The decay $h \to \gamma\gamma$ in the Standard-Model Effective Field Theory

A. Dedes[1]*, M. Paraskevas[1]†, J. Rosiek[2]‡, K. Suxho[1]§ and L. Trifyllis[1]¶

[1] *Department of Physics, Division of Theoretical Physics, University of Ioannina, GR 45110, Greece*

[2] *Faculty of Physics Department, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*

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Abstract

Assuming that new physics effects are parametrized by the Standard-Model Effective Field Theory (SMEFT) written in a complete basis of up to dimension-6 operators, we calculate the CP-conserving one-loop amplitude for the decay $h \to \gamma\gamma$ in general R_ξ-gauges. We employ a simple renormalisation scheme that is hybrid between on-shell SM-like renormalised parameters and running $\overline{\text{MS}}$ Wilson coefficients. The resulting amplitude is then finite, renormalisation scale invariant, independent of the gauge choice (ξ) and respects SM Ward identities. Remarkably, the S-matrix amplitude calculation resembles very closely the one usually known from renormalisable theories and can be automatised to a high degree. We use this gauge invariant amplitude and recent LHC data to check upon sensitivity to various Wilson coefficients entering from a more complete theory at the matching energy scale. We present a closed expression for the ratio $\mathcal{R}_{h\to\gamma\gamma}$, of the Beyond the SM versus the SM contributions as appeared in LHC $h \to \gamma\gamma$ searches. The most important contributions arise at tree level from the operators $Q_{\varphi B}, Q_{\varphi W}, Q_{\varphi WB}$, and at one-loop level from the dipole operators Q_{uB}, Q_{uW}. Our calculation shows also that, for operators that appear at tree level in SMEFT, one-loop corrections can modify their contributions by less than 10%. Wilson coefficients corresponding to these five operators are bounded from current LHC $h \to \gamma\gamma$ data – in some cases an order of magnitude stronger than from other searches. With mild assumptions, we point out a set of possibilities for a field theory content at higher energies which may generate sizeable corrections in $h \to \gamma\gamma$ amplitude. Finally, we correct results that appeared previously in the literature.

*email: adedes@cc.uoi.gr
†email: mparask@grads.uoi.gr
‡email: janusz.rosiek@fuw.edu.pl
§email: csoutzio@cc.uoi.gr
¶email: ltrifyl@cc.uoi.gr

Contents

1 Introduction

The discovery of the Higgs boson [1–3] in year 2012 was made possible mainly because of its decay into two photons [4,5]. The current outcome for this decay channel from LHC (Run-2) with center-of-mass energy $\sqrt{s} = 13$ TeV, integrated luminosity of 36.1 fb^{-1} and Higgs boson mass, $M_h = 125.09 \pm 0.24$ GeV is summarised as the ratio between the experimentally measured value (which may include contributions from new physics scenarios) relative to the Standard-Model (SM) predicted value [6,7]

$$\mathcal{R}_{h\to\gamma\gamma} = \frac{\Gamma(\mathrm{EXP}, h\to\gamma\gamma)}{\Gamma(\mathrm{SM}, h\to\gamma\gamma)}\,. \tag{1.1}$$

The most recent measurements are presented by ATLAS [8] and CMS [9] experiments of LHC,

$$\begin{aligned} \text{ATLAS:} \qquad & \mathcal{R}_{h\to\gamma\gamma} = 0.99^{+0.15}_{-0.14}\,, \\ \text{CMS:} \qquad & \mathcal{R}_{h\to\gamma\gamma} = 1.18^{+0.17}_{-0.14}\,, \end{aligned} \tag{1.2}$$

and are consistent with the SM prediction, with the error margin expected to be reduced in the near future.

If we consider the SM as a complete theory of electroweak (EW) and strong interactions up to the Planck scale, with no other scale involved in between, then the decay amplitude $h \to \gamma\gamma$ arises purely from dimension $d \leq 4$ (renormalisable) interactions. In this case the amplitude is finite, calculable and, since all relevant parameters are experimentally known, it is a certain prediction of the SM. It is this prediction entering the denominator in eq. (1.1). If however, there is New Physics beyond the SM already at a scale Λ which is above, but not far from, the EW scale, say $\Lambda \sim \mathcal{O}(1-10)$ TeV, then its effects can be parametrized by the presence of effective operators with dimension $d > 4$ at scale Λ. These operators together with various parameters (or Wilson coefficients) will then run down to the EW scale and feed the on-shell scattering S-matrix amplitude together with $d \leq 4$ interactions.

All dimension $d \leq 6$ effective operators among SM particles that obey the SM gauge symmetry have been classified in refs. [10,11]. The SM augmented with these effective operators – remnants of unknown heavy particles' decoupling [12] – is called the SM Effective Field Theory, or for a short SMEFT. The quantization of SMEFT has recently been undertaken in ref. [13] in linear R_ξ-gauges with explicit proof of BRST symmetry and where all relevant primitive interaction vertices have been collected.

Within SM, numerous calculations for the $h \to \gamma\gamma$ amplitude exist. The first calculation was performed in ref. [6] in the limit of light Higgs mass ($M_h \ll M_W$), using dimensional regularisation in the 't Hooft-Feynman gauge. Since then, there are other works completing this calculation in linear and non-linear gauges [7, 14, 15], with different regularisation schemes [16–22]. To our knowledge the complete SM one-loop $h \to \gamma\gamma$ amplitude in linear R_ξ-gauges is performed in ref. [23].

In SMEFT[1] there is already a number of papers that calculate the $h \to \gamma\gamma$ amplitude [26,27].[2] The current, state of the art calculation, has been presented by Hartmann and Trott in refs. [29, 30]. The analysis was carried out using the Background Field Method (BFM) [31][3] consistent with minimal subtraction renormalisation scheme ($\overline{\mathrm{MS}}$) and included all relevant (CP-conserving) dimension $d \leq 6$ operators in calculating finite, non-log parts of the diagrams. Our work here is complementary but incorporates some additional features of importance:

[1] For a recent review see, ref. [24] and for pedagogical lectures ref. [25].

[2] Recently, also the next to leading order calculation for $h \to ZZ$ and $h \to Z\gamma$ decay in SMEFT has appeared in the literature [28].

[3] For a more recent approach on BFM-SMEFT see ref. [32].

- a simple calculational treatment in linear R_ξ-gauges based on Feynman rules of ref. [13],
- an analytical proof of gauge invariance (independence on the gauge choice ξ-parameter(s)) of the S-matrix element,
- a simple renormalisation framework which leads to a finite and renormalisation scale invariant amplitude,
- a compact semi-analytical expression highlighting the effect of new operators in the ratio $\mathcal{R}_{h\to\gamma\gamma}$ and corresponding bounds on Wilson coefficients,
- a field content of simple, perturbative, high energy models valid at the energy scale Λ, which, under gentle assumptions, can affect the ratio $\mathcal{R}_{h\to\gamma\gamma}$.

There are quite a few papers addressing a global fit to the Higgs data from LHC Run-1 and Run-2 in the SMEFT framework [33–35]. Our work provides a simple semi-analytic one-loop formula for the ratio $\mathcal{R}_{h\to\gamma\gamma}$ in eq. (1.1) that can be used by these (usually tree level) fits or by analogous experimental analysis at LHC for Higgs boson searches.

Our paper is organised as follows. In section 2 we list operators contributing to the decay $h\to\gamma\gamma$ in SMEFT. Next, in section 3 we develop, in a pedagogical fashion, the renormalisation scheme for calculating the $h\to\gamma\gamma$ amplitude. In section 4 we give analytical expressions for all types of SM and SMEFT contributions to the $h\to\gamma\gamma$ amplitude and to the ratio $\mathcal{R}_{h\to\gamma\gamma}$. Semi-analytical prediction for $\mathcal{R}_{h\to\gamma\gamma}$, depending on the running Wilson coefficients and renormalisation scale μ, are collected in section 5, and supplied with a discussion on numerical constraints of these coefficients. We conclude in section 6. Finally, in Appendix A we collect analytical expressions for the relevant one-loop self-energies and, relevant to $h\to\gamma\gamma$, three-point one-loop corrections in general R_ξ-gauges.

2 Relevant Operators

In EFT, an effect from the decoupling of heavy particles with masses of order Λ is captured by the running parameters of the low energy theory influenced by higher dimensional operators added to SM renormalisable Lagrangian $\mathcal{L}_{\mathrm{SM}}^{(4)}$. The full effective Lagrangian we consider here can be expressed as,

$$\mathcal{L}=\mathcal{L}_{\mathrm{SM}}^{(4)}+\sum_X C^X Q_X^{(6)}+\sum_f C'^f Q_f^{(6)}\,, \tag{2.1}$$

where $Q_X^{(6)}$ denotes dimension-6 operators that do not involve fermion fields, while $Q_f^{(6)}$ denotes operators that contain fermion fields. All Wilson coefficients should be rescaled by Λ^2, for example $C^X\to C^X/\Lambda^2$. We shall restore $1/\Lambda^2$ only in section 4 and thereafter. The prime in C'^f, denotes a coefficient in flavour ("Warsaw") basis of ref. [11] while we use unprimed coefficients in fermion mass basis defined in ref. [13].

The operators involved in the calculation of decay $h\to\gamma\gamma$ are collected in Table 1. They can easily be identified when drawing the Feynman diagrams for $h\to\gamma\gamma$ looking at the primitive vertices listed in ref. [13]. There are 8 classes of such operators $X^3,\varphi^6,\varphi^4D^2,\psi^2\varphi^3,X^2\varphi^2,\psi^2X\varphi,\psi^2\varphi^2D,\psi^4$ where X represents a gauge field strength tensor, φ the Higgs doublet, D a covariant derivative and ψ a generic fermion field. Not counting flavour multiplicities and hermi-

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}'_p e'_r \varphi)$
		$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}'_p u'_r \widetilde{\varphi})$
		$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^* \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}'_p d'_r \varphi)$
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{eW}	$(\bar{l}'_p \sigma^{\mu\nu} e'_r)\tau^I \varphi W^I_{\mu\nu}$	$Q^{(3)}_{\varphi l}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}'_p \tau^I \gamma^\mu l'_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{eB}	$(\bar{l}'_p \sigma^{\mu\nu} e'_r)\varphi B_{\mu\nu}$		
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{uW}	$(\bar{q}'_p \sigma^{\mu\nu} u'_r)\tau^I \widetilde{\varphi}\, W^I_{\mu\nu}$		
		Q_{uB}	$(\bar{q}'_p \sigma^{\mu\nu} u'_r)\widetilde{\varphi}\, B_{\mu\nu}$		
		Q_{dW}	$(\bar{q}'_p \sigma^{\mu\nu} d'_r)\tau^I \varphi\, W^I_{\mu\nu}$		
		Q_{dB}	$(\bar{q}'_p \sigma^{\mu\nu} d'_r)\varphi\, B_{\mu\nu}$		
		ψ^4			
		Q_{ll}	$(\bar{l}'_p \gamma_\mu l'_r)(\bar{l}'_s \gamma^\mu l'_t)$		

Table 1: *A set of $d = 6$ operators in Warsaw basis that contribute to the $h \to \gamma\gamma$ decay amplitude, directly or indirectly, in R_ξ-gauges. We consider only CP-conserving operators in our analysis. The operator Q_φ cancels out completely in the $h \to \gamma\gamma$ amplitude. The operators Q_{ll} and $Q^{(3)}_{\varphi l}$ present themselves indirectly through the translation of the renormalised vacuum expectation value (vev) into the well measured Fermi coupling constant, cf. eq. (3.5). The notation is the same as in refs. [11, 13]. For brevity we suppress fermion chiral indices L, R.*

tian conjugation, in general, there are 16+2 CP-conserving operators.[4] Actually, *not all* operators in Table 1 contribute in the final result for the $h \to \gamma\gamma$ amplitude. The operator Q_φ cancels out completely after adding all contributions. This leaves 17 CP-conserving operators (or Wilson coefficients) relevant to the $h \to \gamma\gamma$ amplitude.

Another classification of various $d = 6$ operators can be devised alongside with their strength [36, 37]. The division is between operators that are potentially tree level generated (PTG operators) and those that are loop generated (LG operators) by the more fundamental theory at high energies (UV-theory) under the assumption that the latter is *perturbatively decoupled*. Under this classification operators relevant for $h \to \gamma\gamma$ amplitude are arranged as follows: LG operators are suppressed by $1/(4\pi)^2$ factors for each loop and may be thought to be sub-dominant corrections with respect to PTG operators. Relevant to $h \to \gamma\gamma$, PTG and LG classes of operators are listed in Table 2 (also for later convenience in section 5.3). On the other hand, a perturbative decoupling of the UV-theory may not necessarily be the case that Nature chooses. In this work, although we do not *assume* any distinction amongst the $d = 6$ operators involved in $h \to \gamma\gamma$ amplitude, we shall be referring to Table 2 as our analysis progresses.

[4] Incorporating the CP-violating operators will not create any problem in the procedure of renormalisation or elsewhere in our analysis. However, these operators are usually strongly suppressed by CP-violating type of observables such as particle Electric Dipole Moments (EDMs) and this is the only motivation for not considering them in this work.

PTG	LG
φ^6 and $\varphi^4 D^2$	X^3
$\psi^2\varphi^3$	$X^2\varphi^2$
$\psi^2\varphi^2 D$	$\psi^2 X\varphi$
ψ^4	

Table 2: *PTG and LG classes of operators shown in Table 1.*

3 Renormalisation

3.1 Parameter initialisation in SMEFT

There is a set of very well measured quantities, to which we rely upon, in relating our calculation for $\mathcal{R}_{h\to\gamma\gamma}$ to the LHC data. This set of experimental values is [38]

$$\begin{aligned} G_F &= 1.1663787(6)\times 10^{-5}\ \mathrm{GeV}^{-2}\,, \\ \alpha_{\mathrm{EM}} &= 1/137.035999139(31) \quad \text{at } Q^2=0\,, \\ M_W &= 80.385(15)\ \mathrm{GeV}\,, \\ M_Z &= 91.1876(21)\ \mathrm{GeV}\,, \\ M_h &= 125.09\pm 0.24\ \mathrm{GeV}\,, \\ m_t &= 173.1\pm 0.6\ \mathrm{GeV}\,. \end{aligned} \tag{3.1}$$

We identify these input values with the ones obtained in SMEFT consistent with the given accuracy of up to $1/\Lambda^2$ expansion terms. Consequently, following ref. [13], for the gauge and Higgs boson masses at tree level, it is enough to set M_W, M_Z and M_h, respectively, equal to

$$\begin{aligned} M_W &= \frac{1}{2}\bar{g}v\,, \\ M_Z &= \frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}\,v\left(1+\frac{\bar{g}\bar{g}' C^{\varphi WB}v^2}{\bar{g}^2+\bar{g}'^2}+\frac{1}{4}C^{\varphi D}v^2\right), \\ M_h^2 &= \lambda v^2-\left(3C^{\varphi}-2\lambda C^{\varphi\Box}+\frac{\lambda}{2}C^{\varphi D}\right)v^4\,, \end{aligned} \tag{3.2}$$

where λ is the Higgs quartic coupling, $\bar{g}', \bar{g}$ are, respectively, the $U(1)_Y$ and $SU(2)_L$ gauge couplings (redefined to obtain canonical form of the gauge kinetic terms, see ref. [13]) and the C-coefficients correspond to operators defined in Table 1. Moreover, the fine structure constant is identified through the Thomson limit ($Q^2=0$) as $\alpha_{\mathrm{EM}}=\bar{e}^2/4\pi$ where $\bar{e}$ is given at tree level by

$$\bar{e}=\frac{\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left(1-\frac{\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}\,C^{\varphi WB}\,v^2\right). \tag{3.3}$$

Similarly, the experimental values for lepton and quark masses, taken as pole masses from ref. [38], are equal to eqs. (3.27) and (3.29) of ref. [13].

The Fermi coupling constant G_F, is identified through the muon decay process. In addition to the W-boson exchange which is modified in SMEFT by the PMNS matrix that is (now) *a non-unitary* matrix containing the operator $Q_{\varphi l}^{(3)}$, G_F is also affected by dipole operators e.g., Q_{eW} or by new diagrams with Z- or Higgs-boson exchange. However, the expression for G_F is simplified by making the approximation of zero neutrino masses and also by *assuming* that

$$C_1\, v^2 \gg C_2\, v\, m_l\,, \tag{3.4}$$

for any generic C_1 and C_2 coefficients entering the muon-decay amplitude and m_l being a charged lepton mass. Only then we identify the Fermi coupling constant of eq. (3.1), within *tree level* in SMEFT, as

$$\frac{G_F}{\sqrt{2}} = \frac{\bar{G}_F}{\sqrt{2}}\Big[1 + v^2(C_{11}^{\varphi l(3)} + C_{22}^{\varphi l(3)}) - v^2 C_{1221}^{ll}\Big]\,, \qquad \text{with} \quad \frac{\bar{G}_F}{\sqrt{2}} \equiv \frac{\bar{g}^2}{8M_W^2} = \frac{1}{2v^2}\,. \tag{3.5}$$

All Wilson coefficients entering in eq. (3.5) are real since they are diagonal elements of Hermitian matrices. In fact, and as a side test of the approximations assumed in eq. (3.4), we have checked that, at tree level in SMEFT, the full S-matrix element for the process $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ is gauge invariant independently of lepton-number conservation. The formula (3.5) agrees with the corresponding one from refs. [24, 28].

3.2 Renormalisation framework

We ultimately want to bring the expression for the amplitude $\mathcal{A}(h \to \gamma\gamma)$, into a form that contains only renormalised parameters that are most closely related to observable quantities, the relevant ones given in eq. (3.1). At tree level in SMEFT, the $h\gamma\gamma$-vertex appears only in association with the unrenormalised (bare) Wilson coefficients, $C_0^{\varphi B}, C_0^{\varphi W}$ and $C_0^{\varphi WB}$ and these are multiplied by the bare vev parameter[5] v_0 (in what follows bare parameters are always denoted with a subscript zero). In order to set the stage, let us for example consider from Table 1 the $d = 6$, CP-invariant operator of the form $X^2\varphi^2$,

$$C_0^{\varphi B}\, \varphi^\dagger\varphi\, B_{\mu\nu}\, B^{\mu\nu}\,, \tag{3.6}$$

where φ is the scalar Higgs doublet and $B_{\mu\nu}$ is the $U(1)_Y$-hypercharge gauge field strength tensor. All fields and coupling constants are unrenormalised quantities in this expression. In what follows, and in order to keep the expressions as simple as possible, we keep working with unrenormalised fields *i.e.,* no usual field redefinition is performed. This is justified, because we are interested in calculating only an S-matrix amplitude rather than a Green function.[6]

After Spontaneous Symmetry Breaking (SSB) in SMEFT (see ref. [13] for details), the expression in eq. (3.6) contains the following term describing the interaction of the Higgs field and two "photons",

$$C_0^{\varphi B}\, v_0\, h\, B_{\mu\nu}\, B^{\mu\nu}\,, \tag{3.7}$$

where h is the Higgs field. We split these bare quantities into renormalised parameters $v, C^{\varphi B}$ and counterterms, $\delta v, \delta C^{\varphi B}$ respectively, as

$$v_0 = v - \delta v\,, \qquad C_0^{\varphi B} = C^{\varphi B} - \delta C^{\varphi B}\,. \tag{3.8}$$

We follow the steps of a simple on-shell renormalisation scheme, first described in SM by Sirlin [41], and introduce new unrenormalised fields A_μ and Z_μ through the linear combinations

$$B_\mu = cA_\mu - sZ_\mu\,, \tag{3.9}$$
$$W_\mu^3 = sA_\mu + cZ_\mu\,, \tag{3.10}$$

[5]In fact this is $\bar{v}_0$ but to order $1/\Lambda^2$ it is replaced with the "unbarred" parameter, v_0.

[6]This is more important than, as it sounds, just a calculational scheme. Certain operators vanish when using equations of motion. Green functions are affected by these operators whereas their S-matrix elements vanish [39, 40].

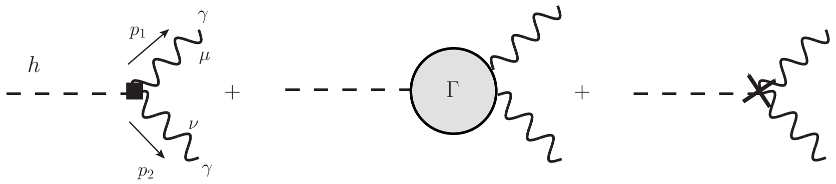

Figure 1: *The sum of three types of diagrams: (left) the SMEFT "tree" contribution with momenta and space-time indices indicated, (center) the 1PI vertex corrections Γ from all operators, SM or not, and (right) the vertex counterterms containing δC and δv. These corrections should be self-explained in eq.* (3.13).

with $c \equiv \cos\theta_W$ and $s \equiv \sin\theta_W$ defined as a ratio of the physical masses of W and Z bosons, like

$$c^2 \equiv \cos^2\theta_W = \frac{M_W^2}{M_Z^2}\,. \tag{3.11}$$

Therefore, the Lagrangian term for the considered operator, $Q_{\varphi B}$, describing (part of) the $h\gamma\gamma$ interaction, reads,

$$c^2\, v\, C^{\varphi B}\left[1 - \frac{\delta C^{\varphi B}}{C^{\varphi B}} - \frac{\delta v}{v}\right] h\, F_{\mu\nu} F^{\mu\nu}\,. \tag{3.12}$$

Note that the vev counterterm arises from pure SM contributions because it multiplies $C^{\varphi B}$, while $\delta C^{\varphi B}$ cancels infinities that arise only from pure SMEFT diagrams *i.e.,* in general, diagrams proportional to other C-coefficients, not necessarily only $C^{\varphi B}$.

Besides operator $Q_{\varphi B}$, counterterms for operators $Q_{\varphi W}$ and $Q_{\varphi WB}$ need to be added, too. Because all these three operators are proportional to the Higgs bilinear combination, $\varphi^\dagger\varphi$, they all contain the vev counterterm as a universal contribution to $h \to \gamma\gamma$ amplitude. The contributions discussed so far are depicted and explained in Fig. 1. By making use of the Feynman rules of ref. [13], their sum is written in momentum space, as

$$\begin{aligned}
4i\,[\,p_1^\nu\, p_2^\mu - (p_1\cdot p_2)\, g_{\mu\nu}\,]\Bigg\{ & c^2\, v\, C^{\varphi B}\left[1 + \Gamma^{\varphi B} - \frac{\delta C^{\varphi B}}{C^{\varphi B}} - \frac{\delta v}{v}\right] \\
& + s^2\, v\, C^{\varphi W}\left[1 + \Gamma^{\varphi W} - \frac{\delta C^{\varphi W}}{C^{\varphi W}} - \frac{\delta v}{v}\right] \\
& - s c\, v\, C^{\varphi WB}\left[1 + \Gamma^{\varphi WB} - \frac{\delta C^{\varphi WB}}{C^{\varphi WB}} - \frac{\delta v}{v}\right] \\
& + \frac{1}{M_W}\overline{\Gamma}^{\rm SM} + \sum_{X\neq \varphi B, \varphi W, \varphi WB} v\, C^X\, \Gamma^X \Bigg\}\,.
\end{aligned} \tag{3.13}$$

One-loop, 1PI vertex contributions proportional to $C^{\varphi B}, C^{\varphi W}$ and $C^{\varphi WB}$ are denoted (up to pre-factors) with $\Gamma^{\varphi B}, \Gamma^{\varphi W}$ and $\Gamma^{\varphi WB}$ in the first three lines of the above equation. The SM contribution, $\overline{\Gamma}^{\rm SM}$, is just the SM-famous result of ref. [6] but with the SM parameters replaced by the SMEFT ones (that is why "barred" Γ), taken from refs. [13, 42]. Furthermore, there are additional one-loop corrections, Γ^X, proportional to Wilson coefficients C^X, like for instance C^W, which are collected in the last line, last term of eq. (3.13).

There are additional diagrams participating in the $h \to \gamma\gamma$ amputated amplitude. These are shown in Fig. 2. The first two classes of diagrams are the Higgs tadpole and its counterterm

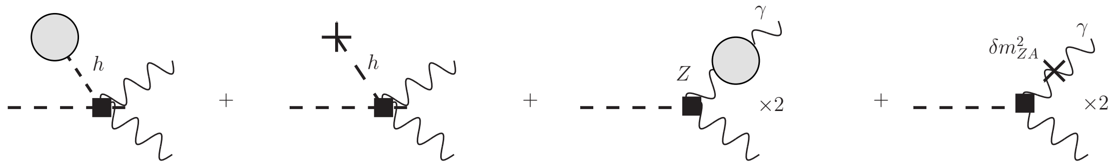

Figure 2: *Tadpole and $Z\gamma$ self-energy contributions with their associated counterterms. Crosses denote SM counterterms and the black boxes indicate pure $d = 6$ operator insertions.*

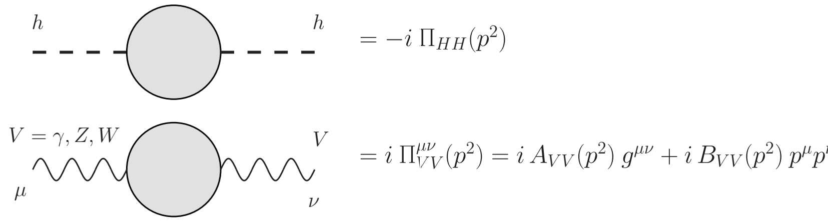

Figure 3: *Definitions for Higgs and vector boson ($V = \gamma, Z, W$), 1PI self-energies.*

contributions. These two diagrams do not enter in our renormalised amplitude because, following the renormalisation scheme of ref. [43], the Higgs tadpole counterterm is adjusted to cancel the 1PI Higgs tadpole diagrams. This guarantees that the vev is unchanged to one-loop order. The last two diagrams in Fig. 2 represent the $Z\gamma$-self energy at $p^2 = 0$, $A_{Z\gamma}(0)$, plus its counterterm, $\delta m^2_{Z\gamma}$. The expression for the counterterm $\delta m^2_{Z\gamma}$ (given below) is gauge invariant independently of the renormalisation condition for the Higgs tadpole. This is practically very useful for proving the gauge invariance of the $h \to \gamma\gamma$ amplitude.

Finally, as usual, by multiplying the amputated graph with the LSZ-factors [44] (see for instance section 7.2 of textbook [45]) for the external Higgs and photon fields,

$$\sqrt{Z_{hh}}\,Z_{\gamma\gamma} = 1 + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0)\,, \tag{3.14}$$

we arrive at the following S-matrix amplitude:

$$\begin{aligned}
i\mathcal{A}^{\mu\nu}(h \to \gamma\gamma) &= \langle \gamma(\epsilon^\mu, p_1),\, \gamma(\epsilon^\nu, p_2)\,|\,S\,|\,h(q)\,\rangle = 4i\,[\,p_1^\nu\,p_2^\mu - (p_1\cdot p_2)\,g^{\mu\nu}\,]\times \\
&\Bigg\{ c^2\,v\,C^{\varphi B}\left[1 + \Gamma^{\varphi B} - \frac{\delta C^{\varphi B}}{C^{\varphi B}} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) + 2\tan\theta_W \frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2}\right] \\
&+ s^2\,v\,C^{\varphi W}\left[1 + \Gamma^{\varphi W} - \frac{\delta C^{\varphi W}}{C^{\varphi W}} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan\theta_W}\frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2}\right] \\
&- sc\,v\,C^{\varphi WB}\left[1 + \Gamma^{\varphi WB} - \frac{\delta C^{\varphi WB}}{C^{\varphi WB}} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan 2\theta_W}\frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2}\right] \\
&+ \frac{1}{M_W}\overline{\Gamma}^{\rm SM} + \sum_{X\neq\varphi B,\varphi W,\varphi WB} v\,C^X\,\Gamma^X \Bigg\}\,.
\end{aligned} \tag{3.15}$$

Eq. (3.15) is our master formula for the renormalised amplitude $\mathcal{A}^{\mu\nu}(h \to \gamma\gamma)$. The definitions for the various self-energies[7] are stated in Fig. 3 and

$$\Pi'_{HH}(M_h^2) \equiv \frac{\partial \Pi_{HH}(p^2)}{\partial p^2}\bigg|_{p^2=M_h^2}, \qquad A_{\gamma\gamma}(p^2) = -p^2\,\Pi_{\gamma\gamma}(p^2) + \mathcal{O}(\alpha_{\rm EM}^2)\,, \tag{3.16}$$

where $\Pi_{\gamma\gamma}(p^2)$ is regular at $p^2 = 0$. All self-energies in eq. (3.15) should arise purely from SM diagrams because we are including terms up to $1/\Lambda^2$ in SMEFT. As noted earlier, the SM counterterm, $\delta m_{Z\gamma}^2$, is *gauge invariant* and is given by [41]:

$$\frac{\delta m_{Z\gamma}^2}{M_Z^2} = \frac{1}{2\tan\theta_W}\,\mathrm{Re}\left[\frac{A_{ZZ}(M_Z^2)}{M_Z^2} - \frac{A_{WW}(M_W^2)}{M_W^2}\right]. \tag{3.17}$$

The quantity $\delta v/v$ is not gauge invariant. Following standard on-shell renormalisation conditions of refs. [41, 43], we write

$$\frac{\delta v}{v} = \mathrm{Re}\left[\frac{A_{WW}(M_W^2)}{2M_W^2}\right] - \frac{\delta g}{g}\,, \tag{3.18}$$

where the counterterm δg of the $SU(2)_L$ gauge coupling is *gauge invariant* and reads as

$$\frac{\delta g}{g} = \frac{\delta e}{e} - \frac{1}{\tan\theta_W}\,\frac{\delta m_{Z\gamma}^2}{M_Z^2}\,. \tag{3.19}$$

Here δe is the electromagnetic charge renormalisation counterterm which is also *gauge invariant.* This is given by eq. (26) of ref. [41]

$$\frac{\delta e}{e} = -\frac{1}{2}\Pi_{\gamma\gamma}^{\rm lept}(0) - \frac{1}{2}\Pi_{\gamma\gamma}^{\rm had}(0) + \frac{7e^2}{32\pi^2}\left[\left(\frac{2}{\epsilon} - \gamma + \log 4\pi\right) - \log\frac{M_W^2}{\mu^2} + \frac{2}{21}\right], \tag{3.20}$$

where μ is the renormalisation scale parameter and $\epsilon \equiv 4-d$. Leptonic and hadronic contributions, $\Pi_{\gamma\gamma}^{\rm lept}(0)$ and $\Pi_{\gamma\gamma}^{\rm had}(0)$, to the photon vacuum polarisation are gauge invariant and the infinite part in the squared brackets should be gauge invariant too. The hadronic contribution from light quarks, $\Pi_{\gamma\gamma}^{\rm had}(0)$, is in principle non-calculable due to strong interaction at zero momenta. A dispersive or other non-perturbative methods should be in order. There is no such problem of course with $\Pi_{\gamma\gamma}^{\rm lept}(0)$.

SM vector boson self-energy contributions can be found in ref. [46]. The Higgs self-energy contribution can be found in refs. [30, 43]. These results have been obtained in the particular case of the 't Hooft-Feynman gauge where $\xi = 1$. Thanks to the set of SMEFT Feynman Rules in general R_ξ-gauges [13], we present in Appendix A all contributions needed in eq. (3.15) with the explicit ξ-dependence. This is necessary for checking the gauge invariance of the amplitude. Finally, the counterterms $\delta C^{\varphi B}, \delta C^{\varphi W}$ and $\delta C^{\varphi WB}$ can be read from refs. [27, 29, 30, 42, 47, 48] where they have been calculated again in 't Hooft-Feynman ($\xi = 1$) gauge. However, in $\overline{\rm MS}$ renormalisation scheme and at one-loop, cancellation of infinities should be independent on the gauge choice as we confirm below.

3.3 ξ-independence

Knowing the gauge invariant and non-invariant parts of various contributions, as described above, is particularly useful for proving the ξ-independence of the amplitude. We first prove gauge invariance

[7] We follow closely the notation of ref. [41].

by means of ξ-independence for the infinite parts proportional to ξ_W or ξ_Z. We find that the combination of $\delta v/v$ and $\Pi'_{HH}(M_h^2)$ in eq. (3.15) is ξ-independent. For the $C^{\varphi B}$ contribution in eq. (3.15), the ξ_W-dependent terms inside $\Pi_{\gamma\gamma}(0)$ and $A_{Z\gamma}(0)$ cancel among each other, as they should since the infinite part of $\Gamma^{\varphi B}$ is ξ-independent by itself. For contributions proportional to $C^{\varphi W}$ ($C^{\varphi WB}$), the ξ_W cancellations take place throughout the self-energy contributions and $\Gamma^{\varphi W}$ ($\Gamma^{\varphi WB}$). Furthermore, diagrams proportional to C^X with $X \neq \varphi B, \varphi W, \varphi WB$, contributing to the last term of eq. (3.15), are gauge invariant on their own. Of course $\overline{\Gamma}^{\rm SM}$ is finite and gauge invariant as it is known from a direct calculation in R_ξ-gauges with dimensional regularisation [23].[8]

We then prove analytically the cancellation of all ξ-dependent finite parts. This was done by first performing a maximal reduction on the related Passarino-Veltman functions [49] and then analytically checking for ξ-dependence among the parametric integrals. This is a highly non-trivial check of the validity of our calculation because the gauge parameter ξ appears everywhere in *both* the SM and SMEFT contributions which are directly related to the $h \to \gamma\gamma$ amplitude. Moreover, this should be also considered as a direct proof for the validity of the expressions for vertices given in ref. [13] in general R_ξ-gauges. Most importantly, the ξ-cancellation shows that the amplitude $\mathcal{A}^{\mu\nu}(h \to \gamma\gamma)$ given in eq. (3.15) is *gauge invariant* as it should be. Needless to say, this is a very encouraging indication towards the correctness of our final result.

As an additional non-trivial check of our calculation, we have also proved gauge invariance for our amplitude before adopting any renormalisation scheme. We confirm that the regularised but yet unrenormalised S-matrix amplitude for $h \to \gamma\gamma$, written in terms of bare parameters, is *gauge invariant.*

3.4 $\overline{\rm MS}$ scheme for Wilson coefficients

All renormalised coefficients, say C, and the counterterms, δC, in eq. (3.15), can be readily written in terms of the $\overline{\rm MS}$-scheme running C-coefficients as

$$C - \delta C = \bar{C}(\mu) - \delta\bar{C}\,, \tag{3.21}$$

where μ is the renormalisation (or subtraction) scale that lays somewhere between the EW scale and the scale Λ, while $\delta\bar{C}$ is a counterterm that subtracts only terms proportional to

$$E \equiv \frac{2}{\epsilon} - \gamma + \log 4\pi\,, \quad \text{with} \quad \epsilon \equiv 4 - d\,, \tag{3.22}$$

in the loop corrections for the Wilson C-coefficients. In $\overline{\rm MS}$ scheme and at one-loop, these counterterms are independent of the choice of the gauge fixing and can be read directly from refs. [42,47,48] to be

$$\begin{aligned}\delta\bar{C}^{\varphi B} &= \frac{E}{16\pi^2}\Bigg\{\left(-3\lambda - Y + \frac{9}{4}\bar{g}^2 - \frac{85}{12}\bar{g}'^2\right)C^{\varphi B} - \frac{3}{2}\bar{g}\bar{g}'C^{\varphi WB}\\ &\quad - \left[\frac{3}{2}\bar{g}'\,\mathrm{Tr}(C'^{eB}\Gamma_e^\dagger) - \frac{5}{6}\bar{g}'N_c\,\mathrm{Tr}(C'^{uB}\Gamma_u^\dagger) + \frac{1}{6}\bar{g}'N_c\,\mathrm{Tr}(C'^{dB}\Gamma_d^\dagger) + \mathrm{H.c.}\right]\Bigg\}\,,\end{aligned} \tag{3.23}$$

$$\begin{aligned}\delta\bar{C}^{\varphi W} &= \frac{E}{16\pi^2}\Bigg\{\left(-3\lambda - Y + \frac{53}{12}\bar{g}^2 + \frac{3}{4}\bar{g}'^2\right)C^{\varphi W} - \frac{1}{2}\bar{g}\bar{g}'C^{\varphi WB} + \frac{15}{2}\bar{g}^3C^W\\ &\quad + \left[\frac{1}{2}\bar{g}\,\mathrm{Tr}(C'^{eW}\Gamma_e^\dagger) + \frac{1}{2}\bar{g}\,N_c\,\mathrm{Tr}(C'^{uW}\Gamma_u^\dagger) + \frac{1}{2}\bar{g}\,N_c\,\mathrm{Tr}(C'^{dW}\Gamma_d^\dagger) + \mathrm{H.c.}\right]\Bigg\}\,,\end{aligned} \tag{3.24}$$

[8] For a strict four-dimensional calculation in *unitary* gauge, see ref. [20].

$$
\begin{aligned}
\delta\bar{C}^{\varphi WB} = \frac{E}{16\pi^2}\Bigg\{&\left(-\lambda - Y - \frac{2}{3}\bar{g}^2 - \frac{19}{6}\bar{g}'^2\right)C^{\varphi WB} - \bar{g}\bar{g}'(C^{\varphi B}+C^{\varphi W}) - \frac{3}{2}\bar{g}'\bar{g}^2 C^{W} \\
&+\Bigg[\frac{1}{2}\bar{g}\ \mathrm{Tr}(C'^{eB}\Gamma_e^\dagger) - \frac{1}{2}\bar{g}\,N_c\,\mathrm{Tr}(C'^{uB}\Gamma_u^\dagger) + \frac{1}{2}\bar{g}\,N_c\,\mathrm{Tr}(C'^{dB}\Gamma_d^\dagger) \\
&-\frac{3}{2}\bar{g}'\ \mathrm{Tr}(C'^{eW}\Gamma_e^\dagger) - \frac{5}{6}\bar{g}'N_c\,\mathrm{Tr}(C'^{uW}\Gamma_u^\dagger) - \frac{1}{6}\bar{g}'N_c\,\mathrm{Tr}(C'^{dW}\Gamma_d^\dagger) + \mathrm{H.c.}\Bigg]\Bigg\}\,,
\end{aligned}
\tag{3.25}
$$

where $\Gamma_{u,d,e}$ is our notation [11,13] for the usual Yukawa couplings in SM, and using Table 4 from ref. [13] the coefficients C'^f are rotated to the fermion mass basis (denoted now as unprimed ones)

$$
Y \equiv \frac{2}{v^2}\sum_{i=1}^{3}(m_{e_i}^2 + N_c m_{d_i}^2 + N_c m_{u_i}^2)\,, \qquad \mathrm{Tr}(C'^{eB}\Gamma_e^\dagger) = \frac{\sqrt{2}}{v}C_{ii}^{eB}m_{e_i}\,, \quad \text{etc.} \tag{3.26}
$$

$N_c = 3$ is the number of colours and m_{f_i} a mass of the SM fermion belonging to the i-th generation. All C-coefficients have been taken real. We have checked explicitly and analytically that the counterterms of eqs. (3.23), (3.24) and (3.25) render the amplitude for $h\to\gamma\gamma$ of eq. (3.15) *finite*, at one-loop and up to $1/\Lambda^2$ in EFT expansion.

3.5 The amplitude

The remaining part of $\mathcal{A}^{\mu\nu}(h\to\gamma\gamma)$ in eq. (3.15) is, at one-loop and up to $1/\Lambda^2$ terms, renormalisation scale invariant: the renormalisation group running of $\bar{C}(\mu)$ coefficients cancels the explicit μ-dependence within various contributions in the RHS of eq. (3.15). Therefore, the amplitude, to be squared in finding the $h\to\gamma\gamma$ decay width, is

$$
i\mathcal{A}^{\mu\nu}(h\to\gamma\gamma) = \langle\gamma(\epsilon^\mu,p_1),\,\gamma(\epsilon^\nu,p_2)\,|\,S\,|\,h(q)\,\rangle = 4i\,[\,p_1^\nu p_2^\mu - (p_1\cdot p_2)\,g^{\mu\nu}\,]\,\mathcal{A}_{h\to\gamma\gamma}\,, \tag{3.27}
$$

where

$$
\begin{aligned}
\mathcal{A}_{h\to\gamma\gamma} = \Bigg\{&c^2\,v\,\bar{C}^{\varphi B}(\mu)\left[1+\Gamma^{\varphi B} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) + 2\tan\theta_W\frac{A_{Z\gamma}(0)+\delta m_{Z\gamma}^2}{M_Z^2}\right] \\
&+ s^2\,v\,\bar{C}^{\varphi W}(\mu)\left[1+\Gamma^{\varphi W} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan\theta_W}\frac{A_{Z\gamma}(0)+\delta m_{Z\gamma}^2}{M_Z^2}\right] \\
&- sc\,v\,\bar{C}^{\varphi WB}(\mu)\left[1+\Gamma^{\varphi WB} - \frac{\delta v}{v} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan 2\theta_W}\frac{A_{Z\gamma}(0)+\delta m_{Z\gamma}^2}{M_Z^2}\right] \\
&+ \frac{1}{M_W}\overline{\Gamma}^{\rm SM} + \sum_{X\neq\varphi B,\varphi W,\varphi WB} v\,C^X(\mu)\,\Gamma^X\Bigg\}_{\rm finite}\,.
\end{aligned}
\tag{3.28}
$$

The subscript "finite" in the final parenthesis means that infinities proportional to E have been subtracted from all contributions in eq. (3.28) such as Γ, Π'_{HH}, Π_{VV}, A_{VV}, *etc.* The $\mathcal{A}_{h\to\gamma\gamma}$ in eq. (3.28) is finite, gauge and renormalisation scale invariant[9] as a physical amplitude must be. In eq. (3.28), $\Gamma^{\varphi B}$, $\Gamma^{\varphi W}$ and $\Gamma^{\varphi WB}$ are given in Appendix A in eqs. (A.2), (A.3) and (A.4). The quantities $\delta v/v$ and $\delta m_{Z\gamma}^2/M_Z^2$ are presented in eqs. (3.18) and (3.17), respectively. All vector boson self-energies in general R_ξ-gauges as well as the quantity $\Pi'_{HH}(M_h^2)$ are also given in Appendix A.

[9] In the sense that $\frac{d}{d\mu}\mathcal{A}_{h\to\gamma\gamma}(\mu) = 0$.

Although all $\bar{C}(\mu)$ coefficients in eq. (3.28) are $\overline{\text{MS}}$ parameters, the weak mixing angle θ_W and the vev v that appear explicitly to multiply Wilson coefficients are defined in terms of physical quantities through eqs. (3.11) and (3.5) [see also eq. (4.16) below]. This is a virtue of our hybrid renormalisation scheme: SM on-shell parameters appear together with $\overline{\text{MS}}$ SMEFT parameters (Wilson coefficients) in the renormalised amplitude. This scheme can easily be applied to every process at one-loop in SMEFT.

From now on, all Wilson coefficients should be considered as running $\overline{\text{MS}}$ quantities, $C \equiv \bar{C}(\mu)$. We remove the "bar" over the $\overline{\text{MS}}$-coefficients letting the argument to denote, or to implicitly imply, the difference.

4 Anatomy of the effective amplitude

In this section we present explicit expressions for the SM contribution, and, contributions proportional to all Wilson coefficients entering the $h \to \gamma\gamma$ amplitude in eq. (3.28), and in Table 1. These coefficients are taken to be real. For clarity, we reinstate explicitly $1/\Lambda^2$ factors in the expressions appeared in this and subsequent sections, so they are no longer incorporated into the definition of C's. Our EFT expansion stops at the order $1/\Lambda^2$ and is one-loop at the $\hbar$-expansion. In our conventions, we denote electromagnetic fermion charges and the third component of particle weak isospin as

$$Q_f = \begin{cases} 0, & \text{for} \quad f = \nu_e, \nu_\mu, \nu_\tau \\ -1, & \text{for} \quad f = e, \mu, \tau \\ 2/3, & \text{for} \quad f = u, c, t \\ -1/3, & \text{for} \quad f = d, s, b \end{cases} \quad \text{and} \quad T_f^3 = \begin{cases} 1/2, & \text{for} \quad f = \nu_e, \nu_\mu, \nu_\tau, u, c, t \\ -1/2, & \text{for} \quad f = e, \mu, \tau, d, s, b \end{cases} . \tag{4.1}$$

The colour factors are $N_{c,e} = 1$ and $N_{c,u} = N_{c,d} = 3$. It is useful to note, when reading the expressions below, that the actual dimensionless EFT expansion parameter is $\frac{1}{G_F \Lambda^2}$. To get a quantitative feeling of its numerical magnitude and to compare with standard loop expansion in the EW gauge couplings, we simply note that it is $\frac{1}{G_F M_W^2} \sim 4\pi$, while for $\Lambda = 1$ TeV one has $\frac{1}{G_F \Lambda^2} \sim \frac{1}{4\pi}$, for $\Lambda = 10$ TeV one has $\frac{1}{G_F \Lambda^2} \sim \frac{\alpha_{\text{EM}}}{4\pi}$ and, finally, for $\Lambda = 100$ TeV one has $\frac{1}{G_F \Lambda^2} \sim \frac{\alpha_{\text{EM}}^2}{\pi^2}$.

4.1 SM and $C^{\varphi WB}$, $C^{\varphi l(3)}$, C^{ll}

The famous "SM" contributions from W and fermion triangle loops are represented by the penultimate term in eq. (3.28). This is

$$\frac{\overline{\Gamma}^{\text{SM}}}{M_W} = \frac{1}{64\pi^2} \frac{\bar{g}^2 \bar{g}'^2}{(\bar{g}^2 + \bar{g}'^2)} \frac{\bar{g}}{M_W} I_{\gamma\gamma} \,, \tag{4.2}$$

with

$$I_{\gamma\gamma} \equiv I_{\gamma\gamma}(r_f, r_W) = \sum_f Q_f^2 N_{c,f} A_{1/2}(r_f) - A_1(r_W) \,, \tag{4.3}$$

and

$$A_{1/2}(r_f) = 2 r_f [1 + (1 - r_f) f(r_f)] \,, \tag{4.4}$$

$$A_1(r_W) = 2 + 3 r_W [1 + (2 - r_W) f(r_W)] \,. \tag{4.5}$$

Here Q_f and m_f are the fermion charge (in the units of proton charge), and mass, respectively, $N_{c,f}$ is the colour factor for fermions (3 for quarks, 1 for leptons) and

$$r_f \equiv \frac{4m_f^2}{M_h^2}\,, \qquad r_W \equiv \frac{4M_W^2}{M_h^2}\,. \tag{4.6}$$

The result is of course finite and is governed by a single function $f(r)$, which reads

$$f(r) = \begin{cases} \arcsin^2\left(\frac{1}{\sqrt{r}}\right), & r \geq 1\,, \\ -\frac{1}{4}\left[\log\left(\frac{1+\sqrt{1-r}}{1-\sqrt{1-r}}\right) - i\pi\right]^2, & r \leq 1\,. \end{cases} \tag{4.7}$$

It is useful for order of magnitude calculations to state that $A_1(r_W) \approx 8.33$, $A_{1/2}(r_t) \approx 1.38$ and $I_{\gamma\gamma} \approx -6.56$ with a negligible imaginary part.

The expression given in eq. (4.2) is *not* exactly the SM contribution for it is written in terms of SMEFT parameters and not in terms of measurable quantities like those listed in eq. (3.1). We therefore rewrite eq. (4.2) in terms of physical quantities using the expression for $\bar{e}$ from eq. (3.3) and G_F from eq. (3.5) that bring in the new coefficients $C^{\varphi WB}$ and $C_{11}^{\varphi l(3)}, C_{22}^{\varphi l(3)}, C_{1221}^{ll}$, respectively,

$$\frac{\overline{\Gamma}^{\rm SM}}{M_W} = \frac{\alpha_{EM}}{16\pi}\left(\frac{8G_F}{\sqrt{2}}\right)^{1/2} I_{\gamma\gamma}\left[1 + 2sc\,\frac{v^2}{\Lambda^2}C^{\varphi WB} - \frac{v^2}{2\Lambda^2}(C_{11}^{\varphi l(3)} + C_{22}^{\varphi l(3)}) + \frac{v^2}{2\Lambda^2}C_{1221}^{ll}\right]. \tag{4.8}$$

Note that the piece before the square brackets on the RHS is the SM contribution to amplitude [up to a Lorentz factor in eq. (3.27)], as it would be calculated in the absence of any higher order operators. Inside the square brackets there are contributions from SMEFT *i.e.,* running Wilson coefficients evaluated at a scale μ. Hence, the precise determination of the $\mathcal{R}_{h\to\gamma\gamma}$ in eq. (1.1) is

$$\mathcal{R}_{h\to\gamma\gamma} = \frac{\Gamma(\text{SMEFT}, h\to\gamma\gamma)}{\Gamma(\text{SM}, h\to\gamma\gamma)} \equiv 1 + \delta\mathcal{R}_{h\to\gamma\gamma}\,, \tag{4.9}$$

where the SM decay width reads, in accordance with standard refs. [23, 50, 51], as

$$\Gamma(\text{SM}, h\to\gamma\gamma) = \frac{G_F\,\alpha_{\rm EM}^2\,M_h^3}{128\sqrt{2}\pi^3}\,|I_{\gamma\gamma}|^2\,, \tag{4.10}$$

with $I_{\gamma\gamma}$ given in eq. (4.3). The SMEFT contributions of eq. (4.8) are encoded in a part of $\delta\mathcal{R}_{h\to\gamma\gamma}$ of eq. (4.9), in terms of measurable quantities s, c and G_F, as

$$\delta\mathcal{R}^{(1)}_{h\to\gamma\gamma} \simeq \frac{4sc}{\sqrt{2}}\frac{1}{G_F\Lambda^2}C^{\varphi WB} - \frac{1}{\sqrt{2}}\frac{1}{G_F\Lambda^2}(C_{11}^{\varphi\ell(3)} + C_{22}^{\varphi\ell(3)}) + \frac{1}{\sqrt{2}}\frac{1}{G_F\Lambda^2}C_{1221}^{\ell\ell}\,, \tag{4.11}$$

where $c^2 = 1 - s^2 = M_W^2/M_Z^2$. Following our EFT expansion assumption, in obtaining eq. (4.11), corrections of $\mathcal{O}(1/\Lambda^4)$ have been consistently ignored.

4.2 $C^{\varphi D}$, $C^{\varphi\Box}$, C^{φ}

A direct calculation shows that the contribution from operators $C^{\varphi\Box}$ and $C^{\varphi D}$ is simply

$$\left(1 + \frac{v^2}{\Lambda^2}C^{\varphi\Box} - \frac{v^2}{4\Lambda^2}C^{\varphi D}\right)(i\mathcal{A}^{\rm SM}) \equiv Z_h^{-1}(i\mathcal{A}^{\rm SM})\,, \tag{4.12}$$

where Z_h is the field redefinition factor for making the kinetic term of the Higgs field canonical in going from SM to SMEFT (see eq.(3.5) of ref. [13]) and $i\mathcal{A}^{\rm SM}$ is the full SM contribution to $h \to \gamma\gamma$ amplitude. There is an explanation for this result based on the quantization of SMEFT presented in ref. [13]. In unitary gauge these operators appear in Higgs boson vertices (hWW and hff) with exactly the same Lorentz structure as in the corresponding SM vertices. On the other hand, in "renormalisable" gauges these operators appear in a complicated way e.g., there are contributions from Goldstone bosons hG^0G^0 that have a non-trivial, non-SM Lorentz structure [13] and eq. (4.12) is not easily seen without performing the actual calculation. However, the result should be independent on the gauge choice as we explicitly confirm. We can view eq. (4.12) in a different way starting from the SM amplitude and perform the redefinition $H = Z_h^{-1} h$ on the single external Higgs boson leg.

As we already mentioned in section 2, the coefficient C^{φ} does not contribute explicitly to the $h \to \gamma\gamma$ amplitude in unitary gauge. Although there are apparent non-trivial contributions from it to vertices in R_ξ-gauges, once again, gauge invariance implies that the amplitude is explicitly independent of C^{φ}. Again, we explicitly verify this situation as well.

In summary, the contribution of operators discussed in this subsection to the ratio (4.9) reads trivially, up to $\sim 1/\Lambda^2$ terms, as

$$\delta\mathcal{R}^{(2)}_{h\to\gamma\gamma} \simeq \sqrt{2}\frac{1}{G_F\Lambda^2}C^{\varphi\Box} - \frac{\sqrt{2}}{4}\frac{1}{G_F\Lambda^2}C^{\varphi D}\,. \tag{4.13}$$

4.3 $C^{e\varphi}$, $C^{u\varphi}$, $C^{d\varphi}$

The relevant diagrams for these operators contain a fermion circulating in the loop. They contribute a ξ-independent piece in the last term of eq. (3.28) which takes the form

$$\Gamma_i^{f\varphi} = -\frac{1}{4\pi^2}\frac{\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}N_{c,f}Q_f^2\frac{v m_{f_i}}{\sqrt{2}M_h^2}[1+(1-r_{f_i})f(r_{f_i})]\,. \tag{4.14}$$

The contribution runs over all charged fermions $f = e, u, d$ with their generation flavours denoted as $i = 1, 2, 3$, *i.e.*, $u_1 = u, u_2 = c, u_3 = t$ etc. The electromagnetic charges Q_f and colour factors $N_{c,f}$, are given in and below eq. (4.1). The function $f(r)$ is defined in eq. (4.7). Turning all parameters into measurable ones in eq. (4.14) we obtain for the $\mathcal{R}_{h\to\gamma\gamma}$ ratio of eq. (4.9)

$$\delta\mathcal{R}^{(3)}_{h\to\gamma\gamma} \simeq -\frac{2^{3/4}}{(G_F M_h^2)^{1/2}}\sum_{f=e,u,d}N_{c,f}Q_f^2\sum_{i=1}^{3}\mathrm{Re}\left[\frac{A_{1/2}(r_{f_i})}{I_{\gamma\gamma}\,r_{f_i}^{1/2}}\right]\frac{1}{G_F\Lambda^2}C^{f\varphi}_{ii}\,, \tag{4.15}$$

with $A_{1/2}(r)$ being a function defined in eq. (4.4) and $I_{\gamma\gamma}$ defined in eq. (4.3). The function inside the square parenthesis peaks at the charm mass and as we shall see below [*cf.* eq. (5.1)] this is the most important contribution in $\delta\mathcal{R}^{(3)}_{h\to\gamma\gamma}$.

All operators we have examined thus far are of PTG type. These operators create *only* finite contributions in the $h \to \gamma\gamma$ amplitude. On contrary, operators that will be examined next will need to be renormalised.

4.4 $C^{\varphi B}$, $C^{\varphi W}$, $C^{\varphi WB}$

The amplitude in eq. (3.28) contains contributions from $Q_{\varphi B}, Q_{\varphi W}$, $Q_{\varphi WB}$ operators[10] appearing already at tree level in SMEFT. These are collected in the first three lines of eq. (3.28), but still

[10] There is an additional contribution from the operator $Q_{\varphi WB}$, arising from eq. (4.8), which must be added in the final amplitude, *cf.* eq. (5.1).

contain the renormalised vev v. This parameter needs to be turned into Fermi coupling constant, G_F, that is a measurable quantity with experimental value given in eq. (3.1). We only need the SM one loop corrections to Δr, which appear through the expression

$$\frac{\bar{G}_F}{\sqrt{2}} = \frac{1}{2v^2}\frac{1}{(1-\Delta r)}\,. \tag{4.16}$$

Note that Δr is a gauge invariant quantity and its form can be found in ref. [41]. This is consistent with our remark in section 3 that the pre-factors of $C^{\varphi B}, C^{\varphi W}$, $C^{\varphi WB}$ in eq. (3.28) are respectively gauge invariant quantities and therefore the whole amplitude is gauge invariant. We then use eq. (3.5) to order $1/\Lambda^2$ *i.e.,* set $\bar{G}_F \to G_F$ in eq. (4.16) and apply the result in eq. (3.28). We find that Δr nicely cancels out when using an alternative expression for $\delta v/v$ derived in ref. [43] in Feynman gauge $\xi = 1$,

$$\frac{\delta v}{v} = \frac{1}{2}\left[\frac{A_{WW}(0)}{M_W^2} + \Delta r - \widetilde{E}\right]_{\xi=1}\,, \tag{4.17}$$

where the parameter $\widetilde{E}$ is given in ref. [43]

$$\widetilde{E}_{\xi=1} = \frac{\alpha_{\rm EM}}{2\pi s^2}\left[2E - 2\log\frac{M_Z^2}{\mu^2} + \frac{\log c^2}{s^2}\left(\frac{7}{4} - 3s^2\right) + 3\right]. \tag{4.18}$$

The quantity $A_{WW}(0)$ is presented in ref. [46] in 't Hooft-Feynman gauge and is recalculated here for completeness in eq. (A.13). By putting eqs. (4.16) and (4.17) in eq. (3.28) we obtain the relevant finite contributions from operators $Q_{\varphi B}, Q_{\varphi W}, Q_{\varphi WB}$, to the physical amplitude $\mathcal{A}_{h\to\gamma\gamma}$

$$\begin{aligned}
&\frac{c^2\, C^{\varphi B}(\mu)}{(\sqrt{2}G_F)^{1/2}\Lambda^2}\left[1 + \Gamma^{\varphi B} - \frac{A_{WW}(0)}{2M_W^2} + \frac{\widetilde{E}}{2} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) + 2\tan\theta_W \frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2}\right]_{\rm finite} \\
&+ \frac{s^2\, C^{\varphi W}(\mu)}{(\sqrt{2}G_F)^{1/2}\Lambda^2}\left[1 + \Gamma^{\varphi W} - \frac{A_{WW}(0)}{2M_W^2} + \frac{\widetilde{E}}{2} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan\theta_W} \frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2}\right]_{\rm finite} \\
&- \frac{sc\, C^{\varphi WB}(\mu)}{(\sqrt{2}G_F)^{1/2}\Lambda^2}\left[1 + \Gamma^{\varphi WB} - \frac{A_{WW}(0)}{2M_W^2} + \frac{\widetilde{E}}{2} + \frac{1}{2}\Pi'_{HH}(M_h^2) - \Pi_{\gamma\gamma}(0) - \frac{2}{\tan 2\theta_W} \frac{A_{Z\gamma}(0) + \delta m^2_{Z\gamma}}{M_Z^2}\right]_{\rm finite}.
\end{aligned} \tag{4.19}$$

This expression takes this particular form *only* in $\xi = 1$ gauge and replaces the first three lines in eq. (3.28). It is important for the reader to notice, that numerically big corrections from Δr have been cancelled out in eq. (4.19). The quantities $\Gamma^{\varphi V}, V = B, W, WB$ are fairly lengthy and are given in the Appendix A together with the self-energies, all in general R_ξ-gauges.

As we already mentioned in the discussion below eq. (3.20), the photon self-energy, $\Pi_{\gamma\gamma}(0)$, contains hadronic contributions from five light quarks *i.e.,* all quarks but the top quark. Therefore, for the related part, $\Pi^{\rm had}_{\gamma\gamma}(0)$, the perturbative formula (A.5) is not reliable. We use instead,

$$\Pi^{\rm had}_{\gamma\gamma}(0) = -\Delta\alpha^{(5)}_{\rm had}(M_Z^2) + \Pi^{\rm had}_{\gamma\gamma}(M_Z^2)\,, \tag{4.20}$$

where now thanks to asymptotic freedom, $\Pi^{\rm had}_{\gamma\gamma}(M_Z^2)$ is a reliable perturbative one-loop calculation for the light quark contributions (see (A.15)) while $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = \Pi^{\rm had}_{\gamma\gamma}(M_Z^2) - \Pi^{\rm had}_{\gamma\gamma}(0)$ is finite and is computed via a dispersion relation that involves experimental data for the ratio $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. A recent analysis [38] gives $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02764 \pm 0.00013$.

The form for $\delta\mathcal{R}^{(4)}_{h\to\gamma\gamma}$ is given semi-analytically below [*cf.* eq. (5.1)]. Since these corrections appear at tree level in SMEFT they are generically the biggest ones from all operators involved in $h \to \gamma\gamma$ amplitude.

4.5 C^W

The contribution from W-loops gives rise to terms proportional to C^W in eq. (3.28). The relevant expression is ξ-independent, and is written as

$$\Gamma^W = \frac{3}{16\pi^2} \frac{\bar{g}^3 \bar{g}'^2}{(\bar{g}^2 + \bar{g}'^2)} [3E + B] \,, \tag{4.21}$$

where E is the infinite piece [see eq. (3.22)] formed as usual in dimensional regularisation, of course removed from eq. (3.28). The integral function B is

$$B \equiv B(r_W) = 2 - r_W f(r_W) + 2J_2(r_W) - 3\log\frac{M_W^2}{\mu^2} \,, \tag{4.22}$$

where the functions $f(r), J_2(r)$ are given in eqs. (4.7) and (A.11), respectively, and μ is the renormalisation scale. The contribution from the operator Q_W in the ratio (4.9) is

$$\delta\mathcal{R}^{(5)}_{h\to\gamma\gamma} \simeq \frac{24\pi\alpha_{\rm EM}}{\sqrt{2}s^2} \left(\frac{1}{\sqrt{2}G_F M_W^2}\right)^{1/2} \mathrm{Re}\left[\frac{B(r_W)}{I_{\gamma\gamma}}\right] \frac{1}{G_F\Lambda^2} C^W \,, \tag{4.23}$$

with $I_{\gamma\gamma}$ defined in eq. (4.3).

4.6 C^{eB}, C^{eW}, C^{uB}, C^{uW}, C^{dB}, C^{dW}

These are again contributions from operators affecting fermion loops and, as such, they are ξ-independent. They are, however, infinite since they involve dipole operators (as one can easily see from ref. [13] there is an extra momentum in the numerator of their corresponding Feynman rules expressions). We obtain the following contribution in the last term of eq. (3.28):

$$\begin{aligned}
\Gamma_i^{fB} &= \frac{1}{4\pi^2} \frac{\bar{g}^2\bar{g}'}{\bar{g}^2 + \bar{g}'^2} N_{c,f} Q_f \frac{m_{f_i}}{\sqrt{2}v} [2E + D(r_{f_i})] \,, \\
\Gamma_i^{fW} &= 2T_f^3 \frac{\bar{g}'}{\bar{g}} \Gamma_i^{fB} \,,
\end{aligned} \tag{4.24}$$

where the function $D(r_{f_i})$ is defined as

$$D(r_{f_i}) \equiv -2\log\frac{m_{f_i}^2}{\mu^2} + 1 - r_{f_i} f(r_{f_i}) + J_2(r_{f_i}) \,. \tag{4.25}$$

Here again f stands for a fermion type, $f = e, u, d$, and $i = 1, 2, 3$ runs over its flavour eigenstates. The relevant contribution from the operators Q_{fB} and Q_{fW} to the ratio $\mathcal{R}_{h\to\gamma\gamma}$ of eq. (4.9) is

$$\delta\mathcal{R}^{(6)}_{h\to\gamma\gamma} \simeq \frac{2M_h}{M_W \tan\theta_W} \sum_{f=e,u,d} N_{c,f} Q_f \sum_{i=1}^{3} \mathrm{Re}\left[\frac{r_{f_i}^{1/2} D(r_{f_i})}{I_{\gamma\gamma}}\right] \frac{1}{G_F\Lambda^2} (C_{ii}^{fB} + 2T_f^3 \tan\theta_W C_{ii}^{fW}) \,. \tag{4.26}$$

Functions $I_{\gamma\gamma}, f(r)$ and $J_2(r)$ are defined in eqs. (4.3), (4.7) and (A.11), respectively.

The expression $\delta\mathcal{R}^{(6)}_{h\to\gamma\gamma}$ in eq. (4.26) has few interesting features. First, it has no suppression whatsoever from $\alpha_{\rm EM}$ as opposed to the behaviour of $\delta\mathcal{R}^{(5)}_{h\to\gamma\gamma}$ in eq. (4.23). Second, it is proportional to the mass of the fermion circulated in the loop and also proportional to $\mathcal{O}(1)$ loop functions ratio. Comparing $\delta\mathcal{R}^{(6)}_{h\to\gamma\gamma}$, which arises from LG operators, with, for example, $\delta\mathcal{R}^{(3)}_{h\to\gamma\gamma}$ of eq. (4.15) which arises from PTG operators and recall Table 2, we see that there is a huge enhancement of the former by a factor of $\mathcal{O}(10)$ in particular for the top-quark. Hence, for the top quark in the loop, this is the biggest correction from all one-loop contributions in SMEFT as we shall see shortly in section 5.

5 Results

5.1 Semi-numerical expression for the ratio $\mathcal{R}_{h\to\gamma\gamma}$

In this section, we sum all contributions to $\mathcal{R}_{h\to\gamma\gamma}$ found in section 4, leaving as unknowns, the renormalisation group running Wilson coefficients, $C = C(\mu)$, the renormalisation scale μ divided by the W-boson mass and the energy scale Λ. Everything we have discussed so far is within the perturbative renormalisation framework explained in section 3. For EFT expansion to be valid, this means that the maximum value of a generic coefficient, C/Λ^2, is at most $\mathcal{O}(1)$. Experimentally, it is suggested from eq. (1.2) that the corrections to $\delta\mathcal{R}_{h\to\gamma\gamma}$ should be at most 15%. Being conservative, and in order to display all "important" contributions from operators in $\delta\mathcal{R}_{h\to\gamma\gamma}$, we present below semi-numerical results for $\delta\mathcal{R}_{h\to\gamma\gamma}$ that are up to $1\% \times C/\Lambda^2$.

With the energy scale Λ *written in TeV units,* we obtain (in Warsaw basis)[11]:

$$\begin{aligned}
\delta\mathcal{R}_{h\to\gamma\gamma} = \sum_{i=1}^{6} \delta\mathcal{R}^{(i)}_{h\to\gamma\gamma} &\simeq 0.06\left(\frac{C^{\ell\ell}_{1221} - C^{\varphi\ell(3)}_{11} - C^{\varphi\ell(3)}_{22}}{\Lambda^2}\right) + 0.12\left(\frac{C^{\varphi\Box} - \frac{1}{4}C^{\varphi D}}{\Lambda^2}\right) \\
&- 0.01\left(\frac{C^{e\varphi}_{22} + 4C^{e\varphi}_{33} + 5C^{u\varphi}_{22} + 2C^{d\varphi}_{33} - 3C^{u\varphi}_{33}}{\Lambda^2}\right) \\
&- \left[48.04 - 1.07\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi B}}{\Lambda^2} - \left[14.29 - 0.12\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi W}}{\Lambda^2} \\
&+ \left[26.62 - 0.52\log\frac{\mu^2}{M_W^2}\right]\frac{C^{\varphi WB}}{\Lambda^2} \\
&+ \left[0.16 - 0.22\log\frac{\mu^2}{M_W^2}\right]\frac{C^{W}}{\Lambda^2} \\
&+ \left[2.11 - 0.84\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uB}_{33}}{\Lambda^2} + \left[1.13 - 0.45\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uW}_{33}}{\Lambda^2} \\
&- \left[0.03 + 0.01\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uB}_{22}}{\Lambda^2} - \left[0.01 + 0.00\log\frac{\mu^2}{M_W^2}\right]\frac{C^{uW}_{22}}{\Lambda^2} \\
&+ \left[0.03 + 0.01\log\frac{\mu^2}{M_W^2}\right]\frac{C^{dB}_{33}}{\Lambda^2} - \left[0.02 + 0.01\log\frac{\mu^2}{M_W^2}\right]\frac{C^{dW}_{33}}{\Lambda^2} \\
&+ \left[0.02 + 0.00\log\frac{\mu^2}{M_W^2}\right]\frac{C^{eB}_{33}}{\Lambda^2} - \left[0.01 + 0.00\log\frac{\mu^2}{M_W^2}\right]\frac{C^{eW}_{33}}{\Lambda^2} + \ldots\,,
\end{aligned} \tag{5.1}$$

where the ellipses denote contributions from the operators Q in Table 1 that are less than $1\% \times C/\Lambda^2$. Terms in the first three parentheses arise from finite loop contributions, $\delta\mathcal{R}^{(1,2,3)}_{h\to\gamma\gamma}$ in eqs. (4.11), (4.13) and (4.15), while all the rest arise from "infinite" diagrams; for these the renormalisation scale μ appears explicitly. All coefficients are running quantities, $C = C(\mu)$, and $\delta\mathcal{R}_{h\to\gamma\gamma}$ should be RGE invariant up to one-loop and up to $1/\Lambda^2$ expansion terms. This can be checked numerically already from the explicit μ-dependence in eq. (5.1) and the β-functions for the C-coefficients calculated in refs. [42,47,48].[12] Furthermore, we remark that in eq. (5.1) and for $\mu = 1$ TeV, the logarithmic parts are of the same order of magnitude as the finite, constant, parts.

[11] Unlike refs. [29, 30] we have made no rescaling of Wilson coefficients with gauge couplings. Of course, the coefficients-$C^{fB,fW}$ are the rotated coefficients in the quark or lepton mass basis adopted in ref. [13] as already noted in section 2.

[12] For this purpose, one can use the numerical codes of refs. [52, 53] or can exploit analytic techniques appeared recently in ref. [54].

Interestingly, for the coefficients in the last three lines of eq. (5.1), the two parts constructively interfere, while for the rest of coefficients they partially cancel.

At the end of the day, only five operators in eq. (5.1) can be bounded by the LHC experimental measurement (1.2) of the ratio $R_{h\to\gamma\gamma}$. Taking $\mu = M_W$, we find

$$\frac{|C^{\varphi B}|}{\Lambda^2} \lesssim \frac{0.003}{(1\text{ TeV})^2}\,, \qquad \frac{|C^{\varphi W}|}{\Lambda^2} \lesssim \frac{0.011}{(1\text{ TeV})^2}\,, \qquad \frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim \frac{0.006}{(1\text{ TeV})^2}\,,$$
$$\frac{|C^{uB}_{33}|}{\Lambda^2} \lesssim \frac{0.071}{(1\text{ TeV})^2}\,, \qquad \frac{|C^{uW}_{33}|}{\Lambda^2} \lesssim \frac{0.133}{(1\text{ TeV})^2}\,. \tag{5.2}$$

All bounded coefficients above are associated with LG operators in Table 2 in a perturbative decoupled UV-theory. Eq. (5.2) seems to be consistent with this observation and $\Lambda \approx 1$ TeV. On the other hand, assuming $|C^{\varphi V}|$ ($|C^{uB,uW}_{33}|$) $\simeq 1$ we obtain $\Lambda \gtrsim 10$ (3) TeV, outside but close to the near-future LHC region. Other operators in eq. (5.1) may contribute at most 15% only when $C = 1$ and $\Lambda = 1$ TeV so their effects are less likely to be observed at present in LHC searches for the $h \to \gamma\gamma$ process.

Operators $Q_{\varphi B}$, $Q_{\varphi W}$ and $Q_{\varphi WB}$ contribute already at tree level in SMEFT and this explains the large value of their coefficients in eq. (5.1). As our calculation shows, taking also into account one-loop corrections, modify their respective tree level contributions to the ratio $\delta R_{h\to\gamma\gamma}$ by 1.3% for $C^{\varphi B}$, by 7.5% for $C^{\varphi WB}$ and by 8.7% for $C^{\varphi W}$ at the renormalisation scale $\mu = M_W$, in agreement with the commonly expected magnitude of the SM-like electroweak one-loop corrections. What is surprising however, is the large loop contribution of dipole operators $Q^{33}_{uB,uW}$. This is basically due to the largeness of the top-quark mass and other features already noted in the discussion below eq. (4.25).

5.2 Other constraints

In the section above, we found that the dominant coefficients in $\mathcal{R}_{h\to\gamma\gamma}$ are those given in eq. (5.2). These coefficients maybe also bounded by observables other than $h \to \gamma\gamma$. It has been noted in refs. [55, 56] that the coefficient $C^{\varphi WB}$ contributes directly to the electroweak S-parameter, one of the parameters that fits Z-pole observables. Its contribution reads

$$\frac{C^{\varphi WB}}{\Lambda^2} = \frac{G_F\,\alpha_{\rm EM}}{2\sqrt{2}sc}\Delta S\,. \tag{5.3}$$

With $\Delta S \in [-0.06, 0.07]$ [35] we obtain $\frac{|C^{\varphi WB}|}{\Lambda^2} \lesssim 0.005$ TeV^{-2} which is of the same order of magnitude as the upper bound we find here in eq. (5.2) from $h \to \gamma\gamma$ measurement. The coefficients $C^{\varphi W}$ and $C^{\varphi B}$ are constrained by LHC Higgs data (giving upper limits on deviations from the SM predictions) or electroweak fits to EW observables. The respective bounds, as they read from refs. [35, 57], are also about the same order of magnitude as in eq. (5.2).

The other two operators in eq. (5.2), Q^{33}_{uB} and Q^{33}_{uW}, are constrained from the $\bar{t}tZ$ production and the latter also by the single top production measurements at LHC. Bounds quoted in ref. [58] are $|C^{uB}_{33}|/\Lambda^2 \lesssim 7.1$ TeV^{-2} and $|C^{uW}_{33}|/\Lambda^2 \lesssim 2.5$ TeV^{-2}. Here, bounds from $h \to \gamma\gamma$ derived in eq. (5.2) are *more than an order of magnitude stronger.*

Restrictions to all other coefficients appeared in eq. (5.1) can be found in various articles in the literature. For example, following ref. [35], $Q_{\varphi D}$ contributes to the T-electroweak parameter and the corresponding bound is, $|C^{\varphi D}|/\Lambda^2 \lesssim 0.03$ TeV^{-2}. This makes its contribution in $h \to \gamma\gamma$ negligible. However, the coefficients $C^{\varphi\Box}$ and C^{W} are not really constrained by fitting the LHC Higgs data. It is obvious from eq. (5.1) that these two coefficients can give $\mathcal{O}(10)$% contributions to $\mathcal{R}_{h\to\gamma\gamma}$ only when one is in the vicinity of EFT validity.

Potentially Tree Generated (PTG) Operators involved in $h \to \gamma\gamma$								
Spin	Field	$C^{\ell\ell}$	$C^{\varphi\ell(3)}$	$C^{\varphi\Box}$	$C^{\varphi D}$	$C^{u\varphi}$	$C^{d\varphi}$	$C^{e\varphi}$
Spin-0	$\mathcal{S}(1,1)_0$			✓		✓	✓	✓
	$\mathcal{S}(1,1)_1$	✓						
	$\phi(1,2)_{\frac{1}{2}}$					✓	✓	✓
	$\Xi(1,3)_0$			✓	✓	✓	✓	✓
	$\Xi_1(1,3)_1$	✓		✓	✓	✓	✓	✓
Spin-$\frac{1}{2}$	$N(1,1)_0$		✓					
	$E(1,1)_{-1}$		✓					✓
	$\Delta_1(1,2)_{-\frac{1}{2}}$							✓
	$\Delta_3(1,2)_{-\frac{3}{2}}$							✓
	$\Sigma(1,3)_0$		✓					✓
	$\Sigma_1(1,3)_{-1}$		✓					✓
	$U(3,1)_{\frac{2}{3}}$					✓		
	$D(3,1)_{-\frac{1}{3}}$						✓	
	$Q_1(3,2)_{\frac{1}{6}}$					✓	✓	
	$Q_5(3,2)_{-\frac{5}{6}}$						✓	
	$Q_7(3,2)_{\frac{7}{6}}$					✓		
	$T_1(3,3)_{-\frac{1}{3}}$					✓	✓	
	$T_2(3,3)_{\frac{2}{3}}$					✓	✓	
Spin-1	$\mathcal{B}(1,1)_0$	✓		✓	✓	✓	✓	✓
	$\mathcal{B}_1(1,1)_1$			✓	✓	✓	✓	✓
	$\mathcal{W}(1,3)_0$	✓	✓	✓	✓	✓	✓	✓
	$\mathcal{W}_1(1,3)_1$			✓	✓	✓	✓	✓
	$\mathcal{L}_1(1,2)_{\frac{1}{2}}$		✓	✓	✓	✓	✓	✓

Table 3: *Dictionary for possible UV-completions with fields that, upon their "integration out", lead to PTG operators affecting the $h \to \gamma\gamma$ amplitude in eq. (5.1). Flavour indices are suppressed. The field notation follows ref. [59].*

5.3 $h \to \gamma\gamma$ relevant UV-models

The question we want to address here is related to possible UV-field theories connected with the Wilson coefficients of eq. (5.1) contributing to the $h \to \gamma\gamma$ amplitude. A possible UV-theory, valid in and above the neighbourhood of the energy scale Λ, contains heavy (w.r.t. the EW scale) fields, which upon their integration out result in a subset of SMEFT operators. Following an analysis based on ref. [59], under the assumption that at energies much smaller than all masses M of the extra particles ($M \sim \Lambda$) the theory is well described by SMEFT, power counting rules result, interestingly, in a limited number of allowed heavy fields with definite quantum numbers and spins 0, 1/2 and 1. Other assumptions include that a candidate UV-theory must be invariant under the

Loop Generated (LG) Operators involved in $h \to \gamma\gamma$											
Spin	Field	$C^{\varphi B}$	$C^{\varphi W}$	$C^{\varphi WB}$	C^{W}	C^{uB}	C^{uW}	C^{dB}	C^{dW}	C^{eB}	C^{eW}
Spin-0	$\mathcal{S}(1,1)_0$	✓	✓								
	$\Xi(1,3)_0$			✓							
Spin-$\frac{1}{2}$	$E(1,1)_{-1}$									✓	
	$\Delta_1(1,2)_{-\frac{1}{2}}$									✓	✓
	$\Sigma_1(1,3)_{-1}$										✓
	$U(3,1)_{\frac{2}{3}}$					✓					
	$D(3,1)_{-\frac{1}{3}}$							✓			
	$Q_1(3,2)_{\frac{1}{6}}$					✓	✓	✓	✓		
	$T_1(3,3)_{-\frac{1}{3}}$								✓		
	$T_2(3,3)_{\frac{2}{3}}$						✓				
Spin-1	$\mathcal{L}_1(1,2)_{\frac{1}{2}}$	✓	✓	✓		✓	✓	✓	✓	✓	✓

Table 4: *Dictionary for possible UV-completions with fields that, upon their "integration out", lead to LG operators affecting the $h \to \gamma\gamma$ amplitude in eq. (5.1). Again, flavour indices are suppressed. The field notation follows ref. [59].*

linearly realised SM-gauge group, is non-anomalous, and it must contain a multiplet with the SM Higgs field in representation $(SU(3)_C, SU(2)_L)_{U(1)_Y} = (1,2)_{\frac{1}{2}}$.

We divide the Wilson coefficients appeared in eq. (5.1) into PTG and LG operators [36] as in Table 2. Then, following the tables in Appendix C of ref. [59], we check which coefficients can originate from integrating out fields with certain quantum numbers. Our results are shown in Tables 3 and 4. There are 5 spin-0 scalars, 13 Weyl fermions with vector-like masses, and 5 spin-1 gauge bosons, that can possibly appear in a UV-theory and affect the $h \to \gamma\gamma$ amplitude through eq. (5.1). Remarkably, the LG coefficients in Table 4 are only a small subset of the PTG ones shown in Table 3. In addition, the C^W-coefficient is absent from both Tables 3 and 4.

Tables 3 and 4, which in connection with Appendix D of ref. [59] relate the Wilson coefficient to the actual couplings of heavy fields, can be used to put bounds on the latter. We illustrate it by presenting an example. Imagine a triplet scalar, $\Xi(1,3)_0$, that is directly found or implied by an experiment with mass M in the TeV-range. According to Tables 3 and 4, at low energies there are contributions from "integrating out" Ξ in PTG coefficients $C^{\varphi\Box}, C^{\varphi D}, C^{u\varphi}, C^{d\varphi}, C^{e\varphi}$ and in a LG coefficient $C^{\varphi WB}$. From eq. (5.1) we obtain that $C^{\varphi\Box}$ and $C^{\varphi WB}$ are multiplied by the biggest pre-factors and therefore play more important role in $h \to \gamma\gamma$ amplitude. The UV-Lagrangian which originates these, is [59]

$$\mathcal{L} = \mathcal{L}_{\text{SMEFT}} - \kappa\varphi^\dagger \Xi^I \tau^I \varphi - \frac{1}{f}\tilde{\kappa}\, \Xi^I W^I_{\mu\nu} B^{\mu\nu}\,, \tag{5.4}$$

where τ^I are Pauli matrices and f is an energy scale with $\Lambda \lesssim 4\pi f$. From Appendix D of ref. [59] we identify

$$\frac{C^{\varphi\Box}}{\Lambda^2} \to \frac{\kappa^2}{2M^4}\,, \qquad \frac{C^{\varphi WB}}{\Lambda^2} \to \frac{1}{f}\frac{\kappa\tilde{\kappa}}{M^2}\,, \tag{5.5}$$

and using our eq. (5.1) we find

$$\kappa \lesssim 1.6 \frac{M^2}{1\ \text{TeV}}\,, \qquad \frac{\kappa\tilde{\kappa}}{f} \lesssim 0.06 \left(\frac{M}{1\ \text{TeV}}\right)^2 . \tag{5.6}$$

If κ takes on its maximal value then $\tilde{\kappa}/f \lesssim 0.004\ \text{TeV}^{-1}$. Of course one can advance a similar analysis in every case of an observable, not necessarily $h \to \gamma\gamma$, that is needed to be explained by a subset of fields affecting eq. (5.1).

5.4 Comparison with literature

As we mentioned in the introduction, the calculation for $h \to \gamma\gamma$ in SMEFT was first performed several years ago in refs. [29,30] and to our knowledge these are the only studies prior to ours here. Our check shows that there are two, numerically important differences. First, all corresponding $\delta\mathcal{R}_{h\to\gamma\gamma}$ in refs. [29,30] are smaller by exactly a factor of four. We think that this is due to a mistake in eq. (26) of ref. [29][arXiv v3]. Second, our eq. (4.15) is *not* in agreement with the corresponding expression of ref. [29]. We believe there is a Yukawa coupling missing for each generation and flavour in the corresponding expression of ref. [29]. Up to the aforementioned differences, we found agreement with $\delta\mathcal{R}^{(1,2,3,5,6)}_{h\to\gamma\gamma}$. As far as $\delta\mathcal{R}^{(4)}_{h\to\gamma\gamma}$ is concerned, a direct comparison of our formulae in eq. (4.19) with the corresponding one in ref. [30] is very difficult. Checking individually quantities appearing in both works, for example, $\delta v/v$ or Π'_{HH}, is meaningless since the calculations in refs. [29,30] were performed in background field gauges while ours in linear R_ξ-gauges. Comparing numerically the correction, $\delta\mathcal{R}^{(4)}_{h\to\gamma\gamma}$, appearing in our eq. (5.1) with a corresponding ratio based on refs. [29,30], we find, upon fixing the factor of four mentioned above, a maximal difference of 5% for $\mu = M_W$, originating from what multiplies the coefficient $C^{\varphi B}$.

6 Conclusions

In our analysis we have calculated the one-loop decay width of the $h \to \gamma\gamma$ process in the SM extended by all CP-conserving gauge invariant operators up to dimension-6 in Warsaw basis. We performed the calculations using the general R_ξ-gauges and a hybrid renormalisation scheme, where we assumed the on-shell conditions for the SM parameters and $\overline{\text{MS}}$ subtraction for the running Wilson coefficients of the higher order operators. We explicitly checked the gauge ξ-parameter cancellation, which provides the very strict test of correctness of our calculations. In addition, we also explicitly proven that at the one-loop and $1/\Lambda^2$ order, the calculated amplitude is independent of the renormalisation scale μ. Our work is complementary to previous analyses [29, 30] of this process using the Background Field Method and comparisons of our results with theirs were made whenever possible. Our master formula for the S-matrix amplitude is given by eqs. (3.27) and (3.28).

We give a complete set of analytical formulae for all classes of SM and SMEFT contributions to $h \to \gamma\gamma$ decay rate, normalised to the SM result as in published LHC searches [see eq. (4.9)]. We also present them in a form of simple and compact semi-analytical expressions depending only on running Wilson coefficients and renormalisation scale μ. Eq. (5.1) summarises all dominant contributions. Such formula can be readily used as additional constraint in experimental or theoretical analyses considering other observables in SMEFT.

We show that numerically largest corrections to the SM prediction can arise from $Q_{\varphi B}$, $Q_{\varphi W}$ and $Q_{\varphi WB}$ operators, contributing already at the tree level, and from Q^{33}_{uB}, Q^{33}_{uW} operators arising at the loop level. Only Wilson coefficients of these operators can be meaningfully constrained using the current precision of the LHC measurements for the $h \to \gamma\gamma$ decay width. In some cases, like C_{33}^{uB} and C_{33}^{uW}, such constraints are already stronger than those from other measurements, in this case for instance from top-quark LHC-physics. Furthermore, we consider possible UV-field theoretic model completions at an energy scale nearby and above Λ. After integration out of all possible heavy fields, tabulated in Tables 3 and 4, we list all possible SMEFT operators that originate and affect the $h \to \gamma\gamma$ amplitude in eq. (5.1).

A general look of our SMEFT calculational framework does not differ from common frameworks calculating electroweak one-loop corrections, like in the renormalisable SM for example. Our work can easily be automatised although we performed as many manual calculations we could for comparisons and cross checks. For example, one can use the SMEFT Feynman rules, given also in a *Mathematica* code, from ref. [13], and existed codes to calculate Feynman diagrams, employ a “traditional” renormalisation prescription from 80’s described also here and, checking gauge invariance at every step, present a concise form of an amplitude in a useful semi-numeric form, as in eq. (5.1). It is worth for pursuing this SMEFT framework further.

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A SMEFT amplitudes and SM self-energies in R_ξ-gauges

We append here the one-loop corrections in general renormalisable gauges for the three-point 1PI functions, $\Gamma^{\varphi B}$, $\Gamma^{\varphi W}$ and $\Gamma^{\varphi WB}$, as well as for the SM vector boson self-energies that are needed for eqs. (3.28) and (4.19). The first, ξ-independent, terms of the equations below refer always to a part in unitary gauge. The *Mathematica* package `FeynCalc` [60, 61] was used for most of our Feynman diagram calculations. To bring Feynman integrals into analytic forms we used the *Mathematica* package `Package-X` [62, 63]. In what follows, we use the mass-ratios

$$r_X \equiv \frac{4M_X^2}{M_h^2} \qquad \text{and} \qquad r_{XY} \equiv \frac{4M_X^2}{M_Y^2}\,. \tag{A.1}$$

For the SMEFT one-loop corrections we have

$$\begin{aligned}\Gamma^{\varphi B} &= \frac{-\lambda}{32\pi^2}\Bigg\{3\bigg(E+2-\frac{\pi}{\sqrt{3}}-\log\frac{M_h^2}{\mu^2}\bigg)+2\bigg(E+2-\log\frac{M_W^2}{\mu^2}-\log\xi_W+J_2(\xi_W r_W)\bigg)\\ &\qquad + E+2-\log\frac{M_Z^2}{\mu^2}-\log\xi_Z+J_2(\xi_Z r_Z)\Bigg\}\,,\end{aligned}\tag{A.2}$$

$$\begin{aligned}\Gamma^{\varphi W} &= \frac{-1}{32\pi^2}\Bigg\{3\lambda\bigg(E+2-\frac{\pi}{\sqrt{3}}-\log\frac{M_h^2}{\mu^2}\bigg)+\bar g^2[6r_W(1-r_W f(r_W))-16(1-r_W)f(r_W)]\\ &\qquad + 2\big(\lambda-\bar g^2(\xi_W+3)\big)\bigg(E-\log\frac{M_W^2}{\mu^2}-\log\xi_W\bigg)\\ &\qquad + 4\lambda-\bar g^2(\xi_W+5)+\frac{6\bar g^2}{\xi_W-1}\log\xi_W+2\lambda J_2(\xi_W r_W)\\ &\qquad + \lambda\bigg(E+2-\log\frac{M_Z^2}{\mu^2}-\log\xi_Z+J_2(\xi_Z r_Z)\bigg)\Bigg\}\,,\end{aligned}\tag{A.3}$$

$$\begin{aligned}\Gamma^{\varphi WB} &= \frac{-1}{32\pi^2}\Bigg\{-\lambda\bigg(E+2+\sqrt{3}\pi-\log\frac{M_W^2}{\mu^2}\bigg)+6\bar g^2\bigg(E-\log\frac{M_W^2}{\mu^2}\bigg)+\frac{2\bar g^2\bar g'^2\big(3\bar g^2+2\lambda\big)}{\lambda(\bar g^2+\bar g'^2)}\\ &\qquad -3\lambda\log\frac{M_h^2}{M_W^2}-\frac{2\bar g^2(3\bar g^2\bar g'^2+2\lambda\bar g^2-4\lambda\bar g'^2)}{\lambda(\bar g^2+\bar g'^2)}r_W f(r_W)+2(\bar g^2-2\lambda)J_2(r_W)\\ &\qquad -\frac{16}{M_h^2}\frac{\bar g^2\bar g'^2}{\bar g^2+\bar g'^2}\sum_f m_f^2 Q_f^2 N_{c,f}[1+(1-r_f)f(r_f)]\\ &\qquad +\lambda\bigg(E+2-\log\frac{M_Z^2}{\mu^2}-\log\xi_Z+J_2(\xi_Z r_Z)\bigg)\\ &\qquad +\big(2\lambda-\bar g^2(\xi_W+3)\big)\bigg(E-\log\frac{M_W^2}{\mu^2}-\log\xi_W\bigg)\\ &\qquad +4\lambda-\frac{\bar g^2}{2}(\xi_W+5)+\frac{3\bar g^2}{\xi_W-1}\log\xi_W+2\lambda J_2(\xi_W r_W)\Bigg\}\,.\end{aligned}\tag{A.4}$$

The SM self-energies are presented (to our knowledge for the first time) also in ref. [64], for general renormalisable gauges, and in ref. [46] for $\xi = 1$. We have recalculated them here for consistency. The results are:

$$\Pi_{\gamma\gamma}(0) = -\frac{1}{48\pi^2}\frac{\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left[21\left(E-\log\frac{M_W^2}{\mu^2}\right)+2-4\sum_f N_{c,f}Q_f^2\left(E-\log\frac{m_f^2}{\mu^2}\right)\right]$$
$$+\frac{1}{32\pi^2}\frac{\bar{g}^2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\left[2(\xi_W+3)\left(E-\log\frac{M_W^2}{\mu^2}\right)+\xi_W+5+\frac{2\xi_W(\xi_W+2)}{1-\xi_W}\log\xi_W\right], \quad \text{(A.5)}$$

$$A_{Z\gamma}(0) = \frac{\bar{g}^3\bar{g}'v^2}{(16\pi)^2}\left[2(\xi_W+3)\left(E-\log\frac{M_W^2}{\mu^2}\right)+\xi_W+5+\frac{2\xi_W(\xi_W+2)}{1-\xi_W}\log\xi_W\right], \quad \text{(A.6)}$$

$$\begin{aligned}
A_{ZZ}(M_Z^2) = \frac{v^2}{768\pi^2}\Bigg\{&\big(59\bar{g}^4-36\bar{g}^2\bar{g}'^2-11\bar{g}'^4\big)E\\
&+\frac{2\big(278\bar{g}^6+29\bar{g}^4\bar{g}'^2-140\bar{g}^2\bar{g}'^4-24\lambda^2(\bar{g}^2+\bar{g}'^2)+36\lambda(\bar{g}^2+\bar{g}'^2)^2-35\bar{g}'^6\big)}{3(\bar{g}^2+\bar{g}'^2)}\\
&+\lambda\left(\frac{32\lambda^2}{\bar{g}^2+\bar{g}'^2}-48\lambda+36(\bar{g}^2+\bar{g}'^2)\right)\log\frac{M_h^2}{\mu^2}\\
&+2\left(\frac{-16\lambda^3}{\bar{g}^2+\bar{g}'^2}+24\lambda^2-18\lambda(\bar{g}^2+\bar{g}'^2)+5(\bar{g}^2+\bar{g}'^2)^2\right)\log\frac{M_Z^2}{\mu^2}\\
&+\big(-69\bar{g}^4+16\bar{g}^2\bar{g}'^2+\bar{g}'^4\big)\log\frac{M_W^2}{\mu^2}\\
&+\frac{(3\bar{g}^2-\bar{g}'^2)(33\bar{g}^4+22\bar{g}^2\bar{g}'^2+\bar{g}'^4)}{\bar{g}^2+\bar{g}'^2}J_2(r_{WZ})\\
&-16\big[4\lambda^2-4\lambda(\bar{g}^2+\bar{g}'^2)+3(\bar{g}^2+\bar{g}'^2)^2\big]J_1(r_Z)\\
&+16(\bar{g}^2+\bar{g}'^2)^2\sum_f N_{c,f}\\
&\quad\times\Bigg\{g_{A,f}^2\left[\left(\frac{3}{2}r_{fZ}-1\right)\left(E-\log\frac{m_f^2}{\mu^2}\right)+2r_{fZ}-\frac{5}{3}+(r_{fZ}-1)J_2(r_{fZ})\right]\\
&\qquad-g_{V,f}^2\left[E-\log\frac{m_f^2}{\mu^2}+r_{fZ}+\frac{5}{3}+\left(\frac{1}{2}r_{fZ}+1\right)J_2(r_{fZ})\right]\Bigg\}\\
&-6\xi_W\bar{g}^2(\bar{g}^2+\bar{g}'^2)\left(E+1-\log\xi_W-\log\frac{M_W^2}{\mu^2}\right)\\
&-3\xi_Z(\bar{g}^2+\bar{g}'^2)^2\left(E+1-\log\xi_Z-\log\frac{M_Z^2}{\mu^2}\right)\Bigg\}, \qquad \text{(A.7)}
\end{aligned}$$

where the axial-vector and vector couplings are defined as $g_{A,f} = \frac{1}{2}T_f^3$ and $g_{V,f} = \frac{1}{2}T_f^3 - \sin^2\theta_w Q_f$, respectively. The neutrino term in $A_{ZZ}(M_Z^2)$ is contained in the fermionic part, and can readily be obtained by taking the limit $m_f \to 0$.

$$
\begin{aligned}
A_{WW}(M_W^2) = \frac{v^2}{768\pi^2}\Bigg\{ & \bar{g}^2\big(59\bar{g}^2 - 9\bar{g}'^2\big)E + \frac{1}{3}\big(556\bar{g}^4 - 75\bar{g}^2\bar{g}'^2 - 3\bar{g}'^4 + 72\lambda\bar{g}^2 - 48\lambda^2\big) \\
& + \frac{4\lambda}{\bar{g}^2}\big(8\lambda^2 - 12\lambda\bar{g}^2 + 9\bar{g}^4\big)\log\frac{M_H^2}{\mu^2} \\
& + \frac{1}{2\bar{g}^2}\big(-69\bar{g}^6 - 53\bar{g}^4\bar{g}'^2 + 17\bar{g}^2\bar{g}'^4 + \bar{g}'^6\big)\log\frac{M_Z^2}{\mu^2} \\
& - \frac{1}{2\bar{g}^2}\big[49\bar{g}^6 + \bar{g}^4(72\lambda - 71\bar{g}'^2) + \bar{g}^2(17\bar{g}'^4 - 96\lambda^2) + \bar{g}'^6 + 64\lambda^3\big]\log\frac{M_W^2}{\mu^2} \\
& - 16\big(3\bar{g}^4 - 4\bar{g}^2\lambda + 4\lambda^2\big)J_1(r_W) + \frac{4(99\bar{g}^6 + 33\bar{g}^4\bar{g}'^2 - 19\bar{g}^2\bar{g}'^4 - \bar{g}'^6)}{\bar{g}^2 + \bar{g}'^2}J_1(r_{WZ}) \\
& + 2\bar{g}^4 \sum_{\ell=e,\mu,\tau}\Bigg\{\left(\frac{3}{4}r_{\ell W} - 2\right)\left(E - \log\frac{m_\ell^2}{\mu^2}\right) + \frac{r_{\ell W}^2}{16} + \frac{1}{2}r_{\ell W} \\
& \qquad\qquad - \frac{10}{3} + \left(\frac{r_{\ell W}^3}{64} - \frac{3}{4}r_{\ell W} + 2\right)\log\left(1 - \frac{M_W^2}{m_\ell^2}\right)\Bigg\} \\
& + \frac{8\bar{g}^2 N_c}{v^2}\sum_{\alpha,\beta}|K_{\alpha\beta}|^2\Bigg\{\left(3M_{d_\beta}^2 + 3M_{u_\alpha}^2 - 2M_W^2\right)E \\
& \qquad + \frac{(M_{d_\beta}^2 - M_{u_\alpha}^2)^2}{M_W^2} + 2(M_{d_\beta}^2 + M_{u_\alpha}^2) - \frac{10}{3}M_W^2 \\
& \qquad + \left[\frac{(M_{d_\beta}^2 - M_{u_\alpha}^2)^3}{2M_W^4} - \frac{3}{2}(M_{d_\beta}^2 + M_{u_\alpha}^2) + M_W^2\right]\log\frac{M_{u_\alpha}^2}{\mu^2} \\
& \qquad + \left[\frac{(M_{u_\alpha}^2 - M_{d_\beta}^2)^3}{2M_W^4} - \frac{3}{2}(M_{d_\beta}^2 + M_{u_\alpha}^2) + M_W^2\right]\log\frac{M_{d_\beta}^2}{\mu^2} \\
& \qquad + \left[\frac{(M_{d_\beta}^2 - M_{u_\alpha}^2)^2}{M_W^4} + \frac{(M_{d_\beta}^2 + M_{u_\alpha}^2)}{M_W^2} - 2\right]J_3(M_{u_\alpha}, M_{d_\beta})\Bigg\} \\
& - 6\xi_W\bar{g}^4\left(E + 1 - \log\xi_W - \log\frac{M_W^2}{\mu^2}\right) \\
& - 3\xi_Z\bar{g}^2(\bar{g}^2 + \bar{g}'^2)\left(E + 1 - \log\xi_Z - \log\frac{M_Z^2}{\mu^2}\right)\Bigg\},
\end{aligned}
\tag{A.8}
$$

where

$$
M_u = \mathrm{diag}(m_u, m_c, m_t)\,, \qquad M_d = \mathrm{diag}(m_d, m_s, m_b)\,, \tag{A.9}
$$

$K_{\alpha\beta}$ is the CKM matrix, and the summation indices in the hadronic contribution run over all the quark generations. The infinite quantity E is given by eq. (3.22), and the functions $J_1(x), J_2(x)$ and $J_3(x)$ are defined through

$$
J_1(x) \equiv \begin{cases} \frac{\sqrt{1-x}}{x}\log\left(\frac{1+\sqrt{1-x}}{\sqrt{x}}\right), & 0 < x \leq 1\,, \\ -2\frac{\sqrt{x-1}}{x}\arctan\left(\frac{\sqrt{x-1}}{1+\sqrt{x}}\right), & x \geq 1\,, \end{cases} \tag{A.10}
$$

$$J_2(x) \equiv \begin{cases} \sqrt{1-x}\Big[\log\Big(\frac{2-x-2\sqrt{1-x}}{x}\Big) + i\pi\Big]\,, & 0 < x \leq 1\,, \\ -2\sqrt{x-1}\,\arctan\Big(\frac{1}{\sqrt{x-1}}\Big)\,, & x \geq 1\,, \end{cases} \tag{A.11}$$

and

$$\begin{aligned} J_3(M_u, M_d) \equiv & \sqrt{\big[(M_d - M_u)^2 - M_W^2\big]\big[(M_d + M_u)^2 - M_W^2\big]} \\ & \times \log\left[\frac{(M_d^2 + M_u^2 - M_W^2) + \sqrt{\big[(M_d - M_u)^2 - M_W^2\big]\big[(M_d + M_u)^2 - M_W^2\big]}}{2M_d M_u}\right]. \end{aligned} \tag{A.12}$$

For completeness we also add here the W-boson one-loop self-energy at zero external momentum, evaluated in Feynman gauge, needed in the master formula (4.19). It reads

$$\begin{aligned} A_{WW}(0) = & \frac{\bar{g}^4 v^2}{64\pi^2}\Bigg\{\left(1 - \frac{\bar{g}'^2}{\bar{g}^2}\right)E + \frac{\lambda}{2\bar{g}^2} - \frac{7\bar{g}'^2}{8\bar{g}^2} + \frac{27}{8} - \frac{3\lambda}{(\bar{g}^2 - 4\lambda)}\log\frac{M_h^2}{\mu^2} \\ & + \left(\frac{17\bar{g}^2}{4\bar{g}'^2} + \frac{3\bar{g}^2}{4(\bar{g}^2 - 4\lambda)} - \frac{1}{2}\right)\log\frac{M_W^2}{\mu^2} - \left(\frac{17\bar{g}^2}{4\bar{g}'^2} - \frac{\bar{g}'^2}{\bar{g}^2} + \frac{5}{4}\right)\log\frac{M_Z^2}{\mu^2}\Bigg\} \\ & + \frac{\bar{g}^2 N_c}{32\pi^2}\sum_{\alpha,\beta}|K_{\alpha\beta}|^2\Bigg[\left(M_{u_\alpha}^2 + M_{d_\beta}^2\right)\left(E - \log\frac{M_{d_\beta}^2}{\mu^2}\right) \\ & \qquad + \frac{M_{u_\alpha}^2 + M_{d_\beta}^2}{2} + \frac{M_{u_\alpha}^4}{M_{u_\alpha}^2 - M_{d_\beta}^2}\log\frac{M_{d_\beta}^2}{M_{u_\alpha}^2}\Bigg] \\ & + \frac{\bar{g}^2}{32\pi^2}\sum_{\ell=e,\mu,\tau} m_\ell^2\left[\left(E - \log\frac{m_\ell^2}{\mu^2}\right) + \frac{1}{2}\right]. \end{aligned} \tag{A.13}$$

Moreover, the derivative of the Higgs self-energy reads

$$\begin{aligned} \Pi'_{HH}(M_h^2) = & \frac{1}{128\pi^2}\Bigg\{(12\bar{g}^2 - 16\lambda)\left(E - \log\frac{M_W^2}{\mu^2}\right) + \frac{6}{\lambda}(\bar{g}^4 + 2\bar{g}^2\lambda - 4\lambda^2) \\ & + \frac{16\lambda^3 - 20\bar{g}^2\lambda^2 + 4\bar{g}^4\lambda + 3\bar{g}^6}{\lambda(\bar{g}^2 - \lambda)}J_2(r_W) \\ & + \big[6(\bar{g}^2 + \bar{g}'^2) - 8\lambda\big]\left(E - \log\frac{M_Z^2}{\mu^2}\right) + \frac{3}{\lambda}\big[(\bar{g}^2 + \bar{g}'^2)^2 + 2\lambda(\bar{g}^2 + \bar{g}'^2) - 4\lambda^2\big] \\ & + \frac{16\lambda^3 - 20\lambda^2(\bar{g}^2 + \bar{g}'^2) + 4\lambda(\bar{g}^2 + \bar{g}'^2)^2 + 3(\bar{g}^2 + \bar{g}'^2)^3}{2\lambda(\bar{g}^2 + \bar{g}'^2 - \lambda)}J_2(r_Z) + 4\lambda(9 - 2\sqrt{3}\pi) \\ & - 16\sum_f N_{c,f}\left(\frac{m_f}{v}\right)^2\left[E - \log\frac{m_f^2}{\mu^2} + 1 + r_f + \left(1 + \frac{r_f}{2}\right)J_2(r_f)\right] \\ & + 4\big(4\lambda - \bar{g}^2\xi_W\big)\left(E - \log\frac{M_W^2}{\mu^2} - \log\xi_W\right) + 4\big(8\lambda - \bar{g}^2\xi_W\big) + 16\lambda J_2(\xi_W r_W) \\ & + 2\big[4\lambda - (\bar{g}^2 + \bar{g}'^2)\xi_Z\big]\left(E - \log\frac{M_Z^2}{\mu^2} - \log\xi_Z\right) + 2\big[8\lambda - (\bar{g}^2 + \bar{g}'^2)\xi_Z\big] + 8\lambda J_2(\xi_Z r_Z)\Bigg\}\,, \end{aligned} \tag{A.14}$$

and the light quark contribution needed in eq. (4.20) is

$$\Pi_{\gamma\gamma}^{\rm had}(M_Z^2) = \frac{\bar{g}^2 g'^2}{12\pi^2(\bar{g}^2+\bar{g}'^2)} \sum_q N_c Q_q^2 \left[E - \log\frac{m_q^2}{\mu^2} + \left(1+\frac{r_{qZ}}{2}\right) J_2(r_{qZ}) + r_{qZ} + \frac{5}{3} \right] . \qquad (A.15)$$

References

[1] P. W. Higgs, *Broken Symmetries and the Masses of Gauge Bosons*, *Phys.Rev.Lett.* **13** (1964) 508–509.

[2] F. Englert and R. Brout, *Broken Symmetry and the Mass of Gauge Vector Mesons*, *Phys.Rev.Lett.* **13** (1964) 321–323.

[3] G. Guralnik, C. Hagen, and T. Kibble, *Global Conservation Laws and Massless Particles*, *Phys.Rev.Lett.* **13** (1964) 585–587.

[4] **ATLAS** Collaboration, G. Aad et al., *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, *Phys.Lett.B* (2012) [arXiv:1207.7214].

[5] **CMS** Collaboration, S. Chatrchyan et al., *Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC*, *Phys.Lett.B* (2012) [arXiv:1207.7235].

[6] J. R. Ellis, M. K. Gaillard, and D. V. Nanopoulos, *A Phenomenological Profile of the Higgs Boson*, *Nucl. Phys.* **B106** (1976) 292.

[7] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin, and V. I. Zakharov, *Low-Energy Theorems for Higgs Boson Couplings to Photons*, *Sov. J. Nucl. Phys.* **30** (1979) 711–716. [Yad. Fiz.30,1368(1979)].

[8] **ATLAS** Collaboration, M. Aaboud et al., *Measurements of Higgs boson properties in the diphoton decay channel with 36 fb^{-1} of pp collision data at $\sqrt{s} = 13$ TeV with the ATLAS detector*, arXiv:1802.04146.

[9] **CMS** Collaboration, A. M. Sirunyan et al., *Measurements of Higgs boson properties in the diphoton decay channel in proton-proton collisions at $\sqrt{s}$ = 13 TeV*, arXiv:1804.02716.

[10] W. Buchmuller and D. Wyler, *Effective Lagrangian Analysis of New Interactions and Flavor Conservation*, *Nucl. Phys.* **B268** (1986) 621–653.

[11] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085, [arXiv:1008.4884].

[12] T. Appelquist and J. Carazzone, *Infrared Singularities and Massive Fields*, *Phys. Rev.* **D11** (1975) 2856.

[13] A. Dedes, W. Materkowska, M. Paraskevas, J. Rosiek, and K. Suxho, *Feynman rules for the Standard Model Effective Field Theory in R_ξ-gauges*, *JHEP* **06** (2017) 143, [arXiv:1704.03888].

[14] B. Ioffe and V. A. Khoze, *What Can Be Expected from Experiments on Colliding e^+e^- Beams with Energy Approximately Equal to 100-GeV?*, *Sov.J.Part.Nucl.* **9** (1978) 50.

[15] M. Gavela, G. Girardi, C. Malleville, and P. Sorba, *A nonlinear R_ξ-gauge condition for electroweak $SU(2) \times U(1)$ Model*, *Nucl.Phys.* **B193** (1981) 257.

[16] D. Huang, Y. Tang, and Y.-L. Wu, *Note on Higgs Decay into Two Photons $H \to \gamma\gamma$*, *Commun.Theor.Phys.* **57** (2012) 427–434, [arXiv:1109.4846].

[17] H.-S. Shao, Y.-J. Zhang, and K.-T. Chao, *Higgs Decay into Two Photons and Reduction Schemes in Cutoff Regularization*, *JHEP* **1201** (2012) 053, [arXiv:1110.6925].

[18] F. Bursa, A. Cherman, T. C. Hammant, R. R. Horgan, and M. Wingate, *Calculation of the One W Loop $H \to \gamma\gamma$ Decay Amplitude with a Lattice Regulator*, arXiv:1112.2135.

[19] F. Piccinini, A. Pilloni, and A. Polosa, *$H \to \gamma\gamma$: a Comment on the Indeterminacy of Non-Gauge-Invariant Integrals*, arXiv:1112.4764.

[20] A. Dedes and K. Suxho, *Anatomy of the Higgs boson decay into two photons in the unitary gauge*, *Adv. High Energy Phys.* **2013** (2013) 631841, [arXiv:1210.0141].

[21] A. L. Cherchiglia, L. A. Cabral, M. C. Nemes, and M. Sampaio, *(Un)determined finite regularization dependent quantum corrections: the Higgs boson decay into two photons and the two photon scattering examples*, *Phys. Rev.* **D87** (2013), no. 6 065011, [arXiv:1210.6164].

[22] A. M. Donati and R. Pittau, *Gauge invariance at work in FDR: $H \to \gamma\gamma$*, *JHEP* **04** (2013) 167, [arXiv:1302.5668].

[23] W. J. Marciano, C. Zhang, and S. Willenbrock, *Higgs Decay to Two Photons*, *Phys. Rev.* **D85** (2012) 013002, [arXiv:1109.5304].

[24] I. Brivio and M. Trott, *The Standard Model as an Effective Field Theory*, arXiv:1706.08945.

[25] A. V. Manohar, *Introduction to Effective Field Theories*, arXiv:1804.05863.

[26] A. V. Manohar and M. B. Wise, *Modifications to the properties of the Higgs boson*, *Phys. Lett.* **B636** (2006) 107–113, [hep-ph/0601212].

[27] C. Grojean, E. E. Jenkins, A. V. Manohar, and M. Trott, *Renormalization Group Scaling of Higgs Operators and $\Gamma(h \to \gamma\gamma)$*, *JHEP* **04** (2013) 016, [arXiv:1301.2588].

[28] S. Dawson and P. P. Giardino, *Higgs Decays to ZZ and $Z\gamma$ in the SMEFT: an NLO analysis*, arXiv:1801.01136.

[29] C. Hartmann and M. Trott, *Higgs Decay to Two Photons at One Loop in the Standard Model Effective Field Theory*, *Phys. Rev. Lett.* **115** (2015), no. 19 191801, [arXiv:1507.03568].

[30] C. Hartmann and M. Trott, *On one-loop corrections in the standard model effective field theory; the $\Gamma(h \to \gamma\,\gamma)$ case*, *JHEP* **07** (2015) 151, [arXiv:1505.02646].

[31] L. F. Abbott, *The Background Field Method Beyond One Loop*, *Nucl. Phys.* **B185** (1981) 189–203.

[32] A. Helset, M. Paraskevas, and M. Trott, *Gauge fixing the Standard Model Effective Field Theory*, arXiv:1803.08001.

[33] C. W. Murphy, *Statistical approach to Higgs boson couplings in the standard model effective field theory*, *Phys. Rev.* **D97** (2018), no. 1 015007, [arXiv:1710.02008].

[34] S. Jana and S. Nandi, *New Physics Scale from Higgs Observables with Effective Dimension-6 Operators*, arXiv:1710.00619.

[35] J. Ellis, C. W. Murphy, V. Sanz, and T. You, *Updated Global SMEFT Fit to Higgs, Diboson and Electroweak Data*, arXiv:1803.03252.

[36] C. Arzt, M. B. Einhorn, and J. Wudka, *Patterns of deviation from the standard model*, *Nucl. Phys.* **B433** (1995) 41–66, [hep-ph/9405214].

[37] M. B. Einhorn and J. Wudka, *The Bases of Effective Field Theories*, *Nucl. Phys.* **B876** (2013) 556–574, [arXiv:1307.0478].

[38] **Particle Data Group** Collaboration, C. Patrignani et al., *Review of Particle Physics*, *Chin. Phys.* **C40** (2016), no. 10 100001 and 2017 update.

[39] H. D. Politzer, *Power Corrections at Short Distances*, *Nucl. Phys.* **B172** (1980) 349–382.

[40] C. Arzt, *Reduced effective Lagrangians*, *Phys. Lett.* **B342** (1995) 189–195, [hep-ph/9304230].

[41] A. Sirlin, *Radiative Corrections in the SU(2)-L x U(1) Theory: A Simple Renormalization Framework*, *Phys. Rev.* **D22** (1980) 971–981.

[42] R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, *JHEP* **04** (2014) 159, [arXiv:1312.2014].

[43] A. Sirlin and R. Zucchini, *Dependence of the Quartic Coupling H(m) on M(H) and the Possible Onset of New Physics in the Higgs Sector of the Standard Model*, *Nucl. Phys.* **B266** (1986) 389–409.

[44] H. Lehmann, K. Symanzik, and W. Zimmermann, *On the formulation of quantized field theories*, *Nuovo Cim.* **1** (1955) 205–225.

[45] M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory.* Addison-Wesley, Reading, USA, 1995.

[46] W. J. Marciano and A. Sirlin, *Radiative Corrections to Neutrino Induced Neutral Current Phenomena in the SU(2)-L x U(1) Theory*, *Phys. Rev.* **D22** (1980) 2695. [Erratum: Phys. Rev.D31,213(1985)].

[47] E. E. Jenkins, A. V. Manohar, and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and lambda Dependence*, *JHEP* **10** (2013) 087, [arXiv:1308.2627].

[48] E. E. Jenkins, A. V. Manohar, and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence*, *JHEP* **01** (2014) 035, [arXiv:1310.4838].

[49] G. Passarino and M. J. G. Veltman, *One Loop Corrections for e^+e^- Annihilation Into $\mu^+\mu^-$ in the Weinberg Model*, *Nucl. Phys.* **B160** (1979) 151–207.

[50] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs hunter's guide*, *Front.Phys.* **80** (2000) 1–448.

[51] A. Djouadi, *The Anatomy of electro-weak symmetry breaking. I: The Higgs boson in the standard model*, *Phys. Rept.* **457** (2008) 1–216, [hep-ph/0503172].

[52] A. Celis, J. Fuentes-Martin, A. Vicente, and J. Virto, *DsixTools: The Standard Model Effective Field Theory Toolkit*, *Eur. Phys. J.* **C77** (2017), no. 6 405, [arXiv:1704.04504].

[53] J. Aebischer, J. Kumar, and D. M. Straub, *Wilson: a Python package for the running and matching of Wilson coefficients above and below the electroweak scale*, arXiv:1804.05033.

[54] A. J. Buras and M. Jung, *Analytic inclusion of the scale dependence of the anomalous dimension matrix in Standard Model Effective Theory*, arXiv:1804.05852.

[55] B. Grinstein and M. B. Wise, *Operator analysis for precision electroweak physics*, *Phys. Lett.* **B265** (1991) 326–334.

[56] M. E. Peskin and T. Takeuchi, *Estimation of oblique electroweak corrections*, *Phys.Rev.* **D46** (1992) 381–409.

[57] A. Falkowski and F. Riva, *Model-independent precision constraints on dimension-6 operators*, *JHEP* **02** (2015) 039, [arXiv:1411.0669].

[58] A. Buckley, C. Englert, J. Ferrando, D. J. Miller, L. Moore, M. Russell, and C. D. White, *Constraining top quark effective theory in the LHC Run II era*, *JHEP* **04** (2016) 015, [arXiv:1512.03360].

[59] J. de Blas, J. C. Criado, M. Perez-Victoria, and J. Santiago, *Effective description of general extensions of the Standard Model: the complete tree-level dictionary*, arXiv:1711.10391.

[60] R. Mertig, M. Bohm, and A. Denner, *FEYN CALC: Computer algebraic calculation of Feynman amplitudes*, *Comput. Phys. Commun.* **64** (1991) 345–359.

[61] V. Shtabovenko, R. Mertig, and F. Orellana, *New Developments in FeynCalc 9.0*, *Comput. Phys. Commun.* **207** (2016) 432–444, [arXiv:1601.01167].

[62] H. H. Patel, *Package-X: A Mathematica package for the analytic calculation of one-loop integrals*, *Comput. Phys. Commun.* **197** (2015) 276–290, [arXiv:1503.01469].

[63] H. H. Patel, *Package-X 2.0: A Mathematica package for the analytic calculation of one-loop integrals*, *Comput. Phys. Commun.* **218** (2017) 66–70, [arXiv:1612.00009].

[64] G. Degrassi and A. Sirlin, *Gauge dependence of basic electroweak corrections of the standard model*, *Nucl. Phys.* **B383** (1992) 73–92.