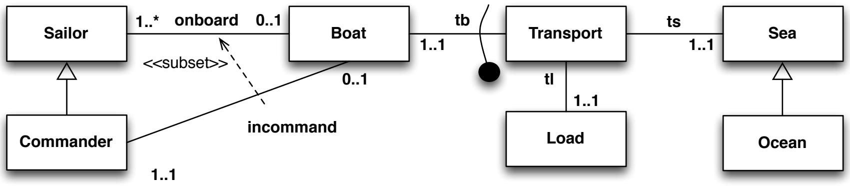
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science

Formal Methods – September 11, 2015

(Time to complete the test: 2 hours)

Exercise 1. Express the following UML class diagram in FOL.



Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

$Sailors$
Dustin
Lubber
Rusty

$Commander$
Alice
Beth

$Boat$
Constitution
Enterprise

$onboard$	
Dustin	Constitution
Lubber	Constitution
Rusty	Constitution

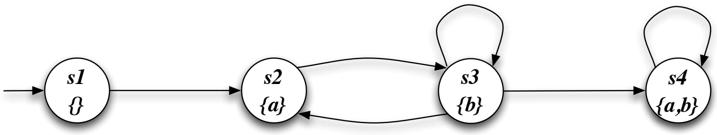
$incommand$	
Alice	Constitution
Beth	Enterprise

Sea
Mediterranean

$Ocean$
Atlantic

1. Check whether the above instantiation, once completed, is correct, and explain why it is or it is not.
2. Express in FOL the following queries and evaluate them over the completed instantiation:
   - (a) Return the commanders that command a boat with at least a sailor who is not a commander.
   - (b) Return the boats that have on board only commanders
   - (c) Return the boats that have on board all the sailors who are not commanders.

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \land \langle next \rangle X) \lor (\neg b \land \langle next \rangle Y))$ and the CTL formula $EF(a \land EX(AGb))$ (showing its translation in Mu-Calculus) against the following transition system:



Exercise 4. Check whether the following Hoare triple is correct, using as *invariant* $i + j \leq 10$.

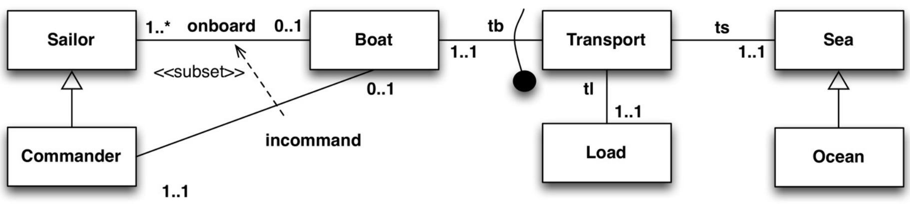
```
{i=0 AND j=10}   while(i<10) do {j=j-1; i:= i+1}   {j=0}
```

Exercise 5. Given the following boolean conjunctive queries:

```
q1() :- edge(r,b), edge(b,g), edge(g,r).
q2() :- edge(i,f), edge(f,v), edge(v,i), edge(i,a), edge(a,v), edge(a,s), edge(s,i).
```

check whether $q1$ is contained into $q2$, explaining the technique used and, in case of containment, showing the homomorphism between the canonical databases.

Exercise 1. Express the following UML class diagram in FOL.



$S(x),\ C(x),\ B(x),\ T(x),\ L(x),\ SEA(x),\ OCEAN(x)$

$ONBOARD(x,y)$

$INCOMMAND(x,y)$

$TB(x,y)$

$TL(x,y)$

$TS(x,y)$

$\forall x,y.\ ONBOARD(x,y) \supset S(x) \wedge B(y)$

$\forall x.\ S(x) \supset 0 \leq \#\{y \mid ONBOARD(x,y)\} \leq 1$

$\forall y.\ B(y) \supset \exists x.\ ONBOARD(x,y)$

$\forall x,y.\ INCOMMAND(x,y) \supset C(x) \wedge B(y)$

$\forall x.\ C(x) \supset 0 \leq \#\{y \mid INCOMMAND(x,y)\} \leq 1$

$\forall y.\ B(y) \supset 1 \leq \#\{x \mid INCOMMAND(x,y)\} \leq 1$

$\forall x,y.\ INCOMMAND(x,y) \supset ONBOARD(x,y)$

$\forall x,y.\ TB(x,y) \supset B(x) \wedge T(y)$

$\forall y.\ T(y) \supset 1 \leq \#\{x \mid TB(x,y)\} \leq 1$

$\forall x,x`,y.\ TB(x,y) \wedge TB(x',y) \supset x = x'$

$\forall x,y.\ TL(x,y) \supset T(x) \wedge L(y)$

$\forall x.\ T(x) \supset 1 \leq \#\{y \mid TL(x,y)\} \leq 1$

$\forall x,y.\ TS(x,y) \supset T(x) \wedge SEA(y)$

$\forall x.\ T(x) \supset 1 \leq \#\{y \mid TS(x,y)\} \leq 1$

$\forall x.\ C(x) \supset S(x)$

$\forall x.\ OCEAN(x) \supset SEA(x)$

Exercise 2. Consider the above UML class diagram and the following (partial) instantiation.

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1. Check whether the above instantiation, once completed, is correct, and explain why it is or it is not.
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1) $\text{SAILOR} = \{ \text{DUST}, \text{LUB}, \text{RUST}, \text{ALI}, \text{BET} \}$

$\text{SEA} = \{ \text{MEDIT}, \text{ATLAN} \}$

$\text{ONBOARD} = \{ \ldots\ldots\ (\text{ALI}, \text{CONST}), (\text{BET}, \text{ENT}) \}$

FOR ALL RELATIONS, ONCE COMPLETED, ALL TUPLES AND CARDINALS ARE OK ✓

2) a. $\exists b, s.\ C(x) \land \text{INCOMMAND}(x, b) \land \text{ONBOARD}(s, b) \land \neg C(s)$

$\{\text{ALICE}\}$

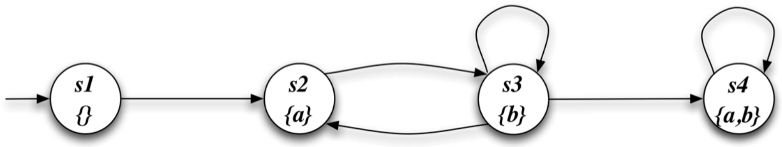
b. $B(x) \land \forall c. (\text{ONBOARD}(c, x) \supset C(c))$

$\{\text{ENT}\}$

c. $B(x) \land \forall s. (\neg C(s) \supset \text{ONBOARD}(s, x))$

$\{ \}$

Exercise 3. Model check the Mu-Calculus formula $\nu X.\mu Y.((a \wedge \langle next \rangle X) \vee (\neg b \wedge \langle next \rangle Y))$ and the CTL formula $EF(a \wedge EX(AGb))$ (showing its translation in Mu-Calculus) against the following transition system:



1) $\nu X. \mu Y. ((a \wedge \langle NEXT \rangle X) \vee (\neg b \wedge \langle NEXT \rangle Y))$

$[X_0] = \{1,2,3,4\}$

$[X_1] = [\mu Y. \ (a \wedge \langle NEXT \rangle X_0) \vee (\neg b \wedge \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREE(NEXT, X_0)) \cup ([\neg b] \cap PREE(NEXT. Y_0)] =$

$= (\{2,4\} \cap \{1,2,3,4\}) \cup (\{1,2\} \cap \phi) = \{2,4\}$

$[Y_2] = ([a] \cap PREE(NEXT, X_0)) \cup ([\neg b] \cap PREE(NEXT. Y_1)] =$

$= (\{2,4\} \cap \{1,2,3,4\}) \cup (\{1,2\} \cap \{1,3,4\} = \{1,2,4\}$

$[Y_3] = ([a] \cap PREE(NEXT, X_0)) \cup ([\neg b] \cap PREE(NEXT. Y_2)] =$

$= (\{2,4\} \cap \{1,2,3,4\}) \cup (\{1,2\} \cap \{1,3,4\} = \{1,2,4\}$

$[Y_4] = [Y_3] = [X_1] = \{1,2,4\}$

$[X_2] = [\mu Y. \ (a \wedge \langle NEXT \rangle X_1) \vee (\neg b \wedge \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREE(NEXT, X_1)) \cup ([\neg b] \cap PREE(NEXT. Y_0)] =$

$= (\{2,4\} \cap \{1,3,4\}) \cup (\{1,2\} \cap \phi) = \{4\}$

$[Y_2] = ([a] \cap PREE(NEXT, X_1)) \cup ([\neg b] \cap PREE(NEXT. Y_1)] =$

$= (\{2,4\} \cap \{1,3,4\}) \cup (\{1,2\} \cap \{4\}) = \{4\}$

$[Y_1] = [Y_2] = [X_2] = \{4\}$

$[X_3] = [\mu Y. \ (a \wedge \langle NEXT \rangle X_2) \vee (\neg b \wedge \langle NEXT \rangle Y)]$

$[Y_0] = \phi$

$[Y_1] = ([a] \cap PREE(NEXT, X_1)) \cup ([\neg b] \cap PREE(NEXT. Y_0)] =$

$= (\{2,4\} \cap \{4\}) \cup (\{1,2\} \cap \phi) = \{4\}$

$[Y_2] = ([a] \cap \text{PREE}(\text{NEXT}, X_1)) \cup ([\neg b] \cap \text{PREE}(\text{NEXT}, Y_1)] =$

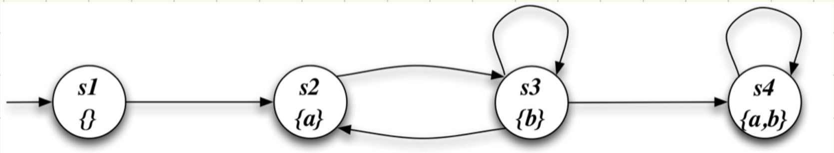
$= (\{2,4\} \cap \{4\}) \cup (\{1,2\} \cap \{4\}) = \{4\}$

$[Y_1] = [Y_2] = [X_3] = \{4\}$

$[X_2] = [X_3] = \{4\}$

$s_1 \in [\nu X. \mu Y. ((a \wedge \langle \text{NEXT} \rangle X) \vee (\neg b \wedge \langle \text{NEXT} \rangle Y))] = \{4\}$? NO!

2) $\underbrace{\text{EF}(\underbrace{a \wedge \underbrace{\text{EX}(\underbrace{\text{AG}\, b}_{\alpha})}_{\beta}}_{\gamma})}_{\delta}$



$[\alpha] = [\text{AG}\ b] = [\nu Z.\ b \wedge [\text{NEXT}] Z]$

$[Z_0] = \{1,2,3,4\}$

$[Z_1] = [b] \cap \text{PREA}(\text{NEXT}, Z_0) =$

$= \{3,4\} \cap \{1,2,3,4\} = \{3,4\}$

$[Z_2] = [b] \cap \text{PREA}(\text{NEXT}, Z_1) =$

$= \{3,4\} \cap \{2,4\} = \{4\}$

$[Z_3] = [b] \cap \text{PREA}(\text{NEXT}, Z_2) =$

$= \{3,4\} \cap \{4\} = \{4\}$ $\qquad [Z_2] = [Z_3] = [\alpha] = \{4\}$

$[\beta] = [\text{EX}\ \alpha] = [\langle \text{NEXT} \rangle \alpha] = \text{PREE}(\text{NEXT}, \alpha) = \{3,4\} = [\beta]$

$[\gamma] = [a \wedge \beta] = [a] \cap [\beta] = \{2,4\} \cap \{3,4\} = \{4\} = [\gamma]$

$[\delta] = [\text{EF}\ \gamma] = [\mu Z\ \gamma \vee \langle \text{NEXT} \rangle Z]$

$[Z_0] = \emptyset$

$[Z_1] = [\gamma] \cup \text{PREE}(\text{NEXT}, Z_0) =$

$= \{4\} \cup \emptyset = \{4\}$

$[Z_2] = [\gamma] \cup \text{PREE}(\text{NEXT}, Z_1) =$

$= \{4\} \cup \{3,4\} = \{3,4\}$

$[z_3] = [\gamma] \cup \mathrm{PREE}(\mathrm{NEXT}, z_2) =$

$= \{4\} \cup \{2,3,4\} = \{2,3,4\}$

$[z_4] = [\gamma] \cup \mathrm{PREE}(\mathrm{NEXT}, z_3) =$

$= \{4\} \cup \{1,2,3,4\} = \{1,2,3,4\}$

$[z_5] = [\gamma] \cup \mathrm{PREE}(\mathrm{NEXT}, z_4) =$

$= \{4\} \cup \{1,2,3,4\} = \{1,2,3,4\}$

$[z_4] = [z_5] = [\delta] = \{1,2,3,4\}$

$T_{s_1} \in \delta$? $\rightarrow$ $s_1 \in [\delta] = \{1,2,3,4\}$? YES!

Exercise 4. Check whether the following Hoare triple is correct, using as *invariant* $\overset{I}{i + j \leq 10}$.

$\overset{P}{\texttt{\{i=0 AND j=10\}}}$ $\texttt{while}(\overset{g}{\texttt{i<10}})$ $\texttt{do}$ $\overset{\delta}{\texttt{\{j=j-1; i:= i+1\}}}$ $\overset{Q}{\texttt{\{j=0\}}}$

1. $P \supset I$ 2. $\{\neg g \wedge I\} \supset Q$ 3. $\{g \wedge I\}\ \delta\ \{I\}$

1. $\{i=0 \wedge j=10\} \supset i+j \leq 10$ ✓

2. $\{i \geq 10 \wedge i+j \leq 10\} \supset j=0$ ✓

3. $\{i<10 \wedge i+j \leq 10\}\ j=j-1; i=i+1\ \{i+j \leq 10\}$

$\{i<10 \wedge i+j \leq 10\} \supset wp(j=j-1; i=i+1)\{i+j \leq 10\}$?

$\{i+j \leq 9\}[j/j-1] = \{i+j \leq 10\}$
$j=j-1;$
$\{i+j \leq 10\}[i/i+1] = \{i+j \leq 9\}$
$i=i+1;$
$\{i+j \leq 10\}$

$\{i<10 \wedge i+j \leq 10\} \supset \{i+j \leq 10\}$?

$\{i+j \leq 10\}$ IS AN INVARIANT

Exercise 5. Given the following boolean conjunctive queries:

```
q1() :- edge(r,b), edge(b,g), edge(g,r).
q2() :- edge(i,f), edge(f,v), edge(v,i), edge(i,a), edge(a,v), edge(a,s), edge(s,i).
```

check whether $q1$ is contained into $q2$, explaining the technique used and, in case of containment, showing the homomorphism between the canonical databases.

BUILD CANONICAL INTERPRETATION

$I_{q_1()}$: $\Delta I_{q_1()} : \{ r, g, b \}$

$EDGE^{q_1()} : \{ <r, b>, <b, g>, <g, r> \}$

$I_{q_2()}$: $\Delta I_{q_2()} : \{ a, i, f, v, s \}$

$EDGE^{q_2()} : \{ <i, f>, <f, v>, <v, i>, <i, a>, <a, v>, <a, s>, <s, i> \}$

QUERY ANSWERING

$I_{q_1()} \models I_{q_2()}$?

$\alpha(i) = r$
$\alpha(f) = b$
$\alpha(v) = g$
$\alpha(a) = b$
$\alpha(s) = g$

$I_{q_1()}, \alpha \models q_2()$ YES!

HOMOMORPHISM

$h(i) = \alpha(i) = r$
$h(f) = \alpha(f) = b$
$h(v) = \alpha(v) = g$
$h(a) = \alpha(a) = b$
$h(s) = \alpha(s) = g$

$(i, f) \in EDGE^{q_2()} \Rightarrow (h(i), h(f)) \in EDGE^{q_1()}$
$(f, v) \in EDGE^{q_2()} \Rightarrow (h(f), h(v)) \in EDGE^{q_1()}$
$(v, i) \in EDGE^{q_2()} \Rightarrow (h(v), h(i)) \in EDGE^{q_1()}$
$(i, a) \in EDGE^{q_2()} \Rightarrow (h(i), h(a)) \in EDGE^{q_1()}$
$(a, v) \in EDGE^{q_2()} \Rightarrow (h(a), h(v)) \in EDGE^{q_1()}$
$(a, s) \in EDGE^{q_2()} \Rightarrow (h(a), h(s)) \in EDGE^{q_1()}$
$(s, i) \in EDGE^{q_2()} \Rightarrow (h(s), h(i)) \in EDGE^{q_1()}$