Feynman Rules for the Standard Model Effective Field Theory in R_ξ-gauges

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Abstract

We assume that New Physics effects are parametrized within the Standard Model Effective Field Theory (SMEFT) written in a complete basis of gauge invariant operators up to dimension 6, commonly referred to as "Warsaw basis". We discuss all steps necessary to obtain a consistent transition to the spontaneously broken theory and several other important aspects, including the BRST-invariance of the SMEFT action for linear R_ξ-gauges. The final theory is expressed in a basis characterized by SM-like propagators for all physical and unphysical fields. The effect of the non-renormalizable operators appears explicitly in triple or higher multiplicity vertices. In this *mass basis* we derive the complete set of Feynman rules, without resorting to any simplifying assumptions such as baryon-, lepton-number or CP conservation. As it turns out, for most SMEFT vertices the expressions are reasonably short, with a noticeable exception of those involving 4, 5 and 6 gauge bosons. We have also supplemented our set of Feynman rules, given in an appendix here, with a publicly available *Mathematica* code working with the `FeynRules` package and producing output which can be integrated with other symbolic algebra or numerical codes for automatic SMEFT amplitude calculations.

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1 Introduction and Motivation

After the discovery of the Higgs boson, the picture of the Standard Model (SM) [1–3] being a spontaneously broken gauge theory at an Electroweak Scale (EW) with $v \sim 246$ GeV has been theoretically established and experimentally confirmed to a significant accuracy. Nevertheless, new physics beyond the SM may be hidden in the experimental errors of measurements that are becoming increasingly accurate at the LHC. Such phenomena can be parametrized in terms of the so-called SM Effective Field Theory (SMEFT) [4–6][1], where, assuming Λ to be the typical energy scale of the SM extension, the observable effects are suppressed by powers of the expansion parameter v/Λ. The SM's weak response to a more fundamental theory (effective or not) living at Λ may be due to the fact that such a scale is far above the EW scale *i.e.,* $\Lambda \gg v$, or because non-renormalizable, UV-dependent couplings are, somehow, small.

Besides the verification of the SM gauge group and content, a renewed interest in the SMEFT arises from the fairly recent completion of all gauge invariant, independent, (mass) dimension-6 operators, first conducted in a study by Buchmüller and Wyler [10] in 1985 and lately amended by the Warsaw university group [11] in 2010. We shall refer to this set of operators as the "Warsaw" basis. In this basis there are 59+1 baryon-number conserving[2] and 4 baryon-number violating operators.

If physics beyond the SM lies not too far from the EW scale, so that is invisible, but also not too close to the EW scale, so that the effective field theory description (EFT) does not fail, then SMEFT observables should encode possible deviations from the SM to order $(v/\Lambda)^2$ no matter what the fundamental (UV) theory is. A serious attempt in calculating such observables should start by first writing down the Feynman rules for propagators and vertices for physical fields, after spontaneous symmetry breaking (SSB) of the effective theory, in a way that consistently renders the theory renormalizable in the "modern" sense - here of absorbing infinities into a finite number of counterterms up to order $(v/\Lambda)^2$. One major criterion for this to be realized is that the gauge boson propagators vanish for momenta $p \to \infty$ as p^{-2} so that the theory satisfies usual power counting rules for renormalizability, as in the SM for example. In 1971, 't Hooft [13] and B. Lee [14] showed that this can be realized in a linear gauge which a year later extended to a larger class of renormalizable gauges by Fujikawa, Lee, Sanda [15], and Yao [16]. This class of renormalizable gauges, called R_ξ-gauges, can be parametrized by one or more arbitrary constants, collectively written as ξ. In addition to the smooth behavior of the propagators, R_ξ-gauges allow for eliminating "unwanted" mixed terms between physical gauge bosons and unphysical (Goldstone) scalar fields in spontaneously broken gauge theories.

To the best of our knowledge, quantization of SMEFT in linear R_ξ-gauges does not exist in the literature thus far. What complicates the picture of quantization in R_ξ-gauges, or as a matter of fact in every other class of gauges, is twofold: a) field redefinitions and reparametrizations and b) mixed field strength operators. A careful treatment of the former to retain gauge invariance is necessary [17] while properly rotating away (but not completely eliminating from vertices) the latter, results in SM-like propagators for physical and unphysical fields. More specifically, in this paper we consider SSB of the "Warsaw" basis theory and present a full set of Feynman rules in R_ξ-gauge in a *mass basis*, with the following features:

[1] For reviews see refs. [7–9].

[2] In counting, we include the lepton-number $d = 5$ violating operator [12] but do not count hermitian conjugated operators and suppress fermion flavor dependence.

- No restriction is made for the structure of flavor violating terms and for CP-, lepton- or baryon-number conservation,
- SMEFT is quantized in R_ξ-gauges written with four different arbitrary gauge parameters, $\xi_\gamma, \xi_Z, \xi_W, \xi_G$ for better cross checks of physical amplitudes.
- Gauge fixing and ghost part of the Lagrangian is chosen to be SM-like and preserve Becchi, Rouet, Stora [18], and Tyutin [19] (BRST) invariance.
- All bilinear terms in the Lagrangian have canonical form, both for physical and unphysical Goldstone and ghost fields; all propagators are diagonal and SM-like.
- Feynman rules for interactions are expressed in terms of physical SM fields and canonical Goldstone and ghost fields.

We are aware that in the literature there are many calculations done already within SMEFT, including several articles with loop calculations usually performed in unitary or non-linear gauges, see for example ref. [9] and references therein. However, we think that a full set of Feynman rules written (and coded in the symbolic computer program) in the R_ξ-gauges, including in addition the most general structure of the flavor violating terms, is something that can largely simplify further such analyses. Especially, having such collection is useful because the number of primary vertices in SMEFT in R_ξ-gauges is huge: 383 without counting the hermitian conjugates (surprisingly, for most SMEFT vertices the Feynman rules are reasonably short, with an exception of self-interactions of 4, 5 and 6 gauge bosons). An explicit diagrammatic representation for all interaction vertices will minimize possible mistakes that arise from missing terms or even entire diagrams in amplitude calculations. Furthermore, implementation of them as a "model file" to the `FeynRules` package [20] produces an output ready to be further used in symbolic or numeric programs for amplitude calculations.

The procedure we followed in deriving the SMEFT Feynman rules consists of the following steps[3]:

1. within the "Warsaw" basis, given for reference in section 2, we perform the SSB mechanism and further field and coupling rescalings with constant parameters which have no effect on the S-matrix elements (up to $\mathcal{O}(\Lambda^{-3})$ corrections). They make all bilinear terms of gauge, Higgs and fermion fields canonical [section 3],
2. we discuss "oblique" corrections to the SM vertices, coming from the constant field and coupling redefinitions when moving from weak to mass basis [section 4],
3. we introduce suitable R_ξ-gauge fixing and ghost terms in the Lagrangian, in a way that renders also the ghost propagators diagonal. The new terms eliminate the "unwanted" gauge-Goldstone mixing and establish BRST invariance. Thus, in the mass basis of SMEFT all quadratic terms of physical (SM particles) and unphysical (Goldstone bosons and ghosts) become SM-like [section 5],
4. we evaluate Feynman rules for all sectors of the theory [Appendix A].

[3]Steps 1 and 2 have been discussed in numerous earlier papers e.g., ref. [21], but we include them here for completeness and consistency.

	fermions					scalars
field	$l'^{j}_{Lp} = \begin{pmatrix} \nu'_{Lp} \\ e'_{Lp} \end{pmatrix}$	e'_{Rp}	$q'^{\alpha j}_{Lp} = \begin{pmatrix} u'^{\alpha}_{Lp} \\ d'^{\alpha}_{Lp} \end{pmatrix}$	u'^{α}_{Rp}	d'^{α}_{Rp}	$\varphi^j = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$
hypercharge Y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Table 1: The SM matter content in the gauge basis. Isospin, colour and generation indices are indicated with $j = 1, 2$, $\alpha = 1...3$ and $p = 1...3$, respectively.

Then, in section 6 and in Appendix B we describe the features of the SMEFT model file for `FeynRules` package and a set of programs generating automatically relevant Feynman rules, both in *Mathematica* and Latex/axodraw format. Our code is publicly accessible from `www.fuw.edu.pl/smeft`. We conclude in section 7. The full list of Feynman rules in R_ξ-gauges is collected in Appendix A.

2 Notation and conventions for the SMEFT Lagrangian

Throughout this article we use the notation and conventions of ref. [11]. However, in order to distinguish between the fields and parameters of the initial, gauge basis and the final, mass basis, we use *primed* notation for fermion fields and their Wilson coefficients in the former, reserving the *"unprimed"* symbols for the physical mass eigenstates basis, where flavor space rotations have been performed. In addition, and not to clutter the notation further as compared to ref. [11], we absorb the theory cut-off scale Λ in the definitions of Wilson coefficients, rescaling them appropriately as $C_X^{(5)}/\Lambda \to C_X^{(5)}$, $C_X^{(6)}/\Lambda^2 \to C_X^{(6)}$.

For completeness and reference, in Tables 2 and 3 we list all, gauge independent, dimension-6 operators of the "Warsaw" basis derived in ref. [11]. The only dimension-5 operator, the lepton-number violating operator [12], reads

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m(l'^k_{Lp})^T\,\mathbb{C}\,l'^n_{Lr} \;\equiv\; (\widetilde{\varphi}^\dagger l'_{Lp})^T\,\mathbb{C}\,(\widetilde{\varphi}^\dagger l'_{Lr})\,, \tag{2.1}$$

where $\mathbb{C}$ is the charge conjugation matrix in notation of ref. [11]. Then the full gauge invariant Lagrangian, up to $\mathcal{O}(\Lambda^{-3})$, takes the form

$$\mathcal{L} = \mathcal{L}^{(4)}_{\rm SM} + C^{\nu\nu}Q^{(5)}_{\nu\nu} + \sum_X C^X Q^{(6)}_X + \sum_f C'^f Q^{(6)}_f\,, \tag{2.2}$$

where $Q^{(6)}_X$ denotes dimension-6 operators that do not involve fermion fields, *i.e.,* operators entitled as $X^3, \varphi^6, \varphi^4D^2, X^2\varphi^2$ columns of Table 2, while $Q^{(6)}_f$ denotes operators that contain fermion fields among other fields *i.e.,* all other operators in Tables 2 and 3. The renormalizable part of the Lagrangian is (we suppress generation indices here),

$$\begin{aligned}\mathcal{L}^{(4)}_{SM} = &-\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) + m^2\varphi^\dagger\varphi - \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2 \\ &+\; i(\bar{l}'_L \not{D} l'_L + \bar{e}'_R \not{D} e'_R + \bar{q}'_L \not{D} q'_L + \bar{u}'_R \not{D} u'_R + \bar{d}'_R \not{D} d'_R) \\ &-\; (\bar{l}'_L\Gamma_e e'_R\varphi + \bar{q}'_L\Gamma_u u'_R\widetilde{\varphi} + \bar{q}'_L\Gamma_d d'_L\varphi)\,.\end{aligned} \tag{2.3}$$

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}'_p e'_r\varphi)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}'_p u'_r\widetilde{\varphi})$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^*\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}'_p d'_r\varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu}G^{A\mu\nu}$	Q_{eW}	$(\bar{l}'_p\sigma^{\mu\nu}e'_r)\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}'_p\gamma^\mu l'_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\,\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	Q_{eB}	$(\bar{l}'_p\sigma^{\mu\nu}e'_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{l}'_p\tau^I\gamma^\mu l'_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu}W^{I\mu\nu}$	Q_{uG}	$(\bar{q}'_p\sigma^{\mu\nu}\mathcal{T}^A u'_r)\widetilde{\varphi}\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}'_p\gamma^\mu e'_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\,\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$	Q_{uW}	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\tau^I\widetilde{\varphi}\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}'_p\gamma^\mu q'_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{q}'_p\tau^I\gamma^\mu q'_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\,\widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}'_p\sigma^{\mu\nu}\mathcal{T}^A d'_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}'_p\gamma^\mu u'_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{q}'_p\sigma^{\mu\nu}d'_r)\tau^I\varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}'_p\gamma^\mu d'_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\,\widetilde{W}^I_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{q}'_p\sigma^{\mu\nu}d'_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}'_p\gamma^\mu d'_r)$

Table 2: Dimension-6 operators other than the four-fermion ones (from ref. [11]). For brevity we suppress fermion chiral indices L, R.

As compared to ref. [11] we slightly change the notation for the gauge group generators while keeping all other conventions identical. The covariant derivative then reads,

$$D_\mu = \partial_\mu + ig'B_\mu Y + igW^I_\mu T^I + ig_s G^A_\mu \mathcal{T}^A\,, \tag{2.4}$$

where the weak hypercharge Y assigned to the fields is given in Table 1. In fundamental representation, the generators for $SU(2)$ read $T^I = \tau^I/2$ with τ^I (I=1,2,3) being the Pauli matrices and for $SU(3)$ read $\mathcal{T}^A = \lambda^A/2$ with λ^A (A=1,...,8) being the Gell-Mann matrices. The field strength tensors are:

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_s f^{ABC}G^B_\mu G^C_\nu\,, \tag{2.5}$$

$$W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu - g\epsilon^{IJK}W^J_\mu W^K_\nu\,, \tag{2.6}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,. \tag{2.7}$$

Finally, we consider the SMEFT accurate up to $\mathcal{O}(\Lambda^{-3})$ corrections and therefore all relations obtained within it are accurate up to this level of approximation. We will implicitly make use of this property in our derivations without making any further notice.

3 Mass eigenstates basis in SMEFT

As usual, in order to identify physical (and unphysical) degrees of freedom in the presence of SSB, one needs to diagonalize the resulting mass matrices for all fields. However, in

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}'_p\gamma_\mu l'_r)(\bar{l}'_s\gamma^\mu l'_t)$	Q_{ee}	$(\bar{e}'_p\gamma_\mu e'_r)(\bar{e}'_s\gamma^\mu e'_t)$	Q_{le}	$(\bar{l}'_p\gamma_\mu l'_r)(\bar{e}'_s\gamma^\mu e'_t)$
$Q_{qq}^{(1)}$	$(\bar{q}'_p\gamma_\mu q'_r)(\bar{q}'_s\gamma^\mu q'_t)$	Q_{uu}	$(\bar{u}'_p\gamma_\mu u'_r)(\bar{u}'_s\gamma^\mu u'_t)$	Q_{lu}	$(\bar{l}'_p\gamma_\mu l'_r)(\bar{u}'_s\gamma^\mu u'_t)$
$Q_{qq}^{(3)}$	$(\bar{q}'_p\gamma_\mu\tau^I q'_r)(\bar{q}'_s\gamma^\mu\tau^I q'_t)$	Q_{dd}	$(\bar{d}'_p\gamma_\mu d'_r)(\bar{d}'_s\gamma^\mu d'_t)$	Q_{ld}	$(\bar{l}'_p\gamma_\mu l'_r)(\bar{d}'_s\gamma^\mu d'_t)$
$Q_{lq}^{(1)}$	$(\bar{l}'_p\gamma_\mu l'_r)(\bar{q}'_s\gamma^\mu q'_t)$	Q_{eu}	$(\bar{e}'_p\gamma_\mu e'_r)(\bar{u}'_s\gamma^\mu u'_t)$	Q_{qe}	$(\bar{q}'_p\gamma_\mu q'_r)(\bar{e}'_s\gamma^\mu e'_t)$
$Q_{lq}^{(3)}$	$(\bar{l}'_p\gamma_\mu\tau^I l'_r)(\bar{q}'_s\gamma^\mu\tau^I q'_t)$	Q_{ed}	$(\bar{e}'_p\gamma_\mu e'_r)(\bar{d}'_s\gamma^\mu d'_t)$	$Q_{qu}^{(1)}$	$(\bar{q}'_p\gamma_\mu q'_r)(\bar{u}'_s\gamma^\mu u'_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}'_p\gamma_\mu u'_r)(\bar{d}'_s\gamma^\mu d'_t)$	$Q_{qu}^{(8)}$	$(\bar{q}'_p\gamma_\mu\mathcal{T}^A q'_r)(\bar{u}'_s\gamma^\mu\mathcal{T}^A u'_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}'_p\gamma_\mu\mathcal{T}^A u'_r)(\bar{d}'_s\gamma^\mu\mathcal{T}^A d'_t)$	$Q_{qd}^{(1)}$	$(\bar{q}'_p\gamma_\mu q'_r)(\bar{d}'_s\gamma^\mu d'_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}'_p\gamma_\mu\mathcal{T}^A q'_r)(\bar{d}'_s\gamma^\mu\mathcal{T}^A d'_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}'^j_p e'_r)(\bar{d}'_s q'^j_t)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d'^\alpha_p)^T\mathbb{C}u'^\beta_r\right]\left[(q'^{\gamma j}_s)^T\mathbb{C}l'^k_t\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}'^j_p u'_r)\varepsilon_{jk}(\bar{q}'^k_s d'_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q'^{\alpha j}_p)^T\mathbb{C}q'^{\beta k}_r\right]\left[(u'^\gamma_s)^T\mathbb{C}e'_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}'^j_p\mathcal{T}^A u'_r)\varepsilon_{jk}(\bar{q}'^k_s\mathcal{T}^A d'_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q'^{\alpha j}_p)^T\mathbb{C}q'^{\beta k}_r\right]\left[(q'^{\gamma m}_s)^T\mathbb{C}l'^n_t\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}'^j_p e'_r)\varepsilon_{jk}(\bar{q}'^k_s u'_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d'^\alpha_p)^T\mathbb{C}u'^\beta_r\right]\left[(u'^\gamma_s)^T\mathbb{C}e'_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}'^j_p\sigma_{\mu\nu}e'_r)\varepsilon_{jk}(\bar{q}'^k_s\sigma^{\mu\nu}u'_t)$		

Table 3: Four-fermion operators (from ref. [11]). For brevity we suppress fermion chiral indices L, R.

SMEFT there is an extra intermediate step involving field rescalings, since SSB also affects the canonical normalization of the kinetic terms. In the following sections we discuss this procedure step by step.

3.1 Higgs mechanism

The relevant operator terms contributing to the Higgs potential are

$$\begin{aligned}\mathcal{L}_{\rm H} &= (D_\mu\varphi)^\dagger(D^\mu\varphi) + m^2(\varphi^\dagger\varphi) - \frac{\lambda}{2}(\varphi^\dagger\varphi)^2 \\ &+ C^\varphi(\varphi^\dagger\varphi)^3 + C^{\varphi\Box}(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi) + C^{\varphi D}(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi)\,. \end{aligned} \tag{3.1}$$

Minimization of the potential results in a "corrected" vacuum expectation value (vev), which reads [21],

$$v = \sqrt{\frac{2m^2}{\lambda}} + \frac{3m^3}{\sqrt{2}\lambda^{5/2}}C^\varphi\,. \tag{3.2}$$

Notice that in all our expressions and Feynman rules that follow we use only this vev. As usual, we next expand the Higgs doublet field around the vacuum,

$$\varphi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\Phi^0) \end{pmatrix} . \tag{3.3}$$

The Lagrangian bilinear terms of the scalar fields are then given by,

$$\begin{aligned} \mathcal{L}_H^{\text{Bilinear}} &= \frac{1}{2}\left(1 + \frac{1}{2}C^{\varphi D}v^2 - 2C^{\varphi\Box}v^2\right)(\partial_\mu H)^2 + \left(\frac{1}{2}m^2 - \frac{3}{4}\lambda v^2 + \frac{15}{8}v^4 C^{\varphi}\right)H^2 \\ &+ \frac{1}{2}\left(1 + \frac{1}{2}C^{\varphi D}v^2\right)(\partial_\mu \Phi^0)^2 + (\partial_\mu \Phi^-)(\partial^\mu \Phi^+). \end{aligned} \tag{3.4}$$

By rescaling the fields as

$$h = Z_h\, H\ , \qquad G^0 = Z_{G^0}\, \Phi^0\ , \qquad G^\pm \equiv \Phi^\pm\ , \tag{3.5}$$

with the constant factors

$$Z_h \equiv 1 + \frac{1}{4}C^{\varphi D}v^2 - C^{\varphi\Box}v^2\ , \tag{3.6}$$

$$Z_{G^0} \equiv 1 + \frac{1}{4}C^{\varphi D}v^2\ , \tag{3.7}$$

one obtains the physical Higgs field h and Goldstone fields $G^0, G^\pm$ with canonically normalized kinetic terms. The tree-level squared mass of the normalized Higgs field h now reads,

$$\begin{aligned} M_h^2 &= 2m^2\left[1 - \frac{m^2}{\lambda^2}\left(3\,C^{\varphi} - 4\,\lambda\,C^{\varphi\Box} + \lambda\,C^{\varphi D}\right)\right] \\ &= \lambda v^2 - (3\,C^{\varphi} - 2\,\lambda\,C^{\varphi\Box} + \frac{\lambda}{2}\,C^{\varphi D})v^4\ . \end{aligned} \tag{3.8}$$

3.2 The gauge sector

The Lagrangian terms which are relevant for gauge boson propagators read,

$$\begin{aligned} \mathcal{L}_{\text{EW}} = &-\frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) \\ &+ C^{\varphi W}(\varphi^\dagger\varphi)W^I_{\mu\nu}W^{I\mu\nu} + C^{\varphi B}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu} + C^{\varphi WB}(\varphi^\dagger\tau^I\varphi)W^I_{\mu\nu}B^{\mu\nu} \\ &+ C^{\varphi D}(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi)\ , \end{aligned} \tag{3.9}$$

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} + C^{\varphi G}(\varphi^\dagger\varphi)G^A_{\mu\nu}G^{A\mu\nu}\ , \tag{3.10}$$

where τ^I are the Pauli matrices. Other, potentially relevant operators of the theory, containing $\widetilde{B}_{\mu\nu}$, $\widetilde{W}^I_{\mu\nu}$ and $\widetilde{G}^A_{\mu\nu}$ influence only CP-violating vertices. Their bilinear terms are total derivatives and do not affect propagators. Therefore, we neglect them in our discussion here.

To simplify the above expressions, it is convenient to introduce "barred" fields and couplings, such as

$$\begin{aligned} \bar{W}^I_\mu &\equiv Z_g W^I_\mu, & \bar{g} &\equiv Z_g^{-1} g\ , \\ \bar{B}_\mu &\equiv Z_{g'} B_\mu\ , & \bar{g}' &\equiv Z_{g'}^{-1} g', \\ \bar{G}^A_\mu &\equiv Z_{g_s} G^A_\mu, & \bar{g}_s &\equiv Z_{g_s}^{-1} g_s\ , \end{aligned}$$

where for our constant, field and coupling rescalings, we choose

$$\begin{aligned} Z_g &\equiv 1 - C^{\varphi W} v^2 \,, \\ Z_{g'} &\equiv 1 - C^{\varphi B} v^2 \,, \\ Z_{g_s} &\equiv 1 - C^{\varphi G} v^2 \,. \end{aligned} \tag{3.11}$$

We note that such transformations do not violate gauge invariance. They preserve the form of the covariant derivative which now reads,

$$D_\mu = \bar{D}_\mu = \partial_\mu + i\bar{g}\bar{B}_\mu Y + i\bar{g}\bar{W}^I_\mu T^I + i\bar{g}_s \bar{G}^A_\mu \mathcal{T}^A \,, \tag{3.12}$$

while the field strength tensors rescale the same way as their respective fields. The particular choice of eq. (3.11) renders the kinetic terms for the electroweak fields canonical, with an exception of the mixed $Q_{\varphi WB}$ operator in eq. (3.9). Furthermore, the last redefinition of eq. (3.11) is sufficient to define massless physical, canonically normalized gluon fields, as

$$g^A_\mu \equiv \bar{G}^A_\mu \,. \tag{3.13}$$

In terms of "barred" electroweak gauge bosons, $\bar{B}_\mu$ and $\bar{W}_\mu$, the bilinear part of the Lagrangian reads,

$$\begin{aligned} \mathcal{L}^{\text{Bilinear}}_{EW} &= -\frac{1}{4}(\bar{W}^1_{\mu\nu}\bar{W}^{1\mu\nu} + \bar{W}^2_{\mu\nu}\bar{W}^{2\mu\nu}) - \frac{1}{4}\begin{pmatrix} \bar{W}^3_{\mu\nu} \\ \bar{B}_{\mu\nu} \end{pmatrix}^\top \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \begin{pmatrix} \bar{W}^{3\mu\nu} \\ \bar{B}^{\mu\nu} \end{pmatrix} \\ &\quad + \frac{\bar{g}^2 v^2}{8}(\bar{W}^1_\mu \bar{W}^{1\mu} + \bar{W}^2_\mu \bar{W}^{2\mu}) \\ &\quad + \frac{v^2}{8} Z^2_{G^0} \begin{pmatrix} \bar{W}^3_\mu \\ \bar{B}_\mu \end{pmatrix}^\top \begin{pmatrix} \bar{g}^2 & -\bar{g}\bar{g}' \\ -\bar{g}\bar{g}' & \bar{g}'^2 \end{pmatrix} \begin{pmatrix} \bar{W}^{3\mu} \\ \bar{B}^\mu \end{pmatrix} \,, \end{aligned} \tag{3.14}$$

where we have defined,

$$\epsilon \equiv C^{\varphi WB}\, v^2 \,. \tag{3.15}$$

From eq. (3.14) one identifies immediately the physical charged gauge bosons $W^\pm_\mu$, as

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(\bar{W}^1_\mu \mp i\bar{W}^2_\mu) \,, \tag{3.16}$$

with the mass

$$M_W = \frac{1}{2}\,\bar{g}\,v \,. \tag{3.17}$$

The neutral gauge boson mass basis is obtained through the congruent matrix transformation [22], producing simultaneously canonical kinetic terms and diagonal masses. It reads,

$$\begin{pmatrix} \bar{W}^3_\mu \\ \bar{B}_\mu \end{pmatrix} = \mathbb{X} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \,, \tag{3.18}$$

with the matrix $\mathbb{X}$ taking the form,

$$\mathbb{X} = \begin{pmatrix} 1 & -\frac{\epsilon}{2} \\ -\frac{\epsilon}{2} & 1 \end{pmatrix} \begin{pmatrix} \cos\bar{\theta} & \sin\bar{\theta} \\ -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix} \,. \tag{3.19}$$

Straightforward calculation leads to a mixing angle [21, 23]

$$\tan\bar{\theta} = \frac{\bar{g}'}{\bar{g}} + \frac{\epsilon}{2}\left(1 - \frac{\bar{g}'^2}{\bar{g}^2}\right) , \tag{3.20}$$

whereas for gauge boson masses we obtain

$$\begin{aligned} M_Z &= \frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}\, v \left(1 + \frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}\right) Z_{G^0} , \\ M_A &= 0 . \end{aligned} \tag{3.21}$$

One can easily verify that the photon remains massless from the vanishing determinant of the mass matrix in eq. (3.14). Note also that the $\mathbb{X}$ transformation affects the trace of this matrix, thus producing the ϵ-dependence for M_Z.

3.3 Gauge-Goldstone mixing

The operators relevant for Goldstone bosons kinetic terms give also rise to Goldstone-gauge boson mixing. They read,

$$\mathcal{L}_H \supset (\bar{D}_\mu\varphi)^\dagger(\bar{D}^\mu\varphi) + C^{\varphi D}(\varphi^\dagger\bar{D}_\mu\varphi)^*(\varphi^\dagger\bar{D}^\mu\varphi) , \tag{3.22}$$

which, in the presence of SSB, generate the "unwanted" terms

$$\begin{aligned} \mathcal{L}_{G-EW} &= -i\frac{\bar{g}v}{2\sqrt{2}}\bar{W}^1_\mu\Big(\partial^\mu\Phi^+ - \partial^\mu\Phi^-\Big) + \frac{\bar{g}v}{2\sqrt{2}}\bar{W}^2_\mu\Big(\partial^\mu\Phi^+ + \partial^\mu\Phi^-\Big) \\ &\quad - \frac{\bar{g}v}{2}Z^2_{G_0}\bar{W}^3_\mu\,\partial^\mu\Phi^0 + \frac{\bar{g}'v}{2}Z^2_{G_0}\bar{B}_\mu\,\partial^\mu\Phi^0 . \end{aligned} \tag{3.23}$$

After expressing $\mathcal{L}_{G-EW}$ in terms of the physical gauge bosons and Goldstone bosons, one arrives to the familiar expression,

$$\mathcal{L}_{G-EW} = iM_W(W^+_\mu\partial^\mu G^- - W^-_\mu\partial^\mu G^+) - M_Z\, Z_\mu\,\partial^\mu G^0 . \tag{3.24}$$

Thus, in mass basis all Wilson coefficients in the bilinear gauge-Goldstone mixing have been absorbed in the definitions of fields and masses. As we discuss in Section 5, such a property essentially allows to adopt the standard R_ξ-gauge fixing also for SMEFT loop calculations.

3.4 Fermion sector

The operators relevant to fermion masses are

$$\begin{aligned} \mathcal{L}_f &= i(\bar{l}'_L\,\not{D}\,l'_L + \bar{e}'_R\,\not{D}\,e'_R + \bar{q}'_L\,\not{D}\,q'_L + \bar{u}'_R\,\not{D}\,u'_R + \bar{d}'_R\,\not{D}\,d'_R) \\ &\quad - (\bar{l}'_L\,\Gamma_e\,e'_R\,\varphi + \bar{q}'_L\,\Gamma_u\,u'_R\,\widetilde{\varphi} + \bar{q}'_L\,\Gamma_d\,d'_R\,\varphi + \text{H.c.}) \\ &\quad + \Big[(\varphi^\dagger\varphi)\,(\bar{l}'_L\,C'^{e\varphi}\,e'_R\,\varphi) + (\varphi^\dagger\varphi)\,(\bar{q}'_L\,C'^{u\varphi}\,u'_R\,\widetilde{\varphi}) + (\varphi^\dagger\varphi)\,(\bar{q}'_L\,C'^{d\varphi}\,d'_R\,\varphi) + \text{H.c.}\Big] \\ &\quad + \Big[C'^{\nu\nu}\,(\widetilde{\varphi}^\dagger\,l'_L)^T\,\mathbb{C}\,(\widetilde{\varphi}^\dagger\,l'_L) + \text{H.c.}\Big] , \end{aligned} \tag{3.25}$$

where $\Gamma_{e,u,d}$ and $C'^{e\varphi}, C'^{u\varphi}, C'^{d\varphi}$ are general complex 3×3 matrices, $C'^{\nu\nu}$ is a symmetric complex 3×3 matrix and primed fields denote the fields in the interaction (gauge) basis (group and generation indices are suppressed).

The fermion kinetic terms remain unaffected by SSB, while the mass terms read

$$\mathcal{L}_{\rm mass} = -\frac{1}{2}\,\nu_L'^{T}\,\mathbb{C}\,M_\nu'\,\nu_L' - \bar{e}_L'\,M_e'\,e_R' - \bar{u}_L'\,M_u'\,u_R' - \bar{d}_L'\,M_d'\,d_R' + \text{H.c.}\,, \tag{3.26}$$

with the 3×3 mass matrices equal to

$$\begin{aligned} M_\nu' &= -v^2 C'^{\nu\nu}\,, & M_e' &= \tfrac{v}{\sqrt{2}}\left(\Gamma_e - C'^{e\varphi}\tfrac{v^2}{2}\right), \\ M_u' &= \tfrac{v}{\sqrt{2}}\left(\Gamma_u - C'^{u\varphi}\tfrac{v^2}{2}\right), & M_d' &= \tfrac{v}{\sqrt{2}}\left(\Gamma_d - C'^{d\varphi}\tfrac{v^2}{2}\right)\,. \end{aligned} \tag{3.27}$$

To diagonalize lepton and quark masses we rotate the fermion fields by the unitary matrices,

$$\psi_X' = U_{\psi_X}\,\psi_X\,, \tag{3.28}$$

with $\psi = \nu, e, u, d$, $X = L, R$ and the "unprimed" symbols denoting the mass eigenstates fields. Then, the singular value decomposition for charged fermion mass matrices results in

$$\begin{aligned} U_{e_L}^\dagger\,M_e'\,U_{e_R} &= \quad M_e = \text{diag}(m_e, m_\mu, m_\tau)\,, \\ U_{u_L}^\dagger\,M_u'\,U_{u_R} &= \quad M_u = \text{diag}(m_u, m_c, m_t)\,, \\ U_{d_L}^\dagger\,M_d'\,U_{d_R} &= \quad M_d = \text{diag}(m_d, m_s, m_b)\,, \end{aligned} \tag{3.29}$$

while the diagonal neutrino mass matrix is obtained through

$$U_{\nu_L}^T\,M_\nu'\,U_{\nu_L} \;=\; M_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\,, \tag{3.30}$$

with all fermion masses now being real and non-negative.

4 Corrections to the SM couplings

Corrections to the interactions described by the dimension-4 SM Lagrangian can come either as genuine new vertices generated by higher order operators, or from the dimension-4 vertices modified by the shifts in the fields and parameters necessary to express them in the mass eigenstates basis. In this section we discuss the second class of ("oblique") corrections.

In terms of physical gauge bosons, the electroweak part of the covariant derivative (its QCD part parametrized in terms of $\bar{g}_s$-coupling is unchanged compared to the SM), reads

$$\begin{aligned} \bar{D}_\mu^{EW} &= \partial_\mu + i\frac{\bar{g}}{\sqrt{2}}\,(T^+ W_\mu^+ + T^- W_\mu^-) \\ &\quad + i(\bar{g}\mathbb{X}_{11}\,T^3 + \bar{g}'\mathbb{X}_{21}\,Y)\,Z_\mu + i(\bar{g}\mathbb{X}_{12}\,T^3 + \bar{g}'\mathbb{X}_{22}\,Y)\,A_\mu\,. \end{aligned} \tag{4.1}$$

The pattern of electroweak symmetry breaking results in a conserved electric charge, identified through the standard relation $Q = T_3 + Y$. The electromagnetic gauge invariance of the broken theory manifests through the "corrected" electroweak unification condition,

$$\bar{e} = \bar{g}'\,\mathbb{X}_{22} = \bar{g}\,\mathbb{X}_{12}\,, \tag{4.2}$$

which couples the photon only to the electric charge while keeping it massless. Using eq. (4.2) and the property $\det \mathbb{X} = 1$ one can always express the covariant derivative in the familiar form,

$$\bar{D}_\mu^{EW} = \partial_\mu + i\frac{\bar{g}}{\sqrt{2}}\,(T^+ W_\mu^+ + T^- W_\mu^-) + i\bar{g}_Z\,(T^3 - \sin^2\bar{\theta}\,Q)Z_\mu + i\bar{e}QA_\mu\,, \tag{4.3}$$

where the modified couplings now read,

$$\begin{aligned} \bar{e} &= \frac{\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left(1 - \frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}\right)\,, \\ \bar{g}_Z &= \sqrt{\bar{g}^2+\bar{g}'^2}\left(1 + \frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}\right)\,. \end{aligned} \tag{4.4}$$

In summary, after redefinitions of fields and couplings in mass basis, corrections to gauge interactions originating from the shift in the gauge and Higgs sector parameters depend only on two additional Wilson coefficients: $C^{\varphi WB}$, responsible for the mixing of electroweak gauge boson kinetic terms, and, $C^{\varphi D}$ appearing through the physical Z^0-boson mass (see eq. (3.21)). Furthermore, the $C^{\varphi D}$ operator breaks the custodial invariance as this is described by the anomalous value of the ρ parameter,

$$\rho = \frac{|J_{C.C}|^2}{|J_{N.C.}|^2} = \frac{\bar{g}^2\,M_Z^2}{\bar{g}_Z^2 M_W^2} = 1 + \frac{1}{2}C^{\varphi D}v^2\,. \tag{4.5}$$

As it is well known, this is strongly constrained by precision EW experiments, at the level of 1% [24]. Consequently, sizable "oblique" corrections in the gauge sector could potentially arise only from the gauge boson kinetic mixing ϵ defined in eq. (3.15).

Another set of "oblique" corrections originates in the flavor sector of SMEFT after diagonalization of the fermion mass matrices [see section 3.4]. In the SM, only the products $U_{u_L}^\dagger U_{d_L}$ and $U_{e_L}^\dagger U_{\nu_L}$ appear in the charged quark and lepton current couplings after flavor rotations: they are identified as the CKM [25] and PMNS [26,27] mixing matrices, respectively. However, in SMEFT the fermion-fermion-$W^\pm$ couplings contain additional contributions from operators, without affecting the fermion bilinear terms of the model. The relevant part of the Lagrangian has the form:

$$\begin{aligned} \mathcal{L}_{c.c.} = &- \frac{\bar{g}}{\sqrt{2}}\,W_\mu^+\,\bar{u}_p\,\gamma^\mu\left\{\left[U_{u_L}^\dagger(\mathbb{1} + v^2 C'^{\varphi q(3)})U_{d_L}\right]_{pr}P_L + \left(\frac{v^2}{2}U_{u_R}^\dagger C'^{\varphi ud}U_{d_R}\right)_{pr}P_R\right\}d_r \\ &- \frac{\bar{g}}{\sqrt{2}}\,W_\mu^+\,\bar{\nu}_p\,\gamma^\mu\,\left[U_{e_L}^\dagger(\mathbb{1} + v^2 C'^{\varphi l(3)})U_{\nu_L}\right]^\dagger_{pr}\,P_L\,e_r + \text{H.c.} \end{aligned} \tag{4.6}$$

As a result, one can identify the physical, although not unitary any more, mixing matrices for quark and leptons, through:

$$K_{\text{CKM}} \equiv K \equiv U_{u_L}^\dagger(\mathbb{1} + v^2 C'^{\varphi q(3)})U_{d_L}\,, \tag{4.7}$$

$$U_{\text{PMNS}} \equiv U \equiv U_{e_L}^\dagger(\mathbb{1} + v^2 C'^{\varphi l(3)})U_{\nu_L}\,. \tag{4.8}$$

In what follows we also redefine the Wilson coefficients of the operators involving fermionic currents, by absorbing into them the fermion flavor rotations from gauge to the mass basis. In

this way we are able to express the mass basis Lagrangian entirely in terms of the "unprimed" fields, Wilson-coefficients K-, and U-mixing matrices. In some cases the redefinitions are not unique, as in the operators involving left fermion $SU(2)$ doublets one can adsorb into the Wilson coefficient either the rotation matrix of the lower or upper constituent of the doublet. We choose it always to be the lower field (e_L or d_L) rotation, as in this way flavor violating K or U matrices appear explicitly in less experimentally constrained u-quark or neutrino couplings (see also discussion in Ref. [28]). Our redefinitions are collected in Table 4.

Finally, Higgs boson interactions with fermions are affected by the transition to the physical mass eigenstates both universally, due to the change of Higgs-boson normalization in eq. (3.6), and in a flavor dependent way, due to the modified relation in eq. (3.27) between fermion masses and the Yukawa couplings. The Higgs-fermion-fermion interaction Lagrangian in mass basis is,

$$\begin{aligned}
\mathcal{L}_{h\psi\psi} = & - \bar{e}\left[\frac{M_e}{v}\left(1-\frac{1}{4}C^{\varphi D}v^2+C^{\varphi\Box}v^2\right)-C^{e\varphi}\frac{v^2}{\sqrt{2}}\right]P_R\, e\, h + \text{H.c.} \\
& - \bar{u}\left[\frac{M_u}{v}\left(1-\frac{1}{4}C^{\varphi D}v^2+C^{\varphi\Box}v^2\right)-C^{u\varphi}\frac{v^2}{\sqrt{2}}\right]P_R\, u\, h + \text{H.c.} \\
& - \bar{d}\left[\frac{M_d}{v}\left(1-\frac{1}{4}C^{\varphi D}v^2+C^{\varphi\Box}v^2\right)-C^{d\varphi}\frac{v^2}{\sqrt{2}}\right]P_R\, d\, h + \text{H.c.}\,, \qquad (4.9)
\end{aligned}$$

with the diagonal fermion mass matrices above, defined in eq. (3.29). Note that the dimension-5 operator in eq. (2.1), induces also a Higgs-neutrino-neutrino vertex but this is highly suppressed since it is proportional to neutrino masses.

5 Gauge fixing and FP-ghosts in R_ξ-gauges

A consistent and convenient, for practical purposes, choice of gauge fixing conditions and ghost sector should fulfill the following requirements:

- Cancel the unwanted Goldstone-gauge boson bilinear mixing, as in SM.
- Lead to SM-like propagators in terms of the effective mass basis parameters and fields.
- Preserve the BRST invariance of the full Lagrangian in the presence of gauge fixing and ghost terms.

Let us notice that the gauge basis Lagrangian in terms of barred couplings and fields, as obtained through eq. (3.11), keeps the same form up to rescaling factors. For the dimension-4 terms it reads,

$$\begin{aligned}
\mathcal{L}^{(4)}_{SM} = & -\frac{1}{4}Z_{g_s}^{-2}\bar{G}^A_{\mu\nu}\bar{G}^{A\mu\nu} - \frac{1}{4}Z_g^{-2}\bar{W}^I_{\mu\nu}\bar{W}^{I\mu\nu} - \frac{1}{4}Z_{g'}^{-2}\bar{B}_{\mu\nu}\bar{B}^{\mu\nu} \\
& + (\bar{D}_\mu\varphi)^\dagger(\bar{D}^\mu\varphi) + m^2\varphi^\dagger\varphi - \frac{1}{2}\lambda(\varphi^\dagger\varphi)^2 \\
& + \; i(\bar{l}'_L\not{\bar{D}}l'_L + \bar{e}'_R\not{\bar{D}}e'_R + \bar{q}'_L\not{\bar{D}}q'_L + \bar{u}'_R\not{\bar{D}}u'_R + \bar{d}'_R\not{\bar{D}}d'_R) \\
& - (\bar{l}'_L\Gamma_e e'_R\varphi + \bar{q}'_L\Gamma_u u'_R\widetilde{\varphi} + \bar{q}'_L\Gamma_d d'_R\varphi)\,, \qquad (5.1)
\end{aligned}$$

while all higher dimensional operators remain unaffected at the considered order. Each term in the "barred" Lagrangian is still manifestly $SU(3)\times SU(2)\times U(1)$ invariant, despite the

$C^{e\varphi} = U_{e_L}^\dagger C'^{e\varphi} U_{e_R}$	$(C^{ll})_{f_1f_2f_3f_4} = (U_{e_L})_{g_2f_2}(U_{e_L})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{e_L})^*_{g_3f_3}(C'^{ll})_{g_1g_2g_3g_4}$
$C^{d\varphi} = U_{d_L}^\dagger C'^{d\varphi} U_{d_R}$	$(C^{ee})_{f_1f_2f_3f_4} = (U_{e_R})_{g_2f_2}(U_{e_R})_{g_4f_4}(U_{e_R})^*_{g_1f_1}(U_{e_R})^*_{g_3f_3}(C'^{ee})_{g_1g_2g_3g_4}$
$C^{u\varphi} = U_{u_L}^\dagger C'^{u\varphi} U_{u_R}$	$(C^{le})_{f_1f_2f_3f_4} = (U_{e_L})_{g_2f_2}(U_{e_R})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{e_R})^*_{g_3f_3}(C'^{le})_{g_1g_2g_3g_4}$
$C^{eW} = U_{e_L}^\dagger C'^{eW} U_{e_R}$	$(C^{qq(1)})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{d_L})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{qq(1)})_{g_1g_2g_3g_4}$
$C^{eB} = U_{e_L}^\dagger C'^{eB} U_{e_R}$	$(C^{qq(3)})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{d_L})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{qq(3)})_{g_1g_2g_3g_4}$
$C^{dG} = U_{d_L}^\dagger C'^{dG} U_{d_R}$	$(C^{dd})_{f_1f_2f_3f_4} = (U_{d_R})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{d_R})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{dd})_{g_1g_2g_3g_4}$
$C^{dW} = U_{d_L}^\dagger C'^{dW} U_{d_R}$	$(C^{uu})_{f_1f_2f_3f_4} = (U_{u_R})_{g_2f_2}(U_{u_R})_{g_4f_4}(U_{u_R})^*_{g_1f_1}(U_{u_R})^*_{g_3f_3}(C'^{uu})_{g_1g_2g_3g_4}$
$C^{dB} = U_{d_L}^\dagger C'^{dB} U_{d_R}$	$(C^{ud(1)})_{f_1f_2f_3f_4} = (U_{u_R})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{u_R})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{ud(1)})_{g_1g_2g_3g_4}$
$C^{uG} = U_{u_L}^\dagger C'^{uG} U_{u_R}$	$(C^{ud(8)})_{f_1f_2f_3f_4} = (U_{u_R})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{u_R})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{ud(8)})_{g_1g_2g_3g_4}$
$C^{uW} = U_{u_L}^\dagger C'^{uW} U_{u_R}$	$(C^{qu(1)})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{u_R})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{u_R})^*_{g_3f_3}(C'^{qu(1)})_{g_1g_2g_3g_4}$
$C^{uB} = U_{u_L}^\dagger C'^{uB} U_{u_R}$	$(C^{qu(8)})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{u_R})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{u_R})^*_{g_3f_3}(C'^{qu(8)})_{g_1g_2g_3g_4}$
$C^{\varphi l(1)} = U_{e_L}^\dagger C'^{\varphi l(1)} U_{e_L}$	$(C^{qd(1)})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{qd(1)})_{g_1g_2g_3g_4}$
$C^{\varphi l(3)} = U_{e_L}^\dagger C'^{\varphi l(3)} U_{e_L}$	$(C^{qd(8)})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{qd(8)})_{g_1g_2g_3g_4}$
$C^{\varphi e} = U_{e_R}^\dagger C'^{\varphi e} U_{e_R}$	$(C^{quqd(1)})_{f_1f_2f_3f_4} = (U_{u_R})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{quqd(1)})_{g_1g_2g_3g_4}$
$C^{\varphi q(1)} = U_{d_L}^\dagger C'^{\varphi q(1)} U_{d_L}$	$(C^{quqd(8)})_{f_1f_2f_3f_4} = (U_{u_R})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{quqd(8)})_{g_1g_2g_3g_4}$
$C^{\varphi q(3)} = U_{d_L}^\dagger C'^{\varphi q(3)} U_{d_L}$	$(C^{lq(1)})_{f_1f_2f_3f_4} = (U_{e_L})_{g_2f_2}(U_{d_L})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{lq(1)})_{g_1g_2g_3g_4}$
$C^{\varphi d} = U_{d_R}^\dagger C'^{\varphi d} U_{d_R}$	$(C^{lq(3)})_{f_1f_2f_3f_4} = (U_{e_L})_{g_2f_2}(U_{d_L})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{lq(3)})_{g_1g_2g_3g_4}$
$C^{\varphi u} = U_{u_R}^\dagger C'^{\varphi u} U_{u_R}$	$(C^{ld})_{f_1f_2f_3f_4} = (U_{e_L})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{ld})_{g_1g_2g_3g_4}$
$C^{\varphi ud} = U_{u_R}^\dagger C'^{\varphi ud} U_{d_R}$	$(C^{lu})_{f_1f_2f_3f_4} = (U_{e_L})_{g_2f_2}(U_{u_R})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{u_R})^*_{g_3f_3}(C'^{lu})_{g_1g_2g_3g_4}$
$C^{\nu\nu} = U_{\nu_L}^\top C'^{\nu\nu} U_{\nu_L}$	$(C^{qe})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{e_R})_{g_4f_4}(U_{d_L})^*_{g_1f_1}(U_{e_R})^*_{g_3f_3}(C'^{qe})_{g_1g_2g_3g_4}$
	$(C^{ed})_{f_1f_2f_3f_4} = (U_{e_R})_{g_2f_2}(U_{d_R})_{g_4f_4}(U_{e_R})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{ed})_{g_1g_2g_3g_4}$
	$(C^{eu})_{f_1f_2f_3f_4} = (U_{e_R})_{g_2f_2}(U_{u_R})_{g_4f_4}(U_{e_R})^*_{g_1f_1}(U_{u_R})^*_{g_3f_3}(C'^{eu})_{g_1g_2g_3g_4}$
	$(C^{ledq})_{f_1f_2f_3f_4} = (U_{e_R})_{g_2f_2}(U_{d_L})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{d_R})^*_{g_3f_3}(C'^{ledq})_{g_1g_2g_3g_4}$
	$(C^{lequ(1)})_{f_1f_2f_3f_4} = (U_{e_R})_{g_2f_2}(U_{u_R})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{lequ(1)})_{g_1g_2g_3g_4}$
	$(C^{lequ(3)})_{f_1f_2f_3f_4} = (U_{e_R})_{g_2f_2}(U_{u_R})_{g_4f_4}(U_{e_L})^*_{g_1f_1}(U_{d_L})^*_{g_3f_3}(C'^{lequ(3)})_{g_1g_2g_3g_4}$
	$(C^{duq})_{f_1f_2f_3f_4} = (U_{u_R})_{g_2f_2}(U_{e_L})_{g_4f_4}(U_{d_R})_{g_1f_1}(U_{d_L})_{g_3f_3}(C'^{duq})_{g_1g_2g_3g_4}$
	$(C^{qqu})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}](U_{e_R})_{g_4f_4}(U_{d_L})_{g_1f_1}(U_{u_R})_{g_3f_3}(C'^{qqu})_{g_1g_2g_3g_4}$
	$(C^{qqq})_{f_1f_2f_3f_4} = (U_{d_L})_{g_2f_2}(U_{e_L})_{g_4f_4}(U_{d_L})_{g_1f_1}(U_{d_L})_{g_3f_3}(C'^{qqq})_{g_1g_2g_3g_4}$
	$(C^{duu})_{f_1f_2f_3f_4} = (U_{u_R})_{g_2f_2}(U_{e_R})_{g_4f_4}(U_{d_R})_{g_1f_1}(U_{u_R})_{g_3f_3}(C'^{duu})_{g_1g_2g_3g_4}$

Table 4: Definitions of the Wilson coefficients multiplying the fermionic currents in the mass basis. We suppress the flavor indices for the two-fermion operators as the contraction is non-ambiguous here. For the four-fermion vertices we assume summation over repeating indices.

presence of Z-factors. Therefore, we may equivalently use this Lagrangian to gauge fix the theory.

Our choice for the gauge fixing term in the electroweak sector reads

$$\mathcal{L}_{GF} = -\frac{1}{2}\boldsymbol{F}^\top \hat{\boldsymbol{\xi}}^{-\mathbf{1}} \boldsymbol{F} \,, \tag{5.2}$$

with the gauge fixing functionals F^i defined through

$$\boldsymbol{F} = \begin{pmatrix} F^1 \\ F^2 \\ F^3 \\ F^0 \end{pmatrix} = \begin{pmatrix} \partial_\mu \bar{W}^{1\mu} \\ \partial_\mu \bar{W}^{2\mu} \\ \partial_\mu \bar{W}^{3\mu} \\ \partial_\mu \bar{B}^{\mu} \end{pmatrix} - \frac{v\hat{\boldsymbol{\xi}}}{2} \begin{pmatrix} -i\bar{g}\frac{\Phi^+ - \Phi^-}{\sqrt{2}} \\ \bar{g}\frac{\Phi^+ + \Phi^-}{\sqrt{2}} \\ -\bar{g} Z_{G^0}^2 \Phi_0 \\ \bar{g}' Z_{G^0}^2 \Phi_0 \end{pmatrix} \tag{5.3}$$

and a 4×4 *symmetric* matrix $\hat{\xi}$ introduced as

$$\hat{\boldsymbol{\xi}} = \begin{pmatrix} \xi_W & & & \\ & \xi_W & & \\ & & \multicolumn{2}{c}{\multirow{2}{*}{$\mathbb{X}\begin{pmatrix} \xi_Z & \\ & \xi_A \end{pmatrix}\mathbb{X}^\top$}} \\ 0 & & & \end{pmatrix} , \tag{5.4}$$

with $\mathbb{X}$ being the 2×2 mixing matrix of the neutral electroweak gauge bosons in eq. (3.19).

With such a choice in gauge basis, the transformations which diagonalize and rescale the electroweak gauge and Goldstone bosons also bring the gauge fixing term in a familiar form. After substituting the mass basis fields into eq. (5.3), we arrive at the expression

$$\begin{aligned} \mathcal{L}_{GF} = \; & - \frac{1}{\xi_W}(\partial^\mu W_\mu^+ + i\xi_W M_W G^+)(\partial^\nu W_\nu^- - i\xi_W M_W G^-) \\ & - \frac{1}{2\xi_Z}(\partial^\mu Z_\mu + \xi_Z M_Z G^0)^2 - \frac{1}{2\xi_A}(\partial^\mu A_\mu)^2 \,, \end{aligned} \tag{5.5}$$

which looks identical to the SM one in the standard linear R_ξ-gauges and has all terms required to eliminate the "unwanted" Goldstone-gauge mixing of eq. (3.24), through a total derivative. As previously mentioned in Section 3.3, such a standard choice for R_ξ-gauges is possible since, in mass basis, all Wilson coefficients of the "unwanted" terms become absorbed in masses and fields.

The gauge fixing conditions violate gauge invariance and we need to introduce a ghost term in the Lagrangian to compensate and restore (the more general) BRST invariance. A convenient and consistent choice for a ghost term takes the form

$$\mathcal{L}_{FP} = \bar{\boldsymbol{N}}^\top \hat{\boldsymbol{E}}(\hat{\boldsymbol{M}}_{\boldsymbol{F}} \boldsymbol{N}) \,, \tag{5.6}$$

where the gauge basis ghost, anti-ghost fields are defined as $N^i = (N^1, N^2, N^3, N^0)$, $\bar{N}^i = (\bar{N}^1, \bar{N}^2, \bar{N}^3, \bar{N}^0)$, respectively and we have also introduced the *symmetric* 4×4 matrix,

$$\hat{\boldsymbol{E}} \;=\; \begin{pmatrix} \mathbb{1}_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & (\mathbb{X}^\top)^{-1}\mathbb{X}^{-1} \end{pmatrix} . \tag{5.7}$$

The gauge fixing functions F^i chosen in eq. (5.3) are linear in the fields and therefore the standard Faddeev-Popov (FP) treatment with determinants applies[4]. The explicit form

[4]In the FP-treatment, it is clear that the matrix $\hat{E}$ factors out from the determinant as $\det(\hat{\boldsymbol{E}}\hat{\boldsymbol{M}}_{\boldsymbol{F}}) = \det(\hat{\boldsymbol{E}})\det(\hat{\boldsymbol{M}}_{\boldsymbol{F}})$, affecting the path integral with an irrelevant constant factor.

of $\hat{M}_F$ can be always obtained by performing an infinitesimal gauge transformation on F^i. However, since we also wish to demonstrate the BRST invariance of the full Lagrangian, we follow instead an equivalent derivation of $\hat{M}_F$ with the help of the BRST-operator, $\boldsymbol{s}$. It reads,

$$\hat{M}_F^{ij} N^j = \boldsymbol{s} F^i \ , \tag{5.8}$$

where *lowercase* Latin indices run in the electroweak space ($\{i,j\}$=1,2,3,0).

Despite the presence of (constant) mixing matrices in the gauge fixing functions, the $\boldsymbol{s}$-operator transforms the fields included in F^i, in a way identical to SM, as

$$\begin{aligned}
\boldsymbol{s}\varphi &= -i\bar{g}' Y\varphi \ N^0 - i\bar{g} T^I \varphi \ N^I \ , \\
\boldsymbol{s}\varphi^\dagger &= +i\bar{g}' \varphi^\dagger \ Y N^0 + i\bar{g}\varphi^\dagger T^I \ N^I \ , \\
\boldsymbol{s}\bar{B}_\mu &= \partial_\mu N^0 \ , \\
\boldsymbol{s}\bar{W}_\mu^I &= \partial_\mu N^I - \bar{g}\epsilon^{IJK} \bar{W}_\mu^J N^K \ .
\end{aligned} \tag{5.9}$$

Then the explicit form for $\hat{M}_F$ has the form,

$$\begin{aligned}
\boldsymbol{\hat{M}_F N} &= \partial^2 \boldsymbol{N} + \bar{g}\overleftarrow{\partial^\mu} \begin{pmatrix} 0 & -\bar{W}_\mu^3 & \bar{W}_\mu^2 & 0 \\ \bar{W}_\mu^3 & 0 & -\bar{W}_\mu^1 & 0 \\ -\bar{W}_\mu^2 & \bar{W}_\mu^1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \boldsymbol{N} \\
&+ \frac{v\bar{g}^2 \hat{\boldsymbol{\xi}}}{4} \begin{pmatrix} H+v & \Phi^0 & \frac{\Phi^+ + \Phi^-}{\sqrt{2}} & \frac{\bar{g}'}{\bar{g}} \frac{\Phi^+ + \Phi^-}{\sqrt{2}} \\ -\Phi^0 & H+v & i\frac{\Phi^+ - \Phi^-}{\sqrt{2}} & \frac{i\bar{g}'}{\bar{g}} \frac{\Phi^+ - \Phi^-}{\sqrt{2}} \\ -Z_{G^0}^2 \frac{\Phi^+ + \Phi^-}{\sqrt{2}} & -iZ_{G^0}^2 \frac{\Phi^+ - \Phi^-}{\sqrt{2}} & Z_{G^0}^2 (H+v) & -\frac{\bar{g}'}{\bar{g}} Z_{G^0}^2 (H+v) \\ \frac{\bar{g}'}{\bar{g}} Z_{G^0}^2 \frac{\Phi^+ + \Phi^-}{\sqrt{2}} & \frac{i\bar{g}'}{\bar{g}} Z_{G^0}^2 \frac{\Phi^+ - \Phi^-}{\sqrt{2}} & -\frac{\bar{g}'}{\bar{g}} Z_{G^0}^2 (H+v) & \frac{\bar{g}'^2}{\bar{g}^2} Z_{G^0}^2 (H+v) \end{pmatrix} \boldsymbol{N}
\end{aligned} \tag{5.10}$$

Once again, the chosen form of eq. (5.6) with the presence of the matrix $\hat{E}$, makes the transformation which diagonalizes the gauge bosons kinetic terms and masses to diagonalize also ghost bilinear terms. By defining ghost and anti-ghost fields in mass basis through the relations

$$\frac{1}{\sqrt{2}}(N^1 \mp iN^2) = \eta^\pm, \quad \frac{1}{\sqrt{2}}(\bar{N}^1 \pm i\bar{N}^2) = \bar{\eta}^\pm \ , \tag{5.11}$$

$$\begin{pmatrix} N^3 \\ N^0 \end{pmatrix} = \mathbb{X} \begin{pmatrix} \eta^Z \\ \eta^A \end{pmatrix}, \quad \begin{pmatrix} \bar{N}^3 \\ \bar{N}^0 \end{pmatrix}^\top = \begin{pmatrix} \bar{\eta}^Z \\ \bar{\eta}^A \end{pmatrix}^\top \mathbb{X}^\top \ , \tag{5.12}$$

all occurrences of the $\mathbb{X}$ matrix in bilinear ghost terms become absorbed, leaving them in a canonical form with squared masses $\xi_W M_W^2$, $\xi_Z M_Z^2$ and zero for the corresponding photon ghost. Again, the ghost propagators are SM-like (see Appendix A.1). Nevertheless, corrections from higher dimensional operators appear explicitly in ghost vertices.

The BRST invariance of the SMEFT action not including the gauge fixing and ghost sector, follows immediately from its gauge invariance. On the other hand, the action of the $\boldsymbol{s}$-operator on ghost and anti-ghost fields which preserves also the BRST invariance in the gauge fixing and ghost sector, is

$$\boldsymbol{s}N^0 = 0 \ , \quad \boldsymbol{s}N^I = \frac{\bar{g}}{2}\epsilon^{IJK} N^J N^K \ , \tag{5.13}$$

$$\boldsymbol{s}\bar{N}^i = F^j (\hat{\boldsymbol{\xi}}^{-\mathbf{1}} \hat{\boldsymbol{E}}^{-\mathbf{1}})^{ji} \ . \tag{5.14}$$

Using eq. (5.8) and eq. (5.14), the property $\hat{\boldsymbol{\xi}}^{-1} = (\hat{\boldsymbol{\xi}}^{-1})^\top$ and the relation $\boldsymbol{s}(\hat{M}_F N) = 0$, which is associated with the nilpotency of BRST-transformations, one obtains

$$\begin{aligned} \boldsymbol{s}\,\mathcal{L}_{GF} &= -\frac{1}{2}\boldsymbol{s}\Big(F^i(\hat{\xi}^{-1})^{ij}F^j\Big) = -F^i(\hat{\xi}^{-1})^{ij}(\boldsymbol{s}F^j) \\ &= -(\boldsymbol{s}\bar{N}^i)\hat{E}^{ij}\hat{M}_F^{jk}N^k = -\boldsymbol{s}\Big(\bar{N}^i\hat{E}^{ij}\hat{M}_F^{jk}N^k\Big) = -\boldsymbol{s}\,\mathcal{L}_{FP}. \end{aligned} \tag{5.15}$$

Hence, the full Lagrangian, now including also the gauge fixing and ghost terms, remains invariant under the BRST-symmetry transformation.

As easily noticed, the BRST transformations on all gauge basis fields, besides anti-ghosts, are identical to SM. Therefore, for this set of fields the BRST-transformation is nilpotent. The gauge fixing functionals F^i, although modified by the presence of new constant (mixing) matrices, are still linear functions of the same fields as in SM (*i.e.,* gauge and Goldstone bosons). Thus, the BRST transformation acting on them is also nilpotent, satisfying $\boldsymbol{s}^2F^i = \boldsymbol{s}(M_F^{ij}N^j) = 0$, which can be always verified explicitly. Finally, we note that the presence of constant matrices in the BRST transformation for anti-ghosts is in practice irrelevant. This is because one can always introduce auxiliary fields in a suitable manner and make BRST transformations on all fields nilpotent without eventually affecting the gauge fixing and ghost terms. The choice

$$\mathcal{L}_{GF} = \boldsymbol{B}^\top\hat{\boldsymbol{E}}\boldsymbol{F} + \frac{1}{2}\boldsymbol{B}^\top\hat{\boldsymbol{E}}\hat{\boldsymbol{\xi}}\hat{\boldsymbol{E}}\boldsymbol{B}\,, \tag{5.16}$$

is equivalent to eq. (5.2) when the equations of motion are taken for the auxiliary fields B^i. Changing only the BRST-transformation for anti-ghosts, into $\boldsymbol{s}\bar{N}^i = B^i$ and introducing the new one $\boldsymbol{s}B^i = 0$ for the auxiliary fields, one can verify that the Lagrangian remains BRST-invariant. Therefore, the BRST-transformation on all fields is now nilpotent, that is

$$\boldsymbol{s}^2 = 0. \tag{5.17}$$

In the QCD-sector, an analogous discussion of the R_ξ-gauges is far more trivial. In terms of barred fields and couplings, the gauge fixing and ghost terms read

$$\mathcal{L}_{GF} + \mathcal{L}_{FP} = -\frac{1}{2\xi_G}F^AF^A + \bar{\eta}_G^AM_F^{AB}\eta_G^B\,, \tag{5.18}$$

with simply,

$$\begin{aligned} F^A &= \partial_\mu g^{A\mu}, \\ M_F^{AB}\eta^B &= \partial^2\eta_G^A + \bar{g}_s\overleftarrow{\partial}_\mu f^{ABC}g^{B\mu}\eta_G^C\,. \end{aligned} \tag{5.19}$$

6 Feynman rules and *Mathematica* implementation

In Appendix A we have collected the Feynman rules for SMEFT propagators and interaction vertices in the R_ξ-gauges. Most of the vertices are reasonably compact and for many processes they can be readily used even for manual calculations. We did not display explicitly only the five and six gluon self-interactions as they are, after symmetrizing in all Lorentz and flavor indices, very long and it is unlikely that they can be used in any calculations without the use of computer symbolic algebra programs.

Apart from the printed version, we have developed a publicly available *Mathematica* code calculating the same set of Feynman rules, such that its output can be directly fed to other symbolic or numerical packages for high energy physics calculations. Our code works within the `FeynRules` package [20] and is constructed as a "model file" for `FeynRules` supplied with set of auxiliary programs performing the field redefinitions described earlier in the paper. In addition, these programs perform some extra simplifications, on top of the ones done by `FeynRules`, like Fierz transformations in four-fermion interactions assuring that all terms in a given vertex are always added with the same ordering of fermion indices. Similarly, there has been made several simplifications in the gluon vertices based on Jacobi identity. Our package contains also routines generating automatically Latex output for SMEFT Feynman rules. If necessary, users can run the SMEFT-code to obtain a subset of vertices for chosen Wilson coefficients, relevant just to their analysis.

The SMEFT package for `FeynRules`, with instructions for the user, can be downloaded from `www.fuw.edu.pl/smeft`. In Appendix B we describe how to install and run our package.

7 Conclusions

It is a central problem in particle physics today to categorize and parametrize New Physics effects that are expected to arise by new effective operators at some scale Λ. In this article we analyzed the structure of Standard Model Effective Field Theory (SMEFT) including non-renormalizable operators up to dimension 6. For the first time in literature we derived the complete set of Feynman rules for this theory quantized in linear R_ξ-gauges.

More precisely, we started from the well known "Warsaw" basis of ref. [11], where the complete set of independent gauge invariant $d \leq 6$ operators is given, and identified the mass eigenstate fields after Spontaneous Symmetry Breaking (SSB). In achieving that goal, we performed constant and gauge invariant field and coupling redefinitions in such a way that all physical and unphysical fields possess canonical kinetic terms. Furthermore, we constructed gauge fixing functionals which in mass basis have a form of the linear R_ξ-gauges used routinely in the SM loop-calculations. A general set of different gauge fixing parameters for each gauge field has been introduced, for completeness and for additional cross-checks of the theory.

In order to restore the broken gauge symmetry after adding the gauge fixing terms, a set of Faddeev-Popov ghosts has been introduced. The ghost Lagrangian has been chosen such that the ghost propagators again have the SM-like structure, while the effect of higher dimensional operators appears explicitly only in their interaction vertices. We proved explicitly that our SMEFT action preserves BRST invariance and provide the reader with pertinent gauge transformations in section 5.

In summary, after establishing all steps described above, the bilinear part of SMEFT Lagrangian and all, physical and unphysical, field propagators expressed in terms of physical masses have exactly the same structure as in the SM (although certain relations of masses and couplings, such as the ρ-parameter for example, are modified by the new operators). The effect of new $d = 5$ and 6 operators *appears explicitly only in triple and higher multiplicity vertices, either as modifications of the SM ones or as genuine new interactions beyond the SM.*

Within the mass basis considered here, we constructed the complete set of Feynman rules in the linear R_ξ-gauges, not resorting to any restriction such as CP- or baryon- lepton-number conservation. The Feynman Rules for the total 383 vertices (not counting the hermitian conjugate ones), which are about four times more than the SM vertices, are given in Appendix A.

All Feynman rules were derived using the `FeynRules` code and a set of auxiliary programs created by the authors to perform field redefinitions, various simplifications and an automatic translation to Latex/axodraw format. All propagators and vertices are both listed explicitly in the Appendix A and provided as a publicly available *Mathematica* package, that can be downloaded from

`www.fuw.edu.pl/smeft`

The reader can consult Appendix B for programming and installation details.

On the practical side, we believe that our SMEFT collection of Feynman rules should significantly facilitate future phenomenological analyses, saving time in deriving from scratch often lengthy expressions in a complicated theory. In addition, our Feynman rules help to avoid possible mistakes and omissions of diagrams, which could easily happen when taking into account only some parts of the full Lagrangian, as this is done in many studies so far. Furthermore, the publicly available SMEFT "model file" for `FeynRules` package that accompanies this article, can be directly used as an input file to other high energy physics computational computer programs, again streamlining the calculation of future SMEFT physical predictions.

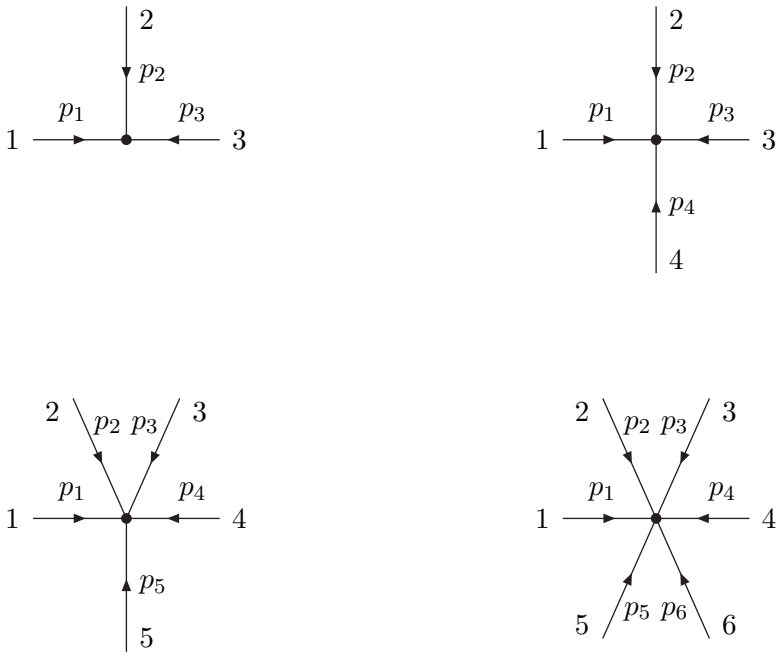
Acknowledgments

AD would like to thank Apostolos Pilaftsis for illuminating discussions about loop calculations in effective field theory. The work of JR and WM is supported in part by the National Science Centre, Poland, under research grants DEC-2015/19/B/ST2/02848, DEC-2015/18/M/ST2/00054 and DEC-2014/15/B/ST2/02157. JR would also like to thank University of Ioannina and CERN for hospitality during his visits. AD and KS would like to thank University of Warsaw for hospitality.

A SMEFT Feynman rules

In this Appendix we list the complete set of Feynman rules for SMEFT in the physical (mass eigenstates) field basis and in R_ξ-gauges.

For all interaction vertices, we assume uniform notation for the particle and momenta numbering. For the four possible topologies of vertex diagrams listed below, external legs are always numbered from the most left horizontal line clockwise. All external momenta are considered to be incoming. We suppress momenta labels in all Feynman diagrams that follow.



In addition to the notation defined in the main paper, we use the following symbols:

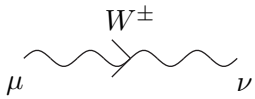
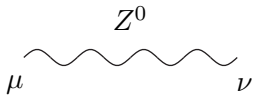
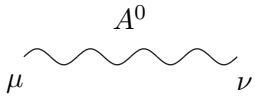
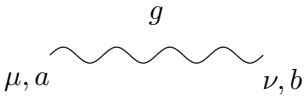
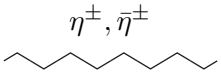
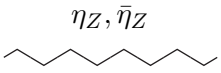
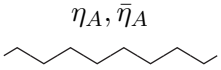
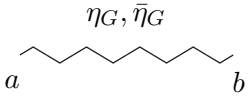
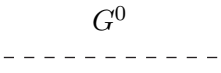
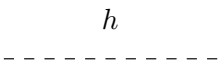
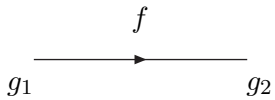
Index type	Symbols
Flavor (generation)	f_i, g_i
Spinor	s_i
Color in triplet representation (quarks)	m_i
Color in adjoint representation (gluons)	a_i, b_i
Lorentz	$\mu_i, \nu_i, \alpha_i, \beta_i, ...$

Finally, $\eta_{\mu\nu}$ denotes the Minkowski metric tensor with signature $(+,-,-,-)$.

An important remark should be made about Lorentz indices contraction. The expressions for the Feynman rules included in this Appendix are lenghty and, to save time and minimize the possibility of misprints, they were generated fully automatically by a specialized *Mathematica* code directly producing Latex output. However, it was difficult to implement in such a *Mathematica* to Latex translator the proper positioning of Lorentz indices, such that upper and lower repeating indices are contracted. Thus, in the expressions in this Appendix one

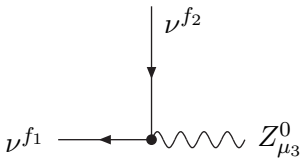
should assume that repeating Lorentz indices are always contracted in a covariant way, even if they are not subscript-superscript pairs.

A.1 Propagators in the R_ξ-gauges

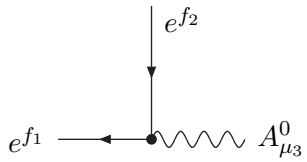
	$-\frac{i}{k^2-M_W^2}\left[\eta^{\mu\nu}-(1-\xi_W)\frac{k_\mu k_\nu}{k^2-\xi_W M_W^2}\right]$
	$-\frac{i}{k^2-M_Z^2}\left[\eta^{\mu\nu}-(1-\xi_Z)\frac{k_\mu k_\nu}{k^2-\xi_Z M_Z^2}\right]$
	$-\frac{i}{k^2}\left[\eta^{\mu\nu}-(1-\xi_A)\frac{k_\mu k_\nu}{k^2}\right]$
	$-\frac{i\delta_{ab}}{k^2}\left[\eta^{\mu\nu}-(1-\xi_G)\frac{k_\mu k_\nu}{k^2}\right]$
	$-\frac{i}{k^2-\xi_W M_W^2}$
	$-\frac{i}{k^2-\xi_Z M_Z^2}$
	$-\frac{i}{k^2}$
	$-\frac{i\delta_{ab}}{k^2}$
	$\frac{i}{k^2-\xi_Z M_Z^2}$
	$\frac{i}{k^2-\xi_W M_W^2}$
	$\frac{i}{k^2-M_h^2}$
	$\frac{i\delta^{g_1 g_2}}{\not{k}-m_f}$

Note that f above stands for any fermion in the theory, $f = \nu, l, u, d$. Apart from Kronecker delta in flavor indices $\delta^{g_1 g_2}$, quark propagators should be multiplied by $\delta^{m_1 m_2}$ in color indices too.

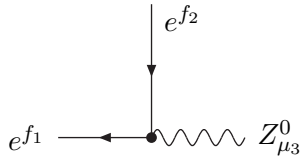
A.2 Lepton–gauge vertices



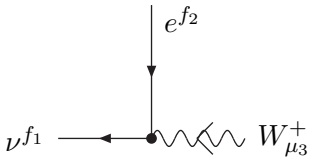
$$
\begin{aligned}
&-\frac{1}{2}i\sqrt{\bar{g}^2+\bar{g}'^2}\delta_{f_1f_2}\gamma^{\mu_3}P_L-\frac{i\bar{g}\bar{g}'v^2}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}C^{\phi WB}\gamma^{\mu_3}P_L\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l1}_{g_1g_2}\gamma^{\mu_3}P_L\\
&-\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l3}_{g_1g_2}\gamma^{\mu_3}P_L
\end{aligned}
$$



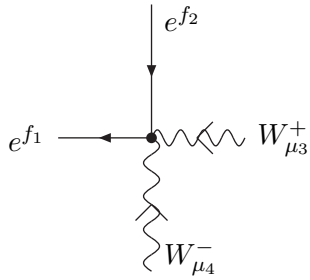
$$
\begin{aligned}
&+\frac{i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\gamma^{\mu_3}-\frac{i\bar{g}^2\bar{g}'^2v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\gamma^{\mu_3}\\
&+\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{eW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&-\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{eB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)
\end{aligned}
$$



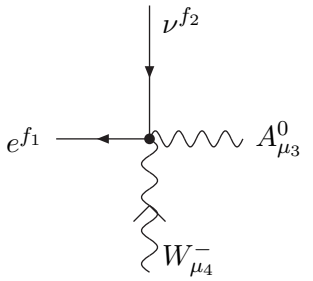
$$
\begin{aligned}
&-\frac{i}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\left(\left(\bar{g}'^2-\bar{g}^2\right)\gamma^{\mu_3}P_L+2\bar{g}'^2\gamma^{\mu_3}P_R\right)\\
&+\frac{i\bar{g}\bar{g}'v^2}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\left(\left(\bar{g}'^2-\bar{g}^2\right)\gamma^{\mu_3}P_L-2\bar{g}^2\gamma^{\mu_3}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{eW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{eB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi l1}_{f_1f_2}\gamma^{\mu_3}P_L+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi l3}_{f_1f_2}\gamma^{\mu_3}P_L\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi e}_{f_1f_2}\gamma^{\mu_3}P_R
\end{aligned}
$$



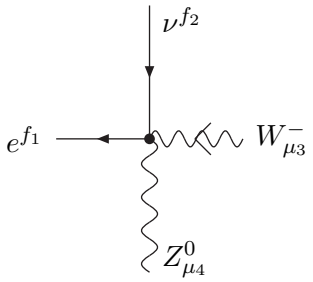
$$
-\frac{i\bar{g}}{\sqrt{2}}U^*_{f_2f_1}\gamma^{\mu_3}P_L-2vp_3^{\nu}U^*_{g_1f_1}C^{eW}_{g_1f_2}\sigma^{\mu_3\nu}P_R
$$



$$
+\sqrt{2}\bar{g}v\left(\sigma^{\mu_3\mu_4}P_LC^{eW*}_{f_2f_1}+C^{eW}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)
$$

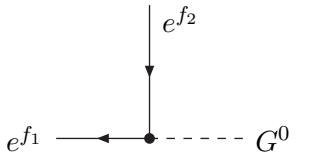


$$+\frac{2\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}\sigma^{\mu_3\mu_4}P_LC^{eW*}_{g_1f_1}$$

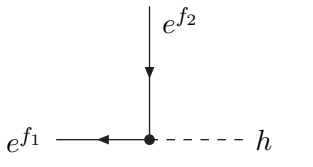


$$-\frac{2\bar{g}^2v}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}\sigma^{\mu_3\mu_4}P_LC^{eW*}_{g_1f_1}$$

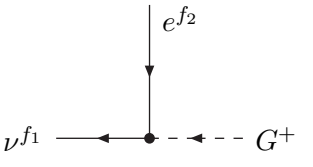
A.3 Lepton–Higgs–gauge vertices



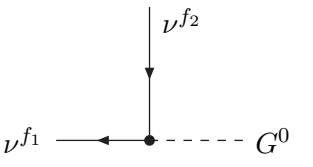
$$+\frac{1}{v}\delta_{f_1f_2}m_{l_{f_1}}\gamma^5-\frac{v}{4}\delta_{f_1f_2}C^{\phi D}m_{l_{f_1}}\gamma^5 - v\not{p}_3P_LC^{\phi l1}_{f_1f_2}-v\not{p}_3P_LC^{\phi l3}_{f_1f_2}-v\not{p}_3P_RC^{\phi e}_{f_1f_2}$$



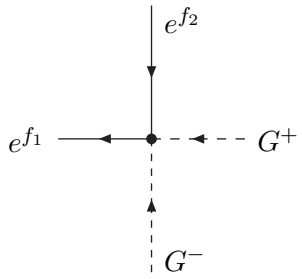
$$-\frac{i}{v}\delta_{f_1f_2}m_{l_{f_1}}-iv\delta_{f_1f_2}C^{\phi\Box}m_{l_{f_1}} + \frac{iv}{4}\delta_{f_1f_2}C^{\phi D}m_{l_{f_1}}+\frac{iv^2}{\sqrt{2}}\left(P_LC^{e\phi*}_{f_2f_1}+P_RC^{e\phi}_{f_1f_2}\right)$$



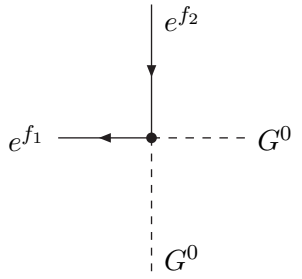
$$-\frac{i\sqrt{2}}{v}m_{l_{f_2}}P_RU^*_{f_2f_1}+i\sqrt{2}vU^*_{g_1f_1}C^{\phi l3}_{g_1f_2}\left(\not{p}_3P_L+m_{l_{f_2}}P_R\right)$$



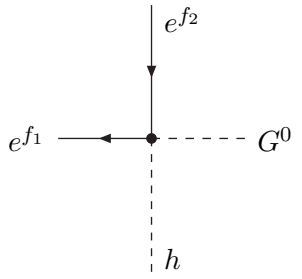
$$-vU_{g_2f_2}\not{p}_3P_LU^*_{g_1f_1}C^{\phi l1}_{g_1g_2}+vU_{g_2f_2}\not{p}_3P_LU^*_{g_1f_1}C^{\phi l3}_{g_1g_2}$$



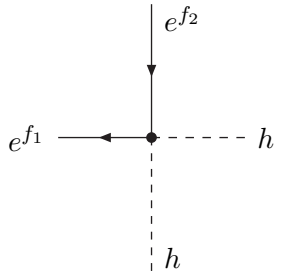
$$+\frac{iv}{\sqrt{2}}\left(P_L C^{e\phi *}_{f_2 f_1} + P_R C^{e\phi}_{f_1 f_2}\right) + i\left(\not{p}_3 P_L - \not{p}_4 P_L\right) C^{\phi l 1}_{f_1 f_2} - i\left(\not{p}_3 P_L - \not{p}_4 P_L\right) C^{\phi l 3}_{f_1 f_2} + i\left(\not{p}_3 P_R - \not{p}_4 P_R\right) C^{\phi e}_{f_1 f_2}$$



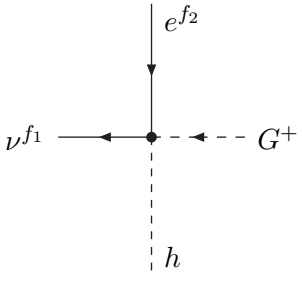
$$+\frac{iv}{\sqrt{2}}\left(P_L C^{e\phi *}_{f_2 f_1} + P_R C^{e\phi}_{f_1 f_2}\right)$$



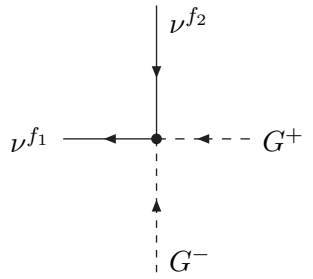
$$+\frac{v}{\sqrt{2}}\left(P_L C^{e\phi *}_{f_2 f_1} - P_R C^{e\phi}_{f_1 f_2}\right) - \left(\not{p}_3 P_L - \not{p}_4 P_L\right) C^{\phi l 1}_{f_1 f_2} - \left(\not{p}_3 P_L - \not{p}_4 P_L\right) C^{\phi l 3}_{f_1 f_2} - \left(\not{p}_3 P_R - \not{p}_4 P_R\right) C^{\phi e}_{f_1 f_2}$$



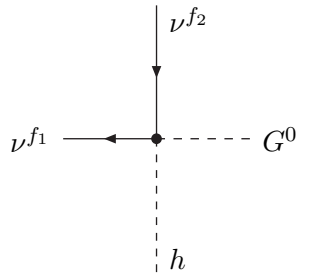
$$+\frac{3iv}{\sqrt{2}}\left(P_L C^{e\phi *}_{f_2 f_1} + P_R C^{e\phi}_{f_1 f_2}\right)$$



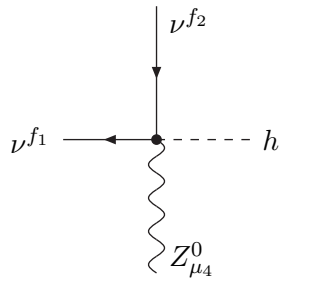
$$+ivP_R U^*_{g_1 f_1} C^{e\phi}_{g_1 f_2} + i\sqrt{2}\left(\not{p}_3 P_L - \not{p}_4 P_L\right) U^*_{g_1 f_1} C^{\phi l 3}_{g_1 f_2}$$



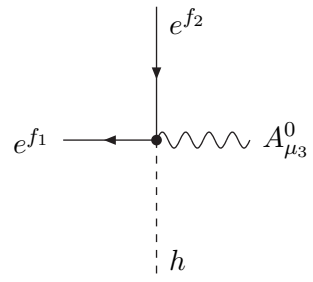
$$+iU_{g_2f_2}\left(\not{p}_3P_L-\not{p}_4P_L\right)U^*_{g_1f_1}C^{\phi l1}_{g_1g_2}+iU_{g_2f_2}\left(\not{p}_3P_L-\not{p}_4P_L\right)U^*_{g_1f_1}C^{\phi l3}_{g_1g_2}$$



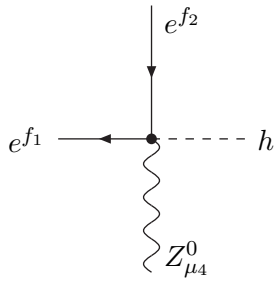
$$-U_{g_2f_2}\left(\not{p}_3P_L-\not{p}_4P_L\right)U^*_{g_1f_1}C^{\phi l1}_{g_1g_2}+U_{g_2f_2}\left(\not{p}_3P_L-\not{p}_4P_L\right)U^*_{g_1f_1}C^{\phi l3}_{g_1g_2}$$



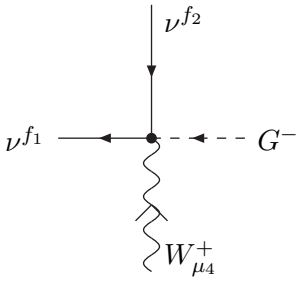
$$+iv\sqrt{\bar{g}^2+\bar{g}'^2}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l1}_{g_1g_2}\gamma^{\mu_4}P_L-iv\sqrt{\bar{g}^2+\bar{g}'^2}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l3}_{g_1g_2}\gamma^{\mu_4}P_L$$



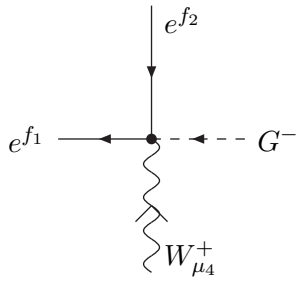
$$+\frac{\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{eW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$
$$-\frac{\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{eB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{eB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



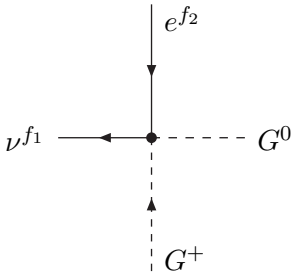
$$
\begin{aligned}
&+\frac{\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^{\nu}\left(C^{eW*}_{f_2f_1}\sigma^{\mu_4\nu}P_L+C^{eW}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^{\nu}\left(C^{eB*}_{f_2f_1}\sigma^{\mu_4\nu}P_L+C^{eB}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)\\
&+iv\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi l1}_{f_1f_2}\gamma^{\mu_4}P_L+iv\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi l3}_{f_1f_2}\gamma^{\mu_4}P_L\\
&+iv\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi e}_{f_1f_2}\gamma^{\mu_4}P_R
\end{aligned}
$$



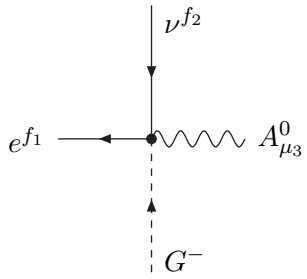
$$-i\bar{g}vU_{g_2f_2}U^*_{g_1f_1}C^{\phi l1}_{g_1g_2}\gamma^{\mu_4}P_L$$



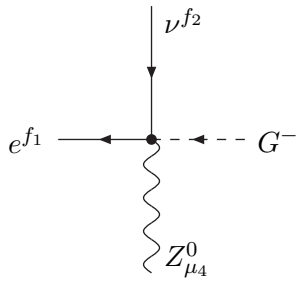
$$-2\sqrt{2}p_4^{\nu}C^{eW*}_{f_2f_1}\sigma^{\mu_4\nu}P_L-i\bar{g}vC^{\phi l1}_{f_1f_2}\gamma^{\mu_4}P_L-i\bar{g}vC^{\phi e}_{f_1f_2}\gamma^{\mu_4}P_R$$



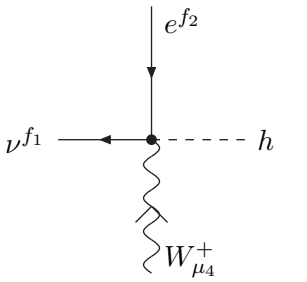
$$-\sqrt{2}\left(\not{p}_3P_L-\not{p}_4P_L\right)U^*_{g_1f_1}C^{\phi l3}_{g_1f_2}$$



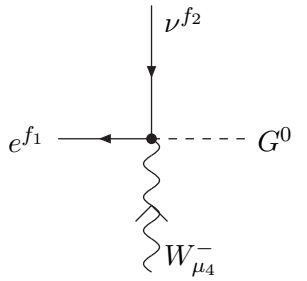
$$-\frac{2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu U_{g_1f_2}\sigma^{\mu_3\nu}P_L C^{eW*}_{g_1f_1} - \frac{2\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu U_{g_1f_2}\sigma^{\mu_3\nu}P_L C^{eB*}_{g_1f_1}$$
$$-\frac{i\sqrt{2}\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}C^{\phi l3}_{f_1g_1}\gamma^{\mu_3}P_L$$



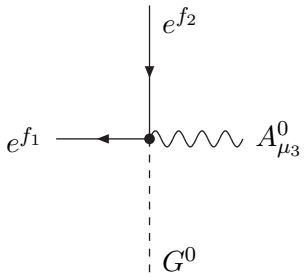
$$-\frac{2\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu U_{g_1f_2}\sigma^{\mu_4\nu}P_L C^{eW*}_{g_1f_1} + \frac{2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu U_{g_1f_2}\sigma^{\mu_4\nu}P_L C^{eB*}_{g_1f_1}$$
$$+\frac{i\sqrt{2}\bar{g}'^2 v}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}C^{\phi l3}_{f_1g_1}\gamma^{\mu_4}P_L$$



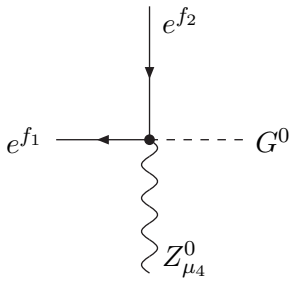
$$-2p_4^\nu U^*_{g_1f_1}C^{eW}_{g_1f_2}\sigma^{\mu_4\nu}P_R - i\sqrt{2}\bar{g}vU^*_{g_1f_1}C^{\phi l3}_{g_1f_2}\gamma^{\mu_4}P_L$$



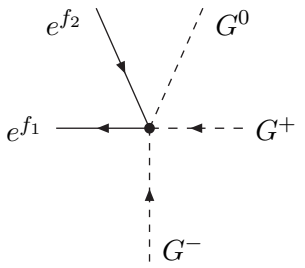
$$+2ip_4^\nu U_{g_1f_2}\sigma^{\mu_4\nu}P_L C^{eW*}_{g_1f_1}$$



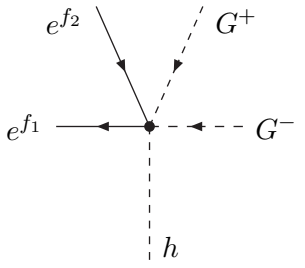
$$-\frac{i\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{eW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L - C^{eW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$
$$+\frac{i\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{eB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L - C^{eB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



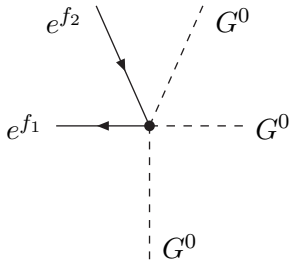
$$-\frac{i\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{eW*}_{f_2f_1}\sigma^{\mu_4\nu}P_L - C^{eW}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)$$
$$-\frac{i\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{eB*}_{f_2f_1}\sigma^{\mu_4\nu}P_L - C^{eB}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)$$



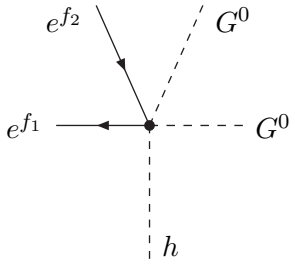
$$+\frac{1}{\sqrt{2}}\left(P_L C^{e\phi*}_{f_2f_1} - P_R C^{e\phi}_{f_1f_2}\right)$$



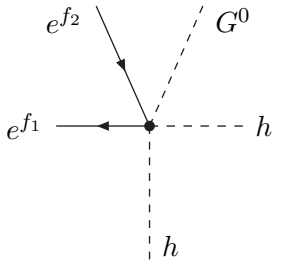
$$+\frac{i}{\sqrt{2}}\left(P_L C^{e\phi*}_{f_2f_1} + P_R C^{e\phi}_{f_1f_2}\right)$$



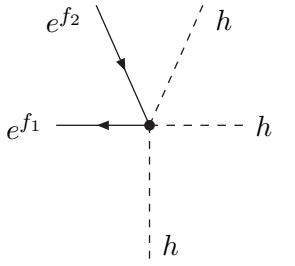
$$+\frac{3}{\sqrt{2}}\left(P_L C^{e\phi *}_{f_2 f_1} - P_R C^{e\phi}_{f_1 f_2}\right)$$



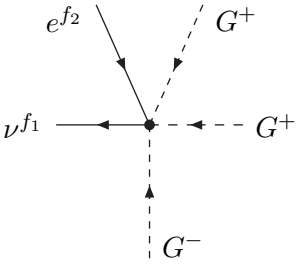
$$+\frac{i}{\sqrt{2}}\left(P_L C^{e\phi *}_{f_2 f_1} + P_R C^{e\phi}_{f_1 f_2}\right)$$



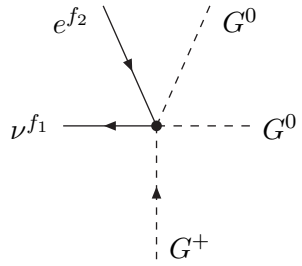
$$+\frac{1}{\sqrt{2}}\left(P_L C^{e\phi *}_{f_2 f_1} - P_R C^{e\phi}_{f_1 f_2}\right)$$



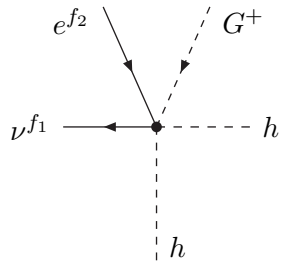
$$+\frac{3i}{\sqrt{2}}\left(P_L C^{e\phi *}_{f_2 f_1} + P_R C^{e\phi}_{f_1 f_2}\right)$$



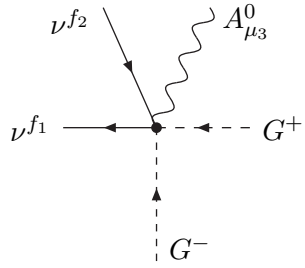
$$+2iP_R U^*_{g_1 f_1} C^{e\phi}_{g_1 f_2}$$



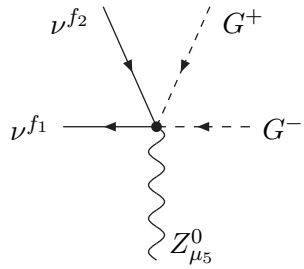
$$+iP_R U^*_{g_1 f_1} C^{e\phi}_{g_1 f_2}$$



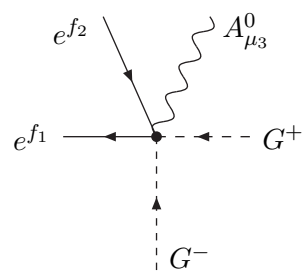
$$+iP_R U^*_{g_1 f_1} C^{e\phi}_{g_1 f_2}$$



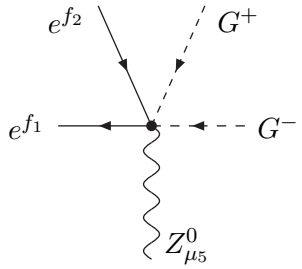
$$-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}} U_{g_2 f_2} U^*_{g_1 f_1} C^{\phi l1}_{g_1 g_2} \gamma^{\mu_3} P_L - \frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}} U_{g_2 f_2} U^*_{g_1 f_1} C^{\phi l3}_{g_1 g_2} \gamma^{\mu_3} P_L$$



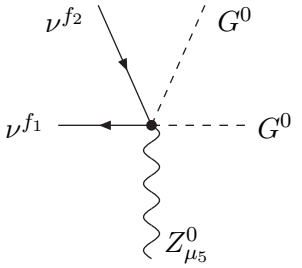
$$+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}} U_{g_2 f_2} U^*_{g_1 f_1} C^{\phi l1}_{g_1 g_2} \gamma^{\mu_5} P_L + \frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}} U_{g_2 f_2} U^*_{g_1 f_1} C^{\phi l3}_{g_1 g_2} \gamma^{\mu_5} P_L$$



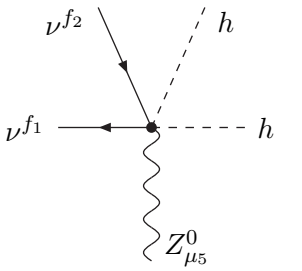
$$-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}} C^{\phi l1}_{f_1 f_2} \gamma^{\mu_3} P_L + \frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}} C^{\phi l3}_{f_1 f_2} \gamma^{\mu_3} P_L - \frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}} C^{\phi e}_{f_1 f_2} \gamma^{\mu_3} P_R$$



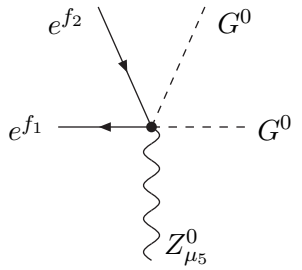
$$+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi l1}_{f_1f_2}\gamma^{\mu_5}P_L-\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi l3}_{f_1f_2}\gamma^{\mu_5}P_L$$
$$+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi e}_{f_1f_2}\gamma^{\mu_5}P_R$$



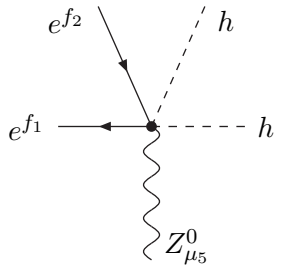
$$+i\sqrt{\bar{g}^2+\bar{g}'^2}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l1}_{g_1g_2}\gamma^{\mu_5}P_L-i\sqrt{\bar{g}^2+\bar{g}'^2}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l3}_{g_1g_2}\gamma^{\mu_5}P_L$$



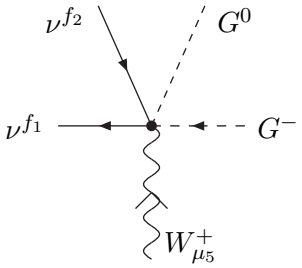
$$+i\sqrt{\bar{g}^2+\bar{g}'^2}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l1}_{g_1g_2}\gamma^{\mu_5}P_L-i\sqrt{\bar{g}^2+\bar{g}'^2}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l3}_{g_1g_2}\gamma^{\mu_5}P_L$$



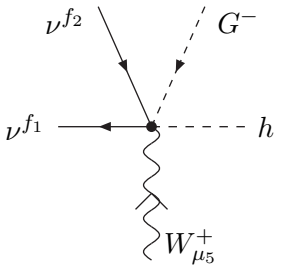
$$+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi l1}_{f_1f_2}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi l3}_{f_1f_2}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi e}_{f_1f_2}\gamma^{\mu_5}P_R$$



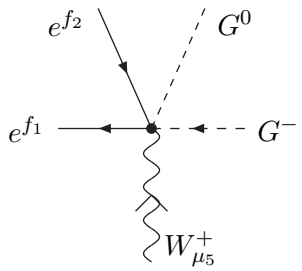
$$+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi l1}_{f_1f_2}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi l3}_{f_1f_2}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi e}_{f_1f_2}\gamma^{\mu_5}P_R$$



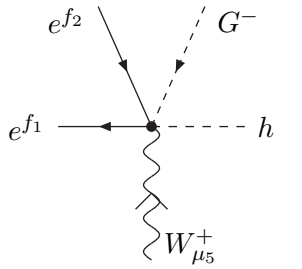
$$+\bar{g}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l1}_{g_1g_2}\gamma^{\mu_5}P_L$$



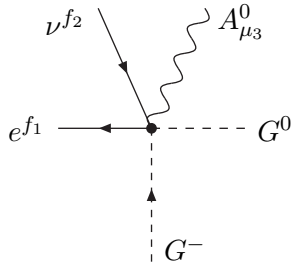
$$-i\bar{g}U_{g_2f_2}U^*_{g_1f_1}C^{\phi l1}_{g_1g_2}\gamma^{\mu_5}P_L$$



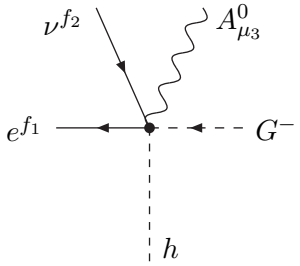
$$+\bar{g}C^{\phi l1}_{f_1f_2}\gamma^{\mu_5}P_L+\bar{g}C^{\phi e}_{f_1f_2}\gamma^{\mu_5}P_R$$



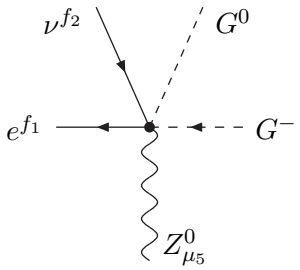
$$-i\bar{g}C^{\phi l1}_{f_1f_2}\gamma^{\mu_5}P_L-i\bar{g}C^{\phi e}_{f_1f_2}\gamma^{\mu_5}P_R$$



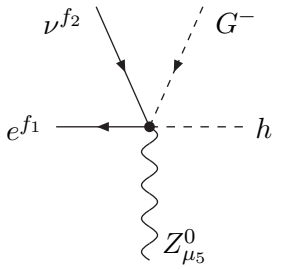
$$+\frac{\sqrt{2}\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}C^{\phi l3}_{f_1g_1}\gamma^{\mu_3}P_L$$



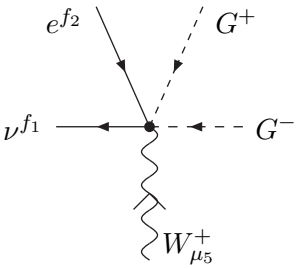
$$-\frac{i\sqrt{2}\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}C^{\phi l3}_{f_1g_1}\gamma^{\mu_3}P_L$$



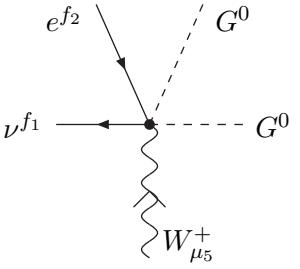
$$-\frac{\sqrt{2}\bar{g}'^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}C^{\phi l3}_{f_1g_1}\gamma^{\mu_5}P_L$$



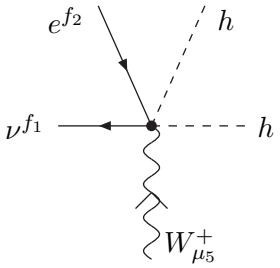
$$+\frac{i\sqrt{2}\bar{g}'^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}C^{\phi l3}_{f_1g_1}\gamma^{\mu_5}P_L$$



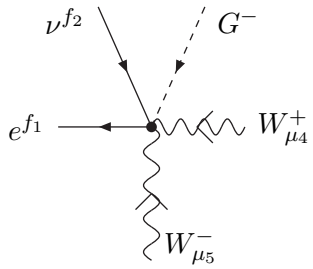
$$-i\sqrt{2}\bar{g}U^*_{g_1f_1}C^{\phi l3}_{g_1f_2}\gamma^{\mu_5}P_L$$



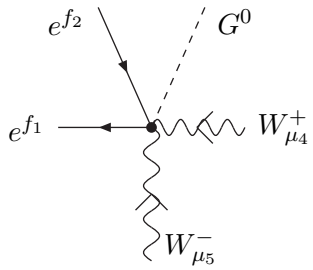
$$-i\sqrt{2}\bar{g}U^*_{g_1f_1}C^{\phi l3}_{g_1f_2}\gamma^{\mu_5}P_L$$



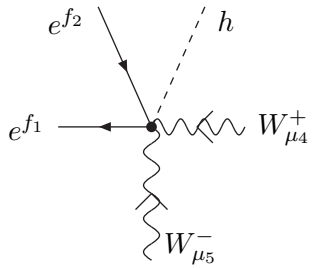
$$-i\sqrt{2}\bar{g}U^*_{g_1f_1}C^{\phi l3}_{g_1f_2}\gamma^{\mu_5}P_L$$



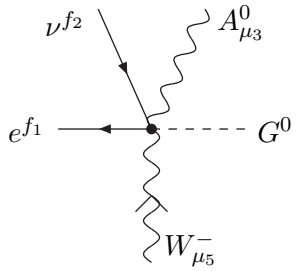
$$-2\bar{g}U_{g_1f_2}\sigma^{\mu_4\mu_5}P_LC^{eW*}_{g_1f_1}$$



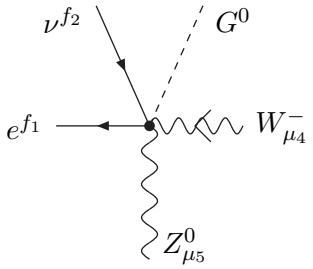
$$-i\sqrt{2}\bar{g}\left(\sigma^{\mu_4\mu_5}P_LC^{eW*}_{f_2f_1}-C^{eW}_{f_1f_2}\sigma^{\mu_4\mu_5}P_R\right)$$



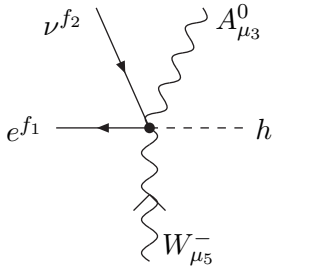
$$+\sqrt{2}\bar{g}\left(\sigma^{\mu_4\mu_5}P_LC^{eW*}_{f_2f_1}+C^{eW}_{f_1f_2}\sigma^{\mu_4\mu_5}P_R\right)$$



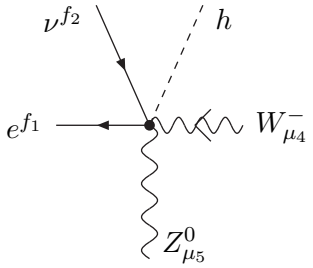
$$-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}\sigma^{\mu_3\mu_5}P_LC^{eW*}_{g_1f_1}$$



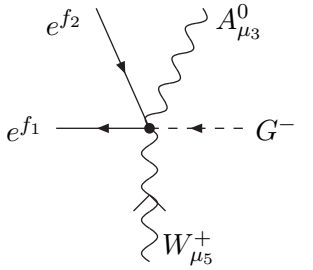
$$+\frac{2i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}\sigma^{\mu_4\mu_5}P_LC^{eW*}_{g_1f_1}$$



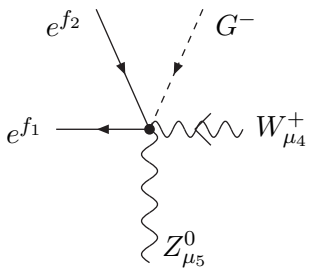
$$+\frac{2\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}\sigma^{\mu_3\mu_5}P_LC^{eW*}_{g_1f_1}$$



$$-\frac{2\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}U_{g_1f_2}\sigma^{\mu_4\mu_5}P_LC^{eW*}_{g_1f_1}$$

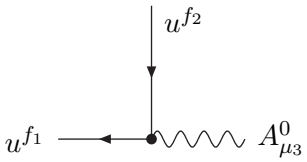


$$-\frac{2\sqrt{2}\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\sigma^{\mu_3\mu_5}P_LC^{eW*}_{f_2f_1}$$

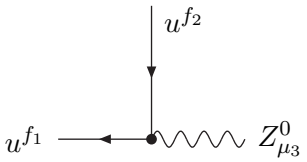


$$+\frac{2\sqrt{2}\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}\sigma^{\mu_4\mu_5}P_LC^{eW*}_{f_2f_1}$$

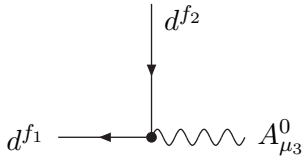
A.4 Quark–gauge vertices



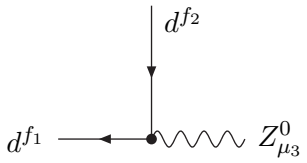
$$
\begin{aligned}
&-\frac{2i\bar{g}\bar{g}'}{3\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\gamma^{\mu_3}+\frac{2i\bar{g}^2\bar{g}'^2v^2}{3\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\gamma^{\mu_3}\\
&-\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{uW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{uW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&-\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{uB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{uB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)
\end{aligned}
$$



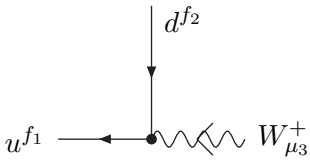
$$
\begin{aligned}
&+\frac{i}{6\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\left(\left(\bar{g}'^2-3\bar{g}^2\right)\gamma^{\mu_3}P_L+4\bar{g}'^2\gamma^{\mu_3}P_R\right)\\
&-\frac{i\bar{g}\bar{g}'v^2}{6\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\left(\left(3\bar{g}'^2-\bar{g}^2\right)\gamma^{\mu_3}P_L-4\bar{g}^2\gamma^{\mu_3}P_R\right)\\
&-\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{uW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{uW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{uB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{uB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}\gamma^{\mu_3}P_L\\
&-\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q3}_{g_2g_1}\gamma^{\mu_3}P_L\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi u}_{f_1f_2}\gamma^{\mu_3}P_R
\end{aligned}
$$



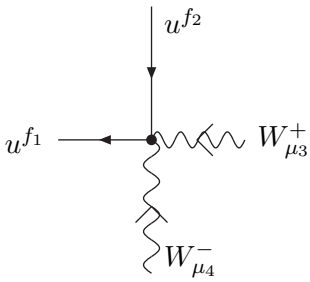
$$
\begin{aligned}
&+\frac{i\bar{g}\bar{g}'}{3\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\gamma^{\mu_3}-\frac{i\bar{g}^2\bar{g}'^2v^2}{3\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\gamma^{\mu_3}\\
&+\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{dW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{dW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&-\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{dB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{dB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)
\end{aligned}
$$



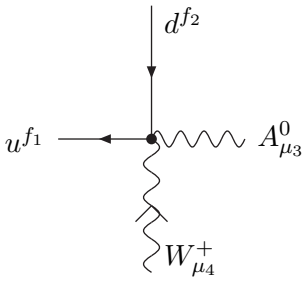
$$
\begin{aligned}
&+\frac{i}{6\sqrt{\bar{g}^2+\bar{g}'^2}}\delta_{f_1f_2}\left(\left(3\bar{g}^2+\bar{g}'^2\right)\gamma^{\mu_3}P_L-2\bar{g}'^2\gamma^{\mu_3}P_R\right)\\
&+\frac{i\bar{g}\bar{g}'v^2}{6\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}\delta_{f_1f_2}C^{\phi WB}\left(\left(\bar{g}^2+3\bar{g}'^2\right)\gamma^{\mu_3}P_L-2\bar{g}^2\gamma^{\mu_3}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{dW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{dW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^{\nu}\left(C^{dB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L+C^{dB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q1}_{f_1f_2}\gamma^{\mu_3}P_L+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q3}_{f_1f_2}\gamma^{\mu_3}P_L\\
&+\frac{1}{2}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi d}_{f_1f_2}\gamma^{\mu_3}P_R
\end{aligned}
$$



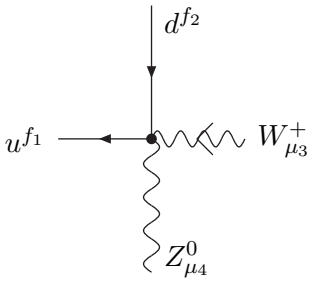
$$
\begin{aligned}
&-\frac{i\bar{g}}{\sqrt{2}}K_{f_1f_2}\gamma^{\mu_3}P_L-2vp_3^{\nu}K_{g_1f_2}\sigma^{\mu_3\nu}P_LC^{uW*}_{g_1f_1}\\
&-2vp_3^{\nu}K_{f_1g_1}C^{dW}_{g_1f_2}\sigma^{\mu_3\nu}P_R-\frac{i\bar{g}v^2}{2\sqrt{2}}C^{\phi ud}_{f_1f_2}\gamma^{\mu_3}P_R
\end{aligned}
$$



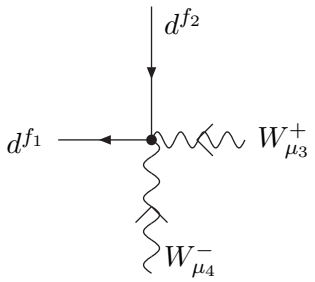
$$
-\sqrt{2}\bar{g}v\left(\sigma^{\mu_3\mu_4}P_LC^{uW*}_{f_2f_1}+C^{uW}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)
$$



$$
-\frac{2\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{g_1f_2}\sigma^{\mu_3\mu_4}P_LC^{uW*}_{g_1f_1}-\frac{2\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_1}\sigma^{\mu_3\mu_4}P_RC^{dW}_{g_1f_2}
$$

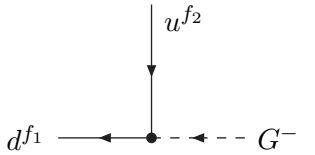


$$+\frac{2\bar{g}^2 v}{\sqrt{\bar{g}^2+\bar{g}'^2}} K_{g_1 f_2} \sigma^{\mu_3\mu_4} P_L C^{uW*}_{g_1 f_1} + \frac{2\bar{g}^2 v}{\sqrt{\bar{g}^2+\bar{g}'^2}} K_{f_1 g_1} \sigma^{\mu_3\mu_4} P_R C^{dW}_{g_1 f_2}$$

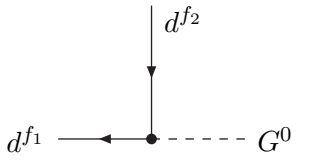


$$+\sqrt{2}\bar{g} v \left(\sigma^{\mu_3\mu_4} P_L C^{dW*}_{f_2 f_1} + C^{dW}_{f_1 f_2} \sigma^{\mu_3\mu_4} P_R \right)$$

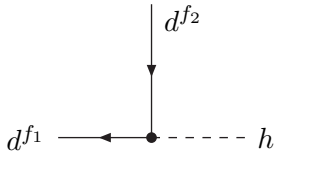
A.5 Quark–Higgs–gauge vertices



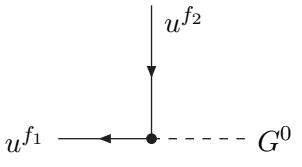
$$\begin{aligned} &-\frac{i\sqrt{2}}{v} K^*_{f_2 f_1} \left(m_{d_{f_1}} P_L - m_{u_{f_2}} P_R \right) \\ &+ i\sqrt{2} v \left(K^*_{f_2 g_2} C^{\phi q 3}_{f_1 g_2} \left(m_{d_{f_1}} P_L - m_{u_{f_2}} P_R \right) - \not{p}_3 P_L K^*_{f_2 g_1} C^{\phi q 3}_{f_1 g_1} \right) \\ &- \frac{iv}{\sqrt{2}} \not{p}_3 P_R C^{\phi u d *}_{f_2 f_1} \end{aligned}$$



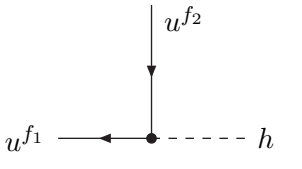
$$\begin{aligned} &+\frac{1}{v} \delta_{f_1 f_2} m_{d_{f_1}} \gamma^5 - \frac{v}{4} \delta_{f_1 f_2} C^{\phi D} m_{d_{f_1}} \gamma^5 \\ &- v \not{p}_3 P_L C^{\phi q 1}_{f_1 f_2} - v \not{p}_3 P_L C^{\phi q 3}_{f_1 f_2} - v \not{p}_3 P_R C^{\phi d}_{f_1 f_2} \end{aligned}$$



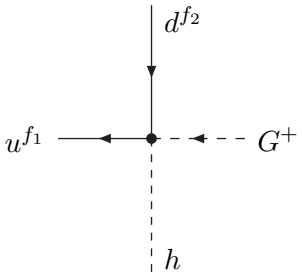
$$\begin{aligned} &-\frac{i}{v} \delta_{f_1 f_2} m_{d_{f_1}} - iv \delta_{f_1 f_2} C^{\phi \Box} m_{d_{f_1}} \\ &+ \frac{iv}{4} \delta_{f_1 f_2} C^{\phi D} m_{d_{f_1}} + \frac{iv^2}{\sqrt{2}} \left(P_L C^{d\phi *}_{f_2 f_1} + P_R C^{d\phi}_{f_1 f_2} \right) \end{aligned}$$



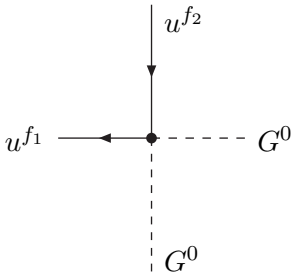
$$-\frac{1}{v}\delta_{f_1f_2}m_{u_{f_1}}\gamma^5+\frac{v}{4}\delta_{f_1f_2}C^{\phi D}m_{u_{f_1}}\gamma^5-vK_{f_1g_2}\not{p}_3P_LK^*_{f_2g_1}C^{\phi q1}_{g_2g_1}$$
$$+vK_{f_1g_2}\not{p}_3P_LK^*_{f_2g_1}C^{\phi q3}_{g_2g_1}-v\not{p}_3P_RC^{\phi u}_{f_1f_2}$$



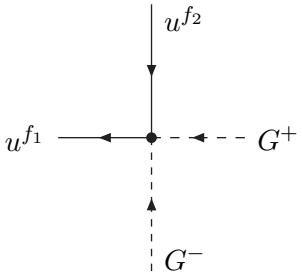
$$-\frac{i}{v}\delta_{f_1f_2}m_{u_{f_1}}-iv\delta_{f_1f_2}C^{\phi\Box}m_{u_{f_1}}$$
$$+\frac{iv}{4}\delta_{f_1f_2}C^{\phi D}m_{u_{f_1}}+\frac{iv^2}{\sqrt{2}}\left(P_LC^{u\phi *}_{f_2f_1}+P_RC^{u\phi}_{f_1f_2}\right)$$



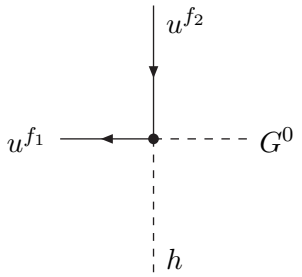
$$+ivP_RK_{f_1g_1}C^{d\phi}_{g_1f_2}-ivP_LK_{g_1f_2}C^{u\phi *}_{g_1f_1}$$
$$+i\sqrt{2}K_{f_1g_1}\left(\not{p}_3P_L-\not{p}_4P_L\right)C^{\phi q3}_{g_1f_2}+\frac{i}{\sqrt{2}}\left(\not{p}_3P_R-\not{p}_4P_R\right)C^{\phi ud}_{f_1f_2}$$



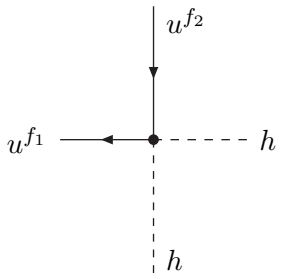
$$+\frac{iv}{\sqrt{2}}\left(P_LC^{u\phi *}_{f_2f_1}+P_RC^{u\phi}_{f_1f_2}\right)$$



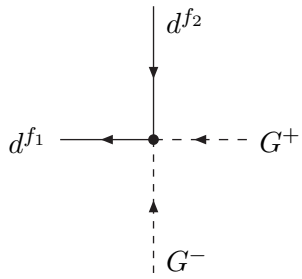
$$+\frac{iv}{\sqrt{2}}\left(P_LC^{u\phi *}_{f_2f_1}+P_RC^{u\phi}_{f_1f_2}\right)+iK_{f_1g_2}\left(\not{p}_3P_L-\not{p}_4P_L\right)K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}$$
$$+iK_{f_1g_2}\left(\not{p}_3P_L-\not{p}_4P_L\right)K^*_{f_2g_1}C^{\phi q3}_{g_2g_1}+i\left(\not{p}_3P_R-\not{p}_4P_R\right)C^{\phi u}_{f_1f_2}$$



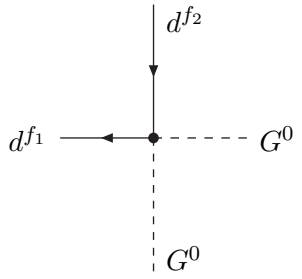
$$-\frac{v}{\sqrt{2}}\left(P_L C^{u\phi *}_{f_2 f_1} - P_R C^{u\phi}_{f_1 f_2}\right) - K_{f_1 g_2}\left(\not{p}_3 P_L - \not{p}_4 P_L\right) K^*_{f_2 g_1} C^{\phi q 1}_{g_2 g_1}$$
$$+ K_{f_1 g_2}\left(\not{p}_3 P_L - \not{p}_4 P_L\right) K^*_{f_2 g_1} C^{\phi q 3}_{g_2 g_1} - \left(\not{p}_3 P_R - \not{p}_4 P_R\right) C^{\phi u}_{f_1 f_2}$$



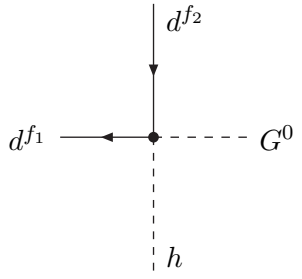
$$+\frac{3iv}{\sqrt{2}}\left(P_L C^{u\phi *}_{f_2 f_1} + P_R C^{u\phi}_{f_1 f_2}\right)$$



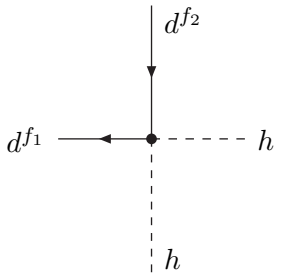
$$+\frac{iv}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} + P_R C^{d\phi}_{f_1 f_2}\right) + i\left(\not{p}_3 P_L - \not{p}_4 P_L\right) C^{\phi q 1}_{f_1 f_2}$$
$$- i\left(\not{p}_3 P_L - \not{p}_4 P_L\right) C^{\phi q 3}_{f_1 f_2} + i\left(\not{p}_3 P_R - \not{p}_4 P_R\right) C^{\phi d}_{f_1 f_2}$$



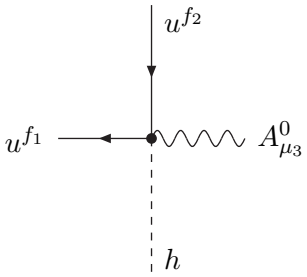
$$+\frac{iv}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} + P_R C^{d\phi}_{f_1 f_2}\right)$$



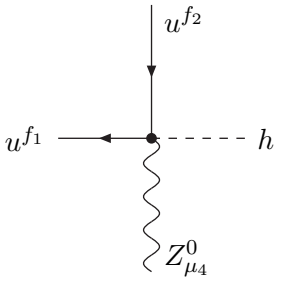
$$+\frac{v}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} - P_R C^{d\phi}_{f_1 f_2}\right) - \left(\not{p}_3 P_L - \not{p}_4 P_L\right) C^{\phi q 1}_{f_1 f_2}$$
$$- \left(\not{p}_3 P_L - \not{p}_4 P_L\right) C^{\phi q 3}_{f_1 f_2} - \left(\not{p}_3 P_R - \not{p}_4 P_R\right) C^{\phi d}_{f_1 f_2}$$



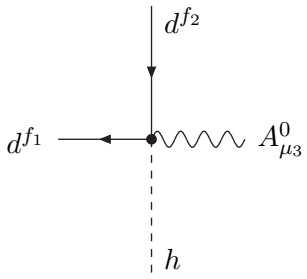
$$+\frac{3iv}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} + P_R C^{d\phi}_{f_1 f_2}\right)$$



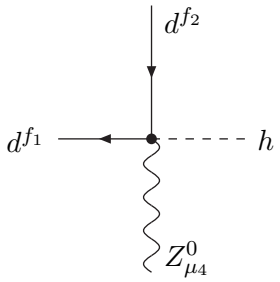
$$-\frac{\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{uW*}_{f_2 f_1}\sigma^{\mu_3\nu}P_L + C^{uW}_{f_1 f_2}\sigma^{\mu_3\nu}P_R\right)$$
$$-\frac{\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{uB*}_{f_2 f_1}\sigma^{\mu_3\nu}P_L + C^{uB}_{f_1 f_2}\sigma^{\mu_3\nu}P_R\right)$$



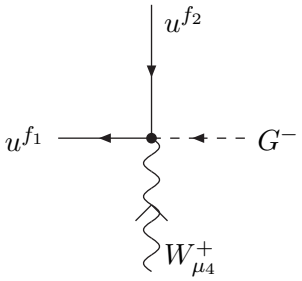
$$-\frac{\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{uW*}_{f_2 f_1}\sigma^{\mu_4\nu}P_L + C^{uW}_{f_1 f_2}\sigma^{\mu_4\nu}P_R\right)$$
$$+\frac{\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{uB*}_{f_2 f_1}\sigma^{\mu_4\nu}P_L + C^{uB}_{f_1 f_2}\sigma^{\mu_4\nu}P_R\right)$$
$$+iv\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1 g_2}K^*_{f_2 g_1}C^{\phi q1}_{g_2 g_1}\gamma^{\mu_4}P_L$$
$$-iv\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1 g_2}K^*_{f_2 g_1}C^{\phi q3}_{g_2 g_1}\gamma^{\mu_4}P_L + iv\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi u}_{f_1 f_2}\gamma^{\mu_4}P_R$$



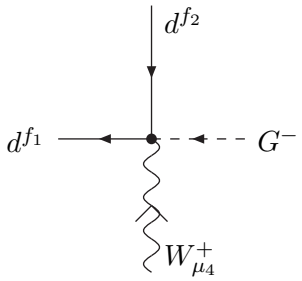
$$+\frac{\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{dW*}_{f_2 f_1}\sigma^{\mu_3\nu}P_L + C^{dW}_{f_1 f_2}\sigma^{\mu_3\nu}P_R\right)$$
$$-\frac{\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{dB*}_{f_2 f_1}\sigma^{\mu_3\nu}P_L + C^{dB}_{f_1 f_2}\sigma^{\mu_3\nu}P_R\right)$$



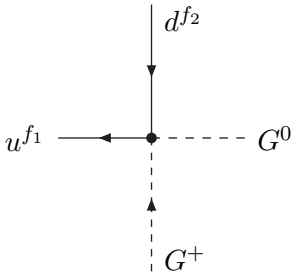
$$
\begin{aligned}
&+\frac{\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{dW*}_{f_2f_1}\sigma^{\mu_4\nu}P_L+C^{dW}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)\\
&+\frac{\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{dB*}_{f_2f_1}\sigma^{\mu_4\nu}P_L+C^{dB}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)\\
&+iv\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q1}_{f_1f_2}\gamma^{\mu_4}P_L+iv\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q3}_{f_1f_2}\gamma^{\mu_4}P_L\\
&+iv\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi d}_{f_1f_2}\gamma^{\mu_4}P_R
\end{aligned}
$$



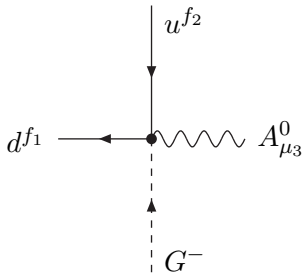
$$+2\sqrt{2}p_4^\nu C^{uW}_{f_1f_2}\sigma^{\mu_4\nu}P_R-i\bar{g}vK_{f_1g_2}K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}\gamma^{\mu_4}P_L-i\bar{g}vC^{\phi u}_{f_1f_2}\gamma^{\mu_4}P_R$$



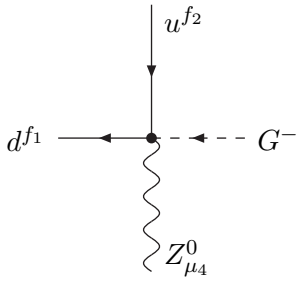
$$-2\sqrt{2}p_4^\nu C^{dW*}_{f_2f_1}\sigma^{\mu_4\nu}P_L-i\bar{g}vC^{\phi q1}_{f_1f_2}\gamma^{\mu_4}P_L-i\bar{g}vC^{\phi d}_{f_1f_2}\gamma^{\mu_4}P_R$$



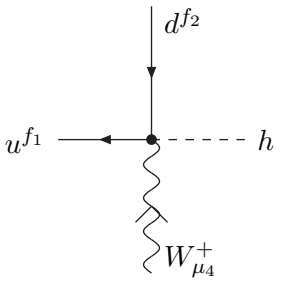
$$-\sqrt{2}K_{f_1g_1}\left(\not{p}_3P_L-\not{p}_4P_L\right)C^{\phi q3}_{g_1f_2}+\frac{1}{\sqrt{2}}\left(\not{p}_3P_R-\not{p}_4P_R\right)C^{\phi ud}_{f_1f_2}$$



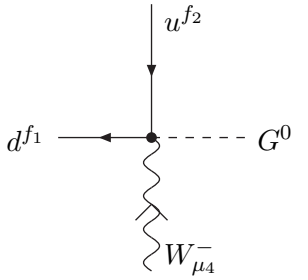
$$-\frac{2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu K^*_{g_1f_1}C^{uW}_{g_1f_2}\sigma^{\mu_3\nu}P_R+\frac{2\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu K^*_{g_1f_1}C^{uB}_{g_1f_2}\sigma^{\mu_3\nu}P_R$$
$$-\frac{2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu K^*_{f_2g_1}\sigma^{\mu_3\nu}P_L C^{dW*}_{g_1f_1}-\frac{2\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu K^*_{f_2g_1}\sigma^{\mu_3\nu}P_L C^{dB*}_{g_1f_1}$$
$$-\frac{i\sqrt{2}\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}K^*_{f_2g_1}C^{\phi q3}_{f_1g_1}\gamma^{\mu_3}P_L-\frac{i\bar{g}\bar{g}'v}{\sqrt{2}\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi ud*}_{f_2f_1}\gamma^{\mu_3}P_R$$



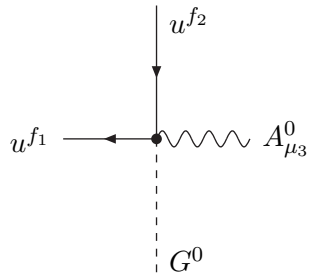
$$-\frac{2\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu K^*_{g_1f_1}C^{uW}_{g_1f_2}\sigma^{\mu_4\nu}P_R-\frac{2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu K^*_{g_1f_1}C^{uB}_{g_1f_2}\sigma^{\mu_4\nu}P_R$$
$$-\frac{2\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu K^*_{f_2g_1}\sigma^{\mu_4\nu}P_L C^{dW*}_{g_1f_1}+\frac{2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu K^*_{f_2g_1}\sigma^{\mu_4\nu}P_L C^{dB*}_{g_1f_1}$$
$$+\frac{i\sqrt{2}\bar{g}'^2v}{\sqrt{\bar{g}^2+\bar{g}'^2}}K^*_{f_2g_1}C^{\phi q3}_{f_1g_1}\gamma^{\mu_4}P_L-\frac{i\bar{g}^2v}{\sqrt{2}\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi ud*}_{f_2f_1}\gamma^{\mu_4}P_R$$



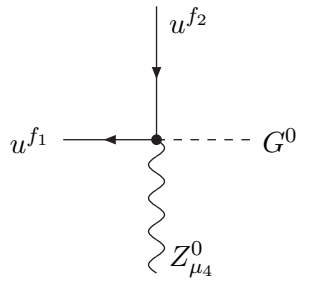
$$-2p_4^\nu K_{g_1f_2}\sigma^{\mu_4\nu}P_L C^{uW*}_{g_1f_1}-2p_4^\nu K_{f_1g_1}C^{dW}_{g_1f_2}\sigma^{\mu_4\nu}P_R$$
$$-i\sqrt{2}\bar{g}vK_{f_1g_1}C^{\phi q3}_{g_1f_2}\gamma^{\mu_4}P_L-\frac{i\bar{g}v}{\sqrt{2}}C^{\phi ud}_{f_1f_2}\gamma^{\mu_4}P_R$$



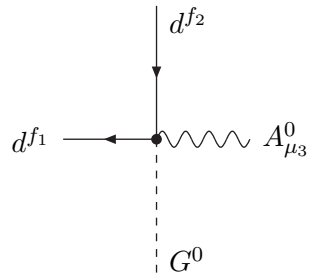
$$+2ip_4^\nu K^*_{g_1f_1}C^{uW}_{g_1f_2}\sigma^{\mu_4\nu}P_R+2ip_4^\nu K^*_{f_2g_1}\sigma^{\mu_4\nu}P_L C^{dW*}_{g_1f_1}-\frac{\bar{g}v}{\sqrt{2}}C^{\phi ud*}_{f_2f_1}\gamma^{\mu_4}P_R$$



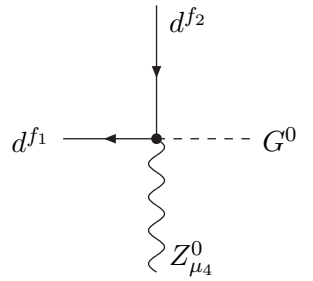
$$-\frac{i\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{uW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L - C^{uW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$
$$-\frac{i\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{uB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L - C^{uB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



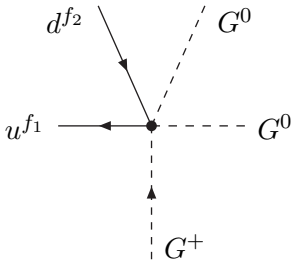
$$-\frac{i\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{uW*}_{f_2f_1}\sigma^{\mu_4\nu}P_L - C^{uW}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)$$
$$+\frac{i\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{uB*}_{f_2f_1}\sigma^{\mu_4\nu}P_L - C^{uB}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)$$



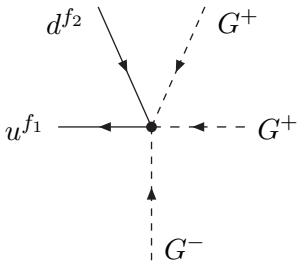
$$-\frac{i\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{dW*}_{f_2f_1}\sigma^{\mu_3\nu}P_L - C^{dW}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$
$$+\frac{i\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_3^\nu\left(C^{dB*}_{f_2f_1}\sigma^{\mu_3\nu}P_L - C^{dB}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



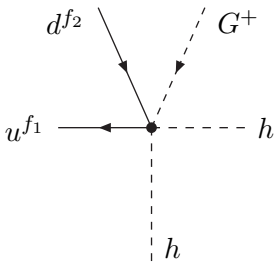
$$-\frac{i\sqrt{2}\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{dW*}_{f_2f_1}\sigma^{\mu_4\nu}P_L - C^{dW}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)$$
$$-\frac{i\sqrt{2}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_4^\nu\left(C^{dB*}_{f_2f_1}\sigma^{\mu_4\nu}P_L - C^{dB}_{f_1f_2}\sigma^{\mu_4\nu}P_R\right)$$



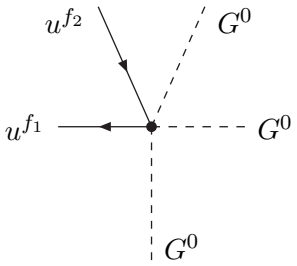
$$+iP_R K_{f_1 g_1} C^{d\phi}_{g_1 f_2} - iP_L K_{g_1 f_2} C^{u\phi *}_{g_1 f_1}$$



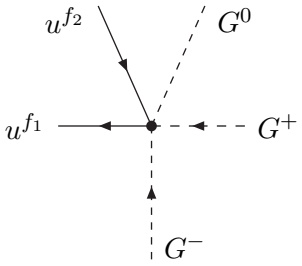
$$+2iP_R K_{f_1 g_1} C^{d\phi}_{g_1 f_2} - 2iP_L K_{g_1 f_2} C^{u\phi *}_{g_1 f_1}$$



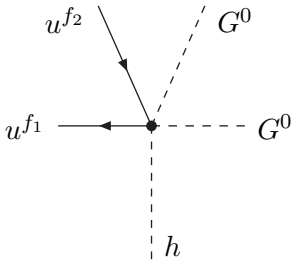
$$+iP_R K_{f_1 g_1} C^{d\phi}_{g_1 f_2} - iP_L K_{g_1 f_2} C^{u\phi *}_{g_1 f_1}$$



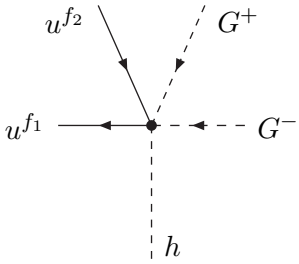
$$-\frac{3}{\sqrt{2}}\left(P_L C^{u\phi *}_{f_2 f_1} - P_R C^{u\phi}_{f_1 f_2}\right)$$



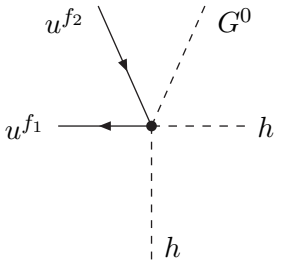
$$-\frac{1}{\sqrt{2}}\left(P_L C^{u\phi *}_{f_2 f_1} - P_R C^{u\phi}_{f_1 f_2}\right)$$



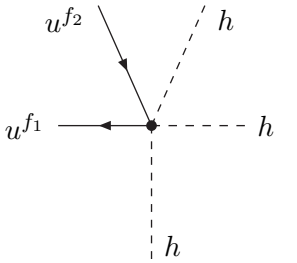
$$+\frac{i}{\sqrt{2}}\left(P_L C^{u\phi *}_{f_2 f_1} + P_R C^{u\phi}_{f_1 f_2}\right)$$



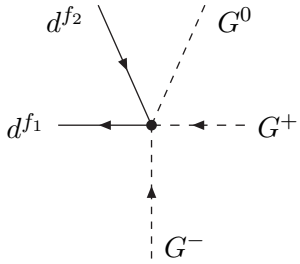
$$+\frac{i}{\sqrt{2}}\left(P_L C^{u\phi *}_{f_2 f_1} + P_R C^{u\phi}_{f_1 f_2}\right)$$



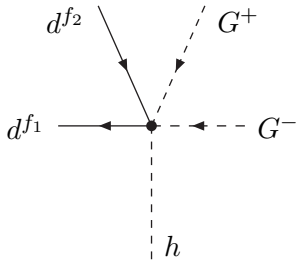
$$-\frac{1}{\sqrt{2}}\left(P_L C^{u\phi *}_{f_2 f_1} - P_R C^{u\phi}_{f_1 f_2}\right)$$



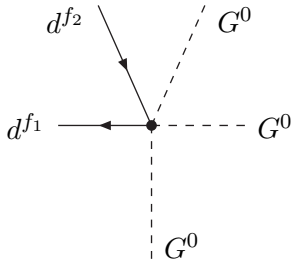
$$+\frac{3i}{\sqrt{2}}\left(P_L C^{u\phi *}_{f_2 f_1} + P_R C^{u\phi}_{f_1 f_2}\right)$$



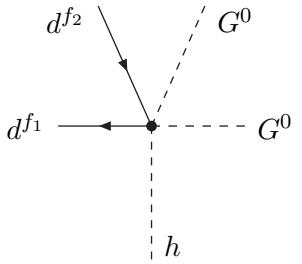
$$+\frac{1}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} - P_R C^{d\phi}_{f_1 f_2}\right)$$



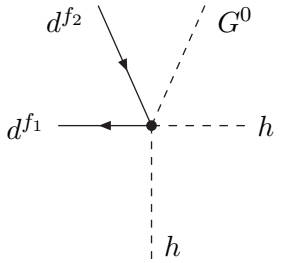
$$+\frac{i}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} + P_R C^{d\phi}_{f_1 f_2}\right)$$



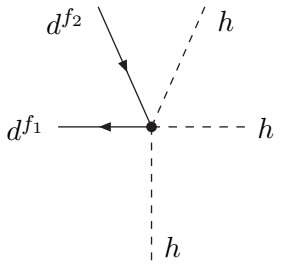
$$+\frac{3}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} - P_R C^{d\phi}_{f_1 f_2}\right)$$



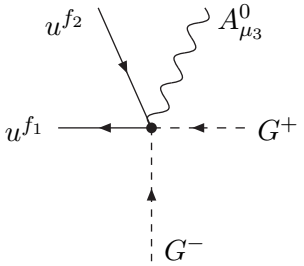
$$+\frac{i}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} + P_R C^{d\phi}_{f_1 f_2}\right)$$



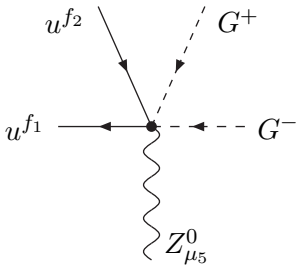
$$+\frac{1}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} - P_R C^{d\phi}_{f_1 f_2}\right)$$



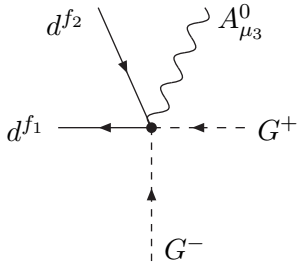
$$+\frac{3i}{\sqrt{2}}\left(P_L C^{d\phi *}_{f_2 f_1} + P_R C^{d\phi}_{f_1 f_2}\right)$$



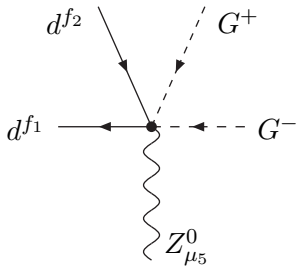
$$-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}\gamma^{\mu_3}P_L$$
$$-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q3}_{g_2g_1}\gamma^{\mu_3}P_L-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi u}_{f_1f_2}\gamma^{\mu_3}P_R$$



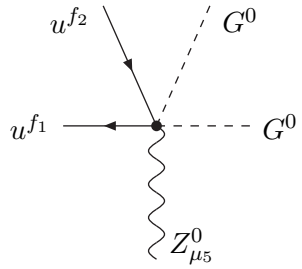
$$+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}\gamma^{\mu_5}P_L$$
$$+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q3}_{g_2g_1}\gamma^{\mu_5}P_L+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi u}_{f_1f_2}\gamma^{\mu_5}P_R$$



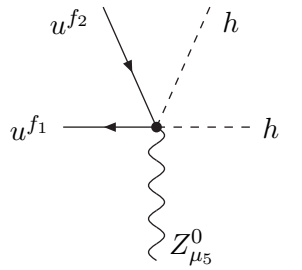
$$-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi q1}_{f_1f_2}\gamma^{\mu_3}P_L+\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi q3}_{f_1f_2}\gamma^{\mu_3}P_L-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi d}_{f_1f_2}\gamma^{\mu_3}P_R$$



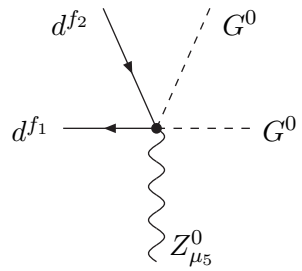
$$+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi q1}_{f_1f_2}\gamma^{\mu_5}P_L-\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi q3}_{f_1f_2}\gamma^{\mu_5}P_L$$
$$+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi d}_{f_1f_2}\gamma^{\mu_5}P_R$$



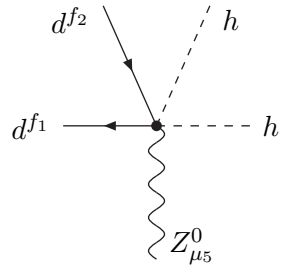
$$+i\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}\gamma^{\mu_5}P_L$$
$$-i\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q3}_{g_2g_1}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi u}_{f_1f_2}\gamma^{\mu_5}P_R$$



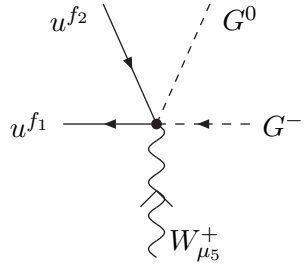
$$+i\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}\gamma^{\mu_5}P_L$$
$$-i\sqrt{\bar{g}^2+\bar{g}'^2}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q3}_{g_2g_1}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi u}_{f_1f_2}\gamma^{\mu_5}P_R$$



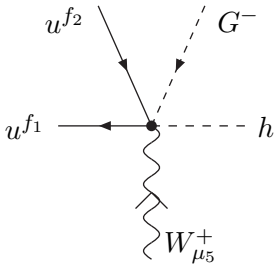
$$+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q1}_{f_1f_2}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q3}_{f_1f_2}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi d}_{f_1f_2}\gamma^{\mu_5}P_R$$



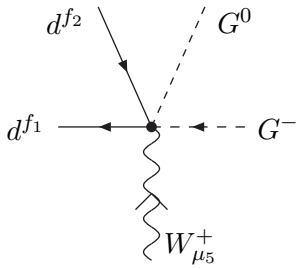
$$+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q1}_{f_1f_2}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi q3}_{f_1f_2}\gamma^{\mu_5}P_L+i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi d}_{f_1f_2}\gamma^{\mu_5}P_R$$



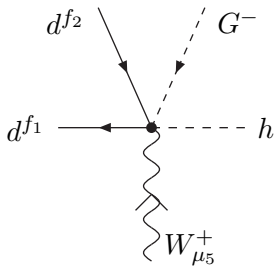
$$+\bar{g}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}\gamma^{\mu_5}P_L+\bar{g}C^{\phi u}_{f_1f_2}\gamma^{\mu_5}P_R$$



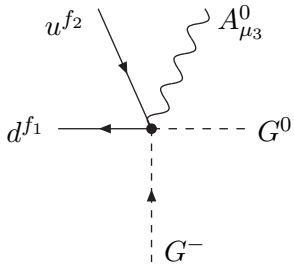
$$-i\bar{g}K_{f_1g_2}K^*_{f_2g_1}C^{\phi q1}_{g_2g_1}\gamma^{\mu_5}P_L - i\bar{g}C^{\phi u}_{f_1f_2}\gamma^{\mu_5}P_R$$



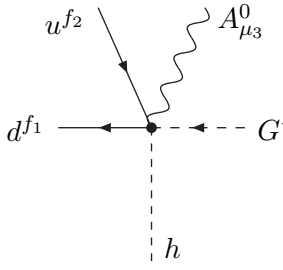
$$+\bar{g}C^{\phi q1}_{f_1f_2}\gamma^{\mu_5}P_L + \bar{g}C^{\phi d}_{f_1f_2}\gamma^{\mu_5}P_R$$



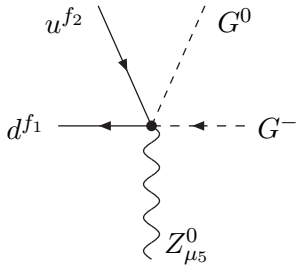
$$-i\bar{g}C^{\phi q1}_{f_1f_2}\gamma^{\mu_5}P_L - i\bar{g}C^{\phi d}_{f_1f_2}\gamma^{\mu_5}P_R$$



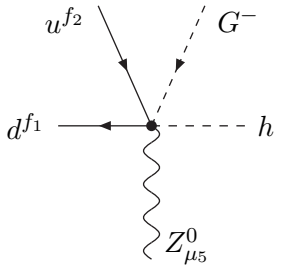
$$+\frac{\sqrt{2}\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}K^*_{f_2g_1}C^{\phi q3}_{f_1g_1}\gamma^{\mu_3}P_L - \frac{\bar{g}\bar{g}'}{\sqrt{2}\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi ud*}_{f_2f_1}\gamma^{\mu_3}P_R$$



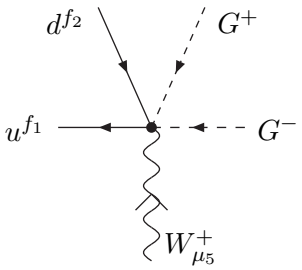
$$-\frac{i\sqrt{2}\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}K^*_{f_2g_1}C^{\phi q3}_{f_1g_1}\gamma^{\mu_3}P_L - \frac{i\bar{g}\bar{g}'}{\sqrt{2}\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi ud*}_{f_2f_1}\gamma^{\mu_3}P_R$$



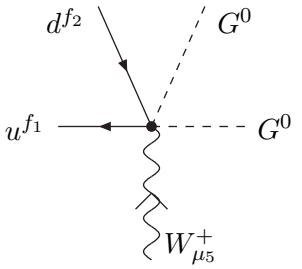
$$-\frac{\sqrt{2}\bar{g}'^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}K^*_{f_2g_1}C^{\phi q3}_{f_1g_1}\gamma^{\mu_5}P_L-\frac{\bar{g}^2}{\sqrt{2}\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi ud*}_{f_2f_1}\gamma^{\mu_5}P_R$$



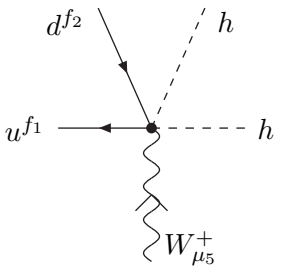
$$+\frac{i\sqrt{2}\bar{g}'^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}K^*_{f_2g_1}C^{\phi q3}_{f_1g_1}\gamma^{\mu_5}P_L-\frac{i\bar{g}^2}{\sqrt{2}\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi ud*}_{f_2f_1}\gamma^{\mu_5}P_R$$



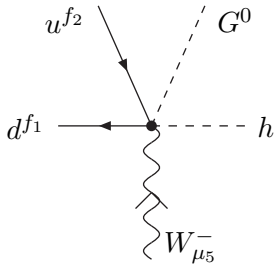
$$-i\sqrt{2}\bar{g}K_{f_1g_1}C^{\phi q3}_{g_1f_2}\gamma^{\mu_5}P_L$$



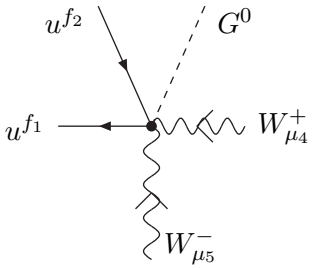
$$-i\sqrt{2}\bar{g}K_{f_1g_1}C^{\phi q3}_{g_1f_2}\gamma^{\mu_5}P_L+\frac{i\bar{g}}{\sqrt{2}}C^{\phi ud}_{f_1f_2}\gamma^{\mu_5}P_R$$



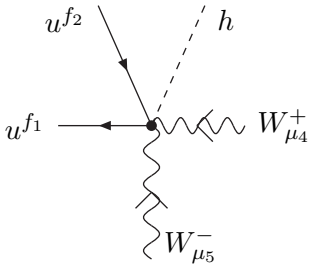
$$-i\sqrt{2}\bar{g}K_{f_1g_1}C^{\phi q3}_{g_1f_2}\gamma^{\mu_5}P_L-\frac{i\bar{g}}{\sqrt{2}}C^{\phi ud}_{f_1f_2}\gamma^{\mu_5}P_R$$



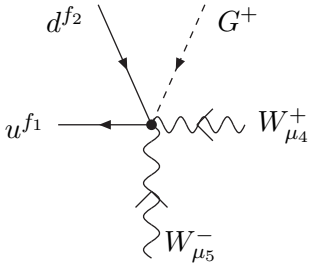
$$-\frac{\bar{g}}{\sqrt{2}}C^{\phi ud*}_{f_2 f_1}\gamma^{\mu_5}P_R$$



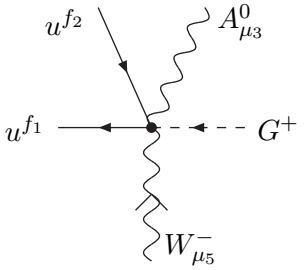
$$-i\sqrt{2}\bar{g}\left(\sigma^{\mu_4\mu_5}P_L C^{uW*}_{f_2 f_1} - C^{uW}_{f_1 f_2}\sigma^{\mu_4\mu_5}P_R\right)$$



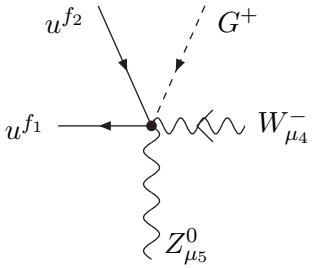
$$-\sqrt{2}\bar{g}\left(\sigma^{\mu_4\mu_5}P_L C^{uW*}_{f_2 f_1} + C^{uW}_{f_1 f_2}\sigma^{\mu_4\mu_5}P_R\right)$$



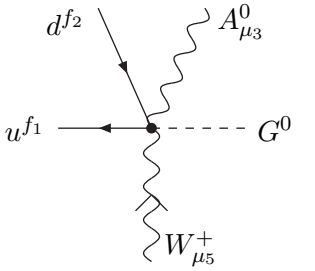
$$-2\bar{g}K_{g_1 f_2}\sigma^{\mu_4\mu_5}P_L C^{uW*}_{g_1 f_1} - 2\bar{g}K_{f_1 g_1}\sigma^{\mu_4\mu_5}P_R C^{dW}_{g_1 f_2}$$



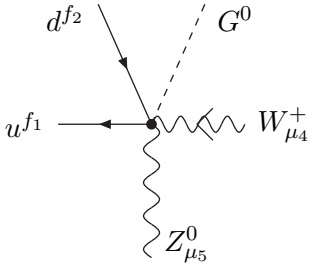
$$-\frac{2\sqrt{2}\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\sigma^{\mu_3\mu_5}P_L C^{uW*}_{f_2 f_1}$$



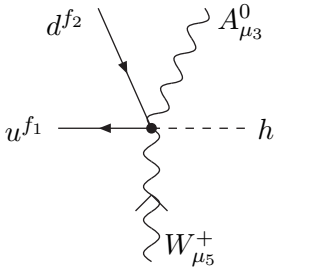
$$+\frac{2\sqrt{2}\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}\sigma^{\mu_4\mu_5}P_L C^{uW*}_{f_2f_1}$$



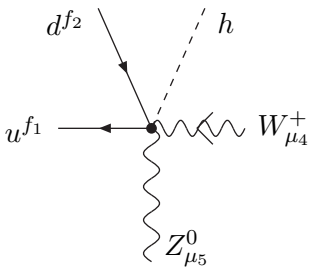
$$-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{g_1f_2}\sigma^{\mu_3\mu_5}P_L C^{uW*}_{g_1f_1}-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_1}\sigma^{\mu_3\mu_5}P_R C^{dW}_{g_1f_2}$$



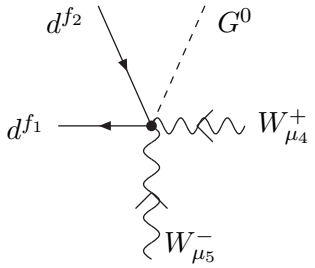
$$+\frac{2i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{g_1f_2}\sigma^{\mu_4\mu_5}P_L C^{uW*}_{g_1f_1}+\frac{2i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_1}\sigma^{\mu_4\mu_5}P_R C^{dW}_{g_1f_2}$$



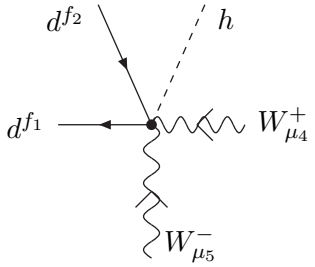
$$-\frac{2\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{g_1f_2}\sigma^{\mu_3\mu_5}P_L C^{uW*}_{g_1f_1}-\frac{2\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_1}\sigma^{\mu_3\mu_5}P_R C^{dW}_{g_1f_2}$$



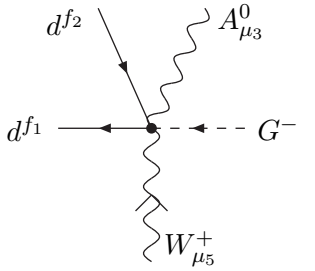
$$+\frac{2\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{g_1f_2}\sigma^{\mu_4\mu_5}P_L C^{uW*}_{g_1f_1}+\frac{2\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}K_{f_1g_1}\sigma^{\mu_4\mu_5}P_R C^{dW}_{g_1f_2}$$



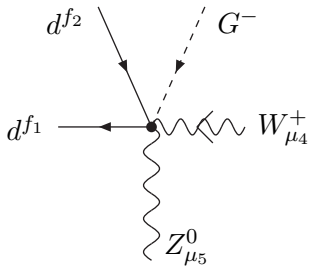
$$-i\sqrt{2}\bar{g}\left(\sigma^{\mu_4\mu_5}P_L C^{dW*}_{f_2f_1} - C^{dW}_{f_1f_2}\sigma^{\mu_4\mu_5}P_R\right)$$



$$+\sqrt{2}\bar{g}\left(\sigma^{\mu_4\mu_5}P_L C^{dW*}_{f_2f_1} + C^{dW}_{f_1f_2}\sigma^{\mu_4\mu_5}P_R\right)$$

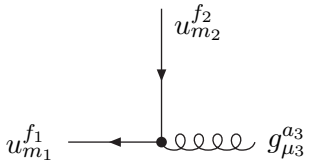


$$-\frac{2\sqrt{2}\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\sigma^{\mu_3\mu_5}P_L C^{dW*}_{f_2f_1}$$

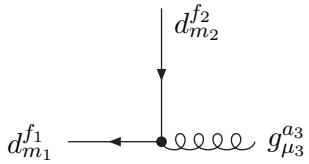


$$+\frac{2\sqrt{2}\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}\sigma^{\mu_4\mu_5}P_L C^{dW*}_{f_2f_1}$$

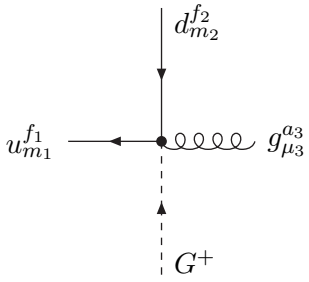
A.6 Quark-gluon vertices



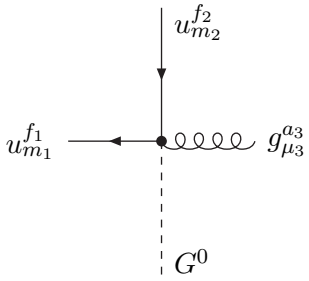
$$-i\bar{g}_s\delta_{f_1f_2}\mathcal{T}^{a_3}_{m_1m_2}\gamma^{\mu_3} - \sqrt{2}vp_3^{\nu}\mathcal{T}^{a_3}_{m_1m_2}\left(C^{uG*}_{f_2f_1}\sigma^{\mu_3\nu}P_L + C^{uG}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



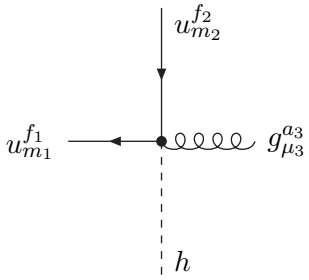
$$-i\bar{g}_s\delta_{f_1f_2}\mathcal{T}^{a_3}_{m_1m_2}\gamma^{\mu_3} - \sqrt{2}vp_3^{\nu}\mathcal{T}^{a_3}_{m_1m_2}\left(C^{dG*}_{f_2f_1}\sigma^{\mu_3\nu}P_L + C^{dG}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



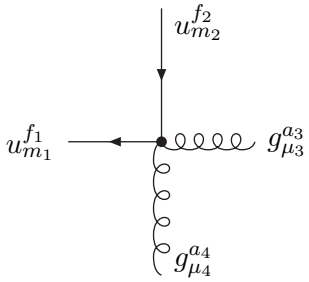
$$+2p_3^{\nu}\mathcal{T}^{a_3}_{m_1m_2}K_{g_1f_2}\sigma^{\mu_3\nu}P_LC^{uG*}_{g_1f_1} - 2p_3^{\nu}\mathcal{T}^{a_3}_{m_1m_2}K_{f_1g_1}C^{dG}_{g_1f_2}\sigma^{\mu_3\nu}P_R$$



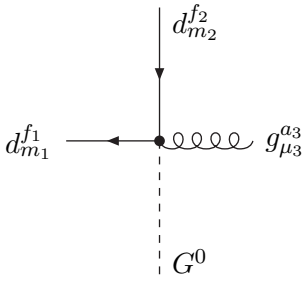
$$-i\sqrt{2}p_3^{\nu}\mathcal{T}^{a_3}_{m_1m_2}\left(C^{uG*}_{f_2f_1}\sigma^{\mu_3\nu}P_L - C^{uG}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



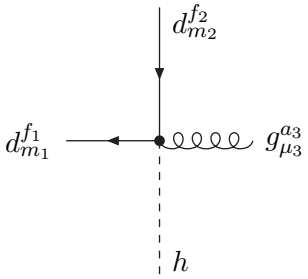
$$-\sqrt{2}p_3^{\nu}\mathcal{T}^{a_3}_{m_1m_2}\left(C^{uG*}_{f_2f_1}\sigma^{\mu_3\nu}P_L + C^{uG}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



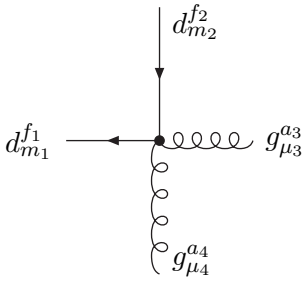
$$-i\sqrt{2}v\bar{g}_sf_{a_3a_4b_1}\mathcal{T}^{b_1}_{m_1m_2}\left(\sigma^{\mu_3\mu_4}P_LC^{uG*}_{f_2f_1} + C^{uG}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)$$



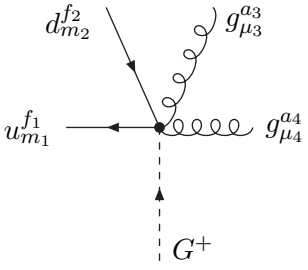
$$+i\sqrt{2}p_3^\nu \mathcal{T}^{a_3}_{m_1m_2}\left(C^{dG*}_{f_2f_1}\sigma^{\mu_3\nu}P_L - C^{dG}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



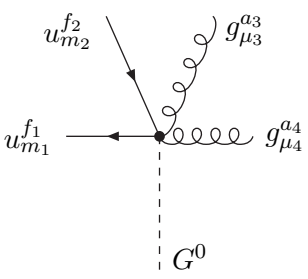
$$-\sqrt{2}p_3^\nu \mathcal{T}^{a_3}_{m_1m_2}\left(C^{dG*}_{f_2f_1}\sigma^{\mu_3\nu}P_L + C^{dG}_{f_1f_2}\sigma^{\mu_3\nu}P_R\right)$$



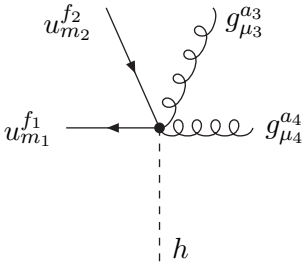
$$-i\sqrt{2}v\bar{g}_s f_{a_3a_4b_1}\mathcal{T}^{b_1}_{m_1m_2}\left(\sigma^{\mu_3\mu_4}P_L C^{dG*}_{f_2f_1} + C^{dG}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)$$



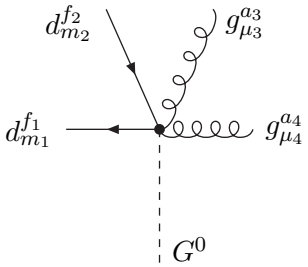
$$\begin{aligned}&+2i\bar{g}_s f_{a_3a_4b_1}K_{g_1f_2}\mathcal{T}^{b_1}_{m_1m_2}\sigma^{\mu_3\mu_4}P_L C^{uG*}_{g_1f_1}\\&-2i\bar{g}_s f_{a_3a_4b_1}K_{f_1g_1}\mathcal{T}^{b_1}_{m_1m_2}\sigma^{\mu_3\mu_4}P_R C^{dG}_{g_1f_2}\end{aligned}$$



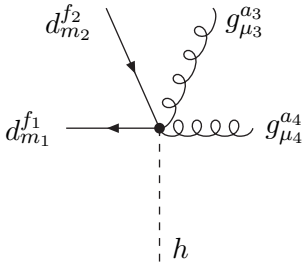
$$+\sqrt{2}\bar{g}_s f_{a_3a_4b_1}\mathcal{T}^{b_1}_{m_1m_2}\left(\sigma^{\mu_3\mu_4}P_L C^{uG*}_{f_2f_1} - C^{uG}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)$$



$$-i\sqrt{2}\bar{g}_s f_{a_3a_4b_1}\mathcal{T}^{b_1}_{m_1m_2}\left(\sigma^{\mu_3\mu_4}P_L C^{uG*}_{f_2f_1} + C^{uG}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)$$

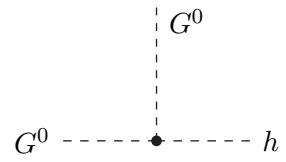


$$-\sqrt{2}\bar{g}_s f_{a_3a_4b_1}\mathcal{T}^{b_1}_{m_1m_2}\left(\sigma^{\mu_3\mu_4}P_L C^{dG*}_{f_2f_1} - C^{dG}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)$$

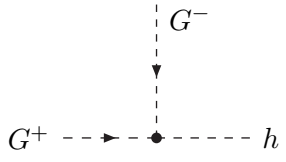


$$-i\sqrt{2}\bar{g}_s f_{a_3a_4b_1}\mathcal{T}^{b_1}_{m_1m_2}\left(\sigma^{\mu_3\mu_4}P_L C^{dG*}_{f_2f_1} + C^{dG}_{f_1f_2}\sigma^{\mu_3\mu_4}P_R\right)$$

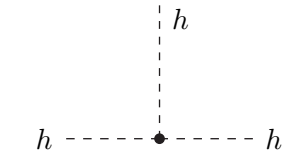
A.7 Higgs-gauge vertices



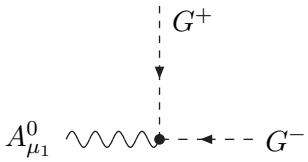
$$\begin{aligned}&-i\lambda v + 3iv^3C^\phi - ivC^{\phi\Box}\left(p_1\cdot p_1 + 2p_1\cdot p_2 + p_2\cdot p_2 + p_3\cdot p_3 + \lambda v^2\right)\\&+\frac{iv}{4}C^{\phi D}\left(3\lambda v^2 - 4p_1\cdot p_2\right)\end{aligned}$$



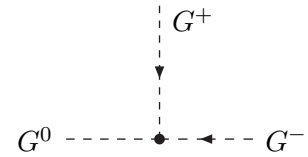
$$\begin{aligned}&-i\lambda v + 3iv^3C^\phi - ivC^{\phi\Box}\left(p_1\cdot p_1 + 2p_1\cdot p_2 + p_2\cdot p_2 + p_3\cdot p_3 + \lambda v^2\right)\\&+\frac{iv}{4}C^{\phi D}\left(\lambda v^2 - 2(p_1\cdot p_3 + p_2\cdot p_3)\right)\end{aligned}$$



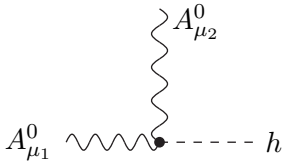
$$
\begin{aligned}
&-3i\lambda v + 15iv^3 C^{\phi} \\
&- ivC^{\phi\Box}\left(3p_1\cdot p_1 + 2p_1\cdot p_2 + 2p_1\cdot p_3 + 3p_2\cdot p_2 + 2p_2\cdot p_3 + 3p_3\cdot p_3 + 9\lambda v^2\right) \\
&+ \frac{iv}{4}C^{\phi D}\left(9\lambda v^2 - 4(p_1\cdot p_2 + p_1\cdot p_3 + p_2\cdot p_3)\right)
\end{aligned}
$$



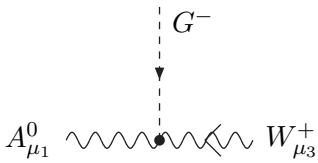
$$
-\frac{i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left(p_2^{\mu_1} - p_3^{\mu_1}\right) + \frac{i\bar{g}^2\bar{g}'^2 v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\left(p_2^{\mu_1} - p_3^{\mu_1}\right)
$$



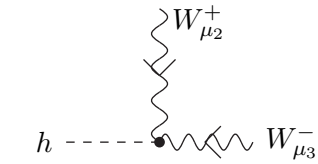
$$
-\frac{v}{2}(p_1\cdot p_2 - p_1\cdot p_3)C^{\phi D}
$$



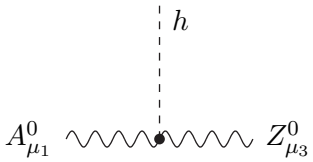
$$
\begin{aligned}
&+\frac{4i\bar{g}'^2 v}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_1^{\mu_2}p_2^{\mu_1} - p_1\cdot p_2\eta_{\mu_1\mu_2}\right) \\
&+ \frac{4i\bar{g}^2 v}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_1^{\mu_2}p_2^{\mu_1} - p_1\cdot p_2\eta_{\mu_1\mu_2}\right) \\
&- \frac{4i\bar{g}\bar{g}' v}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(p_1^{\mu_2}p_2^{\mu_1} - p_1\cdot p_2\eta_{\mu_1\mu_2}\right) \\
&+ \frac{4i\bar{g}'^2 v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1} + \frac{4i\bar{g}^2 v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1} \\
&- \frac{4i\bar{g}\bar{g}' v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}
\end{aligned}
$$



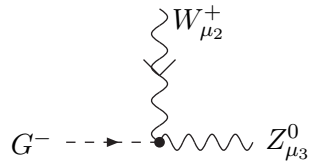
$$
\begin{aligned}
&+\frac{i\bar{g}^2\bar{g}' v}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_1\mu_3} \\
&- \frac{i\bar{g}v}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\left(\eta_{\mu_1\mu_3}\left(\bar{g}'^2\left(4p_1\cdot p_3 + \bar{g}^2v^2\right) + 4\bar{g}^2 p_1\cdot p_3\right)\right. \\
&\left.- 4\left(\bar{g}^2+\bar{g}'^2\right)p_1^{\mu_3}p_3^{\mu_1}\right) + \frac{2i\bar{g}v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_3^{\beta_1}\epsilon_{\mu_1\mu_3\alpha_1\beta_1}
\end{aligned}
$$



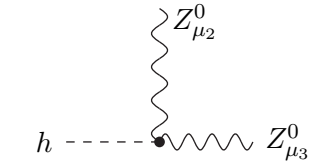
$$
\begin{aligned}
&+\frac{1}{2}i\bar{g}^2 v\eta_{\mu_2\mu_3} + \frac{1}{2}i\bar{g}^2 v^3\eta_{\mu_2\mu_3}C^{\phi\Box} - \frac{1}{8}i\bar{g}^2 v^3\eta_{\mu_2\mu_3}C^{\phi D} \\
&+ 4ivC^{\phi W}\left(p_2^{\mu_3}p_3^{\mu_2} - p_2\cdot p_3\eta_{\mu_2\mu_3}\right) + 4ivC^{\phi\widetilde{W}}p_2^{\alpha_1}p_3^{\beta_1}\epsilon_{\mu_2\mu_3\alpha_1\beta_1}
\end{aligned}
$$



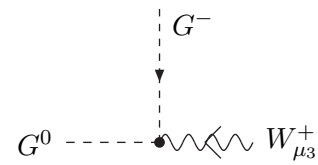
$$
\begin{aligned}
&+\frac{4i\bar{g}\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_1^{\mu_3}p_3^{\mu_1}-p_1\cdot p_3\eta_{\mu_1\mu_3}\right)\\
&-\frac{4i\bar{g}\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_1^{\mu_3}p_3^{\mu_1}-p_1\cdot p_3\eta_{\mu_1\mu_3}\right)\\
&+\frac{2iv\left(\bar{g}'^2-\bar{g}^2\right)}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(p_1^{\mu_3}p_3^{\mu_1}-p_1\cdot p_3\eta_{\mu_1\mu_3}\right)\\
&+\frac{4i\bar{g}\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_1^{\alpha_1}p_3^{\beta_1}\epsilon_{\mu_1\mu_3\alpha_1\beta_1}-\frac{4i\bar{g}\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_1^{\alpha_1}p_3^{\beta_1}\epsilon_{\mu_1\mu_3\alpha_1\beta_1}\\
&+\frac{2iv\left(\bar{g}'^2-\bar{g}^2\right)}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_3^{\beta_1}\epsilon_{\mu_1\mu_3\alpha_1\beta_1}
\end{aligned}
$$



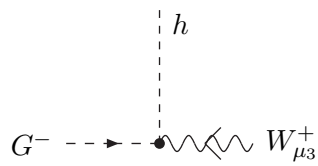
$$
\begin{aligned}
&-\frac{i\bar{g}\bar{g}'^2v}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_2\mu_3}-\frac{1}{4}i\bar{g}v^3\sqrt{\bar{g}^2+\bar{g}'^2}\eta_{\mu_2\mu_3}C^{\phi D}\\
&-\frac{i\bar{g}'v}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\left(\eta_{\mu_2\mu_3}\left(-4\bar{g}^2p_2\cdot p_3-4\bar{g}'^2p_2\cdot p_3+\bar{g}^4v^2\right)\right.\\
&\left.+4\left(\bar{g}^2+\bar{g}'^2\right)p_2^{\mu_3}p_3^{\mu_2}\right)-\frac{2i\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_2^{\alpha_1}p_3^{\beta_1}\epsilon_{\mu_2\mu_3\alpha_1\beta_1}
\end{aligned}
$$



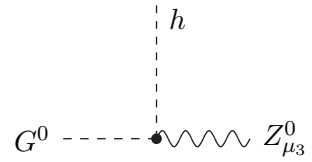
$$
\begin{aligned}
&+\frac{iv}{2}\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_2\mu_3}+\frac{iv^3}{2}\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_2\mu_3}C^{\phi\Box}\\
&+\frac{3iv^3}{8}\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_2\mu_3}C^{\phi D}+\frac{4i\bar{g}^2v}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_2^{\mu_3}p_3^{\mu_2}-p_2\cdot p_3\eta_{\mu_2\mu_3}\right)\\
&+\frac{4i\bar{g}'^2v}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_2^{\mu_3}p_3^{\mu_2}-p_2\cdot p_3\eta_{\mu_2\mu_3}\right)\\
&+\frac{i\bar{g}\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\eta_{\mu_2\mu_3}\left(-4p_2\cdot p_3+\bar{g}^2v^2+\bar{g}'^2v^2\right)+4p_2^{\mu_3}p_3^{\mu_2}\right)\\
&+\frac{4i\bar{g}^2v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_2^{\alpha_1}p_3^{\beta_1}\epsilon_{\mu_2\mu_3\alpha_1\beta_1}+\frac{4i\bar{g}'^2v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_2^{\alpha_1}p_3^{\beta_1}\epsilon_{\mu_2\mu_3\alpha_1\beta_1}\\
&+\frac{4i\bar{g}\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_2^{\alpha_1}p_3^{\beta_1}\epsilon_{\mu_2\mu_3\alpha_1\beta_1}
\end{aligned}
$$



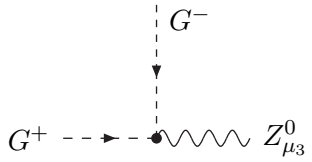
$$
+\frac{\bar{g}}{2}\left(p_1^{\mu_3}-p_2^{\mu_3}\right)+\frac{\bar{g}v^2}{8}C^{\phi D}\left(3p_1^{\mu_3}+p_2^{\mu_3}\right)
$$



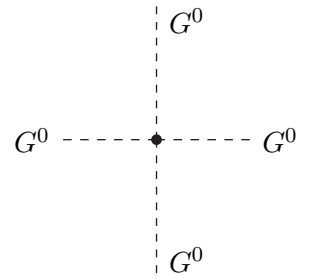
$$
+\frac{i\bar{g}}{2}\left(p_1^{\mu_3}-p_2^{\mu_3}\right)+\frac{1}{2}i\bar{g}v^2C^{\phi\Box}\left(p_1^{\mu_3}-p_2^{\mu_3}\right)-\frac{1}{8}i\bar{g}v^2C^{\phi D}\left(p_1^{\mu_3}-p_2^{\mu_3}\right)
$$



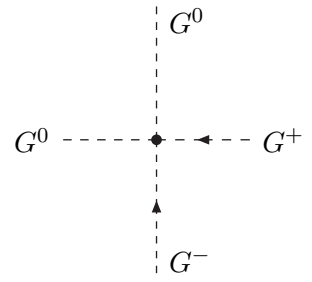
$$-\frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}\,(p_1^{\mu_3}-p_2^{\mu_3})-\frac{1}{2}v^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi\Box}\,(p_1^{\mu_3}-p_2^{\mu_3})$$
$$-\frac{1}{2}v^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi D}p_1^{\mu_3}-\frac{\bar{g}\bar{g}'v^2}{2\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}\,(p_1^{\mu_3}-p_2^{\mu_3})$$



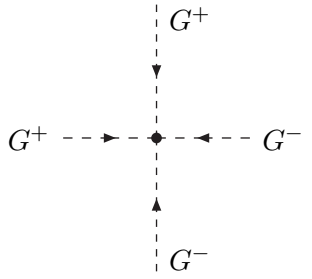
$$+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\,(p_1^{\mu_3}-p_2^{\mu_3})+\frac{1}{4}iv^2\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi D}\,(p_1^{\mu_3}-p_2^{\mu_3})$$
$$-\frac{i\bar{g}\bar{g}'v^2\left(\bar{g}'^2-\bar{g}^2\right)}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\,(p_1^{\mu_3}-p_2^{\mu_3})$$



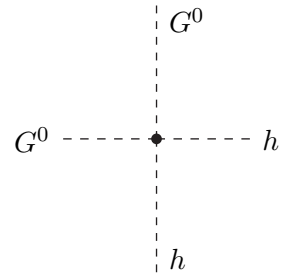
$$-3i\lambda+9iv^2C^{\phi}-i(3p_1\cdot p_1+2p_1\cdot p_2+2p_1\cdot p_3+2p_1\cdot p_4$$
$$+3p_2\cdot p_2+2p_2\cdot p_3+2p_2\cdot p_4+3p_3\cdot p_3+2p_3\cdot p_4+3p_4\cdot p_4)C^{\phi\Box}$$
$$+iC^{\phi D}\left(-p_1\cdot p_2-p_1\cdot p_3-p_1\cdot p_4-p_2\cdot p_3-p_2\cdot p_4-p_3\cdot p_4+3\lambda v^2\right)$$



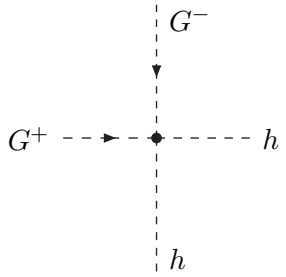
$$-i\lambda+3iv^2C^{\phi}-i(p_1\cdot p_1+2p_1\cdot p_2+p_2\cdot p_2+p_3\cdot p_3+2p_3\cdot p_4+p_4\cdot p_4)C^{\phi\Box}$$
$$+\frac{i}{2}C^{\phi D}\left(-p_1\cdot p_3-p_1\cdot p_4-p_2\cdot p_3-p_2\cdot p_4+\lambda v^2\right)$$



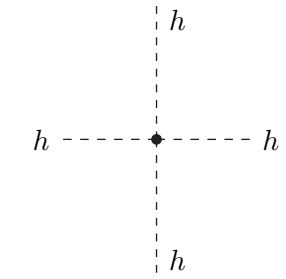
$$-2i\lambda+6iv^2C^{\phi}$$
$$-2i(p_1\cdot p_1+p_1\cdot p_3+p_1\cdot p_4+p_2\cdot p_2+p_2\cdot p_3+p_2\cdot p_4+p_3\cdot p_3+p_4\cdot p_4)C^{\phi\Box}$$
$$-i(p_1\cdot p_3+p_1\cdot p_4+p_2\cdot p_3+p_2\cdot p_4)C^{\phi D}$$



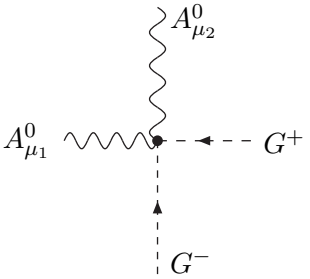
$$
\begin{aligned}
&-i\lambda + 9iv^2 C^{\phi} \\
&\quad - iC^{\phi\Box}\left(p_1\cdot p_1 + 2p_1\cdot p_2 + p_2\cdot p_2 + p_3\cdot p_3 + 2p_3\cdot p_4 + p_4\cdot p_4 + 2\lambda v^2\right) \\
&\quad + iC^{\phi D}\left(-p_1\cdot p_2 - p_3\cdot p_4 + \lambda v^2\right)
\end{aligned}
$$



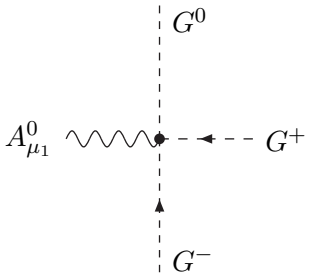
$$
\begin{aligned}
&-i\lambda + 9iv^2 C^{\phi} \\
&\quad - iC^{\phi\Box}\left(p_1\cdot p_1 + 2p_1\cdot p_2 + p_2\cdot p_2 + p_3\cdot p_3 + 2p_3\cdot p_4 + p_4\cdot p_4 + 2\lambda v^2\right) \\
&\quad + \frac{i}{2}C^{\phi D}\left(-p_1\cdot p_3 - p_1\cdot p_4 - p_2\cdot p_3 - p_2\cdot p_4 + \lambda v^2\right)
\end{aligned}
$$



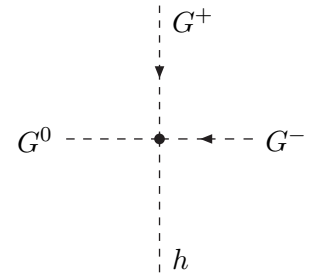
$$
\begin{aligned}
&-3i\lambda + 45iv^2 C^{\phi} - iC^{\phi\Box}\left(3p_1\cdot p_1 + 2p_1\cdot p_2 + 2p_1\cdot p_3 + 2p_1\cdot p_4\right. \\
&\quad \left. + 3p_2\cdot p_2 + 2p_2\cdot p_3 + 2p_2\cdot p_4 + 3p_3\cdot p_3 + 2p_3\cdot p_4 + 3p_4\cdot p_4 + 12\lambda v^2\right) \\
&\quad + iC^{\phi D}\left(-p_1\cdot p_2 - p_1\cdot p_3 - p_1\cdot p_4 - p_2\cdot p_3 - p_2\cdot p_4 - p_3\cdot p_4 + 3\lambda v^2\right)
\end{aligned}
$$



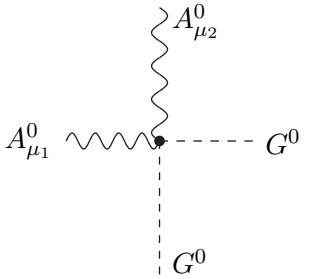
$$
\begin{aligned}
&+\frac{2i\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\eta_{\mu_1\mu_2} + \frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_1^{\mu_2}p_2^{\mu_1} - p_1\cdot p_2\eta_{\mu_1\mu_2}\right) \\
&\quad + \frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_1^{\mu_2}p_2^{\mu_1} - p_1\cdot p_2\eta_{\mu_1\mu_2}\right) \\
&\quad - \frac{4i\bar{g}\bar{g}'}{\left(\bar{g}^2+\bar{g}'^2\right)^2}C^{\phi WB}\left(\eta_{\mu_1\mu_2}\left(\bar{g}'^2\left(p_1\cdot p_2 + \bar{g}^2v^2\right) + \bar{g}^2 p_1\cdot p_2\right)\right. \\
&\quad \left. - \left(\bar{g}^2+\bar{g}'^2\right)p_1^{\mu_2}p_2^{\mu_1}\right) + \frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1} \\
&\quad + \frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1} + \frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}
\end{aligned}
$$



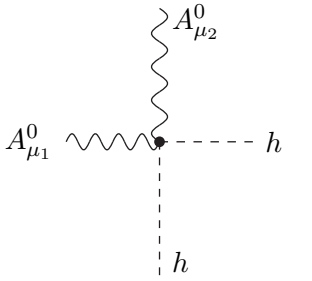
$$+\frac{\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi D}p_2^{\mu_1}$$



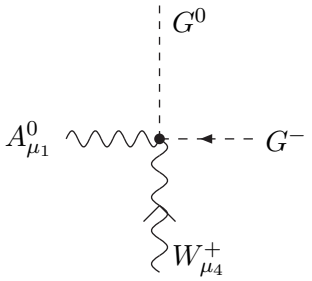
$$-\frac{1}{2}(p_1\cdot p_2 - p_1\cdot p_3 - p_2\cdot p_4 + p_3\cdot p_4)C^{\phi D}$$



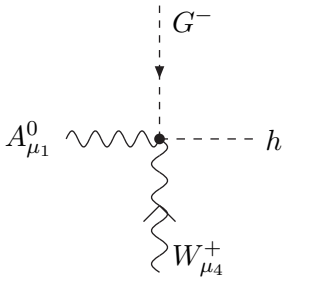
$$\begin{aligned}
&+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)\\
&+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)\\
&-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)\\
&+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}\\
&-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}
\end{aligned}$$



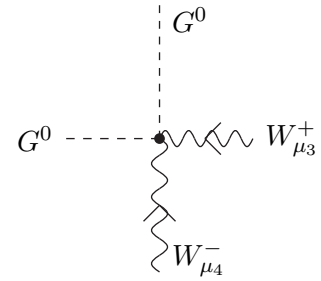
$$\begin{aligned}
&+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)\\
&+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)\\
&-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)\\
&+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}\\
&-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}
\end{aligned}$$



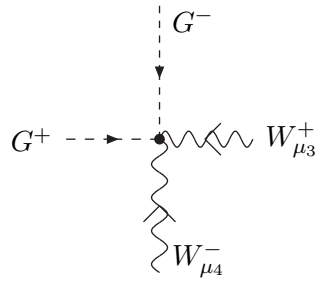
$$
\begin{aligned}
&-\frac{\bar{g}^2\bar{g}'}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_1\mu_4}+\frac{\bar{g}^2\bar{g}'v^2}{8\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_1\mu_4}C^{\phi D}\\
&-\frac{\bar{g}}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\left(4\left(\bar{g}^2+\bar{g}'^2\right)p_1^{\mu_4}p_4^{\mu_1}\right.\\
&\left.-\eta_{\mu_1\mu_4}\left(\bar{g}'^2\left(4p_1\cdot p_4+\bar{g}^2v^2\right)+4\bar{g}^2p_1\cdot p_4\right)\right)\\
&-\frac{2\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}
\end{aligned}
$$



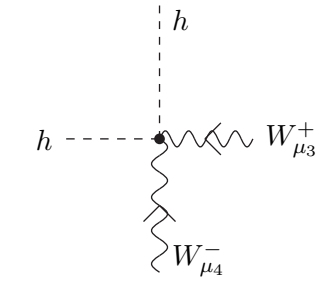
$$
\begin{aligned}
&+\frac{i\bar{g}^2\bar{g}'}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_1\mu_4}+\frac{i\bar{g}^2\bar{g}'v^2}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_1\mu_4}C^{\phi\Box}-\frac{i\bar{g}^2\bar{g}'v^2}{8\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_1\mu_4}C^{\phi D}\\
&+\frac{i\bar{g}}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\left(4\left(\bar{g}^2+\bar{g}'^2\right)p_1^{\mu_4}p_4^{\mu_1}\right.\\
&\left.-\eta_{\mu_1\mu_4}\left(\bar{g}'^2\left(4p_1\cdot p_4+\bar{g}^2v^2\right)+4\bar{g}^2p_1\cdot p_4\right)\right)\\
&+\frac{2i\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}
\end{aligned}
$$



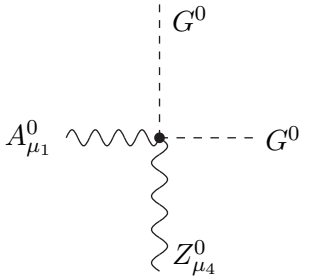
$$
\begin{aligned}
&+\frac{i\bar{g}^2}{2}\eta_{\mu_3\mu_4}-\frac{1}{4}i\bar{g}^2v^2\eta_{\mu_3\mu_4}C^{\phi D}\\
&+4iC^{\phi W}\left(p_3^{\mu_4}p_4^{\mu_3}-p_3\cdot p_4\eta_{\mu_3\mu_4}\right)+4iC^{\phi\widetilde{W}}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}
\end{aligned}
$$



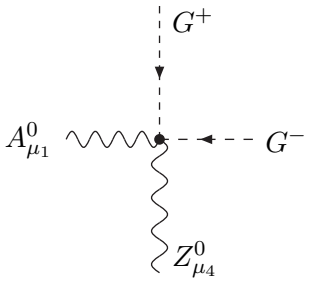
$$
\begin{aligned}
&+\frac{i\bar{g}^2}{2}\eta_{\mu_3\mu_4}+\frac{1}{2}i\bar{g}^2v^2\eta_{\mu_3\mu_4}C^{\phi D}\\
&+4iC^{\phi W}\left(p_3^{\mu_4}p_4^{\mu_3}-p_3\cdot p_4\eta_{\mu_3\mu_4}\right)+4iC^{\phi\widetilde{W}}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}
\end{aligned}
$$



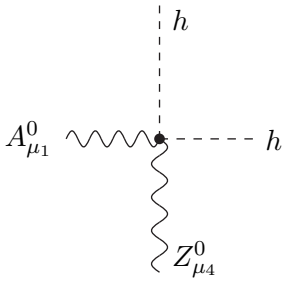
$$+\frac{i\bar{g}^2}{2}\eta_{\mu_3\mu_4}+i\bar{g}^2v^2\eta_{\mu_3\mu_4}C^{\phi\Box}-\frac{1}{4}i\bar{g}^2v^2\eta_{\mu_3\mu_4}C^{\phi D}$$
$$+4iC^{\phi W}\left(p_3^{\mu_4}p_4^{\mu_3}-p_3\cdot p_4\eta_{\mu_3\mu_4}\right)+4iC^{\phi\widetilde{W}}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}$$



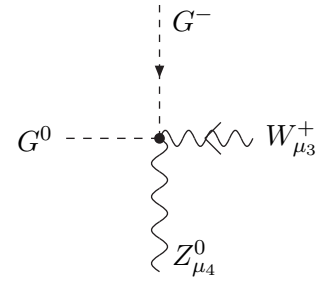
$$+\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_1^{\mu_4}p_4^{\mu_1}-p_1\cdot p_4\eta_{\mu_1\mu_4}\right)$$
$$-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_1^{\mu_4}p_4^{\mu_1}-p_1\cdot p_4\eta_{\mu_1\mu_4}\right)$$
$$+\frac{2i\left(\bar{g}'^2-\bar{g}^2\right)}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(p_1^{\mu_4}p_4^{\mu_1}-p_1\cdot p_4\eta_{\mu_1\mu_4}\right)$$
$$+\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}$$
$$+\frac{2i\left(\bar{g}'^2-\bar{g}^2\right)}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}$$



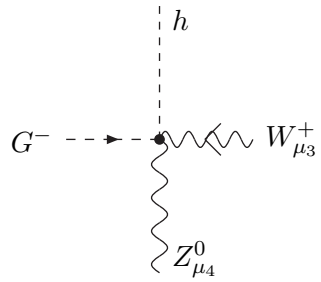
$$-\frac{i\bar{g}\bar{g}'\left(\bar{g}'^2-\bar{g}^2\right)}{\bar{g}^2+\bar{g}'^2}\eta_{\mu_1\mu_4}-\frac{1}{2}i\bar{g}\bar{g}'v^2\eta_{\mu_1\mu_4}C^{\phi D}$$
$$+\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_1^{\mu_4}p_4^{\mu_1}-p_1\cdot p_4\eta_{\mu_1\mu_4}\right)$$
$$-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_1^{\mu_4}p_4^{\mu_1}-p_1\cdot p_4\eta_{\mu_1\mu_4}\right)$$
$$+\frac{2i\left(\bar{g}'^2-\bar{g}^2\right)}{\left(\bar{g}^2+\bar{g}'^2\right)^2}C^{\phi WB}\left(\eta_{\mu_1\mu_4}\left(\bar{g}'^2\left(p_1\cdot p_4+\bar{g}^2v^2\right)+\bar{g}^2p_1\cdot p_4\right)\right.$$
$$\left.-\left(\bar{g}^2+\bar{g}'^2\right)p_1^{\mu_4}p_4^{\mu_1}\right)+\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}$$
$$-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}$$
$$-\frac{2i\left(\bar{g}'^2-\bar{g}^2\right)}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}$$



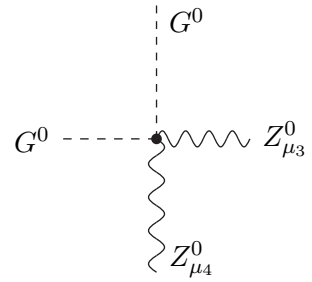
$$
\begin{aligned}
&+\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_1^{\mu_4}p_4^{\mu_1}-p_1\cdot p_4\eta_{\mu_1\mu_4}\right)\\
&-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_1^{\mu_4}p_4^{\mu_1}-p_1\cdot p_4\eta_{\mu_1\mu_4}\right)\\
&+\frac{2i\left(\bar{g}'^2-\bar{g}^2\right)}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(p_1^{\mu_4}p_4^{\mu_1}-p_1\cdot p_4\eta_{\mu_1\mu_4}\right)\\
&+\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}\\
&+\frac{2i\left(\bar{g}'^2-\bar{g}^2\right)}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_1^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_1\mu_4\alpha_1\beta_1}
\end{aligned}
$$



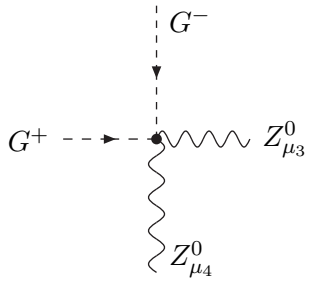
$$
\begin{aligned}
&+\frac{\bar{g}\bar{g}'^2}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_3\mu_4}+\frac{\bar{g}v^2}{8\sqrt{\bar{g}^2+\bar{g}'^2}}\left(2\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_3\mu_4}C^{\phi D}\\
&+\frac{\bar{g}'}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\left(\eta_{\mu_3\mu_4}\left(-4\bar{g}^2p_3\cdot p_4-4\bar{g}'^2p_3\cdot p_4+\bar{g}^4v^2\right)\right.\\
&\left.+4\left(\bar{g}^2+\bar{g}'^2\right)p_3^{\mu_4}p_4^{\mu_3}\right)+\frac{2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}
\end{aligned}
$$



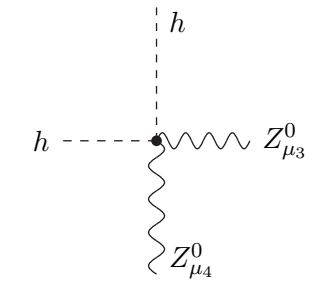
$$
\begin{aligned}
&-\frac{i\bar{g}\bar{g}'^2}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_3\mu_4}-\frac{i\bar{g}\bar{g}'^2v^2}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_3\mu_4}C^{\phi\Box}\\
&-\frac{i\bar{g}v^2}{8\sqrt{\bar{g}^2+\bar{g}'^2}}\left(6\bar{g}^2+5\bar{g}'^2\right)\eta_{\mu_3\mu_4}C^{\phi D}\\
&-\frac{i\bar{g}'}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\left(\eta_{\mu_3\mu_4}\left(-4\bar{g}^2p_3\cdot p_4-4\bar{g}'^2p_3\cdot p_4+\bar{g}^4v^2\right)\right.\\
&\left.+4\left(\bar{g}^2+\bar{g}'^2\right)p_3^{\mu_4}p_4^{\mu_3}\right)-\frac{2i\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}
\end{aligned}
$$



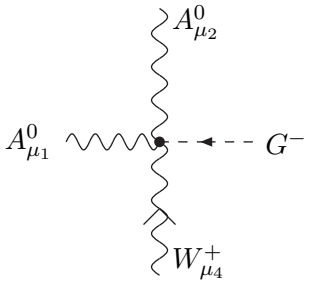
$$
\begin{aligned}
&+\frac{i}{2}\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_3\mu_4}+\frac{iv^2}{4}\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_3\mu_4}C^{\phi D}\\
&+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_3^{\mu_4}p_4^{\mu_3}-p_3\cdot p_4\eta_{\mu_3\mu_4}\right)\\
&+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_3^{\mu_4}p_4^{\mu_3}-p_3\cdot p_4\eta_{\mu_3\mu_4}\right)\\
&+\frac{i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\eta_{\mu_3\mu_4}\left(-4p_3\cdot p_4+\bar{g}^2v^2+\bar{g}'^2v^2\right)+4p_3^{\mu_4}p_4^{\mu_3}\right)\\
&+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}\\
&+\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}
\end{aligned}
$$



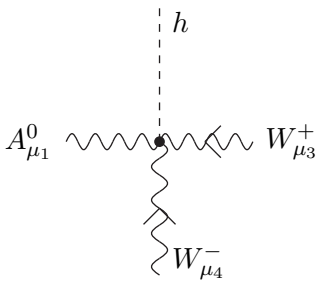
$$
\begin{aligned}
&+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)^2}{2\left(\bar{g}^2+\bar{g}'^2\right)}\eta_{\mu_3\mu_4}+\frac{1}{2}iv^2\left(\bar{g}'^2-\bar{g}^2\right)\eta_{\mu_3\mu_4}C^{\phi D}\\
&+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_3^{\mu_4}p_4^{\mu_3}-p_3\cdot p_4\eta_{\mu_3\mu_4}\right)\\
&+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_3^{\mu_4}p_4^{\mu_3}-p_3\cdot p_4\eta_{\mu_3\mu_4}\right)\\
&-\frac{i\bar{g}\bar{g}'}{\left(\bar{g}^2+\bar{g}'^2\right)^2}C^{\phi WB}\left(\eta_{\mu_3\mu_4}\left(-2\bar{g}'^2\left(2p_3\cdot p_4+\bar{g}^2v^2\right)\right.\right.\\
&\left.\left.-4\bar{g}^2p_3\cdot p_4+\bar{g}^4v^2+\bar{g}'^4v^2\right)+4\left(\bar{g}^2+\bar{g}'^2\right)p_3^{\mu_4}p_4^{\mu_3}\right)\\
&+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}\\
&-\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}
\end{aligned}
$$



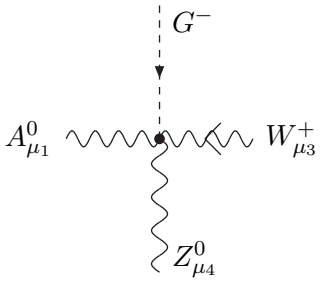
$$
\begin{aligned}
&+\frac{i}{2}\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_3\mu_4}+iv^2\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_3\mu_4}C^{\phi\Box}\\
&+\frac{5iv^2}{4}\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_3\mu_4}C^{\phi D}+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi W}\left(p_3^{\mu_4}p_4^{\mu_3}-p_3\cdot p_4\eta_{\mu_3\mu_4}\right)\\
&+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi B}\left(p_3^{\mu_4}p_4^{\mu_3}-p_3\cdot p_4\eta_{\mu_3\mu_4}\right)\\
&+\frac{i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\eta_{\mu_3\mu_4}\left(-4p_3\cdot p_4+\bar{g}^2v^2+\bar{g}'^2v^2\right)+4p_3^{\mu_4}p_4^{\mu_3}\right)\\
&+\frac{4i\bar{g}^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}+\frac{4i\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{B}}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}\\
&+\frac{4i\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}p_3^{\alpha_1}p_4^{\beta_1}\epsilon_{\mu_3\mu_4\alpha_1\beta_1}
\end{aligned}
$$



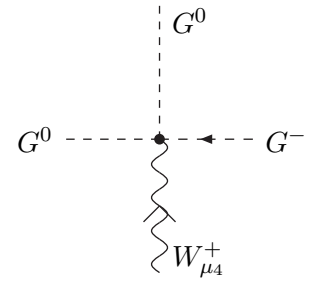
$$-\frac{2i\bar{g}^2\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\eta_{\mu_1\mu_2}p_1^{\mu_4}+\eta_{\mu_1\mu_2}p_2^{\mu_4}-\eta_{\mu_1\mu_4}p_1^{\mu_2}-\eta_{\mu_2\mu_4}p_2^{\mu_1}\right)$$
$$-\frac{2i\bar{g}^2\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}\left(p_1^{\alpha_1}-p_2^{\alpha_1}\right)\epsilon_{\mu_1\mu_2\mu_4\alpha_1}$$



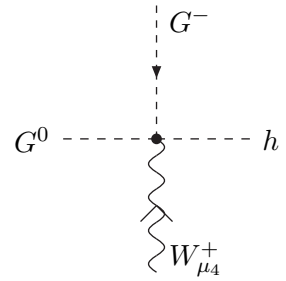
$$-\frac{4i\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi W}\left(\eta_{\mu_1\mu_3}p_1^{\mu_4}-\eta_{\mu_1\mu_3}p_3^{\mu_4}-\eta_{\mu_1\mu_4}p_1^{\mu_3}+\eta_{\mu_1\mu_4}p_4^{\mu_3}\right.$$
$$\left.+\eta_{\mu_3\mu_4}p_3^{\mu_1}-\eta_{\mu_3\mu_4}p_4^{\mu_1}\right)+\frac{2i\bar{g}^2v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}\left(\eta_{\mu_1\mu_3}p_1^{\mu_4}-\eta_{\mu_1\mu_4}p_1^{\mu_3}\right)$$
$$+\frac{4i\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}}\left(p_1^{\alpha_1}+p_3^{\alpha_1}+p_4^{\alpha_1}\right)\epsilon_{\mu_1\mu_4\mu_3\alpha_1}$$
$$-\frac{2i\bar{g}^2v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_1^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_3\alpha_1}$$



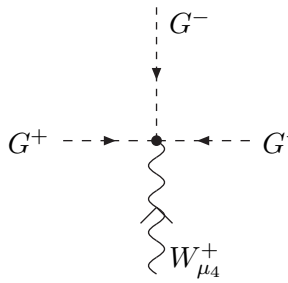
$$+\frac{2i\bar{g}v}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\bar{g}^2\eta_{\mu_1\mu_3}p_1^{\mu_4}-\bar{g}^2\eta_{\mu_1\mu_4}p_1^{\mu_3}+\bar{g}'^2\left(\eta_{\mu_1\mu_4}p_4^{\mu_3}-\eta_{\mu_3\mu_4}p_4^{\mu_1}\right)\right)$$
$$-\frac{2i\bar{g}v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}\epsilon_{\mu_1\mu_4\mu_3\alpha_1}\left(\bar{g}^2p_1^{\alpha_1}+\bar{g}'^2p_4^{\alpha_1}\right)$$



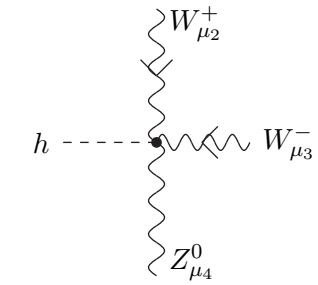
$$+\frac{i\bar{g}v}{2}C^{\phi D}\left(p_1^{\mu_4}+p_2^{\mu_4}\right)$$



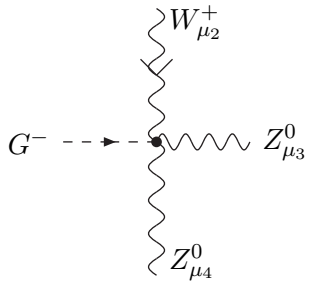
$$+\frac{\bar{g}v}{2}C^{\phi D}\left(2p_1^{\mu_4}-p_3^{\mu_4}\right)$$



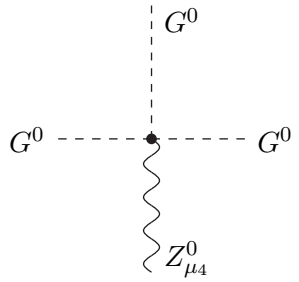
$$-\frac{1}{2}i\bar{g}vC^{\phi D}\left(2p_1^{\mu_4}-p_2^{\mu_4}-p_3^{\mu_4}\right)$$



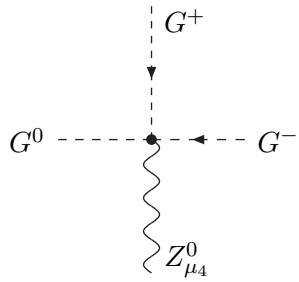
$$\begin{aligned}
&-\frac{4i\bar{g}^2v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi W}\left(\eta_{\mu_2\mu_3}p_2^{\mu_4}-\eta_{\mu_2\mu_3}p_3^{\mu_4}-\eta_{\mu_2\mu_4}p_2^{\mu_3}+\eta_{\mu_2\mu_4}p_4^{\mu_3}\right.\\
&\left.+\eta_{\mu_3\mu_4}p_3^{\mu_2}-\eta_{\mu_3\mu_4}p_4^{\mu_2}\right)-\frac{2i\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}\left(\eta_{\mu_2\mu_4}p_4^{\mu_3}-\eta_{\mu_3\mu_4}p_4^{\mu_2}\right)\\
&-\frac{4i\bar{g}^2v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}}\left(p_2^{\alpha_1}+p_3^{\alpha_1}+p_4^{\alpha_1}\right)\epsilon_{\mu_4\mu_2\mu_3\alpha_1}\\
&-\frac{2i\bar{g}\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_4^{\alpha_1}\epsilon_{\mu_4\mu_2\mu_3\alpha_1}
\end{aligned}$$



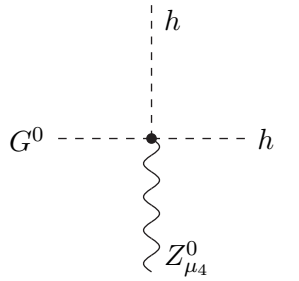
$$\begin{aligned}
&-\frac{2i\bar{g}^2\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\eta_{\mu_2\mu_3}p_3^{\mu_4}+\eta_{\mu_2\mu_4}p_4^{\mu_3}-\eta_{\mu_3\mu_4}\left(p_3^{\mu_2}+p_4^{\mu_2}\right)\right)\\
&-\frac{2i\bar{g}^2\bar{g}'v}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}\left(p_3^{\alpha_1}-p_4^{\alpha_1}\right)\epsilon_{\mu_3\mu_2\mu_4\alpha_1}
\end{aligned}$$



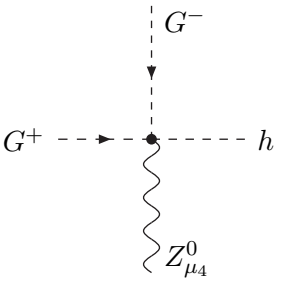
$$-\frac{1}{2}v\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi D}\left(p_1^{\mu_4}+p_2^{\mu_4}+p_3^{\mu_4}\right)$$



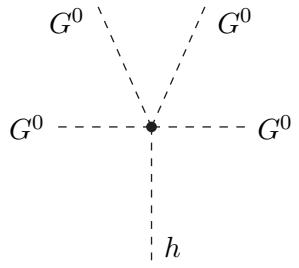
$$-\frac{v\left(\bar{g}'^2-\bar{g}^2\right)}{2\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi D}p_1^{\mu_4}$$



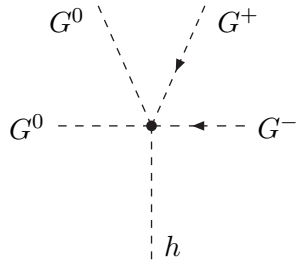
$$-\frac{1}{2}v\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi D}\left(3p_1^{\mu_4}-p_2^{\mu_4}-p_3^{\mu_4}\right)$$



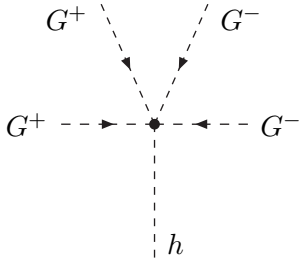
$$+\frac{1}{2}iv\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi D}\left(p_1^{\mu_4}-p_2^{\mu_4}\right)$$



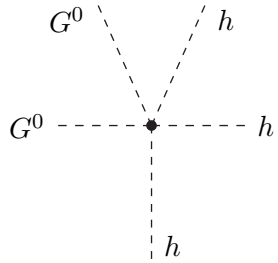
$$+18ivC^{\phi}$$



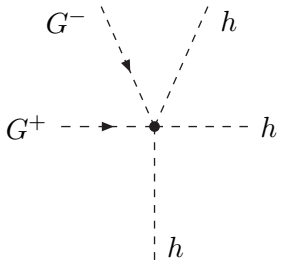
$+6ivC^\phi$



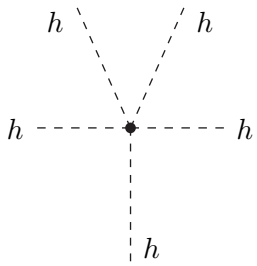
$+12ivC^\phi$



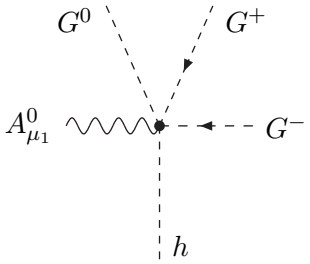
$+18ivC^\phi$



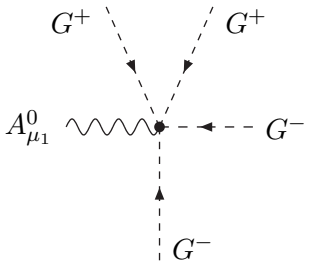
$+18ivC^\phi$



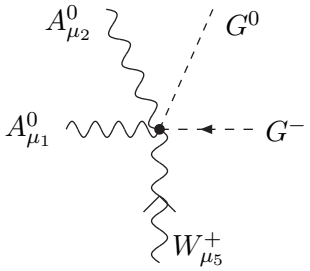
$+90ivC^\phi$



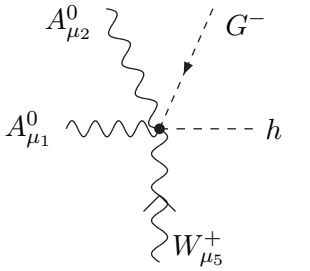
$$+\frac{\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi D}\left(p_2^{\mu_1}-p_5^{\mu_1}\right)$$



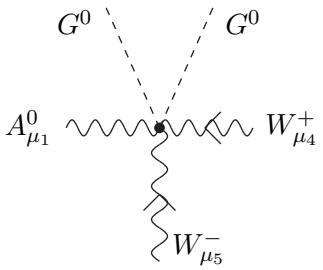
$$-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi D}\left(p_2^{\mu_1}+p_3^{\mu_1}-p_4^{\mu_1}-p_5^{\mu_1}\right)$$



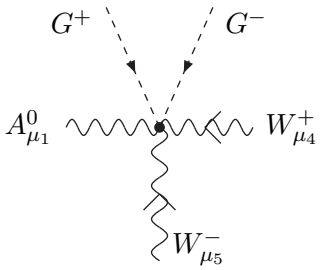
$$+\frac{2\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\eta_{\mu_1\mu_2}p_1^{\mu_5}+\eta_{\mu_1\mu_2}p_2^{\mu_5}-\eta_{\mu_1\mu_5}p_1^{\mu_2}-\eta_{\mu_2\mu_5}p_2^{\mu_1}\right)$$
$$+\frac{2\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}\left(p_1^{\alpha_1}-p_2^{\alpha_1}\right)\epsilon_{\mu_1\mu_2\mu_5\alpha_1}$$



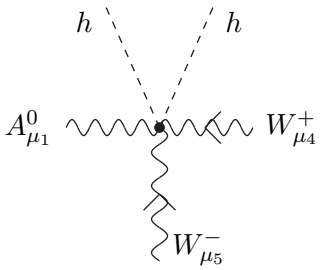
$$-\frac{2i\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\eta_{\mu_1\mu_2}p_1^{\mu_5}+\eta_{\mu_1\mu_2}p_2^{\mu_5}-\eta_{\mu_1\mu_5}p_1^{\mu_2}-\eta_{\mu_2\mu_5}p_2^{\mu_1}\right)$$
$$-\frac{2i\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}\left(p_1^{\alpha_1}-p_2^{\alpha_1}\right)\epsilon_{\mu_1\mu_2\mu_5\alpha_1}$$



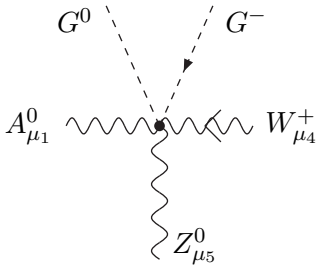
$$
\begin{aligned}
&-\frac{4i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi W}\left(\eta_{\mu_1\mu_4}p_1^{\mu_5}-\eta_{\mu_1\mu_4}p_4^{\mu_5}-\eta_{\mu_1\mu_5}p_1^{\mu_4}+\eta_{\mu_1\mu_5}p_5^{\mu_4}\right.\\
&\left.+\eta_{\mu_4\mu_5}p_4^{\mu_1}-\eta_{\mu_4\mu_5}p_5^{\mu_1}\right)+\frac{2i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}\left(\eta_{\mu_1\mu_4}p_1^{\mu_5}-\eta_{\mu_1\mu_5}p_1^{\mu_4}\right)\\
&+\frac{4i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}}\left(p_1^{\alpha_1}+p_4^{\alpha_1}+p_5^{\alpha_1}\right)\epsilon_{\mu_1\mu_5\mu_4\alpha_1}\\
&-\frac{2i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_1^{\alpha_1}\epsilon_{\mu_1\mu_5\mu_4\alpha_1}
\end{aligned}
$$



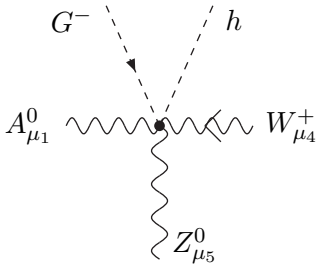
$$
\begin{aligned}
&-\frac{4i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi W}\left(\eta_{\mu_1\mu_4}p_1^{\mu_5}-\eta_{\mu_1\mu_4}p_4^{\mu_5}-\eta_{\mu_1\mu_5}p_1^{\mu_4}+\eta_{\mu_1\mu_5}p_5^{\mu_4}\right.\\
&\left.+\eta_{\mu_4\mu_5}p_4^{\mu_1}-\eta_{\mu_4\mu_5}p_5^{\mu_1}\right)-\frac{2i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}\left(\eta_{\mu_1\mu_4}p_1^{\mu_5}-\eta_{\mu_1\mu_5}p_1^{\mu_4}\right)\\
&+\frac{4i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}}\left(p_1^{\alpha_1}+p_4^{\alpha_1}+p_5^{\alpha_1}\right)\epsilon_{\mu_1\mu_5\mu_4\alpha_1}\\
&+\frac{2i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_1^{\alpha_1}\epsilon_{\mu_1\mu_5\mu_4\alpha_1}
\end{aligned}
$$



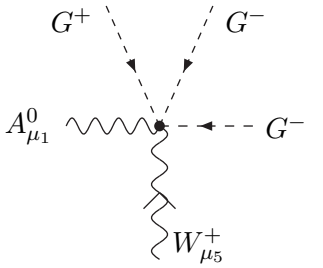
$$
\begin{aligned}
&-\frac{4i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi W}\left(\eta_{\mu_1\mu_4}p_1^{\mu_5}-\eta_{\mu_1\mu_4}p_4^{\mu_5}-\eta_{\mu_1\mu_5}p_1^{\mu_4}+\eta_{\mu_1\mu_5}p_5^{\mu_4}\right.\\
&\left.+\eta_{\mu_4\mu_5}p_4^{\mu_1}-\eta_{\mu_4\mu_5}p_5^{\mu_1}\right)+\frac{2i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}\left(\eta_{\mu_1\mu_4}p_1^{\mu_5}-\eta_{\mu_1\mu_5}p_1^{\mu_4}\right)\\
&+\frac{4i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}}\left(p_1^{\alpha_1}+p_4^{\alpha_1}+p_5^{\alpha_1}\right)\epsilon_{\mu_1\mu_5\mu_4\alpha_1}\\
&-\frac{2i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_1^{\alpha_1}\epsilon_{\mu_1\mu_5\mu_4\alpha_1}
\end{aligned}
$$



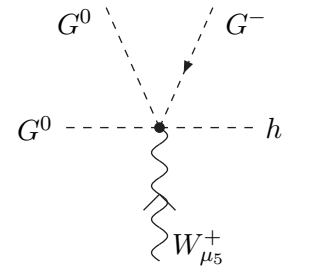
$$-\frac{2\bar{g}}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\bar{g}^2\eta_{\mu_1\mu_4}p_1^{\mu_5}-\bar{g}^2\eta_{\mu_1\mu_5}p_1^{\mu_4}+\bar{g}'^2\left(\eta_{\mu_1\mu_5}p_5^{\mu_4}-\eta_{\mu_4\mu_5}p_5^{\mu_1}\right)\right)$$
$$+\frac{2\bar{g}}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}\epsilon_{\mu_1\mu_5\mu_4\alpha_1}\left(\bar{g}^2p_1^{\alpha_1}+\bar{g}'^2p_5^{\alpha_1}\right)$$



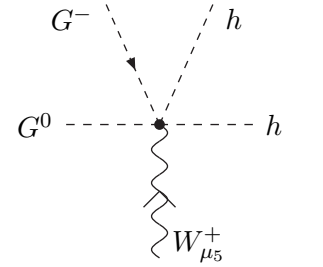
$$+\frac{2i\bar{g}}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\bar{g}^2\eta_{\mu_1\mu_4}p_1^{\mu_5}-\bar{g}^2\eta_{\mu_1\mu_5}p_1^{\mu_4}+\bar{g}'^2\left(\eta_{\mu_1\mu_5}p_5^{\mu_4}-\eta_{\mu_4\mu_5}p_5^{\mu_1}\right)\right)$$
$$-\frac{2i\bar{g}}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}\epsilon_{\mu_1\mu_5\mu_4\alpha_1}\left(\bar{g}^2p_1^{\alpha_1}+\bar{g}'^2p_5^{\alpha_1}\right)$$



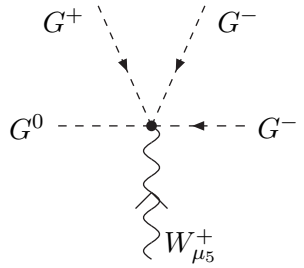
$$+\frac{2i\bar{g}^2\bar{g}'v}{\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_1\mu_5}C^{\phi D}$$



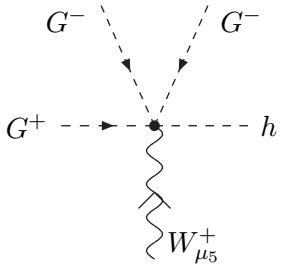
$$+\frac{i\bar{g}}{2}C^{\phi D}\left(p_1^{\mu_5}+p_2^{\mu_5}-2p_4^{\mu_5}\right)$$



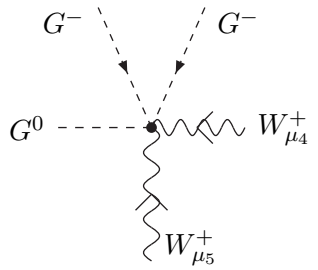
$$+\frac{\bar{g}}{2}C^{\phi D}\left(2p_1^{\mu_5}-p_3^{\mu_5}-p_4^{\mu_5}\right)$$



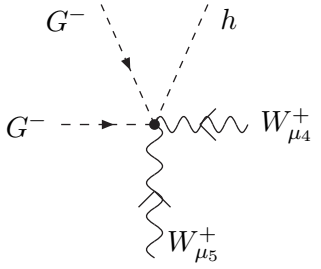
$$+\frac{\bar{g}}{2}C^{\phi D}\left(2p_2^{\mu_5}-p_3^{\mu_5}-p_4^{\mu_5}\right)$$



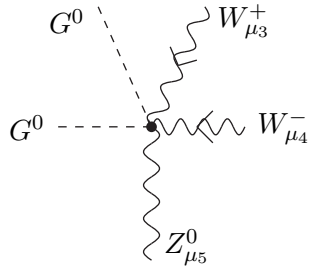
$$-\frac{i\bar{g}}{2}C^{\phi D}\left(2p_1^{\mu_5}-p_2^{\mu_5}-p_3^{\mu_5}\right)$$



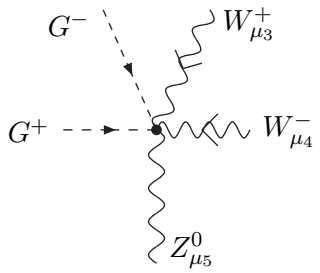
$$-2\bar{g}^2 v\eta_{\mu_4\mu_5}C^{\phi D}$$



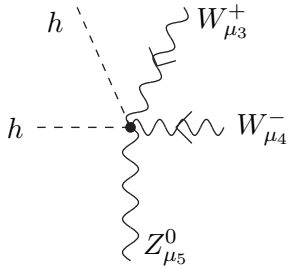
$$+2i\bar{g}^2 v\eta_{\mu_4\mu_5}C^{\phi D}$$



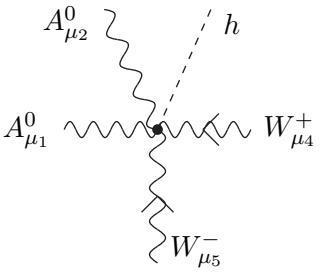
$$\begin{aligned}
&-\frac{4i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi W}\left(\eta_{\mu_3\mu_4}p_3^{\mu_5}-\eta_{\mu_3\mu_4}p_4^{\mu_5}-\eta_{\mu_3\mu_5}p_3^{\mu_4}+\eta_{\mu_3\mu_5}p_5^{\mu_4}\right.\\
&\left.+\eta_{\mu_4\mu_5}p_4^{\mu_3}-\eta_{\mu_4\mu_5}p_5^{\mu_3}\right)-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}\left(\eta_{\mu_3\mu_5}p_5^{\mu_4}-\eta_{\mu_4\mu_5}p_5^{\mu_3}\right)\\
&-\frac{4i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}}\left(p_3^{\alpha_1}+p_4^{\alpha_1}+p_5^{\alpha_1}\right)\epsilon_{\mu_5\mu_3\mu_4\alpha_1}\\
&-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_5^{\alpha_1}\epsilon_{\mu_5\mu_3\mu_4\alpha_1}
\end{aligned}$$



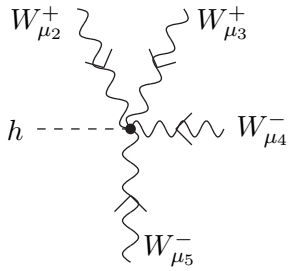
$$
\begin{aligned}
&-\frac{4i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi W}\left(\eta_{\mu_3\mu_4}p_3^{\mu_5}-\eta_{\mu_3\mu_4}p_4^{\mu_5}-\eta_{\mu_3\mu_5}p_3^{\mu_4}+\eta_{\mu_3\mu_5}p_5^{\mu_4}\right.\\
&\left.+\eta_{\mu_4\mu_5}p_4^{\mu_3}-\eta_{\mu_4\mu_5}p_5^{\mu_3}\right)+\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}\left(\eta_{\mu_3\mu_5}p_5^{\mu_4}-\eta_{\mu_4\mu_5}p_5^{\mu_3}\right)\\
&-\frac{4i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}}\left(p_3^{\alpha_1}+p_4^{\alpha_1}+p_5^{\alpha_1}\right)\epsilon_{\mu_5\mu_3\mu_4\alpha_1}\\
&+\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_5^{\alpha_1}\epsilon_{\mu_5\mu_3\mu_4\alpha_1}
\end{aligned}
$$



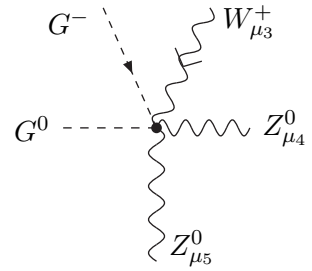
$$
\begin{aligned}
&-\frac{4i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi W}\left(\eta_{\mu_3\mu_4}p_3^{\mu_5}-\eta_{\mu_3\mu_4}p_4^{\mu_5}-\eta_{\mu_3\mu_5}p_3^{\mu_4}+\eta_{\mu_3\mu_5}p_5^{\mu_4}\right.\\
&\left.+\eta_{\mu_4\mu_5}p_4^{\mu_3}-\eta_{\mu_4\mu_5}p_5^{\mu_3}\right)-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}\left(\eta_{\mu_3\mu_5}p_5^{\mu_4}-\eta_{\mu_4\mu_5}p_5^{\mu_3}\right)\\
&-\frac{4i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}}\left(p_3^{\alpha_1}+p_4^{\alpha_1}+p_5^{\alpha_1}\right)\epsilon_{\mu_5\mu_3\mu_4\alpha_1}\\
&-\frac{2i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_5^{\alpha_1}\epsilon_{\mu_5\mu_3\mu_4\alpha_1}
\end{aligned}
$$



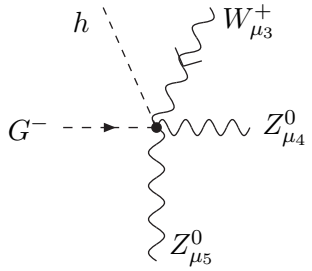
$$
-\frac{4i\bar{g}^2\bar{g}'^2v}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}+\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}-2\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}\right)C^{\phi W}
$$



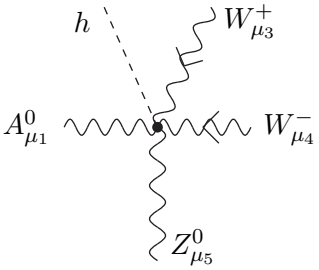
$$
+4i\bar{g}^2v\left(\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}+\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}-2\eta_{\mu_2\mu_3}\eta_{\mu_4\mu_5}\right)C^{\phi W}
$$



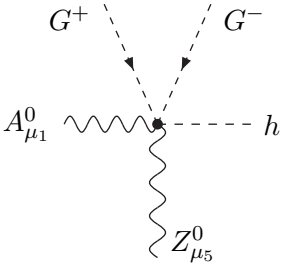
$$+\frac{2\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\eta_{\mu_3\mu_4}p_4^{\mu_5}+\eta_{\mu_3\mu_5}p_5^{\mu_4}-\eta_{\mu_4\mu_5}\left(p_4^{\mu_3}+p_5^{\mu_3}\right)\right)$$
$$+\frac{2\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}\left(p_4^{\alpha_1}-p_5^{\alpha_1}\right)\epsilon_{\mu_4\mu_3\mu_5\alpha_1}$$



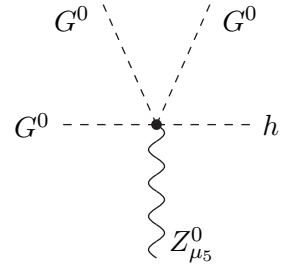
$$-\frac{2i\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi WB}\left(\eta_{\mu_3\mu_4}p_4^{\mu_5}+\eta_{\mu_3\mu_5}p_5^{\mu_4}-\eta_{\mu_4\mu_5}\left(p_4^{\mu_3}+p_5^{\mu_3}\right)\right)$$
$$-\frac{2i\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\phi\widetilde{W}B}\left(p_4^{\alpha_1}-p_5^{\alpha_1}\right)\epsilon_{\mu_4\mu_3\mu_5\alpha_1}$$



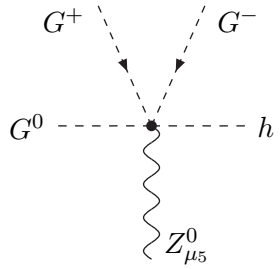
$$+\frac{4i\bar{g}^3\bar{g}'v}{\bar{g}^2+\bar{g}'^2}\left(2\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}-\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}-\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}\right)C^{\phi W}$$



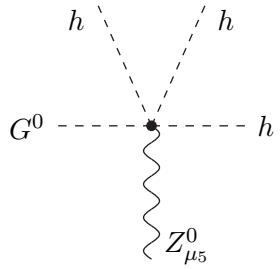
$$-i\bar{g}\bar{g}'v\eta_{\mu_1\mu_5}C^{\phi D}$$



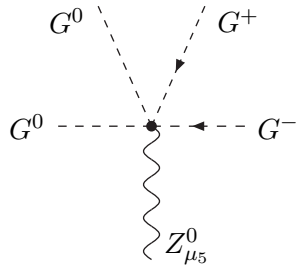
$$-\frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi D}\left(p_1^{\mu_5}+p_2^{\mu_5}+p_3^{\mu_5}-3p_4^{\mu_5}\right)$$



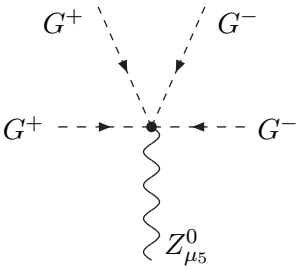
$$-\frac{\bar{g}'^2-\bar{g}^2}{2\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi D}\left(p_1^{\mu_5}-p_4^{\mu_5}\right)$$



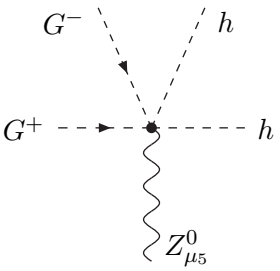
$$-\frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi D}\left(3p_1^{\mu_5}-p_2^{\mu_5}-p_3^{\mu_5}-p_4^{\mu_5}\right)$$



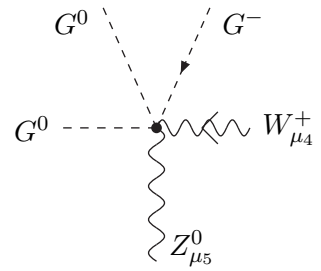
$$+\frac{1}{2}i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi D}\left(p_3^{\mu_5}-p_4^{\mu_5}\right)$$



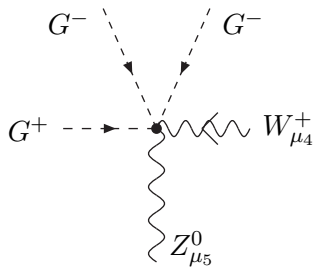
$$+\frac{i\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi D}\left(p_1^{\mu_5}+p_2^{\mu_5}-p_3^{\mu_5}-p_4^{\mu_5}\right)$$



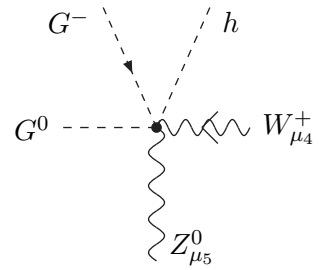
$$+\frac{1}{2}i\sqrt{\bar{g}^2+\bar{g}'^2}C^{\phi D}\left(p_1^{\mu_5}-p_2^{\mu_5}\right)$$



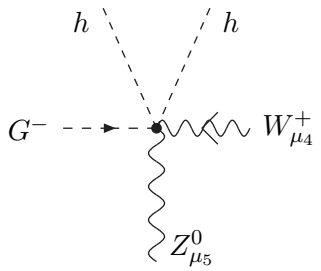
$$-\frac{1}{2}i\bar{g}v\sqrt{\bar{g}^2+\bar{g}'^2}\eta_{\mu_4\mu_5}C^{\phi D}$$



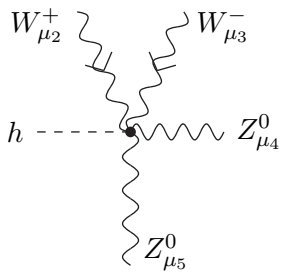
$$-\frac{i\bar{g}v\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_4\mu_5}C^{\phi D}$$



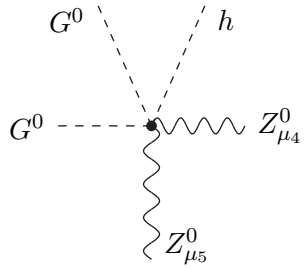
$$+\frac{1}{2}\bar{g}v\sqrt{\bar{g}^2+\bar{g}'^2}\eta_{\mu_4\mu_5}C^{\phi D}$$



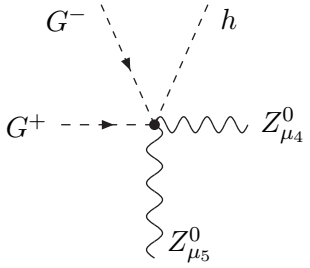
$$-\frac{3}{2}i\bar{g}v\sqrt{\bar{g}^2+\bar{g}'^2}\eta_{\mu_4\mu_5}C^{\phi D}$$



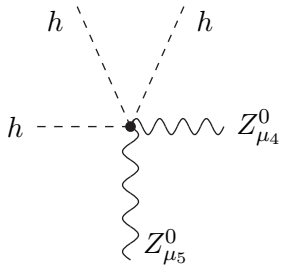
$$-\frac{4i\bar{g}^4v}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}+\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}-2\eta_{\mu_2\mu_3}\eta_{\mu_4\mu_5}\right)C^{\phi W}$$



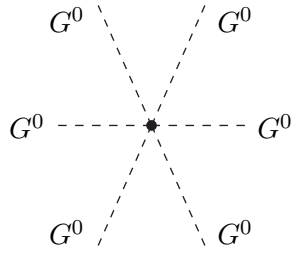
$$+iv\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_4\mu_5}C^{\phi D}$$



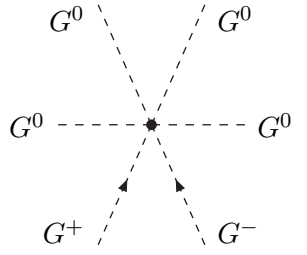
$$+iv\left(\bar{g}'^2-\bar{g}^2\right)\eta_{\mu_4\mu_5}C^{\phi D}$$



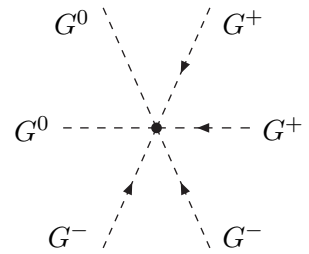
$$+3iv\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_4\mu_5}C^{\phi D}$$



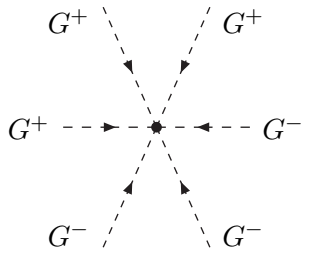
$$+90iC^{\phi}$$



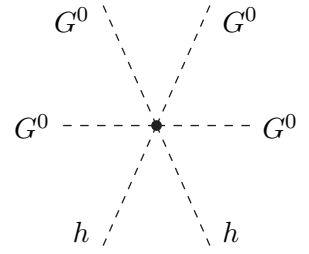
$$+18iC^{\phi}$$



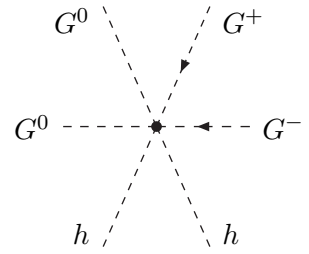
$+12iC^{\phi}$



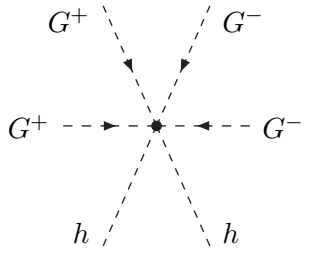
$+36iC^{\phi}$



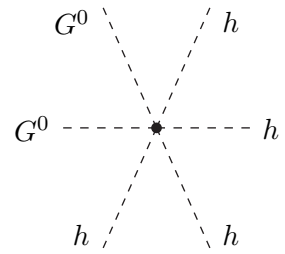
$+18iC^{\phi}$



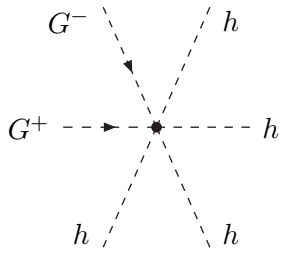
$+6iC^{\phi}$



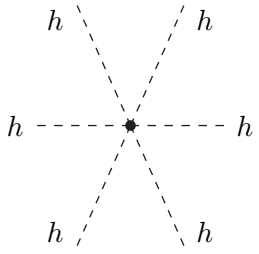
$+12iC^{\phi}$



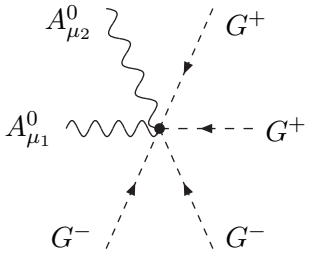
$$+18iC^{\phi}$$



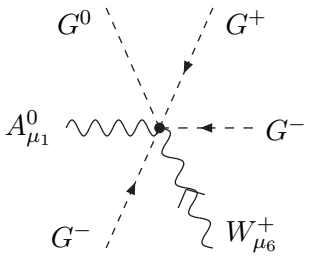
$$+18iC^{\phi}$$



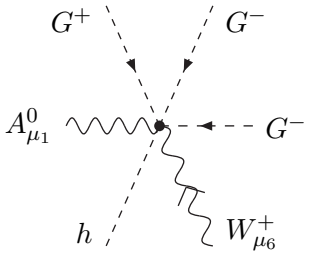
$$+90iC^{\phi}$$



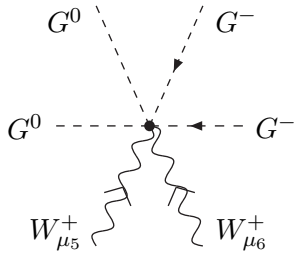
$$+\frac{8i\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\eta_{\mu_1\mu_2}C^{\phi D}$$



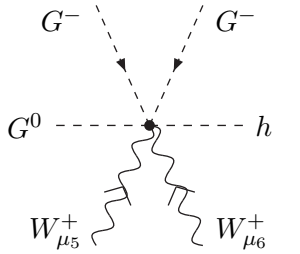
$$-\frac{2\bar{g}^2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_1\mu_6}C^{\phi D}$$



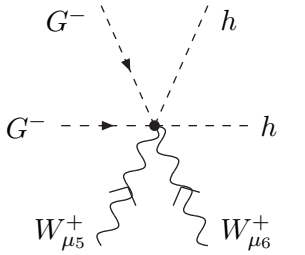
$$+\frac{2i\bar{g}^2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_1\mu_6}C^{\phi D}$$



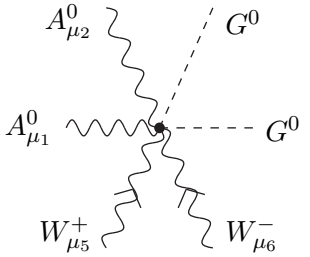
$$-2i\bar{g}^2\eta_{\mu_5\mu_6}C^{\phi D}$$



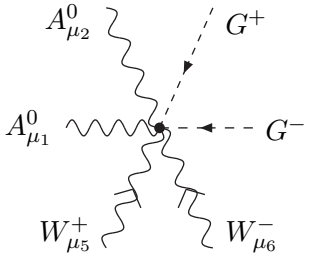
$$-2\bar{g}^2\eta_{\mu_5\mu_6}C^{\phi D}$$



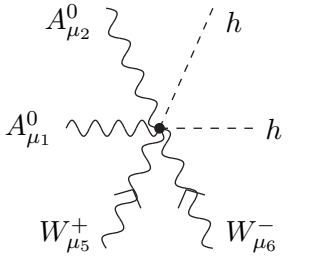
$$+2i\bar{g}^2\eta_{\mu_5\mu_6}C^{\phi D}$$



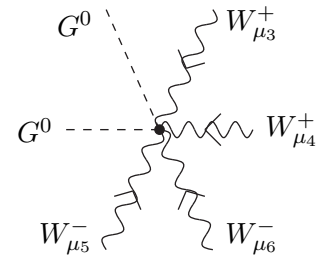
$$-\frac{4i\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_1\mu_6}\eta_{\mu_2\mu_5}+\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_6}-2\eta_{\mu_1\mu_2}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



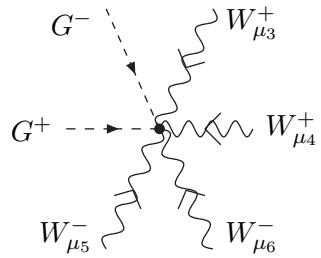
$$-\frac{4i\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_1\mu_6}\eta_{\mu_2\mu_5}+\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_6}-2\eta_{\mu_1\mu_2}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



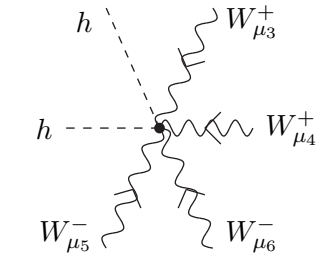
$$-\frac{4i\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_1\mu_6}\eta_{\mu_2\mu_5}+\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_6}-2\eta_{\mu_1\mu_2}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



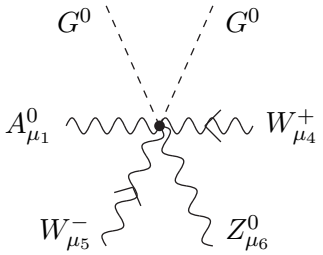
$$+4i\bar{g}^2\left(\eta_{\mu_3\mu_6}\eta_{\mu_4\mu_5}+\eta_{\mu_3\mu_5}\eta_{\mu_4\mu_6}-2\eta_{\mu_3\mu_4}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



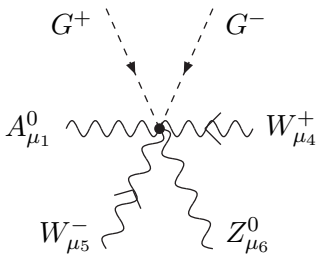
$$+4i\bar{g}^2\left(\eta_{\mu_3\mu_6}\eta_{\mu_4\mu_5}+\eta_{\mu_3\mu_5}\eta_{\mu_4\mu_6}-2\eta_{\mu_3\mu_4}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



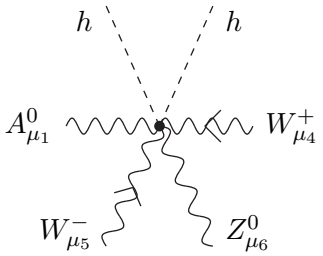
$$+4i\bar{g}^2\left(\eta_{\mu_3\mu_6}\eta_{\mu_4\mu_5}+\eta_{\mu_3\mu_5}\eta_{\mu_4\mu_6}-2\eta_{\mu_3\mu_4}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



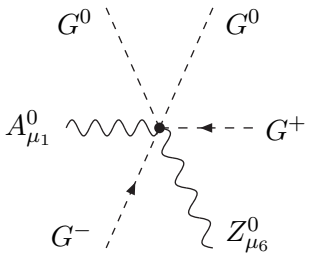
$$+\frac{4i\bar{g}^3\bar{g}'}{\bar{g}^2+\bar{g}'^2}\left(2\eta_{\mu_1\mu_6}\eta_{\mu_4\mu_5}-\eta_{\mu_1\mu_5}\eta_{\mu_4\mu_6}-\eta_{\mu_1\mu_4}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



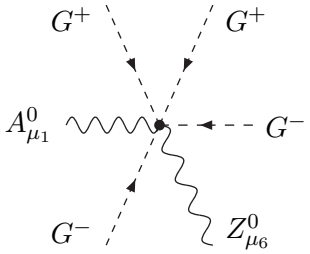
$$+\frac{4i\bar{g}^3\bar{g}'}{\bar{g}^2+\bar{g}'^2}\left(2\eta_{\mu_1\mu_6}\eta_{\mu_4\mu_5}-\eta_{\mu_1\mu_5}\eta_{\mu_4\mu_6}-\eta_{\mu_1\mu_4}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



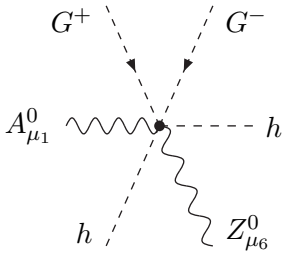
$$+\frac{4i\bar{g}^3\bar{g}'}{\bar{g}^2+\bar{g}'^2}\left(2\eta_{\mu_1\mu_6}\eta_{\mu_4\mu_5}-\eta_{\mu_1\mu_5}\eta_{\mu_4\mu_6}-\eta_{\mu_1\mu_4}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



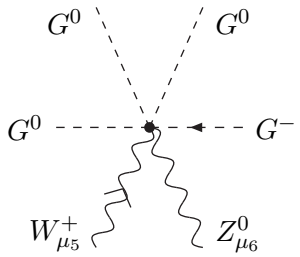
$$-i\bar{g}\bar{g}'\eta_{\mu_1\mu_6}C^{\phi D}$$



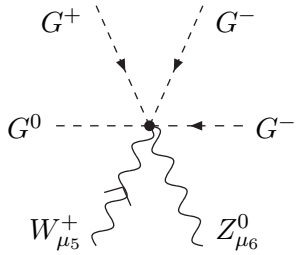
$$-\frac{4i\bar{g}\bar{g}'\left(\bar{g}'^2-\bar{g}^2\right)}{\bar{g}^2+\bar{g}'^2}\eta_{\mu_1\mu_6}C^{\phi D}$$



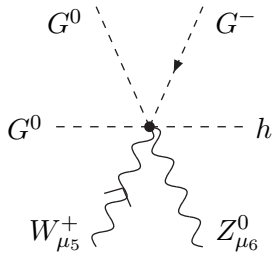
$$-i\bar{g}\bar{g}'\eta_{\mu_1\mu_6}C^{\phi D}$$



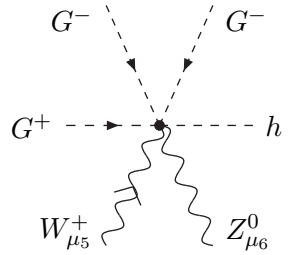
$$+\frac{3}{2}\bar{g}\sqrt{\bar{g}^2+\bar{g}'^2}\eta_{\mu_5\mu_6}C^{\phi D}$$



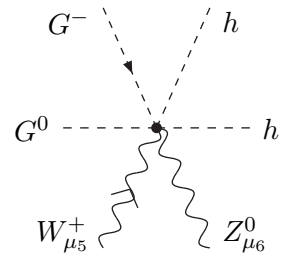
$$+\frac{\bar{g}\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_5\mu_6}C^{\phi D}$$



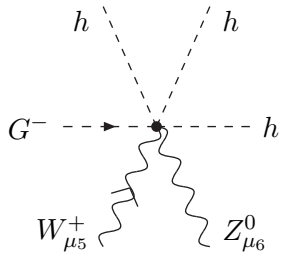
$$-\frac{1}{2}i\bar{g}\sqrt{\bar{g}^2+\bar{g}'^2}\eta_{\mu_5\mu_6}C^{\phi D}$$



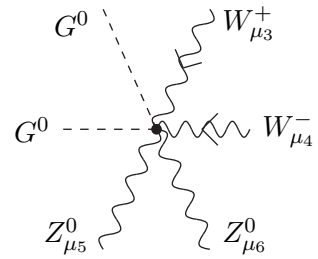
$$-\frac{i\bar{g}\left(\bar{g}'^2-\bar{g}^2\right)}{\sqrt{\bar{g}^2+\bar{g}'^2}}\eta_{\mu_5\mu_6}C^{\phi D}$$



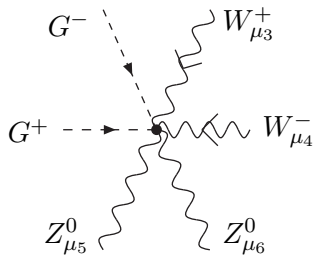
$$+\frac{1}{2}\bar{g}\sqrt{\bar{g}^2+\bar{g}'^2}\eta_{\mu_5\mu_6}C^{\phi D}$$



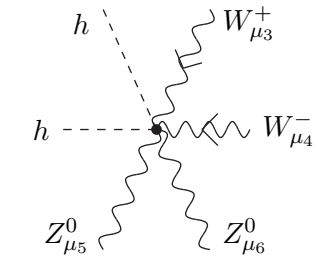
$$-\frac{3}{2}i\bar{g}\sqrt{\bar{g}^2+\bar{g}'^2}\eta_{\mu_5\mu_6}C^{\phi D}$$



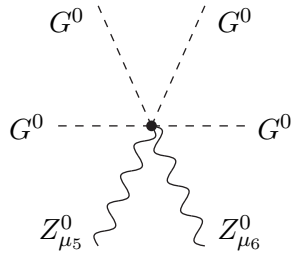
$$-\frac{4i\bar{g}^4}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_3\mu_6}\eta_{\mu_4\mu_5}+\eta_{\mu_3\mu_5}\eta_{\mu_4\mu_6}-2\eta_{\mu_3\mu_4}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



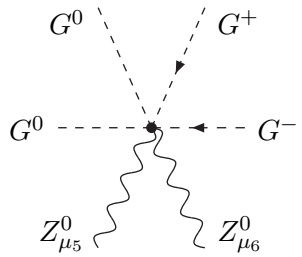
$$-\frac{4i\bar{g}^4}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_3\mu_6}\eta_{\mu_4\mu_5}+\eta_{\mu_3\mu_5}\eta_{\mu_4\mu_6}-2\eta_{\mu_3\mu_4}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



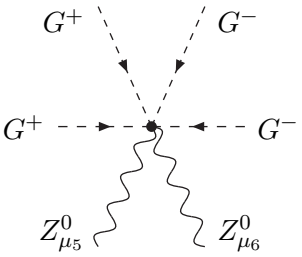
$$-\frac{4i\bar{g}^4}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_3\mu_6}\eta_{\mu_4\mu_5}+\eta_{\mu_3\mu_5}\eta_{\mu_4\mu_6}-2\eta_{\mu_3\mu_4}\eta_{\mu_5\mu_6}\right)C^{\phi W}$$



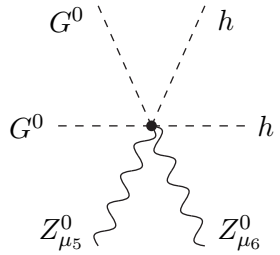
$$+3i\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_5\mu_6}C^{\phi D}$$



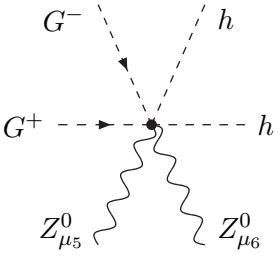
$$+i\left(\bar{g}'^2-\bar{g}^2\right)\eta_{\mu_5\mu_6}C^{\phi D}$$



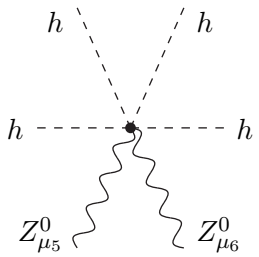
$$+\frac{2i\left(\bar{g}'^2-\bar{g}^2\right)^2}{\bar{g}^2+\bar{g}'^2}\eta_{\mu_5\mu_6}C^{\phi D}$$



$$+i\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_5\mu_6}C^{\phi D}$$

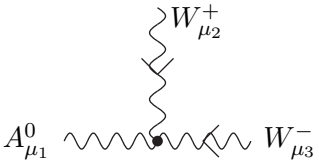


$$+i\left(\bar{g}'^2-\bar{g}^2\right)\eta_{\mu_5\mu_6}C^{\phi D}$$

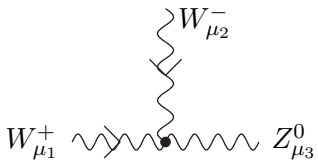


$$+3i\left(\bar{g}^2+\bar{g}'^2\right)\eta_{\mu_5\mu_6}C^{\phi D}$$

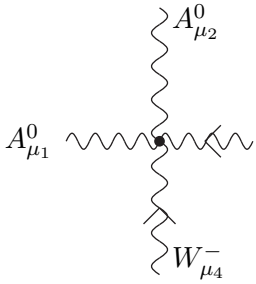
A.8 Gauge-gauge vertices



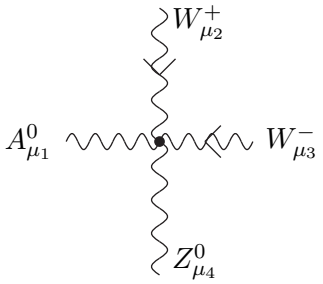
$$\begin{aligned}
&+\frac{i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left(\eta_{\mu_1\mu_2}p_1^{\mu_3}-\eta_{\mu_1\mu_2}p_2^{\mu_3}-\eta_{\mu_1\mu_3}p_1^{\mu_2}+\eta_{\mu_1\mu_3}p_3^{\mu_2}\right.\\
&\left.+\eta_{\mu_2\mu_3}p_2^{\mu_1}-\eta_{\mu_2\mu_3}p_3^{\mu_1}\right)-\frac{6i\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{W}\left(p_1\cdot p_2\eta_{\mu_1\mu_3}p_3^{\mu_2}\right.\\
&-p_1\cdot p_2\eta_{\mu_2\mu_3}p_3^{\mu_1}-p_1\cdot p_3\eta_{\mu_1\mu_2}p_2^{\mu_3}+p_1\cdot p_3\eta_{\mu_2\mu_3}p_2^{\mu_1}\\
&\left.+p_1^{\mu_3}\left(p_2\cdot p_3\eta_{\mu_1\mu_2}-p_2^{\mu_1}p_3^{\mu_2}\right)+p_1^{\mu_2}\left(p_2^{\mu_3}p_3^{\mu_1}-p_2\cdot p_3\eta_{\mu_1\mu_3}\right)\right)\\
&+\frac{i\bar{g}^2v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\left(\bar{g}^2\eta_{\mu_1\mu_2}p_1^{\mu_3}-\bar{g}^2\eta_{\mu_1\mu_3}p_1^{\mu_2}\right.\\
&\left.+\bar{g}'^2\eta_{\mu_1\mu_2}p_2^{\mu_3}-\bar{g}'^2\eta_{\mu_1\mu_3}p_3^{\mu_2}-\bar{g}'^2\eta_{\mu_2\mu_3}p_2^{\mu_1}+\bar{g}'^2\eta_{\mu_2\mu_3}p_3^{\mu_1}\right)\\
&-\frac{2i\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\widetilde{W}}\left(p_1\cdot p_2p_3^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}+p_1\cdot p_3p_2^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}\right.\\
&+p_2\cdot p_3p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}-p_1^{\alpha_1}\left(p_2^{\beta_1}p_3^{\mu_1}+p_3^{\beta_1}p_2^{\mu_1}\right)\epsilon_{\mu_2\mu_3\alpha_1\beta_1}\\
&+p_3^{\beta_1}\left(p_2^{\alpha_1}p_1^{\mu_3}+p_1^{\alpha_1}p_2^{\mu_3}\right)\epsilon_{\mu_1\mu_2\alpha_1\beta_1}\\
&\left.+\epsilon_{\mu_1\mu_3\alpha_1\beta_1}\left(p_2^{\alpha_1}p_3^{\beta_1}p_1^{\mu_2}-p_1^{\alpha_1}p_2^{\beta_1}p_3^{\mu_2}\right)\right)\\
&+\frac{i\bar{g}^2v^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}
\end{aligned}$$



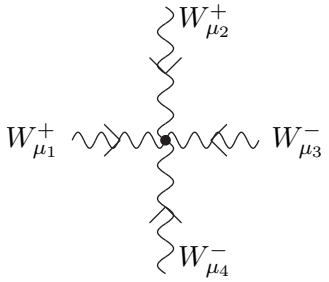
$$
\begin{aligned}
&+\frac{i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left(\eta_{\mu_1\mu_2}p_1^{\mu_3}-\eta_{\mu_1\mu_2}p_2^{\mu_3}-\eta_{\mu_1\mu_3}p_1^{\mu_2}+\eta_{\mu_1\mu_3}p_3^{\mu_2}\right.\\
&\left.+\eta_{\mu_2\mu_3}p_2^{\mu_1}-\eta_{\mu_2\mu_3}p_3^{\mu_1}\right)-\frac{6i\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^W\left(p_1\cdot p_2\eta_{\mu_1\mu_3}p_3^{\mu_2}\right.\\
&-p_1\cdot p_2\eta_{\mu_2\mu_3}p_3^{\mu_1}-p_1\cdot p_3\eta_{\mu_1\mu_2}p_2^{\mu_3}+p_1\cdot p_3\eta_{\mu_2\mu_3}p_2^{\mu_1}\\
&\left.+p_1^{\mu_3}\left(p_2\cdot p_3\eta_{\mu_1\mu_2}-p_2^{\mu_1}p_3^{\mu_2}\right)+p_1^{\mu_2}\left(p_2^{\mu_3}p_3^{\mu_1}-p_2\cdot p_3\eta_{\mu_1\mu_3}\right)\right)\\
&+\frac{i\bar{g}\bar{g}'v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}\left(\bar{g}^2\eta_{\mu_1\mu_3}\left(-p_3^{\mu_2}\right)+\bar{g}^2\eta_{\mu_2\mu_3}p_3^{\mu_1}\right.\\
&\left.+\bar{g}'^2\eta_{\mu_1\mu_2}p_1^{\mu_3}-\bar{g}'^2\eta_{\mu_1\mu_2}p_2^{\mu_3}-\bar{g}'^2\eta_{\mu_1\mu_3}p_1^{\mu_2}+\bar{g}'^2\eta_{\mu_2\mu_3}p_2^{\mu_1}\right)\\
&-\frac{2i\bar{g}}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\widetilde{W}}\left(p_1\cdot p_2p_3^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}+p_1\cdot p_3p_2^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}\right.\\
&+p_2\cdot p_3p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}-p_1^{\alpha_1}\left(p_3^{\beta_1}p_2^{\mu_1}+p_2^{\beta_1}p_3^{\mu_1}\right)\epsilon_{\mu_2\mu_3\alpha_1\beta_1}\\
&+p_3^{\beta_1}\left(p_2^{\alpha_1}p_1^{\mu_3}+p_1^{\alpha_1}p_2^{\mu_3}\right)\epsilon_{\mu_1\mu_2\alpha_1\beta_1}\\
&\left.+\epsilon_{\mu_1\mu_3\alpha_1\beta_1}\left(p_2^{\alpha_1}p_3^{\beta_1}p_1^{\mu_2}-p_1^{\alpha_1}p_2^{\beta_1}p_3^{\mu_2}\right)\right)\\
&-\frac{i\bar{g}\bar{g}'v^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi\widetilde{W}B}p_3^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}
\end{aligned}
$$



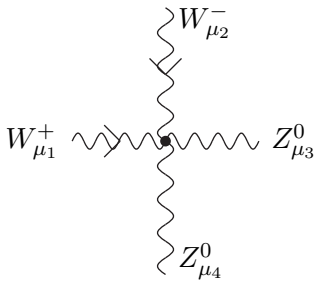
$$
\begin{aligned}
&+\frac{i\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)\\
&-\frac{6i\bar{g}\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^W\left(\eta_{\mu_1\mu_2}p_1^{\mu_4}p_4^{\mu_3}+\eta_{\mu_1\mu_2}p_2^{\mu_4}p_4^{\mu_3}+\eta_{\mu_1\mu_3}p_1^{\mu_4}p_3^{\mu_2}\right.\\
&-\eta_{\mu_1\mu_3}p_2^{\mu_4}p_3^{\mu_2}-\eta_{\mu_1\mu_3}p_1^{\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_3}p_1^{\mu_4}p_4^{\mu_2}-\eta_{\mu_1\mu_4}p_1^{\mu_2}p_4^{\mu_3}\\
&-\eta_{\mu_2\mu_3}p_1^{\mu_4}p_3^{\mu_1}+\eta_{\mu_2\mu_3}p_2^{\mu_4}p_3^{\mu_1}-\eta_{\mu_2\mu_3}p_2^{\mu_1}p_3^{\mu_4}-\eta_{\mu_2\mu_3}p_2^{\mu_4}p_4^{\mu_1}\\
&-\eta_{\mu_2\mu_4}p_2^{\mu_1}p_4^{\mu_3}+p_1^{\mu_3}\left(\eta_{\mu_1\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_4}p_3^{\mu_2}+\eta_{\mu_1\mu_4}p_4^{\mu_2}-\eta_{\mu_2\mu_4}p_4^{\mu_1}\right)\\
&+p_2^{\mu_3}\left(\eta_{\mu_1\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_4}p_4^{\mu_2}+\eta_{\mu_2\mu_4}\left(p_4^{\mu_1}-p_3^{\mu_1}\right)\right)+\eta_{\mu_3\mu_4}p_1^{\mu_2}p_3^{\mu_1}\\
&+\eta_{\mu_3\mu_4}p_2^{\mu_1}p_3^{\mu_2}+\eta_{\mu_3\mu_4}p_1^{\mu_2}p_4^{\mu_1}+\eta_{\mu_3\mu_4}p_2^{\mu_1}p_4^{\mu_2}-p_1\cdot p_3\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}\\
&-p_1\cdot p_4\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}-p_2\cdot p_3\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}-p_2\cdot p_4\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}+p_1\cdot p_3\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\\
&\left.+p_1\cdot p_4\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}+p_2\cdot p_3\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}+p_2\cdot p_4\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\right)\\
&-\frac{2i\bar{g}^3\bar{g}'^3v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^2}\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)C^{\phi WB}\\
&+\frac{2i\bar{g}\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}C^{\widetilde{W}}\left(-\epsilon_{\mu_1\mu_2\mu_3\alpha_1}p_1^{\mu_4}p_3^{\alpha_1}+\epsilon_{\mu_2\mu_4\mu_3\alpha_1}p_2^{\mu_1}p_3^{\alpha_1}+\epsilon_{\mu_1\mu_2\mu_3\alpha_1}p_2^{\mu_4}p_3^{\alpha_1}\right.\\
&+\epsilon_{\mu_2\mu_4\mu_3\alpha_1}p_1^{\alpha_1}p_3^{\mu_1}+\epsilon_{\mu_2\mu_4\mu_3\alpha_1}p_2^{\alpha_1}p_3^{\mu_1}-\epsilon_{\mu_1\mu_2\mu_3\alpha_1}p_1^{\alpha_1}p_3^{\mu_4}+\epsilon_{\mu_1\mu_2\mu_3\alpha_1}p_2^{\alpha_1}p_3^{\mu_4}\\
&-\epsilon_{\mu_1\mu_2\mu_3\alpha_1}p_1^{\mu_4}p_4^{\alpha_1}-\epsilon_{\mu_2\mu_4\mu_3\alpha_1}p_2^{\mu_1}p_4^{\alpha_1}+\epsilon_{\mu_1\mu_2\mu_3\alpha_1}p_2^{\mu_4}p_4^{\alpha_1}-\epsilon_{\mu_2\mu_4\mu_3\alpha_1}p_1^{\alpha_1}p_4^{\mu_1}\\
&-\epsilon_{\mu_2\mu_4\mu_3\alpha_1}p_2^{\alpha_1}p_4^{\mu_1}+\epsilon_{\mu_1\mu_4\mu_3\alpha_1}\left(p_1^{\mu_2}\left(p_3^{\alpha_1}-p_4^{\alpha_1}\right)+\left(p_1^{\alpha_1}+p_2^{\alpha_1}\right)\left(p_3^{\mu_2}-p_4^{\mu_2}\right)\right)\\
&+\epsilon_{\mu_1\mu_2\mu_4\alpha_1}\left(-p_1^{\mu_3}\left(p_3^{\alpha_1}+p_4^{\alpha_1}\right)+p_2^{\mu_3}\left(p_3^{\alpha_1}+p_4^{\alpha_1}\right)+\left(p_2^{\alpha_1}-p_1^{\alpha_1}\right)p_4^{\mu_3}\right)\\
&-\epsilon_{\mu_4\mu_3\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_2}-\epsilon_{\mu_4\mu_3\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_2}+\epsilon_{\mu_4\mu_3\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_2}\\
&+\epsilon_{\mu_4\mu_3\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_2}+\epsilon_{\mu_2\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_3}+\epsilon_{\mu_2\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_3}\\
&+\epsilon_{\mu_2\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_3}+\epsilon_{\mu_2\mu_3\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_4}+\epsilon_{\mu_2\mu_3\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_4}\\
&+\epsilon_{\mu_2\mu_3\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_4}+\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_3}+\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_3}\\
&+\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_3}+\epsilon_{\mu_1\mu_3\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_4}+\epsilon_{\mu_1\mu_3\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_4}\\
&+\epsilon_{\mu_1\mu_3\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_4}-\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_3\mu_4}+\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_3\mu_4}\\
&-\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_3\mu_4}+\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_3\mu_4}+\epsilon_{\mu_1\mu_2\mu_4\mu_3}p_1\cdot p_3\\
&\left.-\epsilon_{\mu_1\mu_2\mu_4\mu_3}p_1\cdot p_4-\epsilon_{\mu_1\mu_2\mu_4\mu_3}p_2\cdot p_3+\epsilon_{\mu_1\mu_2\mu_4\mu_3}p_2\cdot p_4\right)
\end{aligned}
$$



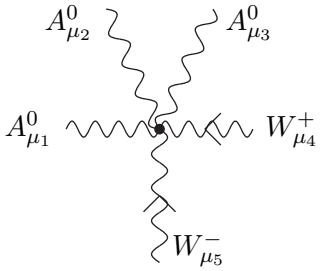
$$
\begin{aligned}
&-\frac{i\bar{g}^3\bar{g}'}{\bar{g}^2+\bar{g}'^2}\left(2\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)\\
&+\frac{6i\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^W\left(\eta_{\mu_1\mu_2}p_2^{\mu_4}p_4^{\mu_3}+\eta_{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}-\eta_{\mu_1\mu_3}p_1^{\mu_2}p_3^{\mu_4}+\eta_{\mu_1\mu_3}p_3^{\mu_4}p_4^{\mu_2}\right.\\
&-\eta_{\mu_1\mu_4}p_1^{\mu_2}p_2^{\mu_3}-\eta_{\mu_1\mu_4}p_2^{\mu_3}p_4^{\mu_2}-\eta_{\mu_1\mu_4}p_3^{\mu_2}p_4^{\mu_3}-\eta_{\mu_2\mu_3}p_2^{\mu_4}p_4^{\mu_1}-\eta_{\mu_2\mu_3}p_3^{\mu_4}p_4^{\mu_1}\\
&+p_1^{\mu_4}\left(\eta_{\mu_1\mu_2}p_2^{\mu_3}+\eta_{\mu_1\mu_3}p_3^{\mu_2}-\eta_{\mu_2\mu_3}\left(p_2^{\mu_1}+p_3^{\mu_1}\right)\right)+\eta_{\mu_2\mu_4}p_2^{\mu_3}p_4^{\mu_1}-\eta_{\mu_2\mu_4}p_2^{\mu_1}p_4^{\mu_3}\\
&+\eta_{\mu_2\mu_4}p_3^{\mu_1}p_4^{\mu_3}+p_1^{\mu_3}\left(\eta_{\mu_1\mu_2}\left(-p_2^{\mu_4}\right)+\eta_{\mu_1\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_4}p_3^{\mu_2}+\eta_{\mu_2\mu_4}p_2^{\mu_1}\right)\\
&+\eta_{\mu_3\mu_4}p_1^{\mu_2}p_3^{\mu_1}+\eta_{\mu_3\mu_4}p_3^{\mu_2}p_4^{\mu_1}+\eta_{\mu_3\mu_4}p_2^{\mu_1}p_4^{\mu_2}-\eta_{\mu_3\mu_4}p_3^{\mu_1}p_4^{\mu_2}-p_1\cdot p_3\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}\\
&-p_2\cdot p_4\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}+p_1\cdot p_2\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-p_1\cdot p_2\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}+p_1\cdot p_3\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\\
&\left.+p_2\cdot p_4\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}+p_3\cdot p_4\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-p_3\cdot p_4\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)\\
&-\frac{i\bar{g}^2\bar{g}'^2v^2\left(\bar{g}'^2-\bar{g}^2\right)}{\left(\bar{g}^2+\bar{g}'^2\right)^2}\left(2\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)C^{\phi WB}\\
&+\frac{2i\bar{g}^2\bar{g}'}{\bar{g}^2+\bar{g}'^2}C^{\widetilde{W}}\left(\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_1^{\mu_4}p_2^{\alpha_1}-\epsilon_{\mu_3\mu_4\mu_2\alpha_1}p_4^{\mu_1}p_2^{\alpha_1}+\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_4^{\beta_1}\eta_{\mu_1\mu_2}p_2^{\alpha_1}\right.\\
&+\epsilon_{\mu_3\mu_2\alpha_1\beta_1}p_4^{\beta_1}\eta_{\mu_1\mu_4}p_2^{\alpha_1}-\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_4^{\beta_1}\eta_{\mu_2\mu_3}p_2^{\alpha_1}-\epsilon_{\mu_1\mu_3\alpha_1\beta_1}p_4^{\beta_1}\eta_{\mu_2\mu_4}p_2^{\alpha_1}\\
&-\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_4^{\beta_1}\eta_{\mu_3\mu_4}p_2^{\alpha_1}-\epsilon_{\mu_3\mu_4\mu_2\alpha_1}p_1^{\alpha_1}p_2^{\mu_1}+\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_1^{\alpha_1}p_2^{\mu_4}\\
&-\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_1^{\mu_4}p_3^{\alpha_1}+\epsilon_{\mu_3\mu_4\mu_2\alpha_1}p_1^{\alpha_1}p_3^{\mu_1}-\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_1^{\alpha_1}p_3^{\mu_4}-\epsilon_{\mu_3\mu_4\mu_2\alpha_1}p_2^{\mu_1}p_4^{\alpha_1}\\
&+\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_2^{\mu_4}p_4^{\alpha_1}+\epsilon_{\mu_3\mu_4\mu_2\alpha_1}p_3^{\mu_1}p_4^{\alpha_1}-\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_3^{\mu_4}p_4^{\alpha_1}+\epsilon_{\mu_3\mu_4\mu_2\alpha_1}p_3^{\alpha_1}p_4^{\mu_1}\\
&+\epsilon_{\mu_1\mu_3\mu_4\alpha_1}\left(p_1^{\mu_2}\left(p_2^{\alpha_1}+p_3^{\alpha_1}\right)+p_1^{\alpha_1}p_3^{\mu_2}-p_3^{\mu_2}p_4^{\alpha_1}-p_2^{\alpha_1}p_4^{\mu_2}-p_3^{\alpha_1}p_4^{\mu_2}\right)\\
&+\epsilon_{\mu_1\mu_4\mu_2\alpha_1}\left(-p_1^{\alpha_1}p_2^{\mu_3}+p_4^{\alpha_1}p_2^{\mu_3}-p_1^{\mu_3}\left(p_2^{\alpha_1}+p_3^{\alpha_1}\right)+p_2^{\alpha_1}p_4^{\mu_3}+p_3^{\alpha_1}p_4^{\mu_3}\right)\\
&-\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_2^{\beta_1}\eta_{\mu_1\mu_2}-\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_2}+\epsilon_{\mu_4\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_2^{\beta_1}\eta_{\mu_1\mu_3}\\
&+\epsilon_{\mu_4\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_3}-\epsilon_{\mu_4\mu_2\alpha_1\beta_1}p_3^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_3}-\epsilon_{\mu_3\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_2^{\beta_1}\eta_{\mu_1\mu_4}\\
&+\epsilon_{\mu_3\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_4}-\epsilon_{\mu_3\mu_2\alpha_1\beta_1}p_3^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_4}-\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_2^{\beta_1}\eta_{\mu_2\mu_3}\\
&-\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_3}-\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_3^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_3}+\epsilon_{\mu_1\mu_3\alpha_1\beta_1}p_1^{\alpha_1}p_2^{\beta_1}\eta_{\mu_2\mu_4}\\
&-\epsilon_{\mu_1\mu_3\alpha_1\beta_1}p_3^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_4}+\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_3\mu_4}-\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_3^{\alpha_1}p_4^{\beta_1}\eta_{\mu_3\mu_4}\\
&\left.-\epsilon_{\mu_1\mu_3\mu_4\mu_2}p_1\cdot p_2+\epsilon_{\mu_1\mu_3\mu_4\mu_2}p_1\cdot p_3+\epsilon_{\mu_1\mu_3\mu_4\mu_2}p_2\cdot p_4-\epsilon_{\mu_1\mu_3\mu_4\mu_2}p_3\cdot p_4\right)
\end{aligned}
$$



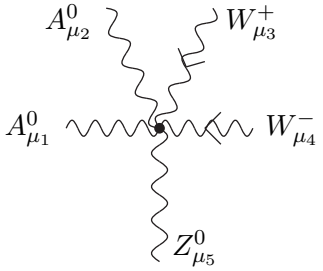
$$
\begin{aligned}
&-i\bar{g}^2\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)+6i\bar{g}C^W\left(\eta_{\mu_1\mu_2}p_1^{\mu_4}p_4^{\mu_3}\right.\\
&+\eta_{\mu_1\mu_2}p_2^{\mu_4}p_4^{\mu_3}+\eta_{\mu_1\mu_3}p_1^{\mu_4}p_3^{\mu_2}-\eta_{\mu_1\mu_3}p_2^{\mu_4}p_3^{\mu_2}-\eta_{\mu_1\mu_3}p_1^{\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_3}p_1^{\mu_4}p_4^{\mu_2}\\
&-\eta_{\mu_1\mu_4}p_1^{\mu_2}p_4^{\mu_3}-\eta_{\mu_2\mu_3}p_1^{\mu_4}p_3^{\mu_1}+\eta_{\mu_2\mu_3}p_2^{\mu_4}p_3^{\mu_1}-\eta_{\mu_2\mu_3}p_2^{\mu_1}p_3^{\mu_4}-\eta_{\mu_2\mu_3}p_2^{\mu_4}p_4^{\mu_1}\\
&-\eta_{\mu_2\mu_4}p_2^{\mu_1}p_4^{\mu_3}+p_1^{\mu_3}\left(\eta_{\mu_1\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_4}p_3^{\mu_2}+\eta_{\mu_1\mu_4}p_4^{\mu_2}-\eta_{\mu_2\mu_4}p_4^{\mu_1}\right)\\
&+p_2^{\mu_3}\left(\eta_{\mu_1\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_4}p_4^{\mu_2}+\eta_{\mu_2\mu_4}\left(p_4^{\mu_1}-p_3^{\mu_1}\right)\right)+\eta_{\mu_3\mu_4}p_1^{\mu_2}p_3^{\mu_1}\\
&+\eta_{\mu_3\mu_4}p_2^{\mu_1}p_3^{\mu_2}+\eta_{\mu_3\mu_4}p_1^{\mu_2}p_4^{\mu_1}+\eta_{\mu_3\mu_4}p_2^{\mu_1}p_4^{\mu_2}-p_1\cdot p_3\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}\\
&-p_1\cdot p_4\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}-p_2\cdot p_3\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}-p_2\cdot p_4\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}+p_1\cdot p_3\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\\
&\left.+p_1\cdot p_4\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}+p_2\cdot p_3\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}+p_2\cdot p_4\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\right)\\
&+2i\bar{g}C^{\widetilde{W}}\left(-\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_1^{\mu_4}p_3^{\alpha_1}-\epsilon_{\mu_3\mu_2\mu_4\alpha_1}p_2^{\mu_1}p_3^{\alpha_1}+\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_2^{\mu_4}p_3^{\alpha_1}\right.\\
&-\epsilon_{\mu_3\mu_2\mu_4\alpha_1}p_1^{\alpha_1}p_3^{\mu_1}-\epsilon_{\mu_3\mu_2\mu_4\alpha_1}p_2^{\alpha_1}p_3^{\mu_1}-\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_1^{\alpha_1}p_3^{\mu_4}+\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_2^{\alpha_1}p_3^{\mu_4}\\
&-\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_1^{\mu_4}p_4^{\alpha_1}+\epsilon_{\mu_3\mu_2\mu_4\alpha_1}p_2^{\mu_1}p_4^{\alpha_1}+\epsilon_{\mu_1\mu_3\mu_2\alpha_1}p_2^{\mu_4}p_4^{\alpha_1}+\epsilon_{\mu_3\mu_2\mu_4\alpha_1}p_1^{\alpha_1}p_4^{\mu_1}\\
&+\epsilon_{\mu_3\mu_2\mu_4\alpha_1}p_2^{\alpha_1}p_4^{\mu_1}+\epsilon_{\mu_1\mu_3\mu_4\alpha_1}\left(p_1^{\mu_2}\left(p_3^{\alpha_1}-p_4^{\alpha_1}\right)+\left(p_1^{\alpha_1}+p_2^{\alpha_1}\right)\left(p_3^{\mu_2}-p_4^{\mu_2}\right)\right)\\
&+\epsilon_{\mu_1\mu_2\mu_4\alpha_1}\left(p_1^{\mu_3}\left(p_3^{\alpha_1}+p_4^{\alpha_1}\right)-p_2^{\mu_3}\left(p_3^{\alpha_1}+p_4^{\alpha_1}\right)+\left(p_1^{\alpha_1}-p_2^{\alpha_1}\right)p_4^{\mu_3}\right)\\
&-\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_2}-\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_2}+\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_2}\\
&+\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_2}-\epsilon_{\mu_2\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_3}-\epsilon_{\mu_2\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_3}\\
&-\epsilon_{\mu_2\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_3}+\epsilon_{\mu_3\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_4}+\epsilon_{\mu_3\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_4}\\
&+\epsilon_{\mu_3\mu_2\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_4}-\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_3}-\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_3}\\
&-\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_3}-\epsilon_{\mu_1\mu_3\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_4}-\epsilon_{\mu_1\mu_3\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_4}\\
&-\epsilon_{\mu_1\mu_3\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_4}+\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_3\mu_4}-\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_3\mu_4}\\
&+\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_3\mu_4}-\epsilon_{\mu_1\mu_2\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_3\mu_4}-\epsilon_{\mu_1\mu_3\mu_2\mu_4}p_1\cdot p_3\\
&\left.+\epsilon_{\mu_1\mu_3\mu_2\mu_4}p_1\cdot p_4+\epsilon_{\mu_1\mu_3\mu_2\mu_4}p_2\cdot p_3-\epsilon_{\mu_1\mu_3\mu_2\mu_4}p_2\cdot p_4\right)
\end{aligned}
$$



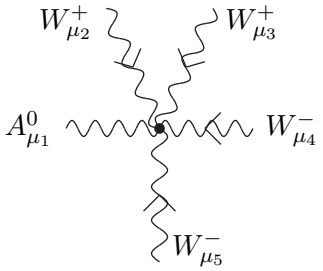
$$
\begin{aligned}
&+\frac{i\bar{g}^4}{\bar{g}^2+\bar{g}'^2}\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)\\
&-\frac{6i\bar{g}^3}{\bar{g}^2+\bar{g}'^2}C^W\left(\eta_{\mu_1\mu_2}p_1^{\mu_4}p_4^{\mu_3}+\eta_{\mu_1\mu_2}p_2^{\mu_4}p_4^{\mu_3}+\eta_{\mu_1\mu_3}p_1^{\mu_4}p_3^{\mu_2}\right.\\
&-\eta_{\mu_1\mu_3}p_2^{\mu_4}p_3^{\mu_2}-\eta_{\mu_1\mu_3}p_1^{\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_3}p_1^{\mu_4}p_4^{\mu_2}-\eta_{\mu_1\mu_4}p_1^{\mu_2}p_4^{\mu_3}\\
&-\eta_{\mu_2\mu_3}p_1^{\mu_4}p_3^{\mu_1}+\eta_{\mu_2\mu_3}p_2^{\mu_4}p_3^{\mu_1}-\eta_{\mu_2\mu_3}p_2^{\mu_1}p_3^{\mu_4}-\eta_{\mu_2\mu_3}p_2^{\mu_4}p_4^{\mu_1}\\
&-\eta_{\mu_2\mu_4}p_2^{\mu_1}p_4^{\mu_3}+p_1^{\mu_3}\left(\eta_{\mu_1\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_4}p_3^{\mu_2}+\eta_{\mu_1\mu_4}p_4^{\mu_2}-\eta_{\mu_2\mu_4}p_4^{\mu_1}\right)\\
&+p_2^{\mu_3}\left(\eta_{\mu_1\mu_2}p_3^{\mu_4}-\eta_{\mu_1\mu_4}p_4^{\mu_2}+\eta_{\mu_2\mu_4}\left(p_4^{\mu_1}-p_3^{\mu_1}\right)\right)+\eta_{\mu_3\mu_4}p_1^{\mu_2}p_3^{\mu_1}\\
&+\eta_{\mu_3\mu_4}p_2^{\mu_1}p_3^{\mu_2}+\eta_{\mu_3\mu_4}p_1^{\mu_2}p_4^{\mu_1}+\eta_{\mu_3\mu_4}p_2^{\mu_1}p_4^{\mu_2}-p_1\cdot p_3\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}\\
&-p_1\cdot p_4\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}-p_2\cdot p_3\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}-p_2\cdot p_4\eta_{\mu_3\mu_4}\eta_{\mu_1\mu_2}+p_1\cdot p_3\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\\
&\left.+p_1\cdot p_4\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}+p_2\cdot p_3\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}+p_2\cdot p_4\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\right)\\
&+\frac{2i\bar{g}^3\bar{g}'^3v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^2}\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)C^{\phi WB}\\
&+\frac{2i\bar{g}^3}{\bar{g}^2+\bar{g}'^2}C^{\widetilde{W}}\left(-\epsilon_{\mu_2\mu_3\mu_1\alpha_1}p_1^{\mu_4}p_3^{\alpha_1}-\epsilon_{\mu_2\mu_3\mu_4\alpha_1}p_2^{\mu_1}p_3^{\alpha_1}+\epsilon_{\mu_2\mu_3\mu_1\alpha_1}p_2^{\mu_4}p_3^{\alpha_1}\right.\\
&-\epsilon_{\mu_2\mu_3\mu_4\alpha_1}p_1^{\alpha_1}p_3^{\mu_1}-\epsilon_{\mu_2\mu_3\mu_4\alpha_1}p_2^{\alpha_1}p_3^{\mu_1}-\epsilon_{\mu_2\mu_3\mu_1\alpha_1}p_1^{\alpha_1}p_3^{\mu_4}+\epsilon_{\mu_2\mu_3\mu_1\alpha_1}p_2^{\alpha_1}p_3^{\mu_4}\\
&-\epsilon_{\mu_2\mu_3\mu_1\alpha_1}p_1^{\mu_4}p_4^{\alpha_1}+\epsilon_{\mu_2\mu_3\mu_4\alpha_1}p_2^{\mu_1}p_4^{\alpha_1}+\epsilon_{\mu_2\mu_3\mu_1\alpha_1}p_2^{\mu_4}p_4^{\alpha_1}+\epsilon_{\mu_2\mu_3\mu_4\alpha_1}p_1^{\alpha_1}p_4^{\mu_1}\\
&+\epsilon_{\mu_2\mu_3\mu_4\alpha_1}p_2^{\alpha_1}p_4^{\mu_1}+\epsilon_{\mu_3\mu_1\mu_4\alpha_1}\left(p_1^{\mu_2}\left(p_3^{\alpha_1}-p_4^{\alpha_1}\right)+\left(p_1^{\alpha_1}+p_2^{\alpha_1}\right)\left(p_3^{\mu_2}-p_4^{\mu_2}\right)\right)\\
&+\epsilon_{\mu_2\mu_1\mu_4\alpha_1}\left(p_1^{\mu_3}\left(p_3^{\alpha_1}+p_4^{\alpha_1}\right)-p_2^{\mu_3}\left(p_3^{\alpha_1}+p_4^{\alpha_1}\right)+\left(p_1^{\alpha_1}-p_2^{\alpha_1}\right)p_4^{\mu_3}\right)\\
&+\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_2}+\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_2}-\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_2}\\
&-\epsilon_{\mu_3\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_2}+\epsilon_{\mu_2\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_3}+\epsilon_{\mu_2\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_3}\\
&+\epsilon_{\mu_2\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_3}+\epsilon_{\mu_2\mu_3\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_1\mu_4}+\epsilon_{\mu_2\mu_3\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_4}\\
&+\epsilon_{\mu_2\mu_3\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_1\mu_4}+\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_3}+\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_3}\\
&+\epsilon_{\mu_1\mu_4\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_3}-\epsilon_{\mu_3\mu_1\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_2\mu_4}-\epsilon_{\mu_3\mu_1\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_4}\\
&-\epsilon_{\mu_3\mu_1\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_2\mu_4}+\epsilon_{\mu_2\mu_1\alpha_1\beta_1}p_1^{\alpha_1}p_3^{\beta_1}\eta_{\mu_3\mu_4}-\epsilon_{\mu_2\mu_1\alpha_1\beta_1}p_2^{\alpha_1}p_3^{\beta_1}\eta_{\mu_3\mu_4}\\
&+\epsilon_{\mu_2\mu_1\alpha_1\beta_1}p_1^{\alpha_1}p_4^{\beta_1}\eta_{\mu_3\mu_4}-\epsilon_{\mu_2\mu_1\alpha_1\beta_1}p_2^{\alpha_1}p_4^{\beta_1}\eta_{\mu_3\mu_4}-\epsilon_{\mu_2\mu_3\mu_1\mu_4}p_1\cdot p_3\\
&\left.+\epsilon_{\mu_2\mu_3\mu_1\mu_4}p_1\cdot p_4+\epsilon_{\mu_2\mu_3\mu_1\mu_4}p_2\cdot p_3-\epsilon_{\mu_2\mu_3\mu_1\mu_4}p_2\cdot p_4\right)
\end{aligned}
$$



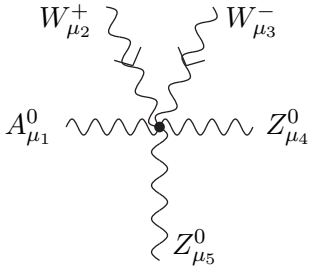
$$
\begin{aligned}
&+\frac{6i\bar{g}^2\bar{g}'^3}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^W\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\left(-p_3^{\mu_5}\right)-2\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}p_1^{\mu_4}\right.\\
&+\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}p_2^{\mu_4}+\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}p_3^{\mu_4}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}p_3^{\mu_5}+\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}p_1^{\mu_3}\\
&-\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}p_2^{\mu_3}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_1^{\mu_4}-2\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_2^{\mu_4}\\
&+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_3^{\mu_4}-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_1^{\mu_3}+\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_2^{\mu_3}\\
&+2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}p_3^{\mu_5}+\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}p_1^{\mu_2}-\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}p_3^{\mu_2}+\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}p_2^{\mu_1}\\
&-\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}p_3^{\mu_1}+\left(2\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)p_1^{\mu_5}\\
&-\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-2\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}+\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)p_2^{\mu_5}+\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}p_1^{\mu_4}\\
&+\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}p_2^{\mu_4}-2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}p_3^{\mu_4}-\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_1^{\mu_2}\\
&+\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_3^{\mu_2}-\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}p_2^{\mu_1}+\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}p_3^{\mu_1}\big)\\
&+\frac{2i\bar{g}^2\bar{g}'^3}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\widetilde{W}}\left(2\eta_{\mu_1\mu_2}p_1^{\alpha_1}\epsilon_{\mu_3\mu_4\mu_5\alpha_1}+2\eta_{\mu_1\mu_2}p_2^{\alpha_1}\epsilon_{\mu_3\mu_4\mu_5\alpha_1}\right.\\
&+2\eta_{\mu_1\mu_2}p_3^{\alpha_1}\epsilon_{\mu_3\mu_4\mu_5\alpha_1}+2\eta_{\mu_1\mu_3}p_1^{\alpha_1}\epsilon_{\mu_2\mu_4\mu_5\alpha_1}+2\eta_{\mu_1\mu_3}p_2^{\alpha_1}\epsilon_{\mu_2\mu_4\mu_5\alpha_1}\\
&+2\eta_{\mu_1\mu_3}p_3^{\alpha_1}\epsilon_{\mu_2\mu_4\mu_5\alpha_1}-\eta_{\mu_1\mu_4}p_2^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_5\alpha_1}+\eta_{\mu_1\mu_4}p_3^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_5\alpha_1}\\
&+\eta_{\mu_1\mu_5}p_2^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_4\alpha_1}-\eta_{\mu_1\mu_5}p_3^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_4\alpha_1}+2\eta_{\mu_2\mu_3}p_1^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_5\alpha_1}\\
&+2\eta_{\mu_2\mu_3}p_2^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_5\alpha_1}+2\eta_{\mu_2\mu_3}p_3^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_5\alpha_1}-\eta_{\mu_2\mu_4}p_1^{\alpha_1}\epsilon_{\mu_1\mu_3\mu_5\alpha_1}\\
&+\eta_{\mu_2\mu_4}p_3^{\alpha_1}\epsilon_{\mu_1\mu_3\mu_5\alpha_1}+\eta_{\mu_2\mu_5}p_1^{\alpha_1}\epsilon_{\mu_1\mu_3\mu_4\alpha_1}-\eta_{\mu_2\mu_5}p_3^{\alpha_1}\epsilon_{\mu_1\mu_3\mu_4\alpha_1}\\
&-\eta_{\mu_3\mu_4}p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_5\alpha_1}+\eta_{\mu_3\mu_4}p_2^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_5\alpha_1}+\eta_{\mu_3\mu_5}p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_4\alpha_1}\\
&-\eta_{\mu_3\mu_5}p_2^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_4\alpha_1}+2\epsilon_{\mu_2\mu_3\mu_4\mu_5}p_2^{\mu_1}+2\epsilon_{\mu_1\mu_2\mu_4\mu_5}\left(p_1^{\mu_3}\right.\\
&\left.\left.-p_2^{\mu_3}\right)-2\epsilon_{\mu_2\mu_3\mu_4\mu_5}p_3^{\mu_1}+2\epsilon_{\mu_1\mu_3\mu_4\mu_5}\left(p_1^{\mu_2}-p_3^{\mu_2}\right)\right)
\end{aligned}
$$



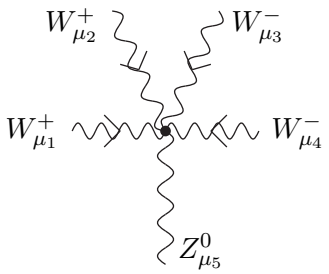
$$
\begin{aligned}
&+\frac{6i\bar{g}^3\bar{g}'^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^W\left(\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}\left(-p_1^{\mu_4}\right)+2\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}p_2^{\mu_4}\right.\\
&-\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}p_5^{\mu_4}-\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}p_2^{\mu_1}+\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}p_5^{\mu_1}+\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}p_1^{\mu_3}\\
&-2\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}p_2^{\mu_3}+\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}p_5^{\mu_3}+\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)p_1^{\mu_5}\\
&+\left(\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\right)p_2^{\mu_5}+2\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_1^{\mu_4}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_2^{\mu_4}\\
&-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_5^{\mu_4}-2\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_1^{\mu_3}+\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_2^{\mu_3}+\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_5^{\mu_3}\\
&-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}p_1^{\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}p_2^{\mu_4}+2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}p_5^{\mu_4}+\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_1^{\mu_2}\\
&-\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_5^{\mu_2}+\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}p_2^{\mu_1}-\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}p_5^{\mu_1}+\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}p_1^{\mu_3}\\
&+\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}p_2^{\mu_3}-2\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}p_5^{\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}p_1^{\mu_2}+\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}p_5^{\mu_2}\big)\\
&+\frac{2i\bar{g}^3\bar{g}'^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\widetilde{W}}\left(2\eta_{\mu_1\mu_2}p_1^{\alpha_1}\epsilon_{\mu_5\mu_3\mu_4\alpha_1}+2\eta_{\mu_1\mu_2}p_2^{\alpha_1}\epsilon_{\mu_5\mu_3\mu_4\alpha_1}\right.\\
&+2\eta_{\mu_1\mu_2}p_5^{\alpha_1}\epsilon_{\mu_5\mu_3\mu_4\alpha_1}-\eta_{\mu_1\mu_3}p_2^{\alpha_1}\epsilon_{\mu_2\mu_5\mu_4\alpha_1}+\eta_{\mu_1\mu_3}p_5^{\alpha_1}\epsilon_{\mu_2\mu_5\mu_4\alpha_1}\\
&+\eta_{\mu_1\mu_4}p_2^{\alpha_1}\epsilon_{\mu_2\mu_5\mu_3\alpha_1}-\eta_{\mu_1\mu_4}p_5^{\alpha_1}\epsilon_{\mu_2\mu_5\mu_3\alpha_1}+2\eta_{\mu_1\mu_5}p_1^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_4\alpha_1}\\
&+2\eta_{\mu_1\mu_5}p_2^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_4\alpha_1}+2\eta_{\mu_1\mu_5}p_5^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_4\alpha_1}-\eta_{\mu_2\mu_3}p_1^{\alpha_1}\epsilon_{\mu_1\mu_5\mu_4\alpha_1}\\
&+\eta_{\mu_2\mu_3}p_5^{\alpha_1}\epsilon_{\mu_1\mu_5\mu_4\alpha_1}+\eta_{\mu_2\mu_4}p_1^{\alpha_1}\epsilon_{\mu_1\mu_5\mu_3\alpha_1}-\eta_{\mu_2\mu_4}p_5^{\alpha_1}\epsilon_{\mu_1\mu_5\mu_3\alpha_1}\\
&+2\eta_{\mu_2\mu_5}p_1^{\alpha_1}\epsilon_{\mu_1\mu_3\mu_4\alpha_1}+2\eta_{\mu_2\mu_5}p_2^{\alpha_1}\epsilon_{\mu_1\mu_3\mu_4\alpha_1}+2\eta_{\mu_2\mu_5}p_5^{\alpha_1}\epsilon_{\mu_1\mu_3\mu_4\alpha_1}\\
&-\eta_{\mu_3\mu_5}p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_4\alpha_1}+\eta_{\mu_3\mu_5}p_2^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_4\alpha_1}+\eta_{\mu_4\mu_5}p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}\\
&-\eta_{\mu_4\mu_5}p_2^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}+2\epsilon_{\mu_2\mu_5\mu_3\mu_4}p_2^{\mu_1}+2\epsilon_{\mu_1\mu_2\mu_3\mu_4}\left(p_1^{\mu_5}\right.\\
&\left.\left.-p_2^{\mu_5}\right)-2\epsilon_{\mu_2\mu_5\mu_3\mu_4}p_5^{\mu_1}+2\epsilon_{\mu_1\mu_5\mu_3\mu_4}\left(p_1^{\mu_2}-p_5^{\mu_2}\right)\right)
\end{aligned}
$$



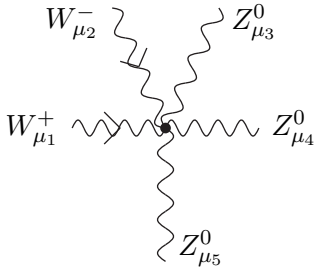
$$
\begin{aligned}
&+\frac{6i\bar{g}^2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^W\left(-2p_4^{\mu_5}\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}+p_2^{\mu_4}\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}+p_3^{\mu_4}\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}\right.\\
&-2p_5^{\mu_4}\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}-2p_2^{\mu_1}\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}-2p_3^{\mu_1}\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}+2p_4^{\mu_1}\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}\\
&+2p_5^{\mu_1}\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}+p_4^{\mu_5}\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-p_2^{\mu_3}\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}+p_5^{\mu_3}\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}\\
&+p_3^{\mu_5}\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)-p_3^{\mu_4}\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}+p_5^{\mu_4}\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}\\
&-p_2^{\mu_3}\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}+p_4^{\mu_3}\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}+p_4^{\mu_5}\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}-p_3^{\mu_2}\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}\\
&+p_5^{\mu_2}\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}+p_2^{\mu_1}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}+p_3^{\mu_1}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}-p_4^{\mu_1}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}\\
&-p_5^{\mu_1}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}+p_2^{\mu_5}\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)-p_2^{\mu_4}\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}\\
&+p_5^{\mu_4}\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}-p_3^{\mu_2}\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}+p_4^{\mu_2}\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}+p_2^{\mu_1}\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}\\
&+p_3^{\mu_1}\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}-p_4^{\mu_1}\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}-p_5^{\mu_1}\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}+2p_2^{\mu_3}\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}\\
&-p_4^{\mu_3}\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}-p_5^{\mu_3}\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}+2p_3^{\mu_2}\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}-p_4^{\mu_2}\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}\\
&\left.-p_5^{\mu_2}\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}\right)-\frac{2i\bar{g}^2\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\widetilde{W}}\left(2\epsilon_{\mu_1\mu_2\mu_3\mu_5}\left(p_2^{\mu_4}-p_3^{\mu_4}\right)\right.\\
&+2\epsilon_{\mu_1\mu_4\mu_2\mu_3}\left(p_2^{\mu_5}-p_3^{\mu_5}\right)+2\epsilon_{\mu_1\mu_4\mu_3\mu_5}p_4^{\mu_2}+2\epsilon_{\mu_1\mu_4\mu_2\mu_5}p_4^{\mu_3}\\
&-2\epsilon_{\mu_1\mu_4\mu_3\mu_5}p_5^{\mu_2}-2\epsilon_{\mu_1\mu_4\mu_2\mu_5}p_5^{\mu_3}-\epsilon_{\mu_4\mu_3\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_1\mu_2}\\
&+\epsilon_{\mu_4\mu_3\mu_5\alpha_1}p_5^{\alpha_1}\eta_{\mu_1\mu_2}-\epsilon_{\mu_4\mu_2\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_1\mu_3}+\epsilon_{\mu_4\mu_2\mu_5\alpha_1}p_5^{\alpha_1}\eta_{\mu_1\mu_3}\\
&-\epsilon_{\mu_2\mu_3\mu_5\alpha_1}p_2^{\alpha_1}\eta_{\mu_1\mu_4}+\epsilon_{\mu_2\mu_3\mu_5\alpha_1}p_3^{\alpha_1}\eta_{\mu_1\mu_4}-\epsilon_{\mu_4\mu_2\mu_3\alpha_1}p_2^{\alpha_1}\eta_{\mu_1\mu_5}\\
&+\epsilon_{\mu_4\mu_2\mu_3\alpha_1}p_3^{\alpha_1}\eta_{\mu_1\mu_5}-2\epsilon_{\mu_1\mu_4\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_2\mu_3}+2\epsilon_{\mu_1\mu_4\mu_5\alpha_1}p_5^{\alpha_1}\eta_{\mu_2\mu_3}\\
&-2\epsilon_{\mu_1\mu_3\mu_5\alpha_1}p_2^{\alpha_1}\eta_{\mu_2\mu_4}-\epsilon_{\mu_1\mu_3\mu_5\alpha_1}p_3^{\alpha_1}\eta_{\mu_2\mu_4}-2\epsilon_{\mu_1\mu_3\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_2\mu_4}\\
&-\epsilon_{\mu_1\mu_3\mu_5\alpha_1}p_5^{\alpha_1}\eta_{\mu_2\mu_4}+2\epsilon_{\mu_1\mu_4\mu_3\alpha_1}p_2^{\alpha_1}\eta_{\mu_2\mu_5}+\epsilon_{\mu_1\mu_4\mu_3\alpha_1}p_3^{\alpha_1}\eta_{\mu_2\mu_5}\\
&+\epsilon_{\mu_1\mu_4\mu_3\alpha_1}p_4^{\alpha_1}\eta_{\mu_2\mu_5}+2\epsilon_{\mu_1\mu_4\mu_3\alpha_1}p_5^{\alpha_1}\eta_{\mu_2\mu_5}-\epsilon_{\mu_1\mu_2\mu_5\alpha_1}p_2^{\alpha_1}\eta_{\mu_3\mu_4}\\
&-2\epsilon_{\mu_1\mu_2\mu_5\alpha_1}p_3^{\alpha_1}\eta_{\mu_3\mu_4}-2\epsilon_{\mu_1\mu_2\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_3\mu_4}-\epsilon_{\mu_1\mu_2\mu_5\alpha_1}p_5^{\alpha_1}\eta_{\mu_3\mu_4}\\
&+\epsilon_{\mu_1\mu_4\mu_2\alpha_1}p_2^{\alpha_1}\eta_{\mu_3\mu_5}+2\epsilon_{\mu_1\mu_4\mu_2\alpha_1}p_3^{\alpha_1}\eta_{\mu_3\mu_5}+\epsilon_{\mu_1\mu_4\mu_2\alpha_1}p_4^{\alpha_1}\eta_{\mu_3\mu_5}\\
&\left.+2\epsilon_{\mu_1\mu_4\mu_2\alpha_1}p_5^{\alpha_1}\eta_{\mu_3\mu_5}+2\epsilon_{\mu_1\mu_2\mu_3\alpha_1}p_2^{\alpha_1}\eta_{\mu_4\mu_5}-2\epsilon_{\mu_1\mu_2\mu_3\alpha_1}p_3^{\alpha_1}\eta_{\mu_4\mu_5}\right)
\end{aligned}
$$



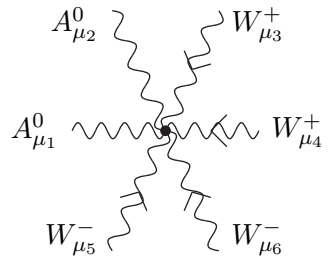
$$
\begin{aligned}
&+\frac{6i\bar{g}^4\bar{g}'}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^W\left(\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}\left(-p_1^{\mu_3}\right)+2\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}p_4^{\mu_3}-\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}p_5^{\mu_3}\right.\\
&-\eta_{\mu_3\mu_5}\eta_{\mu_2\mu_4}p_4^{\mu_1}+\eta_{\mu_3\mu_5}\eta_{\mu_2\mu_4}p_5^{\mu_1}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_1^{\mu_4}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_5^{\mu_4}\\
&-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_1^{\mu_3}-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_4^{\mu_3}+2\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_5^{\mu_3}+\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}p_1^{\mu_2}\\
&-2\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}p_4^{\mu_2}+\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}p_5^{\mu_2}+\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}p_4^{\mu_1}-\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}p_5^{\mu_1}\\
&+\left(\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)p_1^{\mu_5}+\left(\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)p_4^{\mu_5}\\
&-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}p_1^{\mu_4}+\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}p_5^{\mu_4}+\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_1^{\mu_2}+\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_4^{\mu_2}\\
&-2\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_5^{\mu_2}+2\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}p_1^{\mu_3}-\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}p_4^{\mu_3}\\
&\left.-\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}p_5^{\mu_3}-2\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}p_1^{\mu_2}+\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}p_4^{\mu_2}+\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}p_5^{\mu_2}\right)\\
&+\frac{2i\bar{g}^4\bar{g}'}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\widetilde{W}}\left(\eta_{\mu_1\mu_2}p_4^{\alpha_1}\epsilon_{\mu_4\mu_3\mu_5\alpha_1}-\eta_{\mu_1\mu_2}p_5^{\alpha_1}\epsilon_{\mu_4\mu_3\mu_5\alpha_1}\right.\\
&-\eta_{\mu_1\mu_3}p_4^{\alpha_1}\epsilon_{\mu_4\mu_2\mu_5\alpha_1}+\eta_{\mu_1\mu_3}p_5^{\alpha_1}\epsilon_{\mu_4\mu_2\mu_5\alpha_1}+2\eta_{\mu_1\mu_4}p_1^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_5\alpha_1}\\
&+2\eta_{\mu_1\mu_4}p_4^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_5\alpha_1}+2\eta_{\mu_1\mu_4}p_5^{\alpha_1}\epsilon_{\mu_2\mu_3\mu_5\alpha_1}+2\eta_{\mu_1\mu_5}p_1^{\alpha_1}\epsilon_{\mu_4\mu_2\mu_3\alpha_1}\\
&+2\eta_{\mu_1\mu_5}p_4^{\alpha_1}\epsilon_{\mu_4\mu_2\mu_3\alpha_1}+2\eta_{\mu_1\mu_5}p_5^{\alpha_1}\epsilon_{\mu_4\mu_2\mu_3\alpha_1}+\eta_{\mu_2\mu_4}p_1^{\alpha_1}\epsilon_{\mu_1\mu_3\mu_5\alpha_1}\\
&-\eta_{\mu_2\mu_4}p_5^{\alpha_1}\epsilon_{\mu_1\mu_3\mu_5\alpha_1}-\eta_{\mu_2\mu_5}p_1^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_3\alpha_1}+\eta_{\mu_2\mu_5}p_4^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_3\alpha_1}\\
&-\eta_{\mu_3\mu_4}p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_5\alpha_1}+\eta_{\mu_3\mu_4}p_5^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_5\alpha_1}+\eta_{\mu_3\mu_5}p_1^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_2\alpha_1}\\
&-\eta_{\mu_3\mu_5}p_4^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_2\alpha_1}+2\eta_{\mu_4\mu_5}p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}+2\eta_{\mu_4\mu_5}p_4^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}\\
&+2\eta_{\mu_4\mu_5}p_5^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}+2\epsilon_{\mu_4\mu_2\mu_3\mu_5}p_4^{\mu_1}+2\epsilon_{\mu_1\mu_4\mu_2\mu_3}\left(p_1^{\mu_5}-p_4^{\mu_5}\right)\\
&\left.-2\epsilon_{\mu_4\mu_2\mu_3\mu_5}p_5^{\mu_1}+2\epsilon_{\mu_1\mu_2\mu_3\mu_5}\left(p_1^{\mu_4}-p_5^{\mu_4}\right)\right)
\end{aligned}
$$



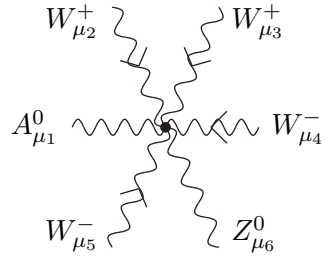
$$
\begin{aligned}
&+\frac{6i\bar{g}^3}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^W\,(-p_3^{\mu_5}\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-p_4^{\mu_5}\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-p_1^{\mu_4}\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}\\
&+p_3^{\mu_4}\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}-p_2^{\mu_1}\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}+p_4^{\mu_1}\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}-p_3^{\mu_5}\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\\
&-p_4^{\mu_5}\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-p_1^{\mu_3}\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}+p_4^{\mu_3}\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}-p_2^{\mu_4}\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}\\
&+p_3^{\mu_4}\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}-p_2^{\mu_3}\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}+p_4^{\mu_3}\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}+2p_3^{\mu_5}\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\\
&+2p_4^{\mu_5}\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}+2p_1^{\mu_2}\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}-p_3^{\mu_2}\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}\\
&-p_4^{\mu_2}\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}+2p_2^{\mu_1}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}-p_3^{\mu_1}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}\\
&-p_4^{\mu_1}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}+p_1^{\mu_5}\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)\\
&+p_2^{\mu_5}\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)+p_1^{\mu_4}\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}\\
&+p_2^{\mu_4}\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}-2p_3^{\mu_4}\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_5}-p_1^{\mu_2}\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}+p_3^{\mu_2}\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}\\
&-p_2^{\mu_1}\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}+p_3^{\mu_1}\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}+p_1^{\mu_3}\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}+p_2^{\mu_3}\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}\\
&-2p_4^{\mu_3}\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_5}-p_1^{\mu_2}\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}+p_4^{\mu_2}\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5})\\
&+\frac{2i\bar{g}^3}{\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\widetilde{W}}\,(2\epsilon_{\mu_1\mu_2\mu_4\mu_5}\left(p_1^{\mu_3}-p_2^{\mu_3}\right)\\
&+2\epsilon_{\mu_3\mu_1\mu_2\mu_5}\left(p_1^{\mu_4}-p_2^{\mu_4}\right)+2\epsilon_{\mu_3\mu_2\mu_4\mu_5}p_3^{\mu_1}+2\epsilon_{\mu_3\mu_1\mu_4\mu_5}p_3^{\mu_2}\\
&-2\epsilon_{\mu_3\mu_2\mu_4\mu_5}p_4^{\mu_1}-2\epsilon_{\mu_3\mu_1\mu_4\mu_5}p_4^{\mu_2}+2\epsilon_{\mu_3\mu_4\mu_5\alpha_1}p_3^{\alpha_1}\eta_{\mu_1\mu_2}\\
&-2\epsilon_{\mu_3\mu_4\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_1\mu_2}+2\epsilon_{\mu_2\mu_4\mu_5\alpha_1}p_1^{\alpha_1}\eta_{\mu_1\mu_3}+\epsilon_{\mu_2\mu_4\mu_5\alpha_1}p_2^{\alpha_1}\eta_{\mu_1\mu_3}\\
&+2\epsilon_{\mu_2\mu_4\mu_5\alpha_1}p_3^{\alpha_1}\eta_{\mu_1\mu_3}+\epsilon_{\mu_2\mu_4\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_1\mu_3}-2\epsilon_{\mu_3\mu_2\mu_5\alpha_1}p_1^{\alpha_1}\eta_{\mu_1\mu_4}\\
&-\epsilon_{\mu_3\mu_2\mu_5\alpha_1}p_2^{\alpha_1}\eta_{\mu_1\mu_4}-\epsilon_{\mu_3\mu_2\mu_5\alpha_1}p_3^{\alpha_1}\eta_{\mu_1\mu_4}-2\epsilon_{\mu_3\mu_2\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_1\mu_4}\\
&+\epsilon_{\mu_3\mu_2\mu_4\alpha_1}p_3^{\alpha_1}\eta_{\mu_1\mu_5}-\epsilon_{\mu_3\mu_2\mu_4\alpha_1}p_4^{\alpha_1}\eta_{\mu_1\mu_5}+\epsilon_{\mu_1\mu_4\mu_5\alpha_1}p_1^{\alpha_1}\eta_{\mu_2\mu_3}\\
&+2\epsilon_{\mu_1\mu_4\mu_5\alpha_1}p_2^{\alpha_1}\eta_{\mu_2\mu_3}+2\epsilon_{\mu_1\mu_4\mu_5\alpha_1}p_3^{\alpha_1}\eta_{\mu_2\mu_3}+\epsilon_{\mu_1\mu_4\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_2\mu_3}\\
&-\epsilon_{\mu_3\mu_1\mu_5\alpha_1}p_1^{\alpha_1}\eta_{\mu_2\mu_4}-2\epsilon_{\mu_3\mu_1\mu_5\alpha_1}p_2^{\alpha_1}\eta_{\mu_2\mu_4}-\epsilon_{\mu_3\mu_1\mu_5\alpha_1}p_3^{\alpha_1}\eta_{\mu_2\mu_4}\\
&-2\epsilon_{\mu_3\mu_1\mu_5\alpha_1}p_4^{\alpha_1}\eta_{\mu_2\mu_4}+\epsilon_{\mu_3\mu_1\mu_4\alpha_1}p_3^{\alpha_1}\eta_{\mu_2\mu_5}-\epsilon_{\mu_3\mu_1\mu_4\alpha_1}p_4^{\alpha_1}\eta_{\mu_2\mu_5}\\
&-2\epsilon_{\mu_1\mu_2\mu_5\alpha_1}p_1^{\alpha_1}\eta_{\mu_3\mu_4}+2\epsilon_{\mu_1\mu_2\mu_5\alpha_1}p_2^{\alpha_1}\eta_{\mu_3\mu_4}+\epsilon_{\mu_1\mu_2\mu_4\alpha_1}p_1^{\alpha_1}\eta_{\mu_3\mu_5}\\
&-\epsilon_{\mu_1\mu_2\mu_4\alpha_1}p_2^{\alpha_1}\eta_{\mu_3\mu_5}+\epsilon_{\mu_3\mu_1\mu_2\alpha_1}p_1^{\alpha_1}\eta_{\mu_4\mu_5}-\epsilon_{\mu_3\mu_1\mu_2\alpha_1}p_2^{\alpha_1}\eta_{\mu_4\mu_5})
\end{aligned}
$$



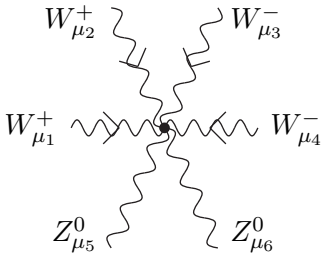
$$
\begin{aligned}
&+\frac{6i\bar{g}^5}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^W\left(\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}p_3^{\mu_4}-\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_3}p_5^{\mu_4}-2\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}p_3^{\mu_1}\right.\\
&+\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}p_4^{\mu_1}+\eta_{\mu_4\mu_5}\eta_{\mu_2\mu_3}p_5^{\mu_1}+\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}p_4^{\mu_3}-\eta_{\mu_1\mu_5}\eta_{\mu_2\mu_4}p_5^{\mu_3}\\
&+\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)p_3^{\mu_5}+\left(\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\right)p_4^{\mu_5}\\
&-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_3^{\mu_4}+\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}p_5^{\mu_4}-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_4^{\mu_3}+\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}p_5^{\mu_3}\\
&-\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}p_3^{\mu_2}-\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}p_4^{\mu_2}+2\eta_{\mu_1\mu_5}\eta_{\mu_3\mu_4}p_5^{\mu_2}+\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}p_3^{\mu_1}\\
&+\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}p_4^{\mu_1}-2\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}p_5^{\mu_1}-\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_3^{\mu_2}+2\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_4^{\mu_2}\\
&-\eta_{\mu_1\mu_4}\eta_{\mu_3\mu_5}p_5^{\mu_2}+\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}p_3^{\mu_1}-2\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}p_4^{\mu_1}+\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}p_5^{\mu_1}\\
&\left.+2\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}p_3^{\mu_2}-\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}p_4^{\mu_2}-\eta_{\mu_1\mu_3}\eta_{\mu_4\mu_5}p_5^{\mu_2}\right)\\
&+\frac{2i\bar{g}^5}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\widetilde{W}}\left(-\eta_{\mu_1\mu_3}p_4^{\alpha_1}\epsilon_{\mu_2\mu_4\mu_5\alpha_1}+\eta_{\mu_1\mu_3}p_5^{\alpha_1}\epsilon_{\mu_2\mu_4\mu_5\alpha_1}\right.\\
&+\eta_{\mu_1\mu_4}p_3^{\alpha_1}\epsilon_{\mu_3\mu_2\mu_5\alpha_1}-\eta_{\mu_1\mu_4}p_5^{\alpha_1}\epsilon_{\mu_3\mu_2\mu_5\alpha_1}+\eta_{\mu_1\mu_5}p_3^{\alpha_1}\epsilon_{\mu_3\mu_2\mu_4\alpha_1}\\
&-\eta_{\mu_1\mu_5}p_4^{\alpha_1}\epsilon_{\mu_3\mu_2\mu_4\alpha_1}+\eta_{\mu_2\mu_3}p_4^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_5\alpha_1}-\eta_{\mu_2\mu_3}p_5^{\alpha_1}\epsilon_{\mu_1\mu_4\mu_5\alpha_1}\\
&-\eta_{\mu_2\mu_4}p_3^{\alpha_1}\epsilon_{\mu_3\mu_1\mu_5\alpha_1}+\eta_{\mu_2\mu_4}p_5^{\alpha_1}\epsilon_{\mu_3\mu_1\mu_5\alpha_1}-\eta_{\mu_2\mu_5}p_3^{\alpha_1}\epsilon_{\mu_3\mu_1\mu_4\alpha_1}\\
&+\eta_{\mu_2\mu_5}p_4^{\alpha_1}\epsilon_{\mu_3\mu_1\mu_4\alpha_1}+2\eta_{\mu_3\mu_4}p_3^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_5\alpha_1}+2\eta_{\mu_3\mu_4}p_4^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_5\alpha_1}\\
&+2\eta_{\mu_3\mu_4}p_5^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_5\alpha_1}+2\eta_{\mu_3\mu_5}p_3^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_4\alpha_1}+2\eta_{\mu_3\mu_5}p_4^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_4\alpha_1}\\
&+2\eta_{\mu_3\mu_5}p_5^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_4\alpha_1}+2\eta_{\mu_4\mu_5}p_3^{\alpha_1}\epsilon_{\mu_3\mu_1\mu_2\alpha_1}+2\eta_{\mu_4\mu_5}p_4^{\alpha_1}\epsilon_{\mu_3\mu_1\mu_2\alpha_1}\\
&+2\eta_{\mu_4\mu_5}p_5^{\alpha_1}\epsilon_{\mu_3\mu_1\mu_2\alpha_1}+2\epsilon_{\mu_1\mu_2\mu_4\mu_5}p_4^{\mu_3}+2\epsilon_{\mu_3\mu_1\mu_2\mu_4}\left(p_3^{\mu_5}-p_4^{\mu_5}\right)\\
&\left.-2\epsilon_{\mu_1\mu_2\mu_4\mu_5}p_5^{\mu_3}+2\epsilon_{\mu_3\mu_1\mu_2\mu_5}\left(p_3^{\mu_4}-p_5^{\mu_4}\right)\right)
\end{aligned}
$$



$$
\begin{aligned}
&-\frac{12i\bar{g}^3\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left(-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_6}\eta_{\mu_3\mu_5}+2\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_6}\eta_{\mu_3\mu_5}\right.\\
&-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_6}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_6}\eta_{\mu_4\mu_5}+2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_6}\eta_{\mu_4\mu_5}\\
&+\eta_{\mu_1\mu_6}\left(2\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}-\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}-\eta_{\mu_2\mu_3}\eta_{\mu_4\mu_5}\right)\\
&-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}\eta_{\mu_4\mu_6}+\eta_{\mu_1\mu_5}\left(2\eta_{\mu_2\mu_6}\eta_{\mu_3\mu_4}-\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_6}-\eta_{\mu_2\mu_3}\eta_{\mu_4\mu_6}\right)\\
&\left.+2\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\eta_{\mu_5\mu_6}+2\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\eta_{\mu_5\mu_6}-4\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\eta_{\mu_5\mu_6}\right)C^W
\end{aligned}
$$

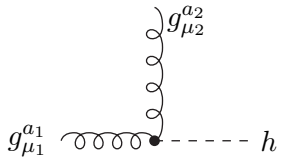


$$
\begin{aligned}
&-\frac{12i\bar{g}^4\bar{g}'}{\bar{g}^2+\bar{g}'^2}\left(-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_6}\eta_{\mu_3\mu_5}-\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_6}\eta_{\mu_3\mu_5}\right.\\
&-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_6}+2\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_6}\eta_{\mu_4\mu_5}+2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_6}\eta_{\mu_4\mu_5}\\
&+2\eta_{\mu_1\mu_6}\left(\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}+\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}-2\eta_{\mu_2\mu_3}\eta_{\mu_4\mu_5}\right)\\
&-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}\eta_{\mu_4\mu_6}-\eta_{\mu_1\mu_5}\left(\eta_{\mu_2\mu_6}\eta_{\mu_3\mu_4}+\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_6}-2\eta_{\mu_2\mu_3}\eta_{\mu_4\mu_6}\right)\\
&\left.+2\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\eta_{\mu_5\mu_6}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\eta_{\mu_5\mu_6}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\eta_{\mu_5\mu_6}\right)C^W
\end{aligned}
$$

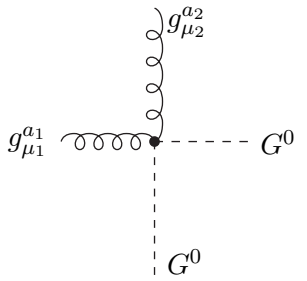


$$
\begin{aligned}
&-\frac{12i\bar{g}^5}{\bar{g}^2+\bar{g}'^2}\left(-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_6}\eta_{\mu_3\mu_5}+2\eta_{\mu_1\mu_2}\eta_{\mu_4\mu_6}\eta_{\mu_3\mu_5}\right.\\
&-\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_6}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_6}\eta_{\mu_4\mu_5}+2\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_6}\eta_{\mu_4\mu_5}\\
&+\eta_{\mu_1\mu_6}\left(2\eta_{\mu_2\mu_5}\eta_{\mu_3\mu_4}-\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_5}-\eta_{\mu_2\mu_3}\eta_{\mu_4\mu_5}\right)\\
&-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_5}\eta_{\mu_4\mu_6}+\eta_{\mu_1\mu_5}\left(2\eta_{\mu_2\mu_6}\eta_{\mu_3\mu_4}-\eta_{\mu_2\mu_4}\eta_{\mu_3\mu_6}-\eta_{\mu_2\mu_3}\eta_{\mu_4\mu_6}\right)\\
&\left.+2\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}\eta_{\mu_5\mu_6}+2\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\eta_{\mu_5\mu_6}-4\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\eta_{\mu_5\mu_6}\right)C^W
\end{aligned}
$$

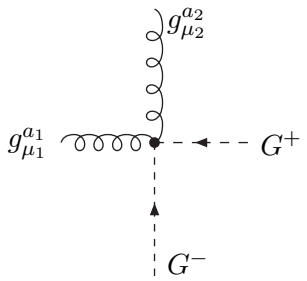
A.9 Higgs-gluon vertices



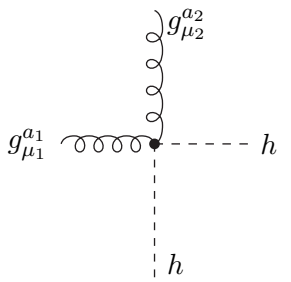
$$+4iv\delta_{a_1a_2}C^{\phi G}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)+4iv\delta_{a_1a_2}C^{\phi\widetilde{G}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}$$



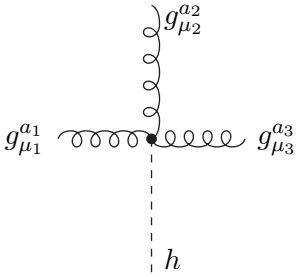
$$+4i\delta_{a_1a_2}C^{\phi G}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)+4i\delta_{a_1a_2}C^{\phi\widetilde{G}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}$$



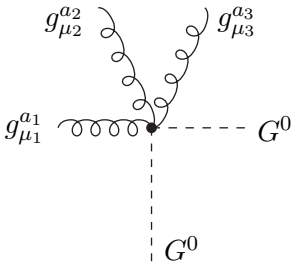
$$+4i\delta_{a_1a_2}C^{\phi G}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)+4i\delta_{a_1a_2}C^{\phi\widetilde{G}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}$$



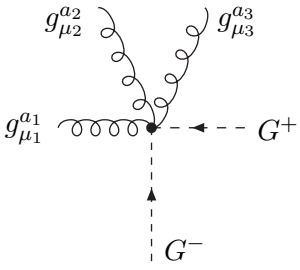
$$+4i\delta_{a_1a_2}C^{\phi G}\left(p_1^{\mu_2}p_2^{\mu_1}-p_1\cdot p_2\eta_{\mu_1\mu_2}\right)+4i\delta_{a_1a_2}C^{\phi\widetilde{G}}p_1^{\alpha_1}p_2^{\beta_1}\epsilon_{\mu_1\mu_2\alpha_1\beta_1}$$



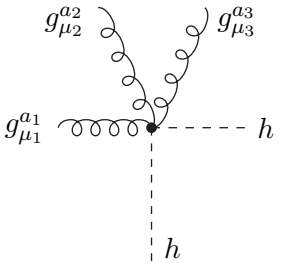
$$+4v\bar{g}_s f_{a_1a_2a_3} C^{\phi G} \left(\eta_{\mu_1\mu_2} p_1^{\mu_3} - \eta_{\mu_1\mu_2} p_2^{\mu_3} - \eta_{\mu_1\mu_3} p_1^{\mu_2} + \eta_{\mu_1\mu_3} p_3^{\mu_2} + \eta_{\mu_2\mu_3} p_2^{\mu_1} - \eta_{\mu_2\mu_3} p_3^{\mu_1}\right) + 4v\bar{g}_s f_{a_1a_2a_3} C^{\phi\widetilde{G}} \left(p_1^{\alpha_1} + p_2^{\alpha_1} + p_3^{\alpha_1}\right) \epsilon_{\mu_1\mu_2\mu_3\alpha_1}$$



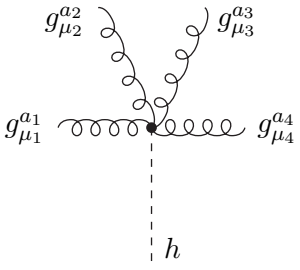
$$+4\bar{g}_s f_{a_1a_2a_3} C^{\phi G} \left(\eta_{\mu_1\mu_2} p_1^{\mu_3} - \eta_{\mu_1\mu_2} p_2^{\mu_3} - \eta_{\mu_1\mu_3} p_1^{\mu_2} + \eta_{\mu_1\mu_3} p_3^{\mu_2} + \eta_{\mu_2\mu_3} p_2^{\mu_1} - \eta_{\mu_2\mu_3} p_3^{\mu_1}\right) + 4\bar{g}_s f_{a_1a_2a_3} C^{\phi\widetilde{G}} \left(p_1^{\alpha_1} + p_2^{\alpha_1} + p_3^{\alpha_1}\right) \epsilon_{\mu_1\mu_2\mu_3\alpha_1}$$



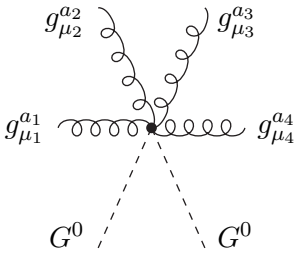
$$+4\bar{g}_s f_{a_1a_2a_3} C^{\phi G} \left(\eta_{\mu_1\mu_2} p_1^{\mu_3} - \eta_{\mu_1\mu_2} p_2^{\mu_3} - \eta_{\mu_1\mu_3} p_1^{\mu_2} + \eta_{\mu_1\mu_3} p_3^{\mu_2} + \eta_{\mu_2\mu_3} p_2^{\mu_1} - \eta_{\mu_2\mu_3} p_3^{\mu_1}\right) + 4\bar{g}_s f_{a_1a_2a_3} C^{\phi\widetilde{G}} \left(p_1^{\alpha_1} + p_2^{\alpha_1} + p_3^{\alpha_1}\right) \epsilon_{\mu_1\mu_2\mu_3\alpha_1}$$



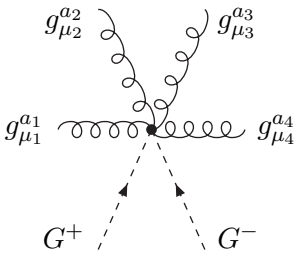
$$+4\bar{g}_s f_{a_1a_2a_3} C^{\phi G} \left(\eta_{\mu_1\mu_2} p_1^{\mu_3} - \eta_{\mu_1\mu_2} p_2^{\mu_3} - \eta_{\mu_1\mu_3} p_1^{\mu_2} + \eta_{\mu_1\mu_3} p_3^{\mu_2} + \eta_{\mu_2\mu_3} p_2^{\mu_1} - \eta_{\mu_2\mu_3} p_3^{\mu_1}\right) + 4\bar{g}_s f_{a_1a_2a_3} C^{\phi\widetilde{G}} \left(p_1^{\alpha_1} + p_2^{\alpha_1} + p_3^{\alpha_1}\right) \epsilon_{\mu_1\mu_2\mu_3\alpha_1}$$



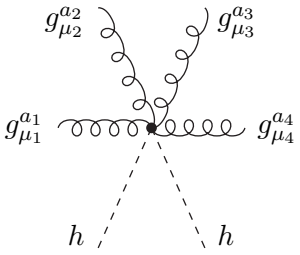
$$-4iv\bar{g}_s^2 C^{\phi G} \left(\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3} - \eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right) f_{a_1a_2b_1} f_{a_3a_4b_1} + \left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3} - \eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right) f_{a_1a_3b_1} f_{a_2a_4b_1} + \left(\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4} - \eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right) f_{a_1a_4b_1} f_{a_2a_3b_1}\right)$$



$$
\begin{aligned}
&-4i\bar{g}_s^2 C^{\phi G}\left(\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)f_{a_1a_2b_1}f_{a_3a_4b_1}\right.\\
&\quad+\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)f_{a_1a_3b_1}f_{a_2a_4b_1}\\
&\quad\left.+\left(\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)f_{a_1a_4b_1}f_{a_2a_3b_1}\right)
\end{aligned}
$$

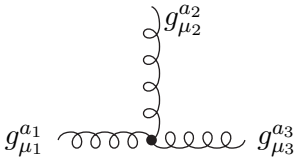


$$
\begin{aligned}
&-4i\bar{g}_s^2 C^{\phi G}\left(\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)f_{a_1a_2b_1}f_{a_3a_4b_1}\right.\\
&\quad+\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)f_{a_1a_3b_1}f_{a_2a_4b_1}\\
&\quad\left.+\left(\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)f_{a_1a_4b_1}f_{a_2a_3b_1}\right)
\end{aligned}
$$



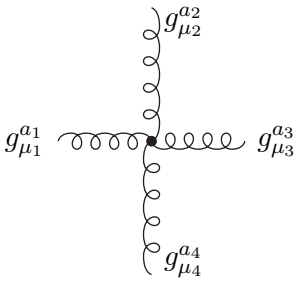
$$
\begin{aligned}
&-4i\bar{g}_s^2 C^{\phi G}\left(\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)f_{a_1a_2b_1}f_{a_3a_4b_1}\right.\\
&\quad+\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)f_{a_1a_3b_1}f_{a_2a_4b_1}\\
&\quad\left.+\left(\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)f_{a_1a_4b_1}f_{a_2a_3b_1}\right)
\end{aligned}
$$

A.10 Gluon-gluon vertices

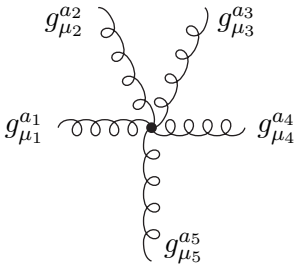


$$
\begin{aligned}
&-\bar{g}_s f_{a_1a_2a_3}\left(\eta_{\mu_1\mu_2}p_1^{\mu_3}-\eta_{\mu_1\mu_2}p_2^{\mu_3}-\eta_{\mu_1\mu_3}p_1^{\mu_2}+\eta_{\mu_1\mu_3}p_3^{\mu_2}\right.\\
&\quad\left.+\eta_{\mu_2\mu_3}p_2^{\mu_1}-\eta_{\mu_2\mu_3}p_3^{\mu_1}\right)+6f_{a_1a_2a_3}C^G\left(p_1\cdot p_2\eta_{\mu_1\mu_3}p_3^{\mu_2}\right.\\
&\quad-p_1\cdot p_2\eta_{\mu_2\mu_3}p_3^{\mu_1}-p_1\cdot p_3\eta_{\mu_1\mu_2}p_2^{\mu_3}+p_1\cdot p_3\eta_{\mu_2\mu_3}p_2^{\mu_1}\\
&\quad\left.+p_1^{\mu_3}\left(p_2\cdot p_3\eta_{\mu_1\mu_2}-p_2^{\mu_1}p_3^{\mu_2}\right)+p_1^{\mu_2}\left(p_2^{\mu_3}p_3^{\mu_1}-p_2\cdot p_3\eta_{\mu_1\mu_3}\right)\right)\\
&\quad+2f_{a_1a_2a_3}C^{\widetilde{G}}\left(p_1\cdot p_2p_3^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}+p_1\cdot p_3p_2^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}\right.\\
&\quad+p_2\cdot p_3p_1^{\alpha_1}\epsilon_{\mu_1\mu_2\mu_3\alpha_1}-p_1^{\alpha_1}\left(p_3^{\beta_1}p_2^{\mu_1}+p_2^{\beta_1}p_3^{\mu_1}\right)\epsilon_{\mu_2\mu_3\alpha_1\beta_1}\\
&\quad+p_3^{\beta_1}\left(p_2^{\alpha_1}p_1^{\mu_3}+p_1^{\alpha_1}p_2^{\mu_3}\right)\epsilon_{\mu_1\mu_2\alpha_1\beta_1}\\
&\quad\left.+\epsilon_{\mu_1\mu_3\alpha_1\beta_1}\left(p_2^{\alpha_1}p_3^{\beta_1}p_1^{\mu_2}-p_1^{\alpha_1}p_2^{\beta_1}p_3^{\mu_2}\right)\right)
\end{aligned}
$$

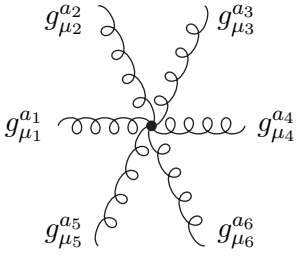
Caution: very long expression, part proportional to $C^{\widetilde{G}}$ in 4-gluon vertex not displayed.



$$
\begin{aligned}
&+i\bar{g}_s^2\left(\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}\right)f_{a_1a_2b_1}f_{a_3a_4b_1}\right.\\
&+\left(\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)f_{a_1a_3b_1}f_{a_2a_4b_1}\\
&\left.+\left(\eta_{\mu_1\mu_3}\eta_{\mu_2\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}\right)f_{a_1a_4b_1}f_{a_2a_3b_1}\right)\\
&-6i\bar{g}_sC^G\left(f_{a_1a_4b_1}f_{a_2a_3b_1}\left(-p_2^{\mu_4}p_3^{\mu_2}\eta_{\mu_1\mu_3}+\eta_{\mu_2\mu_4}p_1\cdot p_4\eta_{\mu_1\mu_3}\right.\right.\\
&+\eta_{\mu_2\mu_4}p_2\cdot p_3\eta_{\mu_1\mu_3}+p_1^{\mu_4}\left(p_4^{\mu_3}\eta_{\mu_1\mu_2}-p_4^{\mu_2}\eta_{\mu_1\mu_3}\right)+p_1^{\mu_3}p_4^{\mu_2}\eta_{\mu_1\mu_4}-p_1^{\mu_2}p_4^{\mu_3}\eta_{\mu_1\mu_4}\\
&+p_2^{\mu_4}p_3^{\mu_1}\eta_{\mu_2\mu_3}-p_2^{\mu_1}p_3^{\mu_4}\eta_{\mu_2\mu_3}-p_1^{\mu_3}p_4^{\mu_1}\eta_{\mu_2\mu_4}+p_2^{\mu_3}\left(p_3^{\mu_4}\eta_{\mu_1\mu_2}-p_3^{\mu_1}\eta_{\mu_2\mu_4}\right)\\
&\left.+p_2^{\mu_1}p_3^{\mu_2}\eta_{\mu_3\mu_4}+p_1^{\mu_2}p_4^{\mu_1}\eta_{\mu_3\mu_4}-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}p_1\cdot p_4-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}p_2\cdot p_3\right)\\
&+f_{a_1a_3b_1}f_{a_2a_4b_1}\left(p_1^{\mu_4}p_3^{\mu_2}\eta_{\mu_1\mu_3}-p_1^{\mu_2}p_3^{\mu_4}\eta_{\mu_1\mu_3}-p_2^{\mu_3}p_4^{\mu_2}\eta_{\mu_1\mu_4}\right.\\
&+p_1^{\mu_3}\left(p_3^{\mu_4}\eta_{\mu_1\mu_2}-p_3^{\mu_2}\eta_{\mu_1\mu_4}\right)-p_1^{\mu_4}p_3^{\mu_1}\eta_{\mu_2\mu_3}+p_2^{\mu_4}\left(p_4^{\mu_3}\eta_{\mu_1\mu_2}-p_4^{\mu_1}\eta_{\mu_2\mu_3}\right)\\
&+p_2^{\mu_3}p_4^{\mu_1}\eta_{\mu_2\mu_4}-p_2^{\mu_1}p_4^{\mu_3}\eta_{\mu_2\mu_4}+p_1^{\mu_2}p_3^{\mu_1}\eta_{\mu_3\mu_4}+p_2^{\mu_1}p_4^{\mu_2}\eta_{\mu_3\mu_4}+\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}p_1\cdot p_3\\
&\left.-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}p_1\cdot p_3+\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}p_2\cdot p_4-\eta_{\mu_1\mu_2}\eta_{\mu_3\mu_4}p_2\cdot p_4\right)\\
&+f_{a_1a_2b_1}f_{a_3a_4b_1}\left(p_1^{\mu_2}p_2^{\mu_4}\eta_{\mu_1\mu_3}+p_3^{\mu_4}p_4^{\mu_2}\eta_{\mu_1\mu_3}-\eta_{\mu_2\mu_4}p_1\cdot p_2\eta_{\mu_1\mu_3}\right.\\
&-\eta_{\mu_2\mu_4}p_3\cdot p_4\eta_{\mu_1\mu_3}-p_1^{\mu_2}p_2^{\mu_3}\eta_{\mu_1\mu_4}-p_3^{\mu_2}p_4^{\mu_3}\eta_{\mu_1\mu_4}-p_3^{\mu_4}p_4^{\mu_1}\eta_{\mu_2\mu_3}\\
&+p_1^{\mu_4}\left(p_2^{\mu_3}\eta_{\mu_1\mu_2}-p_2^{\mu_1}\eta_{\mu_2\mu_3}\right)+p_3^{\mu_1}p_4^{\mu_3}\eta_{\mu_2\mu_4}+p_1^{\mu_3}\left(p_2^{\mu_1}\eta_{\mu_2\mu_4}-p_2^{\mu_4}\eta_{\mu_1\mu_2}\right)\\
&\left.\left.+p_3^{\mu_2}p_4^{\mu_1}\eta_{\mu_3\mu_4}-p_3^{\mu_1}p_4^{\mu_2}\eta_{\mu_3\mu_4}+\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}p_1\cdot p_2+\eta_{\mu_1\mu_4}\eta_{\mu_2\mu_3}p_3\cdot p_4\right)\right)
\end{aligned}
$$

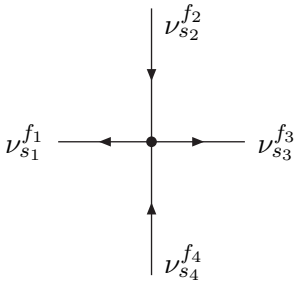


Caution: very long expression, 5-gluon vertex not displayed.

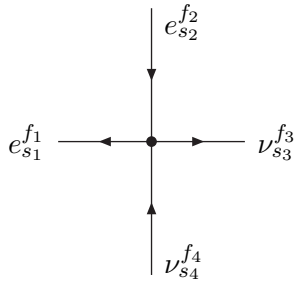


Caution: very long expression, 6-gluon vertex not displayed.

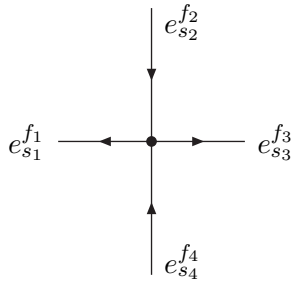
A.11 Four-fermion vertices



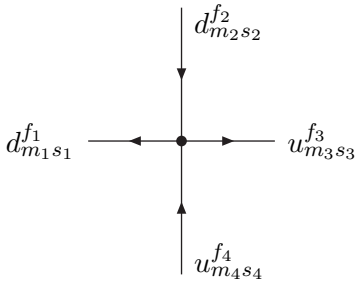
$$+4i(\gamma^\mu P_L)_{s_1 s_2}(\gamma^\mu P_L)_{s_3 s_4} C^{ll}_{g_1 g_3 g_2 g_4} U_{g_4 f_4} U_{g_2 f_2} U^*_{g_3 f_3} U^*_{g_1 f_1}$$



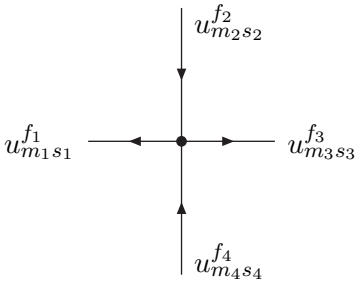
$$\begin{aligned}&+2iU_{g_2 f_4} U^*_{g_1 f_3} (\gamma^\mu P_L)_{s_1 s_2} (\gamma^\mu P_L)_{s_3 s_4} C^{ll}_{f_1 f_2 g_1 g_2} \\ &\quad + iU_{g_2 f_4} U^*_{g_1 f_3} (\gamma^\mu P_L)_{s_3 s_4} (\gamma^\mu P_R)_{s_1 s_2} C^{le}_{g_1 g_2 f_1 f_2}\end{aligned}$$



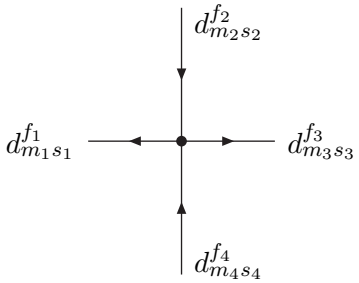
$$\begin{aligned}&+4iC^{ll}_{f_1 f_2 f_3 f_4} (\gamma^\mu P_L)_{s_1 s_2} (\gamma^\mu P_L)_{s_3 s_4} \\ &\quad + 4iC^{ee}_{f_1 f_2 f_3 f_4} (\gamma^\mu P_R)_{s_1 s_2} (\gamma^\mu P_R)_{s_3 s_4} \\ &\quad - i \Big(-C^{le}_{f_1 f_2 f_3 f_4} (\gamma^\mu P_L)_{s_1 s_2} (\gamma^\mu P_R)_{s_3 s_4} \\ &\quad - C^{le}_{f_3 f_4 f_1 f_2} (\gamma^\mu P_L)_{s_3 s_4} (\gamma^\mu P_R)_{s_1 s_2} \\ &\quad + 2(P_L)_{s_3 s_4} (P_R)_{s_1 s_2} C^{le}_{f_1 f_2 f_3 f_4} + 2(P_L)_{s_1 s_2} (P_R)_{s_3 s_4} C^{le}_{f_3 f_4 f_1 f_2} \Big)\end{aligned}$$



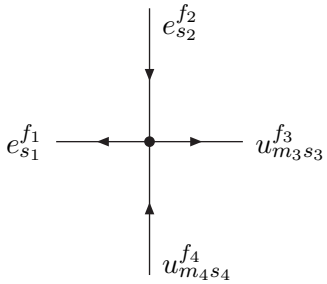
$$
\begin{aligned}
&+\frac{i}{6}\left(3\delta_{m_1m_4}\delta_{m_2m_3}-\delta_{m_1m_2}\delta_{m_3m_4}\right)C^{qu8}_{f_1f_2f_3f_4}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&+2i\delta_{m_1m_2}\delta_{m_3m_4}K_{f_3g_2}K^*_{f_4g_1}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}C^{qq1}_{f_1f_2g_2g_1}\\
&+2iK_{f_3g_2}K^*_{f_4g_1}C^{qq3}_{f_1f_2g_2g_1}\left(2\delta_{m_1m_4}\delta_{m_2m_3}(\gamma^\mu P_L)_{s_1s_4}(\gamma^\mu P_L)_{s_3s_2}\right.\\
&\left.-\delta_{m_1m_2}\delta_{m_3m_4}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}\right)\\
&+i\delta_{m_1m_2}\delta_{m_3m_4}C^{ud1}_{f_3f_4f_1f_2}(\gamma^\mu P_R)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&+\frac{i}{6}\left(3\delta_{m_1m_4}\delta_{m_2m_3}-\delta_{m_1m_2}\delta_{m_3m_4}\right)C^{ud8}_{f_3f_4f_1f_2}(\gamma^\mu P_R)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&+i\delta_{m_1m_2}\delta_{m_3m_4}C^{qu1}_{f_1f_2f_3f_4}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&+i\delta_{m_1m_2}\delta_{m_3m_4}K_{f_3g_2}K^*_{f_4g_1}(\gamma^\mu P_L)_{s_3s_4}(\gamma^\mu P_R)_{s_1s_2}C^{qd1}_{g_2g_1f_1f_2}+\frac{i}{6}\left(3\delta_{m_1m_4}\delta_{m_2m_3}\right.\\
&\left.-\delta_{m_1m_2}\delta_{m_3m_4}\right)K_{f_3g_2}K^*_{f_4g_1}(\gamma^\mu P_L)_{s_3s_4}(\gamma^\mu P_R)_{s_1s_2}C^{qd8}_{g_2g_1f_1f_2}\\
&-i\left(K^*_{f_4g_1}\left(\delta_{m_1m_4}\delta_{m_2m_3}(P_L)_{s_1s_4}(P_L)_{s_3s_2}C^{quqd1*}_{f_2f_3g_1f_1}\right.\right.\\
&\left.-\delta_{m_1m_2}\delta_{m_3m_4}(P_L)_{s_1s_2}(P_L)_{s_3s_4}C^{quqd1*}_{g_1f_3f_2f_1}\right)\\
&+K_{f_3g_1}\left(\delta_{m_1m_4}\delta_{m_2m_3}(P_R)_{s_1s_4}(P_R)_{s_3s_2}C^{quqd1}_{f_1f_4g_1f_2}\right.\\
&\left.\left.-\delta_{m_1m_2}\delta_{m_3m_4}(P_R)_{s_1s_2}(P_R)_{s_3s_4}C^{quqd1}_{g_1f_4f_1f_2}\right)\right)\\
&+\frac{i}{6}\left(K^*_{f_4g_1}\left(\left(\delta_{m_1m_4}\delta_{m_2m_3}-3\delta_{m_1m_2}\delta_{m_3m_4}\right)(P_L)_{s_1s_4}(P_L)_{s_3s_2}C^{quqd8*}_{f_2f_3g_1f_1}\right.\right.\\
&\left.+\left(3\delta_{m_1m_4}\delta_{m_2m_3}-\delta_{m_1m_2}\delta_{m_3m_4}\right)(P_L)_{s_1s_2}(P_L)_{s_3s_4}C^{quqd8*}_{g_1f_3f_2f_1}\right)\\
&+K_{f_3g_1}\left(\delta_{m_1m_4}\delta_{m_2m_3}\left(3(P_R)_{s_1s_2}(P_R)_{s_3s_4}C^{quqd8}_{g_1f_4f_1f_2}+(P_R)_{s_1s_4}(P_R)_{s_3s_2}C^{quqd8}_{f_1f_4g_1f_2}\right)\right.\\
&\left.\left.-\delta_{m_1m_2}\delta_{m_3m_4}\left((P_R)_{s_1s_2}(P_R)_{s_3s_4}C^{quqd8}_{g_1f_4f_1f_2}+3(P_R)_{s_1s_4}(P_R)_{s_3s_2}C^{quqd8}_{f_1f_4g_1f_2}\right)\right)\right)
\end{aligned}
$$



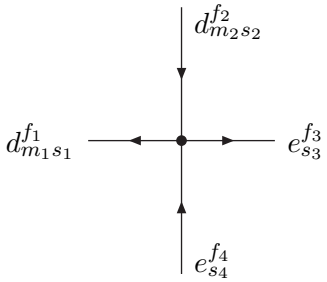
$$
\begin{aligned}
&-\frac{i}{6}\left(3\delta_{m_1m_4}\delta_{m_2m_3}-\delta_{m_1m_2}\delta_{m_3m_4}\right)\left(K_{f_3g_2}K^*_{f_4g_1}C^{qu8}_{g_2g_1f_1f_2}\left(2(P_L)_{s_1s_2}(P_R)_{s_3s_4}\right.\right.\\
&\left.-(\gamma^\mu P_L)_{s_3s_4}(\gamma^\mu P_R)_{s_1s_2}\right)\\
&+K_{f_1g_2}K^*_{f_2g_1}C^{qu8}_{g_2g_1f_3f_4}\left(2(P_L)_{s_3s_4}(P_R)_{s_1s_2}-(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\right)\Big)\\
&+i\delta_{m_1m_2}\delta_{m_3m_4}K_{f_1g_3}K_{f_3g_4}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}\left(K^*_{f_2g_1}K^*_{f_4g_2}\left(C^{qq1}_{g_3g_2g_4g_1}\right.\right.\\
&\left.\left.+C^{qq1}_{g_4g_1g_3g_2}\right)+K^*_{f_4g_1}K^*_{f_2g_2}\left(C^{qq1}_{g_3g_1g_4g_2}+C^{qq1}_{g_4g_2g_3g_1}\right)\right)\\
&+i\delta_{m_1m_2}\delta_{m_3m_4}K_{f_1g_3}K_{f_3g_4}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}\left(K^*_{f_2g_1}K^*_{f_4g_2}\left(C^{qq3}_{g_3g_2g_4g_1}\right.\right.\\
&\left.\left.+C^{qq3}_{g_4g_1g_3g_2}\right)+K^*_{f_4g_1}K^*_{f_2g_2}\left(C^{qq3}_{g_3g_1g_4g_2}+C^{qq3}_{g_4g_2g_3g_1}\right)\right)\\
&+2i\delta_{m_1m_2}\delta_{m_3m_4}\left(C^{uu}_{f_1f_2f_3f_4}+C^{uu}_{f_1f_4f_3f_2}\right)(\gamma^\mu P_R)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&-i\delta_{m_1m_2}\delta_{m_3m_4}\left(K_{f_3g_2}K^*_{f_4g_1}C^{qu1}_{g_2g_1f_1f_2}\left(2(P_L)_{s_1s_2}(P_R)_{s_3s_4}\right.\right.\\
&\left.-(\gamma^\mu P_L)_{s_3s_4}(\gamma^\mu P_R)_{s_1s_2}\right)\\
&\left.+K_{f_1g_2}K^*_{f_2g_1}C^{qu1}_{g_2g_1f_3f_4}\left(2(P_L)_{s_3s_4}(P_R)_{s_1s_2}-(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\right)\right)
\end{aligned}
$$



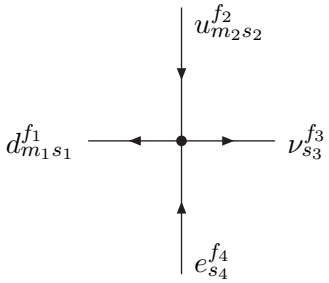
$$
\begin{aligned}
&+2i\delta_{m_1m_2}\delta_{m_3m_4}\left(C^{qq1}_{f_1f_2f_3f_4}+C^{qq1}_{f_1f_4f_3f_2}\right)(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}\\
&+2i\delta_{m_1m_2}\delta_{m_3m_4}\left(C^{qq3}_{f_1f_2f_3f_4}+C^{qq3}_{f_1f_4f_3f_2}\right)(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}\\
&+2i\delta_{m_1m_2}\delta_{m_3m_4}\left(C^{dd}_{f_1f_2f_3f_4}+C^{dd}_{f_1f_4f_3f_2}\right)(\gamma^\mu P_R)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&-i\delta_{m_1m_2}\delta_{m_3m_4}\Big(-C^{qd1}_{f_1f_2f_3f_4}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&-C^{qd1}_{f_3f_4f_1f_2}(\gamma^\mu P_L)_{s_3s_4}(\gamma^\mu P_R)_{s_1s_2}+2(P_L)_{s_3s_4}(P_R)_{s_1s_2}C^{qd1}_{f_1f_2f_3f_4}\\
&+2(P_L)_{s_1s_2}(P_R)_{s_3s_4}C^{qd1}_{f_3f_4f_1f_2}\Big)-\frac{i}{6}\left(3\delta_{m_1m_4}\delta_{m_2m_3}\right.\\
&\left.-\delta_{m_1m_2}\delta_{m_3m_4}\right)\Big(-C^{qd8}_{f_1f_2f_3f_4}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&-C^{qd8}_{f_3f_4f_1f_2}(\gamma^\mu P_L)_{s_3s_4}(\gamma^\mu P_R)_{s_1s_2}\\
&+2(P_L)_{s_3s_4}(P_R)_{s_1s_2}C^{qd8}_{f_1f_2f_3f_4}+2(P_L)_{s_1s_2}(P_R)_{s_3s_4}C^{qd8}_{f_3f_4f_1f_2}\Big)
\end{aligned}
$$



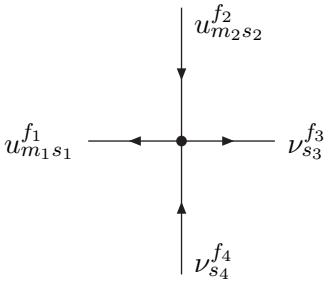
$$
\begin{aligned}
&+iK_{f_3g_2}K^*_{f_4g_1}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}C^{lq1}_{f_1f_2g_2g_1}\\
&-iK_{f_3g_2}K^*_{f_4g_1}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}C^{lq3}_{f_1f_2g_2g_1}\\
&+iC^{eu}_{f_1f_2f_3f_4}(\gamma^\mu P_R)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&+iC^{lu}_{f_1f_2f_3f_4}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&+iK_{f_3g_2}K^*_{f_4g_1}(\gamma^\mu P_L)_{s_3s_4}(\gamma^\mu P_R)_{s_1s_2}C^{qe}_{g_2g_1f_1f_2}\\
&-i\Big((P_L)_{s_1s_2}(P_L)_{s_3s_4}K^*_{f_4g_1}C^{lequ1*}_{f_2f_1g_1f_3}\\
&+(P_R)_{s_1s_2}(P_R)_{s_3s_4}K_{f_3g_1}C^{lequ1}_{f_1f_2g_1f_4}\Big)\\
&-i\Big(K^*_{f_4g_1}(\sigma^{\mu\nu}P_L)_{s_1s_2}(\sigma_{\mu\nu}P_L)_{s_3s_4}C^{lequ3*}_{f_2f_1g_1f_3}\\
&+K_{f_3g_1}C^{lequ3}_{f_1f_2g_1f_4}(\sigma^{\mu\nu}P_R)_{s_1s_2}(\sigma_{\mu\nu}P_R)_{s_3s_4}\Big)
\end{aligned}
$$



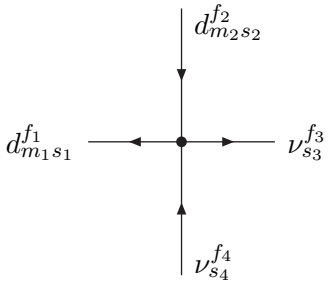
$$
\begin{aligned}
&+iC^{lq1}_{f_3f_4f_1f_2}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}+iC^{lq3}_{f_3f_4f_1f_2}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_L)_{s_3s_4}\\
&+iC^{ed}_{f_3f_4f_1f_2}(\gamma^\mu P_R)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&+iC^{ld}_{f_3f_4f_1f_2}(\gamma^\mu P_L)_{s_3s_4}(\gamma^\mu P_R)_{s_1s_2}+iC^{qe}_{f_1f_2f_3f_4}(\gamma^\mu P_L)_{s_1s_2}(\gamma^\mu P_R)_{s_3s_4}\\
&+i\Big((P_L)_{s_3s_4}(P_R)_{s_1s_2}C^{ledq*}_{f_4f_3f_2f_1}+(P_L)_{s_1s_2}(P_R)_{s_3s_4}C^{ledq}_{f_3f_4f_1f_2}\Big)
\end{aligned}
$$



$$
\begin{aligned}
&+2iK^*_{f_2 g_1} U^*_{g_2 f_3} (\gamma^\mu P_L)_{s_1 s_2} (\gamma^\mu P_L)_{s_3 s_4} C^{lq3}_{g_2 f_4 f_1 g_1} \\
&\quad + i(P_L)_{s_1 s_2} (P_R)_{s_3 s_4} K^*_{f_2 g_1} U^*_{g_2 f_3} C^{ledq}_{g_2 f_4 f_1 g_1} \\
&\quad + i(P_R)_{s_1 s_2} (P_R)_{s_3 s_4} U^*_{g_1 f_3} C^{lequ1}_{g_1 f_4 f_1 f_2} \\
&\quad + iU^*_{g_1 f_3} C^{lequ3}_{g_1 f_4 f_1 f_2} (\sigma^{\mu\nu} P_R)_{s_1 s_2} (\sigma_{\mu\nu} P_R)_{s_3 s_4}
\end{aligned}
$$

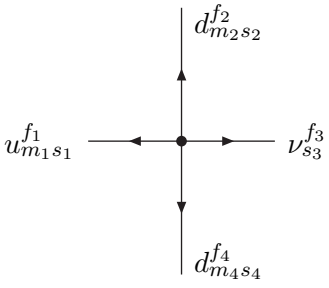


$$
\begin{aligned}
&+iK_{f_1 g_3} U_{g_4 f_4} K^*_{f_2 g_1} U^*_{g_2 f_3} (\gamma^\mu P_L)_{s_1 s_2} (\gamma^\mu P_L)_{s_3 s_4} C^{lq1}_{g_2 g_4 g_3 g_1} \\
&\quad + iK_{f_1 g_3} U_{g_4 f_4} K^*_{f_2 g_1} U^*_{g_2 f_3} (\gamma^\mu P_L)_{s_1 s_2} (\gamma^\mu P_L)_{s_3 s_4} C^{lq3}_{g_2 g_4 g_3 g_1} \\
&\quad + iU_{g_2 f_4} U^*_{g_1 f_3} (\gamma^\mu P_L)_{s_3 s_4} (\gamma^\mu P_R)_{s_1 s_2} C^{lu}_{g_1 g_2 f_1 f_2}
\end{aligned}
$$

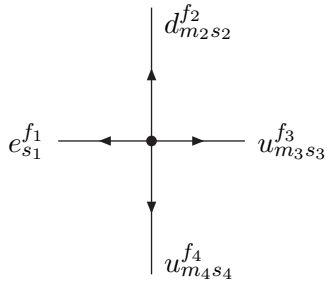


$$
\begin{aligned}
&+iU_{g_2 f_4} U^*_{g_1 f_3} (\gamma^\mu P_L)_{s_1 s_2} (\gamma^\mu P_L)_{s_3 s_4} C^{lq1}_{g_1 g_2 f_1 f_2} \\
&\quad - iU_{g_2 f_4} U^*_{g_1 f_3} (\gamma^\mu P_L)_{s_1 s_2} (\gamma^\mu P_L)_{s_3 s_4} C^{lq3}_{g_1 g_2 f_1 f_2} \\
&\quad + iU_{g_2 f_4} U^*_{g_1 f_3} (\gamma^\mu P_L)_{s_3 s_4} (\gamma^\mu P_R)_{s_1 s_2} C^{ld}_{g_1 g_2 f_1 f_2}
\end{aligned}
$$

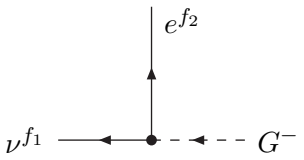
A.12 Lepton and baryon number violating vertices



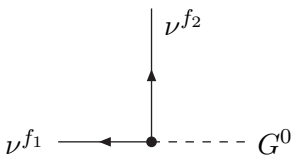
$$
\begin{aligned}
&-i\epsilon_{m_1 m_2 m_4} U^*_{g_1 f_3} \Big((P_L)_{s_1 s_4} (P_R)_{s_3 s_2} C^{duq*}_{f_4 f_1 f_2 g_1} \\
&\quad - (P_L)_{s_1 s_2} (P_R)_{s_3 s_4} C^{duq*}_{f_2 f_1 f_4 g_1} \Big) \\
&\quad - i\epsilon_{m_1 m_2 m_4} K_{f_1 g_1} U^*_{g_2 f_3} \Big((P_R)_{s_2 s_4} (P_R)_{s_3 s_1} C^{qqq*}_{f_4 f_2 g_1 g_2} \\
&\quad - (P_R)_{s_4 s_2} (P_R)_{s_3 s_1} C^{qqq*}_{f_2 f_4 g_1 g_2} + (P_R)_{s_1 s_4} (P_R)_{s_3 s_2} C^{qqq*}_{f_4 g_1 f_2 g_2} \\
&\quad - (P_R)_{s_1 s_2} (P_R)_{s_3 s_4} C^{qqq*}_{f_2 g_1 f_4 g_2} \Big)
\end{aligned}
$$



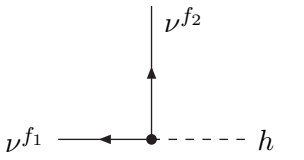
$$
\begin{aligned}
&-i\epsilon_{m_2m_3m_4}\Big((P_L)_{s_4s_2}(P_R)_{s_1s_3}K_{f_3g_1}C^{duq*}_{f_2f_4g_1f_1}\\
&-(P_L)_{s_3s_2}(P_R)_{s_1s_4}K_{f_4g_1}C^{duq*}_{f_2f_3g_1f_1}\Big)\\
&-i\epsilon_{m_2m_3m_4}\Big((P_L)_{s_1s_4}(P_R)_{s_2s_3}K_{f_3g_1}C^{qqu*}_{g_1f_2f_4f_1}\\
&-(P_L)_{s_1s_3}(P_R)_{s_2s_4}K_{f_4g_1}C^{qqu*}_{g_1f_2f_3f_1}\\
&+(P_L)_{s_1s_4}(P_R)_{s_3s_2}K_{f_3g_1}C^{qqu*}_{f_2g_1f_4f_1}\\
&-(P_L)_{s_1s_3}(P_R)_{s_4s_2}K_{f_4g_1}C^{qqu*}_{f_2g_1f_3f_1}\Big)\\
&+i\epsilon_{m_2m_3m_4}\Big(((P_R)_{s_1s_4}(P_R)_{s_2s_3}K_{f_3g_1}K_{f_4g_2}\\
&-(P_R)_{s_1s_3}(P_R)_{s_2s_4}K_{f_4g_1}K_{f_3g_2})\,C^{qqq*}_{g_1f_2g_2f_1}\\
&+P_R\,((P_R)_{s_4s_3}K_{f_3g_1}K_{f_4g_2}-(P_R)_{s_3s_4}K_{f_4g_1}K_{f_3g_2})\,C^{qqq*}_{g_1g_2f_2f_1}\Big)\\
&+i\epsilon_{m_2m_3m_4}\Big((P_L)_{s_1s_4}(P_L)_{s_3s_2}C^{duu*}_{f_2f_3f_4f_1}\\
&-(P_L)_{s_1s_3}(P_L)_{s_4s_2}C^{duu*}_{f_2f_4f_3f_1}\Big)
\end{aligned}
$$



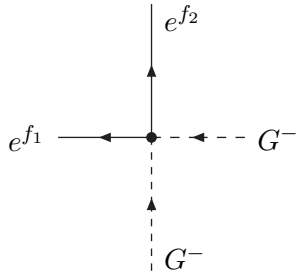
$$-i\sqrt{2}vP_RU_{f_2g_1}C^{\nu\nu*}_{f_1g_1}$$



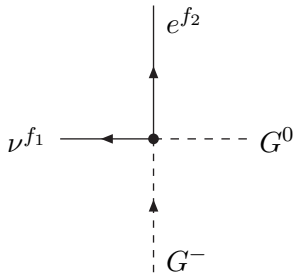
$$+2vP_RC^{\nu\nu*}_{f_1f_2}$$



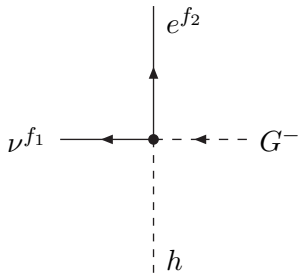
$$+2ivP_RC^{\nu\nu*}_{f_1f_2}$$



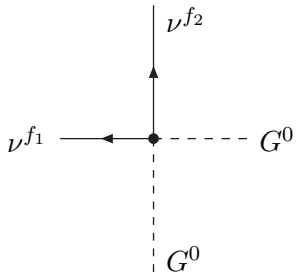
$$+4iP_RU_{f_1g_1}U_{f_2g_2}C^{\nu\nu*}_{g_1g_2}$$



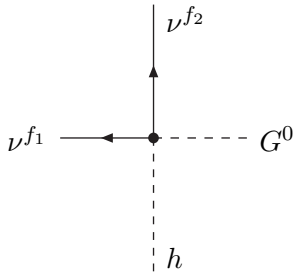
$$-\sqrt{2}P_R U_{f_2 g_1} C^{\nu\nu *}_{f_1 g_1}$$



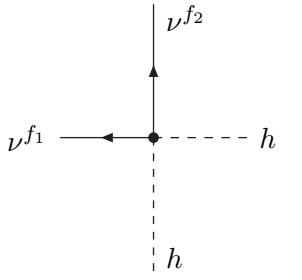
$$-i\sqrt{2}P_R U_{f_2 g_1} C^{\nu\nu *}_{f_1 g_1}$$



$$-2iP_R C^{\nu\nu *}_{f_1 f_2}$$

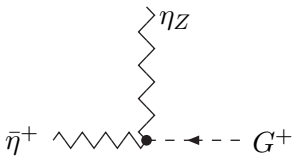


$$+2P_R C^{\nu\nu *}_{f_1 f_2}$$

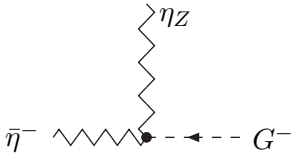


$$+2iP_R C^{\nu\nu *}_{f_1 f_2}$$

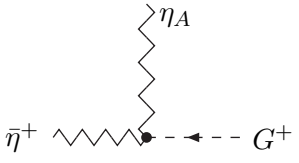
A.13 Ghost vertices



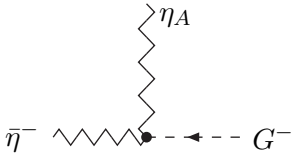
$$-\frac{i\bar{g}v\left(\bar{g}'^2-\bar{g}^2\right)\xi_W}{4\sqrt{\bar{g}^2+\bar{g}'^2}}+\frac{i\bar{g}^2\bar{g}'v^3\left(\bar{g}'^2-\bar{g}^2\right)\xi_W}{4\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}$$



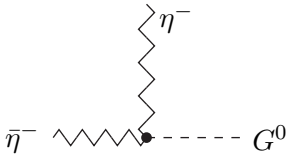
$$-\frac{i\bar{g}v\left(\bar{g}'^2-\bar{g}^2\right)\xi_W}{4\sqrt{\bar{g}^2+\bar{g}'^2}}+\frac{i\bar{g}^2\bar{g}'v^3\left(\bar{g}'^2-\bar{g}^2\right)\xi_W}{4\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}$$



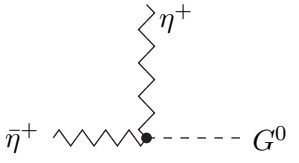
$$+\frac{i\bar{g}^2\bar{g}'v\xi_W}{2\sqrt{\bar{g}^2+\bar{g}'^2}}-\frac{i\bar{g}^3\bar{g}'^2v^3\xi_W}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}$$



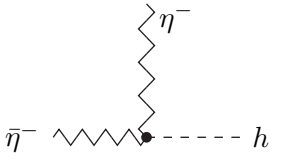
$$+\frac{i\bar{g}^2\bar{g}'v\xi_W}{2\sqrt{\bar{g}^2+\bar{g}'^2}}-\frac{i\bar{g}^3\bar{g}'^2v^3\xi_W}{2\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}$$



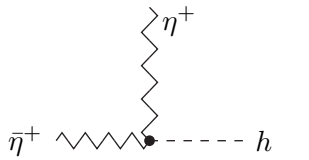
$$+\frac{1}{4}\bar{g}^2v\xi_W-\frac{1}{16}\bar{g}^2v^3\xi_WC^{\phi D}$$



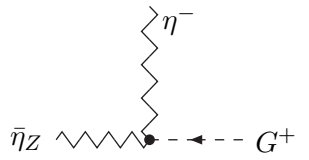
$$-\frac{1}{4}\bar{g}^2v\xi_W+\frac{1}{16}\bar{g}^2v^3\xi_WC^{\phi D}$$



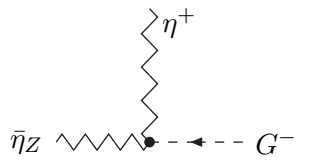
$$+\frac{1}{4}i\bar{g}^2v\xi_W+\frac{1}{4}i\bar{g}^2v^3\xi_WC^{\phi\Box}-\frac{1}{16}i\bar{g}^2v^3\xi_WC^{\phi D}$$



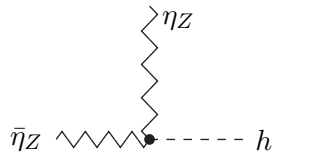
$$+\frac{1}{4}i\bar{g}^2 v\xi_W + \frac{1}{4}i\bar{g}^2 v^3\xi_W C^{\phi\Box} - \frac{1}{16}i\bar{g}^2 v^3\xi_W C^{\phi D}$$



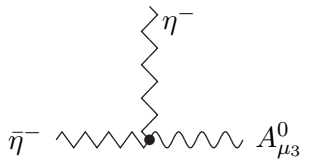
$$-\frac{1}{4}i\bar{g}v\sqrt{\bar{g}^2+\bar{g}'^2}\xi_Z - \frac{1}{8}i\bar{g}v^3\sqrt{\bar{g}^2+\bar{g}'^2}\xi_Z C^{\phi D} - \frac{i\bar{g}^2\bar{g}'v^3\xi_Z}{4\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}$$



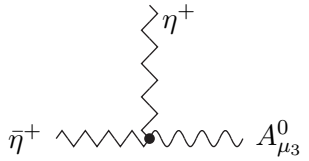
$$-\frac{1}{4}i\bar{g}v\sqrt{\bar{g}^2+\bar{g}'^2}\xi_Z - \frac{1}{8}i\bar{g}v^3\sqrt{\bar{g}^2+\bar{g}'^2}\xi_Z C^{\phi D} - \frac{i\bar{g}^2\bar{g}'v^3\xi_Z}{4\sqrt{\bar{g}^2+\bar{g}'^2}}C^{\phi WB}$$



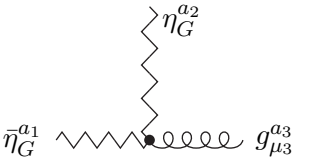
$$\begin{aligned}&+\frac{1}{4}iv\xi_Z\left(\bar{g}^2+\bar{g}'^2\right)+\frac{1}{4}iv^3\xi_Z\left(\bar{g}^2+\bar{g}'^2\right)C^{\phi\Box}\\&+\frac{1}{16}iv^3\xi_Z\left(\bar{g}^2+\bar{g}'^2\right)C^{\phi D}+\frac{1}{2}i\bar{g}\bar{g}'v^3\xi_Z C^{\phi WB}\end{aligned}$$



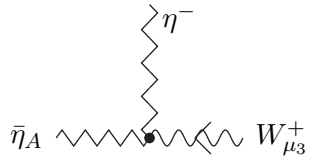
$$+\frac{i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3} - \frac{i\bar{g}^2\bar{g}'^2v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



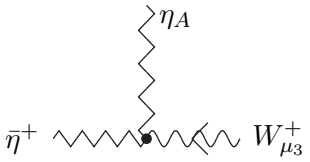
$$-\frac{i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3} + \frac{i\bar{g}^2\bar{g}'^2v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



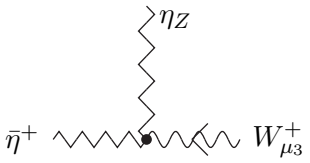
$$-\bar{g}_s f_{a_3a_1a_2}p_1^{\mu_3}$$



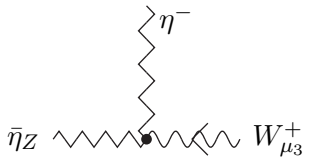
$$-\frac{i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3} - \frac{i\bar{g}^4v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



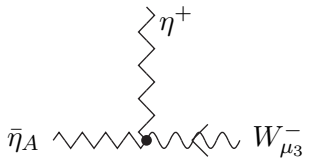
$$+\frac{i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3}-\frac{i\bar{g}^2\bar{g}'^2v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



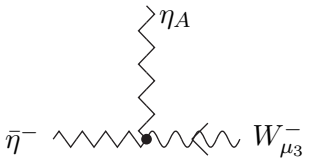
$$+\frac{i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3}+\frac{i\bar{g}\bar{g}'^3v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



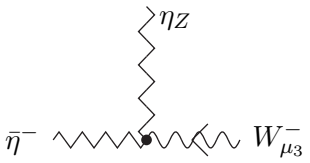
$$-\frac{i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3}+\frac{i\bar{g}^3\bar{g}'v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



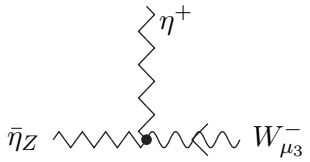
$$+\frac{i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3}+\frac{i\bar{g}^4v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



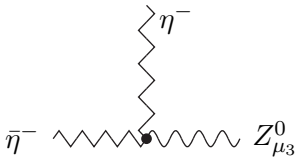
$$-\frac{i\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3}+\frac{i\bar{g}^2\bar{g}'^2v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



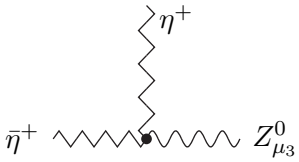
$$-\frac{i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3}-\frac{i\bar{g}\bar{g}'^3v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



$$+\frac{i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3}-\frac{i\bar{g}^3\bar{g}'v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



$$+\frac{i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3}+\frac{i\bar{g}\bar{g}'^3v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$



$$-\frac{i\bar{g}^2}{\sqrt{\bar{g}^2+\bar{g}'^2}}p_1^{\mu_3}-\frac{i\bar{g}\bar{g}'^3v^2}{\left(\bar{g}^2+\bar{g}'^2\right)^{3/2}}C^{\phi WB}p_1^{\mu_3}$$

B SMEFT *Mathematica* package for `FeynRules`

To install and run the code calculating the SMEFT Feynman rules, the user should perform the following steps (tested on Linux systems, with small modifications applicable also on MS Windows or OS-X operating systems):

1. Download and install properly the `FeynRules` package. The latest version can be found at `feynrules.irmp.ucl.ac.be`. SMEFT package has been tested with `FeynRules` v2.3.

2. Download the SMEFT package from `www.fuw.edu.pl/smeft`.

3. Unpack the SMEFT files to `Models/SMEFT` sub-directory of previously installed `FeynRules` package.

4. Update, at the top of master file `smeft_initialize.m`, the variable `$FeynRulesPath`, to reflect the actual location of installation of `FeynRules` on given system.

5. Set the control variable `SetRxiGaugeStatus` to `False` or `True`, requesting respectively evaluation of SMEFT Feynman rules in unitary or R_ξ-gauges (of course only in the second case the Goldstone boson and ghost vertices are produced, at the cost of much longer CPU time).

6. Run in the *Mathematica* notebook the command

   ```
   << smeft_initialize.m
   ```

   It loads `FeynRules` and starts the calculations. These can be time consuming, from minutes on fast computer with unitary gauge selected, to even few hours on a slower machine with R_ξ-gauges.

After running the code, calculated Feynman rules for various classes of interactions are stored in `FeynRules` format in separate disk files (with names of all files displayed on the screen after the calculations are finished) and can be quickly reloaded and reused without rerunning the whole time-consuming code.

The auxiliary programs included in SMEFT package produce also automatically the Latex/axodraw files with all calculated vertices (excluding only the longest five and six gluon

vertices), for easier visual reference. Note however that automatic line breaking in very complicated formulae may be far from perfect and requires, in few cases, manual improvement.

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