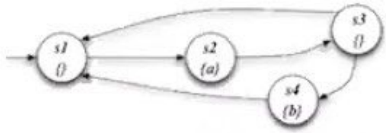
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

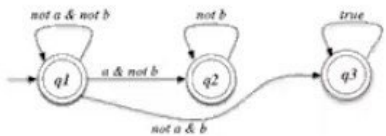
Formal Methods – July 17, 2024

(Time to complete the test: 2 hours)

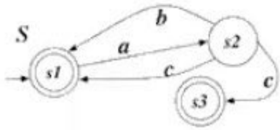
Exercise 1. Model check the Mu-Calculus formula $\nu X.\mu Y.((b \wedge \langle next \rangle X) \vee \langle next \rangle Y)$ and the CTL formula $AG(AFa \wedge EFb \wedge EG\neg b)$ (showing its translation in Mu-Calculus) against the following transition system:

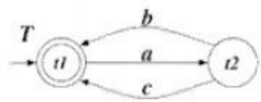


Exercise 2. Model check the LTL formula $\Diamond b \wedge (\neg b \,\mathcal{U}\, a)$ against the transition system of Exercise 1, considering that the Büchi automaton for $\neg(\Diamond b \wedge (\neg b \,\mathcal{U}\, a))$ is the following:



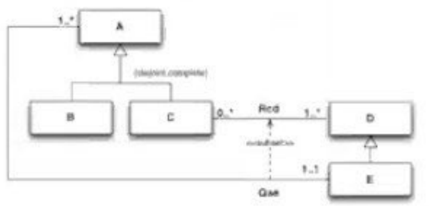
Exercise 3. Consider the following two transition systems:





Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

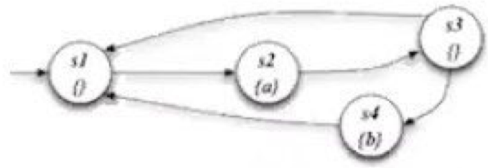
Exercise 4. Express in FOL the following UML class diagram:



Exercise 5. Consider the following predicates: $Supplier(x, y)$, saying that x is a supplier in city y; $Item(x, y)$, saying that item x has color y; and $Sells(x, y, z)$ saying that supplier x sells item y at price z. Express in FOL the following boolean queries, stating which ones are CQs (do not use abbreviations for cardinalities):

1. *Return suppliers in Rome selling a blue item.*
2. *Return suppliers in Rome selling at least two blue items.*
3. *There exists a supplier in Rome selling only blue items*
4. *There exists a supplier in Rome selling all blue items.*
5. *Return the pairs of suppliers such that the first supplier sells at least one item at a cheaper price than the second one.*

Exercise 1. Model check the Mu-Calculus formula $\nu X.\mu Y.((b \wedge \langle next \rangle X) \vee \langle next \rangle Y)$ and the CTL formula $AG(AFa \wedge EFb \wedge EG\neg b)$ (showing its translation in Mu-Calculus) against the following transition system:



$\varphi = \nu X.\mu Y((b \wedge \langle NEXT \rangle X) \vee \langle NEXT \rangle Y)$

$[X_0] = \{1,2,3,4\}$

$[X_1] = [\mu Y((b \wedge \langle NEXT \rangle X_0) \vee \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = ([b] \cap PREE(NEXT, X_0)) \cup PREE(NEXT, Y_0) =$

$= (\{4\} \cap \{1,2,3,4\}) \cup \phi = \{4\}$

$[Y_2] = ([b] \cap PREE(NEXT, X_0)) \cup PREE(NEXT, Y_1) =$

$= (\{4\} \cap \{1,2,3,4\}) \cup \{3\} = \{3,4\}$

$[Y_3] = ([b] \cap PREE(NEXT, X_0)) \cup PREE(NEXT, Y_2) =$

$= (\{4\} \cap \{1,2,3,4\}) \cup \{2,3\} = \{2,3,4\}$

$[Y_4] = ([b] \cap PREE(NEXT, X_0)) \cup PREE(NEXT, Y_3) =$

$= (\{4\} \cap \{1,2,3,4\}) \cup \{1,2,3\} = \{1,2,3,4\}$

$[Y_5] = ([b] \cap PREE(NEXT, X_0)) \cup PREE(NEXT, Y_4) =$

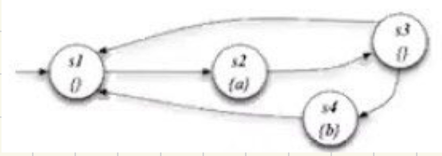
$= (\{4\} \cap \{1,2,3,4\}) \cup \{1,2,3,4\} = \{1,2,3,4\}$

$[Y_4] = [Y_5] = [X_1] = \{1,2,3,4\}$

$[X_0] = [X_1] = \{1,2,3,4\}$

$s_1 \in [\varphi] = \{1,2,3,4\}$? YES!

2) $AG(\underbrace{\underbrace{AFa}_{\gamma} \wedge \underbrace{EFb}_{\beta} \wedge \underbrace{EG\neg b}_{\alpha}}_{\delta})$ with the whole formula $= \varepsilon$



$[\alpha] = [EG\neg b] = [\nu Z. \neg b \wedge \langle NEXT \rangle Z]$

$[Z_0] = \{1,2,3,4\}$

$[Z_1] = [\neg b] \cap PREE(NEXT, Z_0) =$

$= \{1,2,3\} \cap \{1,2,3,4\} = \{1,2,3\}$

$[Z_2] = [\neg b] \cap PREE(NEXT, Z_1) =$

$= \{1,2,3\} \cap \{1,2,3,4\} = \{1,2,3\}$

$[Z_1] = [Z_2] = [\alpha] = \{1,2,3\}$

$[\beta] = [EF\,b] = [\mu Z. b \vee \langle NEXT \rangle Z]$

$[Z_0] = \phi$

$[Z_1] = [b] \cup PREE(NEXT, Z_0) =$

$= \{4\} \cup \phi = \{4\}$

$[Z_2] = [b] \cup PREE(NEXT, Z_1) =$

$= \{4\} \cup \{3\} = \{3,4\}$

$[Z_3] = [b] \cup PREE(NEXT, Z_2) =$

$= \{4\} \cup \{2,3\} = \{2,3,4\}$

$[Z_4] = [b] \cup PREE(NEXT, Z_3) =$

$= \{4\} \cup \{1,2,3\} = \{1,2,3,4\}$

$[Z_5] = [b] \cup PREE(NEXT, Z_4) =$

$= \{4\} \cup \{1,2,3,4\} = \{1,2,3,4\}$

$[Z_4] = [Z_5] = [\beta] = \{1,2,3,4\}$

$[\gamma] = [AF\,a] = [\mu Z. a \vee [NEXT] Z]$

$[Z_0] = \phi$

$[Z_1] = [a] \cup PREA(NEXT, Z_0) =$
$= \{2\} \cup \phi = \{2\}$

$[Z_2] = [\alpha] \cup PREA(NEXT, Z_1) =$
$= \{2\} \cup \{1\} = \{1,2\}$

$[Z_3] = [\alpha] \cup PREA(NEXT, Z_2) =$
$= \{2\} \cup \{1,4\} = \{1,2,4\}$

$[Z_4] = [\alpha] \cup PREA(NEXT, Z_3) =$
$= \{2\} \cup \{1,3,4\} = \{1,2,3,4\}$

$[Z_5] = [\alpha] \cup PREA(NEXT, Z_4) =$
$= \{2\} \cup \{1,2,3,4\} = \{1,2,3,4\}$

$[Z_4] = [Z_5] = [\gamma] = \{1,2,3,4\}$

$[\delta] = [\gamma \wedge \beta \wedge \alpha] = [\gamma] \cap [\beta] \cap [\alpha] = \{1,2,3,4\} \cap \{1,2,3,4\} \cap \{1,2,3\} = \{1,2,3\} = [\delta]$

$[\varepsilon] = [AG\ \delta] = [\nu Z.\ \delta \wedge [NEXT] Z]$

$[Z_0] = \{1,2,3,4\}$

$[Z_1] = [\delta] \cap PREA(NEXT, Z_0) =$

$= \{1,2,3\} \cap \{1,2,3,4\} = \{1,2,3\}$

$[Z_2] = [\delta] \cap PREA(NEXT, Z_1) =$

$= \{1,2,3\} \cap \{1,2,4\} = \{1,2\}$

$[Z_3] = [\delta] \cap PREA(NEXT, Z_2) =$

$= \{1,2,3\} \cap \{1,4\} = \{1\}$

$[Z_4] = [\delta] \cap PREA(NEXT, Z_3) =$

$= \{1,2,3\} \cap \{4\} = \emptyset$

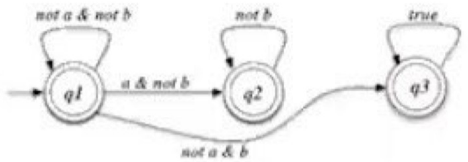
$[Z_5] = [\delta] \cap PREA(NEXT, Z_4) =$

$= \{1,2,3\} \cap \emptyset = \emptyset$

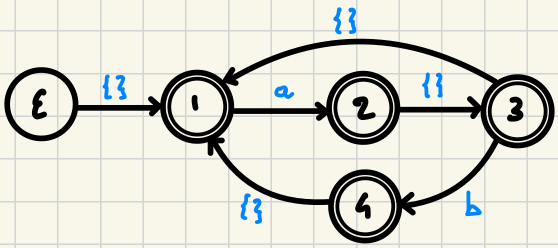
$[Z_4] = [Z_5] = [\varepsilon] = \emptyset$

$\mathcal{T}_{s_1} \in \varepsilon\ ? \rightarrow\ s_1 \in [\varepsilon] = \emptyset\ ?$ NO!

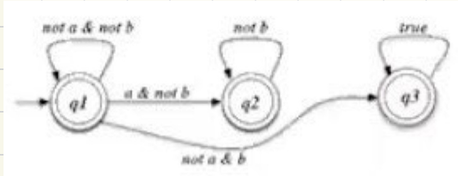
Exercise 2. Model check the LTL formula $\Diamond b \wedge (\neg b \,\mathcal{U}\, a)$ against the transition system of Exercise 1, considering that the Büchi automaton for $\neg(\Diamond b \wedge (\neg b \,\mathcal{U}\, a))$ is the following:



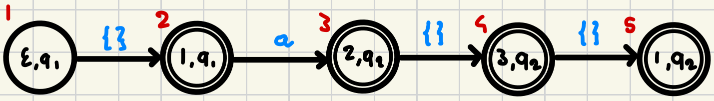
A_φ:



$A_{\neg\varphi}$:



$A_\varphi \cap A_{\neg\varphi}$:



$\varphi = \nu X. \mu Y. (F \wedge \langle NEXT \rangle X \vee \langle NEXT \rangle Y)$

$[X_0] = \{1,2,3,4,5\}$

$[X_1] = [\mu Y. (F \wedge \langle NEXT \rangle X_0 \vee \langle NEXT \rangle Y)]$

$[Y_0] = \emptyset$

$[Y_1] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_0) =$

$= \{1,2,3,4,5\} \cap \{1,2,3,4\} \cup \emptyset = \{1,2,3,4\}$

$[Y_2] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_1) =$

$= \{1,2,3,4,5\} \cap \{1,2,3,4\} \cup \{1,2,3\} = \{1,2,3,4\}$

$[Y_1] = [Y_2] = [X_1] = \{1,2,3,4\}$

$[X_2] = [\mu Y. (F \wedge \langle NEXT \rangle X_1 \vee \langle NEXT \rangle Y)]$

$[Y_0] = \emptyset$

$[Y_1] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_0) =$

$= \{1,2,3,4,5\} \cap \{1,2,3\} \cup \emptyset = \{1,2,3\}$

$[Y_2] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_1) =$

$= \{1,2,3,4,5\} \cap \{1,2,3\} \cup \{1,2\} = \{1,2,3\}$

$[Y_1] = [Y_2] = [X_2] = \{1,2,3\}$

$$[X_3] = [\mu Y.(F \wedge \langle NEXT \rangle X_2 \vee \langle NEXT \rangle Y)]$$

$$[Y_0] = \phi$$

$$[Y_1] = [F] \cap PREE(NEXT, X_2) \cup PREE(NEXT, Y_0) =$$

$$= \{1,2,3,4,5\} \cap \{1,2\} \cup \phi = \{1,2\}$$

$$[Y_2] = [F] \cap PREE(NEXT, X_2) \cup PREE(NEXT, Y_1) =$$

$$= \{1,2,3,4,5\} \cap \{1,2\} \cup \{1\} = \{1,2\}$$

$$[Y_1] = [Y_2] = [X_3] = \{1,2\}$$

$$[X_4] = [\mu Y.(F \wedge \langle NEXT \rangle X_3 \vee \langle NEXT \rangle Y)]$$

$$[Y_0] = \phi$$

$$[Y_1] = [F] \cap PREE(NEXT, X_3) \cup PREE(NEXT, Y_0) =$$

$$= \{1,2,3,4,5\} \cap \{1\} \cup \phi = \{1\}$$

$$[Y_2] = [F] \cap PREE(NEXT, X_3) \cup PREE(NEXT, Y_1) =$$

$$= \{1,2,3,4,5\} \cap \{1\} \cup \phi = \{1\}$$

$$[Y_1] = [Y_2] = [X_4] = \{1\}$$

$$[X_5] = [\mu Y.(F \wedge \langle NEXT \rangle X_4 \vee \langle NEXT \rangle Y)]$$

$$[Y_0] = \phi$$

$$[Y_1] = [F] \cap PREE(NEXT, X_4) \cup PREE(NEXT, Y_0) =$$

$$= \{1,2,3,4,5\} \cap \phi \cup \phi = \phi$$

$$[Y_0] = [Y_1] = [X_5] = \phi$$

$$[X_6] = [\mu Y.(F \wedge \langle NEXT \rangle X_5 \vee \langle NEXT \rangle Y)]$$

$$[Y_0] = \phi$$

$$[Y_1] = [F] \cap PREE(NEXT, X_5) \cup PREE(NEXT, Y_0) =$$

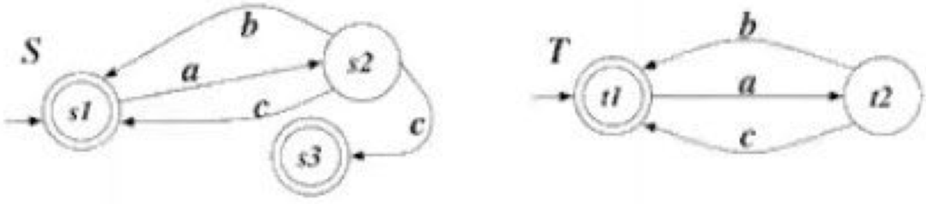
$$= \{1,2,3,4,5\} \cap \phi \cup \phi = \phi$$

$$[Y_0] = [Y_1] = [X_6] = \phi$$

$$[X_5] = [X_6] = \phi$$

$s_1 \in [\varphi] = \phi$? NO!

Exercise 3. Consider the following two transition systems:



Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

TWO TS ARE BISIMILIAR IF THEY HAVE THE SAME BEHAVIOR:

- LOCALLY THEY LOOK INDISTINGUISHABLE
- EVERY ACTION THAT CAN BE DONE ON ONE, CAN ALSO BE DONE ON THE OTHER, AND THEY REACH THE SAME RESULT

$R_0 = S \times T = \{(s_1, t_1), (s_1, t_2), (s_2, t_1), (s_2, t_2), (s_3, t_1), (s_3, t_2)\}$

$R_1 = \{(s_1, t_1), (s_2, t_2), (s_3, t_1)\}$

$R_2 = \{(s_1, t_1), (s_2, t_2)\}$

$R_3 = \{(s_1, t_1)\}$

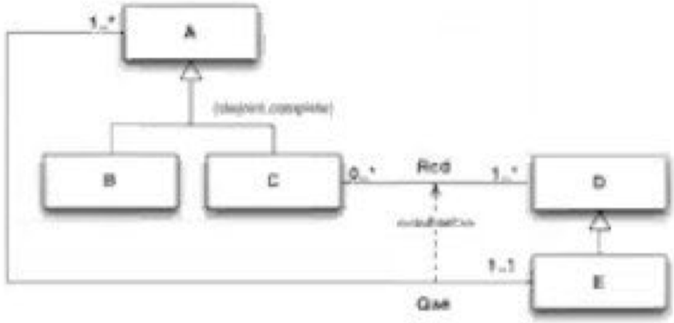
$R_4 = \{\}$

$R_5 = \{\}$

$R_4 = R_5$ GFP FOUND

$(s_1, t_1) \notin$ GFP SO S AND T ARE NOT BISIMILIAR

Exercise 4. Express in *FOL* the following UML class diagram:



$A(x)$, $B(x)$, $C(x)$, $D(x)$, $E(x)$
$R(x,y)$
$Q(x,y)$

$\forall x,y.\ R(x,y) \supset C(x) \wedge D(y)$
$\forall x.\ C(x) \supset \exists y.\ R(x,y)$

$\forall x,y.\ Q(x,y) \supset A(x) \wedge E(y)$
$\forall x.\ A(x) \supset 1 \leq \#\{y \mid Q(x,y)\} \leq 1$
$\forall y.\ E(y) \supset \exists x.\ Q(x,y)$
$\forall x,y.\ Q(x,y) \supset R(x,y)$

$\forall x.\ B(x) \supset A(x) \wedge \neg C(x)$
$\forall x.\ C(x) \supset A(x)$
$\forall x.\ A(x) \supset B(x) \vee C(x)$

$\forall x.\ E(x) \supset D(x)$

Exercise 5. Consider the following predicates: $Supplier(x, y)$, saying that x is a supplier in city y; $Item(x, y)$, saying that item x has color y; and $Sells(x, y, z)$ saying that supplier x sells item y at price z. Express in FOL the following boolean queries, stating which ones are CQs (do not use abbreviations for cardinalities):

1. *Return suppliers in Rome selling a blue item.*
2. *Return suppliers in Rome selling at least two blue items.*
3. *There exists a supplier in Rome selling only blue items*
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1) $\exists x, z.\ S(s, ROME) \land ITEM(x, BLUE) \land SELLS(s, x, z)$ ✓

2) $\exists x, y, z, z'.\ S(s, ROME) \land ITEM(x, BLUE) \land ITEM(y, BLUE) \land SELLS(s, x, z) \land SELLS(s, y, z') \land x \neq y$ ✗

3) $\exists s.\ S(s, ROME) \land \forall x, z.\ (SELLS(s, x, z) \supset ITEM(x, BLUE))$ ✗

4) $\exists s.\ S(s, ROME) \land \forall x.\ (ITEM(x, BLUE) \supset \exists z.\ SELLS(s, x, z))$ ✗

5) $\exists x, x', z, z'.\ SELLS(s, x, z) \land SELLS(s', x', z') \land z < z' \land s \neq s'$ ✗