Respuestas Lab 2

Otoño 2010

1.a) $$\lim_{x\to -3} \frac{x^2+4x+3}{x^2-9} = \lim_{x\to -3} \frac{(x+3)(x+1)}{(x+3)(x-1)} = \lim_{x\to -3} \frac{x+1}{x-1} = \frac{-2}{-4} = \frac{1}{2}$$

1.b) $$\lim_{x\to 4} \frac{\frac{1}{4}-\frac{1}{x}}{4-x}$$

$$= \lim_{x\to 4} \frac{\frac{x-4}{4x}}{4-x} = \lim_{x\to 4} \frac{x-4}{4x(4-x)}$$

$$= \lim_{x\to 4} \frac{(x-4)}{-4x(x-4)} = \lim_{x\to 4} \frac{-1}{4x} = -\frac{1}{16}$$

1.c) $$\lim_{x\to 1} \frac{\sqrt{2x-1}-1}{x-1}$$

$$= \lim_{x\to 1} \frac{(\sqrt{2x-1}-1)}{x-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2x-1}+1}\right) = \lim_{x\to 1} \frac{2x-1-1}{(x-1)(\sqrt{2x-1}+1)}$$

$$= \lim_{x\to 1} \frac{2x-2}{(x-1)(\sqrt{2x-1}+1)} = \lim_{x\to 1} \frac{2(x-1)}{(x-1)(\sqrt{2x-1}+1)} = \lim_{x\to 1} \frac{2}{\sqrt{2x-1}+1}$$

$$= \frac{2}{\sqrt{1}+1} = \frac{2}{2} = 1$$

1.d) $$\lim_{x\to 2} \frac{x^4-16}{8-x^3}$$

$$= \lim_{x\to 2} \frac{(x^2+4)(x^2-4)}{(2-x)(4+2x+x^2)} = \lim_{x\to 2} \frac{(x^2+4)(x+2)(x-2)}{(2-x)(4+2x+x^2)}$$

$$= \lim_{x\to 2} \frac{-(x^2+4)(x+2)(2-x)}{(2-x)(4+2x+x^2)} = \lim_{x\to 2} \frac{-(x^2+4)(x+2)}{4+2x+x^2}$$

$$= \frac{-(4+4)(2+2)}{4+4+4} = -\frac{32}{12} = -\frac{16}{6} = -\frac{8}{3}$$

2.a) $$\lim_{x\to -2^+} \frac{|4-x^2|}{x^2+5x+6}$$

$$= \lim_{x\to -2^+} \frac{|(2-x)(2+x)|}{(x+3)(x+2)} = \lim_{x\to -2^+} \frac{|2-x|\,|2+x|}{(x+3)(x+2)} \quad \dots (*)$$

como $x \to -2^+ \Rightarrow x \geq -2$ y $x < 2$

$\Rightarrow |2-x| = 2-x$ y $|2+x| = 2+x$

$$(*) = \lim_{x\to -2^+} \frac{(2-x)\cancel{(2+x)}}{(x+3)\cancel{(x+2)}}$$

$$= \lim_{x\to -2^+} \frac{2-x}{x+3} = \frac{4}{1} = 4$$

2.b) $$\lim_{x\to 1^-} \frac{|1-x^2| - x + 1}{x^2-1}$$

$$= \lim_{x\to 1^-} \frac{|1-x|\,|1+x| - x + 1}{(x+1)(x-1)} \quad \dots (*)$$

como $x \to 1^- \Rightarrow x \leq 1$ y $x > -1$

$\Rightarrow |1-x| = 1-x$ y $|1+x| = 1+x$

$$(*) = \lim_{x\to 1^-} \frac{(1-x)(1+x) - (x-1)}{(x+1)(x-1)}$$

$$= \lim_{x\to 1^-} \frac{(1-x)(1+x) + (1-x)}{(x+1)(x-1)}$$

$$= \lim_{x\to 1^-} \frac{(1-x)(1+x+1)}{(x+1)(x-1)} = \lim_{x\to 1^-} \frac{-\cancel{(x-1)}(2+x)}{(x+1)\cancel{(x-1)}}$$

$$= \lim_{x\to 1^-} \frac{-2-x}{x+1} = -\frac{3}{2}$$

2.c) $\lim\limits_{x \to 0^+} \frac{1}{x}\sqrt{x^2 - x^4}$

$$= \lim_{x \to 0^+} \frac{1}{x}\sqrt{x^2(1-x^2)} = \lim_{x \to 0^+} \frac{1}{x}|x|\sqrt{1-x^2} \quad \dots (*)$$

Como $x \to 0^+ \Rightarrow x \geq 0$

$\Rightarrow |x| = x$

$$(*) = \lim_{x \to 0^+} \frac{1}{x}\, x\sqrt{1-x^2} = \lim_{x \to 0^+} \sqrt{1-x^2}$$

$$= \sqrt{1} = \underline{1}$$

2.d) $\lim\limits_{x \to 0^+} \frac{\sqrt{x-x^2} - \sqrt{2x}}{\sqrt{x}}$

$$= \lim_{x \to 0^+} \frac{x - x^2 - 2x}{\sqrt{x}} \cdot \frac{1}{\sqrt{x-x^2} + \sqrt{2x}}$$

$$= \lim_{x \to 0^+} \frac{-x - x^2}{\sqrt{x}\left(\sqrt{x(1-x)} + \sqrt{x}\cdot\sqrt{2}\right)}$$

$$= \lim_{x \to 0^+} \frac{x(-1-x)}{x\left(\sqrt{1-x} + \sqrt{2}\right)}$$

$$= \lim_{x \to 0^+} \frac{(-1-x)}{\sqrt{1-x} + \sqrt{2}} = \frac{-1}{1+\sqrt{2}} = \frac{-(1-\sqrt{2})}{1-2} = \underline{1-\sqrt{2}}$$

3) $f(x) = \begin{cases} a^2x^2 + x & \text{si} \quad x < 1 \\ (1-a)x - 1 & \text{si} \quad x > 1 \end{cases}$

• $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} a^2x^2 + x = a^2 + 1$

• $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (1-a)x - 1 = (1-a) - 1 = -a$

Si $\lim\limits_{x \to 1} f(x)$ existe $\Leftrightarrow \lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x)$

Para que $a^2 + 1 = -a \Rightarrow a^2 + a + 1 = 0$

$\Rightarrow a = \dfrac{-1 \pm \sqrt{1-4}}{2} = \dfrac{-1 \pm \sqrt{3}i}{2} \notin \mathbb{R}$

$\therefore \nexists\, a \in \mathbb{R}$ t. $\lim\limits_{x \to 1} f(x) \; \exists$

4) $\lim\limits_{x \to a} f(x) = l$ y $\lim\limits_{y \to l} g(y) = m \Rightarrow \lim\limits_{x \to a} g(f(x)) = m$

Sea $f(x) = 0$ y $g(y) = \begin{cases} 1 & \text{si} \quad x \neq 0 \\ 0 & \text{si} \quad x = 0 \end{cases}$

• $\lim\limits_{x \to 0} f(x) = 0$

• $\lim\limits_{y \to 0} g(y) = 1$

• $\lim\limits_{x \to 0} g(f(x)) = \lim\limits_{x \to 0} 0 = 0$

$\therefore$ El ejercicio es falso

5) $|x - \frac{1}{2}| < \delta \Rightarrow |\frac{1}{x} - 2| < \frac{1}{100}$

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$-\delta < x - \frac{1}{2} < \delta \Rightarrow -\delta + \frac{1}{2} < x < \delta + \frac{1}{2}$

Sea δ_1 .t. $0 < -\delta_1 + \frac{1}{2} < x < \delta_1 + \frac{1}{2} \iff 0 < \delta_1 < \frac{1}{2}$

$$\Rightarrow \frac{1}{\delta_1 + \frac{1}{2}} < \frac{1}{x} < \frac{1}{\frac{1}{2} - \delta_1}$$

$$\Rightarrow \frac{1}{\frac{2\delta_1 + 1}{2}} < \frac{1}{x} < \frac{1}{\frac{1 - 2\delta_1}{2}}$$

$$\Rightarrow \frac{2}{2\delta_1 + 1} < \frac{1}{x} < \frac{2}{1 - 2\delta_1}$$

$$\Rightarrow \frac{2}{2\delta_1 + 1} - 2 < \frac{1}{x} - 2 < \frac{2}{1 - 2\delta_1} - 2$$

$$\Rightarrow \frac{2 - 4\delta_1 - 2}{2\delta_1 + 1} < \frac{1}{x} - 2 < \frac{2 - 2 + 4\delta_1}{1 - 2\delta_1}$$

$$\Rightarrow \frac{-4\delta_1}{2\delta_1 + 1} < \frac{1}{x} - 2 < \frac{4\delta_1}{1 - 2\delta_1}$$

$$\Rightarrow \left|\frac{1}{x} - 2\right| < \max\left\{\left|\frac{-4\delta_1}{2\delta_1 + 1}\right|, \left|\frac{4\delta_1}{1 - 2\delta_1}\right|\right\} \quad \begin{pmatrix} \text{recuerda:} \\ 0 < \delta_1 < 1/2 \end{pmatrix}$$

$$= \max\left\{\frac{4\delta_1}{2\delta_1 + 1}, \frac{4\delta_1}{1 - 2\delta_1}\right\} \quad \ldots (*)$$

Se tiene que $0 < 1 - 2\delta_1 < 1 + 2\delta_1$ si $0 < \delta_1 < 1/2$

$$\Rightarrow 0 < \frac{1}{1 + 2\delta_1} < \frac{1}{1 - 2\delta_1} \Rightarrow \frac{4\delta_1}{2\delta_1 + 1} < \frac{4\delta_1}{1 - 2\delta_1}$$

$\forall \delta_1 \in (0, 1/2)$

$\therefore \ (*) = \dfrac{4\delta_1}{1-2\delta_1}$

$$\Rightarrow \left| \frac{1}{x} - 2 \right| < \frac{4\delta_1}{1-2\delta_1}$$

Ahora queremos $\dfrac{4\delta_1}{1-2\delta_1} < 100$

$$\Rightarrow 4\delta_1 = 100(1-2\delta_1) \Rightarrow 4\delta_1 = 100 - 200\delta_1$$

$$\Rightarrow 204\delta_1 = 100 \Rightarrow \delta_1 = \frac{100}{204} = \frac{25}{51}$$

Respuesta

Sea $\delta = \dfrac{25}{51}$

$$\Rightarrow \left|x - \frac{1}{2}\right| < \frac{25}{51} \Rightarrow -\frac{25}{51} < x - \frac{1}{2} < \frac{25}{51} \Rightarrow \frac{-50+51}{102} < x < \frac{50+51}{102}$$

$$\Rightarrow \frac{1}{102} < x < \frac{101}{102} \Rightarrow \frac{102}{101} < \frac{1}{x} < \frac{102}{1} \Rightarrow \frac{102}{101} - 2 < \frac{1}{x} < \frac{102}{1} - 2$$

$$\Rightarrow \frac{102-202}{101} < \frac{1}{x} - 2 < 100 \Rightarrow -\frac{100}{101} < \frac{1}{x} - 2 < 100$$

$$\Rightarrow -100 < -\frac{100}{101} < \frac{1}{x} - 2 < 100 \Rightarrow \left|\frac{1}{x} - 2\right| < 100$$