Mathematical modeling and Simulation of Brushless DC motor with Ideal Back EMF for a Precision speed control

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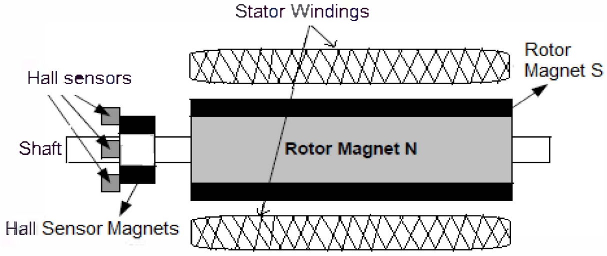
***Abstract*—This paper presents a mathematical model of a three-phase Brushless DC motor based on precise speed control methodology with ideal Back EMF on MATLAB/Simulink platform. In this scheme, equivalent output of three hall sensors determine the rotor position at any instant of time and accordingly switch the six step inverter to drive the motor. The model is based on phase voltage and electromagnetic torque equation. Further analysis has been carried out with transfer function to achieve desired level of performance. Moreover, the ideal Back EMF is also derived from mechanical speed and mechanical angle of the rotor. A wide variation of speed control is accomplished with a PI controller due to its simple control structure and ease of implementation. The final analysis on the speed-torque characteristics for the designed Brushless DC motor show quite satisfactory output for varying (0-20 N-m) load torque applications.**

Keywords—Brushless DC motor, Back EMF, Hall sensors, PI controller, electromagnetic torque.

I. INTRODUCTION

Electrical motors are consuming almost half of all the electrical energy used in the world. Most of the times motors found in all appliances are either single phase induction motors or brushed dc motors, which have low efficiency and high maintenance. Single phase induction motors are less efficient because of the ohmic loss in the rotor. But this type of loss can be reduced in Brushless DC (BLDC) motor due to the absence of brush and mechanical commutation [1]. BLDC motors are one type of synchronous motor, because the rotor magnetic field and stator magnetic field rotate with same frequency. BLDC motors are available in single phase, two phase and three phase configurations. But the three phase configuration is commonly used [2]. BLDC motor is constructed with a permanent magnet rotor and wire wound stator poles. Due to the absence of brush and commutator this motor requires inverter and rotor position sensor. Hall effect sensors are commonly used for sensing rotor position. The inverter uses transistors for low power drives and thyristors for high power drives. The Hall Effect sensor is mounted on the motor shaft [3]. Most of the BLDC motors have three hall sensors [4]. When the rotor magnetic poles pass near the hall sensor, they give a high or low signal, indicating the passing of the north or South Pole near the sensors. Based on the combination of the three hall sensor signals, the exact sequence of commutation can be determined and this signal is sent to the drive circuitry of the inverter circuit [13]. Table I shows the Hall Effect signal values. In response to these signals, the inverter allows the flow of current to stator phase windings in a controlled sequence so that motor produces the desired torque and speed. The construction of a conventional BLDC motor is shown in Fig.1.



BLDC motors are used in industries such as automotive, aerospace, medical, robotics, industrial automation equipment etc [5]. This motor has many advantages over brushed DC motor and induction motor. These are (i) the brushless machines require less maintenance, (ii) Electric noise generation is low, (iii) Brushless motors have low inertia which improves dynamic response, (iv) Better speed versus torque characteristics, (v) Long operating life, (vi) High efficiency [1] follow.

II. MATHEMATICAL MODEL OF BLDC MOTOR

The equivalent circuit of BLDC motor shown in Fig.2 in this motor there is a permanent magnet mounted on the rotor and has three stator phase windings in star connection. The motor is fed by three phase voltage source. Not only sinusoidal but also square wave or even any other wave shaped source voltage can be applied. The modeling of the BLDC motor is based on some assumptions: (i) The motor is not saturated and should be operated with the rated current. (ii) The resistances of the three stator phase windings are equal.

TABLE I. HALL EFFECT SIGNALS

Switching Interval Degree	H1	H2	H3
0-60	1	0	1
60-120	0	0	1
120-180	0	1	1
180-240	0	1	0
240-300	1	1	0
300-360	1	0	0

(iii) Self inductance and mutual inductance are constant. (iv) Iron and stray losses are negligible. (v) Three phases are balanced one. (vi) Uniform air gap. (vii) Hysteresis and eddy current losses are not considered. (viii) Semiconductor switches are ideal.

The model of the armature winding for the BLDC motor is expressed as follows:

$$V_a = Ri_a + L\frac{di_a}{dt} + e_a \tag{1}$$

$$V_b = Ri_b + L\frac{di_b}{dt} + e_b \tag{2}$$

$$V_c = Ri_c + L\frac{di_c}{dt} + e_c \tag{3}$$

where,

$R_a = R_b = R_c = R$, Stator resistance per phase. [Ohm]
$L_a = L_b = L_c = L$, stator inductance per phase. [Henry]
V_a, V_b and V_c are the stator phase voltages.[volt]
i_a, i_b and i_c are stator phase currents.[Ampere]
e_a, e_b and e_c are motor Back EMFs and the expression of e_a, e_b and e_c will be given in equations 5, 6 and 7.

The modeling equation of BLDC motor can be represented in matrix form:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} R+pL & 0 & 0 \\ 0 & R+pL & 0 \\ 0 & 0 & R+pL \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{4}$$

where, p in the matrix presents $\frac{d}{dt}$.

When BLDC motor rotates, each winding generates a voltage known as Back EMF, which opposes the main voltage supplied to the winding according to Lenz's law. The polarity of the Back EMF is opposite direction of the source voltage. It is

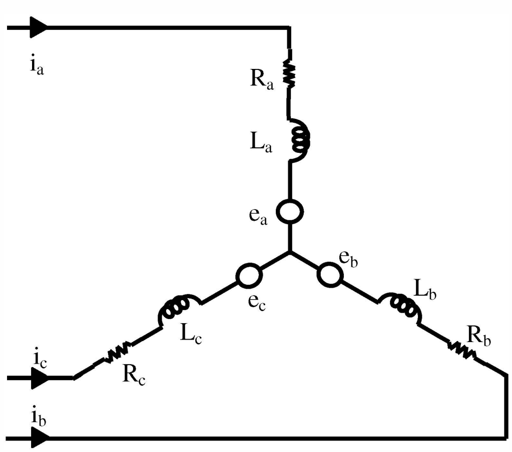


Fig.2. The equivalent circuit of BLDC Motor.

related to the function of rotor position and each phase has 120° phase difference. Back EMF depends mainly on three factors (i) angular velocity of the rotor, (ii) magnetic field generated by the rotor magnets, (iii) the number of turns in the stator windings. Fig. 3 shows the ideal Back EMF and phase current of this motor.

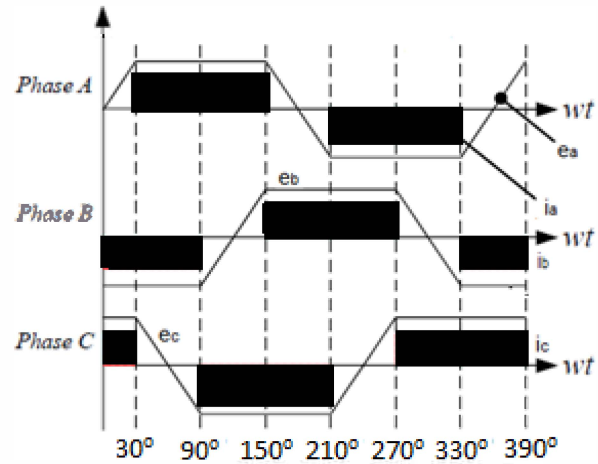


Fig.3. Trapezoidal Back EMF and phase current variation with rotor electrical angle.

Equation of each phase as follows:

$$e_a = k.\omega.f(\theta) \tag{5}$$

$$e_b = k.\omega.f(\theta - \tfrac{2\pi}{3}) \tag{6}$$

$$e_c = k.\omega.f(\theta + \tfrac{2\pi}{3}) \tag{7}$$

where,
k is the Back EMF constant [v/rad/s].
θ is the electrical rotor angle.
ω is the mechanical speed of the rotor [rad/s].

First we consider equation no.(1):

$$V_a = Ri_a + L\frac{di_a}{dt} + e_a$$

Considering Laplace transform of the above equation we find the transfer function

$$V_a(s) - e_a(s) = RI_a(s) + LsI_a(s).$$

So, $$\frac{I_a(s)}{V_a(s) - e_a(s)} = \frac{1}{R + Ls} \tag{8}$$

The Laplace transform of equation (2) and (3) are calculated and it is shown in the form of equation (9) and (10):

$$\frac{I_b(s)}{V_b(s) - e_b(s)} = \frac{1}{R + Ls} \tag{9}$$

$$\frac{I_c(s)}{V_c(s) - e_c(s)} = \frac{1}{R + Ls} \tag{10}$$

Here we have consider four pole configuration.The electrical rotor angle is equal to the mechanical rotor angle multplied by the number of pole pairs:

$\theta = \frac{p}{2}\theta_m$, where θ_m is the mechanical rotor angle.

Therefore, $\frac{d\theta}{dt} = \frac{p}{2}.\frac{d\theta_m}{dt} = \frac{p}{2}.\omega$ (11)

Total torque can be represented the simulation of each phase. So the total torque equation can be defined as:

$$T_e = \frac{i_a e_a + i_b e_b + i_c e_c}{\omega} =$$

$$i_a.K.f(\theta) + i_b.K.f\left(\theta - \frac{2\pi}{3}\right) + i_c.K.f\left(\theta + \frac{2\pi}{3}\right) \quad (12)$$

Moreover the generation of electromagnetic torque can be written as:

$$T_e = J\frac{d\omega}{dt} + T_L + B\omega \quad (13)$$

Fig.5. Three Phase MOSFET - Based Inverter.

where, J is the rotor inertia, [kgm^2], and B is damping constant and T_L is load torque in N-m.

The resultant torque is:

$$T_e - T_L = J\frac{d\omega}{dt} + B\omega$$

Again using Laplace transformation and we will get transfer function:

$$\frac{\omega(s)}{T_e(s) - T_L(s)} = \frac{1}{Js + \theta} \quad (14)$$

III. Schematic Block Diagram of Motor

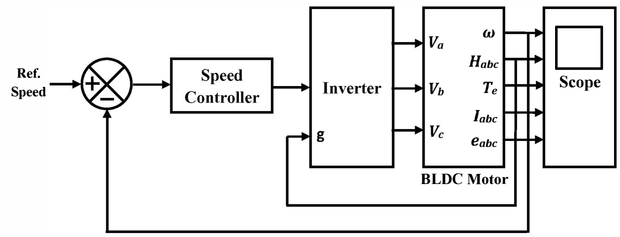


Fig.4. Schematic Block Diagram of BLDC Motor

TABLE II. Switching States

Phase Voltage	Switch Status		Output Voltage
	ON	OFF	
V_a	S_1	S_4	$+V_d/2$
	S_4	S_1	$-V_d/2$
V_b	S_3	S_6	$+V_d/2$
	S_6	S_3	$-V_d/2$
V_c	S_5	S_2	$+V_d/2$
	S_2	S_5	$-V_d/2$

A. Inverter

The three phase inverter has six power switches, two on each phase leg which are connected to a DC voltage source, supplying the input voltage to the three phase BLDC motor. The gating signals given to the inverter circuit are sequenced at 60º intervals of the motor cycle as explained by Table II where each MOSFET conducts for a duration of 120º. The inverter is developed using the following logical equations (15), (16), (17) where S_1 through S_6 represent the logical state of the switches, i.e., logic 1 for ON and logic 0 for OFF.

$$V_a = S_1.\frac{V_d}{2} - S_4.\frac{V_d}{2} \quad (15)$$

$$V_b = S_3.\frac{V_d}{2} - S_6.\frac{V_d}{2} \quad (16)$$

$$V_c = S_5.\frac{V_d}{2} - S_2.\frac{V_d}{2} \quad (17)$$

where, V_d is the DC-link voltage.

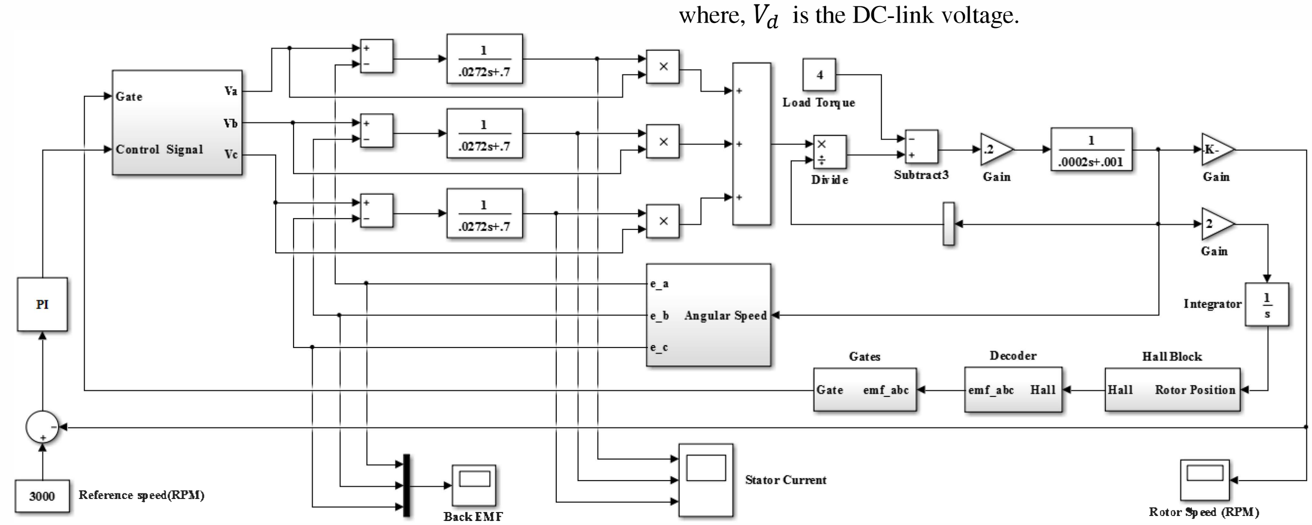


Fig.6. Simulink model of BLDC Motor.

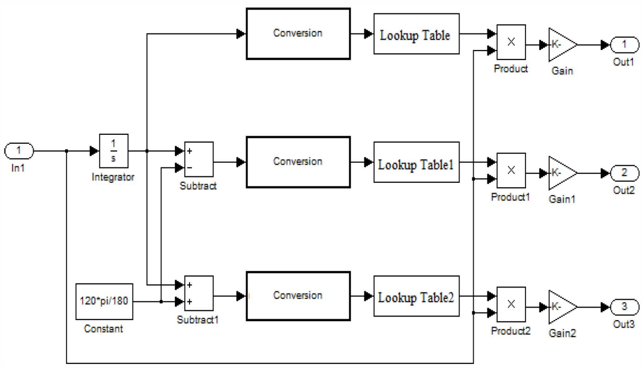


Fig.7. Simulink model of Back EMF generation subsystem.

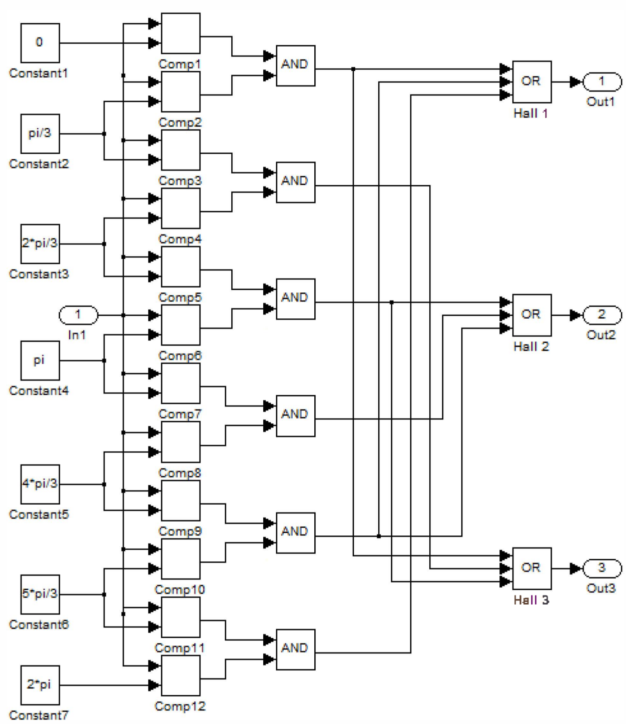


Fig.8. Simulink model of Hall circuit.

IV. Simulation Circuits

The complete Simulink model of the BLDC motor is shown in Fig.6 and Fig.7. In order to implement the sensored drive of BLDC, we have designed a logical subsystem to generate the equivalent outputs of the hall sensor. The logical subsystem is shown in Fig. 8 based on Table II.

V. Simulation Results

The simulation results includes variation of different parameters of BLDC motor like three phase Back EMFs, rotor speed, and electromagnetic torque, with respect to time as shown in Fig. 8, Fig. 9 and Fig.10 respectively. The rotor position varies from 0 to 6.28 radians corresponding to 0° to 360°. The motor ratings and parameters are shown in Table III. Fig. 11 indicates the change in rotational speed with application of varying Load Torque.

TABLE III. RATINGS AND PARAMETERS OF BLDC MOTOR

Motor parameters	Values
No. of poles, P	4
Moment of inertia, J	0.0002 Kg-m^2
Stator Resistance, R	0.7 Ω
Stator Inductance, L	0.0272 H
Rated Speed , N_r	3000 rpm
Damping Constant, B	0.2
Back EMF Constant, K	0.513 volt/rad/sec
Load Torque, T_L	4 N-m

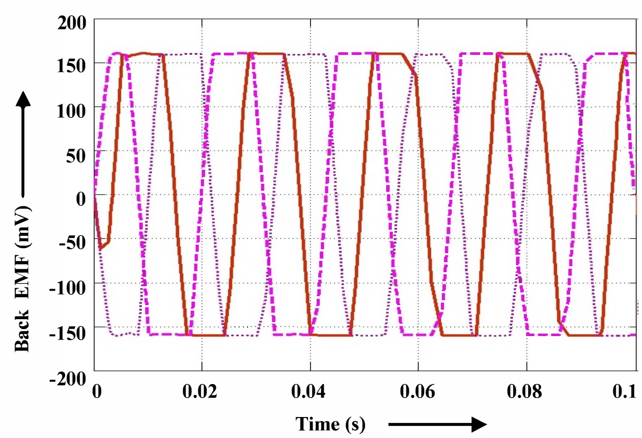


Fig.8. Waveform of Back EMF generated for Phase A, Phase B and Phase C.

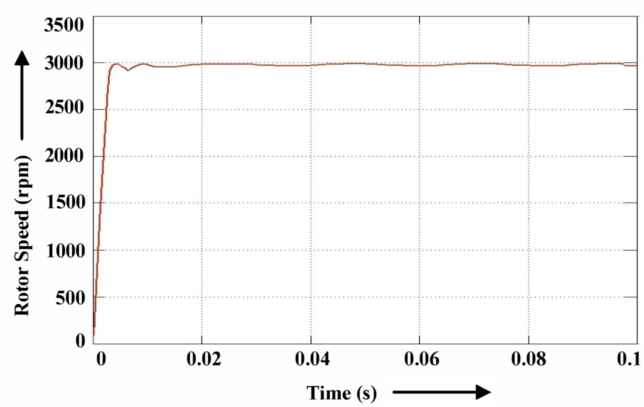


Fig. 9. Rotor speed.

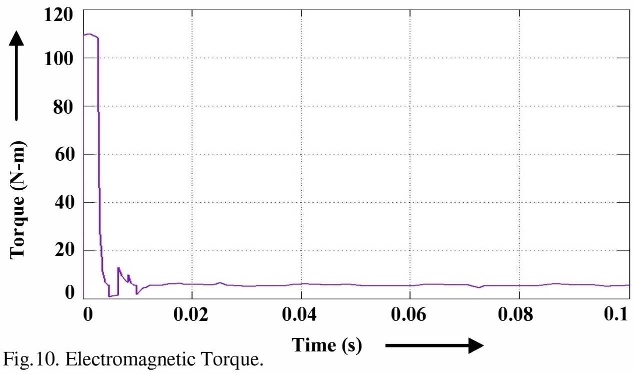


Fig.10. Electromagnetic Torque.

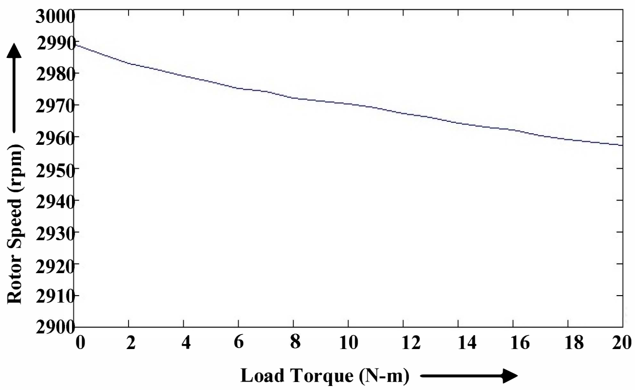


Fig.11. Change in Rotor Speed for variation of Load Torque.

VI. CONCLUSION

This paper deals with the mathematical modeling of three-phase star connected Brushless DC motor along with its precise speed control using PI controller. The main attribute of this model is to achieve the ideal characteristics of the motor as stated before, which is obtained by the developed ideal Back EMF block as shown in Fig.7. The outcome of this work is to achieve desired speed control for a wide range of speed variation applications. This model gives flexibility to the users to change motor parameters and get suitable results.

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