Cognome e nome .... Fila ...... Colonna......

1. Dato il sistema descritto da

$$\begin{aligned}\dot{x}(t) &= \begin{pmatrix} -3 & -1 & 4 \\ -1 & -3 & 4 \\ 0 & 0 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} u(t) \\ y(t) &= \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} x(t) - u(t)\end{aligned}$$

i. calcolarne il polinomio caratteristico e quello minimo;
ii. calcolarne i modi naturali e studiarne eccitabilità ed osservabilità;
iii. studiarne la stabilità interna, esterna ed esterna nello stato zero;
iv. studiarne le proprietà strutturali ed effettuare la scomposizione di Kalman;
v. calcolare la risposta per condizione iniziale $x_0 = (1\ 1\ 1)^T$ ed ingresso $u(t) = t^2 + 2t + 2$.

2. Dato il sistema

$$W(s) = \frac{s(s-1)(s-10)}{(s^2+1)(s^2+100)}$$

se ne traccino i diagrammi di Bode e polare (di Nyquist).

3. Dato il sistema a tempo continuo rappresentato da

$$W(s) = \frac{10}{s^2}$$

se ne calcoli l'equivalente tempo discreto ottenuto campionandolo con tempo di campionamento $T = 0.1$s e si calcoli la risposta forzata all'ingresso (discreto) $u(t) = 2\delta_{-1}(t)$.

4. Si fornisca una realizzazione minima per il sistema rappresentato dalla funzione di trasferimento

$$W(s) = \begin{pmatrix} \frac{s-1}{(s+2)} & \frac{1}{s(s+2)} \\ \frac{1}{s^2} & 0 \end{pmatrix}$$

5. Si fornisca una rappresentazione con lo spazio di stato per un sistema descritto dall'equazione

$$\frac{d^3y(t)}{dt^3} - y(t) = u(t)$$

con $u(t)$ ingresso e $y(t)$ uscita.

1. Dato il sistema descritto da

$$\dot{x}(t) = \begin{pmatrix} -3 & -1 & 4 \\ -1 & -3 & 4 \\ 0 & 0 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} u(t)$$
$$y(t) = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix} x(t) - u(t)$$

i. calcolarne il polinomio caratteristico e quello minimo;
ii. calcolarne i modi naturali e studiarne eccitabilità ed osservabilità;
iii. studiarne la stabilità interna, esterna ed esterna nello stato zero;
iv. studiarne le proprietà strutturali ed effettuare la scomposizione di Kalman;
v. calcolare la risposta per condizione iniziale $x_0 = (1\ 1\ 1)^T$ ed ingresso $u(t) = t^2 + 2t + 2$.

a) CALCOLO PRIMA QUELLO MINIMO DALLA MATRICE MONICA

$$A_m = \begin{pmatrix} -3 & -1 \\ -1 & -3 \end{pmatrix} \qquad \det(A_m - \lambda I) = \begin{pmatrix} -3-\lambda & -1 \\ -1 & -3-\lambda \end{pmatrix} \rightarrow \lambda_1 = -2,\ \lambda_2 = -4$$

$$\lambda_1 = -2 \rightarrow \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} v_1 = 0 \quad v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \ / \ \lambda_2 = -4 \rightarrow \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} v_2 = 0 \quad v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \rightarrow T = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{array}{c} \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \end{array}\begin{array}{l} v_1 \\ v_2 \end{array}$$

$$\Phi(t) = e^{-2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \end{pmatrix} + e^{-4t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

b) AUTOVALORI:

$$\det(A - \lambda I) = 0 \rightarrow \begin{vmatrix} -3-\lambda & -1 & 4 \\ -1 & -3-\lambda & 4 \\ 0 & 0 & -\lambda \end{vmatrix} = -\lambda\left[(-3-\lambda)^2 - 1\right] = -\lambda(\lambda^2 + 6\lambda + 8) \qquad \begin{array}{l} \lambda_1 = 0 \\ \lambda_2 = -2 \\ \lambda_3 = -4 \end{array}$$

AUTOVETTORI:

$$\lambda_1 = 0 \rightarrow \begin{pmatrix} -3 & -1 & 4 \\ -1 & -3 & 4 \\ 0 & 0 & 0 \end{pmatrix} v_1 = 0 \rightarrow v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\lambda_2 = -2 \rightarrow \begin{pmatrix} -1 & -1 & 4 \\ -1 & -1 & 4 \\ 0 & 0 & 2 \end{pmatrix} v_2 = 0 \rightarrow v_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\lambda_3 = -4 \rightarrow \begin{pmatrix} 1 & -1 & 4 \\ -1 & 1 & 4 \\ 0 & 0 & 4 \end{pmatrix} v_3 = 0 \rightarrow v_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$
$$\downarrow$$
$$T = \frac{1}{2}\begin{pmatrix} 0 & 0 & 2 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix}\begin{array}{l} v_1 \\ v_2 \\ v_3 \end{array}$$

ECC E OSS:

$$C v_1 = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0 \qquad v_1 B = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 1$$

$$C v_2 = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 1 \qquad v_2 B = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 1$$

$$C v_3 = \begin{pmatrix} 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1 \qquad v_3 B = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 0$$

	ECC	OSS
λ_1	✓	✗
λ_2	✓	✓
λ_3	✗	✓
	$H(t)$	$\psi(t)$

a) $\Phi(t) = e^{0t}\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}0 & 0 & 1\end{pmatrix} + e^{-2t}\begin{pmatrix}1\\-1\\0\end{pmatrix}\begin{pmatrix}\frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix} + e^{-4t}\begin{pmatrix}1\\1\\0\end{pmatrix}\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & -1\end{pmatrix}$

c) INTERNA:

STABILE SEMPLICEMENTE POICHÈ $Re(\lambda_i) \leq 0$

ESTERNA:

STABILE POICHÈ $Re(\lambda_0) \leq 0$ E $Re(\lambda_{e,0}) < 0$

ESTERNA NELLO STATO ZERO:

$$y(t) = \psi(t-t_0)x(t_0) + \int_{\tau_0}^{t} W(t-\tau)u(\tau)\,d\tau \xrightarrow{t_0=0} y(t) = \int_0^t W(t-\tau)u(\tau)\,d\tau$$

CONSIDERO SOLO GLI $\lambda_{E,0}$ CHE HANNO $Re(\lambda_{E,0}) < 0$ (STABILE)

d) RAGGIUNGIBILITÀ:

$$R = (B \;\; AB \;\; A^2B) = \begin{pmatrix}2 & -2 & 4\\0 & 2 & -4\\1 & 0 & 0\end{pmatrix} \quad rk\,R = 2 \rightarrow \text{DIM}\,\mathcal{R} = 2 \rightarrow \mathcal{R} = \text{SPAN}\left\{\begin{pmatrix}2\\0\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\0\end{pmatrix}\right\}$$

OSSERVABILITÀ:

$$O = \begin{pmatrix}C\\CA\\CA^2\end{pmatrix} = \begin{pmatrix}1 & 0 & -1\\-3 & -1 & 4\\10 & 6 & -16\end{pmatrix} \quad rk\,O = 2 \rightarrow \text{DIM}\,Ker(O) = \text{DIM}\,\mathcal{I} = 3 - rk\,O = 1$$

$$\begin{pmatrix}1 & 0 & -1\\-3 & -1 & 4\\10 & 6 & -16\end{pmatrix} u = 0 \quad \begin{cases}u_a = u_c\\u_a = u_b\end{cases} \quad u = \begin{pmatrix}1\\1\\1\end{pmatrix} \rightarrow \mathcal{I} = \text{SPAN}\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\right\}$$

KALMAN:

$$X_1 = R \cap I \rightarrow \alpha_1 v_1 + \alpha_2 v_2 = \beta_1 w_1 \rightarrow \alpha_1 v_1 + \alpha_2 v_2 - \beta_1 w_1$$

$$(v_1 \;\; v_2 \;\; w_1)\begin{pmatrix}\alpha_1\\\alpha_2\\-\beta_1\end{pmatrix} = 0 \rightarrow \begin{pmatrix}2 & -1 & 1\\0 & 1 & 1\\1 & 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_1\\\alpha_2\\-\beta_1\end{pmatrix} = 0 \quad \begin{cases}\alpha_1 = \alpha_2\\\alpha_2 = \beta_1\\\alpha_1 = \beta_1\end{cases} \rightarrow R \cap I = 0$$

$$\left.\begin{aligned}X_2 &: X_1 \oplus X_2 = R = \begin{pmatrix}-1\\1\\0\end{pmatrix}\\X_3 &: X_1 \oplus X_3 = I = \begin{pmatrix}1\\1\\1\end{pmatrix}\\X_4 &: X_1 \oplus X_2 \oplus X_3 \oplus X_4 = R^3 = \begin{pmatrix}0\\0\\1\end{pmatrix}\end{aligned}\right\}$$

$$T^{-1} = (X_1 \; X_2 \; X_3 \; X_4) = \begin{pmatrix}-1 & 1 & 0\\1 & 1 & 0\\0 & 1 & 1\end{pmatrix} \quad rk = 3$$

$$T = -\frac{1}{2}\begin{pmatrix}1 & -1 & 0\\-1 & -1 & 0\\1 & 1 & -2\end{pmatrix} = \begin{pmatrix}-\frac{1}{2} & \frac{1}{2} & 0\\\frac{1}{2} & \frac{1}{2} & 0\\-\frac{1}{2} & -\frac{1}{2} & 1\end{pmatrix}$$

$$\tilde{A} = TAT^{-1} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} -3 & -1 & 4 \\ -1 & -3 & 4 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ -2 & -2 & 4 \\ 2 & 2 & -4 \end{pmatrix}\begin{pmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -4 \end{pmatrix}$$

$$\tilde{B} = TB = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \qquad \tilde{C} = CT^{-1} = (1 \;\; 0 \;\; -1)\begin{pmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = (-1 \;\; 0 \;\; -1)$$

$$\tilde{D} = D = -1$$

e) $x_0 = (1 \;\; 1 \;\; 1)^T \qquad u(\tau) = \tau^2 + 2\tau + 2$

$$\Phi(\tau) = e^{0\tau}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}(0 \;\; 0 \;\; 1) + e^{-2\tau}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}\left(\tfrac{1}{2} \;\; -\tfrac{1}{2} \;\; 0\right) + e^{-4\tau}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\left(\tfrac{1}{2} \;\; \tfrac{1}{2} \;\; -1\right) =$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} + e^{-2\tau}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} + e^{-4\tau}\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & \frac{1}{2} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_\ell(\tau) = \Phi(\tau)x_0 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + e^{-2\tau}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + e^{-4\tau}\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & \frac{1}{2} & -1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + e^{-2\tau}\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + e^{-4\tau}\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$y_\ell(\tau) = \psi(\tau)x_0 = C\,\Phi(\tau)x_0 = (1 \;\; 0 \;\; -1)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$$

$$W(\tau) = C\Phi(\tau)B + D = (1 \;\; 0 \;\; -1)\, e^{-2\tau}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - 1 = e^{-2\tau}\left(\tfrac{1}{2} \;\; -\tfrac{1}{2} \;\; 0\right)\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - 1 = e^{-2\tau} - 1$$

$$W(s) = \mathcal{L}[W(\tau)] = \frac{1}{s+2} - \frac{1}{s} = -\frac{2}{s(s+2)} \qquad / \qquad U(s) = \mathcal{L}[u(\tau)] = \frac{2!}{s^3} + \frac{2}{s^2} + \frac{2}{s}$$

1. $$Y_{F,}(s) = W(s)\cdot U(s) = -\frac{2}{s(s+2)}\cdot\frac{2!}{s^3} = -\frac{4}{s^4(s+2)} = \frac{R_1}{s^4} + \frac{R_2}{s^3} + \frac{R_3}{s^2} + \frac{R_4}{s^1} + \frac{R_5}{s+2}$$

$$R_1 = \lim_{s\to 0} s^4\, Y_F(s) = \lim_{s\to 0} -\frac{4}{s+2} = -2$$

$$R_2 = \lim_{s\to 0} \frac{d}{ds}\, s^4 \cdot Y_F(s) = \lim_{s\to 0} \frac{d}{ds}\left(-\frac{4}{s+2}\right) = \lim_{s\to 0} \frac{4}{(s+2)^2} = 1$$

$$R_3 = \frac{1}{2}\lim_{s\to 0} \frac{d^2}{d^2s}\, s^4 \cdot Y_F(s) = \frac{1}{2}\lim_{s\to 0} \frac{d^2}{d^2s}\left(-\frac{4}{s+2}\right) = \frac{1}{2}\lim_{s\to 0}\left(-\frac{8}{(s+2)^3}\right) = -\frac{1}{2}$$

$$R_4 = \frac{1}{3!}\lim_{s\to 0} \frac{d^3}{d^3s}\, s^4 \cdot Y_F(s) = \frac{1}{6}\lim_{s\to 0} \frac{d^3}{d^3s}\left(-\frac{4}{s+2}\right) = \frac{1}{6}\lim_{s\to 0}\left(\frac{24}{(s+2)^4}\right) = \frac{1}{4}$$

$$R_5 = \lim_{s\to -2} (s+2)\cdot Y_F(s) = \lim_{s\to -2}\left(-\frac{2}{s}\right) = 1$$

$$y_F(t) = \mathcal{L}^{-1}\left[\frac{R_1}{s^4} + \frac{R_2}{s^3} + \frac{R_3}{s^2} + \frac{R_4}{s^1} + \frac{R_5}{s+2}\right] = \mathcal{L}^{-1}\left[-\frac{2}{s^4} + \frac{1}{s^3} - \frac{1}{2s^2} + \frac{1}{4s} + \frac{1}{s+2}\right] =$$

2. $Y_{F_2}(s) = W(s) \cdot U(s) = -\frac{2}{s(s+2)} \cdot \frac{2}{s^2} = -\frac{4}{s^3(s+2)} = \frac{R_1}{s^3} + \frac{R_2}{s^2} + \frac{R_3}{s} + \frac{R_4}{s+2}$ . . . .

3. $Y_{F_3}(s) = W(s) \cdot U(s) = -\frac{2}{s(s+2)} \cdot \frac{2}{s} = -\frac{4}{s^2(s+2)} = \frac{R_1}{s^2} + \frac{R_2}{s} + \frac{R_3}{s+2}$ . . . .

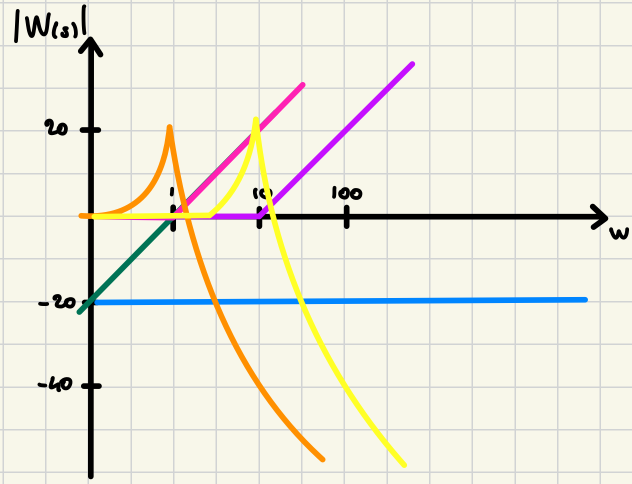
2. Dato il sistema

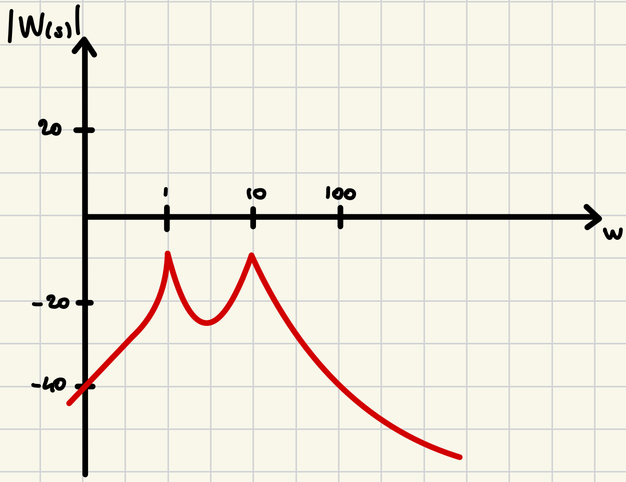
$$W(s) = \frac{s(s-1)(s-10)}{(s^2+1)(s^2+100)}$$

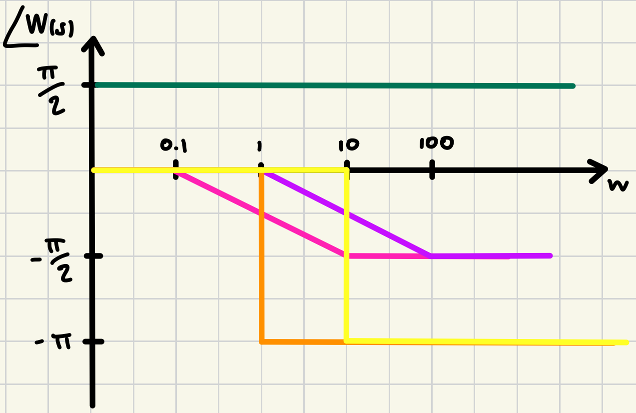
se ne traccino i diagrammi di Bode e polare (di Nyquist).

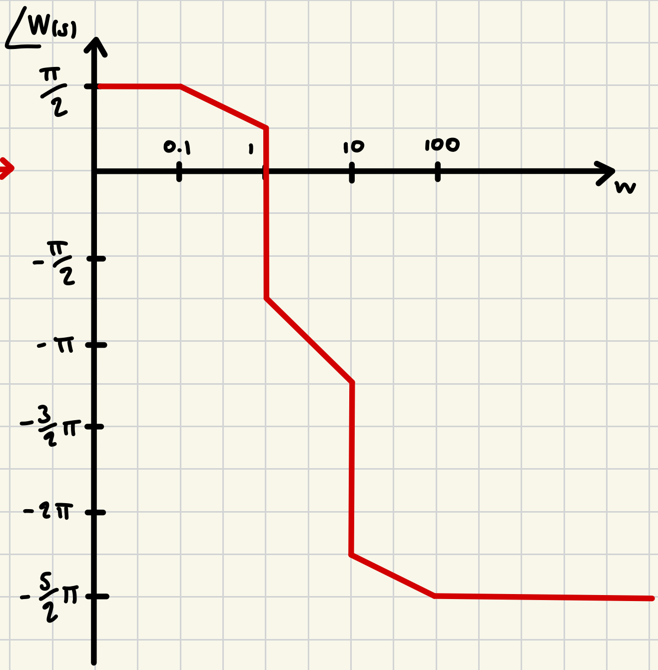
$$W(s) = \frac{s(s-1)(s-10)}{(s^2+1)(s^2+100)} = \frac{1}{10}\,\frac{s(1-s)(1-0.1s)}{(1+s^2)(1+0.01s^2)} \qquad \begin{cases} \omega_{n_1} = 1 \\ \xi = 0 \end{cases} \qquad \begin{cases} \omega_{n_2} = 10 \\ \xi = 0 \end{cases}$$

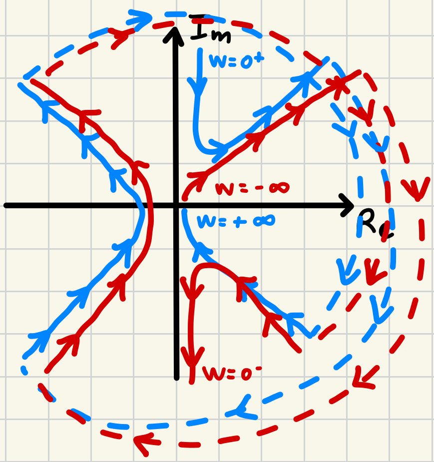
$$20\log|K| = 20\log\left(\frac{1}{10}\right) = -20\log 10 = -20\ dB$$











3. Dato il sistema a tempo continuo rappresentato da

$$W(s) = \frac{10}{s^2}$$

se ne calcoli l'equivalente tempo discreto ottenuto campionandolo con tempo di campionamento $T = 0.1$s e si calcoli la risposta forzata all'ingresso (discreto) $u(t) = 2\delta_{-1}(t)$.

a) $$W(z) = \frac{z-1}{z} \cdot \mathcal{Z}\left[\mathcal{L}^{-1}\left[\frac{W(s)}{s}\right]\Big|_{KT}\right]$$

$$y_F(s) = \frac{W(s)}{s} = \frac{10}{s^3} = \frac{R_1}{s^3} + \frac{R_2}{s^2} + \frac{R_1}{s}$$

$$R_1 = \lim_{s\to 0} s^3 \cdot y_F(s) = \lim_{s\to 0} 10 = 10$$

$$R_2 = \lim_{s\to 0} \frac{d}{ds}\left(s^3 \cdot y_F(s)\right) = 0 \qquad R_3 = \frac{1}{2} \lim_{s\to 0} \frac{d^2}{d^2 s}\left(s^3 \cdot y_F(s)\right) = 0$$

$$y_F(t) = \mathcal{L}^{-1}\left[\frac{R_1}{s^3} + \frac{R_2}{s^2} + \frac{R_1}{s}\right] = \mathcal{L}^{-1}\left[5 \cdot 2! \cdot \frac{1}{s^3}\right] = 5t^2\delta_{-1}(t)$$

$$W(z) = \frac{z-1}{z} \cdot \mathcal{Z}\left[\mathcal{L}^{-1}\left[\frac{W(s)}{s}\right]\Big|_{KT}\right] = \frac{z-1}{z} \cdot \mathcal{Z}\left[5t^2\delta_{-1}(t)\big|_{KT}\right] =$$

$$= \frac{z-1}{z} \cdot \mathcal{Z}\left[5t^2\delta_{-1}(t)\big|_{KT}\right] = \frac{z-1}{z} \cdot \mathcal{Z}\left[5 \cdot \frac{K^2}{100}\delta_{-1}\left(\frac{K}{10}\right)\right] =$$

$$= \frac{z-1}{z} \cdot \frac{1}{20} \cdot \frac{2z}{(z-1)^3} = \frac{1}{10}\frac{1}{(z-1)^2}$$

b) $$u(K) = 2\delta_{-1}(K) \rightarrow U(z) = 2 \cdot \frac{z}{z-1}$$

$$y_F(z) = W(z) \cdot U(z) = \frac{1}{10}\frac{1}{(z-1)^2} \cdot 2 \cdot \frac{z}{z-1} = \frac{1}{5}\frac{z}{(z-1)^3}$$

$$y_F(K) = \mathcal{Z}^{-1}\left[\frac{1}{5}\frac{z}{(z-1)^3}\right] = \mathcal{Z}^{-1}\left[\frac{1}{5}\frac{1}{2}\frac{2z}{(z-1)^3}\right] = \frac{1}{10}K^2\delta_{-1}(K)$$

4. Si fornisca una realizzazione minima per il sistema rappresentato dalla funzione di trasferimento

$$W(s) = \begin{pmatrix} \frac{s-1}{(s+2)} & \frac{1}{s(s+2)} \\ \frac{1}{s^2} & 0 \end{pmatrix}$$

$$W(s) = \begin{pmatrix} \frac{s-1}{s+2} & \frac{1}{s(s+2)} \\ \frac{1}{s^2} & 0 \end{pmatrix} = \begin{pmatrix} -\frac{3}{s+2}+1 & \frac{1}{s(s+2)} \\ \frac{1}{s^2} & 0 \end{pmatrix} = \begin{pmatrix} -\frac{3}{s+2} & \frac{1}{s(s+2)} \\ \frac{1}{s^2} & 0 \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}^{D}$$

$$W'(s) = \begin{pmatrix} -\frac{3}{s+2} & \frac{1}{s(s+2)} \\ \frac{1}{s^2} & 0 \end{pmatrix} = \frac{\begin{pmatrix} -3s^2 & s \\ s+2 & 0 \end{pmatrix}}{s^2(s+2)} = \frac{R_1}{s^2} + \frac{R_2}{s} + \frac{R_3}{s+2}$$

$$R_1 = \lim_{s\to 0} s^2 \cdot W'(s) = \lim_{s\to 0} \frac{\begin{pmatrix} -3s^2 & s \\ s+2 & 0 \end{pmatrix}}{(s+2)} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad rk = 1 \qquad \lambda_1 = 0$$

$$R_2 = \lim_{s\to 0} \frac{d}{ds}\left(s^2 \cdot W'(s)\right) = \lim_{s\to 0} \frac{d}{ds} \frac{\begin{pmatrix} -3s^2 & s \\ s+2 & 0 \end{pmatrix}}{(s+2)} =$$

$$= \lim_{s\to 0} \begin{pmatrix} \frac{-3s^2-12s}{(s+2)^2} & \frac{2}{(s+2)^2} \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ 1 & 0 \end{pmatrix} \qquad rk = 2 \qquad \lambda_2 = 0$$

$$R_3 = \lim_{s\to -2} (s+2) \cdot W'(s) = \lim_{s\to -2} \frac{\begin{pmatrix} -3s^2 & s \\ s+2 & 0 \end{pmatrix}}{s^2} = \begin{pmatrix} -3 & -\frac{1}{2} \\ 0 & 0 \end{pmatrix} \qquad rk = 1 \qquad \lambda_3 = -2$$

$$R_1 = C_{2\times 1} B_{1\times 2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$R_2 = C_{2\times 2} B_{2\times 2} = \begin{pmatrix} 0 & \frac{1}{2} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$R_3 = C_{2\times 1} B_{1\times 2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} -3 & -\frac{1}{2} \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix} \qquad C = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ -3 & -\frac{1}{2} \end{pmatrix} \qquad D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

5. Si fornisca una rappresentazione con lo spazio di stato per un sistema descritto dall'equazione

$$\frac{d^3y(t)}{dt^3} - y(t) = u(t)$$

con $u(t)$ ingresso e $y(t)$ uscita.

$$\frac{d^3y(t)}{dt^3} - y(t) = u(t)$$

$$\begin{cases} y = x_1 \\ \dot{y} = x_2 \\ \ddot{y} = x_3 \end{cases} \quad \begin{cases} \dot{x}_1 = \dot{y} = x_2 \\ \dot{x}_2 = \ddot{y} = x_3 \\ \dot{x}_3 = \dddot{y} = y + u = x_1 + u \\ y = x_1 \end{cases} \rightarrow \begin{cases} \dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u \\ y(t) = (1\ 0\ 0) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \end{cases}$$