Higgs Decays to ZZ and $Z\gamma$ in the SMEFT: an NLO analysis

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(Dated: May 4, 2018)

Abstract

We calculate the complete one-loop electroweak corrections to the inclusive $H \to ZZ$ and $H \to Z\gamma$ decays in the dimension-6 extension of the Standard Model Effective Field Theory (SMEFT). The corrections to $H \to ZZ$ are computed for on-shell Z bosons and are a precursor to the physical $H \to Zf\overline{f}$ calculation. We present compact numerical formulas for our results and demonstrate that the logarithmic contributions that result from the renormalization group evolution of the SMEFT coefficients are larger than the finite NLO contributions to the decay widths. As a byproduct of our calculation, we obtain the first complete result for the finite corrections to G_μ in the SMEFT.

I. INTRODUCTION

The LHC experimental discovery of the Higgs boson, along with the measurement of Higgs properties that are in rough agreement with the Standard Model (SM) predictions, suggests that the SM is a valid effective theory at the weak scale. The lack of new particles up to the TeV scale makes possible the parametrization of possible high scale physics effects in terms of higher dimension operators containing only SM fields [1]. In this paper we study new physics contributions to Higgs decays in the context of the dimension-6 Standard Model Effective Field Theory (SMEFT). When compared with precise theoretical calculations, measurements of Higgs properties serve to constrain the coefficients of the higher dimension operators and restrict possible beyond the SM (BSM) physics at energies $\Lambda >> v$.

We study Higgs decays to Z boson pairs and to $Z\gamma$ in the context of the SMEFT, where new physics is described by a tower of operators,

$$\mathcal{L} = \mathcal{L}_{SM} + \Sigma_{k=5}^{\infty}\Sigma_{i=1}^{n}\frac{\mathcal{C}_i^k}{\Lambda^{k-4}}O_i^k\,. \tag{1}$$

The dimension-k operators are constructed from SM fields and the BSM physics effects reside in the coefficient functions, C_i^k. For large Λ, it is sufficient to retain only the lowest dimensional operators. We assume lepton number conservation, so the lowest dimension relevant operators are dimension-6. Ignoring flavor, there are 59 dimension-6 operators that are $SU(3) \times SU(2) \times U(1)$ invariant combinations of the SM fields [1, 2]. The operators have been classified in several different bases, which are related by the equations of motion [1–4]. In this paper we will use the Warsaw basis of Ref. [2].

A detailed understanding of Higgs properties requires the inclusion of the dimension-6 tree level SMEFT effects, along with radiative corrections in the effective field theory. Measurements at the Higgs mass scale, M_H, that are sensitive to a set of SMEFT coefficients, $\mathcal{C}_i(M_H)$, at leading order will develop logarithmic sensitivity to other coefficients when the renormalization group evolved to the scale Λ, due to the renormalization group mixing of the coefficients [5–9]. There are also finite contributions that may be of the same numerical size as the logarithmic terms.

We compute the one-loop electroweak SMEFT contributions to the decays $H \to ZZ$ and $H \to Z\gamma$. These corrections include the one-loop SM electroweak corrections, along with the one-loop corrections due to the SMEFT operators of Eq. (1). Our results are interesting from a purely theoretical perspective and we present the first complete one-loop SMEFT renormalization of G_μ. The one-loop SMEFT corrections to $H \to b\bar{b}$ [10, 11] and $H \to \gamma\gamma$ [12, 13] are known, as well as a general NLO SMEFT calculation of 2-body Higgs decays [14]. The physical process for $M_H = 125$ TeV is $H \to Zf\overline{f}$ and our calculation is a precursor to the eventual one-loop 3-body SMEFT calculation.

We review the SMEFT framework in Sec. II and our renormalization framework in Sec. III. Sec. IV contains numerical results for $H \to ZZ$ and $H \to Z\gamma$ and a discussion of the phenomenological impact of our calculation. A comparison with the physical off-shell process, $H \to Zf\overline{f}$, is in Appendix A. Appendix B contains numerical fits for the one-loop SMEFT result for $H \to ZZ$ and Appendix C has analytic formulas for the logarithmic contributions to the one-loop SMEFT result for $H \to ZZ$. Lastly, Appendix D contains the one-loop SMEFT calculation of G_μ.

II. SMEFT BASICS

In this section we briefly introduce the SMEFT. We consider the Lagrangian in Eq. (1) truncated at dimension-6,

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}^{(6)}_{\rm EFT}; \qquad \mathcal{L}^{(6)}_{\rm EFT} = \sum_i \frac{\mathcal{C}^{(6)}_i}{\Lambda^2} \mathcal{O}^{(6)}_i, \tag{2}$$

where $\mathcal{L}_{\rm SM}$ is the Standard Model Lagrangian, and $\mathcal{L}^{(6)}_{\rm EFT}$ is the most general $SU(3) \times SU(2) \times U(1)$ invariant Effective Field Theory (EFT) Lagrangian that can be built using only dimension-6 operators. In the following we drop the superscript (6). Only a few operators contribute to the $H \to ZZ$ and $H \to Z\gamma$ decays at tree-level, while more operators contribute at one- loop. In total, 19 of the 59 independent dimension-6 operators of the Warsaw basis enter our calculation,

$$\begin{aligned}
&\mathcal{O}_W, \mathcal{O}_\phi, \mathcal{O}_{\phi\Box}, \mathcal{O}_{\phi D}, \mathcal{O}_{u\phi}, \mathcal{O}_{\phi W}, \mathcal{O}_{\phi B}, \mathcal{O}_{\phi WB}, \mathcal{O}_{uW},\\
&\mathcal{O}_{uB}, \mathcal{O}^{(1)}_{\phi l}, \mathcal{O}^{(3)}_{\phi l}, \mathcal{O}_{\phi e}, \mathcal{O}^{(1)}_{\phi q}, \mathcal{O}^{(3)}_{\phi q}, \mathcal{O}_{\phi u}, \mathcal{O}_{\phi d}, \mathcal{O}_{ll}, \mathcal{O}^{(3)}_{lq}\,,
\end{aligned} \tag{3}$$

where the operators are defined in Tab. I,

$$D_\mu = \partial_\mu + ig' B_\mu Y + ig \frac{\tau^I}{2} W^I_\mu + ig_s T^A G^A_\mu\,, \tag{4}$$

s

$\mathcal{O}_W$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_\phi$	$(\phi^\dagger\phi)^3$	$\mathcal{O}_{\phi\Box}$	$(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$
$\mathcal{O}_{\phi D}$	$(\phi^\dagger D^\mu\phi)^*\,(\phi^\dagger D_\mu\phi)$	$\underset{p,r}{\mathcal{O}_{u\phi}}$	$(\phi^\dagger\phi)(\bar{q}'_p u'_r\widetilde{\phi})$	$\mathcal{O}_{\phi W}$	$(\phi^\dagger\phi)\,W_{\mu\nu}W^{\mu\nu}$
$\mathcal{O}_{\phi B}$	$(\phi^\dagger\phi)\,B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{\phi WB}$	$(\phi^\dagger\tau^I\phi)\,W^I_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\tau^I\widetilde{\phi}\,W^I_{\mu\nu}$
$\underset{p,r}{\mathcal{O}_{uB}}$	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\widetilde{\phi}\,B_{\mu\nu}$	$\underset{p,r}{\mathcal{O}^{(1)}_{\phi l}}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\,\phi)(\bar{l}'_p\gamma^\mu l'_r)$	$\underset{p,r}{\mathcal{O}^{(3)}_{\phi l}}$	$(\phi^\dagger i\overleftrightarrow{D}^I_\mu\,\phi)(\bar{l}'_p\tau^I\gamma^\mu l'_r)$
$\underset{p,r}{\mathcal{O}_{\phi e}}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\,\phi)(\bar{e}'_p\gamma^\mu e'_r)$	$\underset{p,r}{\mathcal{O}^{(1)}_{\phi q}}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\,\phi)(\bar{q}'_p\gamma^\mu q'_r)$	$\underset{p,r}{\mathcal{O}^{(3)}_{\phi q}}$	$(\phi^\dagger i\overleftrightarrow{D}^I_\mu\,\phi)(\bar{q}'_p\tau^I\gamma^\mu q'_r)$
$\underset{p,r}{\mathcal{O}_{\phi u}}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\,\phi)(\bar{u}'_p\gamma^\mu u'_r)$	$\underset{p,r}{\mathcal{O}_{\phi d}}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\,\phi)(\bar{d}'_p\gamma^\mu d'_r)$	$\underset{p,r,s,t}{\mathcal{O}_{ll}}$	$(\bar{l}'_p\gamma_\mu l'_r)(\bar{l}'_s\gamma^\mu l'_t)$
$\underset{p,r,s,t}{\mathcal{O}^{(3)}_{lq}}$	$(\bar{l}'_p\gamma_\mu\tau^I l'_r)(\bar{q}'_s\gamma^\mu\tau^I q'_t)$				

TABLE I: Dimension-6 operators relevant for our study (from [2]). For brevity we suppress fermion chiral indices L and R. $I = 1,2,3$ is an $SU(2)$ index, p and r are flavor indices, and $\phi^\dagger i\overleftrightarrow{D}_\mu\,\phi \equiv \phi^\dagger D_\mu\phi - (D_\mu\phi^\dagger)\phi$.

τ^I are the Pauli matrices and l_L and q_L are the $SU(2)_L$ doublet lepton and quark fields. For simplicity, we assume a diagonal flavor structure for the coefficients $\mathcal{C}$, *i.e.* $\underset{p,r}{\mathcal{C}_i} = \underset{p,r}{\mathcal{C}_i\,\mathbb{1}}$, where p and r are flavor indices. Furthermore, we assume $\underset{e,\mu,\mu,e}{\mathcal{C}_{ll}} = \underset{\mu,e,e\mu}{\mathcal{C}_{ll}} \equiv \mathcal{C}_{ll}$ and $\underset{\mu,\mu,t,t}{\mathcal{C}^{(3)}_{lq}} = \underset{e,e,t,t}{\mathcal{C}^{(3)}_{lq}} \equiv \mathcal{C}^{(3)}_{lq}$.

In general, the presence of the dimension-6 operators changes the structure of the Lagrangian and the correlations between the Lagrangian quantities and the measured observables [15, 16]. In the following we discuss these modifications, as they are relevant to the one-loop SMEFT calculations of $H \to ZZ$ and $H \to Z\gamma$.

A. Higgs sector

In the SMEFT, the Higgs Lagrangian takes the form,

$$\begin{aligned}\mathcal{L} &= (D^\mu\phi)^\dagger(D_\mu\phi) + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \\ &+ \frac{1}{\Lambda^2}\Big(\mathcal{C}_\phi(\phi^\dagger\phi)^3 + \mathcal{C}_{\phi\Box}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \mathcal{C}_{\phi D}(\phi^\dagger D^\mu\phi)^*(\phi^\dagger D_\mu\phi)\Big)\end{aligned} \tag{5}$$

where ϕ is the Higgs doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\phi^0) \end{pmatrix}, \tag{6}$$

and v is the vacuum expectation value (vev) defined as the minimum of the potential,

$$v \equiv \sqrt{2}\langle\phi\rangle = \sqrt{\frac{\mu^2}{\lambda}} + \frac{3\mu^3}{8\lambda^{5/2}}\frac{\mathcal{C}_\phi}{\Lambda^2}. \tag{7}$$

Due to the presence of $\mathcal{O}_{\phi\Box}$ and $\mathcal{O}_{\phi D}$ in Eq. (5), the kinetic terms in the resulting Lagrangian are not canonically normalized. As a consequence we need to shift the fields,

$$\begin{aligned} H &\to H\left(1 - \frac{v^2}{\Lambda^2}(\frac{1}{4}\mathcal{C}_{\phi D} - \mathcal{C}_{\phi\Box})\right), \\ \phi^0 &\to \phi^0\left(1 - \frac{v^2}{\Lambda^2}(\frac{1}{4}\mathcal{C}_{\phi D})\right), \\ \phi^+ &\to \phi^+, \end{aligned} \tag{8}$$

and the physical mass of the Higgs, defined as the pole of the propagator, becomes,

$$M_H^2 = 2\lambda v^2 - \frac{v^4}{\Lambda^2}(3\mathcal{C}_\phi - 4\lambda\mathcal{C}_{\phi\Box} + \lambda C_{\phi D}). \tag{9}$$

As anticipated, the relation between the Lagrangian parameters and the measured observable (M_H) is altered by the presence of the dimension-6 operators [15, 16].

B. Gauge sector

The introduction of the operators in Eq. (3) also alters the form of the kinetic terms for the Lagrangian of the gauge sector. The relevant Lagrangian terms are:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}WI, \mu\nu W^I_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ &+ \frac{1}{\Lambda^2}\left(\mathcal{C}_{\phi W}(\phi^\dagger\phi)W^{I,\mu\nu}W^I_{\mu\nu} + \mathcal{C}_{\phi B}(\phi^\dagger\phi)B^{\mu\nu}B_{\mu\nu} + \mathcal{C}_{\phi WB}(\phi^\dagger\tau^I\phi)W^{I,\mu\nu}B_{\mu\nu}\right), \end{aligned} \tag{10}$$

It is convenient to define "barred" fields, $\overline{W}_\mu \equiv (1 - \mathcal{C}_{\phi W}v^2/\Lambda^2)W_\mu$ and $\overline{B}_\mu \equiv (1 - \mathcal{C}_{\phi B}v^2/\Lambda^2)B_\mu$ and "barred" gauge couplings, $\overline{g}_2 \equiv (1 + \mathcal{C}_{\phi W}v^2/\Lambda^2)g_2$ and $\overline{g}_1 \equiv (1 + \mathcal{C}_{\phi B}v^2/\Lambda^2)g_1$ so that $\overline{W}_\mu\overline{g}_2 = W_\mu g_2$ and $\overline{B}_\mu\overline{g}_1 = B_\mu g_1$. The "barred" fields defined in this way have their kinetic terms properly normalized and preserve the form of the covariant derivative. The masses of the W and Z fields (poles of the propagators) are then expressed in terms of the "barred" couplings [9, 17]:

$$
\begin{aligned}
M_W^2 &= \frac{\overline{g}_2^2 v^2}{4}, \\
M_Z^2 &= \frac{(\overline{g}_1^2+\overline{g}_2^2)v^2}{4} + \frac{v^4}{\Lambda^2}\left(\frac{1}{8}(\overline{g}_1^2+\overline{g}_2^2)\mathcal{C}_{\phi D} + \frac{1}{2}\overline{g}_1\overline{g}_2\mathcal{C}_{\phi WB}\right).
\end{aligned}
\tag{11}
$$

It is interesting to note that the extra terms in the definition of the Z mass are due to the rotation, $(W_\mu^3, B_\mu) \to (Z_\mu, A_\mu)$, that is proportional to $\mathcal{C}_{\phi WB}$ and the shift of ϕ^0 in Eq. (8) that is proportional to $\mathcal{C}_{\phi D}$.

C. Fermion sector

Lastly, we study the fermion sector. We notice that the presence of the dimension-6 operators does not alter the kinetic terms in the Lagrangian, so we concentrate on the mass terms[1]:

$$
\begin{aligned}
\mathcal{L} &= -(y_e \bar{l}_L e_R \phi + y_u \bar{q}_L u_R \tilde{\phi} + y_d \bar{q}_L d_R \phi + h.c.) \\
&+ \frac{1}{\Lambda^2}\left(\mathcal{C}_{e\phi}(\phi^\dagger\phi)(\bar{l}_L e_R \phi) + \mathcal{C}_{u\phi}(\phi^\dagger\phi)(\bar{q}_L u_R \tilde{\phi}) + \mathcal{C}_{d\phi}(\phi^\dagger\phi)(\bar{q}_L d_R \phi) + h.c.\right).
\end{aligned}
\tag{12}
$$

The masses of all fermions are shifted by the interactions of Eq. (12). The lepton and light quark masses do not enter the 1-loop result for $H \to ZZ$ and can be safely set to 0 there. However, the masses of the leptons and lighter quarks contribute logarithmically to the $\gamma\gamma$ wave-function in the one-loop $H \to Z\gamma$ calculation and we retain finite fermion masses there. However, since the lowest order (LO) $H \to Z\gamma$ amplitude is $\mathcal{O}(\frac{v^2}{\Lambda^2})$, the contributions of the terms proportional to $\mathcal{C}_{e\phi}$ and $\mathcal{C}_{d\phi}$ to light fermion masses, m_f, are $\mathcal{O}(\frac{m_f v^4}{\Lambda^4})$ and can be neglected. We concentrate on the definition of the top pole mass,

$$
M_t = \frac{v}{\sqrt{2}}(y_t - \frac{1}{2}\mathcal{C}_{u\phi}\frac{v^2}{\Lambda^2}). \tag{13}
$$

Dimension-6 operators involving fermions also give contributions to the decay of the μ lepton, thus changing the relation between the vev, v, and the Fermi constant G_μ obtained from the measurement of the μ lifetime. Considering only contributions that interfere with

[1] We neglect flavor mixing, so $\mathcal{C}_{u\phi}$ represents generically $\mathcal{C}_{u\phi}$, $\mathcal{C}_{c\phi}$, $\mathcal{C}_{t\phi}$, etc.

the SM amplitude, we obtain the tree level result,

$$
\begin{aligned}
G_\mu &= \frac{1}{\sqrt{2}v^2} - \frac{1}{2\sqrt{2}\Lambda^2}\Big(\underset{e,\mu,\mu,e}{\mathcal{C}_{ll}} + \underset{\mu,e,e\mu}{\mathcal{C}_{ll}}\Big) + \frac{\sqrt{2}}{2\Lambda^2}\Big(\underset{e,e}{\mathcal{C}_{\phi l}}{}^{(3)} + \underset{\mu,\mu}{\mathcal{C}_{\phi l}}{}^{(3)}\Big) \\
&\equiv \frac{1}{\sqrt{2}v^2} - \frac{1}{\sqrt{2}\Lambda^2}\mathcal{C}_{ll} + \frac{\sqrt{2}}{\Lambda^2}\mathcal{C}_{\phi l}^{(3)}, \qquad (14)
\end{aligned}
$$

where in the last equality we assumed flavor universality of the coefficients.

III. RENORMALIZATION

The SM one-loop electroweak corrections to $H \to ZZ$ are well known and we reproduce the results of Ref. [18]. The decay $H \to Z\gamma$ first occurs at one-loop in the SM and analytic results are in Refs. [19, 20].

The calculations of the radiative corrections to $H \to ZZ$ and $H \to Z\gamma$ in the SMEFT proceed in the usual way [21] by the choice of a renormalization scheme, the definition of a suitable set of input parameters and the calculation of the 1PI amplitudes involved. However, since the SMEFT theory is only renormalizable order by order in the (v^2/Λ^2) expansion, we need to add an extra requirement and drop all terms proportional to $(v^2/\Lambda^2)^a$ with $a > 1$. These terms would need counterterms of dimension-8 that are not included in our study. Dropping them makes it possible for us to proceed with our renormalization program. The one-loop SMEFT calculation contains both tree level and one-loop contributions from the dimension-6 operators, along with the full electroweak one-loop SM amplitude. We chose a modified on shell (OS) scheme, where the SM parameters are OS quantities, while the SMEFT coefficients are defined as $\overline{MS}$ quantities.

The tadpole counterterms are defined such that they cancel completely the tadpole graphs [22]. This condition forces us to identify the renormalized vacuum to be the minimum of the renormalized scalar potential at each order of perturbation theory. Notice that, due to this choice, the intermediate quantities defined here are gauge dependent, while the final result is gauge independent as expected.

We choose the G_μ scheme, where we take the physical input parameters to be[2],

$$\begin{aligned} G_\mu &= 1.1663787(6) \times 10^{-5}\mathrm{GeV}^{-2} \\ M_Z &= 91.1876 \pm .0021\mathrm{GeV} \\ M_W &= 80.385 \pm .015 \ \mathrm{GeV} \\ M_H &= 125.09 \pm 0.21 \pm 0.11 \ \mathrm{GeV} \\ M_t &= 173.1 \pm 0.6 \ \mathrm{GeV}\,. \end{aligned}$$

Since the coefficients of the dimension-6 operators are not measured quantities, it is convenient to treat them as $\overline{MS}$ parameters, so the renormalized coefficients are $\mathcal{C}(\mu) = \mathcal{C}_0 -$ poles, where $\mathcal{C}_0$ are the bare quantities. The poles of the coefficients $\mathcal{C}_0$ are obtained from the renormalization group evolution of the coefficients computed in the unbroken phase of the theory in Refs. [7–9]. In general, one can write,

$$\mathcal{C}_i(\mu) = \mathcal{C}_{0,i} - \frac{1}{2\hat{\epsilon}}\frac{1}{16\pi^2}\gamma_{ij}\mathcal{C}_j, \tag{15}$$

where μ is the renormalization scale, γ_{ij} is the one-loop anomalous dimension,

$$\mu\frac{d\mathcal{C}_i}{d\mu} = \frac{1}{16\pi^2}\gamma_{ij}\mathcal{C}_j, \tag{16}$$

and $\hat{\epsilon}^{-1} \equiv \epsilon^{-1} - \gamma_E + \log(4\pi)$ is related to the regulator ϵ for integrals evaluated in $d = 4 - 2\epsilon$ dimensions.

At one-loop, the tree level parameters of the previous section (denoted with the subscript 0 in this section) must be renormalized. The renormalized SM masses are defined by,

$$M_V^2 = M_{0,V}^2 - \Pi_V(M_V^2), \tag{17}$$

where $\Pi_V(M_V^2)$ is the one-loop correction to the 2-point function of either Z or W computed on-shell. The gauge boson 2- point functions in the SMEFT can be found in Refs. [14, 23].

The one- loop relation between the vev and the Fermi constant is defined by the equation,

$$G_\mu + \frac{\mathcal{C}_{ll}}{\sqrt{2}\Lambda^2} - \sqrt{2}\frac{\mathcal{C}_{\phi l}^{(3)}}{\Lambda^2} \equiv \frac{1}{\sqrt{2}v_0^2}(1 + \Delta r), \tag{18}$$

[2] The light quark masses and lepton masses enter into the γ wave-function renormalization for $H \to Z\gamma$ and we take $m_b = 4.78$ GeV, $\mathrm{m_c} = 1.67$ GeV, $\mathrm{m_s} = 0.1$ GeV, $\mathrm{m_d} = 0.005$ GeV, $\mathrm{m_u} = 0.002$ GeV, $\mathrm{m}_\tau = 1.776$ GeV, $\mathrm{m}_\mu = 0.105$ GeV and $\mathrm{m_e} = 0.0005$ GeV. The effects of the light fermions are small here, so a more sophisticated analysis is not warranted.

where v_0 is the unrenormalized minimum of the potential and Δr is obtained from the one-loop corrections to μ decay and is given by

$$\begin{aligned}\Delta r &= 2v^2\mathcal{B} + \mathcal{V}(1+\frac{v^2}{\Lambda^2}\mathcal{C}_{\phi l}^{(3)}) + \mathcal{E}(1+\frac{v^2}{\Lambda^2}(2\,\mathcal{C}_{\phi l}^{(3)} - \mathcal{C}_{ll})) \\ &\quad -\frac{A_{WW}}{M_W^2}(1+2\frac{v^2}{\Lambda^2}\mathcal{C}_{\phi l}^{(3)}) + \frac{1}{16\pi^2}\frac{1}{2\hat{\epsilon}}\frac{v^2}{\Lambda^2}(2\gamma_{\phi l,j}^{(3)}\mathcal{C}_j - \gamma_{ll,j}\mathcal{C}_j). \end{aligned} \tag{19}$$

In Eq. (19), $\mathcal{B}$ is the box contribution, $\mathcal{V}$ is the vertex contribution, $A_{WW} = \Pi_W(0)$ is the W boson self-energy at zero momentum and $\mathcal{E}$ is the sum of the lepton (μ, e, ν_μ, ν_e) wave-function renormalizations. All the quantities are calculated at zero external momenta. Notice that the definition of Δr is modified with respect to the SM result due to the presence of dimension-6 operators in the tree-level relation between G_μ and v given in Eq. (18). Additionally, we absorb the poles of the coefficients $\mathcal{C}$ into the definition of Δr. The renormalization of the vev is then,

$$\begin{aligned} v^2 &= v_0^2 - \delta v^2 \\ \delta v^2 &= \frac{\Delta r}{\sqrt{2}G_\mu}\left(1 - \frac{1}{\sqrt{2}G_\mu\Lambda^2}\mathcal{C}_{ll} + \frac{\sqrt{2}}{G_\mu\Lambda^2}\mathcal{C}_{\phi l}^{(3)}\right). \end{aligned} \tag{20}$$

Analytic expressions for Δr in both the SM and the SMEFT at dimension-6 are given in Appendix C in the R_ξ.

In the following, we indicate with the symbol $\Delta\mathcal{A}^{\mu\nu}$ the sum of the contributions from the renormalization of the vev, the masses, and the SMEFT coefficients described above. The other contributions to the one-loop corrections are the proper one-particle irreducible amplitudes $\mathcal{A}_{1PI}^{\mu\nu}$, the particle reducible contributions $\mathcal{A}_{PR}^{\mu\nu}$ due to the Z/γ mixing which arises in the $H \to ZZ$ process, and the external leg wave-function renormalization $\delta Z_i = -\partial\Pi_i(k^2)/\partial k^2|_{M_i^2}$. The calculation of these contributions is relatively straightforward. We start with the R_ξ Feynman rules for the SMEFT in the Warsaw basis presented in [17] and convert them to a FeynArts [24] model file, using the FeynRules [25] routines, from which we obtain the amplitudes needed for our calculation. We reduce the integrals in terms of Passarino-Veltman integrals [26], using FeynCalc [27, 28] and lastly we use LoopTools [29] to numerically calculate the integrals. Once we compute all the terms that contribute, the one-loop correction can be simply written as

$$\mathcal{A}^{1l,\mu\nu} = \mathcal{A}_{1PI}^{\mu\nu} + \mathcal{A}^{0l,\mu\nu}\frac{1}{2}\sum_i \delta Z_i + \mathcal{A}_{PR}^{\mu\nu} + \Delta\mathcal{A}^{\mu\nu}. \tag{21}$$

We verified that $\mathcal{A}^{1l,\mu\nu}$ is UV and IR finite and we confirmed its gauge invariance by computing the amplitudes for the $H \to ZZ$ and $H \to Z\gamma$ processes in R_ξ gauge.

We conclude this section with a few remarks on the truncation of the expansion in loops and powers of $\frac{v^2}{\Lambda^2}$. To clarify our explanation we consider the generic form of the $H \to ZZ$ amplitude where we reintroduce the notation $\mathcal{C}^{(6)}$ and $\mathcal{C}^{(8)}$ for the coefficients of dimensions 6 and 8 operators:

$$\mathcal{A} \sim \hat{a}_0 g_{SM} + \hat{a}_1 \mathcal{C}^{(6)} \frac{v^2}{\Lambda^2} + \hat{a}_2 \frac{g_{SM}^3}{16\pi^2} + \hat{a}_3 \mathcal{C}^{(6)} \frac{v^2}{\Lambda^2} \frac{g_{SM}^2}{16\pi^2} + \frac{v^4}{\Lambda^4} (\hat{a}_4 (\mathcal{C}^{(6)})^2 \frac{g_{SM}}{16\pi^2} + \hat{a}_5 \mathcal{C}^{(8)}) + \ldots \tag{22}$$

In Eq. (22) we assumed the ordering $g_{SM} > \mathcal{C}^{(6)} \frac{v^2}{\Lambda^2} > \frac{g_{SM}^3}{16\pi^2} > \mathcal{C}^{(8)} \sim (\mathcal{C}^{(6)})^2 \frac{g_{SM}}{16\pi^2}$, and we ordered the terms from largest to smallest according to it. Note that in Eq. (22), we have only included one insertion of dimension-6 operators at tree level, $(\hat{a}_1)$. If we were considering a more complicated process, it would in general be possible to have two (or more) tree level insertions of dimension-6 operators, which would give a contribution of $\mathcal{O}(\frac{v^4}{\Lambda^4})$ that would be of the same order as terms we have included. The squared amplitude then is,

$$\begin{aligned} |\mathcal{A}|^2 \ &\sim\ \hat{a}_0^2 g_{SM}^2 + 2\hat{a}_0 \hat{a}_1 \mathcal{C}^{(6)} \frac{v^2}{\Lambda^2} g_{SM} + \hat{a}_1^2 (\mathcal{C}^{(6)})^2 \frac{v^4}{\Lambda^4} \\ &+\ 2\hat{a}_0 \hat{a}_2 \frac{g_{SM}^4}{16\pi^2} + 2(\hat{a}_1 \hat{a}_2 + \hat{a}_0 \hat{a}_3) \mathcal{C}^{(6)} \frac{v^2}{\Lambda^2} \frac{g_{SM}^3}{16\pi^2} \\ &+\ \frac{v^4}{\Lambda^4} g_{SM} (2(\hat{a}_0 \hat{a}_4 + \hat{a}_1 \hat{a}_3)(\mathcal{C}^{(6)})^2 \frac{g_{SM}}{16\pi^2} + (\hat{a}_0 \hat{a}_4) \mathcal{C}^{(8)}) + \ldots \end{aligned} \tag{23}$$

As explained at the beginning of this section, in order to obtain a result that is finite in the UV in the dimension-6 SMEFT, we need to drop terms that are of order $\frac{v^4}{\Lambda^4}$ in the amplitude $\mathcal{A}$. However Eqs. (22) and (23) show that at the squared amplitude level the requirement is slightly different: while dropping the terms $\sim (\mathcal{C}^{(6)})^2 \frac{v^4}{\Lambda^4}$ would be inconsistent since they are in principle larger than the SM one-loop contributions, we have to drop the terms $\sim (\mathcal{C}^{(6)})^2 \frac{v^4}{\Lambda^4} \frac{g_{SM}^2}{16\pi^2}$ that are of the same order of the contributions from dimension 8 operators. From a practical point of view, however, those terms are for the most part numerically irrelevant, and simply squaring the amplitude (22) after dropping the terms of order $\frac{v^4}{\Lambda^4}$ is a valid option. Lastly, we notice that the condition $\mathcal{C}^{(6)} \frac{v^2}{\Lambda^2} > \frac{g_{SM}^3}{16\pi^2}$ ensures that the fourth term in Eq. (22) is larger than a generic SM 2-loop contribution.

IV. RESULTS

A. $H \to ZZ$

In terms of the physical input parameters, M_W, M_Z and G_μ, the tree level SMEFT amplitude for the on-shell decay $H \to Z^\mu(p_1)Z^\nu(p_2)$ is,

$$\begin{aligned}\mathcal{A}^{0l,\mu\nu} &= \mathcal{A}_{0,SM}\biggl\{\biggl[T^{0l}_{SM}+\frac{1}{\Lambda^2}\Sigma_i\mathcal{C}_iT^{0l}_i\biggr]\biggl(g^{\mu\nu}-\frac{p_2^\mu p_1^\nu}{p_1\cdot p_2}\biggr)\\&\quad+\biggl[T^{0l}_{SM}+\frac{1}{\Lambda^2}\Sigma_i\mathcal{C}_i\hat{T}^{0l}_i\biggr]\biggl(\frac{p_2^\mu p_1^\nu}{p_1\cdot p_2}\biggr)\biggr\}, \qquad (24)\end{aligned}$$

where $\mathcal{A}_{0,SM}=2^{5/4}\sqrt{G_\mu}M_Z^2$, $T^{0l}_{SM}=1$, the sum is over all of the contributing Warsaw basis coefficients $\mathcal{C}_i$, and the tree level SMEFT contribution is,

$$\begin{aligned}\Sigma_iT^{0l}_i\mathcal{C}_i &= \frac{1}{\sqrt{2}G_\mu}\biggl(\frac{c_k}{2}\biggr)-\frac{2p_1\cdot p_2}{\sqrt{2}G_\mu M_Z^2}c_{ZZ}\,,\\ \Sigma_i\hat{T}^{0l}_i\mathcal{C}_i &= \frac{2}{\sqrt{2}G_\mu M_Z^2}c_{ZZ}\,. \qquad (25)\end{aligned}$$

Note that the tree level amplitude depends on only 2 combinations of coefficients,

$$\begin{aligned}c_k &\equiv \frac{\mathcal{C}_{\phi D}}{2}+2\mathcal{C}_{\phi\Box}+\mathcal{C}^{ll}_{\mu ee\mu}-2\mathcal{C}^{(3)}_{\phi l}\,,\\ c_{ZZ} &\equiv \biggl[\mathcal{C}_{\phi W}\frac{M_W^2}{M_Z^2}+(1-\frac{M_W^2}{M_Z^2})\mathcal{C}_{\phi B}+\frac{M_W}{M_Z}\sqrt{1-\frac{M_W^2}{M_Z^2}}\mathcal{C}_{\phi WB}\biggr]. \qquad (26)\end{aligned}$$

The combination c_{ZZ} is limited from global fits to the SMEFT contributions to Higgs decays. The 95% confidence level limit is [15][3]

$$-1.2\biggl(\frac{1\text{ TeV}}{\Lambda^2}\biggr)<c_{ZZ}<1.6\biggl(\frac{1\text{ TeV}}{\Lambda^2}\biggr). \qquad (27)$$

The tree level decay width in the SMEFT is,

$$\begin{aligned}\Gamma(H\to ZZ)^{0l}_{EFT} &= \frac{\beta G_\mu M_H^3}{16\pi\sqrt{2}}\left(12\,x^2-4\,x+1\right)\biggl\{1+\frac{1}{\sqrt{2}G_\mu\Lambda^2}c_k\biggr\}\\&\quad+\frac{3\,\beta M_H}{4\pi}\frac{M_Z^2}{\Lambda^2}c_{ZZ}(2x-1)\\&=\Gamma(H\to ZZ)_{SM}\biggl\{1+\frac{1}{\sqrt{2}G_\mu\Lambda^2}\biggl[c_k+\frac{24x(2x-1)}{12x^2-4x+1}c_{ZZ}\biggr]\biggr\}, \qquad (28)\end{aligned}$$

[3] Note the differing normalization of c_{ZZ} from Ref. [15].

where $\beta=\sqrt{1-\frac{4M_Z^2}{M_H^2}}$ and $x\equiv M_Z^2/M_H^2$. For M_H between $2M_Z$ and 200 GeV the dependence on M_H is minimal [4],

$$\begin{aligned} R^{0l} &\equiv \frac{\Gamma(H\rightarrow ZZ)^{0l}_{EFT}}{\Gamma(H\rightarrow ZZ)^{0l}_{SM}} \\ &\sim 1+\frac{1}{\sqrt{2}G_\mu\Lambda^2}\biggl[c_k-4c_{ZZ}\biggr]+\mathcal{O}\biggl(\mathcal{C}^2\frac{v^4}{\Lambda^4}\biggr)\,. \end{aligned} \tag{29}$$

Using the results of Appendix A, we can compare the on-shell tree level result of Eq. (29) with the off-shell result appropriate for the physical Higgs mass [30–32],

$$\begin{aligned} R^{off} &\equiv \frac{\Gamma(H\rightarrow Zf\overline{f})^{0l}}{\Gamma(H\rightarrow Zf\overline{f})_{SM}} \\ &\sim 1+\frac{1}{\sqrt{2}G_\mu\Lambda^2}\biggl[c_k-.97c_{ZZ}\biggr]+\mathcal{O}\biggl(\mathcal{C}^2\frac{v^4}{\Lambda^4}\biggr)\,. \end{aligned} \tag{30}$$

Comparing Eqs. (29) and (30), we note that the off-shell effects are large [33].

In the SMEFT, the dimension-6 NLO results contain both the complete SM 1−loop electroweak corrections to $H\rightarrow ZZ$ and the one-loop corrections from the dimension-6 SMEFT operators. The complete, renormalized NLO amplitude is,

$$\mathcal{A}^{\mu\nu}_{NLO}=\mathcal{A}^{0l,\mu\nu}+\mathcal{A}^{1l,\mu\nu}\,. \tag{31}$$

The one-loop SMEFT contribution to the amplitude can be expanded as,

$$\begin{aligned} \mathcal{A}^{1l,\mu\nu} &= \mathcal{A}_{0,SM}\biggl\{\biggl(T^{1l}_{SM}+\mathcal{F}_g\log\biggl(\frac{\Lambda^2}{M_Z^2}\biggr)+\frac{(1\ TeV)^2}{\Lambda^2}\sum_i T^{1l}_i\mathcal{C}_i\biggr)\biggl(g^{\mu\nu}-\frac{p_1^\nu p_2^\mu}{p_1\cdot p_2}\biggr) \\ &+ \biggl(\hat{T}^{1l}_{SM}+\mathcal{F}_p\log\biggl(\frac{\Lambda^2}{M_Z^2}\biggr)+\frac{(1\ TeV)^2}{\Lambda^2}\sum_i \hat{T}^{1l}_i\mathcal{C}_i\biggr)\frac{p_1^\nu p_2^\mu}{p_1\cdot p_2}\biggr\}\,, \end{aligned} \tag{32}$$

where the terms $\mathcal{F}_g\log\left(\frac{\Lambda^2}{M_Z^2}\right)$ and $\mathcal{F}_p\log\left(\frac{\Lambda^2}{M_Z^2}\right)$ contain the residual dependence on the renormalization scale, due to our choice of renormalization scheme. Retaining terms to $\mathcal{O}\left(\frac{v^2}{\Lambda^2}\right)$, we parametrize the exact one-loop SMEFT result,[4]

$$\begin{aligned} T^{1l}_i &= a_{0,i}+a_{1,i}\frac{M_H^2}{M_Z^2}+a_{2,i}\biggl(\frac{M_H^2}{M_Z^2}\biggr)^2+a_{3,i}\log\biggl(\frac{M_H^2}{M_Z^2}\biggr)+a_{4,i}\log\biggl(\frac{4M_t^2-M_H^2}{M_Z^2}\biggr) \\ \hat{T}^{1l}_i &= b_{0,i}+b_{1,i}\frac{M_H^2}{M_Z^2}+b_{2,i}\biggl(\frac{M_H^2}{M_Z^2}\biggr)^2+b_{3,i}\log\biggl(\frac{M_H^2}{M_Z^2}\biggr)+a_{4,i}\log\biggl(\frac{4M_t^2-M_H^2}{M_Z^2}\biggr)\,. \end{aligned} \tag{33}$$

[4] This fit is valid for $M_H<2M_t$.

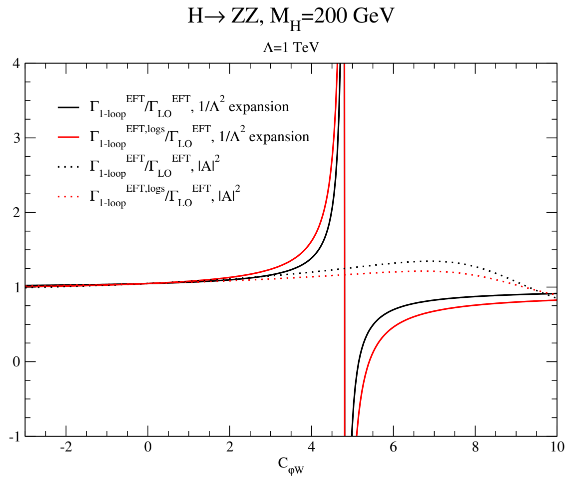

FIG. 1: Dependence of $H \to ZZ$ decay width on $\mathcal{C}_{\phi W}$ in different expansions as explained in the text.

Numerical values for the fit parameters, together with the analytical expressions for $\mathcal{F}_g$ and $\mathcal{F}_p$ are given in Appendix B.

The SM result is easily recovered,

$$\mathcal{A}_{SM}^{1l,\mu\nu} = \mathcal{A}_{0,SM}\left\{T_{SM}^{1l}\left(g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}\right) + \hat{T}_{SM}^{1l}\frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}\right\}, \tag{34}$$

where $T_{SM}^{1l} = a_{0,SM}$ and $\hat{T}_{SM}^{1l} = b_{0,SM}$. We have verified that our one-loop electroweak SM corrections are in agreement with previous results [18].

In Fig. 1, we illustrate the dependence of the decay widths on the terms retained in the expansion of Eq. (23). The curves labelled "$1/\Lambda^2$ expansion" drop the $\hat{a}_1^2$ and $(\hat{a}_0\hat{a}_4 + \hat{a}_1\hat{a}_3)$ contributions in Eq. (23). It is apparent that for large values of the $\mathcal{C}$ (here $\mathcal{C}_{\phi W}$), the expansion is nonsense. For $\mathcal{C}_{\phi W} > 2$, the dimension-6 approximation breaks down and the dimension-6 approximation to the amplitude-squared becomes negative. The curves labeled $\mid \mathcal{A} \mid^2$ contain all of the terms in the square of Eq. (22) (with $\hat{a}_4 \to 0$) and are well behaved for all $\mathcal{C}$. For $\mathcal{C}_{\phi W} < 2$, the full amplitude-squared is well approximated by the terms linear in the coefficient functions and the SMEFT dimension-6 approximation is valid. We have checked that this condition is satisfied for the following plots in this section.

In Fig. 2, we show the contribution relative to the LO SM prediction for representative values of the SMEFT coefficients that contribute at tree level. Choosing all coefficients positive, for the parameters we have chosen, inclusion of the tree level SMEFT coefficients decreases the rate by $\sim 2\%$ (red curve) . The SM one-loop corrections (black curve) increase

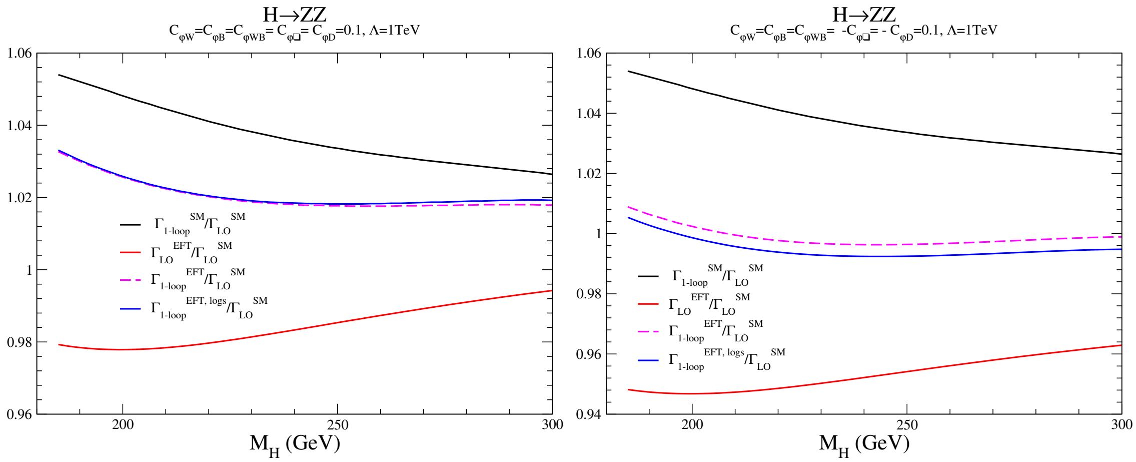

FIG. 2: Dependence of the $H \to ZZ$ decay width on SMEFT coefficients that contribute at tree level. Note that the curve labeled 1−loop EFT is the complete SMEFT result and contains both the one-loop SM result and the one-loop contribution from the dimension-6 SMEFT coefficients. Coefficients not specified are set to 0.

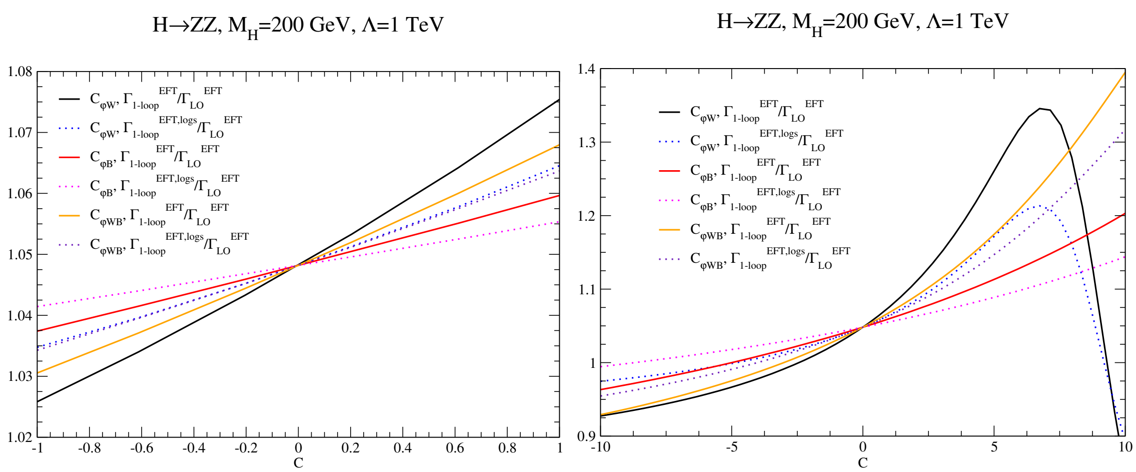

FIG. 3: Dependence of $H \to ZZ$ decay width on SMEFT coefficients. Coefficients are varied one at a time and coefficients not specified are set to 0.

the rate by roughly 5%, leading to a partial cancellation between the 1-loop SM and tree level SMEFT contributions. This makes it clear that global fits to SMEFT coefficients that do not contain the electroweak corrections cannot be more accurate than $\sim \mathcal{O}(5\%)$. The one loop corrections from the the SMEFT operators are much smaller than the SM electroweak corrections. In this example, the complete NLO SMEFT calculation is well approximated by including only the logarithmic contributions from the SMEFT coefficients.

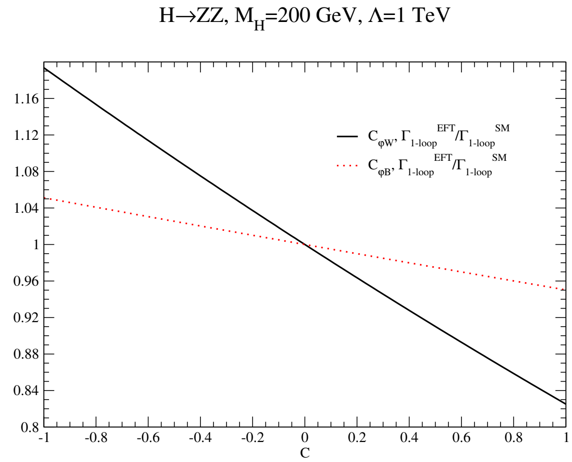

FIG. 4: Same as Fig. 3, but normalized to Γ^{SM}_{1-loop}.

On the RHS of Fig. 2, we flip the sign of the SMEFT coefficients, which increases their numerical significance. Note that the dependence on M_H is rather mild.

In Figs. 3 and 5 , we show the dependence on SMEFT coefficients as a function of the $\mathcal{C}$ for fixed $M_H = 200$ GeV, normalized to the LO SMEFT result. Fig. 3 shows the dependence on coefficients that enter at tree level and in Fig. 4, we replot the same results, normalized to the one-loop SM. The change from the one-loop SM results in the SMEFT is of $\mathcal{O}(5\%)$ for $\mathcal{C}_{\phi B} \sim \mathcal{O}(-1)$ and for $\mathcal{C}_{\phi W} \sim \mathcal{O}(-1)$ this change is $\mathcal{O}(20\%)$. These values are consistent with current fits to LHC Higgs decays to ZZ.

Fig. 5 shows the dependence on a selection of coefficient functions that do not enter at tree level. An interesting feature of our results is that they can be used to obtain limits on coefficients that first enter at loop level. For example, from a global fit [15, 16, 34],

$$\begin{aligned}\mathcal{C}_W &= \frac{\Lambda^2}{v^2}(1.14 \pm .68) \\ &\rightarrow (18.8 \pm 11.2)\left(\frac{\Lambda^2}{1\ \mathrm{TeV}^2}\right). \end{aligned} \tag{35}$$

Such a large value of $\mathcal{C}_W$ would increase the LO SMEFT width to ZZ by $\sim 20\%$ as observed in Fig. 5. Loop corrections to Higgs decays therefore have the possibility of new constraints on the SMEFT coefficients.

Our corrections to the on-shell $H \to ZZ$ one-loop SMEFT result must be considered as a first step in a full SMEFT calculation for $H \to Zf\overline{f}$. Our results suggest, however, that the results of Figs. 3 and 5 can be thought of as K factors that multiply the LO SMEFT off-shell result of Eq. (30). The numerical size of our results implies that a full SMEFT

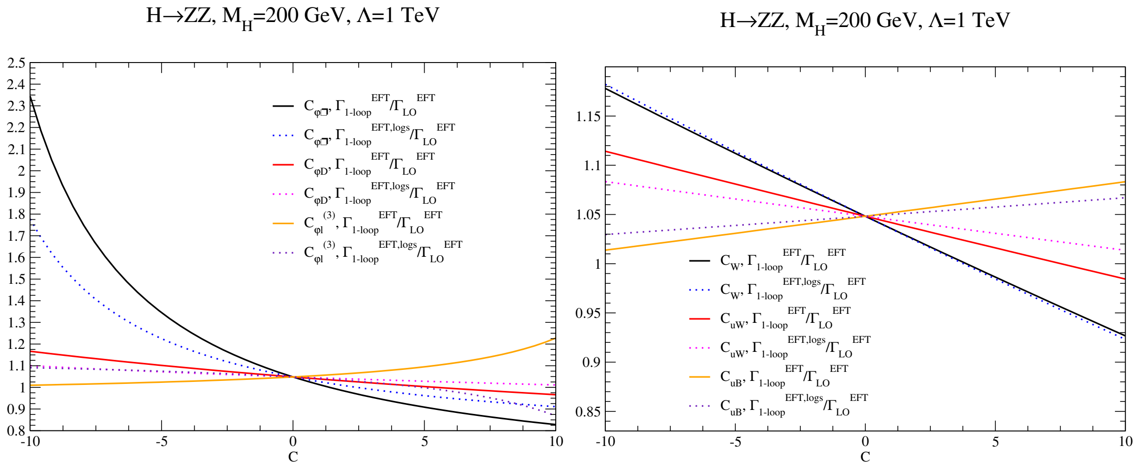

FIG. 5: Dependence of the one-loop corrected SMEFT width on various $\mathcal{C}$ for $M_H = 200\ GeV$.

calculation is needed for $H \to Zf\overline{f}$ in order to obtain reliable results. The SM electroweak corrections for $H \to Zf\overline{f}$ are known and can be implemented using the PROPHECY4F program [35, 36] .

B. $H \to Z\gamma$

The one-loop SMEFT results for $H \to Z^\mu(p_1)\gamma^\nu(p_2)$ can be obtained in a straightforward manner from the results of the previous section. At tree level, there is an SMEFT contribution, while the SM contribution begins at one- loop. The Ward identity for the photon requires $p^\nu \cdot \mathcal{A}_{\mu\nu} = 0$, so the tensor structure is fixed,

$$\mathcal{A}^{\mu\nu}_{Z\gamma} = \left[\mathcal{A}^{0l}_{Z\gamma,EFT} + \mathcal{A}^{1l}_{Z\gamma,SM} + \mathcal{A}^{1l}_{Z\gamma,EFT}\right]\left(g_{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}\right). \tag{36}$$

The tree level SMEFT contribution is

$$\mathcal{A}^{0l}_{Z\gamma,EFT} = -2^{3/4} G_\mu^{-1/2}\left(\frac{M_H^2 - M_Z^2}{\Lambda^2}\right) c_{Z\gamma}\,, \tag{37}$$

with

$$c_{Z\gamma} \equiv \frac{M_W}{M_Z}\sqrt{1 - \frac{M_W^2}{M_Z^2}}\left(\mathcal{C}_{\phi W} - \mathcal{C}_{\phi B}\right) + \left(1 - 2\frac{M_W^2}{M_Z^2}\right)\mathcal{C}_{\phi WB}\,. \tag{38}$$

Numerically, for $M_H = 125$ GeV and $\Lambda = 1$ TeV,

$$\mathcal{A}^{0l}_{Z\gamma,EFT} = 1.5\ \text{GeV}\left(\mathcal{C}_{\phi \text{W}} - \mathcal{C}_{\phi \text{B}} - 1.3\mathcal{C}_{\phi \text{WB}}\right)\left(\frac{1\ \text{TeV}}{\Lambda^2}\right). \tag{39}$$

The SM contribution is well-known,

$$
\begin{aligned}
\mathcal{A}^{1l}_{Z\gamma,SM} &= \left(\frac{M_H^2-M_Z^2}{2}\right)\frac{M_W^2}{\pi^2}G_\mu^{3/2}2^{-1/4}\sqrt{1-\frac{M_W^2}{M_Z^2}}\Big\{\Sigma_f N_c\frac{Q_f M_Z}{M_W}v_f A_{1/2}(\tau_f,\lambda_f) \\
&\quad +A_1(\tau_W,\lambda_W)\Big\} \\
&= 0.209\text{ GeV for }\mathrm{M_H}=125\text{ GeV}\,,
\end{aligned}
\tag{40}
$$

where the sum is over all fermions, $N_c = 1(3)$ for leptons (quarks), $\tau_i = 4M_i^2/M_H^2$, $\lambda_i = 4M_i^2/M_Z^2$, $v_f = 2T_f^3 - 4Q_f\left(1-\frac{M_W^2}{M_Z^2}\right)$, and analytic expressions for the functions A_1 and $A_{1/2}$ can be found in Refs. [19, 20, 37]

We report our one-loop SMEFT corrections to the $H \to Z\gamma$ amplitude, $\mathcal{A}^{1l}_{Z\gamma}$, numerically for $M_H = 125$ GeV,

$$
\begin{aligned}
\mathcal{A}^{1l}_{Z\gamma,EFT} &= \frac{(1TeV)^2}{\Lambda^2}\Big\{-0.038\,\mathcal{C}^{(3)}_{\phi l}+0.00185(\mathcal{C}^{(1)}_{\phi q}-\mathcal{C}^{(3)}_{\phi q}+\mathcal{C}_{\phi u})-0.0126\,\mathcal{C}_{\phi D} \\
&\quad +0.0127\,\mathcal{C}_{\phi\Box}+0.000753\,\mathcal{C}_{u\phi}+0.019\,\mathcal{C}_{ll}+(0.00778-0.0362\log\frac{\Lambda^2}{M_Z^2})\mathcal{C}_{\phi B} \\
&\quad +(-0.00158+0.0154\log\frac{\Lambda^2}{M_Z^2})\mathcal{C}_{\phi W}+(-0.0524-0.0269\log\frac{\Lambda^2}{M_Z^2})\mathcal{C}_{\phi WB} \\
&\quad +(-0.00999+0.0042\log\frac{\Lambda^2}{M_Z^2})\mathcal{C}_{uB}+(0.0669-0.0295\log\frac{\Lambda^2}{M_Z^2})\mathcal{C}_{uW} \\
&\quad +(0.00559-0.0213\log\frac{\Lambda^2}{M_Z^2})\mathcal{C}_{W}\Big\}\,.
\end{aligned}
\tag{41}
$$

(Note that Eq. (41) is in GeV.) An interesting feature of Eq. (41) is the dependence on coefficients not arising at tree level.

The contribution from the SMEFT operators to $H \to Z\gamma$ at 1− loop is much smaller than the SM 1−loop contribution, due to the $\frac{v^2}{\Lambda^2}$ suppression. The contribution to the amplitude, Eq. (41) is shown in Fig. 6

V. CONCLUSIONS

We have computed the one-loop electroweak corrections in the SMEFT to the on-shell $H \to ZZ$ process. Numerically, the logarithmic SMEFT contributions dominate over the finite NLO contributions for most of parameter space. Appendix B contains a numerical fit to the finite NLO SMEFT contributions and Appendix C has analytic results for the

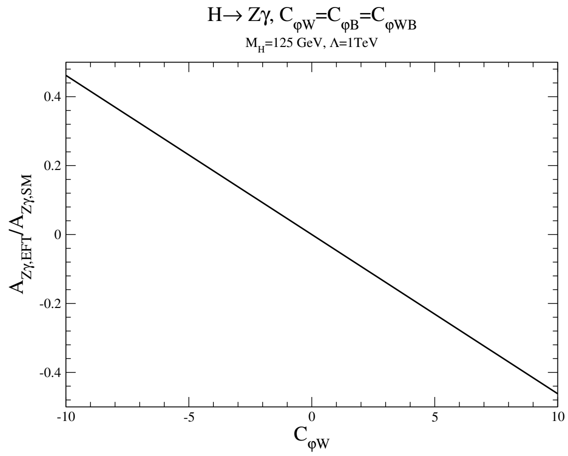

FIG. 6: Contribution to $H \to Z\gamma$ in the SMEFT. The vertical axis is the ratio of the one-loop contribution of Eq. (41) to the SM one-loop result of Eq. (40).

logarithmic contributions to $H \to ZZ$. Our complete result can be obtained from the ancillary files posted with this paper and at https://quark.phy.bnl.gov/Digital_Data_Archive/giardino/zzNLO_18. The calculation of the on-shell decay is a first step toward a full NLO calculation of the physical $H \to Zf\overline{f}$ process and our results can be used to approximate a K factor for the SMEFT $H \to Zf\overline{f}$ decay.

The full $H \to Z\gamma$ SMEFT NLO result is presented as a byproduct of our calculation. Finally, the complete result for the one-loop SMEFT renormalization of G_μ is given for the first time.

Appendix A: Off-shell Production

The decay width for the off-shell decay, $H \to f_1(p_1)f_2(p_2)Z(p_3)$, is

$$\Gamma = \int_0^{(M_H-M_Z)^2} dq^2 \int dm_{23}^2 \frac{|A|^2}{256\pi^3 M_H^3}, \tag{A1}$$

where $m_{ij} = (p_i+p_j)^2$, $m_{12}^2 \equiv q^2$, $m_{12}^2 + m_{23}^2 + m_{13}^2 = M_H^2 + M_Z^2$, $\lambda(M_H^2, M_Z^2, q^2) \equiv q^4 - 2q^2(M_H^2+M_Z^2) + (M_H^2 - M_Z^2)^2$, and $m_{23}^2 \mid_{max,min} \equiv \frac{1}{2}\biggl(M_H^2 + M_Z^2 - q^2 \pm \sqrt{\lambda}\biggr)$. We write the amplitude-squared to $\mathcal{O}(\frac{1}{\Lambda^2})$ in the SMEFT as,

$$|A_{EFT}|^2 = |A_{SM}|^2 + |\delta A_{EFT}|^2 + \mathcal{O}\biggl(\mathcal{C}^2 \frac{v^4}{\Lambda^4}\biggr) \tag{A2}$$

where,

$$
\begin{aligned}
\mid A_{SM}\mid^2 &= 32\,(g_L^2+g_R^2)\,G_F^2\,M_Z^4\left[\frac{2M_Z^2q^2-m_{13}^2q^2-M_H^2M_Z^2+m_{13}^2M_Z^2+m_{13}^2\,M_H^2-m_{13}^4}{(q^2-M_Z^2)^2+\Gamma_Z^2M_Z^2}\right]\\
\mid \delta A_{EFT}\mid^2 &= \mid A_{SM}\mid^2\,\frac{1}{\sqrt{2}G_F\Lambda^2}c_k\\
&\quad +64(g_L^2+g_R^2)\frac{\sqrt{2}G_F}{\Lambda^2}M_Z^4\frac{q^2(q^2+M_Z^2-M_H^2)}{(q^2-M_Z^2)^2+\Gamma_Z^2M_Z^2}\,c_{ZZ}\,. \qquad \text{(A3)}
\end{aligned}
$$

Note that we have not included effects due to possible anomalous $H\to Z\gamma$ vertices here, although they are included in the results of Sec. IV B.

Integrating over dm_{23}^2,

$$
\begin{aligned}
\frac{d\Gamma}{dq^2}\mid_{SM} &= (g_L^2+g_R^2)\,G_F^2\,\sqrt{\lambda(M_H^2,M_Z^2,q^2)}\frac{M_Z^4}{48\pi^3M_H^3}\left[\frac{(12M_Z^2q^2+\lambda(M_H^2,M_Z^2,q^2))}{(q^2-M_Z^2)^2+\Gamma_Z^2M_Z^2}\right]\\
\frac{d\Gamma}{dq^2}\mid_{EFT} &= \frac{d\Gamma}{dq^2}\mid_{SM}\left\{1+\frac{1}{\sqrt{2}G_F\Lambda^2}c_k\right\}\\
&\quad +(g_L^2+g_R^2)\,\frac{G_F}{\Lambda^2}\,\sqrt{\lambda(M_H^2,M_Z^2,q^2)}\frac{M_Z^4}{2\sqrt{2}\pi^3M_H^3}\\
&\quad \cdot\frac{q^2(3M_Z^2+M_H^2)-(M_Z^2-M_H^2)^2+\lambda(M_H^2,M_Z^2,q^2)}{(q^2-M_Z^2)^2+\Gamma_Z^2M_Z^2}c_{ZZ}\,. \qquad \text{(A4)}
\end{aligned}
$$

Finally, integrating over q^2 and setting $M_H = 125$ GeV,

$$
\begin{aligned}
R^{off} &\equiv \frac{\int\ dq^2\ d\Gamma/dq^2\mid_{EFT}}{\int\ dq^2\ d\Gamma/dq^2_{SM}}\\
&\sim\ 1+\frac{1}{\sqrt{2}G_F\Lambda^2}\left\{c_k-.97c_{ZZ}\right\} \qquad \text{(A5)}
\end{aligned}
$$

This is in agreement with Refs. [31, 38, 39]. Note that if we require a minimum $q^2\equiv q^2_{cut}$, the result is altered. Define,

$$
\begin{aligned}
R^{off}(q^2_{cut}) &\equiv \frac{\int_{q^2_{cut}}^{(M_H-M_Z)^2}\ dq^2\ d\Gamma/dq^2\mid_{EFT}}{\int_{q^2_{cut}}^{(M_H-M_Z)^2}\ dq^2\ d\Gamma/dq^2_{SM}}\\
&= 1+\frac{1}{\sqrt{2}G_F\Lambda^2}\left\{c_k+f(q_2^{cut})c_{ZZ}\right\}. \qquad \text{(A6)}
\end{aligned}
$$

The effects of the q^2_{cut} are shown in Fig. 7. Refs. [30, 33] note the the numerically large effect of off-shell Z's, which is seen clearly in the sensitivity to the q^2_{cut}.

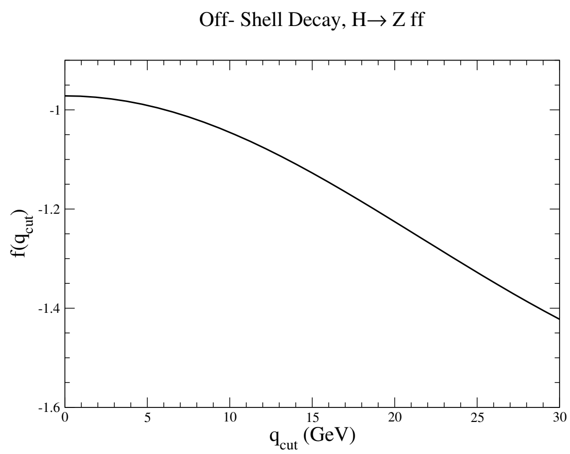

FIG. 7: Dependence of the off-shell decay rate on the q_{cut} defined in Eq. A6.

Appendix B: Numerical values for the on-shell decay $H \to ZZ$

Numerical values for the 1−loop SMEFT on-shell decay $H \to ZZ$ in terms of the parametrization of Eq. 33 are given in Tables 2-3.

	a_0	a_1	a_2	a_3	a_4
SM	-3.09×10^{-2}	-1.41×10^{-2}	5.43×10^{-4}	4.93×10^{-2}	1.57×10^{-2}
$\mathcal{C}_W$	-3.08×10^{-3}	-8.96×10^{-5}	4.76×10^{-5}	1.43×10^{-4}	8.29×10^{-4}
$\mathcal{C}_\phi$	-2.12×10^{-3}	-2.11×10^{-4}	6.59×10^{-6}	1.41×10^{-3}	1.2×10^{-4}
$\mathcal{C}_{\phi\Box}$	2.99×10^{-3}	2.11×10^{-4}	2.9×10^{-5}	-1.14×10^{-4}	1.23×10^{-3}
$\mathcal{C}_{\phi D}$	1.42×10^{-3}	-1.63×10^{-5}	7.09×10^{-6}	4.86×10^{-4}	1.51×10^{-4}
$\mathcal{C}_{u\phi}$	1.56×10^{-3}	-6.49×10^{-6}	1.3×10^{-6}	3.29×10^{-5}	-4.65×10^{-4}
$\mathcal{C}_{\phi W}$	1.14×10^{-2}	8.69×10^{-4}	-1.75×10^{-4}	-1.02×10^{-2}	-7.56×10^{-4}
$\mathcal{C}_{\phi B}$	3.52×10^{-3}	2.64×10^{-5}	-8.25×10^{-5}	-2.02×10^{-3}	-5.46×10^{-4}
$\mathcal{C}_{\phi WB}$	5.88×10^{-3}	-1.63×10^{-3}	-5.93×10^{-5}	1.62×10^{-3}	-8.97×10^{-4}
$\mathcal{C}_{uW}$	-1.72×10^{-3}	7.28×10^{-4}	-4.99×10^{-6}	-5.86×10^{-5}	1.36×10^{-4}
$\mathcal{C}_{uB}$	9.2×10^{-4}	-3.9×10^{-4}	2.67×10^{-6}	3.14×10^{-5}	-7.3×10^{-5}
$\mathcal{C}^{(1)}_{\phi l}$	6.82×10^{-7}	0	0	0	0
$\mathcal{C}^{(3)}_{\phi l}$	3.56×10^{-3}	2.56×10^{-3}	-9.87×10^{-5}	-8.97×10^{-3}	-2.85×10^{-3}
$\mathcal{C}_{\phi e}$	6.26×10^{-5}	0	0	0	0
$\mathcal{C}^{(1)}_{\phi q}$	4.45×10^{-3}	-2.23×10^{-5}	-9.49×10^{-7}	-1.64×10^{-5}	5.43×10^{-5}
$\mathcal{C}^{(3)}_{\phi q}$	-6.06×10^{-3}	2.23×10^{-5}	9.49×10^{-7}	1.64×10^{-5}	-5.43×10^{-5}
$\mathcal{C}_{\phi u}$	-4.71×10^{-3}	5.2×10^{-5}	2.14×10^{-6}	3.67×10^{-5}	-1.2×10^{-4}
$\mathcal{C}_{\phi d}$	6.26×10^{-5}	0	0	0	0
$\mathcal{C}_{ll}$	-3.17×10^{-3}	-1.28×10^{-3}	4.93×10^{-5}	4.49×10^{-3}	1.43×10^{-3}
$\mathcal{C}^{(3)}_{lq}$	8.9×10^{-4}	0	0	0	0

TABLE II: Numerical values for the coefficients defined in Eq. 33 relevant for the on-shell process $H \to ZZ$.

	b_0	b_1	b_2	b_3	b_4
SM	-2.68×10^{-2}	-6.16×10^{-3}	2.83×10^{-4}	2.49×10^{-2}	1.03×10^{-2}
$\mathcal{C}_W$	-1.6×10^{-3}	-4.44×10^{-4}	1.62×10^{-5}	1.76×10^{-3}	3.44×10^{-4}
$\mathcal{C}_\phi$	-2.04×10^{-3}	-1.88×10^{-4}	5.84×10^{-6}	1.26×10^{-3}	1.06×10^{-4}
$\mathcal{C}_{\phi\Box}$	3.2×10^{-3}	7.73×10^{-4}	1.15×10^{-5}	-1.74×10^{-3}	8.63×10^{-4}
$\mathcal{C}_{\phi D}$	1.51×10^{-3}	1.23×10^{-4}	3.13×10^{-6}	8.87×10^{-5}	4.19×10^{-5}
$\mathcal{C}_{u\phi}$	1.52×10^{-3}	-7.6×10^{-6}	1.08×10^{-6}	2.86×10^{-5}	-4.49×10^{-4}
$\mathcal{C}_{\phi W}$	6.37×10^{-3}	1.07×10^{-3}	-3.92×10^{-5}	-5.3×10^{-3}	-8.58×10^{-4}
$\mathcal{C}_{\phi B}$	7.33×10^{-4}	9.47×10^{-5}	-3.39×10^{-6}	-4.85×10^{-4}	-7.1×10^{-5}
$\mathcal{C}_{\phi WB}$	8.71×10^{-4}	-1.19×10^{-4}	6.47×10^{-6}	1.17×10^{-5}	1.92×10^{-4}
$\mathcal{C}_{uW}$	1.09×10^{-5}	-4.3×10^{-8}	-1.47×10^{-8}	-3.18×10^{-7}	1.48×10^{-6}
$\mathcal{C}_{uB}$	-5.85×10^{-6}	2.3×10^{-8}	7.87×10^{-9}	1.7×10^{-7}	-7.93×10^{-7}
$\mathcal{C}_{\phi l}^{(1)}$	6.82×10^{-7}	0	0	0	0
$\mathcal{C}_{\phi l}^{(3)}$	2.8×10^{-3}	1.12×10^{-3}	-5.14×10^{-5}	-4.52×10^{-3}	-1.88×10^{-3}
$\mathcal{C}_{\phi e}$	6.26×10^{-5}	0	0	0	0
$\mathcal{C}_{\phi q}^{(1)}$	4.36×10^{-3}	-3.33×10^{-5}	-1.75×10^{-6}	-3.11×10^{-5}	1.06×10^{-4}
$\mathcal{C}_{\phi q}^{(3)}$	-5.97×10^{-3}	3.33×10^{-5}	1.75×10^{-6}	3.11×10^{-5}	-1.06×10^{-4}
$\mathcal{C}_{\phi u}$	-4.71×10^{-3}	3.33×10^{-5}	1.75×10^{-6}	3.11×10^{-5}	-1.06×10^{-4}
$\mathcal{C}_{\phi d}$	6.26×10^{-5}	0	0	0	0
$\mathcal{C}_{ll}$	-2.79×10^{-3}	-5.6×10^{-4}	2.57×10^{-5}	2.26×10^{-3}	9.4×10^{-4}
$\mathcal{C}_{lq}^{(3)}$	8.9×10^{-4}	0	0	0	0

TABLE III: Numerical values for the coefficients defined in Eq. 33 relevant for the on-shell process $H \to ZZ$.

Appendix C: Analytical expressions of $\mathcal{F}_g$ and $\mathcal{F}_p$

Here we report the explicit results for the coefficients $\mathcal{F}_g$ and $\mathcal{F}_p$ introduced in eq. (32)

$$
\begin{aligned}
\mathcal{F}_g \;=\;& \frac{1}{16\pi^2}\frac{1}{\Lambda^2}\Bigg(\frac{12\sqrt[4]{2}\sqrt{G_\mu}\mathcal{C}_W M_W^3\left(M_H^2-2M_Z^2\right)\left(M_Z^2-6M_W^2\right)}{M_Z^4}\\
&+\; \mathcal{C}_{\phi\Box}(-6M_H^2-12M_t^2+9M_W^2-\frac{2M_Z^2}{3})+\frac{1}{12}\mathcal{C}_{\phi D}\left(-9M_H^2-36M_t^2+8M_W^2-35M_Z^2\right)\\
&+\; \frac{\mathcal{C}_{\phi W}M_W^2\left(M_H^2-2M_Z^2\right)\left(9M_H^2+18M_t^2-56M_W^2+3M_Z^2\right)}{3M_Z^4}\\
&+\; \frac{\mathcal{C}_{\phi B}\left(M_H^2-2M_Z^2\right)\left(M_Z^2-M_W^2\right)\left(9M_H^2+18M_t^2-100M_W^2+85M_Z^2\right)}{3M_Z^4}\\
&+\; \frac{\mathcal{C}_{\phi WB}M_W\left(M_H^2-2M_Z^2\right)\sqrt{M_Z^2-M_W^2}\left(3M_H^2+18M_t^2-42M_W^2+56M_Z^2\right)}{3M_Z^4}\\
&+\; \frac{2\sqrt{2}\,\mathcal{C}_{uW}M_tM_W\left(2M_Z^2-M_H^2\right)\left(8M_W^2-5M_Z^2\right)}{M_Z^4}\\
&+\; \frac{2\sqrt{2}\,\mathcal{C}_{uB}M_t\left(M_H^2-2M_Z^2\right)\left(8M_W^2-5M_Z^2\right)\sqrt{M_Z^2-M_W^2}}{M_Z^4}\\
&+\; \mathcal{C}_{\phi l}^{(1)}\left(8M_Z^2-8M_W^2\right)+\mathcal{C}_{\phi l}^{(3)}\left(6M_t^2-20M_W^2\right)+\mathcal{C}_{\phi e}\left(8M_Z^2-8M_W^2\right)\\
&+\; \mathcal{C}_{\phi q}^{(1)}\left(-12M_t^2+8M_W^2-8M_Z^2\right)+6\,\mathcal{C}_{\phi q}^{(3)}\left(3M_t^2-4M_W^2\right)+4\,\mathcal{C}_{\phi u}\left(3M_t^2+4M_W^2-4M_Z^2\right)\\
&+\; \mathcal{C}_{\phi d}\left(8M_Z^2-8M_W^2\right)-3\,\mathcal{C}_{ll}M_Z^2-6\,\mathcal{C}_{lq}^{(3)}M_t^2\Bigg) \qquad \text{(C1)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_p \;=\;& \frac{1}{16\pi^2}\frac{1}{\Lambda^2}\Bigg(\mathcal{C}_{\phi\Box}(-6M_H^2-12M_t^2+9M_W^2-\frac{2M_Z^2}{3})+\frac{1}{12}\mathcal{C}_{\phi D}\left(-9M_H^2-36M_t^2+8M_W^2-35M_Z^2\right)\\
&+\; 8\,\mathcal{C}_{\phi l}^{(1)}\left(M_Z^2-M_W^2\right)+\mathcal{C}_{\phi l}^{(3)}\left(6M_t^2-20M_W^2\right)+8\,\mathcal{C}_{\phi e}\left(M_Z^2-M_W^2\right)\\
&+\; \mathcal{C}_{\phi q}^{(1)}\left(-12M_t^2+8M_W^2-8M_Z^2\right)+6\,\mathcal{C}_{\phi q}^{(3)}\left(3M_t^2-4M_W^2\right)+4\,\mathcal{C}_{\phi u}\left(3M_t^2+4M_W^2-4M_Z^2\right)\\
&+\; 8\,\mathcal{C}_{\phi d}\left(M_Z^2-M_W^2\right)-3\,\mathcal{C}_{ll}M_Z^2-6\,\mathcal{C}_{lq}^{(3)}M_t^2\Bigg) \qquad \text{(C2)}
\end{aligned}
$$

Appendix D: Analytical expression for Δr

Here we report our result for Δr in R_ξ gauge:

$$
16\pi^2\Delta r = \frac{\sqrt{2}G_\mu}{2}\Delta r_{\rm SM}+\frac{1}{\Lambda^2}\Delta r_{\rm EFT}, \qquad \text{(D1)}
$$

where $\Delta r_{\rm SM}$ and $\Delta r_{\rm EFT}$ are

$$
\begin{aligned}
\Delta r_{\rm SM} \;=\;& \Bigg(\frac{6\,A_0\left(M_W^2\right)\left(M_H^2\left(M_W^2-2M_Z^2\right)+M_W^2\left(3M_Z^2-2M_W^2\right)\right)}{\left(M_H^2-M_W^2\right)\left(M_W^2-M_Z^2\right)}+4A_0\left(M_W^2\xi\right)+\frac{6M_W^2\,A_0\left(M_H^2\right)}{M_H^2-M_W^2}\\
&-\; 12\,A_0\left(M_t^2\right)+\frac{6\,A_0\left(M_Z^2\right)\left(2M_W^2-M_Z^2\right)}{M_W^2-M_Z^2}+2A_0\left(M_Z^2\xi\right)-M_H^2+6M_t^2-2M_W^2-M_Z^2\Bigg) \text{(D2)}
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta r_{\rm EFT} \;=\; & \frac{1}{\hat{\epsilon}}\left(4\,\mathcal{C}_{\phi l}^{(3)}\left(3M_t^2-4M_W^2\right)+24\,\mathcal{C}_{\phi q}^{(3)}M_W^2+\frac{2}{3}\,\mathcal{C}_{\phi\Box}M_W^2-6\,\mathcal{C}_{ll}M_Z^2-12\mathcal{C}_{lq}^{(3)}M_t^2\right)\\
+\; & \Delta r_{\rm SM}\left(\frac{1}{4}\mathcal{C}_{\phi D}-\mathcal{C}_{\phi l}^{(3)}+\frac{1}{2}\mathcal{C}_{ll}\right)+\frac{6M_W^2\left({\rm A}_0\left(M_H^2\right)-{\rm A}_0\left(M_W^2\right)\right)-M_H^4+M_W^4}{M_H^2-M_W^2}\left(\mathcal{C}_{\phi\Box}-\frac{1}{2}\mathcal{C}_{\phi D}\right)\\
+\; & 6\left(M_t^2-2{\rm A}_0\left(M_t^2\right)\right)\left(\mathcal{C}_{\phi l}^{(3)}+\mathcal{C}_{\phi q}^{(3)}-\mathcal{C}_{lq}^{(3)}-\frac{1}{4}\mathcal{C}_{\phi D}\right)+6{\rm A}_0\left(M_Z^2\right)\left(\mathcal{C}_{ll}+c_W^2\left(4C_{\phi l}^{(3)}-\mathcal{C}_{\phi D}\right)\right)\\
-\; & 12M_Z^2\left(c_W s_W\mathcal{C}_{\phi WB}-\frac{5}{3}c_W^2\mathcal{C}_{\phi l}^{(3)}-\frac{5}{12}\mathcal{C}_{ll}-\frac{1}{6}\mathcal{C}_{\phi D}\right)\\
+\; & 12\left({\rm A}_0\left(M_W^2\right)-c_W^2{\rm A}_0\left(M_Z^2\right)\right)\left(\mathcal{C}_{ll}+\mathcal{C}_{\phi l}^{(3)}+\mathcal{C}_{\phi l}^{(1)}\right)\\
-\; & \left({\rm A}_0\left(\xi M_W^2\right)+{\rm A}_0\left(\xi M_Z^2\right)+2\left(M_W^2+M_Z^2\right)\right)\mathcal{C}_{\phi D},
\end{aligned} \tag{D3}
$$

and as usual $c_W=\frac{M_W^2}{M_Z^2}$, $s_W=\sqrt{1-c_W^2}$ and

$$
A_0(x)=-i\frac{(2\pi\Lambda)^{2\epsilon}}{\pi^2}\int\frac{d^dk_1}{(k_1^2-x)}=\frac{x}{\hat{\epsilon}}+x(1-\log(\frac{x}{\Lambda^2})) \tag{D4}
$$

Acknowledgements

We thank Giuseppe Degrassi for discussions. S.D. and P.P.G are supported by the U.S. Department of Energy under Grant Contract de-sc0012704.

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