Formulario Analisi I

Università degli Studi "La Sapienza" di Roma

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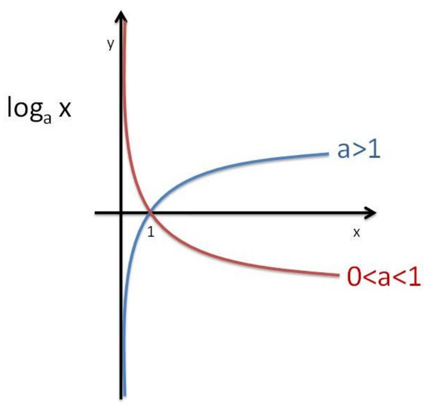
Richiami di matematica elementare

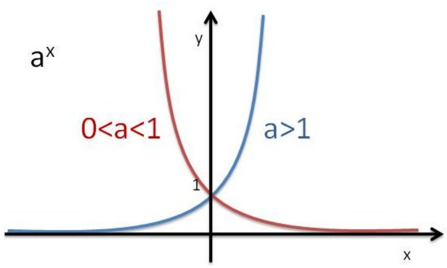
- Proprietà delle potenze ad esponente reale ($x, y \in \mathbb{R}^+$):

$$
\begin{aligned}
&1)x^0 = 1, \quad \forall x \in \mathbb{R}\backslash\{0\}, 1^\alpha = 1, \quad \forall x \in \mathbb{R}\\
&2)x^\alpha \cdot x^\beta = x^{\alpha+\beta}, \quad \forall \alpha, \beta \in \mathbb{R}\\
&3)x^\alpha \cdot y^\alpha = (xy)^\alpha, \quad \forall \alpha \in \mathbb{R}\\
&4)\frac{x^\alpha}{x^\beta} = x^{\alpha-\beta}, \quad \forall \alpha, \beta \in \mathbb{R}\\
&5)\frac{x^\alpha}{y^\alpha} = (\frac{x}{y})^\alpha = (\frac{y}{x})^{\cdot\alpha}, \quad \forall \alpha \in \mathbb{R}\\
&6)(x^\alpha)^\beta = x^{\alpha\cdot\beta}, \quad \forall \alpha, \beta \in \mathbb{R}\\
&7)x^{\frac{1}{n}} = \sqrt[n]{x}, \quad \forall \in \mathbb{N}, \forall x \in \mathbb{R}^+\\
&8)x^{\frac{m}{n}} = \sqrt[n]{x^m} = \sqrt[n]{x}^m, \quad \forall \in \mathbb{N}, \forall x \in \mathbb{R}^+\\
&9)se x < y \Longrightarrow a^x \lesseqgtr a^y se a \gtreqless 1\\
&10)a \leqslant b \Longrightarrow a^x \leqslant b^x, \quad \forall x \in \mathbb{R}_+
\end{aligned}
$$

- Proprietà dei logaritmi ($x, y, a, b \in \mathbb{R}^+, a, b \neq 1$):

$$
\begin{aligned}
&1)a^{log_a x} = x;\\
&2)log_a(a^x) = x;\\
&3)log_a 1 = 0;\\
&4)log_a(xy) = log_a x + log_a y;\\
&5)log_a(\frac{x}{y}) = log_a x - log_a y;\\
&6)log_a(x^\alpha) = \alpha \cdot log_a(x), \forall \alpha \in \mathbb{R};\\
&7)log_a x = \frac{1}{log_x a} = -log_{\frac{1}{a}} x, \quad x \neq 1;\\
&8)log_b x = \frac{log_a x}{log_b x};
\end{aligned}
$$





- Proprietà del modulo o valore assoluto:

$$
\begin{aligned}
&1) \mid x \mid \geq 0, \quad \forall x \in \mathbb{R};\\
&2) \mid x \mid = 0 \Longleftrightarrow x = 0;\\
&3) \mid -x \mid = \mid x \mid, \quad \forall x \in \mathbb{R};\\
&4) \mid x \mid = \sqrt{x^2}, \quad \forall x \in \mathbb{R};\\
&5) \mid x \cdot y \mid = \mid x \mid\mid y \mid, \quad \forall x, y \in \mathbb{R};\\
&6) \mid \frac{x}{y} \mid = \frac{\mid x \mid}{\mid y \mid}, \quad \forall x, y \in \mathbb{R}, y \neq 0;\\
&7) \mid x + y \mid \leq \mid x \mid + \mid y \mid, \forall x, y \in \mathbb{R};\\
&8) \mid\mid x \mid - \mid\mid \leq \mid x - y \mid, \forall x, y \in \mathbb{R};
\end{aligned}
$$

Numeri complessi

- Forma algebrica: $z = x + iy, \forall x, y \in \mathbb{R}; \overline{z} = x - iy, \mid z \mid = \sqrt{x^2 + y^2}, \forall z \in \mathbb{C}; \forall z, w \in \mathbb{C} siha:$

$$
\begin{aligned}
&1)\overline{(z \pm w)} = \overline{z} \pm \overline{w}\\
&2)\overline{(zw)} = \overline{z} \cdot \overline{w}\\
&3)\overline{\frac{z}{w}} = \frac{\overline{z}}{\overline{w}}\\
&4) z \cdot \overline{z} = \mid z \mid^2\\
&5) \mid z \mid \geqslant 0\\
&6) \mid z \mid = 0 \Longleftrightarrow z = 0\\
&7) \mid z \mid = \mid \overline{z} \mid\\
&8) \mid z \cdot w \mid = \mid z \mid \cdot \mid w \mid\\
&9) \mid \frac{z}{w} \mid = \frac{\mid z \mid}{\mid w \mid}, w \neq 0\\
&10) \mid Re(z) \mid \leqslant \mid z \mid, \mid Im(z) \mid \leqslant \mid z \mid, \mid z \mid \leqslant \mid Re(z) \mid + \mid Im(z) \mid\\
&11) \mid z + w \mid \leqslant \mid z \mid + \mid w \mid\\
&12) \mid\mid z \mid - \mid w \mid\mid \leqslant \mid z + w \mid
\end{aligned}
$$

- Forma trigonometrica: $z = \rho(cos\theta + isin\theta), \rho \in \mathbb{R}^+ \theta \in [0, 2\pi), dove \rho = \sqrt{x^2 + y^2}, cos\theta = \frac{x}{\sqrt{x^2+y^2}}, sin\theta = \frac{y}{\sqrt{x^2+y^2}}$
  Se $w = \eta(cos\phi + isin\phi), \eta \in \mathbb{R}^+, \phi \in [0, 2\pi)$ allora:

$$
\begin{aligned}
&1) zw = \rho\eta[cos(\theta + \phi) + isin(\theta + \phi)];\\
&2)\frac{z}{w} = \frac{\rho}{\eta}[cos(\theta - \phi) + isin(\theta - \phi)];\\
&3) z^n = \rho^n[cos(n\theta) + isin(n\theta)]; "Formula di Moivre";\\
&4) \sqrt[n]{z} = \sqrt[n]{\rho}[cos(\frac{\theta + 2k\pi}{n}) + isin(\frac{\theta + 2k\pi}{n})], k = 0, 1, 2, \cdots, (n-1);
\end{aligned}
$$

- Forma esponenziale: $z = \rho e^{i\theta}, \rho \in \mathbb{R}^+, \theta \in [0, 2\pi)$
  Se $w = \eta e^{i\phi}, \eta \in \mathbb{R}^+, \phi \in [0, 2\pi)$ allora:

$$1) zw = \rho\eta e^{i(\theta+\phi)};$$
$$2) \frac{z}{w} = \frac{\rho}{\eta} e^{i(\theta-\phi)};$$
$$3) z^n = \rho^n e^{i(n\theta)};$$
$$4) \sqrt[n]{z} = \sqrt[n]{\rho} e^{\frac{i(\theta+2k\pi}{n}}, k = 0, 1, 2, \cdots, (n-1);$$

LIMITI NOTEVOLI $\forall\theta \in \mathbb{R}$

$\lim_{x\to\pm\infty}((1+\frac{1}{x})^x = e$
$\lim_{x\to\infty}((1+\frac{\alpha}{x})^\theta x = e^{\alpha\theta}$
$\lim_{x\to 0}((1+x)^{\frac{1}{x}} = e$
$\lim_{x\to 0} \frac{log(1+x)}{x} = 1$
$\lim_{x\to 0} \frac{log_a(1+x)}{x} = \frac{1}{loga} \quad \forall a > 0, a \neq 1$
$\lim_{x\to 0} \frac{a^x - 1}{x} = loga$
$\lim_{x\to 0} \frac{(1+x)^\theta - 1}{x} = \theta$
$\lim_{x\to\infty} \frac{x^\theta}{e^x} = 0$
$\lim_{x\to\infty} \frac{x^\theta}{a^x} = 0 \quad \forall a > 1$

$\lim_{x\to\infty} \frac{(logx)^\theta}{x} = 0$
$\lim_{x\to\infty} x^{\frac{1}{x}} = 1$
$\lim_{x\to 0} \frac{sinx}{x} = 1$
$\lim_{x\to 0} \frac{1-cosx}{x^2} = \frac{1}{2}$
$\lim_{x\to 0} \frac{tanx}{x} = 1$
$\lim_{x\to 0} \frac{arcsinx}{x} = 1$
$\lim_{x\to 0} \frac{arctanx}{x} = 1$
$\lim_{x\to 0} \frac{sinhx}{x} = 1$
$\lim_{x\to 0} \frac{tanhx}{x} = 1$

LIMITI NOTEVOLI PER SUCCESSIONI $\forall a, \theta, \beta \in \mathbb{R}$

$\lim_{n\to\pm\infty}((1+\frac{1}{n})^n = e$
$\lim_{n\to\infty}((1+\frac{\theta}{a_n})^{a_n} = e^\theta$
$\lim_{\epsilon_n\to 0}((1+\epsilon_n)^{\frac{1}{\epsilon_n}} = e$
$\lim_{\epsilon_n\to 0} \frac{log(1+\epsilon_n)}{\epsilon_n} = 1$
$\lim_{\epsilon_n\to 0} \frac{log_a(1+\epsilon_n)}{\epsilon_n} = \frac{1}{loga} \quad \forall a > 0, a \neq 1$
$\lim_{\epsilon_n\to 0} \frac{a^{\epsilon_n} - 1}{\epsilon_n} = loga \quad \forall a > 0$
$\lim_{\epsilon_n\to 0} \frac{(1+\epsilon_n)^\theta - 1}{\epsilon_n} = \theta$
$\lim_{a_n\to\infty} \frac{a_n^\theta}{e_n^a} = 0$
$\lim_{a_n\to\infty} \frac{(loga_n)^\theta}{a_n} = 0 \quad \forall a > 1$

$\lim_{n\to\infty} n^{\frac{1}{n}} = 1$
$\lim_{\epsilon_n\to 0} \frac{sin\epsilon_n}{\epsilon_n} = 1$
$\lim_{\epsilon_n\to 0} \frac{1-cos\epsilon_n}{\epsilon_n{}^2} = \frac{1}{2}$
$\lim_{\epsilon_n\to 0} \frac{tan\epsilon_n}{\epsilon_n} = 1$
$\lim_{\epsilon_n\to 0} \frac{arcsin\epsilon_n}{\epsilon_n} = 1$
$\lim_{\epsilon_n\to 0} \frac{arctan\epsilon_n}{\epsilon_n} = 1$
$\lim_{\epsilon_n\to 0} \frac{sinh\epsilon_n}{\epsilon_n} = 1$
$\lim_{\epsilon_n\to 0} \frac{tanh\epsilon_n}{\epsilon_n} = 1$
$\lim_{\epsilon_n\to 0} \frac{cosh\epsilon_n - 1}{\epsilon_n{}^2} = \frac{1}{2}$
$\lim_{\epsilon_n\to 0} \epsilon_n \mid log \mid \epsilon_n \mid\mid^\beta = 0$

FORMULE UTILI

FUNZIONI TRIGONOMETRICHE

Tabella dei valori

gradi	Rad.	senα	cosα	tgα	gradi	Rad.	senα	cosα	tgα
0°	0	0	1	0	180°	π	0	-1	0
30°	$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	210°	$7\pi/6$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	225°	$5\pi/4$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
60°	$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	240°	$4\pi/3$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
90°	$\pi/2$	1	0	N.D.	270°	$3\pi/4$	-1	0	N.D.
120°	$2\pi/3$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	300°	$5\pi/3$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
135°	$3\pi/4$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	315°	$7\pi/4$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
150°	$5\pi/6$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	330°	$11\pi/6$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$

Simmetrie,archi complementari e supplementari

$$sin(-x) = -sin(x) \qquad cos(-x) = cos(x) \qquad tan(-x) = -tan(x)$$

$$sin(\frac{\pi}{2} \pm x) = \mp cos(x) \qquad cos(\frac{\pi}{2} \pm x) = \mp sin(x) \qquad tan(\frac{\pi}{2} \pm x) = \mp cotan(x)$$

$$sin(\pi \pm x) = \pm sin(x) \qquad cos(\pi \pm x) = -cos(x) \qquad tan(\pi \pm x) = \pm tan(x)$$

Formule di addizione

$$sin(x \pm y) = sin(x)cos(y) \pm cos(x)sin(y) \qquad cos(x \pm y) = cos(x)cos(y) \mp sin(x)sin(y)$$

$$tan(x \pm y) = \frac{tgx \pm tgy}{1 \mp tgxtgy}$$

Formule di duplicazione

$$sin(2x) = 2sin(x)cos(x) \qquad cos(2x) = 2(cos(x))^2 - 1 = 1 - (sin(x))^2$$

$$tan(2x) = \frac{2tgx}{1 - tgx^2}$$

Formule di bisezione(scegliere il segno corretto)

$$sin(\frac{x}{2}) = \pm\sqrt{\frac{1 - cos(x)}{2}} \qquad cos(\frac{x}{2}) = \pm\sqrt{\frac{1 + cos(x)}{2}}$$

$$tan(\frac{x}{2}) = \frac{1 - cos(x)}{sin(x)} = \frac{sin(x)}{1 + cos(x)}$$

Formule di prostaferesi

$$sinu + sinv = 2sin(\frac{u+v}{2})cos(\frac{u-v}{2}) \qquad sinu - sinv = 2cos(\frac{u+v}{2})sin(\frac{u-v}{2})$$
$$cosu + cosv = 2cos(\frac{u+v}{2})cos(\frac{u-v}{2}) \qquad cosu - cosv = -2sin(\frac{u+v}{2})sin(\frac{u-v}{2})$$

Formule parametriche

Posto $t = tg(\frac{x}{2})$

$$sinx = \frac{2t}{1+t^2} \qquad cosx = \frac{1-t^2}{1+t^2} \qquad tgx = \frac{2t}{1-t^2}$$

Teorema di Carnot

$$a^2 = b^2 + c^2 - 2bc \cdot cos\alpha$$

DERIVATE ELEMENTARI

TABELLA DERIVATE

$y = k$ $\quad y' = 0$	$y = x$ $\quad y' = 1$
$y = x^n$ $\quad y' = nx^{n-1}$	$y = \{f(x)\}^n$ $\quad y' = n\{f(x)\}^{n-1} f'(x)$
$y = \sqrt{x}$ $\quad y' = \frac{1}{2\sqrt{x}}$	$y = \sqrt{f(x)}$ $\quad y' = \frac{1}{2\sqrt{f(x)}} f'(x)$
$y = \sqrt[n]{x}$ $\quad y' = \frac{1}{n\sqrt[n]{x^{n-1}}}$	$y = \sqrt[n]{f(x)}$ $\quad y' = \frac{1}{n\sqrt[n]{f(x)^{n-1}}} f'(x)$
$y = \sqrt[n]{x^m}$ $\quad y' = \frac{m}{n\sqrt[n]{x^{n-m}}}$	$y = \sqrt[n]{\{f(x)\}^m}$ $\quad \frac{m}{n\sqrt[n]{\{f(x)\}^{n-m}}} f'(x)$
$y = \sin x$ $\quad y' = \cos x$	$y = \sin f(x)$ $\quad y' = \cos f(x)\ f'(x)$
$y = \cos x$ $\quad y' = -\sin x$	$y = \cos f(x)$ $\quad y' = -\sin f(x)\ f'(x)$
$y = tg\ x$ $\quad y' = \frac{1}{\cos^2 x}$	$y = tg\ f(x)$ $\quad y' = \frac{1}{\cos^2 f(x)} f'(x)$
$y = ctg\ x$ $\quad y' = -\frac{1}{\sin^2 x}$	$y = ctg\ f(x)$ $\quad y' = -\frac{1}{\sin^2 f(x)} f'(x)$
$y = \arcsin x$ $\quad y' = \frac{1}{\sqrt{1-x^2}}$	$y = \arcsin f(x)$ $\quad y' = \frac{1}{\sqrt{1-\{f(x)\}^2}} f'(x)$
$y = \arccos x$ $\quad y' = -\frac{1}{\sqrt{1-x^2}}$	$y = \arccos f(x)$ $\quad y' = -\frac{1}{\sqrt{1-\{f(x)\}^2}} f'(x)$
$y = arctg\ x$ $\quad y' = \frac{1}{1+x^2}$	$y = arctg\ f(x)$ $\quad y' = \frac{1}{1+\{f(x)\}^2} f'(x)$
$y = arcctg\ x$ $\quad y' = -\frac{1}{1+x^2}$	$y = axcctg\ f(x)$ $\quad y' = -\frac{1}{1+\{f(x)\}^2} f'(x)$
$y = \log_a x$ $\quad y' = \frac{1}{x}\log_a e$	$y = \log_a f(x)$ $\quad y' = \frac{1}{f(x)} \cdot \log_a e \cdot f'(x)$
$y = \log_a f(x)$ $\quad y' = \frac{1}{f(x)} \cdot f'(x) \cdot \frac{1}{\ln a}$	
$y = \ln x$ $\quad y' = \frac{1}{x}$	$y = \ln f(x)$ $\quad y' = \frac{1}{f(x)} f'(x)$
$y = a^x$ $\quad y' = a^x \cdot \ln a$	$y = a^{f(x)}$ $\quad y' = a^{f(x)} \ln a \cdot f'(x)$
$y = e^x$ $\quad y' = e^x$	$y = e^{f(x)}$ $\quad y' = e^{f(x)} \cdot f'(x)$
$y = x^x$ $\quad y' = x^x(1+\ln x)$	$y = \{f(x)\}^{\varphi(x)}$ $y' = \{f(x)\}^{\varphi(x)} \cdot \left\{\varphi'(x)\ln f(x) + \frac{\varphi(x)}{f(x)} f'(x)\right\}$

Regole di derivazione

$$D(\lambda f(x) + \mu g(x)) = \lambda f'(x) + \mu g'(x)$$

$$D(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

$$D\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

$$Df(g(x)) = f'(g(x))g'(x)$$

$$Df(g(h(x))) = f'(g(h(x)))g'(h(x))h'(x)$$

$$De^{f(x)} = e^{f(x)} \cdot f'(x)$$

$$Dlog \mid f(x) \mid= \frac{f'(x)}{f(x)}$$

$$D[f(x)] * g(x) = [f(x)]^{g(x)}g'(x)log(f(x) + \frac{g(x)f'(x)}{f(x)}$$

Sviluppi di McLaurin delle principali funzioni

Alcuni sviluppi di McLaurin notevoli (si sottintende ovunque che i resti sono trascurabili per $x \to 0$)

e^x	$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + o(x^n)$	$= \sum_{k=0}^{n} \frac{x^k}{k!} + o(x^n)$
$\sinh x$	$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$	$= \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$
$\cosh x$	$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$	$= \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!} + o(x^{2n+2})$
$\tanh x$	$= x - \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^6)$	
$\ln(1+x)$	$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + o(x^n)$	$= \sum_{k=1}^{n} (-1)^{k-1}\frac{x^k}{k} + o(x^n)$
$\sin x$	$= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$	$= \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$
$\cos x$	$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$	$= \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+1})$
$\tan x$	$= x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^6)$	
$\arcsin x$	$= x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \cdots + \left\lvert \binom{-1/2}{n} \right\rvert \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$	$= \sum_{k=0}^{n} \left\lvert \binom{-1/2}{k} \right\rvert \frac{x^{2k+1}}{2k+1} + o(x^{2n+2})$
$\arccos x$	$= \frac{\pi}{2} - \arcsin x$	
$\arctan x$	$= x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$	$= \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2n+2})$
$(1+x)^\alpha$	$= 1 + \alpha x + \binom{\alpha}{2}x^2 + \binom{\alpha}{3}x^3 + \cdots + \binom{\alpha}{n}x^n + o(x^n)$	$= \sum_{k=0}^{n} \binom{\alpha}{k}x^k + o(x^n)$
$\frac{1}{1+x}$	$= 1 - x + x^2 - x^3 + x^4 + \cdots + (-1)^n x^n + o(x^n)$	$= \sum_{k=0}^{n} (-1)^k x^k + o(x^n)$
$\frac{1}{1-x}$	$= 1 + x + x^2 + x^3 + x^4 + \cdots + x^n + o(x^n)$	$= \sum_{k=0}^{n} x^k + o(x^n)$
$\sqrt{1+x}$	$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \cdots + \binom{1/2}{n}x^n + o(x^n)$	$= \sum_{k=0}^{n} \binom{1/2}{k}x^k + o(x^n)$
$\frac{1}{\sqrt{1+x}}$	$= 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \cdots + \binom{-1/2}{n}x^n + o(x^n)$	$= \sum_{k=0}^{n} \binom{-1/2}{k}x^k + o(x^n)$
$\sqrt[3]{1+x}$	$= 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 + \cdots + \binom{1/3}{n}x^n + o(x^n)$	$= \sum_{k=0}^{n} \binom{1/3}{k}x^k + o(x^n)$
$\frac{1}{\sqrt[3]{1+x}}$	$= 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{7}{81}x^3 + \cdots + \binom{-1/3}{n}x^n + o(x^n)$	$= \sum_{k=0}^{n} \binom{-1/3}{k}x^k + o(x^n)$

Si ricordi che $\forall \alpha \in \mathbb{R}$ si pone $\binom{\alpha}{0} = 1$ e $\binom{\alpha}{n} = \frac{\overbrace{\alpha(\alpha-1)\cdots(\alpha-n+1)}^{n \text{ fattori}}}{n!}$ se $n \geq 1$.

INTEGRALI ELEMENTARI

$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$	$\int \frac{1}{x} dx = \log x + c \quad x > \mathbf{0}$
$\int e^x dx = e^x + c$	$\int a^x dx = a^x + \log_a e + c$
$\int \cos x \, dx = \sin x + c$	$\int \sin x \, dx = -\cos x + c$
$\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + c$	$\int \frac{1}{\sin^2 x} dx = -\operatorname{ctg} x + c$
$\int \frac{1}{1+x^2} dx = \operatorname{arctg} x + c$	$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$

$\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c \ (n \neq -1)$	$\int \frac{f'(x)}{x} dx = \log\lvert f(x)\rvert + c$
$\int f'(x) e^x dx = e^{f(x)} + c$	$\int f'(x) a^{fx)} dx = a^{f(x)} + \log_a e + c$
$\int f'(x) \cos f(x) \, dx = \sin f(x) + c$	$\int f'(x) \sin f(x) \, dx = -\cos f(x) + c$
$\int \frac{f'(x)}{\cos^2 f(x)} dx = \operatorname{tg} f(x) + c$	$\int \frac{f'(x)}{\sin^2 f(x)} dx = -\operatorname{ctg} f(x) + c$
$\int \frac{f'(x)}{1+[f(x)]^2} dx = \operatorname{arctg} f(x) + c$	$\int \frac{f'(x)}{\sqrt{1-[f(x)]^2}} dx = \arcsin f(x) + c$