

Steepest Descent Approaches for the Minimization of Open Stacks

Júnior Rhis
Marco Antonio M. Carvalho

Departamento de Computação
Instituto de Ciências Exatas e Biológicas
Universidade Federal de Ouro Preto

March 22, 2016



Outline

- 1 Introduction
 - Formal Definition
 - Example
 - Motivation
- 2 Steepest Descent Approaches
- 3 Computational Experiments
- 4 Conclusions

Steepest Descent Approaches for the MOSP

Introduction

- ▶ The problem of minimizing the maximum number of simultaneous open stacks arises in industrial environments:
 - ▶ A factory needs to supply specific combinations of products that have associated given demands;
 - ▶ A single machine manufactures all the products in batches and at each stage, a single product type is handled;
 - ▶ Whenever a customer places an order for a set of products, a new *open stack* is associated with it, meaning that physical space around the machine is assigned to it until the order is fulfilled - the *stack's closure*.

Steepest Descent Approaches for the MOSP

Introduction

- ▶ There is an implicit assumption of a physical limit on space around the machine because there is not enough free room around the machine to place all customers' orders simultaneously;
- ▶ Moving open stacks requires additional manpower and machinery, introducing risks to product's integrity;
- ▶ In order to better use that physical space, it is necessary to determine the sequence of the products' manufacturing.

Introduction

- ▶ The *Minimization of Open Stacks Problem* (MOSP) is defined on a binary sparse matrix M :
 - ▶ The n rows correspond to the customers's orders and the m columns correspond to each product type available;
 - ▶ Entry $m_{ij} = 1$ if customer i ordered product type j , and $m_{ij} = 0$ otherwise;
 - ▶ Matrix M holds the *consecutive ones property*^a.

^aIn each row all elements between two 1s are considered to have value 1, also called *fill-ins*. Those elements together define a *1-block*.

Formal Definition

- ▶ Given an $n \times m$ sparse binary matrix M , matrix M_π is the matrix obtained from a permutation π of the $\{1, 2, \dots, m\}$ columns of matrix M , and M_π^1 is the matrix obtained from M_π that holds the consecutive ones property;
- ▶ The objective is to find a permutation π of columns of M such that the maximum sum of a column (including the fill-ins)^a in M_π^1 is minimized.

^agiven by Z^π .

Steepest Descent Approaches for the MOSP

Example

Table 1 presents an example of sparse binary matrix M_π and M_π^1 for $\pi=[5, 2, 4, 6, 3, 1]$. $Z^\pi = 3$, on columns 2, 4, 6 and 3. Columns with maximum sum represent the bottleneck of problem and are called *critical columns*.

	1	2	3	4	5	6		5	2	4	6	3	1
1	1	1	0	0	0	0	1	0	1	1	1	1	1
2	1	0	1	0	0	0	2	0	0	0	0	1	1
3	0	0	0	1	1	0	3	1	1	1	0	0	0
4	0	0	0	1	0	1	4	0	0	1	1	0	0
5	0	1	0	0	1	0	5	1	1	0	0	0	0
6	0	0	1	0	0	1	6	0	0	0	1	1	0

Table : M_π and M_π^1 for $\pi=[5, 2, 4, 6, 3, 1]$. Fill-ins are highlighted in bold font.

Steepest Descent Approaches for the MOSP

Motivation

- ▶ MOSP is a NP-hard problem;
- ▶ Its practical application;
- ▶ It models successfully a wide range of equivalent problems in a variety of contexts:
 - ▶ Interval Thickness;
 - ▶ Node Search Game;
 - ▶ Edge Search Game;
 - ▶ Narrowness;
 - ▶ Split Bandwidth;
 - ▶ Pathwidth;
 - ▶ Edge Separation;
 - ▶ Vertex Separation;
 - ▶ Modified Cutwidth;
 - ▶ Programmable Logic Array Folding;
 - ▶ Gate Matrix Layout.

Steepest Descent Approaches for the MOSP

Proposed Methods

- ▶ A graph search for generating an initial solution;
- ▶ A Variable Neighborhood Descent method;
- ▶ A simple Steepest Descent method.

Current State of the Art

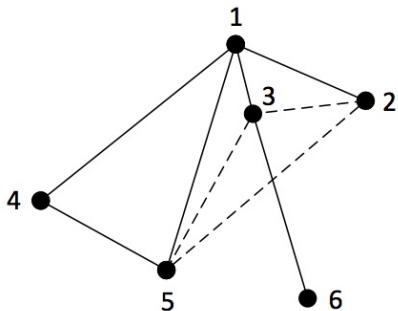
- ▶ Heuristic: Minimum Cost Node Heuristic (2004);
- ▶ Metaheuristic: Biased Random Key Genetic Algorithm (2014);
- ▶ Exact: A^* (2009).

Steepest Descent Approaches for the MOSP

Graph Model

- ▶ Nodes represent customers;
- ▶ Edges represent products – no loops or parallel edges;
- ▶ Each customer induces a clique.

	1	2	3	4	5	6
1	1	0	0	1	1	0
2	1	1	0	0	1	0
3	0	0	1	0	0	1
4	1	1	1	0	0	0
5	0	1	1	0	1	0



Steepest Descent Approaches for the MOSP

Graph Search

- ▶ The main idea is to search for customers, rather than for products directly;
- ▶ Breadth-first search (BFS) in non-decreasing degree order.

Products Sequencing

- ▶ The BFS returns a list of customers;
- ▶ The products sequence is determined in a greedy fashion:
 - ▶ The list of customers is traversed, and once all customers that ordered a specific product are found, it is sequenced.

Steepest Descent Approaches for the MOSP

Variable Neighborhood Descent - VND

- ▶ Classic version – first time applied to the MOSP;
- ▶ Five neighborhood structures:
 - ▶ k -opt of grouped contiguous columns ($k = 2, 3, 4, 5$).
 - ▶ The best neighbor is improved by a insertion move (only bottleneck related columns).
- ▶ All possible movements explored partially
 - ▶ Randomly selected, within a limit (50%).
- ▶ All parameters tuned using the irace package.

Steepest Descent Approaches for the MOSP

Steepest Descent - SD

- ▶ Classic version;
- ▶ Local Search:
 - ▶ Insertion move – only bottleneck related columns;
 - ▶ Target position defined by similarity of nonzero elements.
- ▶ All possible movements explored partially
 - ▶ Randomly selected, within a limit (70%).
- ▶ All parameters tuned using the irace package.

Steepest Descent Approaches for the MOSP

Computational Environment

- ▶ Intel Core i7 3.6 GHz processor;
- ▶ 16 GB RAM;
- ▶ Ubuntu 14.04 LTS;
- ▶ Codes written in C++, compiled with gcc 4.8.4 and the -O3 optimization option.

Methods:

- ▶ Variable Neighborhood Descent (VND);
- ▶ Steepest Descent (SD);
- ▶ Minimum Cost Node heuristic (MCNh).

Instances

Six different data sets from the literature were considered, a total of 475 instances.

- ▶ SCOOP: 24 real world MOSP instances;
- ▶ Faggioli & Bentivoglio: 300 artificial MOSP instances;
- ▶ Challenge: 126 artificial MOSP instances;
- ▶ VLSI: 25 real world Gate Matrix Layout instances.

Steepest Descent Approaches for the MOSP

Average gap from Optimal Solutions (20 runs)

Method	SCOOP	Faggioli & Bentivoglio	Challenge	VLSI
VND	3.76%	2.04%	7.39%	3.93%
SD	2.15%	0.75%	0.6%	1.12%
MCNh	25.27%	13.72%	2.0%	12.92%

Steepest Descent Approaches for the MOSP

Average Running Times (20 runs)

Method	SCOOP	Faggioli & Bentivoglio	Challenge	VLSI
VND	12ms	19.46ms	215ms	64ms
SD	138ms	479ms	18s	4.08s
MCNh	<1ms	<1ms	<1ms	<1ms

Steepest Descent Approaches for the MOSP

Optimal Solutions Found

Method	SCOOP	Faggioli & Bentivoglio	Challenge	VLSI
VND	70.83%	81.33%	6.52%	76.00%
SD	87.50%	93.33%	86.96%	92.00%
MCNh	36.00%	37.00%	78.00%	72.00%

Conclusions

- ▶ This is the first time a VND application on MOSP is reported;
- ▶ The best heuristic in literature performed poorly on 3 problem sets – first time reported;
- ▶ VND is faster, but less accurate;
- ▶ Surprisingly, SD presented a very low gap and a good rate of optimal solutions found for most problem sets;
- ▶ Future work includes:
 - ▶ Variable Neighborhood Search;
 - ▶ Matheuristic – poor models currently.

Acknowledgments

This research has been supported by the National Council for Scientific and Technological Development (CNPq).

Questions?

