## 1 Module Guide: Applied Complex Analysis (Spring Term, 2021)

Module code: MATH96019/MATH97028/MATH97105

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Website: On GitHub, at https://github.com/MarcoFasondini/M3M6AppliedComplexAnalysis

Video lectures: Panopto

**Discussion forum:** Sign up here to Piazza, where you can ask questions (anonymously, if you wish) and find fellow students to work with. The module's Piazza site is here

**Problem classes:** on MS Teams (for now). There are two problem classes on Thursdays in Weeks 2, 4, 6, 8 and 10; one is from 11:00-12:00, the other from 2:00-3:00. The same problem sheet (available on the module website) will be covered in each of the two problem classes.

Office hours: Fridays, 11:00-12:00, on MS Teams (for the time being).

## Reading list:

- 1. M.J. Ablowitz & A.S. Fokas, Complex Variables: Introduction and Applications, Second Edition, Cambridge University Press, 2003
- 2. R. Earl, Metric Spaces and Complex Analysis, 2015
- 3. E. Wegert, Visual Complex Functions: An Introduction with Phase Portraits, Birkhäuser, 2012
- 4. B. Fornberg & C. Piret, Complex Variables and Analytic Functions: An Illustrated Introduction, SIAM, 2019 (this book might become available through the library later this term)

## 1.1 Overview of course

- 1. Complex analysis, Cauchy's theorem, residue calculus
- 2. Matrix-valued functions, with applications to PDEs and fractional-order PDEs
- 3. Singular integrals of the form

$$\int_{\Gamma} \frac{u(\zeta)}{z - \zeta} d\zeta,$$
$$\int_{\Gamma} u(\zeta) \log|z - \zeta| ds$$

with applications to PDEs, airfoil design, etc.

4. Orthogonal polynomials, with applications to Schrödinger operators, solving differential equations.

5. Weiner-Hopf method with applications to integral equations with integral operators

$$\int_0^\infty K(x-y)u(y)dy$$

Central themes:

- 1. Finding "nice" formulae for problems that arise in applications (physics and elsewhere). These can be closed form solution, sums, integral representations, special functions, etc.
- 2. Computational tools for approximate solutions to problems that arise in applications.

Applications (not necessarily discussed in the course):

- 1. Ideal fluid flow
- 2. Acoustic scattering
- 3. Electrostatics (Faraday cage)
- 4. Fracture mechanics
- 5. Schrödinger equations
- 6. Shallow water waves

## 1.2 The Project

There is a project worth 10% of your final mark (and the exam is worth 90%). You can decide whether to do the project individually or in a group of up to four students in total. This project is *open ended*: you (or your group) propose a topic. This could be computationally based (possibly based on the slides), theoretically based (possibly looking at material on the reading list), or otherwise. If you are having difficulty coming up with a proposal, please attend the office hours for advice. You can find examples of projects on the module website.

Timeline:

- 26 Feb: Turn in short description of proposed project (max 2 paragraphs)
- 26 March: Project due