SAPIENZA
UNIVERSITÀ DI ROMA

Fondamenti di Intelligenza Artificiale

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Lab 4: Logica Proposizionale e Resolution

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*The slides have been prepared using the textbook material available on the web, and the slides of the previous editions of the course by Prof. Luigia Carlucci Aiello, Prof. Daniele Nardi and Dott. Fabio Previtali.

Exercises

Tell whether the following formula is valid, satisfiable or unsatisfiable.
If it's satisfiable, provide a model

$$\phi(A, B) = (A \wedge B) \vee (\neg A \wedge \neg B)$$

$\phi(A,B) = (A \wedge B) \vee (\neg A \wedge \neg B)$ 2^n POSSIBILI COMBINAZIONI

		x	y	$\Phi(A,B)$
A	B	$A \wedge B$	$\neg A \wedge \neg B$	$x \vee y$
0	0	0	1	1
0	1	0	0	0
1	0	0	0	0
1	1	1	0	1

SATISFIABLE PERCHÈ HA ALMENO UN MODELLO VERO
NON VALIDA POICHÈ NON HO TUTTI 1

Exercises

Tell which one among the following formulae is a good representation of the sentence.

If John studies and his father works, then his grandfather is happy.

(1) $(Study \wedge Work) \Rightarrow Happy$ ✓
(2) $Study \wedge Work \wedge Happy$
(3) $\neg Study \vee \neg Work \vee Happy$ ✓
(4) $(Study \vee Work) \Rightarrow Happy$

$A \Rightarrow B \equiv \neg A \vee B$ CNF

Exercises

Consider a knowledge base consisting of the conjunction of the following propositions:

$$\neg A \Rightarrow B$$
$$B \Rightarrow A$$
$$A \Rightarrow (C \wedge D)$$

1. Tell whether the knowledge base is consistent. In the positive case provide a model
2. Transform the above propositions into a new knowledge base written in conjunctive normal form
3. Derive $A \wedge C \wedge D$ using Resolution

1 SE ∃ UN INTERPRETAZIONE CHE RENDE TUTTE LE FRASI VERE (UTILI) CONTEMPORANEAMENTE:

A ⇒ B
1
1
0
1

A	B	¬A ⇒ B	B ⇒ A
0	0	0	1
0	1	1	0
1	0	1	1
1	1	1	1

SE ANTECEDENTE = 0 E CONSEGUENTE = 1, A ⇒ B = 1
SE ANTECEDENTE = 1 E CONSEGUENTE = 0, A ⇒ B = 0

A	C	D	C ∧ D	A ⇒ (C ∧ D)
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

MODELLO: A, B, C, D

2 $\neg A \Rightarrow B \rightarrow A \vee B$ $\quad B \Rightarrow A \rightarrow \neg B \vee A$ $\quad A \Rightarrow (C \wedge D) \rightarrow \neg A \vee (C \wedge D) \rightarrow (\neg A \vee C) \wedge (\neg A \vee D)$

3 $C_1 \cup \{L_1\}, C_2 \cup \{\neg L_2\} \rightarrow C_1 \cup C_2$

Th. $A \wedge C \wedge D$ $\quad \neg(A \wedge C \wedge D)$

1 $A \vee B$
2 $\neg B \vee A$
3a $\neg A \vee C$
3b $\neg A \vee D$
4 $\neg A \vee \neg C \vee \neg D$

1 E 2 ⟶ 5 A $\quad$ 5 E 4 ⟶ 8 $\neg C \vee \neg D$

5 E 3a ⟶ 6 C $\quad$ 6 E 8 ⟶ 9 $\neg D$

5 E 3b ⟶ 7 D $\quad$ 9 E 7 ⟶ { }

Exercises

If I leave and go on vacation, then I am happy
If I leave then I go on vacation
I leave

Question: Can I derive, *I go on vacation and I am happy?*

L : LEAVE
V : VACATION
H : HAPPY

1 $(L \wedge V) \Rightarrow H$
2 $L \Rightarrow V$
3 L

1 $\neg(L \wedge V) \vee H \longrightarrow \neg L \vee \neg V \vee H$
2 $\neg L \vee V$
3 L
4 $\neg V \vee \neg H$

Th : $V \wedge H$

NEGHIAMO LA TESI : $\neg(V \wedge H) \rightarrow \neg V \vee \neg H$

2 E 3 $\longrightarrow$ 5 V

5 E 4 $\longrightarrow$ 6 $\neg H$

3 E 1 $\longrightarrow$ 7 $\neg V \vee H$

7 E 6 $\longrightarrow$ 8 $\neg V$

8 E 5 $\longrightarrow$ $\{\}$

Exercises

Consider the following propositional formulae:

$$P \Rightarrow (Q \Leftrightarrow R)$$
$$Q \Rightarrow (P \vee R)$$
$$R \Rightarrow (Q \wedge \neg P)$$

1. Convert them into Conjunctive Normal Form
2. Tell whether or not the resulting set of clauses is satisfiable, in the positive case show a model

1) $P \Rightarrow (Q \Rightarrow R) \wedge (R \Rightarrow Q)$

$\neg P \vee [(\neg Q \vee R) \wedge (\neg R \vee Q)]$

$(\neg P \overset{1a}{\vee} \neg Q \vee R) \wedge (\neg P \overset{1b}{\vee} \neg R \vee Q)$

$Q \Rightarrow (P \vee R)$

$\neg Q \overset{2}{\vee} R \vee P$

$R \Rightarrow (Q \wedge \neg P)$

$(\neg R \overset{3a}{\vee} Q) \wedge (\neg R \overset{3b}{\vee} \neg P)$

$$P \Rightarrow (Q \Leftrightarrow R)$$
$$Q \Rightarrow (P \vee R)$$
$$R \Rightarrow (Q \wedge \neg P)$$

CNF. $(\neg P \overset{x}{\vee} \neg Q \vee R) \wedge (\neg P \vee \neg R \overset{y}{\vee} Q) \wedge (\neg Q \vee R \overset{z}{\vee} P) \wedge (\neg R \overset{w}{\vee} Q) \wedge (\neg R \overset{\varepsilon}{\vee} \neg P)$

2)

P	Q	R	x	y	z	w	ε	TOT
0	0	0	1	1	1	1	1	1
0	0	1						
0	1	0						
0	1	1						
1	0	0						
1	0	1						
1	1	0						
1	1	1						

SODDISFACIBILE

MODELLO: $\neg P\ \neg Q\ \neg R$

Exercises

I'm happy iff I won the lottery or my girlfriend is with me

If it is raining my girlfriend is not with me

It is raining and I am happy

Question: Can I derive, *I am happy iff I won the lottery?*

1 $H \Leftrightarrow (W \vee G) \rightarrow (H \Rightarrow (W \vee G)) \wedge ((W \vee G) \Rightarrow H)$

$(\neg H \vee W \vee G) \wedge ((\neg W \wedge \neg G) \vee H)$

$(\neg H \vee W \vee G) \wedge (\neg W \vee H) \wedge (\neg G \vee H)$

2 $R \Rightarrow \neg G \rightarrow \neg R \vee \neg G$

3 $R \wedge H$

1a $\neg H \vee W \vee G$

1b $\neg W \vee H$

1c $\neg G \vee H$

2 $\neg R \vee \neg G$

3a R

3b H

4a $H \vee \neg W$

4b $W \vee \neg H$

Th: $H \Leftrightarrow W \rightarrow H \Rightarrow W \rightarrow (\neg H \vee W) \wedge (\neg W \vee H)$
$W \Rightarrow H$

$\neg Th: (H \vee \neg W) \wedge (W \vee \neg H)$

NON RISOLVIBILE

Exercises: DPLL

For each of the following formulas, **use the DPLL procedure to determine whether it is satisfiable or unsatisfiable**. Transform each formula ϕ_i into an equivalent set of clauses Δ_i. Give a complete trace of the algorithm, **showing the simplified set of clauses for each recursive call of the DPLL function**. Assume that for each rule DPLL selects variables in alphabetical order (i.e., $A, B, C, D, E, \ldots$), and that the **splitting rule first attempts the value False (F) and then the value True (T)**

(a) $\phi_1 = (\neg A \vee B \vee C) \wedge (\neg B \vee \neg C) \wedge (\neg A \vee \neg C \vee \neg D) \wedge (C \vee \neg D) \wedge (A \vee D) \wedge (A \vee \neg C \vee \neg D)$

(b) $\phi_2 = (\neg A \vee \neg B \vee C \vee \neg E) \wedge (\neg A \vee \neg B \vee C \vee E) \wedge (A \leftrightarrow B) \wedge (B \vee D) \wedge (B \vee C \vee \neg D) \wedge (\neg C)$

(a) $\phi_1 = (\neg A \vee B \vee C) \wedge (\neg B \vee \neg C) \wedge (\neg A \vee \neg C \vee \neg D) \wedge (C \vee \neg D) \wedge (A \vee D) \wedge (A \vee \neg C \vee \neg D)$

SR) A=F $\phi_1 = \{(\neg B, \neg C), (C, \neg D), D, (\neg C, \neg D)\}$ APPLICO UP CON UN LETTERALE

UP) D=T $\phi_1 = \{(\neg B, \neg C), C, \neg C\}$ CONFLITTO → TORNIAMO INDIETRO (BACKTRACKING)

SR) A=T $\phi_1 = \{(B, C), (\neg B, \neg C), (\neg C, \neg D), (C, \neg D)\}$

SR) B=F $\phi_1 = \{C, (\neg C, \neg D), (C, \neg D)\}$

UP) C=T $\phi_1 = \{\neg D\}$

UP) D=F $\phi_1 = \{\}$ MODELLO: $\{\neg D, C, \neg B, A\}$

(b) $\phi_2 = (\neg A \vee \neg B \vee C \vee \neg E) \wedge (\neg A \vee \neg B \vee C \vee E) \wedge (A \leftrightarrow B) \wedge (B \vee D) \wedge (B \vee C \vee \neg D) \wedge (\neg C)$

$(A \Leftrightarrow B) \rightarrow (A \Rightarrow B) \wedge (B \Rightarrow A) \rightarrow (\neg A \vee B) \wedge (\neg B \vee A)$

UP) C=F $\phi_1 = \{(\neg A, \neg B, \neg E), (\neg A, \neg B, E), (\neg A, B), (\neg B, A), (B, D), (B, \neg D)\}$

SR) A=F $\phi_1 = \{\neg B, (B, D), (B, \neg D)\}$

UP) B=F $\phi_1 = \{D, \neg D\}$ CONFLITTO → BACKT

SR) A=T $\phi_1 = \{(\neg B, \neg E), (\neg B, E), B, (B, D), (B, \neg D)\}$

UP) B=T $\phi_1 = \{\neg E, E \ldots.$

DUE CONFLITTI → NON SODDISFACIBILE

Exercises: DPLL

Perform DPLL with clause learning.
Start by using the splitting rule and assign the value F to A. For the next splitting rule, assign T to B.
If you encounter a case where two or more different unit propagation rules are applicable choose the one which gets assigned to T.
Whenever you encounter a conflict, mention which clause can be learned with the clause learning method.

$$\Delta = \{\{A, B, C, D\}, \{\neg A, \neg B\}, \{\neg B, \neg C\}, \{\neg A, \neg D\}, \{A, \neg D\}, \{C, \neg D\}, \{B, \neg C\}, \{\neg B, C\}, \{\neg A, C, D\}\}$$

$$\Delta = \{\{A, B, C, D\}, \{\neg A, \neg B\}, \{\neg B, \neg C\}, \{\neg A, \neg D\}, \{A, \neg D\}, \{C, \neg D\}, \{B, \neg C\}, \{\neg B, C\}, \{\neg A, C, D\}\}, A\}$$

SR) $A=F$ $\Delta = \{(B,C,D), (\neg B, \neg C), \neg D, (C, \neg D), (B, \neg C), (\neg B, C)\}$

UP) $D=F$ $\Delta = \{(B,C), (\neg B, \neg C), (B, \neg C), (\neg B, C), \neg B\}$

SR) $B=T$ $\Delta = \{\neg C, C\}$ CONFLITTO

UP) $B=F$ $\Delta = \{C, \neg C\}$ CONFLITTO

UP) $A=T$ $\Delta = \{\neg B, (\neg B, \neg C), \neg D, (C, \neg D), (B, \neg C), (\neg B, C), (C, D)\}$

UP) $B=F$ $\Delta = \{\neg D, (C, \neg D), \neg C, (C, D)\}$

UP) $C=F$ $\Delta = \{\neg D, \neg D, D\}$ CONFLITTO

NON POSSO PIÙ TORNARE INDIETRO
QUINDI NON SODDISFACIBILE

Exercise

- (a) Transform the following formula in CNF specifying all the steps.

$$\neg((((A \wedge B) \rightarrow C) \wedge (A \vee B \vee C)) \rightarrow ((A \leftrightarrow B) \rightarrow C))$$

- (b) Use resolution to determine if this formula is inconsistent.

$$(\neg A \vee \neg D) \wedge (\neg A \vee D) \wedge (B \vee C \vee \neg D) \wedge (A \vee B) \wedge (\neg C \vee \neg D) \wedge (A \vee \neg B \vee \neg D) \wedge (A \vee \neg B \vee D)$$

- (c) Perform DPLL on the following formula to look for a satisfiable assignment. Assume that DPLL selects variables in alphabetical order (i.e., A, B, ...), and that the splitting rule first attempts the value False (F).

$$\{\{\neg B\}, \{A, B, C, D\}, \{\neg C, \neg D\}, \{C, \neg D\}, \{A, \neg B, D\}, \{A, \neg C\}\}$$

(a) Transform the following formula in CNF specifying all the steps.

TOT

$$\neg((((A \wedge B) \rightarrow C) \wedge (A \vee B \vee C)) \rightarrow ((A \leftrightarrow B) \rightarrow C))$$

$(A \wedge B) \Rightarrow C \rightarrow \neg(A \wedge B) \vee C \rightarrow \neg A \vee \neg B \vee C$

$((A \wedge B) \Rightarrow C) \wedge (A \vee B \vee C) \rightarrow (\neg A \vee \neg B \vee C) \wedge (A \vee B \vee C)$

$A \Leftrightarrow B \rightarrow \begin{matrix} A \Rightarrow B \\ B \Rightarrow A \end{matrix} \rightarrow (\neg A \vee B) \wedge (\neg B \vee A)$

$((\neg A \vee B) \wedge (\neg B \vee A)) \Rightarrow C \rightarrow ((A \wedge \neg B) \vee (B \wedge \neg A)) \vee C$

$\underset{x}{[(\neg A \vee \neg B \vee C)} \wedge (A \vee B \vee C)] \Rightarrow [\underset{y}{((A \wedge \neg B) \vee (B \wedge \neg A))} \vee C] \rightarrow$

$\rightarrow \neg x \vee y \rightarrow (A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge \neg C) \vee (A \wedge \neg B) \vee (B \wedge \neg A) \vee C$

$\neg T_{or} \rightarrow (\neg A \vee \neg B \vee C) \wedge (A \vee B \vee C) \wedge (\neg A \vee B) \wedge (\neg B \vee A) \wedge (\neg C)$

(b) Use resolution to determine if this formula is inconsistent.

$$\overset{1}{(\neg A \vee \neg D)} \wedge \overset{2}{(\neg A \vee D)} \wedge \overset{3}{(B \vee C \vee \neg D)} \wedge \overset{4}{(A \vee B)} \wedge \overset{5}{(\neg C \vee \neg D)} \wedge \overset{6}{(A \vee \neg B \vee \neg D)} \wedge \overset{7}{(A \vee \neg B \vee D)}$$

1 E 2 → ¬A 8

8 E 4 → B 9

9 E 6 → A ∨ ¬D 10

9 E 7 → A ∨ D 11

10 E 1 → ¬D 12

12 E 11 → A 13

13 E 8 → { }

(c) Perform DPLL on the following formula to look for a satisfiable assignment. Assume that DPLL selects variables in alphabetical order (i.e., A, B, ...), and that the splitting rule first attempts the value False (F).

$$\{\{\neg B\}, \{A, B, C, D\}, \{\neg C, \neg D\}, \{C, \neg D\}, \{A, \neg B, D\}, \{A, \neg C\}\}$$

UP) B=F Δ={(A,C,D),(¬C,¬D),(C,¬D),(A,¬C)}

SR) A=F Δ={(C,D),(¬C,¬D),(C,¬D),¬C}

UP) C=F Δ={D, ¬D} INCONSISTENZA

SR) A=T Δ={(¬C,¬D),(C,¬D)}

SR) C=F Δ={¬D}

UP) D=F Δ={}

MODELLO: {¬D, ¬C, A, ¬B}