

### Exercise 1

Consider the following game tree corresponding to a two-player zero-sum game. Max is to start in the initial state (i.e., the root of the tree).

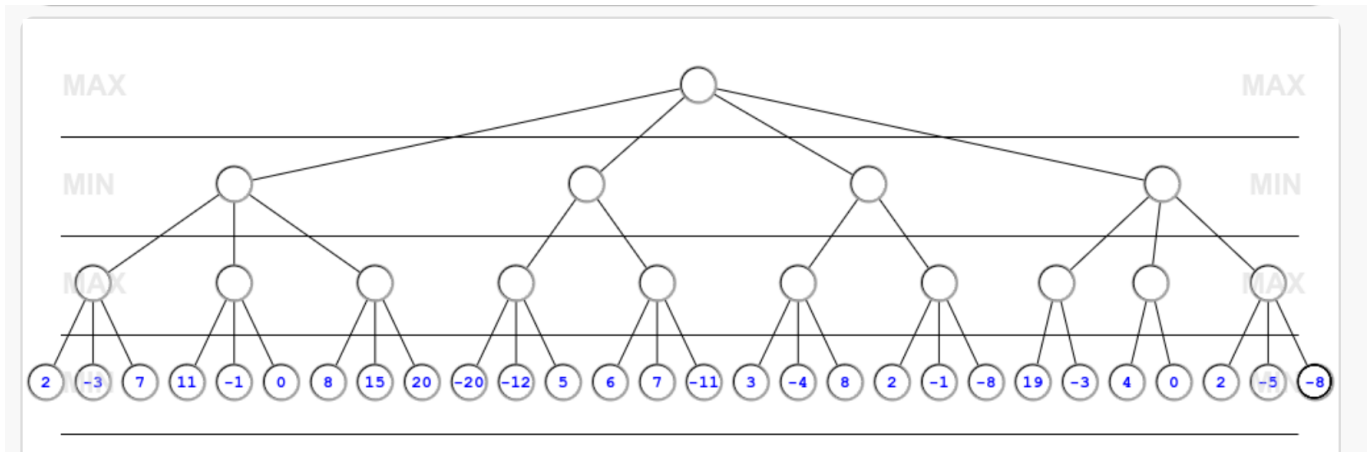


Figure 1: Game tree

- (a) Perform Minimax search on the tree, i.e., annotate all internal nodes with the correct Minimax value. Which move does Max choose?
- (b) Perform Alpha-Beta search on the tree. Annotate all internal nodes (that are not pruned) with the value that will be propagated to the parent node as well as the final  $[\alpha, \beta]$  window before propagating the value to the parent. Mark which edges will be pruned.

The diagram shows a minimax search tree with the following structure and values:

- Root (MAX):**  $\alpha = -\infty$ ,  $\beta = +\infty$
- Level 1 (MIN):**
  - Node 1:  $\alpha = -\infty$ ,  $\beta = +\infty$
  - Node 2:  $\alpha = 7$ ,  $\beta = +\infty$
  - Node 3:  $\alpha = 7$ ,  $\beta = +\infty$
  - Node 4:  $\alpha = 7$ ,  $\beta = +\infty$
- Level 2 (MAX):**
  - Node 1 (under MIN 1):  $\alpha = -\infty$ ,  $\beta = +\infty$
  - Node 2 (under MIN 1):  $\alpha = -\infty$ ,  $\beta = 7$
  - Node 3 (under MIN 1):  $\alpha = -\infty$ ,  $\beta = 7$
  - Node 4 (under MIN 2):  $\alpha = 7$ ,  $\beta = +\infty$
  - Node 5 (under MIN 2):  $\alpha = 7$ ,  $\beta = +\infty$
  - Node 6 (under MIN 3):  $\alpha = 7$ ,  $\beta = +\infty$
  - Node 7 (under MIN 3):  $\alpha = 7$ ,  $\beta = 8$
  - Node 8 (under MIN 4):  $\alpha = 7$ ,  $\beta = +\infty$
  - Node 9 (under MIN 4):  $\alpha = 7$ ,  $\beta = 19$
- Level 3 (Leaf nodes):**
  - Node 1: 2, -3, 7
  - Node 2: 11, -1, 0
  - Node 3: 8, 1, -20
  - Node 4: -20, -12, 5
  - Node 5: 6, 7, -11
  - Node 6: 3, -4, 8
  - Node 7: 2, -1, -8
  - Node 8: 19, -3, 4
  - Node 9: 0, 2, -5, -8

Red diagonal lines indicate pruning after the first child in several MAX nodes. A large red 'Z' is drawn over the final part of the tree.

$$\alpha \geq \beta$$

**Exercise 2**

Consider the following constraint network:  $\gamma = (V, D, C)$ :

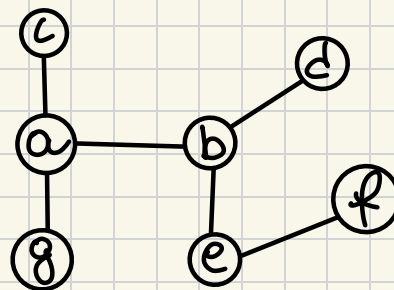
- Variables:  $V = \{a, b, c, d, e, f, g\}$
- Domains: for all  $v \in V, D_v = \{1, 2, 3, 4, 5\}$
- Constraints:  $2a = b - 1, b = d, e = b + 2, 2f = e - 1, a - g = -2, c = 2a + 2$

1. Draw the constraint graph  $\gamma$
2. Run the AC-3 algorithm on the given constraint network. For each iteration, give the content of the data structure M at the start of the iteration, the pair (u, v) removed from M, the domain  $D_u$  of u after the call to Revise ( $\gamma, u, v$ ), and the pairs (w, u) added into M. Note: Initialize M as a lexicographically ordered list (i.e., (a, b) would be before (a, c), both before (b, a) etc., if any of those exist). After initialization, use M as a FIFO queue, i.e., always remove the first (oldest) element from the queue and add new elements to the end of the queue.
3. Give the complete assignment of the variables.
4. Is it possible to apply the AyclicCG algorithm on  $\gamma$  to find a solution? If yes, please explain why. If not, please explain why.

Variables:  $V = \{a, b, c, d, e, f, g\}$

Domains: for all  $v \in V, D_v = \{1, 2, 3, 4, 5\}$

Constraints:  $2a = b - 1, b = d, e = b + 2, 2f = e - 1, a - g = -2, c = 2a + 2$



**VALORI.**  $a=1, b=3, c=4, d=3, e=5, f=2, g=3$

$\mu: \{(\cancel{a,b}), (\cancel{a,c}), (\cancel{a,g}), (\cancel{b,a}), (\cancel{b,d}), (\cancel{b,e}), (\cancel{c,a}), (\cancel{d,b}), (\cancel{e,b}), (\cancel{e,f}), (\cancel{f,e}), (\cancel{g,a})\}$

- |                         |                 |                       |
|-------------------------|-----------------|-----------------------|
| 1) $a = \frac{b-1}{2}$  | $D_a: \{1, 2\}$ | $(*, a)$ GIÀ PRESENTI |
| 2) $a = \frac{c-2}{2}$  | $D_a: \{1\}$    | $(*, a)$ GIÀ PRESENTI |
| 3) $a = g - 2$          | $D_a: \{1\}$    |                       |
| 4) $b = 2a + 1$         | $D_b: \{3\}$    | $(*, b)$ GIÀ PRESENTI |
| 5) $b = d$              | $D_b: \{3\}$    |                       |
| 6) $b = e - 2$          | $D_b: \{3\}$    |                       |
| 7) $c = 2a + 2$         | $D_c: \{4\}$    | $(*, c)$ GIÀ PRESENTI |
| 8) $d = b$              | $D_d: \{3\}$    | $(*, d)$ GIÀ PRESENTI |
| 9) $e = b + 2$          | $D_e: \{5\}$    | $(*, e)$ GIÀ PRESENTI |
| 10) $e = 2f + 1$        | $D_e: \{5\}$    |                       |
| 11) $f = \frac{e-1}{2}$ | $D_f: \{2\}$    | $(*, f)$ GIÀ PRESENTI |
| 12) $g = a + 2$         | $D_g: \{3\}$    | $(*, g)$ GIÀ PRESENTI |

CONTROLLO SOLO  
QUANDO D CAMBIA

IL RIFLESSIVO NON  
SI CONTA

4) SÌ PERCHÈ IL GRAFO È ACICLICO

AL-3 PUÒ ESSERE USATO SIA CON GRAFO CICLICO CHE NON

ACICLICO SOLO CON GRAFO ACICLICO

**Exercise 3**

1. (a) Given the following formula in Propositional Logic:

$$(P \wedge Q) \vee ((P \wedge R) \rightarrow (Q \vee R))$$

determine if it is valid, satisfiable or unsatisfiable. If it is satisfiable, provide a model.

2. (b) If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Given this set of sentences, transform them in a KB in **Propositional Logic** by using an appropriate set of propositional symbols. Determine if it is possible to derive with resolution if the unicorn is “mythical”. What about if it is “magical”? What about if it is “horned”?

**Hint:** Consider “immortal” as “not mortal” or viceversa.

3. (c) Given the following KB:  $\Delta = \{\{\neg B\}, \{A, B, C, D\}, \{\neg C, \neg D\}, \{C, \neg D\}, \{A, \neg B, D\}, \{A, \neg C\}\}$ , apply the DPPL algorithm with clause learning and show the various iterations of the algorithm. Assume that the variables are selected in alphabetical order and the splitting rule attempts the value False first.

1. (a) Given the following formula in Propositional Logic:

$$(P \wedge Q) \vee ((P \wedge R) \rightarrow (Q \vee R))$$

determine if it is valid, satisfiable or unsatisfiable. If it is satisfiable, provide a model.

P	Q	R	<sup>A</sup> $P \wedge R$	<sup>B</sup> $Q \vee R$	<sup>C</sup> $P \wedge Q$	<sup>D</sup> $A \rightarrow B$	$C \vee D$
0	0	0	0	0	0	1	1
0	0	1	0	1	0	1	1
0	1	0	0	1	0	1	1
0	1	1	0	1	0	1	1
1	0	0	0	0	0	1	1
1	0	1	1	1	0	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

SODDISFACIBILE  
POICHÉ TUTTI 1

2. (b) If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Given this set of sentences, transform them in a KB in **Propositional Logic** by using an appropriate set of propositional symbols. Determine if it is possible to derive with resolution if the unicorn is "mythical". What about if it is "magical"? What about if it is "horned"?

Hint: Consider "immortal" as "not mortal" or viceversa.

$M_y$  = UNICORN IS MYTHICAL

$M_a$  = MAMMAL

$M_{ag}$  = MAGICAL

$I$  = IMMORTAL

$H$  = HORNED

1  $M_y \Rightarrow I$     2  $\neg M_y \Rightarrow (M_a \wedge \neg I)$     3  $(I \vee M_a) \Rightarrow H$     4  $H \Rightarrow M_{ag}$

1  $\neg M_y \vee I$     2  $M_y \vee (\neg I \wedge M_a)$     3  $(\neg I \wedge \neg M_a) \vee H$     4  $\neg H \vee M_{ag}$

2a  $M_y \vee M_a$

3a  $\neg I \vee H$

2b  $M_y \vee \neg I$

3b  $\neg M_a \vee H$

Th:  $M_y$  NEGHIAO E AGGIUNGIO 5  $\neg M_y$

5 E 2a:  $M_a$  6

5 E 2b:  $\neg I$  7

6 E 3b:  $H$  8

8 E 4:  $M_{ag}$  9

FALSO

$C_1 \vee (L), C_2 \vee (\neg L)$   
 $C_1 \vee C_2$

Th:  $\Pi_{ag} \rightarrow \neg \Pi_{ag} 5$

$5 \in 4: \neg H 6$        $7 \in 1: \neg My 9$   
 $6 \in 3a: \neg I 7$        $9 \in 2a: \Pi_a 10$       **VERO**  
 $6 \in 3b: \neg \Pi_a 8$        $10 \in 8: \emptyset$

3. (c) Given the following KB:  $\Delta = \{\{\neg B\}, \{A, B, C, D\}, \{\neg C, \neg D\}, \{C, \neg D\}, \{A, \neg B, D\}, \{A, \neg C\}\}$ , apply the DPPL algorithm with clause learning and show the various iterations of the algorithm. Assume that the variables are selected in alphabetical order and the splitting rule attempts the value False first.

UP)  $B \rightarrow F$   $\Delta = \{\{A, C, D\}, \{\neg C, \neg D\}, \{C, \neg D\}, \{A, \neg C\}\}$

SR)  $A \rightarrow F$   $\Delta = \{\{C, D\}, \{\neg C, \neg D\}, \{C, \neg D\}, \{\neg C\}\}$

UP)  $C \rightarrow F$   $\Delta = \{\{D\}, \{\neg D\}\}$  **CONTRADDIZIONE**

UP)  $B \rightarrow F$   $\Delta = \{\{A, C, D\}, \{\neg C, \neg D\}, \{C, \neg D\}, \{A, \neg C\}, \{A\}\}$

UP)  $A \rightarrow T$   $\Delta = \{\{\neg C, \neg D\}, \{C, \neg D\}\}$

SR)  $C \rightarrow F$   $\Delta = \{\{\neg D\}\}$

UP)  $D \rightarrow F$   $\Delta = \{\}$

**SOL:**  $A, \neg B, \neg C, \neg D$

**Exercise 4**

Consider the state space in Figure 3, where S is the initial state and G the goal state. The transitions are annotated by their costs.

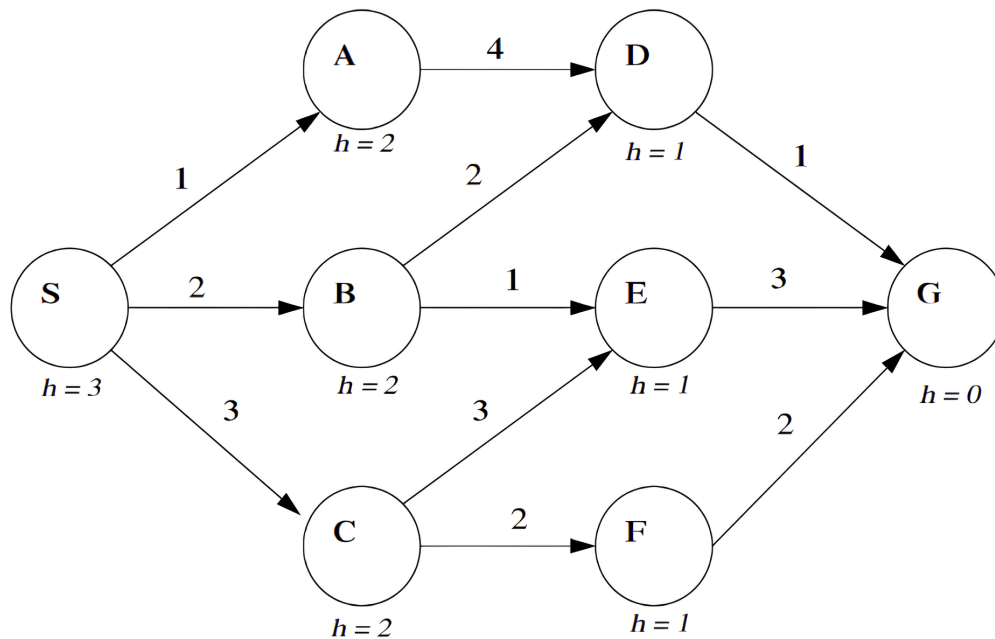
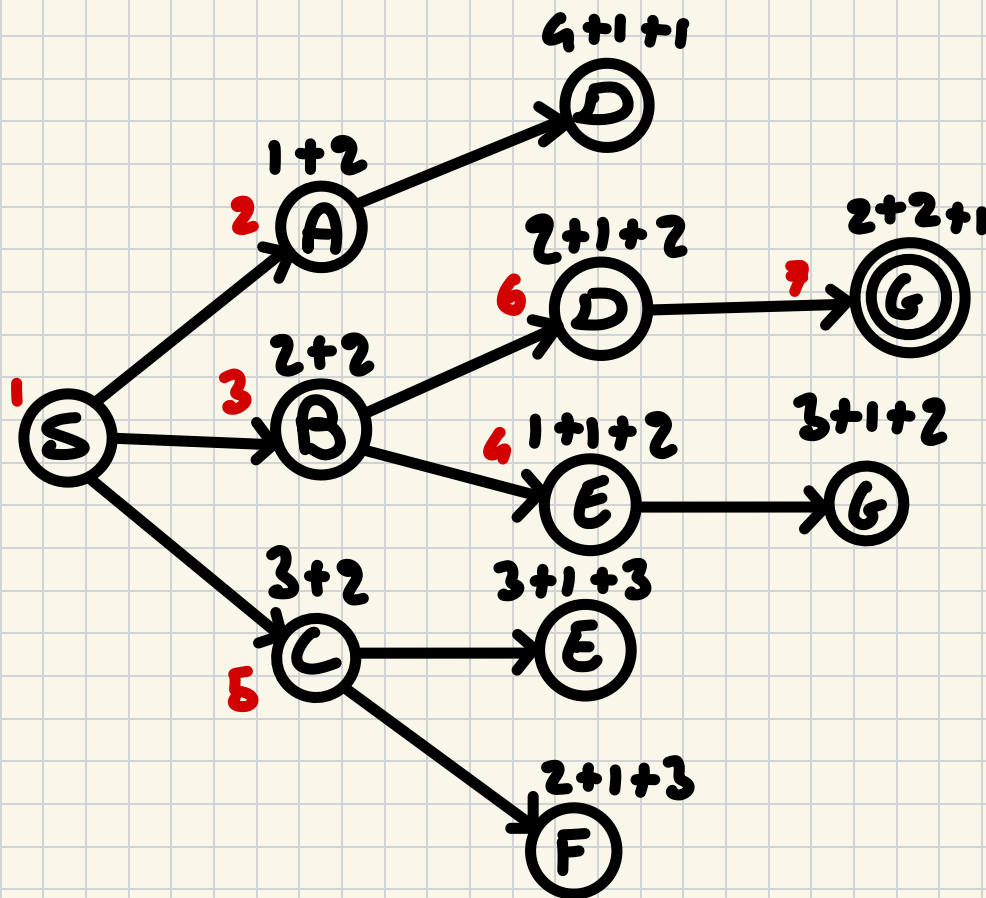


Figure 2: State space

- (a) Run the  $A^*$  search algorithm on this problem. The heuristic for each node is written right below it. Draw the search graph and annotate each node with the  $g$  and  $h$  value as well as the order of expansion. Draw duplicate nodes as well, and mark them accordingly by crossing them out. If the choice of the next state to be expanded is not unique, expand the lexicographically smallest state first. Give the solution found by  $A^*$  search.
- (b) If we had no information about the heuristic, would DFS or BFS get us to the goal? Would the solution returned be optimal?



SI PARTE DA  $\varphi$  MINORE



b) CON DFS E BFS ARRIVIAMO AL GOAL POICHÈ STATE SPACE FINITO E ACICLICO

DFS COMPLETO SE LO STATE SPACE NON È CICLICO

BFS COMPLETO SE LO STATE SPACE È FINITO

SOLUZIONE DI DFS NON È OTTIMALE

SOLUZIONE DI BFS È OTTIMALE SOLO SE TUTTI GLI ARCHI HANNO LO STESSO PESO