Stellar Astrophysics Project

Marco Baroni

Contents

Chapter 1

Introduction

A White Dwarf is one of the more well-known and most standard object in the Universe. They all have a mass below a threshold value of $1.435M_{\odot}$, named Chandrasekhar's mass, and a radius well inscribed in a restricted value. Moreover, they are known to not irradiate through thermonuclear reactions, but instead through a simple thermal compression mechanism, but without changing their volume, mass and composition in any appreciable way.

Given this, a White Dwarf is, at least theoretically, a "standard candle" object for age detection. If a White Dwarf is born from a star that did not underwent particularly disrupting events, it will be born with an almost standard physical characterization (mass, radius, luminosity, temperature and composition) and, like previously said, given that the mass, the radius and the composition are almost constant over time, the only changing features will be luminosity and temperature (e.g. magnitude and color).

In this work we present a study of the age of the globular cluster NGC 6397 (see figure 1.1). The study focuses on the WD branch of the CMD, proposing to analyze the WDs singularly at the point of calculate their ages and thus finding a possible age values of the cluster.
Firstly we corrected the CMD for reddening and extinction and then we changed band from the HST F606W and F814W bands to the Johnson-Cousin system bands V and I. Then, using a theory derived approach we calculated the physical parameters of the WDs. Finally, we calculated the cooling age of each WD using the Mestel's Law and the approximate Main Sequence progenitor age to derive the total age of each WD.

Figure 1.1: HST image of the globular cluster NGC6397

Chapter 2

Creating the dataset

The dataset that we have at disposal is made with the Hubble space Telescope, the magnitude of the objects is given by the F814W filter (analogous to Johnson-Cousin I band), while the color index is given by (F606W - F814W), analogous to the (V-I) color index.
From (Blanco, 1964) we know that the E(V-I) extinction value is given by:

$$E(V-I) = 1.2 \cdot E(B-V) \tag{2.1}$$

And the extinction for NGC6397 is E(B-V) = 0.18. From (Hansen et al. 2007) we know that the distance module is:

$$\mu_0 = 12.13 \pm 0.15 \implies d = 10^{\frac{\mu_0+5}{5}}$$

Having this, we can evaluate the reddening value from the distance module formula, obtaining:

$$R_V = \mu_0 + 5 - 5log_{10}(d) \tag{2.2}$$

Now, having all of this we can obtain the absolute magnitude in F814W and the corrected color index:

$$\begin{cases} M_{F814W} = m_{F814W} - 5log_{10}(d) + 5 - R_V \\ (F606W - F814W)_0 = (F606W - F814W) - E(V-I) \end{cases} \tag{2.3}$$

The associated errors are the accuracy of the HST filter:

$$\Delta m_{F814W} = \Delta(F606W - F814W) = 0.01$$

Then, using the relations from (Harris, 2018) we can obtain the color and magnitude in the Johnson-Cousin system, calling (F606W-F814W)$_0$ = c_{HST}:

$$(V-I)_0 = \begin{cases} \frac{c_{HST}}{0.850 \pm 0.010} & c_{HST} \leq 0.68 \\ \\ \frac{c_{HST}-(0.091\pm0.010)}{0.737\pm0.031} & 0.68 < c_{HST} < 0.975 \\ \\ \frac{c_{HST}+(0.039\pm0.010)}{0.845\pm0.031} & c_{HST} \geq 0.975 \end{cases} \tag{2.4}$$

$$m_I = \begin{cases} F814W & (V-I)_0 \leq 1.2 \\ \\ F814W - (0.130 \pm 0.002) + (0.108 \pm 0.003) \cdot (V-I)_0 & (V-I)_0 > 1.2 \end{cases} \tag{2.5}$$

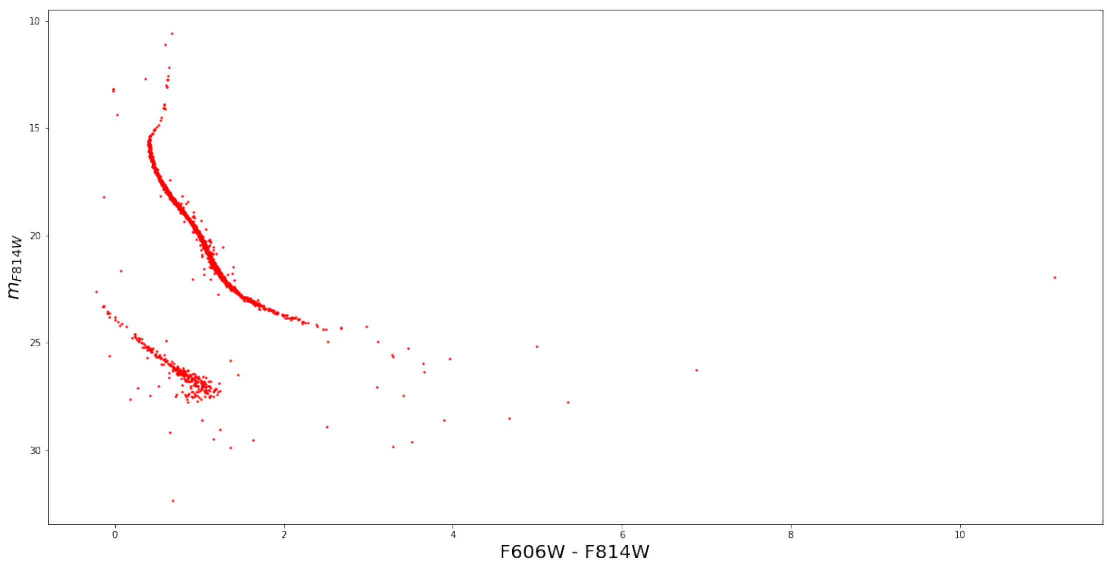

Figure 2.1: Color-Magnitude diagram of NGC 6397 corrected by extinction and reddening.

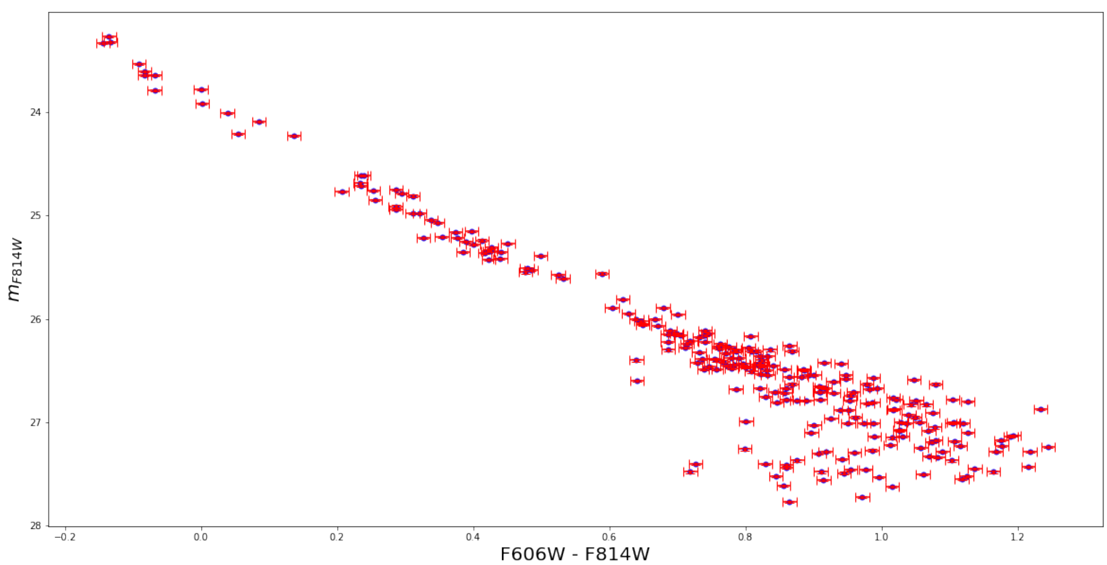

Figure 2.2: Color-magnitude diagram of WDs in NGC 6397 in HST photometric system.

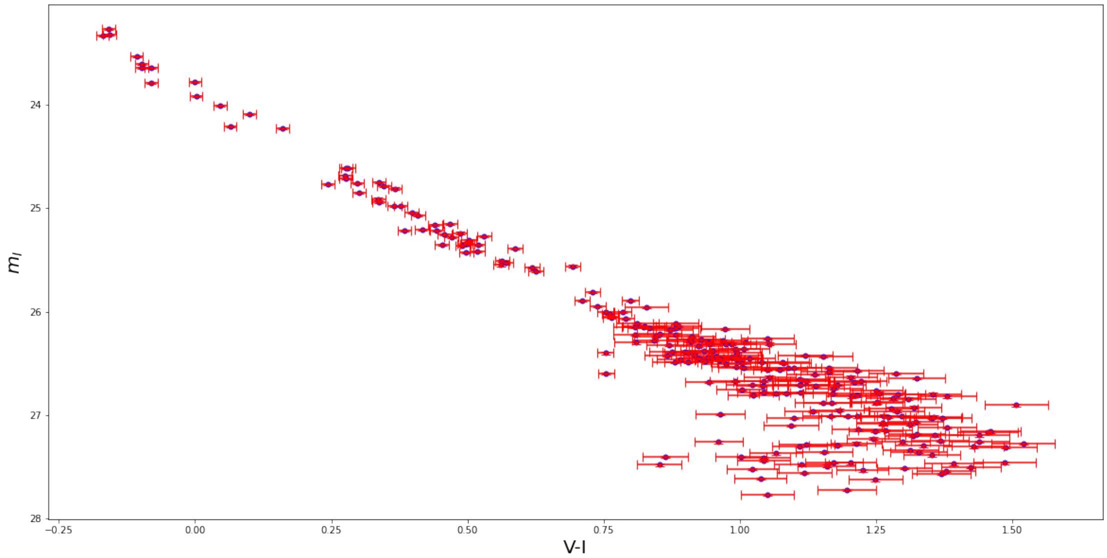

Figure 2.3: Color-Magnitude diagram of WDs in NGC 6397 in the BVI photometric system.

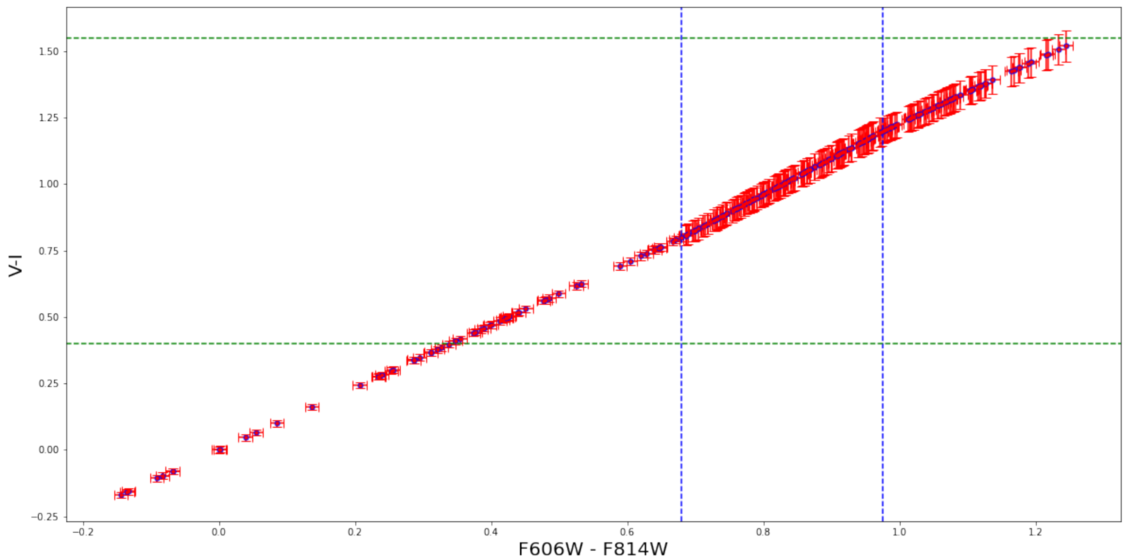

Figure 2.4: Color-color diagram of the correction between the HST colors and BVI colors. The two vertical lines are the slope change points of the relation. The two horizontal lines are the range of validity for the color-temperature relation.

Chapter 3

Physical Parameters of White Dwarfs

We evaluate the effective temperature of the WDs using the relation from (Houdashelt and Bell, 1999):

$$T = (9058.78 \pm 166.72) - (6152 \pm 403.46) \cdot (V - I)_0 + (1987.84 \pm 217.38) \cdot (V - I)_0^2 \tag{3.1}$$

Valid only for $0.4 \leq (V - I)_0 \leq 1.5$.
And we find the radius inverting the relation used in (Hansen et al., 2007):

$$R = 10^{7+\frac{11.63+\mu_0+2.77(V-I)_0-m_I}{5}} \tag{3.2}$$

Then, in black body approximation, the luminosity of the WDs will be:

$$L = 4\pi R^2 \sigma T^4 \tag{3.3}$$

Where $\sigma = 5.67 \cdot 10^{-8}$ W $K^{-4} m^{-2}$ is the Stefan-Boltzmann constant.
After, we obtain the mass of each WD from polytropic models, averaging between a polytrope index n = 1.5 for a degenerate Fermi gas to n = 3 (limiting case) for a degenerate relativistic gas, thus obtaining a mass-radius relation (in Solar units):

$$R = \frac{0.0126}{M_{WD}^{\frac{1}{3}}} \sqrt{1 - \left(\frac{M_{WD}}{M_{Ch}}\right)^{\frac{4}{3}}} \tag{3.4}$$

Where $M_{Ch} = 1.435 M_{\odot}$ is the Chandrasekhar's mass.
By inverting this relation is possible to find the mass of the White Dwarf (always in Solar units):

$$M_{WD} = M_{Ch} \left[\frac{\sqrt{R^4 M_{Ch}^{\frac{4}{3}} + 4} - R^2 M_{Ch}^{\frac{2}{3}}}{2 \cdot (0.0126)^2} \right]^{\frac{3}{2}} \tag{3.5}$$

Lastly, we obtained the mass of the progenitor star by using the relation from K.A.Williams (always in Solar units):

$$M_{prog} = \frac{M_{WD} - 0.358}{0.123} \tag{3.6}$$

The temperature relation (2.1) and the progenitor mass relation (2.6) each creates a gap in the total number of fully analyzed white dwarfs, because of the cutoff values they impose. Specifically, the temperature relation takes into account only a piece of the CMD and the progenitor mass relation does not take care of white dwarfs with mass below the threshold value of $0.358 M_{\odot}$.
In our case only the color-temperature relation diminished the total number of fully analyzed white dwarfs, from **249** to **206**, so we have a net loss of 17.27 % of the sample.
Having the mass of the progenitor we can also estimate the compositional type of white dwarf that we observe, thus permitting us to have estimation about the mass number of the WD's envelope and core, knowing that, meanly:

- $M_{prog} < 0.5 M_{\odot}$: **He-WD**, so A = 4, $\mu = 1$;
- $0.5 M_{\odot} < M_{prog} < 8 M_{\odot}$: **CO-WD**, so A $\approx$ 14 (mean value of A_C and A_O), $\mu = 4$;
- $8 M_{\odot} < M_{prog} < 10 M_{\odot}$: **NeMg-WD**, so A $\approx$ 22 (same as before), $\mu = 14$.

From this further discrimination we see that we have, from the 206 totally analyzed WDs a number of:

- **20 He-WDs**, accounting for the 9.71 % of the fully analyzed sample;
- **186 CO-WDs**, accounting for the 90.29 % of the fully analyzed sample;
- **0 NeMg-Wds**, accounting for the 0 % of the fully analyzed sample.

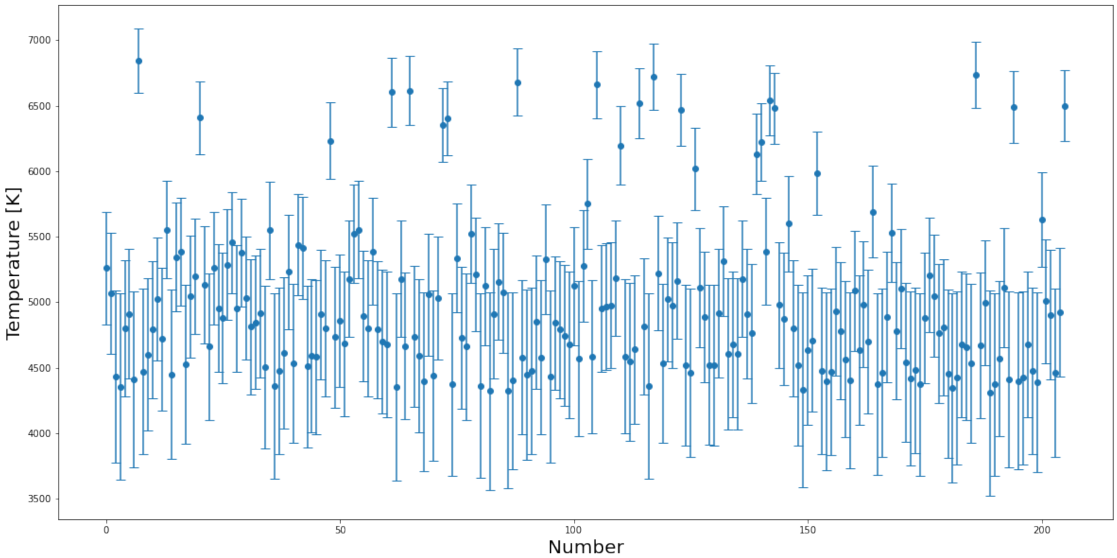

Figure 3.1: Graph of the temperature values of the WDs.

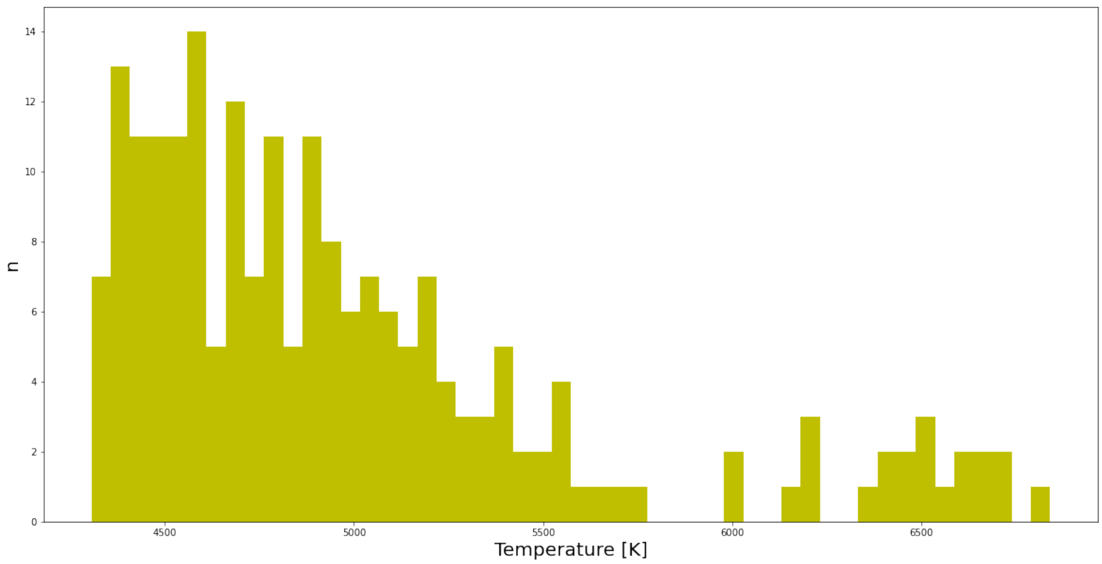

Figure 3.2: Mean temperature distribution of the WDs.

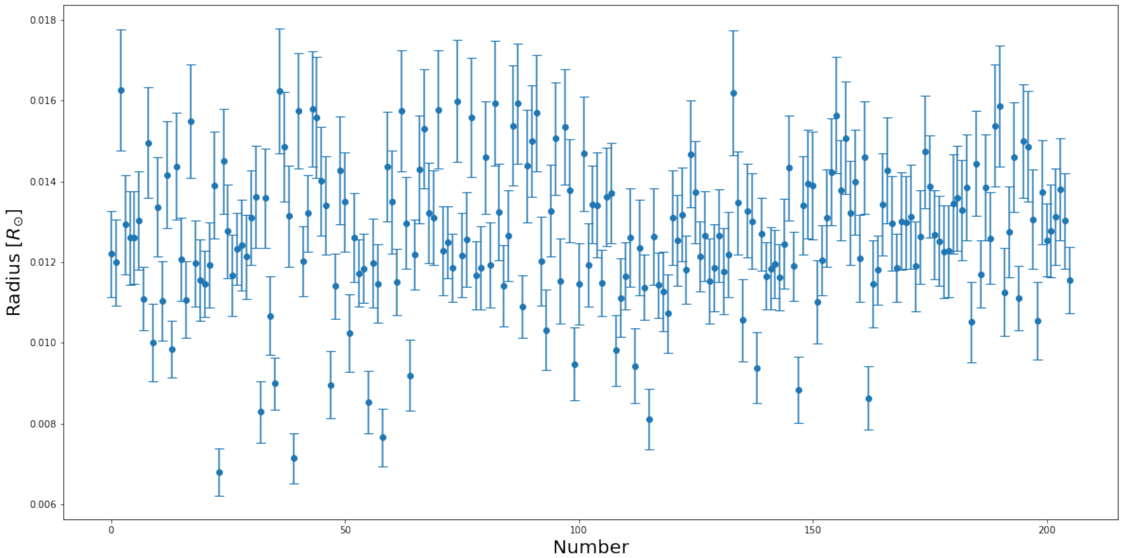

Figure 3.3: Graph of the radius values of the WDs.

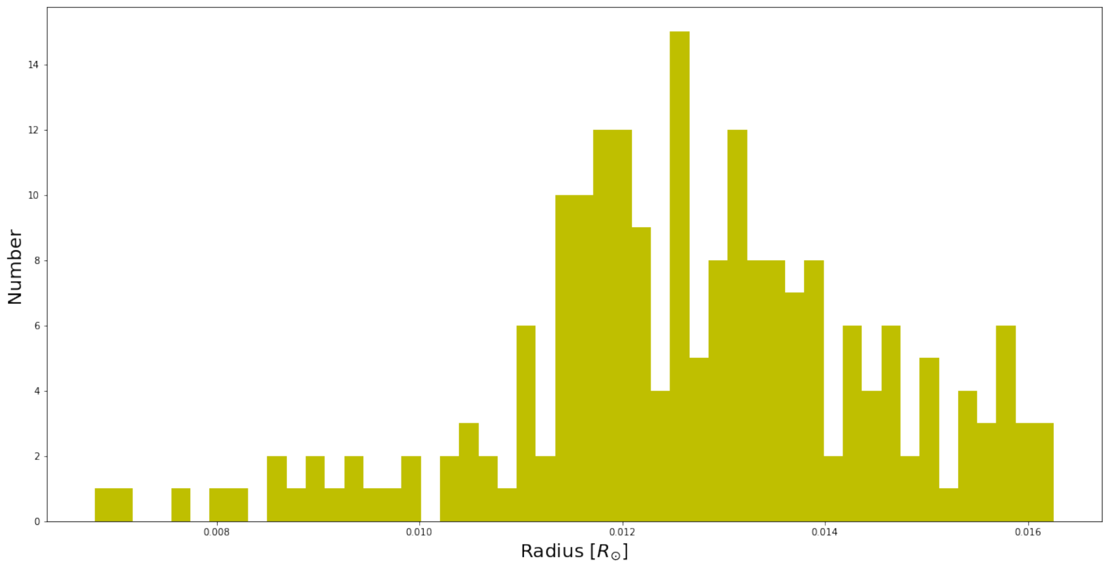

Figure 3.4: Mean radii distribution of the WDs.

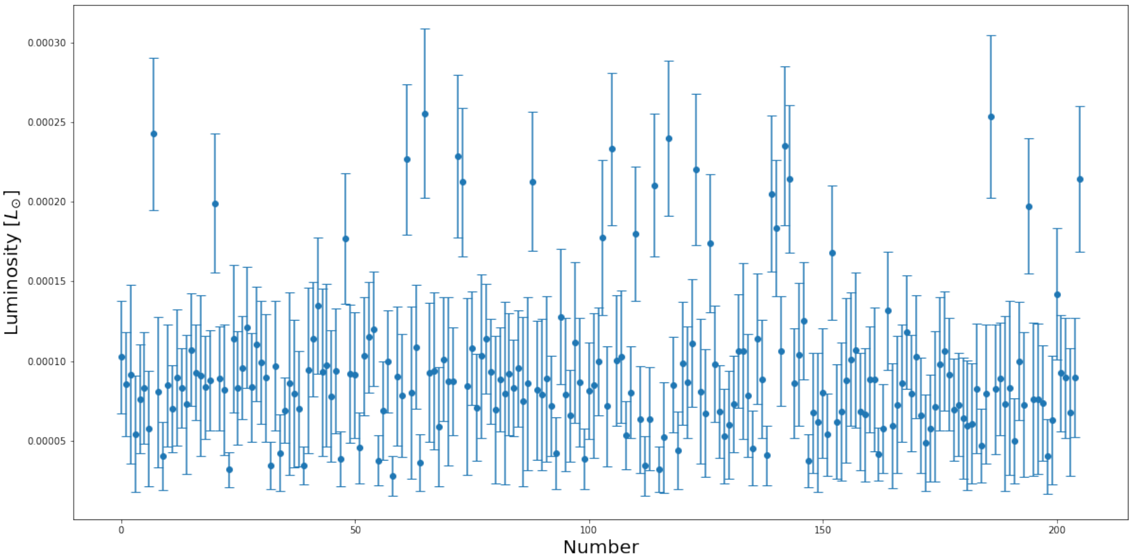

Figure 3.5: Graph of the luminosity values of the WDs.

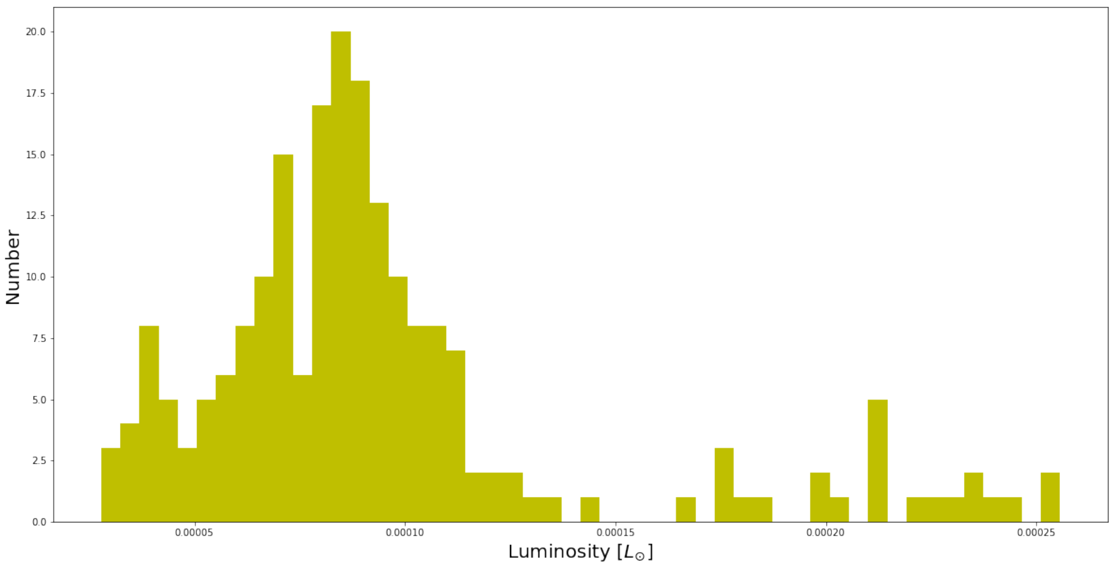

Figure 3.6: Mean luminosity distribution of the WDs.

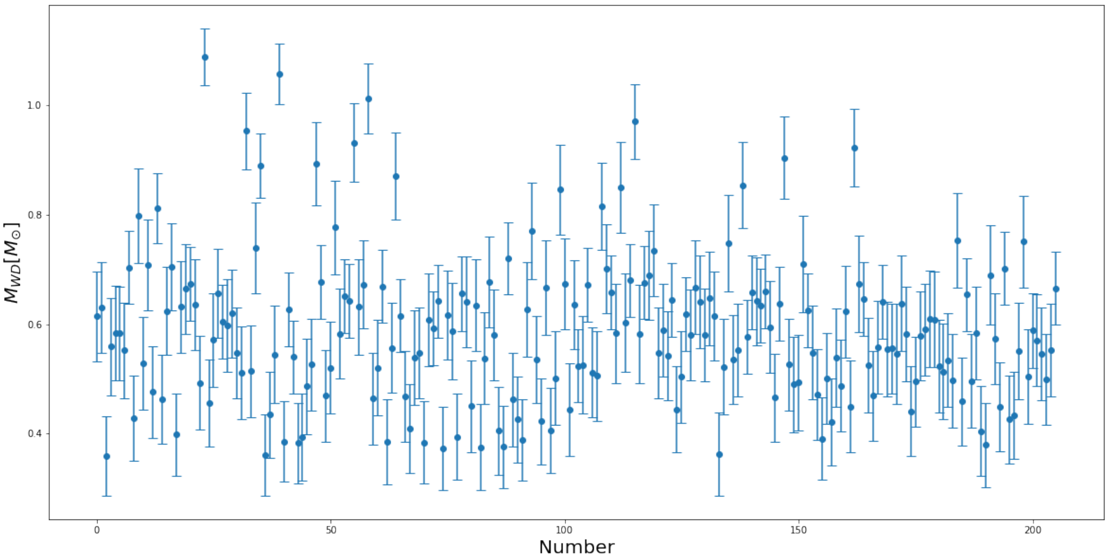

Figure 3.7: Graph of the mass values of the WDs.

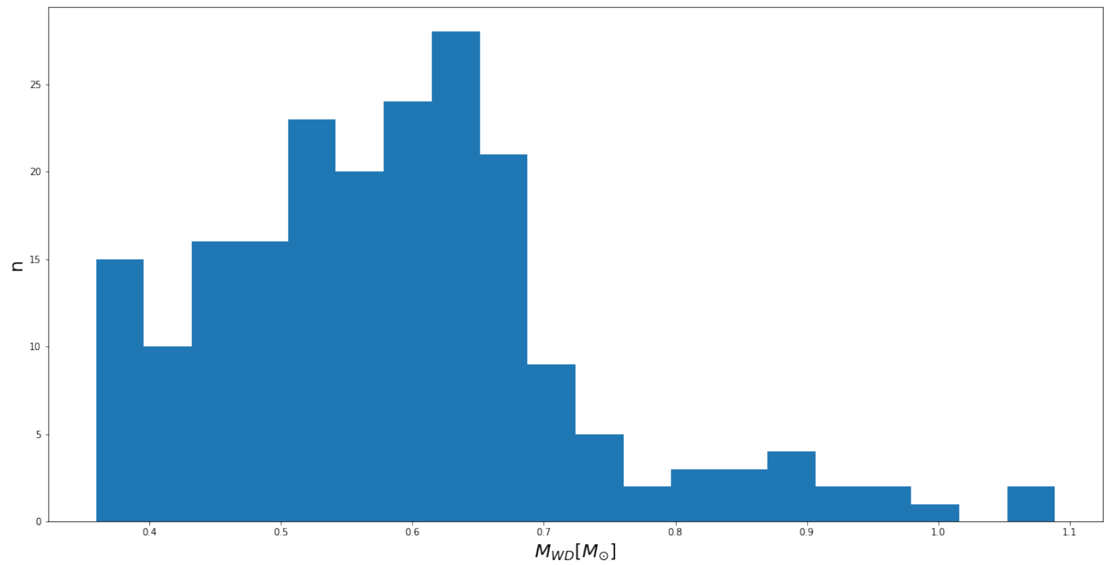

Figure 3.8: Mean masses distribution of the WDs.

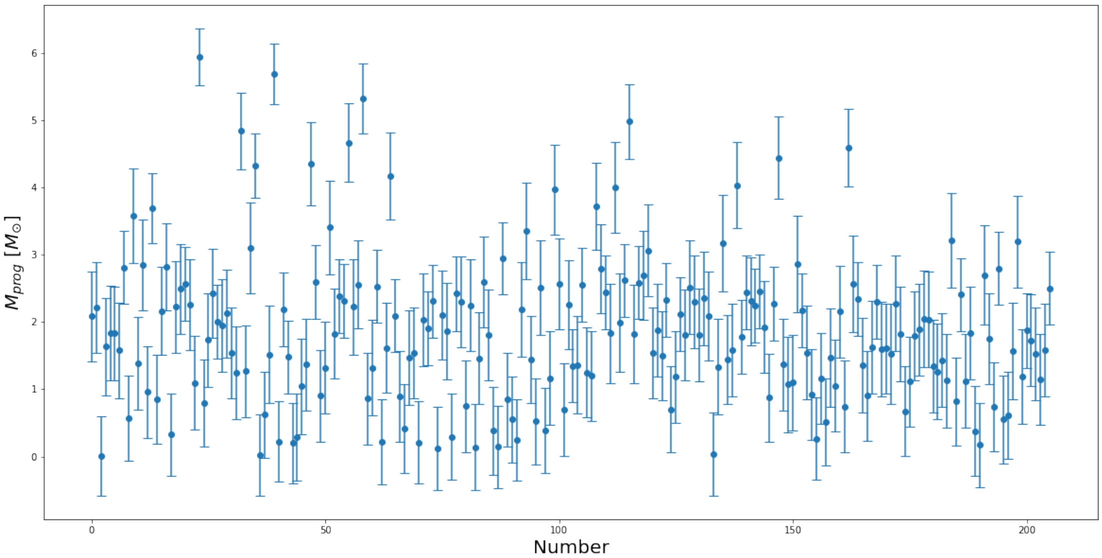

Figure 3.9: Graph of the progenitor mass values of the WDs.

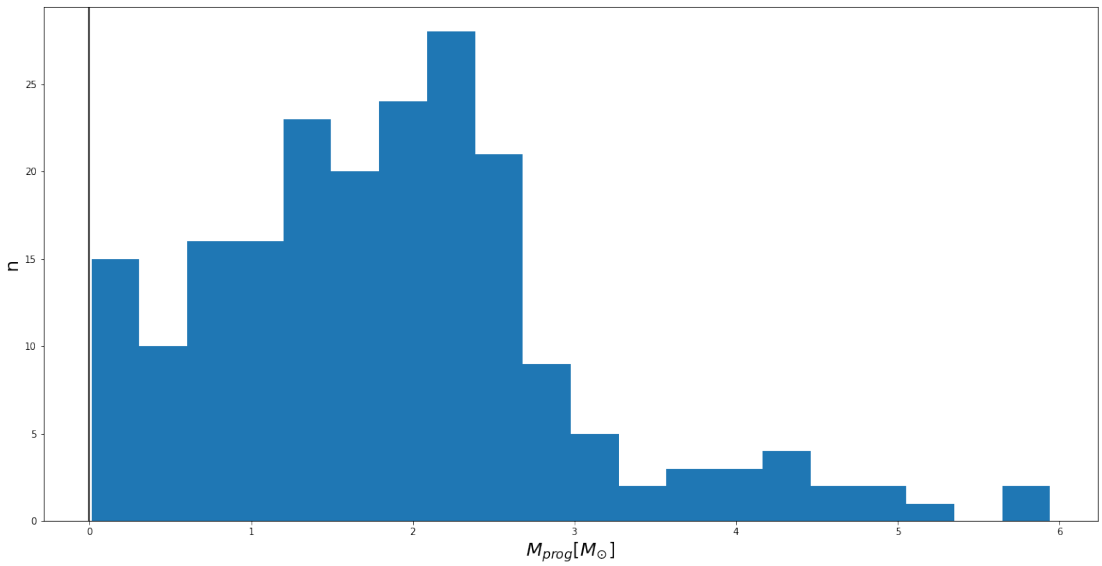

Figure 3.10: Mean progenitor mass distribution of the WDs.

Chapter 4

Age Calculation

The total age of a WD is given by the cooling age and the time for which its progenitor remained in Main Sequence, so:

$$t_{tot} = t_{cool} + t_{MS} \tag{4.1}$$

The cooling time can be calculated from the Mestel's Law:

$$t_{cool} = 0.0088 \cdot \frac{12}{A} \cdot \left(\frac{2}{\mu}\right)^{\frac{2}{7}} \cdot \left(\frac{M_{WD}}{L_{WD}}\right)^{\frac{5}{7}} \quad \text{Gyrs} \tag{4.2}$$

Where A is the mean mass number of the inside of the WD and μ is the mean mass number of the atmosphere of the WD.
The Main Sequence age of the progenitors is derived from (Iben and Tutukov, 1987):

$$t_{MS} = \frac{10}{M_{prog}^{\frac{5}{2}}} \quad \text{Gyrs} \tag{4.3}$$

After these calculations, we had to put some constraints to take into account the possible presence of outliers. To do so we choose a progressive value of threshold h from -10 to 0 at integer steps, so that:

$$t_{tot} - \sigma_{t_{tot}} \geq h \tag{4.4}$$

After we have chosen this threshold, we did the mean value of the ages, knowing that we have a clustering of ages in the near past. This is due to a simple selection effect, so that younger WDs are generally hotter, and so brighter, and are more easily detected. After the mean value we chosen only the WDs that respect 4.4, and then we did the median value of those WDs to find the age at the given threshold.
We choose -10 as last threshold value because for any h < -10 we have found that there are not possible values to find the age of the cluster.

Following there is the graph of the cooling ages, we did not included the main sequence ages graph because of the presence of the outliers (for some examples with h = -1,.5 and -10 see figure A.4, A.5 and A.6).

Table 4.1: Summarizing table of the different results for each threshold value. The percentage of outliers for the He-WD class is 100 % for each chosen threshold.

h [Gyrs]	**%CO outliers**	**%totally analyzed WDs**	**Median Age [Gyrs]**	**Number of old WDs**
0	40.86	53.40	10.86 ± 0.66	7
-1	27.42	65.53	12.02 ± 0.32	4
-2	21.51	70.87	13.19 ± 0.59	3
-3	19.35	72.82	12.90 ± 0.30	3
-4	16.67	75.24	12.90 ± 0.30	3
-5	14.52	77.18	13.54 ± 1.06	2
-6	13.44	78.16	13.54 ± 1.06	2
-7	12.90	78.64	13.75 ± 1.27	2
-8	12.90	78.64	13.75 ± 1.27	2
-9	12.37	79.13	12.78 ±0.97	3
-10	12.37	79.13	12.78 ± 0.97	3
< -11	×	×	×	0

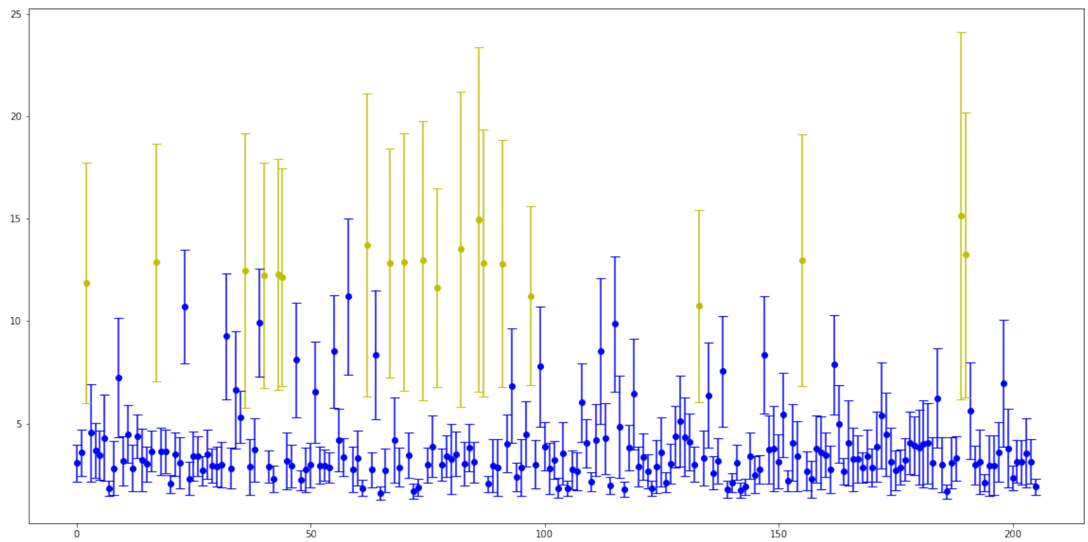

Figure 4.1: Computed cooling ages. In yellow the He-WDs, in blue the CO-WDs.

Chapter 5

Conclusions

From our analysis we inferred a total age of **12.90 ± 0.64 Gyrs**, while the age inferred by Hansen et al. 2007 is 11.47 ± 0.47 Gyrs. While our age is not in accord with the age derived by Hansen et al. 2007, it is perfectly in accord with other estimation, such as Correnti et al. 2018, whom obtained a value of 12.60 ± 0.7 Gyrs.

A better age detection could be obtained using more detailed models for the white dwarf cooling age and for the main sequence age of the progenitor, but, ultimately, this further implementation should give just a better estimation by diminishing the error of the total age, giving to our estimation a certain degree of importance. Moreover, other, more detailed IFMR, could be chosen, but theoretically they should just lead to more refined conclusions and not to completely different ones.

It is important to notice that for any threshold value, each He-WD resulted as a outlier and it is only the number of CO-WDs outliers that decreases, moreover, below the threshold -10 Gyrs, there are too much huge errorbars, thus effectively imposing a limitation on our method, that in all those cases cannot give any results, because it does not find any value that respect (4.4).

Moreover, the CO-WDs that were in any case found as outliers lie in the part of the CMD near the separation between the WD branch and the zone in which binaries MS-WD lies (for some examples with h = -1,-5 and -10 see figure A.1, A.2 and A.3). This give rise to the hypothesis that these WDs maybe are binary systems of a WD and a faint red dwarf main sequence star or, maybe, a huge brown dwarf.

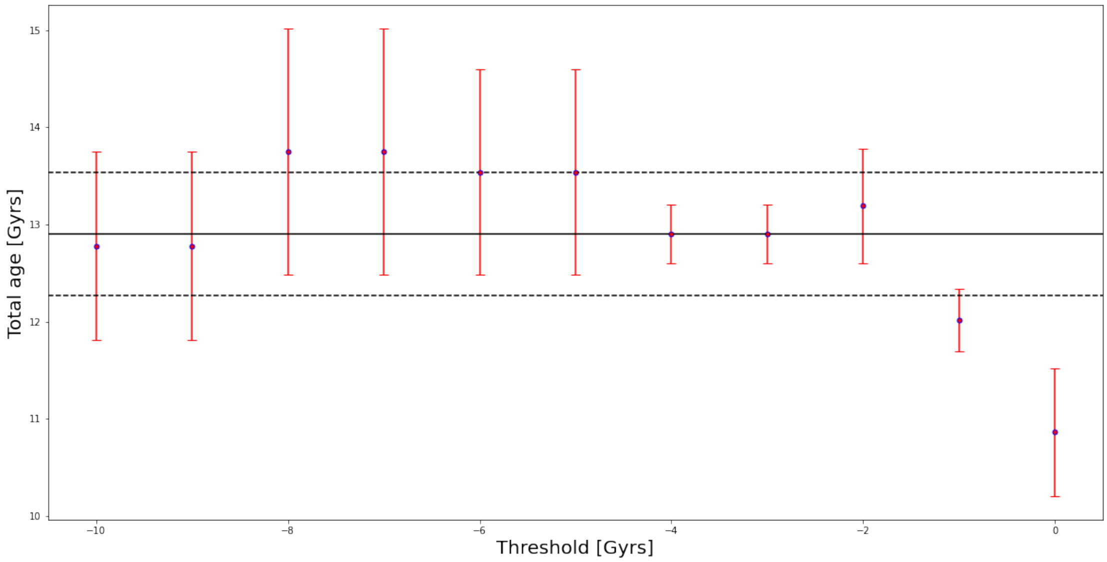

Figure 5.1: Plot of the different cluster age for different threshold values. The solid line is the median of the ages, while the two dotted lines are the upper and lower median error.

Appendix A

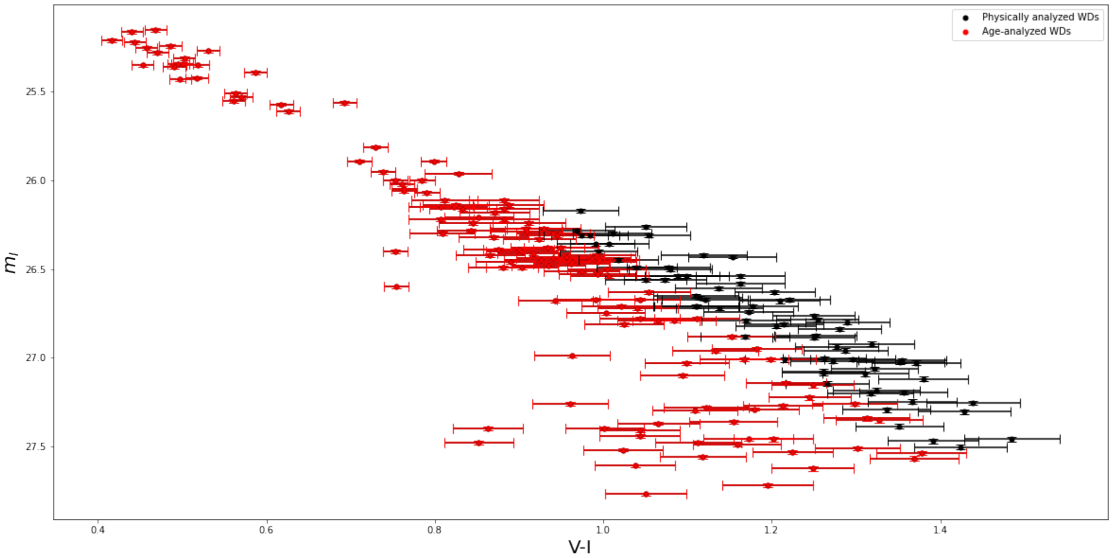

Figure A.1: Example of the CMD of the time-analyzed WDs. The age analysis contains also the physical analysis. Threshold = -1 Gyrs.

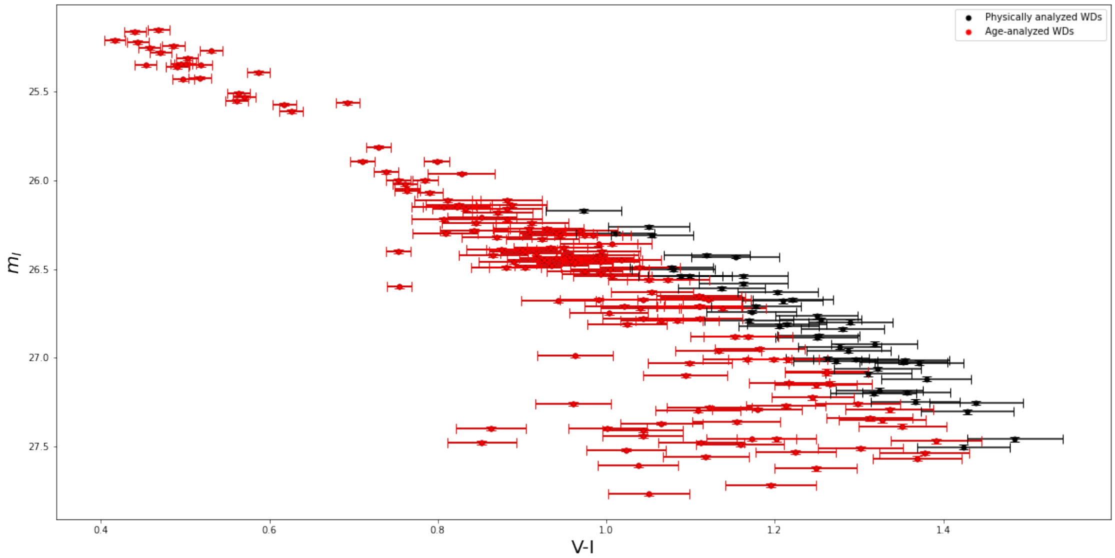

Figure A.2: Example of the CMD of the time-analyzed WDs. The age analysis contains also the physical analysis. Threshold = -5 Gyrs.

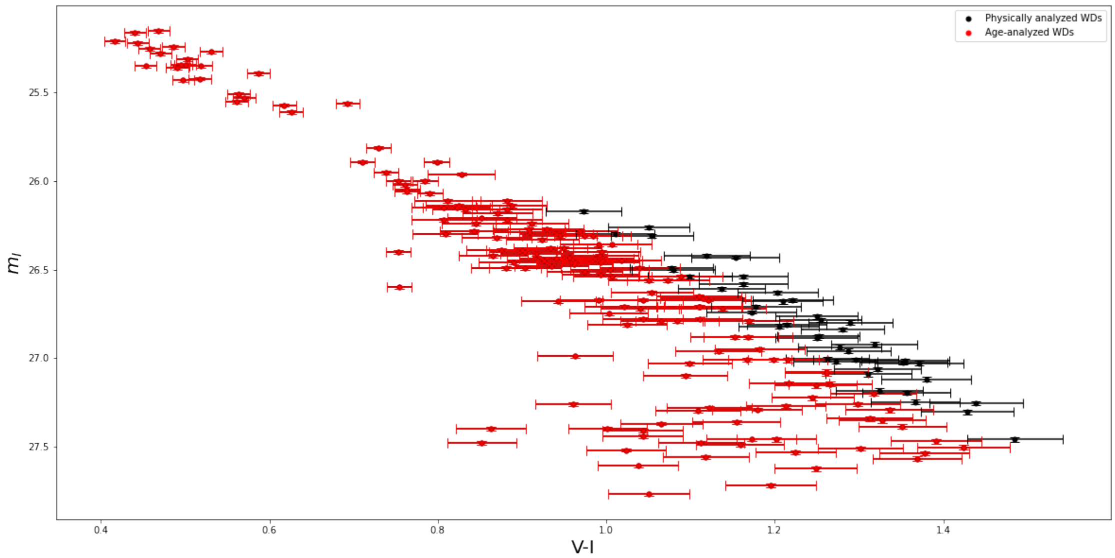

Figure A.3: Example of the CMD of the time-analyzed WDs. The age analysis contains also the physical analysis. Threshold = -10 Gyrs.

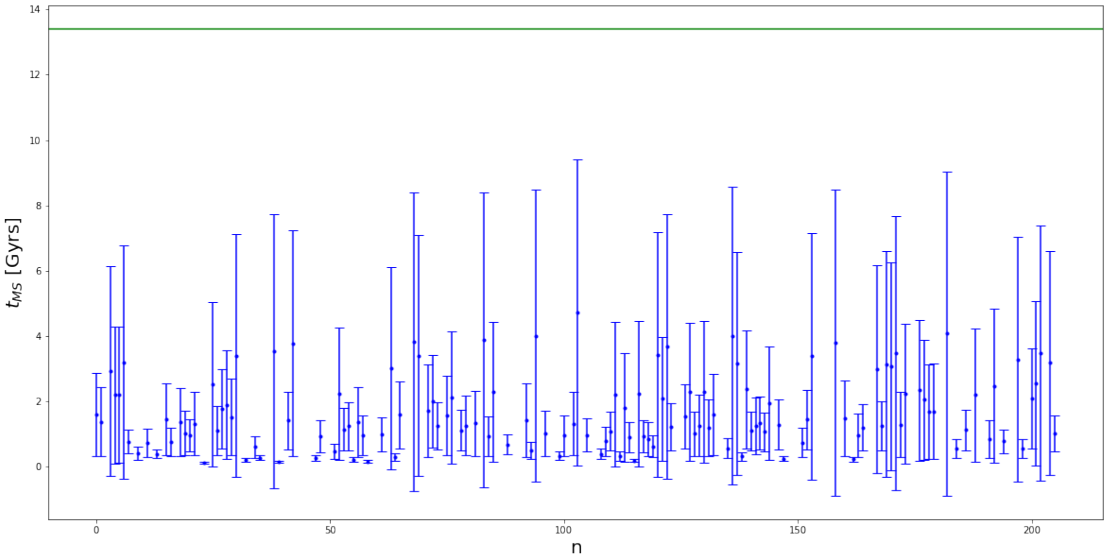

Figure A.4: Computed progenitor ages. In this graph we have deleted the outliers. All the values are colored in blue because all the He-WDs progenitors are classified as outliers. Threshold = -1 Gyrs.

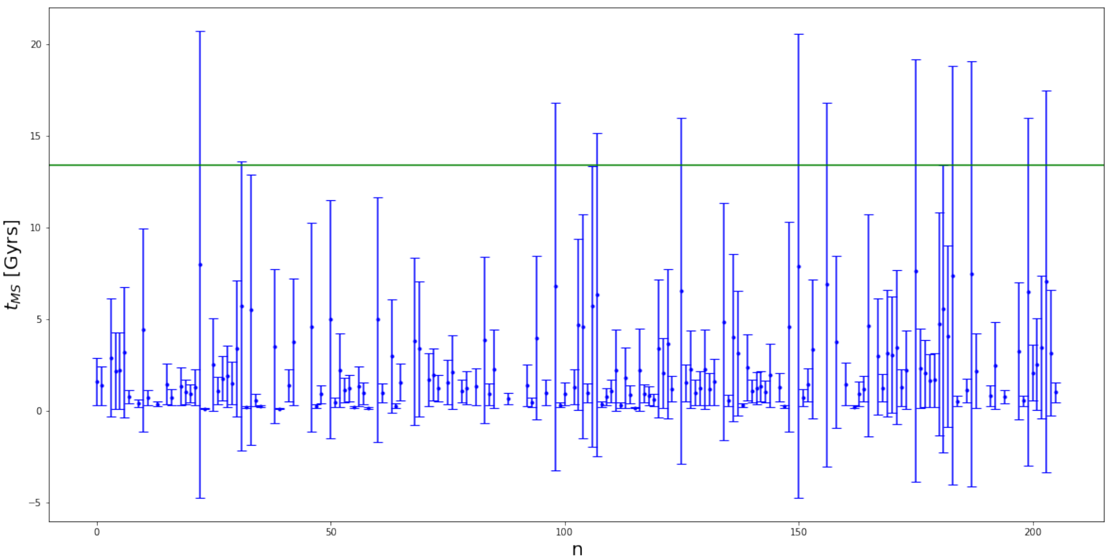

Figure A.5: Computed progenitor ages. In this graph we have deleted the outliers. All the values are colored in blue because all the He-WDs progenitors are classified as outliers. Threshold = -5 Gyrs.

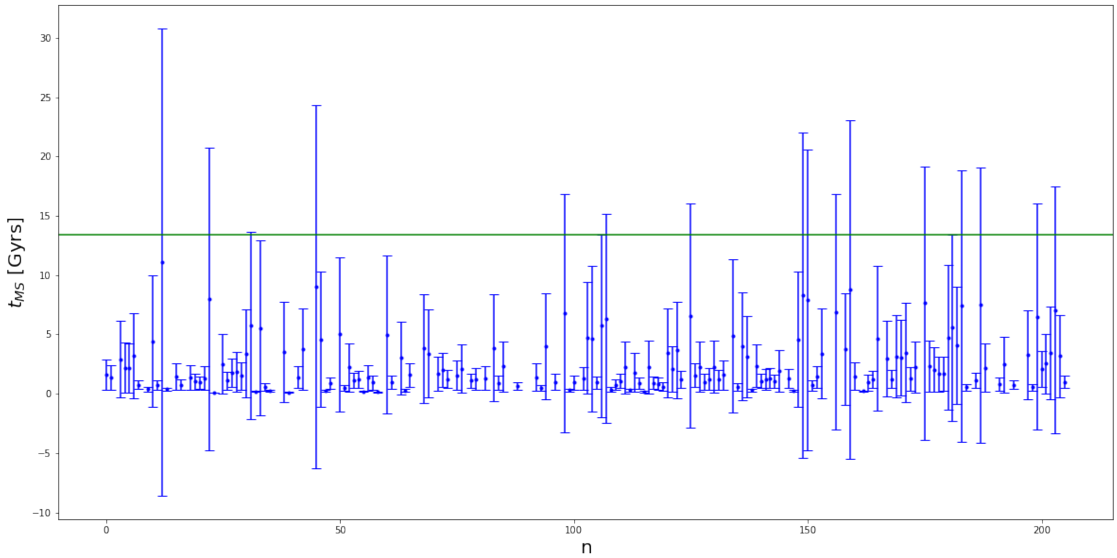

Figure A.6: Computed progenitor ages. In this graph we have deleted the outliers. All the values are colored in blue because all the He-WDs progenitors are classified as outliers. Threshold = -10 Gyrs.

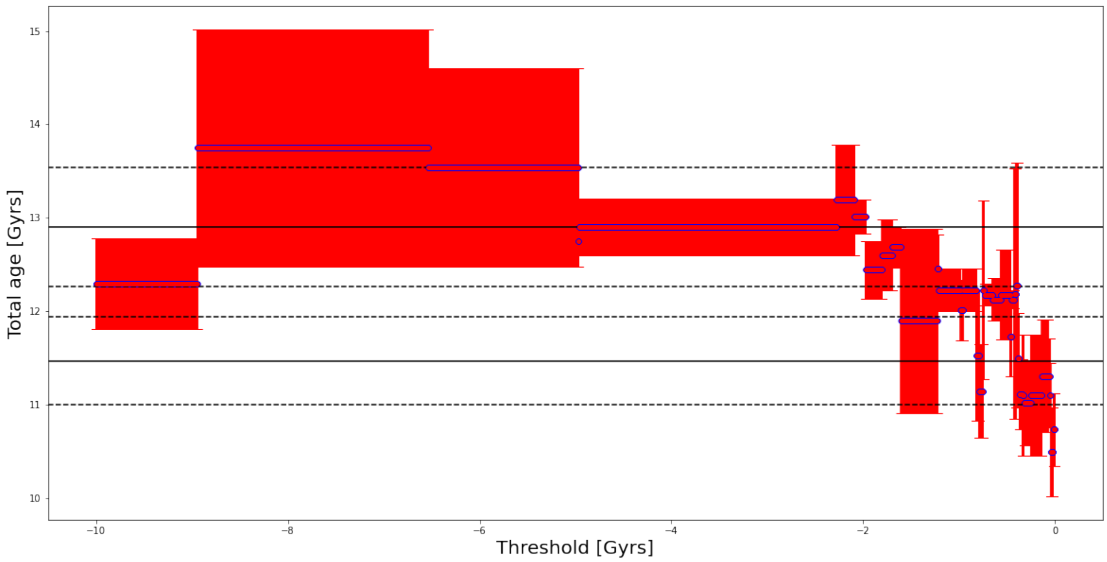

Figure A.7: Extreme example of the threshold method. Here we chose h ∈ [-10 , 0] Gyrs, but at intervals of 0.001 Gyrs. We can see that the median age does not change at all even if the interval between thresholds becomes this small. The second solid black line with the two dotted lines is the age given by Hansen et al. 2007.

Bibliography

[1] Hansen et al. 2007 , *The White Dwarf cooling sequence of NGC 6397*

[2] Kurtis A. Williams , *The White Dwarf initial-final mass relation*

[3] Harris 2018 , *Transformation of HST WFC3/UVIS Filters to the Standard BVI* System

[4] Correnti et al. 2018 *The Age of the Old Metal-poor Globular Cluster NGC 6397 Using WFC3/IR Photometry*

[5] Aricò 2014, *Materia Degenere:Fisica e Applicazioni Astrofisiche*

[6] Houdashelt et al. 1999 , *Improved Color–Temperature Relations and Bolometric Corrections for Cool Stars*