1 Module Guide: Applied Complex Analysis (Spring Term, 2021)

Module code: MATH96019/MATH97028/MATH97105

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Website: On GitHub, at https://github.com/MarcoFasondini/M3M6AppliedComplexAnalysis

Video lectures: Panopto

Discussion forum: Sign up here to Piazza, where you can ask questions (anonymously, if you wish) and find fellow students to work with. The module's Piazza site is here

Problem classes: on MS Teams (for now). There are two problem classes on Thursdays in Weeks 2, 4, 6, 8 and 10; one is from 11:00-12:00, the other from 2:00-3:00. The same problem sheet (available on the module website) will be covered in each of the two problem classes.

Office hours: Fridays, 11:00-12:00, on MS Teams (for the time being).

Reading list:

1. M.J. Ablowitz & A.S. Fokas, Complex Variables: Introduction and Applications, Second Edition, Cambridge University Press, 2003
2. R. Earl, Metric Spaces and Complex Analysis, 2015
3. E. Wegert, Visual Complex Functions: An Introduction with Phase Portraits, Birkhäuser, 2012
4. B. Fornberg & C. Piret, Complex Variables and Analytic Functions: An Illustrated Introduction, SIAM, 2019 (this book might become available through the library later this term)

1.1 The Project

There is a project worth 10% of your final mark (and the exam is worth 90%). You can decide whether to do the project individually or in a group of up to four students in total. This project is *open ended*: you (or your group) propose a topic. This could be computationally based (possibly based on the slides), theoretically based (possibly looking at material on the reading list), or otherwise. If you are having difficulty coming up with a proposal, please attend the office hours for advice. You can find examples of projects on the module website.

Timeline:

- 26 Feb: Turn in short description of proposed project (max 2 paragraphs)
- 26 March: Project due

1.2 Overview of course

1. Complex analysis, Cauchy's theorem, residue calculus
2. Matrix-valued functions, with applications to PDEs and fractional-order PDEs
3. Singular integrals of the form

$$\int_\Gamma \frac{u(\zeta)}{z-\zeta} d\zeta,$$

$$\int_\Gamma u(\zeta) \log |z-\zeta| ds$$

with applications to PDEs, airfoil design, etc.

4. Orthogonal polynomials, with applications to Schrödinger operators, solving differential equations.
5. Weiner-Hopf method with applications to integral equations with integral operators

$$\int_0^\infty K(x-y)u(y)dy$$

Central themes:

1. Finding "nice" formulae for problems that arise in applications (physics and elsewhere). These can be closed form solution, sums, integral representations, special functions, etc.
2. Computational tools for approximate solutions to problems that arise in applications.

Applications (not necessarily discussed in the course):

1. Ideal fluid flow
2. Acoustic scattering
3. Electrostatics (Faraday cage)
4. Fracture mechanics
5. Schrödinger equations
6. Shallow water waves