POLITECNICO DI MILANO

MMSP 2nd Module – Lab1

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Audio signal encoding

EXERCISE 1

Exercise 1 - Problem

1. Load file ‘pf.wav’ and plot it. Check that its values are included between [-1,1].

2. Take only the first 60 sec. of the file and rescale its values between [0,255].

3. Convert each value into its binary representation over 8 bit.

4. Find the entropy of the binary source that has generated the above audio file.

5. Now consider the file as generated by a finite source whose alphabet is [0:255]. Plot the normalized histogram of the file and find the entropy of the above source.

6. Consider now the audio file as generated by a source with memory (let us suppose memory = 1). Find the conditional entropy.

Exercise 1 - Hints

1. Use **audioread** or **wavread** to read the audio file

1. Use **bi2de** for binary representation

1. Use **hist** to estimate pdf / pmf

2. Use **hist3** to estimate joint pdf / pmf

3. Pay attention to $\log_2(0)$

4. Remember entropy properties (Venn diagram)

Image signal encoding

EXERCISE 2

Exercise 2 - Problem

1. Load the image 'lena512color.tiff'. For each image component (R, G, B) display the histogram.

2. Approximate the pdf of each channel as the normalized histogram. Compute the entropy (in bpp) of each channel.

3. Let X be the source represented by the red channel and Y the source represented by the green channel. Compute and plot the joint pdf, p(X,Y).

4. Compute the joint entropy H(X,Y) and verify that $H(X,Y) \leq H(X) + H(Y)$. Why the equality is not satisfied?

5. Suppose to encode Y with H(Y) bits and to send $N = aX+b-Y$ instead of X, where 'a' and 'b' are obtained by linear regression (least squares on $Y=aX+b$). Compute the entropy of N and compare it with the conditional entropy $H(X|Y) = H(X,Y) - H(Y)$.

Exercise 2 - Hints

1. Use **imread** to load image files

2. Images are stored as **uint8**, consider casting to **double**

3. Remember how to solve a linear regression problem using least-squares

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \\ x_N & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix}$$

Discrete memoryless source coding

EXERCISE 3

Exercise 3 - Problem

Consider a discrete random sequence described by the following equation:

$$y(n) = \min(\max(0, \mathrm{round}(\rho\, y(n-1) + w(n))), 15)$$

where w(n) is a Gaussian white noise of variance equal to 1 and ρ=0.95

1. Generate one realization of the process of length = 1,000,000
2. Determine the size of the alphabet of the source
3. Find the entropy H(Y) assuming that y(n) is a discrete memoryless source
4. Let K=ρy(n-1). Compute the joint PDF p(Y,K) and the joint entropy H(Y,K).
5. Compute the conditional entropy H(Y|K). Compare it with H(Y). How many bps are needed to represent the source exploiting inter-symbol redundancy?

Exam track (19th June 2006)

EXERCISE 4

Exercise 4 - Problem

1. Generate N = 10000 samples of a AR(1) random process

$$\rho\, y(n-1) + w(n)$$

where w(n) is a Gaussian white noise of variance   equal to 1 and ρ=0.99 .

1. Clip the sample values in the range [−20,20] and round them to the nearest integer.

2. Compute the entropy H(y) of the source assuming that there is no memory. Compare H(y) with the maximum entropy of a source having the same alphabet.