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# CS-E5740 Complex Networks, Answers to exercise set 1

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Compile with `pdflatex ex_template.tex`

## Problem 1 6/6

a) – Degree of the node with the highest degree: 35
   – Total number of links: 498
   – See Figure 1.

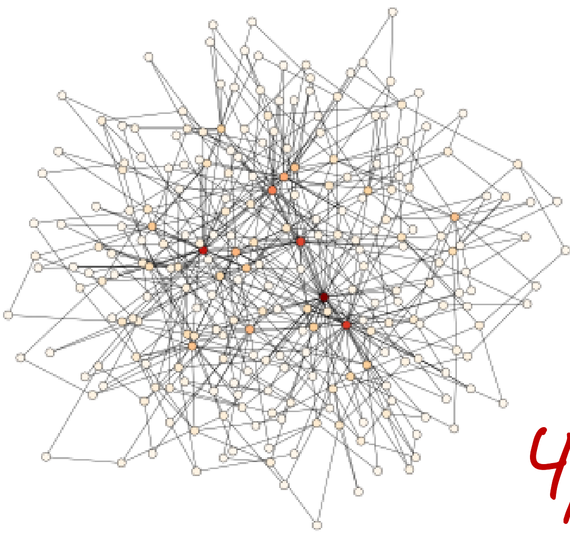

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Figure 1: Barabási-Albert model with $N =$ 2 $50$ and $m = 2$

b) Look at 2

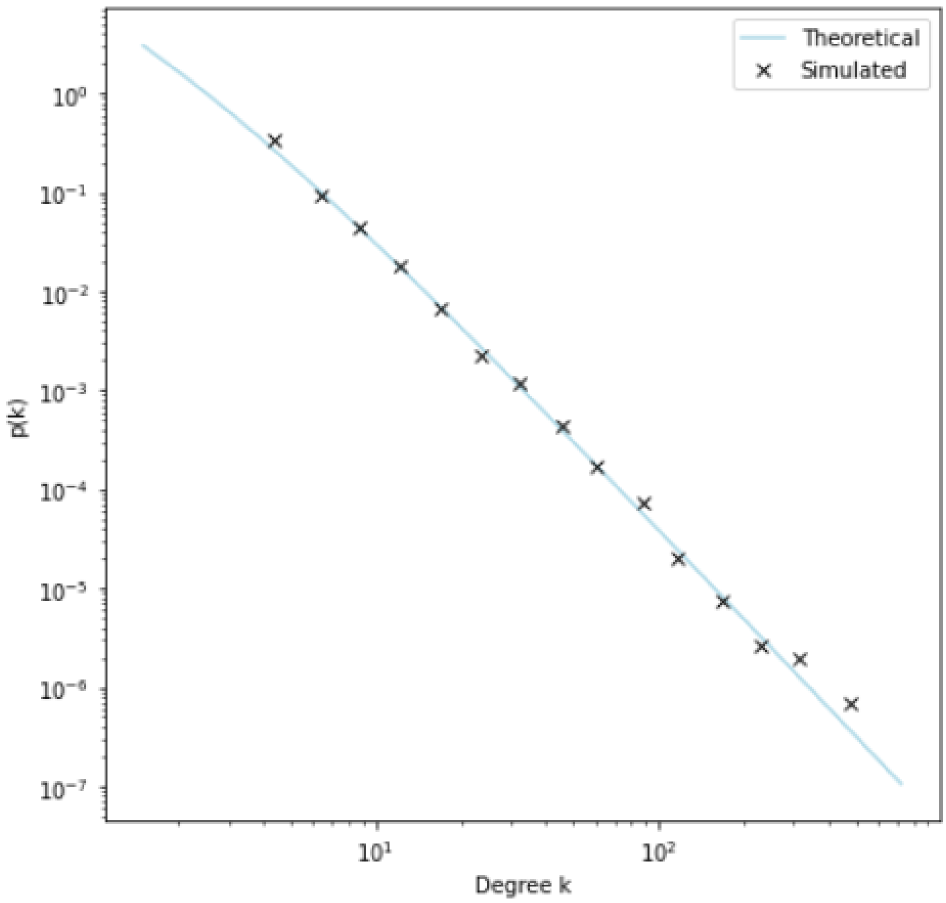
Theoretical
Simulated
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
p(k)
$10^1$
$10^2$
Degree k

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Figure 2:

## Problem 2 6,5/8

a) The sum of the degrees of the nodes of the network increases $2m$ times for each node we add (excluding initial clique), 2 because each time we add an edge that connects 2 nodes.

$$\sum_{j=1}^{N} k_j = 2mN$$

As defined, $f_{k,N}$ is the density of degree $k$, so $Nf_{k,N}$ is the number of verticies of degree $k$. Because we want a specific degree we multiply $Nf_{k,N}$ by $k$. ?

$$k_i = kNf_{k,N}$$ ?

So, we have 3.

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$$\Pi(K) = \frac{K \cdot \cancel{N} f_{K,N}}{2m\cancel{N}} = \frac{K \cdot f_{K,N}}{2m}$$

Figure 3:

b) Because we are adding one node with a degree $k$, we we are adding 1 to $\mu_k^+$. For $k > m$ we have 4

and there are no nodes with deg<m

$$(N+1) f_{K,N+1} - N f_{K,N} = \mu_K^+ - \mu_K^-$$
$$(N+1) f_{K,N+1} = N f_{K,N} + \frac{K-1}{2} f_{K-1,N} - \frac{K}{2} f_{K,N}$$

Figure 4:

For $k = m$ we have 5

Because $K = m \Rightarrow$ ← because of prev. formula/exercise

$$(N+1) f_{m,N+1} = N f_{m,N} + 1 - \frac{m}{2} f_{m,N}$$

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Figure 5:

c) Because we are growing the graph to infinity, small changes like adding a new edge does not effect the overall average degree.

For $k > m$ we have 6

$$(N+1) f_{K,N+1} = N \cdot f_{K,N} + \frac{K-1}{2} \cdot f_{K-1,N} - \frac{K}{2} f_{K,N} \Rightarrow$$
$$\Rightarrow (N+1) f_K = N \cdot f_K + \frac{K-1}{2} \cdot f_{K-1} - \frac{K}{2} f_K \Rightarrow$$
$$\Rightarrow (N+1) f_K - N f_K + \frac{K}{2} \cdot f_K = \frac{K-1}{2} \cdot f_{K-1} \Rightarrow$$
$$\Rightarrow f_K \left(\cancel{N}+1-\cancel{N}+\frac{K}{2}\right) = f_{K-1} \cdot \frac{K-1}{2} \Rightarrow$$
$$\Rightarrow f_K = f_{K-1} \left(\frac{K-1}{\cancel{2}}\right)\left(\frac{\cancel{2}}{2+K}\right) \Rightarrow$$
$$\Rightarrow f_K = f_{K-1} \frac{K-1}{K+2}$$

Figure 6:

For $k = m$ we have 7

$$(N+1) f_{m,N+1} = N f_{m,N} + 1 - \frac{m}{2} f_{m,N} \Rightarrow$$
$$\Rightarrow (N+1) f_m = N f_m + 1 - \frac{m}{2} f_m \Rightarrow$$
$$\Rightarrow (N+1) f_m - N f_m + \frac{m}{2} f_m = 1 \Rightarrow$$
$$\Rightarrow f_m \left(\cancel{N}+1-\cancel{N}+\frac{m}{2}\right) = 1 \Rightarrow$$
$$\Rightarrow f_m \frac{2+m}{2} = 1 \Rightarrow$$
$$\Rightarrow f_m = \frac{2}{2+m}$$

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Figure 7:

d) Simply substituting $k$ with $k+1$ we get

$$f_{k+1} = k/(k+3) \quad \cdot f_k$$

Then recursively we get 8, and it's possible to notice the recursive pattern with $m+2$ and $m+3$.

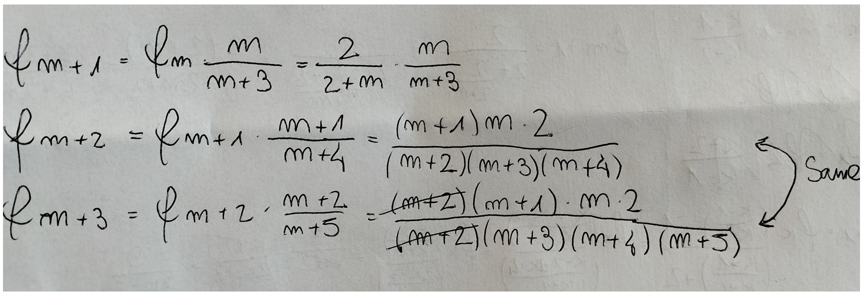


Figure 8:

This way we can say that

$$f_k = 2m(m+1)/k(k+1)(k+2)$$

2k