ISCON(x,y)$

$\forall x,y.\ BONUS(x,y) \supset T(x) \wedge SE(y)$
$\forall x.\ T(x) \supset 0 \leq \#\{y \mid BONUS(x,y)\} \leq 1$
$\forall y.\ SE(y) \supset 0 \leq \#\{x \mid BONUS(x,y)\}$
$\forall x,y.\ BONUS(x,y) \supset ISCON(x,y)$

$\forall x,y.\ RECBY(x,y) \supset A(x) \wedge ART(y)$
$\forall x.\ A(x) \supset 1 \leq \#\{y \mid RECBY(x,y)\} \leq 1$
$\forall y.\ ART(y) \supset \exists x.\ RECBY(x,y)$

$\forall x.\ SE(x) \supset A(x)$

Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

Track

t1
t2
t3
t4
t5
t6

Album

a1
a2
a3

SpEd

s1
s2

Artist

bt
rs

isContained

t1	a1
t2	a1
t3	a1
t1	a2
t4	a2
t5	a2
t5	a3

bonus

t5	s1
t6	s2

recordedBy

a1	bt
a2	bt
a3	rs
s1	rs
s2	bt

1. Check whether the above instantiation, once completed, is correct, and explain why it is or it is not.
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1) $A = \{a_1, a_2, a_3, s_1, s_2\}$

$\forall x,y.\ ISCON(x,y) \supset T(x) \wedge A(y)$

t_1, t_2, t_3, t_4, t_5 ARE TRACKS
a_1, a_2, a_3 ARE ALBUMS $\rightarrow$ CARDINALS OK!

$\forall x,y.\ BONUS(x,y) \supset T(x) \wedge SE(y)$

t_5, t_6 ARE TRACKS
s_1, s_2 ARE SPEC ED $\rightarrow$ CARDINALS OK!

$\forall x,y.\ RECBY(x,y) \supset A(x) \wedge ART(y)$

a_1, a_2, a_3, s_1, s_2 ARE ALBUMS
bt, rs ARE ARTIST $\rightarrow$ CARDINALS OK!

2) a. $\exists t,a,s\ ART(x) \wedge RECBY(a,x) \wedge RECBY(s,x) \wedge ISCON(t,a) \wedge BONUS(t,s)$

$\{rs\}$

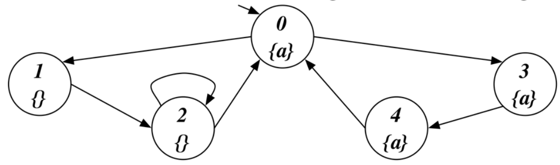
b. $ART(x) \wedge \forall s.(RECBY(s,x) \supset SPECED(s))$

$\{\}$

c. $\exists t.\ T(t) \wedge \forall a.((A(a) \wedge \neg SPECED(a)) \supset ISCON(t,a))$

{FALSE}

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge \langle next\rangle X) \vee ([next]Y))$ and the CTL formula $AF(EG(a \supset AXEX\neg a))$(showing its translation in Mu-Calculus) against the following transition system:



1) $\nu X.\mu Y.((a \wedge \langle NEXT\rangle X) \vee ([NEXT]Y))$

$[X_0] = \{0,1,2,3,4\}$

$[X_1] = [\mu Y.((a \wedge \langle NEXT\rangle X_0) \vee ([NEXT]Y))]$

$[Y_0] = \emptyset$

$[Y_1] = ([a] \cap PREE(NEXT, X_0)) \cup PREA(NEXT, Y_0) =$

$= (\{0,3,4\} \cap \{0,1,2,3,4\}) \cup \emptyset = \{0,3,4\}$

$[Y_2] = ([a] \cap PREE(NEXT, X_0)) \cup PREA(NEXT, Y_1) =$

$= (\{0,3,4\} \cap \{0,1,2,3,4\}) \cup \{3,4\} = \{0,3,4\}$

$[Y_1] = [Y_2] = [X_1] = \{0,3,4\}$

$[X_2] = [\mu Y.((a \wedge \langle NEXT\rangle X_1) \vee ([NEXT]Y))]$

$[Y_0] = \emptyset$

$[Y_1] = ([a] \cap PREE(NEXT, X_1)) \cup PREA(NEXT, Y_0) =$

$= (\{0,3,4\} \cap \{0,2,3,4\}) \cup \emptyset = \{0,3,4\}$

$[Y_2] = ([a] \cap PREE(NEXT, X_1)) \cup PREA(NEXT, Y_1) =$

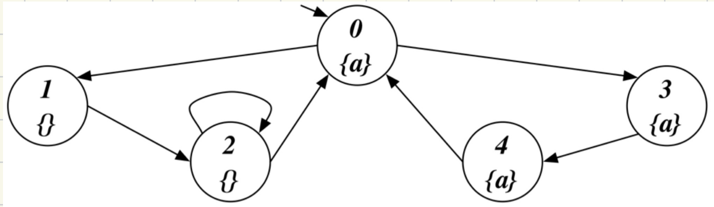
$= (\{0,3,4\} \cap \{0,2,3,4\}) \cup \{3,4\} = \{0,3,4\}$

$[Y_1] = [Y_2] = [X_2] = \{0,3,4\}$

$[X_1] = [X_2] = \{0,3,4\}$

$s_0 \in [\nu X.\mu Y.((a \wedge \langle NEXT\rangle X) \vee ([NEXT]Y))] = \{0,3,4\}$? YES!

2) $AF(\underbrace{EG(\underbrace{a \supset \underbrace{AX\,\underbrace{EX\,\neg a}_{\alpha}}_{\beta}}_{\gamma})}_{\delta})$ with the whole formula ε



$[\alpha] = [EX\,\neg a] = [\langle NEXT \rangle \neg a] = PREE(NEXT, \neg a) = \{0,1,2\} = [\alpha]$

$[\beta] = [AX\,\alpha] = [[NEXT]\,\alpha] = PREA(NEXT, \alpha) = \{1,2,4\} = [\beta]$

$[\gamma] = [a \supset \beta] = [\neg a] \vee [\beta] = \{1,2\} \cup \{1,2,4\} = \{1,2,4\} = [\gamma]$

$[\delta] = [EG\,\gamma] = [\nu Z.\, \gamma \wedge \langle NEXT \rangle Z]$

$[Z_0] = \{0,1,2,3,4\}$

$[Z_1] = [\gamma] \cap PREE(NEXT, Z_0) =$

$= \{1,2,4\} \cap \{0,1,2,3,4\} = \{1,2,4\}$

$[Z_2] = [\gamma] \cap PREE(NEXT, Z_1) =$

$= \{1,2,4\} \cap \{0,1,2,3\} = \{1,2\}$

$[Z_3] = [\gamma] \cap PREE(NEXT, Z_2) =$

$= \{1,2,4\} \cap \{0,1,2\} = \{1,2\}$

$[Z_2] = [Z_3] = [\delta] = \{1,2\}$

$[\varepsilon] = [AF\,\delta] = [\mu Z.\, \delta \vee [NEXT] Z]$

$[Z_0] = \emptyset$

$[Z_1] = [\delta] \cup PREA(NEXT, Z_0) =$

$= \{1,2\} \cup \emptyset = \{1,2\}$

$[Z_2] = [\delta] \cup PREA(NEXT, Z_1) =$

$= \{1,2\} \cup \{1\} = \{1,2\}$

$[Z_1] = [Z_2] = [\varepsilon] = \{1,2\}$

$T_{s_0} \in \varepsilon\,? \rightarrow s_0 \in [\varepsilon] = \{1,2\}\,?$ NO!

Exercise 4. Consider the following two transition systems:

Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

TWO TS ARE BISIMILIAR IF THEY HAVE THE SAME BEHAVIOR:

- LOCALLY THEY LOOK INDISTINGUISHABLE
- EVERY ACTION THAT CAN BE DONE ON ONE, CAN ALSO BE DONE ON THE OTHER, AND THEY REACH THE SAME RESULT

$R_0 = T \times Q = \{(t_1, q_1), (t_1, q_2), (t_1, q_3), (t_2, q_1), (t_2, q_2), (t_2, q_3)\}$

$R_1 = \{(t_1, q_1), (t_2, q_2), \cancel{(t_2, q_3)}\}$

$R_2 = \{\cancel{(t_1, q_1)}, (t_2, q_2)\}$

$R_3 = \{\cancel{(t_2, q_2)}\}$

$R_4 = \{\ \}$

$R_5 = \{\}$

$R_4 = R_5 = \{\}$ T AND Q ARE NOT BISIMILIAR

Exercise 5. Compute the certain answers to the following CQs over the following incomplete database (naive tables), and discuss how you obtained the result:

$$q() \leftarrow contains(x, y), genre(y, z) \qquad q(x, z) \leftarrow contains(x, y), genre(y, z)$$

contains

album	*song*
wywh	$null_1$
$null_2$	$null_3$
$null_4$	$null_5$
$null_6$	$null_3$

genre

song	*type*
$null_1$	progressive
$null_3$	blues
$null_5$	$null_7$

Q_1: $q() \leftarrow CON(x,y), GEN(y,z)$

TRUE BECAUSE THERE EXIST TUPLES (x, y, z) SUCH THAT CONTAINS (x, y) AND GENRE (y, z).

FOR EXAMPLE

x	y	z
wywh	$null_1$	PROG
$null_2$	$null_3$	BLUES
$null_4$	$null_5$	$null_7$

Q_2: $q(x,z) \leftarrow CON(x,y), GENRE(y,z)$

THE CERTAIN ANSWER IS (wywh, PROGRESSIVE). THIS IS THE ONLY TUPLE THAT DOESN'T CONTAIN null VALUES, SINCE THESE CAN TAKE ON ANY VALUE.