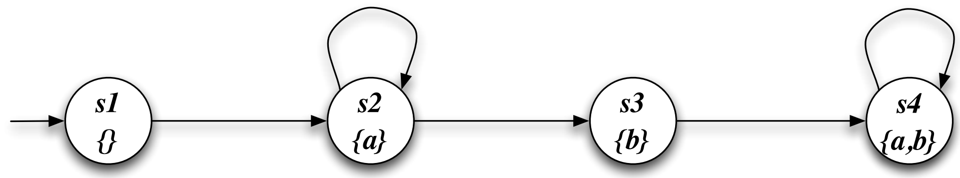
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

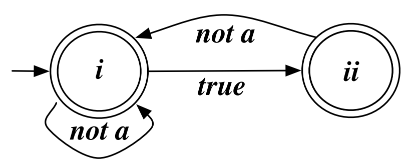
Formal Methods – June 1, 2023

(Time to complete the test: 2 hours)

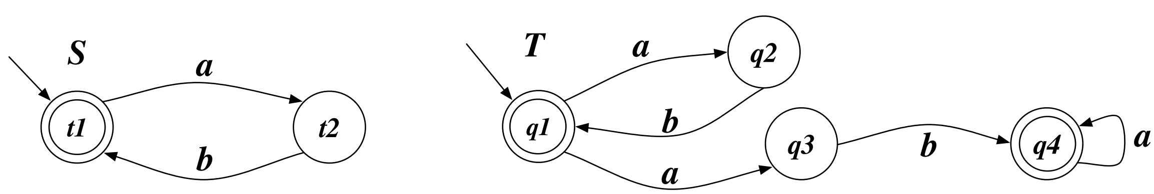
Part 1. Consider the following transition system:



- **Exercise 1.1:** Model check the Mu-Calculus formula $\mu X.\nu Y.(a \vee [next]X) \wedge [next]Y$.
- **Exercise 1.2:** Model check the CTL formula $AFAGEXa$ against the following transition system:
- **Exercise 1.3:** Model check the LTL formula $\Diamond(a \wedge \bigcirc a)$, by considering that the Büchi automaton for $\neg\Diamond(a \wedge \bigcirc a)$ is the one below:



Part 2. Consider the following two transition systems:

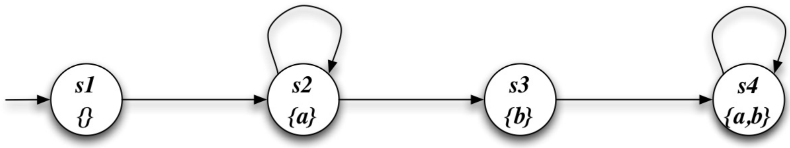


Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

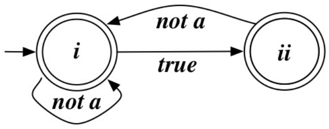
Part 3. Compute the weakest precondition for getting $\{x = 0\}$ by executing the following program:

```
x := 1 - y;
if (x > 1) then
     x := x - y
else x := x + y;
y := y + 1
```

Part 1. Consider the following transition system:



- **Exercise 1.1:** Model check the Mu-Calculus formula $\mu X.\nu Y.(a \vee [next]X) \wedge [next]Y$.
- **Exercise 1.2:** Model check the CTL formula $AFAGEXa$ against the following transition system:
- **Exercise 1.3:** Model check the LTL formula $\Diamond(a \wedge \bigcirc a)$, by considering that the Büchi automaton for $\neg\Diamond(a \wedge \bigcirc a)$ is the one below:



1) $\varphi = \mu X.\nu Y.(a \vee [NEXT]X) \wedge [NEXT]Y$

$[X_0] = \phi$

$[X_1] = [\nu Y.(a \vee [NEXT]X_0) \wedge [NEXT]Y]$

$[Y_0] = \{1,2,3,4\}$

$[Y_1] = ([a] \cup PREA(NEXT, X_0)) \cap PREA(NEXT, Y_0) =$

$= (\{2,4\} \cup \phi) \cap \{1,2,3,4\} = \{2,4\}$

$[Y_2] = ([a] \cup PREA(NEXT, X_0)) \cap PREA(NEXT, Y_1) =$

$= (\{2,4\} \cup \phi) \cap \{1,3,4\} = \{4\}$

$[Y_3] = ([a] \cup PREA(NEXT, X_0)) \cap PREA(NEXT, Y_2) =$

$= (\{2,4\} \cup \phi) \cap \{3,4\} = \{4\}$

$[Y_2] = [Y_3] = [X_1] = \{4\}$

$[X_2] = [\nu Y.(a \vee [NEXT]X_1) \wedge [NEXT]Y]$

$[Y_0] = \{1,2,3,4\}$

$[Y_1] = ([a] \cup PREA(NEXT, X_1)) \cap PREA(NEXT, Y_0) =$

$= (\{2,4\} \cup \{3,4\}) \cap \{1,2,3,4\} = \{2,3,4\}$

$[Y_2] = ([a] \cup PREA(NEXT, X_1)) \cap PREA(NEXT, Y_1) =$

$= (\{2,4\} \cup \{3,4\}) \cap \{1,2,3,4\} = \{2,3,4\}$

$[Y_1]=[Y_2]=[X_2]=\{2,3,4\}$

$[X_3]=[\nu Y.\,(a \vee [NEXT]X_2)\wedge [NEXT]Y]$

$[Y_0]=\{1,2,3,4\}$

$[Y_1]=([a]\cup PREA(NEXT,X_2))\cap PREA(NEXT,Y_0)=$

$=(\{2,4\}\cup\{1,2,3,4\})\cap\{1,2,3,4\}=\{1,2,3,4\}$

$[Y_0]=[Y_1]=[X_3]=\{1,2,3,4\}$

$[X_4]=[\nu Y.\,(a \vee [NEXT]X_3)\wedge [NEXT]Y]$

$[Y_0]=\{1,2,3,4\}$

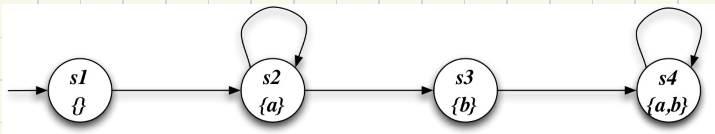
$[Y_1]=([a]\cup PREA(NEXT,X_3))\cap PREA(NEXT,Y_0)=$

$=(\{2,4\}\cup\{1,2,3,4\})\cap\{1,2,3,4\}=\{1,2,3,4\}$

$[Y_0]=[Y_1]=[X_4]=\{1,2,3,4\}$

$[X_3]=[X_4]=\{1,2,3,4\}$ $\qquad$ $s_1\in[\varphi]=?$ YES!

2) $\underbrace{AF\ \underbrace{AG\ \underbrace{EX\ a}_{\alpha}}_{\beta}}_{\gamma}$



$[\alpha]=[EX\ a]=[\langle NEXT\rangle a]=PREE(NEXT,a)=\{1,2,3,4\}=[\alpha]$

$[\beta]=[AG\ \alpha]=[\nu Z.\alpha\wedge[NEXT]Z]$

$[Z_0]=\{1,2,3,4\}$

$[Z_1]=[\alpha]\cap PREA(NEXT,Z_0)=$

$=\{1,2,3,4\}\cap\{1,2,3,4\}=\{1,2,3,4\}$ $\qquad$ $[Z_0]=[Z_1]=[\beta]=\{1,2,3,4\}$

$[\gamma]=[AF\ \beta]=[\mu Z.\,\beta\vee[NEXT]Z]$

$[Z_0]=\phi$

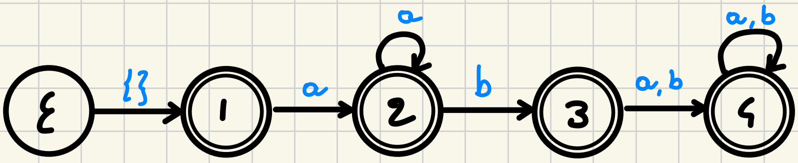
$[Z_1]=[\beta]\cup PREA(NEXT,Z_0)=$ $\qquad$ $s_1\in[\gamma]=?$ YES!

$=\{1,2,3,4\}\cup\phi=\{1,2,3,4\}$

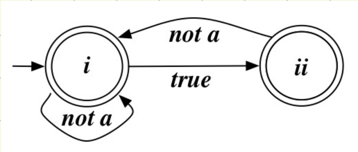
$[Z_2]=[\beta]\cup PREA(NEXT,Z_1)=$

$=\{1,2,3,4\}\cup\{1,2,3,4\}=\{1,2,3,4\}$ $\qquad$ $[Z_1]=[Z_2]=[\gamma]=\{1,2,3,4\}$

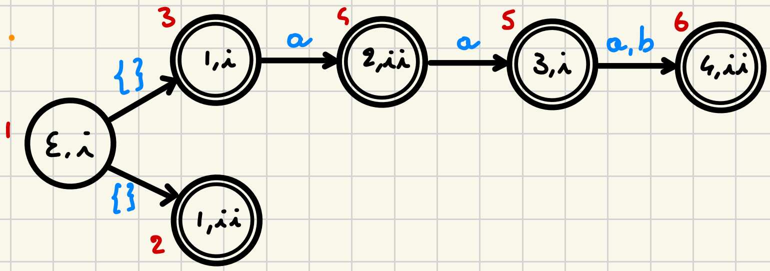
3) A_τ:



$A_{\neg\varphi}$:



$A_\tau \cap A_{\neg\varphi}$:



$\varphi = \nu X. \mu Y. (F \wedge \langle NEXT \rangle X \vee \langle NEXT \rangle Y)$

$[X_0] = \{1,2,3,4,5,6\}$

$[X_1] = [\mu Y. (F \wedge \langle NEXT \rangle X_0 \vee \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_0) =$

$= \{2,3,4,5,6\} \cap \{1,3,4,5\} \cup \phi = \{3,4,5\}$

$[Y_2] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_1) =$

$= \{2,3,4,5,6\} \cap \{1,3,4,5\} \cup \{1,3,4\} = \{1,3,4,5\}$

$[Y_3] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_2) =$

$= \{2,3,4,5,6\} \cap \{1,3,4,5\} \cup \{1,3,4\} = \{1,3,4,5\}$

$[Y_2] = [Y_3] = [X_1] = \{1,3,4,5\}$

$[X_2] = [\mu Y. (F \wedge \langle NEXT \rangle X_1 \vee \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_0) =$

$= \{2,3,4,5,6\} \cap \{1,3,4\} \cup \phi = \{3,4\}$

$[Y_2] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_1) =$

$= \{2,3,4,5,6\} \cap \{1,3,4\} \cup \{1,3\} = \{1,3,4\}$

$[Y_3] = [F] \cap \text{PREE}(\text{NEXT}, X_1) \cup \text{PREE}(\text{NEXT}, Y_2) =$

$= \{2,3,4,5,6\} \cap \{1,3,4\} \cup \{1,3\} = \{1,3,4\}$

$[Y_2] = [Y_3] = [X_2] = \{1,3,4\}$

$[X_3] = [\mu Y.\ (F \wedge \langle NEXT \rangle X_2 \vee \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = [F] \cap \text{PREE}(\text{NEXT}, X_2) \cup \text{PREE}(\text{NEXT}, Y_0) =$

$= \{2,3,4,5,6\} \cap \{1,3\} \cup \phi = \{3\}$

$[Y_2] = [F] \cap \text{PREE}(\text{NEXT}, X_2) \cup \text{PREE}(\text{NEXT}, Y_1) =$

$= \{2,3,4,5,6\} \cap \{1,3\} \cup \{1\} = \{1,3\}$

$[Y_3] = [F] \cap \text{PREE}(\text{NEXT}, X_2) \cup \text{PREE}(\text{NEXT}, Y_2) =$

$= \{2,3,4,5,6\} \cap \{1,3\} \cup \{1\} = \{1,3\}$

$[Y_2] = [Y_3] = [X_3] = \{1,3\}$

$[X_4] = [\mu Y.\ (F \wedge \langle NEXT \rangle X_3 \vee \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = [F] \cap \text{PREE}(\text{NEXT}, X_3) \cup \text{PREE}(\text{NEXT}, Y_0) =$

$= \{2,3,4,5,6\} \cap \{1\} \cup \phi = \phi$

$[Y_0] = [Y_1] = [X_4] = \phi$

$[X_5] = [\mu Y.\ (F \wedge \langle NEXT \rangle X_4 \vee \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = [F] \cap \text{PREE}(\text{NEXT}, X_4) \cup \text{PREE}(\text{NEXT}, Y_0) =$

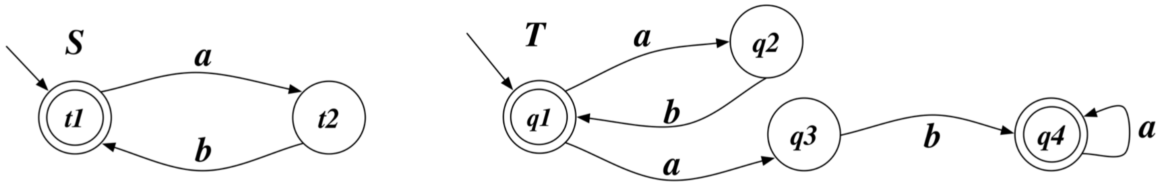
$= \{2,3,4,5,6\} \cap \phi \cup \phi = \phi$

$[Y_0] = [Y_1] = [X_5] = \phi$

$[X_4] = [X_5] = \phi$

$s_1 \in [\varphi] = \phi$? NO!

Part 2. Consider the following two transition systems:



Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

TWO TS ARE BISIMILIAR IF THEY HAVE THE SAME BEHAVIOR:

- LOCALLY THEY LOOK INDISTINGUISHABLE
- EVERY ACTION THAT CAN BE DONE ON ONE, CAN ALSO BE DONE ON THE OTHER, AND THEY REACH THE SAME RESULT

$R_0 = S \times T = \{(t_1,q_1),(t_1,q_2),(t_1,q_3),(t_1,q_4),(t_2,q_1),(t_2,q_2),(t_2,q_3),(t_1,q_4)\}$

$R_1 = \{(t_1,q_1),(t_1,q_4),(t_2,q_2),(t_2,q_3)\}$

$R_2 = \{(t_1,q_1),(t_2,q_2),(t_2,q_3)\}$

$R_3 = \{(t_1,q_1),(t_2,q_2)\}$

$R_4 = \{(t_2,q_2)\}$

$R_5 = \{\}$

$R_6 = \{\}$

$R_5 = R_6 = \{\} \rightarrow (t_1,q_1) \notin R_6$. S AND T ARE NOT BI

Part 3. Compute the weakest precondition for getting $\{x = 0\}$ by executing the following program:

```
x := 1 - y;
if (x > 1) then
      x := x - y
else x := x + y;
y := y + 1
```

$\{(x>1 \wedge x=y) \vee (x \leq 1 \wedge x=-y)\}[x/1-y] = \{(y<0 \wedge y=\frac{1}{2}) \vee (y \geq 0 \wedge 1=0)\}$

X = 1 - Y;

$\{(x>1 \wedge x=y) \vee (x \leq 1 \wedge x=-y)\}$

IF (X > 1) THEN.

$\{x=0\}[x/x-y] = \{x=y\}$

X = X - Y;

$\{x=0\}$

$\{x=0\}[x/x+y] = \{x=-y\}$

ELSE X = X + Y;

$\{x=0\}[y/y+1] = \{x=0\}$

Y = Y + 1;

$\{x=0\}$

wp (d, x = 0) => FALSE