Metamaterials corresponding to noncommutative black holes

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Abstract The propagation of light through the metamaterial analogue to the Noncommutative (NC) spacetime (NC Schwarzschild (NCS) and NC Reissner–Nordström (NCRN) black holes (BH)) is studied and it is compared with particular null geodesic trajectories. We found the properties related to the metamaterial corresponding to these spacetime and main effects observed in the simulations are a deviation in the light path and its corresponding comparison with the theoretical curve of the geodesics. These types simulations can be used to determine the existence of non-commutativity and check whether the astrophysical phenomenon is commutative or not

Resumen Se estudia la propagación de la luz a traves de metamaterial análogo a un espaciotiempo no conmutativo (NC) (agujeros negros NC Schwarzschild (NCS) and NC Reissner–Nordström (NCRN)) y se compara con las soluciones de geodésicas correspondientes. Se encontraron las propiedades que relacionan el metamaterial con el esapcio tiempo y el principal efecto observado en la simulación es la desviación de la trayectoria de la luz y su comparación con la curva teorica de geodésicas. Este tipo de simulaciones pueden ser usadas para determinar la existencia de la no conmutatividad y checar si el fenómeno astrofísico es conmutativo o no.

1. Introduction

The analogy of Maxwell's equations [4] in an anisotropic medium and Maxwell's equations in a vacuum and in a space with curvature is used for the simulations of S, RN and NCS optical BH [2, 3]. All the analysis and simulations already mentioned are reduced to a direct relationship between the term $g_{00} = f(r)$ [4] of a spherically symmetric and static metric, with the optical properties of metamaterial.

2. Metamaterial medium

If we consider a static spacetime metric associated with a spherically symmetric in (t, r, θ, ϕ) coordinates, it can be written in a generic form as $ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2(d\theta^2 + sin^2(\theta)d\phi^2)$, into a flat background to obtain the medium parameters in the Cartesian coordinate system. To do that, we apply a coordinate transformation and we get the permittivity and permeability tensors [4, 2]

$$\epsilon^{ij} = \sqrt{\frac{g_{rr}}{g_{00}}}\left(\delta^{ij} - [1 - f(r)]\frac{x^i x^j}{r^2}\right) \tag{1}$$

3. Geodesics

For a static and spherically symmetric BH we can get a general equation to calculate geodesics [3]

$$\frac{d^2u}{d\phi^2} + f[r(u)]u = \frac{1}{2}\frac{df[r(u)]}{dr}, \tag{2}$$

where $u = \frac{1}{r}$. The corresponding $f(r)$ with metrics NCS [5] and NCRN [1] metrics are given by

$$f(r) = \left(1 - \frac{4m}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)\right), \tag{3}$$

$$f(r) = 1 - \frac{4m}{r\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) + \frac{Q^2}{r^2\pi}\left[\gamma^2\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) - \frac{r}{\sqrt{2\theta}}\gamma\left(\frac{1}{2}, \frac{r^2}{2\theta}\right)\right], \tag{4}$$

where $\gamma(a, x) = \int_0^x dt t^{a-1}e^{-t}$, and with its properties we found

$$\frac{d^2u}{d\phi^2} + u = \frac{6mu^2}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{1}{4u^2\theta}\right) - \frac{m}{2u\sqrt{\pi}\theta^{3/2}}e^{-\frac{1}{4\theta u^2}}, \tag{5}$$

$$\begin{aligned} &\frac{d^2u}{d\phi^2} + u + \frac{Q^2u^2}{\pi\sqrt{\theta}}\left[\gamma\left(\frac{1}{2}, \frac{1}{4u^2\theta}\right)\left[\frac{1}{\sqrt{2}} - e^{-\frac{1}{4u^2\theta}}\right] + \frac{1}{2u\sqrt{\theta}}e^{-\frac{1}{2u^2\theta}} \ldots \right. \\ &\left. \ldots - \frac{1}{2\sqrt{2}}\gamma\left(\frac{1}{2}, \frac{1}{2u^2\theta}\right)\right] = \frac{6mu^2}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{1}{4u^2\theta}\right) - \frac{m}{2u\sqrt{\pi}\theta^{3/2}}e^{-\frac{1}{4\theta u^2}} \end{aligned} \tag{6}$$

To solve Eq. (2) we have performed a regression of the function f(r) in Eq. (3) and Eq. (4) to polynomials of degree n, then it is possible to write; $f(r) = \sum_{i=0}^{n} A_i r^i = A_0 + A_1 r + A_2 r^2 + \ldots + A_n r^n$, and the equation of geodesics takes the for

$$\frac{d^2u}{d\phi^2} + \sum_{i=0}^{n} A_i\left(1 - \frac{i}{2}\right)u^{1-i} = 0 \tag{7}$$

4. Numerical resolution and metamaterials for optical BH

For RN BH, NCS BH and NCRN BH, there may be 1 or 2 divergences corresponding to 1 or 2 horizons respectively. We are interested in solving when there is a single exterior horizon. Furthermore, using a fixed mass and charge, the values of θ and the event horizon for NCS BH and RN BH satisfying the single horizon condition are obtained as shown in Figure 1.a. To solve Eq. (7) if you make the variable changes $Y_1 = u$ and $Y_2 = du/d\phi$, we can rewrite Eq (8) and Eq (9). These equation were solved using the fourth order Runge-Kutta method.

$$Y_2 = \frac{dY_1}{d\phi} \tag{8}$$

$$\frac{dY_2}{d\phi} = Y_1^{1-i}\sum_{i=0}^{n} A_i\left(1 - \frac{i}{2}\right) \tag{9}$$

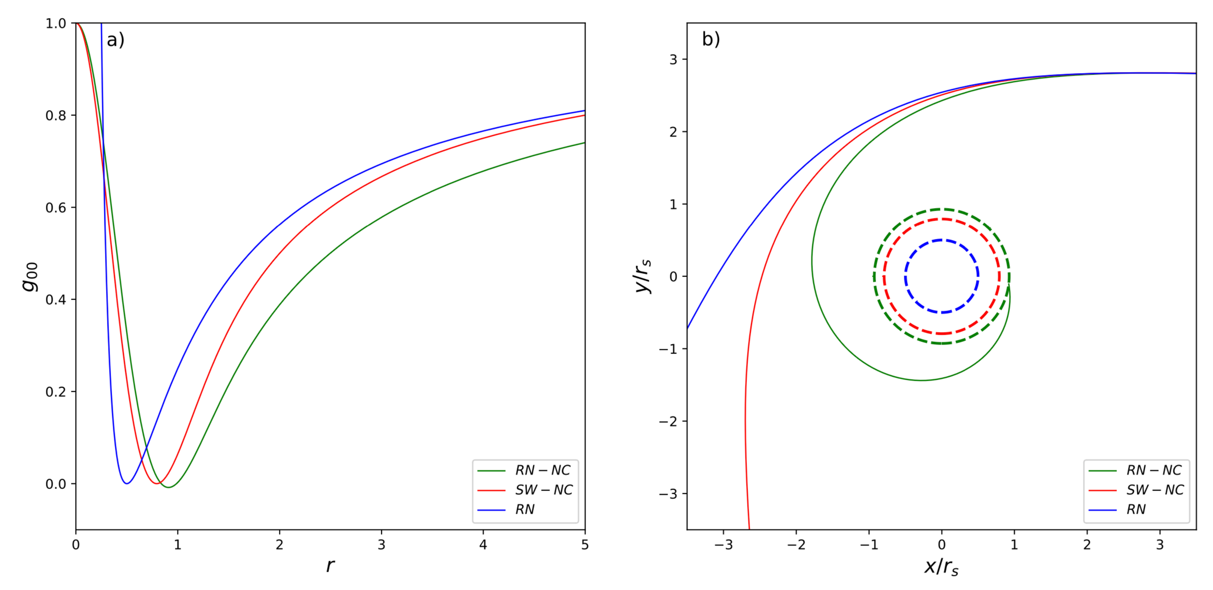

Figure 1: Behavior of the NC metrics Eq. (3), Eq. (4) and RN metric with geometric mass $m = q = 1/2r_s$ (Figure 1.a). Geodesic equations for the BH related to $f(r)$ Figure 1.a, for an impact parameter $b = 2.8r_s$ (Figure 1.b), the geodesics were obtained by solving Eq. (8) and Eq. (9). The values obtained from horizons were; $r_{RNNC} = 0.9277r_s$, $\theta_{RNNC} = 0.0821$, $r_{SWNC} = 0.7926r_s$ and $\theta_{SWNC} = 0.0689$, where r_s is the BH exterior radius.

Using Eq. (1) with the $f(r)$ or the $f(r)$ as a polynomial expansion, the properties $\mu_{ij} = \epsilon_{ij}$ are obtained at each point to mimic the astrophysical phenomenon in a metamaterial. For each simulation (Figure 2) and for the purpose of showing only deflected beams, a square of side $L = 5r_s$ is considered with the absorption properties obtained from the analogy and a circle in the center with absorbency properties that represents the event horizon. The entry of the light beam is controlled so that the ray enters perpendicular to the entry side and with an impact parameter $b = 4r_s$.

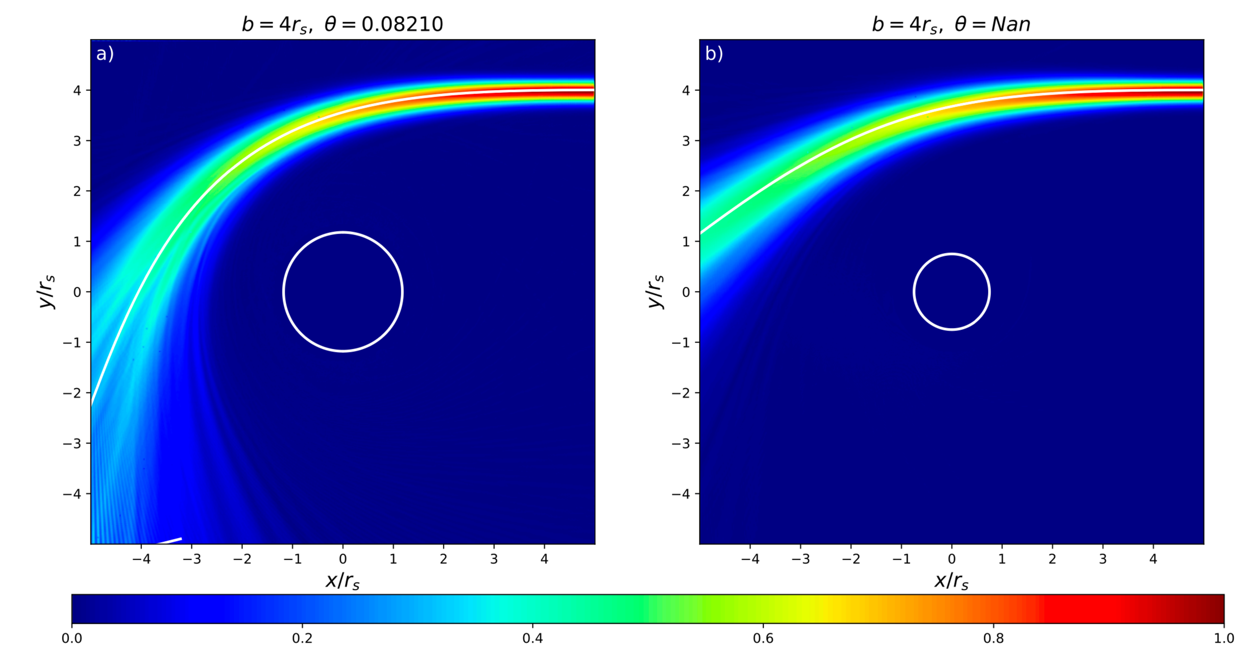

Figure 2: Electric field norm $|\vec{E}|$ corresponding to the propagation of a Gaussian beam for the same impact factor $b = 4rs$. Figure 2.a (NCRN optical BH) and Figure 2.b (RN optical BH) show a comparison between two metamaterials For the simulation parameters $q = m = 1/2r_s$ and $\theta_{RNNC} = 0.0821$ for NCRN BH. The superimposed white line corresponds to the numerical solution for the null geodesics. In this case also Scattering Boundary conditions (SBC) were applied to the four external boundaries.

5. Conclusions

In this work we have presented the optical properties of the metamaterial mimicking a NCRN BH, its equation and solution of null geodesics for comparison with the deflection of a light beam in its corresponding metamaterial. In addition, we show the null geodesic equation and solution of NCS BH in [3]. The main comparison of the work is between the deflection of a light beam in the metamaterial with the corresponding theoretical geodesics (Figure 2). On the other hand, to analyze the effects of non-commutativity it is possible to make a specific comparison of the properties between a standard metric and a non-commutative one (NCRN metric and RN metric) as was done in [3]

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