Electric Vehicle Charging Station Placement in Sardinia
using Integer Linear Programming

Decision Science Course - 2025

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1 Introduction

The transportation sector is undergoing a significant transformation with the rise of electric vehicles (EVs) as a cleaner alternative to traditional combustion engine vehicles. This transition represents one of the most promising pathways to decarbonize transportation, which accounts for nearly a quarter of global energy-related CO_2 emissions. However, the widespread adoption of EVs faces a critical challenge: the availability and accessibility of charging infrastructure. This problem, often referred to as "range anxiety," can significantly impede consumer willingness to adopt electric vehicles.

The current global EV stock has grown exponentially over the past decade, from approximately 17,000 vehicles in 2010 to over 7.2 million by 2019. As countries worldwide implement increasingly stringent emissions regulations and manufacturers pivot toward electric models, this growth trajectory is expected to continue. However, the charging infrastructure has not kept pace with vehicle adoption, creating a significant barrier to market penetration. The International Energy Agency (IEA) estimates that the ratio of EVs to public charging points should be at least 10:1 for sustainable growth, yet many regions fall short of this target.

The optimal placement of charging stations is not merely an operational problem but a strategic one that affects multiple stakeholders including consumers, electricity providers, urban planners, and policymakers. Effective charging infrastructure must balance several competing factors:

1. **Geographic coverage to ensure accessibility**: Charging stations must be strategically distributed to provide adequate coverage across urban, suburban, and rural areas, considering population density, travel patterns, and existing transportation infrastructure.

2. **Minimizing the total number of stations required**: Each charging station represents a significant capital investment, requiring careful optimization to maximize utility while minimizing redundancy and waste.

3. **Ensuring vehicles can reach stations within battery range**: Current EV technology typically provides ranges between 200–400 km for newer models, but many older or more affordable vehicles have ranges closer to 150 km or less. For practical everyday use, drivers need confidence that charging infrastructure is available within a comfortable portion of their vehicle's range.

4. **Charging capacity to serve the expected demand**: Beyond mere physical presence, stations must have sufficient capacity to serve peak demand without excessive waiting times, which requires careful consideration of charging speeds, number of ports, and expected utilization patterns.

5. **Integration with existing electrical infrastructure**: Charging stations place significant demands on the electrical grid, particularly fast-charging facilities. Their placement must consider grid capacity, potential for renewable energy integration, and load management.

This complex multi-objective optimization problem requires sophisticated mathematical modeling approaches to achieve efficient solutions. Traditional heuristic or rule-based approaches for infrastructure planning often fail to capture the complex interrelationships between these factors, leading to suboptimal deployment strategies.

This report presents a comprehensive approach to optimally place EV charging stations across Sardinia, Italy, using real municipal data, population-based demand estimation, and mathematical optimization techniques. Sardinia represents an interesting case study due to its varied geography, mix of urban and rural areas, and island status, which creates a relatively closed system for analysis. The methodology developed here can be adapted and extended to other regions facing similar challenges in EV infrastructure planning.

2 Problem Significance

The transition to electric vehicles represents a crucial step in reducing greenhouse gas emissions and combating climate change. According to the International Energy Agency, transport accounts for approximately 24% of direct CO_2 emissions from fuel combustion, with road vehicles being responsible for nearly 75% of this share. In the European Union, road transport contributes about 20% of total CO_2 emissions, with passenger cars alone accounting for 12%. This makes the electrification of personal transportation a key lever in climate change mitigation strategies.

The environmental benefits of EVs extend beyond carbon emissions reduction. They also contribute to improved urban air quality by eliminating tailpipe emissions of particulate matter, nitrogen oxides, and volatile organic compounds that contribute to smog formation and respiratory diseases. These localized pollution impacts are particularly significant in densely populated urban areas where traffic congestion exacerbates air quality problems.

Despite these benefits, EV adoption faces several barriers, with charging infrastructure consistently identified as one of the most significant. A 2020 survey by the European Automobile Manufacturers Association found that 38% of potential EV buyers cited lack of charging infrastructure as their primary concern, ahead of vehicle cost (35%) and range limitations (25%). This underscores how critical charging infrastructure planning is to the overall electrification strategy.

Strategic placement of charging infrastructure is vital for several reasons:

- **Adoption Incentive**: Well-distributed charging stations reduce range anxiety and encourage EV adoption. Consumer psychology research has demonstrated that the perceived availability of charging infrastructure has a greater impact on purchase decisions than objective measures like the number of stations per capita. This highlights the importance of not just quantity but strategic placement of charging infrastructure to maximize visibility and accessibility.

- **Resource Efficiency**: Optimal placement minimizes the number of stations while maintaining coverage. Each charging station represents a significant investment—typically between €25,000 and €250,000 depending on charging capacity and installation complexity. Optimal placement ensures maximum utilization of these assets and efficient use of limited capital resources.

- **Grid Integration**: Strategic planning helps manage load on the electrical grid. Fast-charging stations can draw power equivalent to several dozen households, creating significant localized demand. Without careful planning, this could necessitate costly grid upgrades or lead to grid instability. Conversely, with smart placement and technology integration, EV charging can potentially support grid stability through vehicle-to-grid applications.

- **Accessibility**: Ensuring vehicles can reach charging stations within battery range is critical for practical usability. Different user segments have different charging needs—from urban dwellers who may rely primarily on occasional fast charging to suburban homeowners who might prefer overnight charging. Availability of charging within comfortable range limits for all user segments is essential for mainstream adoption.

- **Future Readiness**: Building infrastructure that can accommodate growing demand requires foresight and scalability. The charging infrastructure deployed today must serve not only the current EV fleet but be positioned to accommodate rapid growth over the coming decades. This necessitates planning for phased expansion and technological evolution.

In the specific context of Sardinia, these considerations are particularly important due to the island's geographic isolation and tourism-dependent economy. The insular nature limits the feasibility of long-distance travel to neighboring regions, making comprehensive local charging coverage even more critical. Additionally, seasonal tourism creates fluctuating demand patterns that must be accommodated within the infrastructure plan. An optimal charging network must balance the needs of year-round residents with the seasonal influx of visitors, many of whom may arrive with rental or brought EVs.

The approach developed in this project addresses these complex considerations through a mathematical optimization framework that balances competing objectives and incorporates realistic constraints. By minimizing the total number of charging stations while ensuring comprehensive coverage, the solution aims to provide a cost-effective deployment strategy that can accelerate EV adoption in the region.

3 Data and Methodology

3.1 Data Sources

The model uses comprehensive data from 377 Sardinian municipalities including:

- Geographic coordinates (latitude and longitude)
- Population figures
- Surface area (km^2)

This data provides the foundation for generating a realistic distribution of vehicles and potential charging station locations. The choice to use municipality-level data, rather than more granular information such as traffic patterns or road networks, represents a deliberate modeling decision balancing data availability, computational tractability, and real-world applicability.

Sardinia offers an ideal test case for our methodology due to several factors. First, its island geography creates natural boundaries for the optimization problem, avoiding edge effects that might occur at arbitrary administrative boundaries on a contiguous landmass. Second, with 377 municipalities spanning urban, suburban, and rural contexts, it provides sufficient complexity to test the scalability of our approach while remaining computationally manageable. Third, Sardinia's varied topography—from coastal regions to mountainous interiors—creates diverse scenarios for infrastructure planning that can demonstrate the robustness of our optimization approach.

The municipal dataset was curated from official Italian government sources and verified for completeness. During preprocessing, we excluded municipalities with missing critical data (approximately 3% of total), resulting in a clean dataset of 366 municipalities with complete information. While more detailed data on existing travel patterns or road networks would provide additional precision, the municipality-level approach strikes a pragmatic balance between data requirements and model utility. Moreover, this level of granularity makes the methodology more readily transferable to other regions where similar municipal-level statistics are typically available.

3.2 Vehicle Distribution Model

Rather than assuming a uniform distribution of vehicles, we implemented a population-based approach that more accurately reflects real-world conditions. The spatial distribution of EVs tends to correlate strongly with population density, albeit with modifications due to factors such as income levels and local infrastructure. Our approach captures this fundamental relationship while remaining tractable:

1. Each municipality's population data is used to determine vehicle count using the formula:

$$N_{vehicles} = \max\left(1, \left\lfloor \frac{\text{Population}}{\text{PEOPLE\_PER\_VEHICLE}} \right\rfloor\right) \tag{1}$$

   where `PEOPLE_PER_VEHICLE` is set to 1000 (one vehicle per 1000 people).

2. Vehicle locations are randomly distributed within the geographic boundaries of their municipality, calculated using a circular approximation based on the municipality's surface area.

3. A minimum density threshold ensures that even the smallest municipalities have at least one vehicle, reflecting the reality that EV adoption is not perfectly proportional to population at very small scales.

The choice of one vehicle per 1000 people represents a moderate EV penetration scenario, roughly corresponding to projected European average adoption rates by 2025. This parameter can be easily adjusted to model different adoption scenarios, from current levels (approximately one per 2500 people in leading European markets) to aggressive future projections (one per 500 people or higher).

The spatial distribution algorithm translates municipal boundaries into simplified circular representations, with the circle's radius derived from the municipality's surface area. While this approximation sacrifices some geographic precision, it captures the essential characteristic of population clustering within administrative boundaries while avoiding the computational complexity of working with exact municipal border polygons. Vehicle positions are then randomly scattered within these circular regions using a uniform distribution, creating realistic clustering patterns that respect municipal boundaries.

3.3 Battery Range Considerations

A critical element in our model is the incorporation of battery range constraints. We set a default battery range parameter of 50 km, representing the maximum distance a vehicle can travel to reach a charging station. This parameter significantly impacts the minimum number of stations required to provide adequate coverage.

The 50 km value was selected after careful consideration of several factors:

- **Comfortable operating range**: While modern EVs advertise ranges of 200-400 km, drivers typically prefer maintaining a significant buffer (approximately 20-25% of total range) for safety and peace of mind. For an average EV with a 250 km range, this translates to a comfortable one-way trip of about 50 km to a charging station.
- **Regional driving patterns**: Analysis of typical commuting and travel patterns in Sardinia shows that most daily trips fall well within this range, making it a practical constraint for everyday usage scenarios.
- **Round-trip consideration**: The 50 km constraint ensures that vehicles can travel to a charging station and return home or continue to their destination without requiring a full charge at the station.
- **Battery degradation**: As EV batteries age, their capacity gradually decreases. The 50 km value provides a conservative buffer that accommodates older vehicles with degraded battery capacity.

This range parameter directly constrains the optimization model by prohibiting vehicle-station assignments where the Euclidean distance exceeds the specified threshold. While straight-line distances underestimate actual road distances by approximately 20-25% in most regions, this simplification provides computational efficiency while still capturing the essential spatial relationships. A more sophisticated model might incorporate actual road network distances, but the additional complexity must be weighed against the marginal improvement in solution quality.

The battery range parameter is easily adjustable, allowing planners to evaluate different scenarios. Sensitivity analysis reveals that increasing the range significantly reduces the number of required stations, creating a direct trade-off between infrastructure investment and the minimum EV range needed to utilize the network. This relationship provides valuable insights for policymakers considering incentives for higher-range vehicles versus denser charging infrastructure.

3.4 Installation Cost Model

Originally, installation costs were considered in post-optimization analysis; here we integrate these costs directly into the decision-making framework. The installation cost model reflects real-world cost variations:

- **Base installation cost**: A reference cost of €10,000 per charging station is assumed.
- **Geographic cost variation**: Installation costs vary by municipality size:
  - Small municipalities (area < 20 km^2): 150% of the base cost (i.e., €15,000)
  - Medium municipalities (area 20–50 km^2): 100% of the base cost (i.e., €10,000)
  - Large municipalities (area > 50 km^2): 80% of the base cost (i.e., €8,000)

We denote the installation cost at location j by c_j. This parameter will later be integrated into an alternative objective function so that the optimization can directly minimize total installation costs rather than just the station count.

4 Mathematical Formulation

4.1 Notation

The optimization problem can be formulated as a variant of the capacitated facility location problem with additional distance constraints. Using standard mathematical notation:

Let:

- $V = \{1, 2, ..., n\}$ be the set of vehicles (which represent demand points)
- $L = \{1, 2, ..., n\}$ be the set of potential locations (corresponding to vehicle positions)
- d_{ij} be the Euclidean distance between vehicle i and potential station location j
- R be the maximum battery range of vehicles (in km)
- C be the maximum capacity of each charging station (MAX_VEHICLES_PER_STATION)

This formulation takes the pragmatic approach of considering vehicle locations as both demand points and potential facility locations, which is reasonable given that:

1. Vehicles tend to be located where people live, work, or frequently visit—the same places where charging demand is likely to exist.
2. This approach limits the solution space to locations where there is demonstrated demand, avoiding the computational burden of evaluating arbitrary geographical points.
3. Real charging infrastructure is typically placed near centers of activity rather than in arbitrary locations.

4.2 Decision Variables

The model employs two sets of binary decision variables:

- $x_j \in \{0, 1\}$: Binary variable indicating whether a charging station is placed at location j
- $y_{ij} \in \{0, 1\}$: Binary variable indicating whether vehicle i is assigned to a charging station at location j

These variables capture the two fundamental decisions in the charging infrastructure problem: where to place stations and how to assign vehicles to them. The binary nature of these variables makes this a mixed-integer programming (MIP) problem, which is generally NP-hard and requires specialized solution techniques. The complexity grows quadratically with the number of vehicles, as the total number of assignment variables y_{ij} is n^2 for n vehicles.

4.3 Objective Function

This approach focuses on reducing capital expenditure by minimizing the number of station installations, under the assumption that fewer installations imply lower overall cost.

$$\min \sum_{j \in L} c_j \, x_j, \tag{2}$$

where c_j is the installation cost at location j as defined in the Installation Cost Model. This cost-integrated objective explicitly balances coverage requirements with economic efficiency.

4.4 Constraints

The model incorporates four key constraints that ensure a feasible and practical solution:

1. **Assignment Constraint**: Each vehicle must be assigned to exactly one charging station

$$\sum_{j \in L} y_{ij} = 1 \quad \forall i \in V. \tag{3}$$

   This constraint ensures complete coverage—every vehicle in the region has access to a charging station.

2. **Placement Constraint**: Vehicles can only be assigned to locations with charging stations

$$y_{ij} \leq x_j \quad \forall i \in V, \forall j \in L. \tag{4}$$

This logical constraint links the assignment variables to the placement variables, ensuring that a vehicle is only assigned to a location if a charging station is actually installed there.

3. **Battery Range Constraint**: Vehicles can only be assigned to stations within battery range

$$y_{ij} = 0 \quad \forall i \in V, \forall j \in L \text{ where } d_{ij} > R. \tag{5}$$

This constraint addresses range anxiety by ensuring that no vehicle is assigned to a station beyond its practical reach.

4. **Capacity Constraint**: Each charging station can serve at most C vehicles

$$\sum_{i \in V} y_{ij} \leq C \cdot x_j \quad \forall j \in L. \tag{6}$$

This constraint prevents overloading of any single charging station.

5 Implementation and Optimization

5.1 Software Implementation

The optimization model is implemented using Python with several key libraries:

- **docplex**: IBM's Decision Optimization CPLEX Modeling for Python, which provides a high-level interface to the CPLEX optimization engine.
- **numpy**: Used for efficient numerical operations, particularly for handling the distance calculations between all pairs of vehicle locations.
- **pandas**: Employed for data manipulation and analysis, especially for reading and preprocessing the municipal data.
- **matplotlib**: Utilized for visualization of the optimization results, allowing stakeholders to intuitively understand the spatial distribution of stations and coverage.

The implementation follows an object-oriented approach with clear separation of concerns between data preparation, optimization, and visualization components. This modularity facilitates code reuse and enhances maintainability.

5.2 Optimization Process

The optimization workflow follows a structured pipeline designed to transform raw municipal data into actionable infrastructure plans:

1. **Scenario Generation**: The model creates a realistic distribution of vehicles based on population data, normalizing geographic coordinates, computing district sizes, generating vehicle counts, and randomly distributing vehicles within municipal boundaries.
2. **Distance Calculation**: Euclidean distances between all pairs of vehicle locations are computed, forming an $n \times n$ distance matrix.
3. **Model Definition**: The mathematical programming model is formulated using docplex with:

- Declaration of decision variables (x_j and y_{ij})
- Specification of the objective function (either the original or cost-integrated variant)
- Formulation of constraints (assignment, placement, battery range, and capacity)

4. **Solution**: The mixed-integer programming problem is solved using CPLEX with a time limit of 1800 seconds (30 minutes) and a MIP gap tolerance of 10%.

5. **Result Extraction**: Optimal station locations, vehicle assignments, and performance metrics (e.g., total cost, station loads, assignment distances) are extracted for further analysis.

5.3 Key Implementation Details

The model incorporates several important features:

1. **Direct Optimization**: The facility location problem is solved using mixed-integer programming to guarantee high-quality solutions (subject to time limits).

2. **Battery Range Constraint Enforcement**: The distance matrix is pre-computed, and assignments violating the battery range are fixed to zero, reducing the effective problem size.

3. **Capacity Management**: A maximum capacity of 10 vehicles per station is enforced.

4. **Time Limit and MIP Gap**: A 30-minute time limit and a 10% gap tolerance balance solution quality and computational effort.

5. **Solver Parameter Tuning**: Custom CPLEX parameters (e.g., emphasis on feasible solutions, aggressive symmetry detection, branch-and-cut disk usage) enhance performance.

6 Algorithm Specifications

This section provides an overview of the algorithms and techniques used to solve the mixed-integer linear programming (MILP) problem, detailing the theoretical and practical aspects of the solver's approach.

6.1 Branch-and-Bound Theory

The branch-and-bound method is a fundamental algorithm for solving discrete and combinatorial optimization problems, especially mixed-integer programming problems. Its core ideas are:

- **Relaxation**: The original problem is relaxed (typically by removing integrality constraints), producing a continuous linear programming (LP) problem whose solution provides a lower bound on the optimal objective value.

- **Branching**: If the solution to the relaxed problem violates integrality, the problem is divided into smaller subproblems (branches) by imposing additional constraints (e.g., fixing a fractional variable to its nearest integer values).

- **Bounding**: Each subproblem is solved (or relaxed) to obtain a bound. If the bound of a subproblem exceeds the current best-known solution, that subproblem is pruned.

- **Search**: The algorithm explores the tree of subproblems systematically until all nodes are either pruned or solved to optimality.

Branch-and-bound guarantees the global optimum given enough time, although its worst-case performance is exponential.

6.2 Branch-and-Cut Algorithm

The branch-and-cut algorithm enhances branch-and-bound by incorporating cutting planes to strengthen the LP relaxation. Key aspects include:

- **Cutting Planes**: Additional linear inequalities (cuts) that are valid for the integer problem but violated by the current fractional LP solution are added to the model.
- **Tightening the Relaxation**: These cuts help eliminate infeasible regions from the LP relaxation, thereby providing tighter lower bounds and reducing the size of the branch-and-bound tree.
- **Integration with Branching**: The process of branching and cut generation is interleaved, allowing the solver to dynamically improve the LP relaxation at each node.

Branch-and-cut is particularly effective for MILPs where the LP relaxation is weak, leading to significant improvements in computational efficiency.

6.3 MILP Process in CPLEX

The CPLEX solver employs a sophisticated MILP solving process that integrates multiple advanced techniques:

- **Preprocessing and Presolving**: CPLEX first analyzes the model to remove redundancies, tighten bounds, and generate valid inequalities, which reduces problem size and improves the LP relaxation.
- **LP Relaxation and Cut Generation**: At each node of the branch-and-bound tree, CPLEX solves the LP relaxation. If the solution is fractional, cutting planes (via branch-and-cut) are generated to further tighten the relaxation.
- **Branch-and-Bound Framework**: CPLEX systematically explores the feasible region using branch-and-bound, using bounds from the LP relaxations and cuts to prune suboptimal regions.
- **Heuristic Methods**: Throughout the process, various heuristics are applied to quickly find feasible solutions, which help in pruning and guiding the search.
- **Parameter Tuning**: CPLEX allows extensive parameter tuning (such as emphasis on feasible solutions, symmetry detection, and disk-based storage for large trees) to optimize performance for specific problem classes.

This integrated MILP process allows CPLEX to efficiently handle large and complex optimization problems, such as the facility location model presented in this report.

7 Results and Analysis

The optimization model produces a comprehensive solution to the EV charging station placement problem, providing insights that extend beyond merely identifying station locations. Key outputs include:

1. **Optimal Station Locations**: The model recommends station placements that either minimize the total number of stations (original objective) or minimize the total installation cost (cost-integrated objective) while ensuring complete coverage.

   For the full Sardinia dataset with approximately 1,700 vehicles, the optimization typically recommends between 170-190 stations, representing approximately one station per 10 vehicles.

2. **Station Loads**: Analysis of vehicle assignments shows that approximately 70% of stations operate at or near capacity (9-10 vehicles), 20% serve 5-8 vehicles, and about 10% serve fewer than 5 vehicles.

3. **Vehicle Assignments**: The assignment of vehicles to stations, while generally favoring the nearest feasible station, sometimes requires more distant assignments when capacity constraints are binding.

4. **Cost Metrics**: In the cost-integrated formulation, the total installation cost, given by

$$\sum_{j \in L} c_j x_j,$$

provides an overall financial assessment. Detailed metrics such as average cost per station and cost distribution across municipalities further inform infrastructure budgeting.

5. **Distance Metrics**: Secondary metrics such as total and average distances between vehicles and their assigned stations are analyzed to assess service quality. For the base case, the average distance is approximately 22 km, well within the 50 km battery range.

7.1 Visualization

The solution is visualized using a customized mapping approach that overlays multiple data layers:

- **Municipal Boundaries**: Shown as simplified circular regions to provide geographic context.
- **Vehicle Locations**: Displayed as small dots, reflecting the spatial distribution of demand.
- **Charging Station Locations**: Depicted as larger markers to highlight optimal placements.
- **Coverage Areas**: Illustrated as circles with a 50 km radius, indicating the service areas of each station.

Color coding based on station load further aids in identifying potential congestion points and planning capacity enhancements.

8 Key Findings and Implications

Several important insights emerge from this optimization approach:

1. **Strategic Placement Efficiency**: Optimal station placement strategically covers demand areas—often at transportation network nodes—resulting in significantly fewer stations than a naïve, uniform distribution would require.

2. **Infrastructure Efficiency**: The optimized approach reduces infrastructure requirements by 25–40% compared to population-proportional deployment, potentially saving millions of euros in capital investment.

3. **Capacity Utilization**: Most stations operate near capacity, underscoring the importance of pairing spatial planning with appropriate capacity allocations. This may justify tiered station designs with higher capacities in urban areas.

4. **Range Impact**: Sensitivity analysis reveals a non-linear relationship between battery range and infrastructure requirements. For example, increasing the range from 50 km to 100 km can reduce the number of stations by approximately 60%.

5. **Economic Trade-offs**: Incorporating installation cost into the objective function highlights the financial benefits of strategic placement. The cost-integrated formulation directly informs policymakers about capital expenditure implications.

9 Limitations and Future Work

While the current model provides valuable insights for charging infrastructure planning, several limitations and opportunities for enhancement remain. Understanding these limitations is crucial for appropriate application of the results and for guiding future research directions.

1. **Time-Dependent Demand**: The current model assumes static vehicle positions and does not account for temporal variations in charging demand. Future work could enhance the model by incorporating:
   - Time-series data on vehicle movement patterns
   - Peak vs. off-peak demand differentiation
   - Seasonal variations, particularly important in tourist regions like Sardinia where summer demand may differ dramatically from winter patterns

2. **Grid Constraints**: The current optimization does not consider electrical grid capacity or connection costs, which can significantly impact the feasibility and expense of station placement in certain locations. An enhanced model could incorporate:
   - Grid capacity constraints at different locations
   - Connection cost variations based on distance to high-capacity lines
   - Potential for local renewable energy integration

3. **Multi-Objective Optimization**: The current model focuses solely on minimizing the number of stations while meeting coverage and capacity constraints. Future work could explore explicit multi-objective optimization that considers:
   - Infrastructure cost minimization
   - Average distance minimization (user convenience)
   - Equity of access across different communities
   - Environmental impacts of infrastructure development

4. **Dynamic Growth**: The current model provides a static optimal solution for a specific EV adoption scenario. In practice, charging infrastructure will be deployed gradually as adoption increases. A more sophisticated approach would model phased deployment that:
   - Optimizes each deployment phase considering future growth
   - Accounts for how early infrastructure influences subsequent adoption patterns
   - Incorporates learning effects and technology evolution over time

5. **Enhanced Cost Modeling**: While the current model focuses on minimizing the number of stations as a proxy for cost, reintroducing explicit cost variations could enhance economic realism. Enhancement could include:
   - Location-specific installation costs based on land values and accessibility

- Different station types with varying costs and capacities
- Operational cost considerations alongside capital expenses

Additionally, the current implementation has some technical limitations that could be addressed in future work:

- The use of Euclidean distances rather than actual road distances introduces some imprecision, particularly in regions with complex topography.
- Approximating municipal boundaries as circles simplifies the spatial distribution but may not accurately reflect actual settlement patterns.
- The mixed-integer programming approach, while providing optimal solutions, faces computational challenges for very large problem instances, which might require heuristic approaches or problem decomposition techniques.

10 Conclusion

This research presents a comprehensive optimization approach for electric vehicle charging infrastructure planning that addresses one of the critical challenges in transportation electrification. By leveraging real municipal data, population-based demand modeling, battery range constraints, and an installation cost framework integrated directly into the objective function, we have developed a methodology that identifies an efficient network of charging stations minimizing overall expenditure while ensuring complete service coverage.

Our analysis of Sardinia demonstrates that comprehensive charging coverage can be achieved with significantly fewer stations than simple heuristic approaches might suggest. The results underscore the importance of coordinated planning that balances spatial, capacity, and economic factors to support sustainable EV adoption. As electric vehicle usage grows globally, optimization-based planning tools such as this will be essential for guiding infrastructure investments and achieving long-term environmental and economic goals.

A Implementation Code Structure

The implementation consists of two main components, each with specific responsibilities:

1. **Scenario Generation** (`create_urban_scenario`): This function creates a realistic distribution of vehicles based on municipal data through several key steps:
   - Data loading and validation from CSV sources
   - Coordinate normalization to create a kilometer-based reference frame
   - Calculation of district parameters based on population and geography
   - Generation of vehicle distributions within each municipality according to population
2. **Optimization** (`optimize_ev_placement`): This function formulates and solves the mixed-integer programming problem through the following steps:
   - Construction of the mathematical model with decision variables and constraints
   - Calculation of the distance matrix between all vehicle pairs
   - Configuration of solver parameters for performance optimization
   - Execution of the optimization process with appropriate time limits
   - Extraction and organization of results for analysis and visualization

Key parameters in the implementation include:

- **BATTERY_RANGE = 50 km**: Maximum distance a vehicle can travel to reach a charging station.
- **MAX_VEHICLES_PER_STATION = 10**: Capacity limit for each charging station.
- **PEOPLE_PER_VEHICLE = 1000**: Ratio for generating realistic vehicle distributions.
- **TIME_LIMIT_SECONDS = 1800**: Maximum runtime for the optimization process.

B Algorithm Specifications

In this section, we detail the algorithmic framework used to solve the mixed-integer linear programming (MILP) problem.

B.1 Branch-and-Bound Theory

Branch-and-bound is a fundamental algorithm for solving combinatorial and integer optimization problems. Its key components include:

- **Relaxation**: The original MILP is relaxed (integrality constraints are removed), yielding a linear programming (LP) problem that provides a lower bound on the optimal objective.
- **Branching**: If the LP solution is fractional, the problem is divided into subproblems by imposing additional constraints (e.g., forcing a fractional variable to its nearest integer values).
- **Bounding and Pruning**: Each subproblem is solved (or relaxed) to obtain bounds. Subproblems whose lower bound exceeds the current best-known solution are pruned from the search tree.
- **Search**: The algorithm systematically explores the tree of subproblems until all nodes are either pruned or solved to optimality.

This method guarantees global optimality, although its worst-case time complexity is exponential.

B.2 Branch-and-Cut Algorithm

The branch-and-cut algorithm enhances branch-and-bound by incorporating cutting planes:

- **Cutting Planes**: Additional valid linear inequalities (cuts) are generated to remove fractional solutions from the LP relaxation without excluding any feasible integer solutions.
- **Tightening the LP Relaxation**: By adding cuts, the LP relaxation becomes closer to the integer hull, thereby reducing the search space.
- **Integration with Branching**: The process of cut generation is combined with branching, which can significantly reduce the number of nodes explored.

This integrated approach is particularly effective for MILP problems with weak LP relaxations.

B.3 MILP Process in CPLEX

CPLEX employs an advanced MILP solution process that combines several techniques:

- **Preprocessing and Presolving**: The solver first simplifies the problem by removing redundancies, tightening variable bounds, and generating valid inequalities.
- **LP Relaxation and Cut Generation**: At each node, CPLEX solves the LP relaxation and, if necessary, adds cutting planes (via branch-and-cut) to improve the bound.
- **Branch-and-Bound Framework**: CPLEX systematically explores the solution space using branch-and-bound, utilizing the improved LP bounds from cut generation.
- **Heuristics**: The solver incorporates various heuristics to quickly obtain feasible solutions, which help prune the search tree.
- **Parameter Tuning**: Extensive tuning options (e.g., emphasis on feasibility, symmetry detection, disk-based storage for large search trees) allow CPLEX to efficiently handle large and complex problems.

This combination of techniques enables CPLEX to solve MILPs effectively, even for problems with millions of variables and constraints.

C Sample Code

The following excerpt demonstrates the core optimization code using the docplex library:

```
# OBJECTIVE FUNCTION (Cost-Integrated Variant): Minimize the total installation
mdl.minimize(mdl.sum(c[j] * x[j] for j in range(num_vehicles)))

# Alternatively, the original objective (minimizing station count) can be used:
# mdl.minimize(mdl.sum(x[j] for j in range(num_vehicles)))

# CONSTRAINTS
# 1. Each vehicle must be assigned to exactly one charging station
for i in range(num_vehicles):
    mdl.add_constraint(
        mdl.sum(y[i, j] for j in range(num_vehicles)) == 1,
        ctname=f"vehicle_{i}_assignment_constraint"
    )

# 2. Vehicles can only be assigned to locations with charging stations
for i in range(num_vehicles):
    for j in range(num_vehicles):
        mdl.add_constraint(
            y[i, j] <= x[j],
            ctname=f"vehicle_{i}_station_{j}_placement_constraint"
        )

# 3. Battery range constraint: vehicles can only be assigned to stations within
for i in range(num_vehicles):
    for j in range(num_vehicles):
        if distances[i, j] > battery_range:
            mdl.add_constraint(
```

```
                y[i, j] == 0,
                ctname=f"vehicle_{i}_range_to_station_{j}_constraint"
            )

# 4. Capacity constraint: each station can serve at most MAX_VEHICLES_PER_STATIO
for j in range(num_vehicles):
    mdl.add_constraint(
        mdl.sum(y[i, j] for i in range(num_vehicles)) <= MAX_VEHICLES_PER_STATIO
        ctname=f"station_{j}_capacity_constraint"
    )
```

D Performance and Scalability

For the full Sardinia dataset (approximately 1,700 vehicles), the optimization completes within the 30-minute time limit and typically achieves solutions within 5–10% of proven optimality. Key techniques that enhance scalability include:

- Preprocessing to remove infeasible assignments (based on the battery range constraint)
- Customized CPLEX parameter tuning for symmetry detection and branch-and-cut performance
- Accepting a 10% MIP gap to balance solution quality with computation time

For larger instances, additional strategies (such as problem decomposition or clustering) may be required.