CS-E5740 Complex Networks,
Answers to exercise set 2

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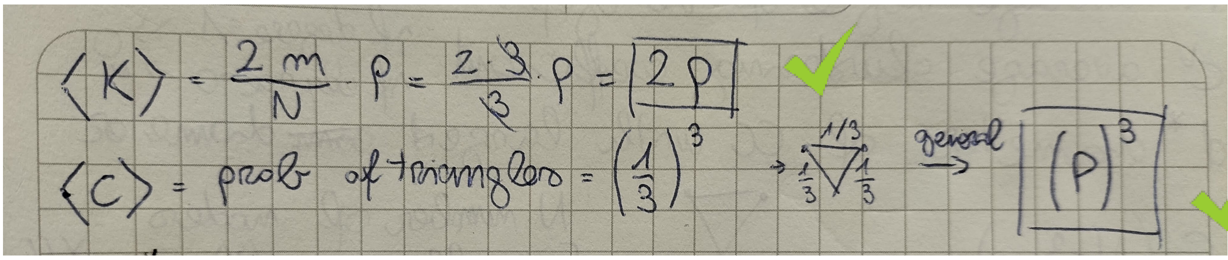
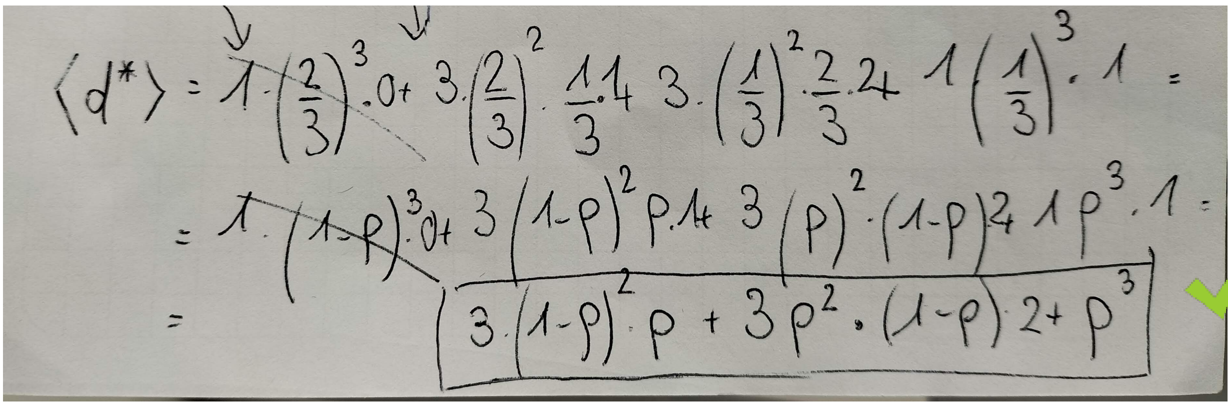
Problem 1



$$\langle k \rangle = \frac{2m}{N} \cdot p = \frac{2 \cdot \cancel{3}}{\cancel{3}} \cdot p = \boxed{2p}$$

$$\langle c \rangle = \text{prob of triangles} = \left(\frac{1}{3}\right)^3 \rightarrow \xrightarrow{\text{general}} \boxed{(p)^3}$$

Figure 1: $\langle k \rangle$ and $\langle c \rangle$ for problem 1



$$\langle d^* \rangle = \cancel{1 \cdot \left(\frac{2}{3}\right)^3 \cdot 0} + 3 \cdot \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} \cdot 1 + 3 \cdot \left(\frac{1}{3}\right)^2 \cdot \frac{2}{3} \cdot 2 + 1 \left(\frac{1}{3}\right)^3 \cdot 1 =$$

$$= \cancel{1 \cdot (1-p)^3 \cdot 0} + 3(1-p)^2 p \cdot 1 + 3(p)^2 \cdot (1-p) \cdot 2 + 1 p^3 \cdot 1 =$$

$$= \boxed{3 \cdot (1-p)^2 \cdot p + 3p^2 \cdot (1-p) \cdot 2 + p^3}$$

Figure 2: $\langle d^* \rangle$ for problem 1

Problem 2

a) The expected value of clustering coefficient equals p because for each node if we count the possible triangles that occur (to get a connection between neighborns) over all the possible connections between them, and the probability for each each to have a connection with any of its neighborns is indipendent for each node in the graph.

b) If N goes to infinity with $\langle k \rangle$ bonded the clustering coefficient will collapse.

c) See plot 3

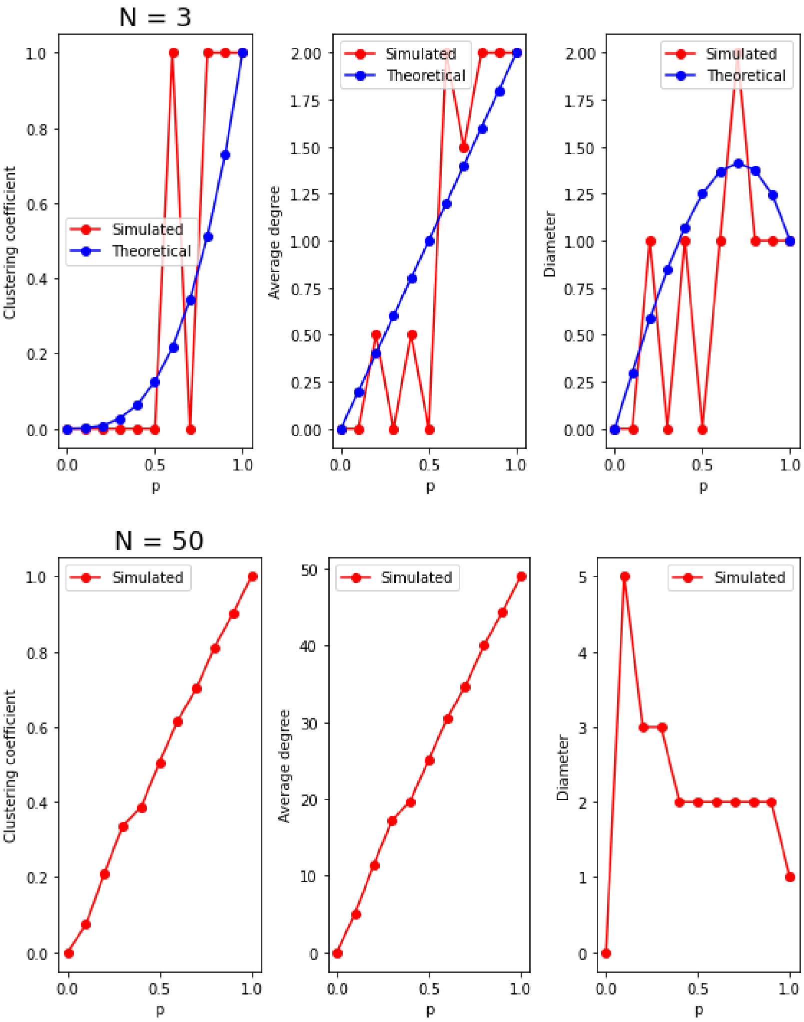


Figure 3: Plot of the quantities for exercise 2c

Problem 3

a) Look at plot 4

- For $n = 30, m = 2, p = 0.2$
  - total number of links: 60
  - number of rewired links: 9
- For $n = 80, m = 2, p = 0.4$
  - total number of links: 160
  - number of rewired links: 65

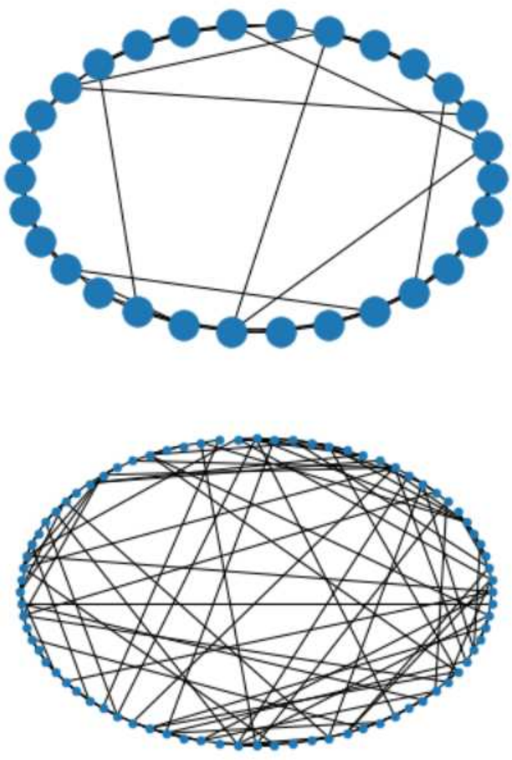


Figure 4: Plot for exercise 3a, on the top we have $N = 30, m = 2, p = 0.2$, and on the bottom we have $N = 80, m = 2, p = 0.4$

b) Look at plot 5

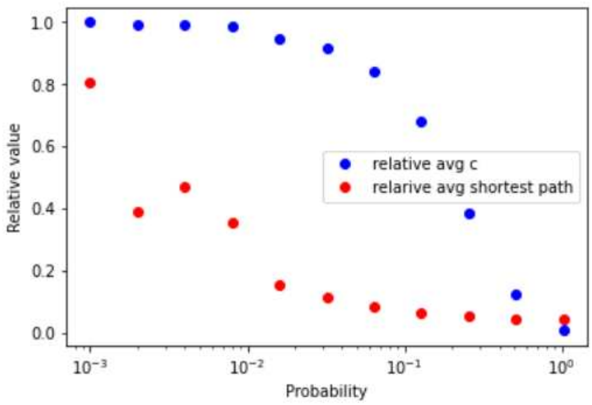


Figure 5: Plot for exercise 3d

1. Yes, both clustering coefficient and path lenght show the trend shown in the lecture slides
2. Because if we take the base case of the Watts-Strogatz small world model we can see that we have a lot of triangles, meaning that for each node its neighborns are connected, meanwile if we change the probability of changing edge, this "triangles" will disappear and thus the clustering coefficient decreases.
3. The relative path length decreases with the probability of each edge changing because we are increasing the "randomness" of the system, thus making it naturally a small worlds. Practlicaly we can see that from each node to each other node the distance decreases because all nodes are better connected and each node does not need to make half of the circle to reach the node opposite to it.

Index of comments

2.1 If <k> is bounded, the network density approaches 0 when N goes to infinity. Therefore, the clustering coefficient also tends to 0.

2.2 Simulated and theoretical plots should follow each other.