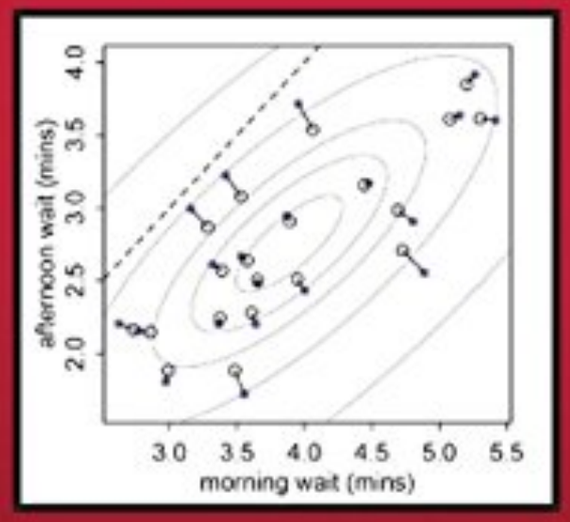
Multilevel regression modeling: concepts, model building, and best practices

Marco Visser
Sean McMahon
Lisa Hülsmann

Texts in Statistical Science

Statistical Rethinking

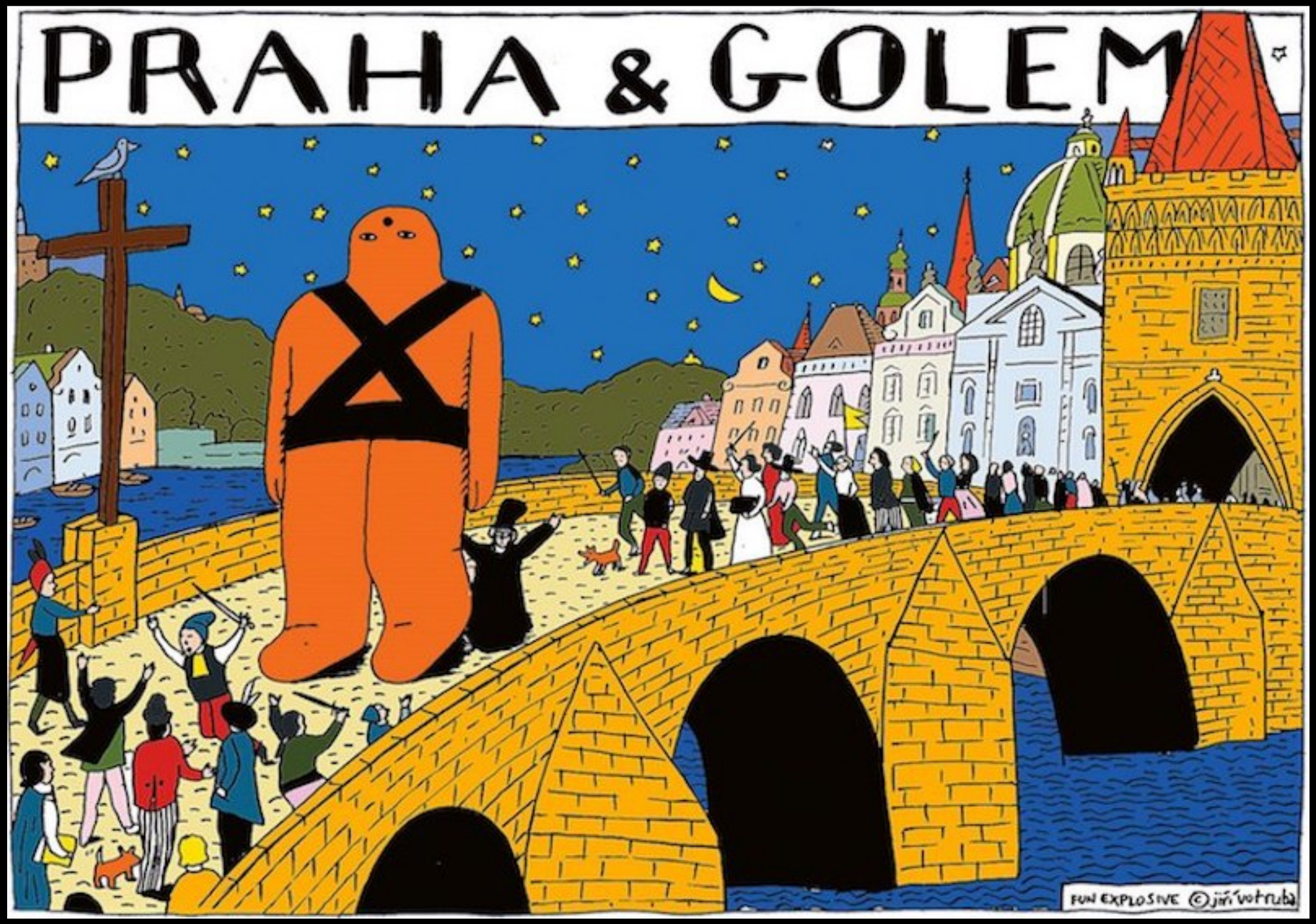
A Bayesian Course with Examples in R and Stan



Richard McElreath

CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK

http://www.tresbohemes.com/2018/04/the-legend-of-the-golem/



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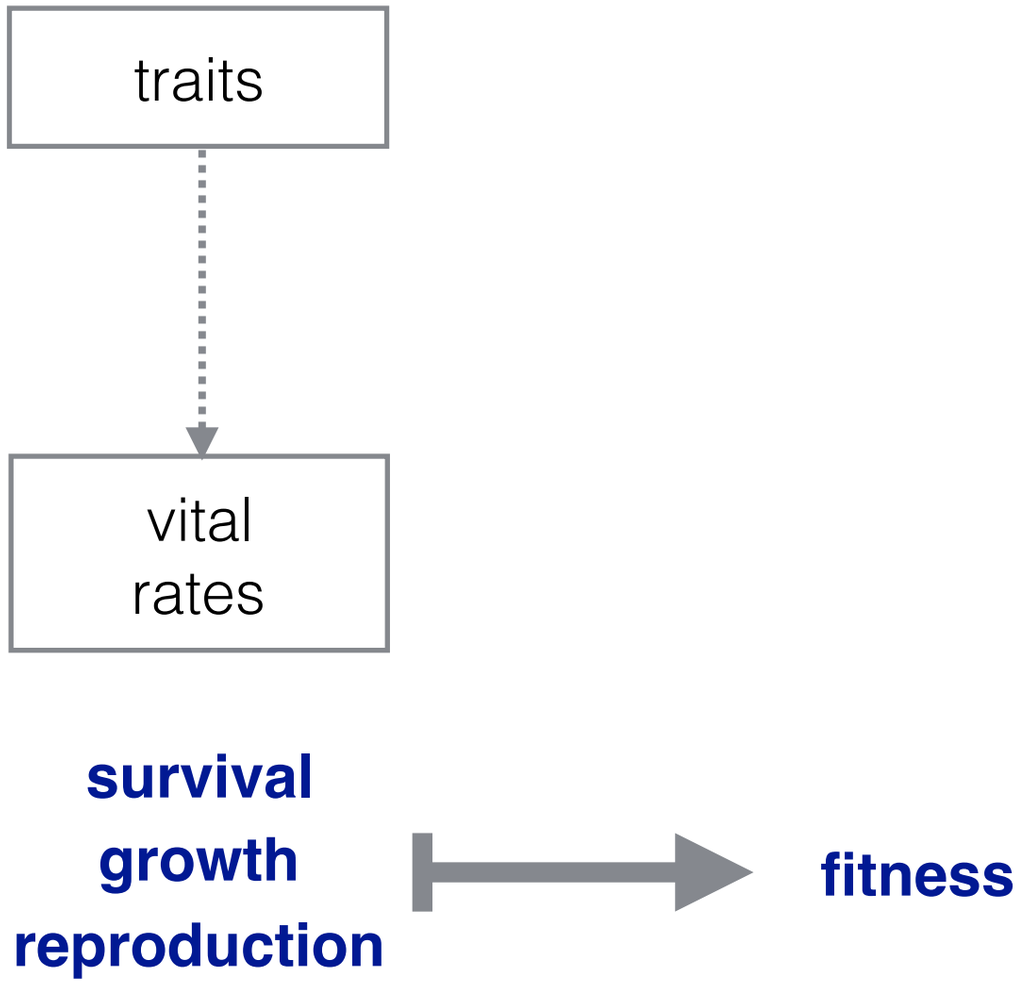
vital
rates

survival
growth
reproduction

vital
rates

survival
growth
reproduction

fitness


traits
vital
rates
survival
growth
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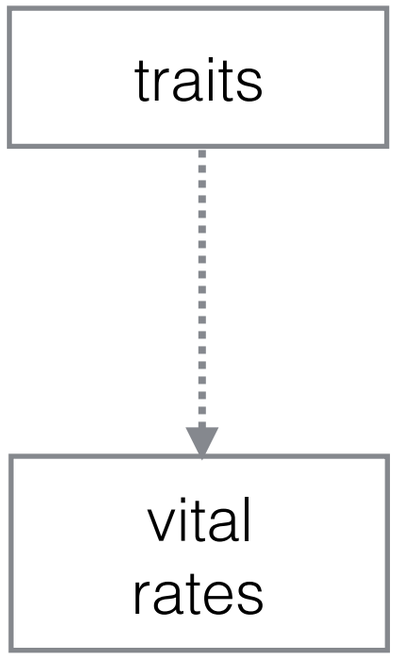
traits
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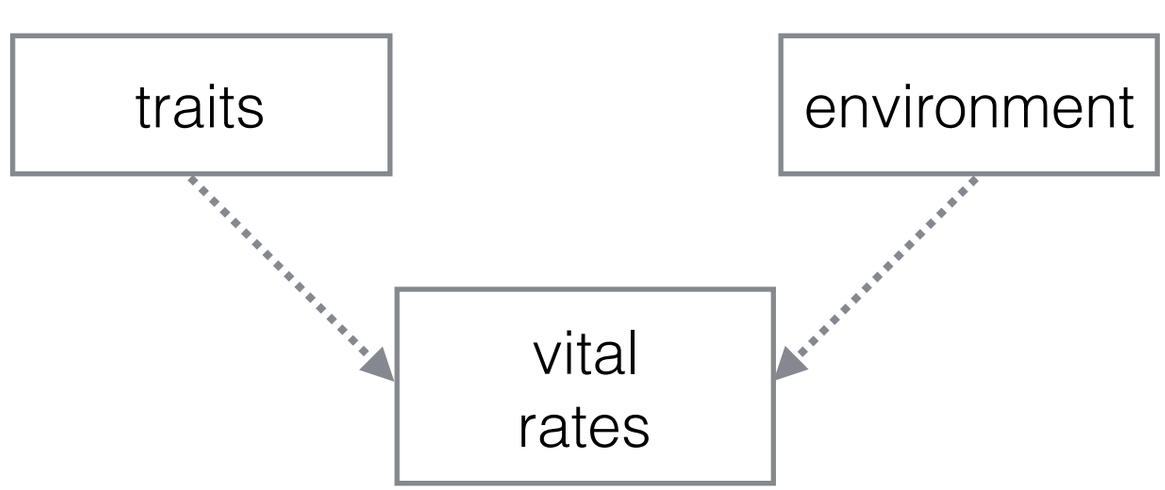
traits
environment
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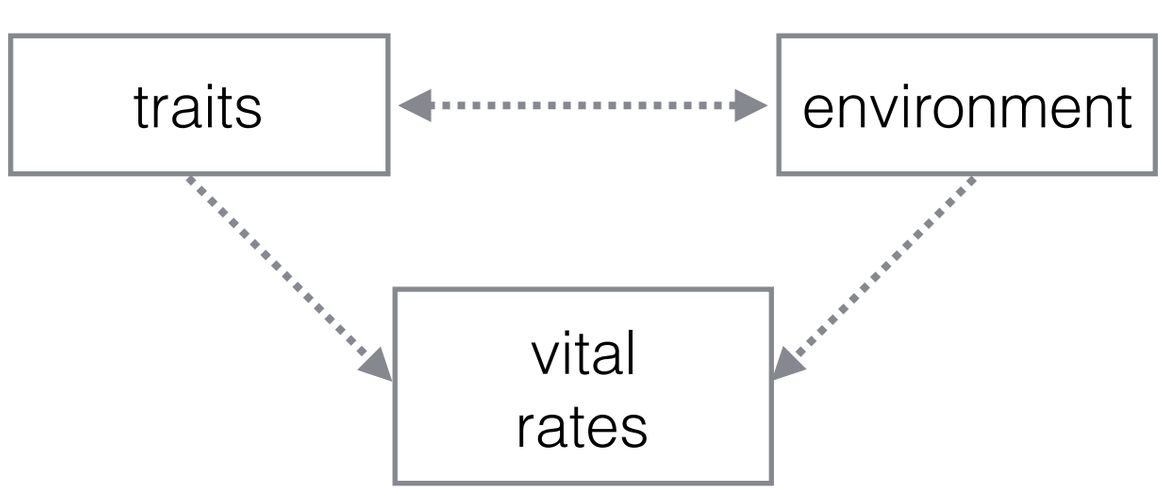
traits
environment
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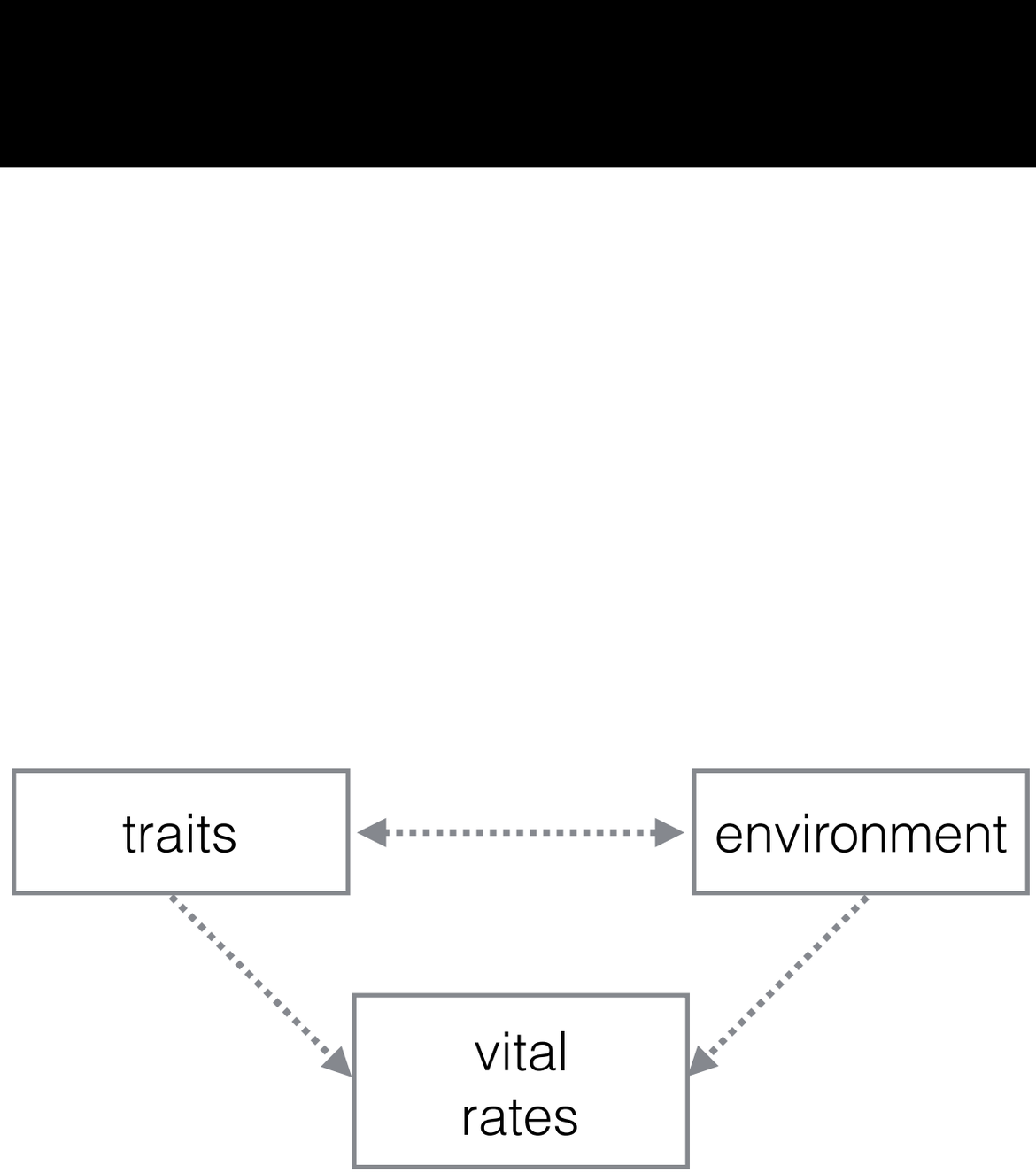
sla
size
height
BVOCs
venation
vessel size
leaf toughness
chemical defenses
mycorrhizal association



survival
growth
reproduction

sla
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vessel size
leaf toughness
chemical defenses
mycorrhizal association

light
water
nutrients
humidity
irradiance
temperature
herbivore density
pathogen presence
microbial community

traits

environment

vital
rates

survival
growth
reproduction

Coefficients:	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.11267	0.01674	6.73	4.00E-11	***
survsm	−0.16849	0.04439	−3.8	1.60E-04	***
survbig	−0.21941	0.08115	−2.7	7.06E-03	**
LMA	−0.18358	0.03696	−4.97	8.90E-07	***
SeedSize	0.30533	0.05517	5.53	4.70E-08	***
WD	−0.27357	0.04192	−6.53	1.50E-10	***
stand_age	0.12056	0.02101	5.74	1.50E-08	***
slope	0.00823	0.01945	0.42	6.72E-01	
light	0.47762	0.01949	24.51	2.00E-16	***
is_small	0.15139	0.02018	7.5	2.30E-13	***
survsm:light	−0.25103	0.04359	−5.76	1.40E-08	***
survsm:WD	−0.52458	0.07362	−7.13	3.00E-12	***
survbig:WD	1.60038	0.18138	8.82	2.00E-16	***
survbig:LMA	1.27205	0.26909	4.73	2.80E-06	***
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LMA:stand_age	0.16952	0.04847	3.5	5.00E-04	***
survsm:slope	−0.10386	0.04215	−2.46	1.40E-02	*
SeedSize:stand_age	−0.06204	0.0421	−1.47	1.41E-01	
LMA:slope	−0.08168	0.04099	−1.99	4.68E-02	*
SeedSize:light	0.15198	0.05032	3.02	2.64E-03	**
WD:light	−0.14239	0.05208	−2.73	6.44E-03	**
survsm:SeedSize	0.34885	0.1104	3.16	1.66E-03	**
SeedSize:WD	−0.13973	0.05405	−2.59	9.97E-03	**
survbig:SeedSize	−1.37951	0.39661	−3.48	5.40E-04	***
LMA:SeedSize	0.31982	0.09057	3.53	4.50E-04	***
SeedSize:is_small	0.11278	0.04363	2.59	9.97E-03	**
survbig:LMA:SeedSize	−1.54015	0.46632	−3.3	1.01E-03	**
LMA:SeedSize:stand_age	0.27	0.126	2.14	3.25E-02	*
survbig:SeedSize:is_small	−0.88274	0.22562	−3.91	1.00E-04	***

Using stepwise model selection on AIC scores, this 3rd order interaction model was selected as the minimum adequate model. ANOVA table statistics: $F_{29,\ 594} = 40.1$, $P < 0.0001$, R-square = 0.64. Ontogenetic variable 'is_small' is 0 for small trees and 1 for large trees.
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doi:10.1371/journal.pone.0016253.t002

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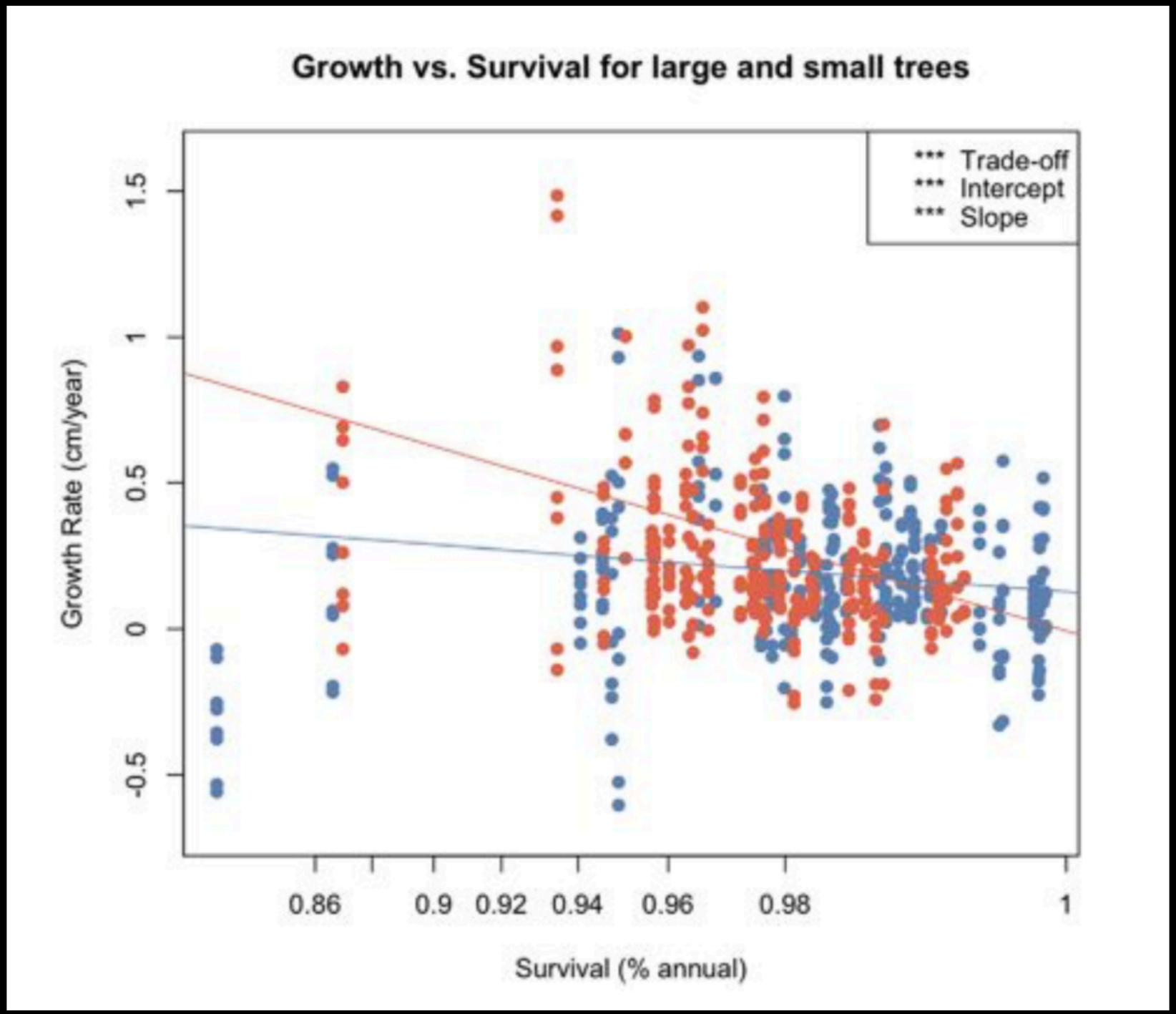
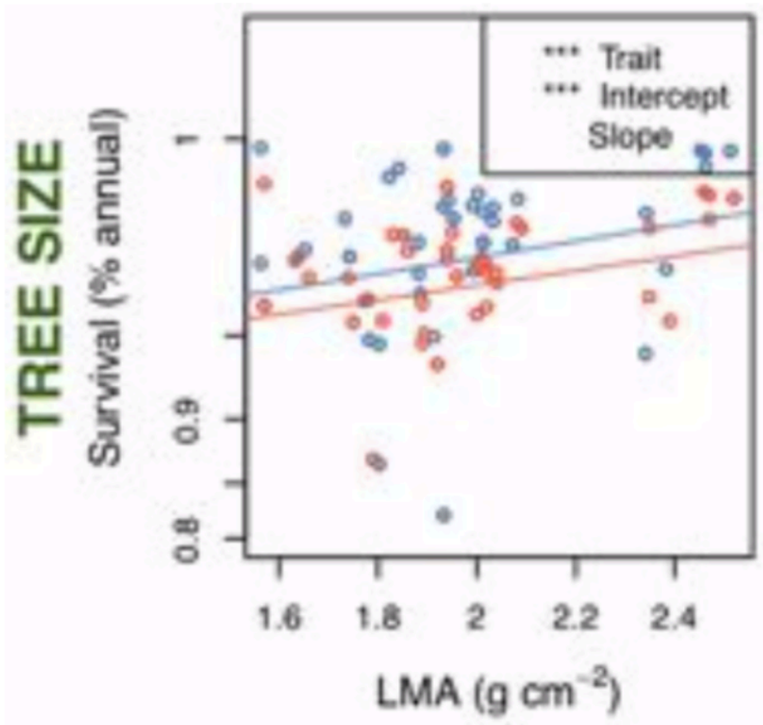
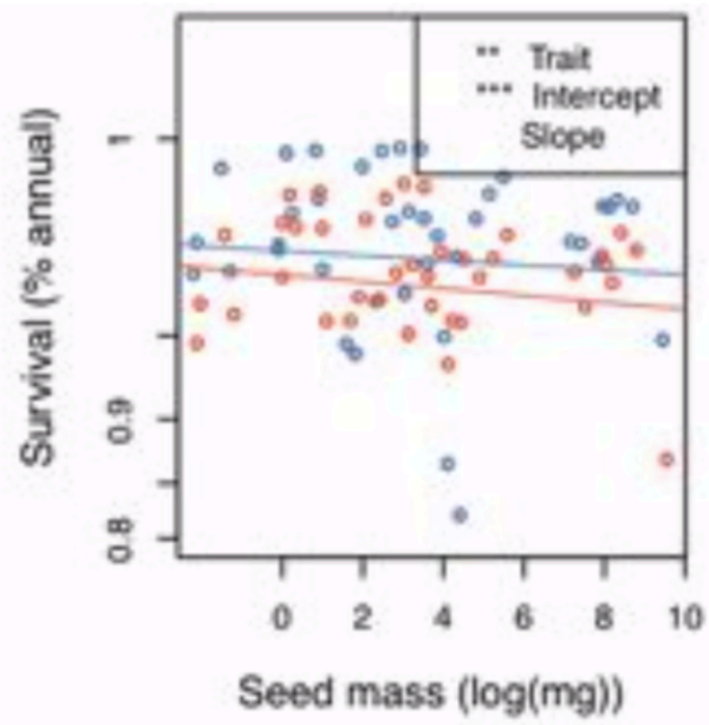
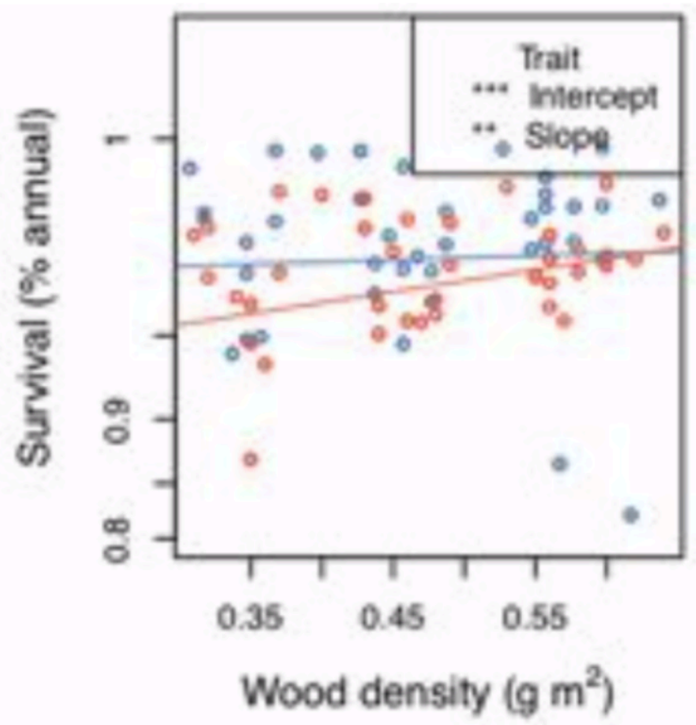
McMahon, Sean M, Charlotte J E Metcalf, and Christopher W Woodall. 2011. "High-Dimensional Coexistence of Temperate Tree Species: Functional Traits, Demographic Rates, Life-History Stages, and Their Physical Context.." PLoS ONE 6 (1): e16253. doi: 10.1371/journal.pone.0016253.


Growth vs. Survival for large and small trees
*** Trade-off
*** Intercept
*** Slope
Growth Rate (cm/year)
1.5
1
0.5
0
-0.5
0.86
0.9
0.92
0.94
0.96
0.98
1
Survival (% annual)

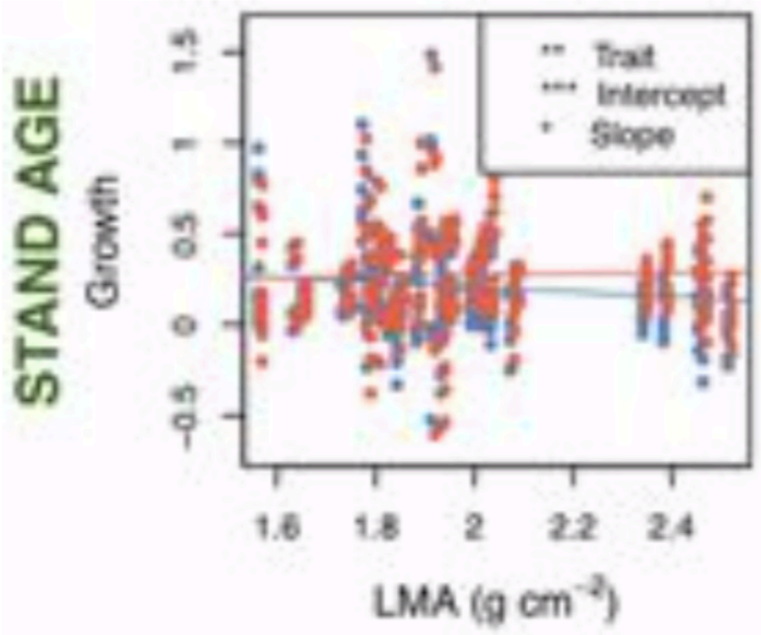
TREE SIZE

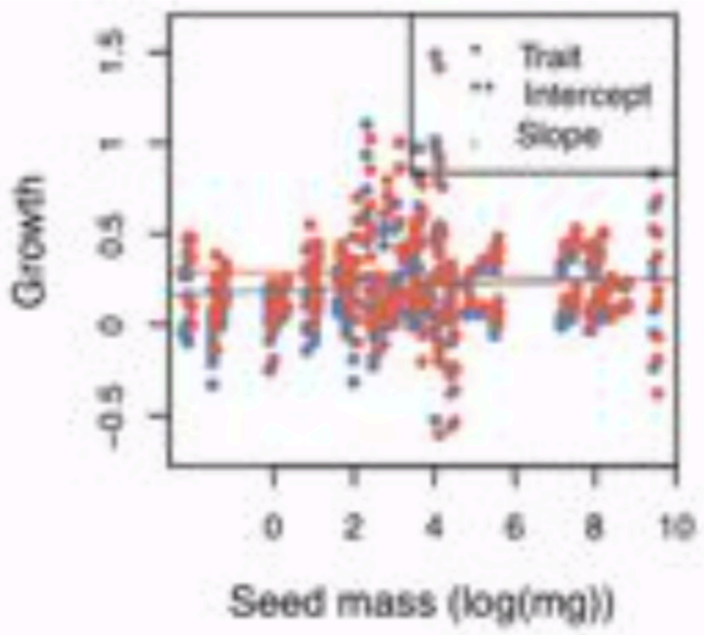

*** Trait
*** Intercept
Slope
Survival (% annual)
1
0.9
0.8
1.6
1.8
2
2.2
2.4
LMA (g cm^{-2})

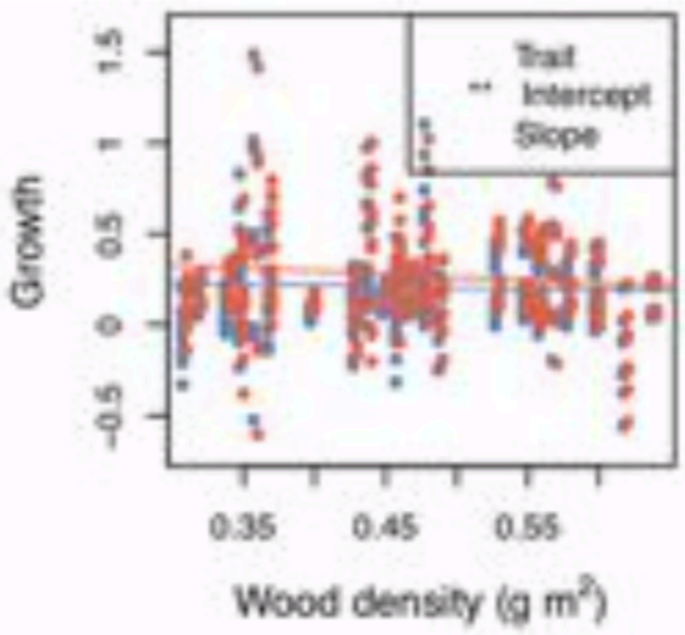

** Trait
*** Intercept
Slope
Survival (% annual)
1
0.9
0.8
0
2
4
6
8
10
Seed mass (log(mg))


Trait
*** Intercept
** Slope
Survival (% annual)
1
0.9
0.8
0.35
0.45
0.55
Wood density (g m^2)

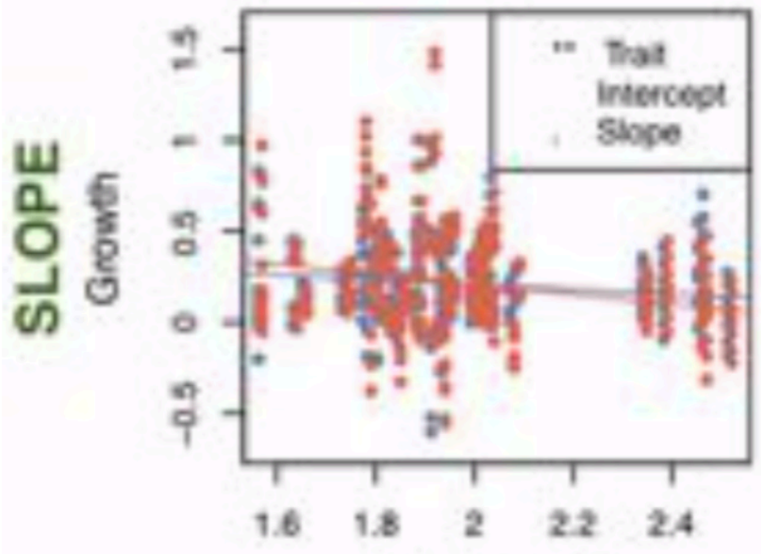
STAND AGE

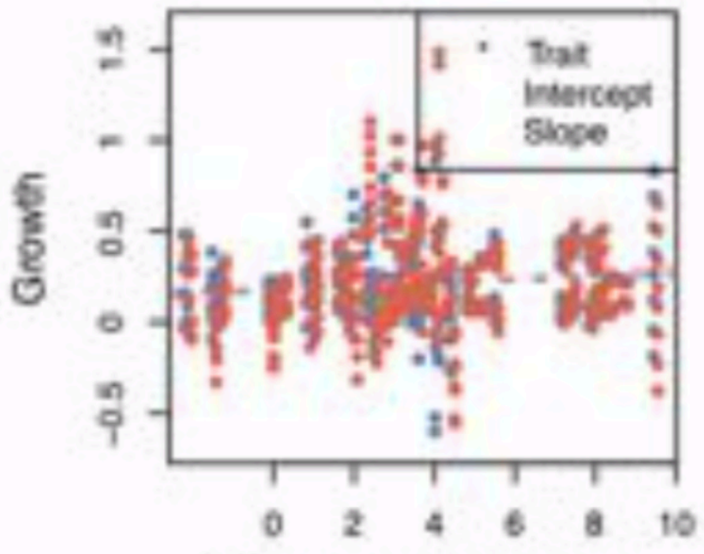


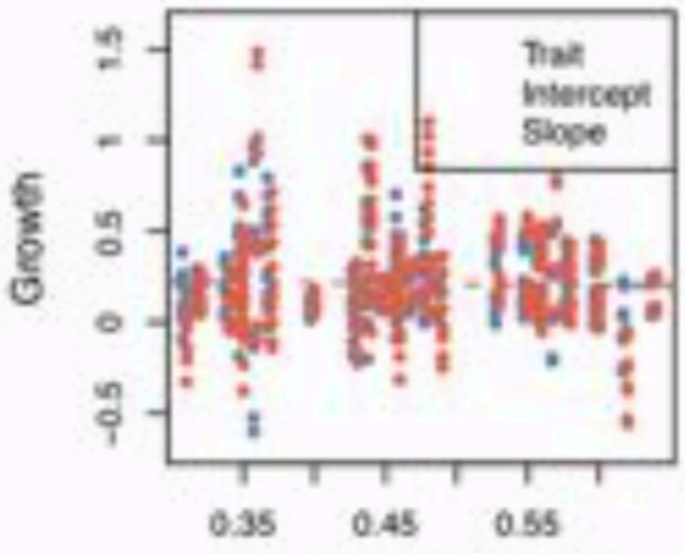




SLOPE







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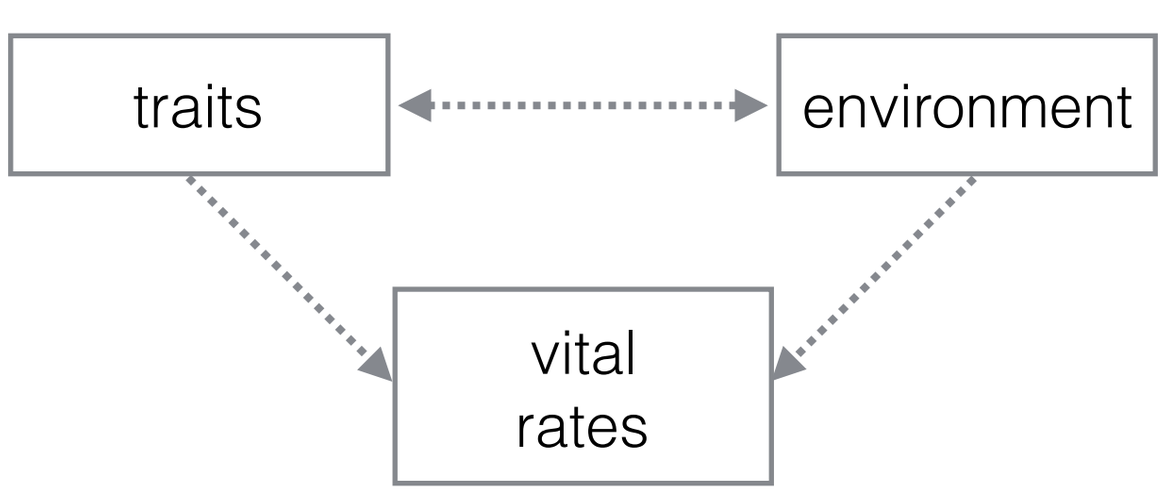
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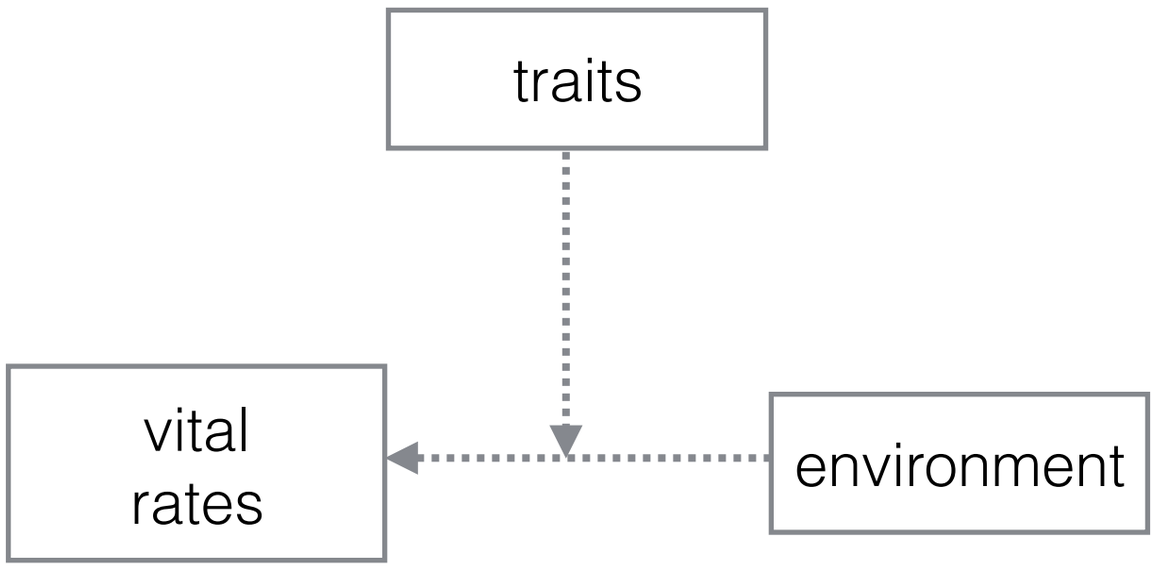
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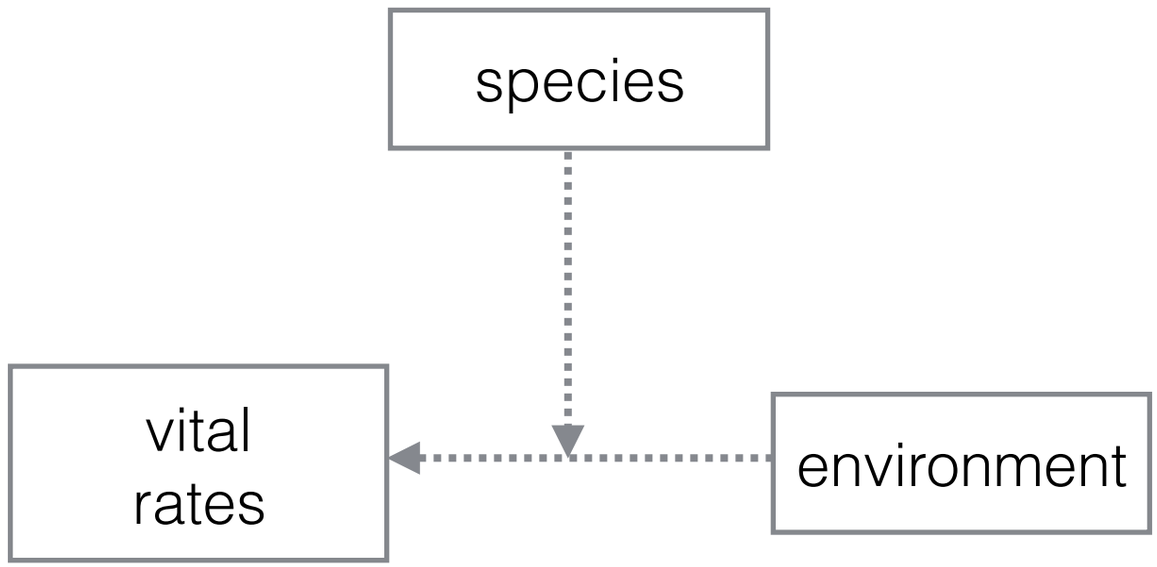
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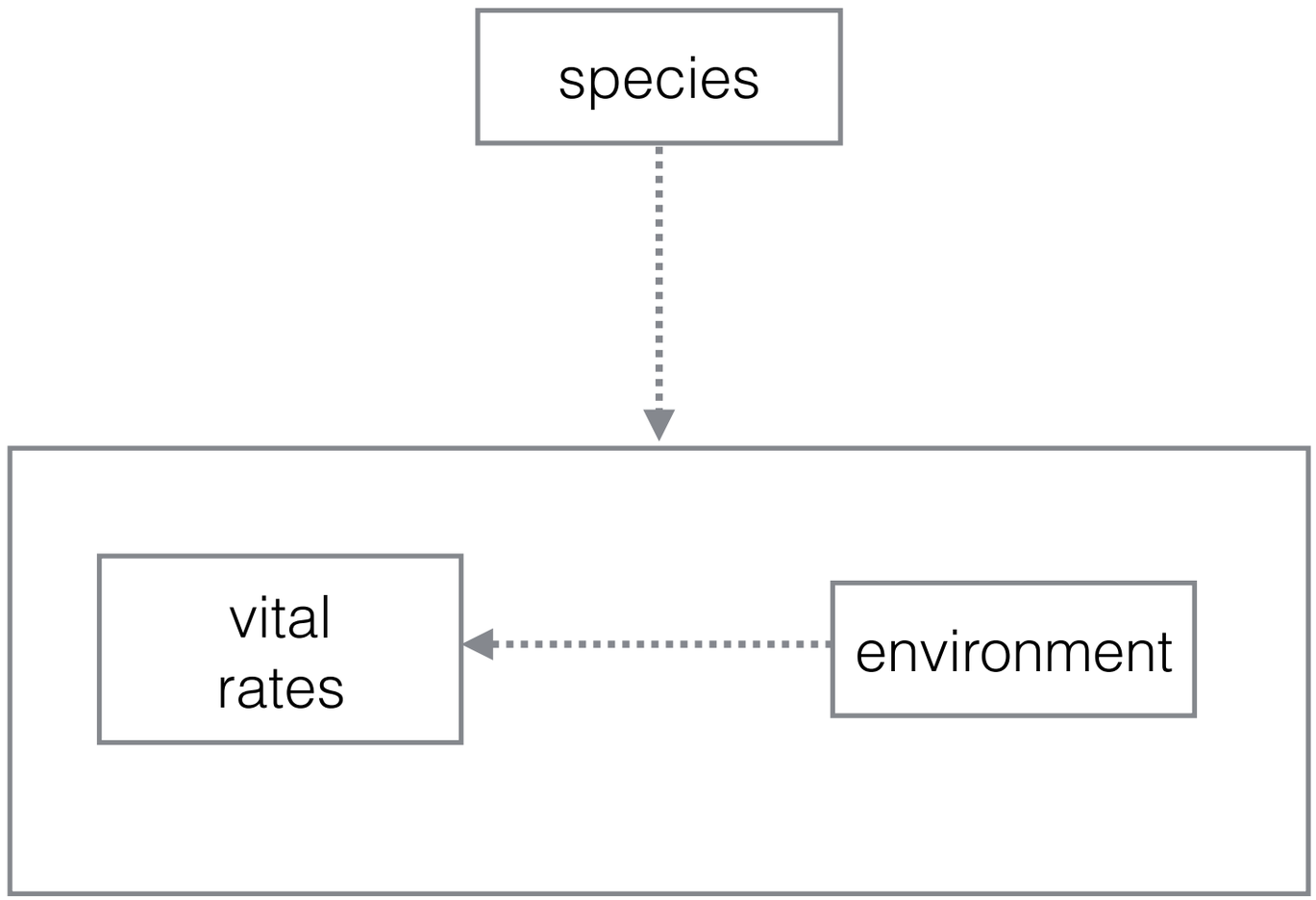
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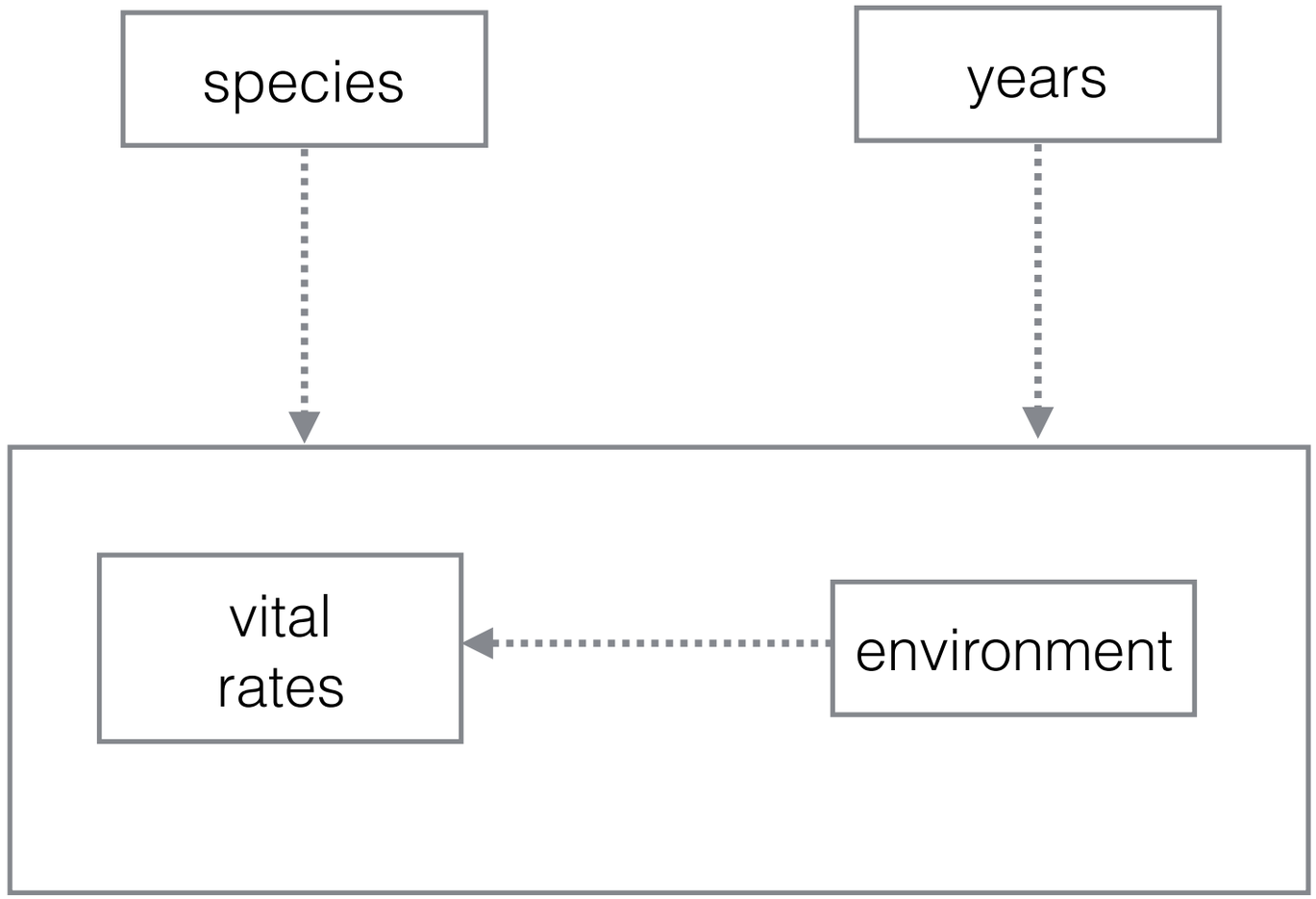
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years
vital
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‘Multilevel regression deserves to be the default form of regression.’

Richard McElreath, *Statistical Rethinking.*