CS-E5740 Complex Networks,
Answers to exercise set 4

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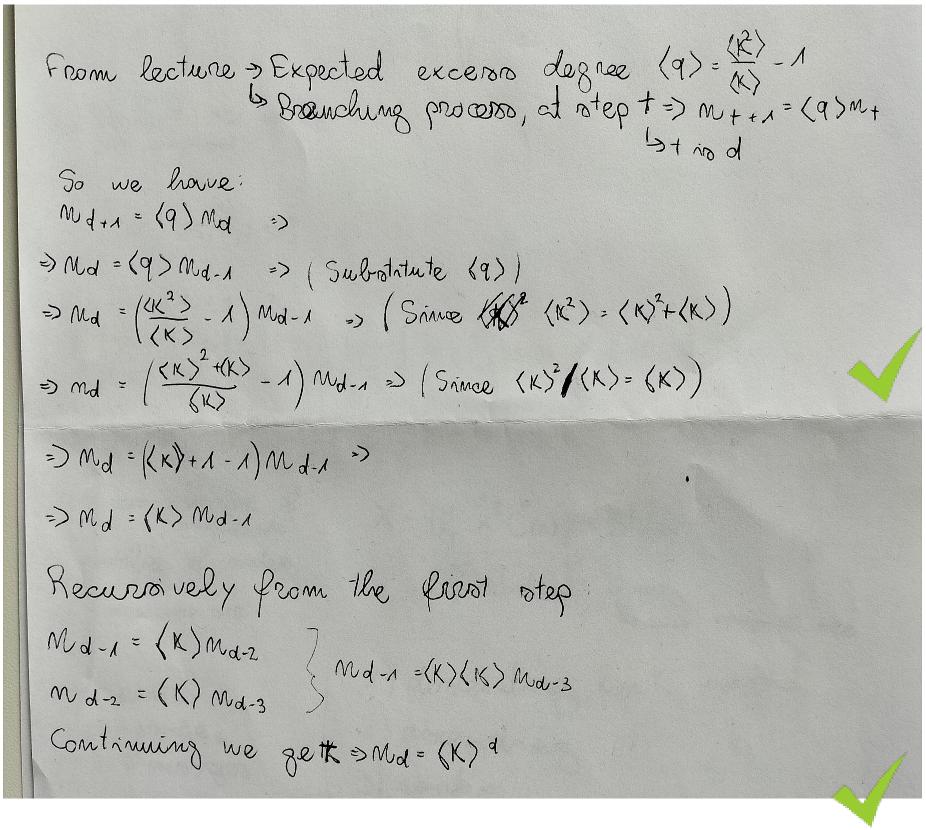
October 13, 2022

Problem 1

a) Look calculation in figure 1.



From lecture ⇒ Expected excess degree $\langle q \rangle = \frac{\langle k^2 \rangle}{\langle k \rangle} - 1$

↳ Branching process, at step $t \Rightarrow n_{t+1} = \langle q \rangle n_t$

↳ t is d

So we have:

$n_{d+1} = \langle q \rangle n_d \Rightarrow$

$\Rightarrow n_d = \langle q \rangle n_{d-1} \Rightarrow$ (Substitute $\langle q \rangle$)

$\Rightarrow n_d = \left(\frac{\langle k^2 \rangle}{\langle k \rangle} - 1\right) n_{d-1} \Rightarrow$ (Since $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$)

$\Rightarrow n_d = \left(\frac{\langle k \rangle^2 + \langle k \rangle}{\langle k \rangle} - 1\right) n_{d-1} \Rightarrow$ (Since $\langle k \rangle^2 / \langle k \rangle = \langle k \rangle$)

$\Rightarrow n_d = (\langle k \rangle + 1 - 1) n_{d-1} \Rightarrow$

$\Rightarrow n_d = \langle k \rangle n_{d-1}$

Recursively from the first step:

$$\left.\begin{array}{l} n_{d-1} = \langle k \rangle n_{d-2} \\ n_{d-2} = \langle k \rangle n_{d-3} \end{array}\right\} n_{d-1} = \langle k \rangle \langle k \rangle n_{d-3}$$

Continuing we get $\Rightarrow n_d = \langle k \rangle^d$

Figure 1: Calculation for exercise 1 section a

From this we get that if $\langle k \rangle$ is < 1, n_d will get increasingly smaller and if > 1 will increase exponentially thus allowing a creation of one single component.

b) See Figure 2

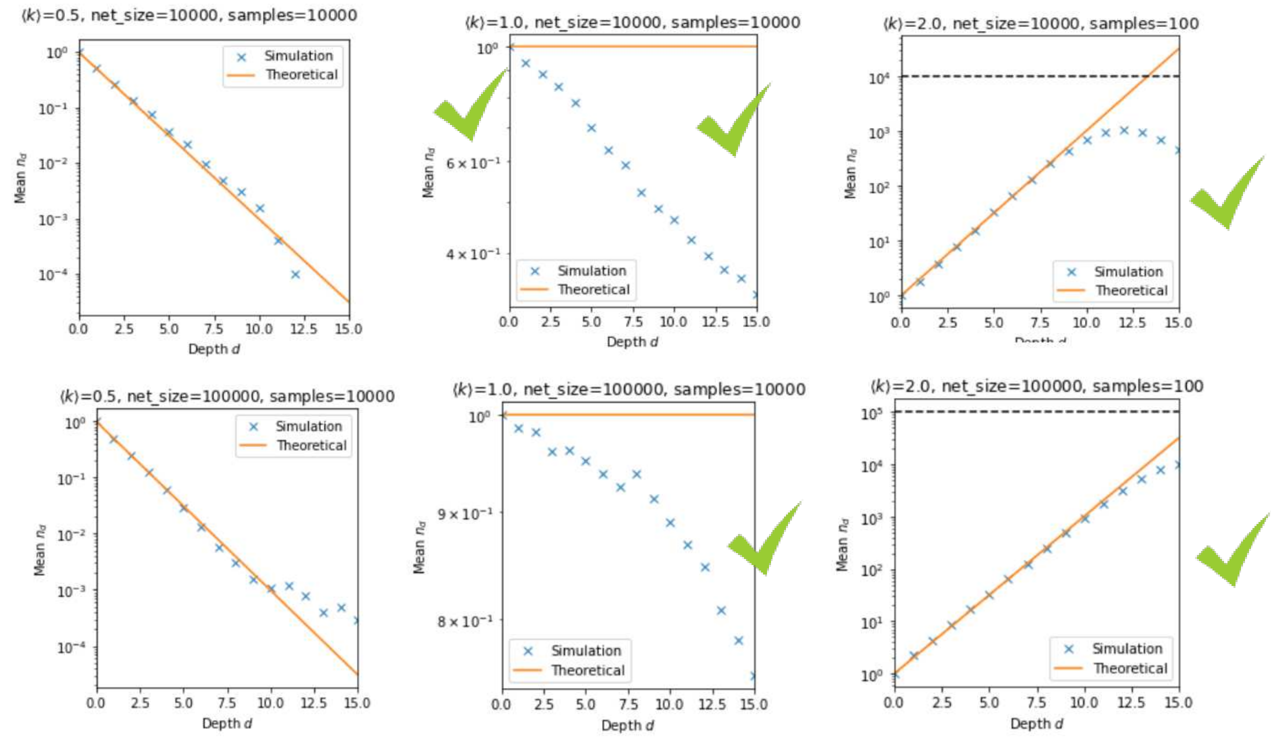


Figure 2: Plot for exercise 1 section b: Mean n_d as a function of d for different values of k

c) See Figure 3

d) See Figure 4

e) See Figure 5. Explaination: the shape of the curve is characterize by a spike in the transition point (our case is $\langle k \rangle = 1$). This implies that if we have a network of an average degree after the peak, a large connected component will start creating, while if it's smaller this won't happen.

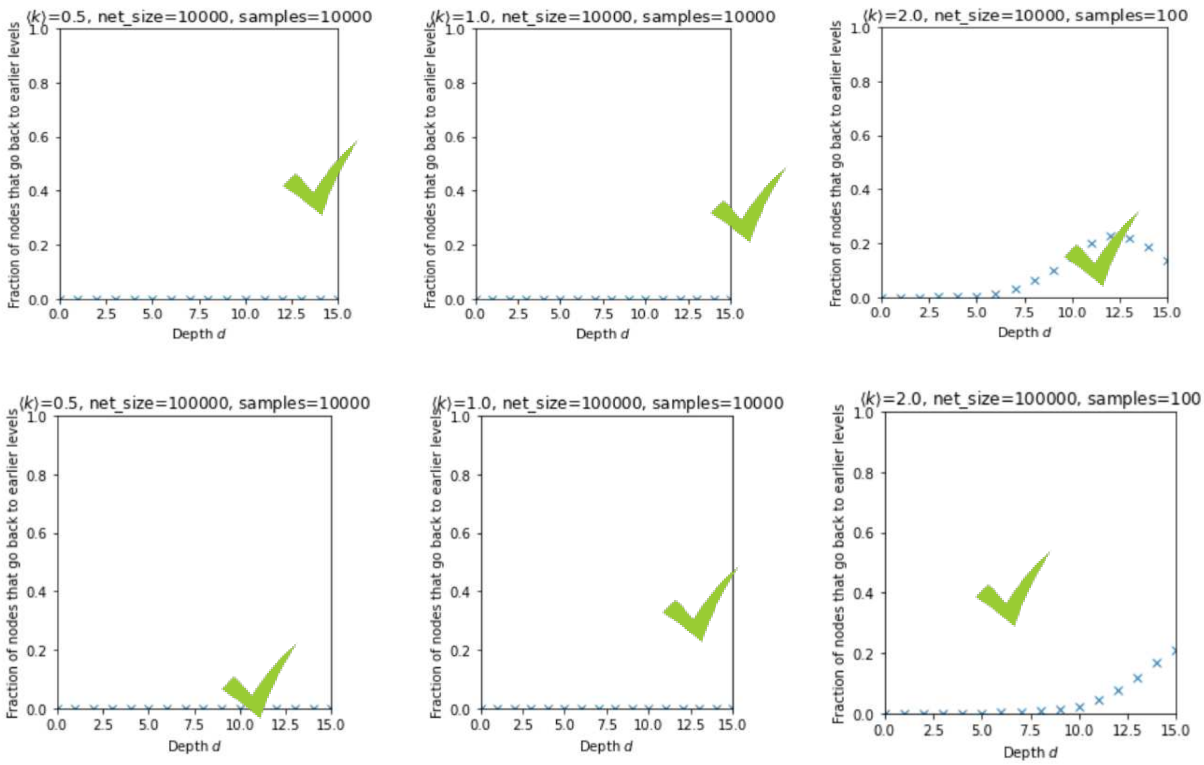


Figure 3: Plot for exercise 1 section c

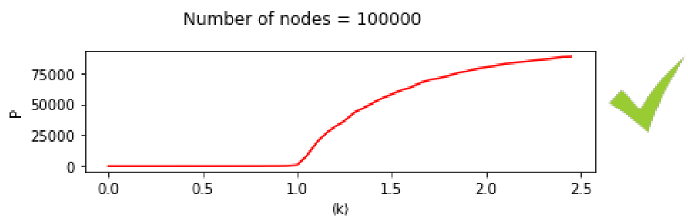


Figure 4: Plot for exercise 1 section d

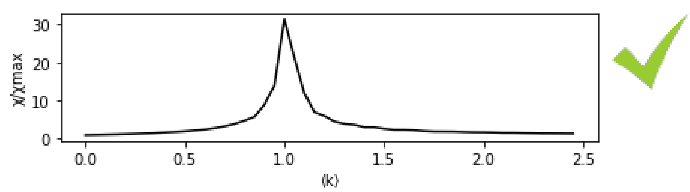


Figure 5: Plot for exercise 1 section e

Problem 2

Plot can be seen in Figure 6. The approach the network most and least vulnerable is when we start targeting connections with small weight, in this way the giant component shirnks the fastest.

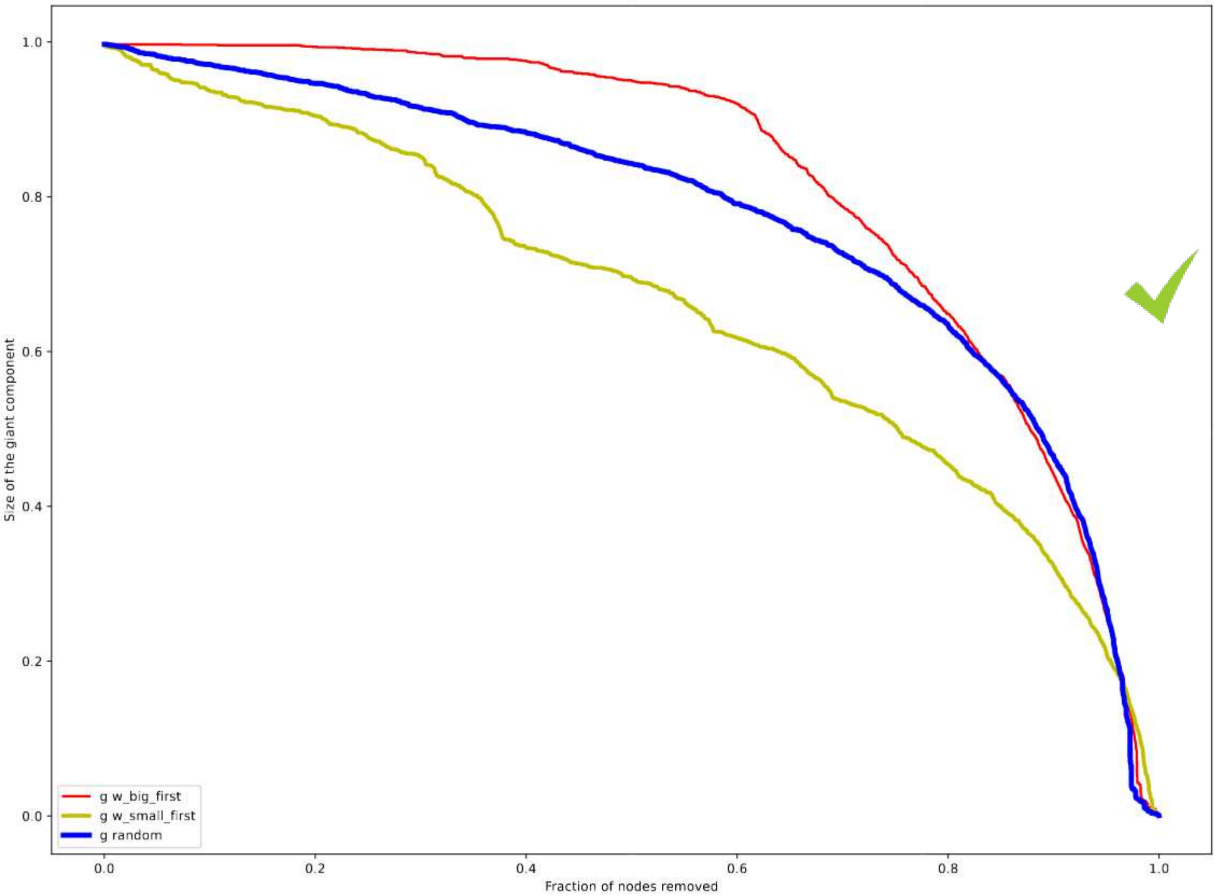


Figure 6: Plot for exercise 2