Higgs Decay to Two Photons at One Loop in the Standard Model Effective Field Theory.

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We present the calculation of the CP conserving contributions to $\Gamma(h \to \gamma\gamma)$, from dimension six operators at one-loop order, in the linear standard model effective field theory. We discuss the impact of these corrections on interpreting current and future experimental bounds on this decay.

I. Introduction. With the first beams circulating in the Large Hadron Collider (LHC) for Run II, and the anticipated data set that will be obtained in the near future, the utility of precise calculations of the properties of the standard model (SM) Higgs boson is clear. Improving the calculations of Higgs properties in the linear SM effective field theory (SMEFT), allows this generalization of the SM to consistently accommodate more precise constraints, or small deviations in the properties of the Higgs.

The experimental precision with which $\Gamma(h \to \gamma\gamma)$ is expected to be measured in Run II is projected to be $\lesssim 10\,\%$ with $\int \mathcal{L}\,dt = 300\,\mathrm{fb}^{-1}$ of the data [1]. This motivates us to calculate contributions to the process $\Gamma(h \to \gamma\gamma)$ due to dimension six operators at one loop. Such contributions can modify this decay at the few percent level. We loop improve the CP conserving operators in this paper, as CP odd operators do not interfere with the SM amplitude.

II. Method of calculation. We use the background field (BF) method with R_ξ gauge fixing [3–5], and we define counterterm subtractions consistent with the modified minimal subtraction renormalization scheme. The renormalization is carried out in $d = 4 - 2\epsilon$ dimensions. The gauge fixing is implemented as in Refs. [2, 7, 8]. We have explicitly checked to see that the dependence on the introduced gauge parameter cancels in the results. The Goldstone bosons of the SM Higgs doublet field, $\phi^\pm, \phi_0$, are defined through the convention

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}i\phi^+ \\ h + \bar{v}_T + \delta v + i\phi_0 \end{pmatrix}. \tag{1}$$

Here, $\bar{v}_T$ is the tree level vacuum expectation value (VEV) in the SMEFT, while δv is the one-loop contribution to the VEV as defined in Ref. [2]. The BF Ward identities are unbroken, resulting in the following relations among the SM counterterms [8]:

$$Z_A Z_e = 1, \qquad Z_h = Z_{\phi_\pm} = Z_{\phi_0},$$
$$Z_W Z_{g_2} = 1. \tag{2}$$

This method leads to several technical simplifications [2]. The Higgs boson is treated as a classical external field consistent with a narrow width approximation. Finite terms enter the calculation due to one-loop renormalization conditions fixing the asymptotic photon and Higgs states, and the gauge couplings. Our results are the one-loop expression for $\Gamma(h \to \gamma\gamma)$ in the SMEFT due to CP conserving operators, including these finite terms.

III. Dimension six operators in the decay. We follow the operator notation and basis of Ref. [10] with φ exchanged for H. The operators are normalized with dimensionful Wilson coefficients including a factor of $1/\Lambda^2$, and each gauge field strength multiplies its corresponding gauge coupling. σ^a are the weak isospin Pauli matrices and the $g_{1,2,3}$ are the SM gauge couplings. Here q, l are the left handed $\mathrm{SU}(2)_\mathrm{L}$ fields, with indices a, b, c. The $\mathrm{SU}(2)_\mathrm{L}$ generators in the covariant derivative are normalized as $\tau^a = \sigma^a/2$. Fermion fields are also labelled with the flavour index p, r, s, t. $\tilde{H}_a = \epsilon_{ab} H^{\dagger,b}$, with the $\mathrm{SU}(2)_\mathrm{L}$ invariant tensor defined by $\epsilon_{12} = 1, \epsilon_{ab} = -\epsilon_{ba}$. We canonically normalize the theory as in Ref. [11] but the corresponding $\bar{g}$ notation is suppressed in this paper. N_c is the number of colours and $Y_p = m_p \sqrt{2}/v$ is the SM Yukawa coupling. We use the normalization $\sigma_{\mu\nu} = i(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)/2$ for the anti-symmetric tensor. The bare operators are defined in terms of bare parameters with the (0) labels suppressed on the fields and couplings. In Ref. [2] the one-loop contribution due to $\mathcal{O}_{HB}^{(0)}, \mathcal{O}_{HW}^{(0)}, \mathcal{O}_{HWB}^{(0)}$

$$\begin{aligned} \mathcal{O}_{HB}^{(0)} &= g_1^2\, H^\dagger H\, B_{\mu\nu}\, B^{\mu\nu}, \\ \mathcal{O}_{HW}^{(0)} &= g_2^2\, H^\dagger H\, W_{\mu\nu}^a\, W_a^{\mu\nu}, \\ \mathcal{O}_{HWB}^{(0)} &= g_1\, g_2\, H^\dagger \sigma^a H\, B_{\mu\nu}\, W_a^{\mu\nu}, \end{aligned} \tag{3}$$

was determined, including the scale independent electroweak finite terms. Here, we complete this calculation to include the full set of operators, using the renormalization group results of Refs. [11–14]. Fig. 1 shows the diagrams that correspond to the remaining "direct" contributions (denoted as a box in the figure). Direct contributions refer to effects in the SMEFT that are not only due to a redefinition of parameters and fields. The canonically normalized effective Lagrangian with mass eigenstate fields is found as in Ref. [11].

$$\mathcal{O}_{eW}^{(0)}{}_{rs} = g_2\,\bar{l}_{r,a}\sigma^{\mu\nu}\,e_s\,\tau_{ab}^I\,H_b\,W_{\mu\nu}^I, \qquad \mathcal{O}_{eB}^{(0)}{}_{rs} = g_1\,\bar{l}_{r,a}\sigma^{\mu\nu}\,e_s\,H_a\,B_{\mu\nu}, \qquad \mathcal{O}_{uW}^{(0)}{}_{rs} = g_2\,\bar{q}_{r,a}\sigma^{\mu\nu}\,u_s\,\tau_{ab}^I\,\tilde{H}_b\,W_{\mu\nu}^I,$$

$$\mathcal{O}_{uB}^{(0)}{}_{rs} = g_1\,\bar{q}_{r,a}\sigma^{\mu\nu}\,u_s\,\tilde{H}_a\,B_{\mu\nu}, \qquad \mathcal{O}_{dW}^{(0)}{}_{rs} = g_2\,\bar{q}_{r,a}\sigma^{\mu\nu}\,d_s\,\tau_{ab}^I\,H_b\,W_{\mu\nu}^I, \qquad \mathcal{O}_{dB}^{(0)}{}_{rs} = g_1\,\bar{q}_{r,a}\sigma^{\mu\nu}\,d_s\,H_a\,B_{\mu\nu},$$

$$\mathcal{O}_{eH}^{(0)}{}_{pr} = H^\dagger H(\bar{l}_p e_r H), \qquad \mathcal{O}_{uH}^{(0)}{}_{pr} = H^\dagger H(\bar{q}_p u_r \tilde{H}), \qquad \mathcal{O}_{dH}^{(0)}{}_{pr} = H^\dagger H(\bar{q}_p d_r H),$$

$$\mathcal{O}_H^{(0)} = (H^\dagger H)^3, \qquad \mathcal{O}_{H\Box}^{(0)} = H^\dagger H \Box (H^\dagger H), \qquad \mathcal{O}_{HD}^{(0)} = (H^\dagger D_\mu H)^*(H^\dagger D^\mu H),$$

$$\mathcal{O}_W^{(0)} = g_2^3\,\epsilon_{abc}W_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}. \tag{4}$$

IV. Renormalization. The divergences present in the calculation cancel once the SM counterterms and the operator counterterms are subtracted. At one loop, the mixing effects induced are taken into account by the renormalization matrix $\mathcal{O}_i^{(0)} = Z_{i,j}\,\mathcal{O}_j^{(r)}$, introduced to cancel the divergences produced by the new operators. We use the indices i, j, k for labeling operators on the dimension six operator basis of the SMEFT. The one-loop operator counterterm matrix $\mathcal{Z}_{i,j}$ is normalized as $Z_{i,j} = \delta_{i,j} + \mathcal{Z}_{i,j}/16\,\pi^2\,\epsilon$. The 3×3 submatrix for the operators $\mathcal{O}_{HB}$, $\mathcal{O}_{HW}$, $\mathcal{O}_{HWB}$ was determined in Ref. [12] and incorporated into the full one-loop calculation in

Ref. [2]. On the basis where $\mathcal{O}_i = (\mathcal{O}_{HB}, \mathcal{O}_{HW}, \mathcal{O}_{HWB}, \mathcal{O}_W, \mathcal{O}_{eB}, \mathcal{O}_{eB}^*, \mathcal{O}_{uB}, \mathcal{O}_{uB}^*, \mathcal{O}_{dB}, \mathcal{O}_{dB}^*, \mathcal{O}_{eW}, \mathcal{O}_{eW}^*, \mathcal{O}_{uW}, \mathcal{O}_{uW}^*, \mathcal{O}_{dW}, \mathcal{O}_{dW}^*)$ and $\mathcal{O}_j = (\mathcal{O}_{HB}, \mathcal{O}_{HW}, \mathcal{O}_{HWB})$, the remaining counterterm matrix $\mathcal{Z}_{i,j}$ (with $i > 3$) that is relevant is given in Eq. 6. We define $\mathcal{N}_j = C_i\,\mathcal{Z}_{i,j}/16\pi^2\epsilon$. The contributions to the $\mathcal{N}_j$ from Eqn. 6 are denoted $\Delta\mathcal{N}_j$. These terms enter through the effective Lagrangian as a tree level counterterm

$$\mathcal{L}_{eff}^{tree} = h\,\bar{v}_T\,(\Delta\mathcal{N}_{HB} + \Delta\mathcal{N}_{HW} - \Delta\mathcal{N}_{HWB})\,e^2\,A_{\mu\nu}A^{\mu\nu} \tag{5}$$
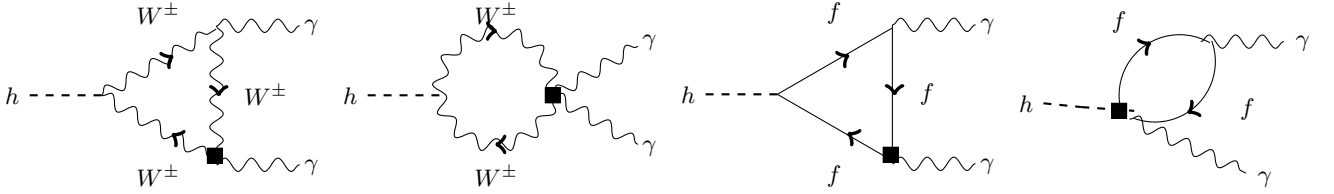

FIG. 1: Diagrams due to $O_W, O_{(e,u,d)B}, O_{(e,u,d)W} + H.c.$ Mirror graphs are not shown.

Only a subset of the operators in Eqn. 4 enter through the renormalization group evolution of these operators, which contribute at tree-level to $\Gamma(h \to \gamma\gamma)$.

$$\begin{pmatrix} 0 & -\frac{15}{2}g_2^4 & \frac{3}{2}g_2^4 \\ -(y_l + y_e)\,Y_e & 0 & -\frac{1}{2}Y_e \\ -(y_l + y_e)\,Y_e^\dagger & 0 & -\frac{1}{2}Y_e^\dagger \\ -N_c\,(y_q + y_u)\,Y_u & 0 & \frac{1}{2}N_c\,Y_u \\ -N_c\,(y_q + y_u)\,Y_u^\dagger & 0 & \frac{1}{2}N_c\,Y_u^\dagger \\ -N_c\,(y_q + y_d)\,Y_d & 0 & -\frac{1}{2}N_c\,Y_d \\ -N_c\,(y_q + y_d)\,Y_d^\dagger & 0 & -\frac{1}{2}N_c\,Y_d^\dagger \\ 0 & -\frac{1}{2}Y_e & -(y_l + y_e)\,Y_e \\ 0 & -\frac{1}{2}Y_e^\dagger & -(y_l + y_e)\,Y_e^\dagger \\ 0 & -\frac{1}{2}N_c\,Y_u & N_c\,(y_q + y_u)\,Y_u \\ 0 & -\frac{1}{2}N_c\,Y_u^\dagger & N_c\,(y_q + y_u)\,Y_u^\dagger \\ 0 & -\frac{1}{2}N_c\,Y_d & -N_c\,(y_q + y_d)\,Y_d \\ 0 & -\frac{1}{2}N_c\,Y_d^\dagger & -N_c\,(y_q + y_d)\,Y_d^\dagger \end{pmatrix}, \tag{6}$$

The resulting renormalized interactions are

$$\mathcal{L}_6^{(0)} = \left(\mathcal{N}_{HB}\,\mathcal{O}_{HB}^{(r)} + \mathcal{N}_{HW}\,\mathcal{O}_{HW}^{(r)} + \mathcal{N}_{HWB}\,\mathcal{O}_{HWB}^{(r)}\right).$$

The expression for the counterterm matrix simplifies by noting $(y_l + y_e) - 1/2 = 2\,Q_\ell$, $(y_q + y_u) + 1/2 = 2\,Q_u$, $(y_q + y_d) - 1/2 = 2\,Q_d$, and $Q_{\ell,u,d} = \{-1, 2/3, -1/3\}$. Defining $A_{\alpha\beta}^{h\,\gamma\,\gamma} = -4\,(p_a \cdot p_b\,g^{\alpha\beta} - p_a^\beta\,p_b^\alpha)$, we find

$$\frac{i\mathcal{A}_{\text{tree}}}{A_{\alpha\beta}^{h\,\gamma\,\gamma}} = \frac{-i\,e^2\,\bar{v}_T}{16\pi^2\epsilon}\left(9\,g_2^4\,C_W + 2\,Q_\ell\,(C_{eB}{}_{rs} - C_{eW}{}_{rs})\,[Y_\ell]_{sr}\right)$$

$$\frac{-i\,e^2\,\bar{v}_T\,N_c}{16\pi^2\,\epsilon}\left(2\,Q_u\,(C_{uB}{}_{rs} + C_{uW}{}_{rs})\,[Y_u]_{sr}\right) \tag{7}$$

$$\frac{-i\,e^2\,\bar{v}_T\,N_c}{16\pi^2\,\epsilon}\left(2\,Q_d\,(C_{dB}{}_{rs} - C_{dW}{}_{rs})\,[Y_d]_{sr}\right) + H.c.$$

V. Direct contributions. In Ref. [2] the one loop contributions to $\Gamma(h \to \gamma\gamma)$ due to the operators in Eqn. 3 were determined. The operators contributing directly in Fig. 1 are $\mathcal{O}_W$, $\mathcal{O}_{(e,u,d)W}^{(0)} + H.c$, $\mathcal{O}_{(e,u,d)B}^{(0)} + H.c$. Calculating the divergent part of these diagrams, we find exact cancellation with Eq. 7. The finite terms after renormalization are then obtained directly.

VI. Indirect contributions. The indirect contributions result from operators redefining fields and parameters within the SM, once the theory is expanded around the VEV. The relevant operators are $\mathcal{O}_H^{(0)}$, $\mathcal{O}_{H\square}^{(0)}$, $\mathcal{O}_{HD}^{(0)}$, and $\mathcal{O}_{(e,u,d)H}^{(0)}$. The divergences associated with these contributions are canceled by the usual SM counterterms, with a reinterpretation of the parameters present in the effective Lagrangian. The operators $\mathcal{O}_{H\square}^{(0)}$ and $\mathcal{O}_{HD}^{(0)}$ contribute to the Higgs and Goldstone boson kinetic terms. Performing the nonlinear field redefinition

$$h \to h\Big(1 + (C_{H\square} - \frac{1}{4}C_{HD})\bar{v}_T^2\Big(1 + \frac{h}{\bar{v}_T} + \frac{h^2}{3\bar{v}_T^2}\Big)\Big),$$
$$\phi_0 \to \phi_0\Big(1 + \frac{1}{3}(C_{H\square} - \frac{1}{4}C_{HD})\phi_0^2\Big),$$
$$\phi_+ \to \phi_+\Big(1 + \frac{1}{2}(C_{H\square} - \frac{1}{2}C_{HD})\phi_-\phi_+\Big),$$
$$\phi_- \to \phi_-\Big(1 + \frac{1}{2}(C_{H\square} - \frac{1}{2}C_{HD})\phi_+\phi_-\Big), \quad (8)$$

modifies the two derivative interactions in the effective Lagrangian and takes the Higgs kinetic term to canonical form. The nonlinear field redefinitions on the Goldstone fields are not required to canonically normalize the theory, but are convenient for later calculations. This redefinition effects the interactions amongst Higgs and Goldstone bosons. In the SMEFT, the SM λ coupling is shifted as

$$\lambda_{SM} \to \lambda\left(1 + \frac{3}{2\lambda}C_H\bar{v}_T^2 - 2C_{H\square}\bar{v}_T^2 + \frac{1}{2}C_{HD}\bar{v}_T^2\right).$$

The vev in the SMEFT is also redefined compared to the SM. We define the vev through the renormalization condition that the one point function of h vanishes to one loop. The tree level redefinition of the vev is $v_{SM} \to \bar{v}_T = v_{SM}\Big(1 - \frac{3}{8\lambda}C_H v_{SM}^2\Big)$. The one-loop contribution to the VEV is given by δv in Eq. (1). Finally, the effective Yukawa coupling also differs from the SM one. The canonical coupling of the Higgs to the fermion fields in the SMEFT, defined through the convention $\mathcal{L} = -h\bar{\psi}'_r[\mathcal{Y}]_{rs}P_L\psi_s + H.c.$ where $\psi' = \{u,d,e\}$ and $\psi = \{q,l\}$, is [11]

$$[\mathcal{Y}]_{rs} = \frac{[M_{\psi'}]_{rs}}{\bar{v}_T}\left[1 + \bar{v}_T^2\left(C_{H\square} - \frac{1}{4}C_{HD}\right)\right] - \frac{\bar{v}_T^2}{\sqrt{2}}C_{\psi'H}^\star_{sr}.$$

Here, $[M_{\psi'}]_{rs}$ is the mass matrix of the fermions in the SMEFT, as defined in Ref. [11]. As indirect contributions

introduce a finite renormalization (i.e., redefine with a rescaling) of the SM results, these terms are expressible in terms of the well-known functions $A_{1,1/2}$ [15–17], where

$$A_1(\tau_p) = 2 + 3\tau_p[1 + (2 - \tau_p)f(\tau_p)], \quad (9)$$
$$A_{1/2}(\tau_p) = -2\tau_p[1 + (1 - \tau_p)f(\tau_p)], \quad (10)$$

where $\tau_p = 4m_p^2/m_h^2$ and

$$f(\tau_p) = \begin{cases} \arcsin^2\sqrt{1/\tau_p}, & \tau_p \geq 1 \\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\tau_p}}{1-\sqrt{1-\tau_p}} - i\pi\right]^2, & \tau_p < 1. \end{cases} \quad (11)$$

VII. Gauge fixing effects. Gauge fixing (GF) has some subtleties in the SMEFT compared to the SM [2]. These subtleties are due to the presence of field redefinitions in the SMEFT taking the theory to canonical form. We apply the nonlinear field redefinitions to the R_ξ GF terms finding the relevant interaction terms,

$$\mathcal{L}_{GF} = -\frac{1}{2}g_2^2\xi\hat{h}\bar{v}_T^3\phi_+\phi_-\left(C_{H\square} - \frac{1}{4}C_{HD}\right) + ... \quad (12)$$

Here, $\hat{h}$ is the classical background Higgs field in the BF method. We also obtain the ghost terms from the variation of the GF terms

$$\mathcal{L}_u = -\frac{1}{2}g_2^2\xi\bar{v}_T^3\left(C_{H\square} - \frac{1}{4}C_{HD}\right)\hat{h}\,u_\pm\bar{u}_\mp \quad (13)$$

where $u_\pm$ are ghost fields. With these redefinitions, the indirect diagrams to calculate are of the same form as in the SM. We have explicitly performed this calculation with these field redefinitions imposed, finding exact cancellation of the gauge parameter as a cross-check. Note that the $W^\pm\phi^\mp\hat{A}$ SM coupling vanishes in the background field method (BFM).

VIII. One-loop $\Gamma(h \to \gamma\gamma)$ in the SMEFT. The result for the impact of CP conserving dimension six operators on the $h \to \gamma\gamma$ amplitude at one loop in the SMEFT is given as follows. We define the new physics amplitude as being composed of the individually gauge invariant components $C_{\gamma\gamma}^{1,NP}$ and f_i as[1]

$$\mathcal{A}^{NP} = \left(C_{\gamma\gamma}^{1,NP} + \frac{C_i^{NP}f_i}{16\pi^2}\right)\bar{v}_T e^2 \mathcal{A}_{\alpha\beta}^{h\gamma\gamma}. \quad (14)$$

The index i runs over all of the operators in Eqs. (3, 4) other than O_{HB} and O_H. For operators with Hermitian conjugates, the expression that appears multiplying an f_i is $\text{Re}(C_i)$, when considering the CP conserving contributions to $\Gamma(h \to \gamma\gamma)$. The coefficient $C_{\gamma\gamma}^{1,NP}$ is the

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[1] Passarino-Veltman decompositions [18] and the tools Form, FormCalc and FeynCalc [19–21] have been implemented to carry out the calculations and independent checks of the results.

one-loop improved set of Wilson coefficients that corresponds to $h \to \gamma\gamma$ at tree level. The tree level expression for this effective Wilson coefficient, in this basis, is given by

$$C_{\gamma\gamma}^{0,NP} = C_{HW} + C_{HB} - C_{HWB}. \qquad (15)$$

The tree level impact of these operators was studied in many works, with Ref. [6] being easily comparable to these results. The one-loop improvement of this coefficient is scheme dependent. The VEV has to be defined at one loop, as do the external Higgs boson (through δR_h), the photon states and the gauge couplings. The renormalization condition for the electromagnetic coupling is fixed in the Thomson limit. The finite terms at one loop in the definition of the electric charge and the photon wave function residue at the physical pole cancel [2] for this process due to the unbroken Ward identities in the BFM; see Eq. (2). This result follows from the theoretical developments in the remarkable Refs. [7–9]. Note that the gauge dependence present in $\delta R_h, \delta v$ and the remaining terms multiplying $C_{\gamma\gamma}^{0,NP}$ cancels in the sum contributing to the amplitude. The one-loop expression is [2]

$$\begin{aligned} C_{\gamma\gamma}^{1,NP} &= C_{\gamma\gamma}^{0,NP}\left(1 + \frac{\delta R_h}{2} + \frac{\delta v}{\bar{v}_T} + (\sqrt{3}\,\pi - 6)\frac{\lambda}{16\,\pi^2}\right) \\ &+ \frac{C_{\gamma\gamma}^{0,NP}}{16\,\pi^2}\left(\frac{g_1^2}{4} + \frac{3\,g_2^2}{4} + 6\,\lambda\right)\log\left(\frac{m_h^2}{\Lambda^2}\right) \qquad (16) \\ &+ \frac{C_{\gamma\gamma}^{0,NP}}{16\,\pi^2}\left(\frac{g_1^2}{4}\,\mathcal{I}[m_Z^2] + \left(\frac{g_2^2}{4} + \lambda\right)(\mathcal{I}[m_Z^2] + 2\,\mathcal{I}[m_W^2])\right). \end{aligned}$$

Here we have renormalized the theory at the scale $\mu^2 = \Lambda^2$, the cutoff scale of the SMEFT. This means that the constraints on Wilson coefficients can be directly interpreted as constraints on the underlying theory generating the operators at the scale $\mu \sim \Lambda$. The expressions for δR_h and δv are somewhat lengthy and are given directly in Ref. [2] for $\xi = 1$. The expression for $\mathcal{I}[m_p]$ for $\tau \geq 1$ is

$$\mathcal{I}[m_p] \equiv \log(\tfrac{\tau_p}{4}) + 2\sqrt{\tau_p - 1}\arctan\left(\frac{1}{\sqrt{\tau_p-1}}\right) - 2 \quad (17)$$

The remaining f_i's are as follows. We find

$$\begin{aligned} f_{HWB} &= \left(-3\,g_2^2 + 4\,\lambda\right)\log\left(\frac{m_h^2}{\Lambda^2}\right) + (4\lambda - g_2^2)\,\mathcal{I}[m_W^2] \\ &- 4\,g_2^2\,\mathcal{I}_y[m_W^2] - 2\,g_2^2\left[1 + \log\left(\frac{\tau_W}{4}\right)\right] \\ &+ e^2\,(2 + 3\,\tau_W) + 6\,e^2(2 - \tau_W)\,\mathcal{I}_y[m_W^2]. \quad (18) \end{aligned}$$

Here the expression for $\mathcal{I}_y[m_p]$ for $\tau \geq 1$ is

$$\mathcal{I}_y[m_p] \equiv \frac{\tau_p}{2}\arcsin^2(1/\sqrt{\tau_p}). \qquad (19)$$

We also find that

$$\begin{aligned} f_{HW} &= -g_2^2\left[3\,\tau_W + \left(16 - \frac{16}{\tau_W} - 6\,\tau_W\right)\mathcal{I}_y[m_W^2]\right], \\ f_W &= -9\,g_2^4\log\left(\frac{m_h^2}{\Lambda^2}\right) - 9\,g_2^4\,\mathcal{I}[m_W^2] - 6\,g_2^4\,\mathcal{I}_y[m_W^2] \\ &+ 6\,g_2^4\,\mathcal{I}_{xx}[m_W^2]\,(1 - 1/\tau_W) - 12\,g_2^4, \qquad (20) \end{aligned}$$

where, for $\tau \geq 1$,

$$\mathcal{I}_{xx}[m_p] \equiv \frac{\tau_p}{\sqrt{\tau_p-1}}\arctan\left(\frac{1}{\sqrt{\tau_p-1}}\right). \qquad (21)$$

The functions f_{HW} and f_W correspond to diagrams with only spin one SM states (in the loops). However, these contributions are not proportional to A_1, which is often termed the "spin one-loop function" for $h \to \gamma\gamma$. Such terminology is misleading when considering the effective theory generalization of the SM. The reason the loop functions differ is due to the higher derivative interactions present in the SMEFT, that are forbidden by the (usual $D \leq 4$) renormalizability of the SM. An interesting consequence of the different loop functions is that a redefinition of the W mass in the SMEFT as an input parameter is not expected to absorb the f_{HW} contribution to the amplitude. Note also that the W mass is unchanged by C_{HW} when taking the theory to canonical form, as in Ref. [11]. For the dipole leptonic operators, we find,

$$\begin{aligned} f_{eB}_{ss} &= 2\,Q_\ell\,[Y_\ell]_{ss}\left[-1 + 2\log\left(\frac{\Lambda^2}{m_h^2}\right) + \log\left(\frac{4}{\tau_s}\right)\right] \\ &- 2\,Q_\ell\,[Y_\ell]_{ss}\left[2\,\mathcal{I}_y[m_s^2] + \mathcal{I}[m_s^2]\right]. \qquad (22) \end{aligned}$$

where the dipole contributions for the quarks follow trivially. Furthermore, we also find that $f_{eW} = -f_{eB}$. Here, $s = \{1, 2, 3\}$ sums over the flavors of the leptons. The remaining dipole f_i's are obtained by obvious replacements following the structure of the terms present in Eq. (7). The indirect contributions are

$$f_{eH}_{ss} = \frac{Q_\ell^2}{2}A_{1/2}(\tau_s), \quad f_{uH}_{ss} = N_c\frac{Q_u^2}{2}A_{1/2}(\tau_s), \quad (23)$$

$$f_{dH}_{ss} = N_c\frac{Q_d^2}{2}A_{1/2}(\tau_s), \qquad (24)$$

$$\begin{aligned} f_{H\Box} &= -\frac{Q_\ell^2}{2}A_{1/2}(\tau_p) - N_c\frac{Q_u^2}{2}A_{1/2}(\tau_r), \\ &- N_c\frac{Q_d^2}{2}A_{1/2}(\tau_s) - \frac{1}{2}A_1(\tau_W), \qquad (25) \end{aligned}$$

and $f_{HD} = -f_{H\Box}/4$. Here p, r, s run over $1, 2, 3$ as flavor indices. We have checked to see that we agree with the indirect corrections also reported in Ref. [22]. Only these indirect results and $C_{\gamma\gamma}^{0,NP}$ are included in eHDECAY [23].

VIII. Numerical results. The contribution of the one-loop improved amplitude to $\Gamma(h \to \gamma\gamma)$ is through

$$\Gamma(h \to \gamma\gamma)_{\text{SMEFT}} = \Gamma(h \to \gamma\gamma)_{\text{SM}} \left| 1 - \frac{8\pi^2 \bar{v}_T \, \mathcal{A}^{NP}}{I^\gamma \, e^2 \, \mathcal{A}^{h\gamma\gamma}_{\alpha\beta}} \right|^2 \quad (26)$$

where $I^\gamma = A_1(\tau_W) + \sum_i N_c Q_i^2 A_{1/2}(\tau_i)$ is the SM contribution, and the sum over i is over all fermions. Note that at next-to-leading order, some contributions do not have the form of the SM amplitude and are "nonfactorizable"; for this reason the SM amplitude explicitly divided out of the second term above will not, in general, cancel. Experimental constraints on SMEFT parameters is a subject of intense study; however, fully general global fits of all of the coefficients that are present in $\mathcal{A}^{NP}$ do not exist. Furthermore, fits generally are determined neglecting the theoretical error in the SMEFT itself. This is a serious challenge to the consistency of constraints [24] when they rise above the percent level (on $C_i \bar{v}_T^2$), for cutoff scales in the TeV range. As such, we consider the case of unknown $C_i \sim 1$ and vary the unknown parameters over $0.8 \leq \Lambda \leq 3$ in TeV units. Note that $\bar{v}_T^2/(0.8\,\text{TeV})^2 \sim 0.1$.

The one-loop improvement of $\mathcal{A}^{NP}$ in the SMEFT can impact the interpretation of experimental constraints on the process $\Gamma(h \to \gamma\gamma)$ in an interesting manner. Taking κ_γ from Ref. [25] to be $0.93^{+0.36}_{-0.17}$, and neglecting light fermion ($m_f < m_h$) effects for simplicity, one finds the one σ range

$$-0.02 \leq \left(\hat{C}^{1,NP}_{\gamma\gamma} + \frac{\hat{C}^{NP}_i f_i}{16\pi^2} \right) \frac{\bar{v}_T^2}{\Lambda^2} \leq 0.02. \quad (27)$$

Here, the hat superscript denotes that the scale $1/\Lambda^2$ has been factored out of a Wilson coefficient. The difference in the mapping of this constraint to the coefficient of $C^{0,NP}_{\gamma\gamma}$ at tree level, and at one loop, should be corrected as if an inferred coefficient of $C^{0,NP}_{\gamma\gamma}$ is to be used in another process as a signal or constraint, or to map to an underlying model.[2] Interpreting a measured deviation from the SM, in terms of the underlying theory, also requires that this correction is taken into account. The correction proportional to $C^{0,NP}_{\gamma\gamma}$ is dominated by the scheme dependent effects defining the VEV in δv, and it is below the few percent level for Λ in the range $[0.8, 3]$ TeV.[3]

The corrections to the inferred bound on $\hat{C}^{0,NP}_{\gamma\gamma}$ due to f_i are of interest. To determine this correction we determine the percentage change on the inferred value of the bounds of $\hat{C}^{0,NP}_{\gamma\gamma}$ using Eq. (27), while shifting the quoted upper and lower bounds by the SMEFT perturbative correction. The envelope of the two percentage variations on the bounds is quoted in the form $[,]$, for values of Λ varying from $[0.8, 3]$ TeV.[4] For one specific choice of signs for C_i and $\kappa_\gamma = \frac{\Gamma(h \to \gamma\gamma)_{\text{SMEFT}}}{\Gamma(h \to \gamma\gamma)_{\text{SM}}}$, we find the results

$$[\Delta \hat{C}^{0,NP}_{\gamma\gamma}]_{HWB} \sim [9, 1] \% \times \hat{C}_{HWB}. \quad (28)$$

The effects of f_{HW} and f_W dependence on the inferred bound on $C^{0,NP}_{\gamma\gamma}$ are

$$[\Delta \hat{C}^{0,NP}_{\gamma\gamma}]_{HW} \sim [6, 0] \% \times \hat{C}_{HW}, \quad (29)$$

$$\left[\Delta \hat{C}^{0,NP}_{\gamma\gamma} \right]_W \sim [21, 2] \% \times \hat{C}_W. \quad (30)$$

The dependence on the cut off scale in f_W is $\sim \log(\Lambda^2)(1/\Lambda^2)$, while it is $\sim 1/\Lambda^2$ for f_{HW}. The impact of the indirect contributions is

$$[\Delta \hat{C}^{0,NP}_{\gamma\gamma}]_{tH} \sim [6, 0] \% \times \text{Re}(\hat{C}_{uH}{}_{33}), \quad (31)$$

$$\left[\Delta \hat{C}^{0,NP}_{\gamma\gamma} \right]_{H\square} \sim [13, 1] \% \times \hat{C}_{H\square}, \quad (32)$$

$$\left[\Delta \hat{C}^{0,NP}_{\gamma\gamma} \right]_{HD} \sim [5, 0] \% \times \hat{C}_{HD}. \quad (33)$$

Here, we have considered only the top quark coupling effects in $A_{1,1/2}$, for simplicity. Finally, consider the effect of the top dipole moment operators, which yields

$$[\Delta \hat{C}^{0,NP}_{\gamma\gamma}]_{dipole} \sim [17, 3] \% \times \left(\text{Re}(\hat{C}_{uB}{}_{33}) + \text{Re}(\hat{C}_{uW}{}_{33}) \right) (34)$$

for Λ in the range $[0.8, 3]$ TeV, falling as $\sim \log(\Lambda^2)(1/\Lambda^2)$. It is interesting to note that Ref. [28] quotes constraints on the dimensionful suppression scale of the dipole operators $\sim$ TeV, as considered here. Adding all of the f_i perturbative corrections in quadrature, assuming $\hat{C}_i \sim 1$, and further correcting the experimental bound for the scheme dependent shift $C^{1,NP}_{\gamma\gamma} \to C^{0,NP}_{\gamma\gamma}$, one finds the net impact of one-loop corrections due to higher dimensional operators on the bound of the tree level Wilson coefficient $\Delta_{\text{quad}} \hat{C}^{0,NP}_{\gamma\gamma} \sim [29, 4] \%$. Similarly, CMS reports $\kappa_\gamma = 0.98^{+0.17}_{-0.16}$ [29], which gives $\Delta_{\text{quad}} \hat{C}^{0,NP}_{\gamma\gamma} \sim [52, 7] \%$. It is possible that these corrections could add up in a manner that is not in quadrature, as this depends on the unknown C_i values. In some cases of weakly coupled and renormalizable UV scenarios, the contributions from $C_{\gamma\gamma}$, C_W, C_{HWB}, C_{tB}, and C_{tW} are all expected to be suppressed by a further loop factor, as the arguments of Ref. [30] apply. Then the indirect

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[2] Possibly through an effective bound on a pseudo-observable [26, 27]

[3] In these illustrative numerical results, we are assuming that the residual gauge dependence in δv is canceled by a one loop extraction of the vev in muon decay.

[4] The shifts underestimate the total impact of NLO corrections to relations between observables. The effect of one loop corrections in a process using a bound on $C^{0,NP}_{\gamma\gamma}$ is neglected.

contributions are expected to be the largest effects. No general statement of this form can be made for strongly interacting theories, or cases where EFTs are present in the UV [31].

We emphasize that the impact of the one-loop corrections listed above is on *current* experimental bounds of $\Gamma(h \to \gamma\gamma)$. If these effects are simply neglected, then it is necessary to add a *theoretical error for the SMEFT* to the experimental error, which can already be dominant in precisely measured processes [24]. As the experimental precision of the measurement of $\Gamma(h \to \gamma\gamma)$ increases, the impact of the neglected corrections directly scales up. Repeating the exercise above with a chosen $\kappa_\gamma^{\mathrm{proj:RunII}} = 1 \pm 0.045$, consistent with projected future bounds (CMS - scenario II [1, 32]) $(\Delta_{\mathrm{quad}} \hat{C}_{\gamma\gamma}^{0,NP})^{\mathrm{proj:RunII}} \sim [167, 21]\,\%$. High luminosity LHC runs are further quoted to have a sensitivity between 2% and 5% in κ_γ [33]. Choosing $\kappa_\gamma^{\mathrm{proj:HILHC}} = 1 \pm 0.03$, one finds $(\Delta_{\mathrm{quad}} \hat{C}_{\gamma\gamma}^{0,NP})^{\mathrm{proj:HILHC}} \sim [250, 31]\,\%$. The Snowmass working group report [33] further quotes TLEP projections with a potential future sensitivity $\kappa_\gamma^{\mathrm{proj:TLEP}} = 1 \pm 0.0145$. Performing the same exercise with this projection $(\Delta_{\mathrm{quad}} \hat{C}_{\gamma\gamma}^{0,NP})^{\mathrm{proj:TLEP}} \sim [513, 64]\,\%$.

VI. Conclusions. We have presented the one-loop result for $\Gamma(h \to \gamma\gamma)$ due to CP conserving operators of dimension six in the SMEFT. The theoretical developments reported in this Letter allow small deviations in the rate of the Higgs decay to two photons to be consistently mapped to underlying new physics models with states that have TeV mass scales. Such models are of great interest as they might provide a more compelling – and less UV sensitive – description of electroweak symmetry breaking. The results of Run I at the LHC also indicate that there is (probably) a mass gap between the electroweak scale and an UV origin of electroweak symmetry breaking, indicating that this mapping must be done if any deviation is discovered in this decay. The calculation reported here can also be used as a model to loop improve many other processes in the SMEFT. The loop corrections reported here are already relevant to precisely interpret experimental bounds on Higgs properties,

and should not be neglected in future studies as they are now known.

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