

# A Robust Digital Position Control of Brushless DC Motor with Dead Beat Load Torque Observer

Jong Sun Ko, *Member, IEEE*, Jung Hoon Lee, *Member, IEEE*, Se Kyo Chung, *Member, IEEE*, and Myung Joong Youn, *Member, IEEE*

**Abstract**—A new control method for the robust position control of brushless DC (BLDC) motor is presented. The linear quadratic controller plus load torque observer is employed to obtain the robust BLDC motor system approximately linearized using the field-orientation method for an ac servo. And the gains are obtained systematically from the discrete state space analysis. In addition, the robustness is also obtained without affecting the overall system response. The load disturbance is detected by a 0-observer of the unknown and inaccessible input, and is compensated by the feedforward without requiring the noisy current information. All these designs are done simply in the state space. Finally, the overall system is controlled by using the microprocessor and the performance of each control algorithm is compared with both the simulation and the experimental results for the two types of the machines, i.e., a BLDC motor and a brushless direct drive (BLDD) motor.

## I. INTRODUCTION

DC motors have been gradually replaced by the BLDC motor since the industry applications require more powerful actuators in small sizes. The advantage of using a BLDC motor is that it can be controlled to have the speed-torque characteristics similar to that of a permanent magnet DC motor. In addition, the BLDC motor has the lower inertia, large power rate, fewer spark problems, and lower noise as compared with the permanent magnet DC servo motor having the same output rating [1], [2]. The disadvantages are the high cost and the more complex controller caused by the nonlinear characteristics [3], [4]. Another problem in a BLDC motor control is that the PI controller is usually employed, which is simple in realization but difficult to obtain a sufficient high performance in the tracking application. It is, however, known that the tracking controller problem using a state variable feedback can be simply solved by the augmentation of the output error as a new state [8], [9]. Even though this method is more complex than a PI controller, it is more efficient to obtain the control gain using the optimal control theory and has no problem compared to the classical controller.

Recently, a direct drive motor system is required for the fast position controller because there is no backlash.

Manuscript received January 3, 1992; revised November 23, 1992.

The authors are with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, Taejeon 305-701, Korea.

IEEE Log Number 9211427.

However, the external force directly imposed by the motor shaft in this system must be quickly rejected to obtain the robustness in the position controller. Also the motor system needs the high torque and fast response time. Therefore, the BLDC motor is the best choice for the robust controller. For the unknown and inaccessible inputs, the observer was studied by J. S. Meditch and G. H. Hostetter [12]. And in [14], K. Ohishi *et al.* presented the load torque observer considering a load torque as the unknown and inaccessible input in the dc servo motor system with a P controller. But this observer contains a current. Generally the measured current is too noisy to be used in the digital controller or the observer. Since the current-regulated PWM inverter in a BLDC motor has a hysteresis band gap caused by the current controller, there is some uncontrollable range of the current resulting in a high-frequency noise.

In this paper, the augmented state variable feedback controller based on the linear quadratic control (LQC) is introduced to the position control of a BLDC motor having a sinusoidal back EMF and is linearized by using the field orientation. In particular, to obtain the robustness against the load variation, the load torque is estimated by a dead beat estimator with no current information using the Luenberger observer and the control signal is compensated by the feedforward terms. Furthermore, this observer needs to sense only one variable, rotor position data, which is not noisy and it is simply realized by Ackerman's rule. For the performance comparison with the BLDC motor, a BLDD motor having a high torque at low speed is also investigated. As a result, the load disturbance can be rejected without affecting the overall system performance under the all operating conditions. This digital control scheme is implemented using the microprocessor.

## II. MODELING OF BLDC MOTOR

### A. Nonlinear Model

Generally, a small horsepower BLDC motor used for a position control is the same as a permanent magnet synchronous machine. The stator is constructed by three phase Y-connection without the neutral and the rotor is made by the permanent magnets. Since each phase has the phase angle difference of 120°, the summation of all

three phase currents becomes zero. The system equations in a  $d-q$  model can be expressed as follows [3]:

$$\dot{i}_{qs} = -\frac{r_s}{L_q}i_{qs} - \frac{L_d}{L_q}\omega_r i_{ds} + \frac{1}{L_q}V_{qs} - \frac{\lambda_m}{L_q}\omega_r \quad (1)$$

$$\dot{i}_{ds} = \frac{L_q}{L_d}\omega_r i_{qs} - \frac{r_s}{L_d}i_{ds} + \frac{1}{L_d}V_{ds} \quad (2)$$

$$L_q = L_{ls} + L_{mq}; \quad L_d = L_{ls} + L_{md} \quad (3)$$

$$T_e = \frac{3}{2} \left( \frac{p}{2} \right) [\lambda_m i_{qs} + (L_d - L_q) i_{qs} i_{ds}] \quad (4)$$

$$= J \left( \frac{2}{p} \right) \frac{d\omega_r}{dt} + B \frac{2}{p} \omega_r + T_L \quad (5)$$

where

- $r_s$  : stator resistance.
- $L_q$  :  $q$  axis stator inductance.
- $L_d$  :  $d$  axis stator inductance.
- $p$  : number of poles.
- $\lambda_m$  : flux linkage of permanent magnet.
- $\omega_r$  : angular velocity of rotor.

#### B. Linearized Model

By means of the field-oriented control, it can make  $i_{ds}$  become zero [5], [6]. Therefore, the system equations of a BLDC motor model can be described as

$$\dot{i}_{qs} = -\frac{r_s}{L_q}i_{qs} + \frac{1}{L_q}V_{qs} - \frac{\lambda_m}{L_q}\omega_r \quad (6)$$

$$\dot{\omega}_r = \frac{3}{2} \frac{1}{J} \left( \frac{p}{2} \right)^2 \lambda_m i_{qs} - \frac{B}{J} \omega_r - \frac{p}{2J} T_L \quad (7)$$

and the torque equation is expressed as

$$\begin{aligned} T_e &= \frac{3}{2} \left( \frac{p}{2} \right) \lambda_m i_{qs} \\ &= k_t i_{qs} \end{aligned} \quad (8)$$

where

$$k_t = \frac{3}{2} \left( \frac{p}{2} \right) \lambda_m.$$

Since the current control is employed in a position control, the system model expressing the position dynamics becomes (7) and the rotor position dynamics becomes

$$\dot{\theta} = \omega_r \quad (9)$$

where  $\theta$  is the rotor position.

For the implementation of the field orientation, each three phase current control command must be generated separately. This command can be obtained by converting the controller current command based on the rotor reference frame to the stator reference frame. The three phase current commands,  $i_{as}$ ,  $i_{bs}$ , and  $i_{cs}$  are, then, tracked by the current regulated PWM (CRPWM) scheme. In this case, the current controller requires the absolute rotor position.

### III. CONTROL ALGORITHM

#### A. Position Controller

The reference is a step value as in a tracking servo problem [7]. The dynamic equation of a given system can be expressed as follows:

$$\dot{x}(t) = Ax(t) + bu(t) \quad (10)$$

$$y = cx(t) \quad (11)$$

where the dimensions of the matrices  $A$ ,  $b$ , and  $c$  are  $n \times n$ ,  $n \times 1$  and  $1 \times n$ , respectively. Usually, a linear quadratic controller is used to solve the regulator problem resulting in a state variable feedback. Applying it to a servo problem, another control value is needed such as

$$u(t) = -Kx(t) + \tilde{u}_c(t) \quad (12)$$

where  $K$  is a feedback gain matrix and  $\tilde{u}_c(t)$  is a compensation input. In case of a regulator,  $\tilde{u}_c(t) = 0$ . It is, however, difficult to find the value of  $\tilde{u}_c(t)$ . Therefore, a new state for the tracking controller is defined as

$$\dot{z} = \theta - \theta_r \quad (13)$$

where  $\theta_r$  is the reference input [9]. It is then evident from (10), (11), and (13) that the open loop tracking system is governed by a state equation of the form as follows:

$$\begin{aligned} \hat{\dot{x}} &= \begin{pmatrix} A & 0 \\ c & 0 \end{pmatrix} \hat{x} + \begin{pmatrix} b \\ 0 \end{pmatrix} u - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \theta_r \\ &= \hat{A}\hat{x} + \hat{b}u - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \theta_r \end{aligned} \quad (14)$$

$$\begin{aligned} y &= [c \quad 0] \hat{x} \\ &= \hat{c}\hat{x} \end{aligned} \quad (15)$$

where  $\hat{x} = [x \ z]^T$ . If  $(\hat{A}, \hat{b})$  is controllable, then

$$\lim_{t \rightarrow \infty} (\theta - \theta_r) = 0 \quad (16)$$

by the control input of

$$u = -kx - k_1 z \quad (17)$$

where  $k$  is a  $1 \times n$  vector and  $k_1$  is a scalar. It is well known that this controller is able to cancel the steady state error caused by the unmeasurable or inaccessible disturbance. Also, if this closed-loop system is asymptotically stable, the overall system is robust to the system parameter variation or the feedback gain variation [8].

The augmented system for the position control of a BLDC motor are expressed as follows:

$$\begin{aligned} \begin{pmatrix} \dot{\omega}_r \\ \dot{\theta} \\ \dot{z} \end{pmatrix} &= \begin{pmatrix} -\frac{B}{J} & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \omega_r \\ \theta \\ z \end{pmatrix} + \begin{pmatrix} k_t \frac{p}{2} \frac{1}{J} \\ 0 \\ 0 \end{pmatrix} i_{qs} \\ &\quad - \begin{pmatrix} \frac{p}{2} \frac{1}{J} \\ 0 \\ 0 \end{pmatrix} T_L - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \theta_r \end{aligned} \quad (18)$$

$$y = (0 \quad 1 \quad 0) \hat{x}. \quad (19)$$

For this system, the rank of a controllability matrix is 3. Therefore the augmented control system is controllable and the steady state value of  $z$  becomes zero by the control input given in the form of

$$u(t) = -\hat{k}\hat{x} \quad (20)$$

where  $\hat{k}$  is a  $1 \times 3$  vector. The block diagram of an augmented state variable feedback control system is shown in Fig. 1.

### B. Digital Control and Proposed Algorithm

The discrete time invariant system equation can be obtained from (10) and (11) under the following conditions:

- 1) Simultaneous sampling of input and output (synchronized A/D and D/A converter);
- 2) negligible conversion time;
- 3) constant value between the sampling time; and
- 4) constant sampling time.

Thus the discrete system equation can be written as follows [10]:

$$x(kh + h) = \Phi x(kh) + \Gamma u(kh) \quad (21)$$

$$y(kh) = cx(kh) \quad (22)$$

where

$$\begin{aligned} \Phi &= e^{Ah} \\ &= \begin{pmatrix} e^{-(B/J)h} & 0 \\ \frac{J}{B}(1 - e^{-(B/J)h}) & 1 \end{pmatrix} \\ \Gamma &= \int_0^h e^{As} ds B \\ &= \begin{pmatrix} -\frac{J}{B}e^{-(B/J)h} + \frac{J}{B} \\ \frac{J}{B}\left(h + \frac{J}{B}e^{-(B/J)h} + \frac{J}{B}\right) \end{pmatrix} k_t \frac{p}{2} \frac{1}{J} \end{aligned}$$

From this equation, the state feedback controller gain can be obtained by the optimal control law minimizing the performance index such that

$$J = \lim_{N \rightarrow \infty} \sum_{i=0}^{N-1} \left\{ \hat{X}_k^T Q \hat{X}_k + u_k^T R u_k \right\} \quad (23)$$

where the weighting matrices  $Q$  and  $R$  are defined as  $\text{diag}[q_{11} \ q_{22} \ q_{33}]$  and 1, respectively. The steady state solution of a LQC can be obtained by solving the following equations as [10]

$$s = \Phi - \Gamma K^T s \Phi - \Gamma K + Q + K^T R K \quad (24)$$

$$K = -(R + \Gamma^T s \Gamma)^{-1} \Gamma^T s \Phi \quad (25)$$

where  $s$  and  $R + \Gamma^T s \Gamma$  are the positive definites. Then the steady state position error is controlled by the controller given as follows:

$$u(k) = -Kx(k). \quad (26)$$

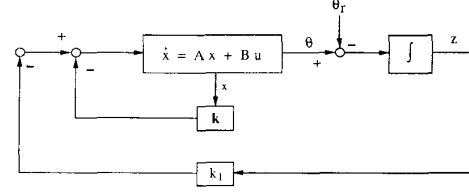


Fig. 1. Block diagram of the augmented state variable feedback controller.

However, a large feedback gain is needed for the fast reduction of an error caused by the disturbance, which results in a very large current command at all operating conditions. Therefore, a new control algorithm is required to reduce the influence of the disturbance at transient state without affecting the predefined overall system performance. If the load torque  $T_L$  is known, it can be expressed, from (8), as

$$T_L = k_t i_{qc2} \quad (27)$$

$$i_{qc2} = \frac{1}{k_t} T_L. \quad (28)$$

Therefore the equivalent  $q$  axis current can be calculated to compensate the load torque. A fast compensation can be obtained by feedforwarding an equivalent  $q$  axis current command for the load torque to the output controller. The control input  $i_{qc}$  is obtained by adding the position controller output  $i_{qc1}$  to the equivalent current  $i_{qc2}$  generated by the estimated load torque and a torque constant  $k_t$  of the motor as follows:

$$u = i_{qc} = i_{qc1} + i_{qc2}. \quad (29)$$

Since the load torque is an unmeasurable or inaccessible disturbance, it is difficult to obtain this value. Therefore the derivation of a load torque by an observer is discussed in the next section.

### C. Load Torque Observer

Generally, it is required to know all the inputs given to the system to estimate a state. But in the real system, there are many cases where some of the inputs are unknown or inaccessible. Almost all these inputs are the internal or external disturbances or the unknown error from an output measurement. For a given system of (10) and (11), the observer is known as follows:

$$\dot{Z} = FZ + Gy + Hu \quad (30)$$

$$w = Ly + MZ \quad (31)$$

where  $F$ ,  $G$ ,  $H$ ,  $L$ , and  $M$  are the real constant matrices and there exists  $T$  such that  $w(t) = Tx(t)$ . Two types of the observer are generally available when  $u(t)$  is unknown and inaccessible [11]. Firstly, a 0-observer is available under the condition that  $u(t)$  is constant ( $\dot{u} = 0$ ) and

there exists  $T$  such that

$$w(t) = T \begin{pmatrix} x(t) \\ u \end{pmatrix}. \quad (32)$$

Secondly, a  $K$ -observer is suggested under the condition that  $u(t) = u_0 + u_1 t^1 + \dots + u_k t^k$  and there exists  $T$  such that

$$w(t) = T \begin{pmatrix} x(t) \\ u(t) \end{pmatrix}. \quad (33)$$

For simplicity, a 0-observer is selected and the augmented system is given as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{u} \end{pmatrix} = \begin{pmatrix} A & b \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} \quad (34)$$

$$y = (c \ 0) \begin{pmatrix} x \\ u \end{pmatrix}. \quad (35)$$

To guarantee the existence of a 0-observer, the observability matrix must have a full rank [11] [12]. An application method to the case where the measurement contains a constant but an unknown bias error is introduced by Bryson and Luenberger [14]. Thus  $T_L$  can be considered as an unknown input in (34) and assumed to be a constant. From (10) and (11), an observer for the load torque estimation is obtained as

$$\dot{T}_L = 0 \quad (36)$$

and the system equation can be expressed as

$$\begin{pmatrix} \dot{\hat{\omega}} \\ \dot{\hat{\theta}} \\ \dot{\hat{T}}_L \end{pmatrix} = \begin{pmatrix} -\frac{B}{J} & 0 & -\frac{p}{2} \frac{1}{J} \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{\omega} \\ \hat{\theta} \\ \hat{T}_L \end{pmatrix} + \begin{pmatrix} k_t \frac{p}{2} \frac{1}{J} \\ 0 \\ 0 \end{pmatrix} i_{qs} + L \begin{pmatrix} \theta - (0 \ 1 \ 0) \begin{pmatrix} \hat{\omega} \\ \hat{\theta} \\ \hat{T}_L \end{pmatrix} \end{pmatrix} \quad (37)$$

where  $L$  is a  $3 \times 1$  feedback gain matrix.

#### D. Implementation of Load Torque Observer

The discrete form of (36) is represented as

$$T_L(k+1) = T_L(k). \quad (38)$$

A load torque observer is obtained considering  $T_L$  as an unknown input. From (38), (21), and (22), the discrete load torque observer can be obtained as follows:

$$\begin{pmatrix} \hat{\omega}(k+1) \\ \hat{\theta}(k+1) \\ \hat{T}_L(k+1) \end{pmatrix} = \begin{pmatrix} a_1 & 0 & a_2 \\ a_3 & 1 & a_4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\omega}(k) \\ \hat{\theta}(k) \\ \hat{T}_L(k) \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ 0 \end{pmatrix} i_{qc}(k) + \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} \left( \theta(k) - (0 \ 1 \ 0) \begin{pmatrix} \hat{\omega}(k) \\ \hat{\theta}(k) \\ \hat{T}_L(k) \end{pmatrix} \right) \quad (39)$$

where  $l_1$ ,  $l_2$ , and  $l_3$  are the elements of  $L$ , and

$$a_1 = e^{-(B/J)h}$$

$$a_2 = -\frac{p}{2J} \left( -\frac{J}{B} e^{-(B/J)h} + \frac{J}{B} \right)$$

$$a_3 = \frac{J}{B} (1 - e^{-(B/J)h})$$

$$a_4 = -\frac{p}{2J} \left( h + \frac{J}{B} e^{-(B/J)h} + \frac{J}{B} \right)$$

$$b_1 = -\frac{p}{2J} \left( -\frac{J}{B} e^{-(B/J)h} + \frac{J}{B} \right) k_t$$

$$b_2 = -\frac{p}{2B} \left( h + \frac{J}{B} e^{-(B/J)h} + \frac{J}{B} \right) k_t.$$

Therefore, the discrete load torque observer can be expressed as

$$\hat{x}(k+1) = \Phi \hat{x}(k) + \hat{\Gamma} u(k) + L(\theta(k) - \hat{y}) \quad (40)$$

$$\hat{y} = \hat{c} \hat{x}(k). \quad (41)$$

The observability matrix  $W_0$  becomes

$$W_0 = \begin{pmatrix} \hat{c} \\ \hat{c} \Phi \\ \hat{c} \Phi^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ a_3 & 1 & a_4 \\ a_1 a_3 + a_3 & 1 & a_2 a_4 + 2a_4 \end{pmatrix}. \quad (42)$$

Since the rank of a matrix  $W_0$  is 3, the discrete load torque observer is observable. To guarantee the less time required to calculate the load torque than the overall system response time and to compensate the load torque at transient state, a dead beat observer is desirable. The dead beat controller is obtained by simply using the pole assignment technique at zero in  $z$ -domain. The characteristic equation becomes

$$z^n = 0 \quad (43)$$

where  $n$  is the order of the system.

It then follows from the Cayley–Hamilton theorem that the system matrix of the closed loop system satisfies the following equation as

$$\Phi_c^n = 0 \quad (44)$$

where  $\Phi_c = \hat{\Phi} - L\hat{c}$  and  $L$  is a feedback gain matrix. This strategy has the property that it will drive all the states to zero in at most  $n$  steps. Since the settling time is  $n \cdot h$ , the design parameter is determined only by the sampling time  $h$ , which is a unique characteristic of a discrete sampled data system [10]. In contrast with this advantage, the control input  $u$  is large and is increased as  $h$  is decreased. Therefore, it is difficult to obtain the desired characteristics by the bounded input, and is sensitive to the measurement error in the fast response observer. However, the proposed load torque observer tracks the real load torque with an unbounded feedback input. This causes the range of the predefined bit number

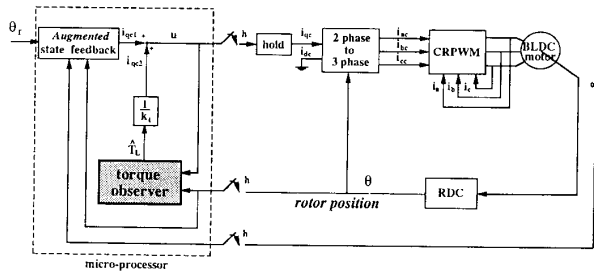


Fig. 2. Block diagram of the proposed digital control system.

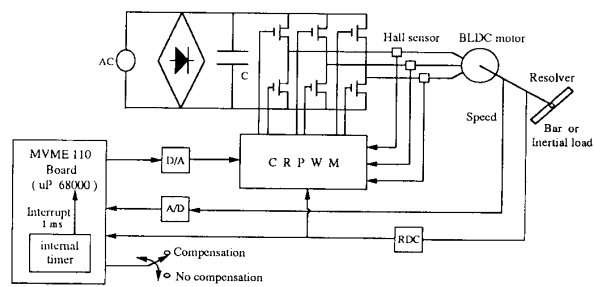


Fig. 3. Schematic diagram of the experimental system.

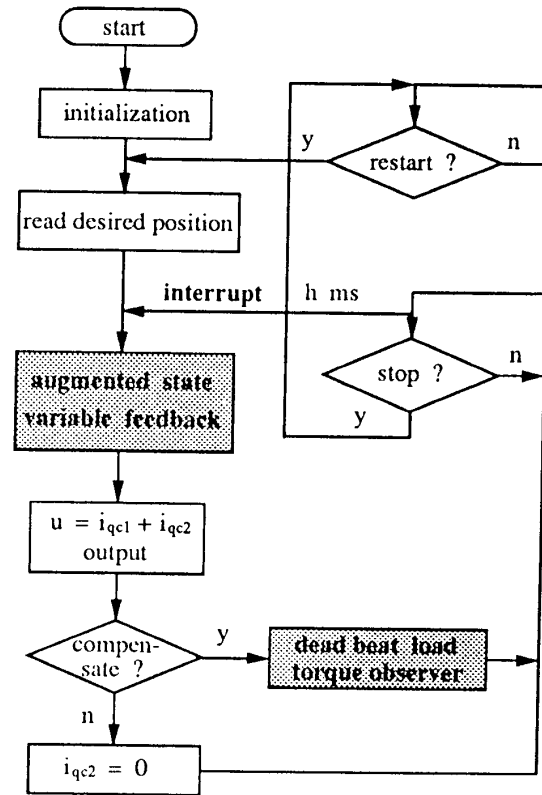
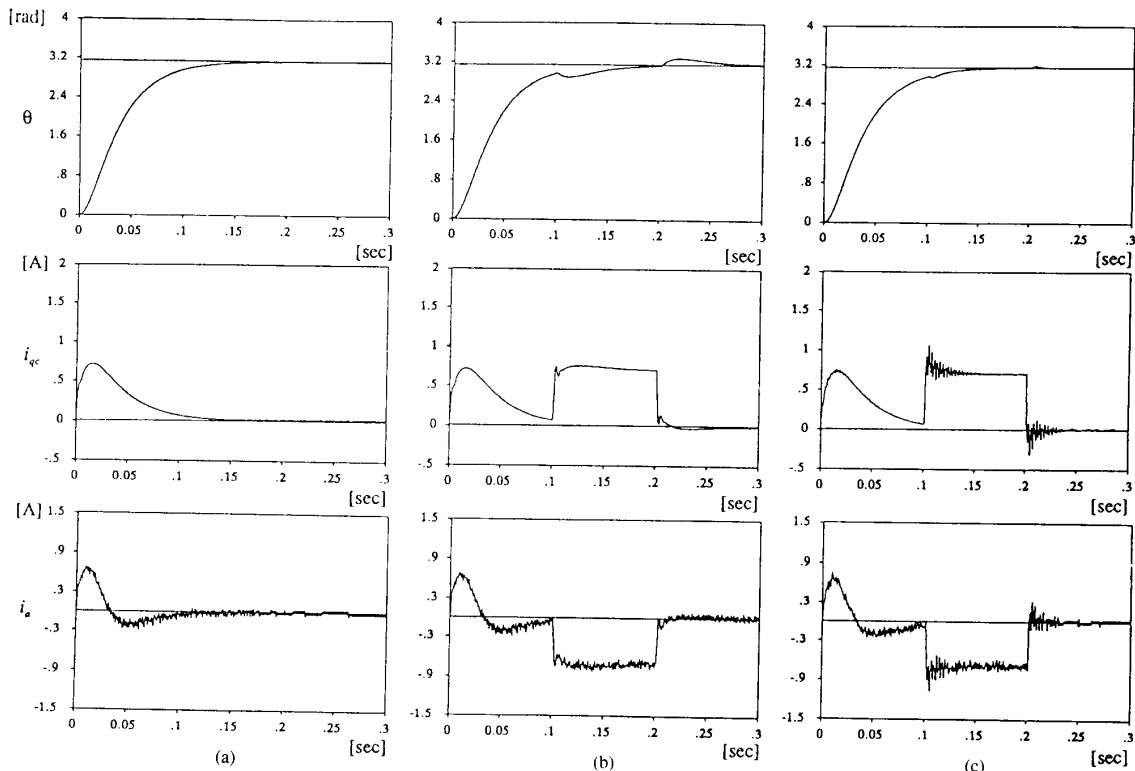


Fig. 4. Flow chart of the control program used in assembly language.

Fig. 5. Simulation results of the rotor position,  $q$  phase current command and  $a$  phase current for BLDC motor, (a) no load response, (b) step-changed load response without compensation, (c) step-changed load response with compensation.

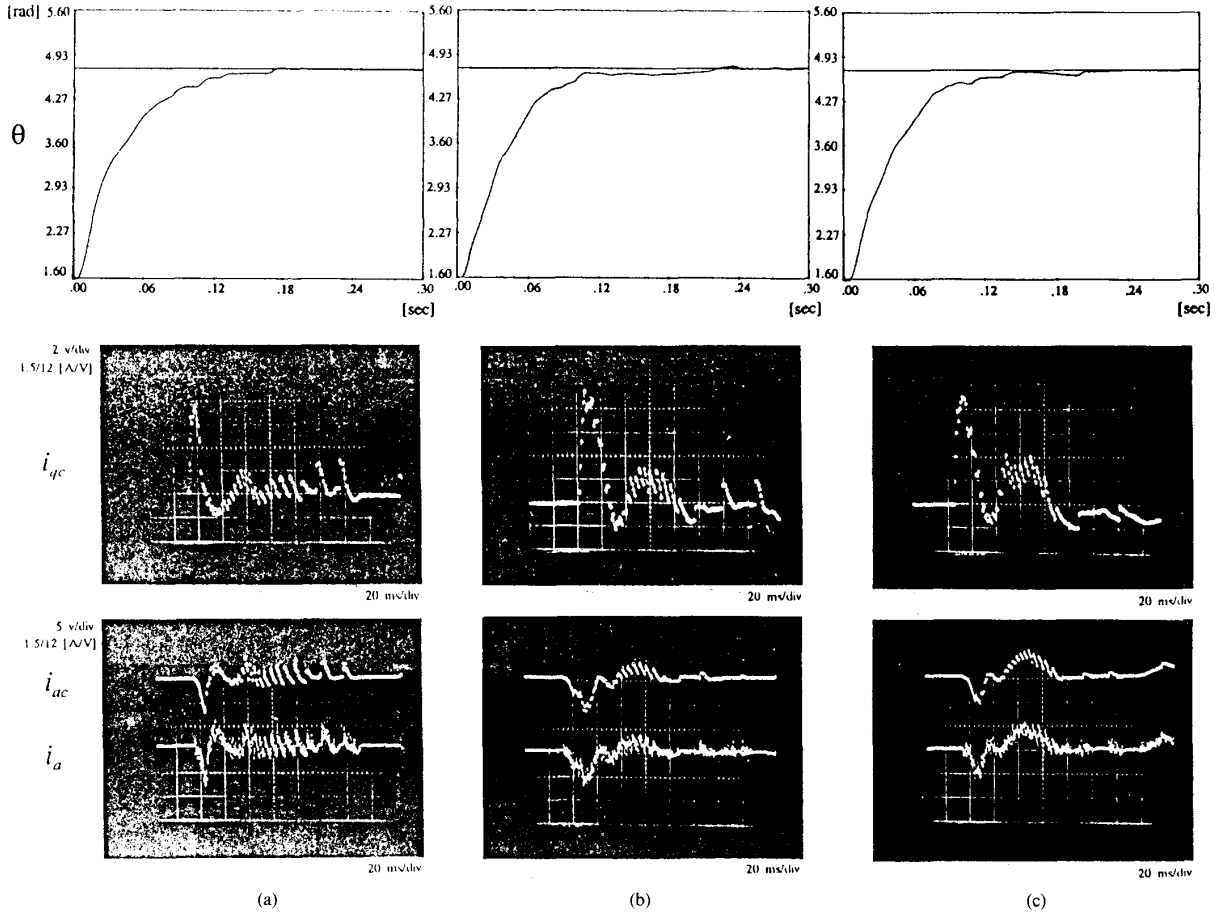


Fig. 6. Experimental results of the rotor position,  $q$  phase current command and  $a$  phase current command with  $a$  phase current for BLDC motor, (a) no load response, (b) bar load response without compensation, (c) bar load response with compensation.

in an internal assembly program sufficiently larger than that of the real control input.

Thus the full state load torque observer can be implemented using only one variable, the position error, instead of using both the position and the speed error. Also, this observer is not influenced by the noise caused by the A/D conversion of the speed. The feedback gain matrix  $L$  can be obtained by the pole placement using Ackermann's formula [10]. If the given system is reachable, a gain matrix  $L$  can be represented as follows:

$$L = P(\Phi)W_0^{-1}[0 \ 0 \ \cdots \ 1]. \quad (45)$$

Because of the dead beat controller, the characteristics of the load torque observer in this system is

$$P(\Phi) = \Phi^3. \quad (46)$$

#### IV. CONFIGURATIONS OF THE OVERALL SYSTEMS

The block diagram of the proposed controller is shown in Fig. 2. This controller is composed of two parts. One is the digital controller part employing the state feedback controller with an optimal gain and the load torque observer realized by a dead beat controller. The load

disturbance is compensated by the feedforward equivalent current command come from the dead beat load torque observer. The other part is the power control unit of a field orientation which includes the position and the current sensors. The digital control part is implemented by the assembly language program employing the hardware of a 68000 microprocessor. The position controller is composed of using the augmented state feedback defined in (14), (15), and (17). For the realization of the augmented state  $z(k+1)$  as defined in (13), the discrete form of this state is approximately obtained by using a trapezoidal rule as

$$z(k+1) = z(k) + \frac{h}{2}(e(k) + e(k-1)) \quad (47)$$

where  $e(k) = \theta(k) - \theta_r$ .

To further reduce the calculation time in the assembly programming, a fixed point calculation is used. The power converter is controlled by the field orientation method, which is composed of a 2 phase to 3 phase converter and a current regulated PWM inverter. The phase converter generates the three phase currents from the current command  $i_{qc}$  using a ROM table and a D/A converter. The

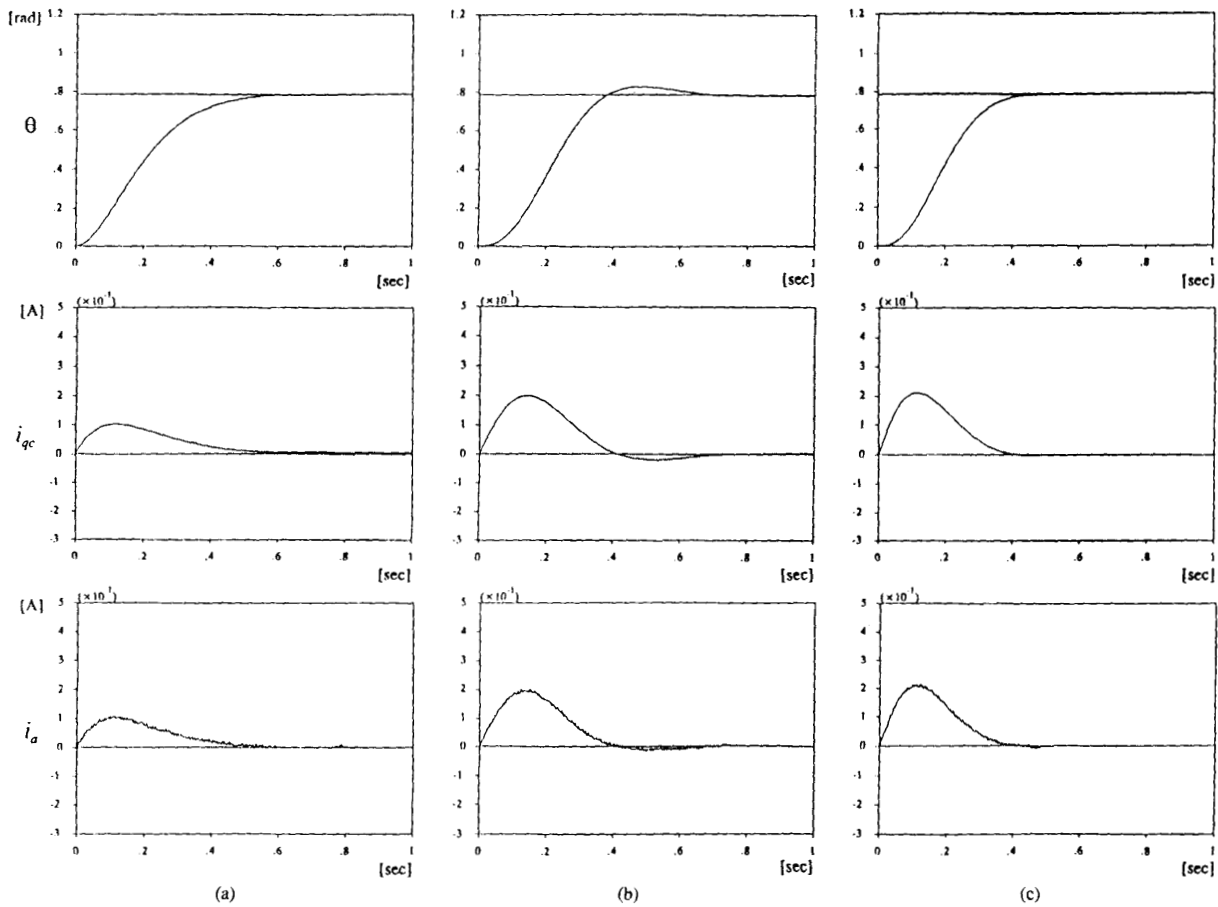


Fig. 7. Simulation results of the rotor position,  $q$  phase current command and  $a$  phase current for BLDD motor, (a) no load response, (b) inertia load response without compensation, (c) inertia load response with compensation.

CRPWM inverter makes the three phase real currents track the three phase command currents using the MOSFET's with the hysteresis band gap [5], [6]. For the position and the current sensing, a resolver to digital converter and three hall sensors are used, respectively.

#### V. SIMULATION AND EXPERIMENTAL RESULTS

The parameters of a BLDC motor used in this simulation and the experiment are given as follows:

Type	Conventional	Direct Drive
Power	:120 w	:120 w
Rated torque	:0.3822 N · m	:11 N · m
Inertia	:1.372 × 10 <sup>-5</sup> kgm <sup>2</sup>	:1.568 × 10 <sup>-3</sup> kgm <sup>2</sup>
Stator resistance	:7.5 ohm	:28.4 ohm
Mechanical time constant	:2.01 ms	:1.1 ms
Electrical time constant	:0.70 ms	:7.4 ms

The proposed digital position controller shown in Fig. 2 is implemented as shown in Fig. 3. The CRPWM inverter is employed as a power converter and the sampling time  $h$  is determined as 1 ms. After some iterations, the weighting matrices are selected as follows:

$$Q = \text{diag}[10^{-1} \quad 10^3 \quad 10^6], \quad R = 1.$$

As a result, the optimal gain matrix becomes  $\hat{k} = [0.02 \quad 3.7098 \quad 89.6631]$  and also the dead beat observer gain becomes  $L = [1819.6 \quad 2.6 \quad -8.7]$ . The total position control system is programmed using the assembly language in a 68 000 microprocessor.

The control flow chart of this system is shown in Fig. 4 with  $i_{qc2}$  being a feedforward value. The simulation results are depicted in Fig. 5. In the no-load case, the feedback gain is obtained by a linear optimal control theory under the conditions that there is no overshoot and the settling time is about 0.1 s. A step load torque of 0.2 Nm is considered in this system to illustrate the overall performance. This torque disturbance is chosen as about a half

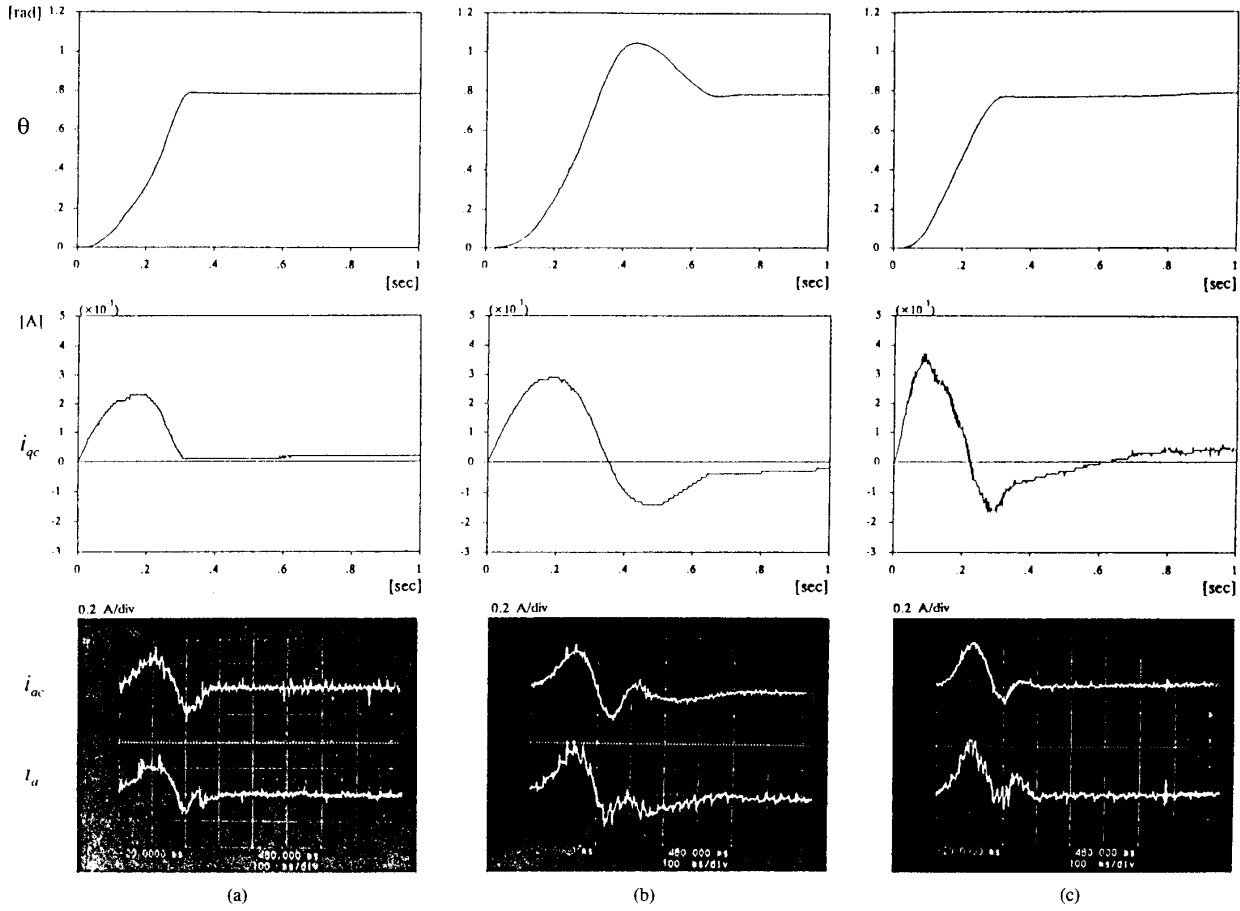


Fig. 8. Experimental results of the rotor position,  $q$  phase current command and  $a$  phase current command with  $a$  phase current for BLDD motor, (a) no load response, (b) inertial load response without compensation, (c) inertial load response with compensation.

the rated torque. As shown in Fig. 5, the step response of the rotor position is sensitive to the change of the load torque  $T_{Load}$ . This load creates the position error about 0.2 rad and the error rejection time is about 0.1 s. However, in the proposed system, the response is almost equal to the no load case under all operating conditions. This result comes from the dead beat torque observer and the feedforward compensation. The  $q$  phase current command  $i_{qc}$  in Fig. 5(c) shows this compensation effect. The peak value of  $i_{qc}$  in Fig. 5(c) is about 1.2 A, but this peak value is at most 0.7 A in Fig. 5(b).

The experimental results are shown in Fig. 6. This figure is obtained by plotting the data stored in the RAM at each sampling time. In this experiment, the load is implemented by using a small bar directly connected to the BLDC motor. Although the effect of the step load change cannot be obtained, the trend of the robustness of this proposed controller can be shown. As can be seen in Fig. 6, there exist some discrepancies caused by the calculation time of a microprocessor and the A/D or D/A conversion time. However, the compensation effects are

obtained by increasing the  $q$  phase current command compared to the no-compensation case. Also, the rotor position plot with a load is more similar to the no load case. In the same way, the results of the BLDD motor are shown in Fig. 7 and 8. Since the BLDD motor has slower system dynamics, the sampling time  $h$  and settling time are determined as 2 and 0.4 ms, respectively. After some iterations for the settling time of 0.4 ms, the weighting matrices are selected as follows:

$$Q = \text{diag}[1 \quad 2 \times 10^2 \quad 10^4], \quad R = 1.$$

Then the optimal gain matrix becomes  $\hat{k} = [0.0059 \quad 0.6579 \quad 3.2602]$  and the dead beat observer gain becomes  $L = [595.1290 \quad 2.1623 \quad -107.0533]$ . Since the BLDD motor has the high torque at low speed, the expressed trends are well depicted.

## VI. CONCLUSIONS

A systematic approach is done for the robust position control of a BLDD motor linearized by a vector control



method based on the field orientation. The LQC plus load torque observer is realized in the digital control system and the discrete state space analysis is done to obtain the gains. Considering the load torque as the unknown and inaccessible input, the robust digital position control system is implemented based on the observer theory without current information. Therefore, the load torque compensator based on the observer theory is not affected by the current noise. And the feedforward can cancel out rapidly the steady state and the transient position error due to the external disturbances such as a various friction and a load torque. But these phenomena are different from those of the disturbance compensation using a high gain effect, which additionally results in the problem of influencing the overall system response. In this proposed controller, the overall system response is not affected by the disturbance compensator.

The total control system is realized by a digital controller where the gain is obtained in  $z$ -domain using the optimal control theory. And the performance of each control algorithm is compared to both the simulation and the experimental results of the two types of the machines, i.e., a BLDC motor and a brushless direct drive (BLDD) motor.

#### REFERENCES

- [1] Electro-craft Corp. U.S.A., "DC motors speed controls servo system," Pergamon, 1972.
- [2] Alexander Kusko and Syed M. Peeran, "Brushless dc motors using unsymmetrical field magnetization," *IEEE Trans. Indus. App.*, vol. IA-23, pp. 319-326, 1987.
- [3] Paul C. Krause, *Analysis of Electric Machinery*. New York: McGraw-Hill, 1984.
- [4] T. Kenjo and S. Nagamori, "Permanent magnet and brushless DC motors," Tokyo: Sogo, 1984, p. 96.
- [5] D. W. Novotny and R. D. Lorenz, "Introduction to field orientation and high performance ac drives," IAS Tutorial Course, 1986.
- [6] B. K. Bose, *Power electronics and AC drives*. New York: MacGraw-Hill, 1986.
- [7] S. G. Tzafestas, *Applied Digital Control*. New York: North-Holland, 1985, p. 51.
- [8] Edward J. Davison, "The output control of linear time-invariant multivariable systems with unmeasurable arbitrary disturbances," *IEEE Trans. Auto. Control*, vol. AC-17, pp. 621-630, 1972.
- [9] B. Porter and A. Bradshaw, "Design of linear multivariable continuous-time tracking systems," *Int. J. Syst. Science*, vol. 5, no. 12, pp. 1155-1164, 1974.
- [10] Karl J. Åström and Björn Wittenmark, *Computer Controlled Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1984.
- [11] J. S. Meditch and G. H. Hostetter, "Observer for systems with unknown and inaccessible inputs," *Int. J. Control*, vol. 19, no. 3, pp. 473-480, 1974.
- [12] Gene Hostetter and J. S. Meditch, "Observing systems with unmeasurable inputs," *IEEE Trans. Auto. Control*, vol. AC-18, pp. 307-308, 1973.
- [13] Arthur E. Bryson, Jr. and David G. Luenberger, "The synthesis of regulator logic using state-variable concepts," *IEEE Proc.*, vol. 58, pp. 1803-1811, 1970.
- [14] K. Ohishi, M. Nakao, K. Ohnishi, and K. Miyachi, "Microprocessor-controlled DC motor for load-insensitive position servo system," *IEEE Trans. Ind. Electron.*, vol. IE-34, pp. 44-49, 1987.



**Jong Sun Ko** (S'89) was born in Korea, on March 20, 1960. He received the B.S. degree from Seoul National University, Seoul, Korea, in 1984, and the M.S. degree from Korea Advanced Institute of Science and Technology (KAIST), Seoul, Korea, in 1989. He is currently working toward the Ph.D. degree in electrical engineering at KAIST.

He has been with Production Engineering R & D Center of Samsung Electronics Co. Ltd. since 1983. His research interests are in the areas on power electronics and control, which include the drive system, digital control, robust control, and factory automation. He is a member of KIEE.



**Jung Hoon Lee** (S'90) was born in Kyungbuk, Korea, on February 1, 1966. He received the B.S. degree from Kyungbuk National University, Taegu, Korea, in 1988, and the M.S. degree from Korea Advanced Institute of Science and Technology (KAIST), Seoul, Korea, in 1990. He is currently working toward the Ph.D. degree in electrical engineering at KAIST.

His research interests are in the areas on power electronics and system control, which include the sliding mode control, electrical drive, and robot control. He is a member of KIEE.



**Se Kyo Chung** (S'91) was born in Korea, on November 26, 1966. He received the B.S. degree from Kyungbuk National University, Taegu, Korea, in 1989, and the M.S. degree from Korea Advanced Institute of Science and Technology (KAIST), Taejeon, Korea, in 1992. He is currently working toward the Ph.D. degree in electrical engineering at KAIST.

His research interests are in the areas on power electronics and control, which include the electrical drive systems and microprocessor-based control applications.



**Myung Joong Youn** (S'76-M'78) was born in Seoul, Korea, on November 26, 1946. He received the B.S. degree from Seoul National University, Seoul, Korea, in 1970, and the MS and Ph.D. degrees in electrical engineering from the University of Missouri-Columbia in 1974 and 1978, respectively.

He was with the Aircraft Equipment Division of General Electric Co. at Erie, PA since 1978, where he was an Individual Contributor on Aerospace Electrical System Program. He has been with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology since 1983 where he is now a professor. His research activities are in the areas on power electronics and control which include the drive system, rotating electrical machine design, and high-performance switching regulators.

Dr. Youn is a member of KIEE and KITE.