CS-E5740 Complex Networks,
Answers to exercise set X

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Compile with `pdflatex ex_template.tex`

Problem 1

a) Adjacency matrix is used to represent the edges in a graph, where 1 is an edge 0 is no connectin

a)

$$\begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 5 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 8 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}$$

b) Edge density (ρ) is the number of edges compared to a fully connected network

$$\text{b)}\ \rho = \frac{2 \cdot m}{N(N-1)} = \frac{2 \cdot 9}{8 \cdot 7} \simeq 0{,}32$$

c) Degree (k) is the number of neighborns of a node

c)

Node number	Degree
1	1
2	1
3	3
4	5
5	2
6	3
7	1
8	2

d) Average degree measures the number of edges over to the number of nodes

d) $$\langle k \rangle = \sum_i \frac{k_i}{N} = \frac{2m}{N} = \frac{2 \cdot 9}{8} = 2.25$$

e) Diameter (d) of a network is the maximum distance found in it, where each node is checked for the shortest path

e) $d \Rightarrow 7 \to 8 \to 4 \to 3 \to 5 = 4$

(Longest path example)

f) Clustering coefficient (c) of a node is the number of neighborns connected together

f)

C_1 excluded $(K_i = 1)$

C_2 excluded

C_3 $\frac{2 \cdot 2}{\cancel{4}3 \cdot \cancel{3}2} = 0,\overline{6}$

C_4 $\frac{2 \cdot 1}{5 \cdot 4} = 0,1$

C_5 $\frac{2 \cdot 1}{2 \cdot 1} = 1$

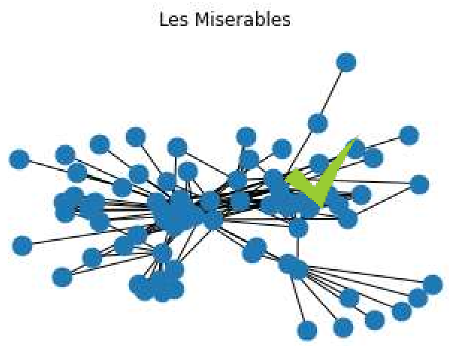
C_6 $\frac{2 \cdot 2}{3 \cdot 2} = 0,\overline{6}$

C_7 excluded

C_8 $\frac{2 \cdot 0}{2 \cdot 1} = 0$

Problem 2

a) Look at plot below.

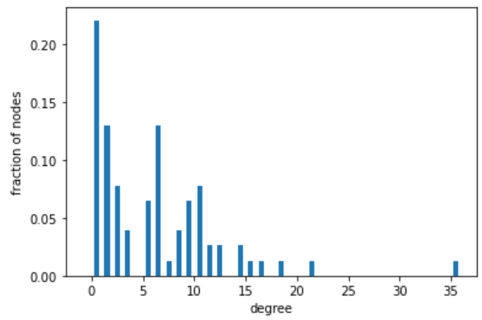


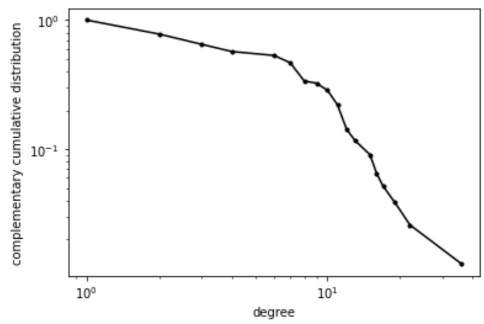
b) D from self-written algorithm: 0.08680792891319207
D from NetworkX: 0.08680792891319207

c) $< l >$ from NetworkX: 2.6411483253588517

d) $< c >$ from NetworkX: 0.5731367499320135

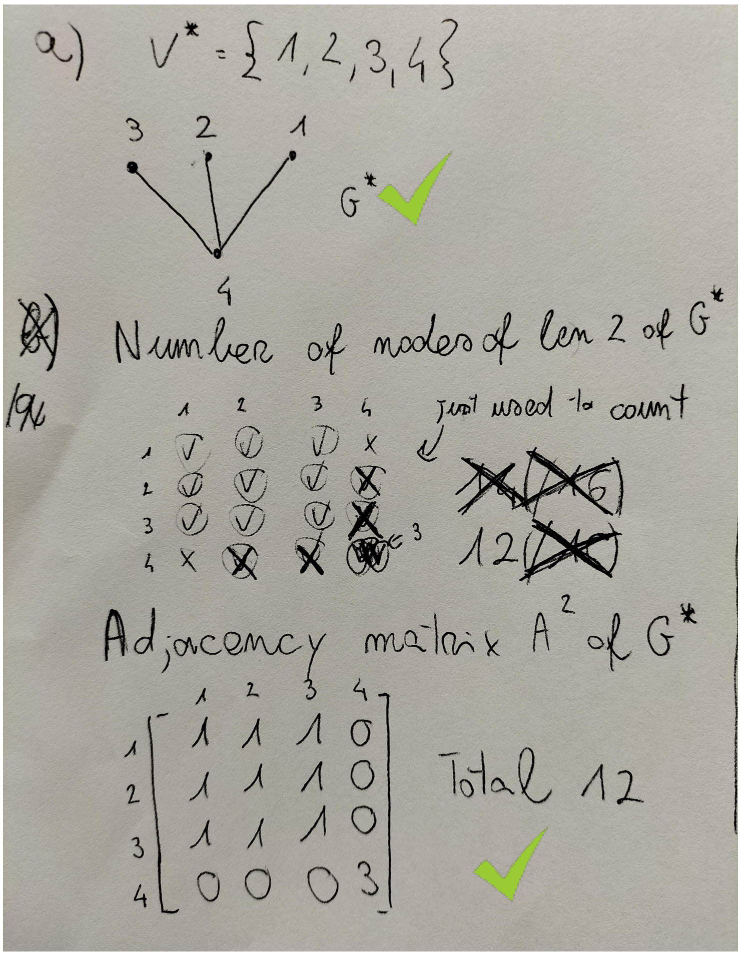
e) Look plot below





Problem 3

a) The total is 12 and it's the same in both cases.


a) $V^* = \{1,2,3,4\}$

G^*

Number of nodes of len 2 of G^*

just used to count

12

Adjacency matrix A^2 of G^*

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \end{array}$$

Total 12

b) Look picture

b)

$$\left.\begin{array}{l} 3 \to 4 \to 3 \to 4 \\ 3 \to 4 \to 2 \to 4 \\ 3 \to 4 \to 1 \to 4 \end{array}\right\} \text{3 walks}$$

$$A^2 \quad \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \end{array}$$

$$A \quad \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

~~$(A^3)_{3,4}$ sum any node connected with step 2~~

$$A^3 = A \cdot A^2$$

$$\Downarrow$$

$$(A^3)_{3,4} = 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 3 = 3$$

c) Look picture

Base case: for $m = 1$ using the definition of adjacency matrix the base case is that if there is an edge between the nodes i and j then we write 1, otherwise 0. So $a_{i,j}^{(1)} = A_{i,j}^{(1)}$.

We assume $a_{i,j}^{(m)}$gives the number of walks of lenght m, we can express a walk of lenght $m + 1$ splitting in in lenght m from v_i to v_k and 1 from v_k to v_j

$$a_{i,j}^{(m+1)} = \sum_{k=1}^{\overset{\text{graph}}{|V|}} \underbrace{A_{kj}}_{\text{len } 1} \underbrace{a_{ik}^{(m)}}_{\text{len } m} = \sum_{k=1}^{\overset{\text{graph}}{|V|}} \underbrace{A_{kj}}_{\text{len } 1} \underbrace{A_{ik}^{(m)}}_{\text{len } m}$$

Index of comments

1.1 Total:
6+6+5=17