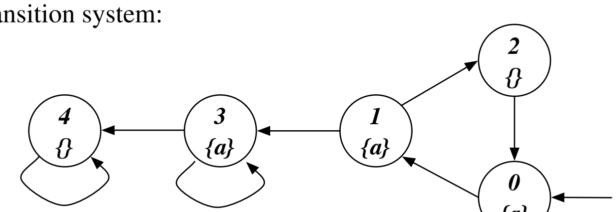
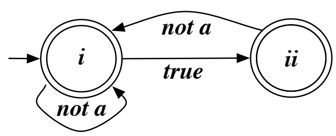
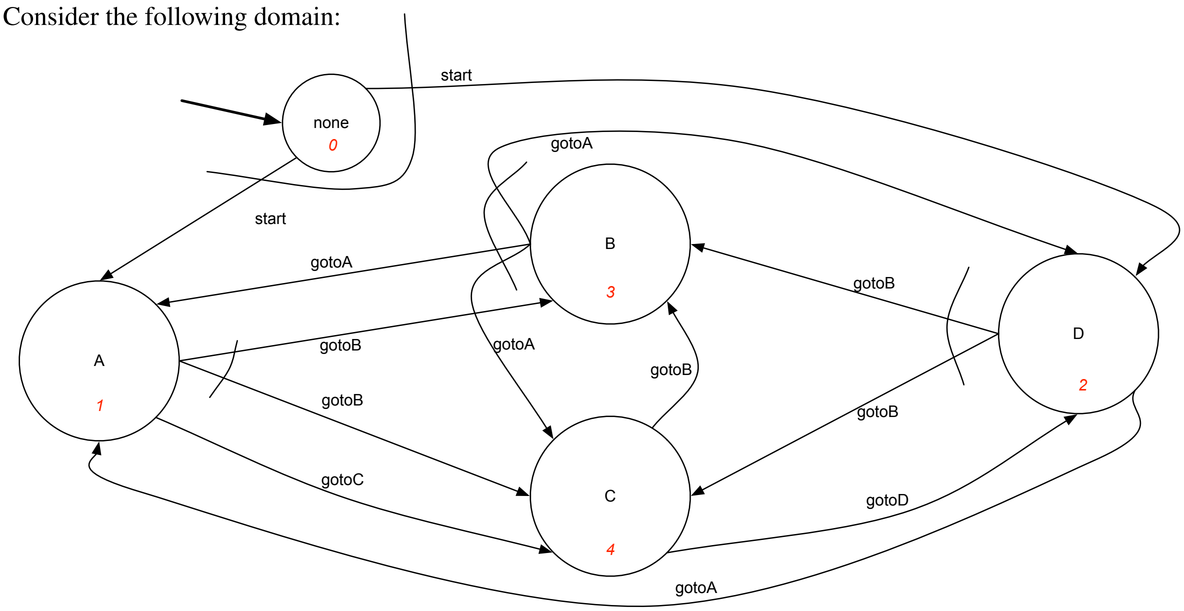

Part 1. Consider the following transition system:



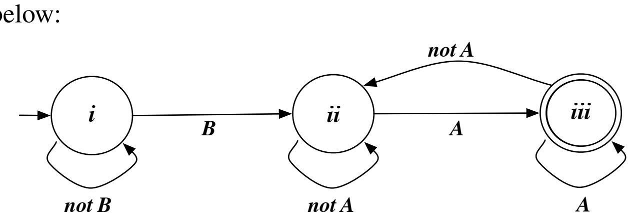
- **Exercise 1.1:** Model check the CTL formula $AF(EFa \wedge (EGa \vee AGa))$, by translating it in Mu-Calculus.
- **Exercise 1.2:** Model check the LTL formula $\Diamond(a \wedge \bigcirc a)$, by considering that the Büchi automaton for $\neg\Diamond(a \wedge \bigcirc a))$ is:



Part 2. Consider the following domain:



- **Exercise 2.1:** Synthesize a strategy (a plan) for realizing the LTLf formula $\Diamond(B \wedge \circ\Diamond(A \wedge \bullet \mathit{false}))$, by considering that the corresponding DFA is the one below:



Part 3.

- **Exercise 3.1:** Given the following conjunctive queries:

```
q1(x) :- edge(x,y), edge(y,y), edge(y,z), edge(z,y).
q2(x) :- edge(x,y), edge(y,z), edge(x,z), edge(x,v), edge(v,z), edge(v,y).
```

check whether $q1$ is contained into $q2$, explaining the technique used and, in case of containment, showing the homomorphism between the canonical databases.

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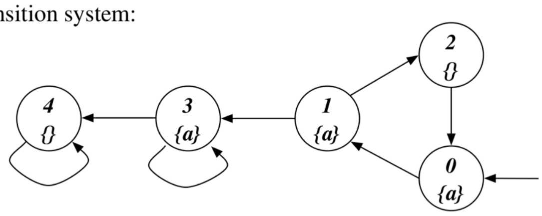

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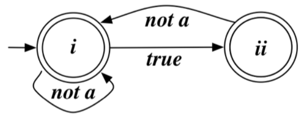


1) $$\underbrace{AF(\underbrace{\underbrace{EF\,a}_{\gamma} \wedge (\underbrace{\underbrace{EG\,a}_{\beta} \vee \underbrace{AG\,a}_{\alpha}}_{\delta})}_{\varepsilon})}_{\eta}$$

$[\alpha] = [AG\ a] = [\nu Z.\ a \wedge [NEXT] Z]$

$[Z_0] = \{0,1,2,3,4\}$

$[Z_1] = [a] \cap PREA(NEXT, Z_0) =$

$= \{0,1,3\} \cap \{0,1,2,3,4\} = \{0,1,3\}$

$[Z_2] = [a] \cap PREA(NEXT, Z_1) =$

$= \{0,1,3\} \cap \{0,2\} = \{0\}$

$[Z_3] = [a] \cap PREA(NEXT, Z_2) =$

$= \{0,1,3\} \cap \{2\} = \phi$

$[Z_4] = [a] \cap PREA(NEXT, Z_3) =$

$= \{0,1,3\} \cap \phi = \phi$

$[Z_3] = [Z_4] = [\alpha] = \phi$

$[\beta] = [EG\ a] = [\nu Z.\ a \wedge \langle NEXT \rangle Z]$

$[Z_0] = \{0,1,2,3,4\}$

$[Z_1] = [a] \cap PREE(NEXT, Z_0) =$

$= \{0,1,3\} \cap \{0,1,2,3,4\} = \{0,1,3\}$

$[Z_2] = [a] \cap PREE(NEXT, Z_1) =$

$= \{0,1,3\} \cap \{0,1,2,3\} = \{0,1,3\}$

$$[Z_1] = [Z_2] = [\beta] = \{0,1,3\}$$

$$[\gamma] = [EF\ \alpha] = [\mu Z.\ \alpha \vee \langle NEXT \rangle Z]$$

$$[Z_0] = \emptyset$$

$$[Z_1] = [\alpha] \cup PREE(NEXT, Z_0) =$$

$$= \{0,1,3\} \cup \emptyset = \{0,1,3\}$$

$$[Z_2] = [\alpha] \cup PREE(NEXT, Z_1) =$$

$$= \{0,1,3\} \cup \{0,1,2,3\} = \{0,1,2,3\}$$

$$[Z_3] = [\alpha] \cup PREE(NEXT, Z_2) =$$

$$= \{0,1,3\} \cup \{0,1,2,3\} = \{0,1,2,3\}$$

$$[Z_2] = [Z_3] = [\gamma] = \{0,1,2,3\}$$

$$[\delta] = [\beta \vee \alpha] = [\beta] \cup [\alpha] = \{0,1,3\} \cup \emptyset = \{0,1,3\} = [\delta]$$

$$[\varepsilon] = [\gamma \wedge \delta] = [\gamma] \cap [\delta] = \{0,1,2,3\} \cap \{0,1,3\} = \{0,1,3\} = [\varepsilon]$$

$$[\eta] = [AF\ \varepsilon] = [\mu Z.\ \varepsilon \vee [NEXT] Z]$$

$$[Z_0] = \emptyset$$

$$[Z_1] = [\varepsilon] \cup PREA(NEXT, Z_0) =$$

$$= \{0,1,3\} \cup \emptyset = \{0,1,3\}$$

$$[Z_2] = [\varepsilon] \cup PREA(NEXT, Z_1) =$$

$$= \{0,1,3\} \cup \{0,2\} = \{0,1,2,3\}$$

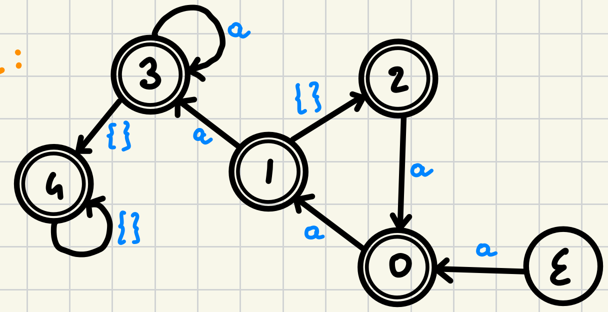
$$[Z_3] = [\varepsilon] \cup PREA(NEXT, Z_2) =$$

$$= \{0,1,3\} \cup \{0,1,2\} = \{0,1,2,3\}$$

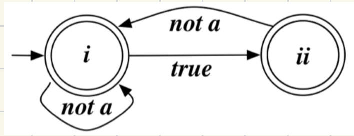
$$[Z_2] = [Z_3] = [\eta] = \{0,1,2,3\}$$

$T s_0 \in \eta$? $\rightarrow$ $s_0 \in [\eta] = \{0,1,2,3\}$? YES !

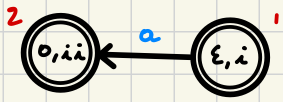
2) A_γ:



$A_{\neg\varphi}$:



$A_\gamma \cap A_{\neg\varphi}$:



$$\varphi = \nu X . \mu Y \left(F \wedge \langle NEXT \rangle X \vee \langle NEXT \rangle Y \right)$$

$$[X_0] = \{1,2\}$$

$$[X_1] = [\mu Y \left(F \wedge \langle NEXT \rangle X_0 \vee \langle NEXT \rangle Y \right)]$$

$$[Y_0] = \emptyset$$

$$[Y_1] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_0) =$$

$$= \{1,2\} \cap \{1\} \cup \emptyset = \{1\}$$

$$[Y_2] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_1) =$$

$$= \{1,2\} \cap \{1\} \cup \emptyset = \{1\}$$

$$[Y_1] = [Y_2] = [X_1] = \{1\}$$

$$[X_2] = [\mu Y \left(F \wedge \langle NEXT \rangle X_1 \vee \langle NEXT \rangle Y \right)]$$

$$[Y_0] = \emptyset$$

$$[Y_1] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_0) =$$

$$= \{1,2\} \cap \emptyset \cup \emptyset = \emptyset$$

$$[Y_0] = [Y_1] = [X_2] = \emptyset$$

$$[X_3] = [\mu Y \left(F \wedge \langle NEXT \rangle X_2 \vee \langle NEXT \rangle Y \right)]$$

$$[Y_0] = \emptyset$$

$$[Y_1] = [F] \cap PREE(NEXT, X_2) \cup PREE(NEXT, Y_0) =$$

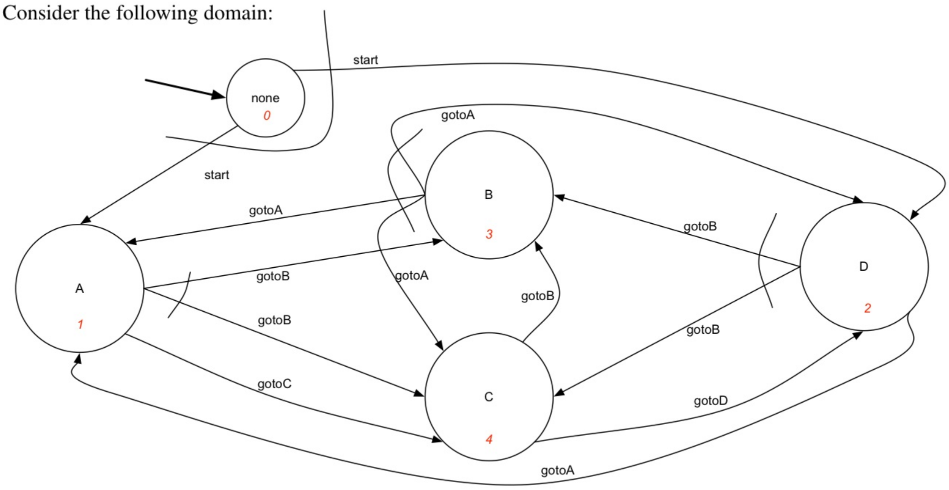
$$= \{1,2\} \cap \emptyset \cup \emptyset = \emptyset$$

$$[Y_0] = [Y_1] = [X_3] = \emptyset$$

$$[X_2] = [X_3] = \emptyset$$

$s_1 \in [\varphi] = \emptyset$? NO!

Part 2. Consider the following domain:



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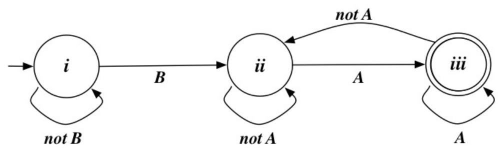
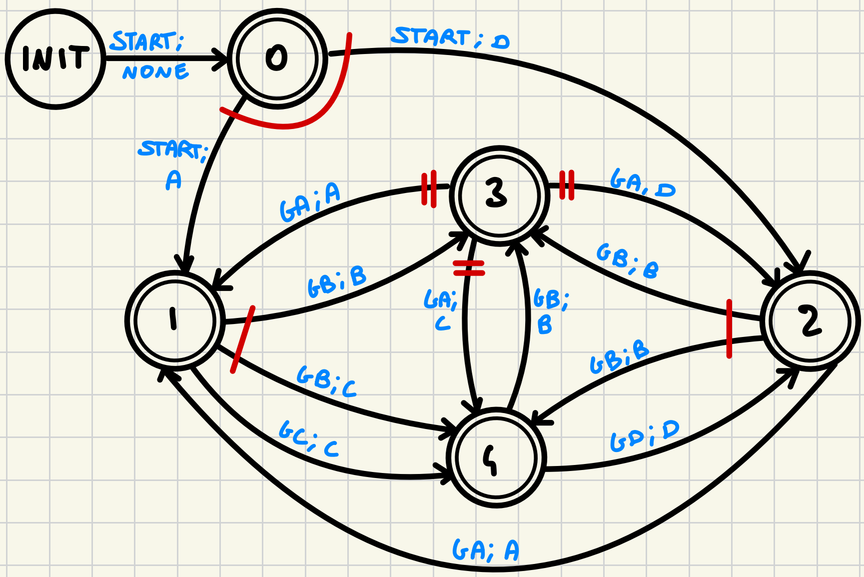
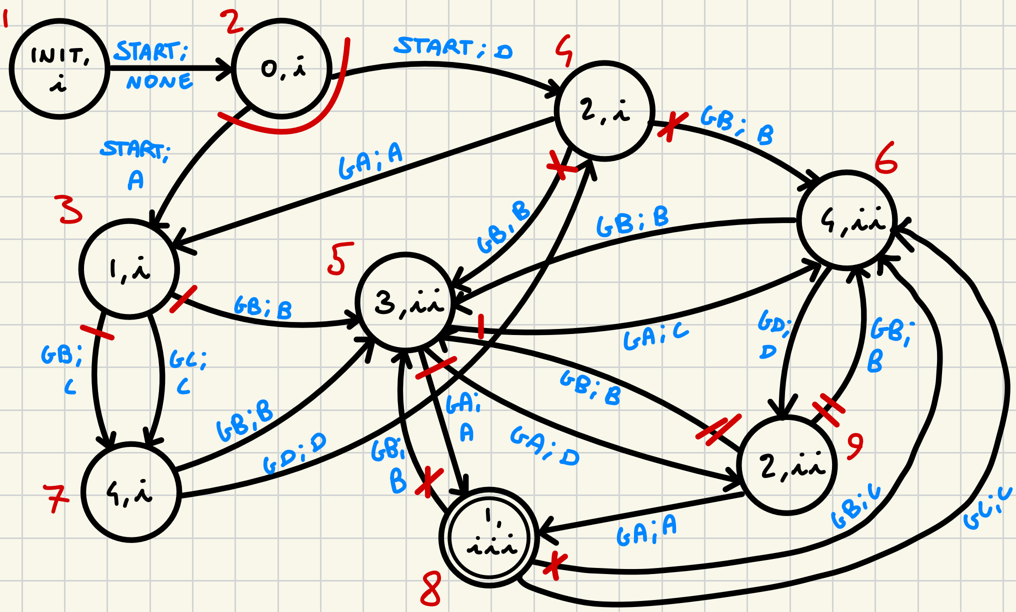


1) A_D :



$A_D \times A_\varphi$:



$W_0 = \{8\}$

$W_1 = W_0 \cup PREADV(W_0) = \{8, 9\}$

$W_2 = W_1 \cup PREADV(W_1) = \{6, 8, 9\}$

$W_3 = W_2 \cup PREADV(W_2) = \{5, 6, 8, 9\}$

$W_4 = W_3 \cup PREADV(W_3) = \{4, 5, 6, 7, 8, 9\}$

$W_5 = W_4 \cup PREADV(W_4) = \{3, 4, 5, 6, 7, 8, 9\}$

$W_6 = W_5 \cup PREADV(W_5) = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$W_7 = W_6 \cup PREADV(W_6) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$W_8 = W_7 \cup PREADV(W_7) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$W_7 = W_8$

$W(1) = \{START\}$	$W_c(1) = START$
$W(2) = \{START\}$	$W_c(2) = START$
$W(3) = \{GB, GC\}$	$W_c(3) = GB$
$W(4) = \{GB\}$	$W_c(4) = GB$
$W(5) = \{GA\}$	$W_c(5) = GA$
$W(6) = \{GD\}$	$W_c(6) = GD$
$W(7) = \{GB\}$	$W_c(7) = GB$
$W(8) = WIN$	$W_c(8) = WIN$
$W(9) = \{GA\}$	$W_c(9) = GA$

$T = (2^X, S, S_0, \rho, W_c)$

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$S_0 = \{1\}$

$\rho(s, x) = \delta(s, (W_c(s), x))$

W_c = ABOVE ↗

Part 3.

- **Exercise 3.1:** Given the following conjunctive queries:

```
q1(x)  :- edge(x,y), edge(y,y), edge(y,z), edge(z,y).
q2(x)  :- edge(x,y), edge(y,z), edge(x,z), edge(x,v), edge(v,z), edge(v,y).
```

check whether $q1$ is contained into $q2$, explaining the technique used and, in case of containment, showing the homomorphism between the canonical databases.

FREEZE

$$q_1(c) \subseteq q_2(c) \quad \begin{cases} q_1(c): E(c,y), E(y,y), E(y,z), E(z,y) \\ q_2(c): E(c,y), E(y,z), E(c,z), E(c,v), E(v,z), E(v,y) \end{cases}$$

BUILD CANONICAL INTERPRETATION

$I_{q_1(c)}$: $\Delta^{q_1(c)}: \{c, y, z\}$

$E^{q_1(c)}: \{\langle c,y\rangle, \langle y,y\rangle, \langle y,z\rangle, \langle z,y\rangle\}$

$I_{q_2(c)}$: $\Delta^{q_2(c)}: \{c, y, z, v\}$

$E^{q_2(c)}: \{\langle c,y\rangle, \langle y,z\rangle, \langle c,z\rangle, \langle c,v\rangle, \langle v,z\rangle, \langle v,y\rangle\}$

QUERY ANSWERING

$I_{q_1(c)} \models q_2(c)$?

$\alpha(y) = y$
$\alpha(z) = y$
$\alpha(v) = y$

$\longrightarrow I_{q_1(c)}, \alpha \models q_2(c)$? YES!

HOMOMORPHISM

$h(c) = c$
$h(y) = \alpha(y) = y$
$h(z) = \alpha(z) = y$
$h(v) = \alpha(v) = y$

$(c,y) \in EDGE^{q_2(c)} \Rightarrow (h(c), h(y)) \in EDGE^{q_1(c)}$

$(y,z) \in EDGE^{q_2(c)} \Rightarrow (h(y), h(z)) \in EDGE^{q_1(c)}$

$(c,z) \in EDGE^{q_2(c)} \Rightarrow (h(c), h(z)) \in EDGE^{q_1(c)}$

$(c,v) \in EDGE^{q_2(c)} \Rightarrow (h(c), h(v)) \in EDGE^{q_1(c)}$

$(v,z) \in EDGE^{q_2(c)} \Rightarrow (h(v), h(z)) \in EDGE^{q_1(c)}$

$(v,y) \in EDGE^{q_2(c)} \Rightarrow (h(v), h(y)) \in EDGE^{q_1(c)}$