An Exactly Solvable Model for Dimension Six Higgs Operators and $h \to \gamma\gamma$

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An exactly solvable large N model is constructed which reduces at low energies to the Standard Model plus the dimension six Higgs-gauge operators $g_1^2 H^\dagger H B_{\mu\nu} B^{\mu\nu}$, $g_2^2 H^\dagger H W^a_{\mu\nu} W^{a\mu\nu}$, $g_1 g_2 H^\dagger \tau^a H W^a_{\mu\nu} B^{\mu\nu}$, and $\epsilon^{abc} W^{a\nu}_{\mu} W^{b\rho}_{\nu} W^{c\mu}_{\rho}$. All other dimension six operators are suppressed by powers of $1/N$. The Higgs-gauge operators lead to deviations from the Standard Model $h \to \gamma\gamma$ and $h \to \gamma Z$ rates. A simple variant of the model can be used to also generate the Higgs-gluon operator $g_3^2 H^\dagger H G^A_{\mu\nu} G^{A\mu\nu}$ which contributes to the Higgs production rate via gluon fusion.

A scalar boson with a mass of $\sim 125\,\mathrm{GeV}$ has recently been discovered at the LHC, and it is important to study its properties in a model-independent way. The Standard Model provides a good description of the LHC data so far, with no evidence for any new particles with masses below $\sim 1\,\mathrm{TeV}$. A widely used starting point for analysis is to assume that the theory at $125\,\mathrm{GeV}$ is the Standard Model including a fundamental scalar doublet, and all new physics effects are characterized by higher dimension operators involving Standard Model fields.

A recent paper [1] considered the impact of dimension-six operators on the Higgs decay rate. The theory considered was the Standard Model plus the dimension six Hamiltonian

$$\mathcal{H}^{(6)} = -\mathcal{L}^{(6)} = c_G \mathcal{O}_G + c_B \mathcal{O}_B + c_W \mathcal{O}_W + c_{WB} \mathcal{O}_{WB} \tag{1}$$

generated by new physics at some scale Λ, where

$$\begin{aligned} \mathcal{O}_G &= \frac{g_3^2}{2\Lambda^2} H^\dagger H G^A_{\mu\nu} G^{A\mu\nu}, & \mathcal{O}_B &= \frac{g_1^2}{2\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu}, \\ \mathcal{O}_W &= \frac{g_2^2}{2\Lambda^2} H^\dagger H W^a_{\mu\nu} W^{a\mu\nu}, & \mathcal{O}_{WB} &= \frac{g_1 g_2}{2\Lambda^2} H^\dagger \tau^a H W^a_{\mu\nu} B^{\mu\nu}. \end{aligned} \tag{2}$$

using the notation of Refs. [2, 3]. These operators give amplitudes which can interfere constructively or destructively with the Standard Model amplitudes for $gg \to h$, $h \to \gamma\gamma$, $h \to Z\gamma$, etc.. The phenomenology of the operators in Eq. (2), including constraints from recent LHC measurements of the Higgs decay rates, and from precision electroweak constraints, was studied in Ref. [1].

This paper constructs an exactly soluble model which generates the dimension six Higgs operators in Eq. (2), with arbitrary coefficients consistent with the effective theory power counting. It also provides an explicit realization of the Lagrangian given in the appendix of Ref. [4].

The set of all dimension-six operators in the Standard Model was classified in Ref. [5]. There are 59 independent ones after redundant operators are eliminated by the equations of motion. The operators not involving fermions are the ones listed in Eq. (2), their CP-odd partners $\widetilde{O}_G, \widetilde{O}_B, \widetilde{O}_W, \widetilde{O}_{WB}$, four pure gauge operators of which two are CP even and two are CP odd,

$$\begin{aligned} O_{G^3} &= f^{ABC} G^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}, & \widetilde{O}_{G^3} &= f^{ABC} \widetilde{G}^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}, \\ O_{W^3} &= \epsilon^{abc} W^{a\nu}_{\mu} W^{b\rho}_{\nu} W^{c\mu}_{\rho}, & \widetilde{O}_{W^3} &= \epsilon^{abc} \widetilde{W}^{a\nu}_{\mu} W^{b\rho}_{\nu} W^{c\mu}_{\rho}, \end{aligned} \tag{3}$$

and three more operators involving the Higgs field,

$$\begin{aligned} O_H &= \left(H^\dagger H\right)^3, \\ O_{H\Box} &= \left(H^\dagger H\right) \partial^2 \left(H^\dagger H\right), \\ O_{HD} &= \left(H^\dagger D_\mu H\right)^\dagger \left(H^\dagger D_\mu H\right). \end{aligned} \tag{4}$$

The exactly soluble model given here is a large-N version of that constructed in Ref. [3]. It produces the CP conserving operators in Eq. (2) with arbitrary order one coefficients, and the operator O_{W^3}, and does not generate any other dimension six operators at leading order in $1/N$.

Consider the Standard Model plus an additional scalar field S^α which is a weak $SU(2)$ doublet with hypercharge Y_S, and transforms as the N dimensional representation of an internal $SU(N)$ global symmetry. S_α is a two component column vector, and $\alpha = 1, \ldots, N$. The theory is given by

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + D_\mu S^{\dagger\alpha} D^\mu S_\alpha - V, \tag{5}$$

the usual Standard Model Lagrangian L_{SM}, the S_α kinetic energy term, and the potential

$$\begin{aligned} V = m_S^2 S^{\dagger\alpha} S_\alpha &+ \frac{\lambda_1}{N} H^\dagger H\, S^{\dagger\alpha} S_\alpha + \frac{\lambda_2}{N} H^\dagger \tau^a H\, S^{\dagger\alpha} \tau^a S_\alpha \\ &+ \frac{\lambda_3}{N} S^{\dagger\alpha} S_\alpha\, S^{\dagger\beta} S_\beta + \frac{\lambda_4}{N} S^{\dagger\alpha} \tau^a S_\alpha\, S^{\dagger\beta} \tau^a S_\beta \end{aligned} \tag{6}$$

where the Standard Model Higgs potential $\lambda \left(H^\dagger H - v^2/2\right)^2$ is part of $\mathcal{L}_{\mathrm{SM}}$. The Lagrangian is the most general renormalizable one consistent with the symmetries. Yukawa couplings of S to the Standard Model fermions are forbidden by $SU(N)$ invariance. We assume that $m_S^2 > 0$, so that $SU(N)$ is not spontaneously broken and the scalar mass m_S is larger than the electroweak scale $v \sim 246$ GeV, so that the new interactions can be treated as higher dimension operators at the electroweak scale.

The large-N limit of the theory is taken in the standard way [6]. One treats the theory in a perturbative expansion in the electroweak couplings $g \sim g_{1,2}$, i.e. one first expands in powers of g and then takes the $N \to \infty$ limit. This is the usual method of computing weak decays in QCD using the $1/N$ expansion [7].[1]

The method of Refs. [6, 8, 9] is used to solve the theory. Add to V the dimension two auxiliary fields Φ and Ψ^a which are real $SU(2)$ singlet and triplet fields with $Y = 0$,

$$V \to V - \frac{\lambda_3 N}{4}\left(\frac{2}{N}S^{\dagger\alpha}S_\alpha + \frac{m_S^2}{\lambda_3} + \frac{\lambda_1}{\lambda_3 N}H^\dagger H - \frac{\Phi}{\lambda_3}\right)^2 - \frac{\lambda_4 N}{4}\left(\frac{2}{N}S^{\dagger\alpha}\tau^a S_\alpha + \frac{\lambda_2}{\lambda_4 N}H^\dagger \tau^a H - \frac{\Psi^a}{\lambda_4}\right)^2 \tag{7}$$

The auxiliarly field equations of motion are

$$\begin{aligned}\Phi &= m_S^2 + \frac{\lambda_1}{N}H^\dagger H + + \frac{2\lambda_3}{N}S^{\dagger\alpha}S_\alpha \\ \Psi^a &= \frac{\lambda_2}{N}H^\dagger\tau^a H + \frac{2\lambda_4}{N}S^{\dagger\alpha}\tau^a S_\alpha\end{aligned} \tag{8}$$

which can be used to eliminate them and give back the original Lagrangian Eq. (6).

In weak coupling, the scalar mass m_S^2 is $\langle\Phi\rangle$. We will therefore use $\langle\Phi\rangle$ as the scale of new physics Λ in Eq. (1) and to normalize the operators in Eq. (2). Using $(H^\dagger\tau^a H)^2 = (H^\dagger H)^2$, the new potential Eq. (7) is

$$\begin{aligned}V &= \frac{N}{2\lambda_3}m_S^2\Phi - \frac{N}{4\lambda_3}\Phi^2 - \frac{N}{4\lambda_4}\Psi^a\Psi^a + \frac{\lambda_1}{2\lambda_3}H^\dagger H\Phi \\ &+ \frac{\lambda_2}{2\lambda_4}H^\dagger\tau^a H\Psi^a + \Phi S^{\dagger\alpha}S_\alpha + \Psi^a S^{\dagger\alpha}\tau^a S_\alpha - \frac{N m_S^4}{4\lambda_3} \\ &- \frac{\lambda_1}{2\lambda_3}m_S^2 H^\dagger H - \left(H^\dagger H\right)^2\left(\frac{\lambda_1^2}{4\lambda_3 N} + \frac{\lambda_2^2}{4\lambda_4 N}\right)\end{aligned} \tag{9}$$

The last term, which is subleading in $1/N$, as well as the cosmological constant term, can be dropped.

The field S_α is now integrated out. This can be done exactly in the large-N limit [6] to give an effective action which is an expansion in powers of H, Φ and Ψ^a. The Higgs field H does not couple directly to S in Eq. (9), so the S functional integral generates terms which only depend on Φ and Ψ^a. The effective action has a derivative expansion in inverse powers of m_S, which will turn into a derivative expansion in inverse powers of $\langle\Phi\rangle$. The infrared divergences are controlled by $\langle\Phi\rangle$, since the theory is in the phase where the $SU(N)$ symmetry is unbroken and the S-sector is massive.

At zero derivatives, one gets the Coleman-Weinberg effective potential [10] in the $\overline{\text{MS}}$ scheme

$$V_{\text{CW}} = \frac{1}{64\pi^2}\text{Tr}\,\left(M^2\right)^2\left[-\frac{3}{2} + \log\frac{M^2}{\mu^2}\right] \tag{10}$$

where

$$\left[M^2\right]_{ab} = \frac{\partial^2 V}{\partial\phi_a\partial\phi_b} \tag{11}$$

and $\phi_a = \text{Re}\,S_{\alpha,i}, \text{Im}\,S_{\alpha,i}$ are the scalar fields. For the interaction in Eq. (9), M^2 has eigenvalues $\Phi \pm \mathbf{\Psi}$, $\mathbf{\Psi}^2 = \Psi^a\Psi^a$, each twice, so that

$$\begin{aligned}V &= \frac{N}{2\lambda_3}\left(m_S^2 + \frac{\lambda_1}{N}H^\dagger H\right)\Phi - \frac{N}{4\lambda_3}\Phi^2 - \frac{N}{4\lambda_4}\Psi^a\Psi^a \\ &+ \frac{\lambda_2}{2\lambda_4}H^\dagger\tau^a H\Psi^a \\ &+ \sum_\pm \frac{N}{32\pi^2}\left(\Phi \pm \mathbf{\Psi}\right)^2\left[-\frac{3}{2} + \log\frac{\Phi \pm \mathbf{\Psi}}{\mu^2}\right]\end{aligned} \tag{12}$$

on integrating out the S_α field. This potential is exact in the large N limit. V is quadratic in $\mathbf{\Psi}$. From the renormalization group (RG) equation for V, one finds that

$$\Phi, \quad \Psi^a, \quad \frac{1}{\lambda_3}\left(m_S^2 + \frac{\lambda_1}{N}H^\dagger H\right), \quad \frac{\lambda_2}{\lambda_4}H^\dagger\tau^a H \tag{13}$$

are μ independent[2] and the exact β-functions are

$$\mu\frac{\mathrm{d}\lambda_3}{\mathrm{d}\mu} = \frac{\lambda_3^2}{2\pi^2}, \qquad \mu\frac{\mathrm{d}\lambda_4}{\mathrm{d}\mu} = \frac{\lambda_4^2}{2\pi^2}. \tag{14}$$

Introduce the parameters $\Lambda_{3,4}$ in place of $\lambda_{3,4}(\mu)$,

$$\frac{1}{\lambda_3(\mu)} = \frac{1}{4\pi^2}\log\frac{\Lambda_3^2}{\mu^2}, \qquad \frac{1}{\lambda_4(\mu)} = \frac{1}{4\pi^2}\log\frac{\Lambda_4^2}{\mu^2}, \tag{15}$$

which are RG invariant. They are the scales at which $\lambda_{3,4}$ have a Landau pole, and at which the scalar theory breaks down. For consistency, we need $\Lambda_{3,4} > \langle\Phi\rangle$. New physics has to enter below $\Lambda_{3,4}$ for the theory to be valid to arbitarily high energies. For example S_α could be scalar fields generated by strong dynamics, or new interactions could enter which make the scalar couplings asymptotically free. We also define [9]

$$\mathfrak{m}_S^2 = \frac{m_S^2}{\lambda_3} \tag{16}$$

which is RG invariant, from Eq. (13).

The effective action can be computed exactly in the large N limit [11–15] in a derivative expansion. The terms which generate operators with dimension ≤ 6 in the Standard Model with coefficients that are non-vanishing in the $N \to \infty$ limit are

$$\begin{aligned}\mathcal{L}_S &= \frac{N}{96\pi^2\left(\Phi\right)}\left[\partial_\mu\Phi\partial^\mu\Phi + D_\mu\Psi^a D^\mu\Psi^a\right] \\ &+ \frac{N}{384\pi^2}\left(\log\frac{\Phi}{\mu^2}\right)\left[W^a_{\mu\nu}W^{a\mu\nu} + 4Y_S^2 B_{\mu\nu}B^{\mu\nu}\right] \\ &+ \frac{N}{192\pi^2\left(\Phi\right)}Y_S\Psi^a W^a_{\mu\nu}B_{\mu\nu} + \mathcal{O}(g^3)\end{aligned} \tag{17}$$

[1] The expansion has terms of order $(g^2N)^r$, so we take the limit $g^2 \to 0$ first, followed by $N \to \infty$. Equivalently, $N \gg 1$, and $g^2N \ll 1$.

[2] RG invariance refers to the dynamics of S_α. The Standard Model fields are treated as external background fields.

where the gauge fields have been normalized so that the covariant derivative is $D_\mu = \partial_\mu + i W_\mu^a T_a + i B_\mu Y$. Φ is a gauge singlet, and has an ordinary derivative. Ψ^a is in the $I=1$ representation of $SU(2)_W$ with $Y=0$, and has a covariant derivative

$$D_\mu \Psi^a = \partial_\mu \Psi^a + i \left(T^c\right)_{ab} W_\mu^c \Psi^b, \quad \left(T^c\right)_{ab} = -i\epsilon_{cab} \tag{18}$$

It is instructive to analyze Eq. (17) at weak coupling. Let

$$\Phi = m_S^2 + \frac{4\sqrt{3}\,\pi m_S}{\sqrt{N}}\sigma, \qquad \Psi^a = \frac{4\sqrt{3}\,\pi m_S}{\sqrt{N}}\Sigma^a \tag{19}$$

The σ and Σ^a are dimension one fields with a canonically normalized kinetic energy term, and

$$\begin{aligned}\mathcal{L}_S &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}D_\mu\Sigma^a D^\mu \Sigma^a \\ &+ \frac{N}{384\pi^2}\left(\log\frac{m_S^2}{\mu^2}\right)\left[W_{\mu\nu}^a W^{a\mu\nu} + 4Y_S^2 B_{\mu\nu}B^{\mu\nu}\right] \\ &+ \frac{\sqrt{N}}{32\sqrt{3}\,\pi m_S}\sigma\left[W_{\mu\nu}^a W^{a\mu\nu} + 4Y_S^2 B_{\mu\nu}B^{\mu\nu}\right] \\ &+ \frac{\sqrt{N}Y_S}{16\sqrt{3}\,\pi m_S}\Sigma^a W_{\mu\nu}^a B_{\mu\nu} + \mathcal{O}(g^3)\,. \end{aligned}\tag{20}$$

The effective action can be computed at weak coupling from the graphs in Fig. 1, which add up to give the gauge invariant structure in Eq. (20).

The one-loop effective action generates kinetic energy terms for σ, Σ^a, given in the first line. The second line gives the threshold correction between the gauge couplings g_h in the theory above m_S and g_l in the theory below m_S,

$$\begin{aligned}-\frac{1}{4g_{l,2}^2(\mu)} &= -\frac{1}{4g_{h,2}^2(\mu)} + \frac{N}{384\pi^2}\log\frac{m_S^2}{\mu^2}, \\ -\frac{1}{4g_{l,1}^2(\mu)} &= -\frac{1}{4g_{h,1}^2(\mu)} + \frac{N}{384\pi^2}4Y_S^2\log\frac{m_S^2}{\mu^2}, \end{aligned}\tag{21}$$

for the $SU(2)$ and $U(1)$ coupling constants. The discontinuity in coupling matches the S_α contribution to the β-functions, which exists above m_S, but not below. The remaining terms are $\sigma W_{\mu\nu}^a W^{a\mu\nu}$, $\sigma B_{\mu\nu}B^{\mu\nu}$ and $\Sigma^a W_{\mu\nu}^a B_{\mu\nu}$ interactions.

The scalar potential Eq. (12) becomes

$$\begin{aligned} V &= -12\pi^2 m_S^2\left(\frac{\sigma^2}{\lambda_3} + \frac{\Sigma^a\Sigma^a}{\lambda_4}\right) \\ &+ \frac{2\sqrt{3}\,\pi}{\sqrt{N}}\left(\frac{\lambda_1}{\lambda_3}\sigma H^\dagger H + \frac{\lambda_2}{\lambda_4}\Sigma^a H^\dagger \tau^a H\right)\end{aligned}\tag{22}$$

which has mass terms for σ and Σ^a (with the wrong sign, but it does not matter, they are auxiliary fields), and couplings of σ and Σ^a to the Higgs doublet. Integrating out the auxilary fields generates the operators Eq. (2) via the graph in Fig. 2

We can now integrate out Φ, Ψ^a exactly, by doing the functional integral using the method of steepest descent [11]. The minimum $\langle\Phi\rangle$ is at

$$\Phi\log\frac{\Phi}{\mu^2} - \Phi + \frac{4\pi^2}{\lambda_3}\left(m_S^2 + \frac{\lambda_1}{N}H^\dagger H\right) - \frac{4\pi^2}{\lambda_3}\Phi = 0. \tag{23}$$

Dropping the $1/N$ term, and using Eq. (15,16), gives[3]

$$\frac{\langle\Phi\rangle}{e\Lambda_3^2}\log\frac{\langle\Phi\rangle}{e\Lambda_3^2} + \frac{4\pi^2\mathfrak{m}_S^2}{e\Lambda_3^2} = 0 \tag{24}$$

in terms of RG invariant parameters. Instead of choosing $\mathfrak{m}_S$ as the Lagrangian parameter, and solving Eq. (24) for $\langle\Phi\rangle$, we can use $\langle\Phi\rangle$ as the free parameter and then determine $\mathfrak{m}_S$ from Eq. (24). $\mathfrak{m}_S \to 0$ as $\langle\Phi\rangle \to 0$. As $\langle\Phi\rangle$ increases, so does $\mathfrak{m}_S$, and $2\pi\mathfrak{m}_S \to \Lambda_3$ as $\langle\Phi\rangle \to \Lambda_3^2$. Eq. (24) implies that $\mathfrak{m}_S$ decreases again for $\langle\Phi\rangle > \Lambda_3$[8, 9], but this is above the Landau pole, and the theory is not valid in this regime.

Evaluating the functional integral around $\langle\Phi\rangle$ gives

$$\begin{aligned}\mathcal{L}_S &= \frac{N}{384\pi^2}\left[\left(\log\frac{\langle\Phi\rangle}{\mu^2}\right)\left(W_{\mu\nu}^a W^{a\mu\nu} + 4Y_S^2 B_{\mu\nu}B^{\mu\nu}\right)\right] \\ &+ \frac{\lambda_1}{2\lambda_3}\left(\lambda_3\mathfrak{m}_S^2 - \langle\Phi\rangle\right)H^\dagger H \\ &+ \frac{\lambda_1}{96\langle\Phi\rangle\lambda_3\log\frac{\Lambda_3^2}{\langle\Phi\rangle}}H^\dagger H\left(W_{\mu\nu}^a W^{a\mu\nu} + 4Y_S^2 B_{\mu\nu}B^{\mu\nu}\right) \\ &+ \frac{\lambda_2 Y_S}{48\langle\Phi\rangle\lambda_4\log\frac{\Lambda_4^2}{\langle\Phi\rangle}}H^\dagger\tau^a H W_{\mu\nu}^a B_{\mu\nu} \\ &+ \frac{N g_2^3}{2880\pi^2\langle\Phi\rangle}\epsilon^{abc}W_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}\,. \end{aligned}\tag{25}$$

There are also terms at higher order in the derivative expansion which have not been computed here. The first term in Eq. (25) gives the strong-coupling version of the threshold correction Eq. (21). The second term is a shift in the Higgs mass proportional to the S_α mass, and can be absorbed into the v^2 term in the Higgs potential in $\mathcal{L}_{\rm SM}$.

Using $\Lambda = \langle\Phi\rangle \sim m_S^2 > v$ as the scale in Eq. (2), we see that we have generated the Standard Model Lagrangian plus the three CP-even dimension six operators in Eq. (1) with coefficients

$$\begin{aligned} c_W &= \frac{(\lambda_1/\lambda_3)}{48\log\frac{\Lambda_3^2}{\langle\Phi\rangle}}\,, \\ c_B &= \frac{(\lambda_1/\lambda_3)Y_S^2}{12\log\frac{\Lambda_3^2}{\langle\Phi\rangle}}\,, \\ c_{WB} &= \frac{(\lambda_2/\lambda_4)Y_S}{24\log\frac{\Lambda_4^2}{\langle\Phi\rangle}}\,, \end{aligned}\tag{26}$$

[3] $e = 2.71828\ldots$.

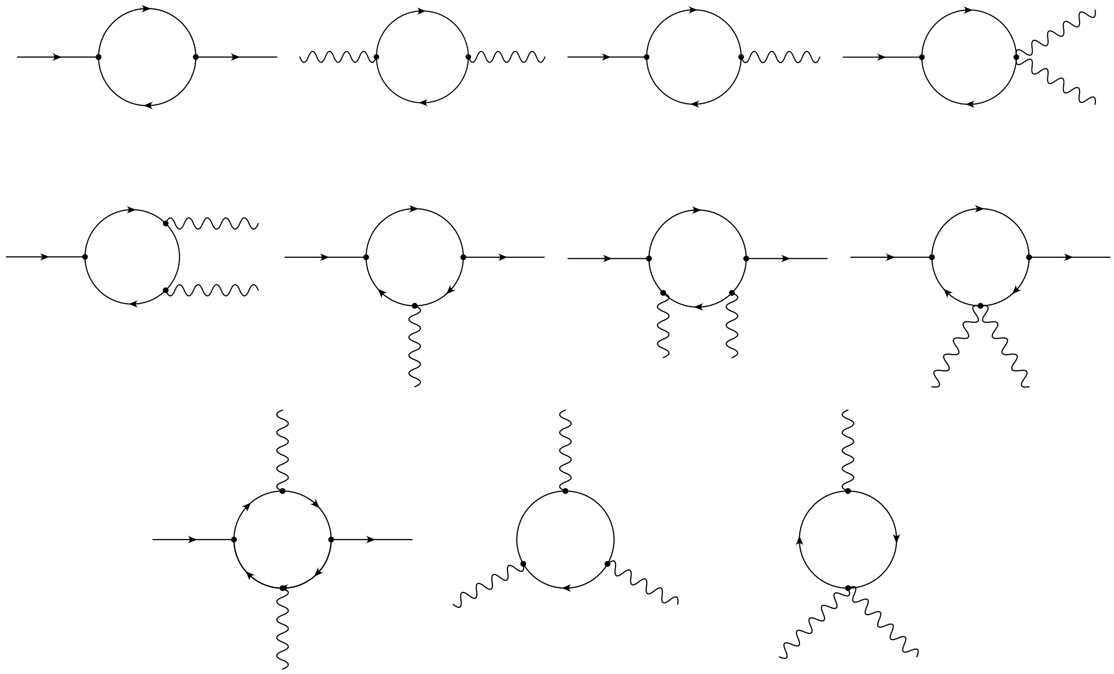

FIG. 1. Graphs contributing to the dimension six effective action. The internal lines are S_α scalar fields. The external lines are Φ, Ψ^a and gauge fields.

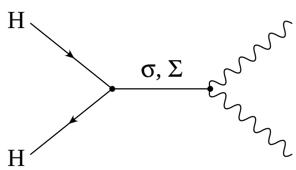

FIG. 2. Graph generating the $h \to \gamma\gamma$ amplitude.

and the O_{W^3} operator with coefficient

$$c_{W^3} = \frac{N g_2^3}{2880\pi^2 \langle \Phi \rangle} . \tag{27}$$

All other dimension six operators are subleading in $1/N$. The ratios (λ_1/λ_3) and (λ_2/λ_4) are RG invariant under S_α dynamics with the Standard Model fields treated as background fields, from Eq. (13).

The linear combinations of coefficients relevant for $h \to \gamma\gamma$ and $h \to \gamma Z$ decays are

$$\begin{aligned} c_{\gamma\gamma} &= c_W + c_B - c_{WB} , \\ c_{\gamma Z} &= c_W \cot\theta_W - c_B \tan\theta_W - c_{WB} \cot 2\theta_W . \end{aligned} \tag{28}$$

The operator c_{WB} is constrained by the S-parameter [16–19],

$$S = -8\pi^2 v^2 \frac{c_{WB}}{\Lambda^2} . \tag{29}$$

From Eq. (26), we see that we can get order unity values of c_W, c_B and c_{WB}. The phenomenology of the Higgs-gauge operators was discussed in detail in Refs. [1–3].

There is one relation that follows from Eq. (26),

$$c_B = 4 Y_S^2 c_W , \tag{30}$$

if we restrict to the model considered here with a single scalar multiplet with hypercharge Y_S. One can construct trivial generalizations of the large N model with multiple heavy scalar fields, which can have different hypercharges, and can also be colored. In this case, one can also generate the gluon term c_G, as in the octet scalar model of Ref. [3], and the $c_B - c_W$ relation no longer holds.

The large N calculation drops terms of order $1/N$, as well as higher order radiative corrections of order $g_2^2 N/(16\pi^2)$. For finite N, the neglected terms are small if $1 \ll N \ll 400$. It would be interesting to explore the full parameter space of scalar couplings and masses where the potential is stable and m_S is below the Landau pole, to determine the allowed region for c_W, c_B and c_{WB}.

The S_α interactions break custodial $SU(2)$ symmetry, since S_α is in a complex representation of $SU(N)$, and the real and imaginary parts of S_α cannot be combined to form an $O(4)$ vector, as is possible for the Higgs field. This does not affect the standard relations such as $M_W = M_Z \cos\theta_W$ that follow from custodial $SU(2)$ symmetry in the Higgs sector. Custodial $SU(2)$ symmetry violation due to S_α interactions only arise from higher dimension operators. One can also study variants of the theory with $SO(N)$ symmetry, or double the S_α fields to have a $O(4) \times SU(N)$ symmetry. In these variants, custodial $SU(2)$ can be incorporated in the S potential.

The model has threshold corrections to the gauge couplings, Eq. (21), which affects gauge unification. This was studied in Ref. [2]. In general, all theories that introduce new dynamics will modify the standard unification scenario, which has perturbative unification with a desert up to the GUT scale.

Finally, the model needs a fine tuning of order 1% to keep m_H small compared to the scale $\Lambda \sim m_S$, since there is contribution to $m_H^2 \propto m_S^2$ in Eq. (25). While not desirable, this is not worse than fine-tunings required in many models proposed to solve the hierarchy problem. The H mass term and dimension six operators have different dependence on the RG invariant parameters, so theories with additional S multiplets can cancel the m_H contribution without cancelling the Higgs-gauge operators, if the parameters satisfy

$$\sum_i \lambda_{1,i} \mathfrak{m}_{S,i}^{2} - \frac{\lambda_{1,i}}{\lambda_{3,i}} \langle \Phi_i \rangle = 0\,. \tag{31}$$

This cancellation condition is not adjusted order-by-order in perturbation theory, since Eqs. (25,31) are exact at leading order in $1/N$. The Higgs mass is then light because it is $1/N$ suppressed.

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