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Naive Dimensional Analysis Counting of Gauge Theory Amplitudes and Anomalous Dimensions

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ABSTRACT: We show that naive dimensional analysis (NDA) is equivalent to the result that L-loop scattering amplitudes have perturbative order $N = L + \Delta$, with a shift Δ that depends on the NDA-weight of operator insertions. The NDA weight of an operator is defined in this paper, and the general NDA formula for perturbative order N is derived. The formula is used to explain why the one-loop anomalous dimension matrix for dimension-six operators in the Standard Model effective field theory has entries with perturbative order ranging from 0 to 4. The results in this paper are valid for an arbitrary effective field theory, and they constrain the coupling constant dependence of anomalous dimensions and scattering amplitudes in a general effective field theory.

Contents

1 Introduction

The LHC runs at 7 and 8 TeV have shown that the Standard Model (SM) with a single Higgs doublet H provides a good description of the data up to an energy scale of a few TeV. A reasonable interpretation of this experimental data is that the SM is a valid Effective Field Theory (EFT) up to an energy scale Λ of order a few TeV, and that any new physics beyond the SM arises at energies $E \geq \Lambda$. This interpretation also is supported by the measurements of rare decay processes in the complementary flavor physics program, which provides independent experimental evidence that the SM is a valid EFT up to energies of at least several TeV.

The leading operators which parametrize arbitrary new physics effects in the SM EFT for energies $E \leq \Lambda$ are of mass dimension six. Recently, the one-loop anomalous dimension matrix γ of the dimension-six operators in the SM was studied as part of an analysis of LHC results such as $h \to \gamma\gamma$ decay [1, 2]. Ref. [2] showed that γ only has terms of order

$$\frac{1}{16\pi^2} \times \left\{ 1, g^2, \lambda, y^2, g^4, g^2\lambda, g^2y^2, \lambda^2, \lambda y^2, y^4, g^6, g^4\lambda, g^6\lambda \right\} , \qquad (1.1)$$

where g is a gauge coupling, y is a Yukawa coupling, and λ is the Higgs self-coupling. Ref. [2] studied this pattern and showed that under a simple rescaling of the dimension-six operators based on naive dimensional analysis (NDA) [3], the one-loop anomalous dimension matrix elements have the form

$$\left(\frac{g^2}{16\pi^2}\right)^{n_1} \left(\frac{\lambda}{16\pi^2}\right)^{n_2} \left(\frac{y^2}{16\pi^2}\right)^{n_3} , \qquad N = n_1 + n_2 + n_3 \qquad (1.2)$$

where $N \equiv n_1 + n_2 + n_3$, the perturbative order of the term, ranges from $N = 0$ to $N = 4$, even though all of the terms are from one-loop diagrams.

In this paper, we derive a formula for the perturbative order N of a scattering amplitude at L loops in an arbitrary EFT. The result is equivalent to the NDA formula in Ref. [3]. Our

result, given in Eq. (2.20), shows that N depends on the number of loops L, as well as on the NDA-weights of the operators, which we define below. The proof is simple — the NDA result of Ref. [3] counts powers of 4π in the denominator. The NDA 4π-counting formula then determines N, since the power of 4π in the denominator of Eq. (1.2) is $2N$. The formula derived is of significant value. As one application, it predicts the structure of anomalous dimension matrices at any loop order in an arbitrary EFT.

2 The NDA Counting Formula

We derive the NDA-weight formula for a general EFT, i.e. a gauge theory with arbitrary higher dimension gauge invariant operators. The EFT Lagrangian is given by

$$\mathcal{L} = \mathcal{L}^{(d \leq 4)} + \sum_{d > 4} \mathcal{L}^{(d)} \tag{2.1}$$

where $\mathcal{L}^{(d \leq 4)}$ denotes the terms in the effective Lagrangian with mass dimension $d \leq 4$ and $\mathcal{L}^{(d)}$ denotes the terms in the effective Lagrangian with mass dimension d. The general form of $\mathcal{L}^{(d \leq 4)}$ is

$$\mathcal{L}^{(d \leq 4)} = -\frac{1}{4}X^2 + \overline{\psi}\,i\slashed{D}\psi + \frac{1}{2}(D\phi)^2 - M\overline{\psi}\psi - \frac{1}{2}m^2\phi^2 - y\phi\overline{\psi}\psi - \kappa\phi^3 - \lambda\phi^4, \tag{2.2}$$

where gauge field strength tensors are denoted by $X_{\mu\nu}$, fermion fields by ψ, scalar fields by ϕ, gauge couplings by g, fermion masses by M, scalar masses by m, Yukawa couplings by y, scalar cubic couplings by κ, and scalar quartic couplings by λ. The covariant derivative is $D^\mu = \partial^\mu + i\sum_k g_k A_k^\mu$ where the sum on k is over all gauge groups. The $d \leq 4$ Lagrangian can also contain topological terms, but these terms are not important for the present discussion and have been omitted.

The NDA formula of Ref. [3] says that a generic term in an effective Lagrangian generated at a scale Λ should be written as

$$f^2\Lambda^2 \left(\frac{\phi}{f}\right)^A \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B \left(\frac{gX}{\Lambda^2}\right)^C \left(\frac{D}{\Lambda}\right)^D, \tag{2.3}$$

with $\Lambda \sim 4\pi f$. Using this normalization, Eq. (2.2) can be written as

$$\mathcal{L}^{(d \leq 4)} = -\frac{1}{4\widehat{C}_X}\frac{f^2 g^2 X^2}{\Lambda^2} + \frac{1}{\widehat{C}_\psi}\overline{\psi}\,i\slashed{D}\psi + \frac{1}{2\widehat{C}_\phi}(D\phi)^2 - \widehat{C}_M\Lambda\overline{\psi}\psi - \frac{1}{2}\widehat{C}_{m^2}\Lambda^2\phi^2$$
$$- \widehat{C}_y\frac{\Lambda\phi\overline{\psi}\psi}{f} - \widehat{C}_\kappa\frac{\Lambda^2\phi^3}{f} - \widehat{C}_\lambda\frac{\Lambda^2\phi^4}{f^2}, \tag{2.4}$$

with rescaled inverse kinetic term coefficients and rescaled couplings given by

$$\widehat{C}_X = \frac{f^2 g^2}{\Lambda^2} = \frac{g^2}{16\pi^2}, \qquad \widehat{C}_\psi = 1, \qquad \widehat{C}_\phi = 1,$$

$$\widehat{C}_M = \frac{M}{\Lambda}, \qquad \widehat{C}_{m^2} = \frac{m^2}{\Lambda^2}, \qquad \widehat{C}_y = \frac{yf}{\Lambda} = \frac{y}{4\pi},$$

$$\widehat{C}_\kappa = \frac{\kappa f}{\Lambda^2} = \frac{1}{4\pi}\frac{\kappa}{\Lambda}, \qquad \widehat{C}_\lambda = \frac{\lambda f^2}{\Lambda^2} = \frac{\lambda}{16\pi^2}, \tag{2.5}$$

setting $f = \Lambda/4\pi$. The kinetic term coefficients are defined as $\widehat{C}^{-1}$, since the particle propagators, which enter the diagrams, are the inverse of the kinetic energy terms. We can also define a second version of Eq. (2.4), in which we do not distinguish between f and Λ by the substitutions $\widehat{C}_i \to C_i$, with the C_i evaluated for $f = \Lambda$

$$
\begin{aligned}
C_X &= g^2, & C_\psi &= 1, & C_\phi &= 1, \\
C_M &= \frac{M}{\Lambda}, & C_{m^2} &= \frac{m^2}{\Lambda^2}, & C_y &= y, \\
C_\kappa &= \frac{\kappa}{\Lambda}, & C_\lambda &= \lambda.
\end{aligned}
\tag{2.6}
$$

We call this normalization convention standard dimensional analysis. In both cases, Λ drops out of all dimension-four operator couplings. The NDA coupling constants $\widehat{C}_i$ differ from the coupling constants C_i of standard dimensional analysis by factors of of 4π.

Similarly, the operators in the effective Lagrangian $\mathcal{L}^{(d)}$ of dimension d can be normalized according to NDA and standard dimensional analysis,

$$
\mathcal{L}^{(d)} = \sum \widehat{C}_{d,w} \frac{O_{d,w}}{f^{2w}\Lambda^{d-4-2w}} = \sum C_{d,w} \frac{O_{d,w}}{\Lambda^{d-4}}
\tag{2.7}
$$

where w, the power of f^2 in the denominator, is defined to be the NDA-weight of the operator, and the NDA coefficient are denoted by $\widehat{C}_{d,w}$. The standard dimensional analysis coefficients $C_{d,w}$ are for the same operator $O_{d,w}$ normalized only using powers of Λ (i.e. with $f \to \Lambda$), but still including a factor of g with each X. As an example, the SM dimension-six operators fall into eight classes,

$$
\begin{aligned}
&\frac{g^3 X^3}{\Lambda^2} & &\frac{\phi^6}{\Lambda^2}, & &\frac{\phi^4 D^2}{\Lambda^2}, & &\frac{g^2 X^2 \phi^2}{\Lambda^2}, & &\frac{\psi^2 \phi^3}{\Lambda^2}, & &\frac{\psi^2 g X \phi}{\Lambda^2}, & &\frac{\psi^2 \phi^2 D}{\Lambda^2}, & &\frac{\psi^4}{\Lambda^2} \\
&\frac{f^2 g^3 X^3}{\Lambda^4}, & &\frac{\Lambda^2 \phi^6}{f^4}, & &\frac{\phi^4 D^2}{f^2} & &\frac{g^2 X^2 \phi^2}{\Lambda^2}, & &\frac{\Lambda \psi^2 \phi^3}{f^3}, & &\frac{\psi^2 g X \phi}{\Lambda f}, & &\frac{\psi^2 \phi^2 D}{f^2}, & &\frac{\psi^4}{f^2}
\end{aligned}
\tag{2.8}
$$

with NDA-weights $w = \{-1, 2, 1, 0, 3/2, 1/2, 1, 1\}$, respectively. The upper row is the usual $C_{d,w}$ normalization, the lower row is the NDA $\widehat{C}_{d,w}$ normalization, and Eq. (2.7) is

$$
\mathcal{L}^{(6)} = C_{6,-1} \frac{g^3 X^3}{\Lambda^2} + \ldots = \widehat{C}_{6,-1} \frac{f^2 g^3 X^3}{\Lambda^4} + \ldots.
\tag{2.9}
$$

The NDA result of Ref. [3] is that an arbitrary graph in the EFT generates a scattering amplitude $\widehat{C}_{d,w}$ corresponding to $O_{d,w}/f^{2w}\Lambda^{d-4-2w}$ with

$$
\widehat{C}_{d,w} \propto \prod_k \widehat{C}_{d_k, w_k},
\tag{2.10}
$$

and an unknown proportionality constant of order unity. Here $\widehat{C}_{d,w}$ and $\widehat{C}_{d_k,w_k}$ are coefficients from either $\mathcal{L}^{(d \leq 4)}$ or $\mathcal{L}^{(d)}$, and there is also a sum over all contributions of the form Eq. (2.10) that contribute. Standard dimensional analysis gives

$$
d - 4 = \sum_k (d_k - 4),
\tag{2.11}
$$

i.e. a graph with two insertions of a dimension-six operator produces a dimension-eight operator, a graph with an insertion of a dimension-six and dimension-two operator produces a dimension-four operator, etc.

Writing the $\mathcal{L}^{(d \leq 4)}$ terms in Eq. (2.10) explicitly gives

$$\widehat{C}_{d,w} \propto \left(\widehat{C}_X\right)^{I_X} \left(\widehat{C}_\psi\right)^{I_\psi} \left(\widehat{C}_\phi\right)^{I_\phi} \left(\widehat{C}_M\right)^{V_M} \left(\widehat{C}_{m^2}\right)^{V_{m^2}} \left(\widehat{C}_y\right)^{V_y} \left(\widehat{C}_\kappa\right)^{V_\kappa} \left(\widehat{C}_\lambda\right)^{V_\lambda} \prod_k \widehat{C}_{d_k,w_k} \, ,$$

$$(2.12)$$

where $I_{X,\psi,\phi}$ are the number of internal X, ψ, ϕ lines, and V_i are the number of vertices of type i. In strongly coupled theories, $\widehat{C}_i$ are order unity, and all terms are equally important. However, in weakly coupled theories, some $\widehat{C}_i$ are small, which gives the usual perturbative expansion. $\widehat{C}_X$ for the photon in QED is $e^2/(16\pi^2)$, the QED expansion parameter. In the NDA rescaled Lagrangian, $e^2/(16\pi^2)$ comes from the photon propagators rather than the vertices, because we are using rescaled fields. Using Eq. (2.6) gives

$$\widehat{C}_{d,w} \propto \left(\frac{g^2}{16\pi^2}\right)^{I_X} \left(\frac{M}{\Lambda}\right)^{V_M} \left(\frac{m^2}{\Lambda^2}\right)^{V_{m^2}} \left(\frac{y}{4\pi}\right)^{V_y} \left(\frac{1}{4\pi}\frac{\kappa}{\Lambda}\right)^{V_\kappa} \left(\frac{\lambda}{16\pi^2}\right)^{V_\lambda} \prod_k \widehat{C}_{d_k,w_k} \, . \quad (2.13)$$

This equation is independent of I_ψ and I_ϕ since $\widehat{C}_\psi$ and $\widehat{C}_\phi = 1$, and each internal gauge field gives a suppresion by $g^2/(16\pi^2)$. The explict Λ terms M/Λ, m^2/Λ^2 and κ/Λ are a reflection that $M, m, \kappa \sim \Lambda$, are natural values. In the SM, M and κ vanish, and $m \ll \Lambda$, the scale of new physics, is the hierarchy problem.

The NDA formula in Eq. (2.13) can be converted to the usual perturbation expansion in terms of the coefficients $C_{d,w}$ of standard dimensional analysis using the relation

$$\frac{C_{d,w}}{\Lambda^{d-4}} = \frac{\widehat{C}_{d,w}}{f^{2w}\Lambda^{d-4-2w}} \quad (2.14)$$

from Eq. (2.7), and $\Lambda = 4\pi f$, to give

$$C_{d,w} \propto \left(\frac{g^2}{16\pi^2}\right)^{I_X} \left(\frac{M}{\Lambda}\right)^{V_M} \left(\frac{m^2}{\Lambda^2}\right)^{V_{m^2}} \left(\frac{y}{4\pi}\right)^{V_y} \left(\frac{1}{4\pi}\frac{\kappa}{\Lambda}\right)^{V_\kappa} \left(\frac{\lambda}{16\pi^2}\right)^{V_\lambda} \left(\frac{1}{4\pi}\right)^{-2w} \prod_k \left(\frac{1}{4\pi}\right)^{2w_k} C_{d_k,w_k} \, .$$

$$(2.15)$$

Eq. (2.15) is the expression one obtains by direct computation using the original Lagrangian Eq. (2.2) and unrescaled operators such as those in the upper row of Eq. (2.8). We know that with the unrescaled theory, an L loop diagram is of order $1/(16\pi^2)^L$. Equating this power of 4π with the powers of 4π in Eq. (2.15) gives our 4π-counting formula

$$2L = 2I_X + V_y + V_\kappa + 2V_\lambda - 2w + 2\sum_k w_k \, . \quad (2.16)$$

Equation (2.13) says that an amplitude for a diagram with insertions of the r operators

$$O_{d_1,w_1}, \ldots, O_{d_r,w_r} \quad (2.17)$$

to produce the final operator $O_{d,w}$ is

$$\propto \left(\frac{g^2}{16\pi^2}\right)^{I_X} \left(\frac{M}{\Lambda}\right)^{V_M} \left(\frac{m^2}{\Lambda^2}\right)^{V_{m^2}} \left(\frac{y}{4\pi}\right)^{V_y} \left(\frac{1}{4\pi}\frac{\kappa}{\Lambda}\right)^{V_\kappa} \left(\frac{\lambda}{16\pi^2}\right)^{V_\lambda}. \tag{2.18}$$

Defining the perturbative order N of a term

$$\left(\frac{g^2}{16\pi^2}\right)^{I_X} \left(\frac{y}{4\pi}\right)^{V_y} \left(\frac{1}{4\pi}\frac{\kappa}{\Lambda}\right)^{V_\kappa} \left(\frac{\lambda}{16\pi^2}\right)^{V_\lambda} \qquad \text{by} \qquad N = I_X + \frac{1}{2}V_y + \frac{1}{2}V_\kappa + V_\lambda, \tag{2.19}$$

we see that the 4π-counting formula Eq. (2.16) gives

$$\boxed{N = L + w - \sum_k w_k \equiv L + \Delta.} \tag{2.20}$$

This is our main result. The perturbative order N is not the same as the loop order L, but is shifted by $\Delta = w - \sum_k w_k$, the difference of the NDA-weight of the final operator and the sum of the NDA weights of the inserted operators.

The NDA formula Eq. (2.20) applies to amplitudes (finite and infinite pieces) at general kinematic points. The amplitude can have inverse powers of momenta through particle propagators. In spontaneously broken theories, it can also have inverse powers of the coupling constants (and fields) since particle masses can be proportional to $g\langle\phi\rangle$ and $y\langle\phi\rangle$, as in the SM. The formula has to be modified in corners of phase-space, such as near threshold, where one can have kinematic enhancements of $1/v$, where the particle velocity v is a dimensionless number not controlled by NDA.

3 Application to the Standard Model

The Lagrangian of the SM EFT has the form

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{SM} + \sum_{d>4} \mathcal{L}^{(d)}, \tag{3.1}$$

where $\mathcal{L}_{SM}$ is the usual SM Lagrangian with terms of mass dimension $d \leq 4$, and $\mathcal{L}^{(d)}$ consists of all terms of mass dimension $d > 4$.

There are a total of 59 independent dimension-six operators in the operator basis [4, 5], assuming the conservation of baryon number. These 59 dimension-six operators divide into eight operator classes [5]. The coupling constant dependence of the one-loop anomalous dimension matrix γ is given in Table 1.

In the SM EFT, Eq. (2.20) can be written in a nicer form. The SM EFT Lagrangian has the symmetry

$$H \rightarrow -H, \qquad q \rightarrow q, \qquad l \rightarrow l, \qquad u \rightarrow -u, \qquad d \rightarrow -d, \qquad e \rightarrow -e, \tag{3.2}$$

		g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
		1	2	3	4	5	6	7	8
g^3X^3	1	g^2	0	0	1	0	0	0	0
H^6	2	$g^6\lambda$	λ,g^2,y^2	$g^4,g^2\lambda,\lambda^2$	$g^6,g^4\lambda$	$\lambda y^2,y^4$	0	$\lambda y^2,y^4$	0
H^4D^2	3	g^6	0	g^2,λ,y^2	g^4	y^2	g^2y^2	g^2,y^2	0
$g^2X^2H^2$	4	g^4	0	1	g^2,λ,y^2	0	y^2	1	0
$y\psi^2H^3$	5	g^6	0	g^2,λ,y^2	g^4	g^2,λ,y^2	$g^2\lambda,g^4,g^2y^2$	g^2,λ,y^2	λ,y^2
$gy\psi^2XH$	6	g^4	0	0	g^2	1	g^2,y^2	1	1
ψ^2H^2D	7	g^6	0	g^2,y^2	g^4	y^2	g^2y^2	g^2,λ,y^2	y^2
ψ^4	8	g^6	0	0	0	0	g^2y^2	g^2,y^2	g^2,y^2

Table 1. The form of the one-loop anomalous dimension matrix γ for the coefficients of dimension-six operators in the operator basis of Ref. [5] when the operators are rescaled according to NDA, see Eq. (3.7). The rows and columns of γ are labelled by the eight operator classes defined in Refs. [2, 5]. There are a total of 59 operators [4, 5] in the eight classes, and we use the notation of Ref. [2]. Not all the entries are directly due to one-particle irreducible graphs. Some of them arise by converting operators to the canonical basis using the equations of motion. This table combines the two tables in Ref. [2] and also includes wave function renormalization.

where q, l are left-handed quark and lepton doublet fields, and u, d, e are right-handed singlet fields, and the symmetry

$$H \to -H, \qquad\qquad y \to -y, \qquad\qquad (3.3)$$

where y are the Yukawa couplings. Operators odd under $H \to -H$ can be written instead using yH, where y is a formal parameter of order the Yukawa coupling, and is included in the NDA formula Eq. (2.3) as a factor of yH/Λ which occurs at most at linear order. Thus the general form of a term in the SM EFT Lagrangian is

$$f^2\Lambda^2 \left(\frac{H}{f}\right)^A \left(\frac{yH}{\Lambda}\right)^{A'} \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B \left(\frac{gX}{\Lambda^2}\right)^C \left(\frac{D}{\Lambda}\right)^D \qquad (3.4)$$

with $A' = 0, 1$.

Operators with $A' = 1$ have their NDA-weight shifted by $-1/2$, and coefficient modified by

$$\widehat{C}_{d,w} \to \frac{fy}{\Lambda}\, \widehat{C}_{d,w-1/2} \qquad\qquad (3.5)$$

so that Eq. (2.20) remains valid. The advantage is that the V_y in Eq. (2.19) is now an even integer, so we obtain instead

$$\left(\frac{g^2}{16\pi^2}\right)^{n_g}\left(\frac{y^2}{16\pi^2}\right)^{n_y}\left(\frac{\lambda}{16\pi^2}\right)^{n_\lambda}, \qquad N = n_g + n_y + n_\lambda, \tag{3.6}$$

for the perturbative order, since there are no κ-vertices in the SM. With this convention, the eight operator classes for dimension-six operators in the SM are

$$\frac{f^2}{\Lambda^4}\, g^3 X^3, \ \frac{\Lambda^2}{f^4}\, H^6, \ \frac{1}{f^2}\, H^4 D^2, \ \frac{1}{\Lambda^2}\, g^2 X^2 H^2, \ \frac{1}{f^2}\, y\psi^2 H^3, \ \frac{1}{\Lambda^2}\, y\psi^2 g X H, \ \frac{1}{f^2}\psi^2 H^2 D, \ \frac{1}{f^2}\psi^4 \tag{3.7}$$

with NDA-weights $w_i = \{-1, 2, 1, 0, 1, 0, 1, 1\}$ for $i = 1, \ldots, 8$, which are all integers. This is the normalization convention of Ref. [2]. We can similarly define weights for the SM Lagrangian terms in $\mathcal{L}^{d\leq 4}$. H^4 has $w = 1$, and H^2 has $w = 0$. The Yukawa operator $\psi^2 H$ has $w = 1/2$ and can be changed to $y\psi^2 H$ with $w = 0$, as in Eq. (3.5). The inverse gauge coupling $1/g^2$ multiplies the $w = -1$ gauge kinetic term.

The anomalous dimension matrix elements γ_{ij}, defined by the renormalization group equation for the dimension-six coefficients

$$\mu\frac{\mathrm{d}\widehat{C}_i}{\mathrm{d}\mu} = \gamma_{ij}\,\widehat{C}_j\,, \tag{3.8}$$

have perturbative weight N_{ij}

$$N_{ij} = L + w_i - w_j \equiv L + \Delta_{ij}\,, \tag{3.9}$$

from L-loop diagrams, which is a special case of Eq. (2.7).

Consider the one-loop anomalous dimension matrix with the rows and columns reordered in decreasing order of NDA weight w_i. The $w = 2$ operator is H^6 with $i \in \{2\}$, $w = 1$ operators are $H^4 D^2$, $y\psi^2 H^3$, $\psi^2 H^2 D$, ψ^4 with $i \in \{3, 5, 7, 8\}$, $w = 0$ operators are $g^2 X^2 H^2$ and $gy\psi^2 X H$ with $i \in \{4, 6\}$, and the $w = -1$ operators is $g^3 X^3$ with $i \in \{1\}$. The one-loop anomalous dimension matrix entries have perturbative order N given by

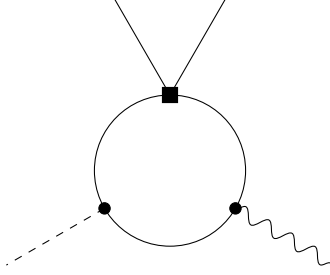| w	operators		$\{2\}$	$\{3, 5, 7, 8\}$	$\{4, 6\}$	$\{1\}$
2	H^6	$\{2\}$	1	2	3	4
1	$H^4 D^2$, $y\psi^2 H^3$, $\psi^2 H^2 D$, ψ^4	$\{3, 5, 7, 8\}$	0	1	2	3
0	$g^2 X^2 H^2$, $gy\psi^2 X H$	$\{4, 6\}$	-1	0	1	2
-1	$g^3 X^3$	$\{1\}$	-2	-1	0	1

$$\tag{3.10}$$

Figure 1. Diagram contributing to the $\psi^2 X H - \psi^4$ anomalous dimension γ_{68} given in Ref. [2]. The solid square is a ψ^4 vertex from $\mathcal{L}^{(6)}$ and the dots are gauge and Yukawa vertices from $\mathcal{L}_{SM}$.

where the rows and columns are labelled by the operator classes, reordered according to decreasing NDA weight.

The anomalous dimensions in Table 1 include terms obtained using the equations of motion. Since equations of motion are obtained by varying the action which obeys NDA, Eqs. (2.3,3.10) are still valid. For example, the penguin operator $O_P = \overline{\psi}\gamma^\nu\psi\, gD^\mu F_{\mu\nu}$ is converted by the equations of motion to four-fermion operators $O_q = g^2\, \overline{\psi}\gamma^\mu\psi\, \overline{\psi}\gamma_\mu\psi$. O_P is a $w = 0$ operator, whereas O_q is a $w = 1$ operator, so equations of motion can shift w. However, O_q is multiplied by g^2, so the value of N is also shifted, and Eq. (3.9) remains valid.

Amplitudes can have inverse powers of coupling, via inverse powers of particle masses, but anomalous dimension diagrams can only depend on non-negative powers of couplings, provided no couplings are included in the operators. This observation implies that the entries in Eq. (3.10) with negative perturbative order vanish. Thus, Eq. (3.10) explains the pattern of couplings found in Table 1. Even though the anomalous dimensions in Table 1 all come from one-loop diagrams, the individual entries range from $N = 0$ to $N = 4$, i.e. they range from tree order to 4-loop order, with the pattern given by Eq. (3.10). The formula also applies to the running of SM coefficients due to dimension-six operators, as given in Ref. [2].

As an illustrative example, Eq. (3.10) shows that γ_{44} computed in Ref. [1] has $N = 1$, and from Table 1, we see that it has terms of order g^2, λ, y^2. Similarly γ_{23} has $N = 2$, with terms of order $g^4, g^2\lambda, \lambda^2$; γ_{24} has $N = 3$ with terms of order $g^6, g^4\lambda$; and γ_{21} has $N = 4$ with terms of order $g^6\lambda$. There are $N = 0$ terms which do not vanish, such as γ_{68} from the graph in Fig. 1 (see Ref. [2] for more details).

All entries with $N \geq 0$ in Eq. (3.10) need not vanish. Which entries vanish depends on the theory under consideration, and its symmetry structure. For example, ψ^4 transforms under flavor symmetries, and only the flavor-singlet ψ^4 operators can contribute to the $H^6 - \psi^4$ anomalous dimension. In addition, this mixing is forbidden at one-loop for the simple reason that one cannot eliminate four ψ fields by a one-loop diagram. The mixing, however, will be present at higher loops. As L increases, so do all the numbers in Eq. (3.10), and eventually, all of them are non-negative so that all the operators can mix at high enough loop-order, if the mixing is not forbidden by symmetries.

4　Conclusions

In conclusion, we have developed a powerful formula based on NDA that constrains the coupling constant dependence of scattering amplitudes and anomalous dimensions in an arbitrary effective field theory. We have used the SM dimension-six operator running as an illustrative example, but the result is general, and holds for arbitrary EFTs. HQET running of $1/m^2$ operators, including non-linear terms from two insertions of $1/m$ operators, was computed in Ref. [6], and satisfies the constraint Eq. (3.9).

Acknowledgments

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