Grundlagen der Rechnerarchitektur Blatt 5

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1 Eine Schaltung für den Weihnachtsbaum

(a) Wahrheitstafel
x_i kodieren Tag und s_i ist Segment i

Tag	x_3	x_2	x_1	x_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7
So 01	0	0	0	0	1	1	1	1	1	1	1
Mo 02	0	0	0	1	0	1	0	1	0	1	0
Di 03	0	0	1	0	0	1	0	1	0	1	0
Mi 04	0	0	1	1	1	1	0	1	0	1	0
Do 05	0	1	0	0	0	1	0	1	0	1	0
Fr 06	0	1	0	1	1	1	1	1	1	1	0
Sa 07	0	1	1	0	1	1	1	1	1	1	0
So 08	0	1	1	1	1	1	1	1	1	1	1
Mo 09	1	0	0	0	1	0	1	0	1	0	0
Di 10	1	0	0	1	1	1	1	1	1	1	0
Mi 11	1	0	1	0	1	0	1	1	1	1	0
Do 12	1	0	1	1	1	0	1	0	1	0	0
Fr 13	1	1	0	0	1	0	1	0	1	0	0
Sa 14	1	1	0	1	1	1	1	1	1	1	0
So 15	1	1	1	0	1	0	1	0	1	0	1
Mo 16	1	1	1	1	1	1	1	0	1	0	0

(b) Kanonische Normalformen

$$\begin{aligned} f_{1,DKNF} =& (\bar{x}_3\bar{x}_2\bar{x}_1\bar{x}_0) + (\bar{x}_3\bar{x}_2x_1x_0) + (\bar{x}_3x_2\bar{x}_1x_0) + (\bar{x}_3x_2x_1\bar{x}_0) + (\bar{x}_3x_2x_1x_0) + (x_3\bar{x}_2\bar{x}_1\bar{x}_0) + (x_3\bar{x}_2\bar{x}_1x_0) \\ &+ (x_3\bar{x}_2x_1\bar{x}_0) + (x_3\bar{x}_2x_1x_0) + (x_3x_2\bar{x}_1\bar{x}_0) + (x_3x_2\bar{x}_1x_0) + (x_3x_2x_1\bar{x}_0) + (x_3x_2x_1x_0) \end{aligned}$$

$$\begin{aligned} f_{2,KKNF} =& (\bar{x}_3 + x_2 + x_1 + x_0) \cdot (\bar{x}_3 + x_2 + \bar{x}_1 + x_0) \cdot (\bar{x}_3 + x_2 + \bar{x}_1 + \bar{x}_0) \\ &\cdot (\bar{x}_3 + \bar{x}_2 + x_1 + x_0) \cdot (\bar{x}_3 + \bar{x}_2 + \bar{x}_1 + x_0) \end{aligned}$$

(c) Algebraische Minimierung

$$
\begin{aligned}
f_{1,DNF} &\overset{P4,P3'}{=} (\bar{x}_2\bar{x}_1\bar{x}_0(\bar{x}_3 + x_3)) + (x_2x_1x_0(\bar{x}_3 + x_3)) + (x_3x_2x_1(\bar{x}_0 + x_0)) + (x_3x_2x_0(\bar{x}_1 + x_1)) \\
&\quad + (x_3x_1x_0(\bar{x}_2 + x_2)) + (x_3\bar{x}_1x_0(\bar{x}_2 + x_2)) + (x_3x_1\bar{x}_0(\bar{x}_2 + x_2)) + (x_2x_1\bar{x}_0(\bar{x}_3 + x_3)) \\
&\quad + (x_2\bar{x}_1x_0(\bar{x}_3 + x_3)) + (x_3x_2\bar{x}_1(\bar{x}_0 + x_0)) + (\bar{x}_3x_1x_0(\bar{x}_2 + x_2)) \\
&\overset{P9',P5}{=} \bar{x}_2\bar{x}_1\bar{x}_0 + x_2x_1x_0 + x_3x_2x_1 + x_3x_2x_0 + x_3x_1x_0 + x_3\bar{x}_1x_0 + x_3x_1\bar{x}_0 + x_2x_1\bar{x}_0 \\
&\quad + x_2\bar{x}_1x_0 + x_3x_2\bar{x}_1 + \bar{x}_3x_1x_0 \\
&\overset{P4,P3'}{=} (x_2x_1(x_0 + \bar{x}_0)) + (x_3x_2(x_1 + \bar{x}_1)) + (x_3x_1(x_0 + \bar{x}_0)) + (x_3x_0(x_1 + \bar{x}_1)) + (x_1x_0(x_3 + \bar{x}_3)) \\
&\quad + (x_2x_0(x_1 + \bar{x}_1)) + \bar{x}_2\bar{x}_1\bar{x}_0 + x_3x_2x_0 \\
&\overset{P9',P5}{=} x_2x_1 + x_3x_2 + x_3x_1 + x_3x_0 + x_1x_0 + x_2x_0 + \bar{x}_2\bar{x}_1\bar{x}_0 + x_3x_2x_0 \\
&\overset{P11'}{=} x_2x_1 + x_3x_2 + x_3x_1 + x_3x_0 + x_1x_0 + x_2x_0 + \bar{x}_2\bar{x}_1\bar{x}_0
\end{aligned}
$$

$$
\begin{aligned}
f_{2,KNF} &\overset{P4',P3}{=} ((\bar{x}_3 + x_2 + x_0) + (x_1\bar{x}_1)) \cdot ((\bar{x}_3 + \bar{x}_2 + x_0) + (x_1\bar{x}_1)) \cdot ((\bar{x}_3 + x_2 + \bar{x}_1) + (x_0\bar{x}_0)) \\
&\overset{P9,P5'}{=} (\bar{x}_3 + x_2 + x_0) \cdot (\bar{x}_3 + \bar{x}_2 + x_0) \cdot (\bar{x}_3 + x_2 + \bar{x}_1) \\
&\overset{P4'}{=} ((\bar{x}_3 + x_0) + (x_2\bar{x}_2)) \cdot (x_3 + x_2 + \bar{x}_1) \\
&\overset{P9,P5'}{=} (\bar{x}_3 + x_0) \cdot (x_3 + x_2 + \bar{x}_1)
\end{aligned}
$$

(d) Karnaugh-Veitch

Segment 3: $f_{3,KV,DNF} = x_3 + x_2x_0 + x_2x_1 + \bar{x}_2\bar{x}_1\bar{x}_0$

	$\bar{x}_0$	x_0	x_0	$\bar{x}_0$	
$\bar{x}_1$	1	0	1	0	$\bar{x}_3$
x_1	0	0	1	1	$\bar{x}_3$
x_1	1	1	1	1	x_3
$\bar{x}_1$	1	1	1	1	x_3
	$\bar{x}_2$	$\bar{x}_2$	x_2	x_2	

Segment 4: $f_{4,KV,KNF} = (\bar{x}_0 + \bar{x}_1 + \bar{x}_3) \cdot (\bar{x}_3 + \bar{x}_2 + x_0) \cdot (\bar{x}_3 + x_1 + x_0)$

	$\bar{x}_0$	x_0	x_0	$\bar{x}_0$	
$\bar{x}_1$	1	1	1	1	$\bar{x}_3$
x_1	1	1	1	1	$\bar{x}_3$
x_1	1	0	0	0	x_3
$\bar{x}_1$	0	1	1	0	x_3
	$\bar{x}_2$	$\bar{x}_2$	x_2	x_2	

(e) Quine McCluskey

Segment 5: $f_{5,QMC} = \bar{x}_2\bar{x}_1\bar{x}_0 + x_2x_1 + x_2x_0 + x_3$

$Q_{4,4} = \{\underline{\bar{x}_3\bar{x}_2\bar{x}_1\bar{x}_0}\}$
$Q_{4,3} = \{\underline{x_3\bar{x}_2\bar{x}_1\bar{x}_0}\}$
$Q_{4,2} = \{\underline{x_3\bar{x}_2x_1\bar{x}_0}, \underline{x_3\bar{x}_2\bar{x}_1x_0}, \underline{\bar{x}_3x_2x_1\bar{x}_0}, \underline{\bar{x}_3x_2\bar{x}_1x_0}, \underline{x_3x_2\bar{x}_1\bar{x}_0}\}$
$Q_{4,1} = \{\underline{\bar{x}_3x_2x_1x_0}, \underline{x_3\bar{x}_2x_1x_0}, \underline{x_3x_2\bar{x}_1x_0}, \underline{x_3x_2x_1\bar{x}_0}\}$
$Q_{4,0} = \{\underline{x_3x_2x_1x_0}\}$

$Q_{3,3} = \{\underline{\bar{x}_2\bar{x}_1\bar{x}_0}\}$
$Q_{3,2} = \{\underline{x_3\bar{x}_2\bar{x}_0}, \underline{x_3\bar{x}_2\bar{x}_1}, \underline{x_3\bar{x}_1\bar{x}_0}\}$
$Q_{3,1} = \{\underline{x_3\bar{x}_2x_1}, \underline{x_3x_1\bar{x}_0}, \underline{x_3\bar{x}_2x_0}, \underline{x_3\bar{x}_1x_0}, \underline{\bar{x}_3x_2x_1}, \underline{x_2x_1\bar{x}_0}, \underline{\bar{x}_3x_2x_0}, \underline{x_2\bar{x}_1x_0}, \underline{x_3x_2\bar{x}_1}, \underline{x_3x_2\bar{x}_0}\}$
$Q_{3,0} = \{\underline{x_2x_1x_0}, \underline{x_3x_1x_0}, \underline{x_3x_2x_0}, \underline{x_3x_2x_1}\}$

$Q_{2,2} = \{\}$
$Q_{2,1} = \{\underline{x_3\bar{x}_2}, \underline{x_3\bar{x}_0}, \underline{x_3\bar{x}_1}\}$
$Q_{2,0} = \{\underline{x_3x_1}, \underline{x_3x_0}, x_2x_1, x_2x_0, \underline{x_3x_2}\}$

$Q_{1,1} = \{\}$
$Q_{1,0} = \{x_3\}$

Segment 6: $f_{6,QMC} = \bar{x}_2x_1\bar{x}_0 + \bar{x}_1x_0 + \bar{x}_3$

$Q_{4,4} = \{\underline{\bar{x}_3\bar{x}_2\bar{x}_1\bar{x}_0}\}$
$Q_{4,3} = \{\underline{\bar{x}_3\bar{x}_2\bar{x}_1x_0}, \underline{\bar{x}_3\bar{x}_2x_1\bar{x}_0}, \underline{\bar{x}_3x_2\bar{x}_1\bar{x}_0}\}$
$Q_{4,2} = \{\underline{\bar{x}_3\bar{x}_2x_1x_0}, \underline{\bar{x}_3x_2\bar{x}_1x_0}, \underline{\bar{x}_3x_2x_1\bar{x}_0}, \underline{x_3\bar{x}_2\bar{x}_1x_0}, \underline{x_3\bar{x}_2x_1\bar{x}_0}\}$
$Q_{4,1} = \{\underline{\bar{x}_3x_2x_1x_0}, \underline{x_3x_2\bar{x}_1x_0}\}$
$Q_{4,0} = \{\}$

$Q_{3,3} = \{\underline{\bar{x}_3\bar{x}_2\bar{x}_1}, \underline{\bar{x}_3\bar{x}_2\bar{x}_0}, \underline{\bar{x}_3\bar{x}_1\bar{x}_0}\}$
$Q_{3,2} = \{\underline{\bar{x}_3\bar{x}_2x_0}, \underline{\bar{x}_3\bar{x}_1x_0}, \underline{\bar{x}_2\bar{x}_1x_0}, \underline{\bar{x}_3\bar{x}_2x_1}, \underline{\bar{x}_3x_1\bar{x}_0}, \bar{x}_2x_1\bar{x}_0, \underline{\bar{x}_3x_2\bar{x}_1}, \underline{\bar{x}_3x_2\bar{x}_0}\}$
$Q_{3,1} = \{\underline{\bar{x}_3x_2x_0}, \underline{\bar{x}_3x_2x_0}, \underline{x_1\bar{x}_1x_0}, \underline{\bar{x}_3x_2x_1}, \underline{x_3\bar{x}_1x_0}\}$
$Q_{3,0} = \{\}$

$Q_{2,2} = \{\underline{\bar{x}_3\bar{x}_2}, \underline{\bar{x}_3\bar{x}_1}, \underline{\bar{x}_3\bar{x}_0}\}$
$Q_{2,1} = \{\underline{\bar{x}_3x_0}, \bar{x}_1x_0, \underline{\bar{x}_3x_1}, \underline{\bar{x}_3x_2}\}$
$Q_{2,0} = \{\}$

$Q_{1,1} = \{\bar{x}_3\}$
$Q_{1,0} = \{\}$

(f) Weniger ist mehr

Segment 1(b): $13 \cdot \text{AND}_4 + \text{OR}_7 + \text{OR}_7 + 23 \cdot \text{NOT}_1 \rightarrow 208$Transistoren
Segment 1(c): $6 \cdot \text{AND}_2 + \text{AND}_3 + \text{OR}_7 + 3 \cdot \text{NOT}_1 \rightarrow 66$Transistoren
$\rightarrow$ Ersparnis um $(1 - \frac{66}{208} \approx 39.8\%)$ durch Minimierung
Segment 2(b): $\text{AND}_5 + 5 \cdot \text{OR}_4 + 11 \cdot \text{NOT}_1 \rightarrow 84$Transistoren
Segment 2(c): $\text{AND}_2 + \text{OR}_2 + \text{OR}_3 + 2 \cdot \text{NOT}_1 \rightarrow 24$Transistoren
$\rightarrow$ Ersparnis um $(1 - \frac{24}{84} \approx 71.4\%)$ durch Minimierung