EFT and Automatic Calculation of Anomalous Dimensions in FeynRules

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ERC miniworkshop

December 5-6, 2013

Still... looking for new physics

- The Standard Model is
  in good agreement with the data
- There is no real indication
  of physics beyond the standard model
- Nevertheless, we keep on searching
- As a phenomenologist, it is hard to stay motivated
  to study only one particular theory

Effective Field Theories describe new physics in a model-independent way

- Integrating out all new physics yields higher dimensional effective operators

$$\mathcal{L} = \mathcal{L}^{(4)}_{\mathsf{SM}} + \mathcal{L}^{(6)} + \ldots$$

- Many possible dimension 6 operators
- Some of them are redundant since they can be related by the equations of motion
- There are 59 independent dim 6 operators
  assuming baryon number conservation
  and flavour indices not included [arxiv:1008.4884]

$$\mathcal{L}^{(6)} = \sum_{i=1}^{59} \frac{c_i}{\Lambda^2} \mathcal{O}_i$$

The scale dependence of the eff couplings is determined by the anomalous dimensions

- The anomalous dimension matrix γ is defined by

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{bmatrix} c_1 \\ \vdots \\ c_{59} \end{bmatrix} = \gamma \begin{bmatrix} c_1 \\ \vdots \\ c_{59} \end{bmatrix}$$

- γ important to make predictions
  - To compute the running of the couplings
  - Operator mixing

- Fully calculated by hand, dixit M.Trott...
  Partial result in [arxiv:1308.2627, 1310.4838]

We want to automatically calculate the anomalous dimension matrix using FeynRules

- Cross check of the results of [arxiv:1308.2627, 1310.4838] and their further work
- The calculation of the counterterms can be reused for Monte Carlo calculations at NLO
- First step is to calculate part of the anomalous dimension matrix by hand

Overview

- How to compute the anomalous dimensions?
- The Background Field Method
- Future Plans

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Consider a subset of operators

The Lagrangian:

$$-\mathcal{L}^{(6)} = c_B \mathcal{O}_B + c_W \mathcal{O}_W + c_{WB} \mathcal{O}_{WB}$$

Where

$$\mathcal{O}_B = \frac{g_1^2}{2\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{O}_W = \frac{g_2^2}{2\Lambda^2} H^\dagger H W^a_{\mu\nu} W^{a\mu\nu}$$

$$\mathcal{O}_{WB} = \frac{g_1 g_2}{\Lambda^2} H^\dagger \frac{\sigma^a}{2} H W^a_{\mu\nu} B^{\mu\nu}$$

Consider a subset of operators

The Lagrangian:

$$-\mathcal{L}^{(6)} = c_B \mathcal{O}_B + c_W \mathcal{O}_W + c_{WB} \mathcal{O}_{WB}$$

With anomalous dimension matrix

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{bmatrix} c_B \\ c_W \\ c_{WB} \end{bmatrix} = \gamma \begin{bmatrix} c_B \\ c_W \\ c_{WB} \end{bmatrix}$$

This anomalous dimension matrix γ has been calculated in [arXiv:1301.2588]

$$\frac{1}{16\pi^2}\begin{bmatrix} \frac{1}{2}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda + 2Y & 0 & 3g_2^2 \\ 0 & -\frac{3}{2}g_1^2 - \frac{5}{2}g_2^2 + 12\lambda + 2Y & g_1^2 \\ 2g_1^2 & 2g_2^2 & -\frac{1}{2}g_1^2 + \frac{9}{2}g_2^2 + 4\lambda + 2Y \end{bmatrix}$$

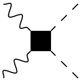
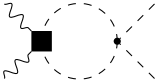
Try to reproduce de part of $\gamma_{B,B}$ proportional to λ

$$\frac{1}{16\pi^2}\begin{bmatrix} \frac{1}{2}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda + 2Y & 0 & 3g_2^2 \\ 0 & -\frac{3}{2}g_1^2 - \frac{5}{2}g_2^2 + 12\lambda + 2Y & g_1^2 \\ 2g_1^2 & 2g_2^2 & -\frac{1}{2}g_1^2 + \frac{9}{2}g_2^2 + 4\lambda + 2Y \end{bmatrix}$$

Scale dependence of c_B enters when renormalizing the Lagrangian

The bare operator contains infinities
At 1-loop, we get

$$c_B \frac{(g_1)^2}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu}$$

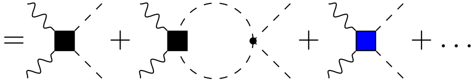
$=$  $+$  $+ \dots$

$$= \text{finite} + \frac{A}{\epsilon} + \dots$$

Scale dependence of c_B enters when renormalizing the Lagrangian

The infinities are cancelled by adding counterterms

$$
\begin{aligned}
Z^{O}_{Bj} c_j \frac{(g_1)^2}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} \\
&= (\delta_{Bj} + \delta Z^{O}_{Bj}) c_j \frac{(g_1)^2}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu}
\end{aligned}
$$



$$
= \text{finite} + \frac{A}{\epsilon} - \frac{A}{\epsilon} + \ldots
$$

δZ^{O}_{ij} depends upon the renormalization scale μ

Obtain the counterterms of the effective couplings

Renormalizing the fields and coupling constants gives

$$Z^O_{Bj} c_j \frac{(g_1)^2}{\Lambda^2} H^\dagger H B_{\mu\nu} B^{\mu\nu}$$

$$= \left[Z^c_{Bj} c_j \right] \frac{\left[Z_{g_1} g_1 \right]^2}{\Lambda^2} \left[Z_H H^\dagger H \right] \left[Z_B B_{\mu\nu} B^{\mu\nu} \right]$$

The renormalization of the effective coupling is given by

$$Z^c_{Bj} = \frac{Z^O_{Bj}}{Z^2_{g_1} Z_H Z_B}$$

Which can be used to calculate the anomalous dimension

The counterterms yield the anomalous dimensions

The counterterms are defined as

$$c_B^{\mathrm{bare}} = Z^c_{Bj} c_j = (\delta_{Bj} + \delta Z^c_{Bj} + \dots) c_j$$

From $\mu \frac{\mathrm{d} c_B^{\mathrm{bare}}}{\mathrm{d}\mu} = 0$ we get

$$\begin{aligned} \mu \frac{\mathrm{d} c_B}{\mathrm{d}\mu} &= -(Z^{-1})^c_{Bj} \mu \frac{\mathrm{d} Z^c_{jk}}{\mathrm{d}\mu} c_k \\ &= -\mu \frac{\mathrm{d}\delta Z^c_{Bj}}{\mathrm{d}\mu} c_j + \cdots \\ &= \gamma_{Bj} c_j \end{aligned}$$

Reproduce the part proportional to λ in $\gamma_{B,B}$

We want to show

$$\gamma_{B,B}\big|_{\gamma} = -\mu \frac{\mathrm{d}\,\delta Z^{c}_{B,B}\big|_{\gamma}}{\mathrm{d}\mu} = 12\lambda$$

The counterterm can be calculated from other counterterms

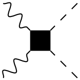
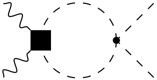
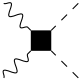
$$Z^{c}_{B,B} = \frac{Z^{O}_{B,B}}{Z^{2}_{g_1} Z_H Z_B}$$

The only relevant λ dependence comes from $Z^{O}_{B,B}$

$$\delta Z^{c}_{B,B}\big|_{\gamma} = \delta Z^{O}_{B,B}\big|_{\gamma}$$

Only one diagram to calculate

Calculation in the $\bar{M}S$ scheme using dimensional regularisation with $d = 4 - 2\epsilon$

$\delta Z^{c}_{B,B}\big|_{\gamma} \cdot$  $= -$  $= \frac{6\lambda}{16\pi^2\epsilon} \cdot$ 

From which since $\mu \frac{\mathrm{d}\lambda}{d\mu} = -2\epsilon\lambda + \dots$

$$\begin{aligned}
\gamma_{B,B}\big|_{\gamma} &= -\mu \frac{\mathrm{d}\ \delta Z^{c}_{B,B}\big|_{\gamma}}{\mathrm{d}\mu} \\
&= -\mu \frac{\mathrm{d}\lambda}{d\mu} \frac{\mathrm{d}\ \delta Z^{c}_{B,B}\big|_{\gamma}}{\mathrm{d}\lambda} \\
&= \frac{12\lambda}{16\pi^2}
\end{aligned}$$

Quod Erat Demonstrandum

$$\frac{1}{16\pi^2}\begin{bmatrix} \frac{1}{2}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda + 2Y & 0 & 3g_2^2 \\ 0 & -\frac{3}{2}g_1^2 - \frac{5}{2}g_2^2 + 12\lambda + 2Y & g_1^2 \\ 2g_1^2 & 2g_2^2 & -\frac{1}{2}g_1^2 + \frac{9}{2}g_2^2 + 4\lambda + 2Y \end{bmatrix}$$

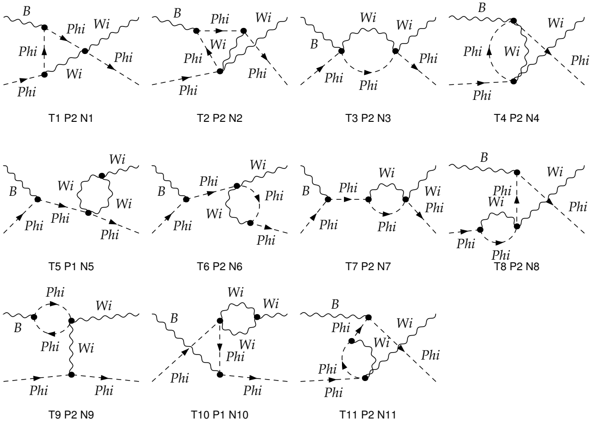
This was easy...

it gets more complicated

try $\gamma_{WB,W}$

$$\frac{1}{16\pi^2}\begin{bmatrix} \frac{1}{2}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda + 2Y & 0 & 3g_2^2 \\ 0 & -\frac{3}{2}g_1^2 - \frac{5}{2}g_2^2 + 12\lambda + 2Y & g_1^2 \\ 2g_1^2 & 2g_2^2 & -\frac{1}{2}g_1^2 + \frac{9}{2}g_2^2 + 4\lambda + 2Y \end{bmatrix}$$

To calculate $\gamma_{WB,W}$ we need e.g. $\delta Z^{O}_{WB,W}$:



We reproduced the red terms

$$\frac{1}{16\pi^2}\begin{bmatrix} \frac{1}{2}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda + 2Y & 0 & 3g_2^2 \\ 0 & -\frac{3}{2}g_1^2 - \frac{5}{2}g_2^2 + 12\lambda + 2Y & g_1^2 \\ 2g_1^2 & 2g_2^2 & -\frac{1}{2}g_1^2 + \frac{9}{2}g_2^2 + 4\lambda + 2Y \end{bmatrix}$$

Two remarks

- Loop corrections might give rise to operators which are not necessarily part of original basis
  $\Rightarrow$ *reduce to the original basis*

- Is this the best way to calculate the anomalous dimensions?

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- Future Plans

In our calculations, the counterterms are not necessarily gauge invariant

- In our calculations we *fixed the gauge* for the whole gauge field
- The *Lagrangian* we are working with is therefore *no longer gauge invariant*
- *Physical quantities* will be *gauge invariant*
- But unphysical quantities like the *counterterms* are *not necessarily gauge invariant*

Gauge invariance is partly restored in the Background Field Method

- The gauge field is split into a *background field* A^a_μ and a *quantum field* Q^a_μ
- We fix the gauge with respect to Q^a_μ
- But the Lagrangian remains *gauge invariant with respect to* A^a_μ
- *Counterterms* remain *gauge invariant*
- Due to gauge invariance, the *counterterms are related*
- This simplifies the equations and makes them less error-prone

We will compare both methods and choose the one best suited for FeynRules

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Future Plans

- I will stay at Cern until the end of February
- Analytical calculations by hand done
  and understood by Christmas
- FeynRules implementation in January - February
- Validation of the code March - ...

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In our calculations, we performed the 'usual' gauge fixing

We fixed the gauge for the whole gauge field Q^a_μ

$$Z[J] = \int \delta Q \det \left[\frac{\delta G^a}{\delta \omega^b} \right]$$
$$\exp i \int d^4x \left[\mathcal{L}(Q) - \frac{1}{2\alpha} (G^a)^2 + J^a_\mu Q^{a,\mu} \right]$$

where

$$\mathcal{L}(Q) = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}$$
$$G^a = \text{ gauge-fixing term, e.g. } G^a = \partial_\mu Q^{a,\mu}$$
$$\delta Q^a_\mu = -f^{abc} \omega^b Q^c_\mu + \frac{1}{g} \partial_\mu \omega^a$$

The Background Field Gauge partly conserves gauge invariance

We split the gauge field: $Q^a_\mu \to A^a_\mu + Q^a_\mu$
We fixed the gauge only for the quantum field Q^a_μ

$$\bar{Z}[J, A] = \int \delta Q \det \left[\frac{\delta G^a}{\delta \omega^b} \right]$$
$$\exp i \int d^4x \left[\mathcal{L}(A + Q) - \frac{1}{2\alpha} (G^a)^2 + J^a_\mu Q^{a,\mu} \right]$$

and use the background gauge

$$G^a = \partial_\mu Q^a_\mu + g f^{abc} \omega^b Q^c_\mu$$