Lab 10 otoño 2019

1) $xy + x^2 + y^2 = 3 \Rightarrow xy + 2 = 3 \Rightarrow xy = 1 \Rightarrow y = \frac{1}{x}$

$\Rightarrow x^2 + \frac{1}{x^2} = 2 \Rightarrow x^4 + 1 = 2x^2 \Rightarrow x^4 - 2x^2 + 1 = 0 \Rightarrow (x^2-1)^2 = 0$

$x^2 = \pm 1 \Rightarrow x = \pm 1 \Rightarrow y^2 + 1 = 2 \Rightarrow y = \pm 1$

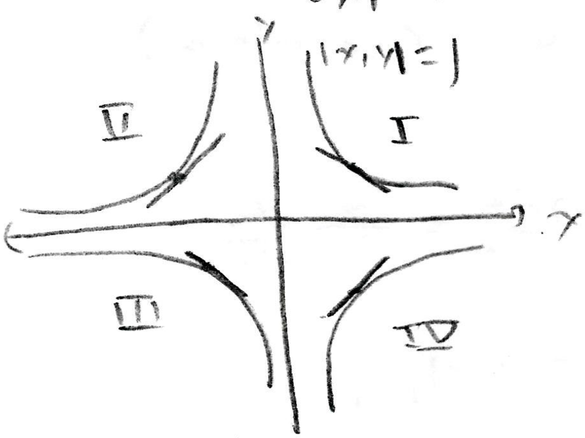
puntos: $(1,1)$, $(-1,-1)$

circulo: $2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

elipse: $2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{(2x+y)}{x+2y}$

iguales en $(1,1)$, $(-1,-1)$

2) $|xy| = 1$, $\left|\frac{dy}{dx}\right| = 1$



en I: $|xy| = xy = 1$

$\Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow \left|\frac{dy}{dx}\right| = \left|-\frac{y}{x}\right| = 1$

$\Rightarrow y = 1,\ x = 1$

$(1,1)$

por simetría

II: $(-1,1)$, III: $(-1,-1)$, IV: $(1,-1)$

3) $16y^2 - 9x^2 = 20 \Rightarrow 16y^2 + 4y^2 - 25 = 20$, $20y^2 = 45$ $y^2 = \frac{45}{20} = \frac{9}{4}$

$\Rightarrow y = \pm\frac{3}{2} \Rightarrow x^2 = \frac{25-4y^2}{9} = \frac{16}{9} \Rightarrow x = \pm\frac{4}{3}$

hipérbola: $32y\frac{dy}{dx} - 18x = 0 \Rightarrow \frac{dy}{dx} = \frac{18x}{32y} = \frac{9x}{16y}$

elipse: $18x + 8y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{4y}{9y}$

Falta verificar la perpendicularidad $\left(a = -\frac{1}{b}\right)$

4) $2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y} = \pm 1$

$\Rightarrow x = -y$ o $x = y \Rightarrow x^2 + x^2 = R^2 \Rightarrow x^2 = \frac{R^2}{2} \Rightarrow x = \frac{R\sqrt{2}}{2}$

$\therefore$ siempre la podemos encontrar

5) $2x - 3y - 3x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{3y-2x}{2y-3x} = 0 \Leftrightarrow 3y = 2x$

$x = \frac{3}{2}y$

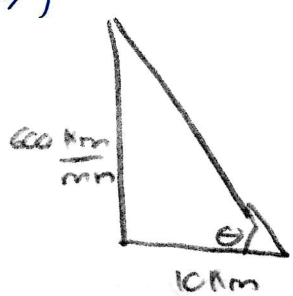
$\Rightarrow \frac{9}{4}y^2 - \frac{9}{2}y^2 + y^2 = 1 \;\; -\frac{5}{4}y^2 = 1$ ∇ No existe

6) $D(t) = \sqrt{x^2 + y^2} = \sqrt{y^6 + y^2}$

$\frac{dD}{dt} = \frac{1}{2}(y^6+y^2)^{-1/2} \cdot \left(6y^5 \cancel{\frac{dy}{dt}}^{1} + 2y\cancel{\frac{dy}{dt}}^{1}\right) = \frac{1}{2}(y^6+y^2)^{-1/2}(6y^5+2y)$

$P_0(8,2)$; $\frac{dD}{dt} = \frac{1}{2}(64+4)^{-1/2}(6 \cdot 32 + 4)$ (se aleja)

7)

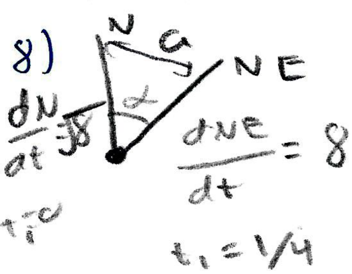


$\tan(\theta) = \frac{y^2}{100} \Rightarrow \sec^2(\theta)\frac{d\theta}{dt} = \frac{y}{50}\frac{dy}{dt}$

$\Rightarrow \sec^2(\theta)\frac{d\theta}{dt}\Big|_{t=1} = \frac{y}{50} \cdot 600 = 12y\Big|_{t=1}$

$\frac{d\theta}{dt}\Big|_{t=1} = \frac{12 \cdot 600}{\sec^2(\theta)|_{t=1}}$

8)



$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$, $a\big|_{t=1} = 8 + 36 - 12\sqrt{8}\frac{\sqrt{2}}{2}$

$= \text{algo}$

$a^2 = b^2 + c^2 - 2bc\frac{\sqrt{2}}{2}$

$2a\frac{da}{dt} = 2b\frac{db}{dt} + 2c\frac{dc}{dt} - \sqrt{2}\left(\frac{db}{dt}c + \frac{dc}{dt}b\right)$

$$\frac{da}{dt} = \frac{2b\frac{db}{dt} + 2c\frac{dc}{dt} - \sqrt{2}\left(\frac{db}{dt}c + \frac{dc}{dt}b\right)}{2a}$$

sustituir y te da el valor.