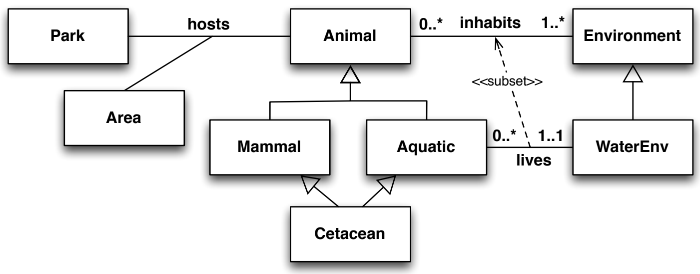
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

Formal Methods – June 17, 2016

(Time to complete the test: 2 hours)

Exercise 1. Express the following UML class diagram in FOL.



Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

$Park$

londonZoo

$Area$

sharkPool
fishPool
SavanaArea

$hosts$

londonZoo	sharkPool	sawshark
londonZoo	fishPool	sawshark

$Aquatic$

sawshark
crocodile

$Cetacean$

dolphin
bluewhale

$WaterEnv$

ocean
lagoon

$lives$

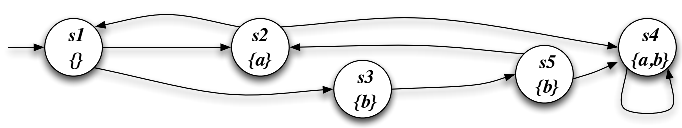
dolphin	ocean
bluewhale	ocean
sawshark	ocean
crocodile	lagoon

$inhabits$

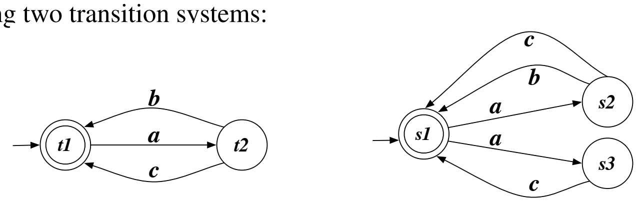
crocodile	ocean

1. Check whether the above instantiation, once completed, is correct, and explain why it is or it is not.
2. Express in FOL and evaluate the following queries:
   (a) Return animals that inhabit at least two environments.
   (b) Return parks that they host only aquatic animals.
   (c) Check if there are parks that host all Cetacean.

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge [next]X) \vee (b \wedge [next]Y))$ and the CTL formula $AF(a \supset EXEGb)$ (showing its translation in Mu-Calculus) against the following transition system:



Exercise 4. Consider the following two transition systems:



Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

Exercise 5. Compute the certain answers to the CQ $q(x) \leftarrow Employee(x), Manages(x, y)$ over the incomplete database (naive tables), by explaining and exploiting the connection with conjunctive query containment:

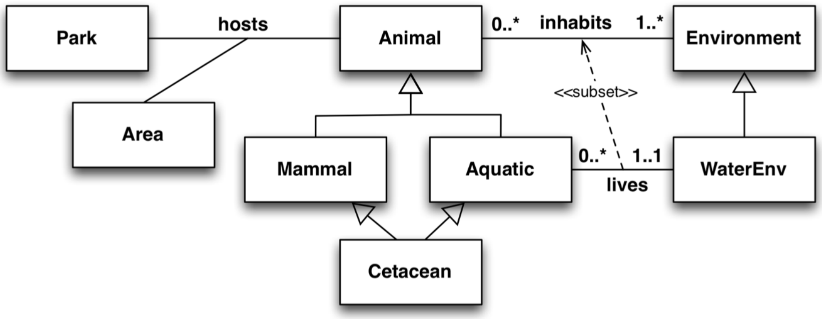
$Employee$

$name$
Smith
$null_1$
Brown

$Manages$

mgr	mgd
Green	Smith
Smith	$null_1$
$null_1$	Brown
Brown	$null_2$

Exercise 1. Express the following UML class diagram in FOL.



$P(x)$, $AREA(x)$, $A(x)$, $M(x)$, $AQ(x)$, $CET(x)$, $ENV(x)$, $WENV(x)$

$HOSTS(x,y,z)$

$INHAB(x,y)$

$LIVES(x,y)$

$\forall x,y,z.\ HOSTS(x,y,z) \supset P(x) \land AREA(y) \land A(z)$

$\forall x,y.\ INHAB(x,y) \supset A(x) \land ENV(y)$

$\forall x.\ A(x) \supset \exists y.\ INHAB(x,y)$

$\forall y.\ ENV(y) \supset 0 \leq \#\{x \mid INHAB(x,y)\}$

$\forall x,y.\ LIVES(x,y) \supset AQ(x) \land WENV(y)$

$\forall x.\ AQ(x) \supset 1 \leq \#\{y \mid LIVES(x,y)\} \leq 1$

$\forall y.\ WENV(y) \supset 0 \leq \#\{x \mid LIVES(x,y)\}$

$\forall x,y.\ LIVES(x,y) \supset INHAB(x,y)$

$\forall x.\ M(x) \supset A(x)$

$\forall x.\ AQ(x) \supset A(x)$

$\forall x.\ M(x) \supset \neg AQ(x)$

$\forall x.\ A(x) \supset M(x) \lor AQ(x)$

$\forall x.\ CET(x) \supset M(x) \land AQ(x)$

$\forall x.\ WENV(x) \supset ENV(x)$

Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

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1. Check whether the above instantiation, once completed, is correct, and explain why it is or it is not.
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1) C = { DOLPHIN, BLUEW, SAWSHARK, CROCO }   LIVES: { ... (CROCO, OCEAN) }

$\forall x,y,z.\ HOSTS(x,y,z) \supset P(x) \wedge AREA(y) \wedge A(z)$

LONDON ZOO IS A PARK
SHARKP, FISHP ARE AREAS → CARDINALS OK !
SAWSHARK IS AN ANIMAL

$\forall x,y.\ INHAB(x,y) \supset A(x) \wedge ENV(y)$

CROCO IS AN ANIMAL → CARDINALS OK !
OCEAN IS AN WENV

$\forall x,y.\ LIVES(x,y) \supset AQ(x) \wedge WENV(y)$

D, B, S, C ARE CETACEANS → CARDINALS OK !
OCEAN, LAGOON ARE WENV

2) a. $\exists e,e'.\ C(x) \wedge INHAB(x,e) \wedge INHAB(x,e') \wedge e \neq e'$

{ }

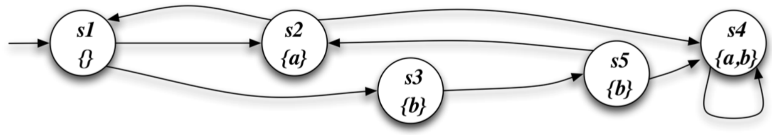
b. $P(x) \wedge \forall a,c\ (HOSTS(x,a,c) \supset AQ(c))$

{ LONDONZOO }

c. $\exists p.\ P(p) \wedge \forall c.\ (C(c) \supset \exists a.\ HOSTS(p,a,c))$

{ FALSE }

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge [next]X) \vee (b \wedge [next]Y))$ and the CTL formula $AF(a \supset EXEGb)$ (showing its translation in Mu-Calculus) against the following transition system:



1) $\nu X.\mu Y((a \wedge [NEXT]X) \vee (b \wedge [NEXT]Y))$

$[X_0] = \{1,2,3,4,5\}$

$[X_1] = [\mu Y((a \wedge [NEXT]X_0) \vee (b \wedge [NEXT]Y))$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREA(NEXT, X_0)) \cup ([b] \cap PREA(NEXT, Y_0)) =$

$= (\{2,4\} \cap \{1,2,3,4,5\}) \cup (\{3,4,5\} \cap \phi) = \{2,4\}$

$[Y_2] = ([a] \cap PREA(NEXT, X_0)) \cup ([b] \cap PREA(NEXT, Y_1)) =$

$= (\{2,4\} \cap \{1,2,3,4,5\}) \cup (\{3,4,5\} \cap \{4,5\}) = \{2,4,5\}$

$[Y_3] = ([a] \cap PREA(NEXT, X_0)) \cup ([b] \cap PREA(NEXT, Y_2)) =$

$= (\{2,4\} \cap \{1,2,3,4,5\}) \cup (\{3,4,5\} \cap \{3,4,5\}) = \{2,3,4,5\}$

$[Y_4] = ([a] \cap PREA(NEXT, X_0)) \cup ([b] \cap PREA(NEXT, Y_3)) =$

$= (\{2,4\} \cap \{1,2,3,4,5\}) \cup (\{3,4,5\} \cap \{1,3,4,5\} = \{2,3,4,5\}$

$[Y_3] = [Y_4] = [X_1] = \{2,3,4,5\}$

$[X_2] = [\mu Y((a \wedge [NEXT]X_1) \vee (b \wedge [NEXT]Y))$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREA(NEXT, X_1)) \cup ([b] \cap PREA(NEXT, Y_0)) =$

$= (\{2,4\} \cap \{1,3,4,5\}) \cup (\{3,4,5\} \cap \phi) = \{4\}$

$[Y_2] = ([a] \cap PREA(NEXT, X_1)) \cup ([b] \cap PREA(NEXT, Y_1)) =$

$= (\{2,4\} \cap \{1,3,4,5\}) \cup (\{3,4,5\} \cap \{4\}) = \{4\}$

$[Y_1] = [Y_2] = [X_2] = \{4\}$

$[X_3] = [\mu Y((a \wedge [NEXT]X_2) \vee (b \wedge [NEXT]Y))$

$[Y_0] = \phi$

$$[Y_1] = ([a] \cap PREA(NEXT, X_2)) \cup ([b] \cap PREA(NEXT, Y_0)) =$$

$$= (\{2,4\} \cap \{4\}) \cup (\{3,4,5\} \cap \phi) = \{4\}$$

$$[Y_2] = ([a] \cap PREA(NEXT, X_2)) \cup ([b] \cap PREA(NEXT, Y_1)) =$$

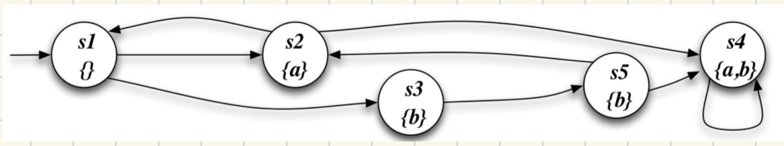
$$= (\{2,4\} \cap \{4\}) \cup (\{3,4,5\} \cap \{4\}) = \{4\}$$

$$[Y_1] = [Y_2] = [X_3] = \{4\}$$

$$[X_2] = [X_3] = \{4\}$$

$$s_1 \in [\nu X. \mu Y. ((a \wedge [NEXT]X) \vee (b \wedge [NEXT]Y))] = \{4\}?$$ **NO!**

2) $\underbrace{AF\,(\underbrace{a \supset \underbrace{EX\,\underbrace{EG\,b}_{\alpha}}_{\beta}}_{\gamma})}_{\delta}$



$$[\alpha] = [EG\ b] = [\nu Z.\ b \wedge \langle NEXT \rangle Z]$$

$$[Z_0] = \{1,2,3,4,5\}$$

$$[Z_1] = [b] \cap PREE(NEXT, Z_0) =$$

$$= \{3,4,5\} \cap \{1,2,3,4,5\} = \{3,4,5\}$$

$$[Z_2] = [b] \cap PREE(NEXT, Z_0) =$$

$$= \{3,4,5\} \cap \{1,2,3,4,5\} = \{3,4,5\} \qquad [Z_1] = [Z_2] = [\alpha] = \{3,4,5\}$$

$$[\beta] = [EX\ \alpha] = [\langle NEXT \rangle\ \alpha] = PREE(NEXT, \alpha) = \{1,2,3,4,5\} = [\beta]$$

$$[\gamma] = [a \supset \beta] = [\neg a] \cup [\beta] = \{1,3,5\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5\} = [\gamma]$$

$$[\delta] = [AF\ \gamma] = [\mu Z.\ \gamma \vee [NEXT] Z]$$

$$[Z_0] = \phi$$

$$[Z_1] = [\gamma] \cup PREA(NEXT, Z_0) =$$

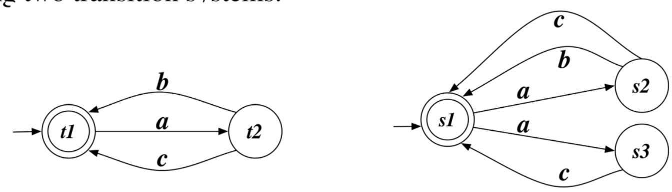
$$= \{1,2,3,4,5\} \cup \phi = \{1,2,3,4,5\}$$

$$[Z_2] = [\gamma] \cup PREA(NEXT, Z_1) =$$

$$= \{1,2,3,4,5\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5\} \qquad [Z_1] = [Z_2] = [\delta] = \{1,2,3,4,5\}$$

$$T s_1 \in \delta\ ? \rightarrow s_1 \in [\delta] = \{1,2,3,4,5\}\ ?$$ **YES!**

Exercise 4. Consider the following two transition systems:



Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

TWO TS ARE BISIMILIAR IF THEY HAVE THE SAME BEHAVIOR:

- LOCALLY THEY LOOK INDISTINGUISHABLE
- EVERY ACTION THAT CAN BE DONE ON ONE, CAN ALSO BE DONE ON THE OTHER, AND THEY REACH THE SAME RESULT

$R_0 = T \times S = \{(t_1, s_1), (t_1, s_2), (t_1, s_3), (t_2, s_1), (t_2, s_2), (t_2, s_3)\}$

$R_1 = \{(t_1, s_1), (t_2, s_2), \cancel{(t_2, s_3)}\}$

$R_2 = \{\cancel{(t_1, s_1)}, (t_2, s_2)\}$

$R_3 = \{\cancel{(t_2, s_2)}\}$

$R_4 = \{\}$

$R_5 = \{\}$

$R_4 = R_5$ GFP FOUND

$(t_1, s_1) \notin$ GFP SO T AND S ARE NOT BISIMILIAR

Exercise 5. Compute the certain answers to the CQ $q(x) \leftarrow Employee(x), Manages(x, y)$ over the incomplete database (naive tables), by explaining and exploiting the connection with conjunctive query containment:

Employee

name
Smith
$null_1$
Brown

Manages

mgr	*mgd*
Green	Smith
Smith	$null_1$
$null_1$	Brown
Brown	$null_2$

$q(x) \leftarrow$ EMPLOYEE(x), MANAGES (x,y)

THE CERTAIN ANSWER IS {SMITH, BROWN}. THESE ARE THE ONLY TUPLE THAT DOESN'T CONTAIN null VALUES, SINCE THESE CAN TAKE ON ANY VALUE.