Formulario

Formule trigonometriche

$\cos^2 x + \sin^2 x = 1$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\sin(2x) = 2 \sin x \cos x$
$\cos(2x) = \cos^2 x - \sin^2 x$
$\sin x + \sin y = 2 \sin(\frac{x+y}{2}) \cos(\frac{x-y}{2})$

$\tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$
$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$
$\sec x = \frac{1}{\cos x}$ $\quad \csc x = \frac{1}{\sin x}$
$1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$
$\cos x + \cos y = 2 \cos(\frac{x+y}{2}) \cos(\frac{x-y}{2})$
$\tan x \tan y = \frac{\sin(x+y)}{\cos x \cos y}$

$\sin x = \frac{2t}{1+t^2}$ per $t = \tan \frac{x}{2}$
$\cos x = \frac{1-t^2}{1+t^2}$ per $t = \tan \frac{x}{2}$
$\tan x = \frac{2t}{1-t^2}$ per $t = \tan \frac{x}{2}$
$\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2}$
$\cos x \cos y = \frac{\cos(x-y) + \cos(x+y)}{2}$
$\sin x \cos y = \frac{\sin(x+y) + \sin(x-y)}{2}$

Limiti notevoli successioni

$a_n \overset{n\to+\infty}{\longrightarrow} 0$

1. $\lim\limits_{n\to+\infty} \frac{\sin a_n}{a_n} = 1$
2. $\lim\limits_{n\to+\infty} \frac{1-\cos a_n}{a_n^2} = \frac{1}{2}$
3. $\lim\limits_{n\to+\infty} \frac{\tan a_n}{a_n} = 1$
4. $\lim\limits_{n\to+\infty} \frac{\arcsin a_n}{a_n} = 1$
5. $\lim\limits_{n\to+\infty} \frac{\arctan a_n}{a_n} = 1$

Altri limiti successioni

1. per $a_n \overset{n\to+\infty}{\longrightarrow} +\infty$,
   $\lim\limits_{n\to+\infty} (1 + \frac{1}{a_n})^{a_n} = e$
2. per $a_n \overset{n\to+\infty}{\longrightarrow} 0$,
   $\lim\limits_{n\to+\infty} \frac{\sqrt{1+a_n} - 1}{a_n} = \frac{1}{2}$
3. per $a_n \overset{n\to+\infty}{\longrightarrow} 0$, $a \in \mathbb{R}$,
   $\lim\limits_{n\to+\infty} \frac{(1+a_n)^a - 1}{a_n} = a$
4. per $a_n \overset{n\to+\infty}{\longrightarrow} 0$,
   $\lim\limits_{n\to+\infty} \frac{\ln(1+a_n)}{a_n} = 1$

Limiti notevoli funzioni

1. $\lim\limits_{x\to 0} \frac{\sin(x)}{x} = 1$
2. $\lim\limits_{x\to 0} \frac{1-\cos(x)}{x^2} = \frac{1}{2}$
3. $\lim\limits_{x\to 0} \frac{\tan(x)}{x} = 1$
4. $\lim\limits_{x\to 0} \frac{\arcsin(x)}{x} = 1$
5. $\lim\limits_{x\to 0} \frac{\arctan(x)}{x} = 1$

Altri limiti notevoli funzioni

1. $\lim\limits_{x\to 0} \frac{\ln(1+x)}{x} = 1$
2. $\lim\limits_{x\to\pm\infty} \left(1 + \frac{1}{x}\right)^x = e$
3. $\lim\limits_{x\to 0} \frac{e^x - 1}{x} = 1$
4. $\lim\limits_{x\to 0} \frac{\sqrt{1+x} - 1}{x} = \frac{1}{2}$
5. $\lim\limits_{x\to 0} \frac{(1+x)^a - 1}{x} = a$, $\forall a \in \mathbb{R}$

Serie geometrica

$$\sum_{k=0}^{\infty} q^k = \begin{cases} +\infty & q > 1 \\ \frac{1}{1-q} & |q| < 1 \\ \nexists & q \leq -1 \end{cases}$$

Serie armonica generalizzata

$$\sum_{k=1}^{\infty} \frac{1}{k^\alpha} = \begin{cases} \text{converge} & \alpha > 1 \\ +\infty & \alpha \leq 1 \end{cases}$$

Serie armonica a segni alterni

$\sum_{k=1}^{\infty} (-1)^k \frac{1}{k^\alpha}$ converge per $\alpha > 0$.

Sviluppi di McLaurin, $x_0 = 0$

$e^x = \sum_{k=0}^{n} \frac{1}{k!} x^k + o(x^n)$ per $x \to 0$

$\log(1+x) = \sum_{k=1}^{n} (-1)^{k+1} \cdot \frac{x^k}{k} + o(x^n)$ per $x \to 0$

$(1+x)^\alpha = \sum_{k=0}^{n} \binom{\alpha}{k} x^k + o(x^n)$ per $x \to 0$

$\sin x = \sum_{k=0}^{n} (-1)^k \cdot \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$ per $x \to 0$

$\cos x = \sum_{k=0}^{n} (-1)^k \cdot \frac{x^{2k}}{(2k)!} + o(x^{2n+1})$ per $x \to 0$

$\arctan x = \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2n+2})$ per $x \to 0$

Derivate

$f(x)$	$f'(x)$
x^α, $\alpha \in \mathbb{R}$	$\alpha x^{\alpha-1}$
e^x	e^x
$\ln \lvert x \rvert$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\lvert x \rvert$	$\text{sgn}\, x$

Integrali

$f(x)$	$\int f(x)dx$
x^α, $\alpha \neq -1$	$\frac{x^{\alpha+1}}{\alpha+1} + C$
$\frac{1}{x}$	$\ln \lvert x \rvert + C$
$e^{\alpha x}$, $\alpha \in \mathbb{R} \setminus \{0\}$	$\frac{e^{\alpha x}}{\alpha} + C$
$\ln x$	$x \ln x - x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\tan x$	$-\ln \lvert \cos x \rvert + C$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x + C$
$\frac{1}{1+x^2}$	$\arctan x + C$