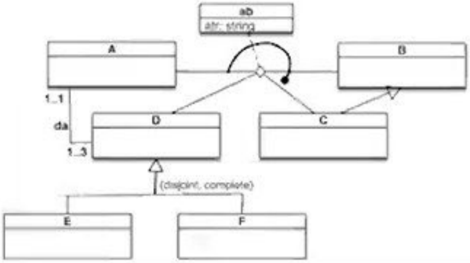
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

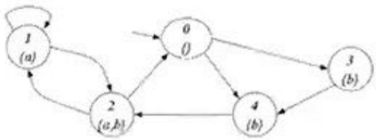
Formal Methods – January 30, 2025

(Time to complete the test online: 2:00 hours)

Exercise 1 Express the following UML class diagram in FOL:

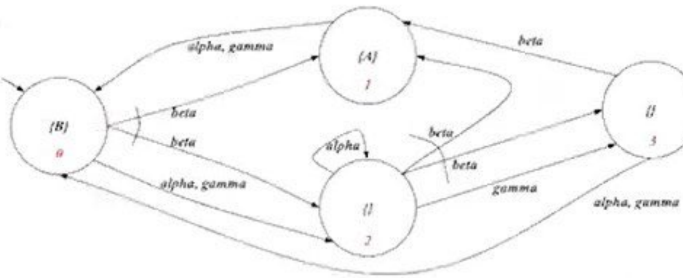


Exercise 2 Consider the following transition system:

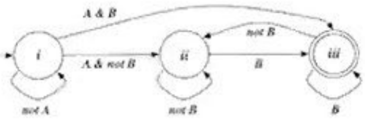


- **Exercise 2.1:** Model check the Mu-Calculus formula: $\nu X.\mu Y.((a \wedge [next]X) \vee (b \wedge \langle next\rangle Y))$
- **Exercise 2.2:** Model check the CTL formula $EF(AG(a \supset AXEXb))$, by translating it in Mu-Calculus.
- **Exercise 2.3:** Model check the LTL formula $\Box\Diamond(b)$, by considering that the Büchi automaton for $\neg\Box\Diamond(b)$ (i.e., $\Diamond\Box(\neg b)$) is the one below:

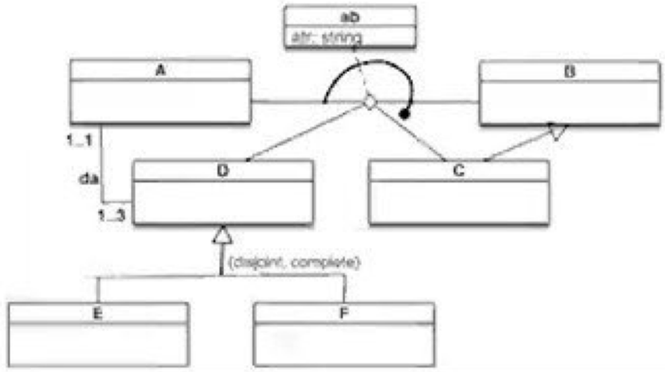
Exercise 3 Consider the following domain:



Synthesize a strategy (a plan) for realizing the LTL$_f$ formula $\Diamond(A \wedge \Diamond(B \wedge \bullet false))$, by considering that the corresponding DFA is the one below:



Exercise 1 Express the following UML class diagram in FOL:



$A(x)$, $B(x)$, $C(x)$, $D(x)$, $E(x)$, $F(x)$
$AB(x,y,z,w)$
$ATR(x,y,z,w,\tau)$
$DA(x,y)$

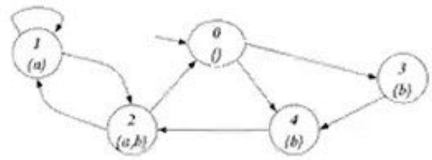
$\forall x,y,z,w.\ AB(x,y,z,w) \supset A(x) \wedge B(y) \wedge C(z) \wedge D(w)$
$\forall x,y,z,z',w,w'.\ AB(x,y,z,w) \wedge AB(x,y,z',w') \supset z = z' \wedge w = w'$
$\forall x,y,z,w,\tau.\ ATR(x,y,z,w,\tau) \supset AB(x,y,z,w) \wedge STRING(\tau)$

$\forall x,y.\ DA(x,y) \supset D(x) \wedge A(y)$
$\forall x.\ D(x) \supset 1 \leq \#\{y \mid DA(x,y)\} \leq 1$
$\forall y.\ A(y) \supset 1 \leq \#\{x \mid DA(x,y)\} \leq 3$

$\forall x.\ C(x) \supset B(x)$

$\forall x.\ E(x) \supset D(x) \wedge \neg F(x)$
$\forall x.\ F(x) \supset D(x)$
$\forall x.\ D(x) \supset E(x) \vee F(x)$

Exercise 2 Consider the following transition system:



- **Exercise 2.1:** Model check the Mu-Calculus formula: $\nu X.\mu Y.((a \wedge [next]X) \vee (b \wedge \langle next \rangle Y))$
- **Exercise 2.2:** Model check the CTL formula $EF(AG(a \supset AXEXb))$, by translating it in Mu-Calculus.
- **Exercise 2.3:** Model check the LTL formula $\Box\Diamond(b)$, by considering that the Büchi automaton for $\neg\Box\Diamond(b)$ (i.e., $\Diamond\Box(\neg b)$) is the one below:

1) $\varphi = \nu X.\mu Y.((a \wedge [NEXT]X) \vee (b \wedge \langle NEXT \rangle Y))$

$[X_0] = \{0,1,2,3,4\}$

$[X_1] = [\mu Y.((a \wedge [NEXT]X_0) \vee (b \wedge \langle NEXT \rangle Y))] =$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREA(NEXT, X_0)) \cup ([b] \cap PREE(NEXT, Y_0)) =$

$= (\{1,2\} \cap \{0,1,2,3,4\}) \cup (\{2,3,4\} \cap \phi) = \{1,2\}$

$[Y_2] = ([a] \cap PREA(NEXT, X_0)) \cup ([b] \cap PREE(NEXT, Y_1)) =$

$= (\{1,2\} \cap \{0,1,2,3,4\}) \cup (\{2,3,4\} \cap \{1,2,4\}) = \{1,2,4\}$

$[Y_3] = ([a] \cap PREA(NEXT, X_0)) \cup ([b] \cap PREE(NEXT, Y_2)) =$

$= (\{1,2\} \cap \{0,1,2,3,4\}) \cup (\{2,3,4\} \cap \{0,1,2,3,4\}) = \{1,2,3,4\}$

$[Y_4] = ([a] \cap PREA(NEXT, X_0)) \cup ([b] \cap PREE(NEXT, Y_3)) =$

$= (\{1,2\} \cap \{0,1,2,3,4\}) \cup (\{2,3,4\} \cap \{0,1,2,3,4\}) = \{1,2,3,4\}$

$[Y_3] = [Y_4] = [X_1] = \{1,2,3,4\}$

$[X_2] = [\mu Y.((a \wedge [NEXT]X_1) \vee (b \wedge \langle NEXT \rangle Y))] =$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREA(NEXT, X_1)) \cup ([b] \cap PREE(NEXT, Y_0)) =$

$= (\{1,2\} \cap \{0,1,3,4\}) \cup (\{2,3,4\} \cap \phi) = \{1\}$

$[Y_2] = ([a] \cap PREA(NEXT, X_1)) \cup ([b] \cap PREE(NEXT, Y_1)) =$

$= (\{1,2\} \cap \{0,1,3,4\}) \cup (\{2,3,4\} \cap \{1,2\} = \{1,2\}$

$[Y_3] = ([a] \cap PREA(NEXT, X_1)) \cup ([b] \cap PREE(NEXT, Y_2)) =$

$= (\{1,2\} \cap \{0,1,3,4\}) \cup (\{2,3,4\} \cap \{1,2,4\}) = \{1,2,4\}$

$[Y_4] = ([a] \cap PREA(NEXT, X_1)) \cup ([b] \cap PREE(NEXT, Y_3)) =$

$= (\{1,2\} \cap \{0,1,3,4\}) \cup (\{2,3,4\} \cap \{0,1,2,3,4\}) = \{1,2,3,4\}$

$[Y_5] = ([a] \cap PREA(NEXT, X_1)) \cup ([b] \cap PREE(NEXT, Y_4)) =$

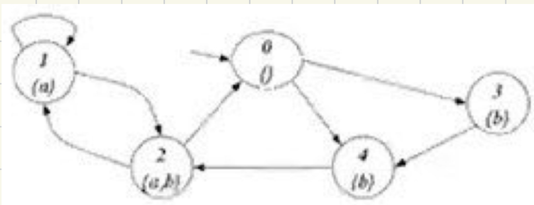
$= (\{1,2\} \cap \{0,1,3,4\}) \cup (\{2,3,4\} \cap \{0,1,2,3,4\}) = \{1,2,3,4\}$

$[Y_4] = [Y_5] = [X_2] = \{1,2,3,4\}$

$[X_1] = [X_2] = \{1,2,3,4\}$

$s_0 \in [\varphi] = \{1,2,3,4\}$? NO!

2) $\underbrace{EF(\underbrace{AG(\underbrace{a \supset \underbrace{AX\,\underbrace{EX\,b}_{\alpha}}_{\beta}}_{\gamma})}_{\delta})}_{\varepsilon}$



$[\alpha] = [EX\, b] = [\langle NEXT \rangle b] = PREE(NEXT, b) = \{0,1,3,4\} = [\alpha]$

$[\beta] = [AX\, \alpha] = [[NEXT]\alpha] = PREA(NEXT, \alpha) = \{0,2,3\} = [\beta]$

$[\gamma] = [a \supset \beta] = [\neg a] \vee [\beta] = \{0,3,4\} \cup \{0,2,3\} = \{0,2,3,4\} = [\gamma]$

$[\delta] = [AG\, \gamma] = [\nu Z.\, \gamma \wedge [NEXT] Z]$

$[Z_0] = \{0,1,2,3,4\}$

$[Z_1] = [\gamma] \cap PREA(NEXT, Z_0) =$

$= \{0,2,3,4\} \cap \{0,1,2,3,4\} = \{0,2,3,4\}$

$[Z_2] = [\gamma] \cap PREA(NEXT, Z_1) =$

$= \{0,2,3,4\} \cap \{0,3,4\} = \{0,3,4\}$

$[Z_3] = [\gamma] \cap PREA(NEXT, Z_2) =$

$= \{0,2,3,4\} \cap \{0,3\} = \{0,3\}$

$[Z_4] = [\gamma] \cap \text{PREA}(\text{NEXT}, Z_3) =$

$= \{0,2,3,4\} \cap \phi = \phi$

$[Z_5] = [\gamma] \cap \text{PREA}(\text{NEXT}, Z_4) =$

$= \{0,2,3,4\} \cap \phi = \phi$

$[Z_4] = [Z_5] = [\delta] = \phi$

$[\varepsilon] = [EF\ \delta] = [\mu Z.\ \delta \vee \langle \text{NEXT} \rangle Z]$

$[Z_0] = \phi$

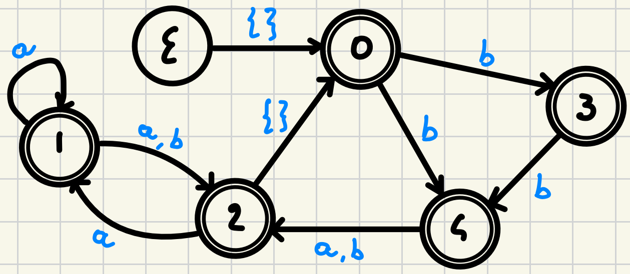
$[Z_1] = [\delta] \cup \text{PREE}(\text{NEXT}, Z_0) =$

$= \phi \cup \phi = \phi$

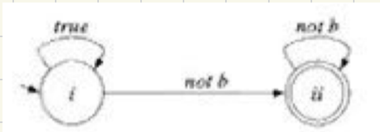
$[Z_0] = [Z_1] = [\varepsilon] = \phi$

$T_{s_0} \in \varepsilon\ ? \rightarrow s_0 \in [\varepsilon] = \phi\ ?$ **NO!**

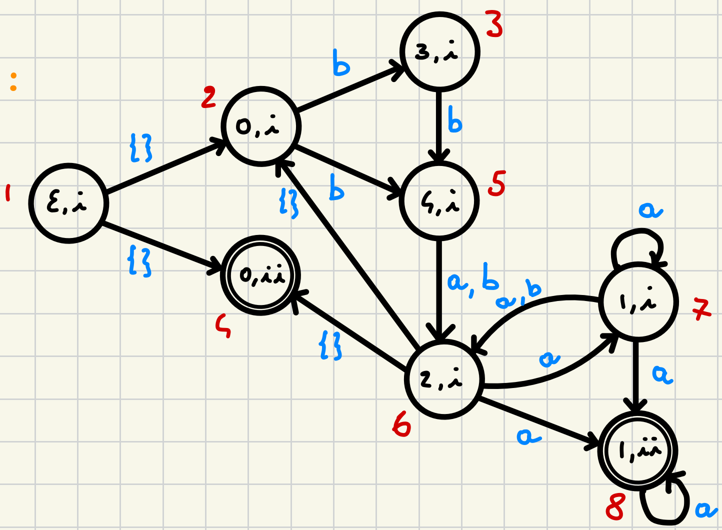
3) A_γ:



$A_{\neg\varphi}$:



$A_\gamma \cap A_{\neg\varphi}$:



$\varphi = \nu X.\ \mu Y.\ (F \wedge \langle \text{NEXT} \rangle X \vee \langle \text{NEXT} \rangle Y)$

$[X_0] = \{1,2,3,4,5,6,7,8\}$

$[X_1] = [\mu Y.\ (F \wedge \langle \text{NEXT} \rangle X_0 \vee \langle \text{NEXT} \rangle Y)]$

$[Y_0] = \phi$

$$[Y_1] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_0) =$$

$$= \{4,8\} \cap \{1,2,3,5,6,7,8\} \cup \phi = \{8\}$$

$$[Y_2] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_1) =$$

$$= \{4,8\} \cap \{1,2,3,5,6,7,8\} \cup \{6,7,8\} = \{6,7,8\}$$

$$[Y_3] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_2) =$$

$$= \{4,8\} \cap \{1,2,3,5,6,7,8\} \cup \{5,6,7,8\} = \{5,6,7,8\}$$

$$[Y_4] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_3) =$$

$$= \{4,8\} \cap \{1,2,3,5,6,7,8\} \cup \{2,3,5,6,7,8\} = \{2,3,5,6,7,8\}$$

$$[Y_5] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_4) =$$

$$= \{4,8\} \cap \{1,2,3,5,6,7,8\} \cup \{1,2,3,5,6,7,8\} = \{1,2,3,5,6,7,8\}$$

$$[Y_6] = [F] \cap PREE(NEXT, X_0) \cup PREE(NEXT, Y_5) =$$

$$= \{4,8\} \cap \{1,2,3,5,6,7,8\} \cup \{1,2,3,5,6,7,8\} = \{1,2,3,5,6,7,8\}$$

$$[Y_5] = [Y_6] = [X_1] = \{1,2,3,5,6,7,8\}$$

$$[X_2] = [\mu Y. (F \wedge \langle NEXT \rangle X_1 \vee \langle NEXT \rangle Y)]$$

$$[Y_0] = \phi$$

$$[Y_1] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_0) =$$

$$= \{4,8\} \cap \{1,2,3,5,6,7,8\} \cup \phi = \{8\}$$

$$[Y_2] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_1) =$$

$$= \{4,8\} \cap \{1,2,3,5,6,7,8\} \cup \{6,7,8\} = \{6,7,8\}$$

$$[Y_3] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_2) =$$

$$= \{4,8\} \cap \{1,2,3,5,6,7,8\} \cup \{5,6,7,8\} = \{5,6,7,8\}$$

$$[Y_4] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_3) =$$

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$[Y_6] = [F] \cap PREE(NEXT, X_1) \cup PREE(NEXT, Y_5) =$

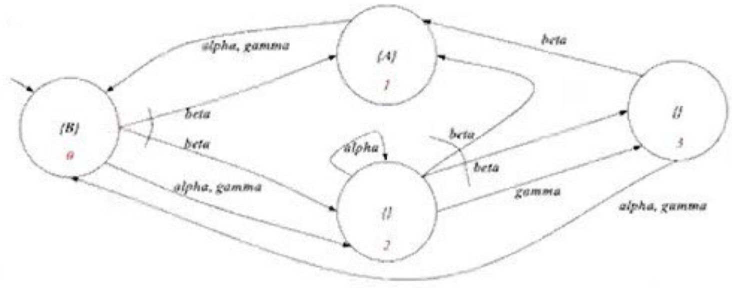
$= \{4, 8\} \cap \{1,2,3,5,6,7,8\} \cup \{1,2,3,5,6,7,8\} = \{1,2,3,5,6,7,8\}$

$[Y_5] = [Y_6] = [X_2] = \{1,2,3,5,6,7,8\}$

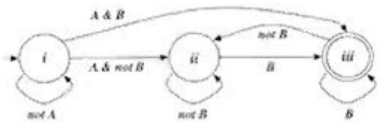
$[X_1] = [X_2] = \{1,2,3,5,6,7,8\}$

$s_1 \in [\varphi] = ?$ YES!

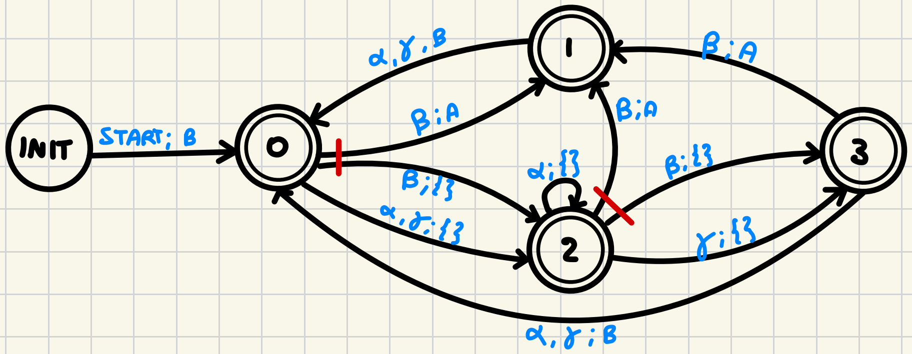
Exercise 3 Consider the following domain:



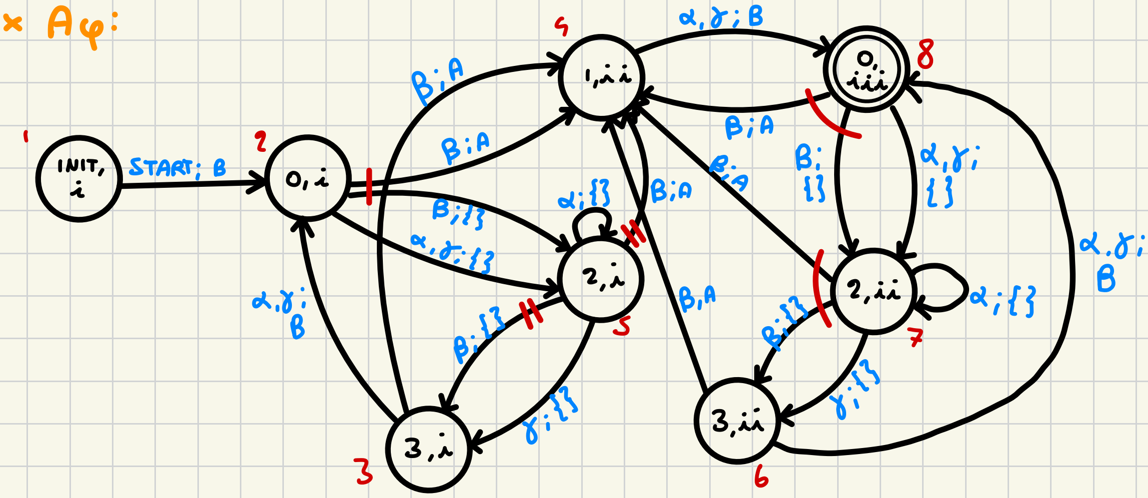
Synthesize a strategy (a plan) for realizing the LTL_f formula $\Diamond(A \wedge \Diamond(B \wedge \bullet false))$, by considering that the corresponding DFA is the one below:



A_D:



$A_D \times A_\varphi$:



$W_0 = \{8\}$

$W_1 = W_0 \cup PREADV(W_0) = \{4, 6, 8\}$

$W_2 = W_1 \cup PREADV(W_1) = \{3, 4, 6, 7, 8\}$

$W_3 = W_2 \cup PREADV(W_2) = \{3, 4, 5, 6, 7, 8\}$

$W_4 = W_3 \cup PREADV(W_3) = \{2, 3, 4, 6, 7, 8\}$

$W_5 = W_4 \cup PREADV(W_4) = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$W_6 = W_5 \cup PREADV(W_5) = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$W_6 = W_5$

$w(1) = \{START\}$
$w(2) = \{\alpha, \beta, \gamma\}$
$w(3) = \{\beta\}$
$w(4) = \{\alpha, \gamma\}$
$w(5) = \{\beta, \gamma\}$
$w(6) = \{\alpha, \gamma\}$
$w(7) = \{\beta\}$
$w(8) = WIN$

$w_c(1) = START$
$w_c(2) = \beta$
$w_c(3) = \beta$
$w_c(4) = \alpha$
$w_c(5) = \beta$
$w_c(6) = \alpha$
$w_c(7) = \beta$
$w_c(8) = WIN$

$T = (2^{X}, S, s_0, \rho, w_c)$

$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$s_0 = \{1\}$

$\rho(s, x) = \delta(s, (w_c(s), x))$

$w_c =$