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Reinforcement Learning for $\mathrm{LTL}_f/\mathrm{LDL}_f$ Goals: Theory and Implementations

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Abstract

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Introduction

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LTL$_f$ and LDL$_f$

In this chapter we introduce the reader to the main important framework for talk about behaviors over time, which gives the foundations for our approach. First we talk about the well known Linear time Temporal Logic (LTL), Propositional Dynamic Logic (PDL) and their main applications; then we go more in deep by presenting a specific formalism, namely *Linear Temporal Logic over Finite Traces* LTL$_f$ and *Linear Dynamic Logic over Finite Traces* LDL$_f$. Finally, we study the translation from LTL$_f$/LDL$_f$ formulas to Deterministic Finite Automata (DFA). We require the reader to be acquainted with classical logic (Shapiro and Kouri Kissel, 2018) and automata theory (Hopcroft et al., 2000).

2.1 Linear time Temporal Logic (LTL)

Temporal Logic (Goranko and Galton, 2015) is a category of formal languages aimed to talk about properties of a system whose truth value might change over time. This is in contrast with atemporal logics, which can only discuss about statements whose truth value is constant.

Linear time Temporal Logic (Pnueli, 1977), or *Linear Temporal Logic* (LTL) is such a logic. It is the most popular and widely used temporal logic in computer science, especially in formal verification of software/hardware systems, in AI to reasoning about actions and planning, and in the area of Business Process Specification and Verification to specify processes declaratively.

It allows to express temporal patterns about some property p, like *liveness* (*p will eventually happen*), *safety* (*p will never happen*) and *fairness*, combinations of the previous patterns (*infinitely often p holds*, *eventually always p holds*).

2.1.1 Syntax

A LTL formula φ is defined over a set of propositional symbols $\mathcal{P}$ and are closed under the boolean connectives, the unary temporal operator $\circ$(*next-time*) and the binary operator $\mathcal{U}$ (*until*):

$$\varphi \quad ::= \quad A \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \circ\varphi \mid \varphi_1 \,\mathcal{U}\, \varphi_2$$

With $A \in \mathcal{P}$.

Additional operators can be defined in terms of the ones above: as usual logical operators such as $\vee$, $\Rightarrow$, $\Leftrightarrow$, *true*, *false* and temporal formulas like *eventually* as $\Diamond\varphi \doteq \mathit{true}\,\mathcal{U}\,\varphi$, *always* as $\Box\varphi \doteq \neg\Diamond\neg\varphi$ and *release* as $\varphi_1 \,\mathcal{R}\, \varphi_2 \doteq \neg(\neg\varphi_1 \,\mathcal{U}\, \neg\varphi_2)$.

Example 2.1. Several interesting temporal properties can be defined in LTL:

- *Liveness*: $\Diamond\varphi$, which means "condition expressed by φ *at some time* in the future will be satisfied", "sooner or later φ will hold" or "eventually φ will hold". E.g., $\Diamond rich$ (eventually I will become rich), $Request \implies \Diamond Response$ (if someone requested the service, sooner or later he will receive a response).

- *Safety*: $\Box\varphi$, which means "condition expressed by φ, *every time* in the future will be satisfied", "always φ will hold". E.g., $\Box happy$ (I'm always happy), $\Box\neg(temperature > 30)$ (the temperature of the room must never be over 30).

- *Response*: $\Box\Diamond\varphi$ which means "at any instant of time there exists a moment later where φ holds". This temporal pattern is known in computer science as *fairness*.

- *Persistence*: $\Diamond\Box\varphi$, which stand for "There exists a moment in the future such that from then on φ always holds". E.g. $\Diamond\Box dead$ (at a certain point you will die, and you will be dead forever)

- *Strong fairness*: $\Box\Diamond\varphi_1 \implies \Box\Diamond\varphi_2$, "if something is attempted/requested infinitely often, then it will be successful/allocated infinitely often". E.g., $\Box\Diamond ready \implies \Box\Diamond run$ (if a process is in ready state infinitely often, then infinitely often it will be selected by the scheduler).

2.1.2 Semantics

The semantics of LTL is provided by (infinite) *traces*, i.e. ω-word over the alphabet $2^{\mathcal{P}}$. More formally, a *trace* π is a *word* on a *path* of a *Kripke structure*.

Definition 2.1 (Clarke et al. (1999))**.** a Kripke structure $\mathcal{K}$ over a set of propositional symbols $\mathcal{P}$ is a 4-tuple $\langle S, I, R, L\rangle$ where S is a finite set of *states*, $I \subseteq S$ is the set of *initial states*, $R \subseteq S \times S$ is the *transition relation* such that R is left-total and $L : S \to 2^{\mathcal{P}}$ is a *labeling function.*

A *path* ρ over $\mathcal{K}$ is a sequence of states $\langle s_1, s_2, \ldots\rangle$ such that $\forall i.R(s_i, s_{i+1})$. From a path we can build a *word* w on the path ρ by mapping each state of the sequence with L, namely:

$$w = \langle L(s_1), L(s_2), \ldots\rangle$$

In simpler words, a trace of propositional symbols $\mathcal{P}$ is a infinite sequence of combinations of propositional symbols in $\mathcal{P}$. Moreover, we denote by $\pi(i)$ with $i \in \mathbb{N}$ the labels associated to s_i, i.e. $L(s_i)$.

Example 2.2. In figure 2.1 is depicted an example of Kripke structure $\mathcal{K}$ over $\mathcal{P} = \{p, q\}$ where:

$$\begin{aligned} S &= \{s_1, s_2, s_3\} \\ I &= \{s_1\} \\ R &= \{(s_1, s_2), (s_2, s_1), (s_2, s_3), (s_3, s_3)\} \\ L &= \{(s_1, \{p, q\}), (s_2, \{q\}), (s_3, \{p\})\} \end{aligned}$$

The path $\langle s_1, s_2, s_3, s_3, s_3 \ldots\rangle$ yields the following trace π:

$$\begin{aligned} \pi &= \langle L(s_1), L(s_2), L(s_3), L(s_3), L(s_3), \ldots\rangle \\ &= \langle\{p, q\}\{q\}, \{p\}, \{p\}, \{p\}, \ldots\rangle \end{aligned}$$

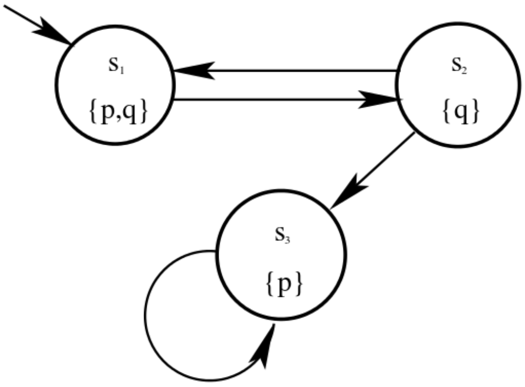


Figure 2.1. An example of Kripke structure.

Definition 2.2. Given a infinite trace π, we define that a LTL formula φ is *true* at time i, in symbols $\pi, i \models \varphi$ inductively as follows:

$$\pi, i \models A, \text{ for } A \in \mathcal{P} \text{ iff } A \in \pi(i)$$

$$\pi, i \models \neg\varphi \text{ iff } \pi, i \not\models \varphi$$

$$\pi, i \models \varphi_1 \wedge \varphi_2 \text{ iff } \pi, i \models \varphi_1 \wedge \pi, i \models \varphi_2$$

$$\pi, i \models \bigcirc\varphi \text{ iff } \pi, i+1 \models \varphi$$

$$\pi, i \models \varphi_1 \mathcal{U} \varphi_2 \text{ iff } \exists j.(j \geq i) \wedge \pi, j \models \varphi \wedge \forall k.(i \leq k < j) \Rightarrow \pi, k \models \varphi_1$$

Similiarly as in classical logic we give the following definitions:

Definition 2.3. A LTL formula is *true* in π, in notation $\pi \models \varphi$, if $\pi, 0 \models \varphi$. A formula φ is *satisfiable* if it is true in some π and is *valid* if it is true in every π. φ_1 *entails* φ_2, in symbols $\varphi_1 \models \varphi_2$ iff $\forall \pi, \forall i. \pi, i \models \varphi_1 \implies \pi, i \models \varphi_2$.

Now we state an important result:

Theorem 2.1 (Sistla and Clarke (1985)). *Satisfiability, validity, and entailment for* LTL *formulas are* PSPACE*-complete.*

Indeed, Linear Temporal Logic can be thought of as a specific decidable (PSPACE-complete) fragment of classical first-order logic (FOL).

2.2 Propositional Dynamic Logic (PDL)

Dynamic Logics (Pratt, 1976b; Troquard and Balbiani, 2015) (DL) are modal logics[1] for representing the states and the events of dynamic systems. We can speak about

[1] *Modal Logic* extends classical logics to include operator expressing *modality* (e.g. "necessarily", "possibly", "usually"). However, the term "modal logic" is used more broadly to cover a family of logics with similar rules and a variety of different symbols. Temporal Logic and Dynamic Logic described in this chapter are examples of modal logics. (Garson, 2016)

the properties that holds in a state (assertion language) and about properties on transitions between states (programming language). Dynamic Logics are indeed called *logics of programs*.

Propositional Dynamic Logic (Fischer and Ladner, 1979) (PDL), probably the most well-known (propositional) logic of programs in computer science, is the propositional counterpart of Pratt's original dynamic logic (Pratt, 1976b), which was a *first-order* modal logic. Basically, this means that from three types of terms, *assertions*, *data* (as in FOL) and *actions* we drop the *data* terms, hence we can reason only about abstract propositions and the actions for modify them.

As we did with LTL, in the following sections we describe syntax and semantics of PDL.

2.2.1 Syntax

A PDL formula φ is defined over a set of propositional symbols $\mathcal{P}$ and a set of atomic programs Π built as follows:

$$\begin{array}{lcl} \varphi & ::= & A \mid \mathbf{0} \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid [\alpha]\varphi \\ \alpha & ::= & \phi \mid \varphi? \mid \alpha_1 + \alpha_2 \mid \alpha_1 ; \alpha_2 \mid \varrho^* \end{array}$$

with $A \in \mathcal{P}$ and $\phi \in \Pi$. We can define classical logic operators $\vee, \Rightarrow, \Leftrightarrow, \mathbf{1}$ as usual, and the *possibility* operator $\langle\ \rangle$ from the *necessity* operator $[\ ]$, namely $\langle\alpha\rangle\varphi \doteq \neg[\alpha]\neg\varphi$. The propositions $[\alpha]\varphi$ and $\langle\alpha\rangle\varphi$ are read "box α φ" and "diamond α φ", respectively.

Notice that φ stands for the propositional component of the logic, while program α stands for the dynamic component. Moreover, notice that propositions and programs are intertwined and cannot be separated: the definition of propositions depends on the definition of programs because of the construct $[\alpha]\varphi$, and the definition of programs depends on the definition of propositions because of the construct $\varphi?$.

Example 2.3. Now we provide some example of compound formulas and programs:

- $[\alpha]\varphi$: "It is necessary that after executing α, φ is true";
- $\langle\alpha\rangle\varphi$ "There exists a computation of α that terminates in a state satisfying φ.
- $\alpha;\beta$: "Execute α, then execute β";
- $\alpha;\cup\beta$: "Choose either α or β nondeterministically and execute it";
- α^*: "Choose α a nondeterministically chosen finite number of times (zero or more);
- $\varphi?$: "Test φ: proceed if true, fail if false".

Example 2.4. To better understand the expressive power of PDL, it is worth to notice this correspondence between basic programming language constructs and PDL formulas:

$$\begin{aligned} \textbf{skip} &\coloneqq \mathbf{1} \\ \textbf{fail} &\coloneqq \mathbf{0} \\ \textbf{if } \varphi \textbf{ then } \alpha \textbf{ else } \beta &\coloneqq \varphi?;\alpha \cup \neg\varphi?;\beta \\ \textbf{while } \varphi \textbf{ do } \alpha &\coloneqq (\varphi?;\alpha)^*;\neg\varphi? \end{aligned}$$

$$\textbf{repeat } \alpha \textbf{ until } \varphi \coloneqq \alpha; (\neg\varphi?; \alpha)^*; \varphi?$$
$$\{\varphi\}\alpha\{\psi\} \coloneqq \varphi \implies [\alpha]\varphi$$

The programs **skip** and **fail** are the program that does nothing (no-op) and the failing program, respectively. The ternary **if-then-else** operator and the binary **while-do** operator are the usual conditional and while loop constructs found in conventional programming languages. The construct $\{\varphi\}\alpha\{\psi\}$ is the Hoare partial correctness assertion (Pratt, 1976a).

2.2.2 Semantics

The semantics for PDL formulas is provided by *Labelled Transition System* (LTS).

Definition 2.4. A *Labelled Transition System* over a set of propositional symbols $\mathcal{P}$ and a set of atomic programs Π is a 3-tuple $\langle S, R_p, V\rangle$ where S is the set of *states*, $R_p : \Pi \to 2^{S\times S}$ is a mapping from atomic programs to a binary relation over S and $V : \mathcal{P} \to 2^S$ is a mapping from propositional symbols to subsets of S.

Example 2.5. In figure 2.2 two examples of LTS defined over $\mathcal{P} = \{p, q\}$ and $\Pi = \{\pi_1, \pi_2\}$ are depicted. For the LTS on the left, $\mathcal{M}_1$, we have:

$$\begin{aligned} S &= \{x_1, x_2\} \\ R_p(\pi_1) &= \{(x_1, x_1)\} \\ R_p(\pi_2) &= \{(x_1, x_2)\} \\ V(p) &= \{x_1\} \\ V(q) &= \{x_2\} \end{aligned}$$

While for the LTS on the right, $\mathcal{M}_2$, we have:

$$\begin{aligned} S &= \{y_1, y_2, y_3, y_4\} \\ R_p(\pi_1) &= \{(y_1, y_2), (y_2, y_2)\} \\ R_p(\pi_2) &= \{(y_1, y_3), (y_2, y_4)\} \\ V(p) &= \{y_1, y_2\} \\ V(q) &= \{y_3, y_4\} \end{aligned}$$

In order to formally define the semantics of a PDL formula φ, we use the following notation:

- $xR(\pi)y$ iff there exists an execution of π from x that leads to y;
- $x \in V(p)$ iff p is true in x.

In order to include all possible propositions and programs, we extend R_p and V inductively as follows:

- $xR_p(\alpha; \beta)y$ iff there exists a state z such that $xR_p(\alpha)z$ and $zR_p(\beta)y$

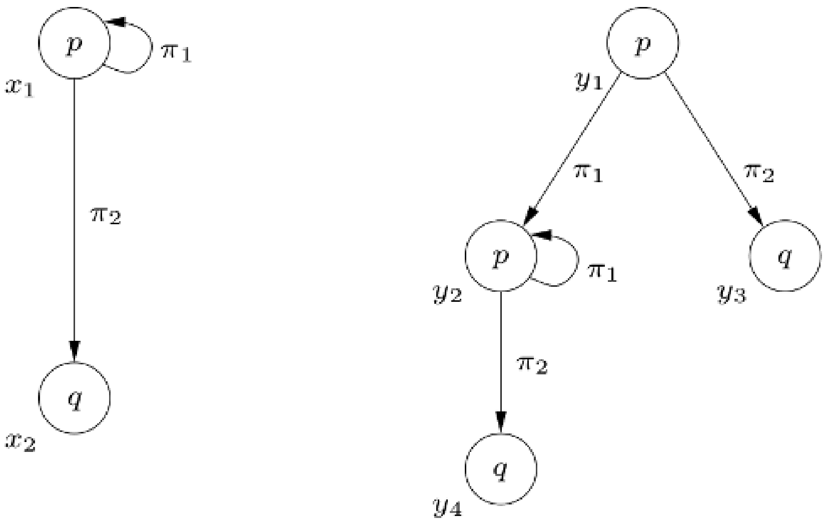


Figure 2.2. Two examples of LTS

- $xR_p(\alpha \cup \beta)y$ iff $xR_p(\alpha)y$ and $xR_p(\beta)y$
- $xR_p(\alpha*)y$ iff there exists an integer n and there exist states $z_0, \dots, z_n$ such that $z_0 = x, z_n = y$ and $\forall.k = 1, \dots, n,\ z_{k-1}R_p(\alpha)z_k$
- $xR_p(\varphi?)y$ iff $x = y \wedge y \in V(\varphi)$
- $V(0) = \emptyset$
- $V(\neg\varphi) = S \setminus V(\varphi)$
- $V(\varphi_1 \wedge \varphi_2) = V(\varphi_1) \wedge V(\varphi_2)$,
- $V([\alpha]\varphi) = \{x | \forall y.y \in S \wedge xR_p(\alpha)y \implies y \in V(\varphi)\}$

Now we give a definition for PDL formula satisfaction as we did in Definition 2.2:

Definition 2.5. Given a LTS $\mathcal{M}$, we define that a PDL formula φ is *true in a state* s, in symbols $\mathcal{M}, s \models \varphi$ iff $s \in V(\varphi)$:

Example 2.6. Considering $\mathcal{M}_1$ and $\mathcal{M}_2$ introduced in Example 2.5, we can give the following statements:

- $\mathcal{M}_1, x_1 \models p$
- $\mathcal{M}_1, x_2 \models q$
- $\mathcal{M}_1, x_1 \models \langle\pi_1\rangle p \wedge \langle\pi_2\rangle q$
- $\mathcal{M}_1, x_1 \models [\pi_1^*]p$
- $\mathcal{M}_2, y_1 \models \langle\pi_1^*; \pi_2\rangle q$
- $\mathcal{M}_2, y_1 \models [\pi_1 \cup \pi_2](q \vee p)$
- $\mathcal{M}_2, y_3 \models [\pi_1 \cup \pi_2]\mathbf{0}$

Definition 2.6. We define *satisfiability*, *validity* and *entailment* for PDL formulas in an analogous fashion as we did for LTL formulas in Definition 2.3.

Now we cite a result about complexity of reasoning in PDL:

Theorem 2.2 (Pratt (1980))**.** satisfiability*,* validity *and* entailment *in* PDL *is* EXPTIME*-complete.*

In (De Giacomo and Massacci, 2000) has been proposed an algorithm more effective in practice, though still running in deterministic exponential time in the worst case.

2.3 Linear Temporal Logic on Finite Traces: LTL$_f$

Linear-time Temporal Logic over finite traces, LTL$_f$, is essentially standard LTL (Pnueli, 1977) interpreted over finite, instead of over infinite, traces (De Giacomo and Vardi, 2013). This apparently trivial difference has big impact: as we will see, some LTL formula has a different meaning if interpreted over infinite traces or finite ones.

2.3.1 Syntax

In fact, the syntax of LTL$_f$ is the same of the one showed in Section **??**, i.e. *formulas* of LTL$_f$ are built from a set $\mathcal{P}$ of propositional symbols and are closed under the boolean connectives, the unary temporal operator $\circ$(*next-time*) and the binary operator $\mathcal{U}$ (*until*):

$$\varphi \quad ::= \quad \phi \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \circ\varphi \mid \varphi_1 \mathcal{U} \varphi_2$$

With $A \in \mathcal{P}$.

We use the standard abbreviations for classical logic formulas:

$$\begin{aligned}
\varphi_1 \vee \varphi_2 &\doteq \neg(\neg\varphi_1 \wedge \neg\varphi_2) \\
\varphi_1 \Rightarrow \varphi_2 &\doteq \neg\varphi_1 \vee \varphi_2 \\
\varphi_1 \Leftrightarrow \varphi_2 &\doteq \varphi_1 \Rightarrow \varphi_2 \wedge \varphi_2 \Rightarrow \varphi_1 \\
\mathit{true} &\doteq \neg\varphi \vee \varphi \\
\mathit{false} &\doteq \neg\varphi \wedge \varphi
\end{aligned}$$

And for temporal formulas:

$$\Diamond\varphi \doteq \mathit{true}\,\mathcal{U}\,\varphi \tag{2.1}$$

$$\Box\varphi \doteq \neg\Diamond\neg\varphi \tag{2.2}$$

$$\bullet\varphi \doteq \neg\circ\neg\varphi \tag{2.3}$$

$$\mathit{Last} \doteq \bullet\mathit{false} \tag{2.4}$$

$$\mathit{End} \doteq \Box\mathit{false} \tag{2.5}$$

As the reader might already noticed, 2.1 and 2.2 are defined as in Section **??**; Equation 2.3 is called *weak next* (notice that on finite traces $\neg\circ\varphi \not\equiv \circ\neg\varphi$); 2.4 denotes the end of the trace, while 2.5 denotes that the trace is ended.

Example 2.7. Here we recall Example 2.1 and we see the impact on *Always*, *Eventually Response* and *Persistence* LTL formulas if interpreted on finite traces (i.e. formulas in LTL$_f$):

- *Safety*: $\Box A$ means that always *till the end of the trace* φ holds;
- *Liveness*: $\Diamond A$ means that eventually *before the end of the trace* φ holds;
- *Response*: $\Box\Diamond\varphi$ on finite traces becomes equivalent to *last point in the trace satisfies* φ, i.e. $\Diamond(Last \wedge \varphi)$. Intuitively, this is true because $\Box\Diamond\varphi$ implies that at the last point in the trace φ holds (because there are no successive instants of time that make φ true); but if this is the case, then what happens at previous points in the trace does not matter because the formula evaluates always to true, since as we just said φ must hold at the last point in the trace, hence the equivalence with $\Diamond(Last \wedge \varphi)$.
- *Persistence*: $\Diamond\Box\varphi$ on finite traces becomes equivalent to *last point in the trace satisfies* φ, i.e. $\Diamond(Last \wedge \varphi)$. Analogously to the previous case, the equivalence holds because $\Diamond\Box\varphi$ implies that at the last point in the trace $\Box\varphi$ holds (and so φ), since we have no further successive instants of time that makes $\Box\varphi$ true. But if this is the case, then what happens at previous points in the trace does not matter because the formula evaluates always to true, since as we just said $\Box\varphi$ (and so φ) must hold at the last point in the trace, hence the equivalence with $\Diamond(Last \wedge \varphi)$.

In other words, no direct nesting of eventually and always connectives is meaningful in LTL$_f$, and this contrast what happens in LTL of infinite traces.

Example 2.8. Another remarkable evidence about the relevance of the assumption about finiteness of traces is provided by the DECLARE approach (Pesic and van der Aalst, 2006).

DECLARE is a declarative approach to business process modeling based on LTL interpreted over finite traces. The intuition is to map finite traces describing a domain of interest (e.g. processes) into infinite traces under the assumption that

$$\Diamond end \wedge \Box(end \Rightarrow \circ end) \wedge \Box(end \Rightarrow \bigwedge_{p\in\mathcal{P}} \neg p) \tag{2.6}$$

which means that the following english statements hold:

- *end* eventually holds ($end \notin \mathcal{P}$);
- once *end* is true, it is true forever;
- when *end* is true all other propositions must be false

In other words, every finite trace π_f is extended with an infinite sequence of *end*, or in symbols $\pi_{inf} = \pi_f\{end\}^\omega$. By construction we have that

$$\pi_{inf} \models \Diamond end \wedge \Box(end \Rightarrow \circ end) \wedge \Box(end \Rightarrow \bigwedge_{p\in\mathcal{P}} \neg p)$$

Despite it seems a nice construction to adapt LTL on finite traces, in fact it is wrong due to the *next* operator: in an infinite trace a successor state always exists, whereas in a finite one this does not hold. There exists a counterexample showing that the interpretation of LTL formulas on finite traces with the construction just explained is **not** equivalent with proper interpretation over finite traces offered by LTL$_f$, i.e. in general:

$$\pi_f\{end\}^\omega \models \varphi \not\Leftrightarrow \pi_f \models_f \varphi \tag{2.7}$$

To see why this is the case, consider the DECLARE "negation chain succession" $\Box(a \Rightarrow \bigcirc\neg b)$ which requires that at any point in the trace, the state after we see a, b is false. Consider also the finite trace $\pi_f = \{a\}$ and the associated infinite trace $\pi_{inf} = \{a\}\{end\}^\omega$ built as explained before. We have that

$$\pi_{inf} \models \Box(a \Rightarrow \bigcirc\neg b)$$

where $\models$ has been defined in 2.2. This is true because there is only one occurrence of a and then *end* holds forever (and so b does not).

But if the same formula is interpreted on finite traces (namely $\models_f$):

$$\pi_f \not\models_f \Box(a \Rightarrow \bigcirc\neg b)$$

because the finite trace a is true at the last instant, but then there is no next instance where b is false, so $\bigcirc\neg b$ is evaluated to *false* and the formula does not hold. The correct way to express "negation chain succession" on finite traces would be $\Box(a \Rightarrow \bullet\neg b)$.

The LTL formulas φ that are insensitive to the problem just shown, i.e. such that

$$\pi_f\{end\}^\omega \models \varphi \text{ iff } \pi_f \models_f \varphi \tag{2.8}$$

holds are defined *insensitive to infiniteness* (De Giacomo et al., 2014). This is another important evidence about the the relevance of the finiteness trace assumption.

2.3.2 Semantics

Formally, a *finite trace* π is a finite word over the alphabet $2^{\mathcal{P}}$, i.e. as alphabet we have all the possible propositional interpretations of the propositional symbols in $\mathcal{P}$. We can see π as a *finite* word on a path of a Kripke structure, similarly as we discussed in Section 2.1.2 (but in that case the traces were *infinite*). Given a finite path $\rho = \langle s_1, s_2, \ldots, s_n\rangle$ on a Kripke structure $\mathcal{K}$, a finite trace π associated to the path ρ is defined as $\langle L(s_1), L(s_2), \ldots, L(s_n)\rangle$.

We use the following notation. We denote the *length* of a trace π as $length(\pi)$. We denote the i_{th} *position* on the trace as $\pi(i) = L(s_i)$, i.e. the propositions that hold in the i_{th} state of the path, with $0 \leq i \leq last$ where $last = length(\pi) - 1$ is the last element of the trace. We denote by $\pi(i, j)$, the *segment* of π, the trace $\pi' = \langle \pi(i), \pi(i+1), \ldots, \pi(j)\rangle$, with $0 \leq i \leq j \leq last$

Definition 2.7. Given a finite trace π, we define that a LTL$_f$ formula φ is *true* at time i $(0 \leq i \leq last)$, in symbols $\pi, i \models \varphi$ inductively as follows:

$$\pi, i \models A,\ \text{for } A \in \mathcal{P} \text{ iff } A \in \pi(i)$$

$$\pi, i \models \neg\varphi \text{ iff } \pi, i \not\models \varphi$$

$$\pi, i \models \varphi_1 \wedge \varphi_2 \text{ iff } \pi, i \models \varphi_1 \wedge \pi, i \models \varphi_2$$

$$\pi, i \models \bigcirc\varphi \text{ iff } i < last \wedge \pi, i+1 \models \varphi \tag{2.9}$$

$$\pi, i \models \varphi_1 \mathcal{U} \varphi_2 \text{ iff } \exists j.(i \leq j \leq \mathit{last}) \wedge \pi, j \models \varphi \wedge$$
$$\forall k.(i \leq k < j) \Rightarrow \pi, k \models \varphi_1 \tag{2.10}$$

Notice that Definition 2.7 is pretty similar to Definition 2.2, except the bounding of indexes in Equation 2.9 and Equation 2.10, to recognize that the trace is ended. Analogously to Definition 2.3 we give the following definitions:

Definition 2.8. A LTL$_f$ formula is *true* in π, in notation $\pi \models \varphi$, if $\pi, 0 \models \varphi$. A formula φ is *satisfiable* if it is true in some π and is *valid* if it is true in every π. φ_1 *entails* φ_2, in symbols $\varphi_1 \models \varphi_2$ iff $\forall \pi, \forall i. \pi, i \models \varphi_1 \implies \pi, i \models \varphi_2$.

2.3.3 Complexity and Expressiveness

Thanks to reduction of LTL$_f$ satisfiability (Definition 2.8) into LTL satisfiability for PSPACE membership and reduction of STRIPS planning into LTL$_f$ satisfiability for PSPACE-hardness, as proposed in (De Giacomo and Vardi, 2013), we have this result:

Theorem 2.3 (De Giacomo and Vardi (2013))**.** *Satisfiability, validity and entailment for* LTL$_f$ *formulas are* PSPACE*-complete.*

About expressiveness of LTL$_f$, we have that:

Theorem 2.4 (De Giacomo and Vardi (2013); Gabbay et al. (1997))**.** LTL$_f$ *has exactly the same expressive power of* FOL *over finite ordered sequences.*

2.4 Regular Temporal Specifications (RE$_f$)

In this section we talk about regular languages as a form of temporal specification over finite traces. In particular we focus on regular expressions (Hopcroft et al., 2000).

A regular expression ϱ is defined inductively as follows, considering as alphabet the set of propositional interpretations $2^{\mathcal{P}}$, from a set of propositional symbols $\mathcal{P}$:

$$\varrho \quad ::= \quad \phi \mid \varrho_1 + \varrho_2 \mid \varrho_1 ; \varrho_2 \mid \varrho^*$$

where ϕ is a propositional formula that is an abbreviation for the union of all the propositional interpretations that satisfy ϕ, i.e. $\phi = \sum_{\Pi \models \phi} \Pi$ and $\Pi \in 2^{\mathcal{P}}$.

We denote by $\mathcal{L}(\varrho)$ the language recognized by a RE$_f$ expression. We interpret these expression over finite traces, introduced in Section 2.3.2.

Definition 2.9. We say that a regular expression ϱ *is true* in the finite trace π ifs $\pi \in \mathcal{L}(\varrho)$. We say that ϱ *is true at instant* i if $\pi(i, \mathit{last}) \in \mathcal{L}(\varrho)$. We say that ϱ *is true between instants* i, j if $\pi(i, j) \in \mathcal{L}(\varrho)$.

Example 2.9. We recall Example 2.2. The trace resulting from path $\langle s_1, s_2, s_3, s_3, \ldots \rangle$, i.e.:

$$\pi = \langle \{p, q\}\{q\}, \{p\}, \{p\}, \{p\}, \ldots \rangle$$

belongs to the language generated by the following regular expression:

$$\varrho_1 = p \wedge q; q; p^*$$

But also by this one:

$$\varrho_2 = \mathit{true}; q + p; \mathit{true}^*$$

Example 2.10. We can express some of the formulas shown in Example 2.7, and many others, in RE$_f$:

- *Safety*: φ^*, equivalent to $\Box\varphi$
- *Liveness*: $true^*; \varphi; true^*$, equivalent to $\Diamond\varphi$;
- *Response* and *Persistence*: as said before, when interpreted on finite traces, they are equivalent to $\Diamond(Last \wedge \varphi)$; hence, they can be rewritten in RE$_f$ as $true^*; \varphi$
- *Ordered occurrence*: $true^*; \varphi_1; true^*; \varphi_2; true^*$, equivalent to $\Diamond(\varphi_1 \wedge \bigcirc\Diamond\varphi_2)$ means that φ_1 and φ_2 happen in order;
- *Alternating sequence*: $(\psi, \varphi)^*$ means that ψ and φ alternate from the beginning of the trace, starting with ψ and ending with φ.

The *Alternating sequence* is an example of formula that has not a counterpart in LTL$_f$. More generally, LTL$_f$ (and LTL) are not able to capture regular structural properties on path (Wolper, 1981).

This observation about expressiveness of RE$_f$ is confirmed by Theorem 6 of (De Giacomo and Vardi, 2013), which is a consequence of several classical results (Büchi, 1960; Elgot, 1961; Trakhtenbrot, 1961; Thomas, 1979):

Theorem 2.5 (De Giacomo and Vardi (2013)). *RE$_f$ is strictly more expressive than LTL$_f$*

More precisely, RE$_f$ is expressive as *monadic second-order logic* MSO over bounded ordered sequences (Khoussainov and Nerode, 2001).

2.5 Linear Dynamic Logic on Finite Traces: LDL$_f$

The problem with RE$_f$ is that, although is strictly more expressive than LTL$_f$, is considered a low-level formalism for temporal specifications. For instance RE$_f$ misses a direct construct for negation and for conjunction. Moreover, negation requires an exponential blow-up, hence adding complementation and intersection constructs is not advisable.

Linear Dynamic Logic of Finite Traces LDL$_f$ (De Giacomo and Vardi, 2013) merges LTL$_f$ with RE$_f$ in a very natural way, borrowing the syntax of PDL, described in Section 2.2. It keep the declarativeness and convenience of LTL$_f$ while having the same expressive power of RE$_f$.

2.5.1 Syntax

Formally, LDL$_f$ formulas φ are built over a set of propositional symbols $\mathcal{P}$ as follows (Brafman et al., 2017):

$$\begin{array}{lcl} \varphi & ::= & tt \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \langle\varrho\rangle\varphi \\ \varrho & ::= & \phi \mid \varphi? \mid \varrho_1 + \varrho_2 \mid \varrho_1; \varrho_2 \mid \varrho^* \end{array}$$

where tt stands for logical true; ϕ is a propositional formula over $\mathcal{P}$; ϱ denotes path expressions, which are RE over propositional formulas ϕ with the addition of the test construct $\varphi?$ typical of PDL. Moreover, we use the following abbreviations for classical logic operators:

$$\begin{aligned}\varphi_1 \vee \varphi_2 &\doteq \neg(\neg\varphi_1 \wedge \neg\varphi_2)\\ \varphi_1 \Rightarrow \varphi_2 &\doteq \neg\varphi_1 \vee \varphi_2\\ \varphi_1 \Leftrightarrow \varphi_2 &\doteq \varphi_1 \Rightarrow \varphi_2 \wedge \varphi_2 \Rightarrow \varphi_1\\ \mathit{ff} &\doteq \neg tt\end{aligned}$$

And for temporal formulas:

$$[\varrho]\varphi \doteq \neg\langle\varrho\rangle\neg\varphi \tag{2.11}$$

$$\mathit{End} \doteq [\mathit{true}]\mathit{ff} \tag{2.12}$$

$$\mathit{Last} \doteq \langle\mathit{true}\rangle\mathit{End} \tag{2.13}$$

$[\varrho]\varphi$ and $\langle\varrho\rangle\varphi$ are analogous to box and diamond operators in PDL; Formula 2.13 denotes the last element of the trace, whereas Formula 2.12 denotes that the trace is ended. Intuitively, $\langle\varrho\rangle\varphi$ states that, from the current step in the trace, there exists an execution satisfying the RE ϱ such that its last step satisfies φ, while $[\varrho]\varphi$ states that, from the current step, all executions satisfying the RE ϱ are such that their last step satisfies φ. It is worth to notice that this interpretation of $[\,]$ and $\langle\,\rangle$ is pretty similar to the one shown in Section 2.2.1, as well as the test construct $\varphi?$ are used to insert into the execution path checks for satisfaction of additional LDL$_f$ formulas.

2.5.2 Semantics

As we did in the previous sections, we formally give a semantics to LDL$_f$ (interpreted over finite traces, like LTL$_f$ and RE).

Definition 2.10. Given a finite trace π, we define that a LDL$_f$ formula φ is *true* at time i ($0 \leq i \leq \mathit{last}$), in symbols $\pi, i \models \varphi$ inductively as follows:

$$\begin{aligned}&\pi, i \models tt\\ &\pi, i \models \neg\varphi \text{ iff } \pi, i \not\models \varphi\\ &\pi, i \models \varphi_1 \wedge \varphi_2 \text{ iff } \pi, i \models \varphi_1 \wedge \pi, i \models \varphi_2\\ &\pi, i \models \langle\phi\rangle\varphi \text{ iff } i < \mathit{last} \wedge \pi(i) \models \phi \wedge \pi, i+1 \models \varphi\\ &\pi, i \models \langle\psi?\rangle\varphi \text{ iff } \pi, i \models \psi \wedge \pi, i \models \varphi\\ &\pi, i \models \langle\varrho_1 + \varrho_2\rangle\varphi \text{ iff } \pi, i \models \langle\varrho_1\rangle\varphi \vee \langle\varrho_2\rangle\varphi\\ &\pi, i \models \langle\varrho_1; \varrho_2\rangle\varphi \text{ iff } \pi, i \models \langle\varrho_1\rangle\langle\varrho_2\rangle\varphi\\ &\pi, i \models \langle\varrho^*\rangle\varphi \text{ iff } \pi, i \models \varphi \vee i < \mathit{last} \wedge \pi, i \models \langle\varrho\rangle\langle\varrho^*\rangle\varphi \text{ and } \varrho \text{ is not } \textit{test-only}\end{aligned}$$

We say that ϱ is *test-only* if it is a RE$_f$ expression whose atoms are only tests, i.e. $\psi?$.

Notice that LDL$_f$ fully captures LTL$_f$. For every formula in LTL$_f$ there exists a LDL$_f$ formula with the same meaning, namely:

LTL$_f$	LDL$_f$
A	$\langle A\rangle tt$

$$\begin{array}{rl}
\neg\varphi & \neg\varphi \\
\varphi_1 \wedge \varphi_2 & \varphi_1 \wedge \varphi_2 \\
\circ\varphi & \langle true\rangle(\varphi \wedge \neg End) \\
\varphi_1 \,\mathcal{U}\, \varphi & \langle(\varphi_1?; true)^*\rangle(\varphi_2 \wedge \neg End)
\end{array}$$

Notice also that every RE$_f$ expression ϱ is captured in LDL$_f$ by $\langle\varrho\rangle End$. Moreover, since also the converse holds, i.e. every LDL$_f$ formula can be expressed in RE (by Theorem 11 in (De Giacomo and Vardi, 2013)), the following theorem holds:

Theorem 2.6 (De Giacomo and Vardi (2013))**.** LDL$_f$ *has exactly the same expressive power of* MSO

Now we show several LDL$_f$ examples.

Example 2.11. Formulas described in Examples 2.7 and 2.10 can be rewritten in LDL$_f$ as:

- *Safety*: $[true^*]\varphi$, equivalent to LTL$_f$ formula $\Box\varphi$
- *Liveness*: $\langle true^*\rangle\varphi$, equivalent to LTL$_f$ formula $\Diamond\varphi$
- *Conditional Response*: $[true^*](\varphi_1 \Rightarrow \langle true^*\rangle\varphi_2)$, equivalent to LTL$_f$ formula $\Box(\varphi_1 \Rightarrow \Diamond\varphi_2)$
- *Ordered occurrence*: $\langle true^*; \varphi_1; true^*; \varphi_2; true^*\rangle End$ equivalent to the RE$_f$ expression $true^*; \varphi_1; true^*; \varphi_2; true^*$
- *Alternating occurrence*: $\langle(\psi;\varphi)^*\rangle End$ equivalent to the RE$_f$ expression $(\psi;\varphi)^*$

Example 2.12. Consider the Example 2.2 and 2.9. ϱ_1 and ϱ_2 are translated into LDL$_f$ as $\langle\varrho_1\rangle End$ and $\langle\varrho_2\rangle End$ respectively.

Other LDL$_f$ formulas satisfiable in the Kripke structure $\mathcal{K}$ depicted in Figure 2.1 are:

- $\langle p\rangle tt$ by every (non-empty) path, since s_1 is the initial state and we have that $\{p,q\} \models p$
- $\langle q\rangle tt$ as the previous case
- $\langle(p;q);(p;q)^*;p;p^*\rangle tt$ by paths of the form $\rho = s_1, s_2, (s_1, s_2)^\omega, s_3, (s_3)^\omega$
- $[true^*]\langle p \vee q\rangle tt$ is satisfied for every path, since for every reachable state either p or q are true;

2.6 LTL$_f$ and LDL$_f$ translation to automata

Given an LTL$_f$/LDL$_f$ formula φ, we can construct a deterministic finite state automaton (DFA) (Rabin and Scott, 1959) $\mathcal{A}_\varphi$ that accept the same finite traces that makes φ true. In order to do this, we proceed in two steps: First we translate LTL$_f$ and LDL$_f$ formulas into (NFA) (De Giacomo and Vardi, 2015). Then the NFA obtained can be transformed into a DFA following the standard procedure of *determinization*.

Now we recall definitions of NFA and DFA:

Definition 2.11. An NFA is a tuple $\mathcal{A} = \langle \Sigma, Q, q_0, \delta, F \rangle$, where:

- Σ is the input alphabet;
- Q is the finite set of states;
- $q_0 \in Q$ is the initial state;
- $\delta \subseteq Q \times \Sigma \times Q$ is the transition relation;
- $F \subseteq Q$ is the set of final states;

Definition 2.12. A DFA is a NFA where δ is a function $\delta : Q \times \Sigma \to Q$

By $\mathcal{L}(A)$ we mean the set of all traces over Σ accepted by $\mathcal{A}$.

In the next two subsections we give some definition that will be used in the algorithm; then we describe the algorithm for the translation and give some example.

2.6.1 ∂ function for LTL$_f$

We give the following definition:

Definition 2.13. The *delta function ∂ for* LTL$_f$ *formulas* is a function that takes as input an (implicitly quoted) LTL$_f$ formula φ in NNF and a propositional interpretation Π for $\mathcal{P}$, and returns a positive boolean formula whose atoms are (implicitly quoted) φ subformulas. It is defined as follows:

$$\begin{aligned}
\partial(A,\Pi) &= \begin{cases} \mathit{true} & \text{if } A \in \Pi \\ \mathit{false} & \text{if } A \notin \Pi \end{cases} \\
\partial(\neg A,\Pi) &= \begin{cases} \mathit{false} & \text{if } A \in \Pi \\ \mathit{true} & \text{if } A \notin \Pi \end{cases} \\
\partial(\varphi_1 \wedge \varphi_2,\Pi) &= \partial(\varphi_1,\Pi) \wedge \partial(\varphi_2,\Pi) \\
\partial(\varphi_1 \vee \varphi_2,\Pi) &= \partial(\varphi_1,\Pi) \vee \partial(\varphi_2,\Pi) \\
\partial(\circ\varphi,\Pi) &= \varphi \wedge \neg \mathit{End} \equiv \varphi \wedge \Diamond \mathit{true} \qquad (2.14)\\
\partial(\varphi_1 \mathcal{U} \varphi_2,\Pi) &= \partial(\varphi_2,\Pi) \vee (\partial(\varphi_1,\Pi) \wedge \partial(\circ(\varphi_1 \mathcal{U} \varphi_2),\Pi)) \\
\partial(\Diamond\varphi,\Pi) &= \partial(\varphi,\Pi) \vee \partial(\circ\Diamond\varphi,\Pi) \\
\partial(\bullet\varphi,\Pi) &= \varphi \vee \mathit{End} \equiv \varphi \vee \Box \mathit{false} \\
\partial(\varphi_1 \mathcal{R} \varphi_2,\Pi) &= \partial(\varphi_2,\Pi) \wedge (\partial(\varphi_1,\Pi) \vee \partial(\bullet(\varphi_1 \mathcal{R} \varphi_2),\Pi)) \\
\partial(\Box\varphi,\Pi) &= \partial(\varphi,\Pi) \wedge \partial(\bullet\Box\varphi,\Pi)
\end{aligned}$$

where *End* is defined as Equation 2.5.

Moreover, we define $\partial(\varphi, \epsilon)$ which is inductively defined as Equation 2.14, except for the following cases:

$$\begin{array}{rcl} \partial(A, \epsilon) & = & \mathit{false} \\ \partial(\neg A, \epsilon) & = & \mathit{false} \\ \partial(\circ\varphi, \epsilon) & = & \mathit{false} \\ \partial(\bullet\varphi, \epsilon) & = & \mathit{true} \end{array} \tag{2.15}$$

Note that $\partial(\varphi, \epsilon)$ is always either *true* or *false*.

2.6.2 ∂ function for LDL$_f$

We give the following definition:

Definition 2.14. The *delta function ∂ for* LDL$_f$ *formulas* is a function that takes as input an (implicitly quoted) LDL$_f$ formula φ in NNF, extended with auxiliary constructs $\boldsymbol{F}_\psi$ and $\boldsymbol{T}_\psi$, and a propositional interpretation Π for $\mathcal{P}$, and returns a positive boolean formula whose atoms are (implicitly quoted) φ subformulas (not including $\boldsymbol{F}_\psi$ or $\boldsymbol{T}_\psi$). It is defined as follows:

$$\begin{array}{rcl} \partial(tt, \Pi) & = & \mathit{true} \\ \partial(\mathit{ff}, \Pi) & = & \mathit{false} \\ \partial(\phi, \Pi) & = & a \\ \partial(\varphi_1 \wedge \varphi_2, \Pi) & = & \partial(\varphi_1, \Pi) \wedge \partial(\varphi_2, \Pi) \\ \partial(\varphi_1 \vee \varphi_2, \Pi) & = & \partial(\varphi_1, \Pi) \vee \partial(\varphi_2, \Pi) \\ \partial(\langle\phi\rangle\varphi, \Pi) & = & \begin{cases} \boldsymbol{E}(\varphi) & \text{if } \Pi \models \phi \\ \mathit{false} & \text{if } \Pi \not\models \phi \end{cases} \\ \partial(\langle\varrho?\rangle\varphi, \Pi) & = & \partial(\varrho, \Pi) \wedge \partial(\varphi, \Pi) \\ \partial(\langle\varrho_1 + \varrho_2\rangle\varphi, \Pi) & = & \partial(\langle\varrho_1\rangle\varphi, \Pi) \vee \partial(\langle\varrho_2\rangle\varphi, \Pi) \\ \partial(\langle\varrho_1; \varrho_2\rangle\varphi, \Pi) & = & \partial(\langle\varrho_1\rangle\langle\varrho_2\rangle\varphi, \Pi) \\ \partial(\langle\varrho^*\rangle\varphi, \Pi) & = & \partial(\varphi, \Pi) \vee \partial(\langle\varrho\rangle\boldsymbol{F}_{\langle\varrho^*\rangle\varphi}, \Pi) \\ \partial([\phi]\varphi, \Pi) & = & \begin{cases} \boldsymbol{E}(\varphi) & \text{if } \Pi \models \phi \\ \mathit{true} & \text{if } \Pi \not\models \phi \end{cases} \\ \partial([\varrho?]\varphi, \Pi) & = & \partial(\mathit{nnf}(\neg\varrho), \Pi) \vee \partial(\varphi, \Pi) \\ \partial([\varrho_1 + \varrho_2]\varphi, \Pi) & = & \partial([\varrho_1]\varphi, \Pi) \wedge \partial([\varrho_2]\varphi, \Pi) \\ \partial([\varrho_1; \varrho_2]\varphi, \Pi) & = & \partial([\varrho_1][\varrho_2]\varphi, \Pi) \\ \partial([\varrho^*]\varphi, \Pi) & = & \partial(\varphi, \Pi) \wedge \partial([\varrho]\boldsymbol{T}_{\langle\varrho^*\rangle\varphi}, \Pi) \\ \partial(\boldsymbol{T}_\psi, \Pi) & = & \mathit{true} \\ \partial(\boldsymbol{F}_\psi, \Pi) & = & \mathit{false} \end{array} \tag{2.16}$$

where $\boldsymbol{E}(\varphi)$ recursively replaces in φ all occurrences of atoms of the form $\boldsymbol{T}_\psi$ and $\boldsymbol{F}_\psi$ by $\boldsymbol{E}(\psi)$.

Moreover, we define $\partial(\varphi, \epsilon)$ which is inductively defined as Equation 2.16, except for the following cases:

$$\begin{array}{rcl} \partial(\langle\phi\rangle\varphi, \epsilon) & = & \mathit{false} \\ \partial([\phi]\varphi, \epsilon) & = & \mathit{true} \end{array} \tag{2.17}$$

Note that $\partial(\varphi, \epsilon)$ is always either *true* or *false*.

2.6.3 The LDL$_f$2NFA algorithm

Algorithm 2.1 (LDL$_f$2NFA) takes in input a LDL$_f$/LTL$_f$ formula φ and outputs a NFA $\mathcal{A}_\varphi = \langle 2^\mathcal{P}, Q, q_0, \delta, F\rangle$ that accepts exactly the traces satisfying φ. It is a variant of the algorithm presented in (De Giacomo and Vardi, 2015), and its correctness relies on the fact that every LDL$_f$/LTL$_f$ formula φ can be associated a polynomial *alternating automaton on words* (AFW) accepting exactly the traces that satisfy φ and that every AFW can be transformed into an NFA (De Giacomo and Vardi, 2013). The proposed algorithm requires that φ is in *negation normal form* (NNF), i.e. with negation symbols occurring only in front of propositions.

The function ∂ used in lines 5, 12 and 15 is the one defined in sections 2.6.1 and 2.6.2; whether we are translating a LTL$_f$ or a LDL$_f$ formula, we use the function ∂ from Definition 2.13 and from Definition 2.14, respectively.

Algorithm 2.1. LDL$_f$2NFA: from LTL$_f$/LDL$_f$ formula φ to NFA $\mathcal{A}_\varphi$

```
 1: input LDL$_f$/LTL$_f$ formula $\varphi$
 2: output NFA $\mathcal{A}_\varphi = \langle 2^\mathcal{P}, Q, q_0, \delta, F\rangle$
 3: $q_0 \leftarrow \{\varphi\}$
 4: $F \leftarrow \{\emptyset\}$
 5: if $(\partial(\varphi, \epsilon) = true)$ then
 6:     $F \leftarrow F \cup \{q_0\}$
 7: end if
 8: $Q \leftarrow \{q_0, \emptyset\}$
 9: $\delta \leftarrow \emptyset$
10: while ($Q$ or $\delta$ change) do
11:     for $(q \in Q)$ do
12:         if $(q' \models \bigwedge_{(\psi \in q)} \partial(\psi, \Pi))$ then
13:             $Q \leftarrow Q \cup \{q'\}$
14:             $\delta \leftarrow \delta \cup \{(q, \Pi, q')\}$
15:             if $(\bigwedge_{(\psi \in q')} \partial(\psi, \epsilon) = true)$ then
16:                 $F \leftarrow F \cup \{q'\}$
17:             end if
18:         end if
19:     end for
20: end while
```

How LDL$_f$2NFA works

The NFA $\mathcal{A}_\varphi$ for an LDL$_f$ formula φ is built in a forward fashion. Until convergence is reached (i.e. states and transitions do not change), the algorithm visits every state q seen until now, checks for all the possible transitions from that state and collects the results, determining the next state q', the new transition (q, Π, q') and if q' is a final state. Intuitively, the delta function ∂ emulates the semantic behavior of every LTL$_f$/LDL$_f$ subformula after seeing Π.

States of $\mathcal{A}_\varphi$ are sets of atoms (each atom is a quoted φ subformula) to be interpreted as conjunctions. The empty conjunction $\emptyset$ stands for *true*. q' is a set of quoted subformulas of φ denoting a minimal interpretation such that $q' \models \bigwedge_{(\psi \in q)} \partial(\psi, \Pi)$ (notice that we trivially have $(\emptyset, p, \emptyset) \in \delta$ for every $p \in 2^\mathcal{P}$).

The following result holds:

Theorem 2.7 (De Giacomo and Vardi (2015))**.** *Algorithm* LDL$_f$2NFA *is correct, i.e., for every finite trace* $\pi : \pi \models \varphi$ *iff* $\pi \in \mathcal{L}(\mathcal{A}_\varphi)$*. Moreover, it terminates in at most an exponential number of steps, and generates a set of states* S *whose size is at most exponential in the size of the formula* φ*.*

In order to obtain a DFA, the NFA $\mathcal{A}_\varphi$ can be determinized in exponential time (Rabin and Scott, 1959). Thus, we can tranform a LTL$_f$/LDL$_f$ formula into a DFA of double exponential size.

Example 2.13. In this example we see a run of the Algorithm 2.1 with the LTL$_f$ formula $\Box A$ (A atomic).

0. Set up:

$$\begin{aligned} q_0 &= \{\Box A\} \\ Q &= \{q_0, \emptyset\} \\ F &= \{q_0, \emptyset\} \qquad \text{(because } \partial(\Box A, \epsilon) = \mathit{true}\text{)} \\ \delta &= \{(\emptyset, \{\}, \emptyset), (\emptyset, \{A\}, \emptyset)\} \end{aligned}$$

1. Iteration: analyze $q = \{\Box A\}$

   - with $\Pi = \{A\}$ we have

$$\begin{aligned} q' &\models \bigwedge_{(\psi \in q)} \partial(\psi, \Pi) \\ &\models \partial(\Box A, \Pi) \\ &\models \partial(A, \Pi) \wedge \partial(\bullet \Box A, \Pi) \\ &\models \mathit{true} \wedge (\text{“}\Box A\text{”} \vee \text{“}\Box \mathit{false}\text{”}) \end{aligned}$$

   Notice that $\mathit{true} \wedge (\text{“}\Box A\text{”} \vee \text{“}\Box \mathit{false}\text{”})$ is a *propositional formula* with LTL$_f$ formulas as atoms. As a minimal interpretation we have both $q' = \{\text{“}\Box A\text{”}\}$ and $q' = \{\text{“}\Box \mathit{false}\text{”}\}$. Since in both cases we have that $\partial(\psi, \epsilon) = \mathit{true}$, at the end of the iteration we have:

$$\begin{aligned} q_0 &= \{\Box A\} \\ Q &= \{q_0, \{\Box \mathit{false}\}, \emptyset\} \\ F &= \{q_0, \{\Box \mathit{false}\}, \emptyset\} \\ \delta &= \{(\emptyset, \{\}, \emptyset), (\emptyset, \{A\}, \emptyset), \\ &\quad (q_0, \{A\}, q_0), (q_0, \{A\}, \{\Box \mathit{false}\})\} \end{aligned}$$

   - with $\Pi = \{\}$ we have

$$\begin{aligned} q' &\models \bigwedge_{(\psi \in q)} \partial(\psi, \Pi) \\ &\models \partial(\Box A, \Pi) \end{aligned}$$

$$\models \partial(A,\Pi) \wedge \partial(\bullet\Box A,\Pi)$$

$$\models \mathit{false} \wedge (\text{“}\Box A\text{”} \vee \text{“}\Box \mathit{false}\text{”})$$

Which is always false. Thus we do not change nothing.

2. Iteration: we already analyzed $q = \{\Box A\}$, so we analyze $q = \{\Box \mathit{false}\}$
   - Both with $\Pi = \{\}$ and $\Pi = \{A\}$ we have that:

$$q' \models \bigwedge_{(\psi\in q)} \partial(\psi,\Pi)$$

$$\models \partial(\Box false,\Pi)$$

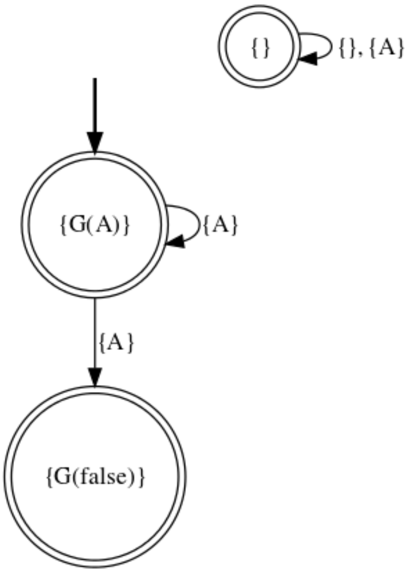
$$\models \partial(false,\Pi) \wedge \partial(\bullet\Box false,\Pi)$$

$$\models \mathit{false} \wedge (\text{“}\Box false\text{”} \vee \text{“}\Box \mathit{false}\text{”})$$

Which is always false. Thus we do not change nothing.

The NFA $\mathcal{A}_\varphi = \langle 2^{\{A\}}, Q, q_0, \delta, F\rangle$ is depicted in Figure 2.3, whereas the associated DFA is in Figure 2.4.

Figure 2.3. The NFA associated to $\Box A$. $G(A)$ stands for $\Box A$

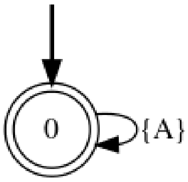


Example 2.14. Analogously to what we did in 2.13, we see a run of the Algorithm 2.1, with the LTL$_f$ formula $\Diamond A$ (A atomic).

0. Set up:

$$\begin{aligned} q_0 &= \{\Diamond A\} \\ Q &= \{q_0, \emptyset\} \\ F &= \{\emptyset\} \qquad (\text{because } \partial(\Diamond A,\epsilon) = \mathit{false}) \\ \delta &= \{(\emptyset,\{\},\emptyset),(\emptyset,\{A\},\emptyset)\} \end{aligned}$$

Figure 2.4. The DFA associated to $\Box A$



1. Iteration: analyze $q = \{\Diamond A\}$
   - with $\Pi = \{A\}$ we have

$$\begin{aligned} q' &\models \bigwedge_{(\psi \in q)} \partial(\psi, \Pi) \\ &\models \partial(\Diamond A, \Pi) \\ &\models \partial(A, \Pi) \vee \partial(\circ \Diamond A, \Pi) \\ &\models \mathit{true} \vee (\text{“}\Diamond A\text{”} \wedge \text{“}\Diamond \mathit{true}\text{”}) \end{aligned}$$

     Since the propositional formula is trivially true, as a minimal interpretation we have $q' = \emptyset$. Considering that the empty conjunction is considered as *true* (as explained in Section 2.6), at the end of the iteration we have:

$$\begin{aligned} q_0 &= \{\Diamond A\} \\ Q &= \{q_0, \emptyset\} \\ F &= \{\emptyset\} \\ \delta &= \{(\emptyset, \{\}, \emptyset), (\emptyset, \{A\}, \emptyset), (q_0, \{A\}, \emptyset)\} \end{aligned}$$

   - with $\Pi = \{\}$ we have

$$\begin{aligned} q' &\models \bigwedge_{(\psi \in q)} \partial(\psi, \Pi) \\ &\models \partial(\Diamond A, \Pi) \\ &\models \partial(A, \Pi) \vee \partial(\circ \Diamond A, \Pi) \\ &\models \mathit{false} \vee (\text{“}\Diamond A\text{”} \wedge \text{“}\Diamond \mathit{true}\text{”}) \end{aligned}$$

     As a minimal interpretation we have $q' = \{\text{“}\Diamond A\text{”} \wedge \text{“}\Diamond \mathit{true}\text{”}\}$. Since $\partial(\Diamond A, \epsilon) \wedge \partial(\Diamond \mathit{true}, \epsilon) = \mathit{false} \wedge \mathit{false} \neq \mathit{true}$, we do not add q' to the accepting states F. Thus we have:

$$\begin{aligned} q_0 &= \{\Diamond A\} \\ Q &= \{q_0, \emptyset, \{\Diamond A \wedge \Diamond \mathit{true}\}\} \end{aligned}$$

$$\begin{aligned}
F &= \{\emptyset\}\\
\delta &= \{(\emptyset,\{\},\emptyset),(\emptyset,\{A\},\emptyset),\\
&\quad(q_0,\{A\},\emptyset),\\
&\quad(q_0,\{\},\{\Diamond A \wedge \Diamond \mathit{true}\})\}
\end{aligned}$$

2. Iteration: we already analyzed $q = \{\Diamond A\}$, so we analyze $q = \{\Diamond A \wedge \Diamond \mathit{true}\}$

   - with $\Pi = \{\}$ we have that:

     $$\begin{aligned}
     q' &\models \bigwedge_{(\psi \in q)} \partial(\psi,\Pi)\\
     &\models \partial(\Diamond A,\Pi) \wedge \partial(\Diamond \mathit{true},\Pi)\\
     &\models [\partial(A,\Pi) \vee \partial(\circ\Diamond A,\Pi)] \wedge [\partial(\mathit{true},\Pi) \vee \partial(\circ\Diamond \mathit{true},\Pi)]\\
     &\models [\partial(A,\Pi) \vee (\text{“}\Diamond A\text{”} \wedge \text{“}\Diamond \mathit{true}\text{”})] \wedge [\mathit{true} \vee (\text{“}\Diamond \mathit{true}\text{”} \wedge \text{“}\Diamond \mathit{true}\text{”})]\\
     &\models \partial(A,\Pi) \vee (\text{“}\Diamond A\text{”} \wedge \text{“}\Diamond \mathit{true}\text{”})\\
     &\models \mathit{false} \vee (\text{“}\Diamond A\text{”} \wedge \text{“}\Diamond \mathit{true}\text{”})
     \end{aligned}$$

     As in the previous iteration, the minimal model is $q' = \{\text{“}\Diamond A\text{”} \wedge \text{“}\Diamond \mathit{true}\text{”}\}$. Hence we add a new transition $(\{\Diamond A \wedge \Diamond \mathit{true}\},\{\}.\{\Diamond A \wedge \Diamond \mathit{true}\})$.

   - with $\Pi = \{A\}$ the delta-expansion is the same, except for the last step, where:

     $$q' \models \mathit{true} \vee (\text{“}\Diamond A\text{”} \wedge \text{“}\Diamond \mathit{true}\text{”})$$

     The formula is always true, hence the minimal model is $q' = \emptyset$ and we add a new transition $(\{\Diamond A \wedge \Diamond \mathit{true}\},\{\}.\emptyset)$.

   The NFA $\mathcal{A}_\varphi$ is then composed by:

$$\begin{aligned}
q_0 &= \{\Diamond A\}\\
Q &= \{q_0,\emptyset,\{\Diamond A \wedge \Diamond \mathit{true}\}\}\\
F &= \{\emptyset\}\\
\delta &= \{(\emptyset,\{\},\emptyset),(\emptyset,\{A\},\emptyset),\\
&\quad(q_0,\{A\},\emptyset),\\
&\quad(q_0,\{\},\{\Diamond A \wedge \Diamond \mathit{true}\})\\
&\quad(\{\Diamond A \wedge \Diamond \mathit{true}\},\{\}.\{\Diamond A \wedge \Diamond \mathit{true}\})\\
&\quad(\{\Diamond A \wedge \Diamond \mathit{true}\},\{\}.\emptyset)\}
\end{aligned}$$

The NFA $\mathcal{A}_\varphi = \langle 2^{\{A\}}, Q, q_0, \delta, F\rangle$ is depicted in Figure 2.5, whereas the associated DFA is in Figure 2.6.

Example 2.15. We list other examples of $\mathcal{A}_\varphi$ given a LTL$_f$/LDL$_f$ formula φ, obtained by Algorithm 2.1:

Figure 2.5. The NFA associated to $\Diamond A$. $F(A)$ stands for $\Diamond A$

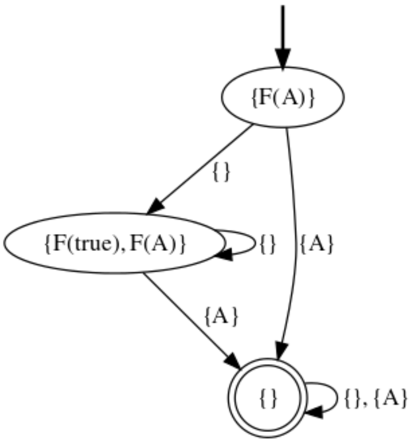
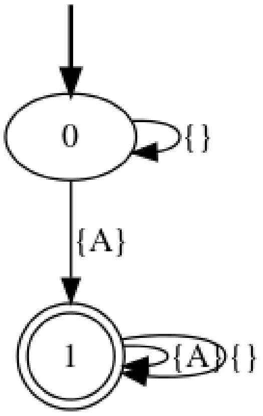


Figure 2.6. The DFA associated to $\Diamond A$



- *Conditional Response*: the LTL$_f$ formula $\varphi = \Box(A \Rightarrow \Diamond B)$ or equivalently the LDL$_f$ formula $\varphi = [true^*](\langle A\rangle tt \Rightarrow \langle true^*\rangle\langle B\rangle tt)$ translates into the automaton depicted in Figure 2.7.
- *Alternating sequence*: the LDL$_f$ formula $\varphi = \langle (A;B)^*\rangle End$ translates into the automaton depicted in Figure 2.8.

2.6.4 Complexity of LTL$_f$/LDL$_f$ reasoning

In this section we study the complexity of LTL$_f$/LDL$_f$ reasoning (i.e. complexity of problems as defined in Definition 2.8.

Theorem 2.8 (De Giacomo and Vardi (2013)). *Satisfiability, validity, and logical implication for* LDL$_f$ *formulas are* PSPACE*-complete*

Figure 2.7. The DFA associated to $\varphi = \Box(A \Rightarrow \Diamond B)$

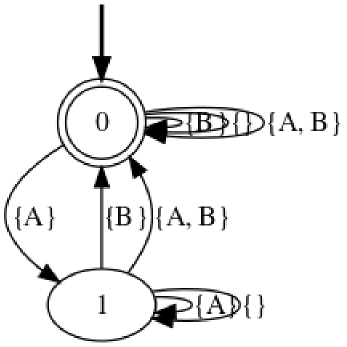
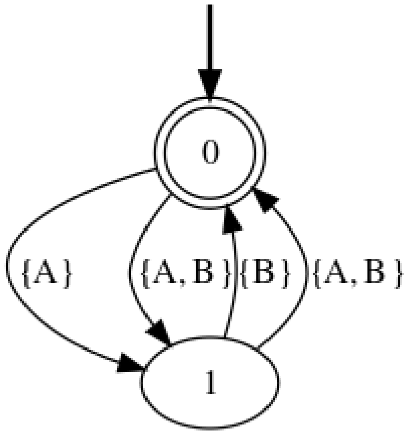


Figure 2.8. The DFA associated to $\varphi = \langle (A; B)^* \rangle End$



Proof. Given a LTL$_f$/LDL$_f$ φ, we can leverage Theorem 2.7 to solve these problems, namely:

- For LTL$_f$/LDL$_f$ satisfiability we compute the associated NFA (as explained in Section 2.6 (which is an exponential step) and then check NFA for nonemptiness (NLOGSPACE).
- For LTL$_f$/LDL$_f$ validity we compute the NFA associated to $\neg\varphi$ (which is an exponential step) and then check NFA for nonemptiness (NLOGSPACE).
- For LTL$_f$/LDL$_f$ logical implication $\psi \models \varphi$ we compute the NFA associated to $\psi \wedge \neg\varphi$ (which is an exponential step) and then check NFA for nonemptiness (NLOGSPACE).

□

2.7 Conclusions

In this chapter we provided the logical tools to face other topics in later chapters. We introduced several formal languages that allowed us to introduce LTL$_f$ and LDL$_f$, focusing on their interesting properties. Moreover, we described in detail the procedure for translation from LTL$_f$/LDL$_f$ formulas to DFAs, which yields an effective way to reasoning about LTL$_f$/LDL$_f$ formulas.

Chapter 3

FLLOAT

3.1 Main features

3.2 Package structure

3.3 Code examples

3.4 License

Chapter 4

RL for LTL$_f$/LDL$_f$ Goals

4.1 Reinforcement Learning

Reinforcement Learning (Sutton and Barto, 1998) is a sort of optimization problem where an *agent* interacts with an *environment* and obtains a *reward* for each action he chooses and the new observed state. The task is to maximize a numerical reward signal obtained after each action during the interaction with the environment. The agent does not know a priori how the environment works (i.e. the effects of his actions), but he can make observations in order to know the new state and the reward. Hence, learning is made in a *trial-and-error* fashion. Moreover, it is worth to notice that in many situation reward might not been affected only from the last action but from an indefinite number of previous action. In other words, the reward can be *delayed*, i.e. the agent should be able to foresee the effect of his actions in terms of future expected reward. Figure 4.1 represent the interaction between the agent and the environment in this setting.

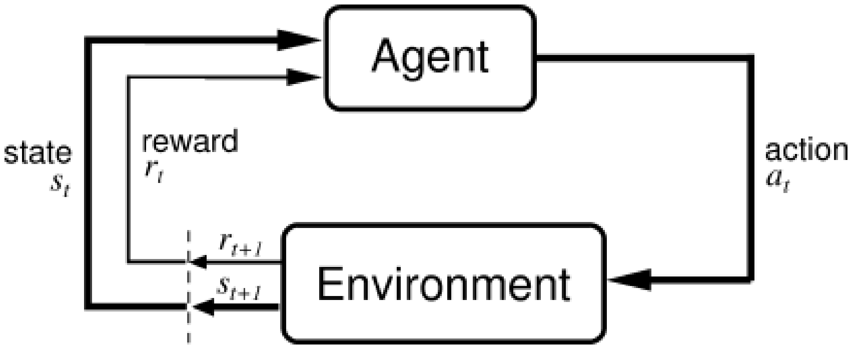


Figure 4.1. The agent and its interaction with the environment in Reinforcement Learning

In the next subsections we introduce some of the classical mathematical frameworks for RL: Markov Decision Process (MDP) and Non-Markovian Reward Decision Process (NMRDP).

4.2 Markov Decision Process (MDP)

A Markov Decision Process (MDP) $\mathcal{M}$ is a tuple $\langle S, A, T, R, \gamma \rangle$ containing a set of *states* S, a set of *actions* A, a *transition function* $T : S \times A \to Prob(S)$ that returns for every pair state-action a probability distribution over the states, a *reward* *function* $R : S \times A \times S \to \mathbb{R}$ that returns the reward received by the agent when he performs action a in s and transitions in s', and a *discount factor* γ, with $0 \leq \gamma \leq 1$, that indicates the present value of future rewards. With $T(s, a, s')$ we denote the probability to end in state s' given the action a from s.

The discount factor γ deserves some attention. Its value highly influences the MDP, its solution, and how the agent interprets rewards. Indeed, if $\gamma = 0$, we are in the pure *greedy* setting, i.e. the agent is shortsighted and look only at the reward that it might obtain in the next step, by doing a single action. The higher γ, the longer the sight horizon, or the foresight, of the agent: the far rewards are taken into account for the current action choice. If $\gamma < 1$ we are in the *finite horizon* setting: namely, the agent is intrinsically motivated to obtain rewards as fast as possible, depending on how γ is far from 1. When $\gamma = 1$ we are in the *infinite horizon* setting, which means that the agent considers far rewards as they can be obtained in the next step. In other words, we may think the agent as *immortal*, since the time the agent spend to reach rewards does not matter anymore.

A *policy* $\rho : S \to A$ for an MDP $\mathcal{M}$ is a mapping from states to actions, and represents a solution for $\mathcal{M}$. Given a sequence of rewards $R_{t+1}, R_{t+2}, \ldots, R_T$, the *expected return* G_t at time step t is defined as:

$$G_t \coloneqq \sum_{t=k+1}^{T} \gamma^{k-t-1} R_k \tag{4.1}$$

where can be $T = \infty$ and $\gamma = 1$ (but not both).

The *value function* of a state s, the *state-value function* $v_\rho(s)$ is defined as the expected return when starting in s and following policy ρ, i.e.:

$$v_\rho(s) \coloneqq \mathbb{E}_\rho[G_t | S_t = s], \forall s \in S \tag{4.2}$$

Similarly, we define q_ρ, the *action-value function for policy* ρ, as:

$$q_\rho(s, a) \coloneqq \mathbb{E}_\rho[G_t | S_t = s, A_t = a], , \forall s \in S, \forall a \in A \tag{4.3}$$

Notice that we can rewrite 4.2 and 4.3 recursively, yielding the *Bellman equation*:

$$v_\rho(s) = \sum_{s'} P(s'|s, a)[R(s, a, s') + \gamma v(s')] \tag{4.4}$$

where we used the definition of the transition function:

$$T(s, a, s') = P(s'|s, a) \tag{4.5}$$

We define the *optimal state-value function* and the *optimal action-value function* as follows:

$$v^*(s) \coloneqq \max_\rho v_\rho(s), \forall s \in S \tag{4.6}$$

$$q^*(s, a) \coloneqq \max_\rho q_\rho(s, a), \forall s \in S, \forall a \in A \tag{4.7}$$

Notice that with 4.6 and 4.7 we can show the correlation between $v^*_\rho(s)$ and $q^*_\rho(s, a)$:

$$q^*(s, a) = \mathbb{E}_\rho[R_{t+1} + \gamma v^*_\rho(S_{t+1}) | S_t = s, A_t = a] \tag{4.8}$$

We can define a partial order over policies using value functions, i.e. $\forall s \in S. \rho \geq \rho' \iff v_\rho(s) \geq v_{\rho'}(s)$. Now we give the definition of optimal policy:

Definition 4.1. An *optimal policy* ρ^* is a policy such that $\rho^* \geq \rho$ for all ρ.

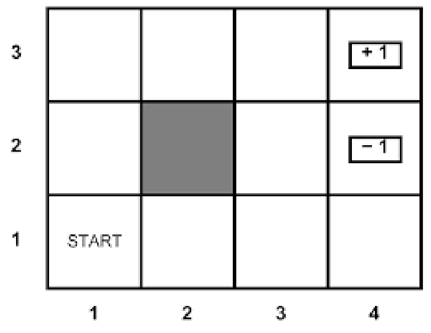
Given an MDP $\mathcal{M}$, a typical reinforcement learning problem is the following: find an optimal policy for $\mathcal{M}$, without knowing T and R. Notice that instead of explicit specification of the transition probabilities and rewards, the transition probabilities are accessed through a simulator that is restarted many times from a fixed or uniformly random initial state $s_0 \in S$. We call this way of structuring the learning process *episodic reinforcement learning.* Usually, in episodic reinforcement learning, we require the presence of one or more *goal states* where the simulation of the MDP ends and the task is considered completed, or a maximum time limit T for the number of action that can be taken by the agent in one single episode, and the overcoming of T determines the end of the episode. Optionally, *failure states* can be also defined, where the episode end similarly to goal states, but the task is considered failed.

Examples

Many dynamic systems can be modeled as Markov Decision Processes.

Example 4.1 (Gridworld). Perhaps the most simple MDP used as a toy example is *Gridworld*, depicted in Figure 4.2. There are 3 × 4 cells, i.e. states of the MDP $S = \{s_{11}, s_{12}, \ldots, s_{34}\} \setminus \{s_{22}\}$. The agent can do four actions: $A = \{Right, Left, Up, Down\}$. The initial state is fixed and is $s_0 = s_{11}$ and the agent can move in any of the adjacent and free cells from the current state. Assuming an episodic task, the goal is to reach s_{34}, and s_{24} represent a failure state. The state transition function T can be *deterministic*, i.e. the agent always succeed in performing actions, or non-deterministic, i.e. the effect of an action is determined by the probabilistic distribution returned by $T(s, a)$. An example of non-deterministic T is to give 90% of success (the agent moves in the chosen direction) and 10% of fail (the agent moves at either the right or left angle to the intended direction). If the move would make the agent walk into a wall (borders of the grid and s_{22}), the agent stays in the same place as before. The reward function $R(s, a, s')$ is defined as -1 if $s' = s_{24}$, as 1 if $s' = s_{34}$, and -0.01 otherwise. The small negative reward given at each transition is a popular mean for reward function design: it is called *step reward* and its purpose is to encourage the agent to finish the episode as fast as possible, with a priority proportional to the absolute value of the reward. The discount factor γ should be strictly higher than 0 because more than one step is needed to reach the goal state.

Figure 4.2. The Gridworld environment



An example of optimal policy is shown in Figure 4.3. As the reader can notice, the arrows represent the action that should be taken in a certain cell, in order to

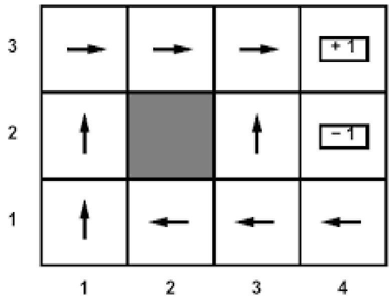


Figure 4.3. An example of optimal policy for the Gridworld environment

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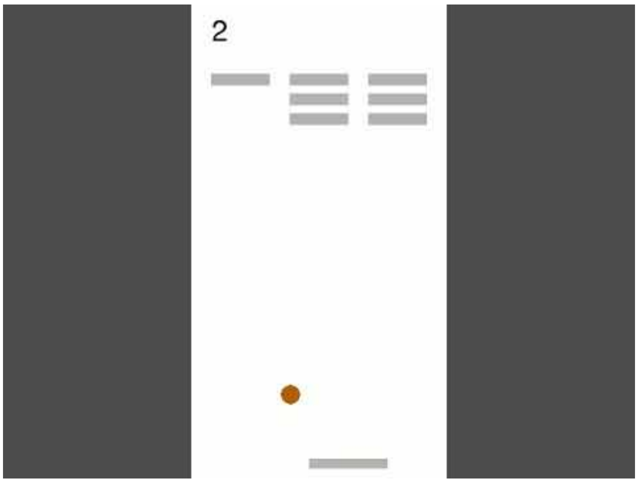

Figure 4.4. A screenshot from the videogame BREAKOUT

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maximize the expected return. We observe that the optimal action in s_{13}, according to the policy, is not the one to take the shortest path to the goal, i.e. the Up action. This is because there is a small probability to ends in s_{24}, the failure state, and be punished with a high negative reward. In terms of expected reward, it is better to take the longer path, at the price of collect small negative rewards, but avoiding the risk to fail miserably.

Example 4.2 (**Breakout**).

4.3 Temporal Difference Learning

Temporal difference learning (TD) (Sutton, 1988) refers to a class of model-free reinforcement learning methods which learn by bootstrapping from the current estimate of the value function. These methods sample from the environment, like Monte Carlo (MC) methods, and perform updates based on current estimates, like dynamic programming methods (DP) (Bellman, 1957). We do not discuss MC and DP methods here.

Q-Learning (Watkins, 1989; Watkins and Dayan, 1992) and SARSA are such a methods. They update $Q(s,a)$, i.e. the estimation of $q^*(s,a)$ at each transition

$(s, a) \rightarrow (s', r)$. The update rule is the following:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\delta \tag{4.9}$$

where δ is the *temporal difference*. In SARSA, it is defined as:

$$\delta = r + \gamma Q(s',a') - Q(s,a) \tag{4.10}$$

whereas in Q-Learning:

$$\delta = r + \gamma \max_{a'} Q(s',a') - Q(s,a) \tag{4.11}$$

TD(λ) is an algorithm which uses *eligibility traces*. The parameter λ refers to the use of an eligibility trace. The algorithm generalizes MC methods and TD learning, obtained respectively by setting $\lambda = 1$ and $\lambda = 0$. Intermediate values of λ yield methods that are often better of the extreme methods. Q-Learning and SARSA that has been shown before can be rephrased with this new formalism as Q-Learning(0) and SARSA(0), special cases of Watkin's Q(λ) and SARSA(λ) respectively. In this setting, Equation 4.9 is modified as follows:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\delta e(s,a) \tag{4.12}$$

Where $e(s,a) \in [0,1]$, the *eligibility of the pair* (s,a), determines how much the temporal difference δ should be weighted. SARSA(λ) is reported in Algorithm 4.1, whereas Watkin's Q(λ) in Algorithm 4.2, both in the variants using *replacing eligibility traces* (see line 9 and line 10, respectively).

Algorithm 4.1. SARSA(λ) (Singh and Sutton, 1996)

```
1:  Initialize $Q(s,a)$ arbitrarily and $e(s,a) = 0$ for all $s, a$
2:  repeat{for each episode}
3:      initialize $s$
4:      Choose $a$ from $s$ using policy derived from $Q$ (e.g. $e$-greedy)
5:      repeat{for each step of episode}
6:          Take action $a$, observe reward $r$ and new state $s'$
7:          Choose $a'$ from $s'$ using policy derived from $Q$
8:          $\delta \leftarrow r + \gamma Q(s',a') - Q(s,a)$
9:          $e(s,a) \leftarrow 1$                                  ▷ replacing traces
10:         for all $s, a$ do
11:             $Q(s,a) \leftarrow Q(s,a) + \alpha\delta e(s,a)$
12:             $e(s,a) \leftarrow \gamma\lambda e(s,a)$
13:         end for
14:         $s \leftarrow s'$, $a \leftarrow a'$
15:     until state $s$ is terminal
16: until
```

4.4 Non-Markovian Reward Decision Process (NMRDP)

For some goals, it might be the case that the Markovian assumption of the reward function R – that reward depends only on the current state, and not on history – does not hold. Indeed, for many problems, it is not effective that the reward is limited to depend only on a single transition (s, a, s'); instead, it might be extended

Algorithm 4.2. Watkin's Q(λ) (Watkins, 1989)

```
Initialize $Q(s,a)$ arbitrarily and $e(s,a) = 0$ for all $s, a$
repeat{for each episode}
    initialize $s$
    Choose $a$ from $s$ using policy derived from $Q$ (e.g. $e$-greedy)
    repeat{for each step of episode}
        Take action $a$, observe reward $r$ and new state $s'$
        Choose $a'$ from $s'$ using policy derived from $Q$ (e.g. $e$-greedy)
        $a^* \leftarrow \arg\max_a Q(s', a)$ (if $a'$ ties for max, then $a^* \leftarrow a'$)
        $\delta \leftarrow r + \gamma Q(s', a^*) - Q(s, a)$
        $e(s,a) \leftarrow 1$                              ▷ replacing traces
        for all $s, a$ do
            $Q(s,a) \leftarrow Q(s,a) + \alpha\delta e(s,a)$
            if $a' = a^*$ then
                $e(s,a) \leftarrow \gamma\lambda e(s,a)$
            else
                $e(s,a) \leftarrow 0$
            end if
            $e(s,a) \leftarrow \gamma\lambda e(s,a)$
        end for
        $s \leftarrow s'$, $a \leftarrow a'$
    until state $s$ is terminal
until
```

to depend on *trajectories* (i.e. $\langle s_0, a_0, \ldots, s_{n-1}, a_{n-1}, s_n\rangle$), e.g. when we want to reward the agent for some (temporally extended) behaviors, opposed to simply reaching certain states.

This idea of rewarding behaviors has been proposed by (Bacchus et al., 1996) where they defined a new mathematical model, namely Non-Markovian Reward Decision Process (NMRDP), and showed how to construct optimal policies in this case.

In the next subsections, we give the main definitions to reason in this new setting. Then we show the solution proposed in (Bacchus et al., 1996).

4.4.1 Preliminaries

Now follows the definition of NMRDP, which is similiar to the MDP definition given in Section 4.2.

Definition 4.2. A Non-Markovian Reward Decision Process (NMRDP) (Bacchus et al., 1996) $\mathcal{N}$ is a tuple $\langle S, A, T, \bar{R}, \gamma\rangle$ where S, A, T and γ are defined as in the MDP, and $\bar{R} : S^* \to \mathbb{R}$ is the *non-Markovian reward function*, where $S^* = \{\langle s_0, s_1, \ldots, s_n\rangle_{n\geq 0, s_i \in S}\}$ is the set of all the possible traces, i.e. projection of trajectories $\langle s_0, a_0, \ldots, s_{n-1}, a_{n-1}, s_n\rangle$

Given a trace $\pi = \langle s_0, s_1, \ldots, s_n\rangle$, the *value of* π is:

$$v(\pi) = \sum_{i=1}^{|\pi|} \gamma^{i-1} \bar{R}(\langle s_0, s_1, \ldots, s_n\rangle) \tag{4.13}$$

where $|\pi|$ denotes the number of transitions (i.e. of actions).

The policy $\bar{\rho}$ in this setting is defined over sequences of states, i.e. $\bar{\rho} : S^* \rightarrow A$. The *value of* $\bar{\rho}$ given an initial state s_0 is defined as:

$$v^{\bar{\rho}}(s) = \mathbb{E}_{\pi \sim \mathcal{N}, \bar{\rho}, s_0}[v(\pi)] \tag{4.14}$$

i.e. the expected value in state s considering the distribution of traces defined by the transition function of $\mathcal{N}$, the policy $\bar{\rho}$ and the initial state s_0.

We are interested in two problems, that we will study in the next sections:

- Find an optimal (non-Markovian) policy $\bar{\rho}$ for an NMRDP $\mathcal{N}$ (Definition 4.2);
- Define the non-Markovian reward function for the domain of interest.

4.4.2 Find an optimal policy $\bar{\rho}$ for NMRDPs

The key difficulty with non-Markovian rewards is that standard optimization techniques, most based on Bellman's (Bellman, 1957) dynamic programming principle, cannot be used. Indeed, this requires one to resort to optimization over a policy space that maps histories (rather than states) into actions, a process that would incur great computational expense. (Bacchus et al., 1996) give the definition of a decision problem *equivalent* to an NMRDP in which the rewards are Markovian. This construction is the key element to solve our problem, i.e. find an optimal policy for an NMRDP.

Equivalent MDP

Now we give the definition of *equivalent* MDP of an NMRDP, and state an important result.

Definition 4.3 (Bacchus et al. (1996))**.** An NMRDP $\mathcal{N} = \langle S, A, T, \bar{R}, \gamma \rangle$ is *equivalent* to an extended MDP $\mathcal{M} = \langle S', A, T', R', \gamma \rangle$ if there exist two functions $\tau : S' \rightarrow S$ and $\sigma : S \rightarrow S'$ such that

1. $\forall s \in S : \tau(\sigma(s)) = s$;
2. $\forall s_1, s_2 \in S$ and $s'_1 \in S'$: if $T(s_1, a, s_2) > 0$ and $\tau(s'_1) = s_1$, there exists a unique $s'_2 \in S'$ such that $\tau(s'_2) = s_2$ and $T'(s'_1, a, s'_2) = T(s_1, a, s_2)$;
3. For any feasible trace $\langle s_0, s_1, \dots, s_n \rangle$ of $\mathcal{N}$ and $\langle s'_0, s'_1, \dots, s'_n \rangle$ of $\mathcal{M}$ associated to the trajectories $\langle s_0, a_0, \dots, s_{n-1}, a_{n-1}, s_n \rangle$ and $\langle s'_0, a_0, \dots, s'_{n-1}, a_{n-1}, s'_n \rangle$, such that $\tau(s'_i) = s_i$ and $\sigma(s_0) = s'_0$, we have $R(\langle s_0, s_1, \dots, s_n \rangle) = R'(s_{n-1}, a_{n-1}, s'_n)$.

Given the Definition 4.3, we give the definition of corresponding policy:

Definition 4.4 (Bacchus et al. (1996))**.** Let $\mathcal{N}$ be an NMRDP and let $\mathcal{M}$ be the equivalent MDP as defined in Definition 4.3. Let ρ be a policy for $\mathcal{M}$. The *corresponding policy* for $\mathcal{N}$ is defined as $\bar{\rho}(\langle s_0, \dots, s_n \rangle) = \rho(s'_n)$, where for the sequence $\langle s'_0, \dots, s'_n \rangle$ we have $\tau(s'_i) = s_i \; \forall i$ and $\sigma(s_0) = s'_0$

From definitions 4.3 and 4.4, and since that for all policy ρ of $\mathcal{M}$ the corresponding policy $\bar{\rho}$ of $\mathcal{N}$ is such that $\forall s. v_\rho(s) = v_{\bar{\rho}}(\sigma(s))$, the following theorem holds:

Theorem 4.1 (Bacchus et al. (1996))**.** *Let ρ be an optimal policy for MDP $\mathcal{M}$. Then the corresponding policy is optimal for NMRDP $\mathcal{N}$.*

The Theorem 4.1 allow us to learn an optimal policy $\bar{\rho}$ for NMRDP by learning a policy ρ over an equivalent MDP, which can be done by resorting on any off-the-shelf algorithm (e.g. see Section 4.3). Moreover, obtaining the corresponding policy for the original NMRDP is straightforward, although in practice is not needed, since it is enough to run the policy ρ over the MDP.

In other words, the problem of finding an optimal policy for an NMRDP reduces to find an optimal policy for an equivalent MDP such that Condition 1, 2 and 3 of Definition 4.3 hold.

4.4.3 Define the non-Markovian reward function $\bar{R}$

To reward agents for (temporally extended) behaviors, as opposed to simply reaching certain states, we need a way to specify rewards for specific trajectories through the state space. Specifying a non-Markovian reward function explicitly is quite hard and unintuitive, impossible if we are in a infinite-horizon setting. Instead, we can define *properties* over trajectories and reward only the ones which satisfy some of them, in contrast to enumerate all the possible trajectories.

Temporal logics presented in Section 2.1 gives an effective way to do this. Indeed, in order to speak about a desired behavior, i.e. fulfillment of properties that might change over time, we can define a *formula* φ (or more formulas) in some suited temporal logic formalism semantically defined over trajectories π, speaking about a set of properties $\mathcal{P}$ such that each state $s \in S$ is associated to a set of propositions ($S \subseteq 2^{\mathcal{P}}$). In this way, a trajectory $\pi = \langle s_0, a_0, \dots, s_{n-1}, a_{n-1}, s_n \rangle$ is rewarded with r_i iff $\pi \models \varphi_i$, where r_i is the reward value associated to the fulfillment of behaviors signified by φ_i.

4.4.4 Using PLTL

In (Bacchus et al., 1996) the temporal logic formalism is *Past Linear Temporal Logic* (PLTL), which is a past version of LTL (Section 2.1). As explained before, using the declarativeness of PLTL, is possible to specify the desired behavior (expressed in terms of the properties $\mathcal{P}$) that should be satisfied by the experienced trajectories and reward only them, hence obtaining a non-Markovian reward function. More formally, given a finite set Φ of PLTL *reward formulas*, and for each $\phi_i \in \Phi$ a real-valued reward r_i, the *temporally extended reward function* $\bar{R}$ is defined as:

$$\bar{R}(\langle s_0, s_1, \dots, s_n \rangle) = \sum_{\phi_i \in \Phi : \langle s_0, s_1, \dots, s_n \rangle \models \phi_i} r_i \tag{4.15}$$

In order to run the actual learning task, (Bacchus et al., 1996) proposed a transformation from the NMRDP to an equivalent MDP with the state space *expaneded* which allows to label each state $s \in S$. The idea is that the labels should keep track in some way the (partial) satisfaction of the temporal formulas $\phi_i \in \Phi$. A state s in the transformed state space is replicated multiple times, marking the difference between different (relevant) histories terminating in state s.

In this way, they obtained a compact representation of the required history-dependent policy by considering only relevant history, and can produce this policy using computationally-effective MDP algorithms. In other words, the states of the NMRDP can be mapped into those of the expanded MDP, in such a way that corresponding states yield same transition probabilities and corresponding traces have same rewards.

4.5 NMRDP with LTL$_f$/LDL$_f$ rewards

In this section we explain how to specify non-Markovian rewards with LTL$_f$/LDL$_f$ formulas (instead of PLTL) and how the associated MDP expansion works (Brafman et al., 2017), analogously to what we saw with PLTL (Section 4.4.4).

The temporally extended reward function $\bar{R}$ is similar to Equation 4.15, but instead of using PLTL formula we use LTL$_f$/LDL$_f$ formulas. Formally, given a set of pairs $\{(\varphi_i, r_i)_{i=1}^m\}$ (where φ_i denotes the LTL$_f$/LDL$_f$ formula for specifying a desired behavior, and r_i denotes the reward associated to the satisfaction of φ_i, and given a (partial) trace $\pi = \langle s_0, s_1, \dots, s_n\rangle$, we define $\bar{R}$ as:

$$\bar{R}(\pi) = \sum_{1 \le i \le m : \pi \models \varphi_i} r_i \tag{4.16}$$

For the sake of clarity, in the following we use $\{(\varphi_i, r_i)_{i=1}^m\}$ to denote $\bar{R}$.

Now we describe the MDP expansion for doing learning in this setting, as proposed in (Brafman et al., 2017). Without loss of generality, we assume that every NMRDP $\mathcal{N}$ is reduced into another NMRDP $\mathcal{N}' = \langle S', A', T', R', \gamma\rangle$:

$$\begin{aligned} S' &= S \cup \{s_{init}\} \\ A' &= A \cup \{start\} \\ T'(s,a,s') &= \begin{cases} 1 & \text{if} \;\; s = s_{init}, a = start, s' = s_0 \\ 0 & \text{if} \;\; s = s_{init} \text{ and } (a \neq start \text{ or } s' \neq s_0) \\ T(s,a,s') & \text{otherwise} \end{cases} \\ R'(\langle s_{init}, s_0, \dots, s_n\rangle) &= R(\langle s_0, s_1, \dots, s_n\rangle) \end{aligned} \tag{4.17}$$

and s_{init} is the new initial state. In other words, we prefix to every feasible trajectory $\mathcal{N}$ the pair $\langle s_{init}, start\rangle$, denoting the beginning of the episode. We do this for two reasons: allow to evaluate formulas in s_0 and make it compliant with the most general definition of the reward, namely $R(s,a,s')$, also when there is no true action that is done (i.e. empty trace).

Definition 4.5 (Brafman et al. (2017))**.** Given an NMRDP $\mathcal{N} = \langle S, A, T, \{(\varphi_i, r_i)_{i=1}^m, \gamma\}\rangle$ (i.e. with non-Markovian rewards specified by LTL$_f$/LDL$_f$ formulas) it is possible to build an $\mathcal{M} = \langle S', A, T', R', \gamma\rangle$ that is *equivalent* (in the sense of Definition 4.3) to $\mathcal{N}$. Denoting with $\mathcal{A}_{\varphi_i} = \langle 2^{\mathcal{P}}, Q_i, q_{i0}, \delta_i, F_i\rangle$ (notice that $S \subseteq 2^{\mathcal{P}}$ and δ_i is total) the DFA associated with φ_i (see Section 2.6), the equivalent MDP $\mathcal{M}$ is built as follows:

- $S' = Q_1 \times \dots \times Q_m \times S$ is the set of states;
- $T' : S' \times A \times S' \to [0,1]$ is defined as follows:

$$\begin{aligned} &Tr'(q_1, \dots, q_m, s, a, q_1', \dots, q_m', s') = \\ &\begin{cases} Tr(s,a,s') & \text{if } \forall i : \delta_i(q_i, s') = q_i' \\ 0 & \text{otherwise;} \end{cases} \end{aligned}$$

- $R' : S' \times A \times S' \to \mathbb{R}$ is defined as:

$$R'(q_1, \dots, q_m, s, a, q_1', \dots, q_m', s') = \sum_{i : q_i' \in F_i} r_i$$

Theorem 4.2 (Brafman et al. (2017)). *The NMRDP $\mathcal{N} = \langle S, A, T, \{(\varphi_i, r_i)\}_{i=1}^m, \gamma\rangle$ is equivalent to the MDP $\mathcal{M} = \langle S', A, T', R', \gamma\rangle$ defined in Definition 4.5.*

Proof. Recall that every $s' \in S'$ has the form $(q_1, \ldots, q_m, s)$. Define $\tau(q_1, \ldots, q_m, s) = s$. Define $\sigma(s) = (q_{10}, \ldots, q_{m0}, s)$. We have $\tau(\sigma(s)) = s$, hence Condition 1 is verified. Condition 2 of Definition 4.3 is easily verifiable by inspection. For Condition 3, consider a possible trace $\pi = \langle s_0, s_1, \ldots, s_n\rangle$. We use σ to obtain $s'_0 = \sigma(s_0)$ and given s_i, we define s'_i (for $1 \leq i \leq n$) to be the unique state $(q_{1,i}, \ldots, q_{m,i}, s_i)$ such that $q_{j,i} = \delta(q_{j,i-1}, s_i)$ for all $1 \leq j \leq m$. Moreover, we require that, without loss of generality, every trajectory in the new MDP starts from s_{init} and now have a corresponding possible trace of $\mathcal{M}$, i.e., $\pi = \langle s'_0, s'_1, \ldots, s'_n\rangle$. This is the only feasible trajectory of $\mathcal{M}$ that satisfies Condition 3. The reward at $\pi = \langle s_0, s_1, \ldots, s_n\rangle$ depends only on whether or not each formula φ_i is satisfied by π. However, by construction of the automaton $\mathcal{A}_{\varphi_i}$ and the transition function T, $\pi \models \varphi_i$ iff $s'_n = (q_1, \ldots, q_m, s_n)$ and $q_i \in F_i$ □

Let ρ' be a (Markovian) policy for $\mathcal{M}$. It is easy to define an *corresponding* policy on $\mathcal{N}$, i.e., a policy that guarantees the same rewards, by using τ and σ mappings defined in Theorem 4.2 and the result shown in Theorem 4.4.

Obviously, typical learning techniques, such as Q-learning or SARSA, are applicable on the expanded $\mathcal{M}$ and so we can learn an optimal policy ρ for $\mathcal{M}$. Thus, an optimal policy for $\mathcal{N}$ can be learnt on $\mathcal{M}$. Of course, none of these structures is (completely) known to the learning agent, and the above transformation is never done explicitly. Rather, the agent carries out the learning process by assuming that the underlying model is $\mathcal{M}$ instead of $\mathcal{N}$ (applying the fix introduced in Definition 4.17).

Observe that the state space of $\mathcal{M}'$ is the product of the state spaces of $\mathcal{N}$ and $\mathcal{A}_{\varphi_i}$, and that the reward R' is Markovian. In other words, the (stateful) structure of the LTL$_f$/LDL$_f$ formulas φ_i used in the (non-Markovian) reward of $\mathcal{N}$ is *compiled* into the states of $\mathcal{M}$.

Why should we use LDL$_f$

LDL$_f$ formalism (introduced in Section 2.5) has the advantage of *enhanced expressive power* over other proposals, as discussed in (Brafman et al., 2017). Indeed, we move from linear-time temporal logics to LDL$_f$, paying no additional (worst-case) complexity costs. LDL$_f$ can encode in polynomial time LTL$_f$, regular expressions (RE) and the past LTL (PLTL) of (Bacchus et al., 1996). Moreover, LDL$_f$ can naturally represent "procedural constraints" (Baier et al., 2008), i.e., sequencing constraints expressed as programs, using "if" and "while", hence allowing to express more complex properties.

4.6 RL for LTL$_f$/LDL$_f$ Goals

In this section we define a particular problem and propose a solution, which is the main theoretical contribution of this work. We call this problem *Reinforcement Learning for* LTL$_f$/LDL$_f$ *Goals.*

4.6.1 Problem definition

the World, the Agent and the Fluents

Let $\mathcal{W}$ be a *world* of interest (e.g. a room, an environment, a videogame). Let W be the set of *world states*, i.e. the states of the world $\mathcal{W}$. A *feature* is a function f_j that maps a world state to the values of another domain D_j, such as reals, finite enumerations, booleans, etc., i.e., $f_j : W \to D_j$. Given a set of features $F = \langle f_1, \ldots, f_d \rangle$, the *feature vector* of a world state w_h is the vector $\mathbf{f}(w_h) = \langle f_1(w_h), \ldots, f_d(w_h) \rangle$ of feature values corresponding to w_h.

Now consider an agent that lives in $\mathcal{W}$. The agent can interact with $\mathcal{W}$ by executing an action a taken from a *set of actions* A. Without loss of generality, we assume that such learning agent has a special action *stop* which deems the end of an episode. Moreover, the agent has its own set of features $F_{ag} = \langle f_1, \ldots, f_d \rangle$, which yields its *representation of the world* S, where $S \subseteq F_{ag}(W)$ and $F_{ag}(W) = \{\mathbf{f}_{ag}(w) | w \in W\}$. Finally, assume that the agent has a *clock* which determines the granularity of its acting. At every clock, the agent can do action a and observe the new state s from the new world state w', namely $s = \mathbf{f}_{ag}(w)$.

We consider arbitrary LTL$_f$/LDL$_f$ formulas φ_i $(i = 1, \ldots, m)$ over a set of fluents $\mathcal{F}$ used for provide a high-level description of the world. We denote by $\mathcal{L} = 2^{\mathcal{F}}$ the set of possible fluents configurations. Given a set of feature F_{goal}, a *configuration of fluents* $\ell_h \in \mathcal{L}$ is formed by the components that assign truth values to the fluents according to the feature vector $\mathbf{f}_{goal}(w_h)$. At every step, the features for fluents evaluations are observed, obtaining a particular configuration $\ell \in \mathcal{L}$. Notice that in general the features for the fluents and for the agent state space may differ. The formula φ_i is selecting sequences of fluents configurations $\ell_1, \cdots, \ell_n$, with $\ell_k \in \mathcal{L}$, whose relationship with the sequences of states $s_1, \ldots, s_n$, with $s_k \in S$ is unknown.

In other words, a subset of features are used to describe agent states s_h and another subset (for simplicity, assumed disjoint from the previous one) are used to evaluate the fluents in ℓ_h. Hence, given a sequence $w_1, \ldots, w_n$ of world states we get the corresponding sequence of sequences learning agent states $s_1, \ldots, s_n$ and simultaneously the sequence of fluent configurations $\ell_1, \ldots, \ell_n$. Notice that we do not have a formalization for $w_1, \ldots, w_n$ but we do have that for $s_1, \ldots, s_n$ and for $\ell_1, \ldots, \ell_n$.

Oversimplifying, we may say that S is the set of configurations of the low-level features for the learning agent, while $\mathcal{L}$ is the set of configuration of the high-level features needed for expressing φ_i.

Markovian assumption of the state transition function

Now we make the following assumption: that is, the agent actions in A induce a transition distribution over the features and fluents configuration, i.e.,

$$T^g_{ag} : S \times \mathcal{L} \times A \to Prob(S \times \mathcal{L}) \tag{4.18}$$

This means that the state transition function T^g_{ag} is *Markovian*, i.e. the probability to end in the next state s' with the next fluents configuration ℓ' depends only from s, ℓ and a (the current agent state, the current fluents configuration and the action taken, respectively).

Such a transition distribution together with the initial values of the fluents ℓ_0 and of the agent state s_0 allow us to describe a probabilistic transition system accounting for the dynamics of the fluents and agent states. In other words, in response to an agent action a_h performed in the current state w_h (in the state s_h

of the agent and the configuration ℓ_h of the fluents), the world changes into w_{h+1} from which s_{h+1} and ℓ_{h+1}. This is all we need to proceed.

Notice that we do not assume nothing about the primitive state transitions function, i.e. the one induced over the agent state space S and the one over the set of possible fluents configurations $\mathcal{L}$. However, it might be the case that the Markovian assumption of the agent state transition function holds, i.e. the probability distribution for the next state s' is fully determined by the current state s and the action a. In that case, the considerations presented in this section still holds, since it is a special case of our general assumptions. We only require that the *joint* transition function in Equation 4.18 satisfies the Markov property.

We are interested in devising policies for the learning agent such that at the end of the episode, i.e., when the agent executes *stop*, the LTL$_f$/LDL$_f$ goal formulas φ_i ($i = 1, \ldots, m$) are satisfied. Now we can state our problem formally.
Problem definition: *We define* RL for LTL$_f$/LDL$_f$ goals*, denoted as*

$$\mathcal{M}_{ag}^{goal} = \langle S, A, R, \mathcal{L}, T_{ag}^g, \{(\varphi_i, r_i)\}_{i=1}^m \rangle$$

with T_{ag}^g, R and r_i unknown,the following problem: given a learning agent $\mathcal{M}_{ag} = \langle S, A, T, R \rangle$, with T and R unknown and a set $\{(\varphi_i, r_i)\}_{i=1}^m$ of LTL$_f$/LDL$_f$ *formulas with associated rewards, find a (non-Markovian) policy $\bar{\rho} : S^* \to A$ that is optimal wrt the sum of the rewards r_i and R.*

Observe that an optimal policy for our problem, although not depending on $\mathcal{L}$, is guaranteed to satisfy the LTL$_f$/LDL$_f$ goal formulas.

4.6.2 Examples

——————————————————-TODO

4.6.3 Reduction to MDP

To devise a solution technique, we start by transforming $\mathcal{M}_{ag}^{goal} = \langle S, A, T_{ag}^g, R, \mathcal{L}, \{(\varphi_i, r_i)\}_{i=1}^m \rangle$ into an NMRDP $\mathcal{M}_{ag}^{nmr} = \langle S \times \mathcal{L}, A, T_{ag}^g, \{(\varphi_i', r_i)\}_{i=1}^m \cup \{(\varphi_s, R(s, a, s'))\}_{s \in S, a \in A, s' \in S} \rangle$ where:

- States are pairs (s, ℓ) formed by an agent configuration s and a fluents configuration ℓ.
- $\varphi_i' = \varphi_i \wedge \Diamond Done$.
- $\varphi_s = \Diamond(s \wedge a \wedge \circ(Last \wedge s'))$.
- T_{ag}^g, r_i and $R(s, a, s')$ are unknown and sampled from the environment.

Formulas φ_i' simply require to evaluate the corresponding goal formula φ_i after having done the action *stop*, which sets the fluent *Done* to true and ends the episode. Hence it gives the reward associated to the goal at the end of the episode. The formulas $\Diamond(s \wedge a \wedge \circ(Last \wedge s'))$, one per (s, a, s'), requires both states s and action a are followed by s' are evaluated at the end of the current (partial) trace (notice the use of *Last*). In this case, the reward $R(s, a, s')$ from $\mathcal{M}_{ag}$ associated with (s, a, s') is given.

Notice that policies for $\mathcal{M}_{ag}^{nmr}$ have the form $(S \times \mathcal{L})^* \to A$ which needs to be restricted to have the form required by our problem $\mathcal{M}_{ag}^{goal}$.

A policy $\bar{\rho} : (S \times \mathcal{L})^* \to A$ *has the form* $S^* \to A$ when for any sequence of n states $\langle s_1 \cdots s_n \rangle$, we have that for any pair of sequences of fluent configurations $\langle \ell'_1 \cdots \ell'_n \rangle$, $\langle \ell''_1 \cdots \ell''_n \rangle$ the policy returns the same action, $\bar{\rho}(\langle s_1, \ell'_1 \rangle \cdots \langle s_n, \ell'_n \rangle) = \bar{\rho}(\langle s_1, \ell''_1 \rangle \cdots \langle s_n, \ell''_n \rangle)$. In other words, a policy $\bar{\rho} : (S \times \mathcal{L})^* \to A$ has the form $\bar{\rho} : S^* \to A$ when it does not depend on the fluents $\mathcal{L}$. We can now state the following result.

Theorem 4.3. *RL for* LTL$_f$/LDL$_f$ *goals* $\mathcal{M}_{ag}^{goal} = \langle S, A, Tr_{ag}^g, R, \mathcal{L}, \{(\varphi_i, r_i)\}_{i=1}^m \rangle$ *can be reduced to RL over the NRMDP* $\mathcal{M}_{ag}^{nmr} = \langle S \times \mathcal{L}, A, T_{ag}^g, \{(\varphi'_i, r_i)\}_{i=1}^m \cup \{(\varphi_s, R(s, a, s'))\}_{s \in S, a \in A, s' \in S} \rangle$*, restricting policies to be learned to have the form* $S^* \to A$*.*

Observe that by restricting $\mathcal{M}_{ag}^{nmr}$ policies to S^* in general we may discard policies that have a better reward but depend on $\mathcal{L}$. On the other hand, these policies need to change the learning agent in order to allow it to observe $\mathcal{L}$ as well. As mentioned in the introduction, we are interested in keeping the learning agent as it is, apart for additional memory.

As a second step, we apply the construction of Section 4.5 and obtain a new MDP learning agent. In such construction, however, because of the triviality of their automata, we do not need to keep track of state φ_s, but just give the reward $R(s, a, s')$ associated to (s, a, s'). Instead we do need to keep track of state of the DFAs $\mathcal{A}_{\varphi_i}$ corresponding to the formulas φ'_i. Hence, from $\mathcal{M}_{ag}^{nmr}$, we get an MDP $\mathcal{M}'_{ag} = \langle S', A', Tr'_{ag}, R' \rangle$ where:

- $S' = Q_1 \times \cdots \times Q_m \times S \times \mathcal{L}$ is the set of states;
- $Tr'_{ag} : S' \times A' \times S' \to [0, 1]$ is defined as follows:

$$Tr'_{ag}(q_1, \ldots, q_m, s, \ell, a, q'_1, \ldots, q'_m, s', \ell') = \\ \begin{cases} Tr(s, \ell, , a, s', \ell') & \text{if } \forall i : \delta_i(q_i, \ell') = q'_i \\ 0 & \text{otherwise;} \end{cases}$$

- $R' : S' \times A \times S' \to \mathbb{R}$ is defined as:

$$R'(q_1, \ldots, q_m, s, \ell, a, q'_1, \ldots, q'_m, s', \ell') = \\ \textstyle\sum_{i: q'_i \in F_i} r_i + R(s, a, s')$$

Finally we observe that the environment gives now both the rewards $R(s, a, s')$ of the original learning agent, and the rewards r_i associated to the formula so has to guide the agent towards the satisfaction of the goal (progressing correctly the DFAs $\mathcal{A}_{\varphi_i}$).

By applying Theorem 4.2 we get that NMRDP $\mathcal{M}_{ag}^{nmr}$ and the MDP $\mathcal{M}'_{ag}$ are equivalent, i.e., any policy of $\mathcal{M}_{ag}^{nmr}$ has an equivalent policy (hence guaranteeing the same reward) in $\mathcal{M}'_{ag}$ and vice versa. Hence we can learn policy on $\mathcal{M}'_{ag}$ instead of $\mathcal{M}_{ag}^{nmr}$.

We can refine Theorem 4.3 into the following one.

Theorem 4.4. *RL for* LTL$_f$/LDL$_f$ *goals* $\mathcal{M}_{ag}^{goal} = \langle S, A, T_{ag}^g, R, \mathcal{L}, \{(\varphi_i, r_i)\}_{i=1}^m \rangle$ *can be reduced to RL over the MDP* $\mathcal{M}'_{ag} = \langle S', A, T'_{ag}, R' \rangle$*, restricting policies to be learned to have the form* $Q_1 \times \ldots \times Q_n \times S \to A$*.*

As before, a policy $Q_1 \times \ldots \times Q_n \times S \times \mathcal{L} \rightarrow A$ *has the form* $Q_1 \times \ldots \times Q_n \times S \rightarrow A$ when any ℓ and ℓ' the policy returns the same action, $\rho(q_1, \ldots, q_n s, \ell) = \rho(q_1, \ldots, q_n s, \ell')$.

The final step is to solve our original RL task on $\mathcal{M}_{ag}^{goal}$ by performing RL on a new MDP $\mathcal{M}_{ag}^{new} = \langle Q_1 \times \cdots \times Q_m \times S, A, T''_{ag}, R'' \rangle$ where:

- Transitions distribution T''_{ag} is the marginalization wrt $\mathcal{L}$ of T'_{ag} and is unknown;
- Rewards R'' is defined as:

$$R''(q_1, \ldots, q_m, s, a, q'_1, \ldots, q'_m, s') = \sum_{i: q'_i \in F_i} r_i + R(s, a, s').$$

- States q_i of DFAs $\mathcal{A}_{\varphi_i}$ are progressed correctly by the environment.

Indeed we can show the following result.

Theorem 4.5. *RL for* LTL$_f$/LDL$_f$ *goals* $\mathcal{M}_{ag}^{goal} = \langle S, A, T', R, \mathcal{L}, \{(\varphi_i, r_i)\}_{i=1}^m \rangle$ *can be reduced to RL over the MDP* $\mathcal{M}_{ag}^{new} = \langle Q_1 \times \cdots \times Q_m \times S, A, T''_{ag}, R'' \rangle$ *and the optimal policy* ρ_{ag}^{new} *learned for* $\mathcal{M}_{ag}^{new}$ *can be reduced to a corresponding optimal policy for* $\mathcal{M}_{ag}^{goal}$.

Proof. From Theorem 4.4, by the following observations. For the sake of brevity, we use $\mathbf{q}$ to denote $q_1, \ldots, q_m$. Notice also that for all $\ell, \ell' \in \mathcal{L}$, $R'(\mathbf{q}, s, \ell, a, \mathbf{q}', s', \ell') = R''(\mathbf{q}, s, a, \mathbf{q}', s')$.

We show that the values of $v_{ag}^{\rho}(q_1, \ldots, q_m, s, \ell)$, i.e. the state value function for $\mathcal{M}'_{ag}$(for simplicity v^{ρ}, unless otherwise stated), for some policy ρ, do not depend on ℓ or, in other words, it is necessary that $\forall \ell_1, \ell_2. v^{\rho}(q_1, \ldots, q_m, s, \ell_1) = v^{\rho}(q_1, \ldots, q_m, s, \ell_2)$. Finally, we notice that $\forall \ell. v_{ag}^{\rho, new} = v_{ag}^{\rho}$

From Equation 4.4 we have:

$$\begin{aligned} & v_{\rho}(\mathbf{q}, s, \ell) = \\ & \sum_{\mathbf{q}', s', \ell'} P(\mathbf{q}', s', \ell' | \mathbf{q}, s, \ell, a)[R'(\mathbf{q}, s, \ell, a, \mathbf{q}', s', \ell') + \gamma v_{\rho}(\mathbf{q}', s', \ell')] = \\ & \sum_{\mathbf{q}', s', \ell'} P(\mathbf{q}', s', \ell' | \mathbf{q}, s, \ell, a)[R''(\mathbf{q}, s, a, \mathbf{q}', s') + \gamma v_{\rho}(\mathbf{q}', s', \ell')] \end{aligned} \tag{4.19}$$

Using the equivalence between R' and R'', as already pointed out. Notice that we can compute $\mathbf{q}'$ from $\mathbf{q}$ and ℓ', hence we do not need ℓ. In other words:

$$P(\mathbf{q}', s', \ell' | \mathbf{q}, s, \ell, a) = P(\mathbf{q}', s', \ell' | \mathbf{q}, s, a)$$

Equation 4.19 becomes:

$$\sum_{\mathbf{q}', s', \ell'} P(\mathbf{q}', s', \ell' | \mathbf{q}, s, a)[R''(\mathbf{q}, s, a, \mathbf{q}', s') + \gamma v_{\rho}(\mathbf{q}', s', \ell')] \tag{4.20}$$

At this point, we see that v^{ρ} does not depend from ℓ, hence we can safely drop ℓ as argument for v_{ρ}, obtaining v_{ag}^{ρ}. Indeed, from 4.20:

$$\sum_{\mathbf{q}', s'} [R''(\mathbf{q}, s, a, \mathbf{q}', s') + \gamma v_{ag}^{\rho, new}(\mathbf{q}, s)] \sum_{\ell'} P(\mathbf{q}', s', \ell' | \mathbf{q}, s, a) = \tag{4.21}$$

$$\sum_{\mathbf{q}',s'} P(\mathbf{q}', s'|\mathbf{q}, s, a)[R''(\mathbf{q}, s, a, \mathbf{q}', s') + \gamma v^{\rho}_{ag}(\mathbf{q}', s')] =$$
$$v^{\rho}_{ag}(\mathbf{q}, s)$$

Where in 4.21 we marginalized the distribution $P(\mathbf{q}', s', \ell'|\mathbf{q}, s, a)$ over ℓ'. From Definition 4.1 of optimal policy, we can reduce an optimal policy ρ^{new}_{ag} to a policy of the form $\rho'_{ag} : Q_1 \times \cdots \times Q_m \times S \to A$ that is optimal for $\mathcal{M}'_{ag}$ (since the state value function of $\mathcal{M}^{new}_{ag}$, after dropping the argument ℓ, and of $\mathcal{M}'_{ag}$ are equivalent). From Theorem 4.4 the thesis. □

It is worth to remark that in the resulting MDP $\mathcal{M}^{new}_{ag}$ the *explicit presence of the fluents configuration ℓ has been removed.* Rather, the dependency is compiled into the expanded state space $Q_1 \times \dots Q_m \times S$, where $Q_1, \dots, Q_m$ are the automata state spaces associated to the formulas φ_i.

Summary

——————————————————-TODO

4.7 Conclusions

In this chapter we introduced the topic of Reinforcement Learning, as well as foundational definitions (MDP, policy ρ, state-value function v_ρ) and algorithms (Q-Learning, Sarsa). Then we presented the notion of NMRDP and a technique to built an equivalent MDP, by specifying non-Markovian reward function with LTL$_f$/LDL$_f$ formalisms. Finally, we defined and studied a new problem, *Reinforcement Learning for* LTL$_f$/LDL$_f$ *goals*, and proposed a reduction that yields an equivalent MDP which can be used to solve the original problem.

Chapter 5

Automata-based Reward shaping

5.1 Reward Shaping Theory

5.1.1 Classic Reward Shaping

5.1.2 Dynamic Reward Shaping

5.2 Reward shaping over $\mathcal{A}_\varphi$

5.3 Reward shaping on-the-fly

Chapter 6

RLTG

6.1 Main features

6.2 Package structure

6.3 Code examples

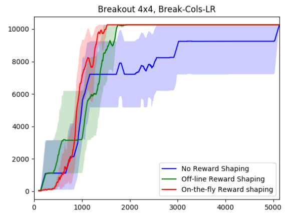
6.4 License

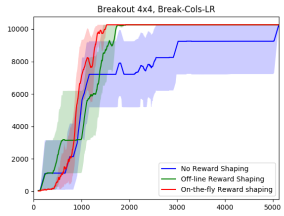
Chapter 7

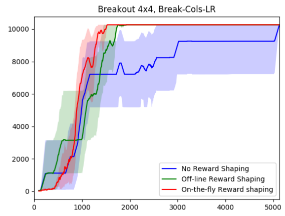
Experiments

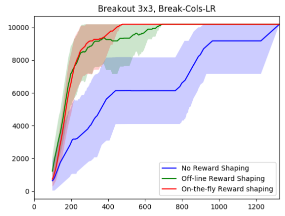
look at experiment introduction in Grzes phd thesis

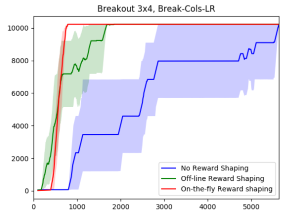
7.1 BREAKOUT

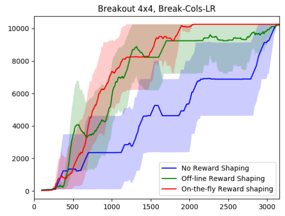












7.2 SAPIENTINO

7.3 MINECRAFT

Chapter 8

Conclusions

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