Respuestas Lab 6 otoño 2019

1) a) $f(x)=\dfrac{1}{\sqrt{1+x}}$ con $x_0 \neq -1$

$$\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}=\lim_{x\to x_0}\frac{\dfrac{1}{\sqrt{1+x}}-\dfrac{1}{\sqrt{1+x_0}}}{x-x_0}=\lim_{x\to x_0}\frac{\dfrac{\sqrt{1+x_0}-\sqrt{1+x}}{\sqrt{1+x}\,\sqrt{1+x_0}}}{x-x_0}$$

$$=\lim_{x\to x_0}\frac{1+x_0-(1+x)}{(x-x_0)(\sqrt{1+x_0}+\sqrt{1+x})\sqrt{1+x}\,\sqrt{1+x_0}}=\lim_{x\to x_0}\frac{-1}{(\sqrt{1+x_0}+\sqrt{1+x})(\sqrt{1+x})(\sqrt{1+x_0})}$$

$$=\frac{-1}{2\sqrt{1+x_0}\,(\sqrt{1+x_0})(\sqrt{1+x_0})}=\frac{-1}{2\,(\sqrt{1+x_0})^3}=-\frac{1}{2}(1+x_0)^{-3/2}$$

1.b) $f(x)=x|x|$ con $x_0\neq 0$

$$\lim_{x\to x_0}\frac{x|x|-x_0|x_0|}{x-x_0}\ \cdots\ (*)$$

Si $x_0<0$

$$(*)=\lim_{x\to x_0}\frac{-x\cdot x+x_0\cdot x_0}{x-x_0}=\lim_{x\to x_0}\frac{x_0^2-x^2}{x-x_0}=\lim_{x\to x_0}\frac{(x_0+x)(x_0-x)}{x-x_0}$$

$$=\lim_{x\to x_0}-(x_0+x)=-2x_0$$

Si $x_0>0$

$$(*)=\lim_{x\to x_0}\frac{x\cdot x-x_0x_0}{x-x_0}=\lim_{x\to x_0}\frac{(x-x_0)(x+x_0)}{x-x_0}=\lim_{x\to x_0}x+x_0=2x_0$$

$$\therefore \lim_{x\to x_0}f(x)=2|x_0|$$

2) $f, g: \mathbb{R} \to \mathbb{R}$ diferenciables, $f'(x) = g(x)$ y $f(x) = g'(x)$ $\forall x \in \mathbb{R}$

Calcula $(f^2(x) + g^2(x))'$

Por regla de la cadena:

$$(f^2(x)+g^2(x))' = (f^2(x))' + (g^2(x))' = 2f(x)f'(x) + 2g(x)g'(x)$$

$$= 2f(x)g(x) + 2g(x)f(x) = 4f(x)g(x)$$

3) Sup. $f(x_0) = a$ y $f'(x_0) = b$. Calcula: $\lim_{h\to 0} \dfrac{f^2(x_0+h) - f^2(x_0)}{h}$

Por definición $\lim_{h\to 0} \dfrac{f^2(x_0+h) - f^2(x_0)}{h} = (f^2(x_0))' = 2f(x_0)f'(x_0)$

$= 2ab$

4) Recta tangente, recta normal de $y = f(x)$ en $P_0 = (x_0, f(x_0))$ con $f(x) = \dfrac{1}{\sqrt{1+x}}$ y $f_2(x) = \dfrac{x|x|}{1}$.

1) $f(x) = \dfrac{1}{\sqrt{1+x}}$ con $x_0 \neq -1$

$f'(x) = \frac{1}{2}(1+x_0)^{-3/2}$

Ecuación de la recta: $y - y_0 = m(x - x_0)$

$y - y_0 = \frac{1}{2}(1+x_0)^{-3/2}(x - x_0) \iff y = y_0 + \frac{1}{2}(1+x_0)^{-3/2}(x - x_0)$

2) $f_2(x) = x|x|$ con $x_0 \neq 0$

$f_2'(x) = 2|x_0|$

$y - y_0 = 2|x_0|(x - x_0) \iff y = 2|x_0|(x - x_0) + y_0$

별 5) Sup $f(0) = 1 = f'(0)$ y $f(2) = 2 = f'(2)$. Determina:

a) $(f^2 \circ (1+f^2))'(0)$

$$((f(1+f^2))^2)'(0) = 2[f(1+f^2)(f(1+f^2))'](0)$$

$$= 2[f(1+f^2)f'(1+f^2)(1+f^2)'](0)$$

$$= 2[f(1+f^2)f'(1+f^2)(2f \cdot f')](0)$$

$$= 2[f(1+f^2(0))f'(1+f^2(0))(2f(0)f'(0))]$$

$$= 2[f(2)f'(2)(2)] = 2[2 \cdot 2 \cdot 2] = 2^4 = \underline{16}$$

b) $[\sqrt{f \circ (1+f^2)}\,]'(0)$

$$= \frac{1}{2}\left\{[f(1+f^2)]^{-1/2}[f(1+f^2)]'\right\}(0)$$

$$= \frac{1}{2}\left\{[f(1+f^2)]^{-1/2}f'(1+f^2)(1+f^2)'\right\}(0)$$

$$= \frac{1}{2}\left\{[f(1+f^2)]^{-1/2}f'(1+f^2)\,2ff'\right\}(0)$$

$$= \frac{1}{2}\left\{[f(1+1)]^{-1/2}f'(2)\,2\cdot 1\right\}$$

$$= \frac{1}{2}[2]^{-1/2} \cdot 2 = 2^{-1/2}$$

6) Sup $f(1)=2$, $f'(1)=3$, $g(2)=4$ y $(g^2 \circ f)'(1)=16$

Determina $g'(2)$.

$$(g^2(f))' = 2g(f)(g(f))' = 2g(f)g'(f)f'$$

$$(g^2(f))'\Big|_{(1)} = 16 = 2g(f(1))\, g'(f(1))f'(1)$$

$$\Leftrightarrow 16 = 2g(2)g'(2)\cdot 3$$

$$\Leftrightarrow 16 = 2\cdot 4\, g'(2)\cdot 3$$

$$\Leftrightarrow g'(2) = \frac{16}{2\cdot 4\cdot 3} = \frac{2}{3}$$