Fake News and Consumer Loyalty

Marco Cerundolo

University of Warwick

Abstract

This project examines one way in which competition between partisan media firms can drive fake news. Consumers in the model are assumed to have a preference for news which is accurate, on the one hand, but also news which matches their beliefs, on the other hand. The consumers' beliefs are endogenous to the model since they update their opinions from listening to the news and choose again which news to listen to in a second period, based on these new beliefs. From this it follows that firms face a trade-off when choosing their level of bias: they want to reduce their bias in order to appeal to more consumers, however, they have an incentive to produce biased news in order to ensure that their consumers return in the future. In particular, the model find that when consumers are sufficiently interested in learning the truth, the firms will be unbiased, whereas when the consumers are sufficiently averse to hearing news they disagree with, the firms' news becomes completely biased. This lies in contrast to the case of a monopolist, which produces unbiased news. Therefore, the model suggests that competition creates the incentive for firms to produce fake news in order to ensure 'consumer loyalty'.

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Introduction

Bias in the news is ubiquitous. The aim of this project is to explain why media bias and fake news exist and how they can be driven by demand. Consumers demand from the news two things which lie in opposition: they demand informative content, however, they also demand news which matches their prior beliefs. The project assumes this is due to a behavioural aversion to dissonant information. Given these consumer demands and a distribution of consumer priors, the project examines the bias that two competing news firms will choose. The firms have a clear incentive to produce unbiased news in order to appeal to the largest number of consumers possible. Therefore, there is a need to explain why, despite this incentive to produce unbiased news, demand can still drive bias.

The project argues that bias can emerge because firms have an incentive to create a loyal customer base, given uncertainty about the true state. Firms in the model can always capture a larger market share by decreasing their bias and therefore appealing to more customers. However, by considering multiple periods of consumption, the model highlights that news firms are not simply trying to secure the largest number of consumers in the present but trying to secure an audience into the future. By being prepared to provide fake news, the firms ensure that consumers will return to their news even if the firm's bias doesn't match the true state. Fake news is therefore a manipulation of the consumers beliefs, and the consumers tolerate this manipulation because it protects them from hearing news they disagree with.

Consider two news firms, one which is left-biased and another which is right-biased. There is a left-wing and right-wing politician and each period one of the politicians will make a mistake. Assume that one of the politicians is superior and is therefore less likely to make a mistake. The firms are in the business of reporting which of the two politicians didn't make a mistake and is therefore more likely to be the superior politician. Since consumers prefer to hear news they agree with, they would rather hear that the politician who they believe to be superior has avoided making a mistake. Given this demand, the firms are in a battle to convince their consumers that the politician whose bias matches their own is the superior one. If they fail to convince their consumers of this, then their consumers will change their opinion about which politician is superior and will start to consume news from the opposing firm. Now suppose that the firms have to choose their level of bias before they learn which politician is superior. This models the fact that a firm's decision of how biased to be is a long-term strategic decision, whereas the political cycle and turnover of leaders if far more frequent.

The model shows that when news firms are in competition, they have an incentive to produce biased news to ensure that their consumers remain aligned with their partisan bias and therefore return in the future. In particular, the model predicts that a monopolist will produce unbiased news whereas in a duopoly the firms will produce fully biased news when the consumers are sufficiently averse to hearing news they disagree with. The explanation for this is that by giving the consumers false information about which politician is superior, the firms increase the expected return of consumers in the future. When consumers place high importance on hearing accurate information, a firm which produces biased news will experience demand only from those with extreme opinions and will therefore have influence over the opinions of only few people. However, as consumers become increasingly averse to hearing news they disagree with, the fall in demand in response to a firm producing biased news is reduced, and so firms find that it is optimal to produce biased news.

Within the literature on demand-driven media bias, the effect of competition is ambiguous [1]. A result which is robust across both theoretical and empirical research is that consumers demand news which matches their prior beliefs [2][3][4][5]. However, the reason for this demand is important in determining the effect of competition. For example, Gentzkow and Shapiro (2006) [6] show that if consumers are only interested in receiving accurate information then competition can reduce the incentive for firms to bias their news towards peoples' priors.

In contrast, when consumers' demand is driven by a behavioural preference for confirmatory news, competition can exacerbate media bias. Both Mullainathan and Shleifer (2005) [7] and Gabszewicz et al. (2001) [8] show that, within standard Hotelling logic, two competing firms will bias towards opposite extremes to gain monopoly power and charge a higher price. However, in the last decade the news has been revolutionised from being predominantly newspaper based to being freely available either online or on TV. Firm profit is now primarily generated through advertisement revenues and so firms want to maximise the number of consumers who listen to their news. However, in this setting the Hotelling model is unable to explain why media firms would choose to become biased, since in the absence of prices they maximise market share by producing unbiased news. Therefore, the novel contribution of this project is that it provides an explanation for how demand can drive competing firms to be more biased, despite their content being freely available.

The Model

The true state of the world is $S \in \{L, R\}$, where $Pr(S = L) = Pr(S = R) = \frac{1}{2}$. The players in the game are two news firms, $F \in \{L, R\}$, one which is left-biased and the other which is right-biased. At the beginning of the game the two firms simultaneously choose their strategy which is a level of bias $\sigma_F \in [0, 1]$. Nature then chooses the true state and in each period of the game, $i \in \{1, 2\}$, the firms observe independent signals of the true state, $s_i \in \{l, r\}$, where the accuracy of the signal is $\pi = Pr(s_i = l|S = L) = Pr(s_i = r|S = R)$ and $\pi \in (\frac{1}{2}, 1)$.[1]

Each period the firms produce a news report $\hat{s}_F \in \{L, R\}$, which is a signal of the true state to the consumers of the news. If the signal s_i that a firm receives matches the firm's bias then they report it truthfully. If however, the signal they observe doesn't match their bias then they report it truthfully only with probability $1-\sigma_F$, and report a signal matching their own bias with probability σ_F. Therefore if $\sigma_F = 0$ then firm F will always truthfully report s_i and if $\sigma_F = 1$ then their report is completely uninformative since they will report news matching their bias irrespective of the signal they observe. The two firms choose the level of bias σ_F that maximises the expected consumption of their news in the second period.

The set of consumers have priors $p_1 = Pr(S = R)$, which are uniformly distributed over $[0, 1]$. We say that a consumer is left-wing if $p_1 < 1/2$, is right-wing if $p_1 > 1/2$ and we denote the bias of the consumer by $b \in \{L, R\}$. Over two periods the consumers myopically choose the news which maximises their expected utility in the current period. The utility from listening to firm F is defined as

$$U_F = \bar{u} - Pr(\hat{s}_F \neq S) - \alpha \left| p - \frac{1}{2} \right| \mathbb{1}\{\hat{s}_F \neq b\}.$$

As can be seen from the function, the consumers receive a fixed level of utility $\bar{u}$ from consuming the news. However, since they are interested in learning the true state of the world, they experience a disutility equal to the probability that the news they are listening to is false. In addition, the consumers are averse to information they disagree with, and so if they observe a signal $\hat{s}_F$ which doesn't match the direction of their bias they experience a loss of $\alpha \left|p - \frac{1}{2}\right|$. Therefore α is a measure of the intensity of aversion to dissonant information, and the weighting $\left|p - \frac{1}{2}\right|$ captures the fact that those with the most extreme views find it most uncomfortable hearing opposing views.

The consumers form a posterior belief p_2 from the signal, $\hat{s}_F$, they observe and given this belief they choose which news firm to observe in the second period. The objective of the firms is to maximise consumption in the second period. The timing of the whole game is as follows:

[1]This set-up abstracts away from the motivating example in the introduction. We assume that each period one of the two politicians makes a mistake and the probability that the inferior politician makes a mistake is π. If the firms then reports which politician didn't make a mistake, then this is equivalent to the set-up presented in the formal model.

1. Both firms simultaneously choose a level of fake-news.
2. The firms observe a signal correlated with the true state and report a signal according to their strategy.
3. Consumers observe the signal from the firm they choose.
4. The consumers update their opinions and choose again which firm to listen to.
5. The firms get a payoff equal to the fraction of consumers who choose their news in the second period.

Consumer Choice

In order to determine the firms' optimal strategy, it is first necessary to analyse which firm a consumer will listen to for a given prior. Let E_F^b denote the expected utility of a consumer with bias b of listening to firm F. In general, we have that

$$E_F^b = \bar{u} - Pr(\hat{s}_F \neq S) - \alpha \left| p - \frac{1}{2} \right| \mathbb{P}\{\hat{s}_F \neq b\}.$$

To simplify the analysis, for the whole of the project we assume that $\bar{u}$ is sufficiently high such that for any level of bias $\sigma_F \in [0, 1]$, consumers who share the same bias as firm F expect positive utility from listening to firm F [2].

A left-wing consumer, with prior $p < \frac{1}{2}$ will choose to listen to the left-biased firm if and only if $E_L^L(p) > E_R^L(p)$. Similarly, a right-wing consumer will listen to the right-biased firm if and only if $E_R^R(p) > E_L^R(p)$. The consumers' expected utility function gives us the following proposition.

Proposition 1. *There exists $\lambda \in [0, 1]$ such that all consumers with prior $p < \lambda$ choose the left-biased firm and all consumers with prior $p > \lambda$ choose the right-biased firm.*

This proposition states that there is a single point on the unit interval where consumers go from preferring left-biased to right-biased news. This is a desireable property due to the fact that it is intuitive but also because it makes the firms' optimisation problem tractable.

As shown in the proof of Proposition 1, when the left-wing firm is more biased than the right-wing firm, λ is the prior of the left-wing consumer who is indifferent between the two firms. Similarly, when the right-wing firm is more biased than the left-wing firm, λ is the prior of the right-wing consumer who is indifferent between the two firms. When the two firms have an equal level of bias, $\lambda = \frac{1}{2}$, and so all consumers listen to the firm which shares their own bias. Therefore λ is a function of the bias that two firms choose, σ_L and σ_L, and also α, the consumers' aversion to dissonant news.

[2] This ensures that when there is a left-wing and a right-wing firm, all consumers will choose to listen to the news.

The firms are interested in who will choose to listen to their news in the second period. If a consumer is such that $p_1 < \lambda$ and so they listen to the left-biased firm in the first period, then if they observe an L-signal, they will have $p_2 \leq p_1 < \lambda$ and so will listen to the left-wing firm again in the second period. However, if the consumer listens to the left-wing firm and the firm reports an R-signal then $p_2 > p_1$ and so their consumption in the second period depends on whether $p_2 < \lambda$ or $p_2 > \lambda$. This in turn depends on the consumer's specific prior belief and the bias of the left-wing firm. The same reasoning holds for a consumer listening to the right-wing firm. Defining the two terms $\lambda_L^1 = \frac{\lambda(1-\pi)}{\pi-\lambda(2\pi-1)}$ and $\lambda_R^1 = \frac{\lambda\pi}{1-\pi(1-\lambda)-\lambda(1-\pi)}$ we have the following proposition.

Proposition 2. *Consider a consumer who listens to the left-wing firm, so $p_1 < \lambda$. Upon observing $\hat{s}_L = R$, if $p_1 < \lambda_L^1$ then $p_2 < \lambda$ and if $p_1 > \lambda_L^1$ then $p_2 > \lambda$. Similarly, consider a consumer who listens to the right-wing firm, so $p_1 > \lambda$. Upon observing $\hat{s}_R = L$, if $p_1 > \lambda_R^1$ then $p_2 > \lambda$ and if $p_1 < \lambda_R^1$ then $p_2 < \lambda$.*

Therefore consumers with priors in the interval $[0, \lambda_L^1)$ will listen to the left-wing firm in the first period and again in the second period regardless of the news that is reported. Whereas those with priors in the interval (λ_L^1, λ) will listen to the left-wing firm in the first period but will only listen to it again in the second period if they report an L-signal. The same idea is true for consumers of the right-wing news in the intervals $(\lambda_R^1, 1]$ and (λ, λ_R^1) respectively.

Firm Strategy

Before considering the strategy of competing firms, let's briefly consider the bias that a monopolist will choose. We have the following simple proposition.

Proposition 3. *A monopolist will choose to be unbiased, with $\sigma = 0$.*

Intuitively, this is because the firm wants to appeal to the most consumers possible and by being biased it will only lead to consumers choosing no longer to consume from it. This result lies in contrast to the case of two competing firms where, as we will see, for α sufficiently high, the firms' news becomes completely uninformative.

The objective of the two competing firms is to choose the level of bias that maximises the number of consumers who listen to their news in the second period. In achieving this goal the firms face two competing incentives. By increasing their bias, σ_F, they reduce the number of consumers who choose to listen to their news in the first period, since the probability they produce fake news, $\mathbb{P}\{\hat{s}_F \neq \mu\}$, increases. This is bad for the firm because as their audience in the first period decreases, they have influence over the opinions of fewer consumers and are therefore influencing the consumption decisions in period two of fewer consumers.

However, the firms also benefit by increasing their bias. This is because as their bias increases, in the event that the signal they observe doesn't match their bias, they are less likely to report it truthfully to their consumers. This is significant because, if for example the left-wing firm reports against its bias with $\hat{s}_L = R$, then all consumers who observe that signal will update their opinions to the right. Those consumers with priors $\lambda_L^1 < p_1 < \lambda$ will have posterior beliefs $p_2 > \lambda$, by Proposition 2, and so in the second period the left-biased firm will have lost those consumers. Therefore, by increasing its bias, the firm increases 'consumer loyalty' in the sense that those who listen to their news in the first period are more likely to return in the second period. The left-biased firm's objective function, which is derived in the proof of Proposition 4 is:

$$E(\Pi^L) = \frac{1}{2}\left[(\sigma_L + 1)\lambda + (1 - \sigma_L)\lambda_L^1 + (1 - \sigma_R)(\lambda_R^1 - \lambda)\right],$$

where Π^Ldenotes the left-wing firm's profit, which is equal to the fraction of consumers who choose their news in the second period. We have the following proposition which identifies that the only possible equilibrium are corner solutions.

Proposition 4. *The only possible symmetric Nash Equilibria are* $(\sigma_L^*, \sigma_R^*) = (0, 0)$ *or* $(1, 1)$.

The task of identifying the equilibrium can now be reduced to checking whether the firms will deviate when the strategy profiles are $(0, 0)$ and $(1, 1)$. We make the following claim.

Claim. *There exists* α^* *such that for all* $\alpha < \alpha^*$ *the unique equilibrium is* $(0, 0)$ *and for all* $\alpha > \alpha^*$ *the unique equilibrium is (1,1).*

Whatever the value of α, when the firms increase their bias they face a fixed marginal increase in the expected return of their customers in the second period. However, when they increase their bias, the marginal fall in demand for their news in the first period is dependent on α. When the firms increase their bias, demand in the first period falls because consumers value the news being accurate. As the consumers' aversion to dissonant information increases, they increasingly value being protected from news they disagree with over hearing accurate news, and therefore an increase in the firm's bias causes the consumer a smaller fall in utility. Therefore, as α increases, the firms are able to increase their bias with a smaller fall in demand in the first period. Once the consumers' aversion passes the point α^*, it suddenly becomes in the firms interest to prioritise ensuring all their consumers return in the second period by having fully biased news.

Unfortunately, the derivation of α^* in terms of π is algebraically intractable. To illustrate the existence of such an α^* we therefore consider two concrete examples.

Example. We want to show that for a small value of α the equilibrium is $(0, 0)$ and for a large α the equilibrium is $(1, 1)$. To make the computations simpler consider the two extreme cases where (i) $\alpha = 0$ and (ii) $\alpha \to \infty$.

(i): Fix $\sigma_R = 0$. If $\sigma_L = 0$ then $\lambda = \frac{1}{2}$ and $E(\Pi^L(0,0)) = \frac{1}{2}$. If in contrast $\sigma_L > 0$ then the firm maximises $E(\Pi^L(\sigma_L, 0)) = \frac{1}{2}(\sigma_L(\lambda - \lambda_L^1) + \lambda_L^1 + \lambda_R^1)$which is maximised at $\sigma_L = 1$such that $E(\Pi^L(1,0)) = \frac{1}{2}(\lambda + \lambda_R^1)$. We derive that for $\sigma_L = 1$ and $\sigma_R = 0$ we have $\lambda = 1 - \pi$ and $\lambda_R^1 = \frac{1}{2}$ and so $E(\Pi^L(1,0)) = \frac{1}{2}(\frac{3}{2} - \pi) < \frac{1}{2}$. Therefore the left-wing firm will choose $\sigma_L = 0$ and so $(0,0)$ is an equilibrium. The fact that $(1,1)$ is not an equilibrium is proved in the appendix.

(ii): Fix $\sigma_R = 1$. The left-wing firm wants to maximise $E(\Pi^L(\sigma_L, 1)) = \frac{1}{2}(\sigma_L(\lambda - \lambda_L^1) + \lambda_L^1 + \lambda)$. As $\alpha \to \infty$, one can intuitively see that consumers will always listen to the firm which shares their own bias, since this minimises the chance of hearing news they disagree with. Therefore $\lambda = \frac{1}{2}$ and $\lambda_L^1 = 1 - \pi$ and so $E(\Pi^L(\sigma_L, 1)) = \frac{1}{2}(\sigma_L(\pi - \frac{1}{2}) + \frac{3}{2} - \pi)$. This is maximised at $\sigma_L = 1$ such that $E(\Pi^L(1,1)) = \frac{1}{2}$ and therefore $(1,1)$ is an equilibrium. The fact that $(0,0)$ is not an equilibrium is proved in the appendix.

To highlight the mechanism which drives the firms to produce biased news, consider the case where there is only a single period of consumption. Both firms try to make their news appeal to the largest number of consumers possible and this is achieved by both firms setting their bias equal to zero. Therefore the unique equilibrium is $(0,0)$, irrespective of how large α is. In contrast, over two periods of consumption the firms will choose to be fully biased when $\alpha > \alpha^*$. This shows that firms in the model bias their news because they are maximising consumption in the future.

Conclusion

This project emphasises the fact that when consumers decide what news to consume as a function of their beliefs, then firms have an incentive to influence their listeners' opinions. Because of this, firms can benefit from distorting the news towards their bias. The key point of the model is that when firms choose their level of bias, they face a trade-off between influencing the opinions of more people or having a tighter control over the opinions of those who listen. This basic idea creates scope for areas of future research and suggests some deeper points.

The model argues that firms produce biased news in order to ensure future consumption. Testing this theory empirically could perhaps be approached by comparing differences in media bias between newspapers which do and do not have subscriptions. The model suggests that if a newspaper changed from being only available through single issues to being only available through a year long subscription, then the firm's incentive to influence its consumers' opinions would fall and therefore so would its bias.

The model could easily be extended to explore the effect that a polarised population has on media bias. The firms in the model have their level of bias constrained because they don't want to reduce the number of consumers who listen to their news in the first period. If society were more polarised, and so the consumers' priors were not uniformly distributed, then firms would experience a smaller fall in demand from increasing their bias. Therefore, the model suggests that a polarised society would have news which is more biased because the firms could have a tighter control over people's opinions without a fall in the number of consumers they influence. Since consumers learn from the news, this suggests a feedback loop in which a polarised population induces the news to be more biased which in turn worsens the polarisation in the population. Addressing externalities in the market for news, such as this one, is an important avenue for further consideration.

Appendix

Proof of Proposition 1. A left-wing consumer is indifferent between the two firms if they have prior p such that $f(p) = E_L^L(p) - E_R^L(p) = 0$. A right-wing consumer is indifferent between the two firms if $g(p) = E_L^R(p) - E_R^R(p) = 0$. We have

$$\begin{aligned}
f(p) =& \alpha \left(\frac{1}{2} - p\right) (Pr(\hat{s}_R = R) - Pr(\hat{s}_L = R)) + Pr(\hat{s}_R \neq S) - Pr(\hat{s}_L \neq S) \\
=& \alpha \left(\frac{1}{2} - p\right) (\sigma_L + (\sigma_R - \sigma_L)(\pi + p(1 - 2\pi))) + \pi(\sigma_R - \sigma_L) + \sigma_L - p(\sigma_R + \sigma_L)
\end{aligned}$$

and

$$\begin{aligned}
g(p) =& \alpha \left(p - \frac{1}{2}\right) (Pr(\hat{s}_R = L) - Pr(\hat{s}_L = L)) + Pr(\hat{s}_R \neq S) - Pr(\hat{s}_L \neq S) \\
=& \alpha \left(p - \frac{1}{2}\right) (-\sigma_L + (\sigma_R - \sigma_L)(p(2\pi - 1) - \pi)) + \pi(\sigma_R - \sigma_L) + \sigma_L - p(\sigma_R + \sigma_L) \\
=& f(p).
\end{aligned}$$

Therefore, since the two functions are equal, we can say more simply that a consumer with prior p is indifferent between the two firms if $f(p) = 0$. We have that for any σ_L and σ_R,

$$f(0) = \left(\frac{\alpha}{2} + 1\right) (\sigma_L + \pi(\sigma_R - \sigma_L)) > 0 \text{ and } f(1) = \left(\frac{\alpha}{2} + 1\right) (\pi(\sigma_R - \sigma_L) - \sigma_R) < 0.$$

By the Intermediate Value Theorem there exists $\lambda \in (0, 1)$ such that $f(\lambda) = 0$ and this root in unique because $f(p)$ is a quadratic. Therefore we have that for all consumers with prior $p < \lambda$, $f(p) = g(p) > 0$ and so they choose the left-wing firm and for all consumers with prior $p > \lambda$, $f(p) = g(p) < 0$ and so they choose the right-wing firm. Additionally, we have that

$$f\left(\frac{1}{2}\right) = (\sigma_R - \sigma_L)\left(\pi - \frac{1}{2}\right).$$

Therefore, for $\sigma_L > \sigma_R, f\left(\frac{1}{2}\right) < 0$ and so $\lambda \in \left(0, \frac{1}{2}\right)$ which means that it is a left-wing consumer who is indifferent between the two firms. Conversely, for

$\sigma_L < \sigma_R, f\left(\frac{1}{2}\right) > 0$ and so $\lambda \in \left(\frac{1}{2}, 1\right)$ which means that it is a right-wing consumer who is indifferent between the two firms.

$\square$

Proof of Proposition 2. We have

$$Pr(S = R|\hat{s}_L = R) = \frac{Pr(\hat{s}_L = R|S = R)Pr(S = R)}{Pr(\hat{s}_L = R)} = \frac{p\pi}{p\pi + (1-p)(1-\pi)}.$$

Setting $Pr(S = R|\hat{s}_L = R) = \lambda$ we derive the formula $\lambda_L^1 = \frac{\lambda(1-\pi)}{\pi-\lambda(2\pi-1)}$. Similarly, we have

$$Pr(S = R|\hat{s}_R = L) = \frac{Pr(\hat{s}_R = L|S = R)Pr(S = R)}{Pr(\hat{s}_R = L)} = \frac{p(1-\pi)}{p(1-\pi) + (1-p)\pi}.$$

Setting $Pr(S = R|\hat{s}_R = L) = \lambda$ we derive $\lambda_R^1 = \frac{\lambda\pi}{1-\pi(1-\lambda)-\lambda(1-\pi)}$.

$\square$

Proof of Proposition 3. Recall that we have assumed $\bar{u}$ to be sufficiently high such that a consumer of a given bias will always listen to a firm with their same bias over listening to no news. From this it follows that when the firm is unbiased all consumers will choose to listen in the first and second periods. If $\bar{u}$ was so high that all consumers will still choose to listen for different values of bias then the firm will be indifferent between those levels of bias. However, if $\bar{u}$ was sufficiently low and the firm deviated from zero bias then there would be some consumers who would prefer to listen to no news. These consumers will have their prior unchanged after the first period and therefore not consume in the second period either. Therefore the firm will choose to be unbiased. $\square$

Proof of Proposition 4. Consider the left-biased firm's maximisation problem: to choose the level of bias σ_L that maximises expected consumption in the second period. Recall that λ and $1-\lambda$ are the fractions of the interval which choose to consume from the left-biased and right-biased firms respectively in the first period. Additionally, when the signal firm F receives doesn't match its bias, σ_F is the probability that it lies and $1-\sigma_F$ is the probability it reports the truth. Conditioning over all possible events, we have that the expected fraction of the unit interval which chooses to listen to the left-biased firm in the second period is

$$\begin{aligned}
E(\Pi^L) &= Pr(S = L)\left[\pi(\lambda + (1-\sigma_R)(\lambda_R^1 - \lambda)) + (1-\pi)(\sigma_L\lambda + (1-\sigma_L)\lambda_L^1)\right] \\
&\quad + Pr(S = R)\left[(1-\pi)(\lambda + (1-\sigma_R)(\lambda_R^1 - \lambda)) + \pi(\sigma_L\lambda + (1-\sigma_L)\lambda_L^1)\right] \\
&= \frac{1}{2}\left[(\sigma_L + 1)\lambda + (1-\sigma_L)\lambda_L^1 + (1-\sigma_R)(\lambda_R^1 - \lambda)\right].
\end{aligned}$$

For (σ,σ) to be an equilibrium, with $\sigma \in (0,1)$, a necessary condition is that $\frac{\partial E(\Pi^L)}{\partial \sigma_L}(\sigma,\sigma) = 0$. We will show that this condition never holds. We have that

$$\frac{\partial E(\Pi^L)}{\partial \sigma_L} = \frac{1}{2}\left[\lambda + (1+\sigma_L)\frac{\partial \lambda}{\partial \sigma_L} - \lambda_L^1 + (1-\sigma_L)\frac{\partial \lambda_L^1}{\partial \sigma_L} + (1-\sigma_R)\left(\frac{\partial \lambda_R^1}{\partial \sigma_L} - \frac{\partial \lambda}{\partial \sigma_L}\right)\right].$$

Setting $\sigma_L = \sigma_R = \sigma$, then $\lambda = \frac{1}{2}$, $\lambda_L^1 = 1-\pi$, $\lambda_R^1 = \pi$ and as calculated at the end of the proof, $\frac{\partial \lambda}{\partial \sigma_L} = \frac{1-2\pi-\alpha}{2\sigma(\alpha+2)}$ and $\frac{\partial \lambda_L^1}{\partial \sigma_L} = \frac{\partial \lambda_R^1}{\partial \sigma_L} = 4\pi(1-\pi)\frac{\partial \lambda}{\partial \sigma_L}$. Therefore setting $\frac{\partial E(\Pi^L)}{\partial \sigma_L}(\sigma,\sigma) = 0$ we have that

$$\begin{aligned}\frac{\partial E(\Pi^L)}{\partial \sigma_L}(\sigma,\sigma) &= \frac{1}{2}\left[\frac{\partial \lambda}{\partial \sigma_L}(2\sigma(1-4\pi(1-\pi)) + 8\pi(1-\pi)) + \pi - \frac{1}{2}\right] \\ &= \frac{1-2\pi-\alpha}{2\sigma(\alpha+2)}(\sigma(1-4\pi(1-\pi)) + 4\pi(1-\pi)) + \frac{\pi}{2} - \frac{1}{4} = 0.\end{aligned}$$

Solving gives $\sigma = \frac{4\pi(1-\pi)(1-2\pi-\alpha)}{(\alpha+2)(\frac{1}{2}-\pi)-(1-4\pi(1-\pi))(1-2\pi-\alpha)} \geq 1$. Therefore, $\frac{\partial E(\Pi^L)}{\partial \sigma_L}(\sigma,\sigma) = 0$ cannot be solved for $\sigma \in (0,1)$. Therefore, there exists no symmetric equilibrium (σ,σ) for $\sigma \in (0,1)$ and so any symmetric equilibrium must be $(0,0)$ or $(1,1)$.

We now show the calculations involved in finding $\frac{\partial \lambda}{\partial \sigma_L}$, $\frac{\partial \lambda_L^1}{\partial \sigma_L}$ and $\frac{\partial \lambda_R^1}{\partial \sigma_L}$. We have that λ is the unique root in $(0,1)$ of $f(p) = 0$, therefore $f(\lambda) = \alpha\left(\frac{1}{2}-\lambda\right)(\sigma_L + (\sigma_R-\sigma_L)(\pi+\lambda(1-2\pi))) + \pi(\sigma_R-\sigma_L) + \sigma_L - \lambda(\sigma_R+\sigma_L) = 0$. Differentiating both sides with respect to σ_L we get

$$\begin{aligned}&\frac{f(\lambda)}{\partial \sigma_L} = \\ &\alpha\left[\left(\frac{1}{2}-\lambda\right)\left(1 + \frac{\partial \lambda}{\partial \sigma_L}(\sigma_R-\sigma_L)(1-2\pi) - (\pi+\lambda(1-2\pi))\right) - \frac{\partial \lambda}{\partial \sigma_L}(\sigma_L + (\sigma_R-\sigma_L)(\pi+\lambda(1-2\pi)))\right] \\ &+ 1 - \pi - \frac{\partial \lambda}{\partial \sigma_L}(\sigma_R+\sigma_L) - \lambda = 0.\end{aligned}$$

Setting $\sigma_L = \sigma_R = \sigma$ such that $\lambda = \frac{1}{2}$ and rearranging for $\frac{\partial \lambda}{\partial \sigma_L}$ we get that $\frac{\partial \lambda}{\partial \sigma_L} = \frac{1-2\pi-\alpha}{2\sigma(\alpha+2)}$. By differentiating directly from the formulas $\lambda_L^1 = \frac{\lambda(1-\pi)}{\pi-\lambda(2\pi-1)}$ and $\lambda_R^1 = \frac{\lambda\pi}{1-\pi(1-\lambda)-\lambda(1-\pi)}$ we get the expressions $\frac{\partial \lambda_L^1}{\partial \sigma_L} = \frac{\partial \lambda_L^1}{\partial \sigma_R} = 4\pi(1-\pi)\frac{\partial \lambda}{\partial \sigma_L}$. □

Proof of Examples. (i): We want to show that for $\alpha = 0$, $(1,1)$ is not an equilibrium. Fix $\sigma_R = 1$. Then by calculating the root of $f(p) = 0$ we find that $\lambda = \frac{\pi(1-\sigma_L)+\sigma_L}{1+\sigma_L}$. Then using this formula for λ and that of λ_L^1 in $E(\Pi^L(\sigma_L,1)) = \frac{1}{2}(\sigma_L(\lambda-\lambda_L^1) + \lambda_L^1 + \lambda)$, we find by plotting the function on a computer that it is maximised at $\sigma_L = 0$. Therefore $(1,1)$ is not an equilibrium.

(ii): We want to show that as $\alpha \to \infty$, $(0,0)$ is not an equilibrium. Fix $\sigma_R = 0$. If the left-wing firm chooses $\sigma_L = 0$ then $E(\Pi^L(0,0)) = \frac{1}{2}$. For any

σ_L it is the case that $\lambda = \frac{1}{2}$ and so $\lambda_R^1 = \pi$. Therefore for $\sigma_L > 0$ the left-wing firm maximises $E(\Pi^L(\sigma_L, 0)) = \frac{1}{2}(\sigma_L(\lambda - \lambda_L^1) + \lambda_L^1 + \lambda_R^1)$ at $\sigma_L = 1$ such that $E(\Pi^L(1, 0)) = \frac{1}{2}(\lambda + \lambda_R^1) = \frac{1}{2}(\frac{1}{2} + \pi) > \frac{1}{2}$. Therefore the left-wing firm will choose $\sigma_L = 1$ so $(0, 0)$ is not an equilibrium.

$\square$

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