

# Modeling optical black holes with neural networks and light trajectories

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**Abstract** We study the generation of a dataset of light beam trajectories passing through a 2D metamaterial. The trajectories are generated via numerical simulation with properties  $\epsilon(r) = \mu(r) = \epsilon_0(A(r_s/r)^B)$  where  $A$  and  $B$  are variables that control the curvature of the light beam, as in [4]. Additionally, an extra parameter  $\theta$  is introduced, which is adjusted so that the light beam always enters perpendicularly to the square metamaterial with dimensions  $l = 10r_s$ , where  $r_s$  is the Schwarzschild (S) black hole (BH) event horizon radius. The trajectories are extracted from the data composing each image and represented by 20 points  $(x, y)$  for training a neural network, with the goal of predicting the values of  $A$ ,  $B$ , and  $\theta$  that reproduce the optical trajectories [1, 2].

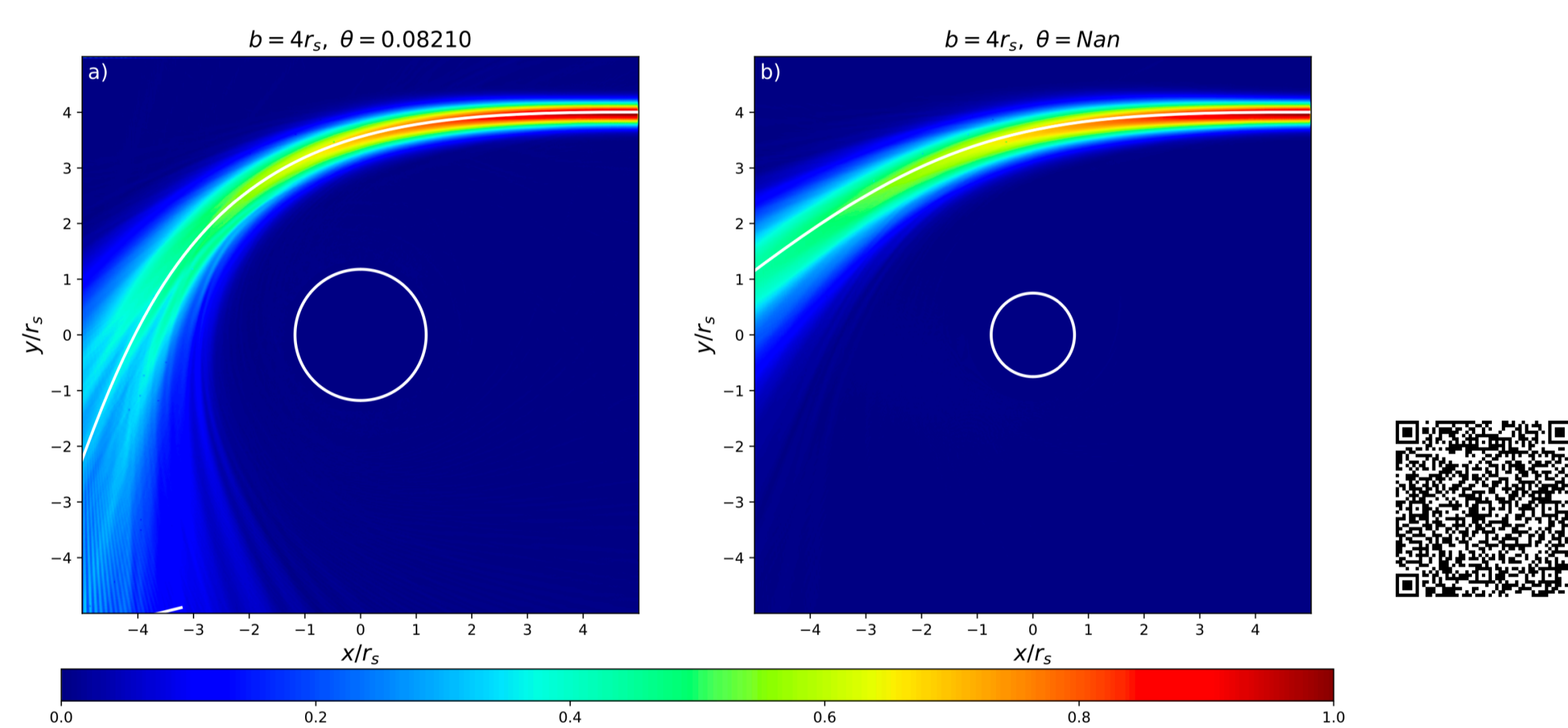
**Resumen** Se estudia la generación de un dataset de trayectorias de un haz de luz atravesando un metamaterial 2D. Las trayectorias se generan mediante simulación numérica con propiedades  $\epsilon(r) = \mu(r) = \epsilon_0(A(r_s/r)^B)$ , donde  $A$  y  $B$  son variables que controlan la curvatura del haz de luz, como en [4]. Además, se introduce un parámetro adicional  $\theta$ , que se ajusta para que el haz de luz siempre entre perpendicularmente al metamaterial cuadrado de dimensiones  $l = 10r_s$ , siendo  $r_s$  el radio del horizonte de eventos de un agujero negro de Schwarzschild. Las trayectorias se extraen de los datos que componen cada imagen y se representan mediante 20 puntos  $(x, y)$  para el entrenamiento de una red neuronal, con el fin de predecir los valores  $A$ ,  $B$  y  $\theta$  que reproduzcan las trayectorias ópticas [1, 2].

## 1. Introduction

The analogy between Maxwell's equations [3] in an anisotropic medium and Maxwell's equations in a vacuum and in a space with curvature is used for the simulations of Reissner–Nordström (RN) and non-commutative Schwarzschild (NCS) optical black holes (OBH) [2]. In order to obtain the medium parameters in the Cartesian coordinate system, we apply a coordinate transformation and we get the permittivity and permeability tensors [3, 1].

$$\epsilon^{ij} = \sqrt{\frac{g_{rr}}{g_{00}}} \left( \delta^{ij} - [1 - f(r)] \frac{x^i x^j}{r^2} \right). \quad (1)$$

With this equation, a material can be characterized from a spacetime metric and mimic the curvature behavior of a photon's trajectory as it passes through that space. For example, for NCRN and RN OBHs, the curvature of the beam as it passes through the metamaterial is shown in Figure 1.



**Figure 1:** Electric field norm  $|\vec{E}|$  corresponding to the propagation of a Gaussian beam for the same impact factor  $b = 4r_s$ . Figure 2.a (NCRN OBH) and Figure 2.b (RN OBH) show a comparison between two metamaterials. For the simulation parameters  $q = m = 1/2r_s$  and  $\theta_{NCRN} = 0.0821$  for NCRN BH. The superimposed white line corresponds to the numerical solution for the null geodesics. In this case also Scattering Boundary conditions (SBC) were applied to the four external boundaries. The QR provides access to the previous poster for reference.

## 2. Preprocessing methodology

For the generation of the training dataset, it is proposed, as in [4], to generate curvature using optical properties that depend on the radius, in the following form:

$$\epsilon(r) = \mu(r) = \epsilon_0 A \left( \frac{r_s}{r} \right)^B, \quad (2)$$

where  $A$  and  $B$  are variables used to generate different trajectories. Additionally, within the simulation model, a parameter is adjusted to ensure that the light beam always enters the metamaterial perpendicularly, as shown in Figure 1.

### 2.1 Important considerations

For the dataset generation, it was initially unknown whether there was any correlation between the variables  $A$ ,  $B$ , and  $\theta$ . Therefore, a small dataset of 50 simulations was generated, where the angle of incidence of the light beam was controlled, as well as its path through specific regions of interest (only deflected beams in which, at the point of maximum deflection, the trajectory never exits through the lower boundary and always exits through the left boundary). In all cases, the light beam entered from a distance of  $4r_s$  from the right boundary of the material.

The model parameters were kept consistent across all simulations (same mesh resolution, same wavelength, and a constant ratio between the mesh size and the wavelength, according to the documentation). Subsequently, using these data and applying a Monte Carlo method, an extended dataset of 300 samples  $(A, B, \theta)$  was generated. This dataset inherently preserved all the above considerations and was used for the numerical simulations.

### 2.2 Correlations between $A$ , $B$ y $\theta$

Before generating the dataset, a correlation analysis between the variables was performed. A significant correlation factor was observed between  $B$  and  $\theta$ , showing that they have a linear relationship. Taking this relationship into

account, the dataset of 300 samples was generated. The other variables show no correlation. Also, the factor  $A$  does not affect the trajectory; it only makes the light beam more concentrated or more dispersed as it passes through the material. In other words Factor  $A$  affects only the spread of the beam and was therefore not used as a target variable.

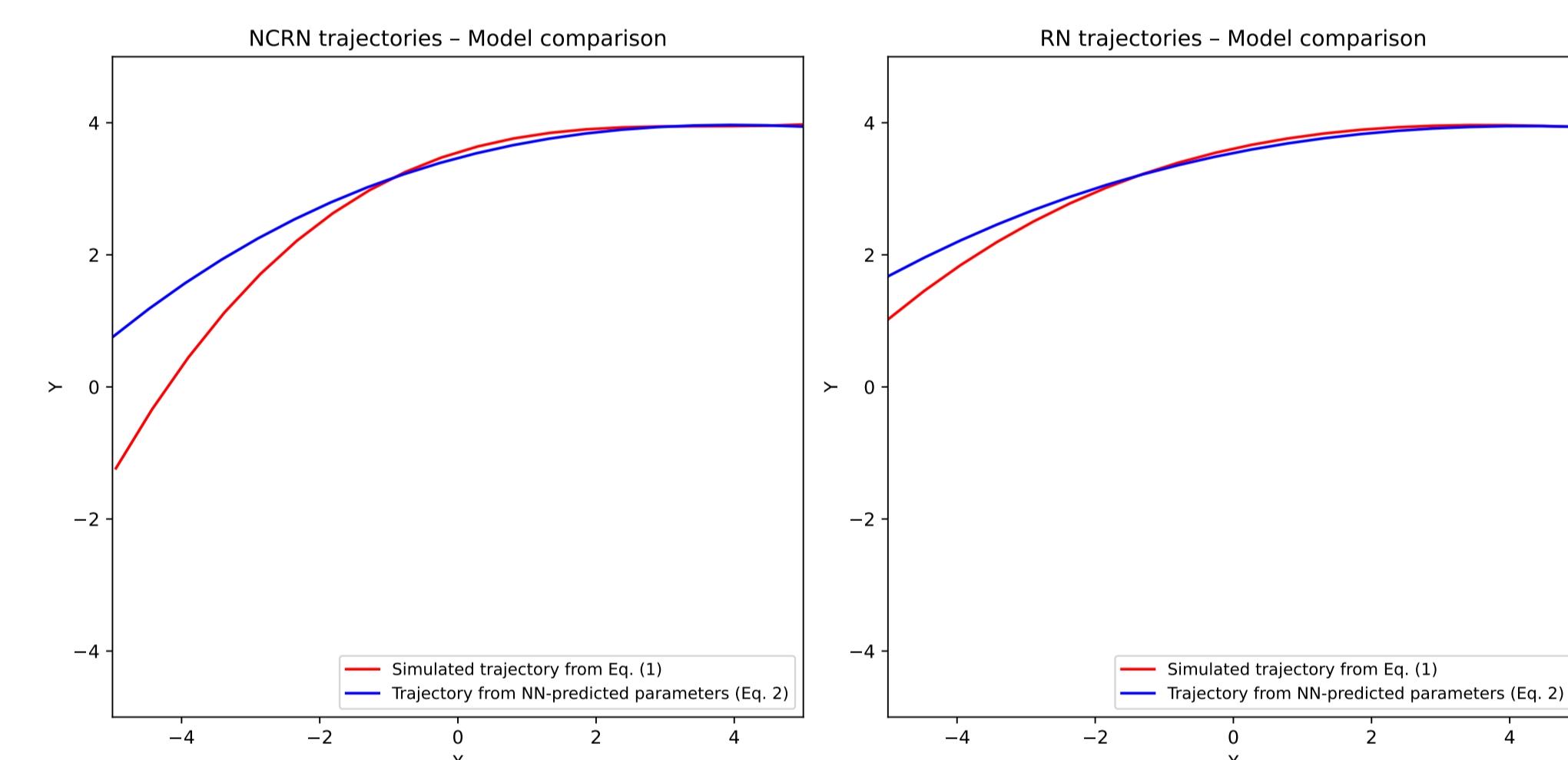
### 2.3 Trajectory extraction process

To obtain the trajectory of the light beam, the data from the image are considered. A threshold for the electric field norm,  $|\vec{E}|$ , is set, and the values of the points above this threshold are extracted. Then, a polynomial regression is performed on this point cloud and evaluated to obtain the trajectory.

## 3. Results

With the considerations mentioned above, the model was trained using only one target variable,  $B$  and we used this value to predict  $\theta$ . Additionally, 20 points of the trajectory  $(x, y)$  were used, resulting in 40 input variables. To avoid overfitting, different configurations of hidden neurons and regularization terms were tested, and the most optimal one was selected: layers (8),  $\alpha = 0.05 \rightarrow R^2 = 0.9294$ , RMSE = 0.2880, MAE = 0.1484.

To determine how separated the trajectories are, the mean squared error (MSE) of the distance between points in  $Y$  with the same  $X$  is calculated for the red and blue lines, obtaining  $MSE_{NCRN} = 0.397025$  and  $MSE_{RN} = 0.040191$ .



**Figure 2:** Visual comparison of the trajectory obtained from the simulation using Eq.1 for the NCRN OBH and RN OBH against the trajectories obtained from the model prediction. It is important to note that the points of the red lines are used as input to the model to predict the target variable  $B$  and subsequently predict  $\theta$  in order to obtain the blue trajectories from Eq.2

## 4. Conclusions

The value of  $MSE_{NCRN}$  indicates a significant deviation, in contrast to  $MSE_{RN}$ , which is visually shown in Figure 2. This discrepancy is likely due to the limited size of the training dataset and the choice of the target variables, as the model was trained using only one target variable instead of three due to their characteristics. Moreover, it is important to note that capturing the full complexity of the physical phenomenon with a single target variable is challenging. For future simulations and minor trajectory adjustments, additional parameters will be introduced in the form  $\epsilon(r) = \mu(r) = \epsilon_0(r_s/r)^B + C(r) + D$ . Additionally, by increasing the training dataset size, it will be possible to apply a different configuration of hidden layers and train more complex neural networks.

## References

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