Color Space and Its Divisions

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Color Order from Antiquity to the Present

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To the memory of Dorothy Nickerson and David L. MacAdam, and to Andreas Brockes for their encouragement to continue the pursuit of color order and color difference

All that is alive tends toward color, individuality, specificity, effectiveness, and opacity; all that is done with life inclines toward knowledge, abstraction, generality, transfiguration, and transparency.

Johann Wolfgang von Goethe

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Preface

Our color experiences are an important component of our visual experiences and as such form a significant aspect of our consciousness. Color experiences are the outcome of processing by the brain of information acquired as a result of interaction of light energy with the three types of retinal cone cells in the eyes. The three cone types filter the spectral complexity of our surroundings and from the three kinds of signals the brain constructs complex experiences that allow us to interact with our surroundings in a purposeful way. Luminance contours are important input for the generation of these experiences but much of the original complexity of the information arriving at the eye is in terms of spectral signatures translated by the brain into colors. It has been estimated that we can distinguish between color experiences that number in the millions.

Given the human predilection for ordering experiences and given the large number of different possible color experiences, it is not surprising that the question of how to bring order to this multiplicity is one that humans thought about since antiquity. Confusion created by the seemingly different result of mixing colored lights and colored materials has complicated the search for answers considerably. Only in the later eighteenth century have color experiences begun to be sorted into three-dimensional arrangements, and three attributes of color experiences have only been defined unambiguously in mid-nineteenth century. Ordering color into a uniform color space means the creation of a geometrical model that is considered to be isomorphic (one-to-one correspondence) with experiences. It is evident that this cannot be a simple effort. Alternately, it can mean ordering color stimuli in a regular way so that the most general psychological ordering principles are observed.

This text presents a history of the significant steps in development of thinking about color order in the Western world from ancient Greece to the present. Not surprisingly, given the complexity of the matter and the fact that it involves a sense, several fundamental questions continue to be unanswered. Among other things this has to do, despite hundreds of years of concentrated effort by many individuals, with the absence of a scientifically satisfactory experimental database of how humans perceive colors and color differences. The text is limited to issues of color space as viewed against a simple achromatic surround. It does not address issues of color appearance under widely varying surround conditions.

Every space is divisible and a given color space is inextricably linked with the definition of the divisions used. A dividing line is drawn between uniform color spaces where distances in all directions are isomorphic to perceived differences of equal magnitude and general color spaces that are ordered according to some other principles. The plural is used because it has become evident that there is no single uniform color space but each such space is related to a quite highly specific set of viewing and general experimental conditions. Uniform color spaces are of particular interest for color quality control purposes in industries manufacturing colorants or colored goods. A considerable variety of general regular spaces are in use, for example, in the graphics industry and in computer display technology.

Regular arrangements of colors fit into euclidean space and can have many different simple geometrical forms, depending on the definition of distance. It is not evident that a uniform color space can have euclidean form (there are no obvious reasons why it should).

The book begins with a general introduction to the subject in Chapter 1. Following the historical account of color order systems in Chapter 2, fundamentals of psychophysics, the branch of science concerned with the relationship between stimuli and experiences, are presented in Chapter 3. Chapter 4 describes the results of perceptual scaling of colors according to attributes. In Chapter 5 these scales are related to scales based on psychophysical modification of physical measurements (reflectance or spectral power distribution measurements). The history until the present of the development of mathematical color space and difference formulas is described in Chapter 6. Three of the color order systems presented in Chapter 2 have been selected for more detailed description of their development, their psychophysical structure, and the problems associated with them in Chapter 7. Chapter 8 contains an analysis of the agreements and discrepancies in psychophysical data describing color at levels of difference ranging from color matching error to large. Chapter 9, finally, draws conclusions and offers an experimental plan for the kind of reliable, replicated perceptual data needed to make progress in this field.

Aside from offering the first extended historical account of this fascinating field, the book contains new analytical results of perceptual and psychophysical color data and a synthesis of data developed for different purposes and under different circumstances. I believe it to be not only of interest to experts and educators in industry and academe but also to neuroscientists and philosophers grappling with problems of awareness and consciousness, to designers, graphic artists, art historians, students of vision, psychology, design, and, last but not least, the general reader with interest in the subject matter.

This book represents the culmination of some forty years of interest in the subject. During these years I had many discussions with fellow devotees in industry, academe, the Inter-Society Color Council, and other organizations that helped shape my knowledge and views. I am grateful to all that helped me to see the issues clearer.

The text was read and commented on in its entirety by Dale Purves. Individual chapters have been read by Larry Hardin and Andreas Schwarz. I am grateful to all three, but any remaining errors are my own.

A note about certain conventions: An author's name with an associated year relates to a complete reference at the back of the book. Comments in angled parentheses are by the author, except in Chapter 2 where they usually contain Greek and Latin color names.

Chapter 1

The Concept of Color Space and Color Solid

1.1 INTRODUCTION

Attempting to understand our place in the world and classifying things and experiences is a well-known human trait. Already ancient Greek philosophers thought about the multitude of color perceptions, but they despaired of finding a system in which to place them. First, colors were logically sorted according to lightness, regardless of hue. Early in the second millennium we begin to find descriptions of tonal scales of individual hues or mixed tones, like flesh color. They were achieved by adding lighter or darker pigments of similar hue, even black or white, to saturated chromatic pigments. Systematic hue circles began to appear in the late seventeenth century. The concept of a three-dimensional logical arrangement of color perceptions began to take shape only in the eighteenth century.

Color space is a three-dimensional geometric space with axes appropriately defined so that symbols for all possible color perceptions of humans or other animals fit into it in an order corresponding to the psychological order. In this space each color perception is represented as a point. The symbolic representations of color perceptions in this space form the color solid. The earliest proposals for color solids had simple geometrical forms: triangular double pyramid, sphere, cone, and so forth. There is, of course, no a priori reason why a systematic arrangement of color perceptions should fit into a simple geometrical solid. What controls the form of the solid is the definition of the axes of the space and their divisions.

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There is ample evidence that the colors we experience in various conditions from a given spectral stimulus can vary widely. According to one view they are determined by empirical rules derived on an evolutionary basis for our species and for each individual. There is strong evidence that the color attributed to an object depends on the nature and complexity of the surround in which the object is seen. In scientific experiments the complexity of surrounds usually is minimized (elementaristic approach).[1] Color experiences from given stimuli under elementaristic conditions depend on the exact conditions and change to a smaller or larger extent as quality and complexity of the surround and lighting change. Only under closely controlled conditions can a color space for the average color normal human observer be represented by spectral stimuli. In these relativized circumstances terms such as *color stimulus* and *object color* have applicability restricted to the experimental conditions and cannot claim the level of universality that has generally been assumed from the eighteenth to the twentieth century. Critics of the idea of color space have pointed to its lack of solid foundation. While this is ultimately true in the end such criticism appears simply to address the fact that at this point in time we do not have an understanding of consciousness. Color perceptions are the result of brain activity; they are subjective and private. As for all other sensory feelings and beliefs we do not know how in a given situation a given light stimulus can result in our seeing an object, and this object to have the appearance of red. It is not clear that humans will ever gain an understanding of this process. Color scientists have over the years built conjectural models based on what must, in the absence of true knowledge, be called coincidental relationships between stimuli as viewed in controlled circumstances and visual perceptions. In a perfect world this is not an acceptable process. Given the lack of fundamental understanding of consciousness it is an empirical approach having produced many reasonably well established, coincidental or otherwise, relationships.

Within the framework of an evolutionary development model, some key questions concern what forces in our early history shaped the development of visual sense and what strategies were implemented during its evolution by neurochemistry to deal successfully with the pressures of these forces. The simplistic color perceptions and attributes on which color scaling is based are doubted by some psychologists as having anything to do with the fundamental perceptual processes embedded in our visual system as a result of interactions with the environment. We appear to be only at the beginning of a process to find answers. Questions such as why color space is (at least) three-dimensional and why there are four psychologically fundamental hues and not more or less have started to be asked only recently.

The issue of a systematic arrangement of color perceptions under simplified viewing conditions is a relatively abstract matter, removed from such considerations. It is probably not surprising that it took shape in the age of Enlightenment with its belief in a universal rational order. In the twentieth century, aside from fundamental considerations of trying to understand our place in the world, the quest was shaped by technical and economic issues of color control of manufactured colored goods.

A color space belongs in the domain of psychology. The description of stimuli that under standard conditions result in perception of colors in that space is an aspect of physics. Together they form the uneasy domain of psychophysics that attempts to connect stimuli with perceptions (see Chapter 3). The stimuli are messages to us from the outside world. An alternative view is that we actively search for them when viewing the world. They enter through the pupils of the eyes and are absorbed by the retinal layer. There they trigger a complex chain of events that result in our perceptions. These events belong into the domain of neuroscience and are part of the conundrum of consciousness.

The number of different color experiences we can have is unknown, but large. Given a particular starting point in color space the finest perceptual division of color space is represented by visual threshold increments deviating from that point in all directions. A color space of given definition can only be expressed in terms of differences within the related color solid against a chosen surround because it is only applicable to those conditions. The smallest difference in a color solid as related to a given starting point, therefore, consists of a pair of different color stimuli displayed against a particular (usually neutral) surround and seen as having a just perceptible difference.

Generally, a color space and the related color solid may be defined as an economic systematic description of subjective color experiences, and as such it is not subject to engineering precision. It is indicative of our visual strategies vis-à-vis the world.

Personal Color Spaces and Color Solids

Each person with normal color vision has individual, personal (relativized) color spaces and related color solids (depending on the conditions under which they were established). Such individual solids vary within limits, based on the detailed implementation in an individual of his/her color vision apparatus. (Relativized personal spaces generally are at least in ordinal if not in interval order compared to that of the average observer.)[2] What it means is that if the reader and the writer sense the spectral power distribution representing a particular object color field in a particular surround and illumination, the resulting experience is likely to be somewhat different. Such a statement assumes that both observers are "color normal" and that color normal individuals have in essence the same fundamental color experiences. It does not consider the possibility, raised by some philosophers, of what is loosely called "spectrum inversion." It cannot be excluded with certainty that, for example, the reader actually experiences as green what the writer experiences as red, regardless of how it is named.

How different the experience resulting from a given spectral power distribution might be in terms of hue can be judged from individual determination of unique hues and, to less extent, from color perceptions judged to be intermediate between unique hues. Unique hues are those four primary hues that do not contain perceptual components of other hues. A unique red hue is neither yellowish nor bluish: it is just red. Color stimuli resulting in unique hue perception vary among color normal observers.[3] This variation depends on the hue in question. It ranges approximately from 5% to 12% of the total hue variation in a hue circle experienced under standard viewing conditions (i.e., approximately two to five Munsell 40-hue steps; see Chapters 2 and 7 for information on the Munsell system). Because of the absence of unambiguous criteria, it is not possible to meaningfully assess the stimulus variability for other hues. It is quite evident that there is also variability in the experience of gray scale steps, in adaptation and constancy response and other visual mechanisms, resulting in considerable variability of individual experience when looking at a given scene of color stimuli. Persons with impaired color vision have implicit color spaces significantly different from those of color normal observers. Their nature cannot be conveyed with certainty. Theoretical considerations of the genetics of color vision indicate that as much as 50% of the female population have the potential for four rather than the normal three cone types even though none has so far been identified as having four cone types.[4] Richer color experiences than those had by standard trichromatic observers have recently been determined for females with the genetic potential for four cone types. In how many ways their color experiences are richer remains to be determined.

Adaptation and Conspicuousness of Differences

Color experiences, in the normal case, result from the impact of light energy on the retina in our eyes. They are known to depend on the absolute level of light energy. This level can differ by a ratio of 1 million to 1 (on a retinal illumination basis). There are mechanical (pupil size) and neurochemical processes to manage such large variation, known under the general term of adaptation.[5] The complete process adjusts the range of incoming light to an output capability with a range of approximately 100:1. The adaptation process works to map the energy pattern in a given viewing situation to the total output range so that contrasts between different areas are seen roughly as the same under a wide range of illumination. At very low intensities of light we see no hued colors and neither do we at very high intensities. Probably because of the importance of very low light levels (night) in the lives of some early ancestors, we have a separate set of receptors for that situation, the rods. Rod signals pass through the same postreceptoral cells into the brain as cone signals. If they have any effect on daylight color vision, it is very small. The response pattern as a function of light intensity of our daylight-level sensors, the cones, is S-shaped, but the response has a considerable range that is approximately linear in the center region. There are issues at low and high levels of response of cones that cannot be of concern in this discussion. This text is largely limited to color spaces and solids represented by reflecting materials at mid levels of illumination, say 500 to 1500 lux.[6]

Earlier, mention was made of the lack of constancy of color perceptions resulting from most stimuli as a function of surround or illumination changes. Chromatic adaptation is a seemingly opposite process. Its purpose appears to be to provide a considerable level of color constancy for reflecting objects with certain spectral signatures. Among terrestrial nonhuman mammals trichromatic color vision is limited largely to fruit eaters and pollinators. For them it is important to recognize their objects of interest in all natural lighting conditions. Color is an important part of the stored memory of the appearance of objects and helps to recognize them rapidly when encountered again. Without chromatic adaptation, colored objects in the natural world might change their appearance significantly over time as a result of changes in ambient illumination. With independent adaptation capabilities for each cone type, likely together with additional processes, our ancestors could recognize the colors of most natural objects as essentially the same regardless of the quality of illumination and surround. This is less true today than it was at a time when there were only natural objects and all light was sunlight, direct, scattered, or reflected. Today we have a large number of artificial colorants and various artificial light sources that have complicated the issue considerably, resulting in smaller or larger changes in the appearance of objects as a function of surround and illuminant. This text does not consider most issues of chromatic adaptation but considers color spaces and related solids only in terms of colored objects as viewed against achromatic backgrounds of varying levels of lightness under a standard light source.

The visual system has developed in a way that favors the conspicuousness of small differences in reflectance signatures of objects. Its cause may have been an escalating battle between camouflage and detection, a matter of life and death. Highest discrimination of small reflectance differences between objects is provided in a surround with reflectance intermediate to those of the objects compared. This results in improved detection of highly camouflaged predators or prey in natural surroundings. The principle also applies when the number of objects with different color increases and/or the differences between them become larger. Best discrimination is provided in this case by an average (i.e., mid-level) achromatic surround. The best surround to view a complete color atlas, by this reasoning, is a mid-level gray.

Mathematical Color Appearance Models

Our complete color experience is much wider than what was just discussed. We view natural scenes, color television, computer monitors, projected slides, projected digital images, the output of many coloration devices under many different light sources, metameric objects under different light sources, and so on. It has become important to be able to predict for an average observer the appearance of colored objects in many different conditions. This is the province of color appearance modeling, evolving rapidly in the last ten years and having developed several mathematical models that are still comparatively simple and correspondingly only modestly accurate. This is expressed to some extent by the fact that several different modeling approaches can result in about the same level of prediction accuracy. An excellent survey of color appearance modeling has recently been provided by MD. Fairchild (1998). As indicated, the present text is concerned with color appearance under limited conditions only.

1.2 DIVISIONS OF COLOR SPACES AND SOLIDS

Color spaces and solids are always expressed in terms of differences of some kind between color perceptions. As will be shown, there are various kinds of differences that have been proposed for color spaces. A kind of color space of particular interest is one in which distances in the solid in all directions are proportional to the magnitude of perceived differences between the related color experiences. Such a space can be built from (or divided into) threshold differences or larger differences. Differences imply scales, and there are several different kinds of scales possible. The primary scales are psychological or perceptual. Such scales are built on the basis of perceptual attributes. A logical expectation is that the perceptual attributes form the axes of the space. For simple observation situations (uniform achromatic surround and defined light source) three attributes are sufficient to define the perceived color of an object. If its dependence on surround and illumination is to be considered quantitatively, additional attributes are required (see Chapter 4). We will find, however, that all possible hue perceptions are best ordered in a circle and that the hue attribute, therefore, is a function of two dimensions of the space.

A psychological color solid and the space into which it fits can be built from a very large number of color samples. It requires picking the appropriate samples to represent the chosen type and size of difference. Once the task is complete, the selected samples represent the solid and the space. This is not a generally satisfactory solution because producing many copies of the solid requires large sheets of uniformly colored materials. Our inability to define color experiences from an object verbally or by some other subjective means with a high degree of accuracy and precision makes it desirable to use objective means of defining the color samples. Weight of colorants in a mixture has been used in the earliest attempts at illustrating color scales, (e.g., F. Glisson, Chapter 2). With the development of photometry in the eighteenth century and colorimetry in the late nineteenth century, physical and psychophysical means of specifying color stimuli became available. It quickly was learned that in a given set of conditions the relationship between measured stimuli and perceptions is not linear, and the next task was to develop models of the relationship between physical properties of stimuli and resulting perceived color. This is the domain of psychophysics. This branch of psychology developed as a quantitative science in the mid-nineteenth century. Both psychological color scaling and psychophysical color modeling continue to be incomplete activities.

The simplest kind of scale applicable to the universe of color experiences is the ordinal scale. It describes the order of entities that form the scale. An example of an ordinal scale is a random series of gray papers, arranged in terms of perceived lightness. The differences between steps are also random in perceived size. A psychophysical example is a gray scale in which the steps differ by 10% in luminous reflectance. It is not a perceptually uniform scale. There are an infinite number of possible color spaces based on such ordinal scales and encompassing three dimensions. Examples are cone sensitivity spaces, the CIE tristimulus X, Y, Z and x, y, Y spaces, the Luther-Nyberg space, or spaces based on color matching functions different from the standard CIE functions (see Chapter 5).[7]

Of historically greater interest have been interval scales of object color perceptions and the psychological space derived from them. Interval scales, as defined in Chapter 3, are scales where the meaning of the size of the interval is the same regardless of where on the scale the interval is located. Interval scales are known as psychometric scales. Typical interval scales are lightness, hue, and chroma scales—or scales of complex color differences—based on perceived total color difference. An example of approximation of a uniform psychological color solid based on interval scales is the Munsell "tree" of colors, a three-dimensional logical arrangement of color chips forming interval scales in certain directions. However, equal geometrical distances in the Munsell system do not correspond in all directions to equal perceived differences.

A different approach has been pursued in the development of the Optical Society of America Uniform Color Scales (OSA-UCS; see Chapters 2 and 7). Here no attributes have been scaled, but uniformity in size of complex chromatic color differences in a triangular grid was determined at approximately constant lightness, as was the magnitude of combined chromatic and lightness differences. A fitted formula was then used to tile the corresponding space so that colors in twelve directions were defined from a central color approximately perceptually equally distant. The result is a space with a square grid pattern for colors of equal perceptual lightness rather than the radial grid of the Munsell system. It turns out that also here a uniform solid has not been formed. Uniformity of color space and the related color solid has been a goal since the earliest attempts at building color appearance spaces. Its importance increased with the capabilities of accurately and inexpensively measuring reflectance properties and the related opportunities for objective color quality control in the mid-twentieth century. This text, while discussing many different kinds of color spaces, pays particular attention to the issues of uniform color spaces.

Ratio scales represent the next higher level of scale complexity. Here not only are the intervals quantitatively the same, but ratios are also fully valid. Historically ratio scales for colors have been controversial. Many observers do not agree that it is possible to make a judgment that a given color is twice as

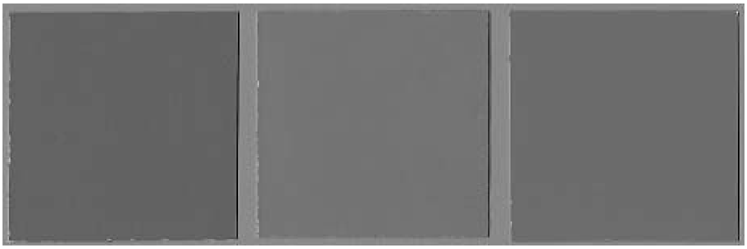

Fig. 1-1 *Images of chips of the OSA-UCS system.* Left*: Color 000;* center*: color 00-4;* right*: color 00-8. (See color plate.)*

red or twice as black as another color. The difficulties involved can be visualized by comparing, say, OSA-UCS chips 000, 00-4, and 00-8 (Fig. 1-1). If the OSA-UCS greenness–redness scale could be considered a ratio scale, the statement describing the *g*-8 chip as twice as red as the *g*-4 chip should apply. But many observers, including the author, are not prepared to agree with such a statement.

Once a color solid has been perceptually developed for a specific set of conditions, the selected stimuli/samples can be defined physically by spectral power or reflectance measurements. The next step is to build a mathematical model connecting the physical with the psychological data in a manner resulting in perfect or near perfect agreement between the two sets. As will be seen in Chapter 6, much effort has been devoted to finding the mathematical definition of a uniform psychophysical color space. There are a number of problems and difficulties with such efforts. They begin with the difficulties or impossibility of creating an euclidean geometrical model of a uniform psychological color space. In addition the physical definition of samples and spectral power distributions, representative of the observed objects, is not without problems.

1.3 UNIFORM AND REGULAR COLOR SPACES

The Oxford English Dictionary's definition of *uniform* in regard to motion or dimensions is "free from fluctuation or variation in respect to quantity or amount." In regard to color space the term *uniformity* has historically had two different uses: (1) Absence of variation in terms of a single concept: perceived color difference between two grades in any direction in space. (2) Absence of variation in terms of attributes that are perceptually significant but do not result in perceptually uniform differences, such as blackness or relative content of unique hues. In the former case the space is uniform in terms of the magnitude of perceived differences but not uniform in terms of blackness or redness. In the latter case the space is uniform only in terms of the chosen attributes but not in terms of perceived differences. It is useful to reserve the term "uniform" for the former situation and use another term, perhaps "regular color space" (Hering space for the Hering-inspired version), for the latter.

As will be seen in Chapter 2, the concept of uniform color solid has a long history. In the seventeenth century Glisson attempted to develop a gray (lightness) scale and three tonal color scales with visually equidistant steps with which to specify the colors of objects. T. Mayer, J. H. Lambert, and P. O. Runge in the eighteenth and nineteenth centuries were already thinking in terms of visually uniform steps between the scale points of their color solids. Mayer appears to have been the first to propose a three-dimensional color solid. H. von Helmholtz was the first to attempt to find the relationship between physical measurements of the stimulus and a perceptually uniform space.

W. Ostwald, apparently through a misunderstanding of Helmholtz's concept of brightness, decided to use Hering's blackness and whiteness as the two attributes that together with chromatic color, form the color perception. He used Hering's equilateral triangular template to arrange all color perceptions of a given hue. In this template, in the tradition of Runge, W. Wundt, and Hering, lightness is not an attribute and chromaticness of all full colors (pure pigment or maximal color) is considered perceptually equal. In regard to chromaticness the result is that the perceptual magnitude of chromaticness steps depends on hue. In addition Ostwald decided that the Weber-Fechner law was applicable regardless of size of color difference, and he scaled the grades in the hue triangle accordingly.

Munsell introduced a radical philosophical departure from the German school by using the three attributes lightness, hue, and chroma. His chromaticness measure, the chroma unit, is in principle of equal perceptual magnitude regardless of hue, and it is defined in terms of (imperfectly defined) constant perceived lightness. While he originally constrained his color solid into the form of a sphere, Munsell soon learned from experiments that when building his system from the central gray midpoint perceptual uniformity was not compatible with the complete color solid having a spherical form. The result was the irregular shape of the Munsell "color tree." Munsell's successors continued to refine the scaling of the three attributes, the last accepted revision being the Munsell Renotations. In the Renotations perceptual data were "smoothed" to some degree in terms of psychophysical data.

A major reason for the development of early forms of color solids, as Chapter 2 will show, was to have a basis for discovering systematic rules of color harmony. This desire was behind the efforts of Runge, O. N. Rood, Munsell, Ostwald, and others (Schwarz, 1999). An American version of Ostwald's system was called *Color Harmony Manual*. Even though claims of having discovered universal rules of color harmony have been discredited, there has been a continuing discussion on the usefulness of the various systems for the purposes of art and design. Ostwald strove to make his system derivable from additive color mixture data, thus developing a system that attempted to combine perceptual psychology, psychophysics, and harmony.

The Swedish Natural Color System (NCS) is a modern interpretation of Hering's ideas. It was derived on a purely psychological basis using presumed innate concepts of Hering type *Vollfarben* (full colors) with unique hues, blackness, and whiteness. Psychophysical measures were used only to specify color grade samples exemplifying the system under a specific set of conditions. The attributes of this Hering or Ostwald type of system are hue, expressed by quadrant in terms of one or two unique hues, blackness, and whiteness (or hue, blackness, and chromaticness). The double-cone geometrical form of systems such as Ostwald's and NCS's appears to imply conventional definitions of the geometrical dimensions. But by placing all full colors on the periphery of the central disk of the double cone and a perceptually uniform gray scale on the central vertical axis, the meaning of the vertical dimension in these systems is not defined. As a result the steps are not uniform in the sense defined above but regular. The practical value of such systems must be found in principles other than uniformity of difference.

The Munsell system on the other hand, as mentioned, is based on the psychological attributes hue, as expressed in terms of five primary hues, value (lightness), and chroma (saturation). Munsell's original intent was to represent a uniform color space. However, by concentrating on planes of constant hue, he neglected the changes in hue difference as a function of chroma and lightness between adjacent constant hue planes. A uniform version of the Munsell system is impossible to fit into a euclidean system as will be shown. By disregarding the issue of relative perceptual size of hue and chroma difference steps, the Munsell system is simply accepted as fitting a polar system. In this system the polar angle, radial distance, and distance from the origin in the third dimension have defined meanings: hue, chroma, and lightness; but the units are of different perceptual size (in the case of hue also as a function of chroma). This was experimentally determined in the 1930s. According to D. Nickerson's index of fading formula (1936), one unit of value difference is equal to two units of chroma difference and, at chroma 5, to three 100-step units of hue difference.

In 1943, based on the then newly available calculations by D. L. MacAdam of the object color limits, Nickerson and S. Newhall calculated two three-dimensional models of the psychological Munsells solid (Fig. 1-2).[8] They are approximations of a uniform psychological solid under two different observational conditions without among other things, considering the matter of the relationship between unit hue and chroma differences. They were described as fulfilling the following requirements: "Dimensional scales . . . calibrated in perceptually uniform steps; units of the several scales . . . equated; the surface of the solid . . . represents all colors of maximum saturation; the difference and volume . . . representative of all colors which are perceptibly different; conditions of stimulation or viewing . . . described; and, finally, the scales . . . stan-

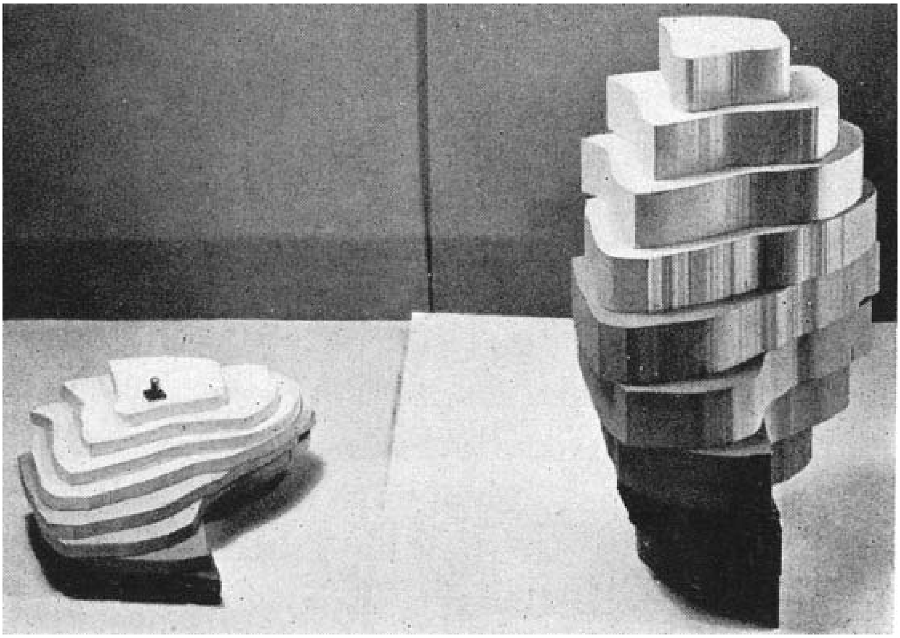

Fig. 1-2 *Models of Nickerson and Newhall's psychological color solid, based on the Munsell system. The two figures represent color solids based on large and small perceived differences.* Left*: At the level of Munsell Book of Color differences.* Right*: At the just noticeable difference level.*

dardized in terms of a generally recognized psychophysical system." B. Bellamy and Newhall (1942) investigated differences at the threshold level and reported them in terms of the Munsell attributes. Their results, surprisingly, indicated one unit of value to be equal to eight units of chroma and, at chroma 6, to 22 units of hue difference, thus indicating a vast change in relative importance as the differences became small. The shorter version of the model represents space proportions when viewing differences of the magnitude of chroma or value steps. The taller version has the vertical dimension increased by a factor 4 to indicate the scales when judging samples differing at the just noticeable difference level. Figure 1-3 illustrates cross sections at the five basic Munsell hues through the solid, with the inner contours representing areas covered by actual Munsell color samples. These figures, seemingly, are the first attempt at realistic (but euclidean) geometrical representation of the universe of (relativized) human object color perceptions in approximations of perceptually uniform spaces.

D. B. Judd and I. H. Godlove, investigating the relationship between hue and chroma differences used by Nickerson in her formula, discovered that it is not possible to map the results onto a flat plane. According to this formula the radial distance covered by the hue differences at any given chroma level is approximately twice that of a circle, meaning the total hue angle is approx-

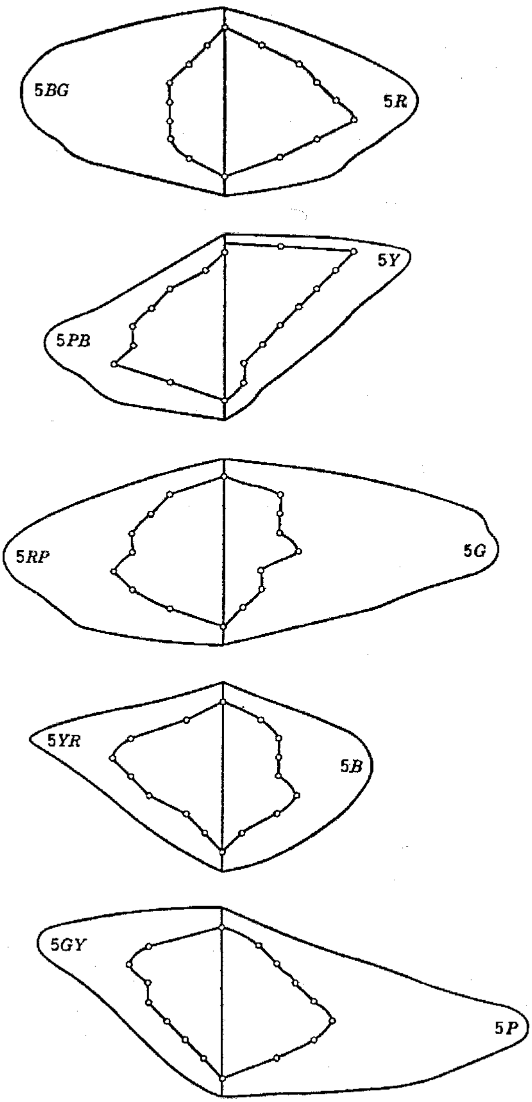

Fig. 1-3 Vertical sections in five different hue planes through the color solid that represents the Munsell system extrapolated to the optimal color limits. The inner borders delineate the space filled by samples of the system.

imately 720 degrees. Judd attributed this result to experimental error, and Godlove (1951) wrote a formula that reduces the magnitude of hue differences so that the Munsell equal lightness psychological data map onto a plane. Toward the end of his life Judd reconsidered his view, and there is now significant additional evidence indicating that the Nickerson formula is approximately correct. Full clarification requires further psychological scaling.

As will be shown in Chapter 5, three sets of extensive chroma spacing data, determined at different times, are not in good agreement, and the implicit chroma scale of the Optical Society of America Unform Color Scales (OSA-UCS) system does not agree with any of the three. The psychophysical chroma scale implicit in the widely used CIELAB color space formula is not in good agreement with any of the above.[9] It is fair to say that we do not have a reliable chroma scale. Similarly there are no data of hue scaling around a hue circle at constant chroma and lightness that can be considered reliable and replicated. Different formulas have been proposed for the weighting of CIELAB hue differences in recent years and formulas optimal for one set of data usually perform significantly less well for another. We also do not have a psychophysical model with known scientific validity for uniform hue spacing.

Similary, elementaristic lightness scales are found to depend on surround conditions and there is poor agreement between perceived lightness of chromatic color patches and measured luminous reflectance.

In 1969 Judd wrote an essay on the subject of ideal color space. His initial definition of an ideal color space was: "Ideal color space is a tridimensional array of points, each representing a color, so located that the length of the straight line between any two points is proportional to the perceived size of the difference between the colors represented by the points." A number of experimental facts, however, led him to conclude that ideal color space by this definition is impossible. He listed these facts as:

1. Evidence for curvature of color space from the MacAdam ellipse data.
2. Superimportance of hue as indicated in the Nickerson formula.
3. Diminishing returns in color difference perception.
4. Influence of surround color.

Accounting for these problems, he offered the following redefinition of the concept of ideal color space: "Ideal color space redefined is a tridimensional array of points, each representing a color, such that all pairs of points separated by any fixed distance correspond to pairs of color perceived to differ by the same amount provided that the appraisal of the perceived size be carried out with optimal surround colors chosen in accord with Schönfelder's law[10] that the surround be the average of the two colors being compared." It is evident that uniformity of difference is the central principle behind both definitions.

In the 1970s R. M. Evans and B. Swenholt extensively investigated psychological color space and concluded that to accommodate achromatic surrounds of varying brightness or lightness, five attributes require consideration and that, therefore, a euclidean map of color space is a simplification applying to one surround only.

In its work the Uniform Color Scales Committee of the Optical Society of America was well aware of the problem of hue superimportance. As mentioned, it abandoned separate scaling of color attributes in favor of scaling complex chromatic and chromatic/lightness differences. Based on a euclidean formula fitted to the visual data it proceeded to tile, according to a proposal by I. Balinkin and C. E. Foss, the implicit space uniformly in twelve directions from a central midgray. The resulting space combines an irregular shape with a crystalline interior structure (see Chapters 2 and 7). As a result hue superimportance was purposely neglected, and the space is not visually uniform in all directions.

In 1981 G. Wyszecki defined uniform color space as follows: "A uniform color space is a geometrical representation of color perceptions in a three-dimensional space in which the distance between any two points can be taken as a measure of the magnitude of the difference between the two color perceptions that are represented by the two points." It appears that the concerns of Judd and Evans about surround and other issues had been shelved.

Today it is evident that there is no single uniform color space and no simple geometrical model of perceptually uniform space that is more than an approximation. In addition different surrounds and different sizes of intervals on which the space may be based result in different geometrical forms of the space and different selections of color chips within the space to represent visually uniform steps.

A goal of sensory psychophysics is to determine the relationship between physical stimuli and psychological response. In the case of colors seen as those of objects, this requires discovering the relationship between the spectral return of light, reflected from objects, and the psychological response of the observer. Given the variability in response of observers, this is usually done for an average color normal observer. Quantitative description of human color vision used in models consists either of functions representing the average sensitivity of the three cones or color-matching functions that predict if two different spectral power distributions are seen as matching by the standard observer for whom the color-matching functions apply. The two sets of functions are considered linearly related. Many color space formulas contain additional suppositions concerning the color vision apparatus, in particular, an opponent color theory. In the last fifty years the CIE has proposed several color space and color difference formulas (see chapter 6). Other formulas have been proposed by other organizations and by individuals. The best of these formulas explain 65% to 80% of the average variation in perceived differences of the visual data on which they are based. Significant further improvement is unlikely without reliable visual hue, chroma and lightness scales and understanding of how size of difference affects the implicit color space. Most formulas for object colors developed in the last twenty years are based on the CIELAB formula, recommended by the CIE in 1976 as a compromise formula for unification of practice. As will be demonstrated, this formula is quite clearly not a good basis for color difference calculation. Improvements in the fit of the formula to visual data since then have been based on statistically determined mathematical fixes. This text presents new, more detailed understanding of the relationship between stimuli conventionally taken as color stimuli and color perceptions associated with objects, with the potential to provide a basis for improved formulas.

1.4 COLOR SPACE, SENSATION, PERCEPTION, AND AWARENESS

The terms sensation and perception[11] have traditionally referred to immediate and direct qualitative experiences such as "hard," "cold," and "green" in the former case and complete psychological processes involving implied meaning, past experience, memory, and strategy in the latter. This view of the visual system descended from ideas by Descartes who described "three grades of sensory response. The first is limited to the immediate stimulation of the bodily organs by external objects. . . . The second grade comprises all the immediate effects produced in the mind . . . such effects include pain, pleasure, hunger, colors, sound, taste . . . The third grade includes all the judgments about things outside us . . ." In recent years it has become quite clear that such a distinction has little connection with reality. In order to form a perception, we now understand that it is necessary to pay attention to a stimulus and thereby become aware of it. Experiments have shown that the visual system continuously senses a large number of stimuli arriving at the retina without the owner of the system becoming aware of them or having recollection of them. It seems useful to use the term perception for what results from a given local stimulus after it has received attention and it has undergone the complete processing resulting in awareness. A step taken in connection with color scaling is to form judgments based on perception.

Color perception and the concept of color space and solid are important components of the not well-defined concepts of awareness and consciousness. Consciousness remains a mystery but is now being investigated intensively by neuroscientists, physicists, psychologists, and philosophers. There is a growing corpus of neurological information regarding the functioning of the visual system. At the same time we have more than 200 years of investigations of psychological color space. But there continues to be a black box into which biologically produced correlates of physical stimuli disappear and out of which color experiences appear. This situation prevents the development of a convincing model of human color vision and has resulted in the use of growing numbers of mathematical variables to fit cone sensitivity or color matching function data to perceptual data. It is generally accepted that all information derived from radiant energy required for us to experience form, color, and motion passes through the filter of the three cones. If this is an ultimate truth, then we must look to neurophysiology of the retina and pathways in the brain to provide more information on the processes as a basis for better models. What we have seems promising and, at the same time, is unsatisfactory. This assessment is based on the unproved assumption that color experiences are directly derivable from the neurophysiological functioning of certain cells in our visual system, an idea that has, more or less directly, informed efforts toward a uniform psychophysical color space for the last century. However, there is a significant group of scientists and philosophers disagreeing with it, and the issue must be considered open.

In the 1940s color space and difference research in the United States profited from the historical curiosity of a search for work not directly connected with war effort (Nickerson, 1977). Early results generated their own momentum, and work continued in the 1950s and 1960s through the efforts of a few dedicated individuals in a committee of the Optical Society of America. In the 1960s, with growing capabilities for industrial reflectance measurement and color calculation, color technologists in colorant producing and using industries around the world became increasingly interested in the possibilities of objective color quality control and provided impetus for new work. These efforts have resulted in the level of success mentioned above, based on reliable reflectance measurement techniques and the developments in color science to be discussed below. Since the mid-1980s the major activity has shifted to academic institutions. More recently lack of funding and new and seemingly more exciting fields of research in color have slowed the pace of color space and difference research work appreciably. It seems that we must rely again on a few dedicated individuals, interested in pushing the frontier in this field for the sake of advancing toward the distant goal of understanding qualitatively and quantitatively, in very limited situations, the relationship between visual stimuli and the resulting color and color difference perceptions.

1.5 PLAN OF THE BOOK

Chapter 2 begins with an attempt at a general definition of the meaning of color space and color solid. It is followed by a historical survey of ideas about color order beginning with ancient Greek philosophers. Given the paucity of surviving documents, our knowledge in this area is likely incomplete. The survey continues through the Middle Ages and the Renaissance into the Age of Enlightenment. Three-dimensional color solids began making an appearance in the eighteenth century. The contributions of psychophysics, starting in the midnineteenth century, to the matter at hand are discussed as are development in understanding of human color vision and of the colorimetric system. Brief discussions of the systems of Hering, Munsell, Ostwald, the German DIN 6164, the Optical Society of America Uniform Color Scales, the Swedish Natural Color System, and others, including systems used in video display, are presented to bring the reader to the present in this multimillennia pursuit of ordering our color perceptions.

Chapter 3 offers a survey of psychophysics as relevant to the color-ordering enterprise. Many of the problems and complexities of psychophysics are touched on including theories of categorization, relationship of differences and magnitudes, uni- and multidimensional scaling methods, and the relationship between psychological and psychophysical color spaces.

The theme of Chapter 4 is perceptual color attributes and how they are scaled. It concentrates on perceptual scaling only. Views regarding color attributes have a history of their own. On the one hand is the physics inspired set of hue, saturation, and brightness or lightness introduced by Newton and Helmholtz, on the other, Herings "natural" system of hue, whiteness, and blackness. A large portion of the chapter is given to data of perceptual scaling of color attributes, including location of unique hues and distances between them. The paucity of extensive sets of global scaling data and the lack (for unknown reasons) of close agreement among those few that exist is commented on.

In Chapter 5 the perceptual scales of Chapter 4 are related to physical definitions of color stimuli such as reflectance or spectral power distribution data. This requires brief discussions of photometry and colorimetry as well as psychophysical spaces such as cone response, tristimulus, and opponent color spaces. The relationship between color matching and color appearance is touched on, as is placement of unique hues in psychophysical spaces and curvature of lines connecting blues of constant hue in CIE-based opponent color diagrams. The chapter closes with a discussion of the number of colors we can distinguish.

Chapter 6 contains all major historical steps in the effort of finding psychophysical formulas attempting to describe uniform color space, beginning with Helmholtz' line element and ending in the present. The chapter ends with a brief comparison of color and spectral spaces as well as a comparison of performance of various formulas against the Munsell system and the RIT-DuPont data exemplifying global color and small color difference data.

Chapter 7 contains more extensive descriptions and comparative analysis of three major color order systems: the Munsell and OSA-UCS system and the NCS system. The former two are attempts at a uniform color space while the latter is an implementation of Herings "natural color system." It is demonstrated that neither constant value planes of the Munsell system nor constant lightness planes of the (experimental results of the) OSA-UCS can be isomorphically plotted on a euclidean plane.

In Chapter 8 color differences have been scaled at many different levels, from color-matching error and threshold differences to small and large suprathreshold differences. In this chapter important data from each category are compared in the cone sensitivity and the tristimulus spaces for agreement and discrepancies. One of the comparisons involves the magnitude of Weber fractions, another the direction of the unit chromatic contours in the spaces. Many issues are found to be unresolved because of lack of reliable, replicated data.

Chapter 9 draws conclusions from the facts of the previous chapters by attempting to answer 13 questions related to color scaling and uniform and regular color spaces. An approximation of a uniform color space at the level of small color differences is shown as a counterpoint to the figure of the Nickerson-Newhall uniform color space in this chapter (Fig. 1-2). A research program is proposed in some detail to establish reliable, replicated data that can be used to determine the properties of uniform color spaces for a given viewing and surround situation at different levels of size of difference.

Chapter 2

Historical Development of Color Order Systems

2.1 COLOR AND COLOR ORDER SYSTEMS

One of the definitions in the Oxford English Dictionary, not without its difficulties, of the word *color* is "The quality or attribute in virtue of which objects present different appearances to the eye, when considered with regard only to the kind of light reflected from their surfaces." The definition is symptomatic of the age-old problems of clearly describing this psychological quality. But we all (those of us with normal color vision) know colors when we encounter them. In 1912 the philosopher B. Russell expressed himself on the subject as follows ". . . truths about the colour do not make me know the colour itself better than I did before . . . I know the color perfectly and completely when I see it. . . ."

The conventional view of colors is that they are the result of standard stimuli associated with them. I. Newton's seven primary hues (R O Y G B I V) are the result of spectral stimuli of particular wavelengths (as seen in a dark surround). In color technology certain pigment or dye combinations result in certain color perceptions when viewed under a standard light in a light booth. That the appearance may change significantly if we change the light source or the surround is described in terms of color inconstancy. The idea that there is a standard stimulus for a given color perception has often been questioned (independent of the issue of metamerism). In the 1960s the inventor of the Polaroid photographic process, E. Land, sharply delineated the limitations of such views. In natural scenes the colors assigned to a given spectral power distribution entering the eye depend on the spectral structure of the total visual field according to rules that are not yet understood. It does not

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seem appropriate to talk about color illusions when the implied stimulus/perception association is violated. Colors perceived in any given situation are the *real* colors. Our mind constructs them on the basis of its interpretation of the total visual field as well as of items in this field that appear to belong together. This view causes severe problems for the idea of a color order system in which the specimens are defined by their reflectances. On the other hand, there appears to be a finite limit in number of color perceptions that the color normal observer can experience when seen as colors of objects. There is a legitimate question as to how one can systematically demonstrate these perceptions. The answer is that we have to select a standard set of conditions in which to view the objects in order to be able to more closely relate stimuli and perceptions. This is the approach taken implicitly or explicitly by all developers of physical color order systems. After the materials and the observation situation have been fixed the variability of color normal observers comes into focus. As a result more recent efforts in development of color order systems always involved average data from many observers. From this perspective, extensive relativization of the viewing circumstances to tightly controlled conditions, relating color perceptions to particular spectral stimuli has a degree of validity. Any claim to general validity is erroneous, however.

There are two situations in which it is meaningful to associate a color perception with a specific stimulus:

1. A specific set of conditions fixes the association, but it is understood that the ensuing color experience can be the result of many other sets of stimuli and surround conditions. All the associations are taken to represent the universe of possible object color perceptions.
2. The surround conditions represent a reasonable and practical limitation for the purposes of controlling the industrial production of materials with reflectance/transmittance properties making the objects seen as colored.

The latter situation deals with two issues:

1. The initial colorant formulation for objects needs to meet criteria of closeness to the reference sample and, perhaps, degree of color constancy (lack of perceived change of color of formulation under different light sources).
2. The degree of perceived difference between the approved formulation sample and subsequent repeated manufacture of the material should be minimal.

In both cases it appears justified to use simplified surround conditions for visual tests. Objective test methods must reflect these conditions.

Color order systems that, as used here, include color appearance systems are logical arrangements of color chips and/or geometrical and numerical arrangements of symbols for such chips. One system (NCS) is based on mental references defining the color appearance. The chips themselves are symbols for color perceptions that can be created when the average color normal person views them under specified conditions of lighting and surround. Color appearance systems, specifically, are atlases of collections of color chips or prints.[1]

The remainder of this chapter presents the historical development of color order systems in the Western world.

2.2 FROM ANCIENT GREECE TO THE MIDDLE AGES

Pre-Platonists

Among the oldest extant uses of color words are those by the poet, theologian, and natural philosopher Xenophanes, active in the sixth century BC. He explained all things to have come from water and earth and commented on the rainbow: "And she whom they call Iris, she too is actually a cloud, purple and flame-red and yellow to behold."[2]

Pythagoras (ca. 582–507 BC) philosopher and founder of a religious brotherhood is credited by his followers as having discovered that the relationship between musical notes could be expressed with numbers, as could any other relationship. Of considerable importance to the Pythagoreans was the *tetractys*, their name for the sum of the first four numbers, regarded as the source of all things. According to Philolaus, a disciple of Pythagoras, he equated colors with the number 5. Plutarch remarks on the views of the Pythagoreans on color: "[They] called the surface of a solid *chroma*, that means color. Additionally, they named the species of color, white, black, red and yellow. They looked for the cause of the differences in color in various mixtures of the elements, the manifold colors of animals, however, in their nutrients as well as the climatic regions."[3]

Empedocles (ca. 492–432 BC), a Sicilian philosopher and politician, is credited with the view that everything in existence is composed of four indestructible elements: earth, fire, water, and air. On color Empedocles is quoted as follows: (after Simplicius, 28) "As when painters decorate temple-offerings with colors—men who, following their intelligence, are well-skilled in their craft—these, when they take many-colored pigments in their hands, and have mixed them in harmony, taking more of some, less of another, create from them forms like all things. . . ." (after Aetios, 132) "Empedokles explained color as that fitting into the pores of the visual organ. The multiplicity of colors, so he said, derive from the fixed mixtures of the elements. That there are four colors, exactly as many as elements: white [*leukhyn*], black [*melan*], red [*erydron*], yellow-green [*ochron*]."[4]

Democritus (ca. 460–? BC) expanded the atomistic theory of his predecessor Leucippus. Among many other works he is reported to have written a text

On colors, not extant. Grammarians credited him as the inventor of then unusual words, among them one with the meaning "change of color." He is reported by Galen to be the author of "By convention there is color, by convention sweetness and bitterness, but in reality only atoms and void." On primary colors and color mixture Democritus is quoted by Theophrastus (97): "According to Democritus there are four primary colors. White [*leukhyn*], he explained, is the smooth, polished. . . . The black color [*melan*], on the other hand, consists of (forms) of an opposite nature: rough, nodulated and uneven. . . . Red [*erydron*] he said to consist of similar but larger (forms), as warmth. . . . The green color [*khloron*], on the other hand, consists of hardness and emptiness, as a mixture of both. . . . The primary colors he said to be based on these forms. Each of them being purer the less it is composed of mixed forms. The other colors he said to derive from mixtures of the (four primary colors)."[5]

Plato

Plato (427?–347 BC), philosopher and mathematician, friend and pupil of Socrates, founder of the Academy, was one of the preeminent figures in Greek philosophy. His influence remains active today. Plato made several statements about colors in his dialogues. Concerning primary colors, and their mixtures his key statement is found in *Timaeus* (67, 68):

> . . . we ought to term white that which dilates the visual ray, and the opposite of this black. . . . in (the eye) the fire, mingling with the ray of the moisture, produces a color like blood, to which we give the name red. A bright hue mingled with red and white gives the color auburn [*xandon*]. The law of proportion, however, according to which the several colors are formed, even if a man knew he would be foolish in telling, for he could not give any necessary reason, nor indeed any tolerable or probable explanation of them. Again, red when mingled with black and white, becomes purple, but it becomes umber [*orphninon*] when the colors are burned as well as mingled and the black more thoroughly mixed with them. Flame color [pyrron] is produced by a union of auburn and dun [*phaion*], dun by an admixture of black and white; and pale yellow [*ochron*] by an admixture of white and auburn. White and bright meeting and falling upon a full black, become dark blue [*kyanoyn*], and when the dark blue mingles with white a light blue [*glaykon*] color is formed as flame color with black makes leek-green [*prasion*]. There will be no difficulty in seeing how and by what mixtures the colors derived from these are made according to the rules of probability.

Plato's color mixture scheme is shown graphically in Fig. 2-1. Note that the classification into primary, and so on, colors is by the author.

Aristotle

Aristotle (384–322 BC) was a student of Plato at the Academy. After teaching Alexander the Great he founded a school in the Lyceum in Athens. Together with Plato, Aristotle had an enormous influence on philosophical thinking

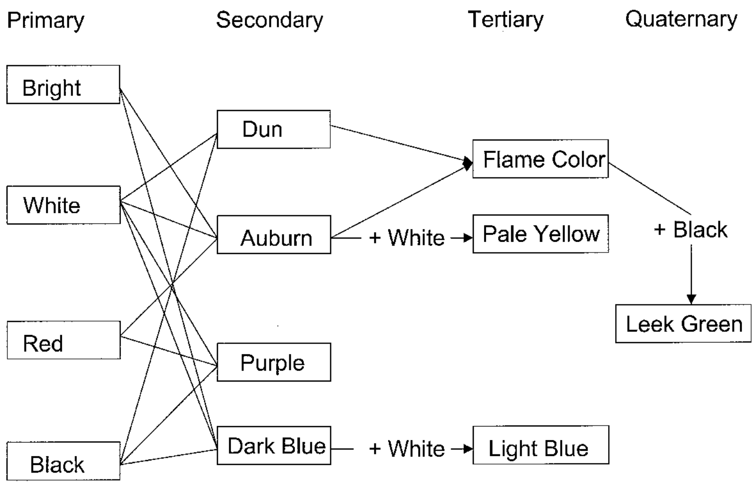


Fig. 2-1 *Plato's color mixture scheme. The four primary experiences are bright, white, red and black. Additions of these in various combinations form the secondary and later stage mixture colors.*

during the next 2000 years. In regard to basic colors and color scales Aristotle expressed himself on more than one occasion and not always in the same way.

In *Sense and Sensibilia* (442a20 ff): "Savors and colors contain respectively about the same number of species. For there are seven species of each, if, as is reasonable, we regard gray as a variety of black [*melanon*] (for the alternative is that yellow [*xandon*] should be classed with white [*leukhon*], as rich with sweet); while crimson [*phoinikoyn*], violet [*aloyrgon*], leek-green [*prasinon*] and deep blue [*kyanoyn*], come between white and black, and from these all others are derived by mixture."

Earlier he stated: (439b20 ff):

On Colors is a text attributed to Aristotle, but by some writers also to Theophrastus, Aristotle's successor at the Lyceum, and believed to represent Aristotle's final thinking on the subject. Here he stated (1–3):

Ptolemy

This Greco-Egyptian mathematician and astronomer (second century AD) is reported to have mentioned in the lost first book of his *Optics* a list of twelve simply seen colors. Eleven of these are listed in a biography of Ptolemy found in the library of the ninth-century Byzantine scholar and patriarch of Constantinople Photius.[6] They are, in no particular order: black, white, orange [*xandos*], dun, gray [*phaion*], yellow [*ochros*], red [*erythros*], blue [*kyanos*], purple [*halurgos*], shining or bright [*lampron*], and dark brown [*orphinon*].

Ancient Greek Thinking on Color Species and Color Order

The idea that there are four material elements and correspondingly four fundamental colors belongs to Empedocles. It proved to be influential for many centuries. The quote about painters attributed to him indicates that naturalistic painting and the mixture of pigments to achieve certain colors were well established in Greece at his time. Democritus who explained the world in atomistic terms is reported by Aristotle to have said that color does not have existence because it is generated by the turning of atoms. Democritus changed the name of the fourth, yellow/green elemental color from Empedocles's *ochron* to *khloron*.

Several other pre-Socratic philosophers are reported to have written texts on natural philosophy and are likely to have discussed senses. But their works are lost. Among them are Heraclitus, Alcmaeon, Anaxagoras, Diogenes, and Leucippus.

From Plato's statements it is evident that his elemental colors bright, white, red and black are not colorants but ideas of color fundamentals. From these in secondary, tertiary, and quaternary mixtures the next eight colors are created (see Fig. 2-1). Additional colors are generated from further mixture. There is clearly an awareness of lightness from the sequences "auburn plus white makes light yellow" and "dark blue plus white makes light blue", as well as "purple burned makes umber". But there is no indication of a systematic arrangement by lightness.

It is apparent that Aristotle was aware of a fundamental difference between mixing lights and mixing pigments. In *Sense and Sensibilia* he offers seven fundamental colors seemingly ordered by brightness. In *On Colors* they appear to be reduced to two, white of air, water and earth, and yellow of fire. Black is the result of transmutation. Here he also showed his awareness of differences in intensity and brightness that can be present in colors of a given hue. Influenced by Pythagoras and his followers, he believed that colors appearing harmonious together are formed by simple ratios. Like Plato he was overwhelmed by the multitude of colors and could not see a way of organizing them systematically.

Ptolemy's list appears derived from Plato's except that orange [*xandos*] is added, as is gray [*phaion*]. Missing is leek green [*prasinon*] and flame color [*pyrrhon*], and there is only one bluish color. It is a simple list without any apparent order.

Pliny (AD 23–79), in a history of Greek art indicates that the painter Polygnotus, active in the fifth-century BC, added light ochre and other pigments to the standard early colors of painting, black, white, and red. He is reported to have painted women in light robes and their jewelry in various bright colors. However, the idea of light and shadow was not yet understood. Pliny puts the invention of cinnabar pigment into the same time period. Light and shade appear to have become common in Greek painting one or two generations later. Invention of further new pigments in the fifth to second centuries BC made more and more naturalistic painting possible and required more extensive practical knowledge in pigment mixture.

Of considerable and ongoing controversy is Pliny's statement that the most refined classical Greek painters only used four colors in their work: white, black, yellow, and red.[7] Blue pigments must have been known to the Greeks through trade with Egypt and other nations in the region. Traces of blue pigments have been discovered on statuary and on temple friezes. Works of the celebrated classical painters, like Apelles, have not survived. What is reputed to be a copy in mosaic of a painting by Philoxenos (4th c. BC) was found in the ruins of the House of the Faun (estimated to have been made ca. 100 BC) in Pompeii. It represents Alexander the Great confronting Darius at the battle of Issus. The colors of its tesserae are muted, beiges, browns, and grays, without any blues or greens, thus being in agreement with a four-color painting theory.

The sacral colors of the ancient Greeks are white, the color of festivities, black, the color of mourning, and various reddish shades from scarlet to violet.[8] White bulls or lambs were often used in sacrifices. Red, as the color of blood, invoked both death and life.

E. Veckenstedt has shown that the classical Greek epic writers used some 140 color words, while the philosophers of the same time period only used approximately 50 words. He also showed that of the words used by the epic writers and the philosophers nearly twice as many tended toward white than toward black (Veckenstedt, 1888). Clearly, in the practical world many color experiences were distinguished by name even though philosophers mentioned only relatively few. At the same time there was considerable confusion as to the exact meaning of given color names, well understandable in the absence of color standards. B. Berlin and P. Kay (1969) have classified Homeric Greek as representing stage IIIb in their scheme of development of basic color terms, namely having the basic terms of black, white yellow, red, and blue. By the time of the later classical epic writers the color palette had become quite extensive, with many terms indicating mixed hues and toned colors. There is no record, however, of a comprehensive effort to sort and order the multitude of colors.

A perennial problem is that of interpreting the meaning of color names. Without having specific examples for the meaning of various names, later interpretations vary considerably. It is evident that throughout the classical history of Greece what developed into color names had originally more general meanings. The situation became more complicated in Roman times as different translators of Greek texts sometimes used different Latin words for a given Greek color word. A particularly confusing example is the word *glaukos*, variously translated as light blue, grayish green, or gray, in one case as flashy. As seen later, in the late Middle Ages it briefly assumed also the meaning of yellow.

Color Order in the Middle Ages

According to the record Romans made no fundamental contributions to the subject of color order. Lucretius when discussing vision in *De rerum natura* (The way things are), his poetic account of Epicurean philosophy, essentially followed the atomic theory of Leucippus and Democritus as commented upon by Epicurus.

In the forth-century AD the translator and commentator Chalcidius (life dates not known) produced the translation of Plato's *Timaeus* dialogue most widely known in the Middle Ages. His text includes commentary. In the commentary on section 67ff of the *Timaeus*, Chalcidius described a five-term scale of basic colors as follows: white–yellow–red–blue–black, apparently the first written record of a reordering of the classical Greek fundamental colors into a single tonal scale with red in the middle.

Avicenna and Averroës

The step to multiple tonal scales is found in documentary evidence related to the Persian philosopher and physician Avicenna (Abu Ali al-Husayn ibn Sina, 980–1037). Avicenna was knowledgeable in Aristotle's writings. In his speculations on the human soul, influenced by Aristotle, he also addressed issues of color vision. In his manuscript, translated into Latin in the twelfth century, *Liber de anima seu sextus de animalibus* (Book of the soul) he may have been the first to describe tonal scales of (more or less) individual hues. Changes along Avicenna's tonal scales[9] involve, in modern terminology, most likely both lightness and chroma. He described three such sequences:

1. From *subpallidum* to *pallidum* (a gray or perhaps yellow scale, the generally accepted translation of *pallidus* is pale, wan, and its meaning as a hue term is in dispute)
2. From *subrubreum* to *rubreum* (a red scale)
3. From green to indigo (a scale involving hue as well as lightness and chroma changes)

The prefix "sub-" has the meaning of below, and the first two scales refer to perhaps grays or browns to yellow, respectively wine red to full red. The third scale travels across an expanse of hue as well as lightness, from a medium green to a dark reddish blue. These appear to be the first instance of a color arrangement that requires at least two dimensions to represent it.

The Spanish-Arabic philosopher and commentator ibn-Rushd (1126?–1198?), known under the Latin name of Averroës, wrote extensive commentaries on Aristotle, among many other texts. In his commentary on *Sense and Sensibility* he introduced terms that were translated into Latin as *remittere*, with a meaning to yield, to abate, and *intendere*, with a meaning to spread, direct. They were applied to describe an interpretation of Aristotle's thesis on colors. Accordingly, as Bacon later expressed it: ". . . as Aristotle means, the causes generating colors are elementary qualities that are augmented and abated in their generations through the power of brightness" (Parkhurst 1990). Applied to chromatic colors, it may mean tonal scales, and to black and white, a gray scale.

Eraclius

Eraclius is believed to have been an Italian monk. Nothing is known about his life dates or his activities. There are several manuscripts in Latin in existence that are copies of two books in rhymes and one in prose, less certainly by him, describing the manufacture of colorants for artistic purposes. Stylistic and content comparisons have resulted in estimates that he lived in the tenth century. In section 50 of the prose manuscript Eraclius[10] discusses the "various kinds and names of the principle and intermediate colors . . .":

> Of colours, some are white and some are black. . . . The intermediate colours are red, green, yellow, purple, prasinus [leek green], azure and indicus [indigo], which are each of them, in themselves, beautiful; but are more so when mixed, because, by their variety, they give beauty to one another. And then, in composition, they have a different hue, . . . colours of different kinds are mixed together, in order that they may partake of the nature of the others as well as their own, and make as many, and beautiful, and pleasing, varieties as possible. In this mixture, and in the order in which one is laid over another in painting, great skill is excercised.

In section 58 he describes how shading and highlights are best achieved for various colors: "And note that you must shade azure with black; and lay on the lights with white lead. . . . If you wish to make a colour like lily green, mix azure with white lead; shade it with azure; lay on the lights with white lead; and when it is dry, cover it over with clear saffron."

It is evident from this that Eraclius was well acquainted with color mixing and painting technology. Mixing of pigments, lasing, and toning in black and white directions were standard procedures. However, his color sequence does not indicate a concern with systematic color arrangement.

Theophilus

Explicit tonal scales going from the full color of a bright pigment in both directions toward white and black were first described by the German Benedictine monk Theophilus (1080?–1125?), also known under the name Rugerus. In approximately 1122 he wrote a treatise, *De diversis artibus* (The various arts) in which he describes for the benefit of brother monks at some length technical details for painting, glass making, and metalworking as practiced in his time. It seems likely that Theophilus was acquainted with the works of Eraclius. In Book I of his manuscript he describes how to mix colorants for painting flesh tones with their shadows and highlights as well as facial and head hair. He also describes how to mix colorants for the painting of draperies in wall and ceiling painting and how to imitate the rainbow in painting. For flesh color he begins with heated lead white, having turned yellowish, to which unheated lead white and vermilion (mercuric sulfide, red pigment) is added until flesh color is attained. More white is added for light faces, for pallid ones green earth instead of vermilion. Theophilus proceeds with first and second shadow colors, progressively darker, with first and second rose colors and a dark red, first and second highlight colors as well as a dark gray used for painting eyes. Thus Theophilus describes scales of colors centered on average flesh color and going into lighter, darker, redder, and greener directions. For example, the second shadow color for flesh is described as follows:

> Afterwards take the (first) shadow color for flesh which has been referred to above, and mix with it more green earth and burnt ochre so that it is a darker shade of the former color. Then fill the middle space between the eyebrows and eyes, under the middle of the eyes, near the nose, between the mouth and chin, on the down or beards of young men, on the half-palms toward the thumb, on the feet above the smaller areas of relief, and on the faces of children and women from the chin right up to the temples.

Similarly, for painting drapery various tonal scales are described, for example, for a greenish yellow hue:

> Mix pure viridian (green) with yellow ochre so that the yellow ochre predominates, and fills the drapery. Add to this color a little sap green and a little burnt ochre and make the drawing. Mix white with the ground-color and paint the first light areas. Add more white, and paint the lighter areas on top. Mix with the above shadow-color more sap green and burnt ochre and a little viridian and make the shadow on the outside. . . . Mix dark blue with white in the above way. Similarly mix black with white. In the same way mix yellow ochre with white and for its shadow add a little burnt ochre.

The colors of the rainbow he describes as follows:

> The band which looks like a rainbow is composed of various colors: namely vermilion and viridian, also vermilion and dark blue, viridian and yellow ochre, and also vermilion and folium [a vegetable red lake]. . . . Then mix from vermilion and white whatever tones you please so that the first contains a little vermilion, the second more, the third still more, the fourth yet more, until you reach pure vermilion. Then mix with this a little burnt ochre, then burnt ochre mixed with black and finally black. . . . You can never have more than twelve of these strokes in each color range. And if you want these many so arrange your combinations that you place a plain color in the seventh row.

Here Theophilus clearly describes twelve-tone tonal scales for various pure color pigments that represent the most intense color available for a given hue. Presumably, the scale colors are to be mixed so that the steps appear approximately even. In this fashion he extends Avicenna's scales in a systematic manner, however, without proposing a formal arrangement to place these in.

Learned discussions concerning color scales continued without adding substantially to what Avicenna, Eraclius, and Theophilus had provided. In the later twelfth century the Sicilian physician Urso de Salerno[11] commented in a discussion about the four element-related basic colors that there were many and often unnamed intermediate colors and rather than name them a good painter could produce them by mixture from the four elemental colors.

Albertus Magnus (ca. 1200–1280) Dominican monk, philosopher, teacher, and saint had considerable interest in optics and the sense of vision. He added to Avicenna's scales a number of colors he believed were missing: *fuscum* (dark colored) to the *pallidum* scale; *croceum* (saffron colored, golden), *purpureum* (purple), and *indicum* (here considered to be a red color) to the red scale; and *viride clarum* (bright green) and *viriditas intensa* (intense greenness) to the green scale.[12]

Grossteste and Bacon

Robert Grosseteste (ca. 1168–1253) was an English Franciscan monk, later to become chancellor of Oxford University and bishop of Lincoln. His extensive writings include works on optics and color. By postulating a direct path from the material world to its essential nature through optics, he legitimized extensive studies of optical phenomena. In *De colore* (On color, ca. 1233), one of four books on optics, he described a seven-color scale (colors unnamed) credited to Aristotle and Averroës. These colors [presumably including black and white] are related to bright light [*lux clara*] by remission and, implicitly, to darkness [*lux obscura*] by intention (Parkhurst, 1990).

Roger Bacon (1214/20–1292), also a Franciscan, Aristotelian philosopher and scientist, lived a generation after Grossteste. He wrote three different, if related, texts with sections on color: *Liber de sensu et sensato* (Book of the senses and sensibilities, ca. 1255, attributed to Bacon), *Opus majus* (part of a projected encyclopedia), and *De multiplicatione specierum* (On the multiplication of species).[13] Bacon attempted to apply the second-century Greek philosopher Porphyry's system of predicables, a logical sequence of attributes—genus, species, difference, property, and accident—to the problem of color. He applied the term genus to a Chalcidian list of five fundamental color properties: whiteness [*albedo*], yellowness [*glaucitas*], redness [*rubedo*], blue-greeness [*viriditas*], and blackness [*nigredo*]. Each of these has a range of color species attached to them: for example, *glaucitas* is found in *lividus, flavus, glaucus, ceruleus, pallidus*, and *citrinus* (see below for translation). Bacon discussed at length Grosseteste's views on color and their relation to lightness and darkness. He used the specific Latin term *gradus* (step, gradation) to describe color variations representing difference in the Porphyry sequence.

In *De sensu et sensato* Bacon included a list of twenty colors, sorted in approximately tonal sequence, he considered important and explained their meaning as derived from older sources (interpretations of Bacon's meaning are in parentheses).

candidus	shining white
albus	white, dead white
lividus	incomplete white (ivory, oatmeal)
flavus	like *lividus* (pale yellow)
glaucus	yellow
ceruleus	between *glaucus* and *citrinus* (color of beeswax)
pallidus	between *citrinus* and *ceruleus* (pale)
citrinus	orange, between *glaucus* and *puniceus*
puniceus	(reddest of the yellows)
rufus	(golden, scarlet)
croceus	saffron colored, blood red
rubeus	true red, the median color between white and black
rubicundus	darker, more bluish than *rubeus*
purpureus	(purple, violet)
viridis	(blue-green)
venetius	between true blue and black (dark blue)
lividus	(dark gray?)
lazulus	(dark blue)
fuscus	(dark colored, no specific hue)
nigrum	black

Bacon was on the threshold of a more complete understanding of color phenomena, without being able to establish a clear system.

Dietrich von Freiberg

The German Dominican monk Freiberg wrote circa 1310 a manuscript *De iride* (About the rainbow) in which he provided the first accurate explanation for the rainbow phenomenon. He argued (against Aristotle) that chromatic colors could not be produced from mixture of white and black: ". . . a mixture of white and black does not produce anything except a remission of the white and black from their perfections" (various grays, as well known by artists; Wuerschmitt 1914). He also used the term *glaucus* to denote yellow, as did Bacon and the English encyclopedist Bartholomew Anglicus in 1250 (Parkhurst, 1990).

Medieval Progress toward a Systematic View of Colors

Chalcidius, the translator and commentator of Plato, introduced a tonally ordered scale of five fundamental colors that was influential until the late

Renaissance (see d'Aguilon and Kircher below). Avicenna's attempt at ordering colors, influenced by Aristotle, resulted for the first time in scales that modulate a given color along a more or less tonal path. Albertus Magnus commented on the need to add additional steps to Avicenna's scales. Roger Bacon's sustained efforts at color order did not result in significant progress. It was the practical Theophilus who, based on needs of painters and artisans in enameling to create life-like images, proposed the toning of a given pigment color in twelve systematic steps toward white and black. If he saw in this approach the possibility of a systematic description of the world of colors, he did not say so in his writings.

2.3 COLOR ORDER IN THE RENAISSANCE

Despite the flowering of the Renaissance on both sides of the Alps and the writing of books on painting (Leon Battista Alberti, 1335, and Cennino Cennini, ca. 1400), there was only modest progress in thinking on the subject of color scales until the sixteenth century. Cennini mentions a list of seven colors (but not as a scale) that appear to derive from Aristotle: "Know that there are seven natural colors, or rather four actually mineral in character, namely, black, red, yellow and green; there are natural colors but need to be helped artificially, as lime-white, blue-ultramarine, azzurite, giallorino" (Cennini, ch. 36).[14] Cennini described and recommended a style of painting in which the full pigment colors represent the darkest colors in a picture and gradations are made exclusively with the addition of white (known as the Cennini style; Hall, 1992). Alberti offered a painter's view on colors. While he discussed (in the Latin version of his manuscript) various views on color of philosophers and experts, in the Italian version he said: "I speak here as a painter. . . . Through the mixing of colors infinite other colors are born, but there are only four true colors—as there are four elements—from which more and more other kinds of colors may be thus created. Red is the color of fire, blue of the air, green of water, and of the earth gray and ash [*bigio et cenericio*]. . . . Therefore, there are four genera of colors, and these make their species according to the addition of dark and light, black or white. They are thus almost innumerable. Therefore the mixing of white does not change the genus of colors but forms the species. Black contains a similar force in its mixing to make almost infinite species of color." The interpretation of Alberti's choice of the earth-related fourth primary chromatic color is difficult. It has been described as a dull yellowish gray (Gavel, 1979). It appears that the association of his primary chromatic colors with the four elements was for Alberti more important than a system that would recognize yellow as primary. However, he clearly envisaged the systematic toning of the primary colors toward black and white.

The painter and scientist Leonardo da Vinci (1452–1519) proposed a sort of tonal scale of six colors: "The simple colors are six, of which the first is white, although some philosophers do not accept white or black in the number of colors, because one is the origin of all colors and the other their absence. But as painters cannot do without them, we include them in the number of the others, and say that in this order white is the first among the simple, yellow is the second, green is third, blue is fourth, red is fifth, and black is sixth."[15] Leonardo's hue sequence clearly considers the hue circle. It is obvious that Leonardo, like Alberti, was also well versed in tonal expansions achievable with the addition of white and/or black pigments to the pigments representing the four simple chromatic colors and their mixtures.

A curious twelve-step scale based on brightness and without regard to a systematic arrangement of hues was proposed by the Italian neoplatonic philosopher Marsilio Ficino (1433–1499) in a letter to a friend.[16] "In light there are many ideas of colors as there are colors in objects. At the lowest degree where it is communicable, there is the idea of black, at the second the idea of brown, at the third dark yellow, at the fourth dark blue and green, at the fifth sky blue and sea green, at the sixth full red, at the seventh light red, at the eighth saffron yellow, at the ninth white, at the tenth the transparent or the shining, at the eleventh the brilliant, and finally, there is the idea of splendor."

In 1528 the Italian philologist and poet Antonio Telesio (1482–1534) published in Venice a small book *Antonii Thylesii de coloribus libellus* (Antonio Telesio's little book on colors). It is reprinted in its entirety in the original Latin in Goethe's *Geschichte der Farbenlehre* (1810). Telesio described twelve basic colors, not in any particular order. Aside from the differently named black only three colors are identical to colors in Bacon's list. For each of these basic colors Telesio offered a historical commentary and several related color names. The list[17] and some related color names are as follows:

CATEGORY NAME	RELATED COLOR NAMES
1. *Coeruleus* (blue, sky blue)	*indicum, cyaneum, venetus, blavus*
2. *Caesius* (bluish gray, gray)	*glaucus, baios, charopon*
3. *Ater* (black, dead black)	*niger, anthracinus, fuscus*
4. *Albus* (white, dead white)	*pallidus, candidus, leucophaeus*
5. *Pullus* (dark colored, grayish black)	*impluviatus*
6. *Ferugineus* (rust colored, dusky)	*hyacinthus*
7. *Rufus* (red, ruddy)	*rutilum, russum, sanguinatus*
8. *Ruber* (red, ruddy)	*purpureus, xerampelinus*
9. *Roseus* (rose colored)	*incarnatus*
10. *Puniceus* (purple)	*spadiceus*
11. *Fulvus* (yellowish brown, tawny)	*croceus, luteus, flammeus*
12. *Viridis* (green)	*prasinus*

The list is remarkable for its focus on desaturated, natural colors and the absence, perhaps influenced by Alberti, of saturated yellow. It is apparent that

Telesio understood *rufus* to be a yellowish red and *ruber* a bluish one. It also shows that Telesio's meaning of *glaucus* has returned to bluish gray.

Hieronymus Cardanus (1501–1576) Italian physician and mathematician offered a further step in a color value scale.[18] The subject of color appears in more than one of his works, primarily in *De gemmis et coloribus* (On gems and colors, 1563). Here Cardanus described an Aristotelian seven-step color scale as follows: "white, yellow, red, green, wine color, blue, and black." White and black are primordial colors, the other five "intermediate." Additional mixed colors fall between these primary colors. What is new in Cardanus's list is that he assigns numbers of brightness to these colors, thus: "we assume that white contains a hundred parts of light, scarlet fifty, black nothing." These colors fix the beginning, middle, and end of the scale. Yellow is described as containing 65 to 78 parts of light, green 62, deep green 40, wine color 30, blue 25, and blackish gray 20. This scale is not influenced by concerns of painters or by issues of color mixing. It is the first-known "quantitative" assessment of the intrinsic brightness/lightness of object colors.

Sigfrid Aronus Forsius

Significant progress was made by the Finnish mathematician, astronomer and clergyman Sigfrid Forsius (1560–1624). In 1611, while in Stockholm, he wrote a manuscript on physics, chapter VII of which is titled "On Vision." There he presented two figures in form of circles. The first has sixteen colors ranging in one direction from white via red to black and in the other direction from white via blue to black. The red scale has the steps white gold, gold, burnt gold, red, purple, brown and violet brown; the blue scale dapple, gray, sky blue, blue, blue green, green, blackish green. Forsius discussed this scale as follows: "Among the colors there are two prime colors, white and black, from which all others have their origin. . . . In the middle between these colors, red since ancient times has been placed on one side and blue on the other one. . . . Gold between white and red, . . . brown between red and black. . . . Then on the other side between white and blue is gray. . . . And on the lower part green between blue and black" (Fig. 2-2*a*).

With this circle Forsius put classical and medieval ideas on tonal color scales into graphical form.[19] However, he was not satisfied with the result and continued:

> But if you want right to consider the origin and relations of the colors, you should start from the five principle middle colors which are red, blue, green, gold, and gray of white and black. And their gradings, they rise either closer to white by their paleness or to black by their darkness; albeit they are (as above has been made known) related to one another as previously shown. Because red rises to white through pale red (pink) and skin color; to black through purple, brown, violet brown and black brown. Similarly gold relates toward white through pale gold, wooden and wheat color; to black through burnt gold and blackish brown.

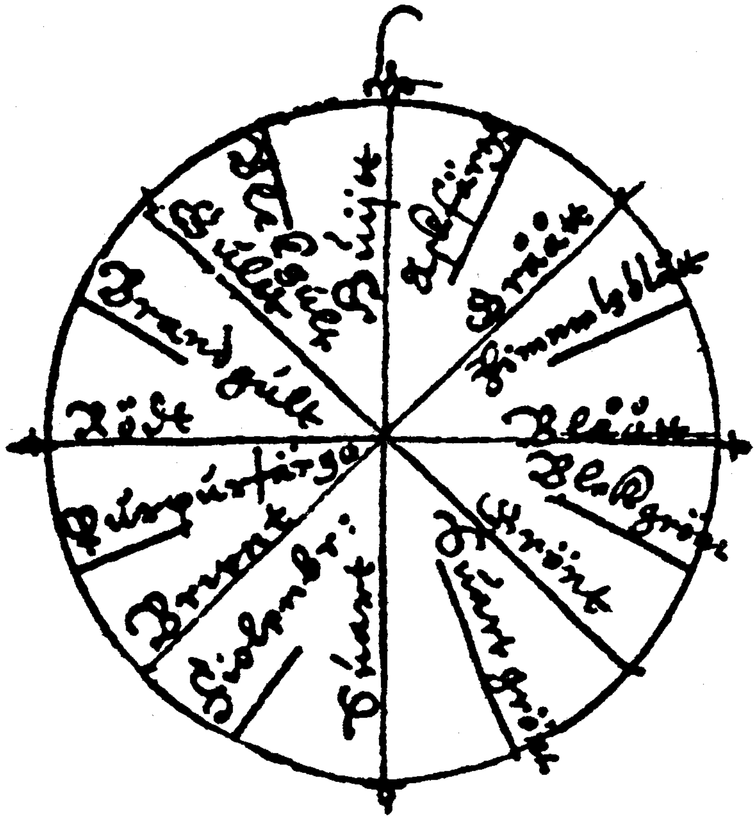


Fig. 2-2a *Classical color order is interpreted 67 Forsius.*

> Equally blue rises to white through sky blue and pale blue, like Dutch cloth; and to black through dark blue like indigo color that has some brownish to it. So rises also green toward white through verdigris and pale green; to black through blackish green. Gray approaches white by the color of light gray, dapple gray and lime: to black by mouse gray, black gray and pale black. And this is the correct relationship of colors that in their number agree with that of the planets as do the lower colors with the five membranes of the eye, and with the five senses. All this can be seen from the accompanying figure. (Fig. 2-2*b*)

Even though some commentators have interpreted the second of Forsius' circles as a color sphere, with white and black at the poles and four primary hues: gold (yellow) and blue, as well as red and green, opposing each other, there is no indication in the text that Forsius had a three-dimensional arrangement in mind. To be interpreted as a sphere with the primary hues in the proper sequence of a hue circle requires an unusual interpretation of Forsius' figure: the left semicircle would be paired with the first right curved line to

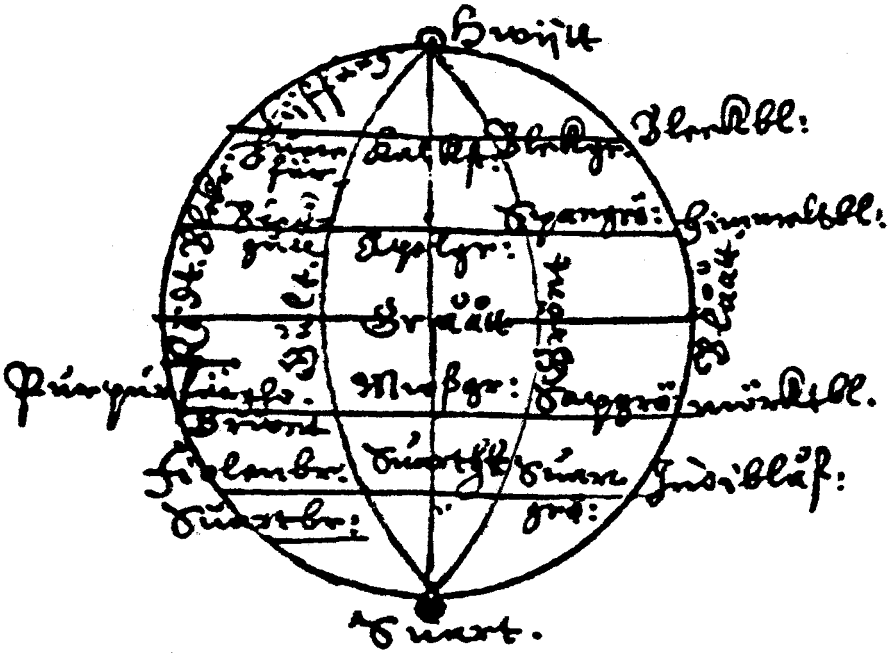


Fig. 2-2b *Forsius' proposal of tonal scales ending in white and black.*

form a circle, and vice versa. How to properly draw a transparent sphere was well known in the seventeenth century from several sixteenth-century and earlier books on perspective. On the other hand, Forsius certainly knew that there are many intermediate hues between his primary hues and with them many more curved lines. How he would have arranged these is not clear. It is evident that Forsius' second figure does not represent a color sphere but is rather a figure that illustrates four chromatic and an achromatic value scale beginning and ending at the same points. His manuscript was not published in book form and thus did not become known outside Sweden.

The Pythagorean ideas on musical harmony as expressed in the tetractys were expanded by the Roman philosopher and statesman Boethius (ca. 475–525). Boethius wrote an extensive treatise on ancient music *De institutione musica*, influential into the sixteenth century. Figure 2-3, from a twelfth-century manuscript of Boethius' work on music, is the representation of an octave, with the basic ratio of 6 and 12.[20] Additional ratios show fifths (*diapente*), fourths (*diatesseron*), and tone. A diagram of a similar style was used by the fourteenth-century French mathematician Nicole Oresme (1323–1382) to show the relationship between various basic polygons.[21] The same type of diagram was used in the 1542 edition of Argyropolus' commentary on Porphyry's *Isagoge* to show

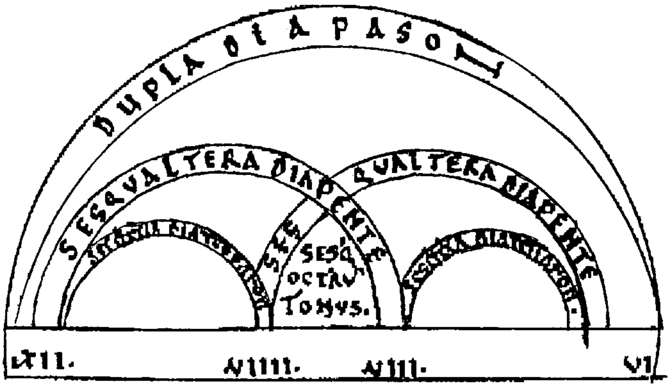


Fig. 2-3 Graphical interpretation of a musical octave (or double diapason) split into dual-tone cords, after Boethius (fifth c.). From a twelfth-century manuscript of Boethius' De institutione musica. *This type of diagram was used to represent several kinds of connections in the first half of the second millennium.*

the relationship between the five types of classification of things (see Parkhurst, 1990). The use of such diagrams was thought to show the simplicity of natural laws as expressed in the classical four areas of study: arithmetic, geometry, music, and astronomy. It is not surprising that classically schooled thinkers also used diagrams of this kind to show relationships between colors.

D'Aguilon, Fludd, and Kircher

François d'Aguilon (Aguilonius, 1567–1617) of the Jesuit order in Brussels is the author of a six volume work on optics *Opticorum libri sex* (1613), famous because of its illustrations by Peter Paul Rubens. In regard to colors d'Aguilon expresses himself incapable of dealing exhaustively with the complexities of color mixing as practiced by artists. He included a relational diagram, in the style of Boethius's diagram, in which he expressed the classical ideas of all colors generated from white and black. The top portion of the figure can be interpreted as a semiquantitative tonal description of the three primary hue colors: yellow close to white and far from black, with blue the opposite and red halfway between white and black. In the bottom area three important mixed hues are shown: gold as a mixture of yellow and red, purple of red and blue, and green of yellow and blue. D'Aguilon's diagram follows Chalcidius's sequence of colors restricting the chromatic primary colors to yellow, red, and blue, the painter's primaries (Fig. 2-4).

Robert Fludd (1574–1637), an English mystic, philosopher, and physician, is known for a re-interpretation of an earlier scale of the colors of urine and as the first creator of a printed color circle (Fig. 2-5).[22] This is a curious, perhaps mystically influenced, construction. It simply connects the white and black ends of a seven-color linear arrangement to form a circle. The contents of these

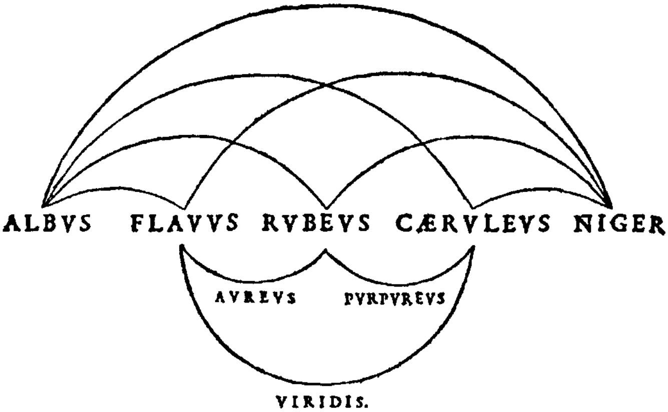


Fig. 2-4 *François d'Aguilon's basic color scale and mixture diagram of 1613. In the upper portion the tonal scales of white, the full colors, and black are shown, and in the lower section important secondary colors resulting from mixtures of the primary chromatic colors yellow, red, and blue.*

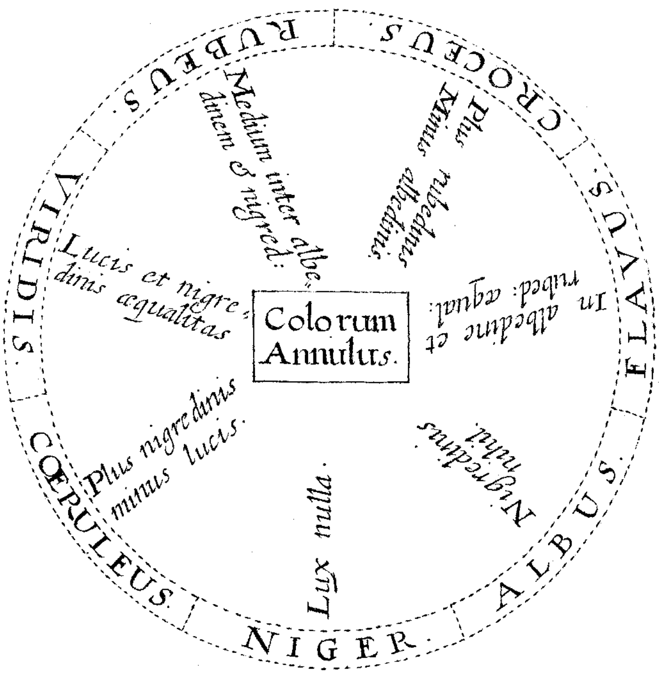
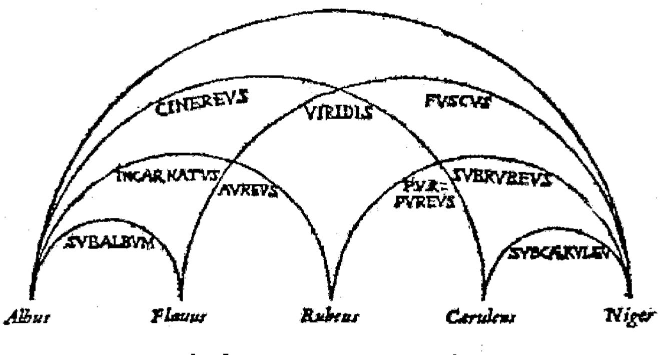


Fig. 2-5 *Color circle of Robert Fludd of 1629, perhaps the first in print.*

colors is described as follows: black: no light; blue: more blackness, less light; green: equality of light and blackness; red: middle between whiteness and blackness; orange: more redness, less whiteness; yellow: equality of whiteness and redness; white: no blackness. The Aristotelian number of seven colors and the designation of red as halfway between black and white are conventional. What is unusual is the description of colors toward black in terms of light and toward white in terms of whiteness.

Athanasius Kircher (ca. 1601–1680), German Jesuit with wide interests in the sciences, author of 44 volumes of writing, professor at the University of Würzburg and later at the College of Rome, wrote a text on light and color: *Ars magna lucis et umbrae* (All there is to know about light and shadow), published in 1646. He also considered, in the Greek tradition, colors to be the result of activity of light and darkness: "Since color is the property of a dark body or, as some say, a shadowed light, the true offspring of light and shadow, we must treat thereof. . . ." In the second chapter, *On the multitudinous variety of colors*, he used a modified version of d'Aguilon's color diagram to illustrate the arrangement of colors (Fig. 2-6). In the cusps formed by semicircles the mixture colors of the two chromatic colors involved are placed: *aureus*, *viridis*, and *purpureus*. Below the peaks of the semicircles are located important tonal



Analogia rerum cum coloribus.

Albus	Flauus	Rubeus	Cæruleus	Niger
Lux pura	Lux tincta	Lux colorata	Vmbra	Tenebræ
Lux	Vmbra tenuissima	Vmbra moderata	Vmbra densa	Tenebræ
Dulce	Dulce temperatum	[illegible]	Acidum	Amarum
Ignis	Aër vel æther	Auroræ medium	Aqua	Terra
Pueritia	Adolescentia	Iouentus	Virilitas	Senectus
Intellectus	Opinio	Error	Pertinacia	Ignorantiæ
Deus	Angelus	Homo	Brutum	Planta
Nete	Parenete	Meſe	Parameſe	Hypathe.

Fig. 2-6 *Athanasius Kircher's color diagram in the Boethius style, 1646. Below are his analogues in other qualities and ideas. Tonal scales are shown as by d'Aguilon, with intermediate tonal colors identified (e.g., cinereus, ash colored, between white and blue). The secondary mixtures are shown at the intersections of the corresponding tonal arches.*

colors resulting from mixture of the chromatic color with white or black. Thus *subalbum* is a mixture of yellow and white while *fuscus* is a mixture of yellow and black. The diagram is interesting because of the attached analogies between colors and various other qualities, properties, and things: light and shade, taste, the four classical elements, human age, intellect, a scale from God to plants, and finally, the strings of the Greek *lyra*.

Francis Glisson's Color Specification System

Glisson (1597?–1677), an English physician and Regius professor at Cambridge, wrote several books on medicine, among them *Tractatus de ventriculo et intestinis* (Treatise of ventricle and intestines), published in London in 1677. Its chapter IX is surprisingly titled *De coloribus pilorum* (On the colors of hair). The main purpose of this chapter seems to have been to present Glisson's ideas about a color specification system that could not only be used to specify the color of hair but of any colored object. Glisson was a supporter of the idea of five fundamental colors: white, black, yellow, red, and blue. Using four visually equally spaced scales, a gray scale, and scales from white to full color of yellow, red, and blue, all object colors could be specified. Glisson used a novel approach in his scale development. He determined the "equivalent strength" of his white and black pigments (lead white, carbon black) by making a mixture that visually fell halfway between the two extremes, that is, a middle gray. To achieve this, he found that he needed to use a weight ratio of 50:1. Since he wanted to have a scale of 24 steps and the middle gray was to be step 12, he multiplied the weights of his two pigments by 12 to arrive at 600 grains of white and 12 grains of black. For the next darker step he used 1 grain of black more and 50 grains of white less, that is, 13 and 550. In this fashion he completed his scale in both directions (Fig. 2-7).

Glisson also provided a schematical figure of the arrangement of the scales (Fig. 2-8). He described the gray scale (or, in his terms, blackness scale) as being straight and the three chromatic scales as being rounded and sideways from or oblique to (*tres . . . scalae obliquae sunt*) the gray scale. He also was aware of intermediate hues but did not indicate how they should be placed. The midpoints, indicated by marks, are the locations of his pure chromatic pigments (orpiment, vermilion, azurite or bice) extending in one direction to white and in the other to black. But Glisson believed that it was not necessary and a waste of effort to make complete chromatic scales: the half scales from white to the full pigment color together with the gray scale would suffice for color specification. Glisson described as an example of a chromatic scale the redness scale, with 12 steps. He had determined the strength of vermilion against lead white to be 1:20 and used the same methodology as in the gray scale. Glisson provided an example of color specification for the golden yellow blossoms of a flower. This color he judged to be equivalent to grade 11 of the yellowness scale, grade 3 of the redness scale, and grade 2 of the gray scale. If necessary, he said, colors could be specified to half steps between his grades.

Scala Nigredinis.

Gradus ejus.	Grana cerussæ.	Grana atramenti fuliginei.	Utriusque proportio minima.
13^{us}.	Simplex Nigredo.		
22^{us}.	100.	gr. XXII.	C. 4 $\frac{6}{11}$ F. 1.
21^{us}.	150.	gr. XXI.	C. 7 $\frac{1}{7}$ F. 1.
20^{us}.	200.	gr. XX.	C. 10. F. 1.
19^{us}.	250.	gr. XIX.	C. 13 $\frac{1}{3}$ F. 1.
18^{us}.	300.	gr. XVIII.	C. 16 $\frac{12}{18}$ F. 1.
17^{us}.	350.	gr. XVII.	C. 20 $\frac{16}{17}$ F. 1.
16^{us}.	400.	gr. XVI.	C. 25. F. 1.
15^{us}.	450.	gr. XV.	C. 30. F. 1.
14^{us}.	500.	gr. XIV.	C. 35 $\frac{5}{7}$ F. 1.
13^{us}.	550.	g. XIII.	C. 42 $\frac{4}{13}$ F. 1.
12^{us}.	600.	gr. XII.	C. 5. F. $\frac{1}{10}$
11^{us}.	650.	gr. XI.	C. 5 $\frac{10}{11}$ F. $\frac{1}{10}$
10^{us}.	700.	gr. X	C. 7. F. $\frac{1}{10}$
9^{us}.	750.	gr. IX.	C. 8 $\frac{1}{3}$ F. $\frac{1}{10}$
8^{us}.	800.	gr. VIII.	C. 10. F. $\frac{1}{10}$
7^{us}.	850.	gr. VII.	C. 12 $\frac{1}{10}$ F. $\frac{1}{10}$
6^{us}.	900.	gr. VI.	C. 15. F. $\frac{1}{10}$
5^{us}.	950.	gr. V.	C. 19. F. $\frac{1}{10}$
4^{us}.	1000.	gr. IV.	C. 25. F. $\frac{1}{10}$
3^{us}.	1050.	gr. III.	C. 35. F. $\frac{1}{10}$
2^{us}.	1100.	gr. II.	C. 55. F. $\frac{1}{10}$
1^{us}.	1150.	gr. I.	C. 115. F. $\frac{1}{10}$
Simplex Albedo, basis scalæ.			

Fig. 2-7 *Pigment mixture chart for Francis Glisson's gray scale. First column: grade of scale; second column: weight of lead white (in grains); third column: weight of carbon black; fourth column: reduced pigment ratio.*

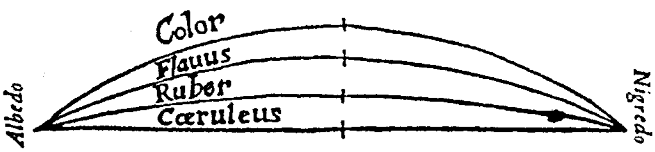


Fig. 2-8 *Glisson's sketch of the arrangement of tonal scales. The blue, red, and yellow scales arch over the horizontal gray scale according to their lightness. The vertical dashes denote the location of the pure pigment.*

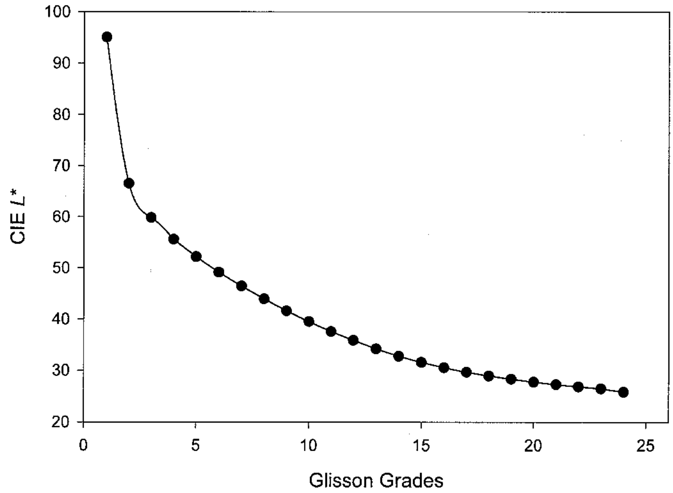


Fig. 2-9 *CIE L* lightness values of reconstructed Glisson gray scale, beginning with white on the left and ending with black on the right. The scale is only approximately uniform in terms of CIE lightness. There is a very large step between white and the first mixture.*

Glisson's gray and red scales have been reconstructed using the specified pigments and weights (Kuehni and Stanziola, 2002). Reflectance curves were measured and tristimulus values, as well as CIE lightness values L^*, calculated. In Fig. 2-9 the differences in CIELAB (see Chapter 6) L^* between the steps of the gray scale are shown. Near both ends the step sizes increase strongly. In between, they gradually decline in size toward black. Only extensive visual scaling would have disclosed this fact. Similar results were obtained for the red scale with, again, the first and last steps being too large and toward full red the steps gradually becoming smaller.

Brenner and Waller

On a different front progress was made toward a color atlas. In 1680 the Swedish painter and archeologist Elias Brenner (1647–1717) published in Stockholm his *Nomenclatura et species colorum* (Nomenclature and species of colors) containing 31 color samples in six groups: white, yellow, red, green, blue, and black, to assist miniature painters. Six years later, improving on Brenner to whom he referred, the member of the Royal Society of London Robert Waller published *A Catalogue of simple and mixt Colours, with a Specimen of each Colour prefixt to its proper Name* (Waller, 1686). It contains in a rectangular chart 112 mixed colors (many named in Latin, Greek, French, and English). The composition of the mixtures is identified by the colorant names of the column and row headings. As Waller explained: "... I have mixt each of the *Simple Yellows* and *Reds* with each of the *simple Blews*, and these *Mixtures* give most of the *mean* Colours, *viz. Greens*, *Purples*, &c." (italics in the original). The column header colorants are Spanish white, azurite, ultramarine, smalt, litmus, indigo, ink black. The row header yellow colorants are (lead white), Naples yellow, gamboge, ochre, orpiment, and umbra, the red colorants minium, burnt ochre, vermilion, carmine, red lake, dragon's blood, red ochre (carbon black). One-to-one mixtures were made to fill in the intersecting 98 fields. Waller's expressed idea was to have a scientific systematic arrangement of colors. He defended mixtures in single weight ratios by indicating that the possible number of mixture ratios was infinite and therefore not practically doable. In some copies of the printed paper the colorant mixtures were dabbed in.

2.4 NEWTON'S COLOR DIAGRAM

It is remarkable that the first explicit, if incomplete, hue circle was the work of Newton. Isaac Newton (1642–1727), celebrated mathematician and physicist, clarified the composition of white light as a mixture of lights of different wavelengths. When these are viewed individually they create various hue experiences, the spectral colors. In his early lectures on optics, deposited as a matter of his job responsibilities as Lucasian professor of mathematics in the library of Cambridge University (*Optica*, 1670–1672) Newton was primarily concerned with the refrangeability of light rays and had little to say about color mixture and logical arrangements of colors. Proposition 3 states: "The colors white and black together with intermediate ashens or grays are generated from rays of every sort confusedly mixed." In proposition 2 Newton described a scale of eleven hues that he considered "prominent primitives." He experimented with overlaid mixtures of prismatic lights by appropriately arranging three prisms and experienced the well-known difficulties in obtaining white from mixture of two prismatic colors. He found, and expressed in proposition 4, that "[p]rimitive colors can be exhibited by the composition of the neighboring colors on each side of them."

In his mature reflections on colors, *Opticks* (1704), Newton introduced a partly scientifically based color circle and color mixture diagram (Fig. 11 of Plate II, Part II, Book I) (Fig. 2-10). He described it (in part) as follows:

> With the Center O and Radius OD describe a Circle ADF and distinguish its circumference into seven parts . . . proportional to the seven musical Tones or Intervals of the eight Sounds, contained in an Eight. . . . Let the first part DE represent a red Colour, the second EF orange, the third FG yellow, the fourth GH green, the fifth AB blue, the sixth BC indico, and the seventh CD violet, And conceive that these are all the Colours of uncompounded Light gradually passing into one another, as they do when made by Prisms. . . . Let p be the center of gravity of the Arch DE [comparably for q, r, s, t, v, x] and about those centers of gravity let

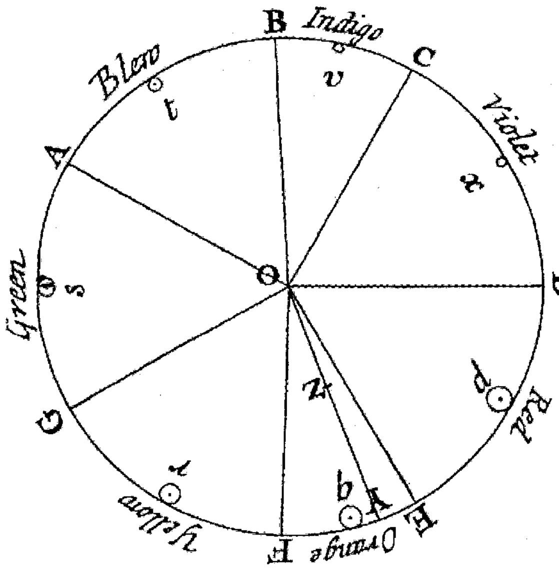


Fig. 2-10 Newton's color circle, 1704.

Circles proportional to the number of rays of each Colour in the given mixture be described. . . . Find the common center of gravity of all those Circles p, q, r, s, t, v, x. Let that center be Z; and from the center of the Circle ADF, through Z to the circumference, drawing the right line OY, the place of the point Y in the circumference shall shew the Colour arising from the composition of all the Colours in the given mixture, and the line OZ shall be proportional to the fullness or intenseness of the Colour, that is, to its distance from whiteness. As if Y fall in the middle between F and G, the compounded Colour shall be the best yellow; if Y verge from the middle toward F or G, the compounded Colour shall accordingly be a yellow, verging toward orange or green. If Z fall upon the circumference the Colour shall be intense and florid in the highest degree; if it fall in the mid way between the circumference and center it shall be but half so intense, that is, it shall be such a Colour as would be made by diluting the intensest yellow with an equal quantity of whiteness; and if it fall upon the center O, the Colour shall have lost all its intenseness and become a white. . . . if the point Z fall in or near the line OD, the main ingredient being the red and violet, the Colour compounded shall not be any of the prismatic Colours, but a purple, inclining to red or violet. . . .

Newton describes with his figure a diagram that is a complete spectral hue circle as well as an additive mixture diagram. At the same time he uses the figure to give a specific example of additive color mixture:

. . . suppose a Colour is compounded of these homogeneal Colours, of violet 1 part, of indico 1 part, of blue 2 parts, of green 3 parts, of yellow 5 parts, of orange

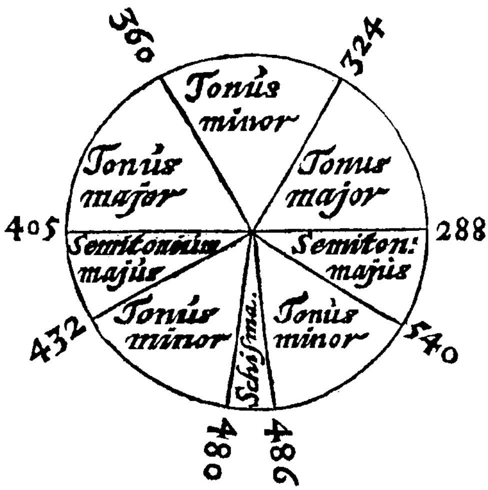


Fig. 2-11 Descartes's circular diapason, 1650. It may have served as a model for Newton's color circle.

> 6 parts, and of red 10 parts. Proportional to these parts I describe the Circles x, v, t, s, r, q, p respectively, that is, so that if the Circle x be 1, the Circle s 3. . . . Then I find Z, the common center of gravity of these Circles, and through Z drawing the line OY the point Y falls upon the circumference between E and F, . . . and thence I conclude, that the Colour compounded of these ingredients will be an orange, verging a little more to red than to yellow. Also I find that OZ is a little less than one half of OY, and thence I conclude, that this orange hath a little less than half the fullness or intenseness of an uncompounded orange . . . this proportion being not of the quantities of mixed orange and white powders, but of the quantities of the lights reflected from them.[23]

The form of Newton's diagram, reflecting his belief in a correspondence between colors and musical tones, may have been influenced by an illustration of the relationship between musical ratios in form of a circle (Fig. 2-11) by the French philosopher and scientist René Descartes (1596–1650).[24] In agreement with the musical scale Newton used the classical number of seven for the basic colors of the spectrum (the Aristotelian seven colors, however, include white and black). He was intrigued by the possibility of parallels between musical harmony and color harmony and pointed out the common reddidshness of shortest and longest wave colors of the spectrum and compared it with the similarity of the tones at the beginning and end of an octave (as reported by Diderot).[25] Newton's work, as is well known, represented a sea change in thinking about colors, opening furious and extended discussions only settled some 200 years later.

2.5 DEVELOPMENT OF THE COLOR CIRCLE

C.B.'s Color Circle

Only four years after the publication of Newton's *Opticks* with its spectral color circle, an anonymous, probably French, author published an image of a hue circle in color. The Dutch printer van Dole had issued several editions of a book on painting miniatures, beginning in 1673. The author was only identified with the initials C.B.[26] To date the identity of the author remains unclear. In 1708 a new edition was published under a new title *Traité de la Peinture en Mignature* (Treatise on miniature painting). It contained, for the first time, images of a seven-hue circle (Fig. 2-12) and of a twelve-hue circle. The author described the circles as follows: "Here are two figures by which one will be able to see how the primitive colors, yellow, red, crimson and blue generate the other colors and which one might call the *Encyclopedia of Colors*. The first figure includes the four primitive colors and three composed from them, and the second includes those same colors with five others which are produced as much from the primitives as of their composites." It is evident that the figures

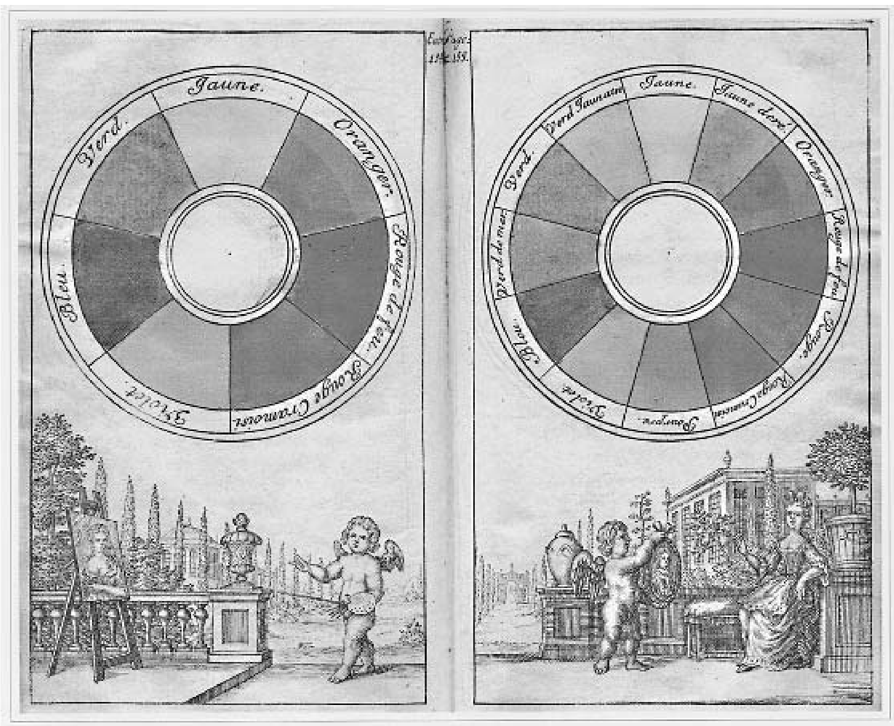


Fig. 2-12 *C.B.'s hand-painted color circles, 1708. Left: The seven-color circle; right: the twelve-color circle. Note that the pigments used for some of the colors have deteriorated. (See color plate.)*

represent pigment-mixing diagrams. In Gage's view (1993) the author used a yellowish and a bluish red primary pigment because only with those could he produce a mix of true red (shown in the twelve-hue circle).

Identity of number of colors between Newton's circle and that of the first circle in the *Traité* may be suggestive, but the anonymous author's selection of the seven colors, related to pigments, is noticeably different from Newton's. The colors of the seven-hue circle are yellow, orange, fire red, crimson, violet, blue, and green. Newton's indigo is missing. A neutral red only makes an entrance in the twelve-color circle. Here the colors are yellow, golden yellow, orange, fire red, red, crimson, purple, violet, blue, sea green, green, yellowish green. We notice that the circle is heavily red-directed: there are seven colors containing redness and only three colors containing greenness. Yellowish and bluish colors are balanced with five each. C.B.'s color circle is the first known explicit and complete (considering also extra spectral purple) circular hue arrangement. By the year 1800 this book had seen 33 editions and thus was widely known, at least in artistic circles.

Castel and Schiffermüller

Louis-Bertrand Castel (1688–1757), a French mathematician, Jesuitical priest, opponent of Newton in matters of color, was deeply interested in a connection between color and music and hoped to construct a color organ.[27] As described in *Projet d'une nouvelle optique* (Project of a new optics) of 1739, by mixing the three basic colors yellow, red, and blue, he constructed a twelve hue color spiral which he correlated with the tones and half tones of musical octaves. Surprisingly, and apparently, as Lambert later reported, as a result of French national preferences, his primary colors were a yellowish red (fire red), a neutral to reddish yellow (*stil de grain*, yellow vegetable dye lake), and "true sky blue." He derived the twelve colors directly from work with the prism in a refutation of Newton's findings of seven basic colors: crimson, red, orange, golden yellow, yellow, olive, green, sea green, blue, "violant," "agathe," and violet. "Higher-octave" color spiral segments where obtained by adding white to his original spiral segment, "lower-octave" colors by adding black. He constructed scales of twelve steps each of additions of white and black for the twelve hues of a hue spiral, ending up with a system of 144 colors. Castel hoped to expand the gradations in all directions and to display the results in a color room that he envisaged to be covered with colored woven bands or wallpaper specimens representing his colors. He attempted to make the gradations uniform in terms of perceived differences and, on the hue spiral, ended up with a different number of steps between his three primary colors. Spirals imply three dimensions and Castel's color room seems the first implicit idea of a color space.

Castel's work was continued and expanded by the Viennese entomologist and brother Jesuit Ignaz Schiffermüller (1727–1806/9). Schiffermüller did not share Castel's enthusiasm for a possible connection between music and color

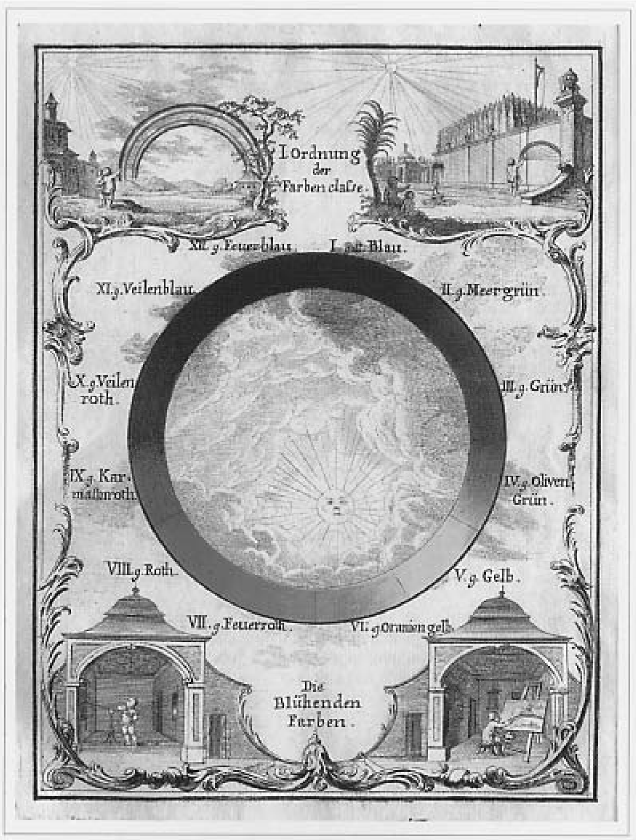


Fig. 2-13 Continuous color circle of Ignaz Schiffermüller, with twelve classes of colors, 1771. (See color plate.)

but was interested in a system that could be used, among other things, to deduce rules of color harmony. This required a physical expression of the system in considerable detail. Castel's central color spiral segment was flattened into a color circle. To specify the twelve hues Schiffermüller used more descriptive hue terms. Again, they were arranged in attempted perceptually equal steps. Between yellow and red there are two intermediate steps: orange and fire red; between red and blue there are four steps: crimson red, violet red, violet blue, and fire blue; between blue and yellow there are three steps: sea green, green, and olive green. Each of the twelve hues is the key representative of a "color class," and Schiffermüller used Roman numerals to identify them, starting with blue (Fig. 2-13). Next Schiffermüller began to create steps toward white and black but did not get beyond three examples in the blue region (Fig. 2-14). His book *Versuch eines Farbensystems* (Attempt to construct a color system) was published in Vienna in 1771, thirteen years after Tobias

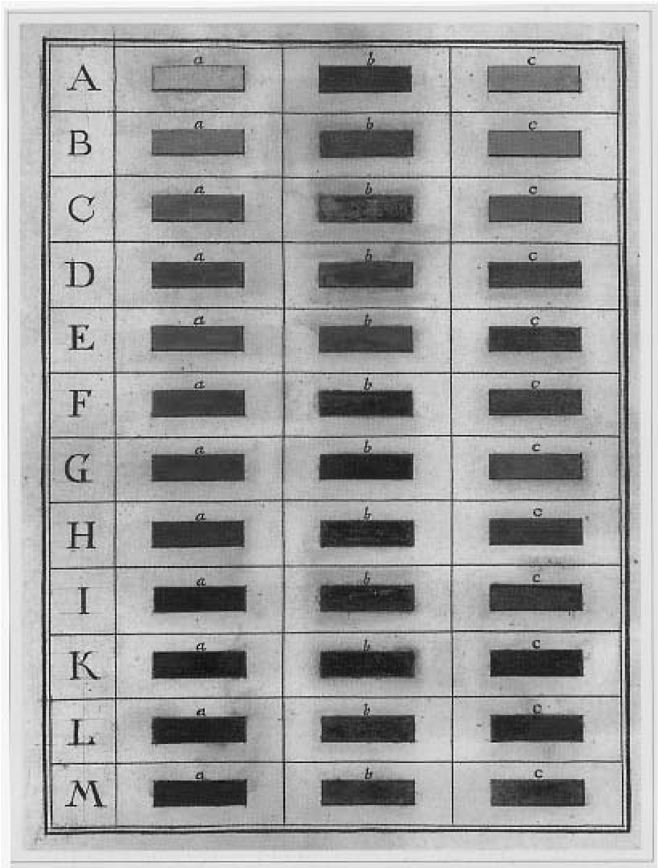


Fig. 2-14 Schiffermüller's tonal scales of three blues, from his 1771 work. Some of the colorations have deteriorated. (See color plate.)

Mayer presented his three-dimensional system in Göttingen and one year before Lambert published his book on the subject of a color pyramid (see below).

Why Color Circles?

One of the questions arising is why, given the linear nature of the visible spectrum, hues began to be uniformly represented in a hue circle. The first extant circular representations of colors appeared in medieval (continuing into the fifteenth century) diagnostical diagrams of the color of urine.[28] Obviously these were not complete hue circles but may have been influential for circular depictions of colors of different hues. The circles by Forsius and Fludd do not represent hue circles. Both of Forsius' two circles have a central vertical axis of the colors of a gray scale, and he placed tonal scales of four primary hues, red, yellow, green, and blue, on circular segments on either side of the gray scale. It is not obvious that the circle has a clear meaning. In case of Fludd it is not unlikely that mystical, perhaps alchemical reasons produced his circle, which begins with white and ends in black next to it.

Newton's choice of a circle for his spectral colors may have been influenced by Descartes's circular diapason, but it was also based on his knowledge of desaturation of spectral light by white light, common for all spectral colors. Thus the radius length is an indicator of the saturation of a color. Newton's placing of the spectral hues on the periphery of the circle in proportion to a musical scale meant that opposite colors were not exactly complementary colors (if close to them). For this reason, and the varying chromatic strength of different hues, opposite colors "when mixed in an equal proportion . . . the Colour compounded of those two shall not be perfectly white, but some faint anonymous Colour." Newton also was aware of compounded purple colors that belong near line D–O in his circle and that, thus, spectral hues and compounded purples can form a closed series. Newton chose a distribution of hues according to a musical scale on his color circle that, with its common white center, also represents geometrical parsimony.

C.B.'s color circle, as shown, is a pigment-mixing diagram based on the idea of yellow, red, and blue as primaries. The circular form, unlikely to have been influenced by Newton, may have been the result of independent realization by a painter that, perceptually, a series of hues derived from three primary pigments and produced by mixing neighboring pigments can return upon itself.

As an entomologist Schiffermüller had interest in systematic color arrangement as a means for classifying butterflies and other colored insects. He also had contact with many artists and was aware of issues in finding harmonious color combinations, devoting a chapter of his book to this question. In addition he thought that the time had arrived to create a complete system of colors. His circle was a first attempt in this direction. Its form was likely derived from the spiral arrangement of hues of different tonal values of Castel and his presumed knowledge of Newton's work and C.B.'s book. By this time the hue circle had become a convention that made intuitive sense. Many future systems would be based on it.

The arguments offered so far for the color circle consist of rationalizations based on likely reasoning and insight available in the seventeenth century. There is a further and perhaps stronger argument. As will be shown later and in Chapter 6, a form of data analysis called multidimensional scaling (see Chapter 3), when applied to spectral colors, results in a geometrical distribution of data points best fit with a circle. In addition mathematical analysis of large collections of spectra of colored objects by various methods, but without consideration of human color vision properties, locates the spectra in spaces where a series of hues approximately forms a circle. A hue circle therefore is a pattern that may have become subconsciously apparent and is the result of one of the strategies of our visual system.

A full-fledged color order system based on a hue circle was delayed because of the growing importance of the idea of three primary chromatic colors, presented as a triangle, that could be used to create all other hues by mixture. The source of the idea of yellow, red, and blue as three primary colors is unknown, and it might have developed from dyeing and painting technology. We have seen these three chromatic primary colors in d'Aguilon's diagram of 1613. The same primary chromatic colors are mentioned by Robert Boyle in his *Experiments and Considerations Touching Colours* (1664). Using "pigments" as generic word for colorants he wrote: ". . . there are but few Simple and Primary Colours (if I may so call them) from whose Various Compositions all the rest do as it were Result. . . . I have not yet found, that to exhibit this strange Variety they [painters] need imploy any more than *White*, and *Black*, and *Red*, and *Blew*, and *Yellow*; these *five*, Variously Compounded, . . . being sufficient to exhibit a Variety and Number of Colours, such as those that are altogether Strangers to the Painters Pallets, can hardly imagine. . . . by these simple compositions again Compounded among themselves, the Skilfull Painter can produce what kind of Colour he pleases, and a great many more than we have yet Names for." Yellow, red, and blue had been celebrated as key colors in several works of the contemporary French painter Nicholas Poussin (1594–1665). Francis Glisson was a supporter of these colors as primary chromatic colors. The inventor of four-color printing (yellow, red, blue, and black), Johann Christoffel Le Blon (1667–1741), published his book on this subject, *Coloritto*, in 1725. As a result, in the first explicit steps toward a three-dimensional color system, the form of a triangular pyramid (tetrahedron) was used with the primary colors yellow, red, blue, and white at the vertices.

2.6 MAYER AND LAMBERT'S COLOR SOLIDS

Tobias Mayer (1723–1762), self-educated German geographer, astronomer, and physicist developed, perhaps through his work with map printers, an interest in color and presented in 1758, in Göttingen, a public lecture on the relationship of colors. A report about the lecture appeared in a local journal. The lecture was published posthumously in 1775 in a collection of works, *Opera inedita Tobiae Mayeri* (Unpublished Works of T. M.) with the title *De affinitate colorum commentatio* (Commentary on the relationship of colors).[29] Here Mayer, for the first time, presented a plan for a systematic color solid, based on three primary chromatic colors. In principle, the samples representing the solid were to be spaced to be visually equidistant. The chosen geometric form of the solid is that of a double tetrahedron. Even though using pigments as a basis for his work, he also had colored lights in mind: "There are three simple or basic colors and no more than that, from mixture of which all others can be generated, which themselves from others, in whatever ratio they might be mixed, cannot be generated in any way: red, yellow and blue. We see them in rainbows, but even more distinctly in rays of the sun captured by a glass prism,

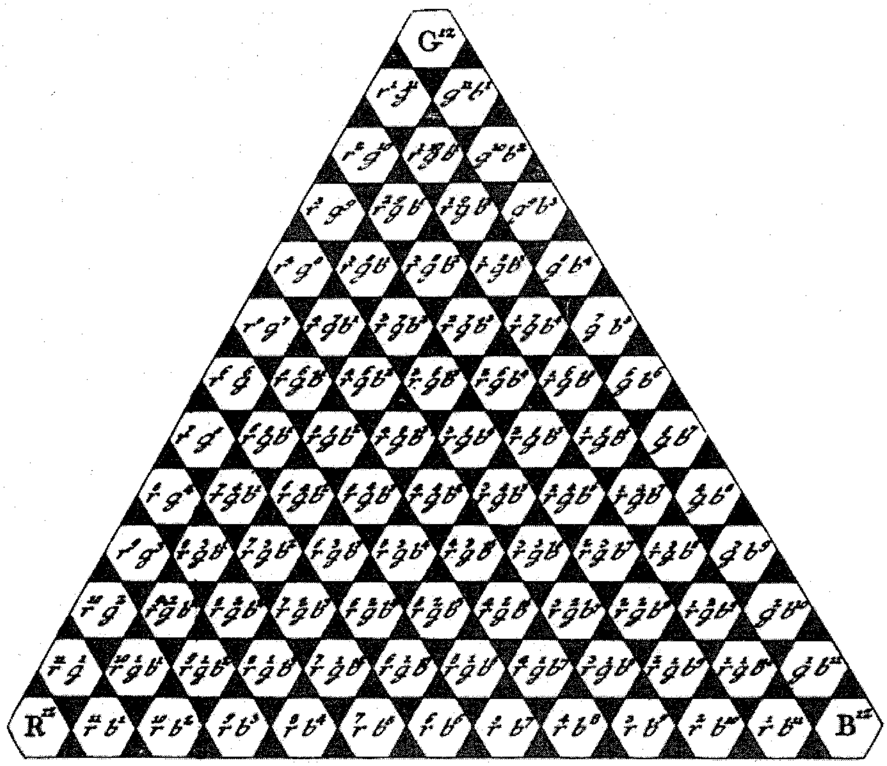


Fig. 2-15 Arrangement of 91 colors of the central plane of Tobias Mayer's double triangular pyramid, with the color designation scheme based on the three primary colors R, G, and B, 1758.

though there they are accompanied and surrounded by secondary colors." Based on experiments with pigments Mayer concluded that there should be twelve steps between each of the three primary colors as well as (just as Glisson) twelve steps toward white from the central plane and twelve steps toward black: "One has to employ such a ratio between colors to be mixed so that it can be expressed with numbers which are not very large. . . . Neither in architecture nor music are proportions greater than twelve generally accepted since such ratios could barely be apprehended by unaided senses." Mayer recognized the principle of threshold differences (he may have been the first to express it relatively unambiguously).

Mayer designated his three primaries with **r**, **g** (for *gelb*, yellow), and **b** and indicated their value in the ratio with superscript numbers. In this manner an equilateral triangle is generated in which there are 91 designated colors (Fig. 2-15). He maintained a constant sum of colorant parts in mixtures with white and black and on the next level where colors were mixed with one part white

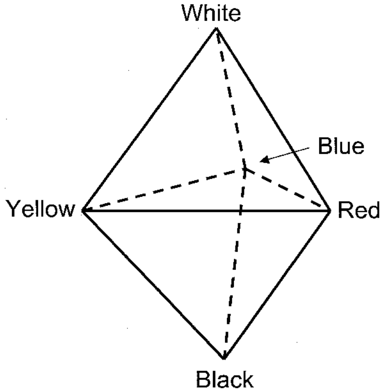


Fig. 2-16 Schematic representation of Tobias Mayer's double triangular pyramid color solid.

each, and with eleven divisions per side, obtained 78 colors. Pursuing this approach he ended up with 364 colors above the middle plane and the same number below, a total of 819 color samples for his double tetrahedron (Fig. 2-16), each with its own designation. Mayer did not envisage an explicit central gray scale made from mixtures of white and black but expected that mixture of identical parts of the three primary colors (colorants) in the central plane would produce neutral grays. In the published paper Mayer did not assign specific pigments to his primary colors. However, in the newspaper report of his presentation in Göttingen the "Mayer coordinates" of several pigments were identified, among them, with the identifier 12 (i.e., primary colors), orpiment (king's yellow), vermilion, and azurite (*Bergblau*). (Note that these are the same pigments as those selected by Glisson.)

It is apparent that Mayer's proposal was one of theory, applicable also to mixture of lights. It is known that he made some experiments with pigment mixture but not to what extent he investigated visually equally spaced scales. There is no indication that he was fully aware of the nonlinear relationship between pigment mixture ratio and the resulting visual experiences, as already Johann Wolfgang von Goethe in his comments indicated (Goethe, 1810). Mayer's publisher, Georg Christoph Lichtenberg, added a commentary where he applied Newton's center of gravity principle to Mayer's proposal. He also printed a reproduction of a color triangle hand painted by Mayer and commented extensively on the difficulties in obtaining a good result. In 1772, three years before Mayer's previously unpublished works appeared in print, the Alsatian mathematician, physicist, and astronomer Johann Heinrich Lambert (1728–1777) issued a small book under the title *Beschreibung einer mit dem Calauischen Wachse ausgemalten Farbenpyramide wo die Mischung jeder Farbe aus Weiss und den drey Grundfarben angeordnet, dargelegt und derselben Berechnung und vielfachen Gebrauch gewiesen wird* (Description of a color pyramid painted with Caulau wax where the mixture of each color from white and the three basic colors is arranged, explained, and its calculation and various uses indicated). He had read the report about Mayer's lecture and cited it in his book *Photometria* (Lambert, 1760). Using Mayer's proposal as a basis Lambert developed his own system. He also employed colorant mixtures and the same identification scheme. Lambert justified the triangular scheme from a presumably visually equidistant color circle in which yellow occupies the 12 o'clock position, the border of red against violet the 4 o'clock and blue the 8 o'clock positions. He searched among the available pigments for a yellow that was neither reddish nor greenish and, correspondingly, for a neutral red and blue and on that basis selected gamboge, carmine (cochineal), and *Berlinerblau* (prussian blue, potassium ferric ferrocyanide).[30] A condition Lambert required, beside neutrality of hue, was that his colorants should approach as nearly as possible the intensity of spectral colors. He commented: "I leave it undecided if in the future colorants will be found which approach the spectral colors even closer than the mentioned carmine, gamboge and Prussian blue." Realizing from practical experience that the coloristic strengths of his three colorants was significantly different, he established them by finding (as Glisson did) middle colors between the three primaries and reported them as two parts carmine to three parts prussian blue to twelve parts gamboge. Mixing his primary colorants in a corresponding ratio (12 parts prussian blue, 12 parts gamboge, and 2 parts carmine) produced a near black when applied to white paper and Lambert did not see a need for mixtures of his colorants with black and discarded the lower half of Mayer's pyramid. Lambert expressed dissatisfaction with Mayer's uniform twelve steps. He had the Prussian court painter Calau experiment to develop originally a six-step, later a nine-step visually equidistant triangle. Mixing the colorants with a near water-soluble wax of Calau's invention and gum resulted in colorations of high chroma and good stability. Since all three colorants had a high degree of transparency, Lambert believed he could avoid adding a white pigment and instead used the white of the paper together with increasing dilutions of the colorant mixtures to achieve the tonal declines toward white. Nevertheless, Lambert found that he had to produce a total of 67 mixtures to color his pyramid of 108 colors so that the result met his visual criteria. Interestingly, near black colors are pushed close to basic blue (colors 11, 12, 19, 20; see Fig. 2-17) at the lowest level rather than at 21, 27, 28 where, based on the gravimetric rule, one would expect them.

Lambert proposed for the grades along the sides of the triangle a color naming system that is only moderately intuitive. He used a system borrowed from the methodology for designating directions around the compass, for example: 1, blue; 10, blue toward red; 18, bluish reddish blue; 25, bluish red toward blue; 31, blue red or red blue; 36, red blue toward red; 40, reddish bluish red; 43, red toward blue; 45, red. For the interior of the triangle he used primarily descriptive color terms such as "chestnut red brown."

Lambert saw his pyramid as a general color atlas of use to merchants, for example, to determine if they had fabrics in stock in the desirable colors. Consumers could use the atlas to decide what color clothing to buy and what

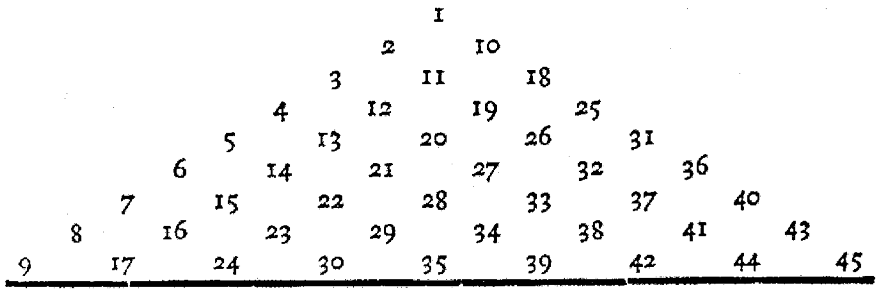


Fig. 2-17 *Identification scheme of primary colors (1, 9, and 45) and color mixes used in the basis plane of Lambert's triangular pyramid color solid, 1772.*

colors to combine: "Caroline wants to have a dress like Selinda's. She memorizes the color number from the pyramid and will be sure to have the same color. Should the color need to be darker or go more in the direction of another color, this will not pose a problem." Lambert believed the pyramid to be of particular use for dyers. If they could find three dyes approximating his three primary colorants, and after having determined their coloristic strength relative to Lambert's primaries, dyers could calculate how much of each dye to use to achieve a given shade in the pyramid. Other potential users were artists who, after sketching, for example, flowers in pencil and writing their color numbers next to them, could reproduce the natural colors in the studio by referring to the corresponding colors in the pyramid.

Lambert built a wooden pyramidal structure, and included an image of it in his book (Fig. 2-18). In this display the 108 selected colors could be properly displayed. For comparison purposes he had Calau paint chips with twelve common artist's colorants along the bottom border: naples yellow, king's yellow, orpiment, azurite, smalt, indigo, lamp black, sap green, chrysocolla, verdigris, vermilion, and florentine lake.

Lambert went about the task with a considerable amount of scientific zeal. Aside from developing an arithmetic of colorant mixture, he explained the result of black from a mixture of his three primary colorants logically as the simultaneous prevention of the activity of each of the three colorants by the other two. His color pyramid is the first attempt to create a geometrical, physical model of object color experiences achievable with his primary colorants, a color solid. The shortcomings of Mayer and Lambert's proposals will become apparent as we proceed.

2.7 COLOR CIRCLES FROM HARRIS TO HENRY

Moses Harris

Color circles continued to be proposed in various formats. In 1766 Moses Harris (1731–1785), an English entomologist and illustrator of insects, dedi-

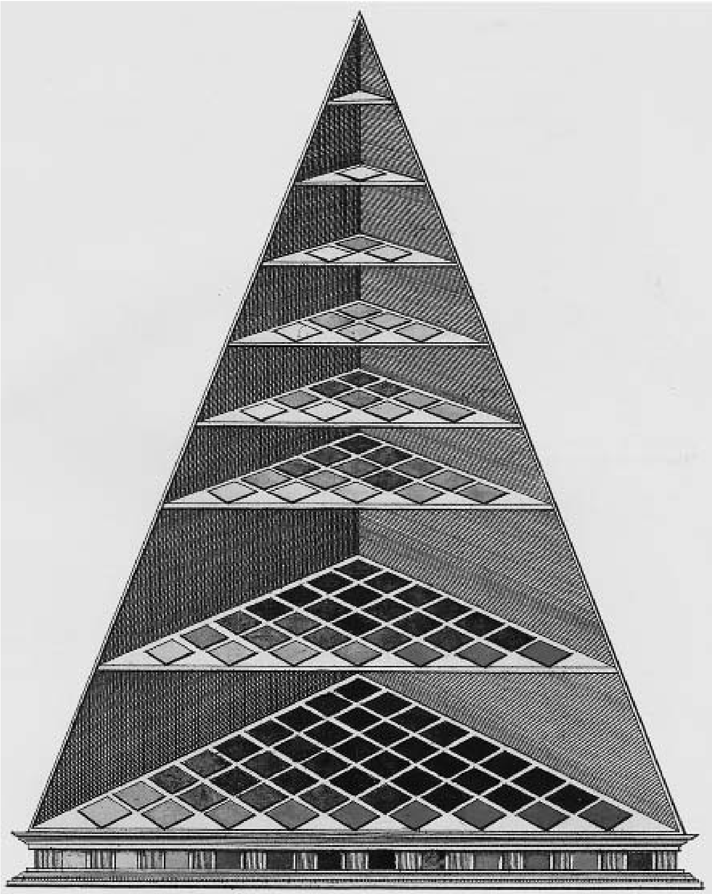


Fig. 2-18 *Depiction of Lambert's color pyramid, 1772. The lowest level contains the 45 colors identified in Fig. 2-17. The higher levels contain reduced sets at higher lightness, ending in white on top of the pyramid. Black is located on the lowest level. The colors displayed on the front of the model represent well-known artist's pigments of the time. (See color plate.)*

cated a small volume titled *The Natural System of Colours* to the then president of the Royal Academy, the painter Joshua Reynolds. It contains copperplates illustrating two color circles, one based on "prismatic" primaries yellow, red, and blue (whose mixture is illustrated as black, however; see Fig. 2-19) and one based on "compound" primaries orange, green, and purple. There are eighteen hues per circle and each hue is illustrated in twenty gradations. The first circle is said to contain only those colors "shewn by the prism." The second one contains "all other colours in nature, not found in the prismatic part." Harris provides specific examples of what he means when using color names, for example: red–vermilion, wild poppy; blue–ultramarine, cornbottle-flower; green–sap-green, leaves of the lime-tree.

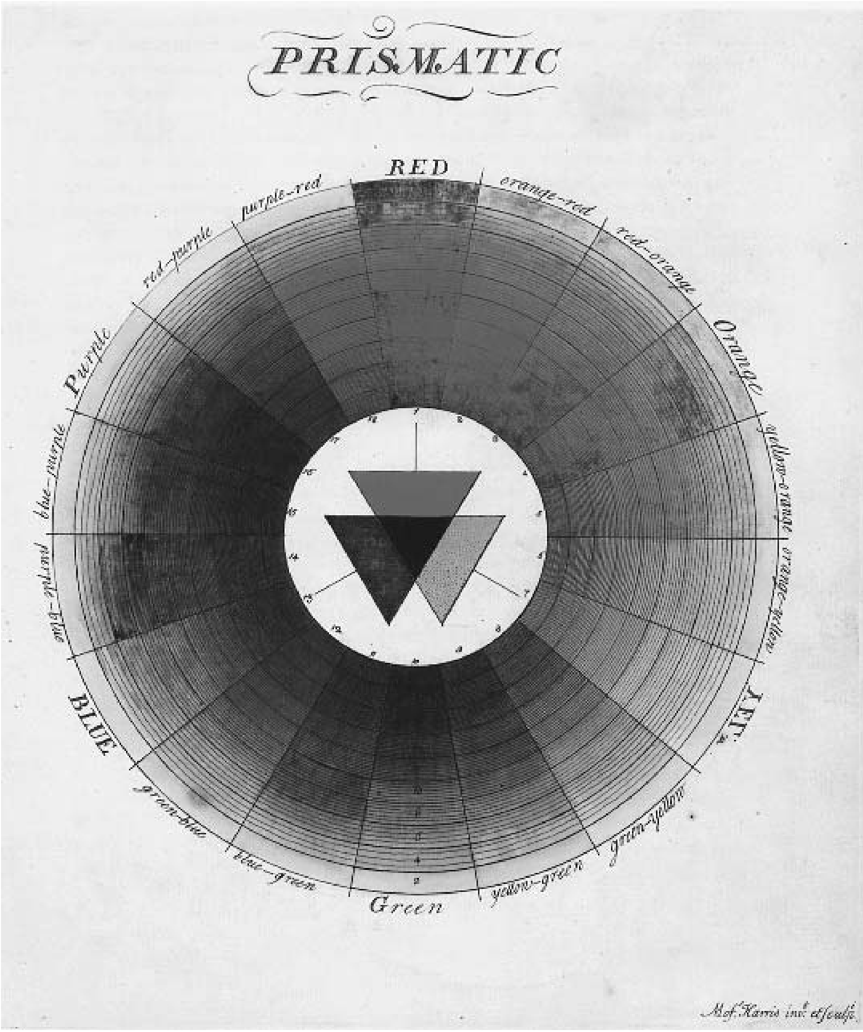


Fig. 2-19 *Prismatic version of Moses Harris's color circle of 1786. Some deterioration of colorants is evident. (See color plate.)*

Concerning the twenty gradations of each hue Harris explained: "The number of colours in this circle are supposed to be eighteen, each of these being divided into twenty parts or degrees of power, from the deepest or strongest to the weakest, . . . so that each of the colours in the innermost or smallest circle contains 20 degrees of power, but each of the outermost but one." They then represent tonal scales from the maximal color near the center of the circle to whitish colors at the periphery. Harris' circle is the first in which maximally contrasting colors are systematically opposed: "Suppose it is required what colour is most opposite or contrary in hue to red, look directly opposite to that colour in the system and it will be found to be green . . . of every colour or teint in the system no one of them contain in their compositions any of the colours of which those on the opposite side are formed. . . .” Harris remarked on the problems of coloring his system properly. Certain colorants (indigo, gamboge, carmine, sap green) are said by him to contain twenty degrees of power while others do not.

Frisch

A color circle with tonal scales in direction of both black and white was offered in 1788 by the Prussian painter Johann Christoph Frisch (1738–1815).[31] Frisch’s immediate purpose was to demonstrate tonal colors in much detail as an aid for painters at a time when subdued coloration was fashionable. Frisch’s color circle, presented only in outline, consists of 40 concentric rings, beginning with black in the center. Only two blackish colors are found on the second ring: blackish red and blue. On ring 3 the range is expanded to eight hues with reduced blackness: yellow, orange, red, purple, violet, blue, sea green, and leaf green. On ring 4 the hues swell to sixteen, with further reduced blackess and on ring 5 to 32, the maximum number. Here the explicit description ends and rings 6 to 10 were to contain tonal values toward white. Rings 11 to 20 are even less well described but apparently were to contain further tonal values ending up in black again at the periphery. Frisch expressed the opinion that there are more equal-sized hue steps between yellow and blue and between red and blue than between yellow and red, and he selected the size of his hue segments accordingly.

Goethe

Because of its importance in aesthetics the color circle of the German poet and natural philosopher Johann Wolfgang von Goethe (1749–1832), introduced in his *Zur Farbenlehre* (On the doctrine of colors) of 1810, should be mentioned here. Goethe’s six-color circle is based on two triangles: the fundamental three colors with red on top, and the intermediary colors. The position of red results from a process Goethe called *Steigerung* (intensification) in which red represents the peak. At the same time it reflects successive contrast: “To recognize quickly which colors are called forth by this contrast one can use the illuminated color circle of our tables that is altogether organized according to natural principles and offers here good service because its opposing colors are those that are demanding each other in the eye. In this manner yellow demands violet, orange blue, purple green and vice versa.” Goethe interpreted his color circle also in regard to principles of color harmony.

Color circles for the primary purpose of demonstrating rules of color harmony have also been developed by the German painter Matthias Klotz (1748–1821) in 1816 (see Section 2.10), the English colorant producer and dealer George

Field (1777–1854) in 1817, the French chemist Michel-Eugène Chevreul (1786–1889) in 1839 (see Section 2.12), the German art historian Friedrich Wilhelm Unger (1810–1876) in 1858, the Austrian physiologist and son of an artist Ernst Brücke (1819–1892) in 1866, the French physiologist Charles Henry (1859–1926) in 1888, as well as others.

2.8 THREE PRIMARY COLOR THEORIES

The idea of three fundamental colors received an important boost in 1801 from the English physicist and physician Thomas Young (1773–1829). He restated an hypothesis first mentioned by the English glassmaker George Palmer in his *Theory of Colours and Vision* (Palmer, 1777).[32] Palmer believed that "The surface of the retina is compounded of particles of three different kinds, analogous to the three rays of light [necessary to mix all colors]; and each of these particles is moved by its own ray." In Young's mature version the eye is provided with distinct sets of nervous fibers: ". . . if we seek for the simplest arrangement, which would enable [the eye] to receive and discriminate the impressions of the different parts of the spectrum, we may suppose three distinct sensations only to be excited by the rays of the three principal pure colours, falling on any given point of the retina, the red, the green, and the violet, while the rays occupying the intermediate spaces are capable of producing mixed sensations, the yellow those which belong to the red and the green" (Young, 1824). However, Young's three primary colors were not those of Boyle, Le Blon, or the dyers and painters, a situation that continued to cause confusion until resolved by Maxwell and Helmholtz (see glossary entry *primary colors*).

In 1809 the English painter and conchologist James Sowerby (1757–1822) published *A new elucidation of colours, original, prismatic, and material: showing their concordance in three primitives, yellow, red and blue, and the means of producing, measuring, and mixing them: with some observations on the accuracy of Sir Isaac Newton*. In this treatise he attempted a synthesis of ideas about colors from lights and from colorants. Sowerby experimented with additive color mixture by placing narrow strips of paintings of his three primary colorants gamboge, carmine, and prussian blue next to each other and viewing them from a distance. As a painter he was interested in the effects of adding white and black pigments to his primary colorants. In this work a two-dimensional color order system progresses from white in the center to the most intense chromatic colors and from there toward black on the periphery.

2.9 RUNGE'S COLOR SPHERE

Phillip Otto Runge (1777–1810), German romantic painter and acquaintance of Goethe, published in his last year *Die Farben-Kugel oder Construction des*

Verhältnisses aller Mischungen der Farben zueinander, und ihrer vollständigen Affinität, mit angehängtem Versuch einer Ableitung der Harmonie in den Zusammenstellungen der Farben (Color sphere or construction of the relationship of all mixtures of colors, and their complete affinity, with an attached essay on the derivation of harmony in color compositions), 1810. Runge made first mention of his color sphere in a letter to Goethe in 1807. The book contains also an essay by Runge's friend Henrik Steffens *Über die Bedeutung der Farben in der Natur* (On the significance of colors in nature).[33]

Runge distinguished between transparent and opaque colors. The color sphere was constructed to contain the totality of opaque colors created from the five elemental colors red, yellow, and blue as well as black and white. The central color circle has blue on top (in agreement with Castell and Schiffermüller) green at 4 o'clock and red at 8 o'clock. Of the twelve segments reddish colors occupy seven, as do yellowish. Bluish colors occupy five and greenish colors only three segments. The circle is described as a rounding out of two opposing equilateral triangles (in agreement with Goethe). The first contains the fundamental colors yellow, red, and blue. The other triangle contains the mixture colors seen as half way between the fundamental colors: orange, violet and green. When all three colors of a triangle are mixed in equal amounts, neutral gray results. A gray axis is erected in the center of the circle so that gray is obtained by mixture of black and white, by mixture of the three primary chromatic colors, or by mixture of two opposing colors in the hue circle. The gray axis and the color circle form a double cone that is extended to the ideal geometric figure of the sphere (Figs. 2-20, 2-21).

> Since all three colors, blue, yellow and red stand at the same distance from white and black, therefore, the center of the color disk in which those three have lost their individuality through equal activity must be in the same relationship and

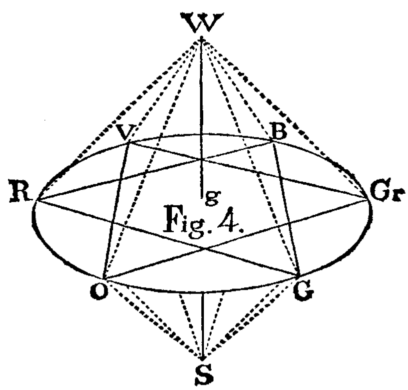


Fig. 2-20 *Runge's basic color triangle (R for red, G for yellow, B for blue) combined with the triangle of secondary colors (orange, violet, green), extended horizontally to a circle and vertically in cone shape toward white and black, 1810.*

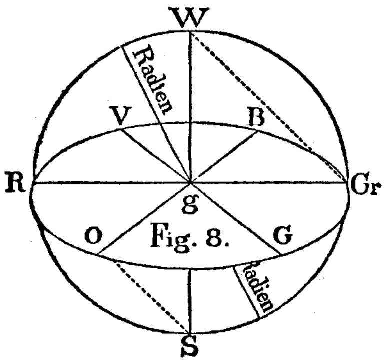


Fig. 2-21 *Runge's double-cone color solid rounded to final spherical form, with white at the top and black at the bottom of the sphere, 1810.*

> in the same distance to white as to black as those three. Since both of these points (the center point between white and black and that of the triangle Blue, Yellow, Red) mathematically coincide it follows that both must be one and the same . . . and that from the identical difference a complete indifference results into which all individual qualities have dissolved. . . . This point, since it is in equal distance to all five elements, is therefore to be seen as the general central point of them all. . . . All mixtures that result from the inclination of a point on the complete color circle toward white or black (a tendency which is common to all these points) will slowly loose themselves toward white and toward black. . . . as the differences of all points of the inclination toward white or black from the central point are radii the points form nothing but circle segments ending in the poles white and black. . . . Thereby, the complete relationship of all five elements, through its differences and inclinations, forms a perfect sphere. Its surface contains all five elements and those of their mixtures which are generated in friendly inclination of their qualities and toward its center all colors of the surface dissolve in equal steps into a balanced gray. . . . Every color is placed in its proper relationship to all pure elements as well as all mixtures and in this manner the sphere is to be seen as a general table by which he who requires various tables in his business, can always find the relationship of the totality of all colors. It now must be evident to the attentive reader that it is not possible to find a plane figure that is a complete table of all mixtures; the relationship can only be presented as a solid.

In a hand-colored copperplate figure views of the sphere from the white and the black poles, a horizontal cross section along the equator and a vertical cross section through the two poles are given (Fig. 2-22). Unlike Lambert's, Runge's idea was not to offer just a color atlas but to present an idealized theoretical construct that not only was meant to represent color relations for the painter but also presumed deeper psychological and mystical relationships gleaned in part from Goethe. As indicated in the title of the publication, Runge used the sphere geometry to also develop his ideas about color harmony.

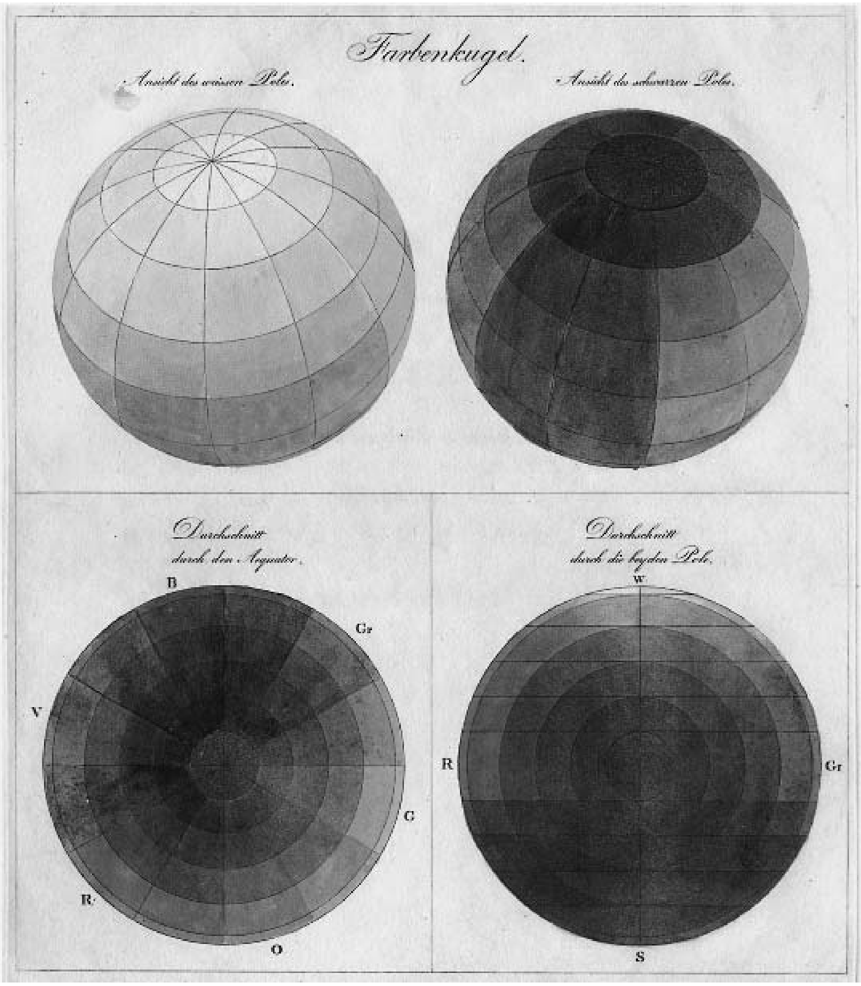


Fig. 2-22 Hand-colored copperplate of Runge's Farben-Kugel (1810). Views toward the white and black poles are on top. The equatorial cross section is on bottom left and the polar cross section on the right. There are four saturation steps between the full color on the surface and the middle gray in the center of the sphere. (See color plate.)

While representing the most complete color order system so far, Runge's proposal still suffers from unresolved issues. The central axis from pole to pole represents a gray scale, but the full colors of the equatorial circle are of various lightnesses so that the meaning of the vertical axis is not explicitly defined. Colorants that in equal mixture produce a neutral gray do not exist, but Runge did not address the problems materializing when mixing colorants. These and other issues have only been resolved in the twentieth century.

2.10 THE CYLINDRICAL SYSTEM OF MATTHIAS KLOTZ

Runge was an admirer of Goethe's *Farbenlehre*, but another of his countrymen, the Bavarian court and theater painter M. Klotz (1748–1816), was highly critical of Goethe's efforts and paid in his country for his views with a lack of recognition of his own efforts in this field.[34] In 1797 he announced in a journal *Aussicht auf eine Farbenlehre* (Prospect for a color doctrine), followed in 1806 with Notification concerning a color doctrine, in 1810 with Explanatory notification, and finally, in 1816 with *Gründliche Farbenlehre* (a thorough doctrine of color, published in 300 copies at the expense of the author). Klotz' book consists of two sections, one called *Chromatik* and the other *Prismatik*, the former concerned with object colors, the latter with lights. As a painter Klotz had extensive experience in pigment mixture and believed in the primacy of yellow, red, and blue. Before Runge, Klotz defined his primary colors in the sense of unique hues, as later used by Hering. Comparable to Lambert, Klotz used white paper as his source of whiteness by diluting the pigment dispersions appropriately. In this manner he prepared nine visually equidistant grades (including white) of the three primary pigments, as well as of a gray scale. The highest degree of saturation was obtained at grade 4 of his scale. The darker grades were termed "oversaturated" by Klotz. He next mixed seven intermediary combinations between each of the primaries at grade level 4, resulting in a total of 24 equidistant hues. Klotz believed that in this color circle complementary colors were placed opposite. Klotz placed neutral midgray in the center of the circle and placed between each of the 24 full hues and the central gray three steps of reduced saturation. In this manner he obtained a color chart of 97 colors, with saturated chromatic colors at the periphery and neutral gray at the center. Lightness varies considerably in this chart. Klotz recognized three modification potentials in his system: *Buntmodifikation* (hue modification), *Brechungsmodifikation* (saturation modification) and *Hell-Dunkelmodifikation* (lightness/darkness modification). Klotz understood the structure of the cylindrical model resulting from these modifications but refrained from explicitly describing a cylinder model because he could not see clearly how to bring object colors and light colors in such a system into agreement.

2.11 THE EARLY DEVELOPMENT OF PSYCHOPHYSICS

Uniformity of spacing of colors in color scales, an atlas, or color solid had been a clear but difficult to implement goal for Glisson, Castel, Mayer, Lambert, Runge, and the just mentioned Klotz. It began to be approached from a different angle as part of general psychological investigations. The idea of a differential threshold for illumination was introduced by the French mathematician Pierre Bouguer (1698–1758) in the posthumous *Traité d'optique sur la gradation de la lumière* (Optical treatise on the gradation of light, 1760). He had experimentally determined the intensity a light must have to make a weaker one disappear. In a particular experiment involving the light of candles he found that an increment of 1/64 of the intensity of the candlelight had to be added to it so that the difference was perceptible. Bouguer concluded that just perceptible differences in brightness were caused by changes in illumination and represent a nearly constant fraction.[35]

The term limen was introduced by the German philosopher Johann Friedrich Herbart (1776–1841). He defined a limen or threshold of consciousness, by arguing that ideas could only emerge above a threshold of activity of consciousness.

Weber and Fechner

Thresholds of human senses soon began to be investigated quantitatively by Ernst Heinrich Weber (1795–1878), German anatomist and physiologist. Weber began with investigating the sense of touch. Later he extended his inquiries to temperature discrimination and visual discrimination of the length of lines. Regarding the perception of weight differences Weber stated in 1834: "For experience has taught us that expert and practiced men feel a disparity of weight if it is not less than 1/30 of the heavier weight, and perceive the disparity to be the same if, in place of ounces, we put drams." Regarding length of lines he found: "The length, therefore, in which the disparity lies, even though it is two times less on the former instance, is, however, recognized just as easily, because in both cases the difference between the compared lines is 1/100 of the longer line." In 1846 he contributed the chapter *Der Tastsinn und das Gemeingefühl* (Sense of touch and common sensibility) to *Wagner's Handwörterbuch III*, part of a multivolume encyclopedia of psychology, which made his findings widely known. Weber did not state his findings in an explicit law.

The German physicist and mystic Gustav Theodor Fechner (1801–1887) began to think about issues of psychophysics before he knew of Weber. In his mystical work *Zend-Avesta* (Fechner, 1851) he stated: "If the strength of the physical activity actually underlying some mental activity at some point in space and time is measured by its energy β (energy understood in the sense of mechanics) and if its change, assuming an infinitely small part of time and space, is named $d\beta$, then the accompanying change in the intensity of the mental activity, to be estimated by feeling or in consciousness, is not proportional to the energy change $d\beta$, but to the relative change $d\beta/\beta$. . . ." In his *Elemente der Psychophysik* (Elements of psychophysics; Fechner, 1860) Fechner introduced the term *psychophysics* and argued that sensation cannot be measured. All that can be measured are stimuli and the amount of stimuli that result in a particular sensation or the difference between two sensations. The smallest difference that results in a noticeable difference is the threshold difference. Fechner saw the just noticeable difference (JND), expressed in terms of stimulus, to be the unit of sensation, with the magnitude of sensation being the sum of JNDs that lead from the absolute threshold to a given sensation. Fechner expressed Weber's findings, meanwhile known to him, for the JND increments as follows:

$$d\gamma = K\frac{d\beta}{\beta}, \tag{2-1}$$

where $d\gamma$ is the difference in sensation, K is a constant, and $d\beta$ is the difference in stimulus β. Fechner assumed that if this equation is valid for the JND, it is also valid for small increments of β, the stimulus: "In fact, if one multiplies $d\beta$ *and* β by any number, so long as it is the same number for both, the proportion remains constant and with it also the sensation difference $d\gamma$. This is Weber's law. If one doubles or triples the value of the variation $d\beta$ without changing the initial value β, $d\beta$ is also doubled or tripled." If equation (2-1) applies, Fechner reasoned, the relation between sensation and stimulus can be expressed in the simplest form as

$$\gamma = \log\beta. \tag{2-2}$$

Fechner saw this formula as a differential formula and integrated it with the result

$$\gamma = K(\log\beta - \log b), \tag{2-3}$$

where b represents a threshold; or expressed differently,

$$\gamma = K\log\left(\frac{\beta}{b}\right). \tag{2-4}$$

Fechner termed the value β/b the fundamental stimulus value. "The magnitude of the sensation γ is . . . proportional to the logarithm of the fundamental stimulus value." Fechner named this formula the *Maasformel* (measurement formula). It expresses the number of JNDs the stimulus is above threshold. "In the measurement formula one has a general dependent relation between the size of the fundamental stimulus and the size of the corresponding sensation. . . . This permits the amount of sensation to be calculated from the relative amounts of the fundamental stimulus and thus we have a measurement of sensation."

Fechner's work resulted in three types of experimental methods applied in psychology: just noticeable differences or the method of limits, the method of right and wrong cases or method of constant stimuli, and the method of average error. Each of these methods found champions in succeeding years and is still employed today.

Fechner introduced the idea of "inner" and "outer" psychophysics. Outer psychophysics considers the relationship between the measurable physical stimulus and the reported psychological response. Fechner understood inner psychophysics to mean the relationship between the excitation of nerve fibers and the mind. Thus he saw the entire process as consisting of physical stimulus–excitation–sensation–reported response. He was particularly interested in where in this chain the cause of the logarithmic relation is found.

Plateau and Delboeuf

In 1872 the Belgian physicist Joseph Antoine Ferdinand Plateau (1801–1883) read a paper before the Belgian Royal Society: *Sur la mesure des sensations physiques, et sur la loi qui lie l'intensité de ces sensations à l'intensité de la cause excitante* (On the measurement of physical sensations and on the law that connects the intensity of sensations to the intensity of the cause of the excitation). Plateau asked several painters to halve the perceptual distance between white and black oil-painted squares and found that that the results were in close agreement. He also concluded from viewing etchings in different lights that luminance ratios, and not differences as Fechner required, were applicable and that the formula connecting luminance with perceived brightness was a power function.

Objections were raised against Fechner that humans have no intrinsic sense of the magnitude of a sensation. This objection was countered by the Belgian physicist Joseph Rémi Léopold Delboeuf (1831–1896) who played an important role in early psychophysics. He showed that observers can judge the size of the interval between two sensations immediately and directly (they can effectively compare perceptual distance between two different gray samples against the perceptual distance between two different, say, red samples). Delboeuf demonstrated that for sensations to be measured, they only have to be arranged on a measurable scale, and that absolute magnitude was not required. This idea of sense-distance was further developed by the American psychologist Edward Bradford Titchener (see below). Perceptual scaling of color space since is usually based on Delboeuf's idea. Delboeuf was a pupil of the Belgian inventor of statistics Adolphe Quetelet (1796–1874) who was the first to apply the Laplace and Gauss normal law of error to human data. Thus Delboeuf was sensitive to the effects of judgment error on the determination of perceptual increments. Delboeuf also made an elaborate experiment in which he had observers perceptually halve various distances between gray samples. His analysis of the results confirmed the applicability of the Weber-Fechner law. As a result Plateau's power law fell into obscurity until the 1930s.

Investigation of a visually equidistant eight-step gray scale by the German psychologist Hermann Ebbinghaus (whom we will encounter again later) in 1887 resulted in measured light intensities not quite representative of the Weber-Fechner law.

2.12 CHEVREUL'S HEMISPHERIC SYSTEM

Design of a hemispheric color order system was offered in 1839 in his book *De la loi du contrast simultané des couleurs* by Michel-Eugène Chevreul

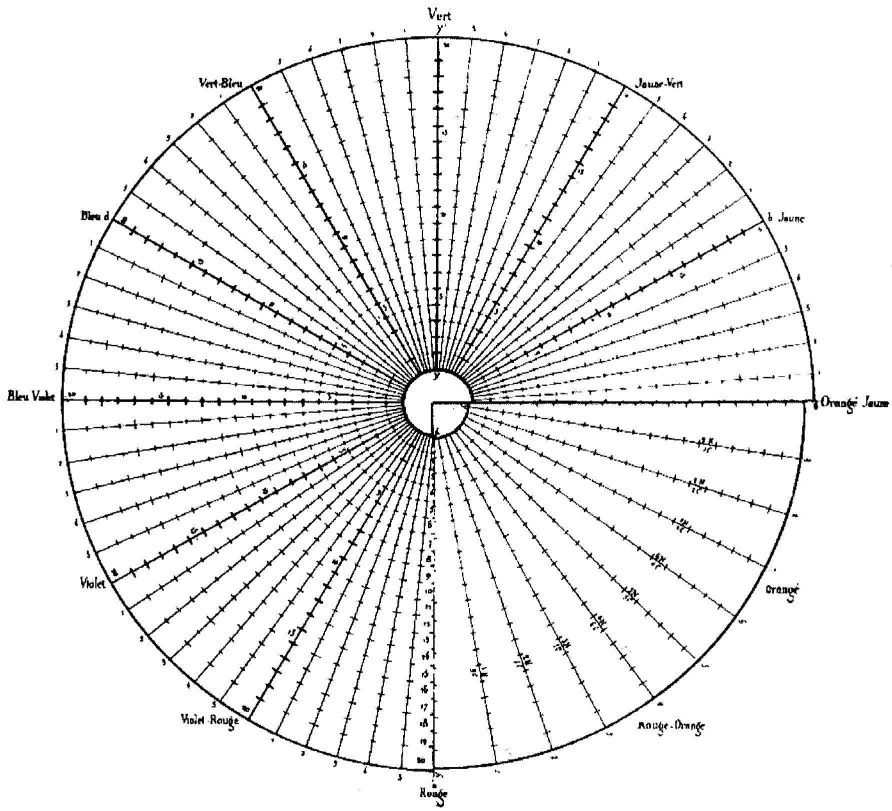


Fig. 2-23 *Chevreul's concept for a 72-hue circle with twenty grades each. White is in the center and black at the circumference of the circle, 1839.*

(1786–1889), a French chemist and director of the *Manufacture Impériale des Gobelins*, the famous tapestry manufacturer. The base plane of Chevreul's system consists of a 72-hue color circle based on the primary colors yellow, red, and blue, separated by equal segments. Twenty grades of lightness of the corresponding hue are located on radial lines from the white center, ending in black on the periphery of the circle (Fig. 2-23). In this ladder the full color of each hue is located at its appropriate level of lightness:[36]

> In each of the scales . . . there is one tone which, when pure, represents in its purity the colour of the scale to which it belongs: therefore I name it the normal tone of this scale. . . . If the tone 15 of the Red scale is the normal tone, the normal tone of the Yellow scale will be a lower number, while the normal tone of the Blue scale will be of a higher number. This depends on the unequal degree of brilliancy and luminousness of the colours.

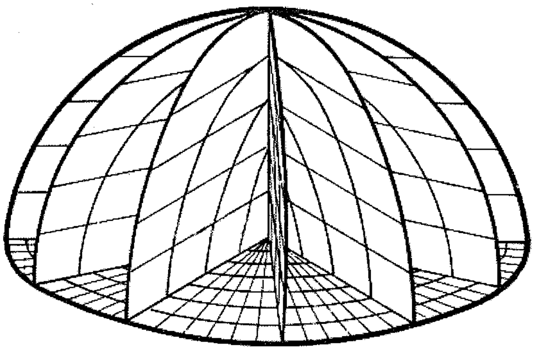


Fig. 2-24 Schematic depiction of the tonal hemisphere raised above Chevreul's color circle. Black covers the hemisphere, with white at its origin.

However, later Chevreul placed all full colors on the same grade, a fact that resulted in considerable confusion about his system by readers of different book versions. Colors toward the center from the "normal tone" are mixed with white, and those toward the periphery with black in appropriate amounts to make the constant hue ladder visually equidistant. Chevreul realized that there are many other colors of a given hue that he had not represented in his plane (the plane being in essence a flat representation of the surface of a color solid). To represent the missing colors Chevreul chose to erect a hemisphere above the plane (Fig. 2-24). The apex of the hemisphere is occupied by black, the line between white at the center of the plane and the vertical axis is formed by a 20-step gray scale. Chevreul then proposed to mix each of the colors of the base plane in ten steps with increasing amounts of black but comments: "It is understood that these proportions relate to the effect of the mixtures upon the eye and not to material quantities of the Red and Black substances." In this manner as the angle of the radial lines increases toward 90° the colors become increasingly blackish. In such a system there would be large visual steps between the ninth line of colors, nearing the vertical center line, and the gray scale of the center line, particularly near the core of the hemisphere. This can be avoided, as Schwarz has pointed out (Schwarz, 1997), if one assumes that Chevreul had in mind mixtures of the base plane colors with the appropriate gray rather than with black so that a smooth transition to the gray scale would result. Calculations show that in neither case explicit redundancy of colors occurs, thus negating later criticism of Ostwald and others in this respect. Assuming the mixtures to be with gray would have the advantage that all colors in a hemispherical layer would have the same lightness throughout, a result that Chevreul likely intended. Chevreul's system was never fully illustrated. Ten 72-hue circles, from the full colors with increasing amounts of black, and twelve constant hue ladders representing the key colors of the base plane were produced under Chevreul's supervision using paper printing techniques (Fig. 2-25; Chevreul, 1864).

2.13 DOPPLER'S SPHERE OCTANT

The Austrian mathematician Christian Doppler (1803–1853), discoverer of the Doppler effect, the change in frequency of energy as a result of relative motion of the source and the observer, developed an interest in color in connection with astronomy. He explained changes in the apparent color of stars, often described by astronomers, as due to the effect that became named after him. To be able to explain the effect systematically, Doppler required a systematic color order system.[37] He described such a system in a paper with the title *Versuch einer systematischen Classification der Farben* (Essay of a systematical classification of colors) published in the Proceedings of the Royal Bohemian Society of Sciences in 1847. Placing his three primary prismatic colors yellow, red, and blue on orthogonal axes with the common origin of black, he erected an achromatic axis separated by identical angles from the three chromatic axes. He believed mixture colors of equal intensity between two chromatic primary colors to be located on circular segments. The resulting space has the form of a sphere octant (Fig. 2-26).

> At the points R, B, G, M are located pure red, blue, yellow and gray (white) of intensity $AB = AR = AG = AM$, the quarter circles BR, BG and GR contain all binary mixtures of violet, green and orange, therefore not yet mixed with gray (white). At the points α, β and γ neutral violet, orange and green are located.—Colors tending toward B, R and G are more bluish, reddish, or greenish. . . . The circle segments MR, MB and MG are the loci for all mixtures of red with gray, blue with gray and yellow with gray. . . . There is an infinite number of concentric spherical sections [$M\alpha$, $M\beta$ and $M\gamma$] or at least as many as there are gradations of white light from black via gray to the most intense white.

Doppler was the first to develop a three-dimensional color system not based on object colors. As a mathematician he was able to describe the relationships between colors in correct mathematical terms (see also Schwarz, 1991).

2.14 YELLOW, RED, AND BLUE, FOR A TIME FIRMLY ESTABLISHED AS PRIMARY COLORS

The speculations of Palmer, the work of Le Blon and his competitors, and the choices of Mayer, Lambert, Runge, and others, helped to establish yellow, red, and blue as the generally accepted primary colors. Young, possibly influenced by Palmer, also had concluded that there are three primary colors but his informed considerations made him eventually select red, green, and violet as the additive primaries.

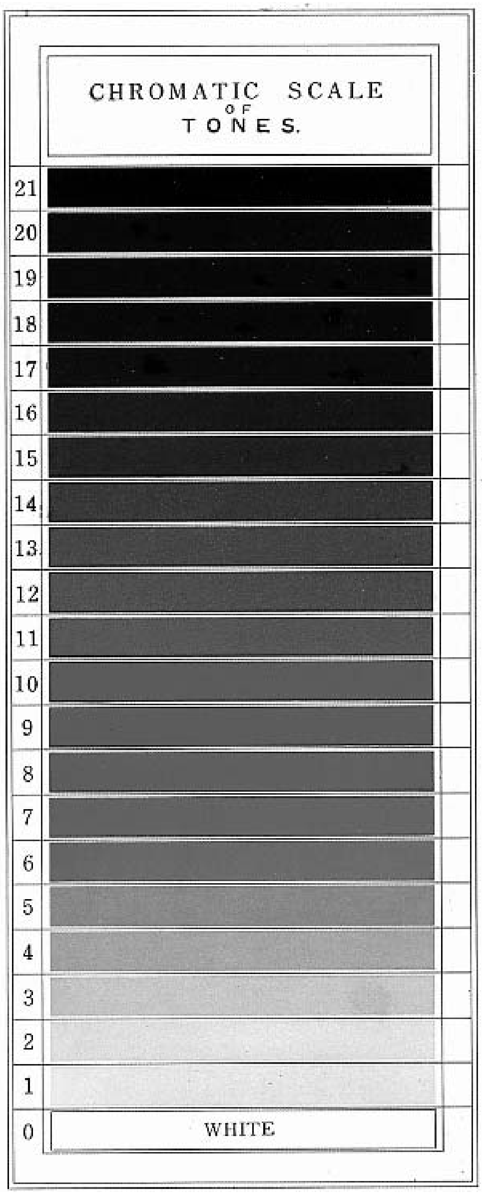


Fig. 2-25 *One of twelve chromatic scales by Chevreul, with local discoloration. The colors range from white through the full color (grade 11) to black. These scales are located on the base plane of the hemisphere. (See color plate.)*

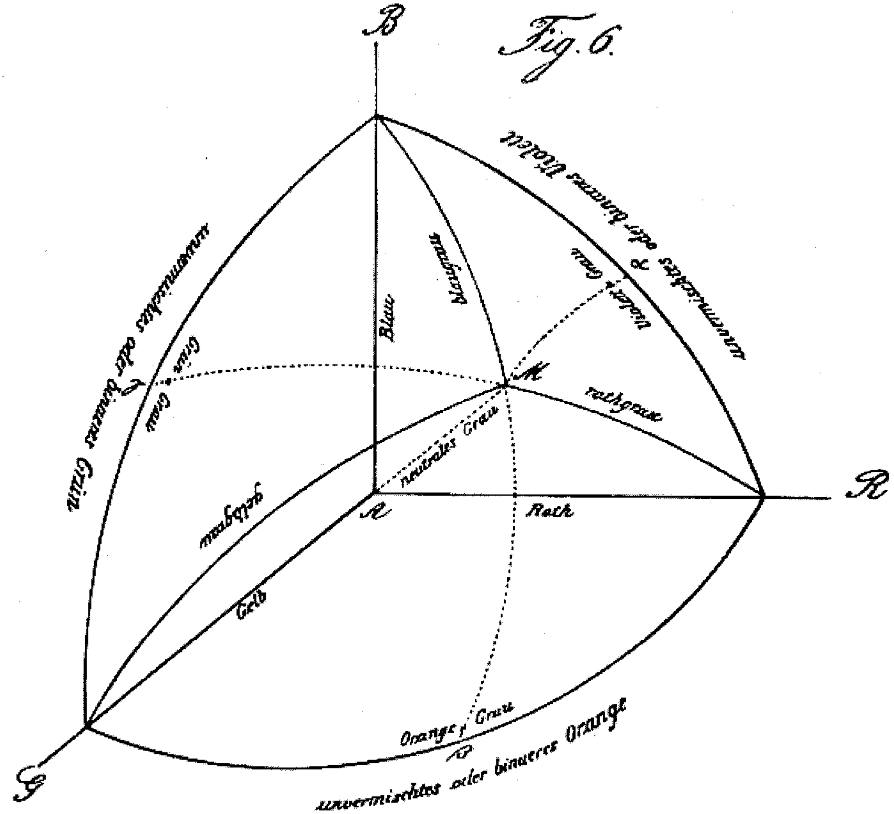


Fig. 2-26 *Sphere octant color space of Christian Doppler, 1847. The axes represent the three primary colors, yellow (G), red (R), and blue (B).*

Hayter

In 1826 the English architect and painter Charles Hayter (1761–1835) self-published a book titled *A new practical treatise on the three primitive colours assumed as a perfect system of rudimentary information*. Hayter's choice of primary colors were yellow, red, and blue, and he referred to Leonardo, Newton, and Young as his predecessors. At the same time Hayter understood that there is a difference "... between the properties of such materials as give their colours to substances suitable to the purposes of art, and the transient effects of light, which must not be considered as belonging to a system of mixing colours for the purpose of painting." Hayter offered six proofs for the primacy of yellow, red, and blue:

> First—That Yellow, Red, and Blue, are entire colours of themselves, and cannot be produced by the mixture of any other colours.... Secondly—Yellow, Red, and

Blue, contain the sole properties of producing all other colours whatsoever, as to colour . . . ; Thirdly—Because, by mixing proper portions of the Three Primitives together, Black is obtained, providing for every possible degree of shadow. Fourthly—And every practical degree of light is obtained by diluting any of the colours . . . by the mixture of white paint. Fifthly—All transient or prismatic effects can be imitated with the Three Primitive Colours, as permanently considered, but only to the same degree of compensation as white bears to light. Sixthly—There are no other materials, in which colour is found, that are possessed of any of the foregoing perfections."

Yellow, red, and blue as universal primaries received strong support for a time from the Scottish physicist David Brewster (1781–1868), best known for his work on light polarization. In his 1831 book *A treatise on optics*, Brewster stated: "Red, yellow, and blue light exist at every point of the solar spectrum. As a certain position of red, yellow, and blue constitute white light, the colour of every point of the spectrum may be considered as consisting of the predominating colour at any point mixed with white light. . . ." He offered spectral curves representing the content of red, yellow, and blue in spectral colors. Brewster's theory of three universal primary colors was widely accepted for some thirty years until Helmholtz and Maxwell demonstrated its errors.

2.15 HELMHOLTZ, GRASSMANN, AND MAXWELL

In 1852 the German physicist Hermann Ludwig Ferdinand von Helmholtz (1821–1894) clarified the difference between additive and subtractive color mixture. In the same paper he published a table of mixtures of pairs of five spectral lights. Only the yellow and blue lights he selected resulted, when appropriately mixed, in white light. Reading Helmholtz's account, the German mathematician Hermann Günther Grassmann (1809–1877), purely on the basis of logical thinking, demonstrated in 1853 that if Newton's theory of compound colors is true it must be possible to match any perceived color with three properly selected color stimuli. These colors must be related to the sensitivities of Palmer's and Young's postulated three sensors. Grassmann then showed that as a result any color of the spectrum must have a complementary color with the mixture of the two in appropriate ratio adding up to white light. Grassmann offered a color circle (Fig. 2-27) demonstrating this fact. The color arrangement in the circle uses the identification of key Fraunhofer lines in the spectrum (capital letters) for orientation and in this fashion making the circle for the first time semiquantitative. Grassmann's circle is an advanced version of Newtons's circle. Less than a year after their publication Helmholtz (1854) experimentally validated Grassmann's theoretical considerations, thus confirming Young's view.

In 1857 the Scottish physicist James Clerk Maxwell (1831–1879) published a paper on his color disk mixture experiments in which he described the results of disk mixtures in an equilateral triangle. Maxwell stated: "From [various]

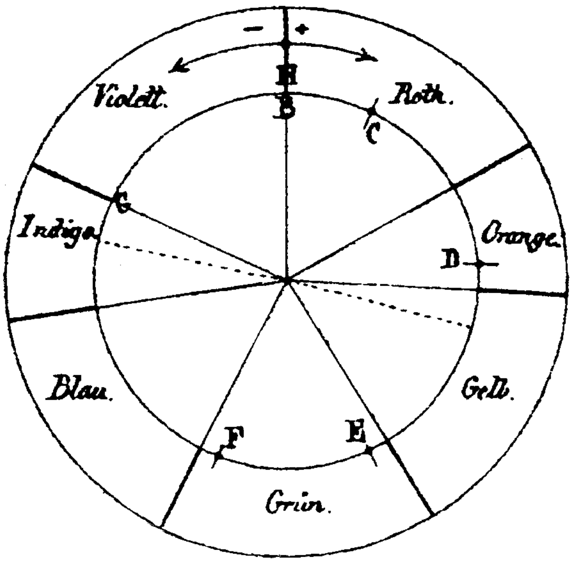


Fig. 2-27 Grassmann's color circle, derived from Newton's circle, 1853. The beginning and end of the spectrum have been moved to the 12 o'clock position. The sector widths are identical to Newton's.

facts I would conclude that every ray of the spectrum is capable of stimulating all three pure sensations [related to Young's three sensors], though in different degrees. The curve, therefore, which we have supposed to represent the spectrum will be entirely within the triangle of colour. All natural or artificial colours, being compounded of the colours of the spectrum, must lie within this curve. . . ." In 1855 he had produced a schematic sketch of such an arrangement. The triangle has at its vertices the three primary colors red, violet, and green. One to one mixtures of these produce intermediate hues carmine, blue, and yellow. The spectral colors are shown on the circle, with white at its center. All real colors must fall on or within the circle (Fig. 2-28).

In the first edition of his *Handbuch der physiologischen Optik* (Treatise on physiological optics) of 1867 Helmholtz offered estimated spectral sensitivity curves for the three Youngian sensors (see Fig. 5-3) and a Maxwell-type triangle with the resulting trace of spectral colors (Fig. 2-29). These excitation curves were later measured first by Maxwell (Fig. 5-4) and later by Helmholtz's assistant Artur König (Fig. 5-5). Here Helmholtz also presented a view of the basis plane, with opposing colors being complementary colors, as well as a view from the top of his color cone which he compared to Lambert's tetrahedron. (In Lambert's "subtractive" pyramid, black is in the center of the basis plane and white is at the top, while in Helmholtz's "additive" cone, white is in the center of the basis plane and black on top; Figs. 2-30 and 2-31.)

In an 1872 paper on color vision Maxwell stated:

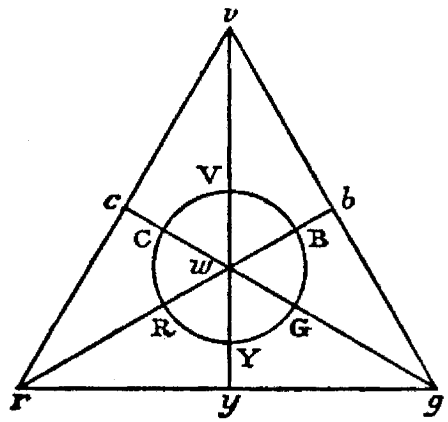


Fig. 2-28 Maxwell's sketch of the fundamental color triangle based on the primaries red, green, and violet at the apices, and white in the center. The prismatic and extraspectral purple colors are located on a circle, with object colors falling within the circle, 1856.

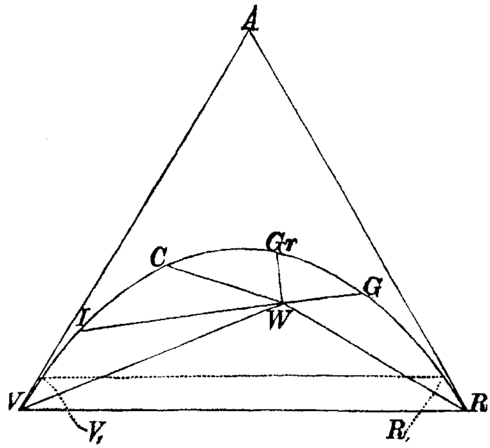


Fig. 2-29 Helmholtz's estimate of the location of the spectral curve (from the right: red, yellow (G), green, cyan, indigo, violet, with white at the intersection point) in Maxwell's primary triangle, 1867.

> Let us . . . suppose the colour sensations measured on some scale of intensity, and a point found for which the three distances, or coordinates, contain the same number of feet as the sensations contain degrees of intensity. Then we may say, by a useful geometrical convention, that the colour is represented, to our mathematical imagination, by the point so found in the room; and if there are several colours, represented by several points the chromatic relations of the colours will be represented by the geometrical relations of the points. This method of expressing the relations of colors is of great help to the imagination.

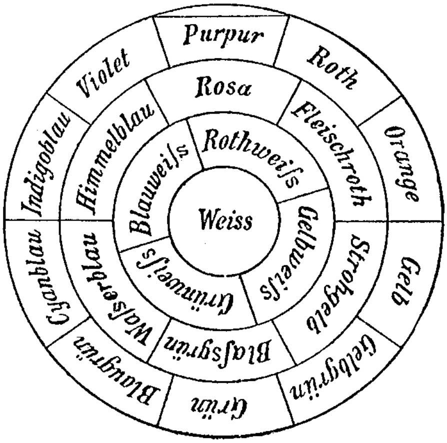


Fig. 2-30 Basis plane of Helmholtz's color cone with white in the center and the saturated colors on the periphery, 1867.

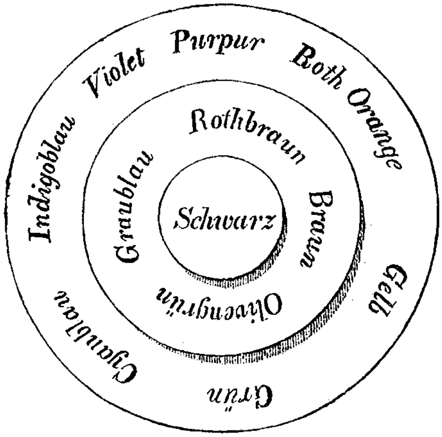


Fig. 2-31 View from the top of Helmholtz's cone, with black on top and tonal colors from black to the saturated colors.

With these statements Maxwell provided the basis for all future mathematical/geometrical models of psychological color space.

2.16 HERING

Helmholtz and his theory of color vision was opposed, at times bitterly, by the German physician and physiologist Karl Ewald Konstantin Hering (1834–1918) (e.g., see Turner, 1994). In the mid-1870s Hering proposed a seemingly much different idea of color vision based not on three but on four primary hues: yellow, red, blue, and green, which he called the natural color system (Hering, 1920). Hering referred to physiologist Hermann Aubert's (1826–1892) statement in his 1865 book *Physiologie der Netzhaut* (Physiology of the retina): "If we want to be clear about color sensations, then the words black, white, red, yellow, green, and blue suffice as main designations and I may therefore treat them as principal sensations or principal colors. . . ." Hering developed the concept of antagonistic assimilation-dissimilation processes to explain brightness perception from black to white. At equilibrium the two processes produce midgray. Two similar antagonistic or opponent processes produce four fundamental colors (*Urfarben*), phenomenologically simple colors: red versus green, and yellow versus blue. Hering declared the hues of the four fundamental colors to be the salient features in a color circle. He used a now classical psychological argument in support:

> All hues can be arranged in the circle so that these primary hues divide it into its four quadrants. All colors in (one) half of this circle are clearly yellowish . . . all those (in the other) have more or less blueness in common. . . . If one imagines this color circle halved so that the dividing line passes . . . through two intermediate colors that are opposite each other, for example, through a violet and one of the greenish yellows, and if the hues in either half are compared, then one finds no one chromatic quality common to all these hues. . . . If the dividing line does not pass through two primary colors we always encounter colors that have no chromatic property in common with certain other colors of the same half and thus they do not have the slightest similarity of hue. In this way we recognize . . . that a rational division of the color circle or grouping of color hues in terms of their internal relations is possible only by using the four specified primary colors. (Fig. 2-32)

Adjacent primary hues (but not those opposed) can form hue mixtures. The hues in their most intense form (C) can be "nuanced" by admixture of whiteness (W) and/or blackness (S). Hering presented as an example an equilateral triangle with the full color, white and black at the corners (Fig. 2-33). He developed arithmetic principles that guide the composition of mixed colors. They are composed of relative proportions of C, W, and S. *Reinheit* (purity) of color can be expressed by $C/(C + W + S)$. Hering was fully aware of the intrinsic lightness of colors:

> If one has a primary blue that is as clear as possible and finds a primary yellow that one cannot say is either lighter or darker than the blue, then anyone with good color vision who has even a little practice in color analysis will also observe that the yellow is less clear than the blue or that it is more or less grayish or blackish. On the other hand, if he has next to the clearest possible yellow a blue that does not look decidedly darker than the yellow, then he will see that the blue is whitish. . . . Moreover, I find a good primary red, that is, the clearest possible, lighter than the clearest primary green available.

Hering did not construct an explicit color order system, and it is not clear how he would have done so, that is, how he would have resolved the issue of the different "brightness" of his primary colors.

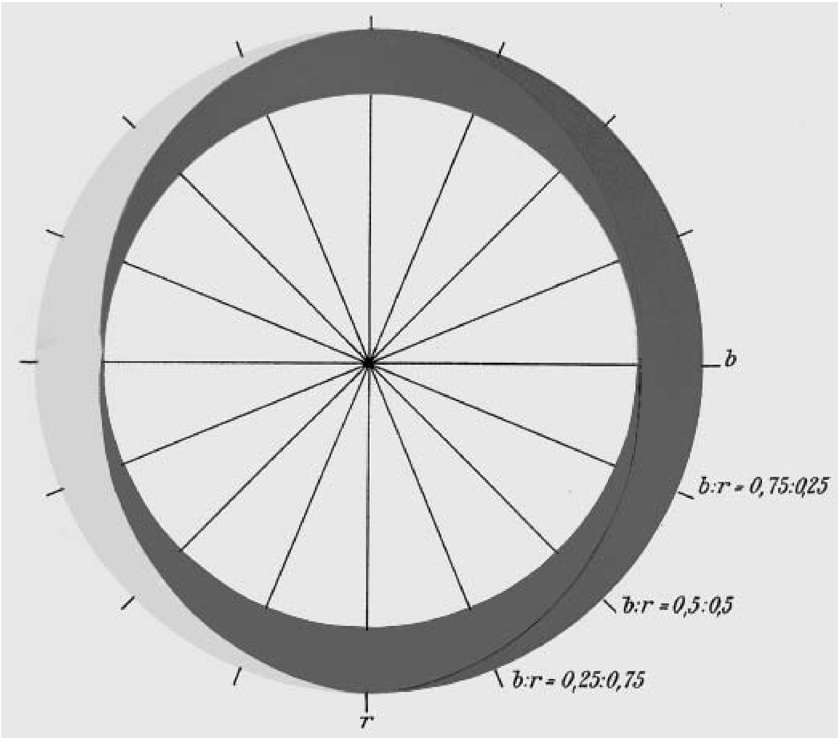


Fig. 2-32 Hering's diagram illustrating the composition of mixed hue perceptions from the unique hues located on the main axes. The fractions of blue and red of three mixed hues are shown bottom right, 1905–1911. (See color plate.)

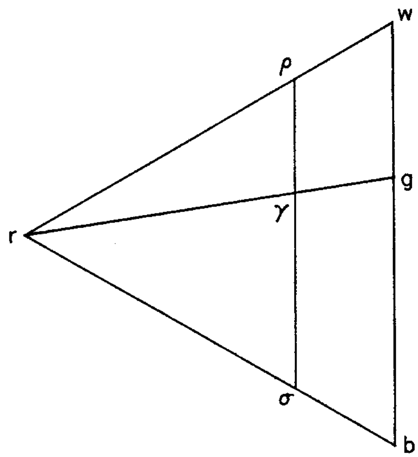


Fig. 2-33 Schematical view of Hering's constant hue triangle, with full color r, white w, and black s on the vertices. Colors along the line parallel to that connecting w and s have constant chromaticness. On the line r to g are located all colors derived from r by veiling with various amounts of g, 1905–1911.

Hering was not satisfied with the Weber-Fechner law, which he found not to apply in his investigations into gray scales. Instead, he proposed a relationship between stimulus intensity and lightness response that equals a hyperbolic function. When the luminous reflectance is plotted on a log scale, this relationship results in an S-shaped function, later found to also apply to cone response kinetics.

Aubert was the first to recognize that Hering's opponent color theory does not have to be contradictory to that of Young and Helmholtz: ". . . if one strictly distinguishes between the process of excitation and the process of sensation."[38] This idea was pursued by Helmholtz and further developed by Schrödinger and Luther (see below). It was the Dutch physiologist Franciscus Donders (1818–1889) and the German physiologist Johannes von Kries (1853–1928) who proposed a combination of the two theories in a "zone" theory, assigning the Young-Helmholtz theory to the cone level and the Hering theory to a later zone on the visual pathway.[39] Despite Donders's and Kries's proposals, the Young-Helmholtz theory of color vision became the leading paradigm until Hering's opponent-color theory was resuscitated in the second half of the twentieth century.

2.17 GEOMETRICAL SYSTEMS OF THE NINETEENTH CENTURY

Cubic Systems

In 1868 William Benson, an English architect, was, in his book *Principles of the science of colors, concisely stated to aid and promote their useful application in the decorative arts,* the first to propose a cubic system. He placed white and black on two opposed corners of the tilted cube with yellow, pink, and sea green on the upper three intermediate corners and red, blue, and green on the lower three (Fig. 2-34). The center of the cube is occupied by a medium gray. Variations of a cubic system were proposed by E. A. Hickethier (1940), and others.

Pyramidal and Cone Systems

A color solid in form of a decagonal pyramid was offered in 1874 by the German physicist Wilhelm von Bezold (1837–1907), the discoverer of the Bezold-Brücke color effect. The hues of his decagon are red, orange, yellow, yellow-green, green, blue-green, blue, violet, and purple. White is located in the center of the base plane and black at the pyramid's apex (Fig. 2-35), that is, it is based on light mixture.

Influenced by Hering, Mayer's triangular double pyramid, was modified in 1897 by the Austrian psychologist Alois Höfler (1853–1928) into a double square pyramid. However, the Hering primaries occupy the corners of the central square of Höfler's construction (Fig. 2-36).

The issue raised by Hering, but not resolved by him, as to how to reconcile the lightness of full colors with his arrangement in an equilateral triangle con-

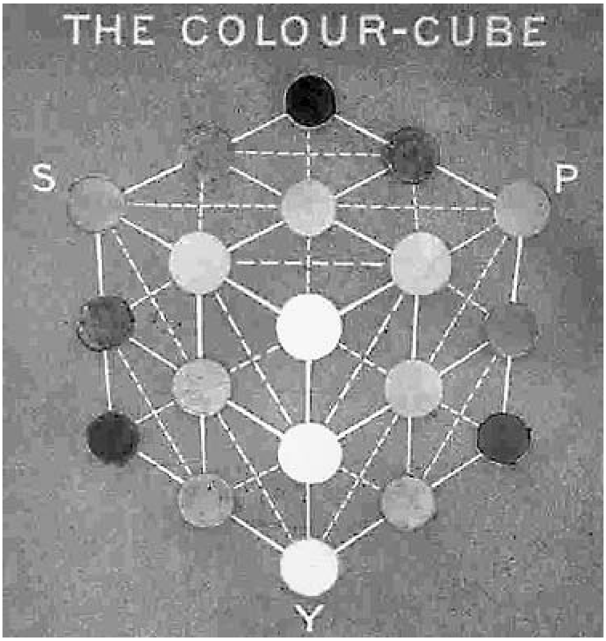


Fig. 2-34 *View toward the top, white corner of Benson's tilted color cube, 1868. The gray scale is hidden behind the white sphere. (See color plate.)*

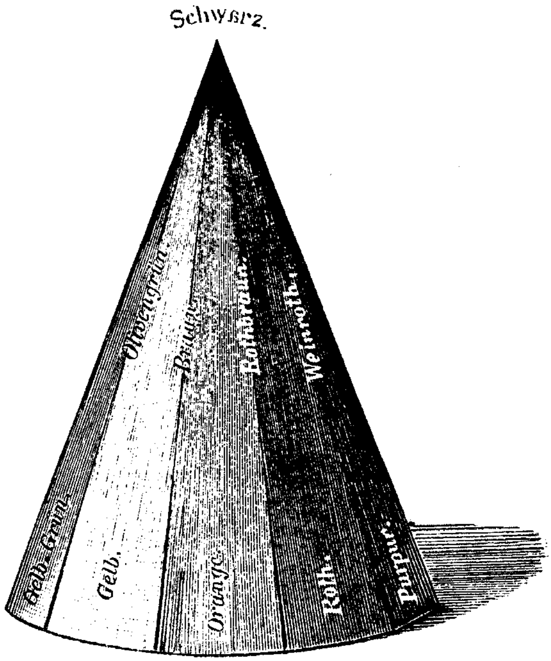


Fig. 2-35 *Bezold's decagonal color pyramid, with black on top and white at the origin, 1874.*

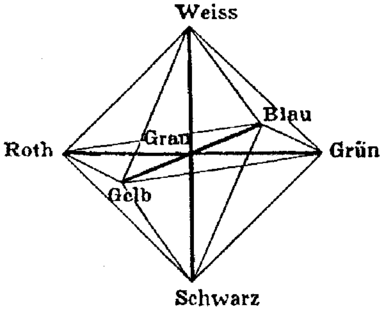


Fig. 2-36 *Höfler's double pyramid based on Hering's unique hues, with the gray scale on the vertical axis, 1897.*

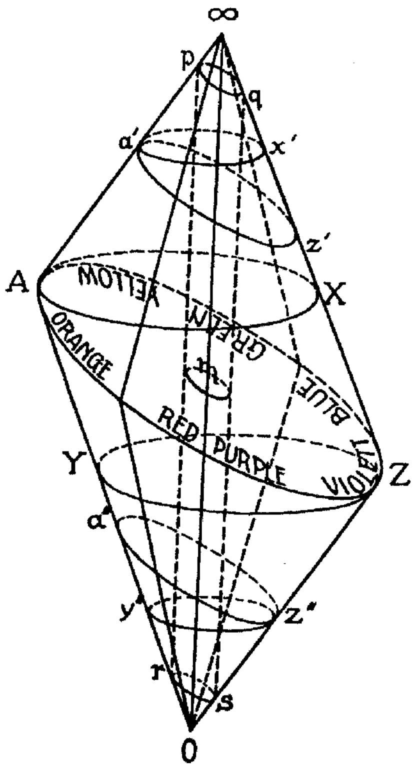


Fig. 2-37 *Tilted double cone of Kirschmann, 1895. Zero and infinite brightness are at the apices. Colors of constant brightness are on horizontal circular planes.*

tinued to be of concern to psychologists. The first attempt to solve it in form of a tilted double cone was offered in 1895 by the German psychologist and student of Wilhelm Wundt (see below) August Kirschmann (1860–1932) (Fig. 2-37). Kirschmann was also concerned with the difference between lights and

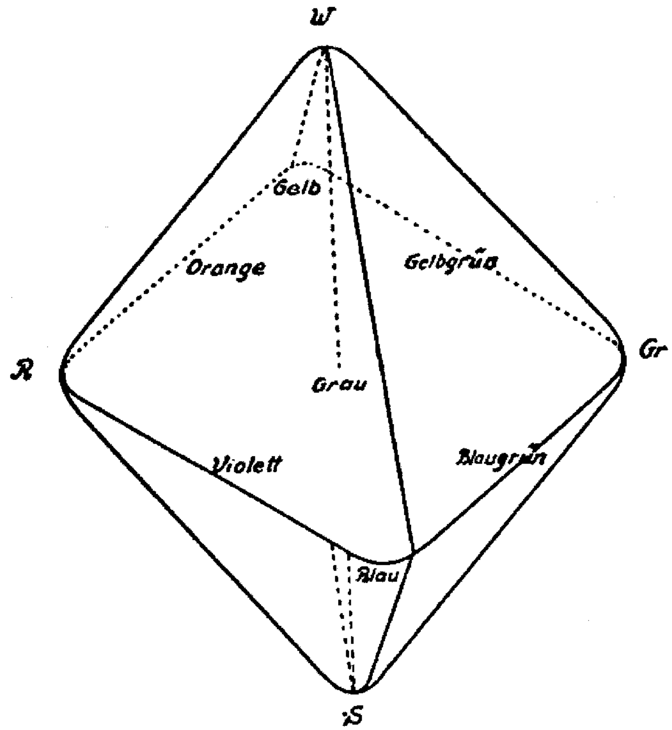


Fig. 2-38 *Ebbinghaus's tilted double pyramid with rounded corners, 1902. The gray scale is located on the vertical axis.*

object colors. His system was to be applicable to both, and he did not put "white" and "black" at the two apices of his tilted double cone but the signs 0 and ∞, explaining: "It is an error to set "white" or "black" at the ends of the axis of a color sphere or a double cone because these expressions do not correctly designate the extremes of the achromatic series of light sensations but instead are ideas of a rather complex nature."

The German psychologist Hermann Ebbinghaus (1850–1909) offered in 1902 a color system in form of a tilted double pyramid. The neutral axis of Ebbinghaus's system is vertical, but the central opponent color plane is tilted along the red–green axis with yellow being closer to white. All surface edges of the solid are rounded because, as Ebbinghaus argued, the transitions between colors are fluid (Fig. 2-38). In 1909 the English-American psychologist Edward Bradford Titchener (1867–1927) offered detailed instructions for building a square tilted double-pyramid model (Fig. 2-39). Neither Höfler, Kirschmann, Ebbinghaus, nor Titchener provided quantitative details of the color arrangements (a painted version of Titchener's pyramid, colored by the

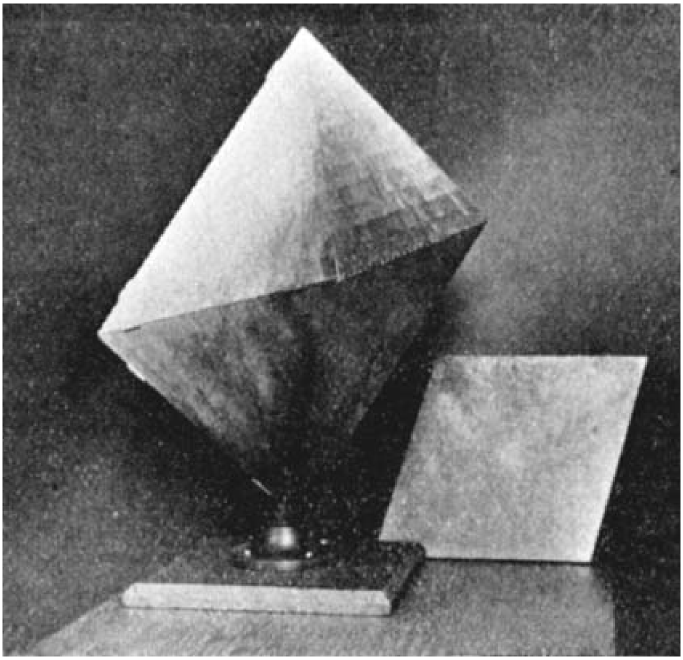


Fig. 2-39 *Titchener's model of the tilted double pyramid. The rhombic central cross section is shown on the right, 1909.*

bird painter Fuentes, is said to be in existence at Cornell University). Their aim was to provide conceptual models of the psychological color solid.

2.18 THE NINETEENTH-CENTURY EXPERIMENTAL PSYCHOLOGISTS

In the footsteps of Fechner and contemporaneously with Maxwell and Helmholtz experimental psychology began to become a science, with particular strength in Germany. The sense of vision was of considerable interest. Experimental psychology claimed Fechner and Helmholtz as its immediate ancestors. The great man of experimental psychology was Wilhelm Max Wundt (1832–1920), a prolific researcher and teacher with many well-known pupils. Under Wundt's supervision considerable research was conducted involving the sense of vision. Wundt discussed color order at several occasions. In 1874, in his *Grundzüge der physiologischen Psychologie*, he proposed a color sphere, not unlike Runge's. But based on the then newly and explicitly investigated color spectrum, yellowish colors assumed a much smaller portion of his hue circle. The eight identified main hues are red, yellow, yellow-green, green, blue-green, blue, violet, and purple. Of the eight sectors reddish and greenish colors

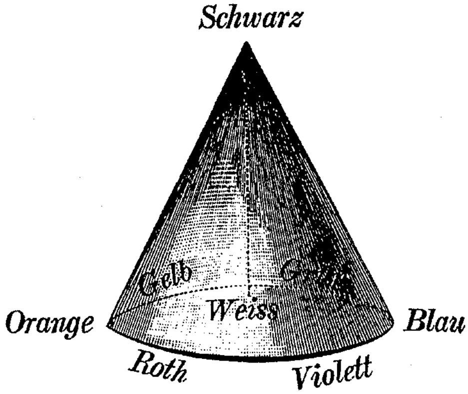


Fig. 2-40 *Wundt's cone with black on top and white on the origin, 1893.*

occupy four, bluish five, and yellowish colors three. In 1892, in the second edition of *Vorlesungen über die Menschen- und Thierseele* (Lectures on the human and animal soul), Wundt offered a Helmholtzian cone with six basic hues: red, orange, yellow, green, blue, and violet (Fig. 2-40). White was located in the center of the base circle and black at the apex of the cone. In *Grundriss der Psychologie* (1896), Wundt offered a double cone.

Further Development of Psychophysics

After Weber and Fechner and their immediate followers, major research activity in psychophysics began to shift to the United States.[40] The Polish-American psychologist Joseph Jastrow worked on the methods of determining the differential limen and recommended that the general limen be replaced by the probable error of visual determination (Jastrow, 1888). Before Jastrow the limen had been defined as the point where 50% of the observers were above and the other 50% below in their evaluation. Jastrow proposed to move the point up based on the statistical error of evaluation, specifically to 75% (the concept of standard deviation was developed only later).

In the same time period the American psychologist James McKeen Cattell (1860–1944) also worked on psychophysical problems. He experimentally pursued the idea that general difference thresholds could be determined by measuring reaction times of observers in psychological tasks. He strengthened the use of statistical methods for the analysis of psychological measurements and became the champion of this new approach that stressed the relationship between the individual and the average observer. In his *On the perception of small differences* (together with his colleague, the philosopher G. S. Fullerton) Cattell criticized Fechner's law on just noticeable differences as being too much dependent on introspection and indicated his preference for Fechner's method of constant stimuli, making use of the error law (Fullerton and Cattell, 1892).

Psychophysics faced fundamental criticism by Kries who argued that numbers applied to sensations are not quantities but merely convenient labels, an issue not yet fully resolved. In the midtwentieth century the American psychologist S. S. Stevens (1906–1973) developed a theory of sensory magnitude based on extensive experimental data. He found that such data could usually be modeled well with power functions. Chapter 3 offers a more detailed view of psychophysics.

2.19 THE MUNSELL SYSTEM

A decisive step forward toward a systematic arrangement of object colors was taken by the American artist Albert H. Munsell (1858–1918).[41] At the age of twenty-one he read Ogden N. Rood's *Modern Chromatics* (then just published), a book that influenced many artists of the period (Rood, 1879). It offered him a solid foundation in color theory and psychophysics of the time. After art studies in Paris and Rome Munsell became lecturer in color composition and artistic anatomy at the Massachusetts Normal Art School in Boston. There he began to concern himself with how best to teach systematic arrangement of colors to his students and how to systematically express pleasing color combinations for artwork. Plotting the colors of one of his pictures in form of a double spiral in 1898 suggested a sphere to Munsell. He began to construct small spheres with color fields on the surfac that, when spun, demonstrated color balance (Fig. 2-41). Early in 1899 Munsell initiated a patent application for his color sphere (granted in the year 1900). In the same year he decided to use the decimal system as a basis of his systematic color arrangement and chose five primary colors: yellow, red, purple, blue, and green. The central vertical axis of the sphere was to be formed by a 100-step lightness scale (named "value scale") and Munsell conceived the then new idea of the chroma attribute for the colors of equal lightness, extending radially from the central axis. By applying these and the hue attribute systematically, Munsell moved away from the tradition of placing the most intense colors of a given hue on the same central plane. This resulted in the vertical and horizontal dimensions of his color solid having unambiguous definitions in terms of the attributes. A portable visual photometer of Munsell's design provided semiquantitative luminous reflectance data for the colors he selected. As early as 1900 Munsell realized that the form of his color solid could not be a sphere because of the varying intensity of different pigments but would need to be a spheroid. By 1904 Munsell had made visual uniformity by attribute a key concept of his systematic color arrangement and had settled on his color terminology. In 1905 he published *A Color Notation*, a description of the system, with print reproductions of color charts (Munsell, 1905). In the same year

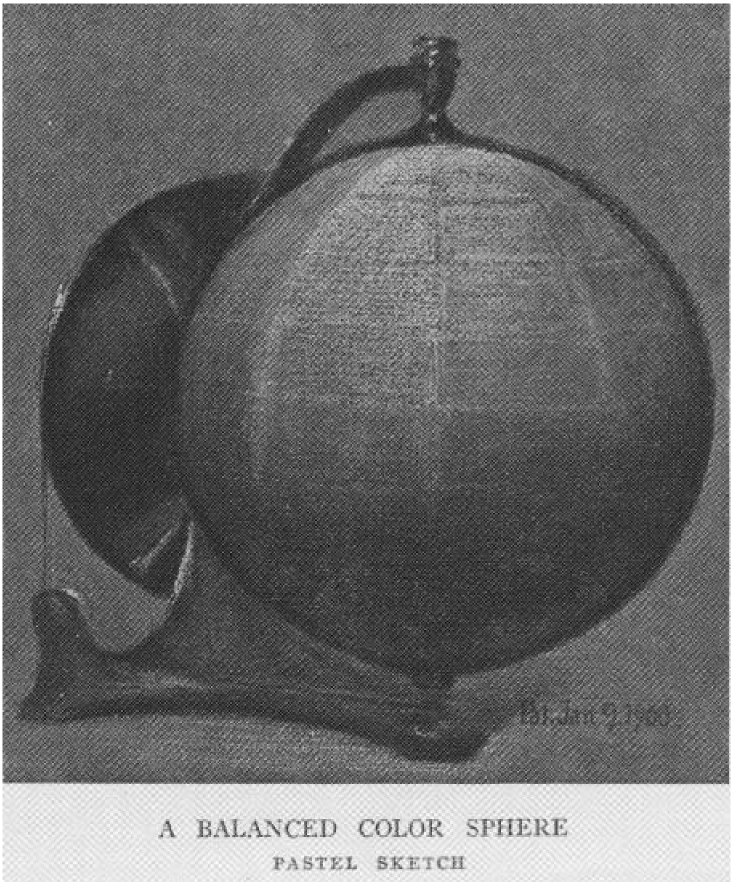


Fig. 2-41 *Artist's rendition of Munsell's balanced color sphere, patented in 1900. The sphere was rotatable to achieve additive color mixture to gray and thereby show the "balance" of the colors on the sphere. The mirror in the back discloses the blue region of the sphere. (See color plate.)*

Munsell met with Ostwald (see below) and exchanged views on color order, but both were to pursue their own views on what such a system was to look like. In 1907 the first version of his *Atlas of the Munsell Color System*, a portfolio of eight plates with painted paper samples, was published, followed in 1915 by an extended version with fifteen plates and a total of 880 color samples (Munsell, 1907, 1915). The two versions offered for the first time detailed internal views of a color solid.

In 1918, shortly before Munsell's death, the Munsell Color Company was formed by Munsell's son and friends. It produced painted paper chips of Munsell colors and in 1929 issued the first version of the *Munsell Book of*

Color, a collection of forty pages, each containing chips of various value and chroma of one given hue. The system is open ended in that the chips represent what is possible with available pigments. Identification of the samples is by hue identifier, value level, and chroma level, for example, 2.5BG5/10 meaning a sample of hue 2.5 blue-green, value 5, and chroma 10. The units of the three visual scales are not identical in perceived magnitude.

Modifications to the system (the so-called Munsell Renotations) were proposed in 1943 by the Optical Society of America Subcommittee on the Spacing of Munsell Colors (Newhall, Nickerson, and Judd, 1943) in form of colorimetric values of the aim colors. They are the basis of the modern system (Fig. 2-42). Expanded editions, based on the Renotations, have been issued in Japan (*Chroma Cosmos 5000* and a condensed version thereof: *Chromaton 707*), and the system has also been implemented in the form of textile samples. For more details and an analysis of the Munsell system, see Chapter 7.

2.20 RIDGWAY'S COLOR ATLAS

In 1912, before the publication of Munsell's 15 plate color *Atlas*, the American ornithologist and botanist Robert Ridgway (1850–1925) published *Color standards and color nomenclature*, an extensive proposal for what amounts to a double-cone color order system. It consists of fifty-three plates each with twenty-seven pigment painted color chips, a total of 1115 chips (excluding multiple whites and blacks). In several respects Ridgway's system is a forerunner of Ostwald's. It is based on Maxwellian disk mixture. His primary disks were produced using the brightest then available synthetic-organic dyes. His central color plane was divided into 71 hue steps, taken in double steps, resulting in thirty-six full color hues. Ridgway considered six hues to be fundamental: red, orange, yellow, green, blue and violet. His fundamental red was located at 644 nm (as he had been assured it should be by the then associate physicist of the U.S. National Bureau of Standards, P. G. Nutting), therefore being a quite yellowish red. His yellow, green, and blue are located near the corresponding generally accepted unique hues. There are 22 steps containing redness, 17 with yellowness, 13 with greenness, and 19 with blueness. The hue steps were meant to be visually equidistant. There are 13 steps between unique red and blue but only six between unique yellow and unique green. Disk mixtures toward white and black for all hues were made in three grades each in constant increments (Fig. 2-43). As Lambert already did Ridgway remarked that the admixtures to yellow have a much different lightness scaling than those to blue. From the central neutral gray scale of nine grades to the full color there are grades of decreasing but always identical admixture of gray. Recent measurements of an atlas have shown the gray scale to be essentially a Fechner type scale and the color circle to be less than uniform. (Aside from possible colorant deterioration it is also necessary to keep in mind the limited color measurement capabilities when Ridgway developed his *Atlas*.)

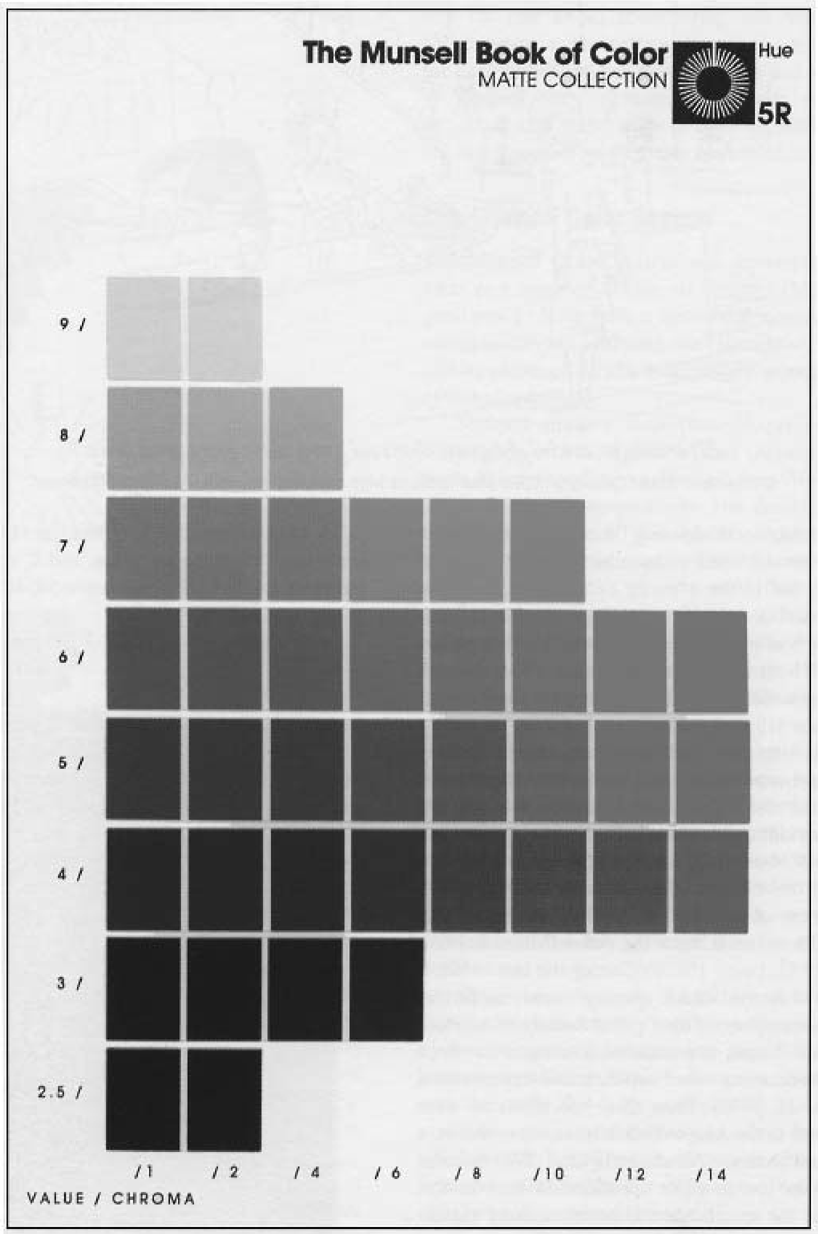


Fig. 2-42 *Constant hue page from a modern version of the Munsell Book of Color. The gray scale is not shown. The chroma scale begins at 1 and continues from 2 at two-grade intervals to chroma 14. Value grades are shown from 2.5 to 9. Courtesy Gretag-Macbeth Company. (See color plate.)*

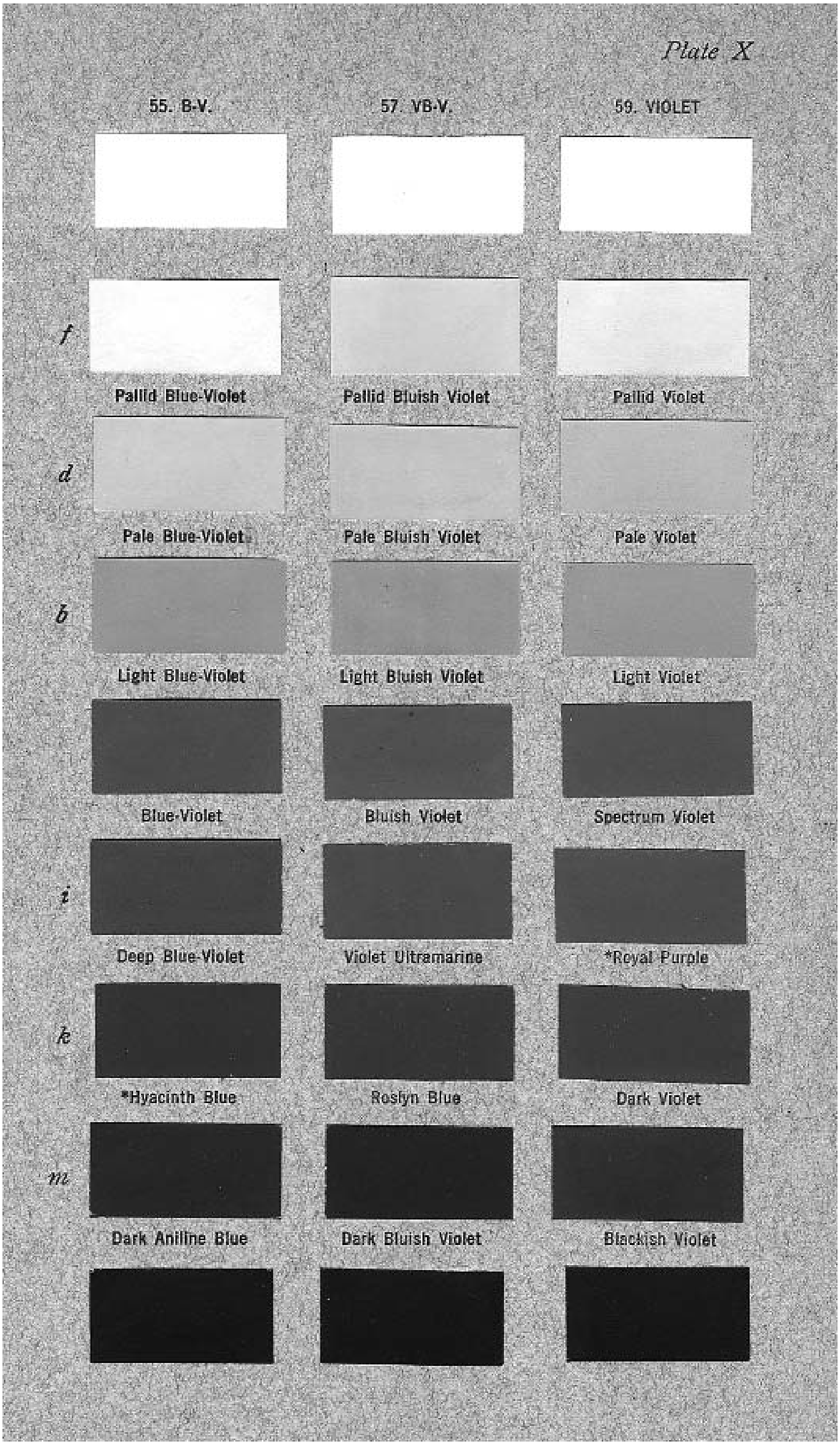


Fig. 2-43 *Page from Ridgway's color atlas showing three reddish blue hues lightened from the central color in three steps toward white and darkened in four steps toward black, 1912. (See color plate.)*

2.21 OSTWALD'S *FARBKÖRPER* (COLOR SOLID)

The German chemist and Nobel prize winner Wilhelm Ostwald (1853–1932) began in 1917 to publish a series of texts on color science, the first of which is called *Mathetische Farbenlehre* (Theory of logical ordering of colors). Here Ostwald described a double-cone color solid of related colors based on additive disk mixture. He scaled the resulting pychophysical solid using the Weber-Fechner law to approximate the implied psychological color order. Ostwald used Hering's color equation: $r + s + w = 1$, where r is the amount of the *Vollfarbe* (full color, colors of highest intensity), s that of black, and w that of white. He interpreted the equation in terms of reflectance data in the visible portion of the spectrum. However, the real colors he used in the disk mixture had, of necessity, in all cases reflectances different from those idealized. Ostwald based his system on three fundamental hues: yellow, red, and blue. The hue circle begins with yellow occupying the 12 o'clock position, proceeding clockwise to red, blue, and green (unlike Munsell who placed red at 12 o'clock and proceeded clockwise toward yellow). Opposing hues are complementary; that is, when optically mixed they result in an achromatic color. Full colors are located conventionally on the periphery of the central plane. All colors of a given hue are placed on an equilateral triangle that is half of a vertical section through the center of the double cone (Fig. 2-44). The central vertical axis is a gray scale. As in Hering's constant hue triangles, lines parallel to the line connecting the full color and white are lines of equal blackness, lines parallel to the line connecting full color and black are lines of equal whiteness. Lines parallel to the line connecting black and white are lines of equal purity. Tonal colors (tints) toward white are located in the upper half of the double cone and tonal colors toward black (shades) in the bottom half.

To bring his psychophysical solid into agreement with psychological scaling, Ostwald applied logarithmic scaling so that 16 grades from black to white and black to full color, as well as white to full color, resulted in 15 visually equidistant steps according to the Weber-Fechner law. In such an arrangement there are 120 chromatic samples in a triangle and Ostwald applied a double-letter system to identify them, in addition to the hue number. The system reveals slices through the color solid in four different directions: equal hue, equal whiteness and blackness, and equal purity. In Europe, the abridged *Farbkörper* illustrating on 12 plates 680 samples (24 hues and 8-step gray scale), when first published in 1919 was a revelation of the multitude of colors, presented systematically (see Fig. 2-45).

Ostwald's system was analyzed extensively by Foss, Nickerson, and Granville in 1944. They demonstrated that, unsurprisingly, it represents a series of compromises and that it fills only a portion of the available gamut of the Rösch-MacAdam limits (see Section 2.23). Perceptual differences between grades in different areas of the solid were found to be of varying size.

Ostwald developed several sets of colorations as atlases of his system, the most elaborate published as *Der Farbenatlas*, with 2500 samples (Ostwald,

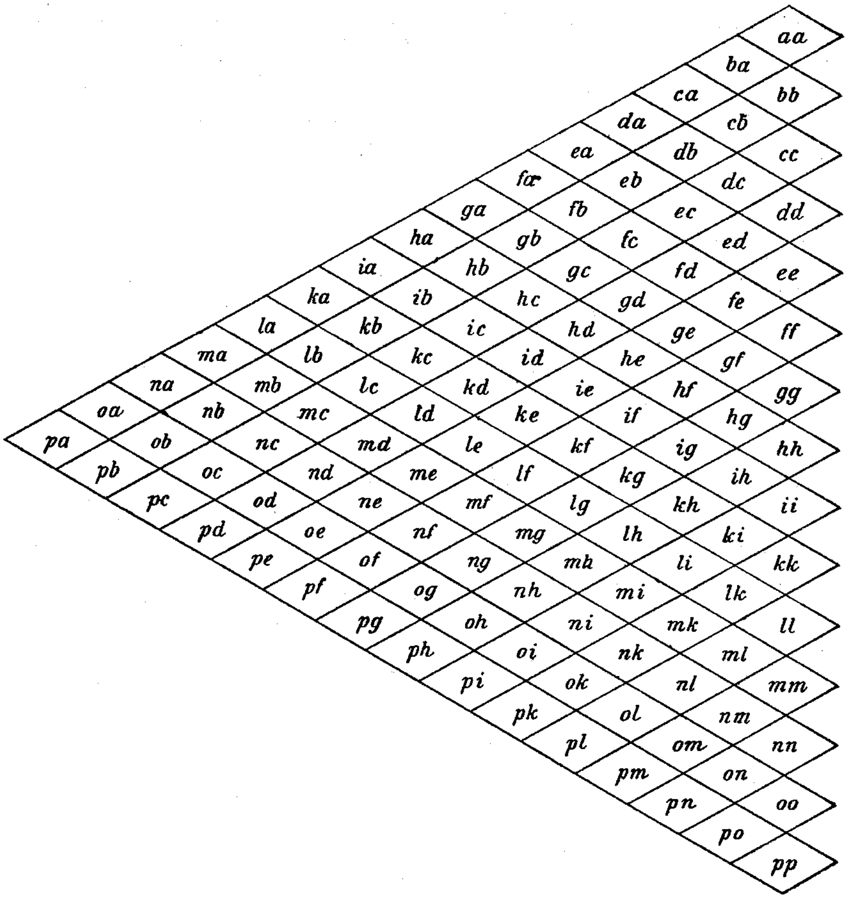


Fig. 2-44 Color identification scheme for colors in a constant hue plane of Ostwald's double-cone color solid (1917). The grey scale is represented on the extreme right by colors aa *to* pp*. The full color is represented by* pa.

1917). An atlas in two different sizes implemented with dyed wool samples was also offered (*Der Wollatlas*). Ostwald used his color solid to demonstrate his rules of color harmony (Ostwald, 1918). Implementations of the Ostwald system after the Second World War are those by Aemilius Müller, who gave up complementarity for improved uniform hue spacing (Müller, 1953) and the *Color Harmony Manual* of the Container Corp. of America (Jacobson, 1942; Granville, 1994), issued in three editions.

2.22 GEOMETRICAL SYSTEMS OF THE TWENTIETH CENTURY

After Munsell and Ostwald several new color order systems were proposed, all built on the ideas of double cones, double pyramids, or cubes. All of these

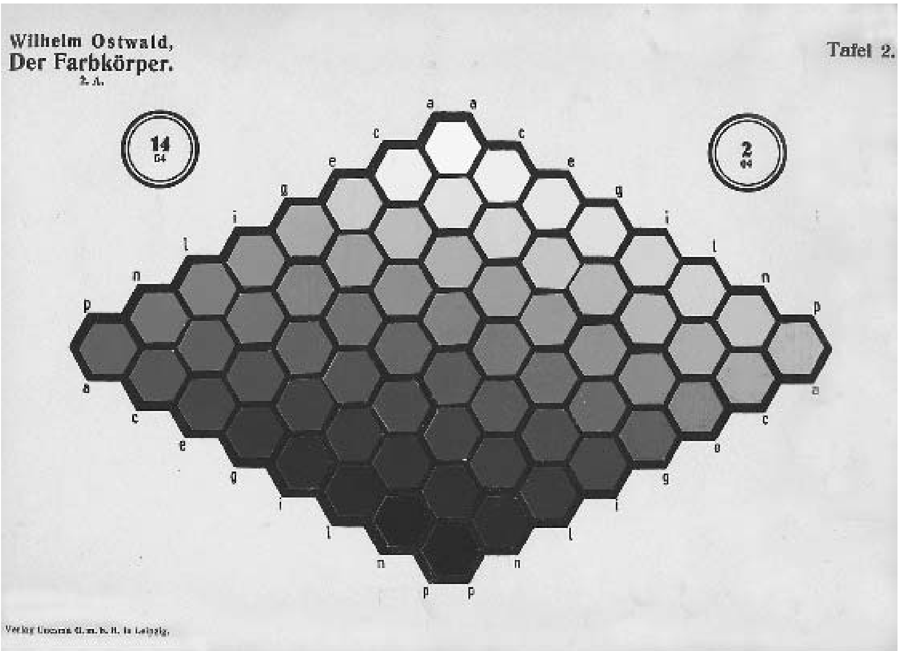


Fig. 2-45 Vertical cross section through Ostwald's double-cone color solid illustrating constant hue colors 1 and 13, with veiling toward white and black. The achromatic scale is at the center. From Farbkörper, undated. (See color plate.)

systems formed more or less idealized geometrical solids and did not offer any fundamentally new ideas. Among them are:

- Rounded double cone with tilted central plane from 1920 by the American art educator Arthur Pope (1880–1974). Similar systems were subsequently developed in Sweden by Tryggve Johansson (1939) and Sven Hesselgren (1953), and by Frans Gerritsen (1975).
- Modified double cone by C. Villalobos-Dominguez and J. Villalobos in 1947, developed in Argentina and with an atlas consisting of 7000 samples produced by halftone printing.
- Double pyramid of 1929 by the American psychologist Edwin G. Boring.
- Tilted cube systems by E. Alfred Hickethier, 1940, issued with 1000 printed samples, and Harald Küppers, 1972, issued with 1400 printed samples.

2.23 RÖSCH-MACADAM COLOR SOLID

In the early twentieth-century color measurement and specification began to make significant progress. In 1931 the *Commission Internationale de*

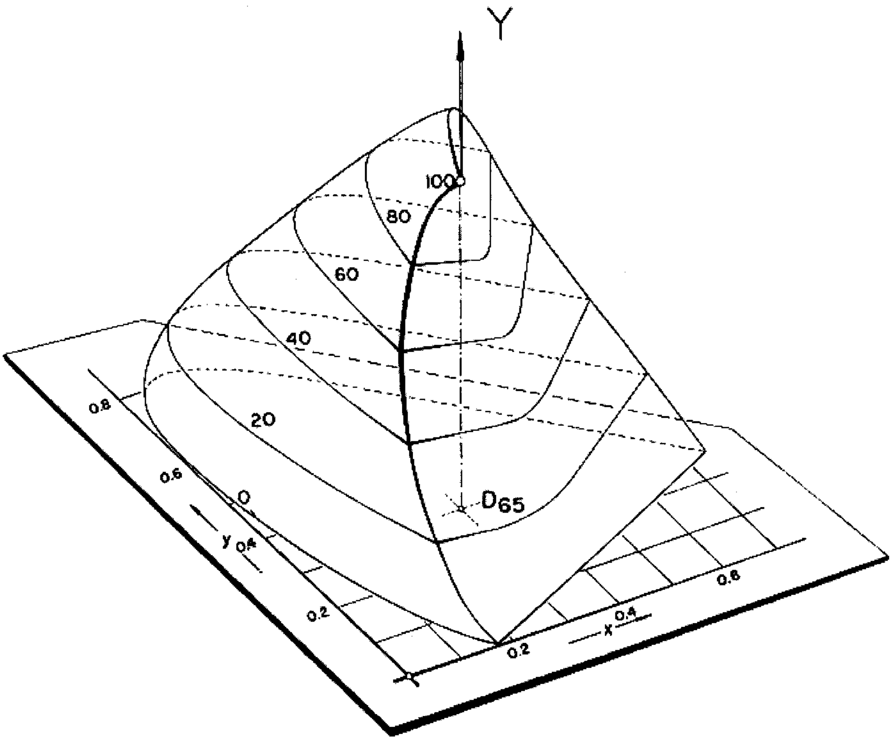


Fig. 2-46 Projective view of the Rösch-MacAdam color solid containing all object colors as viewed by the CIE standard observer in standard daylight D65. The axes represent CIE chromaticity coordinates x and y and luminous reflectance Y. From Wyszecki and Stiles (1982).

l'Éclairage (CIE, International Commission on Illumination) promulgated the 2° standard observer and standard illuminants A, B, and C, making possible the specification of colors as seen by the standard observer under a given illuminant by three numbers, the tristimulus values X, Y, and Z. Alternately, colors can be specified in terms of their chromaticity coordinates x and y in the CIE chromaticity diagram and the luminous reflectance value Y (see Chapter 5). The Austrian physicist Erwin Schrödinger (1887–1961) developed in 1920 a theory of optimal color stimuli. It was further developed by Sigfried Rösch in 1928 and by David L. MacAdam in 1935. MacAdam calculated for the 2° standard observer and illuminants C and A the chromaticity loci of optimal colors as a function of luminous reflectance Y. The resulting solid represents an object color space meant to encompass all possible object colors, specifiable in terms of measured values (Fig. 2-46). Investigations have indicated that this space is not visually uniform. The CIE later recommended more uniform color spaces based on transformations of the tristimulus space.

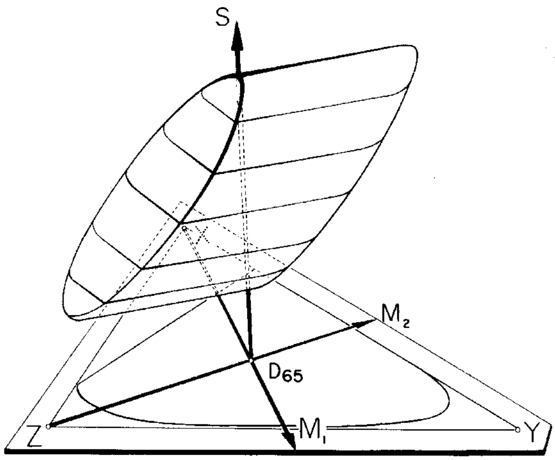


Fig. 2-47 *Luther-Nyberg color solid for object colors viewed in standard daylight D65, based on the color moments M1 and M2 and the color weight S. From Wyszecki and Stiles (1982).*

2.24 THE LUTHER-NYBERG COLOR SOLID

At about the same time, and employing a different interpretation of a colorimetric system, the Austrian physiologist Robert Luther (1868–1945) and the Russian mathematician N. D. Nyberg independently developed a vector color solid based on what Luther called color moments (Luther, 1927; Nyberg, 1929). The two color moments M_1 and M_2 are located at right angle and are defined as $M_1 = Y - X$ and $M_2 = Y - Z$. The color weight S, defined as $S = X + Y + Z$, is located perpendicular to the crossing point of the two color moments. Within the resulting solid all possible object colors are located (Fig. 2-47). The Luther-Nyberg solid is an early linear expression of an opponent color space where the two color moments are interpretable as greenness-redness respectively yellowness-blueness system axes. In more modern systems the color weight is replaced by the luminous reflectance Y. The Luther-Nyberg solid is located in a psychophysical vector space that is not visually equidistant.

2.25 THE GERMAN DIN6164 SYSTEM

A comprehensive proposal for a color order system, based on ideas of Manfred Richter, was made beginning in 1955 in Germany and promulgated as German standard DIN6164. By 1962 a collection of 590 painted paper chips was available. In 1984 a glossy set of 1004 color chips was added. The color solid forms a modified double cone with the distance from the full hue plane

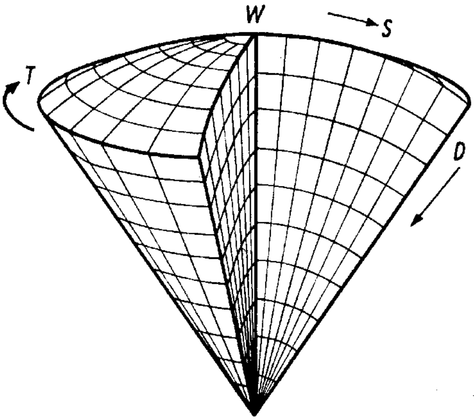


Fig. 2-48 Schematic depiction of the DIN6164 color solid based on attributes hue (T), saturation (S), and darkness degree (D). W denotes the white point. From Richter (1976).

to white much shorter than to black. A schematical view is shown in Fig. 2-48. The three color attributes are hue number T, saturation degree S, and darkness degree D. Colors of constant hue as defined in the system have constant dominant wavelength regardless of degree of saturation, and the perceived hue therefore generally varies slightly along these lines. The 24 simple hues of the system, modified from the Ostwald system, begin with greenish yellow at 12 o'clock and proceed clockwise toward red. They are intended to be visually equidistant. Between unique yellow (approximately DIN color 2) and unique red (DIN 9) there are seven steps, between red and blue (DIN 17) eight steps, between blue and green (DIN 21) four steps, and from green to yellow five steps. Six saturation steps were visually scaled at one lightness level only, and the results were extrapolated to other levels. Colors of equal degree of saturation are located on roughly elliptical contours in the chromaticity diagram, not unlike the equal chroma contours of the Munsell Renotations. The darkness degree D is on a scale of 0 to 10, with twenty steps illustrated. It is based on a formula recommended by Delboeuf in 1872, with experimentally determined values for the constants:

$$D = 10 - 6.1723 \log(40.7h + 1), \tag{2-5}$$

where h is the relative lightness A/A_0, with A_0 representing the lightness of the optimal color of the same hue (at the Rösch-MacAdam limit). White has the value $D = 0$ and black $D = 10$.

The DIN system is based on limited visual data (scaling of hue at one level of saturation, scaling of saturation for eight hues at one level of lightness, scaling of darkness degree to find the applicable constants in the Delboeuf equation) and inter- and extrapolation to other areas. Three compromises between psychological results and ease of expression in a psychophysical system have been made:

1. Expression of hues as constant dominant wavelengths, regardless of saturation.
2. Straight-line extrapolations of points of equal saturation intervals at one level of lightness to other levels.
3. Definition of lightness scale based on optimal colors rather than on psychological experiment.

The system was described by K. Witt (1981), K. Richter and Witt (1986), and G. Derefeldt (1991).

2.26 OPTICAL SOCIETY OF AMERICA UNIFORM COLOR SCALES

Intensive study of uniform color scales during and after the Second World War resulted in new insights about difficulties in representing hue, chroma, and lightness in a euclidean system. The Munsell Renotations were completed in 1943, but there was an obvious, considerable discrepancy between those data and the MacAdam color-matching error data of 1942 (see Chapter 6). In 1947 the Optical Society of America formed a research committee to develop as uniform an atlas with samples of a uniform color solid as possible and a formula to express the implicit space, usable for calculation of color differences from colorimetric data. It was headed from 1947 to 1972 by Dean B. Judd and then by MacAdam. The committee very early on decided to present its results in form of a crystalline internal structure of color space, thus to abandon the Munsellian attributes. In the chosen system twelve equal distances around a center color form a cubo-octahedron (Fig. 2-49). The result is the Optical Society of America Uniform Color Scales (MacAdam, 1974). Overlapping cubo-octahedra make it possible to fill color space with this crystalline infrastructure. The cubo-octahedra are arranged in a manner that results in a grid of squares in a given lightness plane, with all differences along the horizontal and vertical directions of the grid implicitly of equal perceptual size (Fig. 2-50). Grids at the next higher and next lower lightness increment level are offset. This arrangement does not result in explicit lines of equal hue or circles of equal chroma but allows seven different kinds of cleavage planes that provide new vistas within the color solid (Fig. 2-51). In 1974 an atlas with 558 painted samples was made commercially available and the colorimetric aim values for these samples have been published (MacAdam, 1978). The system is open ended, and samples are identified by three rectangular coordinates L, j, g, where L denotes lightness, j approximately yellowness or blueness, and g approximately greenness or redness. Formulas have been

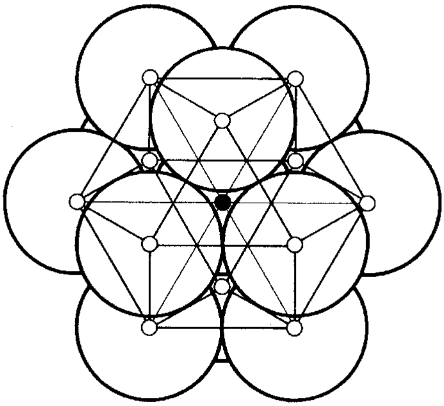


Fig. 2-49 A central color (black dot) surrounded by twelve visually equidistant colors located in the center of spheres and forming a cubo-octahedral structure. From Gerstner (1986).

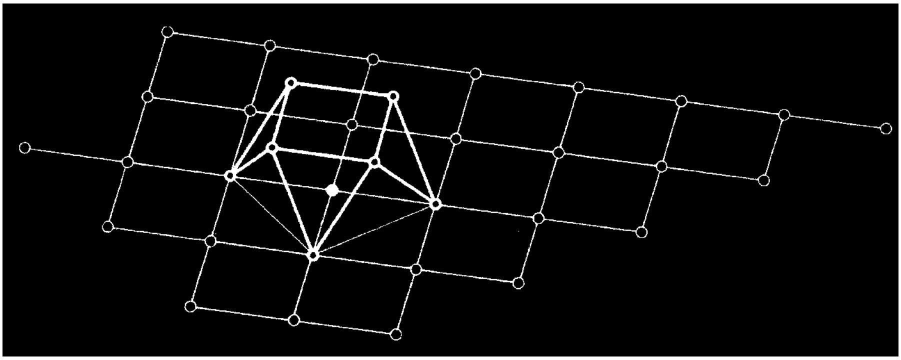


Fig. 2-50 Square grid constant lightness plane of the L, g, j space with the rotated half cubo-octahedron placed over it. Grids at the next higher and lower lightness levels are offset by one half chromatic step, as illustrated by the portion of the cubo-octahedron. From Gerstner (1986).

developed that attempt to represent an accurate psychophysical model of the psychological space (see Chapter 6). They include adjustments of the lightness scale for the crispening effect and for the Helmholtz-Kohlrausch effect (see Sections 5.8 and 8.2 for discussions of lightness and chromatic crispening and Section 5.8 for the Helmholtz-Kohlrausch effect). For more details and an analysis of these scales, see Chapter 7.

2.27 SWEDISH NATURAL COLOR SYSTEM

A system called, after Hering, Natural Color System was developed in Sweden in the 1970s. It traces its sources to Hering and Ostwald, and more recently to

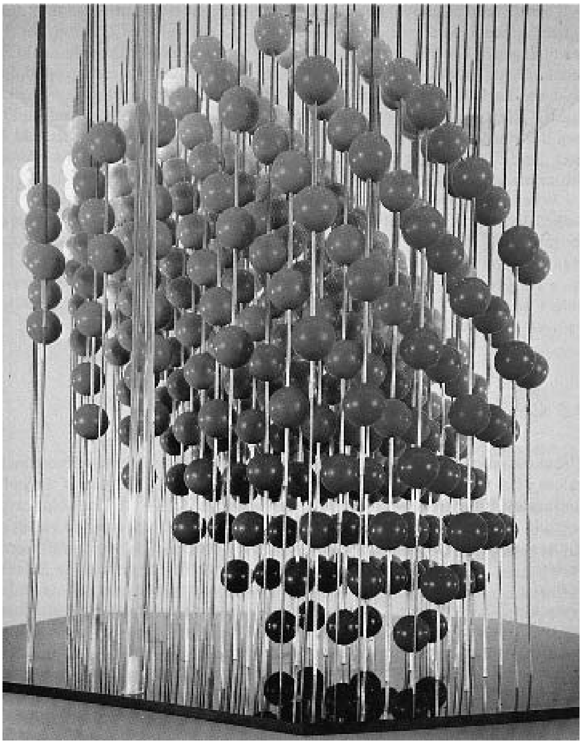


Fig. 2-51 *View of MacAdam's model of the OSA-UCS color solid illustrating the existence of several cleavage planes. See text for more detail. Slide courtesy D. L. MacAdam. (See color plate.)*

efforts by Johansson (1949) and Hesselgren (1954). The fundamental idea was that any color normal observer is able to determine in any object color the "content" of one or two fundamental full colors and of black and white. The three, respectively four, percentages always add up to 100%. In a manner comparable to Ostwald's, the Natural Color System is represented by a double cone. A forty-hue circle of full colors based on experimentally determined unique hues is located at the edge of the double cone (Fig. 2-52). The central axis is represented by a gray scale with ten steps, with white on top. Mixtures of full colors with white or black are located on the surface of the double cone. Mixtures of full colors with white and black are located in the interior. Colors of equal hue are located on triangles bounded by white, black, and the full color (Fig. 2-53). Colors are identified by hue number, NCS chromaticness c, and NCS blackness s. An atlas representing the system has been developed, containing 1750 matt color chips arranged in triangles on forty equal hue planes. Every other plane illustrates colors from $c = 10$ while those between

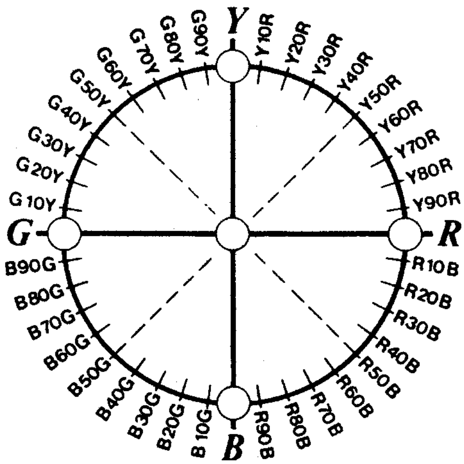


Fig. 2-52 Hue circle of the NCS color solid with hue designation scheme, identifying ten hue steps between unique hues. Courtesy NCS.

begin at $c = 40$. Colorimetric values based on the 2° standard observer of the aim colors and typical color chips have been provided in a table (Swedish Standard, 1982). For more details and an analysis of the NCS system, see Chapter 7.

2.28 UNIVERSAL COLOR LANGUAGE

In 1931 the then just formed (American) Inter-Society Color Council (ISCC) began work under Godlove on a standardized method (in English) to designate colors. An initial list was issued in 1939, and Kelly and Judd issued in 1955 NBS Circular 553 *The ISCC-NBS Method of Designating Colors and a Dictionary of Color Names*. In 1976 the NBS Special Publication 440 *Color: Universal Language and Dictionary of Names* was issued (Kelly and Judd, 1976). The Munsell color solid was subdivided into regions that can be designated by common names. The system is applicable to opaque, translucent, and transparent colored materials. Twenty-nine major color regions contain a total of 267 subregions around centroid colors. Among the twenty-six major regions are those of the primary colors black, white, yellow, red, blue, and green, as well as secondary and tertiary colors such as orange, brown, olive, purplish pink, and violet. The subregions are identified by modifiers, such as vivid, strong, deep, dark, light, and grayish. A supplement contains on eighteen pages a sample of the centroid color of the 267 subregions, with its ISCC-NBS name and Munsell Renotation.

In the 1976 publication Kelly describes six levels of precision of designat-

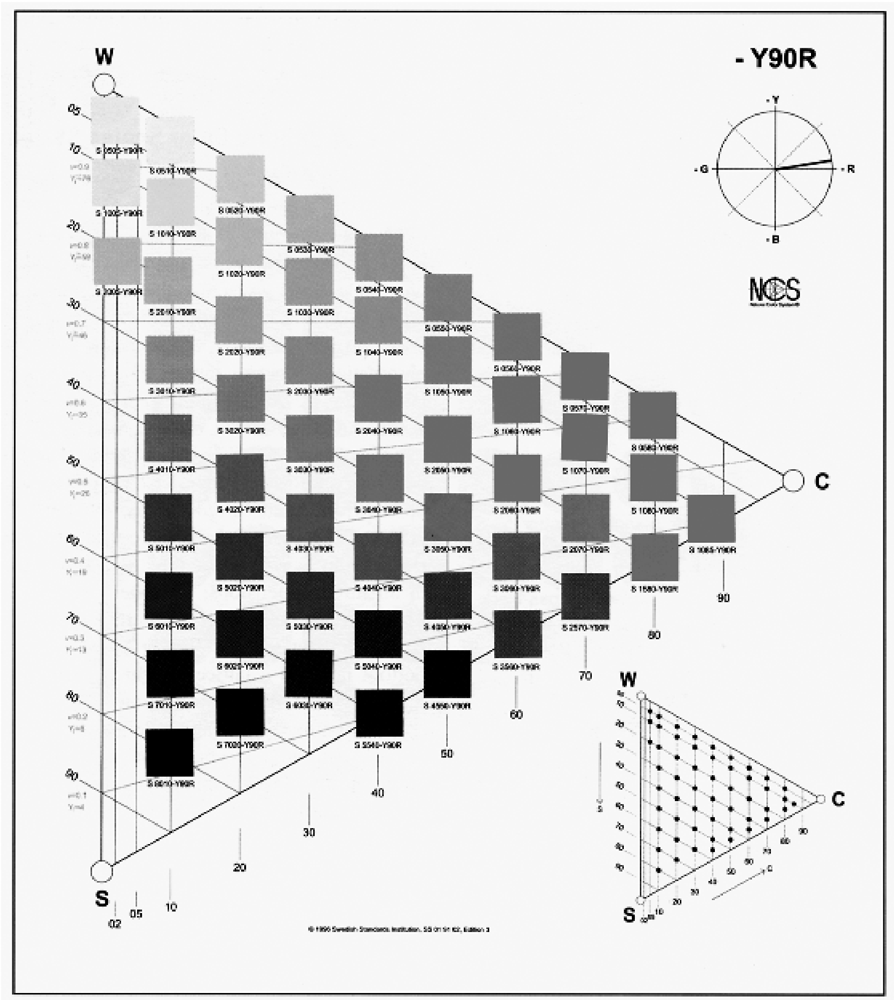


Fig. 2-53 NCS constant hue triangle of hue Y90R with full color C, white W, and black S. Colors of constant blackness s are located on lines parallel to W–C, colors of constant chromaticness c fall on lines parallel to W–S. Courtesy NCS. (See color plate.)

ing colors. Level 1 consists of thirteen generic color names: white, gray and black, yellow, orange, brown, red, pink, purple, blue, green, olive, yellow green. On level 2 sixteen intermediate hue names are added, such as reddish brown, bluish green, and purplish pink. Level 3 contains the 267 subregions. On level 4 the space is further subdivided into more than 1000 colors, as in the *Munsell Book of Color*. Some 100,000 colors can be specified on level 5 by visually interpolating between Munsell chips, such as a color specified as 7.8PB

2.2/12.5. On the final level colors are specified by standard colorimetric data, such as CIE x, y, Y values. Here the number of definable colors is in the millions.

2.29 COLOR MIXING SPACES

In computer and color reproduction technology of the late twentieth century several types of color space are in routine use. In cathode ray tube (CRT) displays, color television, and most computer video displays, color stimuli are generated with three different types of phosphors that can be activated with electron beams. The colors resulting from such activation typically are orange-red, leaf green, and violet, additive color primaries. A large variety of color experiences can be created as a result of their mixture. These stimuli are represented in the RGB color cube. The abbreviations R, G, B designate red, green, and blue, loose verbal designations for additive color primaries used. The cube resembles the Benson cube (see Section 2.15). Colors can be identified in terms of R, G, B units of activation. In the RGB version the standard values of the three components range from 0 to 255. As the cube is rotated so that white and black fall on the vertical axis, a version of a polar coordinate system is imitated and the system is termed HSB (for hue, saturation, brightness) space. In this model hue is expressed as hue angle in degrees, saturation as a percentage from 0% at the achromatic point (gray) to 100% at full saturation, and brightness as a percentage from 0% at black to 100% at white. Achromatic colors have identical values of the three components, chromatic colors have varying values. The phosphors used define the gamut (the maximum chromatic range) of colors that can be achieved. The HSB and RGB spaces depend on the exact definitions of the primary colors used in their creation. They are regular spaces but not uniform as defined in Chapter 1.

Software programs such as Adobe Photoshop® display slices through the HSB space with a continuous gray scale on the left-hand side and the full colors at different lightness on the right. The plane is filled with intermediate colors (Fig. 2-54). Because of the additive mixture the hue of such a plane is not uniform but changes slightly as a function of saturation and brightness.

Another space used in design software is the CMYK space. The letters stand for the subtractive primaries of four-color printing: cyan, magenta, yellow, and black (K to avoid confusion with B). In printing processes the primary colors are defined in terms of standard printing inks. For CRT simulation (e.g., in Adobe's Illustrator® software) CMYK is defined in terms of individual percentages of the four components. A white color has zero values for all components. The gray scale differs in percentage of K. A chromatic color may have percentage values in all four categories. The gamut of CMYK is usually smaller than the gamut for RGB because of the limited chroma of printing primaries.

In programs like Illustrator® and Photoshop® colors can be defined simul-

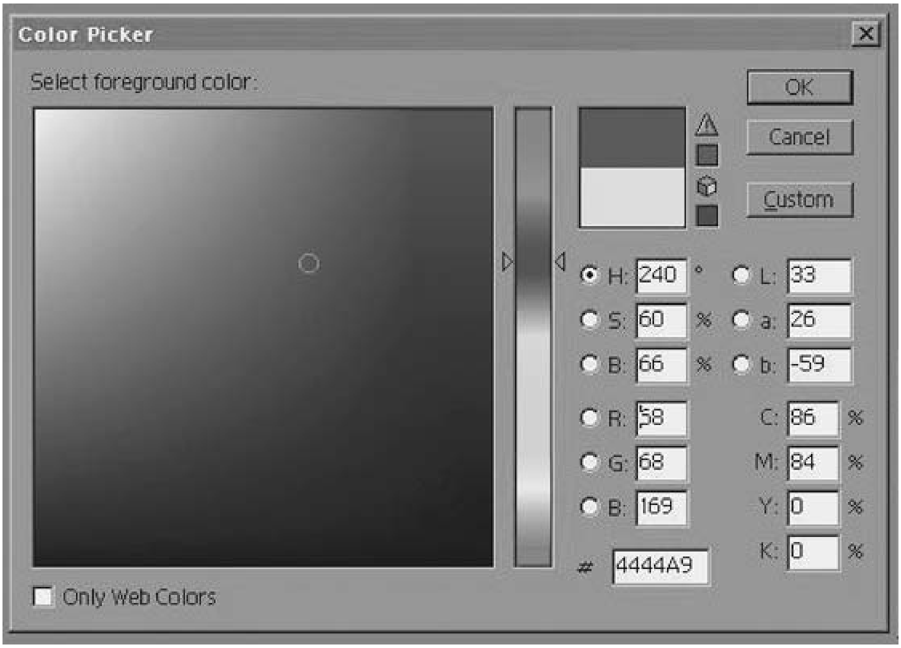


Fig. 2-54 *Color Picker screen from Adobe® Photoshop showing specification of a given reddish blue color in four systems: HSB, Lab (=L*, a*, b*), RGB, and CMYK. (See color plate.)*

taneously in terms of HSB, RGB, CMYK, and CIELAB $L^*a^*b^*$, and they allow input values from any of these systems to display (with a degree of accuracy that depends on many factors) the corresponding color. There are complex issues of color management in reproducibility of such CRT colors in different media (color transparency or opaque color photos, printing on different types of printers, from computer printers to commercial printers).

2.30 SPECTRAL SPACES

Spectral functions can be placed in regular spaces without consideration of human observers. Such spaces have use in machine vision, scanners, and copiers and other applications. Several methods have been applied for this purpose. Among these is principal component analysis (PCA), first used in connection with colors for a subset of Munsell colors by J. Cohen (1964). Another is through training of computerized neural networks. In PCA basis functions are calculated describing the spectral functions with a degree of accuracy that depends on the number of functions. If the number of basis functions is three, they form an orthogonal space in which each spectrum plots as a point. For an extensive set of samples, such as those of the Munsell system or of images of natural scenes, about 90% of the variation in the spectral functions is explained by three basis functions (Lenz et al., 1996; Wachtler et al., 2001). There is an all-positive nearly horizontal function that can be seen as being the spectral equivalent of a lightness function. The next two functions have positive and negative lobes and can be seen as rough approximations of opponent color functions (see Section 6.18). They extract information in spectral functions more efficiently than functions describing the spectral sensitivities of human cones. On the other hand, while both kinds of functions place the reflectances of a Munsell hue circle into ordinal order, (with some exceptions) only the cone and color-matching functions place them approximately in interval order in regard both to hue difference and chroma. There are several mathematical treatments that can be used for PCA and similar operations. Some of these have found use in color constancy and color appearance modeling (Maloney, 1999). However, for the reasons given and others it is not justified to call spectral spaces based on reflectance functions color spaces.

This review of the historical development of thought on color order and of color-ordering systems has concentrated, with few exceptions, on psychological systems that have often found a physical expression in form of atlases. When comparing such atlases, the samples are found to differ. There is a considerable literature on the results of comparisons among the Munsell, NCS, DIN, and OSA-UCS systems (e.g., see Derefeldt, 1991). The results have been summed up by the philosopher of color C. L. Hardin (1988): ". . . the issue is not that there cannot be a consistent scheme of representing phenomenal color, for there can. And, indeed, there is reason to suppose that the various representations can be mapped into each other (though the mapping relations will often be quite complex)." When doing such mapping, one should distinguish between systems that aim to be perceptually uniform (e.g., Munsell and OSA-UCS) and other systems that are regular in some way (e.g., NCS). Accordingly we may expect common principles behind the first group but significant discrepancies between the two groups, as will be seen in Chapter 7.

In discussion of the Munsell system in Chapter 1 we have seen early scaling experiments that resulted in significantly different ratios of hue differences to value and chroma differences depending on the size of the difference. This points to an apparent dependence of color space on the size of the differences selected to represent the space, a matter discussed in detail in Chapter 6.

As this chapter indicates, development of a uniform color space has been a struggle over more than two centuries, a struggle that has not yet been successful. A modern view for the reason is that our color perceptions are empirically determined and thus not in a relationship to stimuli that is expressible by simple logical rules (Purves and Lotto, 2002) or that there are no explicit neural correlates for what we perceive (Dennett, 1991; O'Regan and Noë, 2001). Under conditions of extensive relativity (average observer pool, constant defined lighting, and surround conditions) it may be possible to reduce the visual situation to one in which no ambiguities are present, and empiricism may be limited to a constant set for the observer pool. The modest degree of agreement in different attempts at a visually uniform space in the last century perhaps gives an indication that this is possible. Additional experimental work is required to establish what is feasible. The value of such a space would be limited to the specific conditions under which it was determined. For many applications this can nevertheless be useful.

Chapter 3

Psychophysics

Seeing and hearing have a comparable function: transforming types of ubiquitous physical energy into a format that brains can use in support of the organisms they are a part of. Tasting and smelling rely on the interaction of organic or inorganic chemicals in the environment with special chemical sensors in the corresponding organs. Other, specialized sensations, such as certain feelings of pain or pleasure, presumably result from the generation of electrochemical signals entirely within the body. Psychophysical methods are empirical attempts to discover the connection between stimuli and the resulting sensory experiences by measurement of reported percepts or performance. By categorizing experiences and analyzing the connection between stimuli intensities and the resulting experiences, psychophysics attempts to discover how sensory psychology operates and to help establish linking propositions between neurobiology and psychology.[1] One might expect sensations to be linearly related to stimuli, but this is rarely the case as will become apparent. The mathematical underpinnings of psychophysics are related to measurement theory, a branch of applied mathematics. A postulate of measurement theory is that measurements are not the same as the attribute being measured. By measuring the difference between two colors, we have not revealed anything about the nature of the two colors. W. S. Sarle (1995) defines measurement of an attribute of a set of things as "the process of assigning numbers or other symbols to the things in such a way that relationships of the numbers or symbols reflect relationships of the attribute being measured. A particular way of assigning numbers or symbols to measure something is called a *scale* of measurement." The mathematics of measurement are complex and have been

Color Space and Its Divisions: Color Order from Antiquity to the Present, by Rolf G. Kuehni
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described in the three-volume work *Foundations of Measurement* (Krantz et al., 1971, 1989, 1990).

3.1 FUNDAMENTS OF PSYCHOPHYSICS

The problem at the heart of psychophysics is lack of understanding, and of a theory about, how our feelings and experiences are generated. It is what the philosopher D. J. Chalmers referred to as "the hard problem" of consciousness (Chalmers, 1996). Given the lack of such knowledge attempts to create a fundamental theory of psychophysics are speculative. Fechner faced critique from his contemporary Kries that sensation magnitude is the result of hidden processes (that sensation is cognitively impenetrable) and that, as a result, numbers applied to sensation are not numerical in the mathematical sense but merely convenient labels. Numbers may erroneously suggest that quantitative measurement in the sense of physical measurement has taken place, implying a precision that is not there.

Psychophysics has not answered this attack in a fundamental way. It needs to be seen as an essentially empirical, pragmatic attempt to connect intensity of stimulation with magnitude of sensation or, rather, perception. Measuring or scaling psychological magnitudes or differences involves making judgments. Judgment is defined in the relevant meaning as "the process of forming an opinion or evaluation by discerning and comparing" (*Webster's Collegiate Dictionary*, 10th ed.). The processes by which judgments are formed and their neurological expression are unknown. If S is the experienced sensation and R the judged numerical response to the stimulus intensity I, there is an implicit psychophysical transformation F_1 as follows:

$$S = F_1(I). \tag{3-1}$$

On the other hand, the response of observers, R, is related to intensity by the judgment function, F_2:

$$R = F_2(I). \tag{3-2}$$

The two transformation functions F_1 and F_2 are normally conflated to arrive at the result

$$R = F(I). \tag{3-3}$$

Without knowing the values of either component it is impossible to determine the other (Marks and Algom, 1998). Equation (3-3) is a mathematical expression of the assumption underlying psychophysics.

A recent attempt, among several, to develop a fundamental psychophysical theory has been made by K. H. Norwich (1993). He based it on information theory. More intense stimuli have greater information content than less intense ones. According to Norwich sensation provides a measure of the information content in the stimuli:

$$S = kH, \tag{3-4}$$

where S is a perceptual variable (the sensation magnitude), k is a positive constant, and H is the stimulus information available for conversion. S is taken to be the result of several separate stimuli, and H is calculated as a function of the probability of each of the stimuli involved. While the stimuli may in some cases be easy to define in a physical sense, the sensors (e.g., the cones of human vision) are believed to continually undergo changes at the microscopic level. The Weber-Fechner law as well as Steven's power law have been shown to be special cases of Norwich's law, with the Weber-Fechner law presumably applying at higher levels of information and the power law at lower. By itself Norwich's law is of course not an explanation of the "hard problem."

Psychophysical measurements can be made at different points in the seeming continuum of experience: (1) absolute threshold, (2) difference threshold (just noticeable differences), and (3) scaling of the full continuum. Absolute and difference thresholds have classically been determined by Fechner's methods, described below. In the 1960s the signal detection theory was developed (Green and Swets, 1966), and it breaks Fechner's idea of a discrete sensory threshold into two components: (1) a neurophysiologically determined basic component (usually called discriminability, d', and (2) a cognitively determined decision process (response bias, β). As a result there are four response types possible: hits, misses, false alarms, and correct rejections.

Fechner believed that sensory scales can be derived on basis of discrimination and that just noticeable differences expressed in terms of stimulus increments or decrements can add up to sensory magnitudes. Stevens, on the other hand, postulated that sensory magnitudes can be directly assessed. According to him measurement is the assignment of numbers to objects according to certain rules and results in empirical relations. There are experimental data in support of either approach. Psychophysicists today seem to lean toward the idea that the fundamental process involves taking differences. Establishing sensory scales may be said to involve adjusting distances between stimuli to match an internal or external standard, or to make numerical estimates of sensory differences.

Psychophysical judgments are known to be subject to various contextual effects. For example, the results of the scale halving method are known to depend on if the stimulus samples are presented in bottom-up or top-down sequence (hysteresis effect). For color scaling as for many other types of perception, the level of adaptation strongly affects the resulting scale. As will be seen in Chapters 4 and 5, gray scales depend strongly on the lightness level of the surround.

Stimulus sequence has also been shown to affect scaling results. Among the possible reasons are changes in the adaptation level based on the previously seen stimulus. The range of stimuli displayed and the level of stimulus are also known to affect the result, particularly in case of magnitude estimates. Among midgets and basketball players the meaning of small, medium, and large involves absolute values that are much different.

Context effects are well known. How the lightness of a surface is assessed depends not only on the adaptation level but also on other contextual information such as is it seen in shadow or in direct illumination. Similar effects apply to chromatic surfaces.

Despite its lack of a solid foundation psychophysics persists because it has considerable pragmatic value. The continuing interest in psychophysics may also be due to what L. E. Marks called "the metaphorical imperative" (1978). Accordingly we have a need to express our inner world in terms of metaphors that are more easily comprehended by other humans. Scaling and magnitude determinations of perceptions can be seen as an expression of the metaphorical imperative.

When viewed from the perspective of Fechner's outer and inner psychophysics, mentioned in Chapter 2, the conventional psychophysical enterprise can be represented in the schematic sketch of Fig. 3-1. The relationship between stimulus and conscious experience represents classical psychophysics. The relationship between nerve excitation and inner sensation is in the domain of neurophysiology and as such subject to process variability that can be

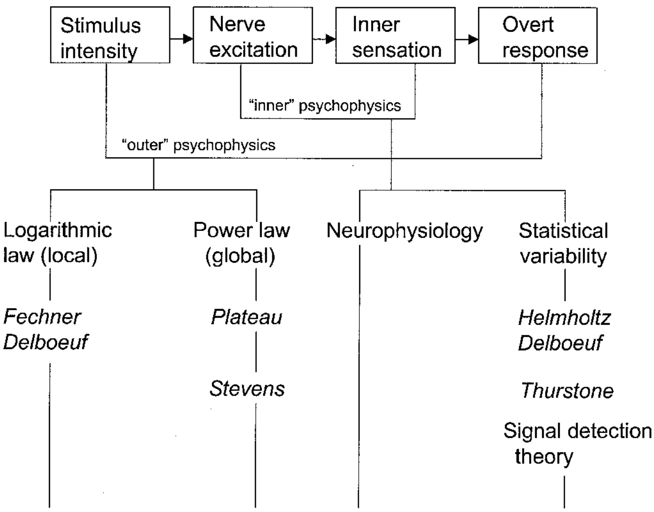

Fig. 3-1 *Conventional psychophysical enterprise as viewed from the perspective of Fechner's inner and outer psychophysics (modified from Murray, 1993). Outer psychophysics attempts to connect the stimulus magnitude with the response magnitude. Inner psychophysics attempts to connect nerve excitation with nonconscious "sensation." It is quantified by recording from visual system cells. Statistical variability and signal detection are taken to be aspects of inner psychophysics.*

treated in different ways, as we have seen. The classical view of psychophysics is that humans react to stimuli. The modern view is that people act on stimuli by interpreting them in a way that contributes toward achieving the goals of the individual.

3.2 CATEGORIES

Categorization is a fundamental trait of humans and perhaps of all animals. To distinguish between edible and poisonous plants, meaning to put them into categories, was an early and imperative task for plant eaters, for example. An important human method of categorization is naming. The process by which we establish categories and sort stimuli into the established categories is clearly complex and at this time largely unknown. Several different theories have been established and vie for recognition.

Stimulus dimensions available for classification are either taken to be continuous (e.g., color) or discrete (four-sided vs. three-sided figures). The structure of categories is either overlapping (hue and chroma) or nonoverlapping (color and shape). In the former case perfect categorization is difficult or impossible while in the latter it is likely. The accuracy of response in categorization experiments is fundamentally either deterministic, meaning equal stimuli result in an equal categorization, or probabilistic: the response of the observer is always based on a (more or less informed) guess. It is obvious that already the stimulus is usually probabilistic. In case of visual stimuli, for example, light is reflected probabilistically off the surface of the object as well as off the eye's cornea, and once the remaining stimulus enters the eye, there is probabilistic activity at all levels of the vision system. Most categorization theories therefore treat categorization data in terms of variability.

Among the theories of category access (what process shapes our category decisions) are those of the necessary and sufficient condition (NSC), the prototype theory, and, related to it, the exemplar theory. According to NSC, categories are described by a series of necessary and sufficient conditions, and the observer tests the sample to see of it meets these conditions. In some situations (e.g., separating squares from triangles), NSC is clearly applicable, but in many others, it is not (e.g., it is difficult to define the necessary and sufficient conditions for a letter symbol to represent an *a*). The prototype theory proposes a (kind of Platonic) prototype for a category whereby objects are classified by their resemblance to the prototype. It implies that we only have the prototype stored in memory and that determining resemblance to it is a cognitive task. It simply delegates the critical categorization to another process. To get out of this difficulty, the exemplar theory was proposed. Accordingly we have all examples of past experiences of the category item stored in memory and compare new examples to the stored ones to categorize them. It seems doubtful, however, that observers need to have seen all reddish colors before they can categorize them as such, for example.

Visual classification is believed to take data from the visual area at the back of the brain to the inferior temporal cortex of the brain located near the temples. This region appears to analyze the data for certain features. It exchanges data with the prefrontal cortex area in the front of the brain an area that may contain category border codes. Categorization is achieved jointly between the latter two areas (Hasegawa and Miyashita, 2002).

In regard to color classification the fundamental question is what process results in setting classification boundaries. If given a thousand randomly different Munsell color chips and asked to categorize them, how would we do it if we had not been exposed to theories and examples before? An obvious attribute for categorization is hue. But it has taken until the seventeenth century to recognize that hue forms a closed circular continuum. Newton has separated spectral colors into seven categories and saw that bluish red and purple colors, not found in the spectrum, can close the spectral array into a circle. Rational classification of hued colors according to whiteness and blackness or lightness and chroma only took place in the nineteenth and early twentieth centuries.

There is the question of the cause of human color categorization in language. As briefly mentioned in Chapter 2, there is a theory by Berlin and Kay that basic color terminology in human languages has followed the same pattern. They identified eleven basic color terms, aside from the six Hering fundamental colors: brown, purple, pink, orange, and gray. The list is notable for its idiosyncrasy: all four chromatic colors are in the yellow-red region of a color circle. In addition the English term pink had in the sixteenth and seventeenth centuries the words yellow or brown attached to it (Merrifield, 1967), and the color of English hunting jackets is called pink. There are no terms for yellow-green, blue-green, and their dark, desaturated versions olive and navy. While there should have been a good number of such colors in natural surroundings in many environments, it has taken systematic categorization of the hue circle to recognize these colors as categories. In C.D.'s extended color circle they are listed as yellowish green and sea blue (see Fig. 2-12). A convincing theory of color categorization that explains the historical development of basic color terms seems to be a long way off.

3.3 DIFFERENCES VERSUS MAGNITUDES

The Weber-Fechner law assumes that just noticeable differences can be considered units of perception. It states that the increment required to result in a just noticeable perceived difference is a constant percentage or fraction of the stimulus magnitude (see Chapter 2). The Weber-Fechner law has been found to be context sensitive. Typical experimental Weber fractions are given in Table 3-1 and an example of the Weber-Fechner law is illustrated in Fig. 3-2.

On a parallel but initially less conspicuous track runs the idea of a power relationship between stimulus and response. Power relationships were

TABLE 3-1 Representative values of Weber fractions for different senses

Sense	Weber Fraction, ΔI/I
Vision (brightness, white light)	1/60
Kinesthesis (lifted weights)	1/50
Pain (thermally aroused skin)	1/30
Audition (tone of moderate loudness)	1/10
Pressure (applied cutaneously)	1/7
Smell (odor of India rubber)	1/4
Taste (table salt)	1/3

Source: Geldard (1962).

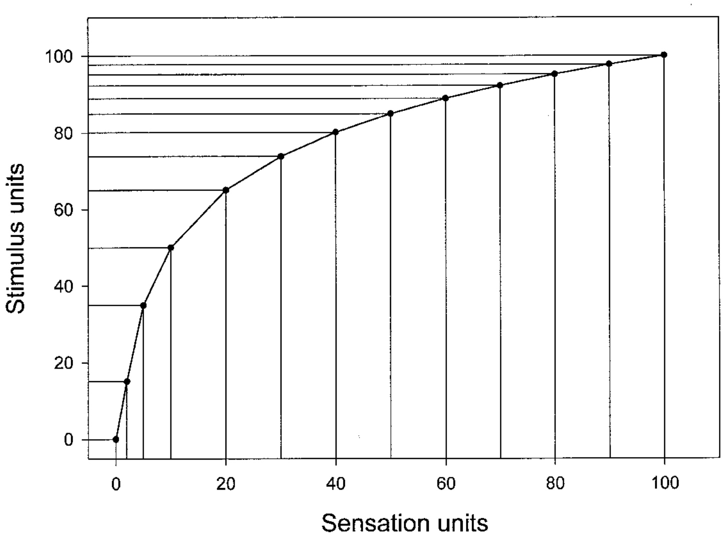

Fig. 3-2 Relation between stimulus and sensation according to the Weber-Fechner law. The logarithmic function illustrates the relationship between an arithmetic (sensation) and a geometric (stimulus) scale.

originally proposed by the mathematicians G. Cramer and D. Bernoulli in their consideration of the utility (subjective value) of money (Bernoulli, 1738). They concluded that the subjective value of an increment of given size of money is larger if it is added to a small amount than when added to a large amount, a relationship that can be expressed with a power function. A power function was proposed by Plateau to express his range partition results in which he had several painter friends paint samples of the psychological midpoint between a black and a white painted samples. From the remarkably uniform results and Plateau's assumption that there were ratios of stimuli and response involved, he concluded that a power relationship could explain them. However, wanting to establish a more detailed gray scale, he asked his friend Delboeuf to develop

TABLE 3-2 Representative exponents of power functions relating subjective magnitude to stimulus magnitude

Sensory Continuum	Measured Exponent	Stimulus Condition
Brightness	0.5	Point source
Brightness	0.5	Brief flash
Brightness	1.0	Point source briefly flashed
Lightness	1.2	Reflectance of gray papers
Visual length	1.0	Projected square
Redness (saturation)	1.7	Red-gray mixture
Taste	1.4	Salt
Smell	0.6	Heptane
Thermal pain	1.0	Radiant heat on skin
Heaviness	1.45	Lifted weights
Electric shock	3.5	Current through fingers

Source: Abbreviated from Stevens (1975). Reprinted by permission of the publisher.

more steps. The results were in better agreement with the Weber-Fechner law, and the power law was largely forgotten until it found its champion in Stevens.

Before Stevens it was J. P. Guilford who proposed in 1932 the first general power law of psychophysics:

$$\Delta I = c_a I^n, \tag{3-5}$$

where I is the stimulus intensity, c_a is the Weber fraction, and n is an exponent. In this equation if $n = 1$, the relationship is logarithmic. Beginning in the 1930s Stevens investigated the relationship between sensory and stimulus magnitudes of many kinds. He concluded that a power law connects all of them, with the power variable anywhere from 0.25 (fourth root) to 1.5:

$$V = bx^p, \tag{3-6}$$

where V is the sensory magnitude, b is a constant, x is the intensity of the stimulus, and p is an exponent. The implication is that the stimulus increments proportional to equal sensory magnitude increments vary less than by equal percentages as predicted by the Weber-Fechner law. Table 3-2 reproduces selected power exponent values from a larger table by Stevens, and Fig. 3-3 graphically illustrates power functions with different exponents.

Stevens and other researchers found that subjects agree quite well on the size of absolute magnitudes of sensory experiences as related to stimulus magnitude. This can be done by direct judgment or magnitude production via cross-modality matching. In the latter case the sensory magnitude is expressed in another medium, for example, by comparing brightness of lights to loudness of sounds or by squeezing a hand dynamometer to express the perceived brightness magnitude.

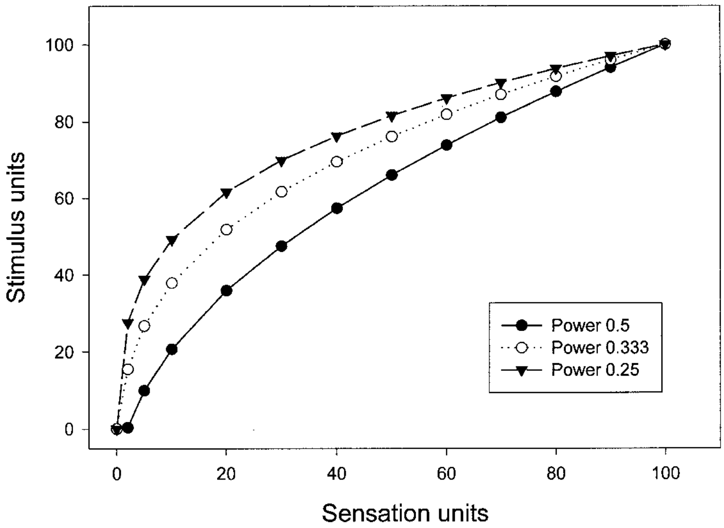

Fig. 3-3 *Relation between stimulus and sensation based on three different power modulations.*

From analysis of his own work and that of other researchers, Stevens concluded that magnitude estimates of sensory experiences tended to have an exponent near 1. In magnitude judgments the observer is free to assign any chosen number to express the magnitude of the perceived stimulus. His own results of magnitude judgments of the steps of a gray scale resulted in a power exponent of 1.2 (Stevens and Galanter, 1957). When making category judgments and difference judgments well above threshold, the power exponent declined in size, and when making difference judgments near threshold, it approached zero and the Weber-Fechner law.

Category scales have set numbers or adjectives and the results of category judgments are usually nonlinearly related to those of magnitude judgments. A gray scale based on equisection or category judgments, namely on differences, results typically in an exponent of 0.25 to 0.5. In his own multiple bisection experiment of brightness Stevens (1953) found an exponent of 0.26. Thresholds are category measurements related to the uncertainty of the response, with a relationship to the stimulus difference most accurately expressed as a logarithmic function. The power function can also be closely matched with an hyperbolic function, first implicitly used by Hering (1874):

$$V = \frac{ax}{1 + kx}, \tag{3-7}$$

where a and k are constants and x is the intensity of the stimulus. In a plot with a linear ordinate and a log abscissa the resulting function has an S shape. Thresholds, and the Weber-Fechner law approximating them represent

TABLE 3-3 Effect of judgment type and size of difference on applicable stimulus modulation

Type of Evaluation	Applicable Power	Example
Magnitude	0.8 and larger	Lightness of gray papers
Paired comparison	0.05–0.75	Small color differences, Munsell chroma scales
Thresholds	0.01–0.33	Color thresholds

(in Fechnerian terms) local psychophysics; the power law is, in terms of differences or magnitudes, representative of more global psychophysics. Table 3-3 provides a comparison.

It is apparent that neither the logarithmic nor the power law provides an explanation for the causes of the psychophysical relationships. They are merely mathematical models. In the case of visual scaling the explanation for the apparent signal compression is buried in the complexity of the visual system and the possible impact of empiricism. While some details about the visual system are known at this time, they are not nearly enough to develop a fully detailed model predictive of all known effects. J. C. Baird has developed a general simulation model that attempts to explain the effects described by the laws from the aggregate action of neural cells with varying thresholds (sensory aggregate model; Baird, 1997). It is not evident that this model applies to visual scaling. K. Richter (1996) has proposed a model that attempts to explain visual psychophysics only in terms of cone receptor saturation. It seems more likely that post-receptor effects also play a role.

Psychophysics in the classical sense depends on so-called linking propositions. These are propositions of how neurophysiology and psychophysics might be connected. Linking propositions can be strong or weak, depending on the level of evidence supporting the proposition. A typical linking proposition is that chromatic perception is supported by neurons with opponent response in the visual area of the brain. Linking propositions related to the concept of a uniform color space are still weak. The modern view of vision is that the brain constructs what we consciously experience from the output of several of its visual modules and using many (and probably many as yet unknown) decision rules to construct the experience from generally ambiguous input at the retinal level. As S. E. Palmer expressed it: "The job of visual perception is to combine external and internal information to make meaningful facts about the environment available to the organism" (Palmer, 1999). Simple links between neural sensitivities to stimuli and psychological measurements or judgments are therefore unlikely except in highly relativized situations.

3.4 PSYCHOPHYSICAL SCALING: LEVELS OF MEASUREMENT

An important aspect of psychophysics is the scaling methods used to assess local and global scaling. Stevens (1946) proposed that onedimensional scales may be placed into a hierarchy with each subsequent type having greater explanatory power than the previous one. This hierarchy is called levels of measurement.

Nominal Scales

They are at the lowest level and refer to names or identifications of items only. The same symbol is assigned to two things if they have the same value of the attribute. An applicable sample for color is names. Colors can be grouped, for example, into blues, browns, pinks, purples, and grays. An appropriate statistic is the number of cases.

Ordinal Scales

Numbers are assigned to things in a way that the order of the numbers reflects the order of the attribute. Items are placed into ascending or descending rank order depending on some kind of magnitude. For colors a typical example is a series of grays that can be ordered according to the concept of blackness. The color nearest to black (sample E in Fig. 3-4) has ordinal scale value 1, that closest to it but lighter (sample A) has value 2, and so on. A series of chromatic colors of varied chromaticness can also be placed into an ordinal chroma scale. Ordinal scales do not contain any information about the sensory magnitude of the steps between the grades on the scale. In Fig. 3-4 a random

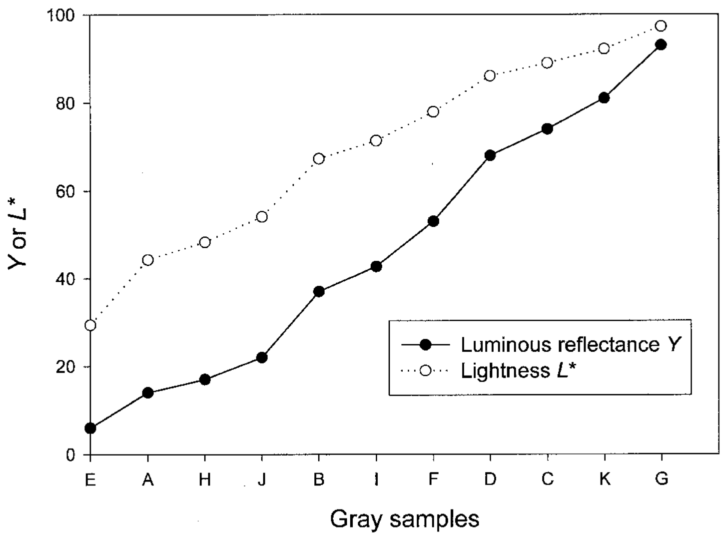

Fig. 3-4 Twelve random samples of gray papers (A–K) are placed into an ordinal scale based on their luminous reflectance value Y *and on an interval scale based on lightness* L*.

collection of gray samples has been ordered according to the luminous reflectance scale (Y). Luminous reflectance places the samples into an ordinal scale where the scale values do not provide any information about the perceptual magnitude of the steps between the grades of the scale. An ordinal scale is subject to logical operations: equal to, greater than, less than; and appropriate statistics are median, percentiles, or rank-order correlation.

Interval Scales

At the next level interval scales provide quantitative information concerning the distances or differences between grades. Here things are assigned numbers so that differences between numbers reflect differences of the attribute. In an interval scale two grades differing by three interval units at the lower end of the scale, and two grades at the higher end of the scale also differing by three interval units are equally distant (e.g., the Celsius and Fahrenheit temperature scales). In interval scales the distance between two percepts is represented with a number according to

$$a = mb + c_0, \tag{3-8}$$

where a is the scale number representing a grade, m is any positive number, b is the distance of the grade from the neighboring grade or the origin, and c_0 is any finite number. Addition of and multiplication with constants are permissible transformations and statistics include mean, standard deviation, or correlation coefficient. Color scales usually are interval scales. In Fig. 3-4 the twelve random gray samples have also been ordered according to the interval scale L^*. The perceptual difference between samples B and I is about half as large as that between samples F and D.

Ratio Scales

These are interval scales that have a natural origin. In this case things are assigned numbers such that differences and ratios between the numbers reflect differences and ratios of the attribute. While in interval scales two numbers could be applied arbitrarily to the scale (m and c_0) only one number can be arbitrarily assigned:

$$a = mb. \tag{3-9}$$

Examples are lengths in meters, duration in seconds, and temperature in degrees Kelvin. As mentioned before, ratio scales in regard to color are controversial. Ratio scales can be multiplied with a constant only and statistics include percent variability.

Absolute Scales

Here things are assigned numbers such that all properties of the numbers reflect analogous properties of the attribute. These measurement levels are part of a continuum of order, with the level of order being weakest for nominal scales and strongest for absolute scales. In practice, depending on the measurement technique used, the same scale can have properties of two measurement levels.

3.5 SCALING METHODS

Many different methods of scaling have been developed in psychophysics. For absolute or difference thresholds the classical methods discussed in this section are those described by Fechner (1860).

Method of Adjustment or Method of Average Error

In this method the observer can adjust the stimulus magnitude and does so until she observes a just noticeable difference. On the surface this is a simple and straightforward method. However, it requires equipment on which the stimulus magnitude (under control of the observer) can be continuously adjusted. The results are direct but also subject to cognitive adjustments and biases.

Method of Limits or Method of Minimal Change

Here the experimenter presents changes in the stimulus magnitude in preset small increments in ascending or descending order, starting with an imperceptible increment in the former case and a clearly perceptible one in the latter. This continues until the observer indicates the perception of a difference in the former case or the absence of a difference in the latter. The JND is taken to be represented by the average of ascending and descending trials.

Method of Constant Stimuli

In this method the experimenter selects several constant pairs of stimuli ranging from below threshold to above threshold. These are then presented several times randomly to the observer. The observer responds with a yes or no, and the JND limit is determined at the 50% or perhaps at the standard deviation level. Alternatively, the experiment is set up so that the observer must respond to a forced choice between two pairs of stimuli. This method has been further varied in the so-called staircase procedure. Here a forced choice is imposed at the lowest level. If the observer responds wrongly, the stimulus difference is increased for another forced choice. The JND limit is approached from both sides in such experiments.

Matching Method

A well-known experiment, believed related to color thresholds, is the determination of color matching error by MacAdam (1942). He constructed an apparatus in which color stimuli could be varied along selected lines in the CIE chromaticity diagram at constant luminosity. A standard stimulus was displayed and the observer adjusted a test field until the two fields matched, that is, had identical appearance. From the variability of repeated tests MacAdam calculated the matching error that he thought to be related to the difference threshold.

These methods are known under the general rubric of confusability scaling. Thurstone stated a law of comparative judgment in 1927. This law considers every perceptual magnitude judgment as a variable datum from a discriminable process that he took to be normally distributed. In this manner the full power of statistics of normal distribution can be applied. Under specific conditions such statistical treatment results in confirmation of the Weber-Fechner law, other conditions result in applicability of Steven's power law.

3.6 UNIDIMENSIONAL SCALING METHODS

Unidimensional scaling involves perceptions that have one attribute only. Several physical dimensions may be involved in generating the attribute. Over the century and a half since the beginning of psychophysics several scaling methods for unidimensional scaling of perceptual distances have been developed. Only those used in color scaling will be briefly discussed.

Partition Scaling

First used in the form of the equisection method by Plateau, partition scaling is a direct estimation method. In the equisection method the perceptual distance between two different stimuli may be halved in several steps. Another version determines equal-appearing intervals.

Ratio Production and Ratio Estimation

As the name implies, the method results in a ratio scale. In ratio production the observer adjusts a magnitude of a perceptual attribute until it equals the perceived double magnitude (or other ratio) of a reference. In ratio estimation the observer estimates the ratio between reference and test stimuli. For reasons discussed previously, this method is rarely applied in color scaling.

Magnitude Production and Estimation

In production the observer is given a stimulus and a number that corresponds to it and is asked to produce a stimulus that is representative of another number. Thus a certain sound may be said to represent the value 10, and the observer is asked to modify the sound stimulus so that the resulting loudness of the sound represents the value 15, say. In magnitude estimation the observer estimates the perceived magnitude of experiences from stimuli and assigns numbers to them. For color scaling a large number of stimuli or the capability of stepwise or continuous stimulus adjustment is required.

Category Scaling

Here the observer is asked to separate large numbers of experiences into categories. The corresponding samples must be similar enough so different observers arrive at different categorization. The variability in judgment by different observers is assumed to follow a standard normal distribution from which an interval scale can be constructed. A typical example is acceptability or pass-fail judgments of small color differences in which the observer is asked to determine if a given sample meets or fails a criterion of acceptability.

Paired Comparison

In this method all samples are presented to the observer in all possible pairs or in all pairs of test against a reference sample. The proportion of times a given sample is judged greater in magnitude of a given attribute is determined. Interval scales are derived from the results under the assumption of statistically normal distribution.

In recent years a form of interval judgment for the purpose of suprathreshold color scaling has been in wide use. In this method sample pairs exhibiting small differences are compared against a reference difference, usually in form of an achromatic pair with a perceptual difference of similar magnitude to those of the sample pairs under estimation. Alternately, an International Standards Organization (ISO) type gray scale that displays achromatic pairs with varying perceptual differences has been used. In such situations what are multidimensional color differences (usually involving hue, chroma, and lightness differences at the same time) are evaluated as if they were unidimensional. The surround lightness and chromaticness affects the perceptual magnitude of the reference pair(s), as well as of the test pairs making the result a function of the surround.

Minimally Distinct Border Scaling

A novel method of scaling color differences was developed in the 1960s and 1970s by R. M. Boynton and P. K. Kaiser (Boynton, 1983). Their idea was that with two perfectly juxtaposed color fields at equal luminance the strength of the resulting border should be an indicator of the chromatic difference in the constant luminance plane. The method was found to be applicable only where the two fields differed in activation of the L and M cones.

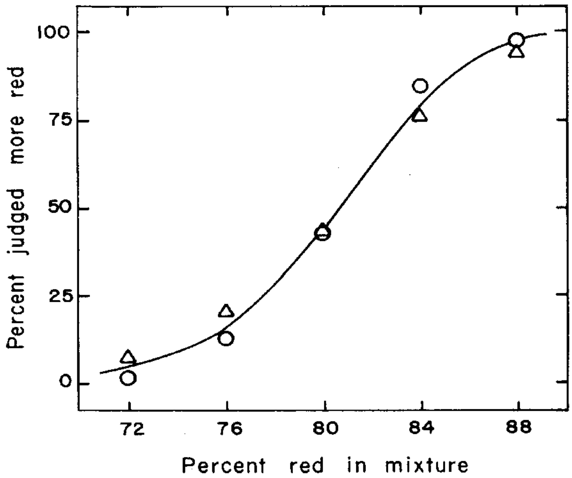

Fig. 3-5 Example of a psychometric function. Samples contain different amounts of a red and a gray stimulus (by spinning disk mixture). Observers judged if a sample contained more red than the one shown previously. Circles indicate results when a reference sample containing 80% red was shown immediately before the test sample, triangles where no reference was shown. From Stevens (1975).

3.7 PSYCHOMETRIC FUNCTION

Psychometric functions are the functions expressing the relationship between subjective and objective measures. The usually normal statistical distribution of individual judgments generally results in nonlinear psychometric functions. An example is pass-fail judgments of color difference. Here the relationship between percent acceptability (% pass) and calculated color difference was found to have a typical S-shaped (sigmoidal) form (see Fig. 3-5 for another example). In such cases it is necessary to linearize the visual scale by using an appropriate method (e.g., Indow and Morrison, 1991). A function that has been used in recent years for this purpose is the cumulative normal distribution (probit) function (e.g., see Berns et al., 1991).

3.8 MULTIDIMENSIONAL SCALING

Multidimensional scaling (MDS), originally developed by W. S. Torgerson in 1952 (1958) and R. N. Shepard (1962), creates geometric models based on similarities, dissimilarities, or proximity. Mathematical analysis is performed on spatial or distance data. However, MDS can also be applied to nondistance data. It can address the nature of the metric of the multidimensional stimulus space implicit in the data, meaning it can extract the unknown dimensionality. Because psychological data usually contain considerable variability differ-

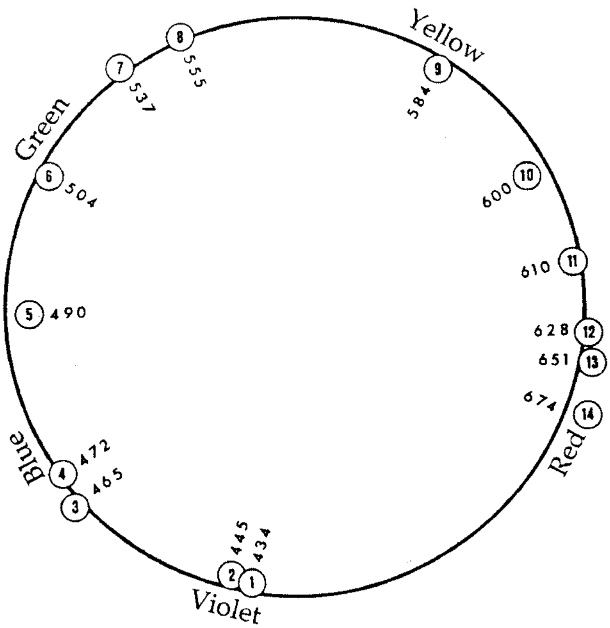

Fig. 3-6 Configuration of MDS analysis of judged differences between fourteen spectral colors (circles with numbers and identified by spectral wavelength in nanometers. A circle segment (closed to a full circle) has been fitted to the points. Presumably on the segment between spectral colors 1 and 14 extraspectral red and purple colors mixed from various ratios of 1 and 14 would be located. From Shepard (1964).

ent MDS techniques often find several different models, all with comparable accuracy of fit. A widely used methodology of MDS is INDSCAL (individual difference scaling; Carroll and Chang, 1970). It is based on euclidean geometry, but evidence has been found that it can "adequately" recover sample configurations even if the true metric is non-euclidean.

In one version of MDS the pairwise distance (in terms of a visual estimate or any other distance measurement) between all items used in the test is established. From these data a similarity or dissimilarity matrix is created as input to the MDS module. Several parameters or assumptions can be changed in MDS and the results are subject to interpretation.

An interesting finding of MDS is that when perceptual distances between fourteen spectral colors are judged by observers the result of the MDS analysis of the data is a two-dimensional structure that is fit best by a circle segment (Fig. 3-6; Shepard, 1994).

MDS has been applied extensively to Munsell data by T. Indow and coworkers (Indow, 1988) and others. Using judgments of magnitude of color difference between many samples of the Munsell system as input into the MDS analysis, Indow recovered the psychological diagram of the Munsell system with good accuracy (see Fig. 4-4 in the next chapter). In principle, scaling of small complex color differences (involving more than one attribute) might be analyzed by MDS to determine the implicit dimensions. This does not appear to have been done so far, perhaps because the dimensionality of the color experience is believed known.

The MDS method is not without its critics (see e.g., Saunders and van Brakel, 2002). Some criticism harks back to Kries original doubts about psychophysics, namely are magnitudes and differences distances that can meaningfully be expressed in numerical or geometrical form.

3.9 PSYCHOLOGICAL AND PSYCHOPHYSICAL SPACES

Complex psychological phenomena such as color vision are usually illustrated in geometrical spaces. Such spaces presuppose an isomorphism between color experiences and the selected geometrical model. The isomorphism may be based on the logic of the four unique hues and their mixtures as well as of hues, black and white or on equality of differences. In most color order systems, as seen in Chapter 2, white is placed on top producing the isomorphic association lighter equals higher. Greener means closer to the location of unique green, and so on. Closer to the centerline means grayer. The specific isomorphism depends on the distance criterion used. In a perceptually uniform (for a specific set of conditions) space equal distances between points represent equal perceived differences. There are two kinds of isomorphism in regard to color space: (1) between the perceptual experiences and the psychological color space and (2) between the psychological and the psychophysical space.

As seen in Chapter 2, historically color solids had simple geometrical forms with euclidean geometry (cones, spheres, pyramids, cubes). The Munsell system indicated that for a "uniform" system the underlying structure might be simple (cylindrical) but the surface of the solid complex. As will be seen in Chapters 7 and 8, a euclidean form of a uniform color space can be ruled out, at least at the global level.

The concept of space is of unlimited complexity. While most humans cannot imagine a space with more than three dimensions mathematicians have developed a number of categories of spaces. C. F. Gauss and N. I. Lobachevski invented hyperbolic geometry and G. F. B. Riemann the elliptic geometry named after him. Riemannian geometry has been used in the past in connection with color scaling (e.g., MacAdam, 1981; Völz, 1998b). In the hyperbolic geometry the sum of the angles in a triangle is less than 180° degrees, in the elliptic geometry greater than 180° degrees. In string theory modern physicists have proposed a universe of nine dimensions.[2]

As will be shown, a uniform color space based on small color differences requires an elliptic geometry. Psychophysical spaces that have no claim on uniformity, such as the Rösch-MacAdam space, fit into euclidean geometry without difficulty. It is evident that any kind of three-dimensional euclidean space form and associated color solid can be used as a regular (but not uniform) psychophysical color space. Modern color difference formulas dealing with some of the complexities of uniform color space currently are based on mathematical modifications of a euclidean space. The exact type of geometry applicable to a uniform color space under well-defined conditions of observation and magnitude of differences remains to be determined.

3.10 PSYCHOPHYSICAL SCALING AS A BASIS OF COLOR SPACE

Color space can be represented in a continuous fashion only with mathematical formulas that, for most people, lack direct comprehension in terms of color experiences. Historically color solids have been illustrated with two- or three-dimensional arrangements of color samples in form of atlases or "color trees." Here psychological and psychophysical scaling is essential if psychological uniformity relative to specific conditions is the goal. Psychophysical scaling has the advantage of providing physical measurement support for the stimuli. On the other hand, they require a psychophysical model that accurately reflects psychological results. A major impetus for psychophysical scaling of threshold and small suprathreshold differences and related mathematical models has been the desire for objective quality control of colored materials.

Psychophysics as a methodology is essential for the construction of a uniform color space. Psychophysical data are subject to considerable variation based on the physics of measurement and observer variability as well as variability in observational context. In the case of color attribute and color difference scaling, it is not yet clear what the major contributors to variability are. Careful experimentation is required to determine the effects of individual contributors.

Chapter 4

Color Attributes and Perceptual Attribute Scaling

4.1 THEORIES OF VISION

Vision is defined as the sense, mediated by the eyes, by which the positions, qualities, and movements of objects are perceived. The generally accepted view is that patterns of light energy are absorbed by light sensitive cells in the eye. The resulting electrochemical signals are passed into the brain where they cause activation of other cell types eventually resulting in sensations and perceptions of form, color, and movement. A representation of the outside world is generated in the retina and eventually in the brain. There are standard relationships between stimulus and resulting perception for which, presumably, neural correlates exist in the brain. Exceptions to these standard relationships are just that. They are caused by limitations in the neural apparatus. Much of the efforts of visual science in the 1980s and 1990s have been attempts at elucidation of these standard relationships and exceptions and finding their neural correlates (e.g., see De Valois, 2000). But parts of this view face serious problems from a slew of perceptual responses that are not explained by standard models.

Already Helmholtz thought that perception is the result of the interaction of the neural messages from the eyes with stored memories from past experiences (Helmholtz, 1866). With the wider acceptance of Darwin's ideas about evolution in the twentieth century, some researchers began to look at senses

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as tools of animals to cope with challenges in their particular ecological niches. In the second half of the twentieth century the American psychologist J. J. Gibson developed what he called an ecological approach to visual perception. He believed that the important rules of vision are found in natural viewing environments and not in fixed laboratory situations. Indeed, both are realities and ultimately must find meshing explanations. But it appears quite clearly wrong to assume that there is a simple standard relationship between states of retinal and early neural cells of the visual system and percepts. There is no doubt that vision has provided its bearers with an important tool to help make decisions affecting their own life and that of their offspring. Vision, implemented in many different ways in the animal kingdom, is undoubtedly shaped by evolutionary forces. It remains to be determined how.

4.2 HISTORICAL DEVELOPMENT OF VIEWS ON ATTRIBUTES

Regardless of how a particular color experience is generated, it can be said to have certain attributes, that is, inherent characteristics. The idea of color attributes has developed slowly over an extended period of time. As shown in Chapter 2, ancient Greeks appear not to have had a clear concept of chromatic color attributes. Only the concepts of light and dark and of hue were familiar to them. The simple linear color scales of the philosophers, originally seemingly in random order, later implied an ordering by lightness, without regard to hue ordering in the sense of the spectrum. In Aristotle's seven-color scale white precedes yellow followed by crimson, violet, green, and blue which precedes black. The red-centered five-color scale, found in descriptions until the sixteenth century, is that of Chalcidius: white–yellow–red–blue–black. From observations in nature and the work of artists the lightening and darkening of full colors (or of pure highly chromatic pigments) was well known in artist's circles of the twelfth century, as the text by Theophilus demonstrates. A painting technique of the early Italian Renaissance, described by Cennini in 1390, involved the use of pure chromatic pigments for the darkest areas and dilutions with a white pigment for lighter areas. In 1435 Alberti advised his fellow painters as follows: ". . . you may change the color with a little white applied as sparingly as possible in the appropriate place within the outlines of the surface, and likewise add some black in the place opposite to it. With such balancing, as one might say, of black and white, a surface rising in relief becomes still more evident. Go on making similar sparing additions until you feel you have arrived at what is required." The full complement of tonal colors began to be exploited by some artists wanting to avoid a certain garishness that can arise when painting with pure chromatic pigments. Chief among them was Leonardo da Vinci who developed the *sfumato* (smoky) style of painting and who is credited by some art historians as having distinguished between lightness (*chiarezza* in Leonardo's terminology) and chroma (*bellezza*) (Ackerman, 1980). Quantitative estimates of the natural lightness of full object colors were provided in print first by Cardanus in 1563, as seen in Chapter 2.

As has been discussed earlier, it is reasonably certain that Forsius and Glisson did not have a concept of a third attribute, beyond hue and lightness.

The concept of saturation as an attribute of prismatic color makes its definitive appearance with Newton. It is clearly based on his experiments with overlays of various wavebands of light. He described points on a radial line in his diagram (see Fig. 2-10) as indicating colors "proportional to the fullness or intenseness of the [prismatic] Colour, that is, to its distance from whiteness." Newton recognized seven primary and an indefinite number of intermediate hues. He described lightness in connection with colorants as follows: "Now considering that these grey and dun Colours may also be produced by mixing whites and blacks, and by consequence differ from perfect whites not in Species of Colours but only in degree of luminousness, it is manifest that there is nothing more requisite to make them perfectly white than to increase their Light sufficiently . . ." (Newton, 1704, p. 112). Here we see for the first time (if not in the same place in the book) mention of three attributes describing color perceptions.

Mayer, who in the mideighteenth century developed the first plan for a three-dimensional color order system, did not distinguish color series from the most saturated colors on the surface of his double triangular pyramid through the interior of the triangle. A possible reason is that in his own efforts with pigments he ended up with a neutral gray quite removed from the gravimetric center of the triangle, according to his own information at r^3 g^2 b^7, indicating that his scheme of determining relative strength of pigments was less than perfect.

Also Lambert did not recognize saturation or chroma as a separate color attribute, even though it can be seen as implicit in his single pyramid. The reason is similar to the one applicable in the case of Mayer. His three primary colors are placed on the same level regardless of their lightness, and his choice of colorants and mixture ratios did not result in black falling onto the gravimetric center of his basis triangle but, again, not far from the blue corner.

The painter Runge provided clear and explicit discussion of the general idea of saturation, but without using such a term. Because he placed the full colors on the circumference of the central horizontal plane of his color sphere, its vertical dimension does not refer to brightness or lightness. But Runge understood clearly the desaturation of a full color, by the admixture of appropriately selected complementary colors or combinations: "When we add to pure green, a product of yellow and blue, the smallest amount of red as the third color, we learn that it simply destroys and dirties the pleasant appearance of green without adding the appearance of redness. Therefore, through a stronger admixture of red, green is dissolved into completely colorless smut, or gray, and assumes a reddish hue only through an even stronger admixture. . . . All diametrically opposed colors and mixtures on the circle are (in the center point of the circle) dissolved (into gray)" (Runge, 1810). In the colored figure of the equatorial cross section of his sphere (see Fig. 2-22) Runge shows four desaturation steps toward the central gray for each full color.[1]

Grassmann, as we have seen in Chapter 2, supplied Newton's diagram with a mathematical foundation based on Helmholtz's and Maxwell's ideas of three fundamental color processes. He also, for the first time, explicitly referred to three attributes that combine to form a color perception: "If, finally, we consider a light of arbitrary composition, the eye can distinguish in it only the three mentioned attributes, that is, any impression by a light can be imitated by mixing a homogeneous color of a certain intensity with white light of a certain intensity. Therefore, we have to distinguish three things in each impression: intensity of color, hue, and intensity of the admixed colorless light" (Grassmann, 1853, pp. 70–71). Grassmann provided a general connection between perceptual color attributes and physical magnitudes.[2]

In his *Physiologische Optik*, preceding the *Handbook*, Helmholtz slightly modified Grassmann's definition:[3]

> Every impression on the eye made by an arbitrarily mixed light can always be represented as a function of three variables that can be expressed in numbers, that is,
>
> 1. Quantity of saturated colored light,
> 2. Quantity of white light that when admixed results in the same color impression,
> 3. Wavelength of the colored light. (Helmholtz 1860)

In the *Handbook* he described the three attributes in the form still in use:

> Accordingly, with all possible combinations of systems of aether-waves of different frequencies of vibration, there is after all a comparatively small number of different states of stimulation of the organ of vision which can be recognized as different colour sensations. First of these are the series of *saturated* colours, composed of the colours on the spectrum, along with purple which links the ends of the series. Each of these hues again may occur more or less pale in different gradations. The paler it is the less saturated it appears. . . . Thus, we have here two kinds of differences between colours, namely, first, differences of *hue*, and second, differences of *saturation*. . . . Lastly, in ordinary speech we are wont to describe differences of luminosity as differences of colour, however, only in case colour is considered as a characteristic of bodies. (italics in the original)

In 1866 Brücke presented a schematic color sphere and a cross-sectional view in which he showed presumed lines of constant saturation (Fig. 4-1). He viewed constant saturation contours as representing the surfaces of spindle shapes, with varying ratios of length to width, ending in the sphere shape at maximum saturation for a given level of lightness.

In Wundt's color sphere of 1874 hue (*Farbton*), saturation (*Sättigung*), and brightness (*Lichtintensität*) are established concepts. His student Kirschmann published in 1895 a paper "Color-saturation and its quantitative relations" in which he described the design of a color disk that, when spun with sufficient speed, displayed what was described as fifteen perceptually equally different saturation steps of the test color of the disk (Fig. 4-2). The heart-shaped figure A is colored with the test color (e.g., an approximation of a Hering type full

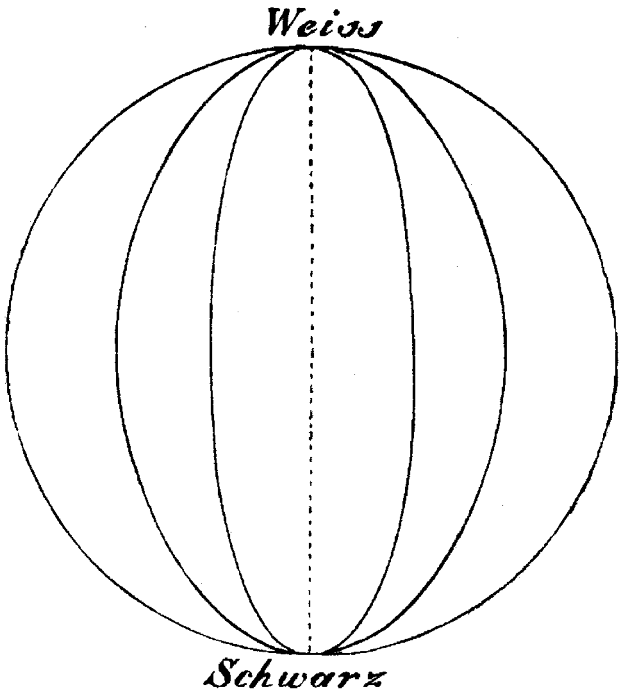

Fig. 4-1 *Brücke's vertical section through the color sphere with white on top and black on the bottom, 1866. The full interior lines are meant to be lines of constant saturation.*

color). The shapes of B and C are designed to produce from black and white the appropriate level of gray to be added to the full color so that all resulting saturation steps have identical lightness (thus representing a chroma scale when seen as object colors). The exact shape of B and C depends on the photometrically measured luminosity of the color of A. With this method Kirschmann investigated the validity of the Weber-Fechner law in regard to saturation.

In Hering's system full colors occupy one corner of equilateral triangles. The remaining colors in a triangle consist of those of the gray scale and the veiled (*verhüllte*) colors with the same hue as the full color. As mentioned in Chapter 2, Hering was fully aware of the varying lightness of full colors but never explicitly described how lightness as an attribute was to fit into his "natural color system." Any color could be described as the sum of one or two primary colors, white and black. Thus, in the majority of cases, colors had four attributes, for example, for a brown yellowness, redness, whiteness, and blackness. Hering abandoned use of the term saturation because he believed its meaning to be "contaminated" by Helmholtz. He defined colors of "equally strong veiling" as those falling on lines parallel to the line connecting white and black in his triangle,[4] those having the same ratio of full color to total color

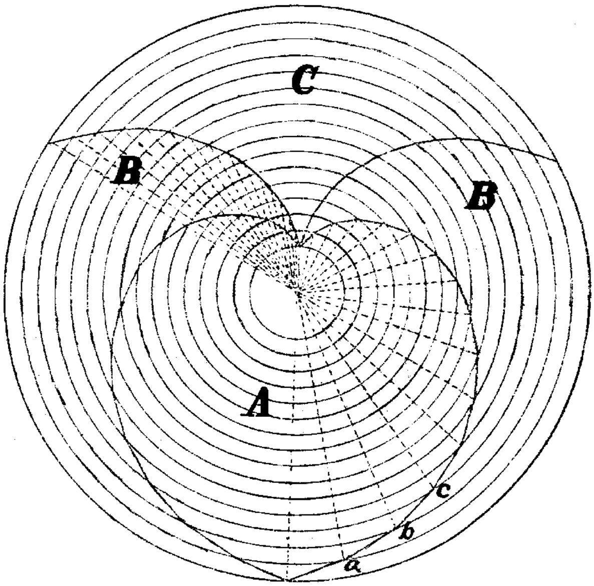

Fig. 4-2 Construction of a disk arrangement for the spinning disk mixture resulting in fifteen concentric circles at equal luminance with perceptually equally (according to the Weber-Fechner law) different steps of saturation of the color displayed on shape A. From Kirschmann (1895).

content. He commented on the difficulty of comparing degree of veiling for differently hued colors and does not seem to have regarded it as a primary attribute of color perception.

Later the Hering pupil E. G. Müller and the psychologist D. Katz suggested replacement of the term saturation with *Eindringlichkeit* (penetrance). In 1917 the psychologist C. Stumpf completely rejected saturation as an attribute and believed it to be a cognitive abstraction for the resemblance of a color perception to its ideal.

Munsell, in his effort to develop a useful teaching tool, first relied on the form of a new kind of color sphere, developed by him in 1898. This sphere differed from Runge's in that the vertical dimension has an unambiguous definition as lightness (value) with the result that on this sphere Hering's full colors are not located on the equatorial plane. Munsell had studied art in Paris around 1880 and, perhaps influenced by the French use of *valeur* in tonal painting, settled on the term value for lightness. The Greek term *chroma* (surface skin, color) was also in use as a general term for color in French art literature at the time of his studies (e.g., George Seurat's *chromo-luminarisme* painting style[5]) and it appeared in the title of Rood's influential *Modern chromatics*. It seems intuitive to use it in a quantitative sense to indicate how much chromatic color is present. Munsell's three color attributes, principally in agreement with those of Helmholtz, were thus named hue, value, and chroma. Munsell, intrigued with the decimal system, selected a 100-part hue circle and five primary colors rather than Hering's four (he thought Hering to be wrong because Hering's claims for the operation of his opponent-color system were not in accord with contemporary knowledge about the retinal structure). His value scale consisted of ten steps. The chroma scale was open ended because Munsell soon found that he could not fit all colors with uniform chroma spacing into a sphere form. As described below and in Chapter 7, scales for all three attributes were intensively investigated before finding their final form in the Munsell Renotations.

Ostwald, working in isolation during the First World War, erroneously believed Helmholtz had made no distinction between what we now call brightness and lightness.[6] For this reason he rejected lightness as an attribute for his system and followed Hering in using full color content, whiteness and blackness. But unlike Hering, he based his hue circle on three primary colors, yellow, red, and blue. Object colors were defined as the sum of full color perceptions and perceptions of white and black. Hering's colors of equally strong veiling were termed by Ostwald *Reingleiche* (colors of equal purity). Analysis of Ostwald's system in 1944 indicated that:

1. The system did not cover the full range of object colors as defined by the Rösch-MacAdam limits.
2. Ostwald's claims for complementarity and visual uniformity in terms of hue differences around the hue circle have not been met and cannot be met.
3. Various compromises have been made by Ostwald in physically implementing his system (Foss, Nickerson, and Granville, 1944).

Basic ideas of Hering were also used by Johannsson and Hesselgren and resulted in the development of the Swedish Natural Color System (see Chapters 2 and 7).

4.3 WHITENESS AND BLACKNESS

In Chapter 2 it was mentioned that white and black have been considered colors for some 3000 years or more. But there has been a certain amount of controversy about this subject. Non-hued visual percepts are often called achromatic colors, that is, colorless colors. Truly achromatic colors differ from chromatic colors in the lack of the key perceptual attribute of hue. While hued colors are well defined, unique blue is a bluish color that is neither greenish nor reddish, neither black nor white are well defined in a similar sense. This becomes apparent if one looks, for example, at a collection of white and black textiles. Generally preferred white and black both have a slightly bluish tinge. But the preferences differ among individuals and cultures. In addition, while not any chromatic stimulus can appear unique red, any neutrally reflecting surface can be made to look white or black or any gray in between depending on illumination and surround. Perceptions of white and black as the achromatic extremes are outputs of our color vision system, just as perceptions of hue and chroma are. It appears entirely justified to consider achromatic colors to be on an equivalent level with chromatic colors.

Helmholtz defined black as the absence of light energy, while Hering, correctly, defined it as an indirect manifestation of light in terms of contrast. When looking at reflectance data of pigmented black and white paint layers, it is obvious that while the white layer reflects most light the black layer reflects very little. But the small amount it reflects is responsible for the fact that a black pigment layer, selectively illuminated with high intensity white light, can be made to look white (as demonstrated by A. Gelb in 1929). Contrast plays a key role in the perception of black, as is well known from the demonstrations of the effect of surround brightness by Evans and from the fact that an unpowered television screen is gray and not black in appearance. The blackness we see in images on the screen must come from contrast effects.

The perception of blackness has been studied in considerable detail (e.g., see Volbrecht and Kliegl 1998). The blackness induction curve is somewhat similar to the heterochromatic flicker brightness curve (see Chapter 5). However, it varies significantly depending on the design of the experiment. If a monochromatic ring surrounds a white center, the agreement with the flicker brightness curve is relatively high. If center and ring are reversed, the function appears to have a chromatic opponent color component added, reminiscent of the Helmholtz-Kohlrausch effect. A detailed model of blackness perception based on cone sensitivities and assumptions about the physiology of color vision has been developed by K. Shinomori and co-workers (Shinomori, 1997).

As we have seen, whiteness as a psychological attribute has been defined in terms of a perceptually uniform gray scale by Hering and Ostwald. The psychophysical description of whiteness has been of interest since the mid-1930s. The definition of the portion of color space that most observers call white or near white, and its description in terms of a psychophysical formula, have been driven by industrial interest in measuring whiteness of papers or textiles. Several formulas were developed around 1960, and in 1979 the CIE recommended a whiteness formula to be used in the interest of uniformity (CIE 1979; for a bibliography until 1976, see Sève, 1979). An improved formula is under development. It is of interest to note that the preferred psychological optimal white is not one representing perfect 100% reflectance across the spectrum, nor an equal energy distribution, but one with a slight bluish tint and achieved with the addition of fluorescent whitening agents. This indicates that whiteness and lightness are different psychological and psychophysical concepts.

4.4 EVANS'S FIVE COLOR ATTRIBUTES

In 1974 R. M. Evans published his interpretation of the results of many years of study of the appearance of color. He concluded that there are five attributes that require consideration for a full description of color appearance. These are brightness, brilliance, lightness, saturation, and hue. Brightness is conventionally defined in terms of luminosity. Brilliance is a term that encompasses the scale of perception from grayish colors, through the zero grayness (G_0) point to fluorent colors (a term proposed by Evans). Object colors are seen to contain grayness up to a certain level of luminous reflectance (dependent on the surround luminous reflectance) where the zero grayness point is located. At this point they are seen as equally light as the surround. When the luminous reflectance increases further, the colors are seen as lights (weaker or stronger, depending on the degree of fluorence). The spectral zero grayness function (Fig. 4-3) bears close resemblance to the saturation discrimination function, a function describing, based on dominant wavelength, the purity of Munsell samples at constant chroma, as well as to MacAdam's color moment per lumen function (MacAdam, 1981). Evans defined lightness as relative luminance and saturation in a form comparable to colorimetric purity (see Glossary). In his system saturation is the perception of the brilliance of the hue component in relation to the total brilliance of the color. Evans defined Munsell chroma as brilliance difference from a gray of the same value. He

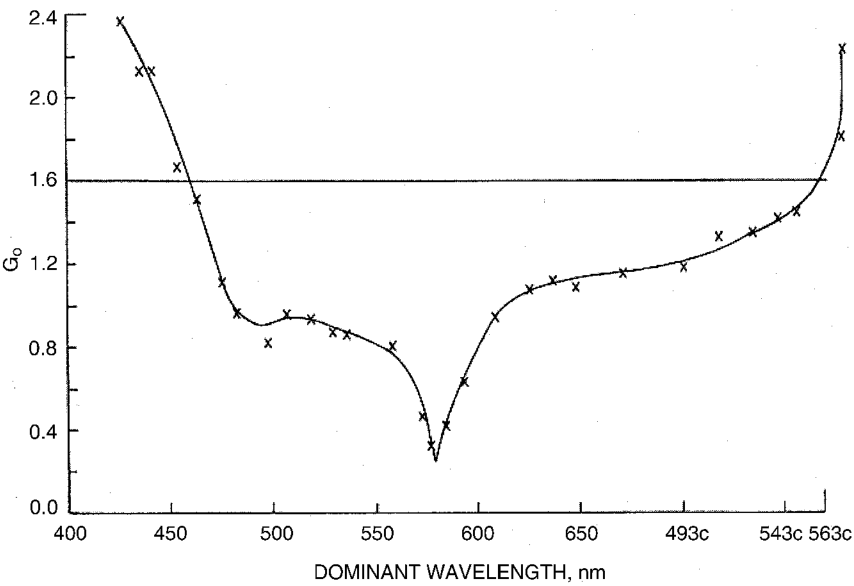

Fig. 4-3 *Zero grayness (G_0) of object colors as a function of dominant wavelength. From Evans (1974).*

described the problems with Judd's original uniform color space definition as due to the need for four dimensions of such a space. When defining uniform color space in terms of surrounds that are always intermediate to the two colors compared, as Judd did in his re-definition, the four-dimensional space can be reduced to three dimensions. This minimizes the required stimulus increment or decrement for color differences. Evans believed the spacing of the Munsell system to be a good approximation of Judd's re-defined uniform color space. The usual approach, however, is to base an attempted uniform color space on a single achromatic surround of a given lightness.

4.5 COMMON COLOR ATTRIBUTE DEFINITIONS

The three psychological Munsell attributes have become generally accepted for object colors. Wyszecki termed the empirical evidence of three attributes being sufficient for an observer with normal color vision to describe any perceived color "overwhelming." His terms for the three attributes were hue, lightness, and chromaticness (Wyszecki, 1981).

Hue

The term hue is defined as "attribute of visual perception according to which an area appears to be similar to one of the colors red, yellow, green, and blue, or to a combination of adjacent pairs of colors considered in a closed ring" (CIE, 1987). Essentially this definition indicates that hues represent the variable one experiences when looking at a Munsell (or other system) hue circle. As mentioned earlier, such hue circles derive their legitimacy from the arrangement of hues in the spectrum, as well as systematic mixtures of stimuli from the beginning and end of the visible spectrum (see Fig. 3-6).

Brightness, Lightness

Lightness is an attribute related to brightness, the definition of which is: "attribute of a visual perception according to which an area appears to emit, or reflect, more or less light" (CIE, 1987). Brightness is generally taken to apply to light sources. Lightness is defined as: "the brightness of an area judged relative to the brightness of a similar area that appears to be white." Lightness thus can be said to be relative brightness and is generally taken to apply to object colors. Brightness and lightness have been controversial since their definitions because they have been defined based on additive functions using an experimental method that is far from natural. As will be shown in Chapter 5, brightness and lightness of chromatic lights, respectively objects, when viewed in natural or strongly relativized conditions have additional components, not contained in the additive function.

Saturation, Chroma

Over the years there has been considerable discussion concerning the term used for the third attribute. Helmholtz's *Sättigung*, or saturation, is now an "attribute of a visual sensation that permits a judgment to be made of the proportion of pure chromatic color in the total sensation" (CIE, 1987). The term chroma is closely related to Munsell chroma and refers to an "attribute of visual sensation that permits a judgment to be made of the amount of pure chromatic color present, regardless of the amount of achromatic color." A differently worded definition is: "attribute of color used to indicate the degree of departure of the color from the gray of the same lightness" (ASTM E284). This definition relates directly to the Munsell system. As discussed next, chroma can be seen as related to an attribute termed "colorfulness," as lightness is related to brightness. Chroma and saturation are identical for two colors having the same hue and lightness. Saturation remains constant regardless of brightness or lightness. Chroma, on the other hand, increases as lightness increases. Saturation relates to an inverted cone arrangement of colors, for example, the DIN system, while chroma refers to a cylindrical arrangement.

The question as to how intuitive the concept of chroma is to the average observer is an interesting one. We have seen that as a well-defined concept, as introduced by Munsell, it is only approximately 100 years old. Seemingly there are no terms of folk psychology that directly refer to it. Empirical evidence of the author with more than 100 untrained observers in experiments where they had to sort Munsell chips of constant hue into a Munsell value/chroma template indicates that many had considerable difficulty to do so because of uncertainty about the concepts of lightness and chroma. A recent investigation has confirmed the higher uncertainty of judgments of assessing color differences as chroma differences, compared to lightness and hue differences (Melgosa et al., 2000).

Hunt's 1977 Proposals

In 1977 R. W. G. Hunt proposed new systematic terminology for color attributes at four levels:

1. Perceptual (psychological)
2. Psychophysical (related to stimulus)
3. Psychometric (interval scales)
4. Psychoquantitative (ratio scales)

Perceptual terms include brightness, lightness, hue, saturation, and perceived chroma. The psychometric terms have the modifier "metric" before them. It changes to "quantitative" for the psychoquantitative terms. The psychoquantitative terms are controversial because of the philosophical problems surrounding them. Hunt also proposed revised definitions for chroma and a new attribute: colorfulness. It is defined as the "attribute of a visual sensation according to which an area appears to exhibit more or less chromatic color." Colorfulness refers to chromatic power perceived regardless of the magnitude of the stimulus. Hunt has used, for example, theatrical lights of varying intensity to demonstrate the meaning of colorfulness. Hunt proposed the definition of perceived chroma as amount of chromatic color in related colors judged in proportion to the average brightness of the surroundings. His proposals for metric hue and chroma are as, for example, defined in the CIELAB formula (see Chapter 6).

4.6 CONFIRMATION OF THREE ATTRIBUTES

The attributes hue, lightness, and chroma for surface colors in simplified viewing conditions have received confirmation from multidimensional scaling experiments. In 1988 Indow reported on the results of nineteen multidimensional difference scaling studies using Munsell color chips. Because of difficulties in comparing chromatic differences of colors with greatly differing hue scaling was only done in overlapping, comparatively small regions of the Munsell chromatic plane. Indow drew the following key conclusions:

1. In a three-dimensional euclidean space the points representing Munsell colors form layers according to the order of Munsell value.
2. In each of the planes colors of the same Munsell hue are located along line segments in the order of Munsell chroma; all lines converge at a single point in the center, corresponding to the gray of the same Munsell value. The circular order of the radial lines agrees with the sequence of Munsell hue.
3. Interpoint euclidean distances between colors are closely related to scaled perceptual distances that were used as the data in the multidimensional scaling experiment.
4. Chroma and value are clearly orthogonal.
5. There are many irregularities in the resulting structure, the main ones being anomaly of hue spacing in sectors B to P, and the first chroma step from neutral is always larger than successive steps.

The result of one multidimensional scaling analysis of Munsell colors at different value levels is shown in Fig. 4-4. Taken together, the findings help support the Munsell system as a systematic perceptual arrangement of colors and the psychological validity of the three attributes.

The perceptual significance of the three Munsell attributes is also indirectly supported by subjecting the reflectance functions of the Munsell chips to dimensionality reduction using a neural network (Usui et al., 1992). The

Fig. 4-4 *Multidimensional scaling analysis of Munsell colors involving samples at four different value levels (From Indow, 1988). The results are shown in an elliptic space. Colors of constant hue are connected with approximately radial lines, colors of equal chroma fall on approximate circles.*

reflectance data of 1596 chips were subjected to a five-layer wineglass type of neural network. Of the three middle-layer units one was found to approximately correspond to value and the other two roughly to opponent responses with red-green and yellow-blue orientation. Lines of constant hue are approximately radial and lines of constant chroma jaggedly oval but placing the hues in proper ordinal order. When reducing the network to two internal layers, yellow and blue colors overlap. On the other hand, of four layers one was found to be redundant, two layers of the four being highly negatively correlated ($r = -0.991$). The most efficient representation involves three internal layers and reconstructs the Munsell color solid roughly as in the X, Y, Z tristimulus space (Fig. 4-5; compare to Fig. 5-30).

Because of the diurnal cycle, to whose effects we are exposed already in the womb, the ideas of bright (or light) and dark are deeply embedded in us. In their simplest form they are achromatic experiences, and all but congenitally blind people know them. As to chromatic colors, the preeminent fact is hue. This is also indicated by our monolexemic names for what we regard as the most significant colors. They generally refer to hue. Level 2 of the Universal Color Language consists of 26 hue-related names (Kelly and Judd, 1976). On

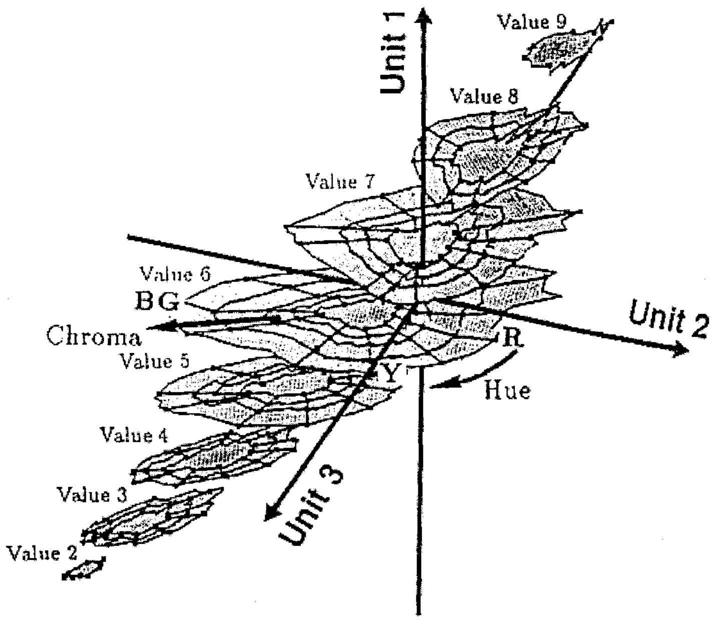

Fig. 4-5 Reconstruction of the Munsell color solid from 1280 reflectance functions of Munsell chips with a five-layer neural network (From Usui et al., 1992). Colors of constant value fall approximately on planes of constant unit 3 values. Constant hue colors fall on approximately radial lines and colors of constant chroma are connected by jagged ellipsoidal contours. The arrangement is similar to one of Munsell colors in the CIE X, Y, Z *space.*

the other hand, the number of modifiers added in level 3 that describe lightness and chroma (e.g., light, strong, vivid, dark, and deep) is only 13. These, together with the 26 hue terms of level 2 and the achromatic terms form the 267 terms of level 3. From this and other facts it is evident that hue is the primary attribute of our normal color experiences. In evolutionary terms light and dark are older, but they encompass one dimension. Hue, on the other hand, has, in a manner of speaking, four dimensions and therefore provides a much larger range of experiences. The general idea of a third attribute of object colors, now named chroma, developed beginning in the nineteenth century with people professionally involved with color, namely painters. In the general population today, even in well-educated segments, it is not yet a commonly understood concept.

The attributes of the Hering-NCS systems are claimed to be intuitive by their creators and to be introspectively determinable with a high degree of precision. The author is not aware of an extensive independent test of this claim. The clear expectation is that in a multidimensional scaling experiment based on uniformity of perceived differences NCS color chips would be placed into a Munsell type color solid.

4.7 CONTRAST VERSUS SIMILITUDE

Given an arrangement of two test fields against a surround sending different spectral power distributions to our eyes, the color experiences resulting from the test fields depend on the spatial proximity of those fields. There are two results possible: either the perceived difference between the test fields becomes larger if the fields are adjacent compared with widely separated or it becomes smaller. In the former case we speak of contrast, in the latter of similitude (also known as the spreading effect; Bezold, 1874). Such effects are usually interpreted as due to lateral interaction between retinal cells (but see Purves and Lotto, 2002). As a result the relationship between stimulus and resulting perception depends on the composition of the visual field.

Contrast has a somewhat different meaning in photography or printing and is defined as the difference between the lightest and the darkest area in the image expressed perceptually or in terms of physical measurements. Aside from lightness contrast there is also chromatic contrast.

Discovery and quantification of contrast as a function of spectral energy is a key aspect of operation of our visual system. As will be shown in Chapter 5, perceptions of white and black do not depend on absolute values of spectral power but are related according to complex, not yet fully understood rules. Similarly it appears that colors are assigned by the visual system to fields of given spectral power, and not by that power but by the relationship of that power to that of neighboring fields according to rules not yet fully understood.

Colors may be said to exist only as contrasts since in a so-called (extreme) *Ganzfeld* where our eyes are individually exposed to uniform light from all directions color experience fades quickly regardless of the spectral power distribution of the light. While contrast effect rules are comparatively simple in case of two fields only, the more articulated the visual field is, the more complex the rules become. Contrast effects resulting in colored shadows were described in the eighteenth century by the Count of Rumford and others. Chevreul famously developed empirical rules of contrast in the nineteenth century. Contrast effects are temporally dependent. When viewing two contrasting fields in a neutral surround, the *simultaneous contrast* is immediately apparent. When the two fields are replaced after some time by the surround color, their images remain still visible, however, with appearances usually changed according to the rules of *successive contrast*.

4.8 NEURAL CORRELATES OF COLOR ATTRIBUTES

Color attributes are phenomenological entities that have as yet no firm basis in neurology. Current answers to the question of neural correlates of color attributes are speculative. According to the opponent color theory, there are six primary object color perceptions: unique yellow, red, blue, green, and white and black. A chromatic object color stimulus invokes one or two of the unique hues, and perhaps white and/or black. It is not yet known what the neural correlates of the unique hue perceptions are. Known opponent color cells in the lateral geniculate nucleus (LGN) and beyond in the cortical visual system do not have outputs correlated with unique hue perceptions, as discussed in Chapter 5 (see also Webster et al., 2000; Valberg, 2001). It appears less and less likely that there are two simple subtractive chromatic opponent color systems as proposed by Hering. Multiple hue detection and discrimination systems have been proposed as an alternative to a two-component opponent color process (e.g., see Krauskopf, 1999). There is a large amount of literature relating to the operation of a neural opponent color system, a subject outside the scope of this text (relevant citations are found in Gegenfurtner and Sharpe, 1999).

Brightness-related signals are transferred from the retina through at least two different neural pathways. Brightness and lightness perception is very complex, for example, as the work by A. L. Gilchrist and co-workers shows (Gilchrist, 1994; Gilchrist et al., 1999), and the Munsell value and similar lightness functions are only simple approximations. Recently Lotto and Purves (1999) have argued that brightness/lightness perception of fields may be guided by our past experiences as a species and as individuals, and interpretation of clues in an image in regard to its likely illumination.

Hue and brightness/lightness perceived in contrast to a surround are affected by a complex set of adaptations whose neural machinery is not well known. As will be seen in Chapter 5, the neural machinery of hue and chroma detection and discrimination is also not known, but signs point to a complex system.

4.9 PSYCHOLOGICAL (PERCEPTUAL) SCALING OF COLOR ATTRIBUTES

This section contains a discussion of various scaling attempts of the three fundamental color attributes hue, chroma, and lightness. The section on hue includes the issue of the number of uniform hue difference steps between unique hues at a given level of chroma. The chroma section includes a discussion of the determination of the magnitude of the first steps from gray, presumably an indication of the coloring power of the unique hues and their combinations. Lightness scaling is briefly touched upon, but most of the discussion of lightness scaling is postponed until Chapter 5.

Hue

Newton's hue circle was shown to be a parsimonious solution to representing the common desaturation effect on all spectral hues. When scaling hue differences of fourteen spectral colors and performing multidimensional analysis of the results, Shepard (1962) obtained data points that are fitted with good accuracy by a circle, with a considerable space between spectral violet and red, as one would expect (see Fig. 3-6). With availability of purple hues in object colors the hue circle is naturally closed, as Hering has pointed out.

Unlike lightness and chroma, hue cannot easily be envisaged by reference to simple numbers. To have a concept of hues based on hue angles as determined in a given system is too abstract to be of practical use. One can have a mental image of what a value 6 gray scale chip looks like, and it may even be possible to have a mental image of a given hue identified by Munsell hue name at chroma 8, but this is no longer possible with any reasonable accuracy for a hue with a hue angle of 165° unless intensive training has taken place.

In general, it is easier to assign unique hue, chroma, and lightness values to a given color stimulus than to mentally synthesize it based on such numbers. Without extensive practical experience I can more easily estimate the content of yellow, red, and gray in a brown than create a mental image of a brown from numbers of yellow, red, and gray given to me. Our color names are relatively vague and most have considerable extension around focal points. Only the unique hues give us a comparatively solid mental reference point (however individually variable as to the required stimulus, as will be seen below). Midpoints between unique hues also can be fixed relatively well, though less well than unique hues. On the other hand, "purplish pink" or "turquoise" can apply to five or more (individually varying) Munsell 40-hue steps. In general, color names are least well defined in the yellow-green segment, probably because of the lack of widely present exemplars during the time period when color terms developed.

As mentioned, division of Newton's hue circle is proportional to the intervals between the musical sounds of an octave, thereby implying a relationship between sound and color (Newton, 1704). Newton's circle did not explicitly include extraspectral (purple) hues. Grassmann's circle is essentially identical to Newton's. Helmholtz's circle, at highest saturation, includes extraspectral hues and is divided into ten equal segments with the middle color of each segment diametrically opposed to its complementary color. Hue circles based on complementary hues being diametrically opposed were common after Helmholtz. However, there is no simple information on which to base the placing of complementary diametrical lines relative to each other. In both his earlier sphere and later cone models Wundt placed complementary hues opposite. In case of the sphere model they were placed in such a manner that the largest segments are occupied by yellow and blue. Rood studied this matter and concluded that there was no objective way of placing the diametrical hue lines relative to each other. His ten-hue chromatic circle has uneven division between the hue lines and is apparently based on a placement using distances along the spectrum scale with arbitrary placement of the nonspectral reds and purples (Rood, 1879). There seem to be only two methods by which to place hues with perceptually meaningful distances into a circle.

Hering was, as mentioned, the first basing a hue diagram on four unique hues. His psychological hue circle (Fig. 2-32) has conceptually equal perceptual increments/decrements of unique hues. As shown below, hue coefficients describing the relative perceptual amounts of unique hues in Hering's system form by definition an × pattern. The NCS system represents a practical implementation of Hering's hue circle.

Munsell's approach to scaling a hue circle was based on uniformity of the size of hue differences along the hue circuit. As a result complementary colors as expressed by optimal colors of Munsell Renotations in the CIE chromaticity diagram do not plot diametrically opposed in the Munsell psychological diagram. We can conclude from this that chromatic circles based on complementary colors are not uniform in terms of perceived hue differences. As we will see below, the Hering/NCS circle also is not uniform in terms of perceived hue differences, indicating that constancy of perceived unique hue increment does not equal uniform size of perceived hue difference in the four quadrants.

An interesting issue in connection with hue scaling is the psychophysical identity of the four unique hues, given an achromatic surround. Historically these have been determined in the spectrum, using optical equipment. Individual results have varied to a surprising degree (e.g., see Ayama, Nakatsue, and Kaiser, 1987). For example, the wavelength of unique green has been determined in various experiments as anywhere from 488 to 561 nm. Similar results have more recently been reported by Webster and co-workers (2000). Unique hues can also be determined, with considerably reduced variability, using object color samples (Hård, Sivik, and Tonnquist, 1997; Kuehni, 2001a). The mean choices of 40 observers that directly selected their unique hues from arrays of Munsell hues were found to be 3.0R, 3.5Y, 2.5BG, 2.75PB. Indow, based on his principal hue scaling method, obtained the following locations: 3.75R, 5Y, 6G, 3PB (Indow, 1999b). Note the discrepancy in the green unique hue. In a recent, as yet unpublished, study using Munsell chip arrays with 75 observers average unique green was found to be 7.0G. Webster's group (Webster et al., 2000) found green to have the largest individual variation among unique hues. This was also the case for Kuehni's original experiment as well as the experiment with 75 observers. It is evident from results with Munsell chips that the range of dominant wavelengths in such experiments is much narrower than the reported range of spectral wavelengths for determinations using optical equipment. Recently it was shown that there is no relationship, as previously surmised, between the ratio of L and M cones in an individual's retinas and the perception of unique yellow (Yamauchi et al., 2002). There is currently no model with good explanatory power for the relationship between stimulus and perceived unique hues.

Conceptually the geometrical image of a perceptually uniform hue circle of constant chroma is a perfect circle with each hue placed at uniform hue angle increments around it (Fig. 4-6). The location of each hue can be described in terms of ordinate (b) and abscissa (a) values relative to the center of the circle. Logically it might be expected that these values represent the content of unique hues in each hue. But, as we will see, this is not the case if hue steps are perceptually uniform. When plotting these absolute coordinates against the hue

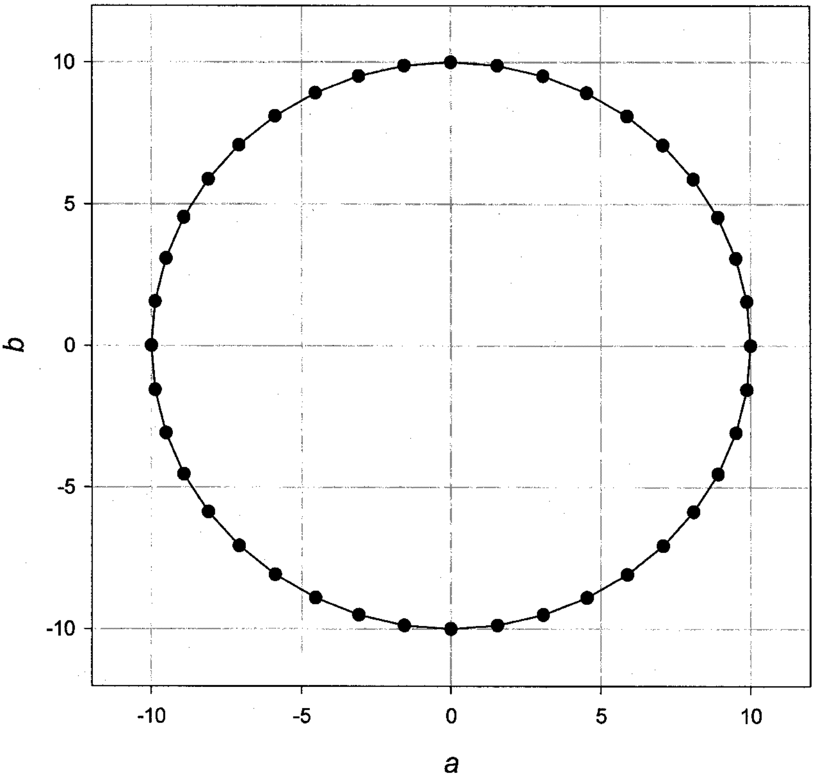

Fig. 4-6 *Idealized uniform chroma circle with forty colors representing identical hue differences in an* a, b *opponent color diagram.*

angle, sine wave curves are obtained (Fig. 4-7). When these are converted to relative values (percentages, called hue coefficients) × pattern functions result (Fig. 4-8). An × pattern in agreement with Fig. 4-8 is also obtained when plotting the forty NCS hue steps, conceptually based on uniform changes in unique hue components. In its psychological diagram the NCS atlas hue circle plots as forty equal hue angle increments of 9°. The Munsell atlas hue steps, involving constant hue differences, plot in the same way. However, this diagram is not based on the four unique hues and has no preferred axis system. In the standard presentation of the Munsell hue circle the x axis is formed by hues 10PB and 10Y, the y axis by 5BG and 5R (as can be seen in Fig. 4-11 below). All four are, to a greater or lesser extent, hues that are mixed in terms of unique hues. But while a step along the NCS hue circle means a 10% change in unique hue content, a step along the Munsell hue circle means a constant hue difference 1/40th of the total hue circuit. As a result the two diagrams are quite different.

The hue scales of the Munsell system up to and including the Munsell Re-renotations are briefly described in Chapter 7. Various hue scaling experiments

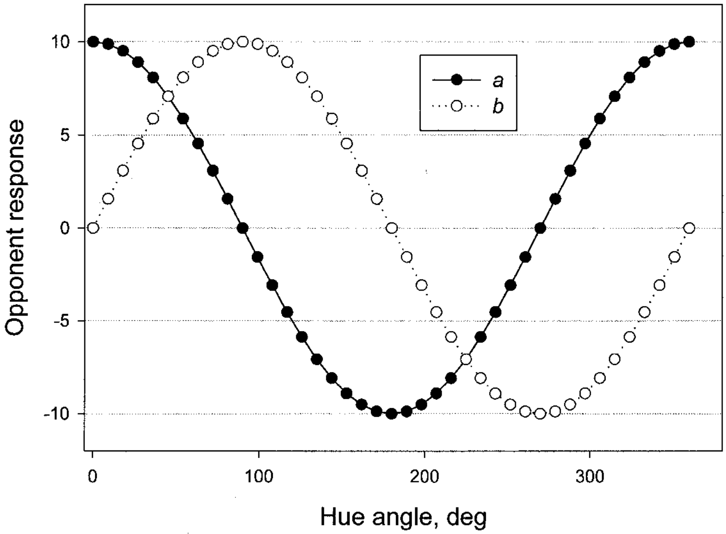

Fig. 4-7 *Plot of* a *and* b *values of the ideal color circle of Fig. 4-6 as a function of hue angle. The points form sinusoidal curves.*

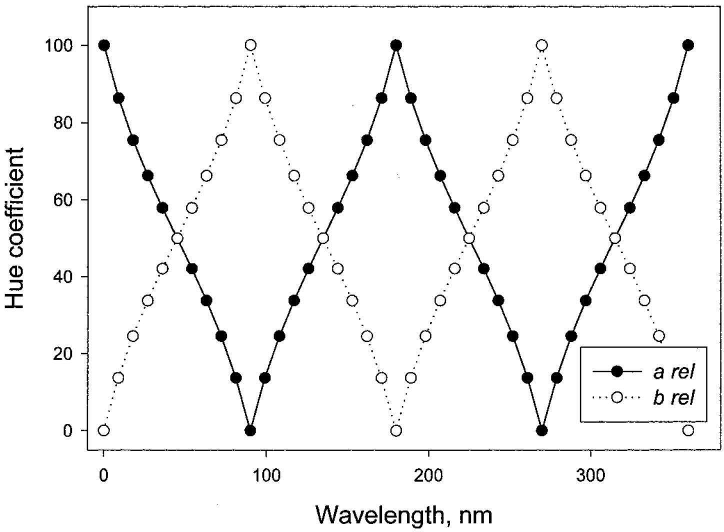

Fig. 4-8 *Hue coefficients (relative* a *and* b *values) as a function of hue angle, calculated from the absolute values of Fig. 4-6.*

using Munsell *Book of Color* chips have been performed by Indow and his colleagues. In 1972 Indow and Ohsumi published the results of multidimensional scaling involving 60 samples. The corresponding hue circle was considerably distorted compared to the Munsell circle. The hue angle between 5PB and 5P, 36° in the psychological diagram, was found to be 80°. In a later more extensive multidimensional scaling experiment involving 176 Munsell chips, a hue circle much more in geometrical agreement with the conceptual Munsell circle was found (Indow and Aoki, 1983). The difference, in the most recent analysis of the results (Indow, 1988), between 5PB and 5P at the highest chroma level is 45°. In a later study Indow and co-workers had observers assess the principal hue components in Munsell chips at different chroma and value levels (Indow, 1999b). They had observers mark on a paper scale the estimated amount of achromatic (gray) color in the total color. Next observers determined the content of one or two principal hues in the color and marked them separately. Average absolute principal hue components at chroma 8 and value 6 are illustrated in Fig. 4-9; There is a reasonable resemblance to Fig. 4-8; however, yellowness is undervalued compared to the other three principal hue components (presumably because observers disagreed on what represents 100% yellow). The resulting implicit, considerably distorted, hue circle is illustrated in Fig. 4-10. Implied chroma varies substantially, as do the hue angle differences. Somewhat different results have been obtained at other levels of chroma and value. The changes in absolute hue components are more linear

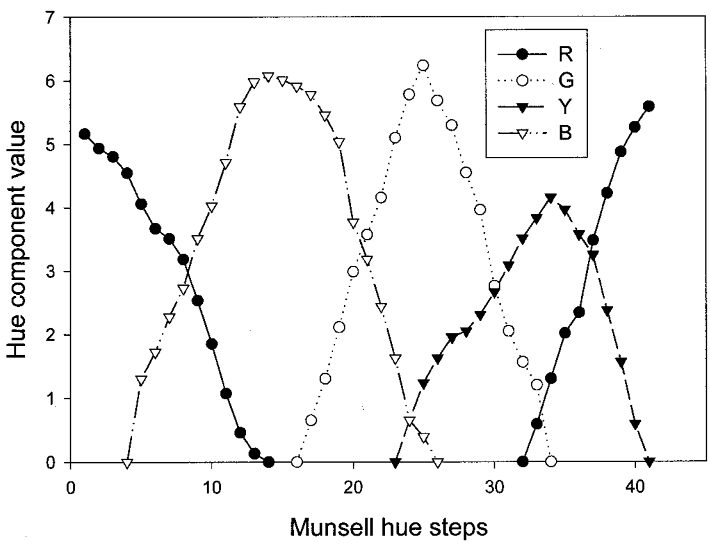

Fig. 4-9 *Average absolute principal hue components of Munsell colors at chroma 8 and value 6 (after Indow, 1999b). The curves have a resemblance to those of Fig. 4-8. The biggest deviation is for yellow, presumably because the observers could not agree on which color represents unique yellow.*

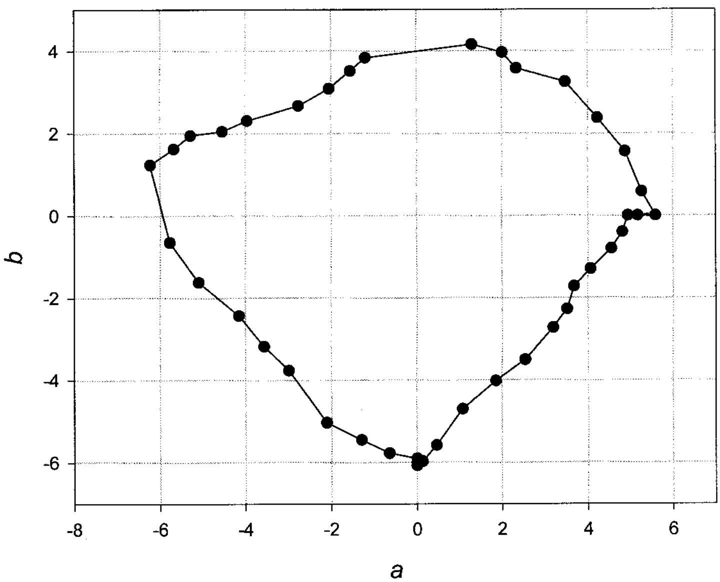

Fig. 4-10 *Hue circle derived from the absolute hue components of Fig. 4-9.*

than sinusoidal between unique hues, indicating that the observers judged the chromatic content of mixed hue colors lower than that of unique hue colors, most severely so for 1:1 mixes (compare Fig. 4-10 and Fig. 4-6).

Number of Uniform Steps between Unique Hues

In the hue circle of his atlas Hesselgren placed different numbers of equal-sized hue steps between his unique hues (Hesselgren, 1952). There are eight steps from blue to red, six from red to yellow, and five each from yellow to green and from green to blue.

Somewhat different results were obtained by Kuehni who investigated the perceptual distance between unique hues. Six observers scaled the Munsell hue circle at value 6, chroma 8 by using the difference between 5PB and 10B as the reference difference. The five Munsell 100-hue steps difference of the reference pair was seen as equal in difference to between 3.0 and 5.8 steps around the hue circle. On average, a total of 22 (rather than the expected 20) hue steps of the size of the reference step was found. In the four sectors between the unique hues the number of equal sized steps was found to be different: R–Y: 5; Y–G: 6; G–B: 4; B–R: 7 (Kuehni, 1999).

When plotting the average unique hues of Kuehni's 40 observers (green adjusted to the results of the 75 observers) on the ideal psychological Munsell hue circle (Fig. 4-11), the yellow and blue unique hues fall on a straight line passing through the center while red and green do not. The lines do not sep-

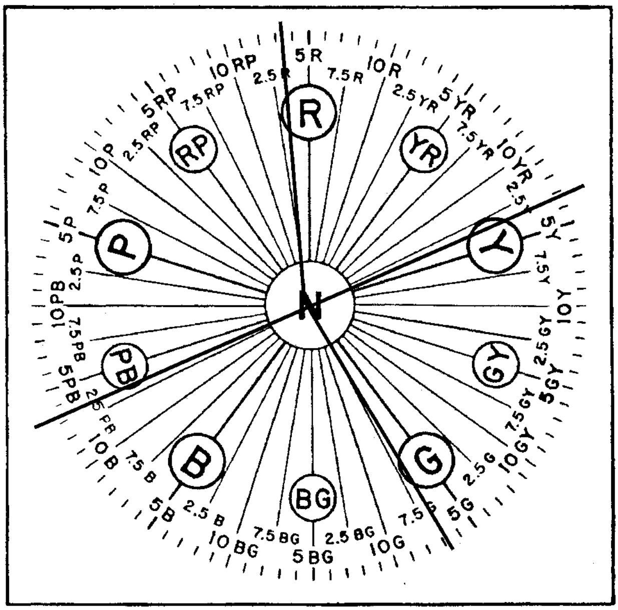

Fig. 4-11 Ideal Munsell hue circle with the average unique hues determined by the author (and an additional experiment for unique green) represented by heavy full lines.

arate the hue circle into four equal segments. Rounded to the nearest half-step on the 40-step Munsell hue scale the number of steps in each sector is as follows: R–Y: 8.0; Y–G: 9.5; G–B: 10.5; B-R: 12.0. If there are four primary hues and all other hues are mixtures of two of them, one might, sensibly and logically expect the primary hues to fall on the axes of a Cartesian diagram. Only then, as we saw above, can the content of primary hues in any given hue be expressed in terms of the axis coordinates. The Munsell system indicates, however, that in such a diagram there is a nonproportional relationship between hue angle and perceived hue difference. For reasons presented in the next chapter we cannot absolutely rely on the Munsell hue differences at a given chroma level having all exactly identical perceived magnitude and additional constant hue difference scaling work is desirable.

A nearly identical figure results when locating the same unique hues in Indow and Aoki's multidimensional euclidean scaling grid of the Munsell hue plane (fig. 6 in Indow 1988). The conclusion here also is that the number of unit-sized hue differences in the four sectors between unique hues differs.

In addition to the experimental results above the following relevant data are available:

1. Based on published information for the DIN color system, it is possible to translate the same unique hues into that system. The number of

TABLE 4-1 Comparison of hue angle segments in the psychological chromatic diagram between unique hues from different psychological experiments

Data set	Angle of segment, Degrees			
	R–Y	Y–G	G–B	B–R
Munsell Renotations	71	84	96	109
Hesselgren	90	75	75	120
DIN system	90	75	60	135
Newhall experiment	63	89	82	126
OSA-UCS	66	83	85	126
Kuehni experiment	82	98	65	115
Qiao et al. experiment	76	76	86	122
Berns function	65	82	87	126
Mean	75.4	82.8	79.5	122.4
CV, %	14.5	9.6	15.2	6.5

uniform hue steps between the unique hues in the DIN system is R–Y: 6; Y–G: 5; G–B: 4; B–R: 9.

2. As part of a preliminary effort during the development of OSA-UCS Newhall scaled a constant chroma circle at value 6 into 40 visually equally sized hue differences (102 observers, 142 pairs, 14,484 judgments; Judd, 1965). Unique hues have been plotted into an optimized model of the Newhall results.
3. They also have been plotted into the g, j diagram of the OSA-UCS system, regarding it as an optimized psychophysical model of the psychological diagram.
4. A hue difference scaling experiment for the purposes of establishing a hue difference weighting function has been performed by Y. Qiao et al. in 1998. The resulting CIELAB weighting function can be integrated and the number of equal hue steps between the unique hues determined.
5. R. S. Berns has proposed a somewhat modified hue function, taking into account suprathreshold small color difference data (Berns, 2001).

In Table 4-1 the hue angle segments in the psychological chromatic diagram between the four unique hues of the various experiments are compared (Kuehni, 2001e). As Table 4-1 shows, there is considerable variation in the results, but the trend is uniform (except for the yellow sector in the Hesselgren and DIN systems). The variation coefficients (CV) indicate greatest agreement for the R–B sector and significant disagreement in the R–Y and G–B sectors. Additional high-reliability hue-scaling experiments to provide a clearer picture of this important issue are very desirable.

In addition a linear model was fitted to the MacAdam color-matching error ellipses resulting in a coefficient of variation of 15% for 100 color differences (four per ellipse; Kuehni 2001d). By this formula the number of hue color matching error steps between the average unique hues was calculated with the following results: R–Y: 52; Y–G: 77; G–B: 72; B–R: 127. The trend is the same as that shown in Table 4-1, but the differences in the segments (after scaling the numbers for a total of 360) are more distinct.

In the NCS system unique hues form by definition equal sized quadrants in their hue circle. The NCS unique hues are well within the observer variability of those of Kuehni's unique hue determination experiment (and the additional experiment on unique green). As mentioned earlier, the implication is that the psychological unit differences are not the same in the different quadrants of that system. This has been mentioned by Tonnquist (1966). It follows that equal numerical changes in perceived content of unique hues do not result in equal perceived hue differences in the four segments.

Chroma

While hue has four comparatively easy to locate psychological markers, the unique hues, chroma has none. Different chroma scales (the results of different experiments) can therefore only be compared in terms of physical or psychophysical scales. Such comparisons will be made in Chapter 6.

Chroma steps have been investigated extensively as part of the development of the Munsell system as described in Chapter 7 and the results codified in the Munsell Renotations. The Renotations represent a considerable change of chroma scaling compared to the 1915 *Atlas* and the 1929 *Book of Color*. Few explicit studies of chroma scaling have appeared since then. In 1957, in preparation for the development of the Optical Society of America Uniform Color Scales (OSA-UCS), Judd, Nickerson, and Nimeroff determined a constant chroma circle at value 6 and chroma 8 (384 chroma differences, 60 observers, 23,040 observations; Judd 1965). As Chapter 5 will show, it differs significantly from the Renotation constant chroma circle. Implicit chroma scales along the chromatic axes were also developed for the OSA-UCS system. The four chroma scales, when compared in a common psychophysical system, are found to differ considerably, as shown in Chapter 5.

In the DIN system saturation rather than chroma was scaled. The constant saturation contours of that system in the CIE chromaticity diagram differ significantly from the constant chroma contours of the Munsell system. DIN saturation is independent of lightness and the relationship between chroma and DIN saturation differs as a function of lightness.

NCS chromaticness is different from Munsell chroma. There has been no recognition in that system of the varying chromatic power of optimal colors as all implied full colors arbitrarily have been set at chromaticness 100. When transforming Munsell colors of constant chroma to NCS notation the resulting chromaticness varies with hue in roughly sinusoidal fashion, as one would expect from the definition of NCS chromaticness.

Chroma scales are also an implied result of Indow's principal hue and chromatic versus achromatic color component determination as well as of his multidimensional scaling work. The scales from the two experimental efforts differ significantly. In the principal hue component work the implied chroma scales are often severely compressed at higher chroma levels, indicating the difficulty observers had in assessing the relative amount of achromatic and chromatic components (see Fig. 4-9). There was also some compression of the chroma scale at high chroma levels in the multidimensional scaling experiment, when compared to the Munsell system. However, when chroma scales of the *Book of Color* atlas are viewed, the higher chroma steps do not generally appear somewhat smaller than those at lower chroma. Indow also appeared to accept the Munsell chroma scales as essentially valid since he fitted a Riemannian model to his MDS data to improve the evenness in regard to chroma. The results raise doubt that chroma can be accurately assessed by determination of principal hue and achromatic components in a color or by multidimensional scaling.

Chromaticness scaling used in all cases, for sensible reasons, an achromatic surround. Chromatic surrounds change chromaticness scaling. However, this subject is outside the scope of this text and an issue for general appearance modeling. As will be argued later in this text, chromaticness depends on the magnitude of the unit chromaticness difference used to scale it (see also the related comment in the next section).

Chromatic thresholds, the first steps from the neutral point, have been studied since the 1920s. Typical saturation thresholds (amount of spectral light to be added to white light to result in a just perceptible color) as determined in 1938 by I. G. Priest and F. G. Brickwedde are shown in Fig. 4-12. Results by other investigators differ in detail but agree in the general features. Such thresholds are, presumably, indicators of chromatic strength of spectral colors. Evans defined spectral chromatic strength as the factor by which the luminance of a stimulus has to be multiplied in a brilliance match to equal the brilliance of the achromatic surround (Evans, 1974). Recall that a brilliance match against the surround is achieved when the stimulus field looks neither grayish nor fluorent against the surround. In 1968 Evans and Swenholt investigated chromatic strength using the Munsell system. In plotting the reciprocal CIE colorimetric purity against the dominant (or complementary) wavelength for chroma circles 2, 4, 6, 8, and 16 at value 5, they obtained curves essentially parallel to each other and reciprocal to the saturation threshold curve (see Fig. 4-13). These curves provide another kind of support for the salience of the chroma attribute.

In an experiment with 35 observers Kuehni investigated the relative sizes of the first steps from gray of the colors nearest to the respective axes in the a^*, b^* diagram in the Munsell as well as the OSA-UCS systems (Kuehni, 2000c). The results indicated that the average observer judged, in the Munsell system at value 6/chroma 8, the first step toward yellow to be about 2.5 times the magnitude of the first step toward blue. In the red/green direction the

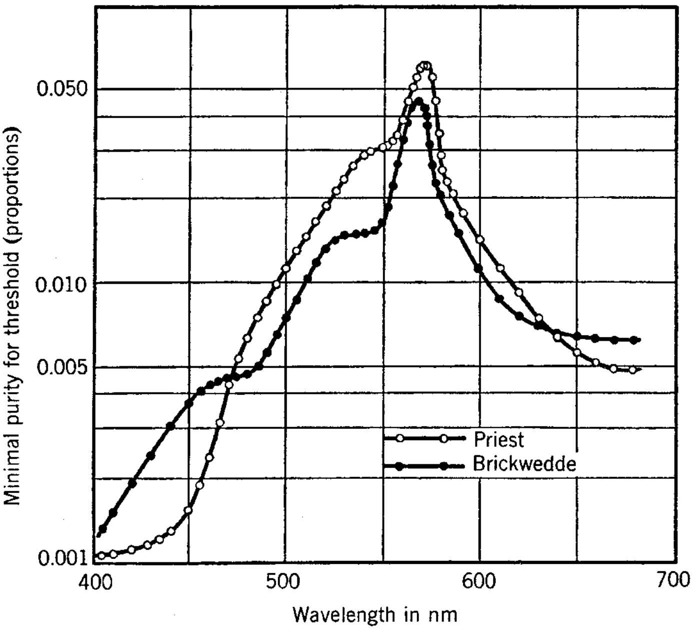

Fig. 4-12 Spectral saturation thresholds determined in 1938 by Priest and Brickwedde. From Osgood (1953).

green step was seen as about 10% larger than the red step. For OSA-UCS the first step toward yellow was seen as about 20% larger than the first step toward blue. In that system the first steps toward red and green were seen as of about equal size. Somewhat different results have been obtained by Indow (2001).

The results of Priest and Brickwedde and many later researchers were obtained monocularly with optical apparatus. M. De Matiello and co-workers investigated the difference between monocular and binocular saturation discrimination at the threshold level as well as for suprathreshold saturation differences (2001). They found the changes in colorimetric purity for threshold steps to be considerably larger for the first step from gray than for the first step from the spectral color. For suprathreshold saturation differences they found power relationships varying by wavelength with exponents from 0.61 to 0.97. Interestingly they found exponents of binocularly determined data to be lower than those obtained for monocular data.

Relationship between Hue and Chroma Differences

This matter created considerable discussion in the 1950s and 1960s (Judd, 1969). As mentioned previously, in 1936 Nickerson developed the first formula, on a psychological basis, that could result in predictions of the perceptual

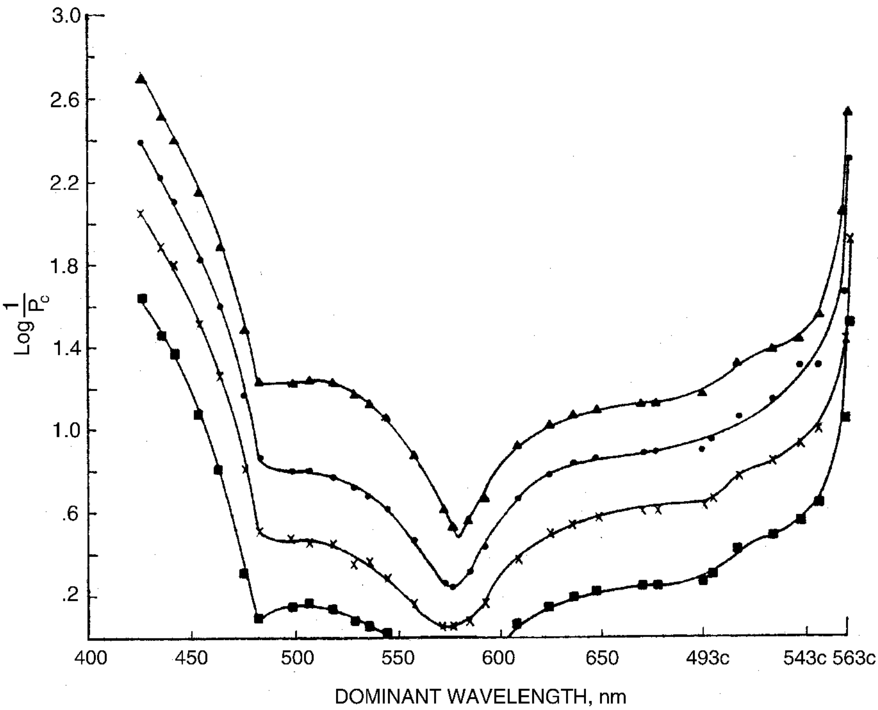

Fig. 4-13 *Reciprocal CIE colorimetric purity as a function of dominant wavelength for Munsell colors at five levels of chroma (represented by different symbols) and value 6. From Evans and Swenholdt (1968).*

magnitude of color differences, the Index of Fading. It was based on the Munsell system and reads as follows:

$$I = \left(\frac{C}{5}\right)(2\Delta H) + 6\Delta V + 3\Delta C, \tag{4-1}$$

where I is the index value, C is Munsell chroma, V is value, and H is hue expressed in terms of the Munsell 100-hue circle. For two colors being compared, perceptual differences according to this formula are equal to the simple sum of weighted differences between the index values. Note that this formula predicts hue differences between colors of neighboring Munsell hues to grow linearly in magnitude as a function of chroma.

As Judd showed, for colors of equal V one hue step is, according to the Nickerson formula, equal to $2C/15 = 0.133C$ chroma steps. When expressing the total hue angle of all 100 hue differences the result is found to be $13.33C$. This is slightly more than twice the theoretical total hue angle of a circle that in a euclidean system is $6.28C$. While initially skeptical of this result and believing it to be due to experimental error, Judd later found support for what he called the "superimportance of hue" in experiments involving large hue differences. If the Nickerson formula is factual, perceptual color space is not euclidean. Judd used a crinkled fan model to account for hue superimportance (see Fig. 7-15).

Hue superimportance was also detected in the experimental basis data for the OSA-UCS system. In 1964 the Committee on Uniform Color Scales of the Optical Society of America completed a basic experiment eventually resulting in the OSA-UCS scales. Forty-three samples were viewed in 102 pairs by 71 observers and compared for size of difference against other pairs. After psychometric scale values were calculated, the correlation between these and various formulas was determined. The correlation coefficient obtained, for example, for the Glasser cube root formula was 0.34 while the Nickerson formula resulted in a value of 0.61. When, instead of sums, the square root of the sum of the squares of the component differences was calculated, the correlation increased to 0.65 (perhaps merely a fortuitous result). An optimal value of 0.80 was obtained with (1) revised Munsell spacings in regard to hue and chroma and (2) use of the following formula, which derives from a formula developed earlier by Godlove as a description of the ideal Munsell system in a euclidean space in place of the Nickerson formula:[7]

$$\Delta E = \left\{\left[f_g f_h (C_1 C_2)^{0.5} \Delta H\right]^2 + (\Delta C)^2 + (4\Delta V)^2\right\}^{0.5}, \tag{4-2}$$

where C_1 and C_2 are the Munsell chromas of the two colors compared, ΔH is the hue difference in Munsell 100-hue steps, ΔV is the difference in Munsell value, and

$$f_g = \frac{[2(1-\cos 3.6\,°\Delta H)]^{0.5}}{\Delta H},$$

$$f_h = \frac{2-k+4(k-1)}{3-\cos 3.6\,°\Delta H},$$

where k is the hue superweight factor with a value from 1 to 2. If the value is 1, then $f_h = 1$. For optimal correlation the value of k was selected as 1.7 (Judd and Nickerson, 1967), this being indicative of hue superimportance. This experiment supported the idea of a relationship between perceptual hue differences and chroma differences that does not fit into a euclidean system. This matter is further discussed in Chapters 7 and 8.

Lightness

The first "qualitative" verbal gray scale, seemingly, is the one by Forsius, with seven grades including black and white (see Chapter 2). Glisson described quantitatively (in terms of weights of black and white pigment to be used) a

24-grade gray scale that is reasonably uniform through the middle portion. Mayer envisaged 23 grades of different lightness in his double pyramid, while Lambert reduced that to 8 grades. Runge's color sphere has a 10-grade gray scale.

In 1729 the French mathematician Bouguer published *Essai d'optique sur la gradation de la lumière* (Optical treatise on the gradation of light) in which he described experiments using shadows of rods made by the light of two candles on a white screen. He found that the distance ratio of the two candles needed to be about 8 : 1 to result in a just noticeable difference between the two shadows. Thus he initiated the study of brightness thresholds. In 1845 his compatriot V. Masson described a white disk on which a black radial line segment was inscribed. When spinning the disk the line segment darkened a ring of the disk (the spinning disk method was invented by Pieter van Musschenbroek in 1768[8]). The width of the line segment as a fraction of the total disk circumference is an indicator of the fractional change in luminosity required to see a difference. Masson found that the ratio, depending on circumstances of illumination and viewing distance, was between 1/50 to 1/120. In 1850 the French astronomer D. F. Arago (1786–1853) repeated the shadow experiment with improved equipment and found a ratio of 1/133. Masson's experiment was repeated in 1858 by Fechner and Volkmann, who found a ratio of 1/100 and by Helmholtz in 1860 who, under optimal conditions, could see a ratio of 1 : 167. In 1888/9 König and Brodhun used Masson's technique to determine brightness as well as chromatic thresholds.

Attacking the problem of a uniform gray scale from another angle Plateau in 1872 asked several painters to paint a color halfway between black and white (as described in Chapter 2), thus performing an equisection. Independently Delboeuf developed a detailed uniform gray scale using successive equisection, as well as by adjusting black and white disk segments to obtain perceptually equal steps.

In 1874 Hering, as part of his psychological color triangle, described a "nuanced" series of grays from black to white that was to have uniform increments/decrements of blackness and whiteness. Ebbinhaus described in 1887 an eight-grade gray scale (see Chapter 5). In 1899 Munsell began to develop his color order system of which an eleven-grade (ten-step) value scale was an integral part. The Munsell value scale underwent development until the 1930s and was finalized in the Munsell Renotations. In the final experiments gray scales were visually measured against a white, gray, and black background. They were found to differ, and the results were averaged into the value scale (see Chapter 5, Section 5.7). Ostwald selected a logarithmic relationship between the stimulus and the response for his gray scale. In the 1960s Kaneko and Takasaki confirmed and further quantified W. Schönfelder's finding that color differences between two stimuli are perceived best if the surround is intermediate between the two stimuli. This makes a gray scale and its steps dependent on the surround. Surround dependence had been found in 1922 by Adams and Cobb and was also investigated by Evans. The effect was included into the definition of the OSA-UCS lightness scale, otherwise based on the Munsell value scale. The NCS whiteness/blackness scale is derived from average judgments of the content of whiteness and blackness in gray samples. The atlas scale has eleven grades (including black and white) resulting in ten steps, the numerical scale has 100 steps.

From the beginning with Bouguer the development of lightness scales has run parallel with the development in photometry. While work with the Masson disk or Delboeuf's equisection method and others did not explicitly require photometry, their results were best interpretable in quantitative terms when compared to the corresponding stimulus strength. The development of lightness scales is therefore further discussed in Chapter 5.

A fundamental issue involving lightness is how brightness and lightness are connected. As has been demonstrated by many researchers over the last 100 years, a source of achromatic color with any given luminance value can, depending on the conditions it is seen in, be perceived as any grade of gray from white to black. The question arises how the human visual system decides to assign a given lightness value to a given luminance value. Among the early researchers investigating this matter was D. Katz who was a student of G. E. Müller. In 1909 he published a book-length paper, *Die Erscheinungsweisen der Farben und ihre Beinflussung durch die individuelle Erfahrung* (translated in 1935 as *The world of colour*) in which he discussed his investigations of different appearance modes of colors, including the relationship of brightness, lightness, and luminance. In a dramatic experiment Gelb showed in 1929 that if a black piece of paper is suspended in midair and illuminated with a strong beam of white light, its appearance is white. If a paper of higher luminous reflectance than that of the black paper is placed next to it, the original paper no longer appears white but a shade of gray or black, depending on the luminous reflectance of the second paper. Thus lightness perception is dependent on the relative luminous reflectance of adjacent fields. A few years later gestalt psychology theorists postulated that the visual system compares the luminance of a target against the weighted average of the luminance of the total scene. In 1948 H. Wallach proposed that lightness is decided by the ratio of the luminances of two adjacent fields. The so-called intrinsic image model of lightness was developed in the last twenty years. It had been found that when a gray paper is successively viewed against backgrounds of different lightness, the ratio of luminous reflectances changes significantly but the perceived lightness of the paper changes little. Similarly there is considerable lightness constancy when viewing the arrangement of paper and background under different kinds of illumination. The intrinsic image model analyzes the image as it appears on the retina according to three components: surface reflectance, illumination, and three-dimensional form clues. However, significant disagreements between the predictions of theory and the experimental results remained in certain situations. As Gilchrist and colleagues (1999) describe it, a fundamental question is that of the anchoring problem. It relates to the question "where to locate the range of luminance values (in a given situation) on the scale of perceived gray shades." Once it is solved, the next question is "how the range of luminance values is distributed on the scale of perceived gray shades." Many researchers have contributed to the current answers: in simple images (two fields of different luminance filling the entire visual space) the highest luminance is seen as white unless the relative area of the darker field is more than half of the total field. Then an area rule applies. As the darker side grows in relative size, it appears lighter and lighter, making the smaller, lighter area appear more and more white, eventually fluorent and then self-luminous.

In more complex images Gilchrist and his group propose to separate the image into components, called frameworks, that belong together according to gestalt principles (an exact definition of framework is difficult, so coplanarity of the elements appears to be a good start). For each local framework the anchoring and area rules of the simple situation approximately apply. The entire visual field is called the global framework. Within a local framework a veridical, or 1:1 scaling between lightness and luminance, is found to apply if the luminance ratio between white and black is less than 30:1. If it is larger, some luminance scale compression occurs. Gilchrist et al. have tested their model on many classical lightness test results and "illusions," such as the Benary effect (1924) or White's illusion (White, 1981) and found good explanatory power. In cases where the information from local frameworks appears contradictory, Gilchrist et al. believe that our visual system makes a compromise between the results or alternately displays one or the other. It is evident that much work remains to be done.

4.10 PERCEPTION OF COLOR DIFFERENCES

A formula that mathematically describes the relationship between attribute component differences and total perceived difference is Nickerson's formula of 1936 encountered above and expressed in terms of the attributes as defined in the Munsell system. The investigation by Bellamy and Newhall (1942) showed that the definition of the formula depends on the size of the differences; that is to say the quantitative relationship between the Munsell attributes changed significantly when tested at the JND level.

In current practice the results of small color difference calculation are usually expressed in terms of total color difference and its hue, chroma, and lightness difference components. The visual small color difference data have in all cases been determined as total differences. The split of the total difference into components is achieved mathematically, based on the assumption that color space is euclidean. The correlation between visual and calculated data has been improved by adding empirical functions. Because of the lack of visual small color difference component data, the degree to which calculated components are in agreement with visual data is unknown. This is an area that requires attention.

Surprisingly it appears that the assumption that perceived, properly scaled attribute differences add up by euclidean summation has never been explicitly tested and confirmed. One problem is that, as seen earlier, there are no reliable, replicated scales of perceptual hue, chroma, and lightness, a prerequisite for such a test.

Another effort in developing a color difference formula based on visually determined components has been made by Indow (1999a, 2002). Using visual judgments of Munsell differences determined for the purpose of multidimensional scaling analysis (as described in Indow, 1988), he attempted to find the form of geometrical space that would result in the best agreement between interpoint distances of colors in that space and of those visually determined. Indow found that the formula he developed resulted in smaller root mean square (RMS) error than that of Adams-Nickerson but larger than the CIE94 formula. This is surprising because the differences judged were rather large, involving Munsell chips. In the same year Indow (1999c) also investigated the accuracy of prediction of Munsell type color differences from the judged differences in their principal hue components and lightness differences. Indow obtained the lowest RMS error with a formula that is the sum of weighted value difference and individually weighted principal hue component differences:

$$\tilde{d}(\Delta V, \Delta\bar{\xi}) = a_V \Delta V + (d_0 + \Sigma a_a \Delta\bar{\xi}_a), \tag{4-3}$$

where $\tilde{d}$ is the predictor of the color difference, ΔV is the Munsell value difference, $\Delta\bar{\xi}_a$ is the difference in a principal hue component, and a_a is the corresponding weight for the principal component difference, with the values $a_R = 0.199$, $a_Y = 0.031$, $a_G = 0.098$, $a_B = 0.136$, $d_0 = 0.610$, and $a_V = 0.459$. The principal hue components are functions of value and chroma.

Also in Indow's work, with different data and using principal hue components, simple addition of the component differences was found to give best results. Because of the normalization of perceived principal hue components to a maximum value of ten different weighting of the individual components is required. In differences the red principal component has the strongest effect. It takes 1.5 units of blue principal hue component (PHC) to equal in a difference one unit red PHC, 2.2 of green PHC, and, adjusted for the lower absolute yellow maximum, 3.4 units yellow PHC.

Indow's work with principal hue components implies, unsurprisingly, that they are the psychological fundamentals constituting hue. At the same time they implicitly constitute an aspect of chroma. The achromatic difference to the maximum value of 10 is also a conflation, in this case of an achromatic portion of chroma and lightness differences. The lightness difference (presumably including the Helmholtz-Kohlrausch effect) is, in addition, separately judged. As mentioned above, Indow's observers, however, had difficulties distinguishing in terms of principal hue components between different Munsell chroma levels at the same lightness.

In this chapter it was demonstrated that for purposes of an object color space uniform in color differences, hue, lightness, and chroma appear to be the essential color attributes. Other attributes can replace lightness and chroma if uniformity in terms of difference is not the guiding principle. Historically two sets of fundamental attributes of color perceptions have been proposed. Hue is common to both. In Hering's approach veiling is described by the relative amounts of blackness and whiteness in the perception. But veiling can also be described in terms of lightness and chroma. It has not been determined how under comparable conditions the two sets of veiling attributes compare. Various experimental scales of constant chroma and equal-sized hue differences vary considerably for unclear reasons. It has also been shown that a psychological color space with the unique hues on the chromatic axes is not uniform because there are varying numbers of equal-sized hue differences in the four quadrants. The relationship between chroma and hue differences appears to change significantly as a function of the size of the differences. In the next chapter these and other issues are investigated in terms of quantitative stimulus differences.

Chapter 5

Psychophysical Scaling of Color Attributes: Stimulus and Perception

In Chapter 4 color scaling was considered from a purely psychological point of view. Once attributes were selected, judgments were made using samples of colored materials to exemplify scales of the attributes in conformance with the geometrical form selected for the color solid. In a similar manner it was also possible to study thresholds and suprathreshold color differences without resort to physical measurements. It was of interest to the early psychophysicists to find the relationship between stimulus magnitude and response. It has been indicated before that such relationships can only be valid for tightly circumscribed conditions of viewing and that they can be significantly different for other conditions. Within these limits it is desirable to characterize both stimulus and resulting perception with a high level of accuracy and repeatability. Since the characterization of the stimulus involves physical measurements, progress was linked to progress in photometry and spectrophotometry.

5.1 REQUIREMENTS FOR A UNIFORM PSYCHOPHYSICAL COLOR SPACE

A formula representing a uniform psychophysical color space for trichromats must transform spectral power distributions or reflectances in such a way that

Color Space and Its Divisions: Color Order from Antiquity to the Present, by Rolf G. Kuehni
ISBN 0-471-32670-4

color scales in three dimensions in good agreement with psychological scales are created. Not only must the colors, for example of a color circle, be placed in the correct ordinal order, but they must also be correctly ordered in regard to saturation or chroma, brightness or lightness, and the unit differences in hue, chroma and lightness, must be the same. As we will see, simple psychophysical spaces such as a form of cone response space, the CIE X, Y, Z space, or linearly related opponent color spaces place colors in proper ordinal sequence; however, distances in these spaces are not proportional to psychological distances. Since the beginning of the twentieth century a search has been on to find the transformation that might accomplish this task. It has become evident that a single formula cannot accomplish it for any kind of color difference, or only perhaps with several adjustable variables. We begin by separately considering the three conventional key attributes of psychological object color order: lightness, hue, and chroma.

5.2 POSTULATED RELATIONSHIP BETWEEN PSYCHOLOGICAL AND PHYSICAL MAGNITUDES

Historically postulated relationships between psychological and physical magnitudes are fraught with difficulties. These are due to the absence of a validated mechanism connecting the psychological experiences of color with the physical measurement of spectral power. The problems begin with the definition of the brightness function. Additive results (i.e., where two stimuli mixed together are seen as resulting in a brightness perception that is the sum of the perceptions of the two component lights) are only obtained if the spectral brightness function is determined according to a particular methodology (see below). Brightness and lightness perceptions thus are complex and dependent on experimental conditions.

The CIE colorimetric system, developed in the late 1920s, is based on experiments establishing what lights are matching. It does not have an explicit connection to the appearance of stimuli. Such a connection was usually assumed, however (e.g., see Adams, 1942). The relative success of this assumption has strengthened the belief in its applicability. Today models of color vision are usually expressed in terms of cone sensitivity functions. These are believed to be linearly related to color-matching functions, and vice versa. Models of color vision have become more sophisticated by considering surrounds and other experimental conditions, but the fundamental assumption of the direct relationship between color matching and color appearance remains.

Brightness and lightness and their differences are expressed in terms of luminance or luminous reflectance, even though this does not consider the spectral brightness component of the Helmholtz-Kohlrausch effect (see below). Only the OSA-UCS system has included this effect. Conventionally hue and chroma are believed to be functions of properly scaled opponent signals derived from CIE tristimulus values, hue the ratio and chroma the euclidean sum (see Section 5.6). These are empirical approximations with a degree of validation from the correlation with visual data, as we will see. To bring such models into close agreement with psychological results requires extensive fine-tuning and application of auxiliary functions, as Chapter 6 will show. The following sections discuss the development of these implied relationships in some detail.

5.3 PHOTOMETRY AND BRIGHTNESS/LIGHTNESS

For the visual sense the earliest quantitative studies comparing stimulus to response involved brightness. Photometry, the quantitative measurement of light, has been of interest since the Renaissance. In its more modern form it was developed by Bouguer and advanced by Lambert and Benjamin Thompson, the Count of Rumford (1753–1814). The initial equipment consisted of simple wedges that allowed visual comparison of two different light sources under controlled conditions. A curious, very early illustration of what appears to be a photometric measurement[1] is that by the celebrated painter Rubens used as the frontispiece for Book V of d'Aguilon's text of 1613 *Opticorum Libri Sex* (Fig. 5-1). Here the effect of distance of light source on the perceived brightness of the patches on the screen is being studied.

Bouguer made quantitative measurements by comparing a test light source against a candle as the reference source. The first step toward making measurements more reliable was to use an artificial reference light source other than a candle. This resulted, beginning in the late eighteenth century, in the

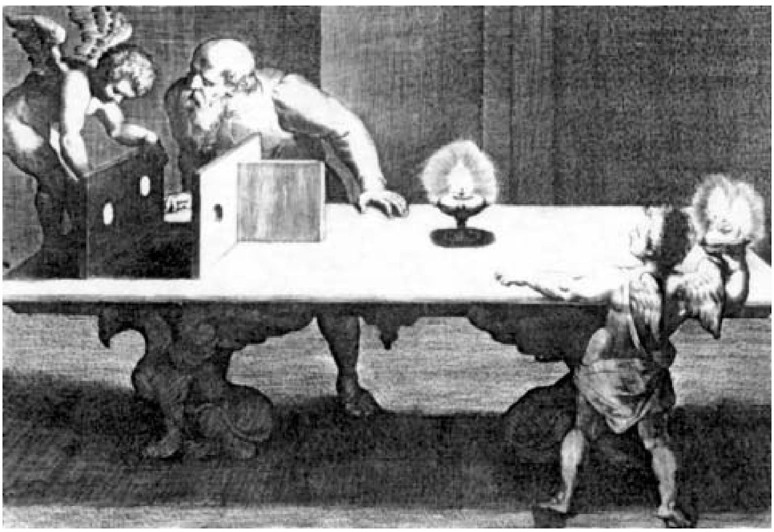

Fig. 5-1 *Rubens's, frontispiece illustration to Book V of d'Aguilon's* Opticorum libri sex *depicting an experiment in measuring light intensity as a function of distance of the lights, 1613.*

development of oil lamps, pentane lamps, and the Hefner lamp, burning various hydrocarbons (used until the 1940s).[2] The human observer began to be replaced by photoelectric cells beginning in the early twentieth century. Lambert had already envisaged this development by describing in *Photometria* in concept a piece of equipment that could measure light analogous to temperature measurement with a thermometer.[3] For such measurements to be in agreement with visual measurements the correspondence between the spectral sensitivity of photoelectric cells and that of the average human had to be established.

The discovery of rods and cones of the human visual system in 1828 by the German physiologist G. R. Treviranus (1776–1837) and the discovery of visual purple in 1877 by F. Boll resulted in extended efforts to determine their purpose and activity.[4] The duplicity theory of the double function of the retina, according to which the cones mediate daylight vision and the rods night vision, was proposed by the German anatomist M. J. S. Schultze (1825–1874) in 1866 and expanded by Helmholtz's student Kries. Measurements of the luminosity of spectral colors at different levels of illumination were made in Helmholtz's laboratory by König and C. Dieterici (1884, 1892). König was able to show the close agreement among an absorption curve he had measured for visual purple, the luminosity curve of a person with monochromatic vision established under daylight conditions, and the luminosity curve for an observer with normal vision determined at very low light levels. This was a clear indication of the existence of the postulated second visual system based on rods and operational for night vision in observers with normal color vision. König and Dieterici obtained a different spectral sensitivity curve for daylight vision. Comparison of the sensitivity of photoelectric cells and of humans indicated large discrepancies, and in order to bring measurements into agreement with average visual results, filters had to be interposed between light source and the photoelectric cell.

A new type of photometer, the flicker photometer, appeared in the early twentieth century and was intensively investigated by H. E. Ives (1912). The original method of obtaining flicker was to rotate a partial disk in front of a light. Depending on the disk sector and the speed of the disk, more or less flicker is observed. If the occluding portion is half of the disk, at the critical flicker frequency at which the flicker disappears the perceived intensity of light is exactly half the intensity when viewing it unobstructed. By matching the brightness of chromatic flickering fields against a steady field of white light, the luminous efficiency of chromatic lights can be measured. Ives's studies showed that brightness measured in this manner is additive. Additivity is not observed when brightness of lights of different color is measured by comparing steady fields. Given the state of computing at the time, additive functions were very desirable. Important determinations of spectral brightness using flicker photometry were reported in 1923 by K. S. Gibson and E. P. T. Tyndall, working at the U.S. National Bureau of Standards. An average sensitivity curve was recommended by them to the CIE and standardized largely unchanged

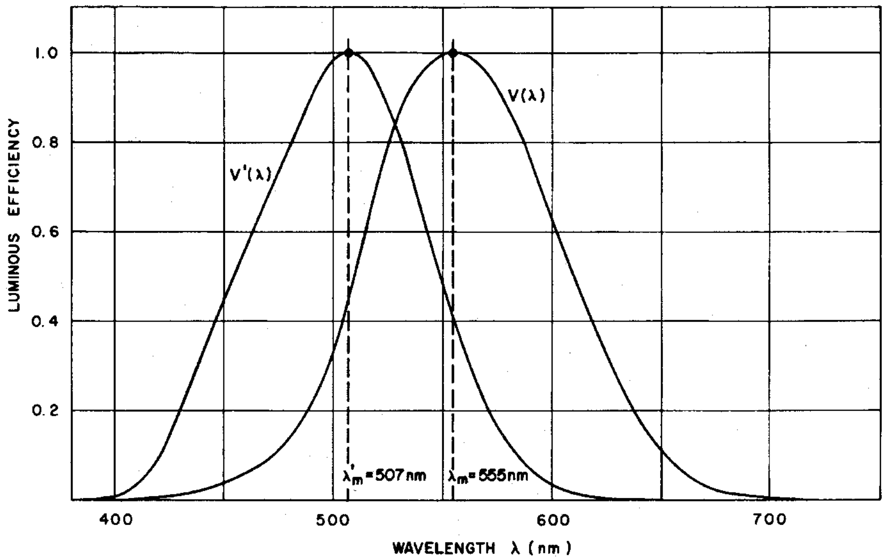

Fig. 5-2 CIE standard scotopic (low light, V'_λ) and photopic (daylight level, V_λ) luminosity curves. The wavelengths of maximum sensitivity are indicated. From Wyszecki and Stiles (1982).

by that organization in 1924 as the CIE standard spectral luminous efficiency function $V(\lambda)$ (CIE, 1924), representing the average photopic observer (daylight vision). A comparable scotopic standard observer (low-light vision) was defined in 1951. The spectral sensitivity functions of the two observers are illustrated in Fig. 5-2. The CIE luminosity function has been criticized after its standardization (e.g., see Stockman and Sharpe, 1999), and in 1988 the CIE offered a revised version based on proposals by Judd in 1951 and by J. J. Vos in 1978 (CIE, 1990). The 1924 function nevertheless is an integral part of the CIE colorimetric system and continues to be widely used.

5.4 THE COLORIMETRIC SYSTEM

The idea of trichromatic color vision was originally put forth by Palmer in 1777 and revived by Young in 1802. Important support was later provided by the work of Helmholtz, Grassmann, and Maxwell, as discussed earlier. In the 1850s Helmholtz estimated the spectral sensitivity of the human color vision system in agreement with Young's theory (Fig. 5-3; Helmholtz, 1860). In 1860 Maxwell provided the first set of measured functions derived using a visual colorimeter (Fig. 5-4). Detailed measurements were made by König and Dieterici in 1892, using an advanced König-Helmholtz spectral colorimeter (Fig. 5-5). Similar measurements were made also by Abney and reported in 1914.

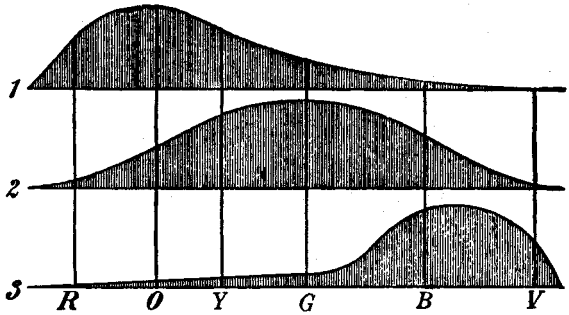

Fig. 5-3 Helmholtz's sketch of estimated spectral sensitivity of three fundamental color vision processes, 1860. The letters refer to major hues of the spectrum, from red (R) on the left to violet on the right.

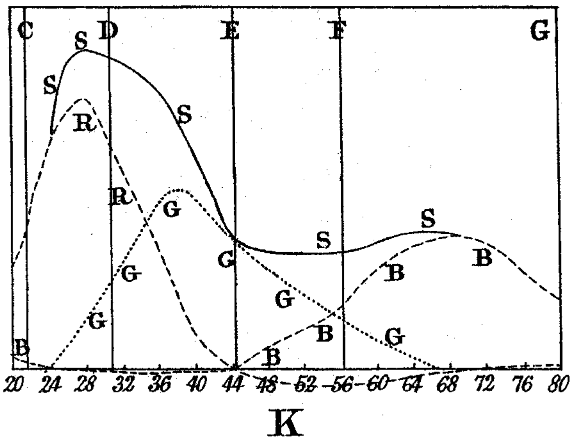

Fig. 5-4 Spectral curves of the three fundamental processes of color vision R, G, and B, and the sum curve S as determined by Maxwell, observer K. The wavelength scale is in arbitrary units; C–G identify Fraunhofer lines in the spectrum, 1860.

Between 1920 and 1930 J. Guild (1930) and W. D. Wright (1928–29) in England independently built improved equipment to measure these functions, and the color-matching functions of various observers were determined. These data were considered by the CIE for standardization. Following a suggestion by Judd, linear transformations were calculated so that two of the implied primaries were located on a line with zero luminance (the alychne). This resulted in the third primary having a spectral function identical to the CIE luminance function. In this form the functions were standardized as the CIE 1931 2° standard observer color-matching functions (applicable to a field of view of 2°, Fig. 5-6; CIE, 1931).[5] In 1964 the CIE 1964 10° standard observer color-matching functions were added, applying to a field of view of 10° (CIE, 1964) and based

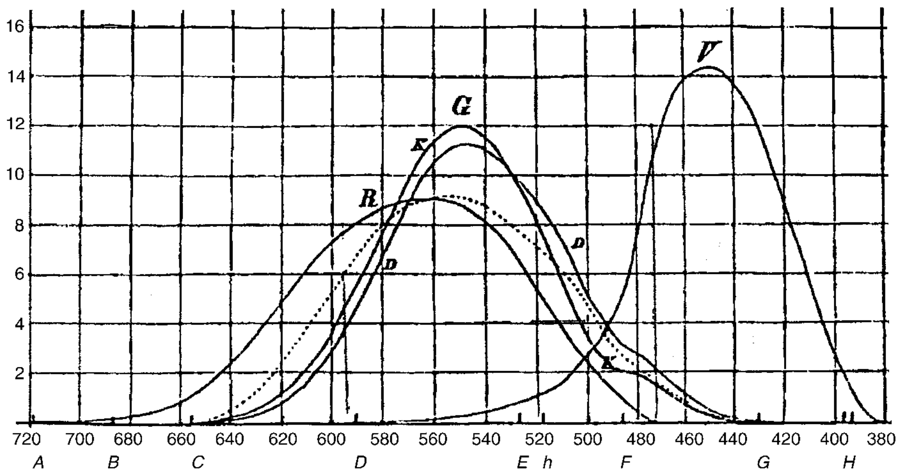

Fig. 5-5 *Measurements of their own fundamental color vision processes* R, G, *and* V *(violet) by König and Dieterici. The dashed line represents the* G *function of an observer with anomalous color vision, 1886. Note that the wavelength scale is in reverse. Letters along the wavelength scale denote major Fraunhofer lines.*

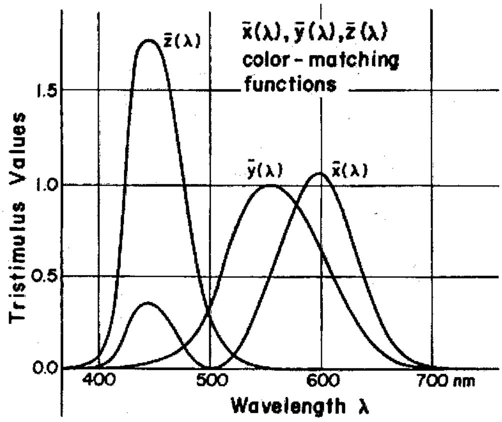

Fig. 5-6 *CIE spectral color matching functions of the 2° standard observer. They have been linearly transformed from the measured functions. From Wyszecki and Stiles (1982).*

on measurements by W. S. Stiles and J. M. Burch (1955) and by N. I. Speranskaya (1959). The two sets of functions differ somewhat.[6] With these definitions of standard observers, as well as definitions of illuminants and of the reflectance factor defining the reflecting properties of object colors, a system was available for quantitative colorimetry. In this system any colored material as viewed under a standard light source by a standard observer is expressed with three figures identified as the tristimulus values X, Y, and Z, where Y is identical to the luminous reflectance for object colors (luminance for lights). The X, Y, Z space is a nonuniform psychophysical color space, taken to be euclidean, relating measured reflectance of a sample viewed in a standard light to a specific point in the space. Tristimulus values in case of object colors are normalized sums of the reflectance function weighted by the spectral power distribution of the light source and the three color matching functions $\bar{x}$, $\bar{y}$, $\bar{z}$. With one dimension in this space representing (flicker) luminance, the other two must represent in some fashion the chromatic components from which hue and chroma are derived. In the CIE colorimetric system the psychophysical definition of hue is the dominant, or complementary wavelength, of saturation colorimetric purity as expressed in the CIE chromaticity diagram, a particular version of chromatic plane defined by the chromaticity coordinates x and y. Their definition is as follows:

$$x = \frac{X}{X+Y+Z} \qquad \text{and} \qquad y = \frac{Y}{X+Y+Z}. \tag{5-1}$$

In the chromaticity diagram spectral colors fall on a horseshoe-shaped function, with saturated nonspectral purple colors falling on a straight line connecting the ends of the spectral function (Fig. 5-7). All possible object and light colors fall on or within the outline of the diagram. The chromaticity diagram is also not visually uniform. A color space can be constructed with the axes x, y, Y. It is commonly used to represent color relationships. An example is the Rösch-MacAdam solid illustrating on its surface optimal object colors for a given standard observer/illuminant combination (see Fig. 2-46). The X, Y, Z space and the chromaticity diagram allow the determination of quantitative relationships between colors in a linear system, but without consideration of surrounds. The locus of spectral colors in X, Y, Z space is illustrated in Fig. 5-8.

The CIE colorimetric system has not escaped criticism and has been even ridiculed.[7] Despite its weaknesses it remains a central aspect of color science and its technical applications. Among the reasons are its noticeable technical successes, the lack of better proposals, and resistance against change. True progress, if possible, requires knowledge of the contents of the black box connecting physical stimuli with the experience of color.

5.5 CONE RESPONSE SPACE

Physiology has identified three cone types in the human retina, in agreement with the postulates of trichromatic vision (but recall the comment about females with the genetic potential for four cone types in Chapter 1). These three cone types are presumably the only transducers of information responsible for color vision (rods may contribute to color vision under certain limited circumstances). Along the visual path in the brain many transformations of

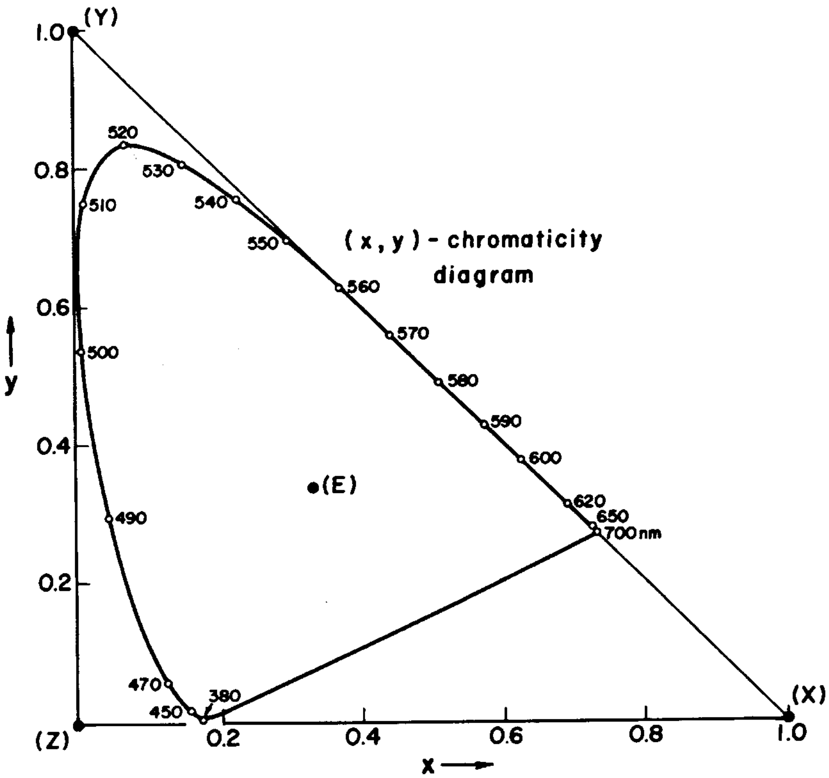

Fig. 5-7 CIE chromaticity diagram with spectral trace and the nonspectral purple colors for the 2° standard observer. E *represents the equal energy illuminant. From Wyszecki and Stiles (1982).*

the information generated in the retina take place, all derived from cone responses. Cone sensitivity and cone response to specific light energy are therefore important pieces of information in an attempt to link physical stimulus and psychological response. Attempts to measure spectral absorption functions of cone pigments began in the middle of the twentieth century, with refinement continuing into the present. However, light radiation undergoes changes in the eye before being absorbed by the cones of the retina, changes that need to be accounted for. They are due to the media in the eye from the outer surface to the cones. For a comparison between stimulus and perception, cone response functions taking into account intraocular absorptions are therefore essential. In vivo measurements of human cone response are not practical, and the data currently accepted are based on absorption measurements of excised eyes, on measurements made on primates, and on the assumption that cone response functions are linearly related to color-matching functions. Proposals for standard cone sensitivity functions have been made

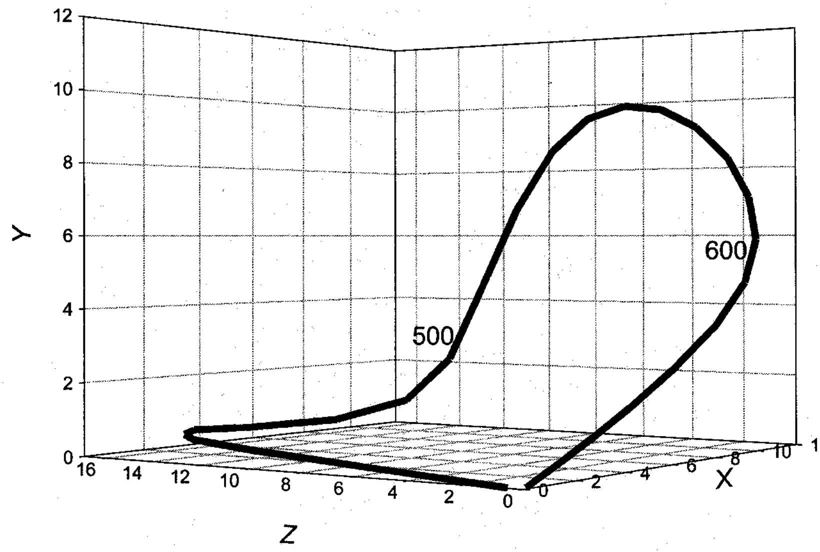

Fig. 5-8 *Locus of spectral colors in the CIE* X, Y, Z *space, beginning at 380 nm and ending at 750 nm (wavelengths 500 and 600 nm are identified).*

by V. C. Smith and J. Pokorny (1975), by Vos (1978), by A. Stockman et al. (1993), and by Stockman and L. T. Sharpe (1998). The latest proposals are based on the Stiles-Burch color-matching functions. There are relatively small differences between the three sets of data that for the current purposes do not seem to be of significance, however, even though, as discussed below, they could have a noticeable effect on correlation with hue difference perception. Cone response functions are reported in different ways: linear or logarithmic scales, function peaks at unity, or identical area under the curves. Figure 5-9*a* and *b* illustrate the Smith-Pokorny functions (as derived from the CIE 2° observer data) in linear scale as well as in logarithmic scale. In the past the three functions were often called *R*, *G*, and *B*, for red, green, and blue. In recent years, as more details about the opponent color system became known, it has been realized that this is not an appropriate way of identifying them. They are now called *L*, *M*, and *S* (for long, medium, and short waveband sensitivity). The three functions can be thought of as constituting the axes of a (nonuniform) psychophysical color space. Figure 5-10 illustrates the spectral trace in the orthogonal *L*, *M*, *S* space defined by the Smith and Pokorny functions. None of the axes of this space has a direct relationship to a color attribute. An unresolved issue is the effect of field size on the cone sensitivity functions. If, as said above, cone sensitivity functions are linearly related to color-matching functions, the cone sensitivity functions must be different for the 2° and 10° standard observers. There are reasons why this could be so. Some vision scientists believe that the same transformation

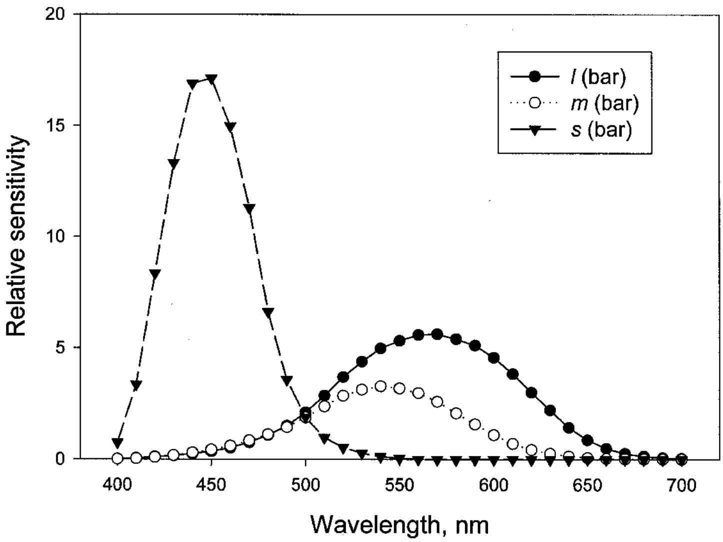

Fig. 5-9a *Spectral cone sensitivity functions according to Smith and Pokorny, calculated for the CIE 2° standard observer, with identical area under the curves, linear sensitivity scale.*

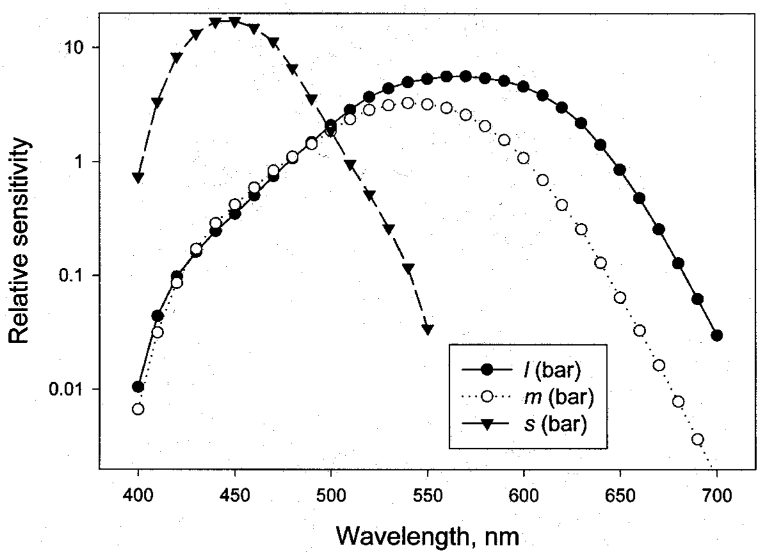

Fig. 5-9b *Spectral cone sensitivities from Fig. 5-9*a *in logarithmic sensitivity scale.*

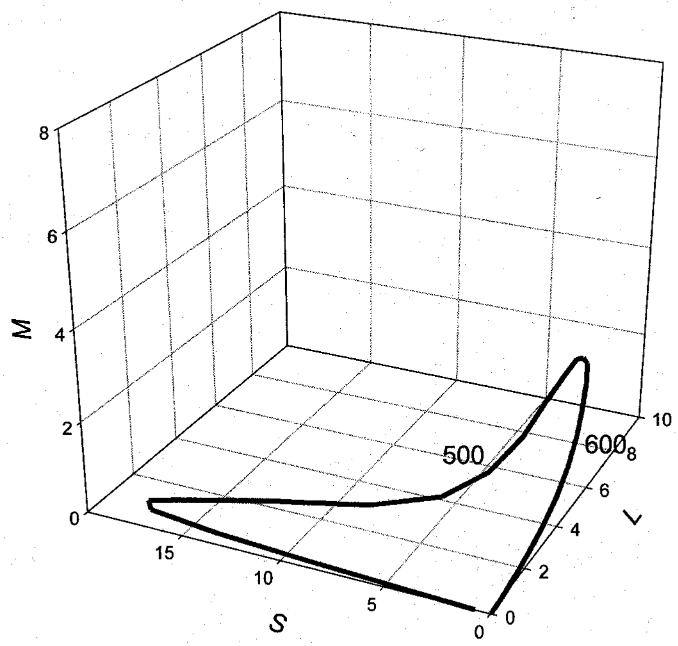

Fig. 5-10 *Spectral trace in the L, M, S space based on the cone sensitivity functions of Fig. 5-9a. The wavelengths 500 and 600 nm are identified.*

formulas are applicable for the 2 and 10 standard observers (Smith, 1999).

Cone responses are essentially in linear relationship with the absorbed light energy up to a luminance of approximately 1000 cd/m^2 (a cloudy sky at noon). Above that, for example, in bright sunlight, saturation effects occur resulting in relatively reduced response.

Individual Variation in Cone Functions and Smoothness of Functions

In recent years a significant effort is under way to develop a new set of cone functions taking into account the latest information on genetic variability of cone expression and using modern technological means (Stockman and Sharpe, 1998, 1999).[8] Genetic research has indicated that the *L* and *M* cones can be expressed in different ways and that some people have a hybrid *L/M* cone (Sharpe et al., 1999). Wavelengths of peak response for *L* cones vary in individuals from approximately 554 to 563 nm. There is less variation in *M* cone expression, with peak wavelengths varying from approximately 532 to 538 nm, and only one normal expression of the *S* cone. There are differences between the sexes in the prevalence of genetic expressions of cones. As mentioned in Chapter 1, it is known that up to 50% of females have the genetic potential for four cone types. So far no individual actually having four cone types has been identified. All these issues touch on the question of a statistically meaningful average observer and how different sets of data may be influenced by the genetic variability in its observer pool. Such observer pools are likely to also differ in the degree of ocular yellowing and distribution of macular pigment, both affecting in particular the *S* cone sensitivity function. Another issue in connection with cone functions is the question of their intrinsic smoothness. CIE color-matching functions are smooth, and cone functions calculated from these by linear transformation are also smooth. Directly measured individual cone sensitivity functions differ somewhat in shape and appear to have in some cases more or less pronounced "dents" (see Fig. 5-5 for an example). Some of these appear to be due to the degree of macular absorption in individuals, as already König surmised.

5.6 OPPONENT COLOR SPACE

Another version of a psychophysical space is the opponent color space. It attempts to provide a psychophysical model for Hering's opponent color hypothesis of color vision and to describe the zone following the cone absorption level in a multiple-zone model. With an opponent color space the conceptual leap is made that there is a connection between functions predicting which color stimuli produce identical color experiences (color-matching functions) and color appearance. Such a connection is not a priori a certainty. It will be shown that such a space can be a rough euclidean approximation of a uniform color space. However, aside from the issue of euclidean nature there are other discrepancies of detail arguing for a considerably more complex picture of reality than a simple opponent color space can provide.

As mentioned before, according to Hering's hypothesis there are three opponent processes that are responsible for our color experiences: a greenness-redness, a yellowness-blueness, and a whiteness-blackness process. Hering had developed physiological ideas of how such a process could be implemented. He engaged in an epic struggle with Helmholtz pitting the opponent color theory against what is known as the Young-Helmholtz trichromatic theory. After conceptual proposals by Donders and Kries, Helmholtz showed in 1896 how the two zone components could be expressed in mathematically equivalent form, but voiced doubts about the need for a second zone step. The psychologist and Hering supporter G. E. Müller developed a complex three-stage theory in 1896 to be championed in the 1960s by Judd. Schrödinger (1887–1961) showed in 1926 again, using the König and Dieterici spectral sensitivity data, how the trichromatic and the opponent color theory can be combined. From the sensitivity data he calculated spectral opponent color response functions (Fig. 5-11). In 1927 Luther in Germany and, independently, in 1929 Nyberg in Russia proposed what came to be known as the Luther-Nyberg color solid (see Section 2.22, Fig. 2-47). It is based on a (nonuni-

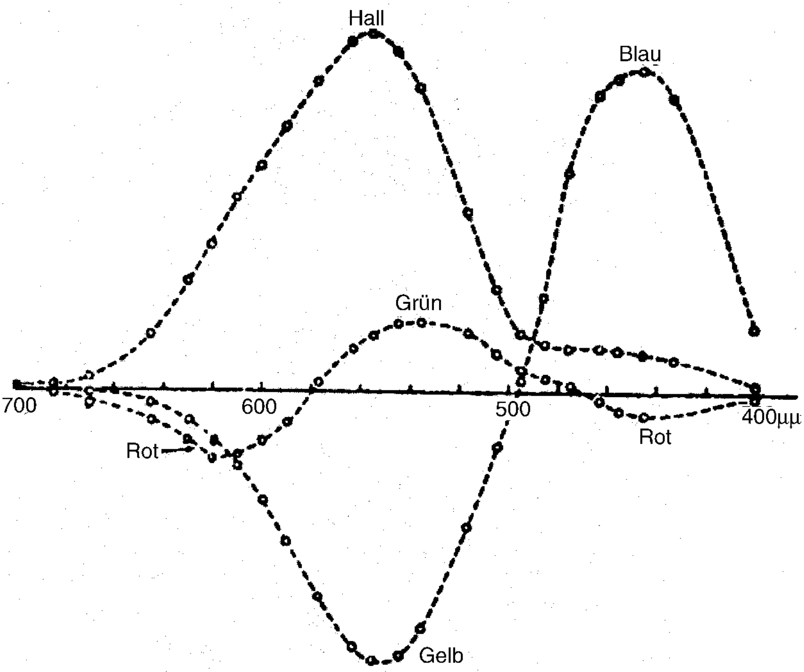

Fig. 5-11 Schrödinger's opponent color and brightness functions calculated from König and Dieterici's fundamental functions (Fig. 5-5). Note that the wavelength is reversed from the customary presentation, 1926. Hell *refers to the brightness function. The other two functions represent yellowness-blueness (*Gelb, Blau*) and greenness-redness (*Grün, Rot*).*

form) linear opponent color space with two color moments and a color weight.

In 1923 the American physicist E. Q. Adams proposed a zone theory in which the output from three cone types is modulated and subtracted to result in opponent color signals. Adams expanded on his proposal in 1942 where he showed, using CIE tristimulus data, that a theory such as he had proposed could offer a good model for the Munsell system (Fig. 5-12). There he specifically proposed that the output of the three cone types should be considered identical to the CIE tristimulus values and the modulation of the output identical to that of the Munsell value function. He constructed a "chromance" diagram by subtracting modulated Y values from modulated X and Z values. In 1944 Nickerson and K. Stultz offered a color space and color difference formula based on Adams's proposals which became influential in technology (more details in Chapter 6).

At the same time support for an opponent color step in color vision began to develop on a different front. Around 1940 physiologists began to measure signals by inserting microelectrodes into retinal cells. The Swedish physiologist R. Granit measured spectral response curves in ganglion cells in the retina

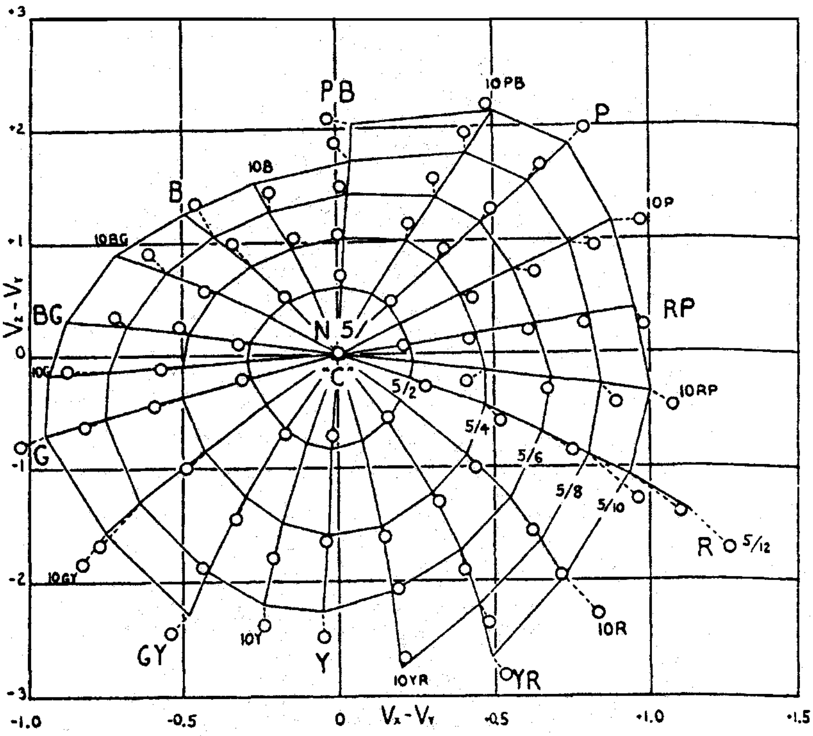

Fig. 5-12 Munsell colors at value 5 of the 1929 Book of Color *in Adams's 1942 opponent color chromatic diagram. Lines represent the smoothing proposal by Newhall (1940).*

of frogs. His student G. Svaetichin found chromatic opponent cells in fish retinas in 1956.[9] In the 1960s American researchers made measurements from cells of the lateral geniculate nucleus of monkeys and found various kinds of cells with opponent color sensitivity (De Valois, 1965; Wiesel and Hubel, 1966).

On the psychological front the American psychologists D. Jameson and L. Hurvich began in the mid-1950s to use hue cancellation as an experimental method and deduced opponent chromatic response functions from the results. In such experiments, for example, a "blue" stimulus is added to a yellowish color until the perceived yellowness is canceled or neutralized. They were able to show that their personal experimental functions could be reasonably approximated using CIE standard observer color-matching functions (Fig. 5-13) in a manner employed by Adams and by Judd in his interpretation of the Müller theory (Hurvich and Jameson, 1955; see Chapter 6 for more details). In 1964 Hurvich and Jameson also translated into English Hering's posthumously published *Grundzüge der Lehre vom Lichtsinn* (Outlines of a theory of the light sense; Hering 1905–1911).

If hue changes depend directly on the chromatic responses shown in Fig. 5-13, it is apparent that they vary considerably as a function of wavelength.

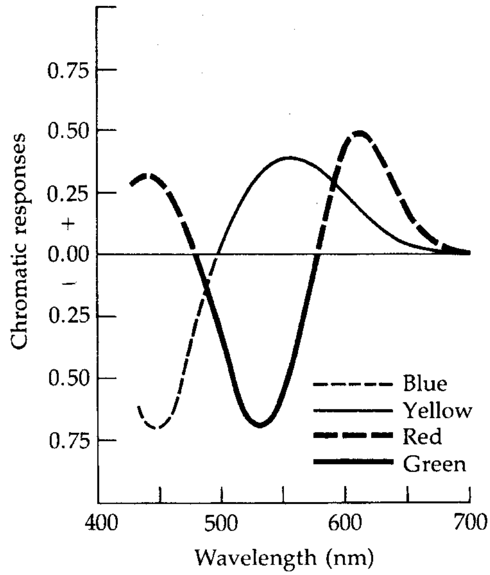

Fig. 5-13 Opponent color functions of the CIE 2° standard observer calculated by Jameson and Hurvich (1955).

Near 475 nm the greenness-redness signal is at zero, indicating the presumed location of unique blue for the CIE standard observer. The blueness signal on both sides of that point changes rapidly, as does the greenness-redness signal, indicating rapid hue changes per nanometer increment. This is indeed the case as wavelength discrimination experiments show. At approximately 500 nm is the neutral point of the blueness-yellowness signal (unique green, but see Section 5.10). Also here both signals change relatively rapidly indicating continued high sensitivity to wavelength changes. By this conjecture the complete change from unique blue to unique green takes place within about 25 nm. At about 578 nm the greenness-redness signal passes through its second neutral point resulting in the display of unique yellow. While the yellowness signal changes relatively slowly in this region, the greenness-redness signal changes rapidly on both sides of the neutral point indicating rapid changes toward greenish yellow and reddish yellow away from the neutral point. This is again supported by a high rate of change in wavelength discrimination experiments. In the regions of approximately 440, 530, and 610 nm and above the rate of change is significantly lower, resulting in reduced perceived wavelength discrimination. While a reasonable agreement exists between the form of opponent functions and experimental wavelength discrimination, there is no solid agreement between rate of change data behind these functions and experimental hue difference data, as will be shown below. There is in addition the matter of the wide ranges of wavelengths of the blue, green, and yellow unique hues as determined by individual observers.

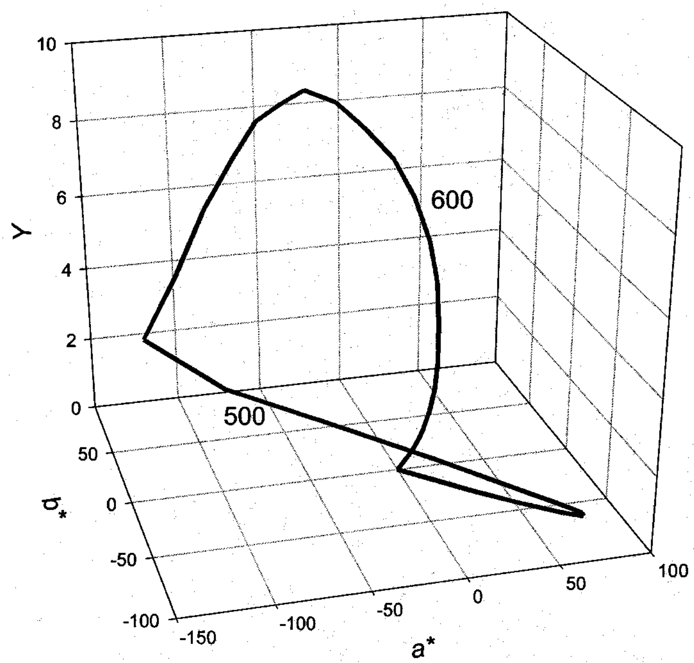

Fig. 5-14 Spectral trace in the Y, a*, b* *space, 10° observer. The wavelengths of 500 and 600 nm are identified.*

As Chapter 6 will show, most color vision models developed since the mid-nineteenth century are zone models and a simple version of a zone model color space was recommended in 1976 by the CIE as the CIELAB formula (CIE 1976; see Chapter 6). In this space, considered euclidean and with a cartesian chromatic diagram, colors are identified by chromatic coordinates a^* and b^* and a lightness coordinate L^*. There is also a polar coordinate interpretation where chroma is defined as the radial distance from the origin and hue by the hue angle, thus bringing the system conceptually in agreement with a psychological system of the Munsell type. Experimental work before and since then has indicated that there are surround effects requiring consideration and that CIELAB space is at best a rough approximation of a large difference uniform color space. Figure 5-14 illustrates the spectral colors in the Y, a^*, b^* space.

An important matter usually not considered in models until very recently is that of the interaction of cone responses between test field(s) and surround. Four types of opponent cells involving L and M cone output comparison have been identified (Wässle et al., 1994). All four involve comparison of output of cones from the center of the receptive field of the opponent cell with those from the surround. A similar situation applies to the opponent cells comparing output from S cones with that of L plus M cones. These mechanisms appear to be responsible for test field/surround effects, such as contrast and assimilation, possibly crispening, and others. Correct treatment of these effects is essential for a uniform color space model.

In recent years experimental data have begun to raise doubts about a simple subtractive opponent color system such as proposed by Hurvich and Jameson. New experimental work points to four independent chromatic dimensions and perhaps to a multiplicity of hue detection mechanisms (see below).

5.7 HOW ARE THE *L, M, S* AND *X, Y, Z* COLOR SPACES RELATED?

On a mathematical basis, according to Smith and Pokorny (1975), the two spaces are related by a set of linear transformation equations as follows:

$$\begin{aligned}\bar{l} &= (0.15516\bar{x} + 0.54308\bar{y} - 0.03287\bar{z}),\\ \bar{m} &= (-0.15516\bar{x} + 0.45692\bar{y} + 0.03287\bar{z}),\\ \bar{s} &= 1.00000\bar{z},\end{aligned} \tag{5-2}$$

where $\bar{x}$, $\bar{y}$, $\bar{z}$ are CIE color-matching functions, originally the Judd and Vos modified 2° observer functions. In this transformation $\bar{l} + \bar{m}$ add up to equal $\bar{y}$ (equivalent of the luminosity function). As mentioned, the same equations have been applied to both CIE 2° and 10° observer data. Slightly different equations have been proposed by other authors.

The inverse relationship is as follows:

$$\begin{aligned}\bar{x} &= 2.920\bar{l} - 3.445\bar{m} + 0.208\bar{s},\\ \bar{y} &= \bar{l} + \bar{m},\\ \bar{z} &= 1.000\bar{s}.\end{aligned} \tag{5-3}$$

Both the L, M, S space and the CIE tristimulus space are viewed as orthogonal. It is evident that the two spaces cannot be orthogonal in the same reference frame, but a case can be made that they represent different realities and each may be orthogonal by itself. There is, however, the unanswered basic question if the assumption of orthogonality is valid for either space.

Based on their recordings from parvocellular neurons in the LGN of macaques A. M. Derrington, J. Krauskopf, and P. Lennie (DKL) in 1984 proposed a color space (nonuniform) representative of the signal output of different cell types (see Chapter 6). They located opponent cells that report in terms of L–M or the reverse and in terms of $(L + M) - S$ or the reverse. When they plotted the corresponding axes in the CIE chromaticity diagram, they

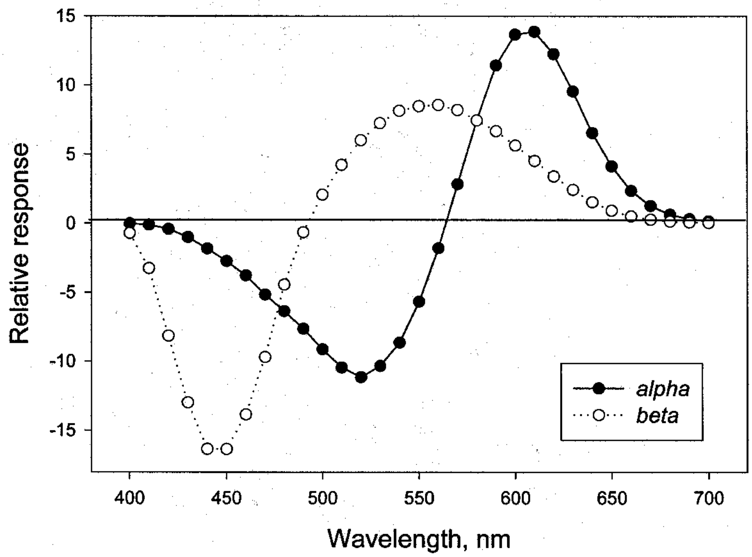

Fig. 5-15 *Balanced opponent functions* α *and* β *calculated from CIE color-matching functions. These functions are supported by recordings in the lateral geniculate nuclei of macaques.*

found that the axes do not represent average unique hues. This can be demonstrated by calculating balanced (equal areas under the four curve segments) opponent functions based on cell input reported by DKL:

$$\alpha = 5.673(\bar{l} - 1.98842\overline{m}),$$

$$\beta = \bar{l} + \overline{m} - \bar{s}. \qquad (5\text{-}4)$$

The resulting two functions α and β are illustrated in Fig. 5-15. The β function is identical to $\bar{y} - \bar{z}$ and therefore has the same form as the Hurvich-Jameson yellowness-blueness opponent color function. The α function is different from the greenness-redness function because it lacks positive values in the short-wave range, considered indicative of the reappearance of redness at short wavelengths. The α function can therefore not be a greenness-redness expressing function (in the sense that green and red are represented by the unique hues), and the DKL space is not a space in alignment with the psychological space (e.g., see Lee 1999). It is important to be aware that the measured functions apply to macaques and that they are from the lateral geniculate nucleus: details in the human visual system may differ and additional transformations in the cortex are known to take place. Recently it has been found that in the primary visual cortex at the back of the brain, most cells receive input from all the different opponent cell types in the lateral geniculate nuclei (De Valois et al., 1997). On that basis a mechanism for S cone input into the greenness-redness opponent system appears to exist. Equation (5-3) indicates that the definition in terms of cone sensitivities of the $\bar{x}$ color-matching function includes S cone input. It is interesting to note that axis rotation required to provide a result equal to a balanced linear opponent chromatic diagram (based on CIE 1931 tristimulus values) can be achieved by the following equation (Kuehni, 2000a):

$$a = f(\alpha - 0.56\beta),$$
$$b = \beta, \tag{5-5}$$

where a and b are linear opponent color functions, f is a normalization constant with the value 0.989. The factor for β is close to 0.5. It will be shown later that this factor varies considerably for optimal modeling of various sets of visual data. The subtractive step is reminiscent of the second transformation in the G. E. Müller theory (see Chapter 6).

5.8 EXPRESSING PSYCHOLOGICAL SCALES IN PSYCHOPHYSICAL SPACES

Historically the X, Y, Z space (and its predecessors) has been used as a basis to express psychological scales in a psychophysical space. The fact that one of its dimensions is aligned with brightness and the other two therefore with chromaticness seemed a good reason to expect reasonable agreement if the functions expressing $\bar{x}$ and $\bar{z}$ are correctly selected. They were selected so that they had only positive values (the experimentally determined color-matching functions have stretches of negative values). There is, of course, no a priori reason why a color-matching space should be in agreement with, or easily modifiable into, a color appearance space, and as will be seen later, there are important discrepancies.

Lightness Scales Expressed in Terms of Luminous Reflectance

Seemingly the first lightness scale that through reconstruction can be expressed in terms of luminous reflectance is Glisson's gray scale. Figure 2-9 shows the progression of CIELAB L^* lightness values (derived from luminous reflectance) as a function of the grades of the reconstructed scale.

Insufficient quantitative data are available to deduce lightness scales in terms of luminous reflectance for the Mayer, Lambert, Runge, or Chevreul systems. As discussed in Chapter 4, Plateau in 1872 concluded that ratios were applicable, based in part on the apparent preservation of the contrasts in etchings in different lights, and that as a result the relationship would be linearized by a power function. Delboeuf, in an elaborate experiment of halving the visual distances between two grays independently, concluded that a logarithmic formula provided the best fit to his data. Plateau received confirmation in 1887 from experiments by Breton who found a square root power relationship to be applicable. But, as we have seen, the Weber-Fechner law was the generally accepted paradigm for some 100 years.

Ebbinghaus's eight-grade gray scale had the following relative luminance values: 100, 44.4, 21.1, 10.3, 5.8, 4.4, 2.0, 1.0 (Ebbinghaus, 1902). This series is linearized optimally with a 0.02 power. The resulting scale is not quite uniform and does not quite represent a Fechner type log scale. Ridgeway's gray scale incorporated into his 1912 color atlas had, by definition, a logarithmic step relationship as did Ostwald's gray scale of 1917.

Priest, Gibson, and McNicholas investigated the relationship between Munsell gray scale steps in the 1915 atlas and luminous reflectance values (Priest et al., 1920). They found that the relationship is best described by a square root power function. Taking the 1919 National Bureau of Standards measurements of the neutral samples of the 1915 *Atlas of the Munsell Color System*, as well as the 1926 diffuse measurements of the same samples (Gibson and Nickerson, 1940; see Fig. 7-5), the optimal linearization (minimal coefficient of variation) is found to be based on an 0.42 power. However, the function plotting reflectance against value step is not smooth. It is composed of two segments intersecting between value steps 4 and 5. The lower segment is optimally linearized with a power of 0.25, the upper with one of 0.48. This appears to be an indication of lightness crispening (see below), as one would expect, and points to a surround with a luminous reflectance of approximately $Y = 20$.

In 1922 Adams and his colleague P. W. Cobb studied the effect of surround on perceived brightness and developed a formula (in the version reformulated by Judd) that recognizes the effect of the luminous reflectance of the surround on the results:

$$V = R\left(\frac{Rb + 100}{R + Rb}\right), \tag{5-6}$$

where V is lightness on a scale of 0 to 100; R is luminous reflectance in percentage of the sample, and Rb luminous reflectance of the surround.

A. E. O. Munsell, L. L. Sloan, and I. H. Godlove continued work on the Munsell value scale and described in 1933 results based on a combination of "just noticeable differences" as well as scale-halving experiments. Their scale is similar to the Munsell *Atlas* and 1929 *Book of Color* scales. It is optimally linearized by a 0.34 power but consists of two branches with the lower linearized with an 0.50 power and the upper with an 0.20 power. Godlove (1933) compared it to the Adams-Cobb data and fit the following empirical formula to the Munsell data, a formula described as "intermediate between those of the Fechner-Delboeuf [logarithmic] and Plateau-Munsell [power function] equations":

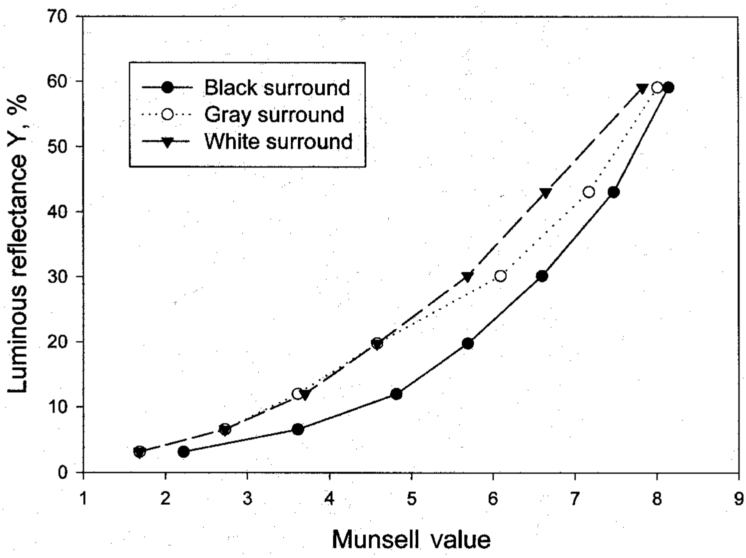

Fig. 5-16 *Results of visual determination of Munsell value of Munsell value scale samples viewed against a white, mid-level gray, and black background. After Newhall et al. (1943).*

$$V^2 = mR + nR^2, \tag{5-7}$$

where V is the Munsell value, m and n are constants, and R is luminous reflectance.

In the studies leading up to the Munsell Renotations the value scale was evaluated against a black, a middle gray, and a white background. The results differed significantly (Fig. 5-16). The optimal power for the results against the black background is cube root against the white background square root. As expected, the results against the gray background show two branches, indicating the lightness crispening effect. The committee working on the Renotations decided to average the results in some fashion, and the final, smoothed curve was mathematically described with a quintic function developed by Judd:

$$Y = 1.2219V - 0.23111V^2 + 0.2395V^3 - 0.021009V^4 + 0.0008404V^5, \tag{5-8}$$

where V is the Munsell value (Newhall et al., 1943). Glasser and colleagues showed in 1958 that this function is closely matched by a cube root function.

Figure 5-17 illustrates lightness scales based on logarithmic (Weber-Fechner) and cube root functions as well as an Adams-Cobb function with a surround luminous reflectance of 35%. It indicates that power functions have less modulating effect than the Weber-Fechner function and that an Adams-Cobb function with a surround luminous reflectance of 35% is roughly comparable to a cube root function.

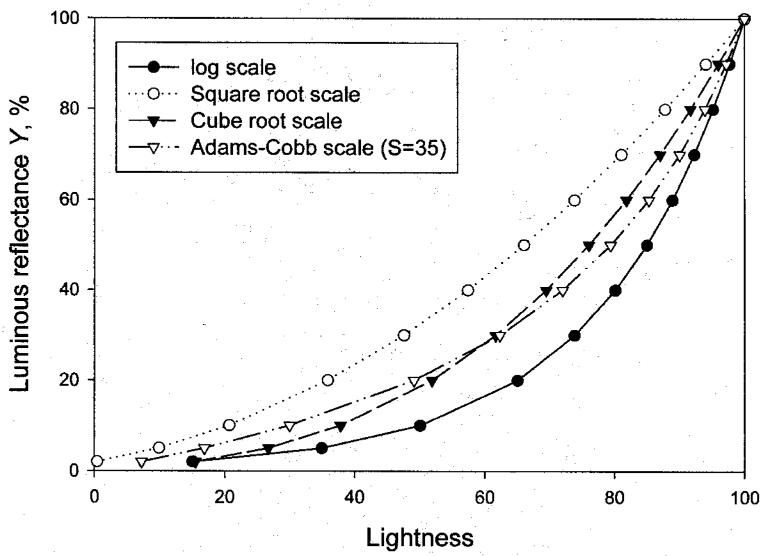

Fig. 5-17 *Luminous reflectance* Y *plotted as a function of lightness according to logarithmic, cube root, square root scaling, as well as the Adams-Cobb lightness scale for a surround of luminous reflectance* Y = *35.*

As mentioned before, in 1933 Schönfelder investigated the effect of surround variation on the precision of color matching. He found that precision is highest if the surround color is intermediate to the standard and the match fields. This applies not only to lightness but also to chromatic differences, and we will encounter what Judd called Schönfelder's law again later. Schönfelder's results were in 1938 confirmed by K. J. W. Craik for achromatic colors and in 1952 by W. R. J. Brown for chromatic colors.

In 1964 Jameson and Hurvich investigated brightness scaling, and they introduced a subtractive constant that was explained as correcting for simultaneous contrast effects:

$$V = bx^p - V_0, \tag{5-9}$$

where V_0 is the subtractive constant. In the same year T. Kaneko investigated lightness of achromatic colors in form of color chips as a function of surround lightness. He found, in agreement with Schönfelder, that differences between gray scale grades were perceived largest if the surround lightness falls between the lightnesses of the samples being compared. When the sample lightness is higher or lower than the surround lightness, a larger increment in luminous reflectance is required for a perceived difference of the same magnitude. He also found that the effect depends to some degree on the size of the test fields compared to the surround. With larger test fields the crispening effect was reduced; that is to say, the magnitude of the lightness crispening effect appears to be a function of the test field size, the effect being strongest for small fields. Kaneko constructed gray scales valid for a given surround lightness (Kaneko, 1964). Adams and Cobb had also noticed the described effect, but they believed it to be limited to threshold level differences. Kaneko found it also to apply for much larger lightness differences. It was H. Takasaki who gave the effect the name *lightness crispening*. Kaneko developed a formula consisting of two simultaneous equations to model his results.

Takasaki conducted in 1966 extensive investigations of the effect of surround lightness on perceived lightness of test fields. He confirmed contrast as well as crispening effects. His complex formula was shown by C. C. Semmelroth in 1970 to be reducible to

$$V = R^m + k\,|R - Rb|^n, \tag{5-10}$$

where V is the perceived lightness, R is the luminous reflectance of the sample, and Rb that of the surround, m and n are exponents, and k is a constant. The equation applies for situations where R is equal or larger than Rb. In the opposite case the +sign is replaced with a –sign. For various experimental data sets $m = 0.4$, $n = 0.2$, and $k = 0.65$ have been found to provide good fits.

In 1975 Judd and Wyszecki constructed an instructive graph of the effect of lightness crispening on perception of value differences in the Munsell value scale, as described by equation (5-10) (Fig. 5-18). Explanatory example: the lightness difference between two Munsell samples $V_s = 2$ and $V_s = 3$ when viewed against a surround of value 7 is approximately 1.65 – 0.7 = 0.95 units. When viewing the samples against a surround of value 3, the corresponding difference is 3.0 – 1.2 = 1.8 units, or nearly twice perceived size. More recently lightness crispening has been measured and described by Whittle (1992).

The effect of surround on perceived lightness is demonstrated with the results of an experiment with 22 observers. On a video screen in a dark room, successively against black, white, and gray ($L^* = 45$) backgrounds, the observers adjusted the luminances of eight separated fields so that all steps looked perceptually equidistant. Grade 4 had against the gray surround the same lightness as the surround itself. For the other two surrounds the luminance of grade 4 was adjusted so that it had the same lightness appearance as in the gray surround. The average result in terms of CIELAB L^* lightness is illustrated in Fig. 5-19. The inflection of the crispening effect at the luminance of the surround is clearly visible. It is apparent that the L^* scale is not applicable to any surround lightness. Similar results were obtained with chromatic fields. Lightness crispening has been investigated by Lübbe (1999) and identified in small color difference data by Kuehni (2001b). In agglomerations of data from different experiments made with different surrounds, lightness crispening is obscured. In several individual data sets it is clearly detectable. It was, for a single surround, measured by Chou and colleagues (2001). For small color difference formulas, several different weightings of the CIELAB lightness scale have been proposed since the mid-1970s, as shown in Chapter

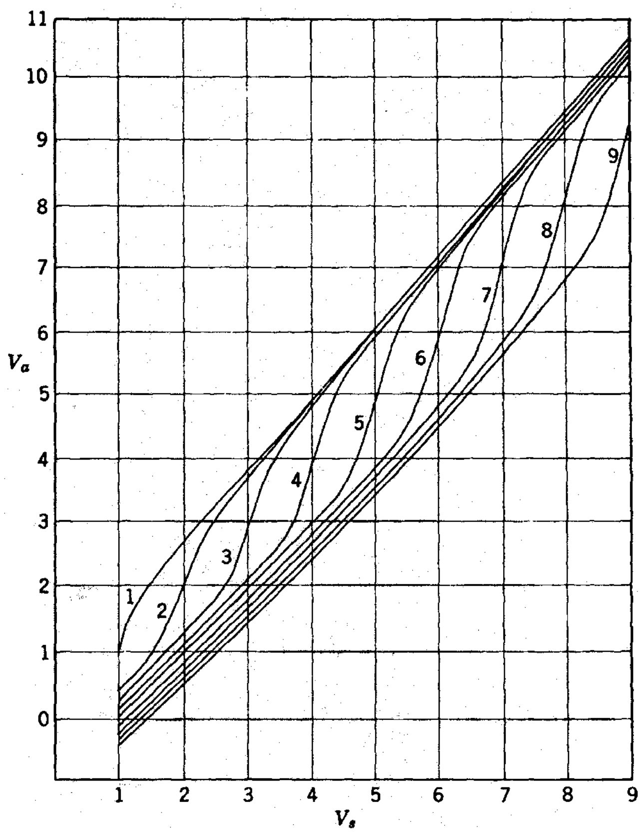

Fig. 5-18 Graph developed by Judd and Wyszecki from a table by Semmelroth (1971) depicting the effect of lightness crispening on perception of value differences in the Munsell Renotation value scale. From Judd and Wyszecki (1975). V_s *is the nominal Munsell value,* V_a *is the background adjusted Munsell value.*

6. Surprisingly few formulas consider surround lightness explicitly (Lübbe, 1999; Kuehni, 2001d) where it has improved the correlation between visual and calculated color differences (Kuehni, 2001b).

Some researchers recently began to use the term *masking* for the crispening effect. Stimuli producing a strong contrast to the surround are said to produce masking and require larger increments for detection than those at lower contrast (Eskew et al., 1999). Aside from luminance or luminous reflectance masking there is also chromatic masking, discussed below. In such experiments masking is reported to produce threshold elevations that can be approximated with a power law with exponents from 0.5 to 1.0. Such masking experiments typically involve only brief exposures to the stimuli and other controls that minimize adaptation and other effects. The results therefore cannot be expected to closely match those of experiments involving object color samples with unrestricted exposure.

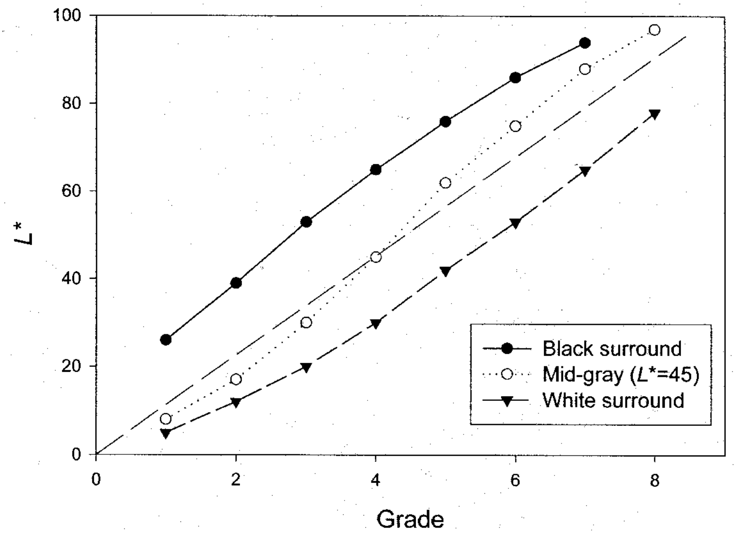

***Fig. 5-19** Average settings expressed in* L* *values of 22 observers of the luminance of eight grades of achromatic colors against three surrounds so that steps between the grades appear equidistant.*

The CIELAB formula of 1976 enshrined the Munsell value function in a cube root format to connect luminous reflectance with perceived lightness. The OSA-UCS formula, as will be seen in Chapter 6, used the Munsell value function as a basis but with adjustments that consider lightness crispening and the Helmholtz-Kohlrausch effect, the latter to be discussed presently.

The gray scale of the Swedish Natural Color System was obtained by assessment of a series of gray samples and their percentages of innate ideas of the observers of ideal white and black. The Adams-Cobb formula with a surround luminous reflectance of $Y = 56$ was found to fit the experimental data well (Hård et al., 1996).

It should be mentioned here that a parallel process, chromatic crispening, exists. According to this effect the smallest stimulus increment for a criterion chromatic difference is required if the chromaticities of the two test samples straddles the chromaticity of the surround (see Chapter 8).

The work of Gilchrist and colleagues (1999) and of Purves and colleagues (see Purves and Lotto, 2002) in regard to the complex relationship between luminance and brightness/lightness was mentioned in Chapter 4. In the Munsell and similar scales the luminance ratio between white and black is close to 100:1, and according to the Gilchrist group's findings, luminance scale compression takes place under these conditions, just as it is found in most data of that kind.

Heterochromatic Brightness/Lightness

When luminance measurements of colored stimuli began to be made routinely, it became evident that the perceptual brightness of constant luminance stimuli was higher than that of an achromatic color with equal luminance. The chromatic stimulus appears to glow compared to the achromatic one. Expressed conversely for object colors, highly chromatic objects having the same perceived lightness as a gray object have a reduced luminous reflectance. Colors of different hues have a different degree of color glow. The result of these findings is that modulated luminous reflectance is not a good predictor of perceived lightness of chromatic stimuli. Helmholtz remarked upon this effect (1860), and it was investigated by V. A. Kohlrausch in 1935, among others, and has become known as the Helmholtz-Kohlrausch effect (HKE). The implication is that an effect in addition to that described by the luminous efficiency function contributes to perceived lightness. The effect is often quantitatively described by the *B/L* ratio, where *B* is the luminance or luminous reflectance of the reference stimulus, usually achromatic, and *L* that of the test stimulus (e.g., see Wyszecki and Stiles, 1982). It is of considerable importance for the photometry of colored lights but also for the definition of a uniform color space.

Interestingly and surprisingly, in the Munsell 1915 atlas the majority of the most highly chromatic chips have a luminous reflectance that is higher than that of the gray with the same visually judged Munsell value. In the Renotations value was defined in terms of luminous reflectance and all colors of a given value have the same luminous reflectance value. The Re-renotations incorporate the HKE and colors of the same value differ in luminous reflectance. The same applies to the OSA-UCS system where the HKE also has been incorporated. In the 1950s and 1960s important experimental studies of the effect were made. Among these is the Wyszecki-Sanders work of 1964 with 20 observers using a visual colorimeter, and an experiment by the Committee on Uniform Color Scales of the Optical Society reported by Wyszecki (1967). The latter experiment was to confirm or modify the findings of Wyszecki and Sanders and involved a total of 76 observers viewing chromatic tiles against a series of achromatic tiles. In analyzing their experimental data in the CIE chromaticity diagram, Wyszecki and Sanders found the resulting iso-brightness to luminance (B/L) contours to vary in irregular fashion (Fig. 5-20). The effect is weakest for yellow and strongest for purple colors. The tile experiment of the OSA committee produced similar contours. Wyszecki and Sanders fitted an analytical formula in the CIE chromaticity diagram that predicts the B/L ratio from the chromaticity coordinates of the color. By that formula the correlation coefficient between experimental and calculated results for their data was found to be 0.92. In 1991 Fairchild and Pirotta found that the magnitude of the implicit additivity failure depends on the luminance level of the surround.

More recently experimental results and interpretations have been reported by Nayatani and co-workers. Their investigations revealed that the magnitude

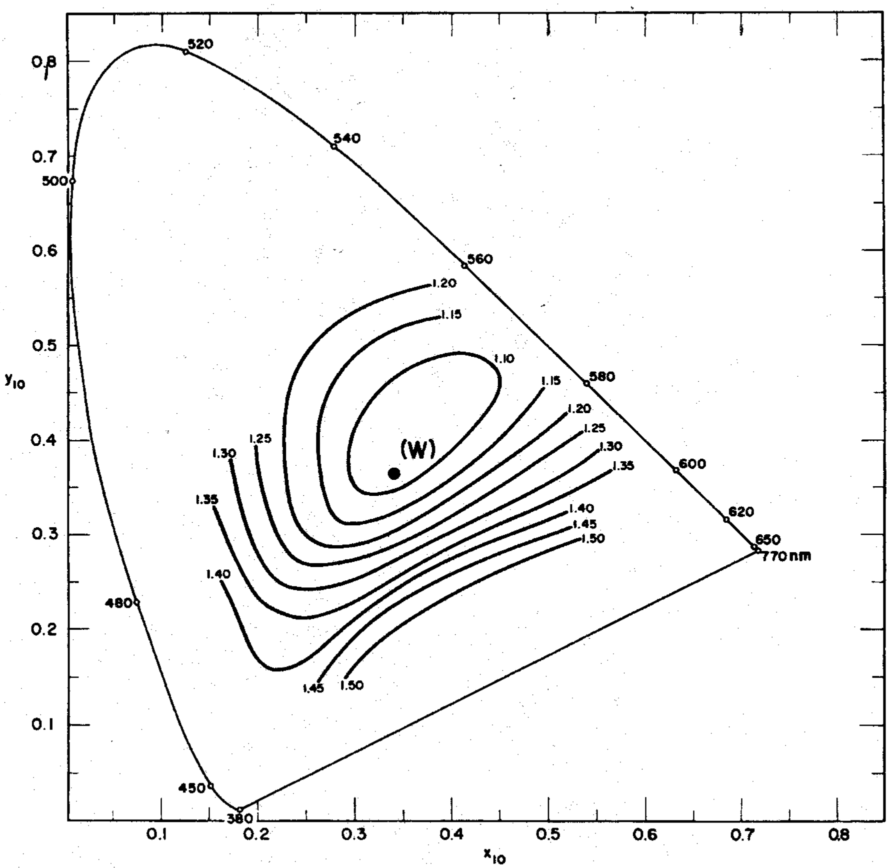

Fig. 5-20 Iso-brightness to luminance (B/L) contours in the CIE chromaticity diagram as determined by Wyszecki and Sanders (1964), the result of the Helmholtz-Kohlrausch effect. W identifies the white point. The effect is weakest for yellow and strongest for purple colors.

of HKE depends on the experimental situation (Nayatani et al., 1994a,b). In 1997 Nayatani offered a formula that predicted the Wyszecki-Sanders data with a correlation coefficient of 0.89. It involves the determination of a spectral function that resembles the chromatic threshold function and Evans's function of zero grayness. In 1998 Nayatani proposed a formula where the spectral chromatic strength function was calculated in three sections from Hurvich and Jameson type opponent color functions: below 500 nm it is calculated as the square root of the sum of the squares of the *B* and *R* lobes of the opponent function, between 500 and 580 nm from the *G* lobe, and above 580 nm from the *R* lobe.

In 2000 Kuehni reported analysis results for the Wyszecki-Sanders and the Wyszecki 1967 data (Kuehni, 2000d), using a linear opponent color system. For the former a correlation coefficient of 0.95 was obtained for predicting the B/L ratios using a formula where the perceived brightness was calculated by adding, by quadrant, a portion of the corresponding linear *a* and/or *b* opponent color value as follows:

$$Y_A = Y_C + 0.23\,|a| \qquad \text{if } b \text{ is positive,}$$

$$Y_A = Y_C + 0.20\,(a^2 + b^2)^{0.5} \qquad \text{if } a \text{ is negative and } b \text{ is negative,}$$

$$Y_A = Y_C + 0.30\,|a| \qquad \text{if } a \text{ is positive and } b \text{ is negative.} \qquad (5\text{-}11)$$

Y_C is the luminosity value of the sample. For all yellowish colors a portion of the positive, respectively negative, lobes of the *a* function is added; for bluish-greenish colors a portion of the geometrically summed two opponent functions is added; for reddish-bluish colors a slightly larger portion of positive lobes of the *a* function is added. To optimally predict the tile data of 1967, larger contributions of opponent color signals in a somewhat different vector composition was required:

$$Y_A = Y_C + 0.43\,(a^2 + b^2)^{0.5} \qquad \text{if } b \text{ is negative,}$$

$$\mathrm{Y}_A = Y_C + 0.28\,(a^2 + b^2)^{0.5} \qquad \text{if } b \text{ is positive.} \qquad (5\text{-}12)$$

This formula predicts the committee data with a correlation coefficient of 0.88. The corresponding effective luminosity function is compared with the CIE luminosity function in Fig. 5-21. When inverted for comparison to the chro-

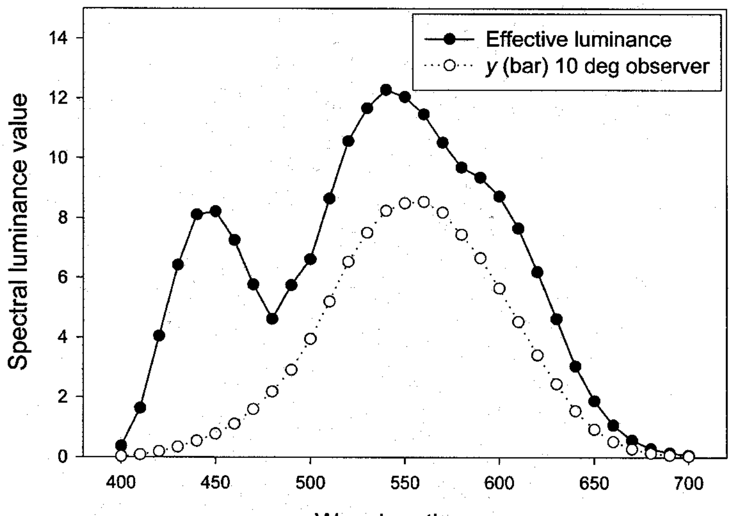

Fig. 5-21 Spectral effective luminous reflectance for the OSA-UCS Helmholtz-Kohlrausch effect data, as modeled by the author (2000d), compared to the 10° observer ȳ function.

matic strength function, its long wave trough is found to be shifted toward lower wavelengths. The reason for the difference of the apparent contribution of opponent color signals to perceived brightness in the two experiments is not clear. However, these results provide implicit support for the idea that the opponent color system makes a contribution to perceived brightness/lightness of chromatic colors.

Psychophysical Scaling of Chroma

In their representation in the psychological diagram, Munsell colors of equal chroma are located on concentric circles. In the CIE chromaticity diagram, they lie on ovoid-shaped contours with the major axis in the direction of approximately dominant wavelength 575 nm, meaning in the yellow-blue direction (see Fig. 7-13). There have been many efforts to find models for chroma spacing that are in better agreement with the psychological spacing. A significant improvement is obtained when plotting them in an opponent color diagram. In an Adams type opponent color space, such as the CIELAB space, the contour (see Fig. 5-24a) is improved but continues to be unbalanced in the yellow-blue direction. In 1946 Saunderson and Milner offered an analytical solution modifying the Adams chromatic diagram by changing the scaling of the yellow-blue axis. In an analysis in 1949 by Burnham this adjustment was found to represent the best fit to the Munsell system. However, he offered no explanation of its need.

What Munsell called chroma has been identified in more recent literature as contrast (at constant luminous reflectance). Contrast has been scaled in terms of contrast threshold steps (e.g, see Eskew et al., 1999). But systematic scaling comparable to the psychological scaling discussed below is lacking.

Constant Chroma Circles

As indicated in Chapter 4, since the establishment of the Munsell Renotations only a few systematic determinations of constant chroma around a hue circle have been made. The Re-renotations, a modification of the Renotations for certain findings obtained in connection with the development of the OSA Uniform Color Scales, have chroma scales significantly different from those of the Reenotations, and the constant chroma contour has a different shape and angle of the major axis (see Chapter 7). In 1957–8 Nickerson, Judd, and Nimeroff, also in preparation for OSA-UCS, developed a circle of constant chroma samples at value 6 (Judd, 1965), as mentioned in the previous chapter. It again differs noticeably in shape and angle of the major axis of the ovoid from those of the Munsell system. Chroma scales are also implicit in the OSA-UCS system, assuming that chroma can be calculated as the euclidean sum of g and j. The formulas for g and j represent the committee's euclidean fit to average judgments by 49 to 76 observers of color differences between 128 nearest neighbor pairs of a triangular grid of colors. The resulting ovoid shape is again significantly different. There is no objective argument for any one of the four scales being most accurate. None of the four psychological chroma scales is well represented by the CIELAB formula. The four "constant" chroma contours are illustrated in Fig. 5-22 a and b in the balanced cone sensitivity opponent diagram [5.67 (L – 1.9884M) vs. $L + M - S$], indicating the significant differences. The four scales and the scale implicit in CIELAB have been modeled with cone sensitivity based opponent color functions. The results are listed in Table 5-1, which also contains the coefficient of variation in percent (COV) of the calculated chroma values. The equations indicate that the contours differ primarily in the contribution of the b function to the a function, namely the implied degree of reappearance of red at the short wave end of the spectrum, and weightings that differ by quadrant. UCS and CIELAB resemble each other in their weightings of the b scale, induced by the common cube root power modulation. The fact that the constants for CIELAB are significantly different and the coefficient of variation is much higher indicates that it is not a good representation for any of the four visually based contours.

As we saw in Chapter 4, indirect chroma scales derived from judgments of the content of unique hues in Munsell chips (Indow's work) have not resulted in meaningful scales. As a result we have comparatively poor agreement between various direct and indirect chroma scaling results and no good fit by the formula currently recommended by the CIE. This applies also to the CIE94 formula whose only difference in regard to chroma is that it has an adjustment for chromatic crispening and the basic ovoid shape of the unit difference contour. As will be discussed later, chromatic crispening is likely limited to comparatively small differences.

TABLE 5-1 Cone sensitivity based opponent color models fitted to "uniform" chroma circles

Data	a	b
Munsell Renotations	$a = 0.926\,[5.67(L - 1.9884M) - 0.50b]$ COV = 5.6%	$b+ = L + M - S$ $b- = 0.90(L + M - S)$
Munsell Re-renotations	$a+ = 0.646\,[5.67(L - 1.9884M) - 0.25b]$ $a- = 0.807\,[5.67(L - 1.9884M) - 0.25b]$ COV = 1.6%	$b+ = 1.09(L + M - S)$ $b- = L + M - S$
Nickerson et al.	$a+ = 0.953\,[5.67(L - 1.9884M) - 0.20b]$ $a- = 1.030\,[5.67(L - 1.9884M) - 0.20b]$ COV = 4.4%	$b = L + M - S$
OSA-UCS	$a+ = 0.908\,[5.67(L - 1.9884M) - 0.40b]$ $a- = 0.976\,[5.67(L - 1.9884M) - 0.40\mathrm{b}]$ COV = 4.7%	$b+ = 1.33(L + M - S)$ $b- = 0.75(L + M - S)$
CIELAB	$a+ = 0.676\,[5.67(L - 1.9884M) - 0.25b]$ $a- = 5.67(L - 1.9884M) - 0.25b$ COV = 8.8%	$b+ = 1.33(L + M - S)$ $b- = 0.65(L + M - S)$

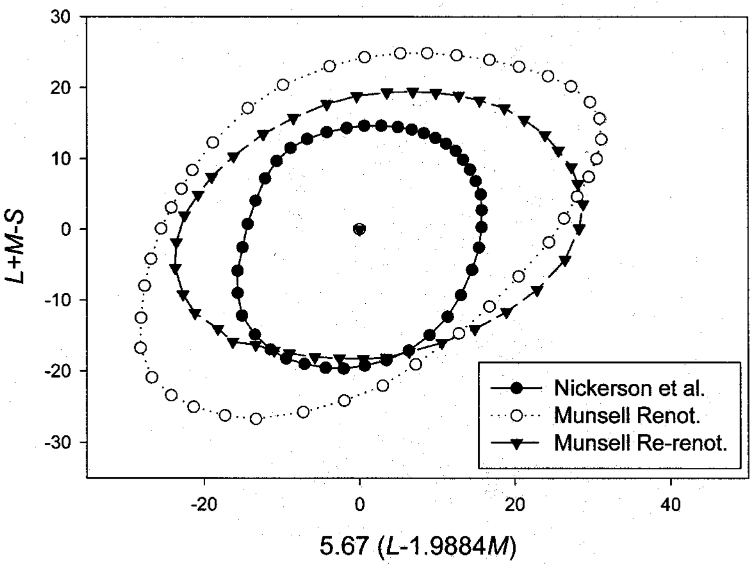

Fig. 5-22a *Constant chroma contours determined in three different experiments in the balanced cone opponent diagram: Nickerson et al. (Judd, 1965), Munsell Renotations, and Munsell Re-renotations.*

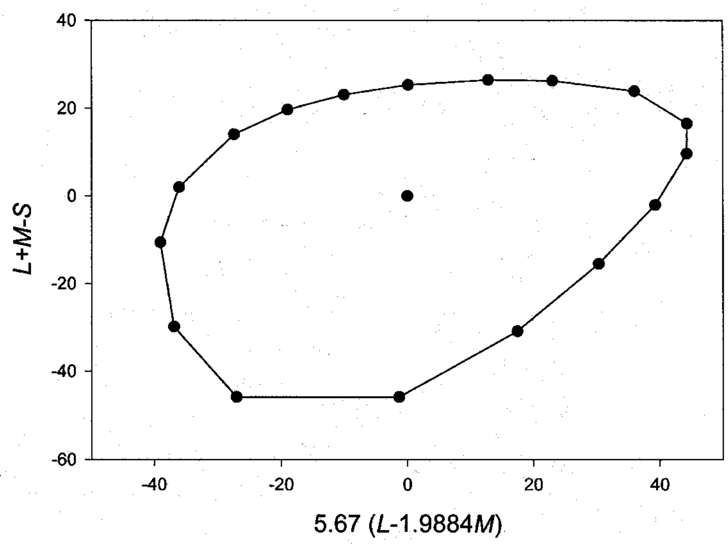

Fig. 5-22b *Constant chroma contour implicit in the OSA-UCS system, in the balanced cone opponent diagram.*

TABLE 5-2 Optimal powers to linearize relations between psychophysical and psychological chroma scale values at or near the axes

Data	*a*+	*a*–	*b*+	*b*–
Munsell Renotations	0.15	0.19	0.42	0.06
COV, %	6.5	4.7	13.1	4.3
Munsell Re-renotations	0.45	0.75	0.70	0.38
COV, %	2.2	6.4	11.1	6.1
OSA-UCS	0.70	0.84	0.58	0.33
COV, %	0.3	0.4	1.8	0.2

Relation of Constant Chroma Circles at Different Chroma Levels

Psychophysical constant chroma contours may be established at different levels of chroma. If they are established so that they are perceptually equally spaced, the question arises as to how the psychophysical values are related for different chroma levels at constant hue and constant luminous reflectance. As was mentioned earlier, an important proposal was made in 1942 by E. Q. Adams. He suggested that the CIE tristimulus values should be scaled in accordance with the Munsell value function. Later the Munsell function was replaced by a power function, and the current CIE recommendation in the CIELAB color space and difference formula employs a cube root function. The same power function was also used in the development of the OSA-UCS formula. However, analysis of the Munsell Renotations, Re-renotations, and OSA-UCS indicates that the effective optimal power is usually different from cube root and differs by semi axis (Kuehni 2000b). Table 5-2 lists the optimal powers to be applied to tristimulus values for the three data sets.

The results, while indicating considerable differences in the optimal powers for the three sets of data, show a degree of relatedness of the Re-renotations to OSA-UCS, as indeed attempted by Judd. The low COV values of OSA-UCS are due to the systematic tiling of the *j, g* diagram once a formula had been fitted to the basic experimental data (see Chapter 7 for details). Given the uncertainty in the related uniform chroma circles and the variability in powers, it is also here not apparent which system, if any, is accurate.

Interrelation between Chroma, Hue, and Lightness Differences

The uncertain relationship between combined hue and chroma differences, on the one hand, and the chromatic and lightness differences, on the other, was discussed in the previous chapter. Color space and difference formulas generally assume euclidean summation. However, the Nickerson and Indow formulas (in Chapter 4) argue for linear addition. The fact that the CIELAB formula is not a good model for any of the experimental chroma and hue scales brings up the issue of conflated component differences calculated from that formula. Figure 5-23 illustrates two colors A and B located on a quarter circle

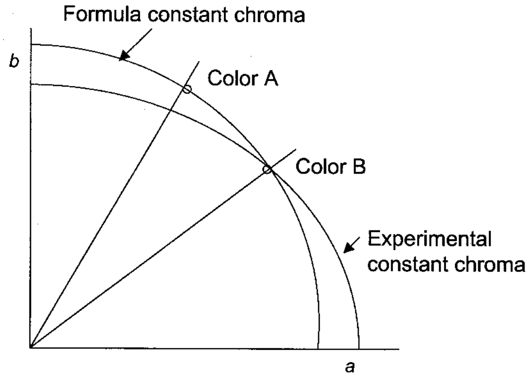

Fig. 5-23 Schematic depiction of quadrant 1 of an opponent color diagram with constant chroma contours calculated from a formula and determined by an experiment.

in the a^*, b^* diagram and therefore representing a hue difference only. However, if the true constant chroma contour is represented as indicated in the figure, as might be possible based on the equations in Table 5-1, the difference between A and B is in reality a mixed difference consisting in part of a hue and in part of a chroma difference. The degree of this conflation also depends on the size of the unit hue difference along the constant chroma contour. We cannot expect to have a reliable hue scale unless we have a reliable chroma scale. This fact was recognized by Judd and his co-workers in their work in the late 1950s. A reliable chroma scale, at the same time, depends on a reliable determination of constant perceptual lightness planes.

Additional conflation can result from the different effective powers shown in Table 5-2. It is evident that we cannot rely on hue and chroma differences calculated from CIELAB to have an accurate relationship to the perceptual data of the three data sets. Conflation between lightness and chromatic differences in CIELAB related formulas is also a fact because chromatic and achromatic colors of equal perceived lightness are not on the same value plane due to the Helmholtz-Kohlrausch effect. A further difficulty is that, as will be seen Chapter 7, a euclidean global color space uniform in terms of hue differences and in terms of chroma differences cannot be uniform in both. At the same time it remains to be seen how uniformly and repeatably observers judge component differences in complex small suprathreshold differences. Such experiments are overdue.

The shape of unit difference contours in a uniform color space is by definition spherical. But that space is not automatically euclidean, as will be shown in Chapters 7 and 8. In a euclidean chromatic diagram the unit contour has trapezoid, or perhaps egg-shaped, rather than circular form. In the recent color vision literature various kinds of contours have provided best fits to threshold data in cone contrast space: superellipses, rhombi, and rectangles. For the same experimental conditions often different contours provided the best fit to data of different observers (e.g., see Sankeralli and Mullen, 1996).

Psychophysical Hue Scaling

The intent of the psychological Munsell chromatic diagram is to be uniform in terms of hue differences. On the other hand, as seen before, in the psychological Munsell diagram unique hues do not fall on diagram axes, and these axes are therefore not psychologically meaningful. It is useful to determine the location of unique hues in the CIELAB diagram to learn if there is a better agreement with the system axes than in the Munsell diagram. This is illustrated in Fig. 5-24a and b, and we find that for the 2° observer two average unique hues fall close to the system axes.

There are significant discrepancies for the green and red unique hues. The CIELAB formula and some formulas recommended later for small color differences imply equal hue angle differences for equal hue differences. CIE94 (see Chapter 6) weights only the magnitude of the calculated hue difference. Various functions adjusting hue differences as a function of hue angle have been introduced into other formulas, including the new CIEDE2000 (see next

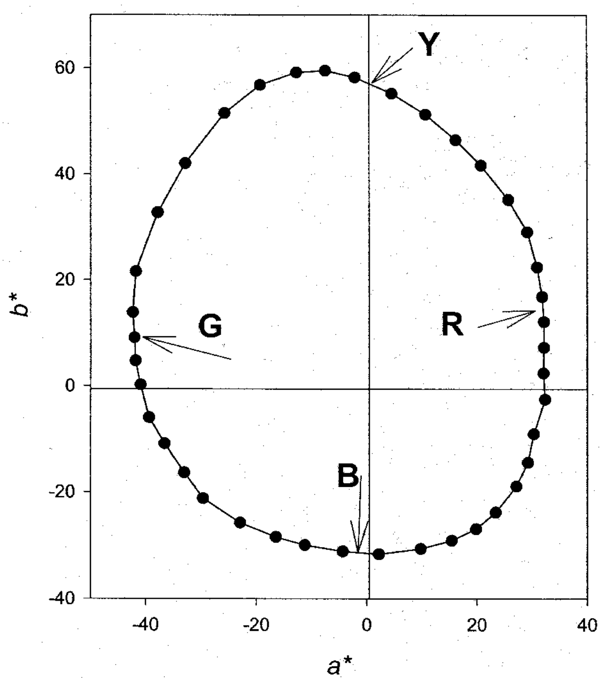

Fig. 5-24a Munsell Renotation colors of value 6, chroma 8 in the a*, b* *diagram of the CIE 2° standard observer and the equal energy illuminant. The letters indicate the locations of the average unique hues determined in the experiment by the author (2001a).*

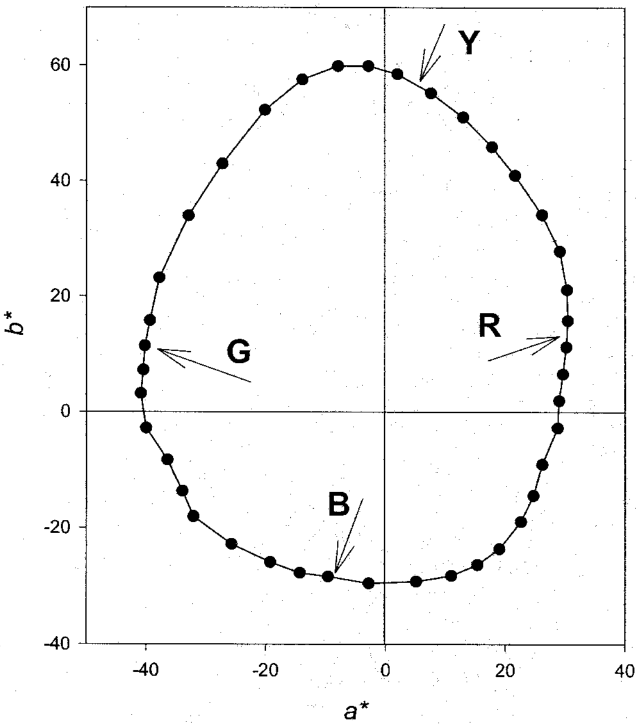

Fig. 5-24b *Colors of Fig. 5-24a illustrated in the* a*, b* *diagram of the CIE 10° standard observer, with locations of average unique hues.*

chapter). These functions not only embody the fundamental variability (as related to uniform hue difference) in hue angle difference but also correction for the error added by the CIELAB formula.

We can surmise from Table 4-1 that the various described experiments have somewhat different hue scales. We find this to be the case when comparing hue angle differences in the optimized cone sensitivity based opponent diagrams of Table 5-1 around the hue circuit for the Munsell Renotations, the Re-renotations and the Newhall data. These are the only data sets where we have continuous consistent hue scaling around a psychologically scaled constant chroma circle (see Fig. 5-25 for the Newhall data). The results indicate little or no discernible agreement except for the region of hue angles 0 to 100 for the Newhall and Re-renotation data. While the changes in hue angle difference appear to be random in the Renotations, there are systematic minima near hue angles 75° and 275° in the Re-renotations. The Newhall data have a systematic development in roughly the first half of the hue circuit. Overall, the lowest level of disagreement is between the Renotations and the Newhall data. All three sets of hue angle differences have slightly positive slopes relative to the hue angle—Renotations 0.0099, Newhall 0.0076, Re-renotations 0.0048—indicating a tendency for the increment to become larger as the hue angle increases.

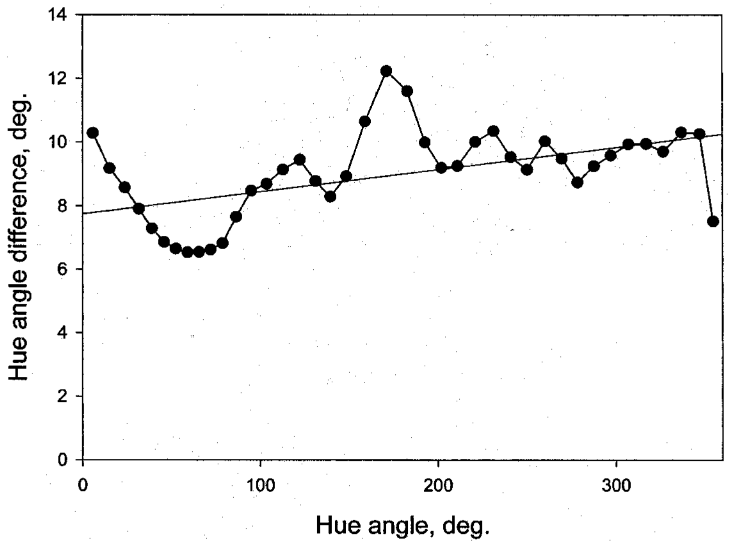

Fig. 5-25 *Hue angle differences in degrees as a function of hue angle between uniform hue steps on a constant chroma contour in the optimized cone sensitivity based opponent diagram, Newhall data. The straight line is a linear regression line.*

Partial scaling of hue circles has been done by Qiao et al. (1998) and several color vision scientists (e.g., see Krauskopf and Gegenfurtner, 1992). It is also implicit in small color difference data (but assuming that a total color difference can be split into components based on the euclidean model and that the CIELAB formula describes visually uniform chroma circles). Differences in stimulus increments needed at different hue angles may be the result of different specific mechanisms being responsible for hue identification and discrimination.

5.9 COLOR MATCHING AND APPEARANCE SCALING

Color-matching functions are expressive of the response of the visual system to the color-matching task. This task does not necessarily have a direct connection to color appearance scaling, even though both presumably are derived from the cone responses. The fact that CIE opponent color diagrams roughly match a Hering type psychological diagram has been suggestive of such a connection.

As mentioned, opponent axes for the 2° observer differ from those for the 10° observer. The differences between the 2° and the 10° color-matching functions are thought to be primarily due to the effect of the macula in the eye (e.g., see Oleari, 1999). In addition the distribution of cone types differs as a function of field size, and the optical density of the cones may change (Pokorny et al., 1976). Macula is a layer of matter containing yellow-colored carotenoids and other compounds. It may have several purposes, one being protecting cell layers of the more central area of the retina by absorbing short wavelength light. The macula has the form of an irregular spot centered on the fovea, the most sensitive region of the retina. However, the central region of the fovea is free of macular material. There is the interesting situation of the central fovea believed to have no *S* cones and being macula free.[10] Informal tests, with rare exceptions, have not resulted in any discernible hue differences when simultaneously viewing the same colored paper with a gray mask with 2° and 10° openings. Small hue changes have been noted in case of some complex grays and some purples. Chroma is often slightly enhanced for small fields compared to larger ones, perhaps as a result of increased contrast at the lower field size.[11] The 2° observer geometry results in the image being focused to a good extent on the macula-free central fovea. The 10° observer geometry, on the other hand, results in a considerable filtering effect by the macula expressed in the different functions. Nevertheless, appearance of colored materials against a neutral surround seems largely unaffected by field size from 2° to 10°.

From the differences between the two sets of observer functions one can predict that metameric matches viewed in a 2° field generally will not match in a 10° field. In regard to appearance, the "blue" system axis falls on dominant wavelength approximately 477 nm for the 2° observer and approximately 470 nm for the 10° observer. It means that if unique hue for the 2° observer falls on the negative *b* axis, that axis for the 10° observer represents a slightly reddish blue. For the "yellow" axis the values are 578 nm, respectively 573 nm, implying in case of the 10° observer a slightly greenish yellow (as demonstrated in the OSA-UCS system). The 10° blue-yellow axis can be rotated by a linear transformation so that it is much closer to the perceptual unique blue and yellow. The $\bar{x}$ modification required is approximately as follows:

$$\bar{x}_{10,\mathrm{mod}} = 0.9\bar{x}_{10} + 0.1\bar{z}_{10}. \tag{5-13}$$

However, such a change also results in significant changes in predicted chroma of a hue circle. The difference between the two CIE standard observers therefore is not just a matter of axis rotation but represents a more fundamental change. It is evident that very specific forms of color-matching functions would be required to reduce reflectances to geometric patterns that closely resemble the geometrical pattern of the psychological space. However, it is questionable if such a transformation exists. There is evidence, to be discussed later, that such a simple approach of combining facts of color matching with facts of color appearance is insufficient. Thus appearance is not in good agreement with the changes implied by the color-matching functions.

5.10 PLACEMENT OF THE RED AND GREEN UNIQUE HUES IN THE OPPONENT COLOR DIAGRAM

The dominant wavelengths of objects seen as having unique red hue are not spectral; their complementary wavelengths fall for most observers on the

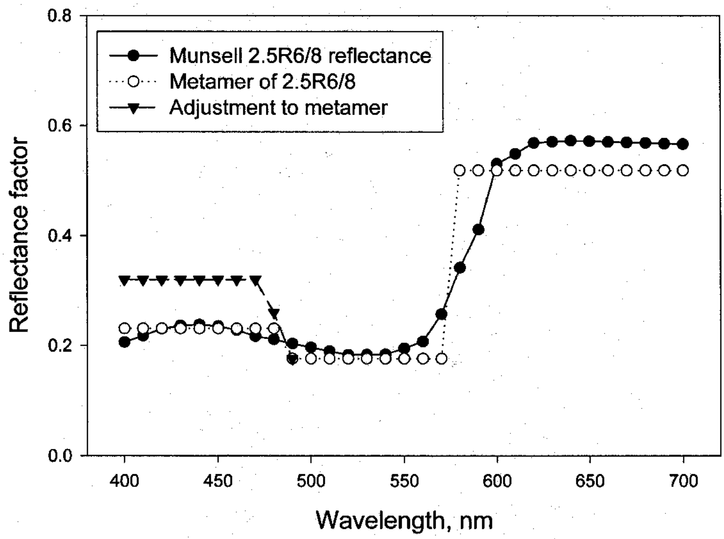

Fig. 5-26 Spectral reflectance functions of a Munsell color chip (2.5R6/8) nearly representing average unique red hue and a modified red. Open circles represent a simplified metamer of 2.5R6/8. Inverted triangles show the short waveband adjustment required to place the color on the positive a* *axis of the CIELAB chromatic diagram.*

purple line in the CIE chromaticity diagram. This is indicated by the fact that objects with red appearance have dominant wavelengths not only in the long wave but also in the shortwave region. Conceptually unique red is located where blueness and yellowness are in balance. The reflectance function of an object resulting in unique red hue appearance in standard daylight against an achromatic surround has a small short wave and a larger long wave loop. The real and an idealized reflectance curve representing Munsell color 2.5R6/8, closest in hue at that level of chroma and value to average unique red, is illustrated in Fig. 5-26. While for the 2° observer the tristimulus values are identical, there is a calculated color difference of 0.9 CIELAB units between them for the 10° observer. When calculating the position of this hue in the a^*, b^* opponent color diagram, we find that its hue angle is not near 0° but at 18° (15° for the 10° observer). The Munsell hue located at 0° for both observers is 5RP, for most observers a distinctly bluish red. Instead of near zero, the b^* value for 2.5R6/8is 12.1, indicating that there is too much implied yellowness or not enough blueness present to balance the two. One can determine how much higher the shortwave reflectance would have to be (illustrated in Fig. 5-26) to result in a b^* value of zero. It is relatively a 40% increase. It is possible to shift the weight of the $b+$ opponent function on the spectral scale while maintaining its total weight in such a way that b^* has a considerably smaller value. But as a result dramatic imbalances of the chromas of a Munsell hue circle are obtained. A similar situation applies to unique green. In fact there does not appear to be a form of color-matching functions that causes objects of all four unique hues to fall on the system axes while maintaining near constant chroma for the hue circle. This situation may point to an ultimate incommensurability between color matching and color appearance. The implication is that opponent signals of the type recorded in the LGN and visual area V4 of the brain undergo additional nonlinear changes in the "black box" between area V4 and the appearance of colors in our consciousness.

5.11 CURVATURE OF LINES OF CONSTANT HUE BLUE COLORS

Another indication of the discrepancy between color matching and color appearance is the curvature in *a, b* diagrams of lines of constant hue of bluish colors. The curvature is evident when plotting the near-unique hue 5PB at value 4 and several chroma levels in the *a**, *b** diagram. It is even more distinctly noticeable when plotting the bluish colors of OSA-UCS in the 10° observer based *a**, *b** diagram. Further indication is the rotation of unit color difference ellipses of bluish colors in counterclockwise direction in the *a**, *b** diagram (Fig. 5-27). This effect can be fixed mathematically by using a hue angle based function (see Luo, Cui and Rigg, 2001). The optimal function has to be different for the two CIE observers.

A clockwise rotation takes place when linearly transforming the $\bar{x}$ color-matching function by subtracting a small amount of $\bar{z}$ and replacing it with the corresponding amount of the original $\bar{x}$:

$$\bar{x}_{\mathrm{mod}} = (\bar{x} + m\bar{x}) - m\bar{z}, \tag{5-14}$$

where m is a factor with a value of 0.15 or less and depends on the visual angle occupied by the sample; that is, the optimal value for m differs for the two CIE standard observers (Kuehni, 1999). The effect of this equation is to adjust the relative size of the two loops of the $\bar{x}$ function by slightly reducing the short-wave and enlarging the long wave loop. We have seen above that adjustments of this type (expressed in terms of cone sensitivity) are also important for optimal fitting of constant chroma data. The effect is also evident in the short-wave red loop of the OSA-UCS *g* function (see Fig. 7-17b). P. W. Trezona and R. P. Parkins (1998) pointed out that the size of the two loops in the CIE color-matching functions is somewhat arbitrary. Application of the revised function rotates not only bluish but also (if less so) yellowish colors in the *a**, *b** diagram.

5.12 MUNSELL COLORS IN THE *L*, *M*, *S* AND *X*, *Y*, *Z* SPACES AND THE A, B DIAGRAM

For a better understanding of how the three psychophysical color spaces are related, it is instructive to compare a selection of Munsell colors (Kuehni,

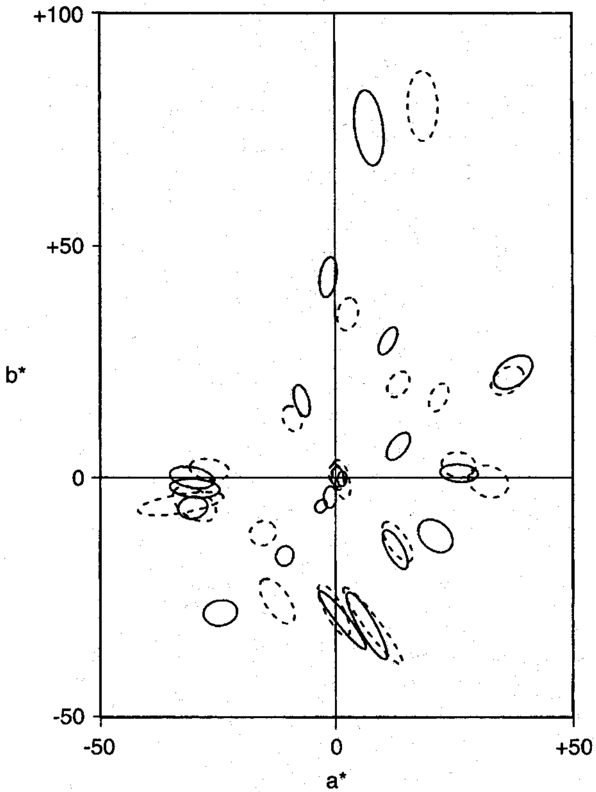

Fig. 5-27 *Ellipses in the* a*, b* *diagram fitted to selected Luo and Rigg (*solid*) and to the RIT-DuPont (*dashed*) small color difference data. Note the rotation of ellipses near the negative* b* *axis. From Melgosa et al. (1997).*

2001c). For this purpose a "Celtic cross" figure is selected consisting of a hue circle at chroma 8 and colors at increasing chroma from the neutral point of hues falling nearest the axes of the *a, b* diagram at values 3, 6, and 8. It is understood that the hue and chroma scaling of these colors cannot be taken as perfect, but there is no doubt that the corresponding samples are a reasonable approximation of a uniform hue circle and uniform chroma scales. In the psychological chromatic diagram the three Celtic crosses coincide forming a single cross. Figure 5-28 illustrates these colors in the three-dimensional psychological diagram. After converting the CIE tristimulus values (adjusted to reflect an equal energy illuminant) to *L, M, S* values using the Smith-Pokorny transformation, the colors are plotted in the *L, M, S* space in Fig. 5-29. The three crosses form sections through an elliptical funnel. Circles in the perceptual diagram are strongly elongated along the *S* axis in the *L, S* and *M, S* planes. The chroma steps along the *S* axis become increasingly larger as *S* increases. Because of the definition of *Y* in terms of *L* and *M* the slices of the elliptical

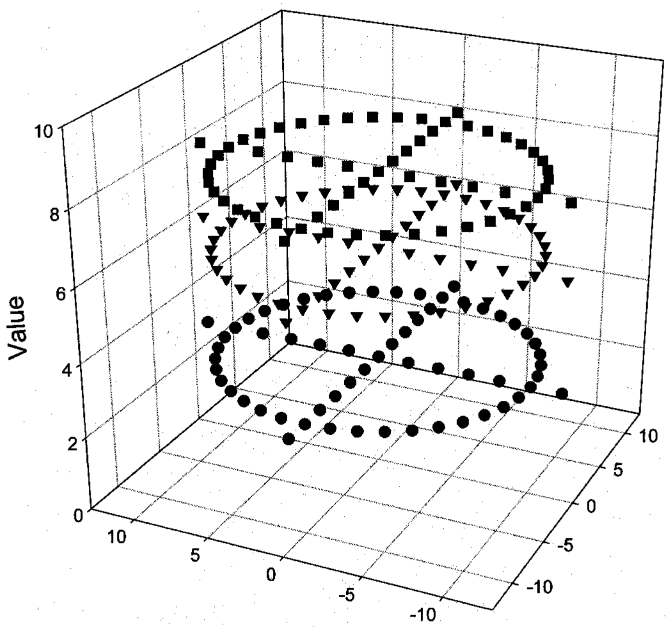

Fig. 5-28 *Representation of Munsell colors at values 3, 6, and 8 and chroma 8, and chroma scales on the axes (Celtic crosses) in the conceptual Munsell color space where hue, value, and chroma scales are not matched.*

funnel are not perpendicular to the equal energy axis but slanted. It is evident that a series of constant value/constant chroma Munsell hue circles has a complex form in the L, M, S space without a simple relationship to any of the three dimensions.

The situation improves when plotting the same colors in the X, Y, Z space (Fig. 5-30). Since in this space one of the dimensions is luminous reflectance, colors of a given Munsell value fall on planes perpendicular to the Y axis. The chroma axes of the cross are well aligned with the X and Z axes of the space. Scaling of X against Z is required to make the contours circular. This is evident if we plot the three crosses in a normalized linear opponent color diagram where $a = 2.272\ (X - Y)$ and $b = Y - Z$ (Fig. 5-31). The funnel of hue circles is now parallel to the Y axis. In order to convert the funnel to a cylinder in agreement with the psychological cylinder and at the same time create uniform chroma scales, we must determine the applicable psychophysical function(s). As mentioned above, in 1942 Adams proposed that the Munsell value function was applicable to all three tristimulus values, and this proposal has been enshrined in the CIELAB formula. However, the three Munsell Celtic crosses require power modulation that is different for the four semi axes (Table 5-2), and additional modulation to convert the cone in tristimulus space with good accuracy into a cylinder (more on this subject in Chapter 7)

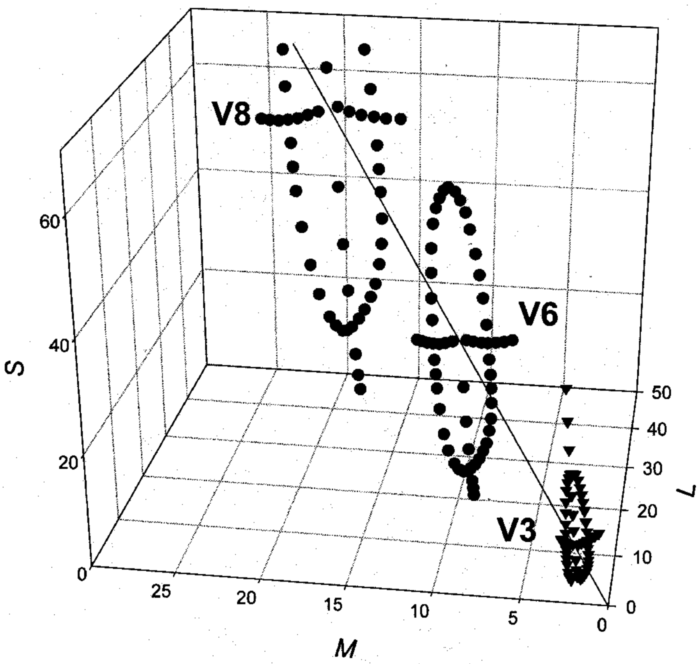

Fig. 5-29 Actual Munsell colors approximating the Celtic crosses of Fig. 5-28 in the L, M, S *cone sensitivity space; 2° observer, equal energy illuminant. The straight line represents the equal energy locus.*

5.13 SUPRATHRESHOLD SMALL COLOR DIFFERENCES

Differences in all sets of such data, as will be seen in the next chapter, have been judged as total differences. In nearly all cases the total differences, as calculated by the CIELAB formula, involve differences in all three attributes. The historical process of fitting psychophysical formulas to such data is described in Chapter 6. CIELAB did not result in good correlation against visual judgments in various data sets, and efforts since the mid-1970s usually involved modification of CIELAB to improve correlation. The most widely used formulas today (in the United States) are CMC and CIELAB. The CIE has issued recommendations of new formulas in 1994 and 2001. As in the case of hue and chroma scaling data sets, different sets of small color difference judgments vary considerably. A formula fitted to one set of data often does not fit another set well. Some of the reasons may have to do with different experimental conditions, often inadequately described. Other reasons may involve the composition of the observer pools used in the experiments and, possibly, cognitive components in the judgments of the observers. Clarification of the reasons behind the psychological variability is important for accurate model building.

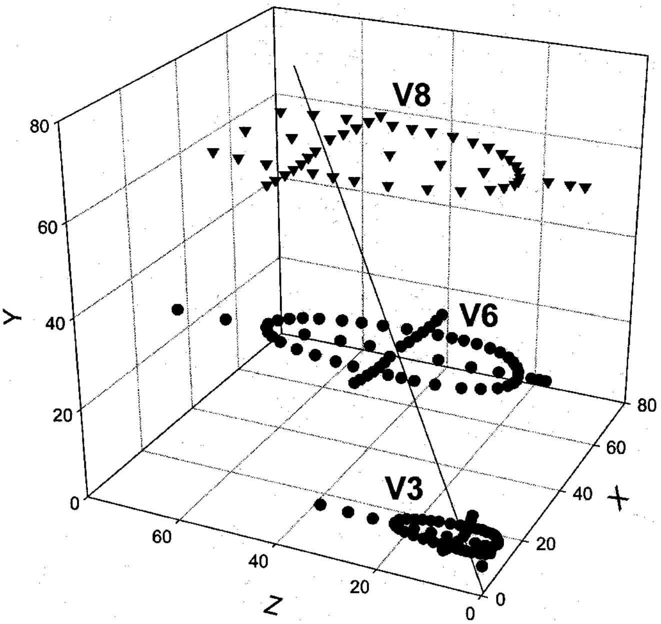

Fig. 5-30 *Colors of Fig. 5-29 in the 2° observer* X, Y, Z *tristimulus space, equal energy illuminant. The straight line represents the equal energy locus.*

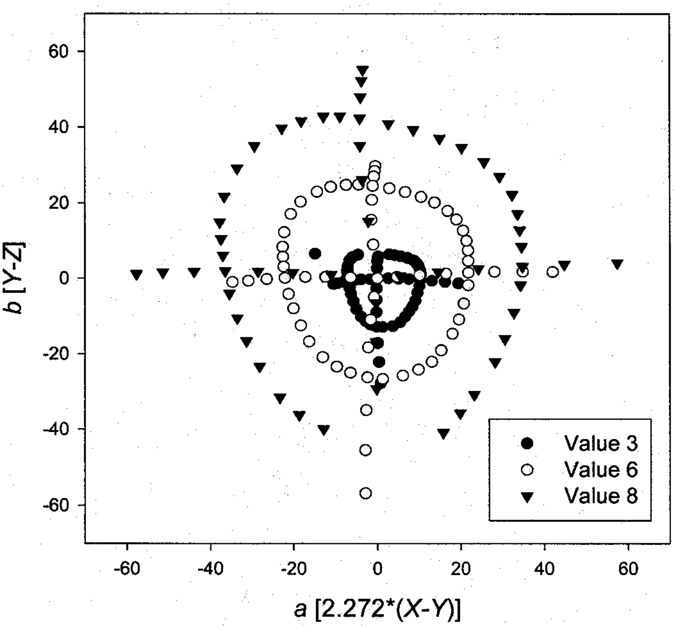

Fig. 5-31 *Colors of Fig. 5-29 projected onto a balanced linear opponent color diagram based on tristimulus values (2° observer, equal energy illuminant).*

5.14 DIFFERENCE THRESHOLD MEASUREMENTS

As will be discussed in the next chapter early measurements of color discrimination thresholds expressed in psychophysical systems were made by Wright (1941) and by Stiles (1946). Newer measurements important for the purposes of this book, using sophisticated computer control of visual displays with tight controls of adaptation, are those by Krauskopf and Gegenfurtner (1992), Strohmeyer et al. (1998), Sankeralli and Mullen (1999), and Smith, Pokorny, and Sun (2000). In recent years threshold measurements using object color samples have been made by Richter (1985) and by Witt (1987).

Typical measurements in vision science involve the so-called pedestal technique. A color stimulus is displayed briefly (one second or less) against a surround that controls adaptation. A small stimulus, typically at constant luminance, is added to or subtracted from the base pedestal stimulus and randomly displayed against it. The observer indicates if she detected a difference between the two displayed fields. The results are now generally reported in a Derrington, Krauskopf, and Lennie type chromatic diagram (see Section 6.17) with its axes scaled in terms of the threshold at surround chromaticity. Among the many results are the following:

1. Thresholds are always smallest in terms of incremental stimulus when the stimuli being compared have luminances or chromaticities straddling those of the surround. This is confirmation of the results of Schönfelder (1933).
2. When measuring chromatic thresholds against a neutral surround, the stimulus increment required for a criterion response increases with increasing contrast between surround and the reference pedestal, except along the system axes.

In this kind of experiment only increments along the $(L + M) - S$ axis increase, while those along the $L - M$ axis do not. Close to surround the increment actually slightly declines in size compared to that exactly at surround (Sankeralli and Mullen). Threshold contours fitted to color centers at some distance from the surround chromaticity are ellipselike, pointing in the general direction of the diagram origin (Krauskopf and Gegenfurtner, 1992; see Fig. 8-14). Such findings have resulted in the idea of multiple hue detection mechanisms throughout the chromatic plane, a crucial issue in regard to modeling of hue discrimination. Mechanisms of such a type are believed to be of cortical nature, but no specific neurophysiological mechanism has been proposed yet. The idea is controversial as seen from arguments by Krauskopf (1999) and by Eskew et al. (1999).

The outcome may depend on the outcome of another controversy. Derrington et al. (1982) found that most parvo cells in the lateral geniculate nucleus are tuned to their cardinal chromatic axes (not unique hues), indicating no hue angle dependent selectivity. Later, in visual area V1 of the cortex, such clear orientation of cell response is absent, even though there is a slight tendency in this direction (Lennie, 1999). The same applies to areas V2 to V4. De Valois et al. (1997) found that LGN cells of all types have input into later cells with color sensitivity. This may explain the differences in threshold measurement results along the cardinal axes compared to away from them. These results point to the possibility of a rather complex hue detection mechanism that may depend on averaging computations among many cells with specific hue tuning.

5.15 HOW MANY COLORS CAN WE DISTINGUISH?

This is an interesting question for which several answers have been given in the past. The first determinations of this kind addressed the limited issue of the number of discriminable steps in the spectrum. Initial experimental attempts to answer this question were made by E. Mandelstamm, reported in 1867. Measurements considered valid for many years were those by W. Dobrowolsky (1872). Kries computed in 1882 from Dobrowolsky's data 208 discriminably different steps. Based on König and Dieterici's experiments of 1884 the number increased to 235. However, there is the issue of conflation of hue, saturation, and brightness differences when looking at spectral colors. In addition the results depend on the experimental setup. When correcting for brightness differences, L. A. Jones in 1917 found 128 hue steps. Many determinations of wavelength discrimination have been made since then, notably by Wright and Pitt in 1934 and Bedford and Wyszecki in 1958, with quite similar results. MacAdam (1947), on basis of his color-matching error data, calculated a number of 250 just noticeable differences in the spectrum, not far different from that of König and Dieterici.

But color, as we know, is not just hue but also brightness/lightness and saturation/chroma. In 1896 Titchener estimated the total number of perceptibly different colors at about 33,000. By 1939 the number, as estimated by Boring, had risen to 300,000. Both Titchener and Boring did not distinguish between light colors and object colors. In the same year Judd (1939) estimated the number of perceptible object colors as 10 million. This figure has become much quoted. In 1943, in connection with their development of a psychological color solid (see Figs. 1-2 and 1-3), Nickerson and Newhall calculated a rounded number of 7,500,000 at the just noticeable difference level (applicable to the model on the right side of Fig. 1-2) and a number of 1,875,000 under less favorable viewing conditions (applicable to the model on the left side). This number applies to a color solid reaching out to the object color limits.

MacAdam in 1947, based on his color matching error data of 1942, limited himself to determining the number of perceptually different colors in a constant luminance plane. Using three standard deviations of the matching error as the JND limit, he calculated the number of colors distinguishable in a constant luminance plane, determined under the conditions of his color-matching error experiment, in a rounded number, as 17,000.

More recently the question has appeared again. In 1981 A. Hård and L. Sivik (in contrast to the 10 million colors Judd mentioned as discriminable) estimated the number of distinct colors that can be identified with a degree of certainty as 10 to 20,000. In 1995 Indow estimated the number of colors that can be discriminated from each other as approximately 7 million without providing an explanation for the estimate. In 1998, using the CIELAB formula as a basis and assuming the limit of object color distinction to be 1 CIELAB unit of total color difference, M. R. Pointer and G. G. Attridge calculated the number of distinguishable object colors to be 2.28 million. They also calculated the theoretical limit of different object colors displayable on a video monitor to be 16.78 million.

Based on a single medium gray background/surround the number must be significantly smaller than 2.3 million because of lightness and chromatic crispening effects. At the same time the JND limit near the surround is likely smaller than 1 CIELAB unit. The CIE94 color difference formula predicts that one CIELAB chroma unit at metric chroma zero is only approximately 0.2 chroma units at metric chroma 100. Unless there is a widely variable surround the number of perceptibly different object colors is more likely about 1 million. Nevertheless, it is a remarkable number of different percepts based on the output from three cone types.

There is, however, an additional issue. The number of 1 million or 2.3 million applies to the situation of placing a net defining JNDs over a much finer net defining visible stimuli. Once a starting position is defined, one can perceive 1 million other colors compared to the reference color. But the reference color can be varied within the just noticeable difference limen around it. Applying the JND net to a new reference point, we experience 1 million slightly different colors. The number of colors we can experience may, after all, be more in the range of what can be produced on a video monitor.

In this chapter it has been shown that the fitting of psychophysical models to uncertain psychological data has added another layer of problems. In the absence of full understanding of the human visual mechanism, such modeling is mainly empirical. The currently neurophysiologically supported (in macaque) opponent space at the LGN level is not in agreement with the human psychological opponent color space. But a comparatively simple adjustment can improve the fit. Interestingly different experimental psychological constant chroma circles can be modeled closely with only changes in the level of this adjustment and modifications in scaling of the semi axes. It is necessary to investigate the causes of differences in constant chroma evaluations: uncontrolled surround effects, variability in the personal color-matching functions of the observer pool, perhaps other reasons. Different sets of hue scaling data around a constant chroma circle also have provided significantly different results. In both cases development of reliable, replicated data seems urgent.

Chapter 6

Historical Development of Color Space and Color Difference Formulas

A color space formula is a set of mathematical equations describing a color space model in terms of cone sensitivities or tristimulus values and spectral power distributions. A color difference formula is a mathematical expression resulting in numbers proportional to perceived difference between points in the space. Historically development of color space and color difference formulas has taken a standard association between a color stimulus (as represented by a family of metamers) and a color perception as a given. Recognition of the need for exact specification of the conditions of viewing the stimuli is a relatively recent phenomenon. Most of the formulas discussed below betray no such recognition. As such they need to be understood as lacking fundamentality and being limited to implicit or explicit assumptions/representations of viewing conditions.

6.1 LINE ELEMENTS

The line element is the so-called first fundamental form of a regular surface.[1] For small color differences it is given explicitly by the Riemannian metric

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ISBN 0-471-32670-4

which determines the arc length of a curve on a surface. Connected with the line element is the geodesic: the curve that locally minimizes the length of a curve. It depends on the geometry of the space to which it applies: in plane geometry the geodesic is a straight line, on a sphere a segment of a great circle (e.g., the equator). In connection with color Wyszecki and Stiles (1982) defined the line element as "a measure of distance in a postulated space in which perceived colors are represented by points or vectors." Historically the spaces related to given line elements have been based on color fundamentals R, G, and B or cone sensitivities. The differences between colors in these spaces have been defined with increments, originally based on the Weber-Fechner law and later on more complex relationships.

Helmholtz and Schrödinger

Interest in a stimulus intensity based model of uniform color space, a model of outer psychophysics in Fechner's sense, grew since the end of the nineteenth century. In 1886 König wrote that it should be easy to construct a color mixture diagram in a manner that equal distances in all directions would be proportional to equal numbers of discrimination steps. What he had in mind is a model that would partition the diagram in agreement with the Weber-Fechner law. The first quantitative version was introduced by Helmholtz in 1896 based on ideas discussed in 1891. He assumed three fundamental color vision processes R, G, and B and the uniform applicability of the Weber-Fechner law:

$$\frac{dR}{R} = \frac{dG}{G} = \frac{dB}{B} = \text{constant}, \tag{6-1}$$

where dR is the increment/decrement in magnitude of the stimulus described by the fundamental process R, and comparable for the other two stimuli. Assuming in addition a euclidean space, we can calculate the threshold differences as

$$ds = \left[\left(\frac{dR}{R}\right)^2 + \left(\frac{dG}{G}\right)^2 + \left(\frac{dB}{B}\right)^2\right]^{0.5}, \tag{6-2}$$

where ds is the distance between two neighboring points in the R, G, B space representing a threshold difference. Helmholtz attempted to make his model fit experimental data of König and Dieterici by appropriately shaping his three visual processes with linear transformations from color-matching functions. All of the resulting functions had two maxima, and none was in agreement with the experimental luminous efficiency function. Helmholtz was unaware of and did not consider contributing factors important for an accurate line element.

A more complex line element was proposed in 1920 by Schrödinger in an attempt to correct perceived shortcomings of the Helmholtz model. Schrödinger recognized that Helmholtz's form of the line element results in a luminance function that is much different from experimental functions. His proposal has the following form:

$$(ds)^2 = \frac{1}{l_R R + l_G G + l_B B}\left[\frac{l_R (dR)^2}{R} + \frac{l_G (dG)^2}{G} + \frac{l_B (dB)^2}{B}\right], \tag{6-3}$$

where l_R , l_G, and l_B are constants corresponding approximately to brightnesses of the three fundamental processes derived from the luminous efficiency function, R, G, B are the fundamental processes as experimentally determined by König and Dieterici. The line element obeys the Weber-Fechner law while being additive. Schrödinger's proposal did not receive much practical attention, perhaps because of the difficulties of executing such calculations on a routine basis.

Stiles

In 1946 Stiles offered a version of a line element based on extensive investigation of two-color thresholds. Importantly Stiles found that different Weber fractions apply to the three visual processes rather than the common fraction assumed by Helmholtz and Schrödinger. Stiles's line element has the following form:

$$(ds)^2 = \left[\frac{\zeta(R)}{\rho} dR\right] + \left[\frac{\zeta(G)}{\gamma} dG\right]^2 + \left[\frac{\zeta(B)}{\beta}\right]^2, \tag{6-4}$$

where $\zeta(R) = 9/(9+9R)$ and comparably for G and B; ρ, γ, β are proportional to the Weber fractions, with values of 1.28 for ρ, 1.65 for γ, and 7.25 for β, indicating that the "blue" process is much less sensitive than the "red" and "green" processes. The three processes were defined by linear transformation from the 1931 CIE color-matching functions. Stiles's line element was tested extensively in succeeding years and was found to provide good approximations to several sets of visual data, including the luminous efficiency function, wavelength discrimination, and chromatic thresholds.

Luneburg's Line Element for Visual Space

Mention should be made here of a line element for the human visual space developed in 1947 by the mathematician R. K. Luneburg. Visual space refers to the space segment covered by our eyes when looking ahead. Visual space and color space are two different aspects of our visual sense, and there is no immediate reason for color space to be in agreement with visual space. The fact that a similar geometry appears to apply to both situations may be coincidental. Based on comparisons between physical measurements and related judgments of distance, Luneburg determined visual space to be Riemannian (i.e., elliptical) with constant curvature. He proposed the following line element:

$$ds^2 = \frac{d\alpha^2 + d\beta^2 + d\gamma^2}{\left[1 + \frac{1}{4K(\alpha^2 + \beta^2 + \gamma^2)}\right]^2} \tag{6-5}$$

where α, β, γ are orthogonal sensory coordinates and K denotes Gaussian total curvature. If $K = 0$, the space is euclidean; if negative, the space is hyperbolic, and if positive, it is elliptic.

Walraven, Bouman, Vos

In 1966 P. L. Walraven and M. A. Bouman offered a new proposal for a line element of color employing photon noise methodology (De Vries-Rose behavior; De Vries, 1948) in place of the empirical Weber-Fechner law. With the exception of the photon noise methodology replacing the Weber-Fechner law the proposal was identical to that of Schrödinger. Testing against new wavelength discrimination and other data did not provide fully satisfactory results.

Walraven continued work with a new co-worker, H. Vos, and they developed the Vos-Walraven line element (Vos and Walraven, 1972, 1991). The model underwent several modifications. In its original form it was based on a model of retinal color processing which included a Helmholtz type cone absorption step taking input of the R, G, B cones at a ratio of 32 : 16 : 1, with cone response compression. In a summation zone the compressed R and G signals form the yellow signal, and all three together the luminance signal. Separately, R and G signals are balanced to form an antagonistic Hering type red-green signal. Similarly the yellow signal is balanced against the B signal, forming the yellow-blue output signal. Finally the luminance signal is balanced against a surround luminance signal, resulting in a brightness contrast signal. Vos and Walraven concluded that opponent processing does not play a role at the threshold level and their line element is essentially of the Helmholtz type. It is defined as follows (Vos, 1979):

$$(ds)^2 = \left\{\frac{dR}{\left[R\left(1 + \frac{R}{R_0} + \frac{R^2}{R_1^2}\right)\right]^{0.5}}\right\}^2 + \left\{\frac{dG}{\left[G\left(1 + \frac{G}{G_0} + \frac{G^2}{G_1^2}\right)\right]^{0.5}}\right\}^2 + \left(\frac{dB}{\left[B\left(1 + \frac{B}{B_0} + \frac{B^2}{B_1^2}\right)\right]^{0.5}}\right)^2 \tag{6-6}$$

where R, G, B are the cone signals at the ratio of 32 : 16 : 1, subscript 0 indicates the number of quanta for which saturation occurs, and subscript 1 the number of quanta for which supersaturation occurs. The model thus can account for various levels of saturation of the cone system and in this manner can predict, in addition to color discrimination at the threshold level, the Bezold-Brücke and the Stiles-Crawford effects. While the model is a modified version of the Helmholtz line element, it considers the ratio of cones that gives rise to different Weber fractions for the three cone types as well as saturation and supersaturation effects, making it not just applicable to a middle range of luminance but to the complete range.

The Helmholtz, Schrödinger, Stiles, and Vos-Walraven line elements were believed to be more or less uniform in terms of thresholds. However, as will be shown in Chapter 8, they are regular color spaces. In his paper Schrödinger suggested that geodesics, the lines indicating the paths of smallest numbers of threshold differences, are lines of constant hue. MacAdam originally determined geodesics based on his color-matching error data mechanically, by stretching threads across a model representing his ellipses (MacAdam, 1981). Geodesics can be calculated, for example, by using linear programming (Jain, 1972). For an example of constant hue and saturation geodesic lines in the CIE chromaticity diagram calculated from MacAdam's 1965 geodesic chromaticity diagram (MacAdam, 1981), see Fig. 6-1. With known hue, chroma, and lightness geodesics, it is possible to develop formulas transforming nonuniform cone sensitivity or tristimulus spaces into a space uniform in terms of the assumptions of the model involved.

6.2 PROJECTIVE TRANSFORMATIONS

Another approach to a uniform color space was based on the idea that the CIE 1931 x, y chromaticity diagram could be linearly transformed to result in a modified diagram in which distances were proportional to visual distances. In 1932 D. B. Judd offered an early version representing his own threshold and other published data (Fig. 6-2) and based on color-matching functions recommended by the Optical Society of America in 1922. The diagram contains radial lines of constant dominant wavelength and ovoids of constant colorimetric purity. In 1935 Judd published a modified version of a uniform chromaticity diagram in the form of a Maxwell triangle (Fig. 6-3). In a different reference frame the diagram became in 1960 the basis for the CIE u, v diagram. In 1935 Judd also introduced the symbol ΔE to denote a color difference. In the following year Judd published a graph of the CIE 1931 chromaticity diagram with ellipses that represent circles of equal size in his 1935 diagram (Fig. 6-4). These ellipses are intended to represent uniform threshold color differences, enlarged 100 times. They illustrated for the first time explicitly the perceptual nonuniformity of the CIE chromaticity diagram.

In 1937 MacAdam modified Judd's 1935 diagram with simplified coefficients resulting in a rectangular coordinate diagram. In 1939 F. C. Breckenridge and W. R. Schaub developed the rectangular uniform chromaticity scale diagram (RUCS), a transformed version of Judd's 1935 diagram.

Fig. 6-1 *Geodesic lines of constant hue and chroma in the CIE chromaticity diagram, calculated from MacAdam's 1965 geodesic chromaticity diagram (Fig. 6-9).* Solid lines: *constant hue;* dashed lines*: constant saturation. From MacAdam (1981).*

In the same year Judd, Scofield, and Hunter proposed the α, β diagram and a color space related to it. In 1942 this linear transformation of the CIE chromaticity diagram became the basis of the National Bureau of Standards (NBS) formula, with the difference units designated as NBS units or *judds* (after D. B. Judd).[2] The formula is as follows:

$$\Delta E = f_g \left\{ \left[221 Y^{0.25} (\Delta\alpha^2 + \Delta\beta^2)^{0.5} \right]^2 + \left[k\Delta(Y^{0.5}) \right]^2 \right\}^{0.5}, \tag{6-7}$$

where f_g is a factor adjusting for glossiness and is defined as $f_g = Y/(Y + K)$, with K usually taken as 2.5; k, having normally a value of 10, adjusts the lightness difference to the chromatic difference

$$\alpha = \frac{2.4266x - 1.3631y - 0.3214}{1.0000x + 2.2633y + 1.1054},$$

$$\beta = \frac{0.5710x + 1.2447y - 0.5708}{1.0000x + 2.2633y + 1.1054}. \tag{6-8}$$

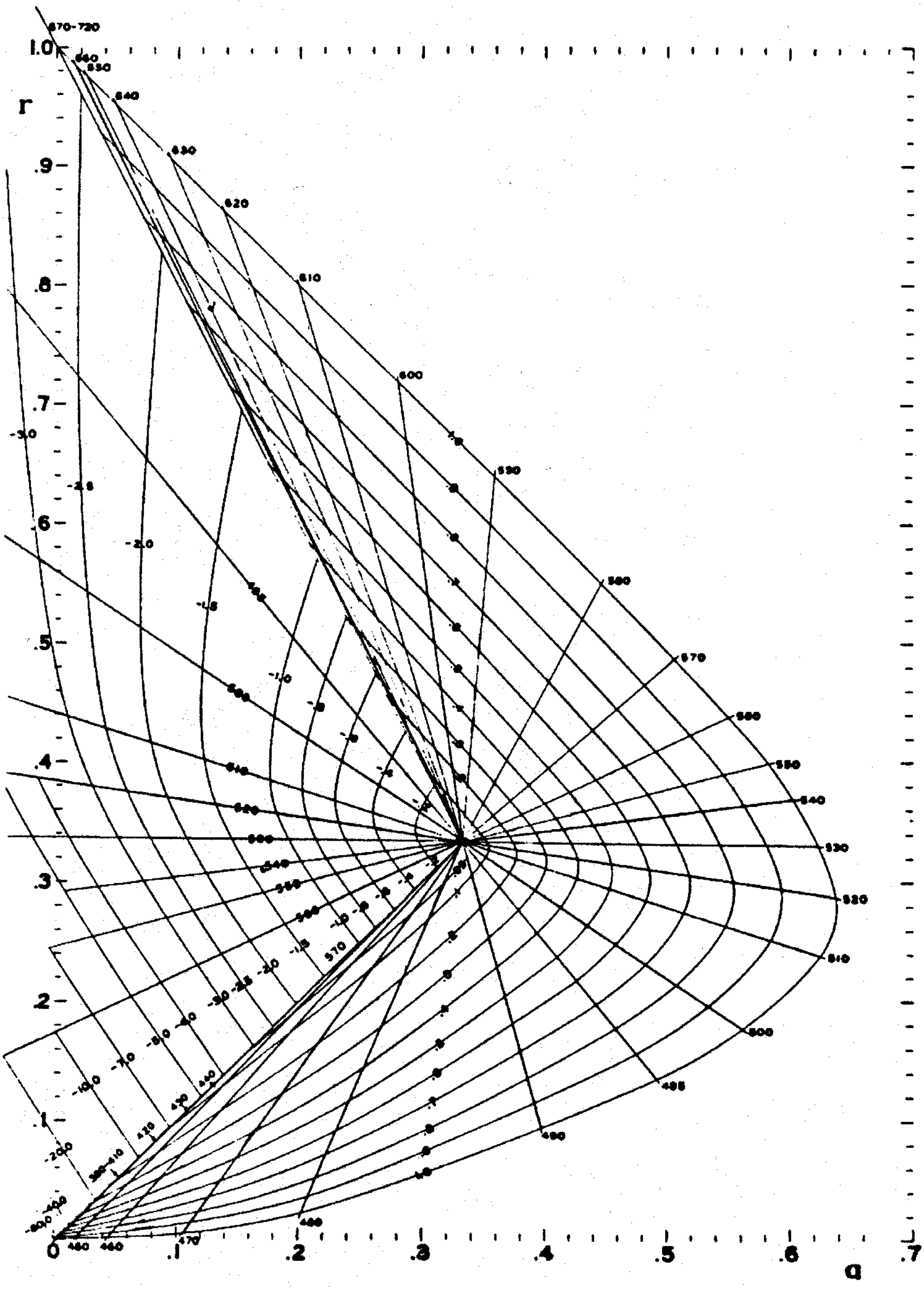

Fig. 6-2 Judd's projective transformation of the CIE chromaticity diagram of 1932 meant to result in proportionality of distances with visual distances. The numbers on the spectral trace represent wavelength in nanometers.

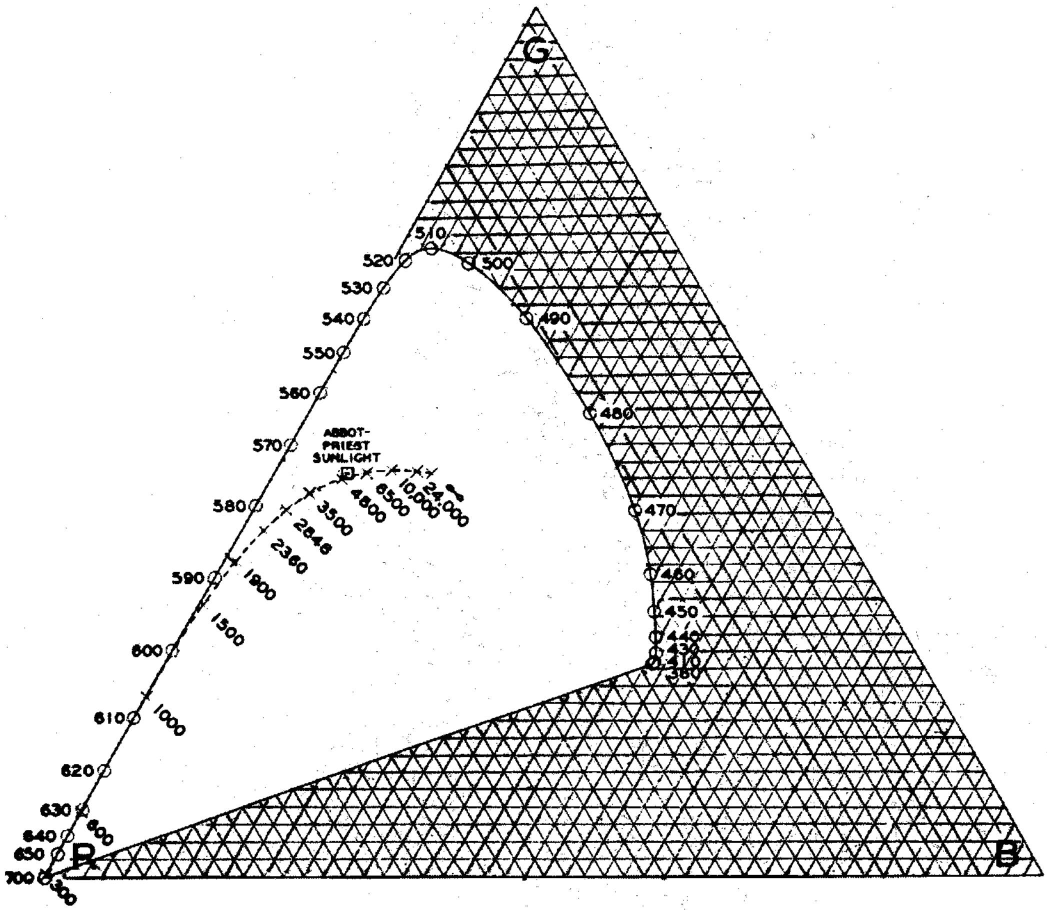

Fig. 6-3 *Judd's 1935 uniform chromaticity diagram embedded in the Maxwell triangle. On the dashed curve are colors of a black body at the indicated absolute temperatures in Kelvin. Compare with Fig. 2-29.*

In 1944 D. Farnsworth proposed a simplified transformation. It provides for minimum deviation from circularity of Munsell hues at chroma 10 and value 5, centered on illuminant C. In 1959 Y. Sugiyama and T. Fukuda optimized a transformation such that both the Munsell system and the MacAdam ellipses were reasonably well represented. In 1963 Wyszecki proposed a transformation formula that became the basis of the CIE 1964 (U*V*W*) system:

$$U^* = 13\,W^*(u - u_0),$$

$$V^* = 13\,W^*(v - v_0),$$

$$W^* = 25\,Y^{1/3} - 17, \tag{6-9}$$

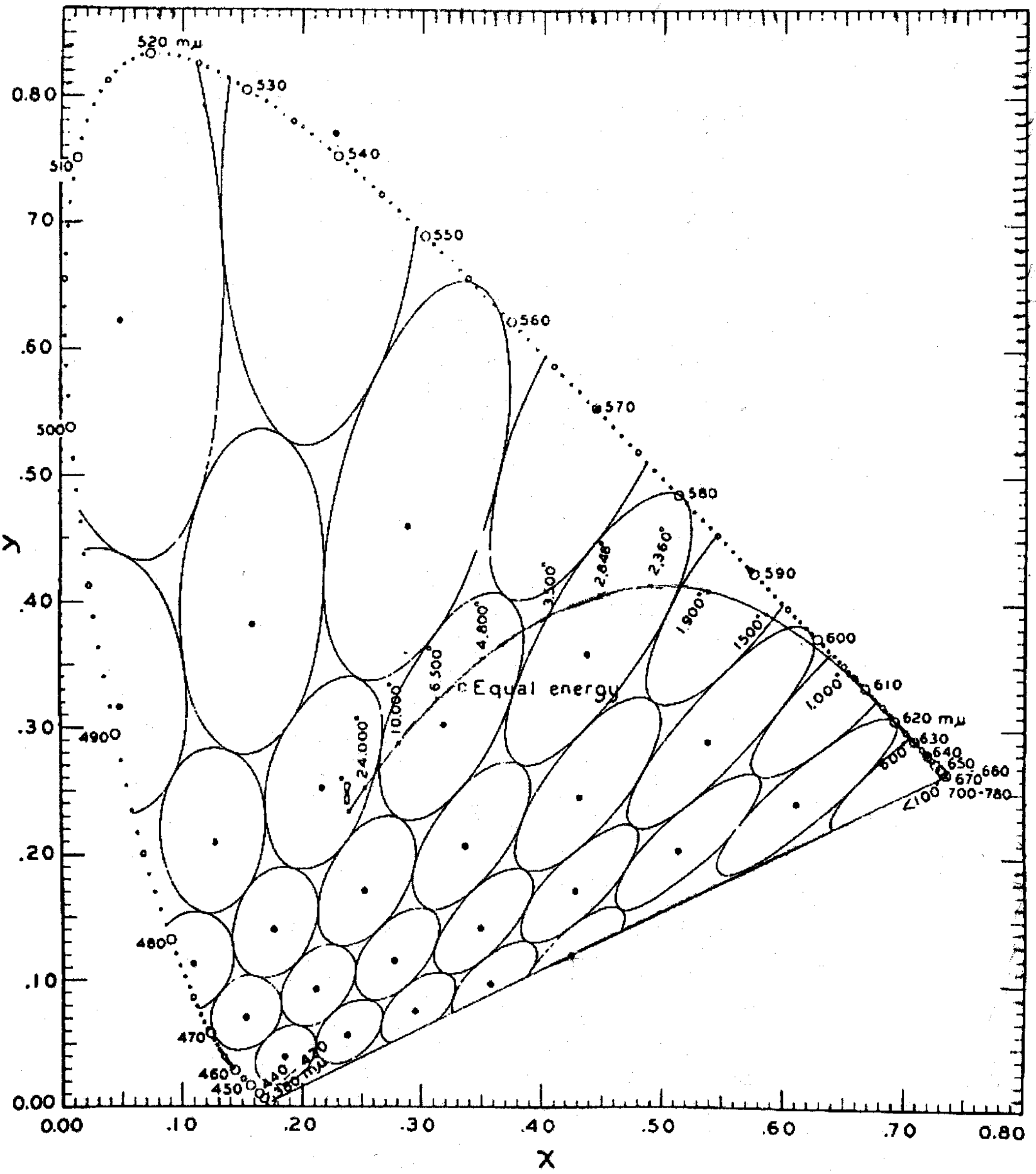

Fig. 6-4 *CIE chromaticity diagram with ellipses that represent circles of equal size in Judd's 1935 diagram. From Judd (1936).*

where $u = 4\ X/\ (X + 15Y + 3Z)$, $v = 6\ Y/(X + 15Y + 3Z)$, X, Y, Z are CIE tristimulus values, u_0, v_0 are the values of the variables u and v for the achromatic color at the origin of the U^*, V^* diagram. The total color difference was calculated as the square root of the sum of the squares of the differences in the three dimensions. This formula has been superseded by the CIELUV formula.

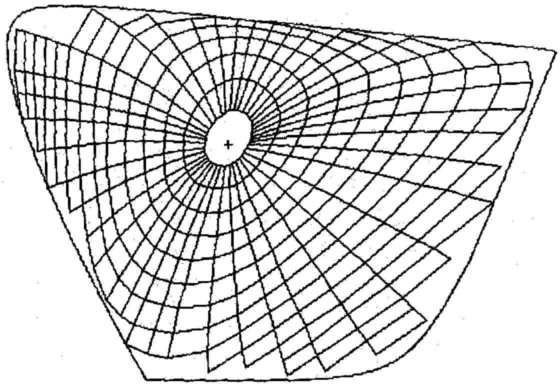

Fig. 6-5 *Munsell Renotation colors at value 5 as represented in the CIELUV projective transformation formula. From Mahy et al. (1994).*

The development of projective transformation came to a conclusion with the recommendation in 1976 by the CIE of the 1976 $L^*u^*v^*$ transformation (CIELUV). This is a minor modification of MacAdam's 1937 diagram (expansion of v by a factor 1.5). A cross section of the Munsell space at value 6 as represented by this formula is shown in Fig. 6-5.

Projective transformation employs the following basic formulas:

$$x' = \left(\frac{a_{11}x + a_{12}y + a_{13}}{a_{31}x + a_{32}y + a_{33}} \right), \qquad y' = \left(\frac{a_{21}x + a_{22}y + a_{23}}{a_{31}x + a_{32}y + a_{33}} \right), \tag{6-10}$$

where x', y' are the transformed chromaticity coordinates and x, y are the CIE chromaticity coordinates; a_{xx} are transformation coefficients. Several of the formulas are in a simplified form requiring only five coefficients. The coefficients of the above and other linear transformation formulas are given in Table I (6.4) of Wyszecki and Stiles (1982). Linear transformations have the advantage, and are thus of interest to optical and lighting engineers, that the system remains additive and mixtures of two lights fall on a straight line connecting their locations in the diagram. The CIELUV formula will be further discussed below.

6.3 FITTING MODELS TO THE MUNSELL SYSTEM

Another avenue pursued involved attempts to develop a uniform color space model from analysis of the structure of the Munsell system. This system had been extensively investigated by the Subcommittee on Uniform Spacing of Munsell Colors of the Optical Society of America. It published a final report in 1943, containing colorimetric specifications of the smoothed Munsell Renotation colors. In this form the Munsell system was taken to represent a good approximation of a visually uniform psychological color space.

In 1936 Nickerson proposed the first formula with the goal of predicting perceptually equal color differences, the Nickerson Index of Fading (see equation 4-1). The formula has a purely psychological basis, since no physical measurements are involved in its application. Its immediate purpose was to provide a measure of fading by light of dyed textiles. Component differences were combined by simple addition. The formula recognized the dependence in a polar system of the hue difference on chroma and also indicated that in the Munsell system unit differences in the three attributes are of different perceptual magnitude. As a result such a system is not uniform, as it doesn't represent distances in all directions proportionally to differences in its euclidean space (see Chapter 7).

In 1942 Adams plotted Munsell colors at several values in what he called a "chromance" diagram, representing a linear opponent color diagram normalized to the equal energy point. The two dimensions were defined as $X - Y$ and $Z - Y$. By applying the Munsell-Sloan-Godlove lightness formula (a power 0.5 modulation; see Chapter 5) not only to the luminous reflectance value Y but also to the other two tristimulus values, Adams converted the chromance diagram to a chromatic value diagram. Adams interpreted the subtractions as representations of his 1923 theory of color vision that took the color-matching functions to be cone responses. In this theory Adams had assumed an inhibitory effect of the output of the postulated Y cone on the outputs of the X, respectively Z cones. He represented this inhibitory effect with subtractions. He called the outputs V_X, V_Y, and V_Z. Adams calculated chromatic values of a high chroma Munsell hue circle and found those related to Z to be larger than those related to X and Y. To have scales of maximum value 10 in all three dimensions he proposed to multiply the V_X values by 1.90 and the V_z values by 0.72. The result of these operations showed reasonably circular contours in the chromatic value diagram for Munsell colors of the 1929 *Book of Color* and the preliminary smoothed values of the committee (see Fig. 5-12). Adams did not propose a color difference formula based on his V_X, V_Y, V_Z space, but assuming that it is a euclidean space, such a formula is obvious. Adams's chromatic value system proved to be influential in coming years.

Aside from the Munsell data, in flux until the release of the Renotations in 1943, there were few object color sample sets with statistically supported visual data. In 1941 Balinkin reported on a set of five pale green tiles. One pair of tiles was designated as the standard difference, and sixty observers estimated the differences of all possible pairs against the reference pair. The data were used in several studies of color difference formulas. In 1944 Nickerson and her co-worker Stultz investigated the usefulness of color difference calculation for color quality control work. The visual data consisted of category judgments (five categories) by twelve observers of painted textile samples, primarily in two color regions: yellowish brown and olive green. In this study they used, among others, a formula based on Adams's chromatic value space as follows:

$$\Delta E_A = \left[(\Delta V_Y)^2 + \{\Delta(V_X - V_Y)\}^2 + \{0.4\Delta(V_Z - V_Y)\}^2\right]^{0.5}, \tag{6-11}$$

where V_X ,V_Y, V_Z are the Adams chromatic values of the samples, now interpreted in terms of the revised Munsell value function. This formula became known as the Adams-Nickerson color difference formula and proved of enduring value. In their evaluations Nickerson and Stultz found the formula only marginally better than formulas derived by Judd but considerably easier to calculate. The experiments also indicated considerable individual variability in judgment.

In 1946 J. L. Saunderson and B. L. Milner proposed a modification of the Adams chromatic value system to obtain closer agreement with the Munsell system. Contours of constant chroma are somewhat eccentric in the Adams chromatic diagram, and the Saunderson-Milner solution corrected for the eccentricity using an empirical trigonometric method. The Saunderson-Milner color space model was described as the "Zeta" space, based on their use of the Greek letter. It is defined as

$$\begin{aligned} \zeta_1 &= (V_X - V_Y)(9.37 + 0.79\cos\Theta), \\ \zeta_2 &= kV_Y, \\ \zeta_3 &= (V_Z - V_Y)(3.33 + 0.87\sin\Theta), \end{aligned} \tag{6-12}$$

where Θ is the angle calculated from $\tan\Theta = 0.4(V_Z - V_Y)/(V_Z - V_Y)$, and k is a constant depending on the observation conditions. Color differences are calculated as the square root of the sum of the squares of the differences in the three ζ values. A somewhat different procedure with a comparable effect was proposed in 1952 by Godlove.

Assuming that two concentric circles of five equally spaced Munsell hues are a good representation of psychologically uniform space, Burnham in 1949 investigated the performance of ten formulas (including the CIE x, y and x, z diagrams) and found the Saunderson-Milner Zeta space to perform best. However, all of the formulas resulted in deviations that were in visual terms statistically significant.

6.4 JUDD'S MODEL OF MÜLLER'S THEORY OF COLOR VISION

In 1949 Judd began to develop a model of G. E. Müller's theory of color vision (to be discussed in more detail later in this chapter). Development continued into the 1960s and the model was influential in Friele's treatment of MacAdam's data resulting in the FMC I and FCM II formulas to be discussed below. The Müller-Judd space is further discussed in Section 6.17.

6.5 COLOR DIFFERENCE THRESHOLDS AND MATCHING ERROR

Threshold differences were first investigated along the spectrum because exact setting of spectral differences was technically relatively easy. However, spectral differences are complex in visual terms since every difference is composed of hue, saturation, and brightness components. Wavelength discrimination began to be investigated at the turn of the twentieth century. A classical investigation is that by W. D. Wright and F. H. G. Pitt (1934). Wright subsequently also investigated threshold differences along straight lines in the CIE chromaticity diagram. The results of both investigations are illustrated in Fig. 6-6 (Wright, 1941). Wright's important data were overshadowed by MacAdam's extensive color matching error data of a single observer, published a year later (1942). As discussed in Chapter 3, in MacAdam's work two hemi-fields were displayed against a dark surround in a specially constructed colorimeter. In one-half a standard color was displayed. The color of the second half could be adjusted by the observer along straight lines passing through the standard color in the CIE chromaticity diagram, in a constant luminance plane. Using a single knob, the observer adjusted the color of the test field by the method of adjustment until a visual match between the two hemi-fields was achieved. The match was approached along a given line from both sides. Color matches along several lines were repeatedly set for each of 25 standard colors. From the visual data (some 20,000 observations) MacAdam calculated the standard error of color matching for his single observer. The result, fitted with ellipses, is illustrated in Fig. 6-7. MacAdam also determined that the threshold differences around his standard colors were approximately twice the standard deviation of the color-matching errors. Subsequently comparable but three-dimensional contours involving also brightness differences were determined for additional observers, and the results indicated that observers vary significantly in this task (Brown and MacAdam, 1949; Brown, 1957). The MacAdam ellipses rapidly became a key set of data used as test data for line elements and color difference formulas. Color-matching error was explained in 1949 in terms of cone activity by Y. LeGrand (see Chapter 8). The ellipses obtained good confirmation in an experiment by R. M. Boynton and N. Kambe (1980). Additional determinations of color-matching error were made by G. Wyszecki and G. Fielder (1971). Determinations of achromatic and chromatic thresholds using industrially relevant conditions were performed by K. Richter (1985) and by Witt (1987, 1990). For a comparison of such data, see Chapter 8. As mentioned in the previous chapter, thresholds and color-matching error data of limited groups of colors and using various methodologies have been performed in recent years by several researchers.

MacAdam's Empirical Line Element

Using a method suitable for describing line elements independent of the form of the implied space (i.e., applicable to euclidean and non-euclidean space),

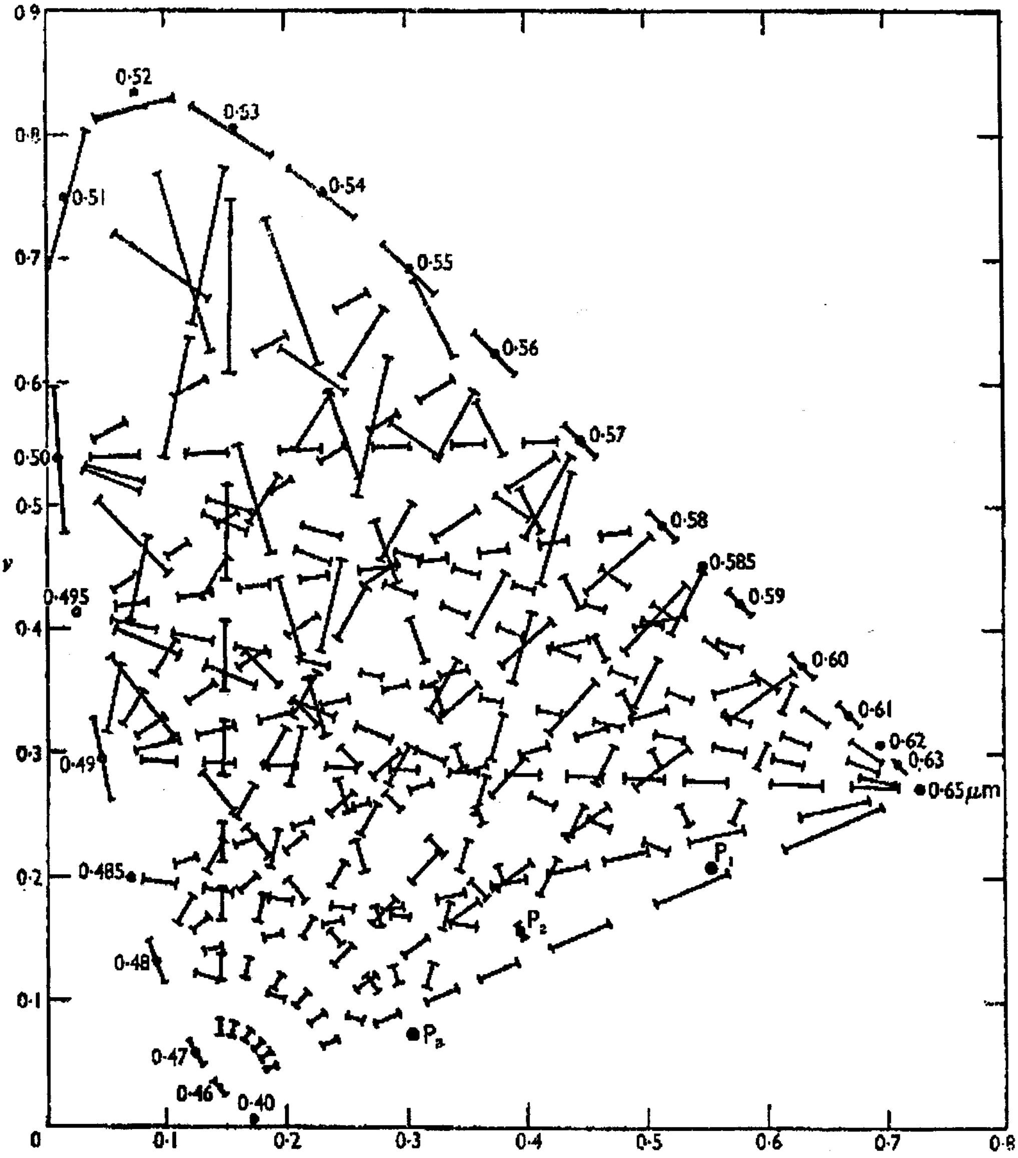

Fig. 6-6 Perceptually equal steps between spectral colors as well as colors in the interior of the chromaticity diagram, as determined by Wright (1969).

MacAdam calculated g_{ik} values from the following equation that describes his ellipses in geometrical terms:

$$(ds)^2 = g_{11}(dx)^2 + 2g_{12}dxdy + g_{22}(dy)^2 = 1, \tag{6-13}$$

Fig. 6-7 *Color matching error ellipses of observer PGN for 25 colors at the center of the ellipses, enlarged 10 times, in the CIE 2° chromaticity diagram. From MacAdam (1942).*

where ds is the distance between the center of an ellipse and a point on its contour and x, y are CIE chromaticity coordinates. MacAdam drew interpolating lines connecting his data points. Results are shown in Fig. 6-8*a–c*. Knowing the g_{ik} values for a particular location in the chromaticity diagram by reading them from tables or graphs with interpolation between neighboring values makes possible the calculation of color differences using eq. 6-13 and applying the square root. Alternately, the chromaticity diagram could be modified locally to convert the ellipses to circles of equal size. Sets of charts that simplified the calculation of color differences by this method have been available in the 1950s and 1960s. From the results of such calculation it was apparent that linear transformation of the CIE chromaticity diagram does not yield a uniform chromaticity diagram in which the MacAdam ellipses form circles of equal size.

By 1950 the situation presented itself as follows: On the one hand, there were line elements, either derived from theoretical considerations using best esti-

Fig. 6-8a–c *Contours of the g_{11}, $2g_{12}$, and g_{22} ellipse parameter functions in the CIE chromaticity diagram for the ellipses of Fig. 6-7.*

mates of cone sensitivity functions, or empirically as in the case of the MacAdam ellipses. On the other hand, there were linear transforms of the CIE chromaticity diagram as well as the Adams chromatic value diagram and modifications thereof in attempts to predict the Munsell data. In terms of psychophysics there were thus color space models based on accumulation of threshold differences (local psychophysics extended to global psychophysics) and models based on global scaling data.

Color Difference Formulas Derived from the MacAdam Data

In 1961 L. F. C. Friele began devising a formula with which MacAdam's ellipses could be described with good accuracy (Friele, 1961, 1965). The formula was slighly modified and simplified by MacAdam and later by K. D. Chickering and became known as the Friele-MacAdam–Chickering or FMC I formula.

Fig. 6-8a–c *(Continued)*

The Roman numeral distinguished it from a later version, FMC II, with a different treatment of lightness. (For an informative discussion of the development of these formulas see MacAdam 1981.) Friele based his approach on the three-stage color vision theory by G. E. Müller (1930). The CIE tristimulus values were converted to cone sensitivity functions P, Q, and S. Opponent color differences and lightness differences were calculated from these in two steps.

FMC II formula

$$P = 0.724X + 0.382Y - 0.098Z,$$

$$Q = -0.48X + 1.37Y + 0.1276Z,$$

$$S = 0.686Z,$$

$$\Delta C_{rg} = \frac{(Q\Delta P - P\Delta Q)}{(P^2 + Q^2)^{0.5}},$$

$$\Delta C_{yb} = \frac{S\Delta L_1}{(P^2 + Q^2)^{0.5}} - \Delta S,$$

Fig. 6-8a–c *(Continued)*

$$\Delta L_1 = \frac{(P\Delta P + Q\Delta Q)}{(P^2 + Q^2)^{0.5}},$$

$$\Delta L_2 = \frac{0.279\Delta L_1}{a},$$

$$\Delta C_1 = \left[\left(\frac{\Delta C_{rg}}{a}\right)^2 + \left(\frac{\Delta C_{yb}}{b}\right)^2\right]^{0.5},$$

$$a^2 = \frac{0.0000173(P^2 + Q^2)}{[1 + 2.73P^2Q^2/(P^4Q^4)]},$$

$$b^2 = 0.0003098(S^2 + 0.2015Y^2),$$

$$\Delta C = K_1 \Delta C_1,$$

$$\Delta L = K_2 \Delta L_2,$$

$$K_1 = 0.054 + 0.46Y^{1/3},$$
$$K_2 = 0.465K_1 - 0.062. \tag{6-14}$$

The total color difference is calculated from the chromatic and the lightness difference as

$$\Delta E = \left[(\Delta C)^2 + (\Delta L^2)\right]^{0.5}. \tag{6-15}$$

The two K functions have the purpose of adjusting the size of the implied ellipse as a function of luminous reflectance and to adjust the lightness value to be in reasonable agreement with Munsell lightness. They are shown above in the simplified form proposed by MacAdam.

A uniform (in terms of color-matching error) color space model is implicit in this difference formula, but it has not been stated explicitly. The formula is an elaborate fitting of the MacAdam ellipses in a cone-based opponent color framework. It represents a compromise in regard to size of the ellipse as a function of luminous reflectance. MacAdam had found that the color-matching error ellipses were affected only to a small extent (less than 20%) by changes within reasonable levels in luminosity. The K_1 function made them change in size in line with cube root compression. In his work with Brown, MacAdam had determined that ellipsoids generated from threshold determinations including luminance differences had one of their three axes parallel to the luminance axis. In the Adams chromatic value space, on the other hand, corresponding ellipsoids are tilted toward the neutral point of the chromatic diagram. One of the advantages of the FMC formula is that it implicitly adjusts for the Helmholtz-Kohlrausch effect.

Nonlinear Transformation of the CIE Chromaticity Diagram

MacAdam continued efforts to represent his ellipses in a chromaticity diagram in which they would appear as circles of equal size. He abandoned a cone sensitivity based solution and concentrated on mathematical methods. Using at first a paper and scissor method and then stepwise linear regression he developed a geodesic chromaticity diagram, illustrated in Fig. 6-9.

Using four linear transforms of the CIE chromaticity diagram, MacAdam, 1965 proposed the following chromaticity coordinates for this diagram:

$$\xi = 3751a_1^2 - 10a_1^4 - 520b_1^2 + 13295b_1^3 + 32327a_1b_1 - 25492_1^2 b_1$$
$$-41672a_1b_1^2 + 10a_1^3b_1 - 5227a_1^{0.5} + 2952a_1^{0.25}$$

$$\eta = 404b_2 - 185b_2^2 + 52b_2^3 + 69a_2(1 - b_2^2) - 3a_2^2b_2 + 30a_2b_2^3, \tag{6-16}$$

Fig. 6-9 *MacAdam's geodesic uniform chromaticity diagram, 1965. Lines of constant value of chromaticity coordinates x and y are drawn in the interior.*

where

$$a_1 = 10x/(2.4x + 34y + 1),$$

$$a_2 = 10x/(4.2y - x + 1),$$

$$b_1 = 10y/(2.4x + 34y + 1),$$

$$b_2 = 10y/(4.2y - x + 1).$$

In evaluations (to be discussed below) against a growing number of sets of visual color difference data involving object color samples, formulas based on MacAdam's ellipses performed relatively poorly, and the development of color space and color difference model formulas generally proceeded along different routes.

More recently a nonlinear transformation was proposed by C. Oleari (2001). It uses an angular transformation of the CIE chromaticity diagram, with dilatation, to form the δ, υ chromaticity diagram in which the MacAdam ellipses approach circles of equal size. When this diagram is applied to the CIE 10° observer and the OSA-UCS colors are plotted in it, they are aligned along slanted lines which Oleari believes are due to the difference in the two and ten degree observers, which he corrected with an additional dilatation.

6.6 FURTHER DEVELOPMENT OF FORMULAS BASED ON OPPONENT COLOR SYSTEMS

Realizing the opportunities arising from instrumental quality control of colored materials, R. S. Hunter began to build filter tristimulus colorimeters in 1948. These were relatively inexpensive instruments that could determine CIE tristimulus values directly and with good accuracy from four measurements per sample. Hunter, in addition, provided for output from the instruments directly in terms of color difference values. Hunter proposed the following formulas for this purpose:

$$L = 10Y^{0.5},$$
$$a_L = \frac{17.5(1.02X - Y)}{Y^{0.5}},$$
$$b_L = \frac{7.0(Y - 0.847Z)}{Y^{0.5}}, \tag{6-17}$$

where X, Y, Z are the CIE tristimulus values for illuminant C taken as percentages. The total color difference was given as the square root of the sum of the squares of the differences in L, a_L, and b_L. These instruments were quite popular and continued to be used into the 1980s. Kuehni showed that unit chromatic contours derived from the formula were not in good agreement with ellipses fitted to small color difference data (1982).

The calculation of the Munsell value function from luminous reflectance with the quintic formula was troublesome, and efforts were made to find a simpler solution. It was found that properly scaled cube roots of luminous reflectance resulted in good agreement with the quintic formula. In 1958 L. G. Glasser, A. H. McKinney, C. D. Reilley, and P. D. Schnelle proposed a cube root model of color space:

$$L^* = 25.29G^{1/3} - 18.38,$$
$$a^* = K_a(R^{1/3} - G^{1/3}),$$
$$b^* = K_b(G^{1/3} - B^{1.3}), \tag{6-18}$$

where $R = 1.02X$, $G = Y$, $B = 0.847Z$, $K_a = 106.0$, and $K_b = 42.34$. For closer agreement with the Munsell data the K_a and K_b constants were adjusted by quadrant. The total color difference was calculated as the square root of the sum of the squares of the differences in L^*, a^*, and b^*. With square and cube root tables and/or then modern electronic calculators, color differences from these formulas could be calculated comparatively rapidly.

In the 1960s, then, there were several competing formulas that were used in industry to assess quality of coloration. Among these were the FMC II, the Adams-Nickerson or the cube root, Saunderson-Milner, CIE 1964, and the Hunter *L,a,b* formulas.

6.7 NEW SMALL COLOR DIFFERENCE DATA

The unsatisfactory situation in regard to industrial color quality control using formulas available by 1960 resulted in several new sets of visual data and considerable activity in testing these data and developing formula modifications. The following sets of data were developed from the 1950s until 1980:

1. Davidson and Friede (1953). Data consisted of 287 pairs of wool textile samples around 19 standards, evaluated by 8 observers in terms of acceptability as a production match to the standards. The results were expressed as acceptability percentages.
2. Robinson (1969). Data consisted of a single blue paint standard with 31 samples around it evaluated by 132 observers with the results expressed as acceptability percentages.
3. Thurner and Walther (1969). The data consisted of 500 textile samples around 27 standards, assessed for acceptability by 16 textile colorists.
4. Kuehni (1971a). 113 samples around three standards, pigments on polyester/cotton fabric, were assessed for acceptability by 10 observers.
5. Kuehni/Metropolitan Section (1971b). Data consisted of 180 textile samples around 10 standards, assessed for acceptability by 16 observers.
6. HATRA (Jaeckel 1975). 854 textile samples around 12 standards were assessed for acceptability by 24 to 32 observers.
7. MMB (Morley, Munn and Billmeyer 1975). 555 pairs of painted samples around 19 standards were assessed by 20 observers according to four categories from no difference to very large difference. Psychometric scales were constructed from the visual data.
8. VVVR (Friele 1978). 20 glossy painted samples each around 10 standards were assessed by 14 observers using a pair-comparison method and by 15 to 25 observers using the acceptability method. Psychometric scales were constructed.
9. ISCC (Kuehni and Marcus 1979). 180 samples around six standards were assessed, four consisting of matte paint on cardboard and two of dyed textiles. They were assesses by 26 to 37 observers using two assessment methods: (a) subjective estimates on a scale with a maximum value of 10; (b) acceptability as a commercial match. Psychometric scales were constructed from the subjective estimates.

Most of these data sets were established by industrial specialists interested in a color difference formula for quality control purposes with a reliability at least equal to that of an average observer.

In 1970 the Society of Dyers and Colourists in England recommended the use of the Adams-Nickerson formula (called ANLAB) for application in the textile industry based on extensive evaluations of formulas against various sets of visual data. It had been shown to be equal or marginally better than a dozen other formulas under consideration. (In 1973 Kuehni found that 13 different formulas were in use in U.S. color-related industries.) Levels of correlation between visual and calculated data for all of these, including ANLAB, were unsatisfactory. Formulas performed poorer than the average observer. D. Strocka showed in 1971 that a simple formula based on circles of equal size in the CIE chromaticity diagram and a simple lightness formula resulted in levels of correlation comparable to that obtained with the best formulas.

Looking for causes of the low level of correlation between visual data and formula, K. MacLaren (1971) used multiple linear regression in the directions of lightness, metric chroma, and hue angle. He was able to show that the magnitude of perceptually equal color differences, when calculated with formulas such as Adams-Nickerson, increased with increasing metric chroma and lightness.

6.8 ELLIPSE AND ELLIPSOID FITTING

Strocka's results with the circular formula were puzzling but indicated that the actual unit difference contours in the CIE chromaticity diagram could not be circles, since the correlation with the circular formula would have been greater. The implication was that the ellipsoidal contours implicit in a formula such as Adams-Nickerson did not match the ellipsoidal contours implicit in the visual data well, perhaps in direction as well as in size. In 1971 Kuehni graphically fitted unit ellipses in the CIE chromaticity diagram to various sets of visual data (Fig. 6-10; Kuehni 1971a). The results indicated that the major axes of the resulting ellipses were usually tilted 20° to 30° clockwise compared to the MacAdam ellipses and that ellipses tended to increase in size as chroma increased (as McLaren had found).

Kuehni asked MacAdam to fit the parameters of his xi-eta equation (see above) to fitted ellipses. The resulting, then unpublished, equations are as follows:

$$\begin{aligned}\xi = &-4067a^2 - 133a^4 - 1675b^2 - 38280b^3 + 46479ab - 12064a^2b \\ &- 2806ab^2 - 37a^3b + 843a^{0.5} - 10254a^{0.25},\end{aligned}$$

where $a = 10x/(2.5x + 34y + 1)$ and $b = 10y/(2.5x + 34y + 1)$;

$$\begin{aligned}\eta = &\, 343b - 125b^2 - 41b^3 + 31a + 41a^3 + 4a^4 - 4ab^2 - 75a^2b^2 \\ &- 85a^2b + 137ab^3, \end{aligned} \tag{6-19}$$

where $a = 10x/(4.3y - x + 1)$ and $b = 10y/(4.3y - x + 1)$.

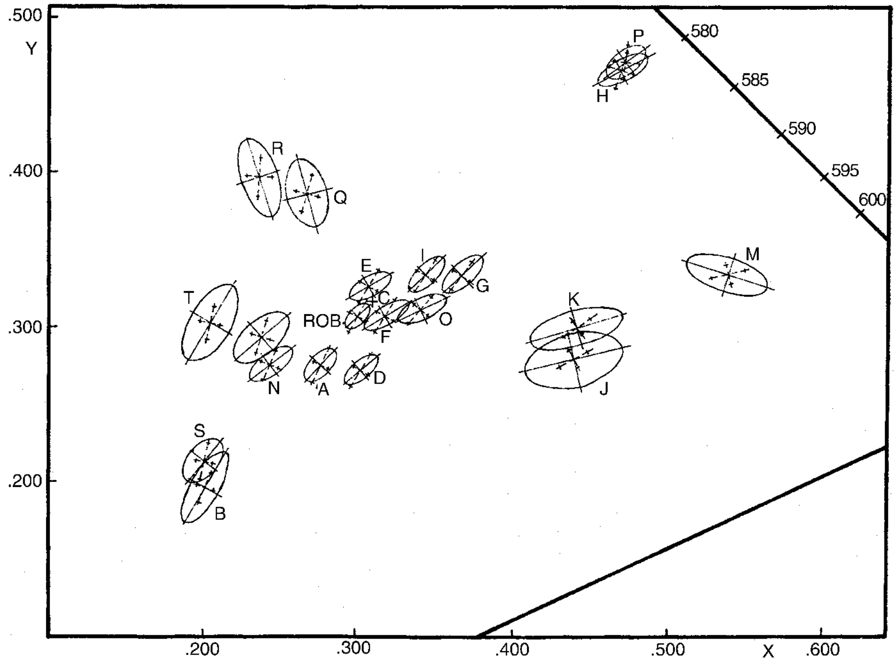

Fig. 6-10 Ellipses graphically fitted to small suprathreshold color difference data by Robinson 1962 (Rob), and by Davidson and Friede 1953 (A–T). From Kuehni (1971a).

The lightness scale used together with the formulas above was the cube root scale. The combined formulas resulted in a correlation with visual data significantly higher than the cube root and ANLAB or the circle formula.

In 1972 Kuehni refined the ellipse fitting method by systematically varying ellipse parameters until highest correlation for the samples around a standard were obtained. In 1975 R. M. Rich, F. W. Billmeyer, and W. G. Howe employed a computer algorithm using the maximum likelihood function to fit ellipses to visual data with greater statistical validity. Mathematical ellipse or ellipsoid fitting has since become a standard tool of investigation of color differences.

6.9 CONTROVERSIES OF DETAIL

A number of controversies abounded in the late 1960s to early 1970s. One of these involved the question of change in ellipse size as a function of lightness. MacAdam's findings using a visual spectrophotometer had, as mentioned earlier, indicated only small changes in the ellipse's size with changes in luminance. Formulas, such as Adams-Nickerson, on the other hand, implied significant changes in unit contours as a function of lightness.

As shown in Section 8.7 the final answer to this question has not been found yet.

Similarly Brown and MacAdam had found that the third axis of color-matching error ellipsoids was aligned parallel with the luminance axis. Formulas such as Adams-Nickerson have implied third axes that are tilted toward the white point of the color space. Work based on small suprathreshold differences reported by W. Schultze and L. Gall in 1971 indicated no tilt of the ellipsoids. The comparative success of power modulation-based formulas indicated that tilt is appropriate.

The dependence of the ellipse's size in a constant luminous reflectance plane on metric chroma has already been mentioned. In addition there was the controversy of perceptibility judgments versus acceptability judgments. While in perceptibility experiments presumably purely psychophysical judgments are obtained (but see Chapter 3), acceptability experiments can, and sometimes do, include additional cognitive overlays. Perceptibility experiments are usually difference magnitude estimations against a reference pair. Acceptability experiments also involve difference magnitude estimation, however, against an internal standard of acceptability in a commercial situation. Biases in the latter case are possible based on specific situations; that is, in a chromatic diagram the limit contour of acceptable color differences may not be positioned symmetrically around the standard. E. Allen and B. Yuhas (1984) and, more recently, Berns (1996) have shown how this situation can be mathematically treated. Another issue is that acceptability tolerances for identically colored materials may vary significantly depending on the context. However, experiments have shown that tolerance contours from acceptability judgments and unit difference contours from perceptibility judgments of the same sample pairs, when determined in the absence of a specific context, are symmetrical; that is to say, acceptability judgment in that situation is guided by perceptibility (Kuehni, 1975; McLaren, 1976; Mahy, Van Eycken, and Oosterlinck, 1994).

6.10 DEPENDENCE OF CALCULATED COLOR DIFFERENCE ON METRIC CHROMA

Multiple linear regression as well as fitting of ellipses and ellipsoids had clearly indicated a dependence of the chromatic differences (in the equal luminance plane) on metric chroma. In 1972 Kuehni proposed a modification to the Glasser et al. (see above) cube root color difference formula that adjusted the size of the calculated total color difference as a function of the radial difference of the standard from the illuminant point in the CIE chromaticity diagram:

$$\Delta E = \left[(\Delta C)^2 + (\Delta L)^2\right]^{0.5}, \tag{6-20}$$

where

$$\Delta C = \left[(\Delta a)^2 + (\Delta b)^2\right]^{0.5} \big/ F,$$

$$F = 1.0 + 6S,$$

$$S = \left[(x - 0.3100)^2 + (y - 0.3162)^2\right]^{0.5}.$$

S is applicable for illuminant C. The need for such an adjustment was ascribed to Takasaki's chromatic crispening effect. For a combined set of data the formula provided modest improvement over the FMC II or the unmodified Glasser formula.

McLaren's work with multiple linear regression had shown that it was useful to consider separately three components of the total color difference, assumed to be related in a euclidean manner: metric lightness, metric chroma, and metric hue differences. Metric chroma was calculated as the square root of the sum of the squares of the cartesian coordinates in the Adams-Nickerson chromatic diagram and metric lightness and metric chroma were subtracted in the euclidean manner from total color difference to result in metric hue difference:

$$\Delta H = \left[(\Delta E)^2 - (\Delta L)^2 - (\Delta C)^2\right]^{0.5}. \tag{6-21}$$

While the *L, a, b* view of the system is the cartesian coordinate view of the euclidean space, that of *L, C, H* is the polar coordinate view.

Using new experimental data, and expanding on McLaren's multiple linear regression, R. McDonald in 1974 found that he could obtain significant improvement in correlation between visual and calculated data by adjusting the total color difference as a function of metric chroma as follows:

$$\mathrm{DE}_a = \frac{\mathrm{DE}}{1 + 0.022C}, \tag{6-22}$$

where $\mathrm{DE_a}$ is the equivalent color difference at the achromatic point and C is the chroma value.

6.11 THE CIE 1976 *L*a*b** AND *L*u*v** SPACES

Because of the mentioned large number of different formulas used in industry, the CIE organization in the early 1970s became increasingly aware of the need to promote uniformity of practice in industrial color control. However, the emerging picture was less than clear, and superiority had been claimed for various formulas and in respect to frequently changing sets of visual data. After much discussion a compromise was found, and in 1976 two formulas were recommended for study and "in the interest of uniformity of usage" (CIE, 1976). One formula is a simplified version of the Adams-Nickerson formula, abbreviated CIELAB, the other the earlier mentioned modified version of MacAdam's 1937 linear transformation of the CIE chromaticity diagram, abbreviated CIELUV. The two formulas are defined as follows:

CIELUV

$$L^* = 116\left(\frac{Y}{Y_n}\right)^{0.333} - 16,$$

$$u^* = 13L^*(u' - u'_n),$$

$$v^* = 13L^*(v' - v'_n),$$

$$\Delta E^*_{uv} = \left[(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2\right]^{0.5},$$

$$u' = \frac{4X}{X + 15Y + 3Z},$$

$$v' = \frac{9Y}{X + 15Y + 3Z}, \qquad (6\text{-}23)$$

where

$$u'_n = \frac{4X_n}{X_n + 15Y_n + 3Z_n},$$

$$v'_n = \frac{9Y_n}{X_n + 15Y_n + 3Z_n},$$

and X_n, Y_n, Z_n are the tristimulus values of the nominally white object color stimulus. The optimal object color solid and the spectral trace derived from this formula are illustrated in Fig. 6-11.

CIELAB

$$L^* = 116\left(\frac{Y}{Y_n}\right)^{0.333} - 16,$$

$$a^* = 500\left[\left(\frac{X}{X_n}\right)^{0.333} - \left(\frac{Y}{Y_n}\right)^{0.333}\right],$$

$$b^* = 200\left[\left(\frac{Y}{Y_n}\right)^{0.333} - \left(\frac{Z}{Z_n}\right)^{0.333}\right],$$

$$\Delta E_{ab}^* = \left[(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2\right]^{0.5}, \qquad (6\text{-}24)$$

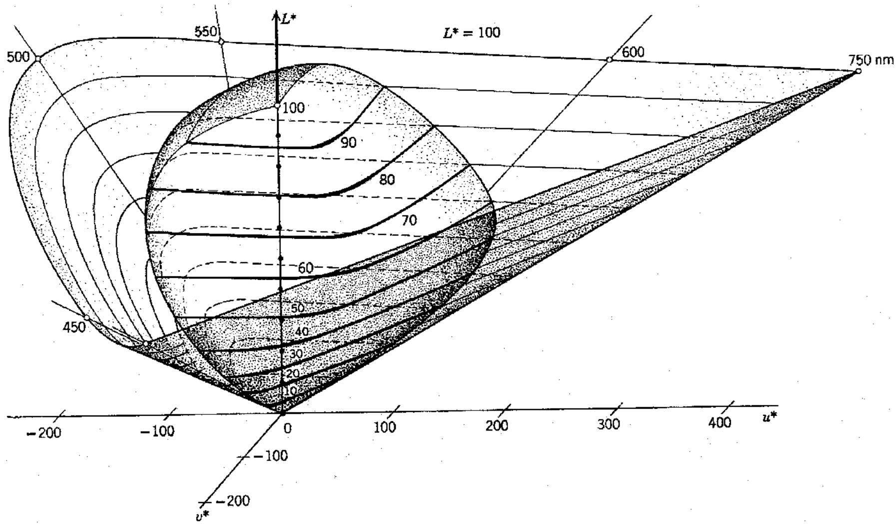

Fig. 6-11 Projective view of the L, u*, v* object color space for the CIE 10° standard observer and illuminant D65 (inner contour) and the spectrum locus. From Judd and Wyszecki (1975).*

where X_n, Y_n, Z_n are the tristimulus values of the nominally white object color stimulus. A slightly different calculation for L^*, a^*, and b^* applies for low tristimulus value ratios. For X/X_n, Y/Y_n, and $Z/Z_n > 0.01$, instead of cube roots, factors f are applied to the ratios, determined as follows: $f(Y/Y_n) = 7.787\ (Y/Y_n) + 16/116$ and comparably for Y and Z. This adjustment is valid for the L^*, a^*, and b^* scales.

Differences can also be calculated in a polar coordinate version, where

Metric chroma	$C^* = [(a^*)^2 + (b^*)^2]^{0.5}$
Hue angle	$h_{ab} = \arctan\ (b^*/a^*)$

From lightness and metric chroma differences, the total color difference ΔE is calculated as follows:

$$\Delta E = \left[(\Delta L^*)^2 + (\Delta C^*)^2 + (\Delta H^*)^2\right]^{0.5}, \tag{6-25}$$

where $\Delta H^* = [(\Delta E)^2 - (\Delta L^*)^2 - (\Delta C^*)^2]^{0.5}$ and $\Delta C^* = [(\Delta a^*)^2 + (\Delta b^*)^2]^{0.5}$. The optimal object color solid and the spectral trace derived from this formula are illustrated in Fig. 6-12.

As mentioned before, the former formula is primarily of interest to lighting engineers as it provides for additivity of light mixtures. The latter was recommended for use with object colors. CIELAB is a simplification of, but no advancement over, the Adams-Nickerson formula, and it was apparent at the

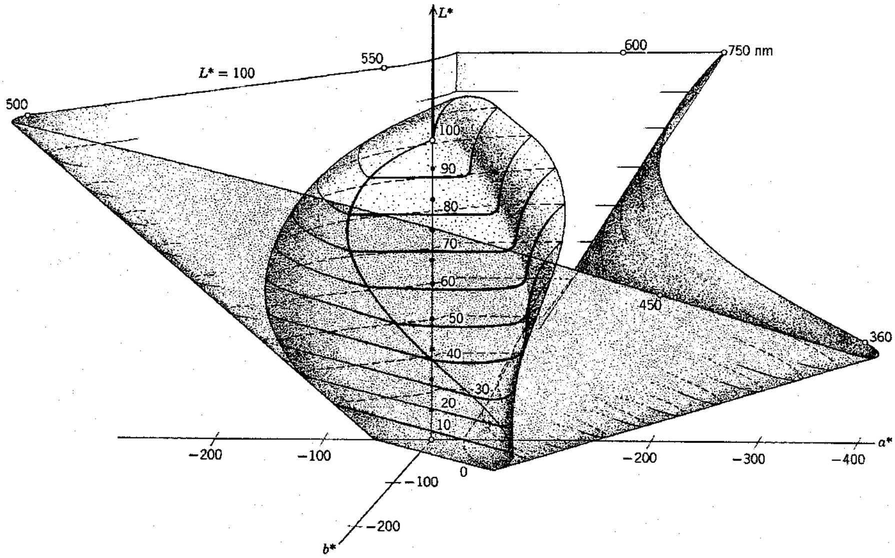

Fig. 6-12 Projective view of the L, a*, b* object color space for the CIE 10° standard observer and illuminant D65 (inner contour) and the spectrum locus. From Judd and Wyszecki (1975).*

time of its recommendation that further modification was necessary to improve the correlation of calculated with average visual color differences.

6.12 FRIELE'S FCM FORMULA

In 1978 Friele published a new formula optimized against various sets of visual data. It was a modification of his earlier effort resulting in the FMC metric. Friele had created psychometric scales for the various sets of acceptability data that he used in addition to perceptibility data. The formula provided significant improvements in correlation for these data, compared to CIELAB.

FCM

$$\begin{aligned} R &= 0.760X + 0.401Y - 0.124Z, \\ G &= -0.484X + 1.381Y + 0.079Z, \\ B &= 0.847Z. \end{aligned} \tag{6-26}$$

For R > G,

$$T = 125Y^{1/3}\left(\frac{1 - Y^{1/3}}{R^{1/3}}\right).$$

For R < G,

$$T = \frac{0.760}{-0.484} 125Y^{1/3}\left(\frac{1-Y^{1/3}}{G^{1/3}}\right).$$

$$D = \int_B^Y \left[\left(\frac{0.085x^{4/3}}{Y^{2/3}}\right)^2 + (0.055Y^{2/3})^2\right]^{-1/2} dx. \tag{6-27}$$

$$L = 18Y^{1/3},$$

$$a = T - 0.3D,$$

$$b = D - 0.3|T|,$$

$$\text{Chroma} = (a^2 + b^2)^{1/2},$$

$$\tau = \frac{0.024R^{4/3}}{Y^{2/3}},$$

$$\delta = \left[\left(\frac{0.085B^{4/3}}{Y^{2/3}}\right)^2 + (0.055Y^{2/3})\right]^{1/2}.$$

$$c^2 = T^2 + D^2,$$

$$\alpha = \arctan\left(\frac{D^2}{T^2}\right),$$

$$f = 1.6[1 - \exp(-0.0015c^2)]\sin 2\alpha + \exp(-0.0015c^2),$$

$$\Delta L = 6Y^{-2/3}\Delta Y$$

$$\tau\Delta T = 0.760\left(\Delta X - \frac{X}{Y}\Delta Y\right) - 0.124\left(\Delta Z - \frac{Z}{Y}\Delta Y\right),$$

$$\tau\Delta D = -0.847\left(\Delta Z - \frac{Z}{Y}\Delta Y\right),$$

$$\Delta E = \frac{2.5}{1 + 0.01Y}\left[(f\Delta L_1)^2 + (\Delta T^2) + (\Delta D)^2 - f\Delta T\Delta D\right]^{0.5}. \tag{6-28}$$

Parameter f_1 determines the weight of the lightness difference relative to the chromatic difference. It ranges from 0.4 to 1.0 for different data sets. The model represents an implementation of the Müller three-stage theory of color vision mentioned earlier. *R, G*, and *B* represent the first, photopigment stage, and *L, T*, and *D* the second stage opponent-response functions at the receptor level. To get to the third, opponent stage at the "optic nerve" level, *T* and *D* are modified, resulting in the rectangular coordinate system *L, a, b*. In this system, somewhat comparable to the CIELAB space, the unit difference contours are ellipsoids, rather than spheres, and the contours have to be normalized for the effect of chromatic crispening and the ellipse shape of the unit contour with the parameters τ and δ. Friele also implemented the controversial findings by Schultze and Gall (1971) that unit small color difference ellipsoids are not tilted toward the achromatic axis of color space.

The FCM formula, despite the improved correlation with visual data, did not receive any significant practical use, perhaps because of its complexity and the fact that its results were soon equaled by further optimization of the CIELAB formula.

6.13 RICHTER'S LABHNU2 FORMULA

In the late 1970s K. Richter proposed a series of formulas as good models for the Munsell and the OSA-UCS systems (Richter, 1980). Of these only the non-linear LABHNU2 formula will be mentioned:

$$a' = \frac{[(x/y) + (1/6)]^{2/3}}{15},$$

$$b' = \frac{[(z/y) + (1/6)]^{1/3}}{12}, \tag{6-29}$$

where *x, y, z* are CIE chromaticity coordinates. The CIELAB lightness formula is used for lightness difference calculation

6.14 WEIGHTING OF METRIC LIGHTNESS, CHROMA, AND HUE DIFFERENCES

JPC79 Formula

As mentioned above, in 1976 McLaren optimized weights for the three color difference components based on the Adams-Nickerson formula. He found that the optimum weights varied by set of visual data. McDonald continued to pursue individual adjustment of the three color difference components and developed new industrial visual data. In 1980 he proposed a formula employing continuous weight adjustment for all three metric components. It was based on an extensive set of 640 samples around 55 standards distributed over a significant portion of the object color solid. The samples consisted of dyed spun polyester sewing thread and the visual evaluations (acceptability judgments) were performed by eight industrial color matchers. An additional set of some 8500 judgments against 600 color standards by a single observer in an industrial dyehouse was also included. McDonald's color difference formula, called the JPC79 formula, is based on the ANLAB formula as follows:

$$\Delta E_{\mathrm{JPC79}} = \left[\left(\frac{\Delta L}{S_L} \right)^2 + \left(\frac{\Delta C}{S_C} \right)^2 + \left(\frac{\Delta H}{S_H} \right)^2 \right]^{0.5}, \tag{6-30}$$

where ΔL, ΔC, and ΔH are, respectively, the lightness, chroma, and hue differences calculated from ANLAB,

$$S_L = 0.08195L/(1 * 0.01765L),$$

$$S_C = [0.0638C/(1 + 0.0131C)] + 0.638,$$

$$S_H = S_C T,$$

$$T = 1 \text{ if } C = 0.38, \text{ otherwise}$$

$$T = 0.38 + |0.4\cos(h + 35)|, \text{ unless } h \text{ is between } 164° \text{ and } 345°, \text{ then}$$

$$T = 0.56 + |0.2\cos(h + 168)|.$$

The JPC79 formula represents the prototype of several further enhancements fitting formulas with the help of various functions to ellipsoids optimized to average visual data. It represents a practical, empirical approach to dealing with the non-euclidean nature of a uniform color space.

CMC (l : c) Formula

This formula was slightly modified from JPC79 and based on CIELAB component differences by the Color Measurement Committee of the Society of Dyers and Colourists in England (Clark, 1984). It has been standardized in England and in the United States, and is recommended by the International Standards Organization (ISO). It is defined as follows:

$$\Delta E_{\mathrm{CMC}} = \left[\left(\frac{\Delta L^*}{l S_L} \right)^2 + \left(\frac{\Delta C^*}{c S_C} \right)^2 + \left(\frac{\Delta H^*}{S_H} \right)^2 \right]^{0.5}, \tag{6-31}$$

where

$$S_L = 0.04097L^*/(1 + 0.01765L^*) \text{ unless } L^* < 16, \text{ then } S_L = 0.511,$$

$$S_C = [0.0638C^*/(1 + 0.0131C^*) + 0.638,$$

$$S_H = S_C(TF + 1 - F),$$

$$F = \left\{ (C^*)^4 / \left[(C^*)^4 + 1900 \right] \right\}^{0.5},$$

$$T = 0.38 + |0.4\cos(h + 35)|,$$

and l and c are additional weights adjusting the relative weight of lightness and chroma differences. Analysis has shown that for flat surface materials (paints, plastics) viewed in sharp juxtaposition values of 1 in both cases are appropriate, while for textile materials, the value $l = 2$ was found to improve correlation. CMC found wide international usage in industry and continues to be used in many firms.

6.15 NEW SETS OF VISUAL DATA

In 1978 the CIE issued a set of guidelines for coordinated research on color difference equations (Robertson, 1978). In these guidelines particular emphasis was placed on five color centers: gray, yellow, red, blue, and green. As a result several experimenters provided data for these centers but new data for many other centers were also established.

Luo and Rigg

As a prelude to their formula fitting (see below) R. Luo and B. Rigg assembled 13 previously published sets of perceptibility and acceptability data (Luo and Rigg, 1986). They fitted separate ellipsoids in x, y, Y space to each subset and compared these. The main difference for ellipsoids in the same neighborhood of global color space appeared to be a size factor (see Fig. 6-13 showing chromatic ellipses). This is understandable as the implicit acceptability tolerances of different observer groups are likely to be different and different scaling techniques had been used in the perceptibility judgments. The relative sizes were adjusted by judging 400 pairs of samples representing 70 color centers against a gray scale. Luo and Rigg also deleted certain subsets that they found to be internally inconsistent and, as a result of additional experiments, modified certain ellipses. No consideration was given to differences in surround in various data sets. The result was 132 ellipses that were considered to be reliable and formed a reasonably regular pattern in the CIE chromaticity diagram. Luo and Rigg found little difference between perceptibility and acceptability ellipses. The data are available on the University of Derby Web site (Derby, 1999).

Cheung and Rigg

In support of the CIE effort M. Cheung and Rigg established in 1986 suprathreshold small color difference data for the suggested five centers. The samples consisted of dyed wool fabric, with 59 to 82 pairs per center, assessed by 20 observers against a standard difference pair under artificial daylight and tungsten light against a neutral gray background of $Y = 13$. The samples had CIELAB color differences from 1 to 9 units from the respective standards.

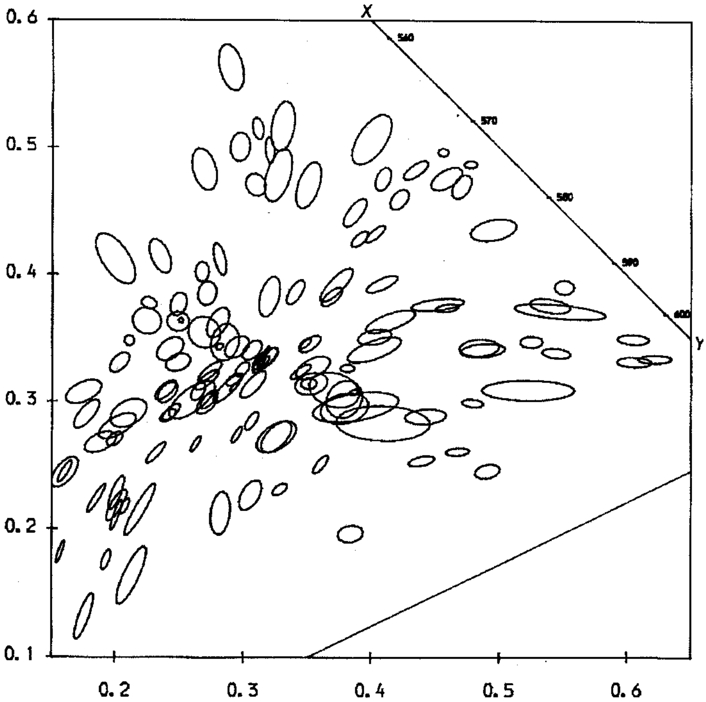

Fig. 6-13 Ellipses fitted to unadjusted data of the Luo and Rigg data set, enlarged 1.5 times, in a portion of the CIE chromaticity diagram. From Luo and Rigg (1986).

Badu and Rigg Large Difference Data

In connection with a Ph.D. thesis, A. Badu prepared in 1986 a set of data with relatively large differences. Samples were prepared to form a grid of approximately equilateral triangles on the a^*, b^* diagram. Some samples also involved lightness differences. There were 408 sample pairs, 238 consisting of nylon dyeings and 170 of glossy paint samples. They were visually evaluated by a group of 20 observers using a gray scale as standard.

Witt (1987–1990)

In 1987 K. Witt reported on the results of a threshold experiment using 50 to 64 sample pairs in four of the five CIE color regions prepared with high gloss acrylic paint. Some 24 observers determined at least four times (some observers 10 times) under simulated daylight and against a surround of Y = 20 if they could perceive a difference between the pairs. Ellipses were calculated from the resulting thresholds for individual observers and for sets of 4 to 22 observers with from 22 to 118 repetitions. Considerable intra- and interobserver variability was noted. Average color differences of the thresholds were approximately 0.4 CMC (2:1) units.

In 1990 Witt reported a further experiment involving the five CIE color centers. In addition to determining thresholds, observers were asked to assess perceptible differences in terms of hue, chroma, and lightness. Another purpose of the experiment was to determine the effects on the thresholds of changing the surround lightness and of a gap between the samples. Interestingly better agreement was found between the chromatic ellipse axis and the hue judgment axis in the CIE chromaticity diagram than in the a^*, b^* diagram. A small surround lightness effect and a larger gap effect were found.

RIT-DuPont Data

In an effort to develop a set of high-reliability small color difference perceptibility data D. Alman of DuPont and Berns of RIT (Rochester Institute of Technology) collaborated in 1990 and published the RIT-DuPont data (Berns et al., 1991). They are based on 156 acrylic lacquer spray-painted sample pairs on primed aluminum panels representing 19 color centers. The sample pairs did not have a common reference and were arranged so as to represent specific vector directions in CIELAB color space. The sample pairs, presented edge-to-edge on unprimed aluminum panels, were evaluated by 50 observers against a near gray standard pair presented identically. Probit analysis was used to establish psychometric scales, and the colorimetric values were set so that visual differences equal to that caused by a 1.0 ΔE CIELAB lightness difference resulted. Ellipsoids in CIELAB space have later been optimized to the visual data. The ellipses in the a^*, b^* plane show fair agreement with those of Luo and Rigg (see Fig. 5-29). Both show ellipses more or less pointed toward the origin of the diagram, except for blue colors. The data have been published and are also available on the University of Derby Web site.

Leeds Data

Two sets of data were established by D. H. Kim and J. H. Nobbs at Leeds University in 1997. They consist of matte painted samples. Sample set PC consists of 152 pairs evaluated juxtaposed by 15 observers against a near neutral reference pair. Sample set GS consists of 204 sample pairs evaluated against a gray scale by 12 observers. Color differences ranged from 0.4 to 3.7 CIELAB units in the two sets. The data are available on the University of Derby Web site.

Pointer and Attridge Large Difference Data

In 1997 Pointer and Attridge published the results of evaluation using a set of samples with relatively large differences (from approximately 2 to 20 CIELAB units). The samples were prepared by varying red, green, and blue exposures on photographic paper. Samples were mounted in gray, commercial 35 mm slide mounts, and displayed against an identically mounted near neutral standard pair. There are four subsets (value 5, saturated, light, and dark) each with six to eight colors in the same categories (red, green blue, cyan, magenta, yellow, skin, and neutral). The total set consists of 384 pairs. Comparison of pairs of samples and standard were made by nine observers against a neutral pair with a designated visual color difference of 10 units. Sixty percent of the sample pairs have less than 6 units of CIE94 color differences; that is, the set contains a large component of small color differences. The data are available from the authors.

Witt (1999)

In 1999 Witt published a set of data involving the five CIE standard color regions. For each region there are approximately 30 painted samples. These have been evaluated by from 10 to 13 observers in approximately 85 different combinations of two each, thus providing a detailed evaluation in each region of color difference vectors in two and three dimensions. Comparisons were made against a specially prepared gray scale (near logarithmic) found to be visually uniform, with the result expressed in steps and third or quarter fraction steps of the reference scale. Color differences ranged from approximately 0.3 to 8 CIELAB units. Means and standard deviations of the visual judgments were calculated. The coefficient of variation of the visual results ranged from 15% to 60%. The data have been published in Witt's paper and are available on the University of Derby Web site.

Guan and Luo Large Difference Data

These authors published in 1999 results of evaluations of a set of relatively large differences (6 to 21 CIELAB units). The samples form a grid of approximate squares in the a^*, b^* diagram at three different levels of L^* (40, 50, and 60). Samples were compared by ten observers not only in the a^*, b^* directions but also on diagonals, involving changes in both a^* and b^*. The samples consist of wool dyeings and they were evaluated in 292 pairs against an eight-step gray scale, also consisting of wool dyeings.

6.16 NEW FORMULAS

BFD (l:c)

Using the above-described composite data set, Luo and Rigg (1987) optimized a formula within the general CMC framework as follows:

$$\Delta E_{\mathrm{BFD}} = \left[\left(\frac{\Delta L}{l} \right)^2 + \left(\frac{\Delta C^*}{cD_C} \right)^2 + \left(\frac{\Delta H^*}{D_H} \right)^2 + R_T \left(\frac{\Delta C^*}{D_C} \right) \left(\frac{\Delta H^*}{D_H} \right) \right]^{0.5}, \quad (6\text{-}32)$$

where

$$D_C = 0.035\overline{C}*/(1+0.0365\overline{C}*)+0.521,$$

$$D_H = D_C(GT'+1-G),$$

$$G = \left\{(\overline{C}*)^4\Big/\left[(\overline{C})^4+14000\right]\right\}^{0.5},$$

$$T' = 0.627+0.055\cos(\overline{h}-254)-0.040\cos(2\overline{h}-136°)+0.070\cos(3\overline{h}-32°),$$

$$+\,0.049\cos(4\overline{h}+114°)-0.015\cos(5\overline{h}-103°),$$

$$R_T = R_H R_C,$$

$$R_H = -0.260\cos(\overline{h}-308°)-0.379\cos(2\overline{h}-160°)-0.636\cos(3\overline{h}+254°),$$

$$+\,0.226\cos(4\overline{h}+140°)-0.194\cos(5\overline{h}-280°),$$

$$R_C = \left\{(\overline{C}*)^6\Big/\left[(\overline{C})^6\right]+70000\right\}^{0.5},$$

$$L = 54.6\log_{10}(Y+1.5)-9.6.$$

ΔC^* and ΔH^* are CIELAB chroma and hue differences. The overbar indicates the mean value between standard and sample. Constants l and c are comparable to the corresponding constants in the CMC equation.

While D_C is a slight modification of S_C in CMC D_H is a complex function that corrects for experimental variation in the relationship between hue angle difference and visual hue difference around the hue circle. An additional term is added to accommodate ellipses of bluish colors not directed toward the origin.

SVF Formula

Also in 1986, a different approach was taken by T. Seim and A. Valberg. They fitted a formula to the Munsell Renotation data based on cone sensitivities. In the first step tristimulus values are converted to cone sensitivities that are then centered on white (S_1 comparable to L, S_2 comparable to M and S_3 comparable to S). The lightness value V is obtained with a hyperbolic formula that represents a close fit of the polynomial Munsell value formula:

$$V_Y = 40v_1(Y), \tag{6-33}$$

where

$$v_1(Y) = \frac{(Y-0.43)^{0.51}}{(Y-0.43)^{0.51}+31.75}.$$

Following Adams, the same model is applied to the two chromatic dimensions. Opponent color signals are calculated in two steps:

$$p_1 = v_1(S_1) - v_1(Y),$$
$$p_2 = v_1(Y) - v_1(S_3) \quad \text{if } S_3 \le Y,$$
$$p_2 = v_2(Y) - v_2(S_3) \quad \text{if } S_3 > Y \tag{6-34}$$

where

$$v_2(Y) = \frac{[(Y/k(V_Y)) - 0.1]^{0.86}}{[(Y/k(V_Y)) - 0.1]^{0.86} + 103.2},$$
$$k(V_Y) = 0.140 + 0.175V_Y.$$

The final opponent-color functions are calculated as

$$F_1 = 700\ p_1 - 54\ p_2,$$
$$F_2 = 96.5\ p_2.$$

Color differences are calculated as

$$\Delta E_{SVF} = \left[(\Delta F_1)^2 + (\Delta F_2)^2 + 2.3(\Delta V_Y)^2\right]^{0.5}.$$

Unlike in the Müller-Judd approach the p level does not involve the subtraction of cone responses but the subtraction of Y from the response of L in one case and the response of S in the other. As a result the yellow-blue axis is established in the final form (except for scaling) at the p level. The green-red axis needs to be rotated; thus semicolon the subtractive form of F_1. A different hyperbolic function is applied to $S > Y$ to account for the apparent change in the modulation in the Munsell system of the S function of yellowish and bluish colors, which we will encounter again later.

The formula provides a reasonably good fit to the Munsell system even though the yellowish and bluish constant hue lines are significantly curved. The formula was also applied to the OSA-UCS data where it is less successful because the implicit modulations in this system are different from those implicit in the Munsell system. Seim and Valberg applied the formula also to small suprathreshold color difference data with reportedly good results.

CIE94

In the early 1990s CIE technical committee 1–29 investigated three data sets considered reliable in regard to the relationship between ΔL^* and L^*, ΔC^*, and C^*, and ΔH^* and hue angle h (Berns, 1993). Considerable scatter was found in all three cases. As a result the committee optimized simple weights for CIELAB lightness, chroma, and hue differences and issued the formula known as CIE94:

$$\Delta E^*_{94} = \left[\left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C^*}{k_C S_C}\right)^2 + \left(\frac{\Delta H^*}{k_h S_h}\right)^2\right]^{0.5}, \tag{6-35}$$

where

$$S_L = 1,$$

$$S_C = 1 + 0.045C^*,$$

$$S_H = 1 + 0.015C^*.$$

The k constants are additional weights on the lightness, chroma, and hue differences. Under reference conditions they all equal 1. For textile applications k_L typically has a value of 2. No weighting of lightness differences was made because the scatter in the combined experimental data (without consideration of surround lightness) was too large for a statistically meaningful weight.

Kim and Nobbs Weights for CIELAB

In 1997 Kim and Nobbs proposed new lightness, chroma, and hue difference weights for the CIELAB formula based on an analysis of five data sets including the Luo and Rigg, RIT-DuPont, and Leeds data:

$$\Delta E_{\mathrm{LCD}} = \left[\frac{(\Delta L^*/S_L)^2}{K_L^2} + \frac{(\Delta C^*/S_C)^2 + (\Delta H^*/S_H)^2 + S_R \Delta C^* \Delta H^*}{K_{CH}^2}\right]^{0.5}, \tag{6-36}$$

where

$$S_L = 1 - 0.01L^* + 0.0002(L^*)^2, \text{ if } L^* < 50, \text{ then } S_L = 1.0,$$

$$S_C = (1 + 0.045C^*)S_{CH},$$

$$S_H = (1 + 0.015C^*)S_{HH},$$

$$S_R = \left[-C^*/(2 + 0.07C)^3\right]\sin(2\Delta\Theta),$$

$$\Delta\Theta = 30\ \exp\left\{-[h - 275/25]^2\right\},$$

$$K_L = K_{CH} = 1.0 \text{ for nontextile samples, } K_L = 1.5 \text{ for textile samples.}$$

Complex sinusoidal S_{CH} and S_{HH} functions have been developed that improved the correlation between calculated and visual data for some data sets but not others:

$$S_{CH} = 1 + 0.07\ \sin(h) - 0.16\ \cos(2h + 250) - 0.05\ \cos(3h) - 0.03\ \cos(4h),$$

$$S_{HH} = 1 + 0.03\ \cos(h + 60) + 0.12\ \cos(2h) + 0.12\ \cos(3h) - 0.07\ \cos(4h - 45).$$

Integrating Weights

Proposals to integrate the weights on chroma and hue differences in CIE94 directly into the calculation of modified a^* and b^* values were made by E. Rohner and D. C. Rich in 1996, by H. G. Völz in 1998 and 1999, and by K. Thomsen in 2000 (see also DIN99 below). In Thomsen's version,

$$a^{*\prime} = a^* f(C^*),$$
$$b^{*\prime} = b^* f(C^*), \tag{6-37}$$

where

$$f(C^*) = \frac{\ln(1+0.0531301C^*)}{0.0500951C^*}.$$

The two numerical factors have been optimized against CIE94 from 200,000 positions at constant L^* in the a^*, b^* diagram. The maximum discrepancy found by this method, compared to conventional calculation using CIE94, is 10.5%. Given the variability of the visual data behind the S_X weights, this is not problematical. The only direct advantage of such an approach is that euclidean relations are maintained. However, the additional optimization parameters added in CIEDE2000 (see below) would require new, more complex integration efforts to maintain euclidean relationship with questionable ultimate value, as all are based on the uncertain fundament of CIELAB. In addition CIEDE2000 is only applicable from threshold to 6 to 8 units of total difference.

Guan and Luo Large Difference Formula

In 1999 Guan and Luo fitted a formula to their large difference data mentioned above and to other large difference data sets. The formula, named GLAB by the authors, follows the CIE94 format with the following weights:

$$S_L = 0.76, \quad S_C = 1 + 0.016\, C_{\mathrm{sb}}, \quad S_H = 1.0,$$

where $C_{sb} = (C^*_{Std} C^*_{Spl})^{0.5}$; that is, the chroma value used is the average of those of the standard and the sample being compared. The factors K_L, K_C, K_H were taken as 1. From this proposal it is evident that different formulas and/or different modification functions are required for small and large differences.

Lübbe Adjustment for Surround Lightness

In 1999 Lübbe proposed a formula for adjusting the L^* scale for the effects of surround

$$L^{*\prime} = L^* - f(L^*_u, L^*)(L^*_u - L^*)$$

where $L^{*\prime}$ is the adjusted lightness value, L^* the original value, and L^*_u the original L^* value of the surround. f is an experimentally determined factor depending on the lightness of the test and the surround fields. It results in S-shaped functions of the relationship between L^* and $L^{*\prime}$.

DIN99 Formula

A new color space and difference formula was developed in 1999 in Germany (DIN 6176 2000). Its purpose is to apply the integration method proposed by Rohner and Rich (1996) to produce an euclidean space and difference formula with good performance against various sets of small color difference data. The formula went through several stages of development, and presently there are four versions in existence. Only the latest (and best performing) formula DIN99d is given here. It is based on the CIELAB formula and the X tristimulus values of reference, and test colors have been adjusted by the procedure proposed by Kuehni (1999).

$$
\begin{aligned}
X' &= 1.12X - 0.12Z,\\
L_{99\mathrm{d}} &= 325.22\ln(1+0.0036L^*),\\
e &= a^*\cos(50^\circ) + b^*\sin(50^\circ),\\
f &= 1.14[-a^*\sin(50^\circ) + b^*\cos(50^\circ)],\\
G &= (e^2+f^2)^{0.5},\\
C_{99\mathrm{d}} &= 22.5\ln(1+0.06G),\\
h_{99\mathrm{d}} &= \arctan\left(\frac{f}{e}\right) + 50^\circ,\\
a_{99\mathrm{d}} &= C_{99\mathrm{d}}\cos(h_{99\mathrm{d}}),\\
b_{99\mathrm{d}} &= C_{99\mathrm{d}}\sin(h_{99\mathrm{d}}),\\
\Delta E_{99\mathrm{d}} &= \frac{1}{k_E}(\Delta L_{99\mathrm{d}}^2 + \Delta a_{99\mathrm{d}}^2 + \Delta b_{99\mathrm{d}}^2)^{0.5}.
\end{aligned}
\tag{6-38}
$$

The formula has been tested against the same data used in testing CIEDE2000 (see below) and performs somewhat better than CMC and CIE94 but slightly inferior to CIEDE2000. Its error against the combined test data set is 35% (compared to 38% for CIE94 and 33% for CIEDE2000) (Cui et al., 2002).

Kuehni Optimization of the CIE94 Formula

In 2001 Kuehni published a report on his analysis of the RIT-DuPont, Witt, Leeds GS and PC, as well as the Pointer-Attridge large color-difference data (Kuehni, 2001b). As a result of the analysis, and limiting changes to those applicable to all visual data sets individually, he recommended a modified CIE94 formula. The first step consists of an adjustment of the $\bar{x}$ color-matching function as follows:

$$\bar{x}_{\mathrm{mod}} = 1.1\bar{x} - 0.1\bar{z}. \qquad (6\text{-}39)$$

The factor 1.1 is applicable to the CIE 10° observer data. It has a value of 1.06 for the 2° observer data. This adjustment results in a rotation of the b^* axis that better aligns the unit ellipses of blue colors with lines of constant hue angle.

The lightness scale is revised to make it dependent on surround lightness, resulting in a true opponent color scale. An S_C type adjustment for the lightness crispening effect is introduced:

$$L^\wedge = 116\left[\left(\frac{Y}{Y_0}\right)^{0.333} - \left(\frac{Y_s}{Y_0}\right)^{0.333}\right],$$

$$\Delta L = \frac{\Delta L^\wedge}{S_L}, \qquad (6\text{-}40)$$

where Y_S is the luminous reflectance of the surround and Y_0 that of the illuminant,

$$S_L = 1 + 0.010L^\wedge.$$

These changes resulted in relative improvements in prediction from 5% to 20% depending on the data set. Other changes were found to result in improved correlation with visual data for individual sets but not for all sets. Surprisingly, when used as a last step in the optimization, hue angle dependent functions to adjust the size of the hue difference component such as investigated by Kim and Nobbs (1997) or the function used in CIEDE2000 (see below), were found to have no meaningful positive effect on the correlation for any of the data sets investigated.

Kuehni found that the introduction of nonsystematic size adjustment factors for individual color groups, ranging from approximately 0.75 to 1.5, improved correlation significantly in all cases. This indicates considerable variation among and within data sets in how observer groups judge the size of color differences in specific locations in color space, compared to the prediction by the formula. The source of this variation has not yet been systematically investigated.

CIEDE2000

In 2001 Luo, Cui, and Rigg, based on extensive analysis of several sets of perceptual color difference data, proposed a modified BFD formula they initially called M2b. It is based on CIELAB with the following analytically arrived modifications:

$$L' = L^*,$$

$$a' = (1+G)a^*,$$

$$b' = b^*,$$

$$C'_{ab} = \left[(a')^2 + (b')^2\right]^{0.5},$$

$$h'_{ab} = \tan^{-1}\left(\frac{b'}{a'}\right),$$

$$G = 0.5\left\{1 - \left[\frac{\left(\overline{C}^*_{ab}\right)^7}{\left(\overline{C}^*_{ab}\right)^7 + (25)^7}\right]^{0.5}\right\}, \tag{6-41}$$

where $\overline{C}^*_{ab}$ is the arithmetic mean of the C^*_{ab} values for a pair of samples.

$$\Delta L' = L'_b - L'_s,$$

$$\Delta C'_{ab} = C'_{ab,b} - C'_{ab,s},$$

$$\Delta H'_{ab} = \left[2\left(C'_{ab,b}C'_{ab,s}\right)^{0.5} \sin\left(\frac{\Delta h'_{ab}}{2}\right)\right],$$

where $\Delta h'_{ab} = h'_{ab,b} - h'_{ab,s}$.

The subscripts b and s refer to the comparison sample and the standard sample, respectively.

$$\Delta E = \left[\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'_{ab}}{k_C S_C}\right)^2 + \left(\frac{\Delta H'_{ab}}{k_H S_H}\right)^2 + R_T\left(\frac{\Delta C'_{ab}}{k_C S_C}\right)\left(\frac{\Delta H'_{ab}}{k_H S_H}\right)\right]^{0.5}, \tag{6-42}$$

where

$$S_L = 1 + \frac{0.015(\overline{L}' - 50)^2}{\left[20 + (\overline{L}' - 50)^2\right]^{0.5}},$$

$$S_C = 1 + 0.045\overline{C}'_{ab},$$

$$S_H = 1 + 0.015\overline{C}'_{ab}T,$$

with $T = 1 - 0.17\cos(\overline{h}'_{ab} - 30°) + 0.24\cos(2h'_{ab}) + 0.32\cos(3\overline{h}'_{ab} + 6°) - 0.20\cos(4\overline{h}'_{ab} - 63°)$. $R_T = -\sin(2\Delta\Theta)R_C$ and

$$\Delta\Theta = 30\exp\left\{-\left[\frac{\left(\overline{h}'_{ab} - 275^{\circ}\right)}{25}\right]^2\right\},$$

$$R_C = 2\left(\frac{\overline{C}'^{7}_{ab}}{\overline{C}'^{7}_{ab} + 25^7}\right)^{0.5}.$$

$\overline{L}'$, $\overline{C}'_{ab}$, and $\overline{h}'_{ab}$ are the arithmetic means for a pair of samples of the respective individual values.

This formulation corrects for the slanted ellipses near the negative b^* axis, contracts the a^* axis near the neutral point so that ellipses located in that area can be treated as such, introduces a new lightness weighting function, and uses the hue difference weighting function T proposed by Berns at the Warsaw CIE meeting in 1999 (Berns, 2000). The new lightness difference weight is adjusted, without explicitly stating so, for a surround with $L^* = 50$. For the combined set of all data the new formula has an error of prediction of 33% compared to 38% for CIE 94 and CMC. The formula performs about equally well as CIE 94 for the RIT-DuPont data set and inferior to BFD for the Luo-Rigg (now BFD) set. It is marginally better with the Witt set and distinctly better with the Leeds and a new set of data, BIT, involving CRT display colors. The formula was adopted as CIEDE2000. Unit difference ellipses in the a^*, b^* diagram generated by this formula are illustrated in Fig. 6-14.

Sections 6.5 to 6.16 have focused on the development of industrially important color difference formulas for use in color quality control. This development has proceeded largely independent of development of models of color vision and their implications for color spaces. Such efforts in a few cases go much beyond those of industrial color difference formulas in that they attempt to encompass all aspects of color vision while color difference formulas tend to only reflect one set of test conditions: daylight illumination and a single, achromatic surround. Visual data sets involving physical samples are expensive to prepare, and visual evaluations of existing ones require different formula optimizations, for reasons that are not clear. Less expensive color difference evaluation using monitor colors is in its infancy, and the monitor conditions that produce identical results by the same observers to those obtained with comparable physical samples in simplified conditions are not yet known. In the area of formula fitting the CIE recommendation of the CIELAB formula had a strong effect in that since then most formulas, for better or worse, have been based on the foundation of CIELAB. As will be discussed in Chapter 9, progress over what is available now with CIEDE2000 depends on new systematic visual data and on a color space model more in agreement with facts than CIELAB.

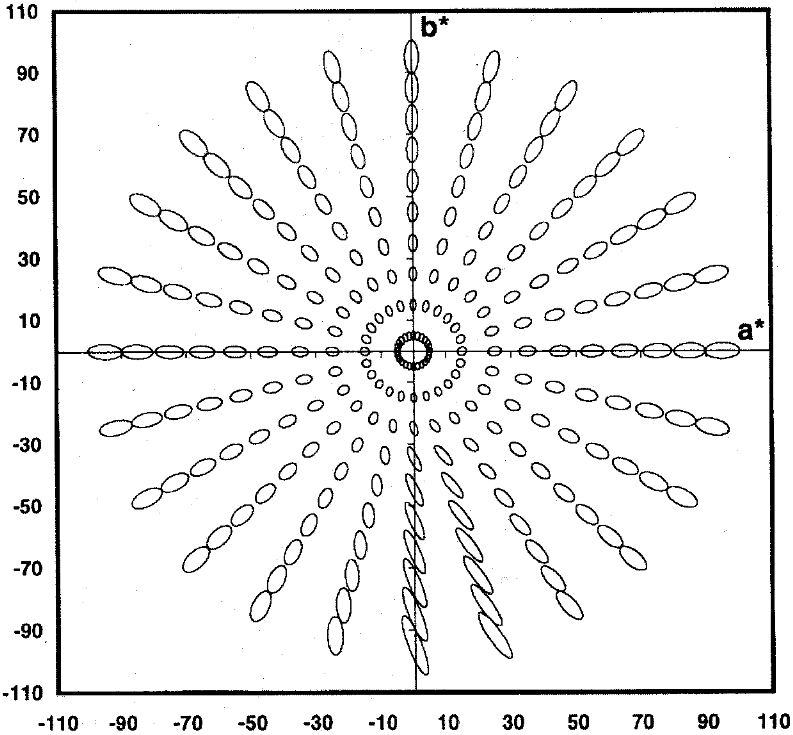

Fig. 6-14 Unit ellipses in the CIELAB a*, b* diagram as calculated from the CIEDE2000 formula. From Luo, Cui, and Rigg (2001).

6.17 COLOR SPACE FORMULAS AND COMPREHENSIVE MODELS OF COLOR VISION

Since the 1950s several comprehensive models of color vision have been proposed, some of which have had an impact on mathematical formulations of color space and color differences. A detailed discussion of this subject is outside the scope of this text and only a brief overview will be presented.

Müller-Judd

G. E. Müller (1850–1934) was a German psychophysicist much interested in color vision. He adopted Hering's theory of three reversible photochemical substances and attempted to bring an opponent color model in line with psychological facts. Müller developed the concept of cortical gray as representing the neutral state of color vision. In a series of articles published in 1896–97 titled *Zur Psychophysik der Gesichtsempfindungen* (On the psychophysics of the visual sense) he described a three-stage process of color vision and later expanded on the subject in a book-length treatise *Über die*

Farbenempfindungen (On color sensations, 1930). The first stage represents cone absorptions, followed by an intermediate differential second stage of coding and final neural coding in the third stage, resulting in chromatic and achromatic opponent color signals. The first two stages of the process are today loosely supported by neurophysiology, while the third stage continues to remain a hypothesis.

Beginning in 1949 and continuing until the end of his life, Judd worked on a model of the Müller theory, attempting to express it in terms of CIE tristimulus values. Three cone responses are derived from tristimulus values as follows:

$$\begin{aligned} P_1 &= 3.1956X + 2.4478Y - 0.6434Z, \\ P_2 &= -2.5455X + 7.0492Y + 0.4963Z, \\ P_3 &= 5.0000Z. \end{aligned} \tag{6-43}$$

In the second stage the cone signals are converted to intermediate opponent color signals as follows:

$$\begin{aligned} \alpha_1 &= P_1 - P_2, \\ \alpha_2 &= 0.015P_1 + 0.3849P_2 - 0.4000P_3. \end{aligned} \tag{6-44}$$

In the third stage these signals are converted to the final chromatic opponent color signals as follows:

$$\begin{aligned} \beta_1 &= \alpha_1 - 0.6265\alpha_2, \\ \beta_2 &= \alpha_2 + 0.1622\alpha_1. \end{aligned} \tag{6-45}$$

The achromatic final signal is equal to the CIE tristimulus value Y. The chromatic signal calculation can be reduced to expressions of tristimulus values with the following result:

$$\begin{aligned} \beta_1 &= 6.325(X - Y), \\ \beta_2 &= 2.004(Y - Z), \\ \beta_3 &= Y. \end{aligned} \tag{6-46}$$

At the third stage this model bears similarity to the Adams zone theory model and the Jameson-Hurvich model to be presented below. Judd used his implementation of the Müller theory to predict results of color vision impairment and good agreement with wavelength discrimination data of protanopes, tritanopes and normal dichromats was obtained (Judd and Yonemura, 1970). The efforts by Friele to use the Müller framework to develop color difference formulas were discussed above. The Müller framework has also been used by S. L. Guth (see section below).

Adams

As mentioned earlier, Adams offered a zone theory of color vision in 1923 and a chromatic value diagram based on it in 1942. He equated cone sensitivity with tristimulus values and applied the Munsell value function, representing the modulation of the output of the "cones," to all three tristimulus values. The chromatic signals were calculated as follows:

$$a = 1.9(V_X - V_Y),$$
$$b = 0.72(V_Z - V_Y), \tag{6-47}$$

where V_X, V_Y, and V_Z are square root Munsell value type functions of the CIE tristimulus values X, Y, and Z that Adams equated with cone responses. The constants were arrived at by comparing the ranges of chromatic value resulting from the color-matching functions and by bringing them into agreement with the Munsell value scale. The ratio is 2.64, larger than the theoretical value for a balanced linear system and larger than the value of 2.5 of the CIELAB space. As seen earlier, this model was used in modified form as a basis for the Adams-Nickerson color space and difference formula and the various formulas derived from it.

Hurvich and Jameson

In the early 1950s, as mentioned, Hurvich and Jameson developed an interest in the Hering theory by then widely disregarded by color science (except in Germany). They experimentally determined chromatic response functions by evaluating the amounts of certain chromatic stimuli required to cancel the hue of other stimuli. A typical result is shown in Fig. 6-15 and can be seen as in agreement with the Hering theory. The two spectral functions can be interpreted to represent a greenness-redness system (filled circles) and a yellowness-blueness system (open circles). Hurvich and Jameson took these functions to be linearly related to CIE color-matching functions, and their functions for white adaptation for the 1931 standard observer were shown in the previous chapter (Fig. 5-13). The general validity of such functions rests on the assumption that they are additive. Tests by Larimer and co-workers (1974–5) indicated the greenness-redness system to be additive and the yellowness-blueness system approximately so. The chromatic functions of Fig. 5-13 are approximated by the equations

$$a = \bar{x} - \bar{y},$$
$$b = 0.4\bar{y} - 0.4\bar{z}, \tag{6-48}$$

where $\bar{x}$, $\bar{y}$, $\bar{z}$, are the CIE 1931 color-matching functions. These functions can be expressed in relative terms as hue coefficients (Fig. 6-16), and the latter

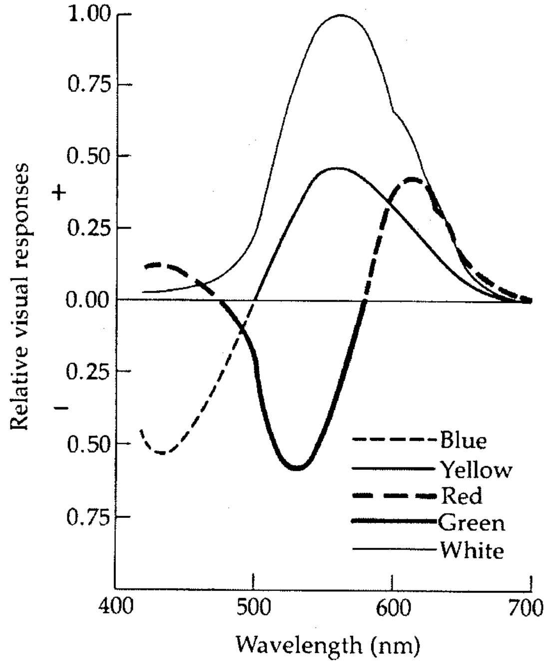

Fig. 6-15 Spectral opponent color (and lightness) functions determined from hue cancellation experiments, of a single observer. From Hurvich (1981).

were found to be in good agreement with the results of spectral hue naming experiments. Except for the weights, they are identical to the final Müller-Judd functions and in basic agreement with the Adams functions. Jameson also expressed the chromatic functions in terms of König type cone fundamentals (1972). The Hurvich-Jameson functions provide good support, derived through the paradigm of hue cancellation, for the basic ideas of Hering, Müller-Judd, and Adams.

Hurvich and Jameson proposed a polar chromatic diagram they termed a psychological diagram. It is derived by multiplying a spectral power distribution with their two chromatic functions a and b, based on their own experimental data. The two functions form the axes of a polar diagram (see Fig. 6-17) where saturation is calculated as the euclidean sum of the two chromatic responses. Conceptually unique hues fall on the axes of this diagram. However, their chromatic functions resemble the functions of equation (6-48), and average experimental unique red and green have significant positive b values. The diagram is essentially identical to a linear opponent diagram in polar coordinate form based on CIE color-matching functions. For a description of the work of Jameson and Hurvich related to the Hering system, see Hurvich (1981).

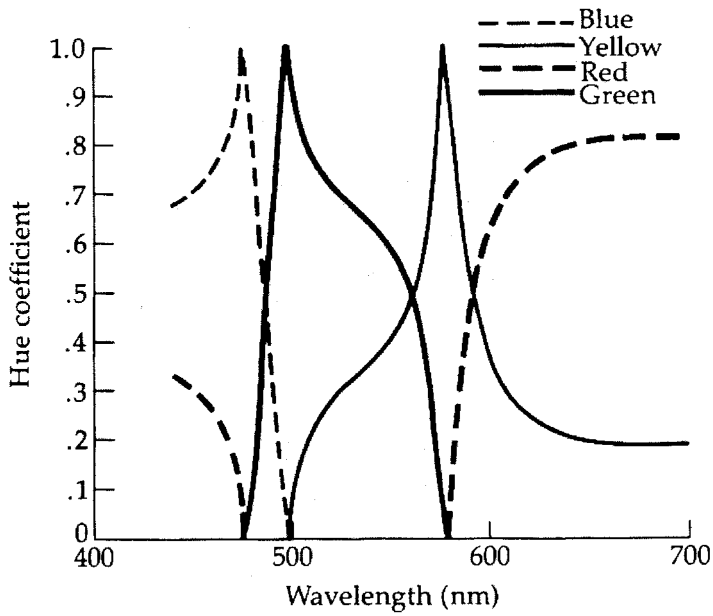

Fig. 6-16 *Spectral hue coefficient functions for an average observer (Hurvich, 1981). Compare with Figs. 4-8 and 4-9.*

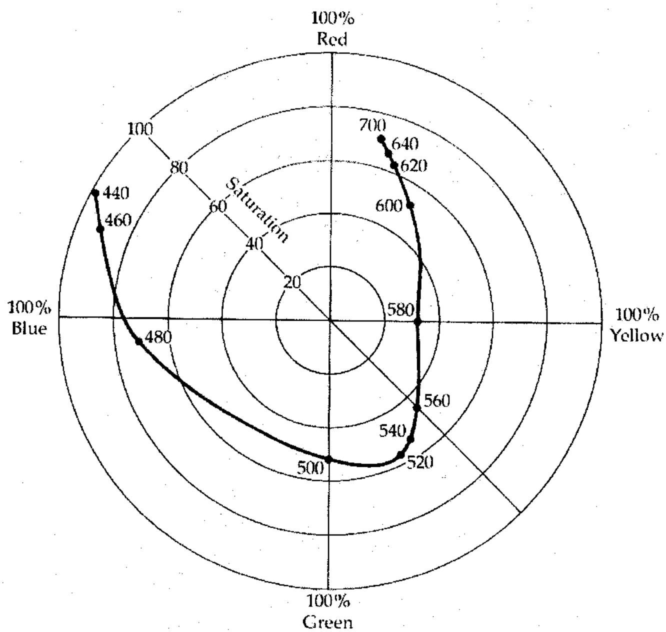

Fig. 6-17 *The polar hue/saturation diagram of Hurvich and Jameson (Hurvich, 1981). Spectral colors fall on the heavy line in the interior.*

Guth ATD Color Vision Model

A comprehensive color vision model has been developed by Guth and coworkers since the early 1970s (Guth, 1972, 1991). It is described as a modernized form of the Müller model. It was revised several times since the original proposal, and the most recent version is known as ATD 01 (Guth, 2001). Since different sets of visual data require different parameters and parameter functions for best fit, there is no single version applicable to all aspects of color vision. Guth does not consider ATD to be a model but "a quantitative theory of color vision."

CIE tristimulus values of the test and the surround are converted to modulated cone responses according to

$$\begin{aligned} L &= (0.16X + 0.56Y - 0.034Z)^{0.70}, \\ M &= (-0.40X + 1.16Y + 0.084Z)^{0.70}, \\ Z &= (0.017Y + 0.27Z)^{0.70}. \end{aligned} \tag{6-49}$$

Output from the three cone types is subjected to gain control by multiplying them with their attenuation factors:

$$\begin{aligned} &\frac{\sigma}{\sigma + L + k(L_a - L)}, \\ &\frac{\sigma}{\sigma + M + k(M_a - M)}, \\ &\frac{\sigma}{\sigma + S + k(S_a - S)}, \end{aligned} \tag{6-50}$$

where $\sigma = 200$ and $k = 5.5$, and subscript a denotes the surround (adapting field). The equations apply only if the cone response of the surround is larger than that of the test field; otherwise, the difference between the test and surround cone responses in the calculation of the attenuation factors is taken as zero. The resulting cone responses after gain control (subscript g) are used to calculate uncompressed responses for stage 1 and stage 2 mechanisms as follows:

$$\begin{aligned} A_{1i} &= 3.75L_g + 2.64M_g, \quad A_{2i} = 0.09A_{1i}, \\ T_{1i} &= 6.90L_g - 6.90M_g, \quad T_{2i} = 0.60T_{1i} + 0.75D_{1i}, \\ D_{1i} &= -0.83L_g + 1.00S_g, \quad D_{2i} = -D_{1i}, \end{aligned} \tag{6-51}$$

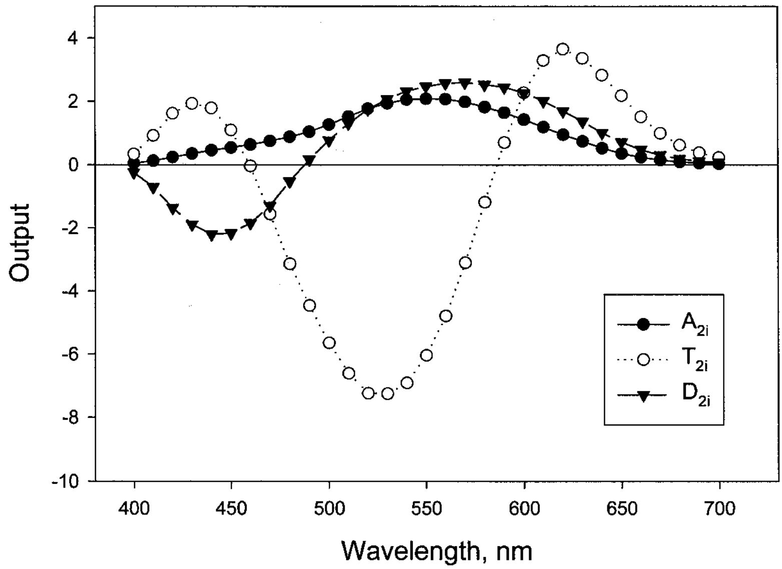

Fig. 6-18 *Spectral functions of Guth's A, T, D color space.*

where subscripts 1 and 2 refer to the mechanism stages and subscript i to the initial uncompressed response. The uncompressed stage 2 opponent color responses are illustrated in Fig. 6-18 for comparison to the Jameson-Hurvich functions and other functions below.

In the next step the final compression for the second-stage ATD values are calculated as follows (the compressed values for the stage 1 ATD values are calculated in the same manner):

$$\begin{aligned} A_2 &= A_{2i}(200 + |A_{2i}|), \\ T_2 &= T_{2i}(200 + |T_{2i}|), \\ D_2 &= D_{2i}(200 + |D_{2i}|). \end{aligned} \tag{6-52}$$

At stage 1 the brightness signal is equal to the vector sum of the compressed achromatic and two chromatic components. Lightness is calculated as the brightness compared to the brightness of the reference white (scale 100). Colorfulness, chroma, and saturation are all calculated according to

$$Co = C = Sa = \left(T_2^2 + D_2^2\right)^{0.50} A_2.$$

Hue H is expressed as hue angle and calculated as $H = \arctan\,(D_2/T_2)$.

Color differences are calculated differently, according to their magnitude. Small color differences (MacAdam ellipses, small suprathreshold differences) are calculated as the square root of the sum of the squares of the three component vector differences at the compressed stage 1 level, while large color differences are calculated in the same manner at the compressed stage 2 level.

Many kinds of data have been predicted, with varying success, with different versions of the model. It has not found industrial use as a uniform color space formula. As a color appearance model the ATD95 version has received faint praise by Fairchild (1998).

OSA-UCS Space Formula

In connection with the development of the OSA-UCS system (see Chapters 2 and 8) the committee also developed a formula to describe the system in terms of the CIE 10° observer data and illuminant D65 (MacAdam, 1974). Tristimulus values are converted to cone sensitivity values that are much different from those of Smith and Pokorny:

$$\begin{aligned} R_{10} &= 0.799X_{10} + 0.4194Y_{10} - 0.1648Z_{10}, \\ G_{10} &= -0.4493X_{10} + 1.3265Y_{10} + 0.0927X_{10}, \\ B_{10} &= -0.1149X_{10} + 0.3394Y_{10} + 0.717Z_{10}. \end{aligned} \tag{6-54}$$

Two chromaticness and a lightness coordinate are calculated as follows:

$$\begin{aligned} g &= C(-13.7R^{1/3} + 17.7G^{1/3} - 4B^{1/3}), \\ j &= C(1.7R^{1/3} + 8G^{1/3} - 9.7B^{1/3}), \\ L &= 5.9\left[\frac{Y_0^{1/3} - 2}{3 + 0.042(Y_0 - 30)^{1/3}}\right], \end{aligned}$$

where

$$C = \frac{1 + 0.042(Y_0 - 30)^{1/3}}{Y_0^{1/3} - 2/3},$$

$$Y_0 = Y(4.4934x^2 + 4.3034y^2 - 4.276xy - 1.3744x - 2.5643y + 1.8103).$$

Variable C adjusts chromaticness for the lightness crispening effect by a modified Semmelroth (1970) formula. Variable Y_0 adjusts lightness and chromaticness for the Helmholtz-Kohlrausch effect with a formula modified from that of Sanders and Wyszecki (1958). (Both factors are discussed in Chapter 5.) The committee explicitly stated that the formula is not to be used for color difference calculation of small color differences. The spectral R, G, B and g, j functions of the space are illustrated in Fig. 7-17*a* and *b*.

Cohen's Fundamental Color Space

Beginning in 1982 J. B. Cohen and W. E. Kappauf began to describe what Cohen later termed fundamental color space (FCS; Cohen and Kappauf, 1982; Cohen, 2001). They found that all metamers of a particular spectral power distribution (SPD) can be shown to consist of a common component (the fundamental) and a variable component (metameric black), as predicted by Wyszecki in 1953. Mathematically the fundamentals can be extracted from an SPD with the help of an orthogonal projector Cohen called matrix *R*. Matrix *R* projects spectral power distributions into the FCS, the mathematical space of all possible fundamentals. The axes of FCS can be arbitrarily selected, but Cohen proposed two preferred reference frames, the canonical frame where one of the orthonormal functions is the equal energy function and the frame where the three axes are orthogonal spectral vectors (455, 513, and 584 nm). Since the CIE luminous reflectance function is different from the equal energy and any of the orthogonal vector functions, neither configuration is comparable to the CIE *X, Y, Z* space.

There is no reason why FCS should be perceptually uniform, and it is not. Burns and co-workers (1990) projected the spectral power distributions of the color chips of the 1929 Munsell *Book of Color* as viewed under illuminant *C* into two different versions of FCS. Only when using the luminosity function as one of the three axes of FCS did the samples arrange themselves into layers that correspond to Munsell value (Fig. 6-19). When the space is viewed from the top (along the luminosity axis), they do not form concentric circles as one might expect but a somewhat elliptical cloud. Data plotting confirms the elliptical nature of the constant chroma contours. The spectral orthonormal *F* functions for this situation are illustrated in Fig. 6-20. The two chromatic functions are significantly different from opponent color functions such as shown in Fig. 6-15.

The Cardinal Planes Space of Derrington, Krauskopf, and Lennie

As mentioned in Chapter 5, in 1984 Derrington and co-workers reported on their findings regarding chromatic mechnisms in the lateral geniculate nuclei (LGN) of macaque monkeys. Given the close genetic relationship to humans results from macaques are considered relevant to human color vision. Measuring parvocellular activity in the LGN of their test objects the authors identified two types of cells with opponent chromatic activity: $R - G$ and $B - (R + G)$ (or the reverse). Taking cone cells of the macaques to be comparable to those of humans, they analyzed the results in terms of Smith and Pokorny's fundamental cell responses and identified three cardinal planes in a space taken to be euclidean. The constant luminance plane has a constant $R + G$ (cone response) axis and perpendicular to it a constant B axis. Perpendicular to the resulting plane is the luminance axis (Fig. 6-21). Colors are defined by their azimuth Φ and elevation Θ in the space. Derrington et al. determined the

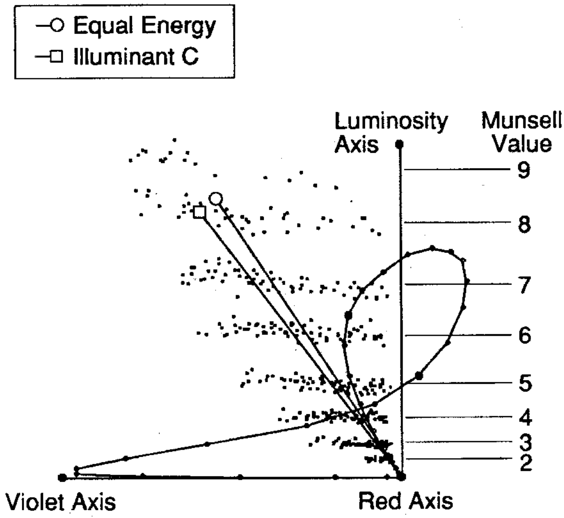

Fig. 6-19 *Colors of the 1929* Munsell Book of Color *as viewed under CIE illuminant C in Cohen's RLV fundamental color space (Burns et al., 1990). The spectral trace is also shown as are the vectors of the equal energy illuminant and illuminant C.*

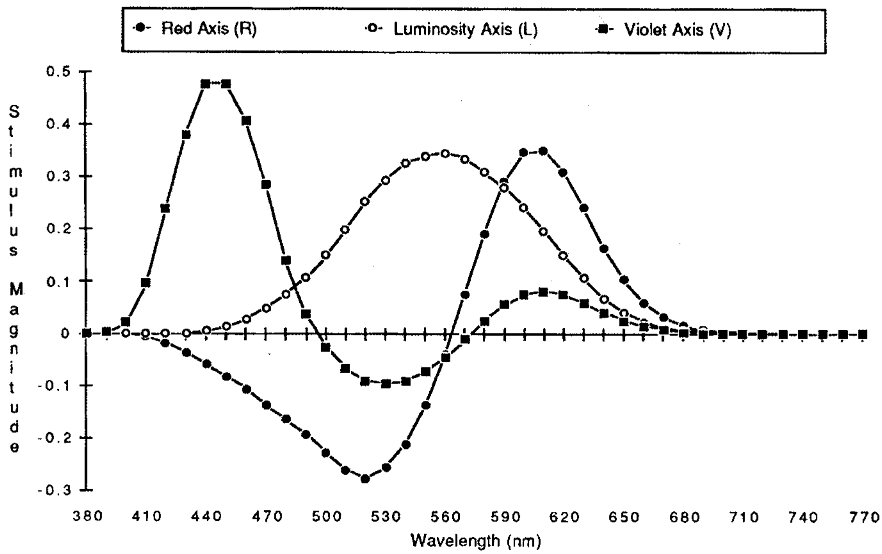

Fig. 6-20 *Spectral functions representing the red, violet and luminosity axes of the RLV version of Cohen's fundamental color space (Burns et al., 1990).*

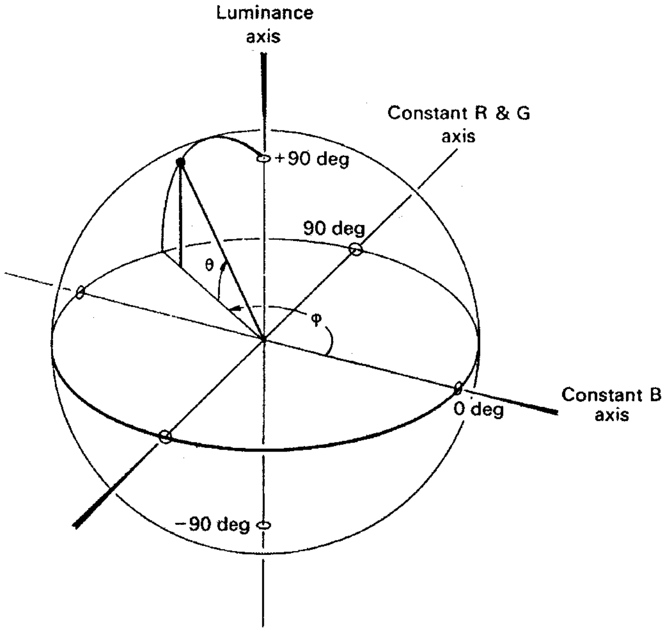

Fig. 6-21 Schematic view of the DKL color space based on cone sensitivity data. A color (black dot) is represented by its azimuth, Φ, and its elevation, Θ (Derrington et al., 1984.)

location of the chromatic axes in the CIE chromaticity diagram (Fig. 6-22). This space has direct neurophysiological support from measured cell activity in the LGN. The axes are neither in agreement with average unique hues nor with those of the CIE-based opponent color chromatic diagram.

Color vision models that imply a color space have also been developed in connection with color appearance models. An inclusion of two of these (Hunt and Nayatani) in the Mahy et al. (1994) evaluation of uniform color spaces indicates that they represent the Munsell value 5 plane and the OSA-UCS systems with significant deviations. Such systems, as mentioned earlier, are considered outside the scope of this text.

De Valois and De Valois

In 1993 R. L. and K. K. De Valois presented a paper titled *A multi-stage color model*. They had been involved in some of the earliest experiments identifying opponent color type cells in the LGN of macaques in the 1960s. They were cognizant of the discrepancy between the output of opponent cells in the LGN and psychological scaling of color space touched on above. The multi-stage model was developed to account for such discrepancies. The De Valois model consists of four stages. In the first stage, three cone types provide responses to light striking them, and the authors used the Smith-Pokorny functions to

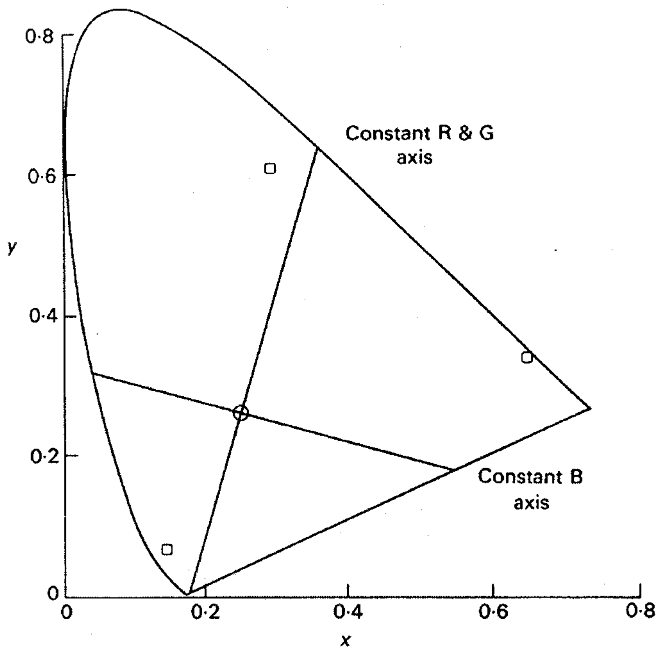

Fig. 6-22 Axes of constant R&G and constant B of the DKL space in the CIE chromaticity diagram. The squares represent the chromaticities of the phosphors used in the experiment.

describe cone sensitivity. They assumed proportions in the retina of L, M, and S cones in a ratio of 10:5:1. In the second stage, cone opponency signals are generated. Two kinds of surround are considered here: an indiscriminate surround based on the output of horizontal cells in the retina and the other a cone type specific surround. The response functions of three cone opponent cell types, illustrated in Fig. 6-23, are derived as follows:

$$\begin{aligned} L_0 &= L-(L+M+S), \\ M_0 &= M-(L+M+S), \\ S_0 &= S-(L+M+S). \end{aligned} \tag{6-55}$$

These cells also have mirror image copies. The cells are considered to carry both luminance and color information at different spatial frequencies.

In the third stage, in the parvocellular pathway, the authors' proposal posits combinations of signals in a way that separates luminance and color information. Accordingly, for example, $L_0 - M_0$ sums color and cancels luminance, while $L_0 + M_0$ sums luminance and cancels color. The response functions of the third stage are illustrated in Fig. 6-24. They resemble somewhat the Hurvich-Jameson opponent color functions but are not balanced. They are calculated, in the indiscriminate version, according to

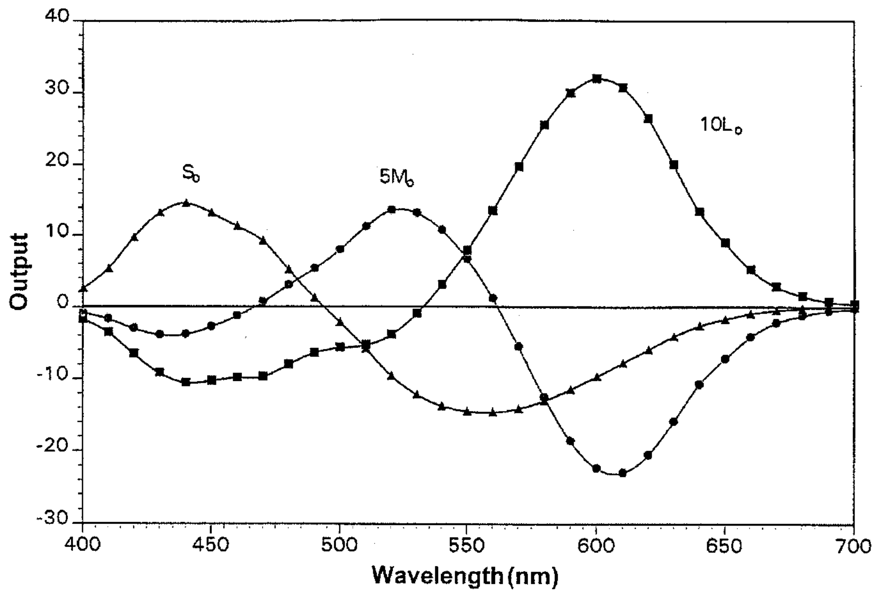

Fig. 6-23 Three cone opponency signals of the second stage of the De Valois and De Valois color model (indiscriminate surround). From De Valois and De Valois (1996).

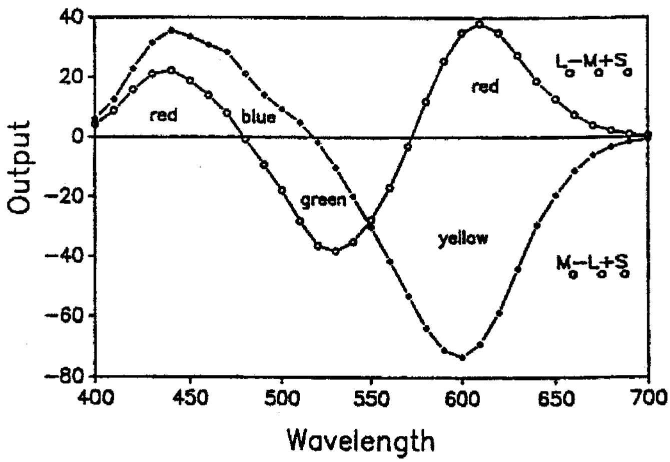

Fig. 6-24 Third-stage chromatic functions of the De Valois and De Valois color model (indiscriminate surround). From De Valois and De Valois (1993).

$$RG = 18L - 23M + 5S,$$

$$YB = -26L + 19M + 7S. \tag{6-56}$$

For the fourth stage, the De Valois proposal has complex color selective cells resulting from the summing of the responses of simpler cells earlier in the pathway. The authors believe that simple opponency such as posited by Hering and Hurvich and Jameson ends at the LGN level and that already in area V1 of the brain's visual system there are cells that fire to stimulation from specific spectral regions and not to those of others. "The chromatic opponency at this stage is between, not within, individual cells." At this stage two cell types each, such as "red" and "blue" or "red" and "yellow," can fire at the same time to the same stimulus, resulting in mixed hues. In the form presented here, the De Valois model does not result in a close representation of the Munsell system.

The De Valois model is of particular interest because it considers a wealth of neurophysiological data accumulated over the last fifty years. It represents informed assumptions in regard to how the visual system operates (up until area V4) and is in this respect different from Guth's early models that optimized the Müller structure to psychophysical data, as the authors point out in a commentary in 1996.

In 1997 De Valois et al. reported on a hue-naming experiment whose results they explained in terms of a modified version of their model where in the third stage they subtracted modified amounts of second stage S_0 from L_0 and M_0 (roughly comparable to the change from the α, β diagram to the *a, b* diagram; see Section 5.7) They also found the red and the green systems to likely receive non-symmetrical inputs from cones or LGN cells. The De Valois proposal is a recent example of a color vision model that assumes a relatively simple relationship between proposed outputs of cells in the visual area of the cortex and color perceptions.

The Opponent Color System: Asymmetrical?

In 1999 E. J. Chichilnisky and B. A. Wandell added data to the thesis that the opponent color system might be asymmetrical. They conducted an experiment in which square color stimuli of 2.5° visual angle on a CRT were briefly flashed against various backgrounds. Observers were asked to identify if the stimuli were greenish or reddish, yellowish or bluish. One type of their presentation of the results is in cone contrast space as differences in *L, M*, and *S* as opposed to the corresponding values of the surround. Significant differences were found between the three observers in the experiment, but all showed classification boundaries that were bent, usually in a shallow bowl shape. The bowl shape did not in all cases pass through the origin; that is, "opponent classifications are not based exclusively on the difference between test stimulus and background light." An example of the results of the redness-greenness classification boundary against a greenish appearing surround is illustrated in Fig. 6-25. The shape of the bowl usually has various bulges, indicating not just global but also local nonlinearities of the boundaries between unique hues. The results were fitted with what the authors call an increment-decrement opponency model, indicating that increments and decrements in cone absorptions from those of the surround were treated differently. (Nonsymmetrical effects of increments and decrements are known from other experiments, as mentioned in Chapter 5.) A

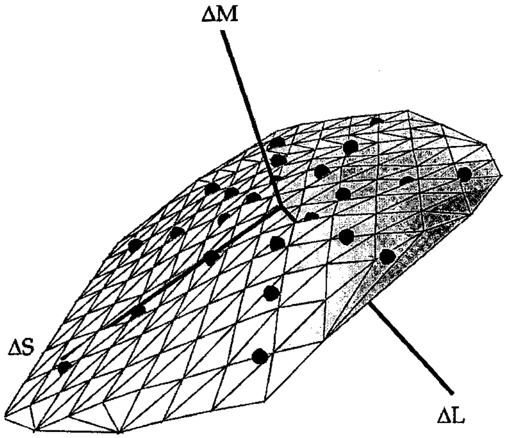

Fig. 6-25 *Redness/greenness classification boundary against a greenish surround in the L, M, S cone contrast space. From Chichilnisky and Wandell (1999).*

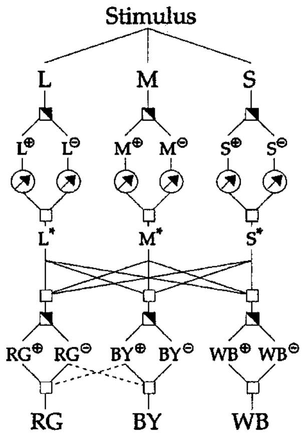

Fig. 6-26 *Schematic view of the color vision model of Chichilniski and Wandell to account for their hue classification results. Output from cones L, M, S is split into increment/decrement portions that are separately modulated to form the intermediate cone signals L*, M*, S*. These are linearly combined to form incremental and decremental pre-opponent signals. They, in turn, are again linearly combined to form the opponent signals RG, BY, and WB. From Chichilnisky and Wandell (1999).*

schematic view of the model is found in Fig. 6-26. Linear cone signals *L, M, S* are are treated separately as increments or decrements with respect to a neutral point. These are scaled through gain control depending only on the surround. The resulting intermediate cone signals L^*, M^*, and S^* are combined linearly and then separated into pairs of increment and decrement pre-opponent signals. These are combined linearly to create the final opponent color signals, whose signs determine the opponent color invoked. Concerning the neural location of opponent color null surfaces the authors comment: "The non-planarity of opponent color classification boundaries raise the possibility that either individual parvocellular LGN neurons are not the neural substrate of perceptual opponency, or that significant nonlinearities in neural responses were overlooked." Increments and decrements have also been found to be treated differently in spatial patterns (Bäuml, 2002).

The relevance of these findings for a model of a global object color space viewed under conditions much different from those of the described experiment remains to be determined. There is the likelihood that all models using simple subtractions of cone absorptions or tristimulus values are much simplified approximations of the real mechanisms of color vision. The issue of the strategy pursued by evolution in the development of the primate visual system assumes key importance as well as if perception ultimately can be modeled from implicit cone responses.

6.18 IS THE OPPONENT COLOR SYSTEM "SOFT-WIRED"?

It is an accepted supposition of color psychology that reddish-greenish colors and yellowish-bluish colors are not possible. However, this has been questioned since 1983 when H. D. Crane and T. P. Piantanida published a paper "On seeing reddish green and yellowish blue." More recently Billock and coworkers (2001) reported on experiments where observers viewed fields of opposing colors at personal equiluminance for each observer under image stabilization (the image impacted the same retinal region at all times). Four out of seven observers experienced under these conditions homogeneous mixtures of red and green or yellow and blue (some only after several trials). The authors proposed a "winner take all" multi-stage model of vision in which opponency is not hardwired into certain types of cells but is the result of the combined output of many cells. As a result one or the other of the unique hues (the winner) takes over. This soft assignment of opponent hues can be defeated in certain observation conditions resulting in mixed opponent hue perceptions.

Models of human color vision must be able to explain vast amounts of different experimental data. All models are empirical because the translation of chemical signals in area V4 to perceptual experiences is unknown. There is a likelihood that the strategies generally pursued so far will not succeed in explaining all visual data because the visual system may pursue empirical strategies that do not directly follow from the identified mechanisms in the retina, the LGN and the early visual areas but are the statistical results of past experiences, possibly elaborated beyond the classical visual areas.

6.19 SPECTRAL SPACES

As briefly mentioned in Chapter 2, regular spaces of reflectance or spectral power data can also be formed by content analysis and dimensionality reduction different from that implied in the color matching functions. When comparing the reflectance functions of Munsell chips of constant chroma and value, regularly spaced around the hue circuit, they are found to change in a regular fashion. In Fig. 6-27*a* through *d* the reflectance functions of twenty (every second) Munsell chips at constant value and chroma representing a hue circle are shown. Given the possibility of many metamers for each chip, this fairly regular change is in part due to the limited number of pigments with their own specific spectral function shape and in part is a reflection of the (relativized) hue change between chips. Lenz and co-workers (1996) have calculated principal components of the reflectance functions of 1269 Munsell chips (including those of Fig. 6-27) by forming a correlation matrix and determining its eigenvectors. The first three eigenvectors are illustrated in Fig. 6-28. It is evident that the first vector, accounting for over 80% of the correlation between the functions in the total set, has all positive values and represents in a general way the average height of the reflectance function, roughly indicative of lightness. The second and third eigenvectors have both positive and negative values and have a crude resemblance to opponent color curves (compare to Fig. 5-13). The functions are orthogonal to each other. When the twenty reflectances of Fig. 6-27 are plotted, they do not form a circle on a flat plane in the resulting space but an irregular three-dimensional contour as shown in Fig. 6-29*a* and *b*. The drop lines in Fig. 6-29*a* provide an indication of the variation in terms of the first eigenvector. It is evident that this space orders Munsell colors in an ordinal manner only in that it places the chips in a sequence that is in agreement with the perceptual sequence. Colors varying in chroma or value (not shown) are also placed relative to each other in proper ordinal order. However, the space is far from uniform in terms of the Munsell system. Ordinal order can be also obtained with many arbitrary continuous functions and does not in itself indicate a useful color space. Other dimensionality reduction methods do not place such color series always in ordinal order (Ramanat et al., 2003). Eigenvector spaces depend on the color stimuli and the sample distribution. Eigenvectors determined from different collections of color samples will be different. In addition metameric color samples do not plot in the same location in eigenvector spaces. For these reasons it is not appropriate to call such spaces color spaces.

6.20 PERFORMANCE COMPARISON OF VARIOUS FORMULAS

Over the last hundred years color discrimination data have been established at four different levels of difference:

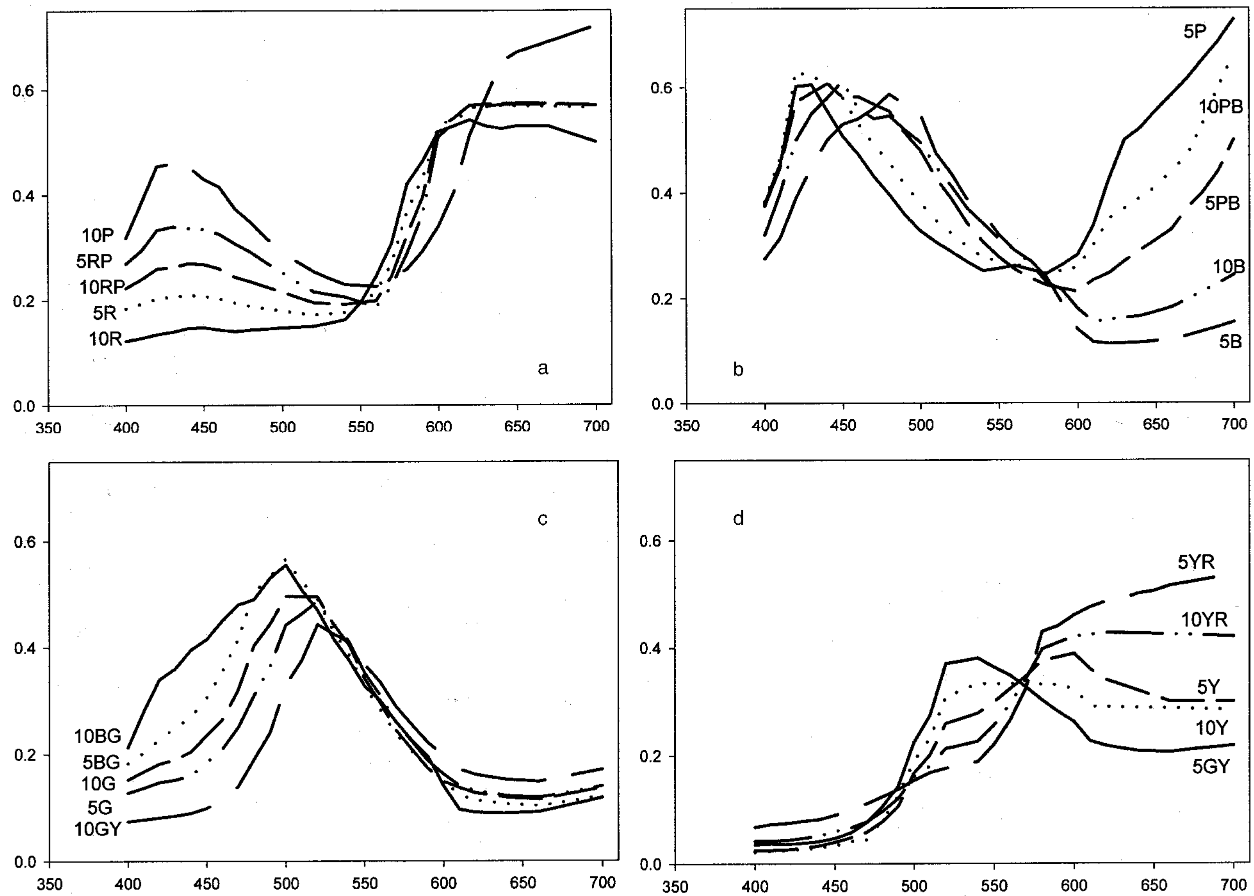

Fig. 6-27a–d *Sequential plot of spectral reflectance functions of every second Munsell chip around the hue circle at value 6 and chroma 8, indicating the comparatively regular changes in reflectance. Note the common crossovers between 550 and 600 nm.*

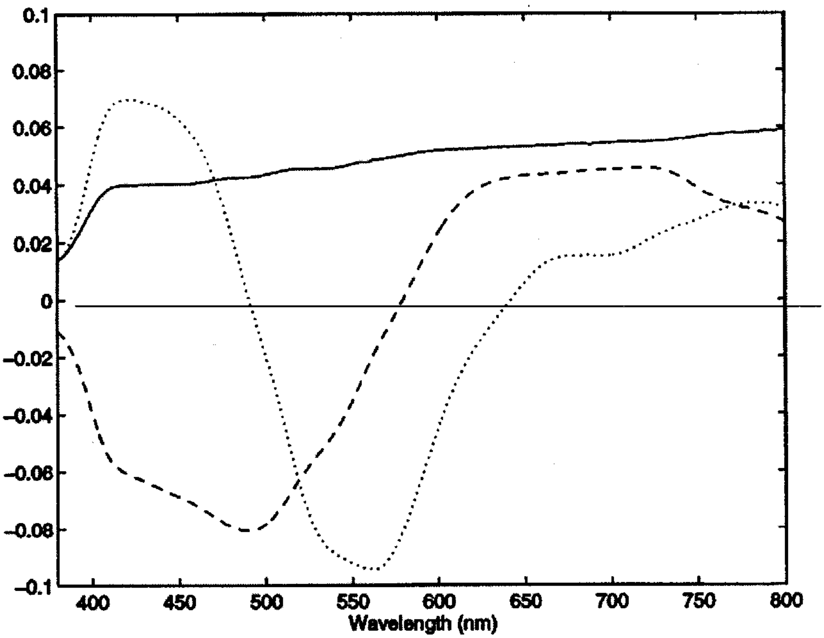

Fig. 6-28 First three eigenvectors representative of the first three principal components, fitted to a database of reflectance functions of 1269 Munsell chips. Solid line*: eigenvector 1;* dashed line*: eigenvector 2;* dotted line*: eigenvector 3. From Lenz et al. (1996).*

1. Subthreshold. These are color-matching error data, as determined by MacAdam and others.
2. Threshold or just noticeable difference data. Examples are luminance difference thresholds, wavelength discrimination data, purity threshold data, and the Wright, Richter, and Witt threshold data.
3. Suprathreshold small differences. These include the RIT-DuPont, Witt data, and others.
4. Large color difference data. Examples are the Munsell and OSA-UCS data, and the Guan and Luo data.

Ideally all these data sets would be describable with high accuracy using a single model/formula. It is obvious from past discussions that this is impossible. When comparing various models of color spacing with typical data sets, significant variation in results is obtained. In a few cases formulas have been optimized against a particular data set, and not surprisingly, these formulas tend to perform better than others against these data. Typical examples are the FMC formulas optimized against the MacAdam data, the OSA-UCS formula optimized against the corresponding large difference data, and the BFD formula optimized mainly against the Luo and Rigg modified data set.

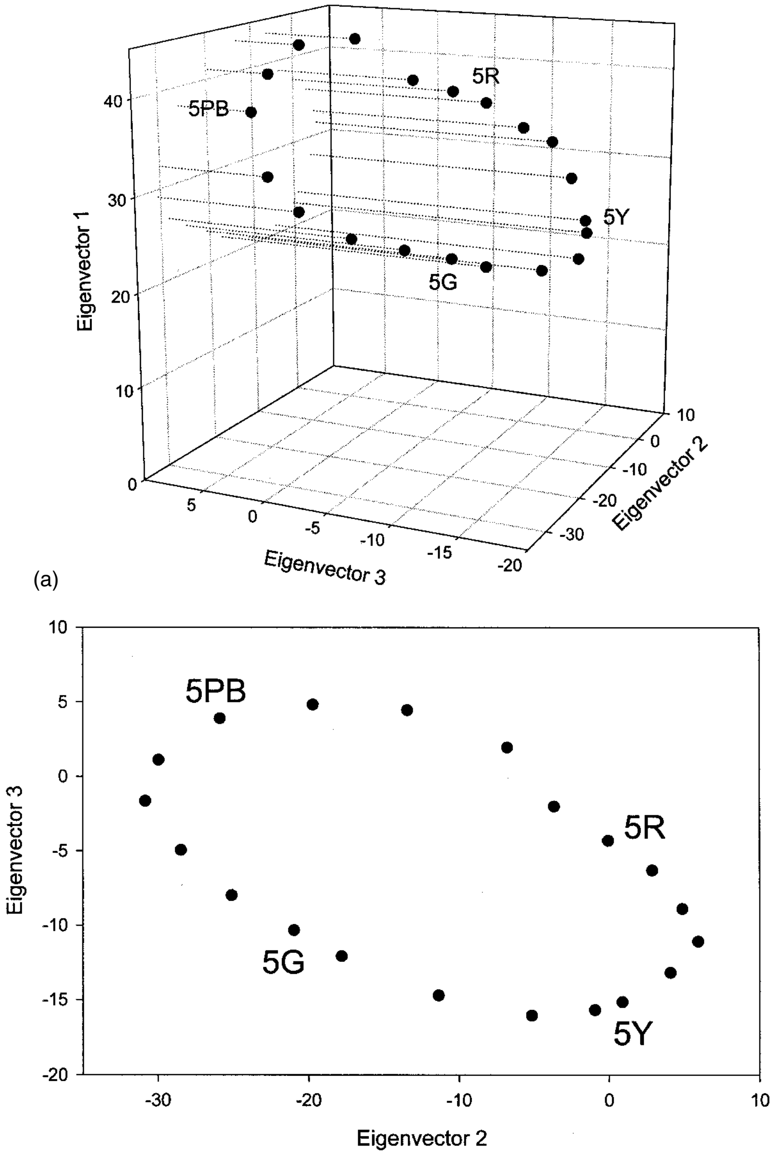

Fig. 6-29a and b *Plots of the twenty Munsell colors of Fig. 6-27 in the space formed by the three eigenvectors of Fig. 6-28 and in the diagram formed by the second and third eigenvectors. Hue designations of four colors are shown for identification. It is apparent that the space orders the Munsell colors regularly in an ordinal sense, but it is far from uniform.*

TABLE 6-1 Prediction accuracy of three color difference formulas for various sets of color difference data

Data set	Coefficient of Variation, %		
	CIELAB	CMC	BFD
MacAdam, 24 ellipses	34.5	32.2	27.7
Wyszecki-Fielder/GW, 28 ellipses	26.9	20.2	18.8
Witt threshold, 5 ellipses	39.3	39.1	42.6
Luo-Rigg, 131 ellipses	25.1	16.2	13.6
Cheung-Rigg, 5 ellipses	31.6	39.1	42.6
Munsell value, 232 differences	7.3	29.0	21.5
Munsell hue, 365 differences	64.6	50.7	38.4
Munsell chroma, 356 differences	20.5	35.5	41.8

Source: Adapted from Melgosa et al. (1992).

In 1992 Melgosa and co-workers compared a number of data sets using the CIELAB, CIELUV, CMC, and BFD formulas. From their extensive data only a few examples are shown in Table 4-1. Results, in form of coefficients of variation for the difference between visual and calculated differences, are bewildering. Some conclusions can be drawn from the table:

1. There are systematic differences between the MacAdam data, on the one hand, and the Wyszecki-Fielder/observer GW data, on the other.
2. Witt threshold data are poorly predicted by any of the three formulas.
3. CMC and BFD are significant improvements over CIELAB for the Luo-Rigg data but not for the Cheung-Rigg data.
4. CIELAB is a good predictor of Munsell value but a poor predictor of Munsell hue differences, while CMC and BFD are poorer in predicting value differences but better (if still poor) in predicting hue differences. CIELAB is a better predictor of Munsell chroma differences than the other two formulas (in agreement with findings in Chapter 8 of absence of chromatic crispening in color differences of the size of Munsell system differences).

Another view of the progress of color difference formulas in predicting average visually judged small color differences and of color space formulas in modeling color solid data is offered in Table 6-2 and in Figs. 6-30 and 6-31. Typical data sets have been used for this purpose. The small color difference data set selected is the RIT-DuPont set with 156 difference pairs in 18 color centers. When the data were established, considerable care was taken to control significant aspects of the experiment. The nature of the experiment resulted in all color differences in this data set having a visual difference of one unit and therefore should have calculated differences of one unit. An appropriate statistic of the performance of color difference formulas for this data set is, again, the coefficient of variation. In Table 6-2 coefficients of

TABLE 6-2 Comparison of performance of some color-difference formulas against the RIT-DuPont data set

Formula	Coefficient of Variation, %
CIELAB	35
CIELUV	39
SVF	38
BFD	28
CIE94	21
CIEDE2000	20
Kuehni optimization of CIE94	18

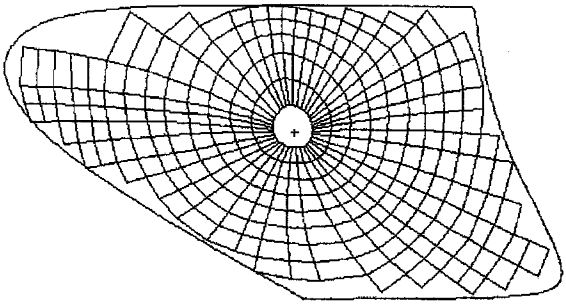

Fig. 6-30 *Munsell Renotation colors at value 5 plotted in the chromatic diagram of Richter's LABHNU formula. From Mahy et al. (1994).*

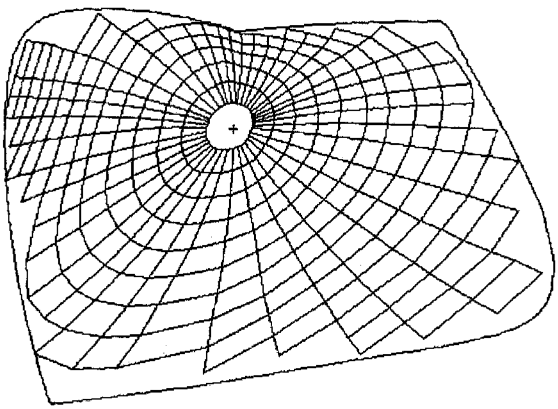

Fig. 6-31 *Munsell Renotation colors at value 5 plotted in the chromatic diagram of Hunt's 1991 color appearance model. From Mahy et al. (1994).*

variation for color differences calculated by various formulas for the set are listed.

The variation in calculated differences for the data set selected is now approximately half of that for the 1976 CIE formulas. The largest improvement was achieved, as indicated earlier, by applying variable weights to the implicit lightness, chroma, and hue differences to deal with chromatic crispening and the non-euclidean nature of uniform color space (in the absence of euclidization procedure).

The Munsell system was selected for the purpose of comparison of the performance of color space formulas. Constant value, respectively metric lightness, planes are illustrated. To be in perfect agreement with the perceptual results, a color space formula should represent an equal value plane of the Munsell system as a series of concentric circles with equal radial spacing and radial lines that are equidistant from each other in terms of hue angle. In the evaluation of Mahy et al. (1994) the formulas (then known) most accurate in modeling the Munsell system were the LABHNU formula of Richter (a somewhat different version from the one documented above, but found in the same reference), SVF, and the CIELAB formula (for OSA-UCS it is the OSA formula and SVF). A significant further improvement will be shown in Chapter 7 (compare Fig. 7-6). Formulas such as CIE94 cannot be used as color space formulas because of the variable weights on the difference components, unless the weights are integrated such as in the Rohner and Rich, Völz, Thomsen, or DIN99 proposals (see above). It is evident, however, that they would perform poorer than the formulas above against the Munsell and OSA-UCS systems because most of the integrated weight of S_C relates to chromatic crispening, absent in the Munsell system and OSA-UCS.

We have seen a rich tapestry of efforts to develop psychophysical models of color space and color differences applicable to differences of various size. But as discussed in Chapter 4, we do not have extensive reliable, replicated data of which we can be confident that they accurately describe color spacing for a given situation of illumination, surround, size of differences, and the truly average observer. Various data sets developed at different times vary considerably for generally unknown reasons and are described optimally by different formulas. It is important to develop data sets that can be reproduced in different locations by observer groups of comparable and known relation to a truly average standard observer (that may still need to be developed). If this proves to be impossible, it may be due to irreproducible cognitive input into the judgments. Truly reliable data sets and further knowledge of the human color vision system may make it possible to develop a psychophysical basis model, perhaps non-euclidean and certainly nonlinear, that can, with various parameters, accurately describe the tiling of color space uniform under closely defined observation conditions. There is a considerable way to get there and it is evident that CIEDE2000 is a mere milestone on that path.

Chapter 7

Major Color Order Systems and Their Psychophysical Structure

In this chapter only the Munsell, OSA-UCS, and Swedish NCS systems will be discussed. The former two are the most important attempts to create psychologically uniform systems, the latter uses the presumed natural approach of Hering (see Chapter 2) to create a color order system, having a regular structure, but not one uniform in terms of size of perceived differences. There are several other newer color order systems extant, but they neither make claims for uniformity nor for regularity according to new, significant psychological attributes.

The issue of viewing conditions for these systems has been attended to in different ways. As discussed previously, the Munsell system is illustrated as if the chips at each value level would be viewed against an achromatic surround of the same value. Chips of two adjacent value levels are illustrated as if viewed against an achromatic surround of intermediate value. The actual atlas displays the chips on white paper (historically of various degrees of whiteness), thus resulting in distortions of the value scale, particularly at lower values. The OSA-UCS system is defined for an achromatic surround of luminous reflectance $Y = 30$ ($L = 0$). The atlas samples are in transparent jackets. NCS, finally, has been established against an immediate achromatic surround of $Y = 78$ in a light booth painted with an achromatic gray of $Y = 54$. The atlas displays samples on white paper. Both latter systems only result in the intended color experiences when viewed against the appropriate surrounds. Munsell and NCS are defined for

Color Space and Its Divisions: Color Order from Antiquity to the Present, by Rolf G. Kuehni
ISBN 0-471-32670-4

CIE daylight C and the 2° standard observer. OSA-UCS is defined for daylight D65 and the 10° standard observer.

7.1 THE MUNSELL COLOR SYSTEM

Development of the System

Albert H. Munsell was educated as an artist and art instructor. His initial interest in color order (beginning in the 1880s) resulted from the need for an educational tool for instruction in color order for school children and art students as well as a tool for objectively expressing harmonious color relations.[1] His initial concept employed a sphere. The inspiration for the sphere form (Munsell became aware of Runge's book only in 1899) came from a child's ball with colored segments and from plotting the colors of one of his paintings in form of a double spiral, suggesting a sphere (Munsell, 1918). Munsell, having convinced himself that Hering's approach could not be correct, decided to base his hue circle on five primary colors: red, yellow, green, blue, and purple. The main motivation was to be able to express his system in a decimal framework. Munsell, not burdened with detailed knowledge about earlier attempts at systematic color order, unhesitatingly decided to make lightness ("value" in his artistically influenced term) a key attribute. His early sphere models, for which he had received a patent in 1900, placed hues of equal value on the equator, that is, in addition to middle green and red, darkened yellow, and lightened blue and purple. Munsell invented the term "chroma" to designate the radial dimension from the neutral gray center to the equatorial colors. After plotting "intensity" against "luminosity" values of painted pigments, measured by Abney, Munsell realized in the year 1900 that on the basis of attributes hue, value, and chroma, a uniform color solid as represented by available pigments could not fit into a sphere but would form an irregular "spheroid." As a result he abandoned the idea of his color solid fitting into an ideal geometrical solid. Munsell named the irregularly shaped solid a "color tree." He maintained the color sphere as an educational tool until the end of his life, and it remained a part of the descriptive literature of the system until the Second World War (Fig. 7-1). Munsell began preparing an equal value chart of painted paper chips in 1901 and proceeded with charts at other value levels. In 1902 he sketched a model based on constant hue planes and began to assemble corresponding hue charts (Fig. 7-2). During the same years Munsell decided that the system should be visually uniform, that is, steps along the three attributes should, within an attribute, be of equal perceptual size. Having differences of equal perceptual magnitude within all three attributes was contemplated by Munsell but somehow never implemented. In 1904, when preparing a publication describing the system and as part of a patent application, Munsell also had settled on his hue, value, and chroma terminology and color identification scheme. Over a span of five years Munsell had developed all key concepts of the system as they stand today. In 1905 he published his conceptual description of the system

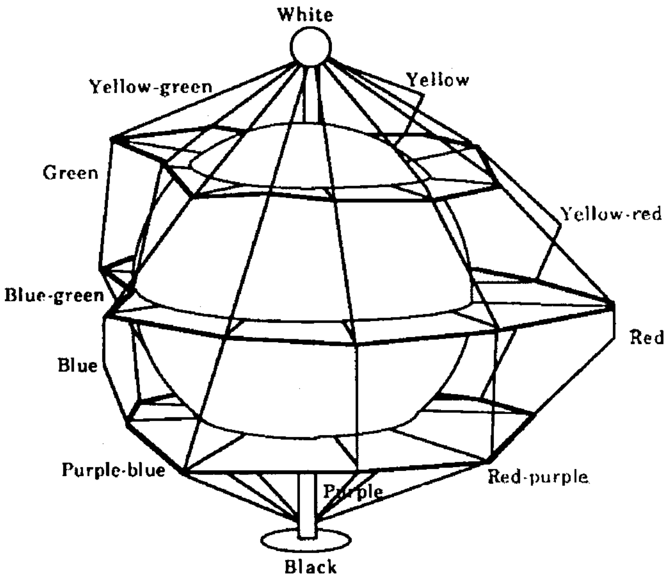

Fig. 7-1 *Munsell's irregular "color tree" enveloping the original sphere. From Derefeldt (1991).*

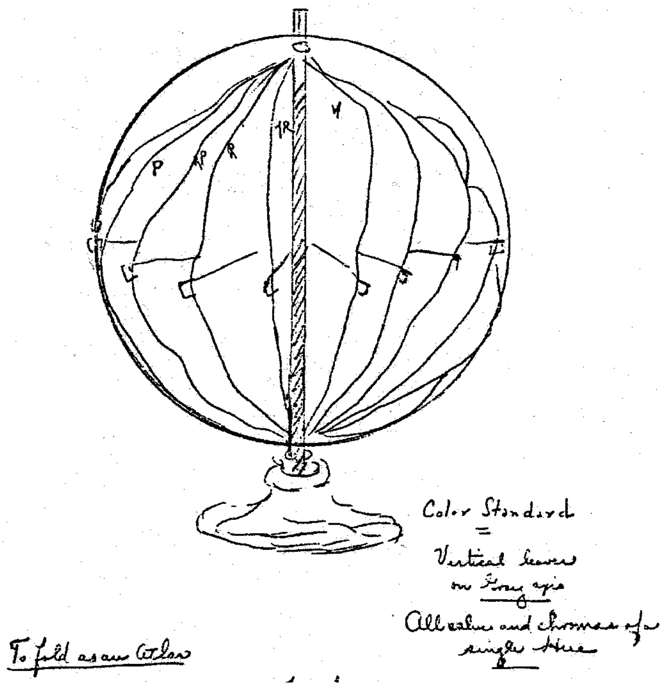

Fig. 7-2 *Trace of Munsell's sketch in his color Diary of an irregular "color tree" with constant hue leafs, dated March 20, 1902.*

under the title *A color notation* (Munsell, 1905). He was granted in 1906 a patent for the system, and he received copyright protection for the charts. In 1907 the first version of the *Atlas of the Munsell color system* was published, containing eight charts with painted samples (Munsell, 1907). A second, enlarged version of the *Atlas* appeared in 1915 (Munsell, 1915). It consists of 15 charts, five constant hue and seven constant value charts, as well as three charts of general descriptive nature.[2]

In the year of Munsell's death, 1918, the Munsell Color Company was formed and continued operating under his son. Scientific support was obtained from the National Bureau of Standards, and the company soon moved from Boston to Baltimore to be closer to the Bureau. One of the young researchers at the Bureau was D. B. Judd. Research on uniform spacing of hue, value, and chroma was continued, and in 1929 the company issued the first version of the *Munsell Book of Color*. The *Munsell Book of Color* is internationally today perhaps the most widely known color order/appearance system and is commercially available in a matte and a glossy chip edition, with a supplementary collection of near-neutral color chips. There is also a textile fabric edition.

The history of the Munsell system has been described by Nickerson (1940, 1969, 1976), Berns and Billmeyer (1985), and Kuehni (2002). A modern description of the system is that by Long and Luke (2001).

The 1915 Atlas

The atlas consists of a text page with four sketches illustrating the color sphere, the color tree, and schematically the position of the vertical and horizontal section charts. The first chart (H) consists of chips of the five primary and the five intermediate hues at values 2 to 8. Chart V consists of the ten-step gray scale and chips of the highest chroma colors of the five primary hues at their appropriate value level. Chart C illustrates chroma scales of the five primary hues shown at their value level. Five charts illustrating vertical cross sections through the solid with always two hues illustrated per chart follow. Finally, there are seven star-shaped horizontal section charts illustrating ten hues at a given value level in all chroma steps possible with the pigments in use.

Fifteen of these chips were measured at the National Bureau of Standards in 1919, and CIE tristimulus values were calculated at a later date. Additional 70 samples were measured in 1926, again with tristimulus values calculated later (Gibson and Nickerson, 1940). When plotted, they do not follow a systematic pattern. Recently measurements of the 58 value 6 color chips of a copy of the 1915 atlas were made by the author. They are illustrated in Fig. 7-3 in the $a^\wedge, b^\wedge$ diagram (fitted to the Munsell Renotations; see equation 7-1 below). The arrangement is reasonably regular. In a few cases the highest chroma level is insufficiently spaced from the previous level. The luminous reflectance values of the samples were found to vary from approximately 36 to 41. Reflectances of the value (gray) scale samples had been measured in 1916 and 1926. The results of the value scale measurements of the 1915 *Atlas* are com-

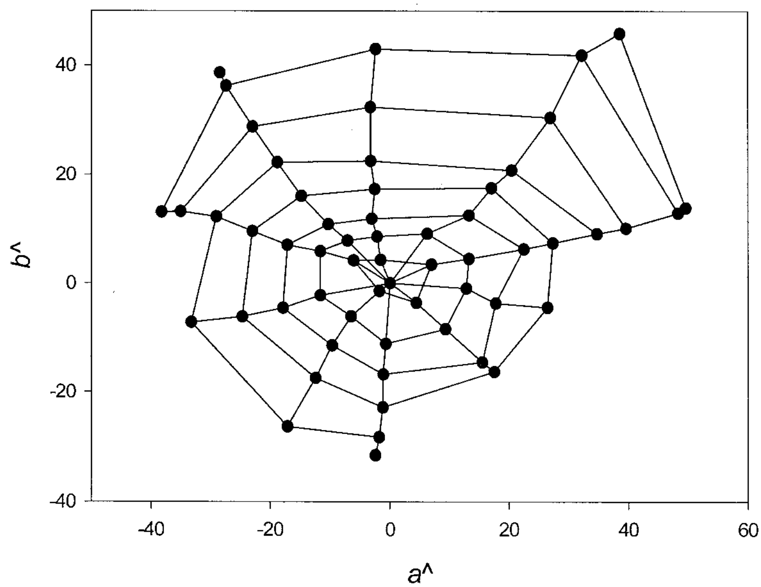

Fig. 7-3 *Location in the* a^, b^ *chromatic diagram of the fifty colors of chart 50 (value 5) of the 1915 Atlas, 2° observer and equal energy illuminant. Radial lines connect colors of constant hue, circular lines colors of constant chroma.*

pared in Fig. 7-4 against measurements of the value samples of the 1929 *Book of Color* (see below). The scales are somewhat uneven and show breaks in continuity: the 1915 *Atlas* value scale between value steps 4 and 5, the *Book of Color* scale between steps 5 and 6. They are indicative of the lightness crispening effect. The 1915 *Atlas* measurements are optimally linearized with a single function by applying a power of 0.43 to the luminous reflectance values.

1929 Book of Color

Based on research conducted together with the National Bureau of Standards, the Munsell Color Company issued in 1929 a revised and extended version of its catalog of sample chips. To distinguish it from the *Atlas*, the new version was called *Munsell Book of Color* (BOC). It consists of twenty hues in values typically from 2 to 8 and chromas from 2 to (in a single case) 14, with a total of 400 samples.

Their reflectances were measured against magnesium oxide in 1935 by J. T. Glenn and J. T. Killian, with MacAdam's help, and the resulting colorimetric data were published in 1940 (Glenn and Killian, 1940). All samples at value 6 have been plotted in Fig. 7-5 in the $a^\wedge$, $b^\wedge$ diagram. In comparing this figure with Fig. 7-3, we find considerable differences between the two versions, par-

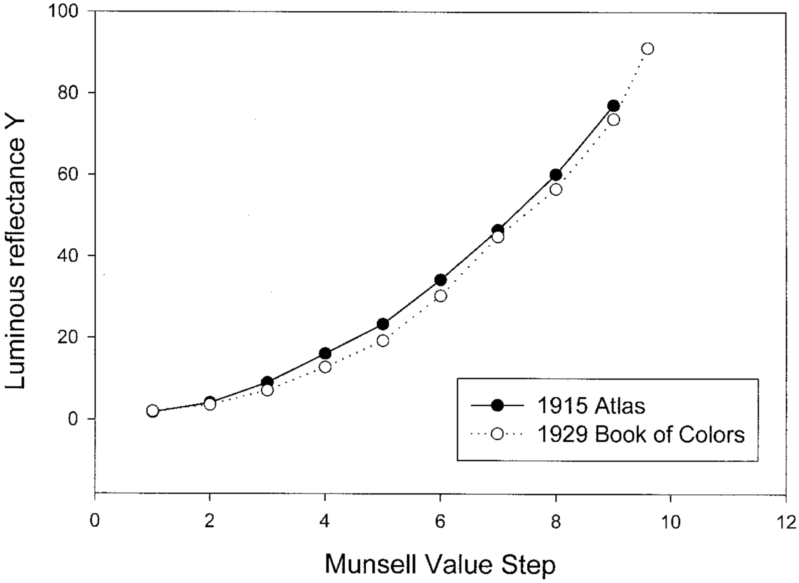

Fig. 7-4 *Luminous reflectance Y of the value scales of the 1915 Munsell* Atlas *(*solid line*) and the 1929* Book of Color *(*dotted line*), 2° observer.*

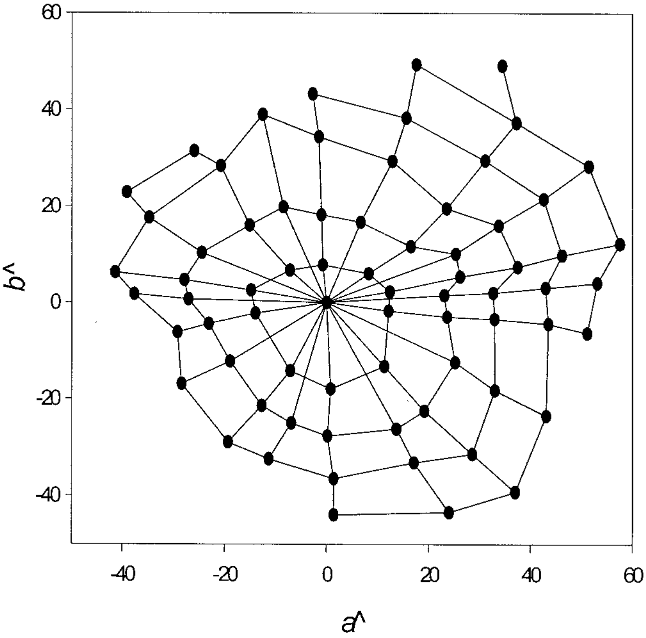

Fig. 7-5 *Location in the* a^, b^ *chromatic diagram of the colors of the value 6 plane of the 1929 Munsell* Book of Color*, 2° observer and equal energy illuminant.*

ticularly in regard to chroma. The luminous reflectances of the value scale of the *Atlas* shown in Fig. 7-4 require, for optimal linearization over the whole range, a power of 0.40 but show evidence of lightness crispening.

The Munsell Renotations

In the mid-1930s the Optical Society of America formed its Subcommittee on the Spacing of Munsell Colors, chaired by Newhall. In 1940 it released a preliminary report describing the psychophysical methods used and preliminary smoothing results for chroma scales (Newhall, 1940). The plotted Glenn and Killian measurements of the 1929 BOC samples and other evidence had indicated jaggedness of constant chroma contours in the CIE chromaticity diagram and, to some extent, of the constant hue lines. In support of the subcommittee's work, 41 subjects had made estimates of the magnitude of differences between gray scale chips of the 1929 BOC to be used for a redefinition of the value scale. All visual estimates were made against a white, a midgray, and a black background (see Fig. 5-16). The report lists averages and ranges of the judgments. A significant and systematic effect of the surround on value judgments was noted. A change in continuity of the value function indicative of the lightness crispening effect in both the ΔR versus R function (where R is luminous reflectance) determined by just noticeable difference measurement and R versus value function from consecutive halving of the scale had also been reported by Munsell et al. (1933), as mentioned in Chapter 5.

In 1943 the committee released its final report (Newhall et al., 1943). Minor irregularities in the mean revised experimental spacing results were smoothed in the CIE chromaticity diagram. The lightness crispening effect in the value scale against the gray surround was also smoothed away. Trends in hue and chroma were extrapolated to the object color limits of the x, y, Y psychophysical color space. The resulting *Renotations*, expressed for the CIE 2° observer and illuminant C, define aim values for 2746 chromatic and nine achromatic colors. Approximately 65% of these have been physically realized as color chips in two volumes of the BOC. The committee used Judd's definition of the value function in form of a smooth quintic equation relating value to luminous reflectance (see equation 5-8). The Munsell Renotation data have also been published in Wyszecki and Stiles (1982).

Aim colors at every second of the 40 defined hues of the Renotations (5 and 10) up to chroma 14 at value 6 are plotted in Fig. 7-6 in the $a^\wedge$, $b^\wedge$ diagram. The changes compared to the 1915 *Atlas* and the 1929 BOC are clearly apparent.

A software package (Leonardo 2000) has been released containing reflectance functions for the Renotation colors achievable with pigments. It allows the calculation of colorimetric aim points for both standard observers and any defined illuminant. It has a display facility that reproduces on the monitor any such generated Munsell aim color (Brill et al., 2001).

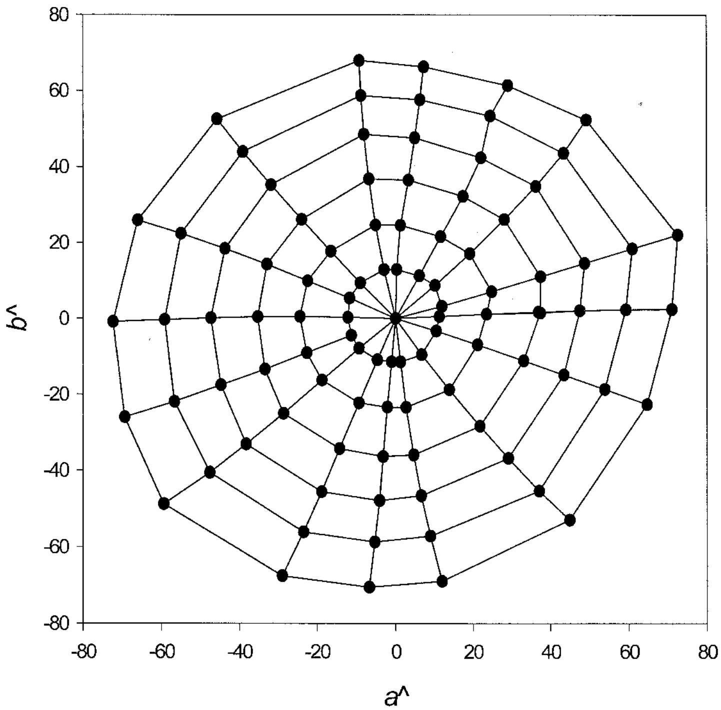

Fig. 7-6 Munsell Renotation aim colors for the 2° observer and equal energy illuminant. Every second hue (5 and 10) at value 6 and chroma 0–12 in the a^, b^ *chromatic diagram. Compare with Figs. 6-30 and 6-31.*

The Munsell Re-renotations

During the 1950s and 1960s the Committee on Uniform Color Scales of the Optical Society of America, under the leadership of Judd, conducted a series of experiments with the goal of developing an improved uniform color solid and associated space formula. Judd summarized these experiments in four reports (Judd, 1955, 1957, 1965, 1967). Judd and Nickerson were interested in the impact of the results of these experiments on the Munsell system. As discussed below, the committee pursued a different path, ultimately resulting in the Society's *Uniform Color Scales*. In 1967 Judd and Nickerson published a National Bureau of Standards report in which they proposed revised Renotations that reflected the new experimental findings. They specified a total of 2874 chromatic colors, in a complete revision of the Munsell system (Judd and Nickerson, 1967). Even though only a small number of corresponding samples were produced for experimental purposes, it is instructive to review the *Re-renotations* to gain insight into the work of the committee. The same aim colors (by name) of the Renotations used for Fig. 7-6 are illustrated for the Re-renotations in Fig. 7-7. It is evident that they represent a significant change.

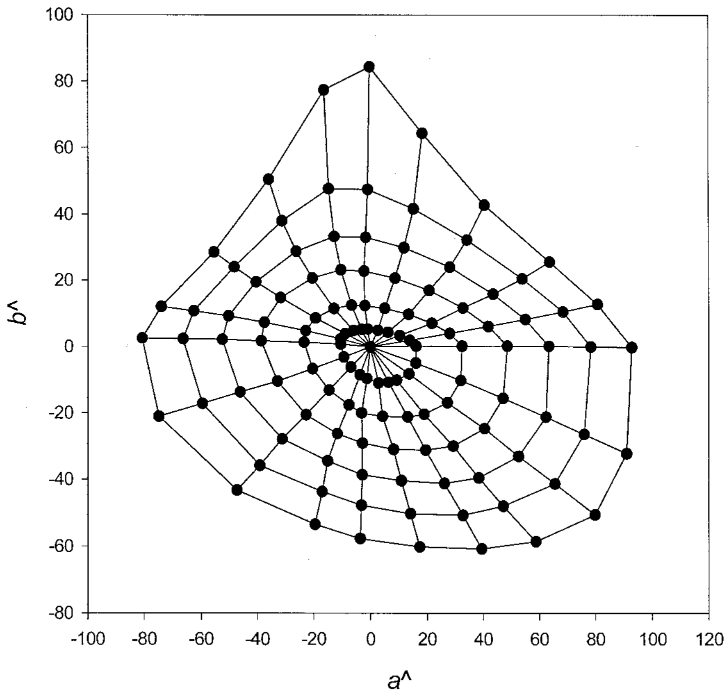

Fig. 7-7 *Munsell Re-renotation aim colors for the 2° observer and equal energy illuminant. Every second hue (5 and 10) at value 6 and chroma 0–12 in the* a^, b^ *chromatic diagram. Compare with the Renotation colors of Fig. 7-6.*

Munsell Psychological Space and Psychophysical Representation

As already sketched in Chapter 2, the ideal form of the Munsell system is cylindrical with a uniformly spaced hue circle, chroma, and value scale. We have seen earlier that this form does not account for the superimportance of hue nor for the irregular distribution of unique hues. It also is not uniform in hue terms in that the hue difference between two adjacent planes of constant hue at constant lightness varies as a function of chroma but not in the way the constant hue angle between the two planes indicates (see below). A conceptual view of the Munsell system is shown in Fig. 7-8. From the discussion above it is evident that this figure is inaccurate in regard to hue difference in terms of the basic goals of uniformity of difference, at least within an attribute. To project the reflectance functions into a space in such a way that the cylindrical form of Fig. 7-8 is duplicated has been a long-standing goal. Many attempts have been made since the definition of the Munsell Renotations, as has been seen in Chapter 6. The more successful ones are based on a form of opponent color system. Such systems usually make use of the CIE colorimetric system, the best-known example being the CIELAB color space formula. The CIELAB formula is not a very good representation of the Munsell

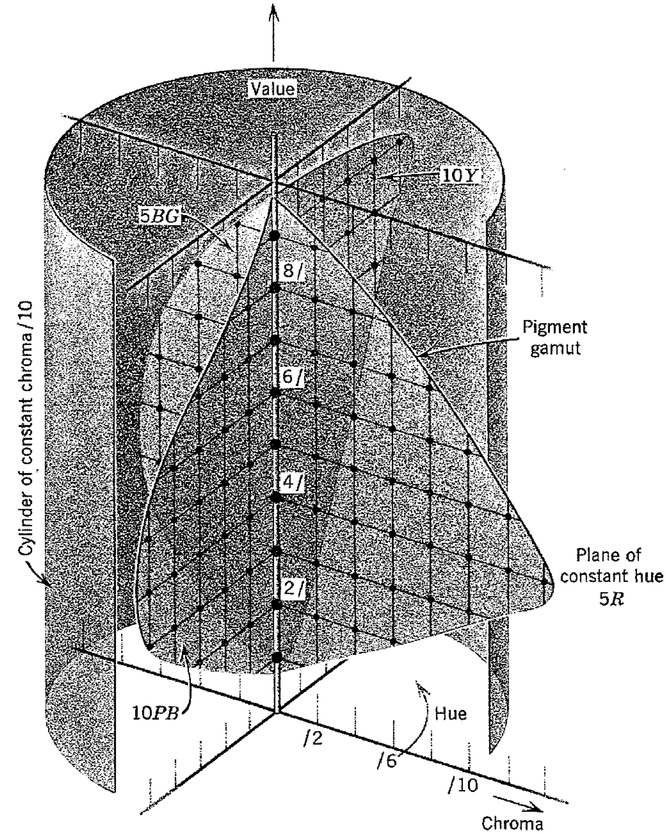

Fig. 7-8 *Schematic view of the cylindrical organization of the Munsell system, illustrating four constant hue planes and the organization of samples on these planes. The chromatic plane is not representative of the perceptual relations in the Munsell system. From Judd and Wyszecki (1975).*

psychological space as illustrated in Fig. 7-9 for the colors of Fig. 7-6. Among other things, it also has the same weakness in regard to hue difference as the psychological conceptual system of Fig. 7-8.

Among the assumptions implicit in most psychophysical models are: (1) colors are defined by two opponent color coordinates a and b and luminous reflectance and (2) unit color difference contours can be expressed in terms of distances in a and b. The CIELAB formula makes the additional assumption that the power modulation of the tristimulus values used to calculate a^*, b^*, and L^* is uniform and its value is cube root. As discussed before, this assumption was made by Adams in 1942 and is still in use. The b^* axis resulting from application of the 2° observer is a reasonable representation of unique yellow and blue, but as we have seen, the a^* axis does not coincide with unique green and unique red. Disregarding this difficulty, we can take the

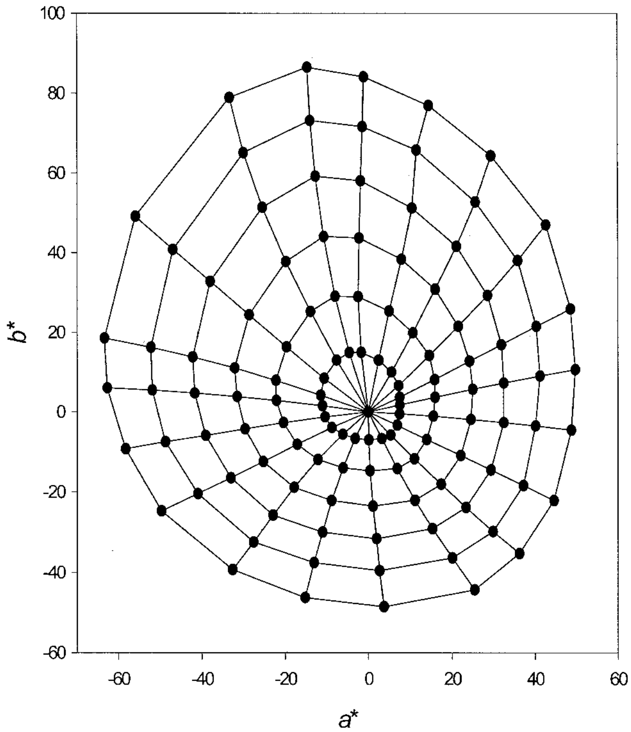

Fig. 7-9 Representation of the colors of Fig. 7-6 in the CIELAB a*, b* *diagram, 2° observer and equal energy illuminant.*

a^* scale to approximately represent greenness-redness and the b^* scale yellowness-blueness. In the four quadrants of the a^*, b^* diagram the number and spacing of hue difference steps of equal size varies.

The power modulation implicit in the Renotations can be investigated by checking it along the axes. Because adaptation negates most of the effect of different broadband white light illuminants, tristimulus values are, for purposes of simplification, normalized to the equal energy illuminant. By definition, colors along the chromatic axes at constant luminous reflectance change in X, respectively Z, only. When plotting these values, we find that the power modulation required to optimally linearize the progression of tristimulus values of the corresponding nearest Renotation colors differs by semi axis (Kuehni, 2000b). In Fig. 7-10*a* and *b* the progression of X, respectively Z, values of Munsell Renotation colors at value 6 along or very near to the opponent axes is illustrated as a function of chroma. The results indicate discontinuous functions, separated by the neutral gray. The optimized power functions for the four segments are as follows (as shown in Table 5-2): a – (green) colors 0.19, a + (red) colors 0.15, b – (blue) colors: 0.06, b + (yellow) colors 0.42. All are significantly different from a cube root function. The equations used for

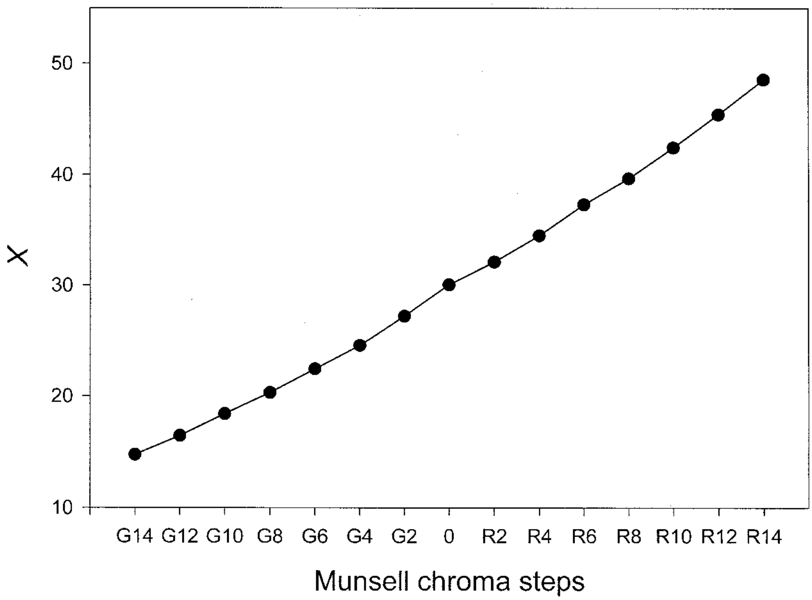

Fig. 7-10a *Progression of* X *tristimulus values of Munsell colors nearest to the* a *opponent color axis as a function of Munsell chroma, value 6, 2° observer, and equal energy illuminant.*

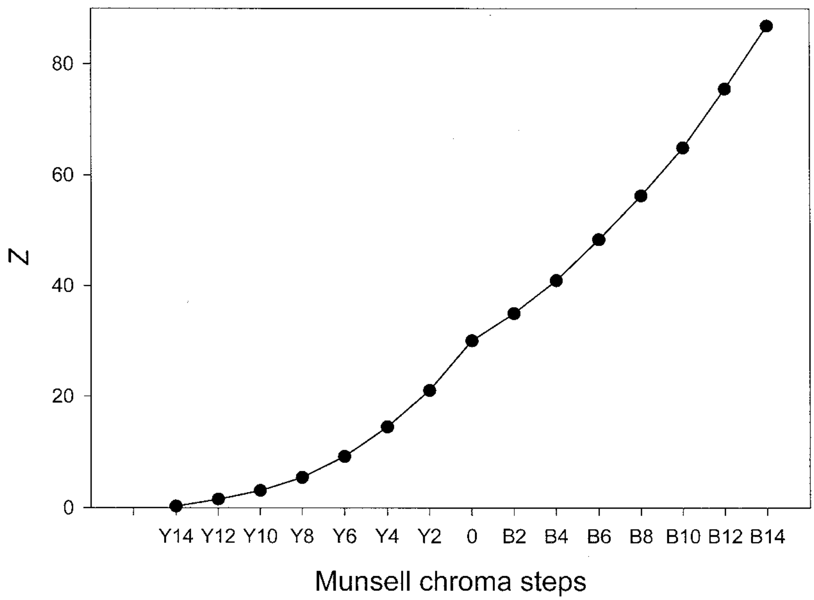

Fig. 7-10b *Progression of* Z *tristimulus values of Munsell colors nearest to the* b *opponent color axis as a function of Munsell chroma, value 6, 2° observer, and equal energy illuminant.*

calculating the chromatic coordinates of the colors of Figs. 7-3, 7-5, 7-6, and 7-7 are as follows:

$$a^\wedge + = 3500\left[\left(\frac{X}{X_0}\right)^{0.15} * \left(\frac{Y}{Y_0}\right)^{0.85} - \left(\frac{Y}{Y_0}\right)\right],$$

$$a^\wedge - = 2000\left[\left(\frac{X}{X_0}\right)^{0.19} * \left(\frac{Y}{Y_0}\right)^{0.81} - \left(\frac{Y}{Y_0}\right)\right],$$

$$b^\wedge + = 1400\left[\left(\frac{Y}{Y_0}\right) - \left(\frac{Z}{Z_0}\right)^{0.06} * \left(\frac{Y}{Y_0}\right)^{0.94}\right],$$

$$b^\wedge - = 485\left[\left(\frac{Y}{Y_0}\right) - \left(\frac{Z}{Z_0}\right)^{0.42} * \left(\frac{Y}{Y_0}\right)^{0.58}\right], \tag{7-1}$$

where X, Y, Z are the CIE tristimulus values of the Munsell colors for the 2° observer and illuminant C and X_0, Y_0, Z_0 are the tristimulus values of illuminant C. The resulting $a^\wedge$ and $b^\wedge$ values are on average of the same size as those obtained with the CIELAB formula. Without a further, nonlinear scaling factor the formula is only applicable to value 6.

From the Cone Shape in Tristimulus Space to the Cylinder in "Uniform" Psychophysical Space

As was seen in Chapter 5, constant chroma circles at different value levels approximately form an inverted, slanted cone in X, Y, Z tristimulus space. When translated into the linear opponent color space, an inverted cone remains but is aligned with the vertical Y dimension of a, b, Y space. Conceptually application of a constant power to tristimulus values efficiently converts this cone into a cylinder. When plotting Munsell Renotation colors closest to the b axis of the opponent color space (close to unique yellow and blue for the average observer) colors of the same chroma at different value levels are found to fall on a straight line (at higher values) that turns into a curved line at lower values (Fig. 7-11*a*). The same applies to colors closest to the a axis (Fig. 7-11*b*). However, the pattern is different. Much less of the available area is filled by the bluish green-to-bluish red colors than by the yellow-to-blue colors. When extrapolating the initially straight lines they are found not to intersect in one point. Intersection in one point applies if the chroma spacing at a given level of value is linearized by the same power as the value scale. But as we have seen, the chroma scales in the four axis directions from the central gray colors are optimally linearized by different powers. The Munsell Renotations have curiously uneven first steps from gray in the four directions. The validity of these steps could not be confirmed (Kuehni 2000c). Figure 7-11*a* and *b* indicate the constraints on the existence of certain colors at certain value levels.

Figure 7-12 illustrates the Weber fractions of Z at various value levels. The surprising result is two nonlinear functions of mixed composition, the upper

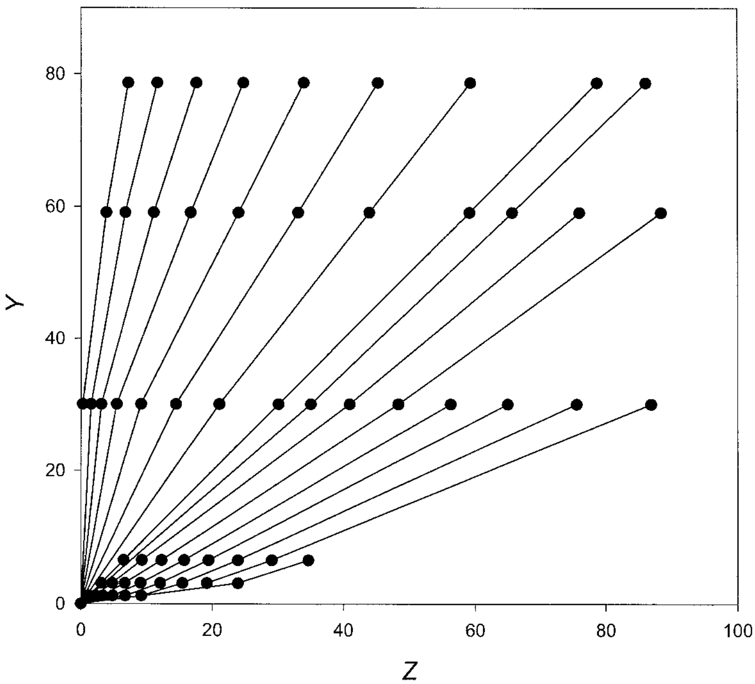

Fig. 7-11a *Plot of Munsell colors nearest to the* a *axis at six value levels (from the top 9, 8, 6, 3, 2, 1) in the* X, Y *diagram, 2° observer and equal energy illuminant. Colors of constant chroma at different value levels are connected by lines.*

consisting primarily of higher value colors and the left half of the lower curve of lower-value colors. As a result of the different powers required for chroma linearization, a cylinder is not obtained without additional adjustment. Empirically the application of a factor of $0.133Y^{0.59}$ was found to provide the required correction for the Renotations (Kuehni, 1999).

Curvature of Constant Hue Lines in the CIE Chromaticity Diagram

In most cases Renotation colors of constant hue are found to fall on curved lines in the CIE chromaticity diagram (Fig. 7-13). This is the result of the so-called Abney effect (Abney, 1910). When varying amounts of an achromatic light are added to a monochromatic light the resulting hue is not constant but changes as a function of the relative contents of chromatic and achromatic light. When such lights or corresponding object colors are represented in a uniform chromatic diagram constant hue lines become straight. Mathematical/geometrical relationship between the two methods of presentation provides an explanation. In a conceptual example an *a*^, *b*^ diagram is illustrated (Fig. 7-14*a*) in which the colors along the four axes required different powers

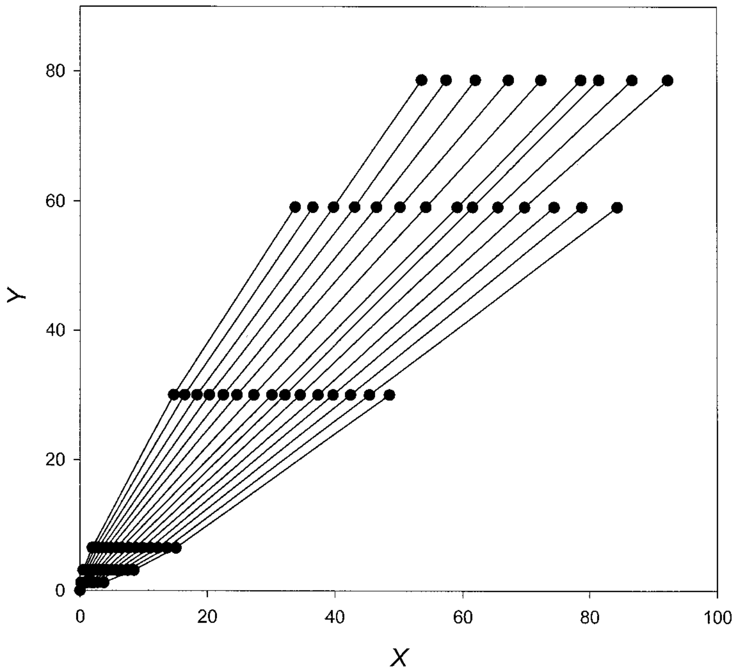

Fig. 7-11b *Plot of Munsell colors nearest to the b axis at six value levels (from the top 9, 8, 6, 3, 2, 1) in the Z, Y diagram, 2° observer and equal energy illuminant. Colors of constant chroma at different value levels are connected by lines.*

for optimization (a^+: power 0.75, a^–: power 0.5, b^+: power 0.25, b^–: power 0.333).

Calculating backward to a linear a, b diagram (Fig. 7-14b), we find the axis colors spaced differently and the intermediate colors connected with lines of different curvature. Curvature of the lines of intermediate color is also obtained if the colors along an axis are optimally linearized by a single power. If the same power is required for both axes, the line at 45° is straight while the one at 135° has a curvature. When calculating the corresponding chromaticity coordinates (Fig.7-14c) curvature is also obtained. Geometry requires that in order to have straight lines after linearization with the appropriate power non-axis lines need to be curved both in the linear a, b and the x, y diagrams in a way that depends on the powers involved. Thus in the x, y diagram most lines connecting constant hue colors of the Renotations are curved.

Unit Difference Contours in the Munsell System

Given the information about the relative magnitude of hue, value, and chroma differences in the Munsell system by Nickerson and by Bellamy and Newhall

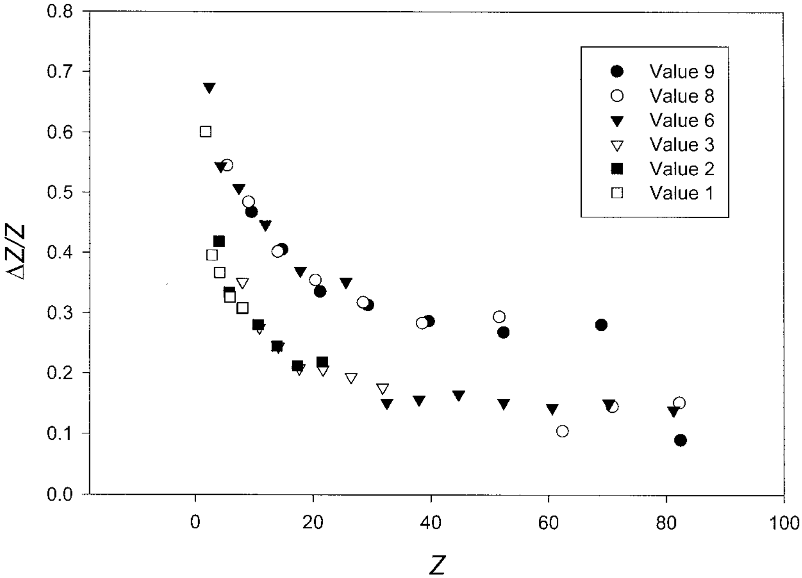

Fig. 7-12 *Weber fractions ΔZ/Z as a function of tristimulus value* Z *of Munsell colors nearest to the* b *axis at six value levels, 2° observer and equal energy illuminant.*

and that colors falling on two neighboring constant hue lines vary in hue differences between them, we can make estimates of the shape of unit chromatic differences in the Munsell system. At the level of differences of the magnitude of Munsell chroma steps, at chroma 5 three Munsell 100 hue steps equal two chroma steps. On this basis most unit contours in the psychological diagram are of a rectangular or oval nature (or perhaps trapezoidal) with the major axis of the contours aligned with constant hue lines. Based on Nickerson's relationship at chroma 5 we can calculate a ratio between major and minor axis of approximately 2:1. Using the ratio of Bellamy and Newhall at threshold the ratio is approximately 2.8:1. This indicates that unit contours at Munsell sample size differences are somewhat less elongated than at threshold level, that is, smaller hue differences require a relatively smaller stimulus increment than larger hue differences. Judd showed in 1968 that, as a result of the unit difference contour, a uniform perceptual color space cannot be mapped isomorphically into a geometric space. A euclidean model of a uniform space at the level of Munsell differences is impossible. Judd's answer to this question was a "crinkled fan" (Fig. 7-15). While a sector of this fan can be spread out a complete circular crinkled fan cannot. As a result it is apparent that the Munsell system is but a regular approximation of a uniform color solid.

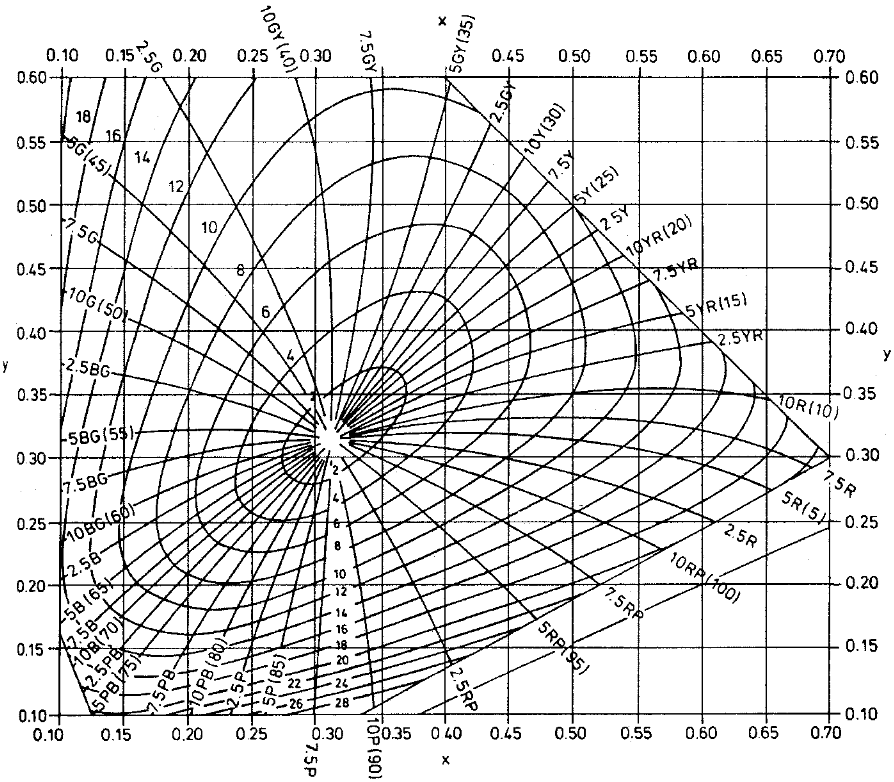

Fig. 7-13 Portion of the CIE chromaticity diagram with radial lines of constant Munsell hue and ovals of constant chroma, 2° observer and illuminant C. From Agoston (1987).

7.2 OPTICAL SOCIETY OF AMERICA UNIFORM COLOR SCALES (OSA-UCS)

Development of the System

We have already encountered some aspects of the development of OSA-UCS in the discussion of the Munsell Re-renotations. In the 1940s the issue of a uniform color space had the attention of the American National Research Council and a list of industries and governmental departments that could profit from the outcome had been established (Judd, 1955). In 1947 the Optical Society of America decided to undertake this research and a committee under the chairmanship of Judd (at the National Bureau of Standards) was appointed. He chaired it until shortly before his death in 1972 whereupon MacAdam assumed chairmanship. Membership varied somewhat during the nearly 30 years of its existence but never consisted of more than fifteen people (Nickerson, 1977). Based on a paper from 1941 by Balinkin, describing regular

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Fig. 7-14a *Hypothetical colors falling on the axis lines and intermediate angles in an* a^, b^ *diagram that by semi axis required different powers for linearization.*

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Fig. 7-14b *Back-calculated linear opponent color diagram of the same colors showing curvature of lines connecting intermediate angle colors.*

tetrahedral tiling of a uniform color space, Foss made already in the first formal meeting of the committee in 1947 a proposal to tile the committee color space in this manner. As L. Silberstein had pointed out in 1942, if one makes the size of color differences in a triangle equal in magnitude, there can only be six nearest neighbors in the chromatic plane for the system to be euclidean. With more than six neighbors the space assumes hyperbolic shape, with less than six Riemannian. Silberstein recommended testing hexagonal arrays of colors

Fig. 7-14c *Colors from Fig. 7-14a in the CIE chromaticity diagram showing a different curvature of intermediate angle color lines.*

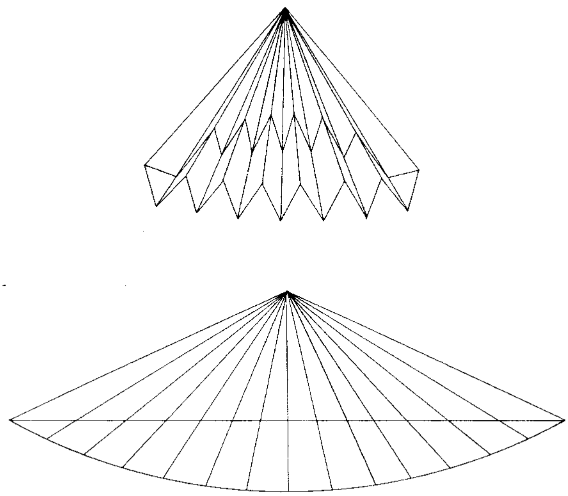

Fig. 7-15 *Segment of Judd's "crinkled fan" uniform color difference surface which considers the unit uniform difference contour of the Munsell system as a ratio between major and minor axes of 2 : 1. The straight line across the open fan is a secant. It is also shown on the crinkled version. When the crinkled fan is circular (representative of the total Munsell hue circle), it cannot be spread out (Judd, 1968).*

to determine if color space is euclidean. As we will see, in its experimental work the committee decided to follow this recommendation.

At the time there were three schools of thought in the committee:

1. The experimental facts of color difference are established, and no further work in this respect is required.
2. The experimental facts are very complex and appear to prove that a uniform euclidean color space is not possible. The best goal of the committee is to develop a simple color difference formula.
3. Uniformity, or its lack, is best exemplified by a set of color chips.

As it progressed the committee developed new experimental data, found that a uniform euclidean space is not possible, and developed what it considered to be the best approximation of a uniform euclidean system possible, exemplified by a set of chips and a formula.[3]

By the end of 1954, 38 color studies related to uniform color scaling had been reviewed (Judd, 1955). In 1952 a decision was made to assemble a 500-chip uniform three-dimensional sample set in a regular rhombohedral arrangement based on the Munsell Renotations, adjusted for its discrepancies with the results of the MacAdam (1942) and Brown-MacAdam (1949) color-matching error data. In preparation Nickerson, Judd, and Nimeroff in 1955–56 developed a constant saturation locus at value 6 (see Fig. 5-22). Newhall determined hue spacing on this locus in 1957–58, and Howett determined chromaticness spacing for near grays at values 3, 6, and 8 as well as chromaticness spacing of the Munsell value 6 plane.

Following Silberstein's recommendation, a color scaling experiment involving 43 color samples at near-constant luminous reflectance in a triangular grid pattern was then undertaken. Two auxiliary experiments were also performed: (1) determination of the relative size of chroma steps at value 6 to value steps at the same value level; (2) determination of the value of the gray having the same perceived lightness as the 43 samples with nominal value 6 (test of the Helmholtz-Kohlrausch effect). The 43 samples were arranged to form contiguous triangles in the chromatic plane, none involving the achromatic gray (Fig. 7-16). They are fundamental for the subsequent development of the Uniform Color Scales. From this arrangement 107 (mixed hue and chroma) differences between nearby colors resulted. They were evaluated by a total of 76 observers (different portions of the total set by different numbers of observers and under presumably somewhat different viewing conditions) in a two-category forced choice experimental paradigm where the observers determined if the difference between given samples A and B was larger or smaller than the difference between samples B and C.[4] From the results psychometric scale values were calculated at the NBS. The scale values were evaluated against four color difference formulas: CIE U*V*W*, Glasser-Reilly cube root, Munsell Renotation (modified Nickerson Index of Fading), and MacAdam-Friele 1965 (see Chapter 6). The highest correlation coefficient

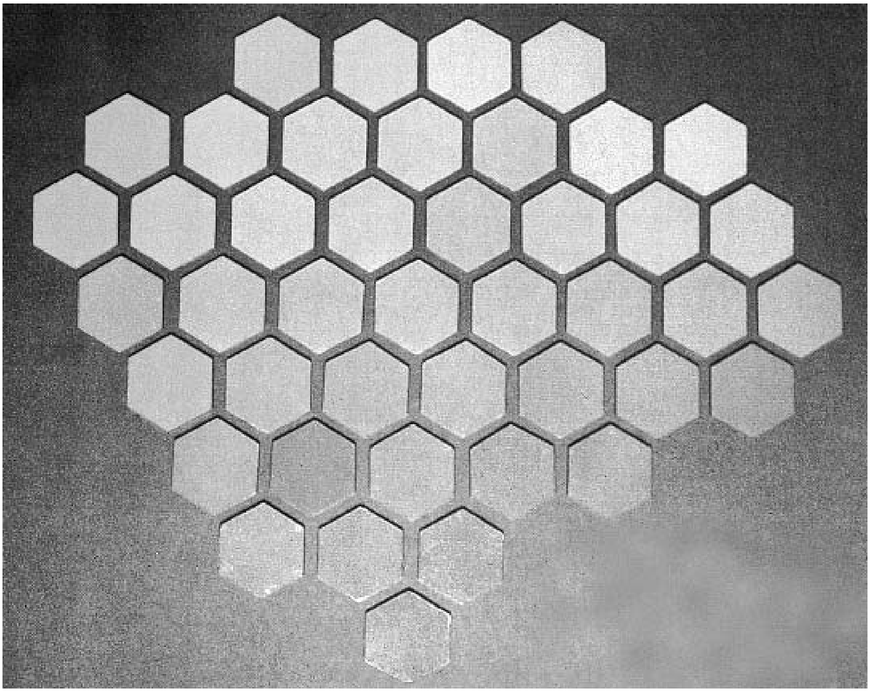

Fig. 7-16 *Image of the 43 hexagonal enamel color plates used by the OSA-UCS committee to establish the fundamental perceptual data for the system. Note that there is no achromatic color among them. (See color plate.)*

between visual and calculated color differences was a disappointing 0.45 and was achieved with the Munsell formula. A series of iterative steps (12 revisions, 33 steps) was subsequently taken to modify that formula as well as the Munsell Renotation aim data to improve the correlation between formula and committee data to 0.80. The resulting revised aim data are the Re-renotations of the NBS report discussed above.

Reilly developed modifications of the cube root formula to improve correlation with the visual data, and MacAdam proceeded from the MacAdam-Friele space to a nonlinear transformation space based on geodesics. Reilly found that correlation was significantly improved by applying a superhue weight of 2.3. Key results obtained from analysis of the visual data and comparison with colorimetric data were as follows:

1. Additivity fails by as much as 30%: ratio judgments of the differences between neutral gray and constant chroma chromatic colors in opposite direction found them to be larger by up to 30% than the direct difference between the two terminal colors.
2. Psychometric scales were found to be nonlinearly related (power 0.37 to 0.80) to *chromaticity coordinate* differences.

It had become evident that it was impossible to present the results of the evaluation of the 43 samples of the value 6 plane on a flat surface. Thus the results were in this respect in agreement with the Munsell system and findings of the MacAdam color-matching error data. The committee decided to proceed along two lines:

1. Prepare a set of color chips representing the closest a euclidean system can come to a uniform color solid and thereby to demonstrate "the reality of the various geometrical difficulties embodied in a set of painted colors" (Nickerson, 1977).
2. Develop a euclidean mathematical formula that best describes the experimental findings. At its thirteenth meeting in 1967 the committee voted to adopt the following statement: "We will affirm to the world that no regular rhombohedral lattice sampling of color space, with a fixed background, can exist; we will produce the best approximation to such a lattice for neutral value 6 background that we can design; and we will specify some or all of the perceived sizes of the differences in our lattice."

The interest of design-oriented people in the novel scales revealed by the crystalline structure was also a consideration for continuing the effort.

The test of the magnitude of the Helmholtz-Kohlrausch effect using a set of gray tiles to compare against the 43 chromatic tiles gave results in reasonable agreement with the Sanders and Wyszecki (1957) determination using light colors (see Chapter 5 for a quantitative comparison of the two sets of data). The committee developed a formula to describe the results as a function of the CIE chromaticity coordinates.

To establish additional experimental facts of the relationship between chromatic and lightness differences, another four sets of four samples each were prepared. Each set formed a regular tetrahedron in the tentative "uniform" color space, one set being yellow at $Y = 90$, another red at $Y = 10$, the third green at $Y = 30$, and the last blue at $Y = 6.6$. Differences were compared within and across sets. A table with the final data lists the observed scale differences as determined from the psychometric scales, the difference computed with the committee's formula and the error (MacAdam, 1974).

Optimized Formula

With the visual results in hand several independent efforts to arrive at an optimized mathematical formula were made. In 1970 two formulas were in contention as the best: a version of MacAdam's nonlinear transformation formula (xi, eta formula; see Chapter 6) and Reilly's modified cube root formula. MacAdam's formula was found not to extrapolate well beyond the experimental data. The committee was interested in a formula that extends reasonably to the limits of object color perception (the MacAdam limits). Reilly's formula was found to do a better job in this respect. Various parameters of

Reilly's formula were now optimized to provide the best correlation obtainable with such a formula (see equation 6-54). The committee selected a set of color fundamentals R, G, B to fit the visual data that imply as yet unknown neurophysiological processes to achieve the fit, illustrated in Fig. 7-17*a*. Figure 7-17*b* shows the committee's opponent color functions j and g. In Fig. 7-17*c* the linear form of these functions is compared to balanced linear opponent functions a and b derived from the 10° observer functions. For ease of comparison the conventional functions have been multiplied by a factor of 8. The results indicate systematic deviations, particularly in higher implied redness and lower blueness of bluish-reddish colors (short wavelengths).

The lightness scale is a modified Semmelroth scale based on a surround of $Y = 30$ and is illustrated in Fig. 7-18. Lightnesses of chromatic colors are further adjusted by a correction for the Helmholtz-Kohlrausch effect in the form of a modified Sanders-Wyszecki formula with calculation from CIE chromaticity coordinates (it should be noted that experimental determination of the HKE had been made only at one level of luminous reflectance).

The fit of the formula to the visual data involving 43 samples is shown graphically in Fig. 7-19 (MacAdam, 1974). The gaps represent cases where the calculated difference exceeds the average perceived differences, and the rectangles the opposite. It is evident that while there is a general impression of inhomogeneity, there is a degree of systematic deviation. Individual differences have errors of the calculated difference of up to 70%. For a mean observed difference of 2.483 units for all 174 differences included by the committee the root mean square error (RMS) using the committee's final formula

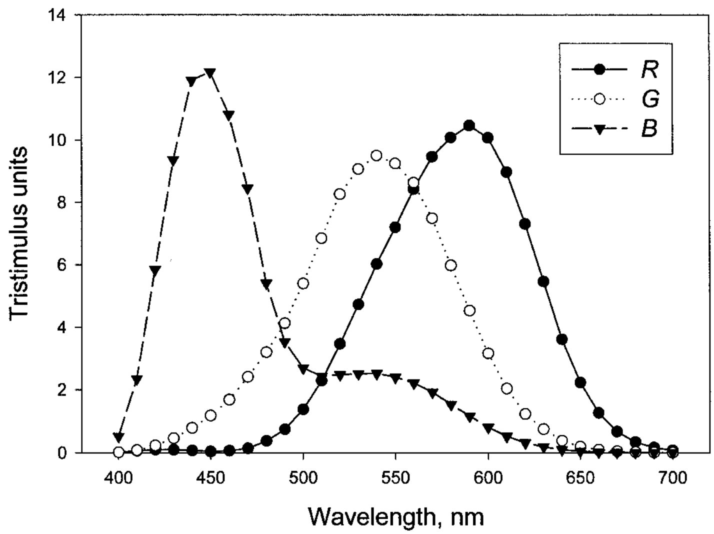

Fig. 7-17a *Spectral* R, G, B *functions selected by the Committee on Uniform Color Scales for the formula fitted to the experimental data, 10° observer and illuminant D65.*

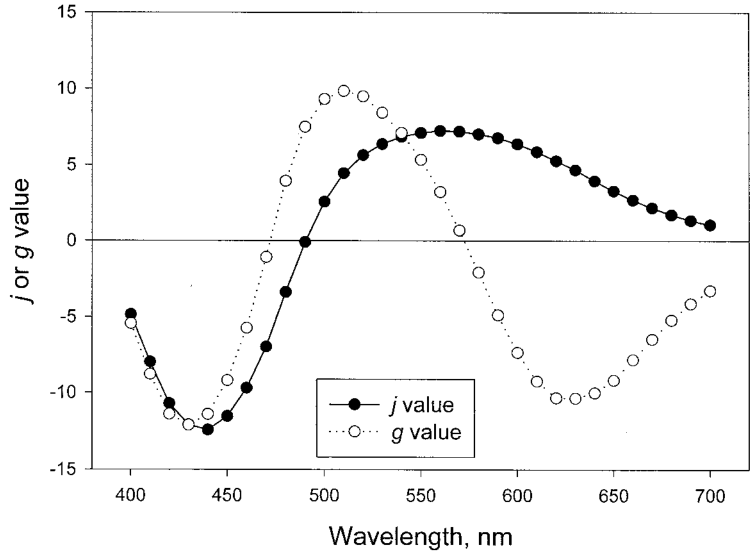

Fig. 7-17b *Spectral opponent color functions* j *and* g *as defined by the Committee, 10° observer and illuminant D65.*

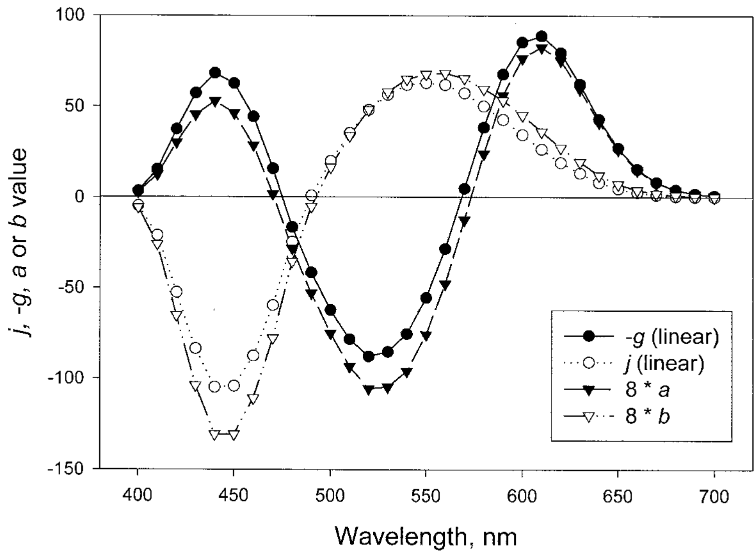

Fig. 7-17c *Comparison of linear spectral opponent functions of the OSA-UCS system with balanced linear functions* a *and* b *derived from 10° observer data. For ease of comparison the linear* g *function has been inverted and the* a *and* b *functions multiplied with the factor 8.*

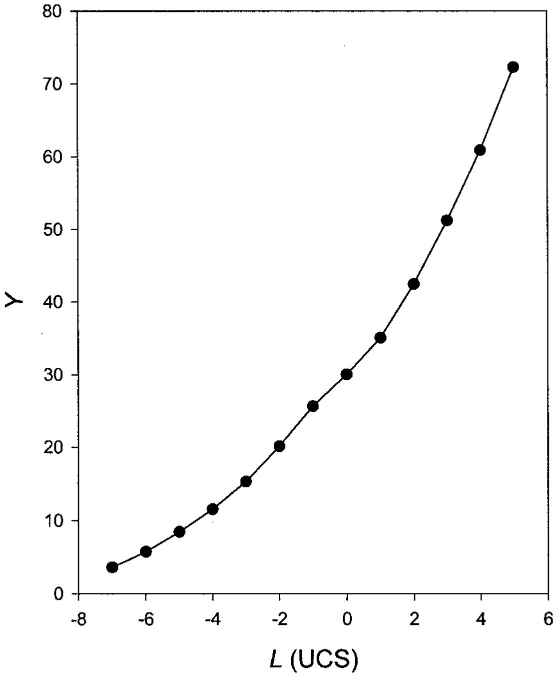

Fig. 7-18 *OSA-UCS lightness* L *as a function of luminous reflectance* Y*, with lightness crispening for surround* L = *0.*

was calculated as 0.421, or 17%. The correlation coefficient between calculated and observed values is 0.45.

The system was described by MacAdam in 1974. An extensive report of the work of the committee, envisaged by Judd, was never published. It is surprising to note that the issue of hue superimportance and its implication for the accuracy of OSA-UCS is absent from the system description. Judd covered hue superimportance in considerable detail in his 1968 paper on ideal color space but without reference to OSA-UCS. In his book *Color measurement* of 1981 MacAdam made the following statement: ". . . the committee forced the data into a euclidean form, despite clear indications that the judgment data required noneuclidean representation." The issue is absent in Wyszecki and Stiles's, Derefeldt's, or Berns's recent description of OSA-UCS (Berns, 2000).

OSA-UCS Atlas

With the formula established, the committee proceeded to sample the implied space according to the rules of regular rhombohedral sampling. Rather than use triangular sampling of the constant lightness plane, it was decided to rotate the unit cubo octahedron to obtain a square grid in the constant lightness

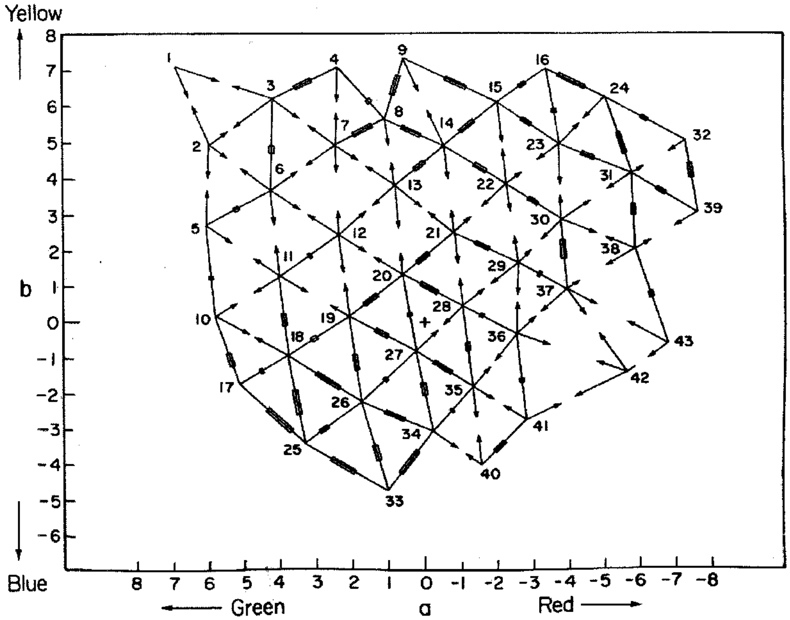

Fig. 7-19 *Plot of 102 chromatic differences between 43 color samples in the chromatic diagram of the Committee on Uniform Color Scales. Numbers identify the samples. Lines with arrows and bars indicate the size of the average visual judgments. The colorimetric difference is too small in case of arrows, too large in case of bars. From MacAdam (1974).*

plane. The twelve distances from the central point M to any of the points on the surface of the cubo octahedron are the same (Fig. 7-20; see also Figs. 2-49 and 2-50). By implication, the distances between the four points each (I, J, K, L and A, B, C, D) are also the same. The basic cubo octahedron of the system is illustrated in Fig. 7-20 and the lattice, doubly expanded in all three dimensions, is illustrated in Fig. 7-21. Preceding and succeeding constant lightness planes are offset by one *j*, *g* unit. An illustrated description of the space lattice used was offered by Foss in 1978 (see Fig. 7-21).

A three-dimensional model of the system was manufactured by MacAdam. He mounted spheres, dipped in the formulated paints, in cubo octahedral configuration as shown in Fig. 2-51. This figure, as well as Fig. 7-20, illustrates the existence of seven cleavage planes that allow bisection of the color solid in various directions. The cleavage planes are formed by the following points in the figure:

Plane 1	E, F, G, H (constant lightness plane)
Plane 2	B, F, I, K, H, D
Plane 3	A, F, I, L, H, C
Plane 4	I, J, G, D, C, E
Plane 5	K, L, G, B, A, E

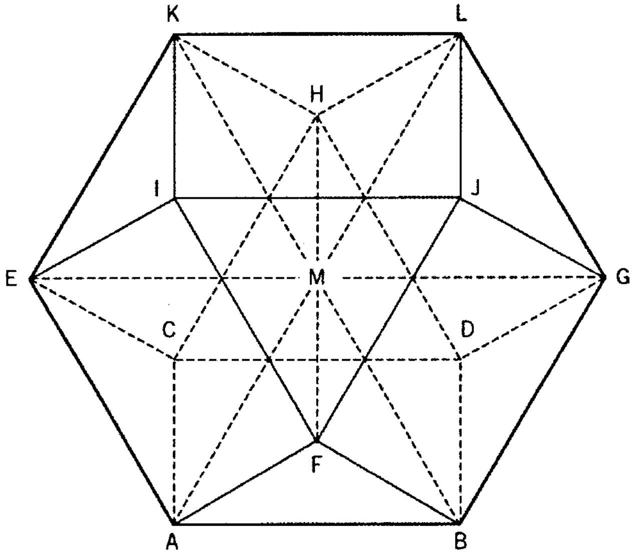

Fig. 7-20 *Cubo octahedral arrangement of twelve colors equally distant from central color M. Colors I, J, K, and L represent the square forming the square grid pattern of the OSA-UCS system at constant lightness. Compare with Figs. 2-49 and 2-50.*

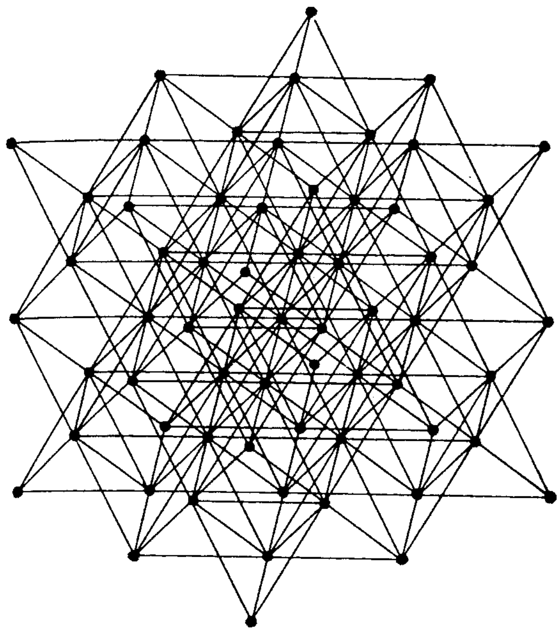

Fig. 7-21 *Basic cubo octahedral unit lattice doubly expanded in all three dimensions. From Foss (1978).*

Plane 6 I, A, D, L
Plane 7 J, B, C, K

In 1974 MacAdam stated: "The regular-rhombohedral system of color sampling adopted by the Optical Society Committee on Uniform Color Scales provides the maximum possible number and variety of uniform color scales and exhibits the maximum possible variety of relationships among colors. The committee hopes that artists and designers will find it useful in devising new and beautiful arrangements of colors."

An identification system based on three numbers was devised to designate each color. The letter j (for the French *jaune*, yellow) roughly indicates yellowness-blueness, the letter g approximately greenness-redness, and the letter L lightness. For the samples of the atlas j ranges from –6 (blue) to +12 (yellow), g from –10 (red) to +6 (green), L from –7 (dark) to +5 (light). Simple mathematical relationships connect the colors of each cleavage plane. In studying systematic color scales, the committee noticed the paucity of near neutral samples and decided to add a series of such samples at half steps, centered on $L = 0$ (the pastel set). The 134 samples of that set range from $L = -1.5$ to $L = 1.5$.

The Optical Society published in 1977 an atlas containing a total of 558 glossy color chips (424 of the regular and 134 of the pastel set), representing the regular rhombohedral system. Colorimetric specifications of the aim colors in terms of CIE chromaticity coordinates and luminous reflectance for the 10° observer and illuminant D65 have been provided by MacAdam (see also Wyszecki and Stiles, 1982).

A detailed description of the regular rhombohedral structure of the OSA-UCS system is found in Agoston (1987). A standard practice for using the system has been issued as ASTM procedure E1360. A software package, Color Cleaver®, is available that displays approximate OSA-UCS scales on a color monitor (Luke, 2001).

A Revised OSA-UCS

In 1990 MacAdam published a revision of OSA-UCS based on 2000 judgments by two young color normal observers. A major advantage claimed by MacAdam for the new observations is that for the first time samples of all twelve lightness planes were evaluated by the same observers. As a result of regression calculations he proposed redefined R and B values as well as redefined j and g values:

$$\begin{aligned} R &= 0.9285X + 0.3251Y - 0.1915Z, \\ B &= -0.2032X + 0.60Y + 0.5523Z, \\ g &= C(12.7R^{1/3} + 19G^{1/3} - 6.3B^{1/3}), \\ j &= C(-1.3R^{1/3} + 17G^{1/3} - 15.7B^{1/3}). \end{aligned} \tag{7-2}$$

These changed values amount to a significant revision. MacAdam also calculated revised aim colors for a regular rhombohedral system based on the revised coordinates.[5] A limited number of corresponding samples were prepared in Denmark by Ransing. However, in a handwritten letter dated March 22, 1995, MacAdam stated: "Incidentally, I have found some errors in that (1990) JOSA paper, and withdraw it entirely." (MacAdam, 1995) An extensive visual confirmation of the OSA-UCS scales is missing to date (but see Indow, 2001).

Comparisons of the Munsell Renotations with OSA-UCS have been made by Wyszecki (1954), Nickerson (e.g., 1978), and Agoston (1987).

How Uniform Is OSA-UCS?

Disregarding hue superimportance in the formula resulted in reduced correlation and, presumably, perceptual nonuniformity of the differences between the samples. In the late 1980s an Inter-Society Color Council Project Committee (44) performed scaling near the achromatic point of the OSA-UCS scales. They found the first steps from gray in the four axis directions of the even numbered lightness levels to be too large compared to more chromatic steps, perhaps indicative of a small chromatic crispening effect. The results of the committee have not been published, and it has been dissolved.

Recently Indow has begun to psychologically measure OSA-UCS using comparison against a Munsell value step. Preliminary results (Indow, 2001) indicate a surprising level of disagreement between the committee results and those of Indow. The disagreement appears to be random in nature. Visual differences along constant j and g lines in the $L = 0$ constant lightness plane vary from 0.63 to 1.55, a factor of nearly 2.5. The coefficient of variation of all of Indow's judgments against the system is 23%. The first steps from gray were found to be larger in the red and blue directions as compared to the green and yellow directions, a result not in agreement with that by Kuehni (2000c).

As already Reilly had found, considerable improvement of fit over that achieved by the committee's formula is possible by considering hue superimportance. Considerable reduction in the RMS error (when compared to visual data) is obtained by separately weighting by quadrant the hue and chroma differences. They are calculated for differences between sample pairs of applying CIELAB methodology to the OSA-UCS total differences. Chroma and hue differences were calculated assuming a euclidean space. The optimization was limited to the 104 differences involving the original 43 tiles. Optimal results by the method described were obtained when multiplying in quadrants 1–3 chroma differences with the factor 0.6 and hue differences with the factor 1.3. In quadrant 4 the factors 1.3 for chroma differences and 1.8 for hue differences proved optimal. The higher values but smaller ratio for quadrant 4 (greenish-bluish colors) indicate that here the visual differences between sample pairs were seen as relatively larger than in the other three quadrants, for reasons that are not obvious. The factors in quadrants 1–3 indicate a superhue weight of 2.2 (close to the one obtained by Reilly, mentioned earlier). The resulting average total color difference was nearly identical to the committee's value for these data. For the reduced data set (104 differences) the committee's RMS error is 20.2%, with a result of 12.7% after the described optimization. The correlation coefficient is improved from a value of 0.45 to 0.81. The results indicate that due to superimportance of hue the unit chromatic difference contours in the OSA-UCS basis data in the *j*, *g* diagram are also elongated in a radial direction, in agreement with the Munsell and small color difference data.

More exactly the ratio of the multiplication factors in quadrants 1–3 is 2.17, comparable to the ratio of 2.0 for Munsell colors at chroma 5 (2.8 at the JND level). In quadrant 4 it is 1.38. Colors falling on diagonals at 45 and 135° are chroma differences only. Colors such as *j*4, *g*0 and *j*0, *g*4 are essentially hue differences from *j*2, *g*2, a color on the diagonal. If hue superimportance has been suppressed in the physical system, one would expect the essentially hue differences to be perceptually larger than the chroma differences. Informal evaluations confirm this. A formal experiment is required. The situation implies that in the final system the visual distances along diagonals in a square anchored at the neutral point are not identical as they should be in a uniform system.

As a result we find that the apparent paradox of circular unit chromatic contours in OSA-UCS compared to the elongated unit contours in the Munsell system and in small color difference data is due to the suppression of hue superimportance in OSA-UCS. In reality the visual data in all three cases are in general agreement. The suppression has also resulted in unequal visual distances along diagonal lines in the *j*, *g* diagram. As a result of these nonuniformities the major value of OSA-UCS should today perhaps be seen in its various types of more or less uniform scales and their aesthetics.

OSA-UCS Psychological Space and Psychophysical Representation

The form of the OSA-UCS psychological space is that of a cube. The chromatic plane is defined by a regular grid of squares. While lightness is an explicit attribute, hue and hue difference is not. Chroma is only explicitly defined along the two major axes and the diagonals. All other steps in the chromatic plane are varying sums of hue and chroma differences. As mentioned before, unlike Munsell, the OSA-UCS system is defined against a specified surround of $Y = 30$ and constant lightness of chromatic samples does not imply constant luminous reflectance because the latter has been adjusted to reflect the Helmholtz-Kohlrausch effect. The locations of unique hues in OSA-UCS as shown in Table 4-1 indicate that this system is considerably different from a Hering system in terms of implied hue spacing.

Plots of X against Y values (Fig. 7-22*a* and *b*) of colors along the axis where $Z = 0$ and Z against Y where $X = 0$ reveal a picture similar to that of Fig. 7-11*a* and *b*. However, the spacing is more uniform and continuous throughout

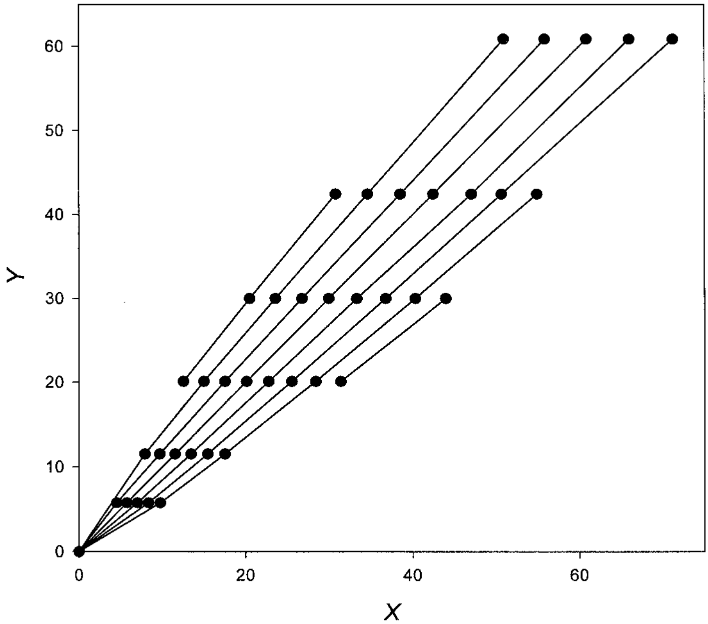

Fig. 7-22a *OSA-UCS colors falling on the g axis at six levels of lightness* L *in the* X, Y *diagram. Achromatic colors fall on the diagonal, 10° observer and illuminant D65. Colors of constant chroma at different value levels are connected by lines. Compare with Fig. 7-11*a.

the range as one would expect from partitioning of the scale by a formula. Here the first steps from gray were found to be in good agreement with the average judgment of 35 observers (Kuehni, 2000b). As mentioned earlier, the optimized linearizing powers are 0.84 for the green colors and 0.70 for the red colors. They are 0.58 for the yellow colors and 0.33 for the blue colors. The average increments in X and Z have the same ratio of 1:2.4 in OSA-UCS and Munsell, but the average increment is 30% larger in OSA-UCS than in Munsell. Figure 7-23 illustrates the Weber fractions for Z (comparable to Fig. 7-12) at six levels of lightness for colors on the j axis. They have a greater regularity than those for the Munsell system for reasons mentioned.

7.3 THE SWEDISH NATURAL COLOR SYSTEM (NCS)

The designation "natural color system" has been used by Hering (*Das natürliche Farbsystem*; Hering, 1905). To quote Hering: "For a systematic grouping of colors the only thing that matters is *color* itself. Neither the qualitative (frequency) nor quantitative (amplitude) physical properties of the radiations are relevant."[6] Thus Hering developed a system on perceptual basis only. He postulated two achromatic percepts white and black and their ratio W/B representing proportions. From this follows the expression for whiteness

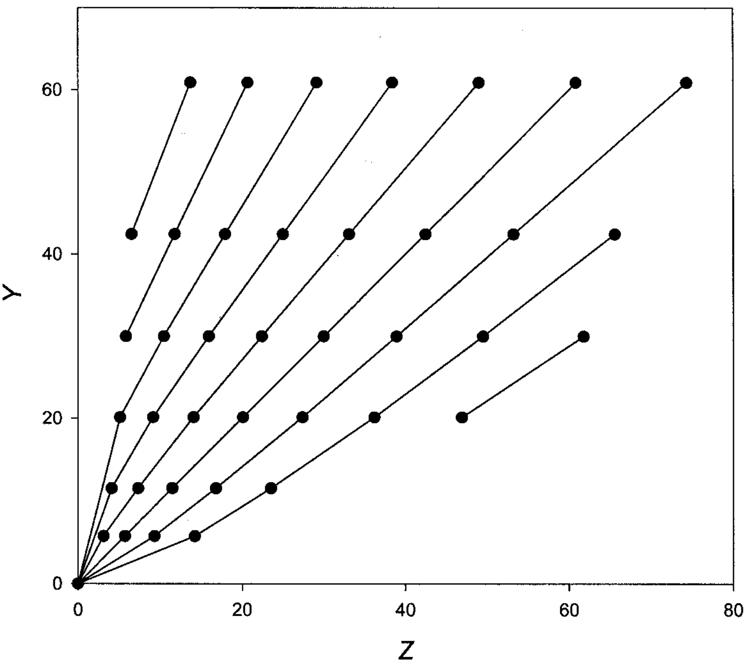

Fig. 7-22b *OSA-UCS colors falling on the* j *axis at six levels of lightness* L *in the* Z, Y *diagram. Achromatic colors fall on the diagonal, 10° observer and illuminant D65. Colors of constant chroma at different value levels are connected by lines. Compare with Fig. 7-11*b.

$H = W/(W + B)$. Hering then described the development of a perceptually equally spaced gray scale, based on the degree of similarity of a given step to white and black. Most chromatic object colors, according to Hering, appear veiled by the presence of whiteness, grayness, or blackness. "Chromatic colors that do not obviously show such veiling I shall call unmasked chromatic colors. . . ." Unmasked and veiled chromatic colors can be ordered according to hue in a color circle. Hering defined four unique hues (*Urfarben*) and placed them in such a manner on the hue circle that they divide it into its four quadrants. Nearly one-half of this circle contains yellowish colors, nearly the other bluish, and so for greenish and reddish colors. The colors in a quadrant can then be defined by the relative amounts of the two adjacent unique hues in a way similar to that described for achromatic colors. Hering placed the unmasked (or "full," *Vollfarbe*) color, black, and white at the corners of an equilateral triangle into which all veiled colors of the hue of the unmasked color can be systematically arranged.

Hering believed that the clarity of chromatic colors could not be determined exactly; it can only be said of two colors which one is less veiled. "In my view, every color that we actually see is more or less veiled. . . ." Any object color perception can be described by its chromatic and achromatic compo-

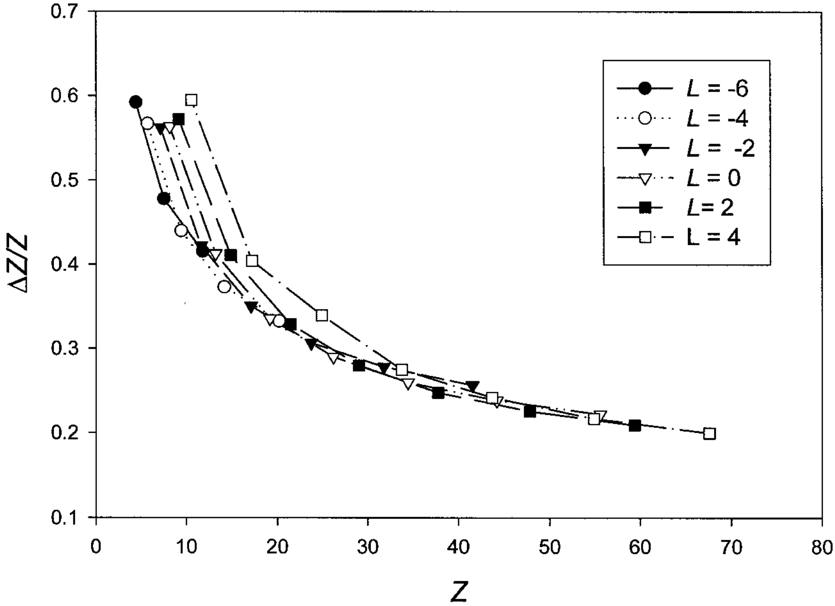

Fig. 7-23 *Weber fractions of* Z *for OSA-UCS colors falling on the* j *axis, at six levels of lightness* L, *10° observer and equal energy illuminant. Compare with Fig. 7-12.*

nents: "It can be said that in each clearly veiled chromatic color both a chromatic and a black-white component can be distinguished. . . ." Regarding lightness/brightness of chromatic colors Hering said: "It is not such a simple matter for chromatic colors, whose brightness or darkness is determined not only by their black-white components, but also in part by their chromatic components. . . . I have . . . ascribed an *intrinsic brightness* to yellow and red and an *intrinsic darkness* to blue and green." Hering's equilateral constant hue triangle therefore does not express anything about lightness or darkness of its colors but only about the degree of veiling of its full color with white and/or black.

As discussed in Chapter 2, Ostwald departed from Hering in attempting a psychophysical definition while maintaining the geometrical form. Restricting himself to colors that could be achieved with colorants, his full colors of necessity fell short of Hering's full colors. The form of his color solid is based on an arrangement of Hering type constant hue triangles in a hue circle, resulting in a double cone solid. But his hue circle is not based on unique hues but on yellow, red, and blue primaries.

In the same chapter mention is made of the efforts of two Swedish researchers, Johannson and Hesselgren, who tried to find a compromise between Hering's double cone and a color solid based in its third dimension on a lightness scale. Johansson accepted hue, saturation, and value or lightness as the three properties uniquely characterizing any color perception. He suggested adding to these clearness or brilliancy and efficiency or vividness. Hesselgren's color atlas of 1952 relied on his idea of subjective color standardization. Regularity of spacing in that atlas was not considered to be completely successful, and in 1964 the newly founded Swedish Color Center Foundation began work toward a revision of the Hesselgren atlas by making new psychophysical experiments. In 1966 Tonnquist reported on the determination of unique hues and a comparison between hue circles based on unique hues ("symmetrical") and those based on equality of perceived hue differences. At the time the plan was to revise the atlas to be in principal agreement with Johansson's version of a natural color system. Eventually, a decision was made to revert back to Hering's original ideas and develop the system accordingly (Hård et al., 1996).

In a pilot study forty subjects were divided into two groups, one of which assessed a series of color samples according to the Hering principles by reference to six samples representing the six elementary Hering colors. The other group assessed them without reference to samples but based on their internal concepts of the six elementary colors. While there were considerable absolute differences in the average percentages between the two sets of results, rank order showed a high level of correlation. The interobserver variability was found to be low and comparable for both groups. Encouraged by these results the Swedish Natural Color System (NCS) was developed on the basis of judgments without reference samples.

From information provided one can estimate that in the basic experiment about fifty observers visually assessed approximately 200 samples. Confidence intervals at the 95% level of the means of the percentage judgments were found to be less than 5 units on 100 unit scales each. Blackness was then compared to luminous reflectance, chromaticness to the euclidean distance from the neutral point in the CIE chromaticity diagram, and hue to the hue angle in the same diagram. From interpolation on these psychophysical scales some 900 samples were produced, and these were assessed in arrays by five to six trained observers for smoothing purposes ("beauty test for acceptance").[7]

The structure of the system is illustrated schematically in Fig. 7-24. It consists of a double cone, formed in the atlas by forty equilateral constant hue triangles. A gray scale forms the centerline connecting the two apices.

Fitting of Psychophysical Scales

A hyperbolic formula was fitted by least square method to the visually established gray scale and was found to be in agreement with the Adams-Cobb formula for a surround of $Y = 56$: $w = Y(56 + 100)/(Y + 56)$. This was found to be in reasonable agreement with the luminous reflectance of the light booth interior used in the judgments ($Y = 54$), even though the luminous reflectance of the immediate surround had $Y = 78$.

Other relationships to psychophysical scales were found to be much less direct. Lines connecting samples with constant luminous reflectance were

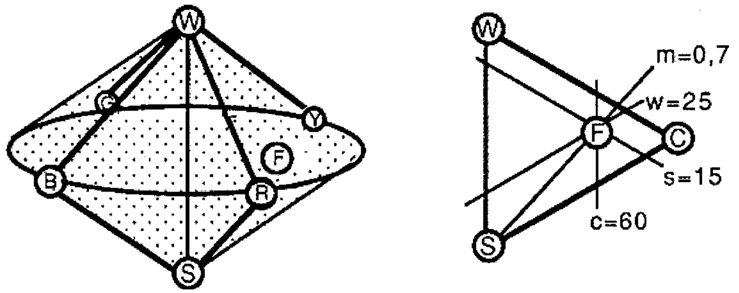

Fig. 7-24 *Schematic views of NCS.* Left*: double cone with circle of full colors.* W*: white;* S*: black;* F*: location of a given color in the double cone.* Right*: Constant hue triangle with full color C. The location of color F is determined by its blackness* s *and chromaticness* c*. From the two values whiteness* w *can be derived. From Hård et al. (1996).*

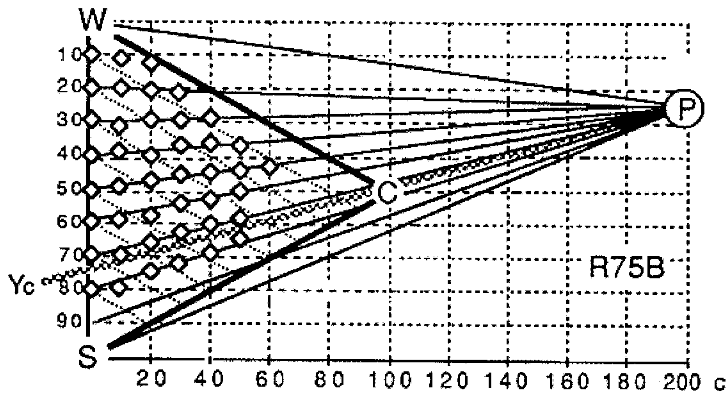

Fig. 7-25 *Lines connecting colors of constant luminous reflectance for NCS hue R75B.* Y_C *denotes the intrinsic blackness of full color C. From Hård et al. (1996).*

plotted into constant hue NCS triangles and found to approximately intersect at a point outside the triangle (Fig. 7-25). The position of this intersection point varies widely by hue. Dominant wavelength of constant hue colors, as one would expect, varies for most colors as a function of chromaticness and lightness.

In the CIE chromaticity diagram or the a^*, b^* diagram NCS constant hue lines are not straight, and constant chromaticness lines do not form circles, pronouncedly so at higher chromaticness (Fig. 7-26). One of the reasons is Hering's arbitrary decision to value chromaticness of all full colors identically at 100. In terms of chroma they vary significantly. Meaningful hue angle differences between hue steps, such as presented in Fig. 5-25 for the Newhall data set, cannot be calculated because of the very considerable variation in lightness of the full colors. The work of Billmeyer and Bencuya (1987) indicate the considerable difference in apparent hue spacing of the Munsell Renotations

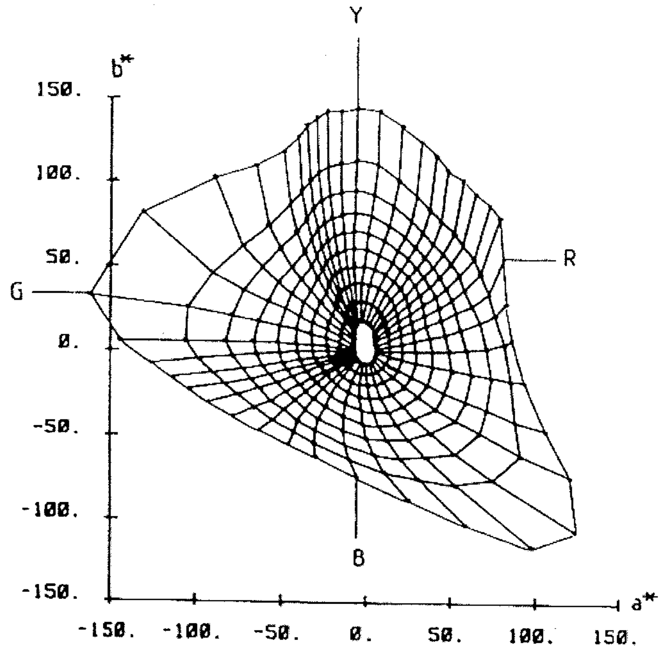

Fig. 7-26 *NCS colors of constant hue (radial lines) and constant chromaticness (ovoids) in the CIELAB* a*, b* *diagram. Capital letters indicate the four NCS unique hues. From Derefeldt and Hedin (1987).*

compared to NCS. As discussed in Chapter 5, depending on what is true constant chroma, the Renotations may contain some hue-chroma conflation. In the case of NCS where constant hue steps at constant chromaticness involve changes in hue, chroma, and lightness, hue steps likely involve a considerable degree of conflation. The disagreement in apparent hue angle differences is therefore not surprising. Indow has recently also begun to investigate the psychological spacing of NCS (Indow, 2001). Of interest is the visual judgment of the hue circle at constant blackness and chromaticness. A complete circle was visually judged at s = 20 and c = 60 and a partial circle at s = 10 and c = 70. Based on summation of differences the segment angles in the inner circle between the four unique hues of the system are R-Y 93°, Y-G 85°, G-B 82°, and B-R 100°, These figures deviate considerably from those of Table 4-1.

In a manner comparable to that shown above for the Munsell Renotations and OSA-UCS, the colors falling near the axes in the X, Y, Z space are shown in Fig. 7-27*a* and *b*. These figures demonstrate the significant difference between attempted uniform color solids and NCS. While the systematic arrangement of the samples is apparent it is of an entirely different nature than that of roughly uniform solids. Developers of NCS have written computer

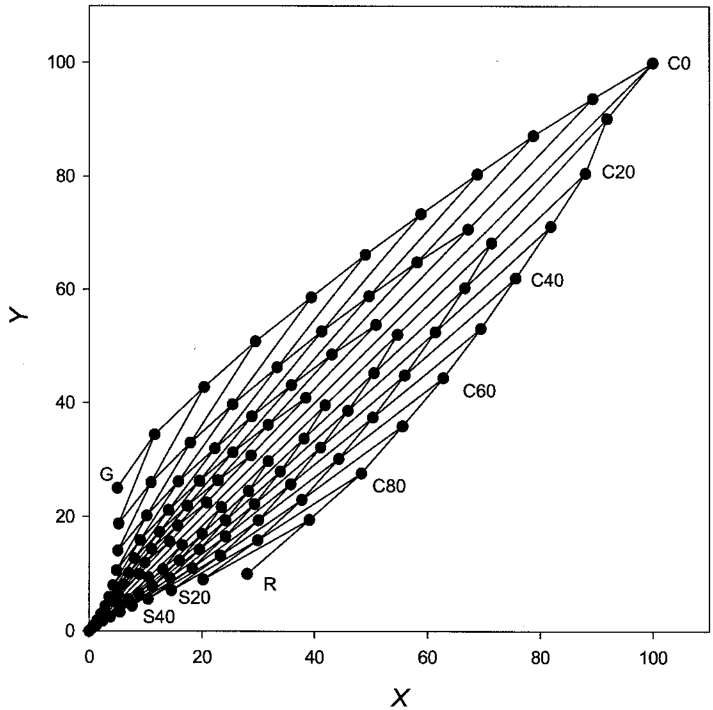

Fig. 7-27a *NCS colors falling closest to the a opponent axis in the* X, Y *diagram. Achromatic colors fall on the diagonal. Lines converging on the diagram origin connect colors of constant NCS chromaticness, curved lines connect colors of constant blackness, 2° observer and equal energy illuminant. Compare with Figs.7-11*a *and 7-22*a.

software that converts, by iterative methods CIE colorimetric values, into NCS values, and vice versa.

NCS Color Terminology and Atlas

Around the 40-step hue circle the hues (other than the unique hues) are identified in terms of their two constituting unique hues, for example, R70B, with the meaning of a hue constituted of 30 parts redness and 70 parts blueness. For veiled colors, according to Hering, the sum of the components of one or two unique hues, whiteness and blackness, is always 100. Chromaticness c is expressed as the percentage of chromatic components in the total color. The full specification of an NCS color is expressed as in the following example: 4030-R70B, indicating a color of blackness s = 40, chromaticness c = 30, and hue as defined above. The system and applications have been described by Hård and Sivik (1981) and by Hård et al. (1996a, b).

The 1995 edition of the atlas contains 1750 color chips of 13 × 15 mm size arranged on forty pages (see Fig. 2-53). It is available in a matte and a glossy

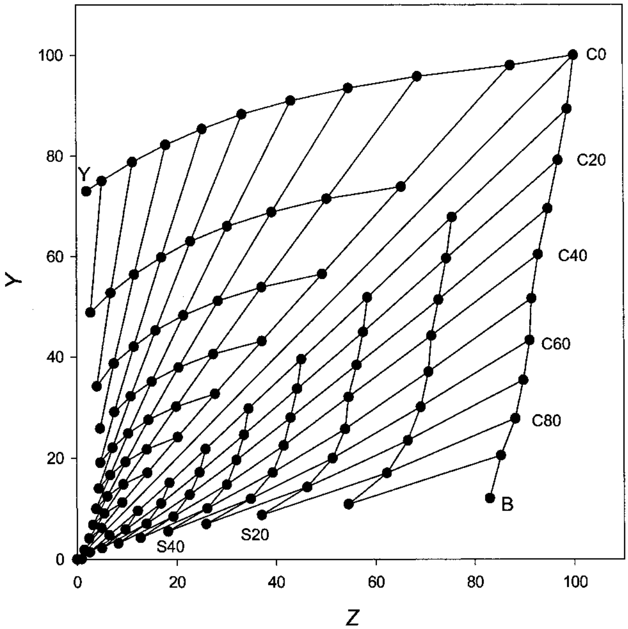

Fig. 7-27b *NCS colors falling closest to the* b *opponent axis in the* Z, Y *diagram. Achromatic colors fall on the diagonal. Lines converging on the diagram origin connect colors of constant NCS chromaticness, curved lines connect colors of constant blackness, 2° observer and equal energy illuminant. Compare with Figs. 7-11*b *and 7-22*b.

version. Swedish Standard SS 01 91 03 contains the aim color specifications of the atlas and many interpolated and extrapolated colors (16,800) in terms of CIE tristimulus values and chromaticity coordinates for the CIE 2° observer and illuminant C.

NCS System and NCS Atlas

The NCS system in its essence represents Hering's idea of describing any object color perception by its chromatic content and veiling with blackness and/or whiteness. To this idea the developers of NCS added a color identification methodology and the claim that any observer with normal color vision can assess the chromatic, black and white contents of any object color with good accuracy.

The atlas represents a curiously dissonant position. On the one hand, a claim for the system is that it is independent of a specific reference frame and that it can be used successfully in any kind of environmental situation. The user, it is claimed, can make quite accurate judgments of the content of the six primary color experiences of any perceived color in any situation. On the other hand, colorimetric specification and quality control was required to meaningfully embody the system in the atlas.

The NCS atlas is a material example of the NCS system instantiated for a particular surround and lighting condition: achromatic surround of $Y = 56$ and CIE illuminant C, with the samples of a size to be, at normal viewing distance, in agreement with the CIE 2° standard observer. Strictly speaking, for other situations the atlas is less or not valid.

7.4 THE "FRAGILITY" OF COLOR ATLASES

Color atlases, such as those described above are fragile in a metaphorical sense as well as in reality. The variability of observer responses to color stimuli and the fact that less than ideal colorants are used to color atlas chips make color atlases fragile in a metaphorical sense. The three described atlases all have been prepared with the help of colorimetric tools. In the case of the Munsell system the Renotations are defined in terms of colorimetric data from which they can be instantiated by matching tristimulus values. The aim values have been arrived at by plotting, smoothing, and interpolating results of visual evaluations. In case of the OSA-UCS atlas a formula has been fitted to a set of visual data, and that formula has been used to tile the space according to the principle of interlocking cubo octahedra. The resulting L, j, g values have been converted back to tristimulus values that are the aim points for the generation of atlas chips. A similar process of defining colors has taken place in generating the NCS chips. A quality issue relatively easily definable and controllable is that of the agreement between aim color and atlas color in terms of colorimetric coordinates.

For more recent atlas editions an aim has been to select pigments that not only are color fast but that result in colorations reasonably color constant when the atlas is viewed in different phases of daylight or artificial daylight sources. None of the three systems makes any but the most general claims in this respect. The available selection of suitable pigments is not such that a significant degree of demonstrated color constancy under different light sources is possible. Atlases have been formulated for CIE daylight C or D65. There are no artificial light sources that closely match these phases in spectral power distribution. Artificial daylight sources are today usually based on fluorescent lamps, and while they match the correlated color temperature of a particular daylight phase, their spectral power distributions are often significantly different. So-called triband lamps (CIE illuminant F11) are particularly different. As a result the appearance of the chips can change significantly, and the original goal of uniformity or regularity of spacing is no longer met or met less well.

The issue of surrounds has been discussed earlier, but a color atlas is valid in its appearance strictly only for the surround color for which it has been designed. Distortions in scaling take place if the surround is different in lightness or chromaticity from the design surround. Distortions due to surround can be relatively large because of crispening effects.

Since a numerical standard observer has been used in the design of atlases, individuals with normal color vision viewing an atlas experience smaller or greater distortions when viewing it, based on the difference between their color vision apparatus and that implied for the standard observer, and likely due to additional reasons. An idea of the magnitude of the changes in experience can be gleaned from the range of colors selected as unique hues by a group of observers (see Chapter 1). In addition observers have varying perceptions of equality of lightness, chroma, and hue steps to a degree that is unknown. Different assessments of constant chroma circles by observer groups in past extensive visual experiments, and different assessments of the number of constant size hue steps between unique hues, as discussed in Chapters 4 and 5, give an indication of the magnitude of these problems.

Aside from these conceptual fragilities there are also material fragilities. Of particular importance here is the resistance of pigments to degradations of various kind. Pigments, binders, and other materials are selected to make modern atlases long lasting. However, much depends on the kinds and lengths of exposures atlases suffer in use.

In this chapter three major color order systems have been compared, two attempts at a uniform color solid and the third a Hering type color solid. The considerable differences between the former two and the latter are evident from views of the distribution of samples in the X, Y and Z, Y planes at the system axes. Hue superimportance is implied in the Munsell system (as a result of the unequal magnitude of hue and chroma scale intervals). It is nonuniform in its euclidean form and non-euclidean when uniform. The system considers neither the Helmholtz-Kohlrausch effect nor the surround lightness. The OSA-UCS system, not arranged according to hue and chroma attributes (in fact, not directly expressive of meaningful chromatic attributes), does not in its final form indicate hue superimportance. However, analysis of the basis data clearly shows the presence of this effect. Hue superimportance was suppressed in the final formula used to tile color space according to this system to make the space euclidean. As a result the system is not uniform. The suppression explains the apparent difference in unit difference chromatic contours between the Munsell system and suprathreshold small color difference data, on the one hand (of generally oval shape), and OSA-UCS, on the other (circular). The circular unit contour is not in agreement with the perceptual facts of the OSA-UCS basis data. OSA-UCS, however, is built under consideration of the Helmholtz-Kohlrausch effect and of a specific achromatic surround. The NCS solid has been developed based on Hering's principles. Because the distances in the solid do not represent constant perceptual distances, there is no problem fitting the solid into a double cone. With minor changes it could be fitted into a sphere. We have arrived at an understanding that the three systems in effect represent regular color solids, none of them being able to make a strong claim for perceptual uniformity.

Chapter 8

From Color-Matching Error to Large Color Differences

Color differences have been scaled from the level of color-matching error, through threshold and industrially important suprathreshold small color differences, large differences of the size in OSA-UCS, all the way to quadrant-sized differences. In all cases considered here the difference judgments have been made against a simple achromatic surround. In previous chapters we have seen that there are systematic changes in terms of stimulus increments, depending on the magnitude of the difference. In this chapter color differences of varying sizes are compared in terms of stimulus increments to assess commonalities and divergences.

8.1 A COMMON BASIS FOR COMPARISON

It is of interest to compare such data both in L, M, S cone sensitivity space as well as the CIE tristimulus space. Aside from scale differences the main difference between the two is that in case of the former comparison is made at the point of interaction of light energy with cones, while in the latter case it is made in a system that implicitly accounts for the reappearance of red at the shortwave end of the spectrum and recognizes luminance and chromaticness as important color attributes. Conversion to the L, M, S space has been cal-

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culated using the Smith-Pokorny definition of cone sensitivities. All comparisons are made assuming an equal energy light source.

Historical color-matching error (CME) and suprathreshold small color difference data have usually been expressed in form of ellipses or ellipsoids in the CIE chromaticity diagram or the x, y, Y space. More recently they typically have been expressed in the L^*, a^*, b^* space or a cone contrast diagram. Systematic studies of threshold differences and color-matching error have started with the work of Wright (1941) and MacAdam (1942). MacAdam continued the CME work together with Brown (1949), followed by Brown's work (1957) with twelve observers. Additional CME data were contributed by Wyszecki and Fielder (1971a). These authors also reported on a different kind of experiment where a chromatic difference was displayed and the observer had to select a third color so that the chromatic differences between the three colors were equally large (1971b). The variability in the setting of the third color was then determined (CDM data). Richter reported in 1985 on an extensive determination of threshold data. For the purposes of comparative analysis it has been assumed that CME data represent object colors at a luminous reflectance of $Y = 30$.

As mentioned earlier, a collection of suprathreshold small color difference data based on several experiments using object color samples was assembled, in parts modified, and normalized by Luo and Rigg in 1986 (L-R data). A portion of these data, limited in luminous reflectance to the range $Y = 25$–35, have been used in this analysis. Other sets of suprathreshold small color difference data, such as the RIT-DuPont (R-D) data, were not considered here because their range of luminous reflectance values is large, with a small amount of data near $Y = 30$. Some of the data, including the R-D data, were used to compare lightness scaling. There are several sets of published large color difference data: Munsell, OSA-UCS, data by Wyszecki and Wright (1965) developed in connection with a field trial for a color difference formula, and the more recent Guan and Luo (1999) data. Color difference formulas as representative approximate models of small and large color difference experiments, CIELAB and CIE 94, have been used for comparison purposes.

Observer groups for the various experiments were obviously different, and there were also many differences in observing conditions. Inconsistencies between different data sets of the same type presumably are due to different observer groups and/or different observation/test conditions. They also may be due to effects so far not considered. Among these may be, as mentioned earlier, the issue of significant variation in the apparent composition of the lightness signal as well as the difference in sensitivity of yellow-blue and green-red opponent systems found for different observers, the uncertainty of constant chroma contours and constant hue differences (and very likely individual variations thereof), and the resulting partial conflation of lightness, chroma, and hue differences.

8.2 CHROMATIC AND LIGHTNESS CRISPENING EFFECTS

In 1949 Le Grand analyzed the MacAdam ellipses in terms of increments in two slightly different sets of three color fundamentals, R, G, and B. His results indicated that for either set the experimental ellipses, when projected onto the R, B plane, were nearly aligned with the axes of that plane, with the longer semi axis aligned with the B axis and the shorter one with the R axis. When plotting the increments representing the semi axis length as a function of the value of the fundamental he found a function (similar to Fig. 8-1*a*) linear over a large portion for B increments and a V-shaped one (similar to Fig. 8-1*b*) for R increments. Boynton and Kambe reported comparable results in 1980 from investigation of the size of incremental thresholds along lines of constant values of fundamentals (note that these were not CME data). The increments in B could be modeled as a constant fraction of B over a large range. Increments in R also confirmed the V-shape centered on the achromatic point, discovered by LeGrand. Nagy et al. (1987) investigated CME data (MacAdam, Brown-MacAdam, and Wyszecki-Fielder) and essentially confirmed the Boynton and Kambe (and thereby the Le Grand) results. They fitted a Fechnerian formula to the ΔS versus S function and a more complex formula relating L and M and containing a component of S to the ΔL versus L function. The authors explained the V-shape of the latter function as "opponent interactions between the two long-wavelengths cone mechanisms." As an

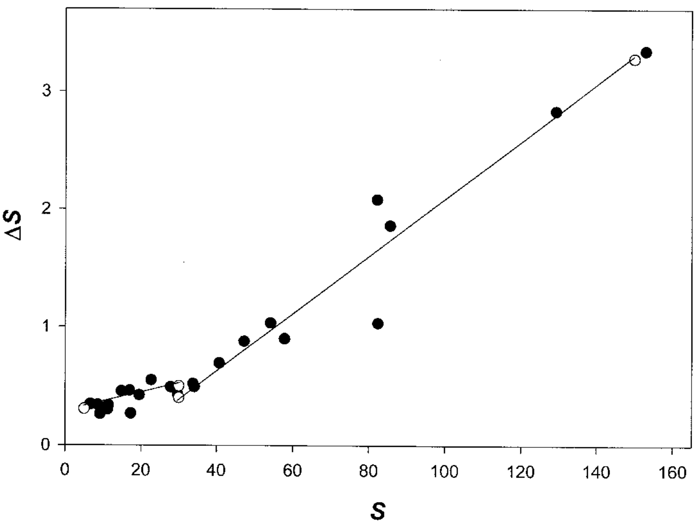

Fig. 8-1a *Increments in* S *as a function of* S *for the MacAdam ellipses (excluding ellipse 1). Open circles represent points calculated by linear regression. They are connected by linear regression lines.*

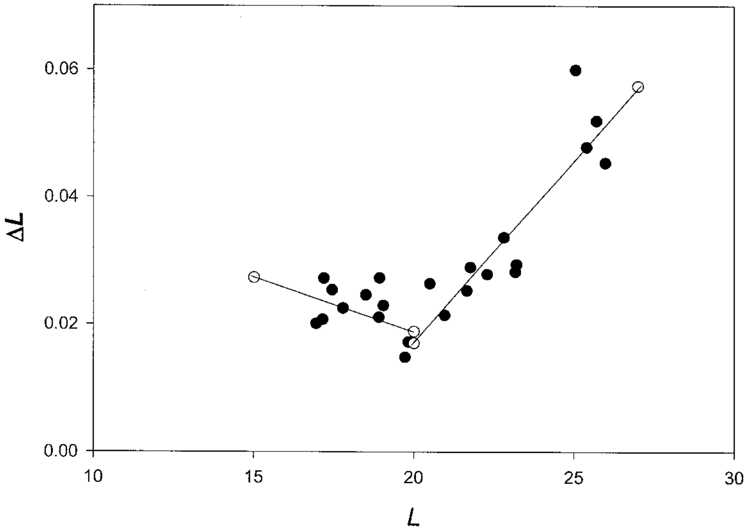

Fig. 8-1b *Increments in* L *as a function of* L *for the MacAdam ellipses (excluding ellipse 1). Open circles represent points calculated by linear regression. They are connected by linear regression lines.*

aside, in a previous investigation Boynton et al. (1983) found that individual observers differ in the relative sensitivity of their two opponent color systems by more than a factor of 3.

Schönfelder, Kaneko, Takasaki, and others, as discussed in Chapter 4, have found that threshold or unit larger difference increments determined against an achromatic surround require the smallest stimulus increments/decrements at the surround chromaticity. The required stimulus changes increase in magnitude as the distance from the achromatic point increases. In case of lightness differences the smallest change is also required at the lightness of the surround.

Tristimulus values for the equal energy illuminant representing CME and color difference data have been converted to Smith-Pokorny cone sensitivity data and plotted as ΔS against S and ΔL against L (Kuehni, 2001d), and linear models were fitted separately to both sides of the achromatic point. As mentioned, in the case of CME data, determined in visual colorimeters, the simplifying assumption was made that they are object colors with a luminous reflectance of $Y = 30$. In case of the Brown-MacAdam and Brown data with variable luminosity, this was considered justified by MacAdam's finding that the size of CME ellipses varies by less than 20% as a function of luminosity in a medium range of luminosities. Figures 8-1*a* and *b* illustrate the increment functions for the MacAdam ellipses (minus ellipse 1).[1] There is a fair amount of variation and the regression lines do not intersect exactly at the achromatic point. It is evident that both cases can be seen as representing V functions with different angles of opening of V. Such a situation is what one would expect from the functioning of the chromatic crispening effect. Increments are, in terms of stimulus, smallest at the surround chromaticity (achromatic point) and grow, absolutely or at least relatively, in both directions away from that point. The magnitude of the effect is different for *L* and *S*, in part due to the fact that colors of constant luminous reflectance can differ much more in *S* than in *L* or *M*, as we have seen earlier. (Because of the large degree of overlap with *L*, the function for *M* is very similar to that of *L* and not shown.)

Comparable results have been obtained for all CME and suprathreshold small color difference data. All slopes for *L* of colors where *L* is smaller than that of the achromatic color are negative. In case of *S* it is negative for observer GW in the Wyszecki-Fielder data. It is much less positive in all other cases than that of the increments for colors with *S* larger than that of the neutral point (except for the Brown-MacAdam data [observer WRJB] where there is no apparent chromatic crispening effect involving *S*). A negative slope for *S* less than that of the achromatic point is implied in the CIE 94 color difference formula. Figure 8-2*a* and *b* illustrates the two functions calculated from a series of CIE 94 ellipses in the a^*, b^* diagram with a constant total color difference.

The results indicate that the chromatic crispening effect is active, as one might expect, in every set of CME, threshold and suprathreshold small color difference data investigated. Its activity is implied in the S_C function of the CIE 94 formula and comparable functions in other color difference formulas. The angle of opening of the V-shaped functions ΔL versus L and ΔS versus S for various sets of data are given in Table 8-1. The angles are only comparable within one type of function (the Richter data are not directly comparable because the average Y is 16.5, rather than 30 as in all other data sets).

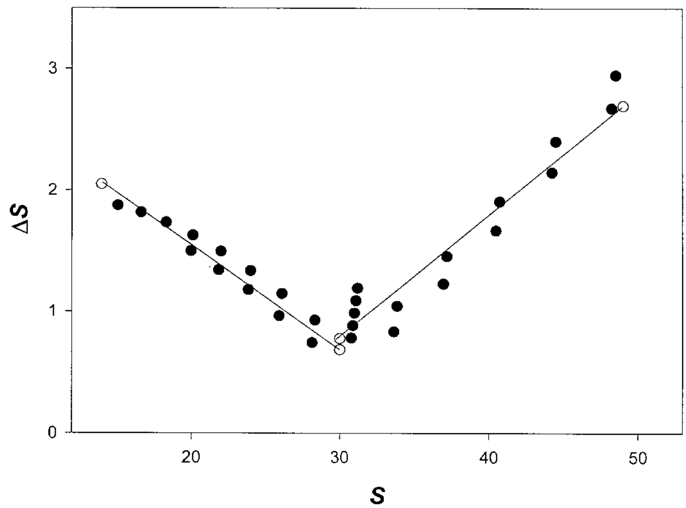

Fig. 8-2a *Increments in* S *as a function of* S *for a series of ellipses according to the CIE94 formula in the* a*, b* *diagram.*

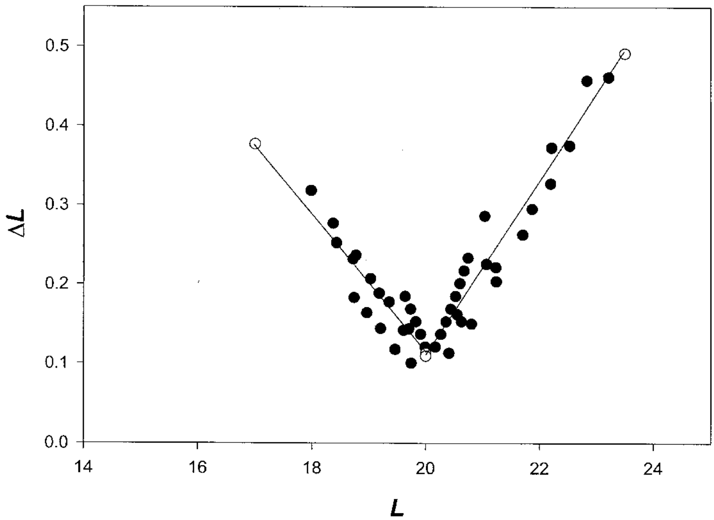

Fig. 8-2b *Increments in* L *as a function of* L *for a series of ellipses according to the CIE94 formula in the* a*, b* *diagram.*

TABLE 8-1 Comparison of angles of opening of the V-shaped functions of Δ*L* versus *L* and Δ*S* versus *S* in various data sets

Data set	Angle, deg *L* Function	Angle, deg *S* Function
MacAdam	110	142
Brown-MacAdam (observer WRJB)	147	180
Brown (weighted averages)	126	124
Wyszecki-Fielder CME (observer AR)	74	149
Wyszecki-Fielder CDM (observer GW)	81	141
Luo-Rigg ellipses (Y = 25–35 only)	110	116
Richter (Y = 16.5)	144	144
CIE 94	73	106

The results show that the V functions implied in CIE 94 are more sharply angled than those of all experimental data sets. This indicates that the S_C function of that formula adjusts for more than the chromatic crispening effect. The implication is that unit small color difference contours in the a^*, b^* diagram adjusted for chromatic crispening are oval in form. The additional effect of S_C is to convert the elongated contours to circles of equal size, as will be seen later.

Using ellipses in the a^*, b^* diagram fitted to the Luo and Rigg and the RIT-DuPont (R-D) data (Melgosa et al., 1994, 1997), one can determine the average change in chromatic ellipse size at an approximately constant luminous reflectance. The average axis length (between L^* = 56 and 64, n = 30) increases by a factor 4.4 from chroma 0 to chroma 100 as a result of the chromatic crispening effect.

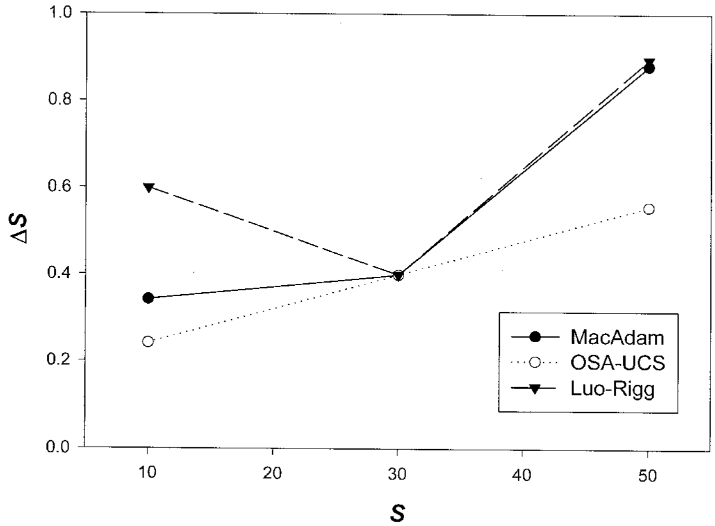

Fig. 8-3a *Regression lines of* ΔS *versus* S *of the selected Luo-Rigg (*Y = *25–35) and OSA-UCS data normalized at the neutral point to those of the MacAdam data.*

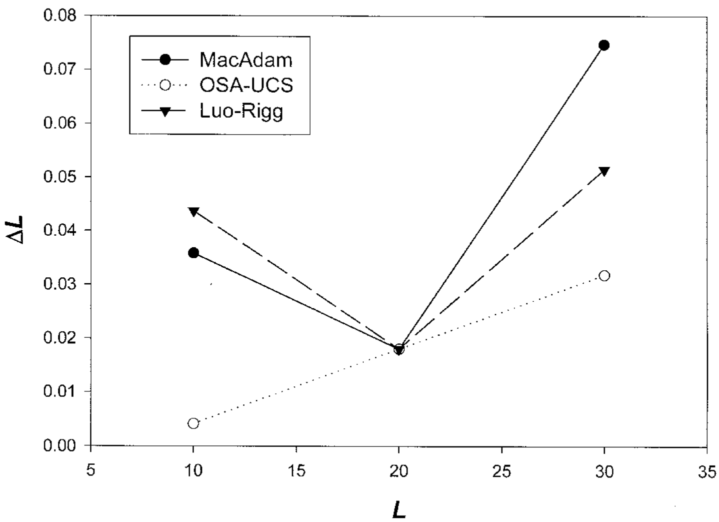

Fig. 8-3b *Regression lines of* ΔL *versus* L *of the selected Luo-Rigg and OSA-UCS data normalized at the neutral point to those of the MacAdam data.*

In Fig. 8-3*a* and *b* the regression lines of the ΔL versus L and the ΔS versus S functions of the L-R and the OSA-UCS data sets have been normalized at the neutral point to those of the MacAdam data. Interestingly, for both cone types the L increments are larger for colors both greenish and yellowish for

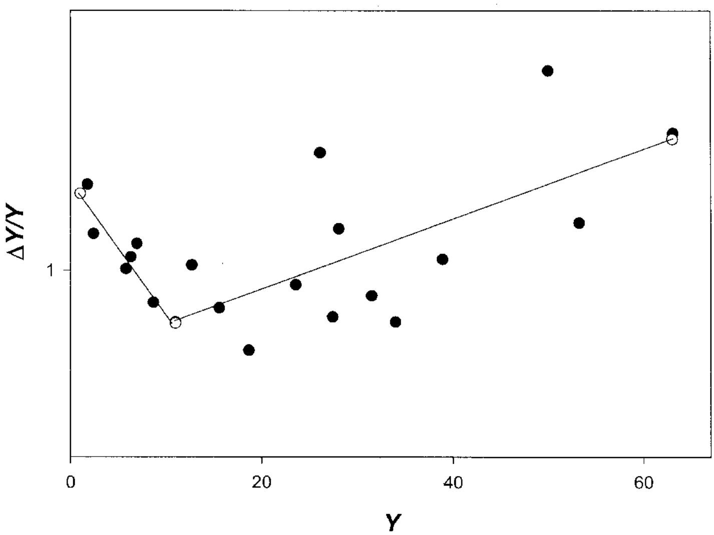

Fig. 8-4 $\Delta Y/Y$ *as a function of* Y *for the RIT-DuPont data. Open circles represent points calculated by linear regression. They are connected by linear regression lines. Surround luminous reflectance* Y = *11.*

the L-R data compared to the MacAdam data, and vice versa for the colors both reddish and bluish.

A generally comparable situation applies to lightness crispening. We have seen in Chapters 5 and 7 that lightness crispening is implicit in all experimentally determined lightness scales. When plotting ΔL versus L of the luminous reflectance scale (or ΔY versus Y) we can expect to find a similar V shaped form as in the case of chroma crispening, if the surround luminous reflectance falls between the extremes of that of the test colors. Since in all CME data the surround luminosity was lower than that of the least luminous test field a V-shaped function is not expected in the data with variable luminosity, as indeed is not the case. In the R-D data, where the surround luminous reflectance was $Y = 11$, the effect is present but small, best illustrated by plotting the Weber fraction of Y versus Y (Fig. 8-4). In terms of ΔL versus L we can find the effect when plotting this function for OSA-UCS lightness that has lightness crispening for $Y = 30$ built into the formula. The crispening effects indicate that the Weber-Fechner law is not applicable to color differences throughout the range of size where the crispening effects are applicable.

8.3 CHROMATIC CRISPENING FADES AS A FUNCTION OF THE SIZE OF THE DIFFERENCE

When plotting ΔL versus L and ΔS versus S for colors of the Munsell and the OSA-UCS systems along the axes in the a^*, b^* diagram, we find no chroma crispening effect, and there is a continuous increase with a positive slope.

The same applies for constant size chroma differences as calculated by the CIELAB formula. When optimizing the divisor in S_C for these color series, it is found to be near 1 (Kuehni, 2001c). It is 1 when optimizing it to the hue and chroma difference optimized OSA-UCS formula for the basis data of that set (see Chapter 7). Guan and Luo (1999) optimized a modified CIE94 formula to various sets of large color difference data, and found different optimal S_C and S_H factors. They recommended for large differences a formula in which the S_C divisor is $1 + 0.016C^*$ and the S_H divisor is 1 (compared to S_C divisor $1 + 0.045C^*$ and S_H divisor $1 + 0.015C^*$ in CIE 94, GLAB, see chapter 6). An explanation for these findings is that chromatic crispening fades as the size of the chromatic difference increases. This is surprising and different from lightness where lightness crispening is present from color-matching error to large differences.

In the absence of detailed experimental data one can estimate the relative magnitude of the adjustment for the chromatic crispening effect in small color difference perceptual data and the effect of converting ellipses to circles in the S_C divisor. If we assume the length of the major axis at the neutral point to be 1.5 (the average of 7 ellipses with $C^* < 2$ is 1.55) and the increase in axis length as a function of the chromatic crispening effect (see above) a factor 4.4, the total value of the S_C divisor should be 6.6 at metric chroma 100. It is found to be only 5.5 in CIE94, a reasonable result given the variability in the visual data.

It seems likely that this change in implied S_C is located on a continuous function, as estimated in Fig. 8-5. Systematic experiments are required to clarify the shape of the function.

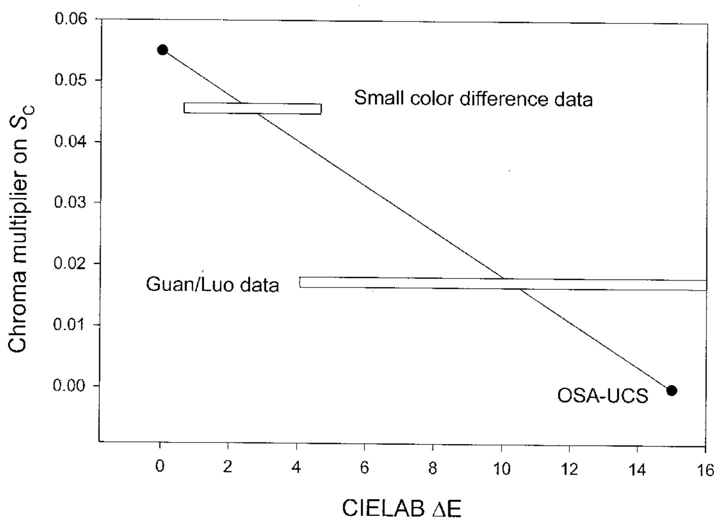

Fig. 8-5 Chromatic chrispening as represented by optimized S_C *divisor as a function of size of CIELAB* Δ*E differences for three sets of data and linear estimate of the function relating the two. The boxes represent the approximate ranges of differences in the data sets.*

8.4 SIZE AND RATIO OF UNIT INCREMENTS

Calculation of the linear regression lines of incremental data allows comparison of the size of the unit increments. Because of the different magnitude of the chromatic crispening effect in different data sets meaningful comparison is only possible of the implied first step from the achromatic point. These are compared in Table 8-2 in the direction of higher L respectively S values.

Several observations and conclusions can be drawn from the data in this table:

1. MacAdam and Brown-MacAdam data are very similar.
2. Observers GW and AR differ significantly in their sensitivity to sub threshold L differences.
3. The Brown average data differ significantly, either as a result of the weighted averaging process or for other reasons, from the other CME data.
4. The color difference matching error is elevated by approximately a factor 2 from the color-matching error.
5. The Richter threshold data indicate a higher L increment compared to CME data but a lower S increment, resulting in a significantly lower ratio.
6. The selected L-R data also have a lower ratio and even more pronouncedly the CIE94 and the CIELAB formulas.

TABLE 8-2 Comparison of unit *L* and *S* increments for the first step from the neutral color toward higher *L* and *S* values and their ratio for various data sets based on linear regressions, $Y = 30$

Data set	L Increment	S Increment	Ratio
CME data			
MacAdam	0.0168	0.40	23.8
Brown-MacAdam	0.0232	0.60	25.8
Wyszecki-Fielder/GW	0.0191	0.72	37.9
Wyszecki-Fielder/AR	0.0300	0.78	26.0
Brown	0.0410	0.21	5.1
Wyszecki-Fielder/CDM	0.076	1.53	19.7
Threshold and small color difference data			
Richter (extrapolated to $Y = 30$)	0.036	0.38	10.6
Luo-Rigg data	0.045	0.63	14.0
Large difference data			
Munsell	0.373	6.00	16.1
OSA-UCS	0.529	8.22	15.5
Color difference formulas			
CIE94/DE = 1.0	0.043	0.31	7.2
CIELAB/DE = 1.0	0.103	0.66	6.4

TABLE 8-3 Comparison of unit increment values of *X* and *Z* for the first step from neutral for selected data, based on regression calculation at *Y* = 30

Data set	*X* Increment	*Z* Increment	Ratio
CME data average (excluding Brown)	0.136	0.625	4.6
Luo and Rigg	0.279	0.629	2.3
Color difference matching data	0.471	1.50	3.2
Munsell data	2.35	6.03	2.5
OSA-UCS	3.23	8.22	2.5
CIELAB formula, DE = 1.0	0.639	0.658	1.0
CIE94 formula, DE = 1.0	0.265	0.310	1.2

7. Munsell and OSA-UCS have similar ratios, and the OSA-UCS steps are approximately 1.4 times larger than two Munsell chroma steps, as we have seen in Chapter 7.

The average *L* increment of the CME data (excluding the Brown data) is 0.022, the *S* increment 0.625, for a ratio of 28.4. For the L-R data selection, the ratio is 14.8. It is evident that a significant change is happening between imperceptible color-matching errors and small color differences: while *S* increments are nearly the same it is *L* increments that become larger in small color differences by approximately a factor of 2. The ratio remains essentially the same when the differences are large. The color difference matching error has a ratio smaller than that of color-matching error but larger than that of color differences. The Richter threshold data with a ratio of 10.6 are an exception to the picture, due to a very low *S* increment.

When data are viewed, in the CIE tristimulus system a more familiar picture emerges. Table 8-3 contains a selection of the data of Table 8-2 expressed in terms of *X* and *Z* (the latter identical to *S*).

Small and large color difference data have comparable ratios of approximately 2.5. The CIELAB formula, by definition, has a ratio of 1, due to the factor of 2.5 between the multipliers of a^* and b^* (500/200). CIE94, due to the effect of S_C, has a slightly larger ratio. Unsurprisingly, the key difference remains the same: while in the CME and small color difference data the *Z* increments are the same, the *X* increment increases by an approximate factor of 2 in the small and large color difference data, compared to CME data. It should be recalled that the multiplier balancing the linear opponent color functions *a* and *b* is 2.3 to 2.4, depending on the standard observer. The implication is that color difference perception regards the balanced *a* and *b* systems as equivalent.

On the other hand, color-matching errors appear based only on the sensitivity limits of the cones. (Here the findings of Boynton and Kambe, 1980, of thresholds along constant *S* and constant *L*/2*M* lines are not in agreement, since they largely duplicated, if with larger Weber fractions, MacAdam's results.) Color-matching error data and color difference data are, it appears, incommensurable, and geodesics based on color-matching error data do not predict geodesics of color difference data. The Richter threshold data, as one would expect, fit into this picture as indicated by an, albeit low, ratio of 1.7.

8.5 DIRECTION OF UNIT CHROMATIC CONTOURS IN THE *L*, *M*, *S* AND *X*, *Y*, *Z* SPACES

The difference between CME and color difference data is further clarified when comparing the shape of their chromatic contours in *L*, *M*, *S* and *X*, *Y*, *Z* spaces. For this purpose the intersection points of the major and minor axes of the unit contours are translated into these spaces and views in certain directions are created. The exercise is limited to representative data: the MacAdam ellipses and the Wyszecki-Fielder (observer GW) data as representative for CME data and the Luo-Rigg data for small suprathreshold color differences. Figures 8-6 to 8-8 illustrate the ellipses, each represented by five points (the endpoints of the long and short ellipse axes in the a^*, b^* diagram and the centerpoint), for the three cases in the *L*, *S* view. A number of observations

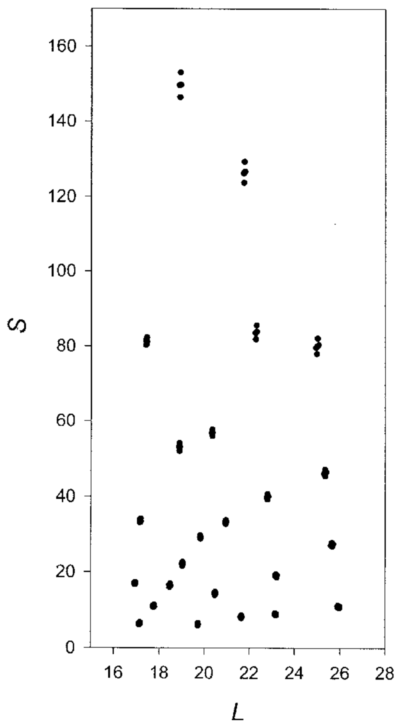

Fig. 8-6 MacAdam ellipses represented by the center point and the four points of the ellipse intersections with the major and minor axes, in the L, S *cone response diagram, equal energy illuminant.*

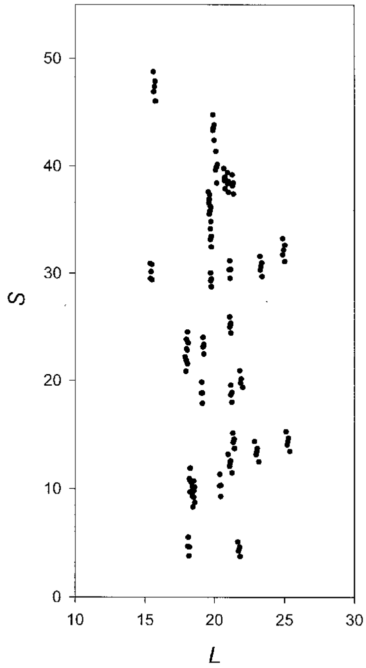

Fig. 8-7 *Selected Luo-Rigg ellipses represented by the center point and the four points of the ellipse intersections with the major and minor axes, in the L, S cone response diagram, equal energy illuminant.*

TABLE 8-4 Angles of major ellipse axis against abscissa in the *L*, *S* and *X*, *Z* diagrams

Data set	Number of Angles		Mean Angle, deg	Range, deg	COV, %
	<90	>90			
L, S diagram					
MacAdam	16	9	89.8	86.2–96.0	0.1
Wyszecki-Fielder/GW	5	23	92.6	87.0–101.0	3.9
Wyszecki-Fielder/AR	8	20	91.0	87.9–99.9	2.1
Luo-Rigg (Y = 25–35)	5	26	93.1	86.1–99.4	3.5
X, Z diagram					
MacAdam	22	3	77.4	57.4–115.0	16.8
Wyszecki-Fielder/AR	21	7	87.9	65.1–136.5	19.8
Luo-Rigg	9	22	96.8	56.9–130.6	19.2

can be made from the results. The MacAdam ellipses are well aligned with the axes of the diagram; that is, they are well described by differences in L and S. Table 8-4 contains data concerning the angles against the abscissa of the major ellipse axis for some data sets in both the L, S and the X, Z diagrams.

The essentially vertical alignment of the MacAdam ellipses in the L, S

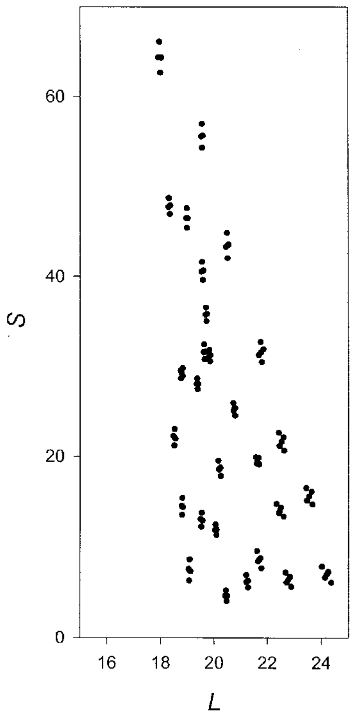

Fig. 8-8 *Wyszecki-Fielder (observer GW) ellipses represented by the center point and the four points of the ellipse intersections with the major and minor axes, in the* L, S *cone response diagram, equal energy illuminant.*

diagram is confirmed by the statistics. Both Wyszecki-Fielder observers show a bias toward angles larger than 90°, primarily due to a counterclockwise rotation of contours in quadrant 4 (yellowish-reddish colors; Fig. 8-8). This effect is stronger in the GW data. The bias in the L-R data is comparable to that of observer GW.

Because of the rotation in space and the rescaling of the X compared to the L axis, the picture looks different in the X, Z diagram, as shown in Figs. 8-9 and 8-10. Here most MacAdam ellipses have a strong bias toward a smaller angle, and the variability in angles has increased significantly. The AR data have a similar tendency and the variability in angles is even larger. The L-R data continue to have a bias toward angles larger than 90°, also with a high variability. Here the tendency of many ellipses is to point in the direction of the neutral point (at $X, Z = 30$). The X, Z diagram is a not a normalized version of an opponent color diagram, and in a normalized diagram the tendency is even clearer. The implication is that these ellipses are aligned along constant hue lines and represent the larger increment required for a unit chroma difference compared to a unit hue difference of the same perceptual size. This tendency is also apparent in one quadrant for observer GW, less so for observer AR. In

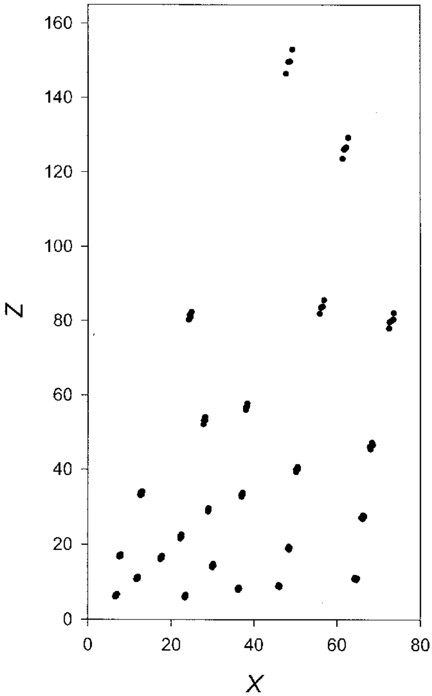

Fig. 8-9 *MacAdam ellipses represented by the center point and the four points of the ellipse intersections with the major and minor axes, in the* X, Z *diagram, equal energy illuminant.*

the Munsell system the contours, as we have seen in Chapter 7, are uniformly aligned along radial lines. In the X, Z diagram this is more or less the same.

The conclusions one can draw is that under the conditions of the MacAdam experiment and/or as a result of the vision properties of that observer the color-matching error is caused by limitations in cone sensitivity only. In the conditions of the Wyszecki-Fielder experiment yellowish-reddish colors of quadrant 4 point toward the neutral point, but the ratio between the two axes remains average for CME data. An opponent color system appears active in case of the L-R suprathreshold data. Here in addition the L, respectively X, unit increment is increased compared to the S or Z increment in accordance with global requirements for uniformity.

8.6 THE PARADOX OF HUE DIFFERENCES

Pages of the Munsell or NCS atlases contain by design colors of constant hue at different levels of lightness and chromaticness. One might assume that the hue differences between corresponding colors on two adjacent pages are all of identical size, since they all bear the same respective hue names (i.e., that hue

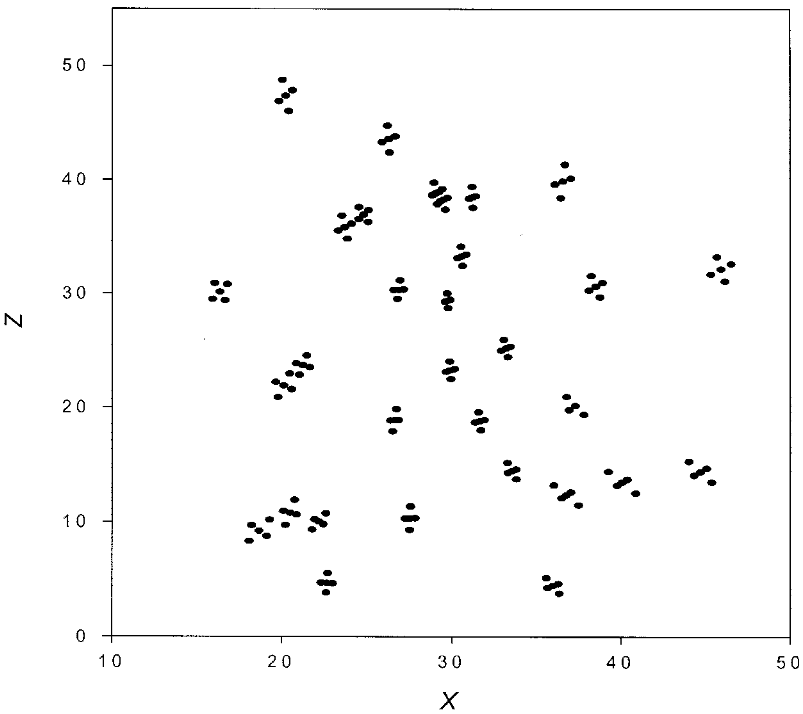

Fig. 8-10 Selected Luo-Rigg ellipses represented by the center point and the four points of the ellipse intersections with the major and minor axes, in the X, Z *diagram, equal energy illuminant.*

differences are independent of chroma and lightness differences). Diagrams such as the Munsell psychological chromatic diagram or the CIELAB diagram show a much different picture. The CIELAB formula predicts that the hue difference between two colors of identical chroma and lightness, and differing only in hue, varies by a factor of 10 between metric chroma of 10 and 100. A comparable result is implicit in the Nickerson Index of Fading. The paradox is that two pairs of colors with the same hue names at two different chroma levels with the same hue angle difference have a calculated Nickerson or CIELAB hue difference differing by a factor of 10. (It should be noted that in the CIELAB formula hue superimportance is not considered and the ratio between unit hue and chroma difference does not conform to the visual ratio.) If this is valid, the implication is that at metric chroma 100 there should be 10 times more distinguishable hue differences compared to metric chroma 10. Hue, of course, fades at the achromatic point and hue differences disappear also. The author is not aware of any study other than Nickerson's where the size of perceived hue differences between sets of Munsell colors of constant hue as a function of chroma and lightness was investigated explicitly. The Munsell BOC illustrates all forty hues down to chroma 2 and half of them to

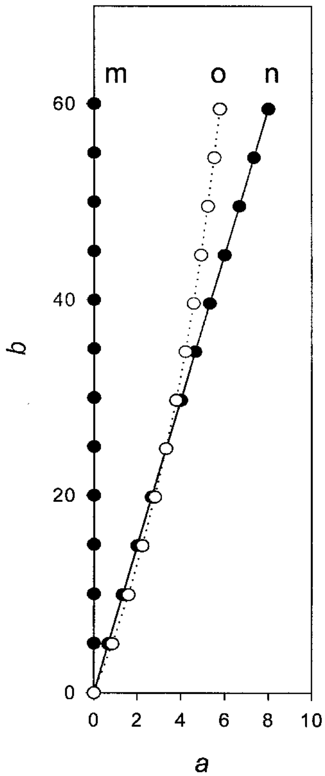

Fig. 8-11 *Lines connecting two hypothetical series (m and n) of colors of constant hue in a perfect hue circle. Line o represents the colors of line n after application of the S_H hue difference weight in the CIE94 color difference formula, normalized at metric chroma 25.*

chroma 1. The only (implied) quantitative psychophysical data we have in this matter are those used to determine the S_H factor in the CIE94 and similar formulas. It should be recalled that it is based on fitting a formula to elliptical contours in the a^*, b^* diagram so that they become circles of equal size. No hue difference judgments have been made in its support.

When plotting the hue differences implicit in CIE94 and similar formulas as a function of metric chroma (Fig. 8-11) we find a different picture applicable at the small color difference level. Figure 8-11 schematically illustrated two series of colors each of constant hue but varying in chroma. The hue angle between members of the two series is constant. Line *o* shows the effect of the S_H factor, normalized at $C^* = 25$. The result is in better agreement with informal evaluations of the Munsell hue circle: perceived hue difference between constant hue pairs fades to zero at the achromatic point but more gradually than the CIELAB formula implies. If the hue difference between line *m* and the normalizing point on line *o* is 1.0, it is 1.4 at chroma 10 and 0.30 at chroma 100. That is to say, colors on neighboring equal hue lines differ nonlinearly in

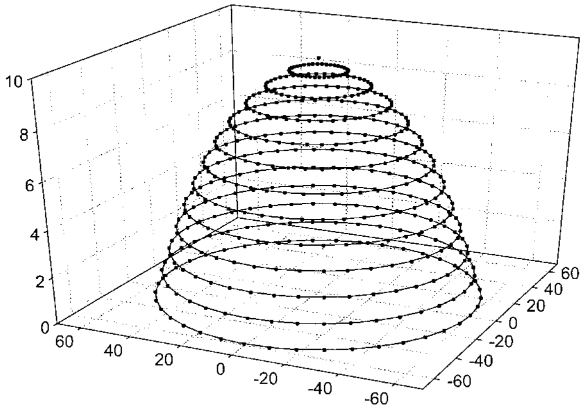

Fig. 8-12 *Uniform chromatic Riemannian plane constructed from segments between lines* m *and* o *in Fig. 8–11.*

hue difference, from metric chroma 10 to 100 approximately by a factor 4.6 rather than the factor 10 implicit in the euclidean chromatic diagram. The S_H factor should be verified with direct hue difference judgments.

There are two important conclusions to be drawn from this result:

1. Constant hue angle difference between colors of two hues does not imply constant perceived hue difference.
2. As a consequence a color space based on S_H cannot be euclidean because uniform slices created by S_H do not add up to form a flat circular plane. When forming a complete hue/chroma plane from segments between lines *m* and *o* of Fig. 8-11, a Riemannian plane (Fig. 8-12) is obtained.

8.7 UNIT DIFFERENCE CONTOURS AROUND THE HUE CIRCLE

Hue Angle versus Ellipse Angle

Color difference formulas derived from CIELAB imply that (with exception of colors of near unique blue hue) colors of constant hue lie on radial lines in the a^*, b^* diagram. While this is reasonably well the case for constant hue lines of the Munsell system, it is less so for those of NCS. As a first approximation we can assume, however, that this rule applies. The question arises if the conjecture that hue and chroma difference perception mechanisms shape unit difference contours of color difference data is well supported by data at the small color difference level. To assess this matter, 139 ellipses fitted to L-R (Melgosa et al., 1994) as well as R-D data (Melgosa et al., 1997) were investigated (11 with metric chroma <5 were excluded). The correlation coefficient between the CIELAB hue angle of the center color and the ellipse angle Θ

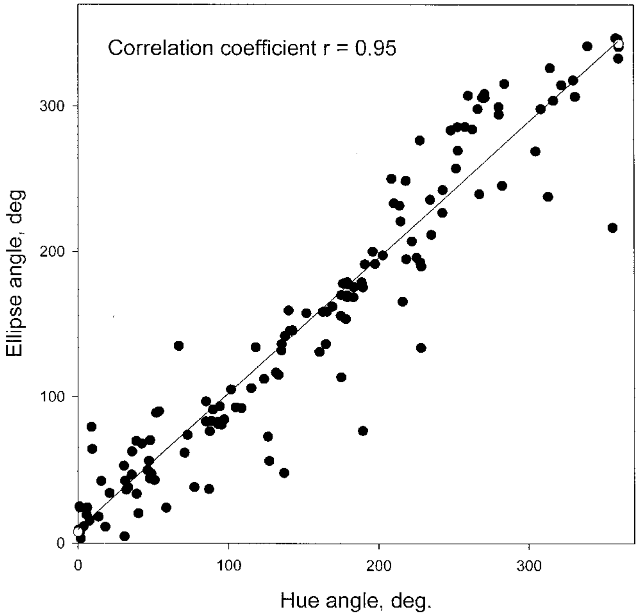

Fig. 8-13 *Scatter diagram of angle of major ellipse axis (ellipse angle) versus hue angle for the combined set of Luo and Rigg and RIT-DuPont ellipses with linear regression line.*

was calculated as 0.95. As the scatter diagram of Fig. 8-13 indicates, significant discrepancies appear to be random. When investigating the R-D data separately ($n = 16$), the correlation coefficient is found to be 0.88. After deleting two outliers that indicate longer axes in the hue direction than the chroma direction, the correlation coefficient increases to 0.98. Angles of the 11 near-neutral ellipses for the complete data set are found to range from 53 to 136.

While it is evident that CIELAB is not a good basis model for a uniform chromatic diagram and there is the fact that transformation of ellipses fitted in the CIE chromaticity diagram into ellipses in a^*, b^* involves a certain amount of error (Melgosa et al., 1994), the surprisingly high correlation coefficient appears to provide solid support for the conjecture of unit contours being aligned with constant hue lines. The conjecture is found to apply also to unit contours in the Munsell system and, as seen in Chapter 7, in OSA-UCS. A tendency in this direction was also found in quadrant 4 of the Wyszecki-Fielder color-matching error data. The alignment of fitted small color difference contours in the chromatic diagram has resulted in color difference calculation being performed in a polar coordinate rather than a rectangular system.

Additional data have in recent years been provided by J. Krauskopf and K. R. Gegenfurtner (K-G, 1992) and by M. J. Sankeralli and K. T. Mullen (1999). K-G determined visual thresholds around sixteen colors placed equidistant

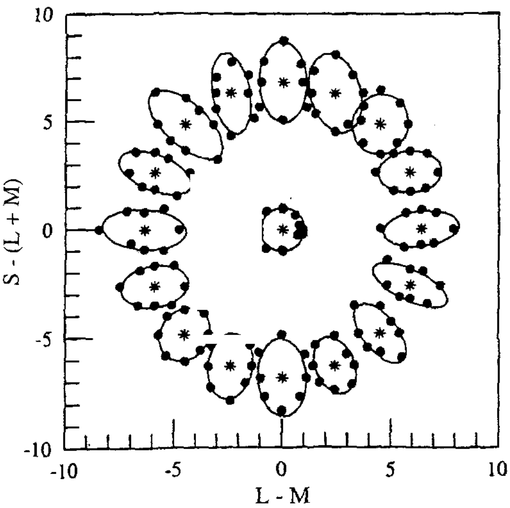

Fig. 8-14 *Visual thresholds around sixteen colors placed equidistant from the white point in the center of the cone opponent diagram. From Krauskopf and Gegenfurtner (1992).*

from the white point in a cone threshold diagram (see Fig. 8-14). In this diagram the scales of the two axes have been adjusted so that a circular contour is formed by the thresholds surrounding the white point. Most of the resulting fitted threshold contours are ellipses more or less pointing toward the achromatic point of the diagram. The fact that they do not closely do so in all cases is an aspect of the controversy concerning the number of hue detection mechanisms in the human visual system (as briefly discussed in Chapter 5). Sankeralli and Mullen (1999) determined unit chromatic contours at the 45° and 135° angles of a normalized cone based diagram with the axes $L - M$ and $S - (L + M)/2$ in the observer's personal isoluminant plane. The more or less elliptical contours are aligned well with the corresponding hue angles.

Ellipse Shape, Size, and Relation to Hue Angle, Metric Chroma, and Metric Lightness

In Chapter 7 it was shown that the ratio of contour axis length for the Munsell colors at moderate chroma has been determined as approximately 2:1 (2.8:1 at JND level). On average, a ratio of about 2:1 applies to the OSA-UCS data (see Section 7.2). Based on a regression of axis length versus metric chroma of the combined Luo and Rigg and RIT-DuPont set, the average major and minor axis lengths are found to be $a = 1.7$, $b = 0.80$ at $C^* = 0$ and $a = 4.1$ and $b = 1.92$ at $C^* = 100$ (ratio of 2.14:1). For seven near-neutral colors the ratio is found to be 1.55:1. The value of 2.14:1 is less than that found by Bellamy and Newhall. The average ratio of the Krauskopf and Gegenfurtner contours is found to be 1.7:1. The values for the Sankeralli and Mullen contours differ

TABLE 8-5 Correlation coefficients for the relationship between ellipse area and metric lightness L^*, Luo and Rigg and RIT-DuPont data

Metric Chroma C*	Correlation Coefficient	Number of Samples
10–20	0.06	21
20–30	–0.19	25
30–40	–0.64	27
31.00–34.99	–0.63	15
40–50	–0.10	21
60–70	0.19	11

I calculated these rusults from the L–R and R–D data.

by observer, with a ratio of 2.4:1 for one and 1.9:1 for the other. Undoubtedly, the experimental conditions also affect the ratio.

When determining the relationship between the length of the major axis and metric chroma for the L-R and R-D data, a correlation coefficient of 0.77 is found ($n = 150$), indicating good correlation. An only slightly lower value is found when comparing ellipse area against metric chroma (0.74). This level of correlation is another expression of the activity of the chromatic crispening effect. On the other hand, no correlation was found when comparing ellipse area and hue angle. In the limited metric chroma range of $C^* = 30 - 40$ ($n = 27$) the correlation coefficient is –0.01.

An unexpected result was obtained when comparing ellipse area against metric lightness L^* as a function of metric chroma. The results are found in Table 8-5. They indicate a considerable negative correlation between lightness and ellipse area for metric chroma values between approximately 25 and 45. That is, in this chroma range the ellipses become smaller as lightness increases. Based on the regression the average ellipse area in the metric chroma range of 30 to 40 is 18.1 at metric lightness 20 and 8.7 at metric lightness 80, a ratio of approximately 2:1. It is not evident what the cause of this finding is, and it requires further investigation and formula fitting.

As discussed in Chapter 4, there is little doubt that lightness and hue are fundamental color attributes. Chroma (contrast) is the necessary third attribute to represent all possible object colors systematically. Regardless of the mechanism we assume for the generation of hue (two process or multi-process) it seems to be a matter of fact that in terms of a psychophysical presentation, we are more sensitive to stimulus increments if they signal hue changes than if they signal chroma changes. In a psychological or psychophysical diagram the outcome is an elongated unit contour. In practical terms, there appear to be two independent systems: one that assesses changes in the ratio of two opponent color signals (assuming a two-process hue detection system) and the other changes in the size of the vector sum of the opponent color system (indicative of contrast). Both are affected by surround. The two seemingly operate independently of each other and are not connected in a euclidean sense. A euclidean space can only be achieved (and only for small color differences) with the help of euclidization operations such as the one by Thomsen.

In Chapter 5 we observed that there are different numbers of constant size hue differences between unique hues. As a result, in a chromatic diagram in which constant hue differences occupy equal hue angles (e.g., the Munsell system), unique hues do not fall on the diagram axes. On the other hand, in a diagram where the unique hues fall on the axes, equal hue angle differences do not indicate equal hue differences. To understand this situation, we require knowledge of the mechanism resulting in unique hues and of the hue difference mechanism.

8.8 GLOBAL DIFFERENCES

Differences of the Munsell or OSA-UCS step size are not the largest differences that can be compared in a color space. It is possible to think of more global kinds of differences by posing questions like:

1. Are the hue differences between the four unique hues the same?
2. Are the differences between colors falling on axes (however chosen) in the Munsell psychological diagram and the central gray of the same magnitude, and is gray in fact exactly in the middle between such colors?

The first question has already been answered in terms of the varying number of hue differences between the unique hues. Questions of the second kind were of interest to Judd and discussed by him in his article *Ideal color space* (1969). Based on his experiments with Munsell chips, he concluded that the superimportance of hue gradually fades as hue differences become very large.

Of Judd's observers a majority decided that the shortest path between opposing axis colors was not through gray but through a desaturated, hued color halfway between the opposing colors being compared. Judd worked out a function that accomplishes this fade for the modified Godlove difference formula (equation 4-2). But he indicated that there appears to be no geometrical model corresponding to his formulation, "at least, I have not been able to think of one." He concluded that also from this viewpoint ideal color space was impossible. Additional complications appear to arise when considering very large color differences.

8.9 HOW FUNDAMENTAL ARE THE VARIOUS KINDS OF DATA?

A goal of vision science is to uncover fundamental color vision processes. For this reason experimental conditions tend to be rigorously controlled to expose only one assumed process at a time to experimental variability. Exposures to test stimuli tend to be very short to minimize changes in adaptation due to the test stimulus.

Color-scaling experiments, on the other hand, usually involve unlimited exposure times. Experimental conditions usually varied between different experiments and the degree of adaptation (in experiments involving small or large suprathreshold experiments) were not considered nor decisively controlled. As a result it is unlikely that fundamental processes have been determined but rather that the results represent overlapping fundamental processes (if it is possible to identify fundamental processes in this manner), however, often representative of practice in visual color quality control and of more natural viewing conditions.

In natural viewing conditions adaptation tends to change continuously as a result of what is being viewed, and the observer controls exposure times. If I harvest ripe strawberries in a strawberry patch, as I change my gaze, the visual system is adapted in various degrees to, among other things, combinations of the brownish-grayish color of soil, the green leaves of the strawberry plants, and, if there are plenty of ripe strawberries, the redness of the fruit. It is known that some adaptation processes proceed very rapidly in time while others are slower. Perceived color differences between two adjoining strawberries are a result of the total momentary adaptation situation.

It remains to be seen how large the differences are between the two kinds of observation conditions. In addition the observer groups in different experiments most always are different. Variations in the results of different observers are well known. They can be due to variations in their cone response functions or other components of their color vision system. The degree of linear transformability of individual color-matching functions remains to be determined as well if meaningful transformable means can be calculated from individual results. As discussed earlier, there is the possibility that certain judgments may be influenced by evolutionary experiences.

It is important to keep in mind that the following conclusions are valid only for a general viewing condition where the surround is achromatic and in luminance/lightness somewhere between the upper and lower extremes of the targets used in testing. At the color-matching error level we have found variation among observers and, possibly, experimental conditions. The MacAdam data are well represented by elongated rectangles in the L, S diagram. Both Wyszecki-Fielder observers here investigated (GW and AR) produced ellipses pointing already to some degree in the direction of the neutral point, most strongly so in the fourth quadrant (this applies also to the third observer). The impression is created that under the conditions of the MacAdam experiment, or at least for that observer, CME is not influenced by specific properties of the opponent color system but is based only on cone response limitations. Under the conditions of the Wyszecki-Fielder CME experiment, rotation of unit ellipses in the direction of the neutral point begins to be noticeable particularly in one quadrant. The same applies to the color difference matching results. At the level of thresholds the opponent color system appears fully engaged (change in X to Z ratio, general alignment of unit contours in the direction of the neutral point) and remains so through large color differences.

It is not obvious what the cause of the rotation of a specific portion of the contours in the Wyszecki-Fielder CME and CDM data is. The MacAdam and Brown-MacAdam experiments employed monocular viewing. In both cases the surround was neutral. In the case of the MacAdam experiment, it was at half the luminance of the test fields; in case of Brown-MacAdam, it was dark. Ellipse rotation toward the neutral point is also noticeable in the five most highly saturated yellowish red colors of observer WRJB. In the Brown experiment a binocular colorimeter was used with a neutral surround slightly less luminous than the least luminous of the test fields. Ellipse rotation here is evident for the two most highly saturated yellowish red colors. The Wyszecki-Fielder CME experiments used two fields of a seven-field binocular colorimeter. The neutral surround was at half the luminance of the test field. As is mentioned in Wyszecki-Stiles, ". . . both eyes . . . could wander over the field with no strict fixation. . . . These conditions . . . approximated most closely those of ordinary viewing." The surrounds in all cases were large compared to the test fields. As mentioned, ellipse rotation is evident in the yellow-to-red segment for all three observers.

Data analysis in this chapter has been pursued based on a conventional view of color vision. From the results we can draw a number of conclusions regarding the various divisions of a common psychophysical color space (e.g., CIELAB), based on the magnitude of differences involved. Before going into details it should be mentioned again that the experimental basis at the various levels tends to be different. Color-matching errors and thresholds are usually determined using visual colorimeters. Small suprathreshold, medium, and large differences have usually been determined using color samples of some kind viewed in varied surroundings or, more frequently, in a light booth, usually in an achromatic surround.

The results of observer PGN in the MacAdam experiment indicate that hue discrimination was not explicitly active but that they are strictly a result of the sensitivity limitations of the three cone types. For both observers, in the Wyszecki-Fielder experiment, hue discrimination appears partially engaged (a number of ellipses point to the origin), but not all other aspects of the opponent color system (the average ratio between L and S increments is the same as in the MacAdam experiment and twice as large as in the Luo-Rigg and large color difference data). This implies engagement of different aspects of the system under different conditions. Lightness and chromatic crispening effects are engaged for CME data.

Based on the Richter data, at the threshold level all identified components are active. Suprathreshold small color difference unit ellipses, as we have seen, have a quite strong tendency to point to the origin. This effect is geometrically enhanced in the X, Z diagram, and even more if the X axis scaling is expanded by a factor 2.3 for numerical balance with Z. Power modulation, lightness, and chromatic crispening apply at this level. The L or X increments have doubled compared to CME data, indicating increments in L or X that are not guided by cone sensitivity but by another mechanism, an opponent color system.

In differences of Munsell and OSA-UCS magnitude the opponent color system and hue discrimination are active, explicitly so in case of Munsell and implicitly in case of OSA-UCS, as we have seen. Lightness crispening is active as seen in the pre-1943 Munsell data and as recognized by inclusion of the effect in OSA-UCS. Chromatic crispening, however, has now faded. The result indicates that the processes guiding uniform tiling of color space are different at different levels of size of differences.

Chapter 9

Conclusions and Outlook

In our travels through psychological and psychophysical color spaces it was shown that a uniform color space can only be defined for a narrow set of conditions and that there is a multiplicity of uniform spaces, each representative of a specific observation situation. It also has become clear that there are an infinite number of regular color spaces in which colors are ordered in systematic sequence (and in the same ordinal order as the perceptual space) but are not uniform. Color solids can fit into any desired euclidean space form if the concept of uniformity is absent. This text is limited to the case of achromatic surrounds, a situation that is not expressive of most natural viewing conditions but represents a useful elementaristic experimental simplification. It is also in agreement with usual industrial observation conditions for the purpose of color quality control. On this journey we have learned that we have only a fraction of the information required to build accurate models of human color vision that speak to the problem of uniform color space.

There are various uses of a color solid, and they have resulted in several different major kinds of spaces. There is the uniform color space, classically defined by Judd, in which equal geometrical distances in any direction (given standard viewing conditions) represent equal perceived differences. There is Hering's "natural color system" in which colors are ordered by their content of unique hues as well as blackness and whiteness. Here lightness is not an attribute and chromaticness is uniform for all maximal colors. There are nonuniform color spaces used in communication and color management, such as the RGB, HSB, or CMYK spaces. Finally there are a number of psy-

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chophysical color space models that order colors and generally have no claims of uniformity, such as the CIE tristimulus space, the x, y, Y space, the Luther-Nyberg space, and many others. For those without claim to uniformity, the general designation regular color space has been proposed. In addition there are spectral spaces derived from mathematical dimensionality reduction of spectral functions of color stimuli not involving cone or color-matching functions.

For a period of some 150 years there was belief that all object color perceptions, uniformly spaced, could fit into a simple geometrical solid. Munsell was the first to show that if uniformity of differences is a goal this belief is misplaced. Since then it has become evident that a Munsell level uniform color solid cannot fit into a euclidean space.

The following is a discussion of a number of significant issues related to color spaces and solids with stated or proposed answers (depending on the strength of current evidence).

9.1 WHAT ARE COLOR SPACES AND HOW CAN THEY BE JUSTIFIED?

Color spaces and color solids represent attempts to show a multitude of possible color perceptions in a systematic fashion. This is an effort that faces several philosophical and scientific problems. We do not know how we perceive colors as part of our consciousness. From an extreme point of view (see Saunders and van Brakel 2002) this dooms the entire enterprise. We can all agree that there is no generally valid direct relationship between spectral power entering our eyes, and thereby between reflectance functions of objects, and our experiences of color. Experimental work in color scaling under elementary conditions and work in color technology indicate, however, that for such conditions there is a reasonably solid relationship between members of a class of metamers and the resulting color experience as judged by an average color normal observer. Without it, colorant formulation and color technologies of various kinds (color photography, television, color printing, etc.) would not work as well as they do. Is viewing color fields under elementaristic conditions a true reflection of natural color experiences? The answer, obviously, is no. However, the evolutionary development of human color vision should not be confused with its current average operation. Arguments can be made that (in industrial countries) we experience color today more often in unnatural as well as quasi elementaristic conditions (looking at a video screen or color television) than in natural conditions (except when on vacation). Does it mean our color experiences are degraded? Does the color vision system of young children now develop differently from that of our grandparents? We do not know but it is unlikely to be so.

The hue sequence of the generally accepted hue circle derives its justification from the spectrum and the mixture in various ratios of narrow band energy from the two ends of the spectrum. For the reason that color experience is private we can only assume that one person experiences the colors of spectral lights approximately the same way the next one does (assuming they do not have impaired color vision). That photometry can produce an ordinal scale of gray objects viewed under elementaristic conditions is without doubt. Perceived brightness of chromatic objects, as was shown, is not in agreement with photometric results, for reasons that we do not know. Speculative models can, however, result in a high level of agreement with average visual data. Quantitative relationship at the ordinal level for elementary conditions also exists quite clearly between spectral power distributions of metameric sets and the resulting perceived saturation or chroma. All these relationships have high rank order correlations. Such information has provided the basis for simplistic color models attempting to relate under elementaristic conditions physical data of lights or materials with the resulting average perceived color. For the models to be quantitatively useful they must have a higher degree of complexity. There is no doubt, and the history of chapters 2 and 6 clearly indicates it, that often (in an absolute sense always) large houses of cards are erected in the building of these models. But as the history of science shows this is how progress toward reaching goals of understanding is made. Will humans understand their color vision system in 50 or 200 years, or never? Time will tell.

The curious thing (or perhaps not) is the considerable practical level of success that color science has gained in coloration technology, color device technology and color management with the kind of house of cards models just mentioned. None of these is perfect and perhaps never will be. One good reason is that there is just too much variation in individual experience from a given stimulus set. These relative successes have given color scientists and technologists a level of confidence that the conjectured relationships might not be totally coincidental. Coloration of textiles, paints and plastics can now be controlled at levels approaching just noticeable differences. Metamerism for a limited number of light sources can be avoided and it is even possible to obtain a good idea if a matched material sample will or will not have a similar appearance under different lights.

All existing color spaces and solids result in color experiences that are at least in ordinal order of perceived color. (It is not known if this applies universally to color normal observers.) To produce an atlas that is an accurate representation of a perceptually uniform color solid for a given set of conditions is impossible at the level of large differences. But this does not seem to be important as long as it is understood in what way the atlas configuration deviates from the space implied by its samples (as we know we cannot produce accurate large scale flat geographical maps). Such an atlas has not yet been produced. The best color difference formulas for small color differences have limited accuracy for reasons that have not yet been investigated in detail. The models are likely too simplistic but, as discussed before, there are also many issues of observer panels and observation conditions. The technical value of improved formulas makes their pursuit worthwhile.

9.2 WHAT CAUSES THE PERCEPTION OF COLORS AND THEIR DIFFERENCES?

Colors are psychological experiences. To explain them reductively with neurological processes (if that is possible) requires linking propositions that attempt to create a credible link between subjective experiences and objectively measured data related to neural responses or physical measures of reflectance (or spectral power). Today the chain of such arguments built to explain perception of colors is far from complete. But there is unanimous support for cones as the sensing devices for light entering the eye at daylight level. The path of the electrochemical signals generated by the cones as a result of light absorptionn leads through retinal cell layers into the brain and passes through the lateral geniculate nuclei as major way stations to visual areas V1 to V4 in the back of the brain. Area V4 is strongly implicated in color vision, and several kinds of cells have been identified as responding to color stimuli (up to date details can be found in Gegenfurtner and Sharpe, 1999, and De Valois, 2000). Here any degree of certainty ends. V4 signals may be said to disappear into a black box out of which color experiences appear in an unknown process. S. Zeki and A. Bartels (1999) believe that conscious color perceptions are generated in area V4 and may be directly related to (or are) the electrochemical signals in V4. Color categorization and difference perceptions, as discussed in Chapter 3, appear to involve additional regions in the front of the brain.

There is growing awareness that, to some extent, our perceptions of forms and colors are formed through empirical interpretation of the ambiguous light signals reaching our retinas, in a form of neural networks, based on our millennia-old experiences as a species and possibly also our experiences as individuals. A complete vision theory must be able to explain all perceptual results. *It is assumed that by limiting ourselves to simple targets and simple relativized observation conditions, it might be possible, for these conditions, to reduce much of the link to between, say, neural events in V4 and perceptions.*

In this path and chain of events several types of cells have been identified that combine in various ways the output of the three cone types over smaller or larger areas in the retina. Some ganglion cells and cells in the LGN have opponent character somewhat akin to that suggested by Hering. However, as seen earlier, there is no direct link between the output of these cells and unique hues. The implication is that there are additional steps required, for example, the subtraction of a fraction of the β opponent signal from the α opponent signal (Chapter 6). In addition the salience of the unique hues may be due to as yet unknown brain processes, and they may only be indirectly related to a and b opponent signals. Color cells in V4 do no longer appear to have simple opponent character, and opponency here may be between cell groups rather than within cells (De Valois, 2000).

The nature of the relationship between color matching and color appearance is not clear. Color-matching functions change with the size of the test fields. But there is little change in appearance of most colored materials as a function of field size. The output of a particular cell type correlates well with luminance as measured under specific test conditions and is likely to supply raw data for brightness and lightness perception. As mentioned in Chapter 5, chromatic perceptions carry with them a brightness component independent from flicker luminance, giving rise to the Helmholtz-Kohlrausch effect. Brightness discrimination, presumably, is the result of higher-level comparison of the brightnesses of two contrasting fields, influenced by how our visual system interprets the scene in which the two fields appear.

Color perceptions fill areas of the visual field surrounded by contrasting contours. A case can be made that color exists only as contrasts, since in the absence of contrasts, in a so-called *ganzfeld* view where the eye is covered by translucent, spherical, colored cups, color fades after brief exposure. This implies that signals from both sides of contours are compared along the visual path and color experiences are generated as the result of complex computations of the brain taking into account many aspects of the visual data provided by the eyes.

The fundamental nature of hue and lightness perception is not in doubt. The degree of contrast of any chromatic perception against a neutral surround, namely saturation or chroma, appears to be in terms of fundamentality a step below those primary perceptions, perhaps because achromatic surrounds as well as saturation or chroma scales are rare in natural scenes.

As discussed in Chapter 6, in simplified conditions hue perceptions are believed to be formed either as the result of a ratioing process where opponent signals are compared or as the result of perhaps weighted averaging of the output of a number of hue detection processes. Hue discrimination, presumably, is the result of a comparison of two fields with different opponent signal ratios or different averages of multiple hue processes (see Section 9.11).

While hue perceptions may be the result of a comparison of the ratio of opponent signals of a field to that of its surround, chroma perceptions may be the result of the absolute level of opponent signals generated by a field to that of the surround. Brightness perceptions, in turn, may be the result of a comparison of the luminance signal combined with a fraction of the opponent signals (if any) of the test field compared to the same combination of the surround. All three are affected by specialized processes that modify perceived hue and enhance or reduce contrast.

By limiting the visual field to a simple set of conditions, the effect of many of these specialized processes appear to remain relatively constant. In this case the variables may be limited, for object colors, to the reflectance functions of the surround and of the two test fields, as long as the display is viewed under a constant light source and in a standard configuration.

9.3 WHY IS OUR BASIC COLOR EXPERIENCE THREE-DIMENSIONAL AND WHY ARE THERE FOUR UNIQUE HUES?

These are very fundamental questions in regard to color space. Our immediate experience of the world as expressed through visual and tactile clues is three-dimensional. As discussed before, the geometry of our *visual* space is three-dimensional but with a Riemannian geometry. Further the fact of three cone types involved in color vision argues, on the surface, for a three-dimensional expression of color experiences. But, as discussed in Chapters 4 and 5, our psychological color space is not directly related to the cone sensitivity space. There are important transformations, some of which have only been guessed at so far. In relativized standard conditions the sum of L and M cone signals is related to the flicker brightness experience. Perceived heterochromatic brightness has a more complex basis. Hue experiences may be related to opponent color signals and are correlated with them in ordinal and even roughly interval order. Saturation scales result from additive mixture of stimuli, causing complementary chromatic, as well as mixtures, causing chromatic and achromatic experiences. The three cone types are not the immediate cause of our elementary three attribute color experience. The cause may be complex cone signal derived mechanisms one of which appears to result in hue, another in saturation and to some extent brightness experience, while a third contributes the major portion of the brightness experience.

In 1964 Judd and co-workers showed that a large variety of measurements of terrestrial daylight can be expressed with three basis functions. (However, such reconstruction is only reasonably accurate for correlated color temperatures down to 5000 K, and the corresponding CIE procedure is only applicable to that limit. This limits the applicability to "white" light and excludes the more strongly colored lights of sunrises and sunsets or colored light reflected from vegetation, etc.) Shepard (1992, 1994) believes the three basis functions of natural daylight to be responsible for the existence of our three cone types and, thereby, for our three-dimensional psychological color experience. This conjecture appears to be anthropocentric and thus not to explain the reduced number of cone types in many other vertebrates as well as the spectrally shifted cone sensitivities of some vertebrate and invertebrate animals. In the animal kingdom numbers of color-related cone types with different spectral sensitivities in a species range up to ten (for mantis shrimp; Cronin and Marshall, 1989).

An alternative conjecture bases the number of cone types in a given genus, their spectral sensitivities, and the resulting refinement of the information in the brightness and opponent color systems on specific pressures in their ecological niches and the accidents of random gene mutations. The energy of sun radiation peaks in the visible region, a good reason for placing receptor sensitivities between approximately 320 to 760 nm. While cone sensitivities found in nature are limited to this range, the number in a genus may be an empirical compromise between visual acuity (sharpness of vision) and need (at one time or another) for spectral discrimination.

The question why four basic hues (and not more or less) seems not yet satisfactorily answered. Hering has not posed it but merely stated the fact that there are four basic hues and black and white. This number appears also to be somewhat accidental. If basic hues are a result of opponent signals, then three cone types, in theory, make six basic hues possible. But because of the high degree of spectral overlap of the *M* and *L* cones, there would be a considerable amount of redundancy in the information. Four basic hues were sufficient to support the survival of our genus.

In the matter of the number of basic hues an intriguing conjecture was recently offered by Lotto and Purves (2002). They claim that we have four basic hues in response to the need "to solve the four-color [topological] map problem" as well as "to order spectra according to their physical similarities and differences."

It is evident that broad band sensors can only provide effective discrimination with overlap of the sensitive regions. Thus, having a short wavelength sensitive detector added with some overlap to a midwavelength sensitive detector improved discrimination capabilities in the visible region. Dichromats can discriminate more than monochromats. Adding a third, overlapping sensor improved discrimination further. Information manipulation was now required to avoid considerable redundancy of information and to normalize perceptions at different levels of lightness. Based on three cone types the minimal opponent system has four basic hues. It can be considered evolutionarily sufficient (since that is what we have), but it is certainly not optimal. There is a large spectral range (from about 580 to 730 nm, nearly 45% of the total visual spectral range) where there is comparatively little discrimination. On the short wavelength end the discrimination ability has been significantly improved (perhaps by up to 20%) by the reappearance of redness. Mathematical analysis shows that filtering of spectral data through the color-matching functions is not optimal in terms of extraction of the information from the spectral data and ordering of spectra according to their similarities. Three principal components and other techniques do a considerably better job. But cone absorption is a process with biological limitations, and our visual system represents a compromise between what is desirable and what is biologically possible. The question is which evolutionary pressure was larger, that of improving discrimination capability while maintaining a high level of visual acuity or the unambiguous solution of the four color map problem?

9.4 WHAT ARE THE FUNDAMENTAL PERCEPTUAL COLOR ATTRIBUTES?

The conventional, but not uncontroversial, view is that they relate to lightness and hue perception, as well as to intensity of coloration. In terms of evolutionary development brightness/lightness is the oldest perceptual variable. It antedates the development of color vision. Its current implementation in humans appears to be of complex nature, fine-tuned to allow useful interpretation of the visual field in terms of light, shadow, and forms. But today it is not the most salient attribute. This position belongs to hue. Color is defined primarily by hue because of the dramatic nature of our hue experiences. Natural hue experiences can be subdued in a late fall Northern landscape but are riotous in a tropical forest. Artificial coloration has expanded the possibilities of strong and varied hue experiences. The importance of hue is further supported by the fact that a seemingly elaborate (but in detail unknown) apparatus has been put in place to make possible our ability to make fine hue distinctions. Hue can be described psychologically in terms of the four Hering unique hues, as Hering, the developers of the Swedish NCS system, Indow, and others have shown.

Hering was clearly aware of brightness/lightness but did not accord it status of a fundamental color attribute. Instead, he used blackness and whiteness as modulators of hue, with a derived attribute of chromaticness. Such a system has steps, in terms of uniform perceived differences, of considerably varying size in all directions. Multidimensional analysis of the NCS system using magnitude of perceived difference between steps would undoubtedly arrange it in a Munsell type configuration with the samples placed at widely varying distances. Hering type systems that disregard lightness as an attribute can fit conceptually into a simple geometrical solid, as they are not concerned with uniform differences.

Judging the size of perceptual color differences between objects against a standard surround—a conventional psychological task—leads to a system based on hue, lightness, and chroma attributes. If hue and lightness are given fundamental status for colors of objects in relativized conditions, the third perceptual attribute is chroma, as Munsell discovered. A systematic arrangement based on uniformity of difference in such a system does not fit into a simple geometrical solid, nor can it be modeled in a euclidean space, as additionally discussed below. It apparently has never been tested competitively under controlled conditions which of the attributes, blackness/whiteness or chroma/lightness, can be experimentally measured with less variability.

While both types of system are uniform in some sense, it is proposed to reserve the term "uniform" for uniformity of perceptual distance only and to use the general term "regular" for others.

9.5 HOW ARE HUE, CHROMA AND LIGHTNESS PERCEPTIONS COMBINED?

In Nickerson's Index of Fading the three components, properly weighted, are simply added. Indow has come to a comparable result based on his perceptual formula. The OSA-UCS data provide no information in this regard because the basis data for the set involved mixed hue and chroma differences. Suprathreshold small differences have only been evaluated as mixed hue, lightness, and chroma differences, and an untested assumption of their euclidean summation has been made (Chapter 5).

Boynton and Kambe (1980) have raised the possibility of hexagonal chromatic contours. The chromatic discrimination contours of Krauskopf and Gegenfurtner (1992) have been fitted with ellipses and circles, but many may be equally well described by polygons. Superelliptical contours were calculated by Teufel and Wehrhahn (2000) for threshold data. Rhombic chromatic threshold contours in cone contrast space have been fitted by Strohmeyer et al. (1998) and by Giulianini and Eskew (1998). Sankeralli and Mullen (1999) tested hue increment identification using combined hue and contrast increments and concluded that hue increment identification ignores differences in contrast (chroma) and that the two types of discrimination may be inherently separate.

There is strong evidence that in a polar coordinate, psychophysically "uniform" chromatic diagram (uniform hue and uniform chroma scales), the unit difference contours are elongated in the direction of chroma, indicating that hue difference discrimination is intrinsically more sensitive in terms of stimulus increments than contrast discrimination. Explicit evidence is found in the work of vision scientists as well as in threshold and suprathreshold object color difference data, including the Munsell system. Elongated unit contours are implicit in the basis data of OSA-UCS, a fact that has been suppressed in the final system. Such evidence is the basis of Judd's term "superimportance of hue."

The Nickerson and the Bellamy and Newhall evaluations indicate unit contours to become slightly more elongated when the differences become smaller (2:1 at the level of Munsell chips vs. 2.8:1 at JND). On average, for small suprathreshold chromatic differences the ratio is found to be approximately 2.2:1 (Chapter 8). The shape is generally taken to be elliptical. Conceptually it is unlikely to fit the simple geometrical form of an ellipse.

How chromatic and lightness differences are combined is also a not a question that is resolved. Recent neurological investigations show that "interactions between color and luminance variations can be complex and not easily predictable" (De Valois, 2000). In terms of color difference formulas the general assumption has been that the sum is euclidean. Based on his color-matching error work MacAdam maintained that the resulting ellipsoids have horizontal alignment (e.g., MacAdam, 1981; Section 8.4). Threshold and suprathreshold color differences are fitted better with formulas that result in an ellipsoid tilted at higher lightness in the direction of the neutral axis, as is the case with CIELAB and related formulas.

9.6 WHAT CAUSES THE PERCEPTION OF THE MAGNITUDE OF COLOR DIFFERENCES?

At this time the answer to this question, in neurophysiological terms, is not known. From findings in Chapter 8 we can conclude that at the color-matching error level differences are largely due to sensitivity limits of the cones. This is fully the case for the MacAdam data and largely for the Wyszecki-Fielder data. At the threshold level, based on the Richter data, an opponent color system seems fully engaged, as evidenced by the lower *S/L*, respectively *Z/X*, ratios. This is the case for larger color differences up to and, likely, exceeding the size of OSA-UCS differences. Here power relationships connect stimulus and perceived difference. The applicable powers and the weighting constants (different by semi axis) are found to vary widely in different experiments for reasons that are not clear but may involve the design of the experiments or the observer panels. Different optimal powers and weights appear to indicate independence of at least four chromatic processes. The immediate reasons for the power relationship are not known but assumed to be saturation effects of various cell types.

The relative enlargement of unit *L* increments against unit *S* increments for threshold and larger differences compared to CME data signals a sea change, the change from the Helmholtzian cone level to the higher zone, opponency level. The higher intrinsic sensitivity of the *L* and *M* cones compared to *S* is suppressed. Regardless of the input from cones, here one unit of *a* is equal to one unit of *b*, with the positive and negative functions in balance. The relative number of *L*, *M*, and *S* cones per unit area in the retina does not seem to play any role in this situation. The new ratio of implicit *L* versus *S* response of the first step from gray is valid throughout the range of differences from threshold to OSA-UCS sized (and very likely larger) differences.

9.7 CHRISPENING EFFECTS

Crispening effects impose significant modulation on the relationship between stimulus and perceived colors or color differences. They are the result of an environmentally useful adaptation that makes minor spectral differences of objects against surround especially well discernible. As discussed in Chapter 4, stimulus increments/decrements resulting in a criterion difference perception are smallest if the test field stimuli straddle that of the surround color. The more the surround differs in hue, chroma, or lightness from the test colors, the larger will be the required stimulus increment for a criterion difference response between the test colors. Highest chromatic discriminability is thus available in a chromatically relatively homogeneous environment, and comparably for lightness discrimination. Like lightness crispening, chromatic crispening may depend on test field size. Detailed quantitative data are not available in both cases.

Crispening effects result in V-shaped functions of unit increment versus stimulus (*L*, *M*, *S* or *X*, *Y*, *Z*) with the vertex pointing at the surround point (Chapter 8). In these functions different observers and, presumably, different observation conditions result in different angles between the legs of the V function. Functions for brightness and lightness data and for the *S* (or *Z*) cone have larger angles than functions for the *L* and *M* cones.

Lightness crispening in simple surround conditions is controlled by the luminous reflectance of the surround and, probably to a lesser extent, by the test field size. Increments for a criterion response become increasingly larger as the test field luminous reflectances differ from that of the surround. Color space and difference formulas so far have generally not considered this effect. If achromatic reference pairs are used the magnitude of difference perceived between them is also subject to the lightness crispening effect, that is, the psychological magnitude of the difference between two gray reference samples depends on the surround and the relative size of the fields. This issue affects comparability of different data sets based on different surrounds. The lightness crispening effect is active from threshold size differences to differences of the size found in the Munsell value scale and, likely, larger differences.

The chromatic crispening effect is, on a relative basis, about equally distinct for L and M compared to S. The L increment required for criterion response quadruples approximately over a change in L of 10 units from the neutral point in direction of increasing L, less so for decreasing L, as the increment is a sum of the fundamental increment and the chromatic crispening increment. As has been shown in Chapter 8, it is most strongly active at the level of CME and threshold differences and appears to gradually fade as chromatic differences become larger. It is absent at the level of Munsell double-chroma steps or OSA-UCS steps.

Chromatic crispening makes it possible to euclidize the Riemannian space implicit in small color difference data. The approximate factor 2 of the ratio of the longer (chroma-related) to the shorter (hue-related) diameter of the unit contour has been included in the S_C weight adjusting for chromatic crispening. Since this weight is a continuous function of chroma it can be integrated in the euclidization of formulas like CIE94.

9.8 PERCEPTUAL INCREMENT MAGNITUDE AS A FUNCTION OF STIMULUS INCREMENT MAGNITUDE

Despite the findings of Plateau the question of increment magnitude for a criterion response as a function of stimulus magnitude was dominated for the second half of the nineteenth century by Fechner and the Weber-Fechner law indicating the required increment to be a constant fraction of the stimulus. Study of lightness scales from black to white in the early twentieth century and additional investigations of other sensory magnitudes pointed to the applicability of power modulation. Different experimental conditions resulted in different optimal power modulations. The findings in regard to crispening discussed in Chapter 8 indicate that the Weber-Fechner law is generally not applicable to color differences. It is also apparent that simple power functions, as proposed by Stevens, are not representative of color differences from subthreshold to medium size.

In global psychophysical color space scaling efforts the Munsell value scale, with its suppression of the lightness crispening effect (and its fit with cube root modulation) and the proposal by Adams in 1942 to apply cube root modulation also to the X and Z tristimulus values, proved influential and continues to be the official paradigm of the CIE. This situation has been influenced in the past by computational difficulties and then by habit. The evidence derived from the Munsell Renotations and the Re-renotations is that different powers are optimal for the four chromatic semi axes. Conceptually it would be surprising if the complex processes of the visual and related cognitive systems could be accurately modeled with a single and simple power modulation. This matter has not yet been thoroughly investigated in regard to threshold and suprathreshold small color difference data.

Experimentally determined incremental and decremental stimulus amounts lack a solid foundation in neurophysiology. The prevailing explanation involves cone response saturation effects. It is unlikely to be the complete story. The situation is complicated by the fact that psychophysical models need to account for three effects in the chromatic plane:

1. Basic nonlinearity of the tristimulus space. Unit contours in the normalized X, Z diagram are elongated. They are largest and most elongated when Z values are large. The contours are smallest for yellowish-greenish colors. Power modulation of the scales linearizes them and makes the ovoid contours of small color difference data at a given chroma approximately equal in size. It makes the ovoid contours of Munsell data approximately equal in size throughout the diagram.
2. The chromatic crispening effect changes the size of contours of small color difference data as a function of chroma. The power-modulated (i.e., normalized in the four quadrants) contours differ in size by a factor of 4.4:1 as a function of chroma in a metric chroma range from 0 to 100 (Chapter 8).
3. The unit contour in a euclidean diagram is fundamentally elongated and the elongation may be a function of the size of the differences, as seen in the previous section.

In formulas such as CIE94 and CIEDE2000 the cube root formula addresses point 1. The weight on chroma differences and the chroma-related weight on hue differences addresses points 2 and 3.

9.9 HOW WELL DO FORMULAS PREDICT PERCEIVED COLOR DIFFERENCES?

As seen in Chapter 6, correlation between average visual data and formulas can be improved to some extent with empirical modification based on regularities observed in the data. Such modification is often specific to a given data set. When looking at agglomerations of data approximately two-thirds of the variation in average perceived difference is predicted by the best formulas. The performance factors PF/3 (a performance measure approximately equivalent to error percentage) of the combined data set used in the development of CIEDE2000 as well as of various subsets assembled from different original data sets (hue, chroma, and lightness differences, as well as others) are uniformly from 30 to 34. Similar values have been obtained with that formula for specific data sets such as BFD, Witt, and BIT. The only exception is the RIT-DuPont data with a value of 19. While the PF/3 results for the BFD data improve from 56 for CIELAB to 37 for CIEDE2000, the improvement for RIT-DuPont is only from 22 to 19.

Progress in the accuracy of prediction can only be made if the source(s) of the error component representing one-third of the total variation of the formula can be determined. In an analysis of five sets of data for several of them significant increases in correlation were obtained by adjusting the size of the perceived difference by size factors seemingly nonsystematic for each color subset (Kuehni, 2001b). No agreement in these factors for given color regions in the different sets could be found. The factors ranged from approximately 0.7 to 1.5. This indicates that the size of average judged difference in a given color region varies significantly in different experiments despite efforts at normalisation. At this point the changes appear to be random, but detailed investigation into the potential causes has not yet been made. When the remaining major discrepancies in the RIT-DuPont set are analyzed, they appear to involve primarily complex differences with hue, chroma, and lightness components, hinting at non-euclidean addition. Among other potential causes are observation conditions such as lighting and surround, composition of observer pool, subjective strategies of the observers, method of averaging individual results, choice of reference difference, and nature of dividing line between samples.

9.10 IS UNIFORM COLOR SPACE EUCLIDEAN?

Psychophysical models provide fits of greater or lesser degree to psychologically uniform color difference data. But euclidean depictions of psychological color space are not uniform, unless (for small color differences) achieved by a special euclidization step. The Munsell chromatic diagram is approximately uniform in terms of hue (at one chroma level) and in terms of chroma, but not both. If the scales are adjusted to equality the chromatic plane can only be illustrated as a crinkled fan. It is not known how far reaching Nickerson's investigation of the relationship of hue and chroma differences were as a function of chroma. If the S_H weight is a truer representation of facts than an assumption of a constant ratio between hue and chroma differences as in the Nickerson formula, the uniform color space is Riemannian. As discussed, a portion of S_C

(together with S_H) eliminates elongation of the unit contour. It is not clear that it actually does so because the complete hue circle has not been scaled relative to chroma in connection with color difference formulas that have such weights.

Euclidean structure has been denied by Judd (1968) in case of a space based on a constant achromatic surround with arguments involving the superimportance of hue and crispening effects. An additional argument was called "diminishing returns in color difference perception." MacAdam had pointed out in 1963 that if there are three color samples A, B, and C selected along a geodesic so that samples A and C appear equally distant from sample B, samples A and C are seen as having less than twice the difference between A and B or B and C. In other words, the applicable scale is an interval scale but not a ratio scale. This effect may be connected with the fading of chromatic crispening for larger differences and the changing of the unit contour elongation ratio as a function of size of the difference.

In 1995 Wuerger and co-workers reported on tests of euclidean color geometry. In one test they used proximity judgments to determine the angle between intersecting lines in color space, and three kinds of tests for the additivity of such angles were made. All three failed to support a euclidean structure. In a second experiment the increase in variability of judgments of similarity with the distance between test and reference stimulus was determined. Also this experiment failed to support euclidean structure.

The euclidean addition of hue, chroma, and lightness differences does so far not seem to have been tested explicitly at the level of suprathreshold small or large differences and under conditions representative of industrial practice (no control of local adaptation). Among the reasons are the difficulty in preparing accurate, well defined samples and the lack of replicated uniform chroma, hue, and lightness (including the Helmholtz-Kohlrausch effect) scales.

As mentioned, euclidean structure with luminous reflectance as one dimension also can be expected to fail because visual lightness perception of chromatic samples involves not only luminous reflectance but also the presumed contribution of the opponent color system in the Helmholtz-Kohlrausch effect. Resulting constant psychological lightness planes are not orthogonal to the achromatic lightness axis.

The question arises if it is useful and veridical to appropriately transform, for small color differences, a^* and b^* scales by integration so that the resulting diagram is of euclidean nature, as Thomsen and others have shown and as implemented in the DIN99 formula. Given the many remaining problems, some addressed in CIEDE2000, the result is likely less than satisfactory and not useful for large color differences. Another version of integration used in the past is projective transformation, extensively employed by MacAdam. However, his best effort in devising a nonlinear transformation for the OSA-UCS basis data did not result in correlation improved over that obtained with a cube root based euclidean formula.

9.11 UNIQUE HUES AND UNIFORM COLOR SPACE

A psychological euclidean color space with the unique hues on the axes is not uniform in terms of differences because hue differences representing identical hue angle differences are of different perceptual magnitude in the four quadrants (Chapter 4). In addition there is the matter of the elongated unit contours.

There is poor agreement between the cone contrast chromatic diagram axes and unique hues. There is only slightly better agreement in case of a chromatic diagram based on opponent functions derived from CIE color matching functions. But there are two major issues here also:

1. The implied unique hues of the two standard observers, taken as the crossover points of the opponent functions, are considerably different. This implies that the primaries represented by the system axes are of more or less mixed hues for both standard observers.
2. In terms of both standard observer opponent color functions objects seen on average as unique red and unique green have a considerable positive *b* ("yellowish") component.

No form of transformed color matching and related opponent functions seems to exist where the four average perceived unique hues fall on the axes and a constant perceived chroma circle is represented as a circle (Chapter 5).

The processes behind unique hues are unknown and may have a location different than opponent processing. But even when known, there remains a systematic discrepancy between a space based on unique hues and their standard increments and decrements and a uniform space.

9.12 EVIDENCE FOR THE OPERATION OF AN OPPONENT COLOR SYSTEM

Aside from the neurophysiological evidence and the purely psychological evidence in the Hering sense, there is considerable indirect psychophysical evidence. In a cone-sensitivity space an approximately perceptually uniform system such as the Munsell system is placed in alignment not in agreement with any of its axes (see Fig. 5-29). Rough alignment with the axes is only obtained in a tristimulus space and constant chroma circles at different lightness levels only become concentric if the chromatic diagram is based on opponent functions (Fig. 5-31). In other words, a reasonably accurate model that places the information contained in reflectance functions of the Munsell system into a corresponding psychophysical model requires opponent color functions.

Implicit evidence for an opponent system also comes from the fact that cone increments required for the perception of threshold and suprathreshold differences are, in case of L and M cones, not related to their intrinsic sensitivity as expressed in color-matching error but represent increments of approximately double relative magnitude. Such enlarged increments are consistent with the operation of an opponent system.

Perceived brightness/lightness of chromatic colors is not in agreement with power-modulated flicker luminance/luminous reflectance. But it can be modeled closely with addition of a portion of opponent color activation to flicker luminance (Chapter 5), indicating that perceived brightness/lightness of chromatic colors is not a dimension of tristimulus color-matching space.

9.13 OPPONENT SIGNALS: THE SOURCE OF HUE AND CHROMA PERCEPTIONS?

In Chapter 7 evidence was offered that it is not possible to represent the Munsell system as a uniform euclidean color solid because of the superimportance of hue. Because of chromatic crispening the effect of hue superimportance can be integrated in small color difference data to end up in a euclidean system. As mentioned, this may not be possible at the size of Munsell color differences where chromatic crispening no longer applies. This fact may be a consequence of our assumption that output of hue and chroma discrimination processes is caused by the same opponent color signals. Such mechanisms, while believed reducible to increments of opponent color responses, represent separate entities. Hue and chroma perception occupy the same two dimensions in a chromatic diagram. They would have to be balanced in terms of the relative perceptual magnitude of the unit stimulus increments to fit into a euclidean system. But as seen in Chapter 7, they are not.

Can opponent color signals be the immediate source of hue and chroma perceptions? In other words, is the hue system based on opponent signals only? As seen in Chapter 4, when implied opponent signals of an ideal hue circle are plotted as a function of hue angle they form offset sinusoidal curves. Hues that represent unit hue differences from selected reference colors along the constant chroma circle plot on a line parallel to the sinusoidal line of the circle (Fig. 9-1). For differences to be hue differences only, the following rule applies: the sum of a and b of the test must be identical to that of the reference; that is to say, differences in a and b between reference and test must be opposite in direction and identical in magnitude. Colors that do not follow this rule result in either chroma or mixed hue and chroma differences. For a pure chroma difference, differences in a and b between reference and test must not only be of the same magnitude but also the same direction. Mixed hue and chroma unit differences of a given magnitude are defined by a and b values that are bounded by the pure hue and pure chroma differences. In such a scheme it is difficult to see where the elongation factor 2 of unit perceived difference contours originates. It seems likely that different mechanisms are responsible for hue and chroma perception (certainly also lightness percep-

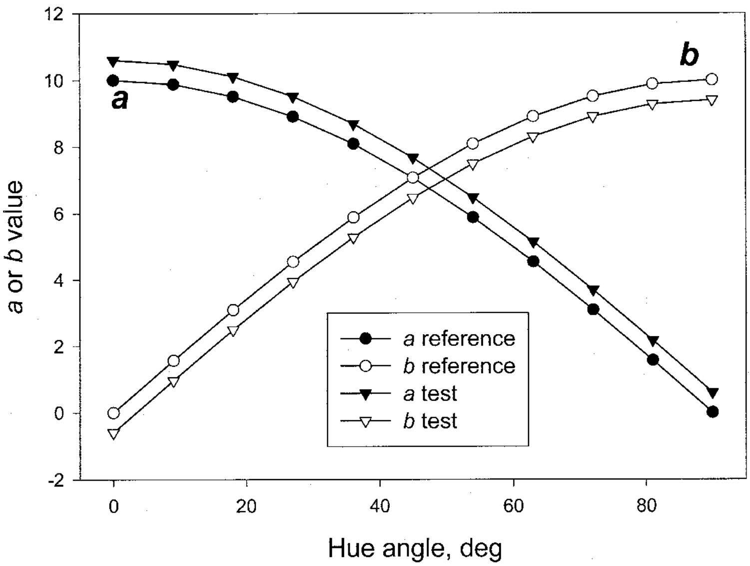
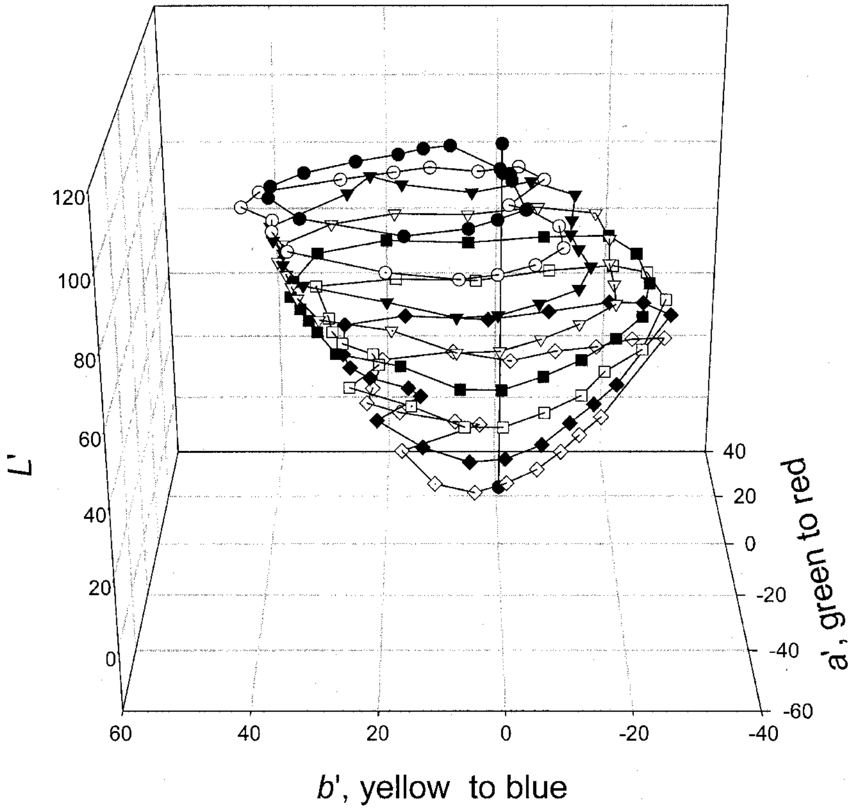


Fig. 9-1 Values of an ideal, euclidean, constant chroma first quadrant with identical hue differences between adjacent colors. Circles represent the reference color, and triangles the test color.

tion) and that additional processing takes place beyond simple opponency. The real hue and chroma perception and hue and chroma difference perception processes may be more complex as hinted at by the results of Chichilnisky and Wandell (Chapter 6).

9.14 THE APPROXIMATE SHAPE OF A UNIFORM COLOR SOLID

Given the difficulties alluded to in earlier sections and the lack of sufficiently detailed experimental data it is not possible to create an accurate geometrical model of a uniform object color solid. An approximation of the surface of such a space based on small color differences was calculated using simplifications and assumptions. At eight different levels of luminous reflectance twenty maximal object colors around the hue circle were determined. The luminous reflectances were adjusted to reflect the Helmholtz-Kohlrausch effect using the corresponding formula from OSA-UCS. These values were transformed into the L^*, a^*, b^* space and the a^* and b^* values euclidized using the Thomsen formula. As was mentioned in Section 9.9, it is not clear if this calculation adjusts also fully for the fundamental elongation of the unit contour. The results are points on the surface of a color solid uniform according to the conditions implied and are shown in three projections in Fig. 9-2*a*, *b*, and *c*. View 9-2*a* shows the solid facing the yellow to blue dimension and view 9-2*b* facing



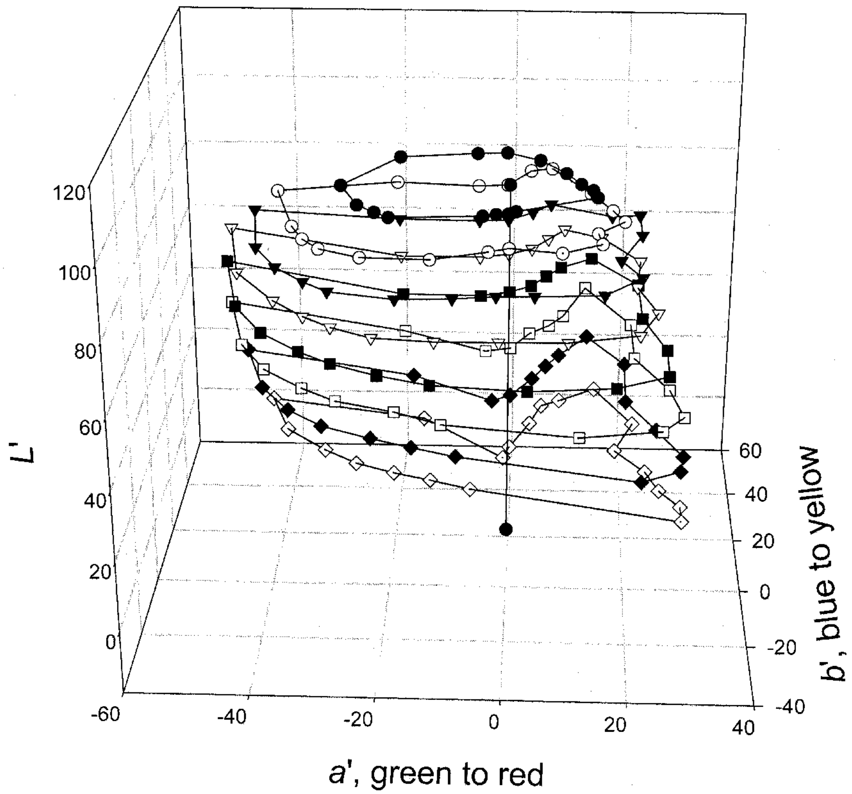
(a)

Fig. 9-2 *View of a color solid approximately uniform at the level of small color differences. The vertical line connects achromatic colors from luminous reflectance 0 to 100. The different symbols represent colors of a given luminous reflectance at the object color limit. They have been adjusted for the Helmholtz-Kohlrausch effect. (*a*) View of the yellow-to-blue axis; (*b*) view of the green-to-red axis; (*c*) view from the top onto the chromatic plane.*

the green to red dimension. Achromatic colors fall on the straight vertical line. The third view shows the solid from the top. Its irregular nature is clearly evident. It can be compared to Fig. 1-2.

9.15 A RESEARCH PROGRAM

Analysis of published data in regard to color attribute scaling and color difference scaling shows significant discrepancies in results between different efforts, despite large numbers of samples, observers, and observations. There are several potential reasons for such discrepancies: different observer panels, different visual context of samples and surround, different light sources, and



(b)

Fig. 9-2 (Continued)

unrecognized contextual clues that can affect the results due to empirical interpretation of the visual field.

Independent replication of experimental results is a standard requirement in science. The degree to which experimental color scaling results can be independently replicated needs to be established. Given the known and perhaps some unknown sensitivities of results to experimental conditions, it seems imperative that independent replication involve essentially identical viewing conditions: the same samples, the same visual context, and the same or very similar (implicit or explicit) light source. Today duplication of this kind is probably best achieved with simulated samples on a computer monitor. The same program creating the displays can be used in different locations, and the general surrounds can be duplicated with masks surrounding the monitor screen. Illumination of the room in which the monitor is viewed may be important for control of the complete adaptation situation. Monitor calibration is a critical issue that needs to be satisfactorily resolved. Variability of results from different observer panels in several locations can then be established. The results may indicate if the variability we see in past results is normal or if, given

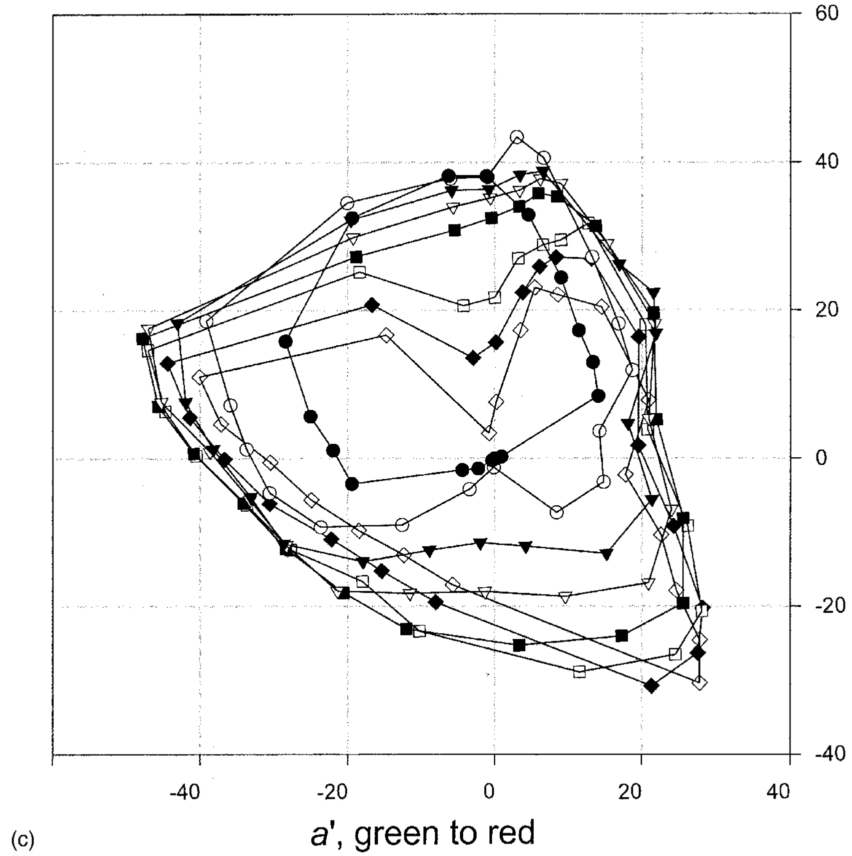


Fig. 9-2 (Continued)

tightly controlled test conditions, the variability can be substantially reduced and how many observers are needed to obtain a reliable average. Alternately, identical physical samples could be used in several locations with closely duplicated observation conditions but different observer panels. Global scaling with Munsell sized differences should be followed by scaling of smaller differences. Given the clear indications of hue superimportance, it appears that global scaling should be based on lightness, chroma, and hue scaling. Consideration has to be paid to the question of the reference difference used to ensure that lightness, chroma, and hue steps are of identical perceptual magnitude. In closed scales such as lightness and hue, selection of a particular size of reference difference may result in a number of steps that is not an integer.

Lightness Scaling

Once an intermediate lightness for the achromatic surround and a reference difference have been chosen, a 10- or 20-step gray scale can be established.

Planes of perceived iso-lightness in a linear a, b, Y space can then be determined at different levels of Y of the gray reference. This is not unproblematic because of the findings of Nayatani and co-workers (1994) that the magnitude of the Helmholtz-Kohlrausch effect depends on the experimental design. The reason for the significant differences reported by these researchers should be elucidated and the experimental design selected accordingly. Iso-lightness planes need to be determined at different levels of lightness. It is unlikely that the assumption of Wyszecki and Sanders and the OSA-UCS committee that planes of iso-lightness as a function of Y are parallel over a wider range of Y is valid.

Chroma Scaling

Once planes of constant lightness have been established chroma can be scaled at different levels of chroma and different levels of perceived lightness. Again, the work may be easiest to accomplish using monitor display colors that are adjustable essentially in one attribute direction only. As the observer adjusts fields for identical perceived chroma, the program should automatically make adjustments for the previously established constant lightness and follow approximate lines of constant hue as reported in the literature. Once 20 to 40 hues around the hue circle have been scaled for chroma, constant chroma contours can be established by interpolation of data points. Such contours need to be established minimally at 5 to 7 levels of chroma at several levels of perceived lightness.

Hue Scaling

With chroma scaling complete, constant chroma contours can be scaled for constant hue increments along the contours. Fine adjustments can be made to result in an integer number of differences. Again, the program should make automatic adjustments for perceived lightness and chroma as the observer makes changes of the perceived hue of the test field.

Many issues need to be decided. For example, should all three attributes be scaled separately for each observer, or should average perceived lightness and then average perceived chroma be used for the subsequent step?

Once results of the best achievable level of replication have been obtained, efforts to fit the visual data with psychophysical formulas can be undertaken. It remains to be seen if formula fitting achieves satisfactory agreement or if, in the end, lookup tables are required.

Complex Color Difference Scaling

With global scaling at the Munsell level complete efforts can be undertaken to determine the addition process required for complex color differences: Is the addition of hue, chroma, and lightness differences simple, euclidean, or some other geometry? This issue has to be studied for pairwise complex differences as well as for differences ranging through all three attributes. The replicated findings at the Munsell level of difference need to be confirmed or modified for small suprathreshold differences.

The apparent increase in chromatic crispening as the size of difference decreases requires quantification, both in regard to hue and chroma. Clarification is also required of the apparent nonlinear effect of lightness on the size of perceived chromatic differences, as discussed in Section 8.6. Additional quantification of the lightness crispening effect in small color difference evaluation is also desirable.

The form of the functions relating attributes and size of differences and the answer to the question of type of addition will shape the form of an equation for small suprathreshold color differences. Also here it remains to be seen if an analytical formula is useful or if the information should be used in form of lookup tables. The same question applies to a universal color difference formula. Can such a formula be meaningfully expressed with equations with changing parameters?

Robustness of Formula

The robustness of a formula developed in such a manner needs to be established. We have already seen that without several adaptable parameters such a formula is only strictly applicable to a narrow set of conditions in regard to samples, surround, and lighting. The question to be resolved is how much conditions can change before the accuracy of the formula is unacceptably reduced. This may involve sample size and structure, separating line, surround quality and structure, spectral power distribution, and intensity of the light source. Can the formula be meaningfully adapted to apply for incandescent or triband lamps?

9.16 KINDS OF SPACES

This text has made clear that the idea of a single fundamental color space is misplaced. As found in practice there are different kinds of color spaces and solids applicable with a degree of accuracy to different situations. Changes in the applicable conditions can quickly further reduce the accuracy of a formula to a significant degree.

Color spaces can be placed in a kind of hierarchy of complexity and meaning. The structural requirements for these kinds of spaces differ.

Uniform Color Appearance Spaces and Solids

These are spaces that for a specific relativized set of viewing conditions illustrate object color samples not only embodying qualitative systematicity in arrangement of colors according to three fundamental attributes but also quantitative systematicity according to the principle of visual uniformity of difference in all directions. Such a space, as we have seen, is non-euclidean in nature but (at least for small color differences) can be euclideanized. For spaces of this kind a high degree of correlation between psychological and psychophysical data is important. Their primary use is for quality control purposes. There is currently no sample set that can be said to satisfactorily express a uniform color solid for the average color normal observer and the applicable set of experimental conditions. Thus all currently existing physical sample sets must be classified as regular, rather than uniform. As a result there is no psychophysical space or color difference formula with satisfactory performance either, as seen earlier. It would be of considerable interest to know how high a degree of correlation can be obtained for an individual observer, given knowledge of that observer's personal cone sensitivities, luminance function, and other properties of the visual system. It might then become apparent to what degree individual experiences shaped our personal color vision apparatus in a way that cannot be systematically expressed with the kinds of simplistic color vision models currently at our disposal. Should the effect be of significant magnitude a color difference formula with much improved performance for the average observer may be impossible. Our current level of knowledge is insufficient to come to such a conclusion.

Regular Color Appearance Spaces

Much of what has been said in the previous subsection is applicable here also. In case of the NCS system differences in principal hue components, blackness and chromaticness are not linearly related to perceived magnitude of difference. Close agreement between the NCS psychological space and a psychophysical space is not important, and in fact no complete mathematical formulation has been attempted so far. Whenever translations seem necessary, they are made with the help of tables. Regular as well as uniform color solids are equally reflective of and variable with the experimental conditions. OSA-UCS, Munsell, and DIN are roughly uniform but euclidean. Ostwald and NCS have also quantitative systematicity but according to principles other than uniformity. Colorcurve, HSB, RGB, CYMK, and other systems have value as arrangements with ordinal systematicity with different kinds of stimulus increments: tristimulus increments in case of Colorcurve, electron gun output increments in case of HSB and RGB as related to video display, primary print color increments in case of CMYK. In the limited region of a color solid that can be covered with colorants or monitor phosphors, they are indicative of the variety of color experiences of the average color normal observer under the limitations of the system. OSA-UCS with its crystalline structure can display a wide variety of color scales varying simultaneously in all three perceptual attributes and changing in color in dramatic ways without, as shown, being uniform.

Psychophysical (Regular) Spaces

For spaces such as Luther-Nyberg, CIELUV, CIELAB, and others there are smaller or greater claims for uniformity, but they are just regular spaces with different principles of tiling. A particular example is the Derrington-Krauskopf-Lennie space with tiling according to macaque cell response in the lateral geniculate nucleus.

Color-Matching Spaces

Color-matching spaces include many examples, such as the Rösch-MacAdam space, the CIE tristimulus space, and the Cohen fundamental color space (with many possible configurations). The last derives its name from the decomposition of spectral power distributions into the so-called fundamental of each reflectance function, meaning the portion of each spectral function free of metameric black. Each related color perception is represented as a vector, just as in the other two spaces mentioned. Average visual metamers plot in the same location in all three cases. These spaces make no claims for quantitative systematicity in terms of appearance.

Spectral Spaces

Spectral spaces do not involve data related to human color vision and, therefore, should not be termed color spaces. They are based on finding components representing a given reduced dimensionality implicit in the spectral functions, such as by principal component analysis. Some three-dimensional spectral spaces place spectra of the Munsell colors in correct ordinal order, but the dimensions have no perceptual meaning and differences between colors in such spaces have no meaningful relationship to perceived differences. Visual metamers plot in different locations in such spaces.

Among the many kinds of color spaces the uniform space has the highest degree of systematicity if equal sense distance has the importance that psychophysics assigns to it. What degree of accuracy and repeatability is possible for a uniform color space for the average observer and a specific set of experimental conditions remains to be determined. Unless one wants to raise a charge of bias of some kind for the RIT-DuPont data, they show that significant improvement compared to many other data sets may be possible based on careful experimentation. Additional improvement resulting from a fuller understanding of the color vision process may also be feasible. Problems and uncertainties increase the more complete the color vision model is to be in its explanatory power, for example, as color appearance models indicate.

Given the view that vision is an animal type's best way to deal with the ambiguities of the information provided by the energy streams that we call light, it is evident that finding the one color space representative of human color vision is a problem without solution. Taxonomy, the classification of objects and phenomena, is an important human activity. Just as any other human activity it is open ended. We have found that a color classification free of context is of necessity very vague. To make it less vague, we must specify the context but the classification becomes applicable only to that context. To achieve the highest level of classification (a uniform color space), the context must be highly defined in all respects.

After some 300 years of effort the conclusion is that color appearance systems such as the Munsell system are general indicators of the multitude of color experiences that can be generated, for example, with pigments painted on paper and viewed in daylight. To reach the borders of our possible experiences work with monochromatic lights is necessary. But it is evident that claims for a given relationship between stimulus and resulting experience are only applicable to closely circumscribed conditions of viewing. A uniform color space (not euclidean) defined in terms of stimuli is only possible for a closely circumscribed set of viewing conditions and in theory only applicable to one observer.

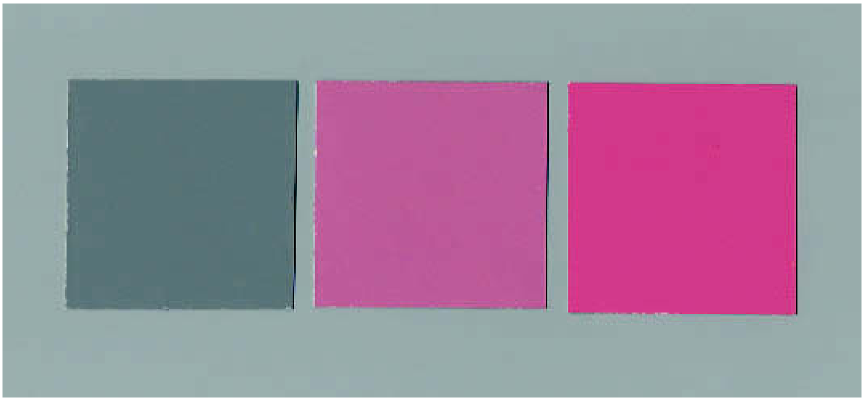


Fig. 1-1 *Images of chips of the OSA-UCS system.* Left*: Color 000;* center*: color 00-4;* right*: color 00-8.*

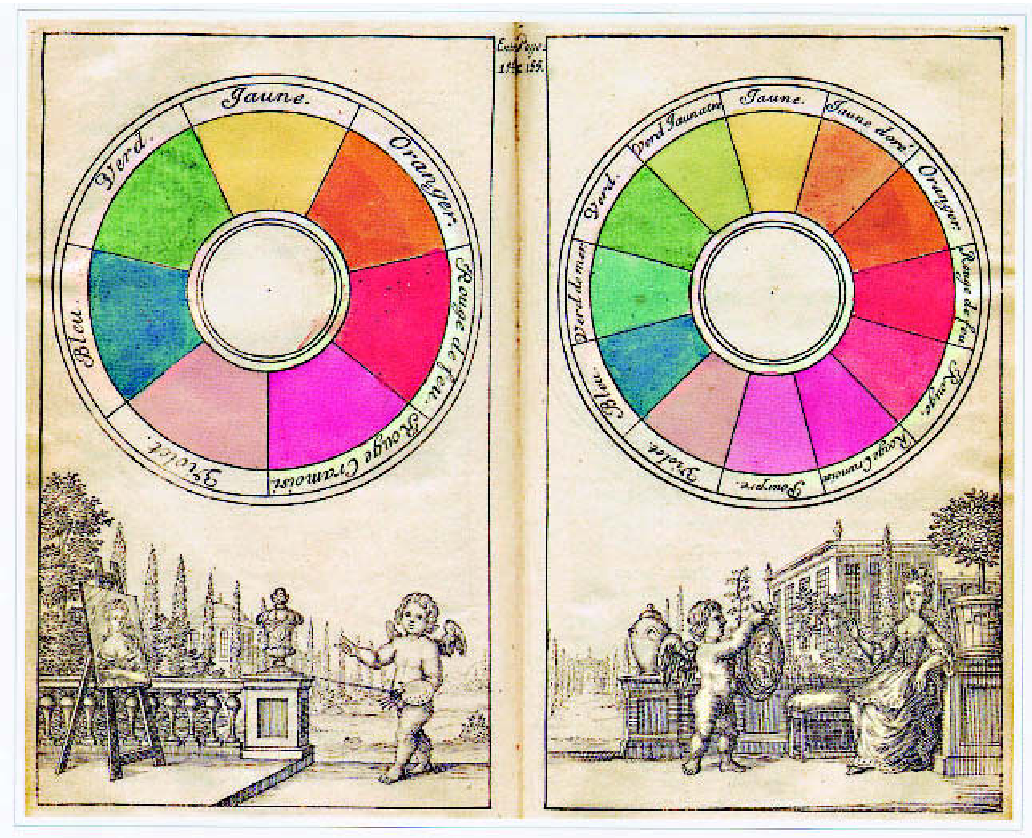


Fig. 2-12 *C.B.'s hand-painted color circles, 1708. Left: The seven-color circle; right: the twelve-color circle. Note that the pigments used for some of the colors have deteriorated.*

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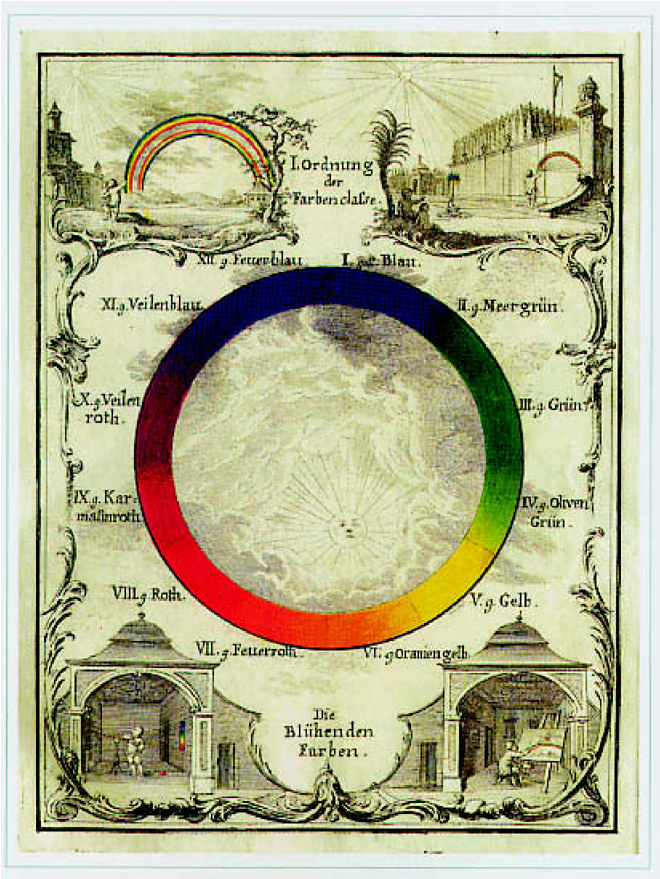


Fig. 2-13 *Continuous color circle of Ignaz Schiffermüller, with twelve classes of colors, 1771.*

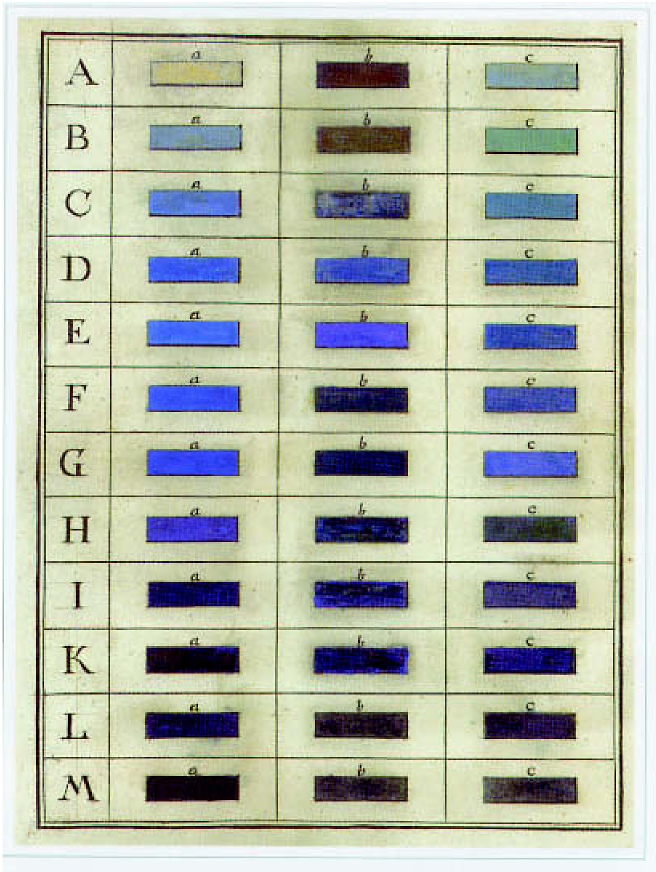


Fig. 2-14 *Schiffermüller's tonal scales of three blues, from his 1771 work. Some of the colorations have deteriorated.*

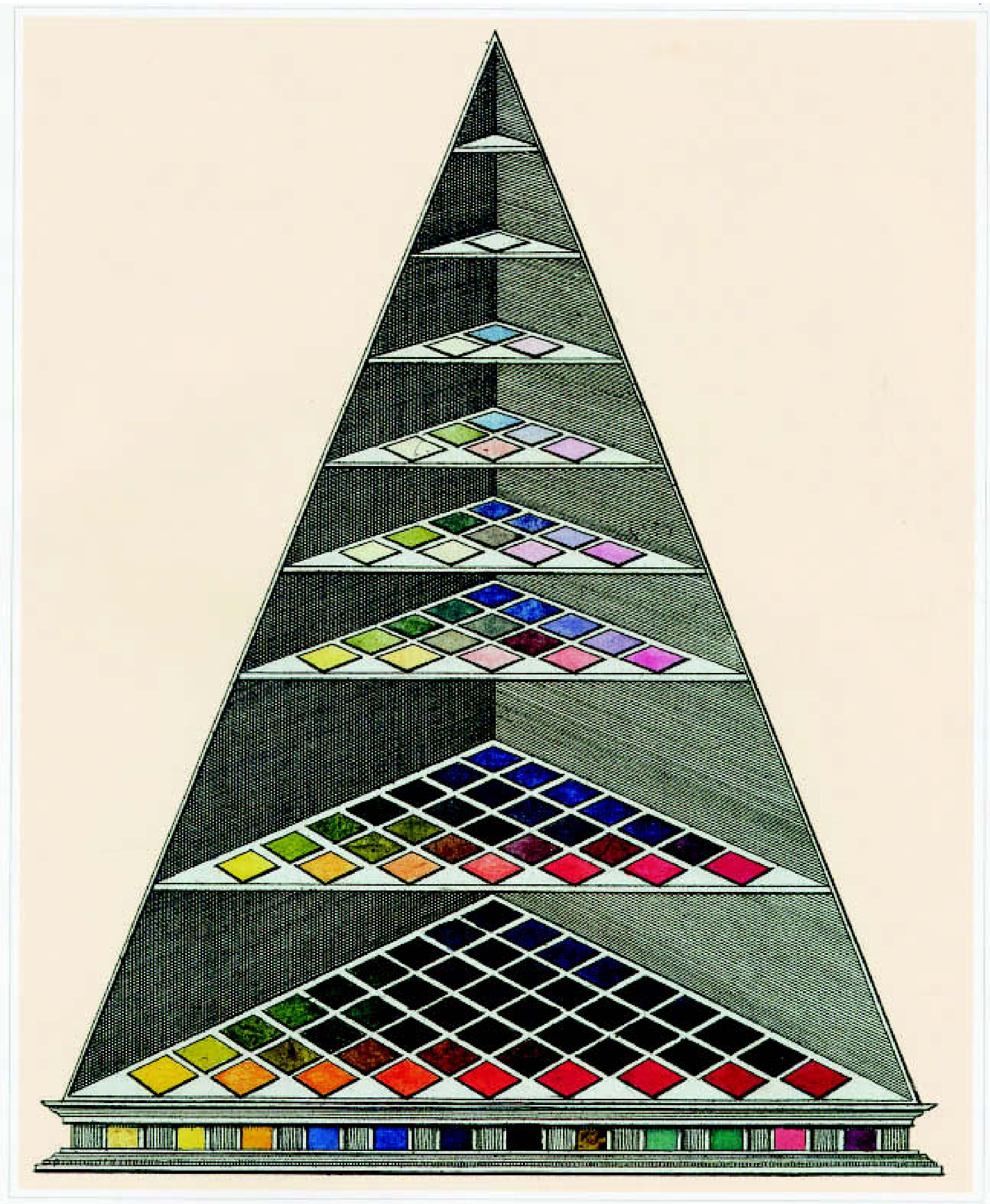


Fig. 2-18 *Depiction of Lambert's color pyramid, 1772. The lowest level contains the 45 colors identified in Fig. 2-17. The higher levels contain reduced sets at higher lightness, ending in white on top of the pyramid. Black is located on the lowest level. The colors displayed on the front of the model represent well-known artist's pigments of the time.*

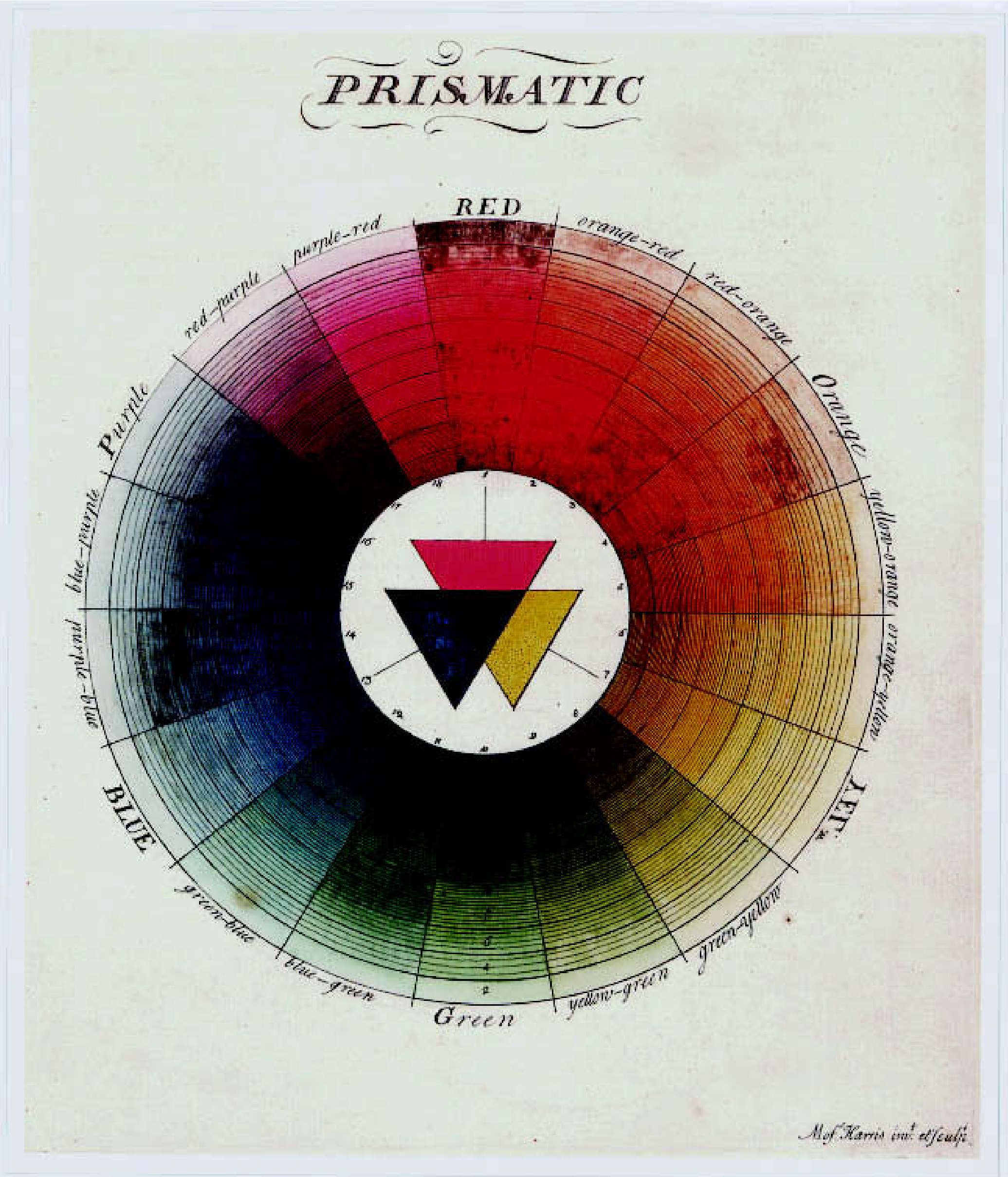


Fig. 2-19 *Prismatic version of Moses Harris's color circle of 1786. Some deterioration of colorants is evident.*

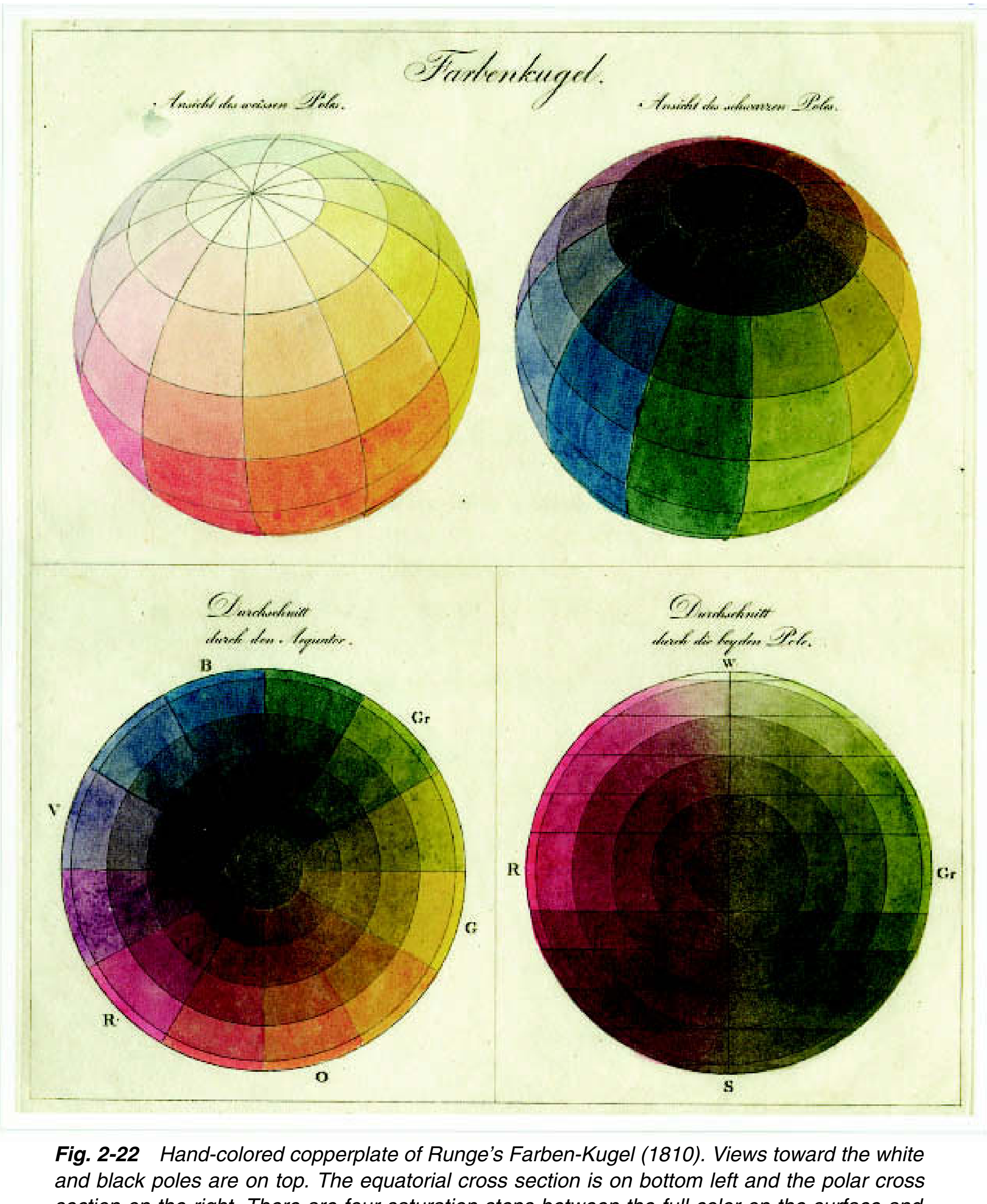


Fig. 2-22 *Hand-colored copperplate of Runge's Farben-Kugel (1810). Views toward the white and black poles are on top. The equatorial cross section is on bottom left and the polar cross section on the right. There are four saturation steps between the full color on the surface and the middle gray in the center of the sphere.*

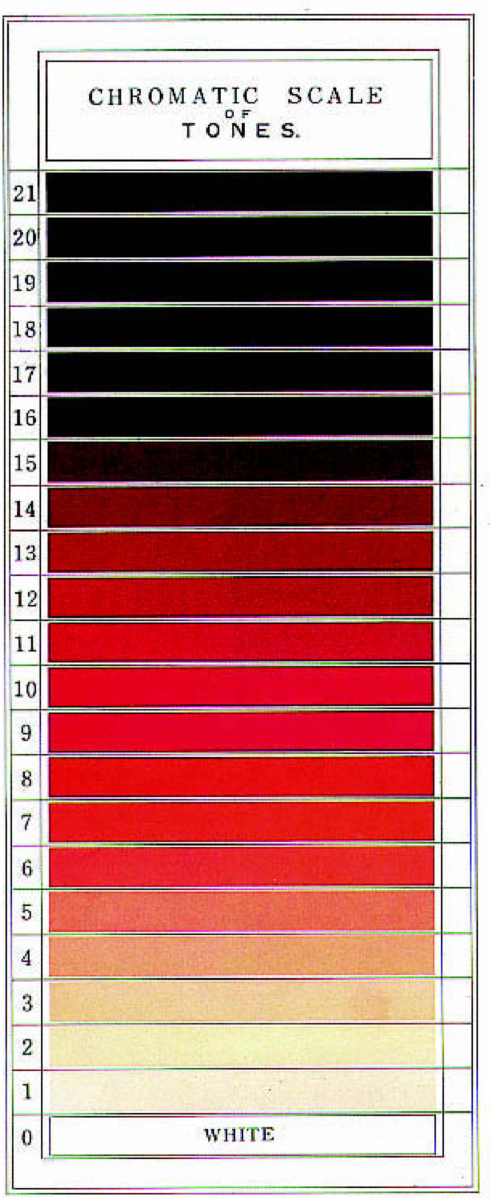


Fig. 2-25 *One of twelve chromatic scales by Chevreul, with local discoloration. The colors range from white through the full color (grade 11) to black. These scales are located on the base plane of the hemisphere.*

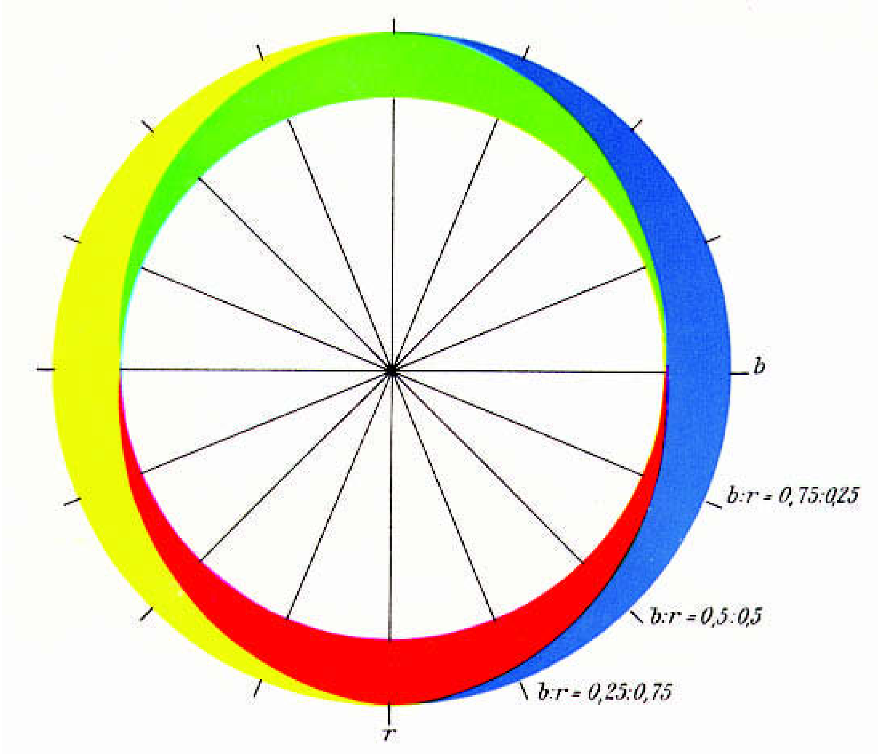


Fig. 2-32 *Hering's diagram illustrating the composition of mixed hue perceptions from the unique hues located on the main axes. The fractions of blue and red of three mixed hues are shown bottom right, 1905–1911.*

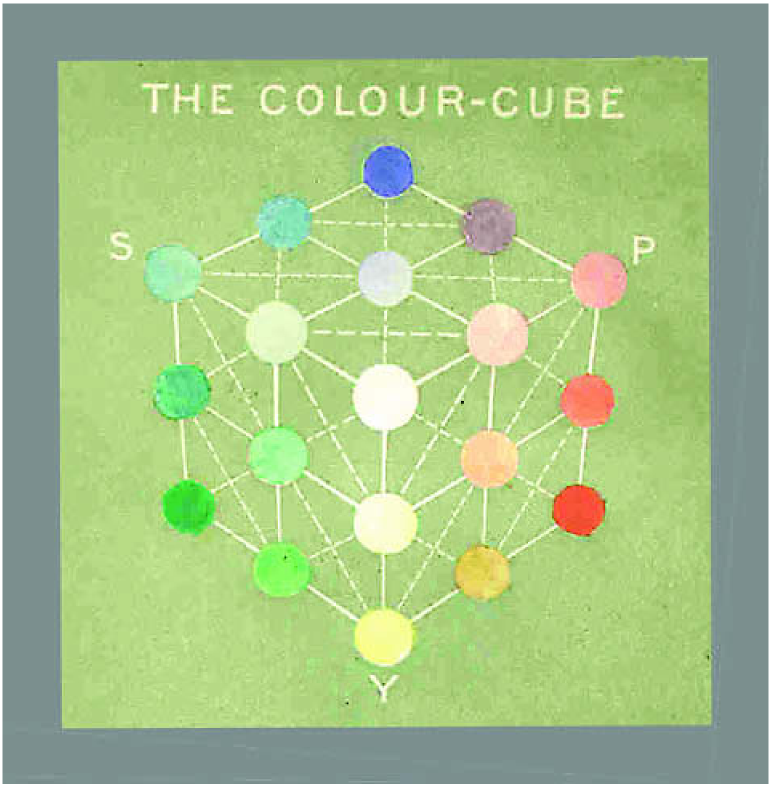
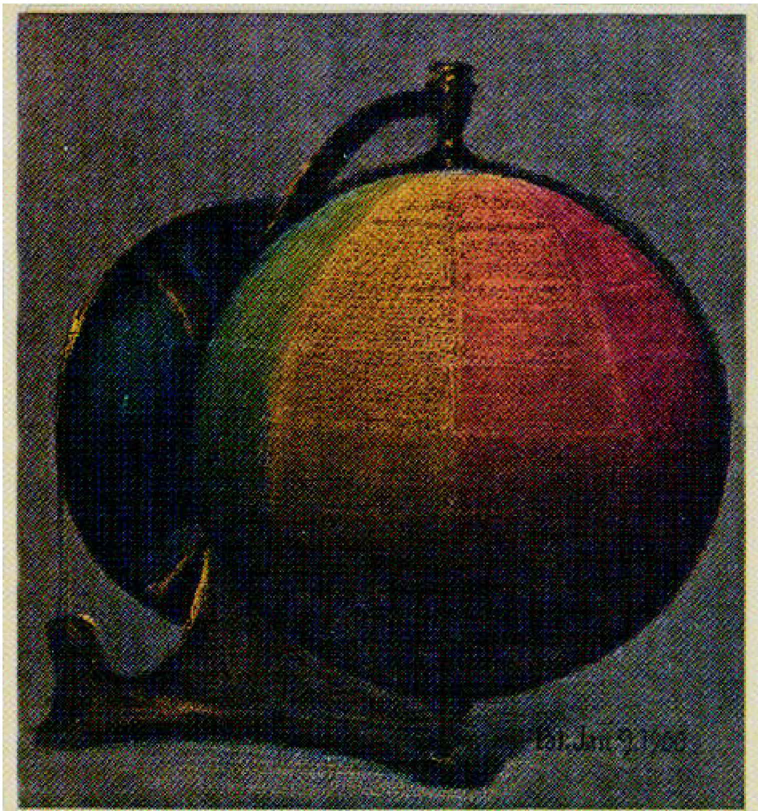


Fig. 2-34 *View toward the top, white corner of Benson's tilted color cube, 1868. The gray scale is hidden behind the white sphere.*



A BALANCED COLOR SPHERE

PASTEL SKETCH

Fig. 2-41 *Artist's rendition of Munsell's balanced color sphere, patented in 1900. The sphere was rotatable to achieve additive color mixture to gray and thereby show the "balance" of the colors on the sphere. The mirror in the back discloses the blue region of the sphere.*

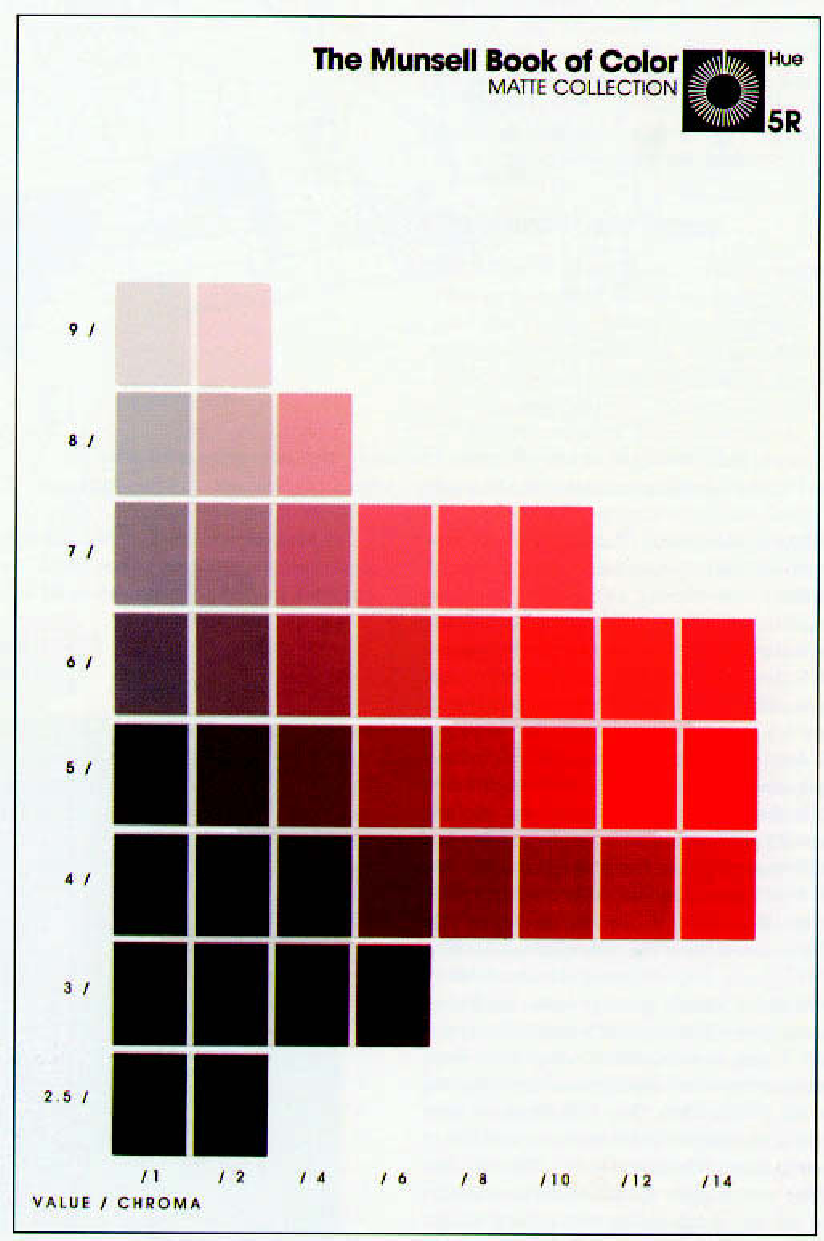


Fig. 2-42 *Constant hue page from a modern version of the Munsell Book of Color. The gray scale is not shown. The chroma scale begins at 1 and continues from 2 at two-grade intervals to chroma 14. Value grades are shown from 2.5 to 9. Courtesy Gretag-Macbeth Company.*

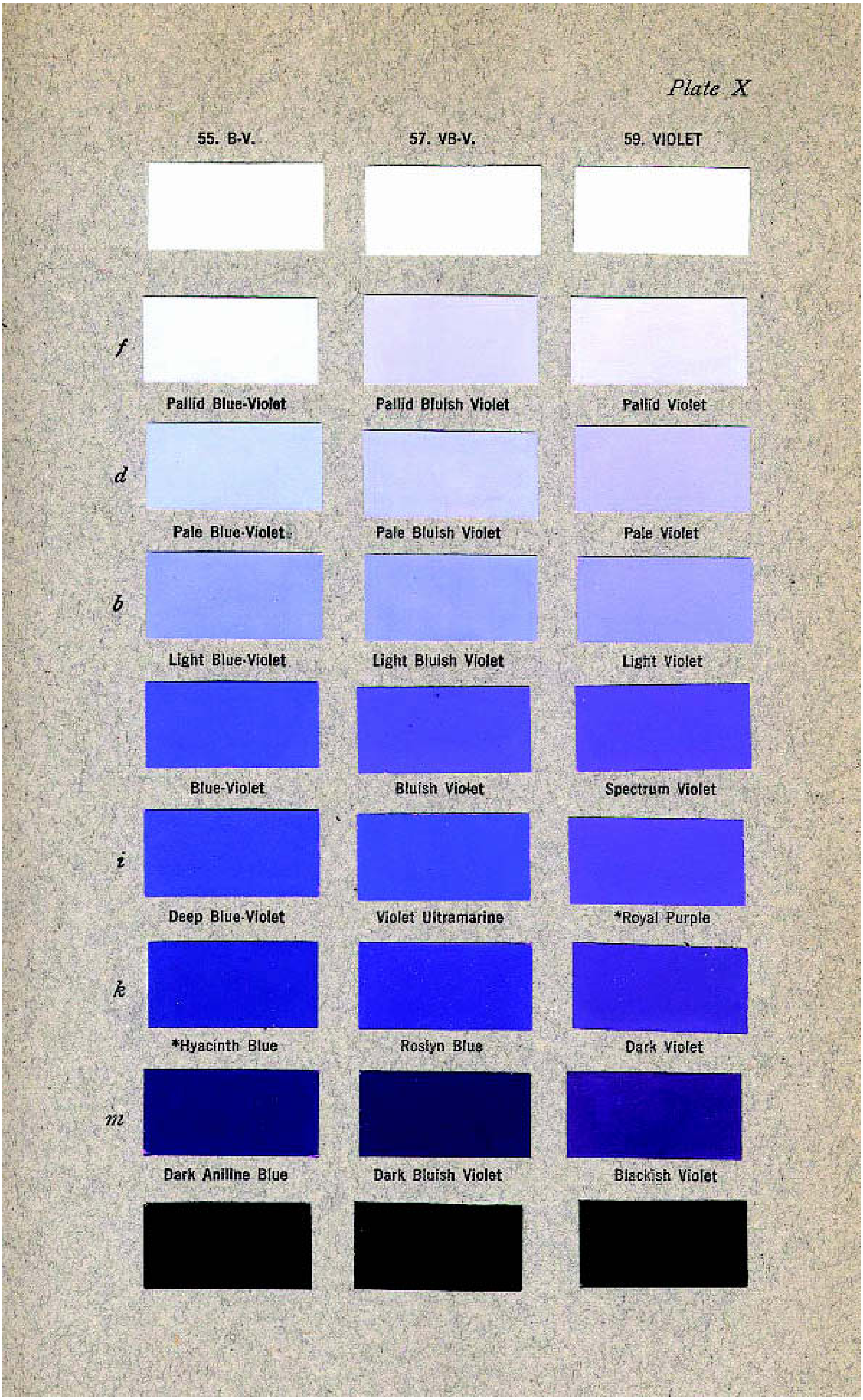


Fig. 2-43 *Page from Ridgway's color atlas showing three reddish blue hues lightened from the central color in three steps toward white and darkened in four steps toward black, 1912.*

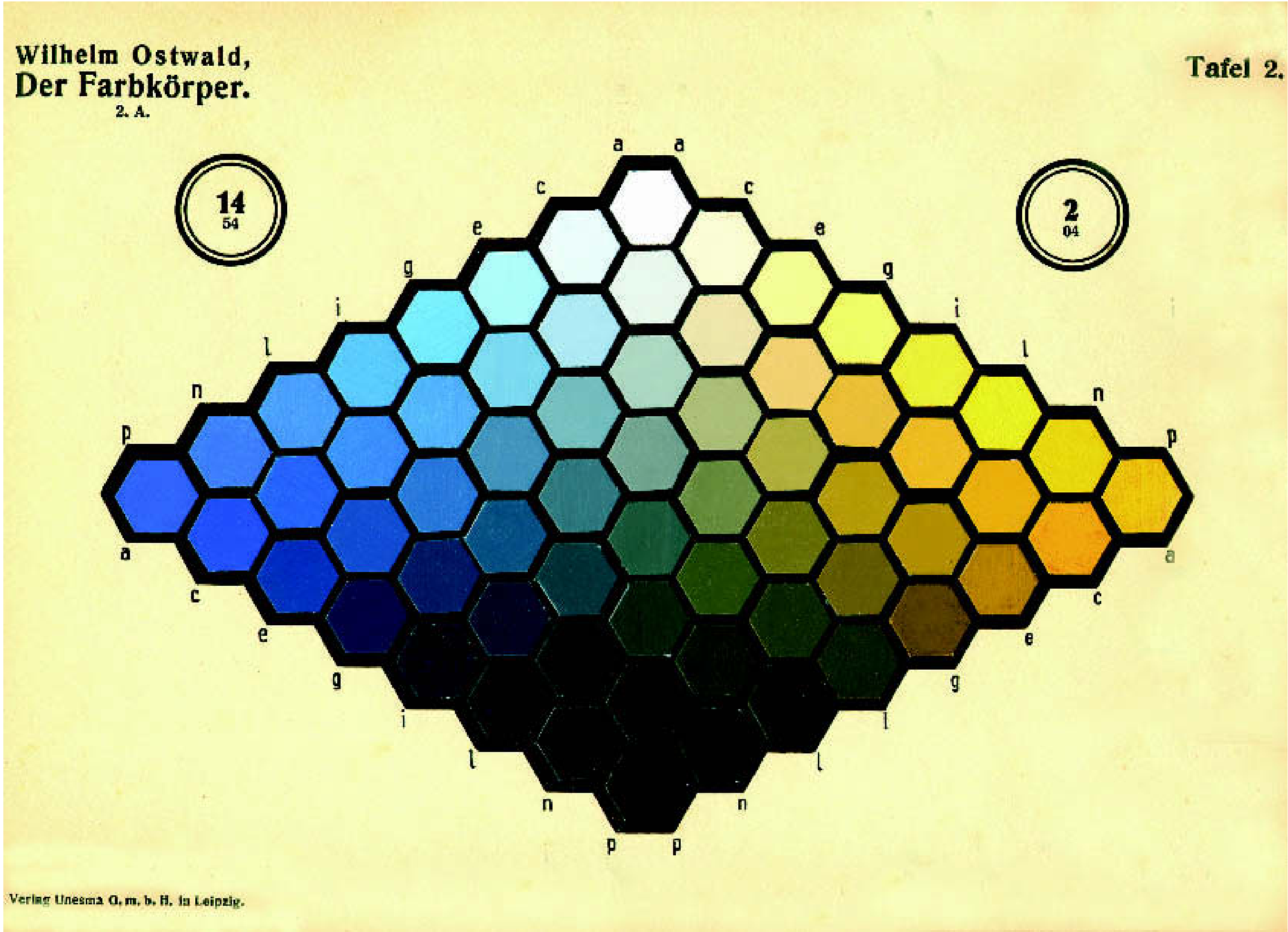


Fig. 2-45 *Vertical cross section through Ostwald's double-cone color solid illustrating constant hue colors 1 and 13, with veiling toward white and black. The achromatic scale is at the center. From Farbkörper, undated.*

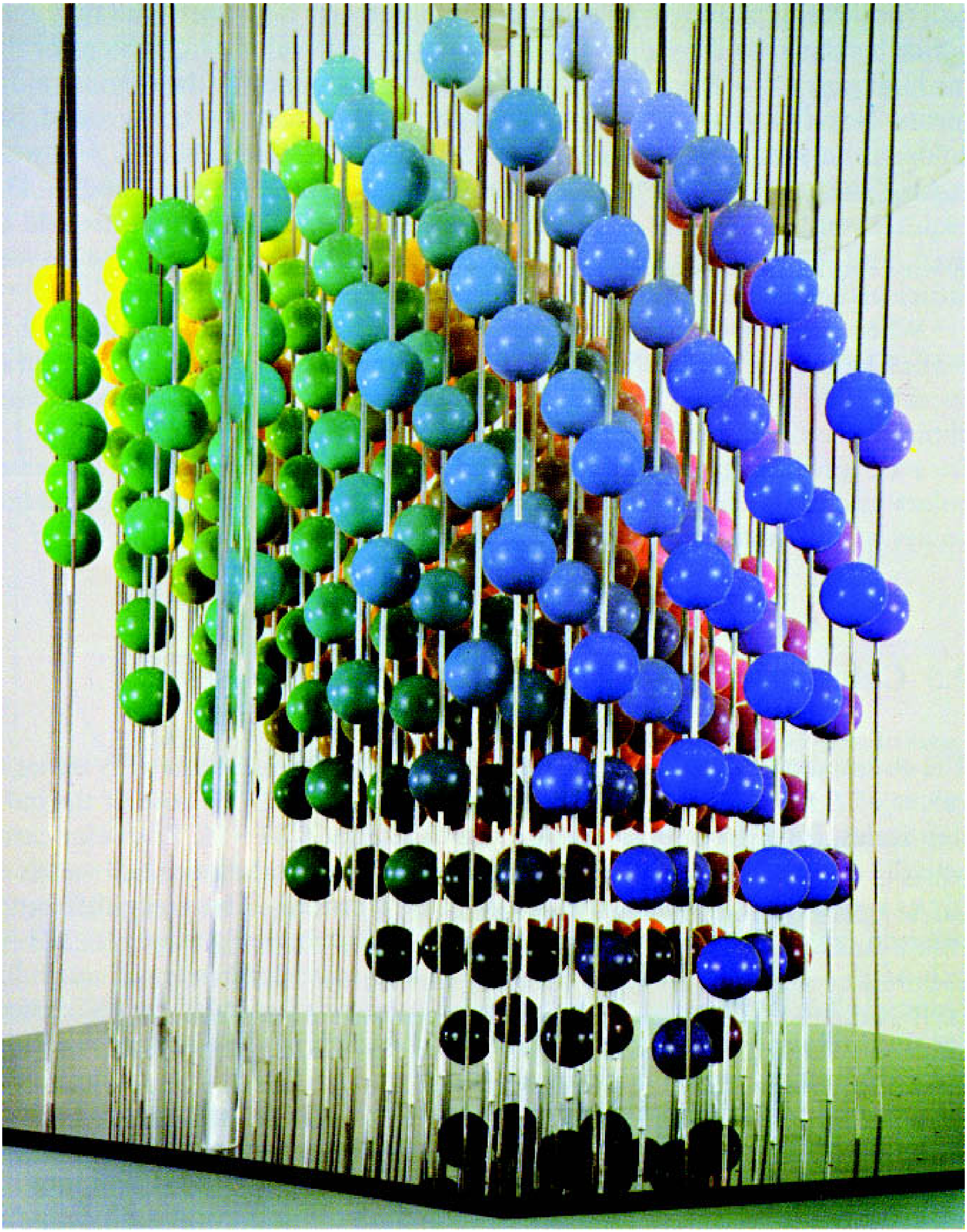


Fig. 2-51 *View of MacAdam's model of the OSA-UCS color solid illustrating the existence of several cleavage planes. See text for more detail. Slide courtesy D. L. MacAdam.*

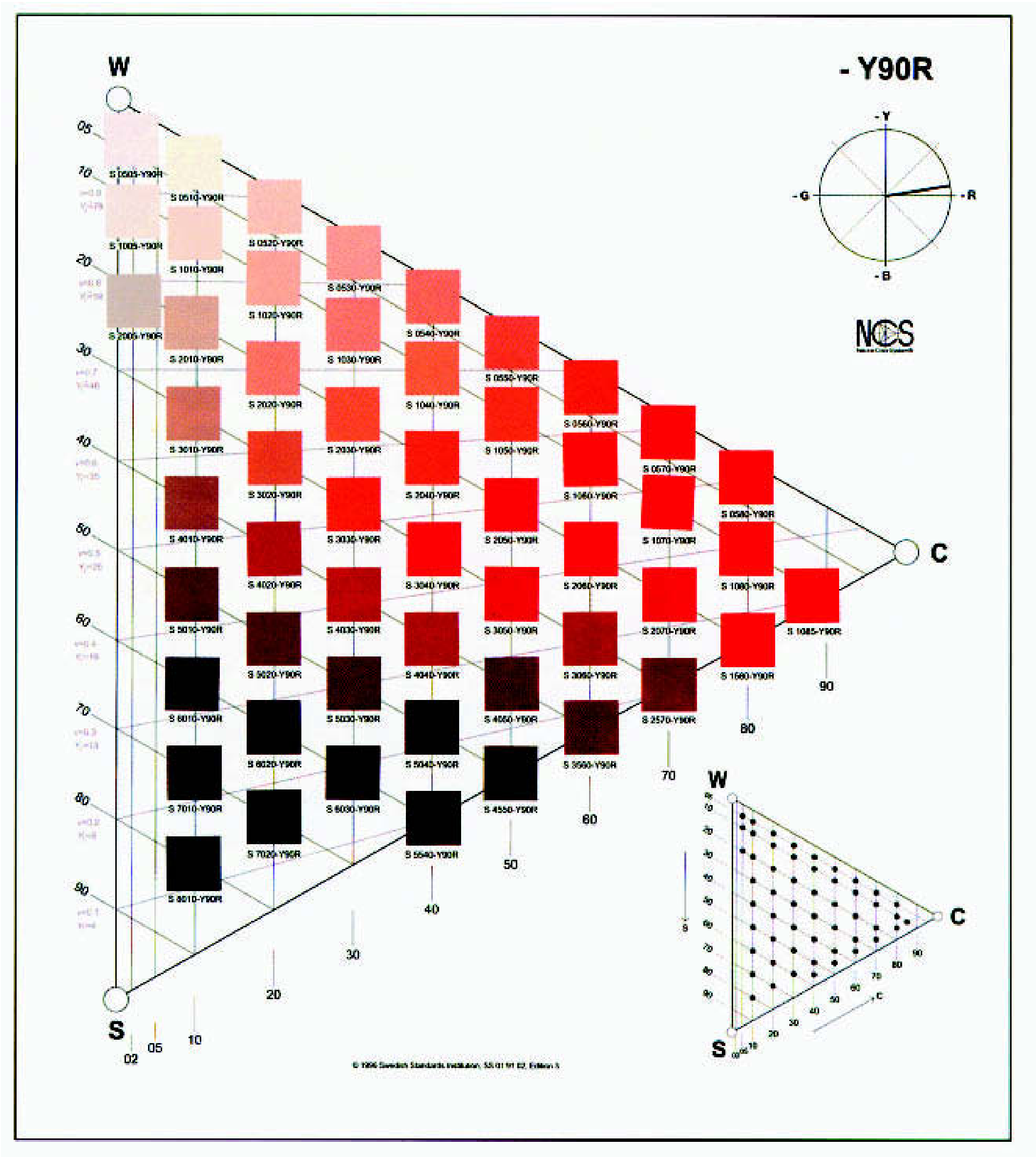


__Fig. 2-53__ NCS constant hue triangle of hue Y90R with full color C, white W, and black S. Colors of constant blackness s are located on lines parallel to W–C, colors of constant chromaticness c fall on lines parallel to W–S. Courtesy NCS.

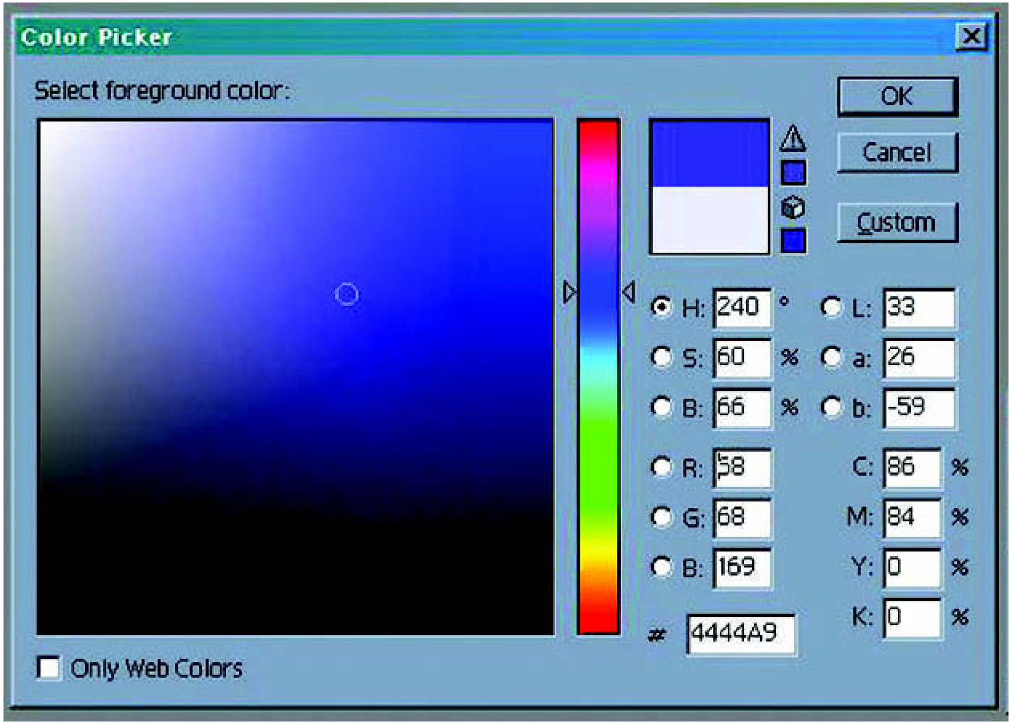


Fig. 2-54 *Color Picker screen from Adobe® Photoshop showing specification of a given reddish blue color in four systems: HSB, Lab (=L*, a*, b*), RGB, and CMYK.*

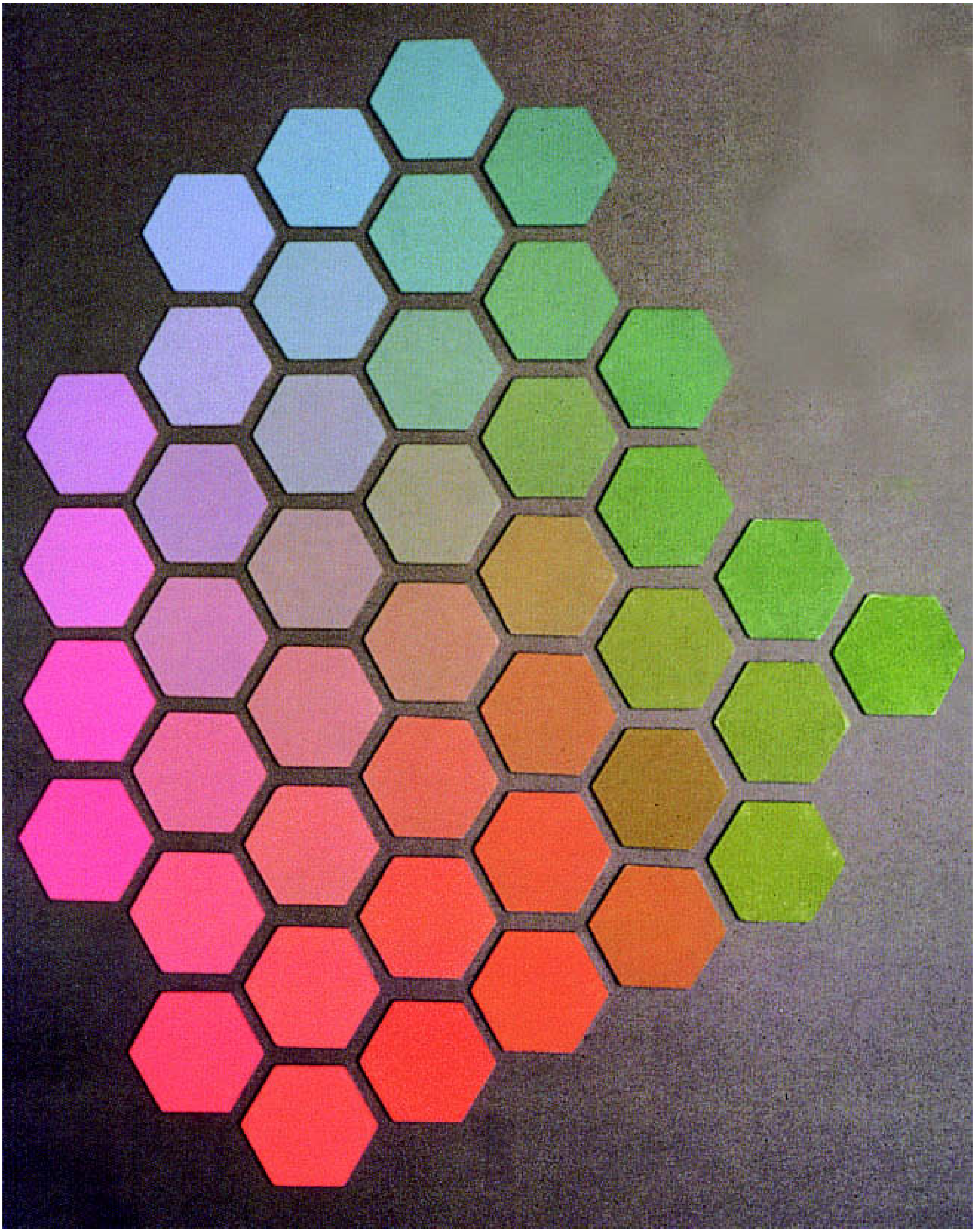


Fig. 7-16 *Image of the 43 hexagonal enamel color plates used by the OSA-UCS committee to establish the fundamental perceptual data for the system. Note that there is no achromatic color among them.*

Notes

Chapter 1

1. For empirical rules, see Purves and Lotto (2002). For an elementaristic approach, see Mausfeld (1998). For relativized conditions, see McLaughlin (2002). On the change in appearance of objects, consider the works of Impressionist painters such as those of Monet who distinctly varied the coloration of his Haystack series and views of the Cathedral of Rouen depending on time of day and/or weather.
2. On the ordinal order of personal spaces, it is well known that only a small percentage of color normal observers perform error-free in the Farnsworth-Munsell 100-hue test where colored caps with small differences in hue must be sorted in correct sequence.
3. For unique hue variation, see Kuehni (2001a). In the *Munsell Book of Colors* the complete hue circle has been divided into 40 perceptually equally different hue steps.
4. On the four cone types in women, tetrachromacy is well established in the animal kingdom. The possibility of tetrachromacy in human females was raised by Jordan and Mollon (1993). Jameson et al. (2001) have investigated the color experiences of females with the genetic potential for having four cone types. The estimate is from Neitz et al. (1998).
5. On adaptation, see, for example, Fairchild (1998). Current usage of the term in the vision science community is limited to processes beginning in the retinal layer. On retinal illumination, actual levels of illumination at the retina are extremely difficult to determine. Instead values based on luminance arriving at the eye, assumed pupil size as well as other assumptions are calculated. The unit of retinal illuminance is the troland.

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6. Lux is the photometric unit of illuminance, 1 lux = 1 lumen per m^2; lumen is the unit of luminous flux. A lumen is equal to the flux emitted in a unit-solid angle from a uniform point source of one (standard) candle.
7. CIE is the French acronym for International Commission on Illumination, an international body concerned with technical aspects of lighting and color. For more details on the Munsell color system, see Chapters 2 and 7. The Munsell system is perhaps the best-known color appearance system.
8. The Nickerson-Newhall psychological color solid models are located at the Hagley Museum and Library in Wilmington, DE. The models have been manufactured by Nickerson's assistant K. F. Stultz.
9. CIELAB is a color space and difference formula recommended by the CIE; see Chapter 6.
10. On Schönfelder's law, see Schönfelder 1933.
11. The broad use of the word *sensation* in the historical psychophysical literature makes it impossible to use uniform terminology for these two terms.

Chapter 2

1. A review of color appearance spaces was provided by G. Wyszecki in 1960 and G. Derefeldt in 1991. Information is also provided on the Web site www.colorsystem.com.
2. Xenophanes as quoted in Freeman (1957).
3. Pythagoras as quoted in Mansfeld (1986), translation by the author.
4. Quote from Empedocles in Mansfeld (1986), translation by the author.
5. Quote from Democritus in Mansfeld (1986), translation by the author.
6. Photius as quoted in Gage (1993).
7. Pliny as quoted by Heinrich Meyer in Goethe, *Geschichte der Farbenlehre*, 1810, translation by the author.
8. Ancient Greek sacral colors, *Oxford Classical Dictionary*, 3d ed. New York: Oxford University Press 1996.
9. On Avicenna scales, see Gage 1993.
10. On Eraclius, see Merrifield 1967.
11. On Urso de Salerno, see Gage (1993).
12. Albertus Magnus quoted in Gage (1993).
13. For an analysis of Bacon's work on color, see Parkhurst (1990).
14. For Cennini quote, see Cennini (1933).
15. Leonardo, Codex Urbinas latinus 1270, see McMahon (1966).
16. For Ficino list, see Barasch (1978).
17. For Telesio list, see Goethe (1810), translation by the author.
18. For Cardanus list, see Barasch (1978).
19. Forsius translation and figures from Feller and Stenius (1970) and Parkhurst and Feller (1982).
20. Boethius and figure, see Murdoch (1984).

21. On Oresme, see Murdoch (1984).
22. On Fludd, see Parkhurst and Feller (1982).
23. Newton was not explicit about how he arrived at seven primary colors. It is possible that he saw seven major hues in the spectrum. It is somewhat surprising that turquoise or sea blue is not one of his primary colors, given its prominence in the spectrum. On the other hand, seven was a classical number, and there is the connection to the musical scale that Newton referred to. Perhaps he was consciously or unconsciously influenced by such associations. However, in a recent paper Jameson et al. (2001) show that 52 trichromats, when viewing the spectrum with both eyes, delineate it on average into 7.4 hue ranges.
24. Descartes's figure in Gage (1993).
25. Diderot quoted from Diderot (1798).
26. On C.B., see Parkhurst and Feller (1982) and Gage (1993).
27. On Castel, see Gage (1993) and Schwarz (1999).
28. On color of urine diagrams, see Gage (1993).
29. For a German translation of Mayer's paper, see Lang (1980). For an English translation, see Fiorentini and Lee (2000).
30. The name *gamboge* is from the word Cambodia, it is a yellow-colored gum resin from trees of the genus *Garcinia*, growing in southeast Asia.
31. For more on Frisch, see Schwarz (1999).
32. The English glassmaker George Palmer (1740–1795) had an interest in color and was active at times in France. In 1777 he published in English a pamphlet titled *Theory of Colours and Vision*. It was translated in the same year into French. Walls (1956) considers Palmer's theory "just as complete as Young's, and nowise inferior to it. . . ." Palmer's theory was reviewed in *Lichtenberg's Magazin* in Göttingen, Germany, in 1781. Thomas Young was a medical student in Göttingen from 1795–96 and knew Lichtenberg who had considerable interest in vision. It is not known if Young learned of Palmer via Lichtenberg. In 1786 Palmer also published (in French) an account of color vision deficiencies.
33. Runge translations by the author.
34. On Matthias Klotz, see Schwarz (1999).
35. On early development of psychophysics, see Boring (1929).
36. Chevreul translation from the English edition of 1854.
37. On Doppler, see Schwarz (1992).
38. Aubert quote from Aubert (1876).
39. See Donders (1881). See also Turner 1994.
40. On further development of psychophysics, see Boring (1929).
41. On Munsell system development, see Munsell (1918) and Kuehni (2002a).

Chapter 3

1. On linking propositions, see, for example, Teller and Pugh (1983).
2. For the nine-dimensional universe, see, for example, Greene (1999).

Chapter 4

1. Runge translations by the author.
2. Grassmann translation by the author.
3. Helmholtz (1909, Vol II, p. 130).
4. Hering definition of constant veiling from Hering (1964, pp. 51–52).
5. *Chromo-luminarisme*, a term invented by the French painter George Seurat (1859–1891) to designate his early style of neoimpressionist painting. The term *chromolithographe* was first mentioned in French literature in 1837.
6. On Ostwald's view of Helmholtz's brightness definition, see Schwarz (1995).
7. Godlove formula from Judd (1969).
8. Pieter van Musschenbroek (1692–1761) was the inventor of the Leyden flask, a form of electrical capacitor. The law of disk mixture was developed by Plateau in 1853 and the technique perfected by Maxwell (Boring, 1942).

Chapter 5

1. For a short history of photometry, see Walsh (1958).
2. On the Hefner lamp, see Walsh (1958).
3. Lambert comment in Lambert (1760).
4. On Treviranus and Boll, see Polyak (1957).
5. For a lively description on the CIE standard observer development, see Wright (1996).
6. The density and distribution of cone types varies throughout the retina. The macular spot has an irregular distribution and is absent in the central area of focus of the normal eyes optics. To account for the average observer, for these differences two different standard observers have been specified by the CIE, one applying to a visual field subtending 2° and the other 10°.
7. For a trenchant critique of the CIE colorimetric system, see Cohen 2001. For a jab see the comment by the eminent visual physiologist W. A. H. Rushton: "The CIE triangle is brilliantly ingenious as an aid to the calculation of chromaticities which can be upheld in a court of law where colour specification is in dispute. But the triangle is monstrous as an indication of what is going on in the mechanism of vision. It displays all colours as a mixture of three primary lights, none of which have an existence that can be easily imagined. One of the three primaries is bright: it is pure green from which is subtracted a lot of red, which it does not contain. The other primaries are quite dark; they have strong colour but zero luminance. These do not seem to me ingredients that lead to clarity in our conception of colour mechanisms and I am astonished that some physiologists and many psychologists employ them to instruct the young and bewilder the old." (*Journal of Physiology* 1972; 220:178)
8. Many people have contributed to the elucidation of the genetic basis of color vision. Among the pioneers were J. Nathans, R. and S. Yokoyama, and others. For a succinct history, see Sharpe et al. (1999).
9. On Granit and Svaetichin, see Polyak (1957).

10. The notion of a central fovea free of *S* cones was first proposed by Artur König in 1894. Since then it was confirmed in some experiments but remains controversial.
11. The effect of field size on observed appearance in unpublished results by the author.

Chapter 6

1. For an extended discussion on the concept of line element, see Wyszecki and Stiles (1982, p. 654ff). See MacAdam (1981) on Schrödinger's and Stiles's line elements.
2. The terms NBS unit or judd have not gained widespread use.

Chapter 7

1. On Munsell system development, see Munsell (1918) and Kuehni (2002a).
2. The supplier of the Munsell *Book of Colors* is GretagMacbeth LLC, New Windsor, NY.
3. On the committee experiments, see Judd and Nickerson (1967) and Judd (1955, 1957, 1965, 1967).
4. This was a forced choice experiment in which the observer could only answer in one of two ways, "larger" or "smaller."
5. MacAdam revision of OSA-UCS, personal communication by J. T. Luke.
6. Hering translations by Hurvich and Jameson; see Hering (1905–1911).
7. On the "beauty test for acceptance," see Hård et al. (1996a).

Chapter 8

1. MacAdam's ellipse 1 applies to a highly saturated reddish blue. It is more highly saturated than any other color used in any color-matching error experiment. The implicit *S* cone absorption value is very high and not in agreement with that of all other ellipses. It has been left out of the analysis for that reason.

Glossary

Acceptability Judgment of the perceived size of a color difference against an internal standard of acceptability as a color match; used in color quality control.

Achromatic color A perceived color without hue: white, gray, black.

Adaptation, visual Modification of the visual response to stimuli due to the effects of the immediate surround and the total visual field of simultaneous or preceding stimuli. There is brightness as well as chromatic adaptation.

Additive Produced by addition; specifically that the physical sum of two visual stimuli is seen as the psychological sum in the sense of matching color perceptions.

Aim color A color specification that is the target to be achieved by a color chip, typically in a systematic collection.

Antagonistic Opposition in physiological action, specifically referring to neurons with opponent color character.

Attribute An inherent characteristic; there are two sets of widely accepted primary color attributes for object colors: (1) Hue, chroma, and lightness; (2) hue, whiteness, and blackness.

Attribute measurement Process of assigning numbers or other symbols to things in a manner that their relationship reflects the relationships of the attribute being measured.

Bezold-Brücke effect A sensory effect named after German scientists, according to which the hue sensation caused under normalized viewing conditions by light of all but three wavelengths changes with changing intensity.

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Blackness Degree of resemblance of a visual field to the fundamental color contrast perception of black. A fundamental color attribute in the Hering system.

Brightness Attribute of a visual perception according to which an area appears to emit, or reflect, more or less light. Differences in brightness range from bright to dim.

Chroma The attribute of a visual sensation permitting the judgment of the degree to which a chromatic, related color differs from the achromatic color of the same lightness.

Chromaticness Attribute of a visual sensation according to which the perceived color of an area appears to be more or less chromatic.

Chromatic plane A plane in which all color perceptions, systematically ordered, of colors seen as equally bright or light are located.

Chromaticity diagram A two-dimensional diagram in which colors can be plotted according to their chromaticity coordinates, resulting in different locations for colors of different hue and chromaticness.

CIE colorimetric system A color specification system developed by the International Commission on Illumination (the acronym is derived from the organization's French name Commission Internationale de l'Éclairage).

Cleavage plane Cleavage is the tendency of crystalline materials to break under strain along defined lines. A cleavage plane is a surface in a crystalline structure revealed after an actual or imagined break. In a color solid it contains colors that stand in simple mathematically definable relationship to each other.

Coefficient of variation A measure of the change in data; the standard deviation of the data expressed as a percentage of the data mean.

Colorant A material that changes the absorption characteristics of other materials: dyes or pigments, certain metal salts.

Color atlas A systematically arranged collection of colored chips that are symbols of the colors of a color solid. The chips only illustrate the intended space when viewed under prescribed conditions by an average color normal observer.

Color appearance Appearance is the sense impression or aspect of a thing; color appearance is the aspect of a colored field that distinguishes it from the comparable aspect of another field that has a different color appearance. Visual appearance includes visual aspects other than color, such as glossiness, transparency, and opacity.

Color appearance models Mathematical models attempting to describe the color appearance of objects as seen by the average observer under different illumination and in different surrounds.

Color circle A circular arrangement of hues in their spectral order, with nonspectral purple colors connecting the shortwave and the long wave ends of the spectrum; usually illustrated with high chroma pigment colorations.

Colorimeter Optical instrument for the investigation of color vision; in technology also an instrument that measures the reflectance of materials through three filters duplicating the color-matching functions of a standard observer.

Colorimetry The branch of color science concerned with the numerical specification of color stimuli.

Colorimetric purity A measure of saturation related to color stimuli and expressed in the CIE chromaticity diagram. Its relationship to perceived saturation in some standard conditions is complex.

Color difference The perceived difference between two non-identical fields of color.

Color difference formula A mathematical formula that allows the calculation from stimuli of the difference between two color fields in a given surround, as perceived by an average observer.

Color, full Translation of Hering's term *Vollfarbe*, the mental image of a color at its highest chromaticness; the color with a particular hue at the highest level of chroma on the MacAdam limit.

Color harmony The combination of color elements in objects of art or craft so that the effect is perceived as harmonious, in concord.

Color-matching error Stimulus variability in repeated matches of a standard color.

Color-matching functions Three spectral functions describing the amounts of three primary lights required to result in color perceptions matching those obtained from spectral lights.

Color metric A metric describes the mathematical structure of a geometrical space; a color metric applies to a color space, specifically a uniform color space.

Color order Systematic arrangement of color perceptions in terms of attributes and geometrical or mathematical models thereof.

Color, primary Colloquial term used in different circumstances: (1) One of three lights whose color appearance cannot be matched by the other two used with the other two to match the appearance of any other light; (2) one of three colorants used in color order systems or in color reproduction, such as yellow, red, and blue or yellow, magenta, and cyan; (3) one of the four Hering *Urfarben* or fundamental hue perceptions of yellow, red, blue, and green.

Color, related Color perception caused by light reflected from an object in the presence of other objects. The perceived color depends on the perceived color of surrounding objects.

Color, unrelated Color perceived to belong to an area seen in isolation from other areas.

Color solid Subset of color space containing, in a given experimental situation, all possible color experiences of the observer under consideration.

Color space Three-dimensional coordinate system within which color experiences can be represented as points with unique positions. The term color space should be limited to psychological spaces or psychophysical spaces based on cone sensitivity or color-matching functions.

Color stimulus A stimulus is something that excites an organism, or one of its components to functional activity. An external color stimulus normally consists of light of one or more wavelengths, viewed against a surround of different spectral composition.

Color zone theory Originally the merger of the Young-Helmholtz and the Hering theory of color vision; more generally any model of color vision consisting of two or more stages of processing.

Cones Cone-shaped light-sensitive cells in the retina. There are three types of cones differing in spectral sensitivity in the normal human retina.

Cone sensitivity functions Spectral functions that describe the response of the three cone types to light energy arriving at the surface of the retina.

Cone contrast diagram A diagram for illustrating the results of contrast experiments in terms of cone activation, such as $\Delta M/M$ versus $\Delta L/L$.

Contrast The difference between things having similar nature; specifically, the degree of difference between two adjacent fields of color. Perceptually contrast is expressed in terms of perceived difference, psychophysically in cone activation (in a cone contrast diagram) or in colorimetric terms.

Correlation coefficient A number indicating the degree of association between two sets of data.

Crispening Describes the fact that smallest increments in stimuli are necessary for a criterion perceptual difference response if the surround color is intermediate to the colors of the two fields compared, both in luminance and chromaticity.

Criterion response Perceptual response at the level of the selected criterion; the criterion is a standard on which the judgment is based.

Detection Discovery or determination of the existence or presence of something; specifically, for example, the determination of presence of redness in a perceived color.

DeVries-Rose behavior Increase of the Weber fraction with the square root of luminance instead of being constant.

Diapason The entire compass of musical notes.

Discrimination The process by which two stimuli differing in some aspect result in different responses of some sort.

Dominant wavelength (of a color stimulus) Wavelength of the monochromatic stimulus that, when additively mixed with the appropriate amount of achromatic stimulus, results in a color match with the test stimulus.

Empirical Originating in observation or experience.

Equal energy light source A theoretical light source that has a relative spectral power distribution of 1.0 across the spectrum.

Euclidean Relating to, or based on the geometry of the Greek mathematician Euclid; specifically that the three color attribute differences in a complex difference sum as the square root of the sum of the squares of the individual attribute differences.

Field of view The size of the retinal image expressed in solid angle. The CIE has specified a 2° and a 10° standard observer.

Flicker Variation in brightness or hue perceived upon stimulation by intermittent or temporally nonuniform light.

Fluorent Appearance of chromatic fields when their luminance or luminous reflectance is higher than that of the surround, but not as much as to make them appear luminous.

Fluorescence A form of luminescence, property of certain inorganic and organic molecules to reemit absorbed ultraviolet or visible light energy in the visible region of the spectrum.

Ganzfeld A situation in which the entire visual field is identical in composition. There are different degrees of *ganzfeld* mentioned in literature. In one situation, the observer has her head in a uniformly light-emitting sphere. It is still possible to see the nose and other facial features and contrast is thereby possible. In another case, the observer has, say, identical colored ping-pong balls with a section removed taped to her eyelids so that no facial or other contrasting feature can be seen. Here the *ganzfeld* is complete.

Geodesic The shortest line between two points on a given surface. The curvature of the line depends on the geometry of the space.

Gestalt psychology The study of perception and behavior based on the individual's response to configurational wholes, stressing the uniformity of the psychological events and rejecting analysis into discrete aspect.

Grade A position in a scale of ranks or qualities; specifically a fixed point in a color scale.

Gray scale A series of grades representing an achromatic color scale, usually with visually equidistant steps between neighboring grades.

Helmholtz-Kohlrausch effect Describes the fact of heterochromatic brightness matching that chromatic colors are perceived as brighter than achromatic colors of the same luminance. The effect is dependent on the dominant wavelength of the color.

Heterochromatic Of mixed chromatic appearance.

Hue Attribute of a visual perception according to which an area appears to be similar to one of the colors yellow, red, blue, or green or to a combination of adjacent pairs of these colors considered in a closed ring.

Hues, unique The four hues of the color circle that can not be matched with colors other than themselves; the psychological primary hues yellow, red, blue, and green. Unique red is a red hue that is neither yellowish nor bluish, for example.

Hue superimportance Refers to the fact that a smaller stimulus increment is required for a criterion difference response if it represents a hue difference than if it represents a chroma or saturation difference of the same perceived magnitude.

Illuminant An illuminating device; technically a set of numbers representing the spectral power distribution of a light source.

Isomorphism A one-to-one correspondence between mathematical sets; specifically, mapping of objects of color experience to objects in a geometrical space so that a one-to-one correspondence is obtained.

Just noticeable difference (JND) Threshold difference; the initial perceptual difference that can be seen when one of two originally identical fields of color changes in any given direction.

Lateral geniculate nucleus A mass of cells in the brain along the visual passageway between the retina and the visual area at the back of the brain.

Lattice A regular geometrical arrangement of points over an area or in a space; specifically related to the arrangement of colors in a color space.

Lightness Perceptual attribute of related colors according to which a color field appears to emit equal or less light compared to a white field. Lightness can be understood as relative brightness.

Line element The first fundamental form of a regular surface. It is defined by the Riemannian metric. In connection with color the term is used to describe a certain kind of color space defined by (weighted) increments of color fundamentals.

Linear model Of the first degree with respect to variables; having a graph that is a straight line.

Linear regression A functional relationship between two or more variables in which the variables are linearly related.

Linking proposition Postulated link between two sets of facts that are only indirectly related.

Luminance Luminous flux of a light beam emanating from a surface in a given direction, per unit solid angle.

Luminous reflectance Luminance of the surface of an object compared to the luminance of the surface of a perfectly reflecting diffuser, illuminated with the same light source and viewed at the same angle. Also known as luminance factor Y.

Macula "Yellow spot," an irregularly formed ring-like area of yellowish pigment in the central region of the retina. The fovea is located in the central area of the macular ring, free of macular pigment.

Magnitude, sensory A numerical or symbolic quantitative measure of the result of a sensory perception.

Magnocellular Relates to layers in the lateral geniculate nucleus in which relatively large cells are located believed to relay information necessary for motion perception.

Masking The reduction or suppression of one percept by the presence of another.

Maximal color See full color.

Metamers Two or more differing spectral power distributions resulting in identical color perceptions for an observer. Also used for objects with different reflectance functions seen as having identical color when viewed in standard conditions under a given light source.

Monochromatic Light of a single wavelength or a very narrow band of wavelengths seen as having identical color.

Monolexemic Describes a word consisting of a single meaningful linguistic unit.

Neuron Cell in the nervous system specialized in the transmission of electrical signals.

Neurophysiology Organic processes and phenomena of the nervous system.

Object color Apparent color of an object. The color of an object can vary depending on the surround and contextual conditions in which it is viewed.

Opponent color theory A theory according to which color perception is based on unique hues forming opposing pairs: red-green, yellow-blue, as well as the non-hued pair black and white.

Orthogonal To intersect or lie at right angles.

Parabolic Refers to a type of curved line resulting from slicing a cone at a certain range of angles.

Parvocellular Relates to layers in the lateral geniculate nucleus in which relatively small cells are located believed to relay information necessary for brightness and color perception.

Perception The subjective, conscious awareness of any aspect of the external or internal environment.

Photometry The measurement of light as related to the average human observer.

Power law A mathematical, exponential relationship between two variables; specifically between a physical stimulus and the perception resulting from it.

Psychometric function Plots the relative frequency of judgments “smaller than,” “equal,” and “larger than” relative to the magnitude of the stimulus.

Psychophysics The study of mental processes by quantitative methods; specifically the reports of human subjects of the perceptions resulting from carefully measured light stimuli.

Reflection The process by which a smooth surface returns electromagnetic radiation, specifically light. In reflection the radiation is returned by a simple optical law: the angle of reflection equals the angle of incidence.

Relativize To treat or describe as not absolute or independent.

Retina A layer coating the inside of the camera type eye, containing the light-sensitive rod and cone cells and cells connected to them. The retina is continuous with the optical nerve.

Riemannian geometry Non-euclidean geometry with positive curvature in which the parallel line postulate is replaced by the postulate that every pair of straight lines intersects.

Rods Rod-shaped light-sensitive cells in the retina, specialized to operate primarily at low light levels resulting in brightness perception only.

Root mean square error The square root of the arithmetic mean of the squares of the deviations of the various items from the arithmetic mean of the whole; also termed standard deviation.

Saturation Attribute of a visual perception which permits a judgment to be made of the degree to which a chromatic stimulus differs in appearance from that of an achromatic stimulus, regardless of their brightness.

Sensation Mental process due to bodily stimulation, now distinguished from awareness of the result of the process.

Spectral spaces Spaces created from reflectance or spectral power distribution data by dimension reduction techniques other than those involving color matching or cone sensitivity functions.

Spectrophotometer An instrument for measuring the relative intensities of light in different spectral regions.

Stimulus An agent that directly influences the activity of a living organism or one of its parts; specifically electromagnetic radiation within the visible band.

Suprathreshold Exceeding the threshold; specifically a difference that is larger than a threshold difference.

Symbolic A formal system of notation representing relationships.

Tetractys The pythagorean name for the sum of the first four integers regarded as the source of all things.

Threshold Visual, the lowest level or increment of stimulus resulting in a visual perception or a difference perception.

Trichromacy Relates to the theory that human color vision is based on the activity of three cone types.

Tristimulus values The scalar values of the amounts of three primary lights required to match a given light. The CIE tristimulus values X, Y, and Z refer to non-real lights $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$.

Tone A tint or shade of color, typically achieved by adding white and/or black pigments to highly chromatic pigments.

Value Munsell's term for the grades of a perceptually uniform gray scale.

Vision Process by which the extended visual system extracts information from light energy to help generate appropriate response behavior.

Weber fraction Proportionality constant between the stimulus increment and the absolute value of the stimulus.

Weighting To apply a statistically or otherwise determined weighting factor to a variable.

Whiteness Attribute of a diffusing surface permitting, when viewed under a standard light source, the judgment of similarity to a standard white surface viewed in the same light.

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