Application of the Background-Field Method to the electroweak Standard Model

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Abstract:

Application of the background-field method yields a gauge-invariant effective action for the electroweak Standard Model, from which simple QED-like Ward identities are derived. As a consequence of these Ward identities, the background-field Green functions are shown to possess very desirable theoretical properties. The renormalization of the Standard Model in the background-field formalism is studied. A consistent on-shell renormalization procedure retaining the full gauge symmetry is presented. The structure of the counterterms is shown to greatly simplify compared to the conventional formalism. A complete list of Feynman rules for the Standard Model in the background-field method is given for arbitrary values of a quantum gauge parameter including all counterterms necessary for one-loop calculations.

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1 INTRODUCTION

The current theoretical understanding of elementary particle physics is based on gauge theories, which are constructed following the principle of gauge invariance. While the classical Lagrangian is manifestly gauge-invariant, one is forced to fix a gauge in order to quantize the theory. In the conventional formulation, the gauge symmetry is spoiled in intermediate steps of calculations and can only be restored at the very end by projecting on physical degrees of freedom.

To avoid the explicit breaking of gauge symmetry, the background-field method (BFM) [1, 2] was developed. By decomposing the usual gauge field into a quantum field and a background field one can impose the gauge fixing necessary for quantization while keeping the gauge invariance of the effective action. The BFM proved to be a valuable tool in gauge theories facilitating computations both technically and conceptually. It has found many applications in gravity and supergravity [3] and also in QCD, e.g. for the calculation of the β-function [2, 4]. The equivalence of the S matrix in the BFM to the conventional one has been proven in Ref. [5]. In the recent formulation of string motivated rules for more efficient computations in gauge theories, the BFM plays an important role [6]. An application to the electroweak one-loop process $Z \to 3\gamma$ was presented in Ref. [7]. The advantages of calculating S-matrix elements within the BFM are mainly due to the fact that the gauge fixing of the background fields is completely independent of the quantum gauge fixing. The choice of an appropriate background gauge can simplify practical calculations considerably. However, for spontaneously broken gauge theories the BFM has hardly been used. There exists no complete formulation of the BFM for the electroweak Standard Model (SM). In particular, the renormalization has not been worked out in detail.

Recently it was shown [8, 9] that application of the BFM in QCD and the electroweak SM yields Green functions with very desirable theoretical properties. They fulfill simple QED-like Ward identities and, in comparison to their counterparts in the conventional R_ξ-gauge formalism, often have an improved asymptotic, UV, and IR behavior. The issue of obtaining Green functions with suitable properties has found considerable interest in the literature during the last years [10, 11, 12, 13]. It is especially important for applications dealing with off-shell Green functions. These become relevant when higher-order contributions are resummed in order to define running coupling constants or to take into account finite-width effects in resonance regions. Furthermore, off-shell formfactors are frequently discussed, e.g. for the neutrino or for the top quark. Off-shell self-energies are often used to parametrize electroweak radiative corrections.

Most previous attempts for the construction of Green functions suitable for these purposes aimed on eliminating their gauge-parameter dependence within a special class of gauges, usually the R_ξ gauges. To this end new "Green functions" were constructed by rearranging contributions between self-energies, vertex and box diagrams. In particular, the pinch technique (PT) [12, 13] provides a definite prescription for obtaining gauge-parameter independent quantities at one-loop order. They were found to fulfill simple Ward identities and possess other desirable theoretical properties. Despite these successes, there are a number of problems related to the PT approach. The extension of the PT to higher orders is rather involved [14], and even at one-loop order the PT is not applicable

in a straightforward manner to all possible Green functions. In addition to these technical difficulties, the PT has also conceptual problems. Strictly speaking, the resulting building blocks of the S matrix should not be called Green functions since their field theoretical meaning has not been clarified. The process independence of the new "Green functions" constructed within the PT has not been proven. Moreover, the simple Ward identities and other desirable features have not been derived within the PT but only verified for specific one-loop examples.

In Refs. [8, 9] it was shown that on the basis of the BFM these theoretical problems are resolved. The results obtained within the PT in QCD and the SM were shown to coincide with the special case $\xi_Q = 1$ of the BFM results, where ξ_Q is a quantum gauge parameter associated with the gauge fixing of the quantum fields[1]. The BFM vertex functions are directly derived from the effective action in all orders of perturbation theory and are evidently process-independent. The validity of QED-like Ward identities is a direct consequence of the gauge invariance of the effective action. Furthermore, one can show that the Ward identities of the BFM directly imply other desirable properties of the Green functions.

From the formulation of the BFM it follows that the Ward identities and the desirable features of Green functions hold for all values of the quantum gauge parameter ξ_Q. This fact is of importance in view of the former treatments [10, 11, 12, 13] which focus on the elimination of the gauge-parameter dependence. The analysis in the BFM shows that not the requirement of gauge-parameter independence is the criterion leading to Green functions with suitable properties but the Ward identities following from gauge invariance. The ambiguity of the vertex functions quantified in the BFM by their dependence on ξ_Q is also inherent in the former treatments where it corresponds to the ambiguity in choosing different prescriptions for eliminating the gauge-parameter dependence.

Owing to the aforementioned properties, the BFM is a well suited formalism for applications in the electroweak SM concerning both the discussion of off-shell quantities and a technically and conceptually simplified evaluation of S-matrix elements. The purpose of this paper is to provide the tools necessary for applying the BFM in the SM and to investigate consequences of the explicit gauge invariance present in the BFM formulation. In particular, an explicit on-shell renormalization of the SM in the BFM is worked out in accordance with the gauge invariance of the effective action. The gauge invariance implies relations between the renormalization constants for parameters and fields and greatly simplifies the renormalization.

The outline of the paper is as follows. In section 2 we write out the classical Lagrangian in order to define our conventions and perform the quantization of the SM in the BFM. The properties of the resulting gauge-invariant effective action and the construction of the S matrix are discussed. In section 3 we derive the Ward identities of the theory. For several examples the differences to the conventional formalism are discussed. In section 4 the renormalization of the SM in the BFM is worked out. Section 5 illustrates how desirable properties of the BFM vertex functions can directly be related to the Ward identities. In the appendix, a complete list of Feynman rules for the SM in the BFM is

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[1]The agreement between the BFM results for $\xi_Q = 1$ obtained in QCD and the corresponding PT results was also noted in Ref. [15].

given for an arbitrary value of the quantum gauge parameter. All counterterms necessary for one-loop calculations are included.

2 THE GAUGE-INVARIANT EFFECTIVE ACTION FOR THE STANDARD MODEL

2.1 The classical Lagrangian

In order to define the relevant quantities, we begin with the classical Lagrangian $\mathcal{L}_{\mathrm{C}}$ of the (minimal) electroweak SM. It consists of the Yang-Mills, the Higgs and the fermion part

$$\mathcal{L}_{\mathrm{C}} = \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}}. \tag{1}$$

The Yang-Mills part is given as

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} \left(\partial_\mu \hat{W}_\nu^a - \partial_\nu \hat{W}_\mu^a + g_2 \varepsilon^{abc} \hat{W}_\mu^b \hat{W}_\nu^c \right)^2 - \frac{1}{4} \left(\partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu \right)^2, \tag{2}$$

where the isotriplet $\hat{W}_\mu^a$, $a = 1, 2, 3$, is associated with the generators I_{W}^a of the weak isospin group $\mathrm{SU(2)_W}$ and the isosinglet $\hat{B}_\mu$ with the weak hypercharge Y_{W} of the group $\mathrm{U(1)_Y}$. For later convenience we denote the classical gauge and Higgs fields with a caret. The Higgs part has the form

$$\mathcal{L}_{\mathrm{H}} = \left(\hat{D}_\mu \hat{\Phi} \right)^\dagger \left(\hat{D}^\mu \hat{\Phi} \right) - V(\hat{\Phi}) \tag{3}$$

with the covariant derivative

$$\hat{D}_\mu = \partial_\mu - ig_2 I_{\mathrm{W}}^a \hat{W}_\mu^a + ig_1 \frac{Y_{\mathrm{W}}}{2} \hat{B}_\mu. \tag{4}$$

In (3), $\hat{\Phi}(x)$ denotes the complex scalar $\mathrm{SU(2)_W}$ doublet field of the minimal Higgs sector with hypercharge $Y_{\mathrm{W}}^{\hat{\Phi}} = 1$

$$\hat{\Phi}(x) = \begin{pmatrix} \hat{\phi}^+(x) \\ \hat{\phi}^0(x) \end{pmatrix}, \tag{5}$$

and the Higgs potential reads

$$V(\hat{\Phi}) = \frac{\lambda}{4} \left(\hat{\Phi}^\dagger \hat{\Phi} \right)^2 - \mu^2 \hat{\Phi}^\dagger \hat{\Phi}. \tag{6}$$

We write the fermionic part as (neglecting quark mixing as throughout this paper)

$$\begin{aligned} \mathcal{L}_{\mathrm{F}} = &\sum_k \left(\overline{L}_k^{\mathrm{L}} i \hat{\slashed{D}} L_k^{\mathrm{L}} + \overline{Q}_k^{\mathrm{L}} i \hat{\slashed{D}} Q_k^{\mathrm{L}} \right) \\ &+ \sum_k \left(\overline{l}_k^{\mathrm{R}} i \hat{\slashed{D}} l_k^{\mathrm{R}} + \overline{u}_k^{\mathrm{R}} i \hat{\slashed{D}} u_k^{\mathrm{R}} + \overline{d}_k^{\mathrm{R}} i \hat{\slashed{D}} d_k^{\mathrm{R}} \right) \\ &- \sum_k \left(\overline{L}_k^{\mathrm{L}} G_k^l l_k^{\mathrm{R}} \hat{\Phi} + \overline{Q}_k^{\mathrm{L}} G_k^u u_k^{\mathrm{R}} \hat{\tilde{\Phi}} + \overline{Q}_k^{\mathrm{L}} G_k^d d_k^{\mathrm{R}} \hat{\Phi} + \mathrm{h.c.} \right). \end{aligned} \tag{7}$$

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The left-handed fermions of each lepton (L) and quark (Q) generation are grouped into $SU(2)_W$ doublets (the color index is suppressed)

$$L_k^L = \omega_- L_k = \begin{pmatrix} \nu_k^L \\ l_k^L \end{pmatrix}, \qquad Q_k^L = \omega_- Q_k = \begin{pmatrix} u_k^L \\ d_k^L \end{pmatrix}, \tag{8}$$

the right-handed fermions into singlets

$$l_k^R = \omega_+ l_k, \qquad u_k^R = \omega_+ u_k, \qquad d_k^R = \omega_+ d_k, \tag{9}$$

where $\omega_\pm = (1 \pm \gamma_5)/2$ are the projectors on right- and left-handed fields, respectively, k is the generation index, and ν, l, u and d stand for neutrinos, charged leptons, up-type quarks and down-type quarks, respectively. The weak hypercharge Y_W is assigned according to the Gell-Mann Nishijima relation

$$Y_W = 2(Q - I_W^3), \tag{10}$$

where Q is the electric charge operator. In (7), G_k^l, G_k^u and G_k^d denote the Yukawa couplings, $\hat{\tilde{\Phi}} = \left(\hat{\phi}^{0*}, -\hat{\phi}^- \right)^T$ is the charge-conjugated Higgs field, and $\hat{\phi}^- = \left(\hat{\phi}^+ \right)^*$.

The physical gauge-boson fields are obtained via

$$\hat{W}_\mu^\pm = \frac{1}{\sqrt{2}} \left(\hat{W}_\mu^1 \mp i \hat{W}_\mu^2 \right), \qquad \begin{pmatrix} \hat{Z}_\mu \\ \hat{A}_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} \hat{W}_\mu^3 \\ \hat{B}_\mu \end{pmatrix}, \tag{11}$$

where

$$c_W = \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} = \frac{M_W}{M_Z}, \qquad s_W = \sin\theta_W = \sqrt{1 - c_W^2}, \tag{12}$$

and θ_W is the weak mixing angle. The electromagnetic coupling is given by

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}; \tag{13}$$

further relations between the physical parameters and the parameters in $\mathcal{L}_C$ can be found in Ref. [16].

2.2 Quantization in the background-field method

In the conventional formalism directly the fields appearing in the classical Lagrangian are quantized. A gauge-fixing term is added to $\mathcal{L}_C$ which breaks the explicit gauge invariance.

Instead, when going from the classical to the quantized theory in the BFM [1, 2], the fields $\hat{V}$ of $\mathcal{L}_C$ are split into classical background fields $\hat{V}$ and quantum fields V,

$$\mathcal{L}_C(\hat{V}) \to \mathcal{L}_C(\hat{V} + V). \tag{14}$$

The quantum fields are the variables of integration in the functional integral. A gauge-fixing term is added which only breaks the gauge invariance of the quantum fields but retains the gauge invariance of the effective action with respect to the background fields.

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In order to avoid tree-level mixing between the gauge bosons and the corresponding unphysical Higgs bosons, we use a generalization of the 't Hooft gauge fixing to the BFM [17]

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi_Q^W}\left[(\delta^{ac}\partial_\mu + g_2\varepsilon^{abc}\hat{W}_\mu^b)W^{c,\mu} - ig_2\xi_Q^W\frac{1}{2}(\hat{\Phi}_i^\dagger\sigma_{ij}^a\Phi_j - \Phi_i^\dagger\sigma_{ij}^a\hat{\Phi}_j)\right]^2$$
$$-\frac{1}{2\xi_Q^B}\left[\partial_\mu B^\mu + ig_1\xi_Q^B\frac{1}{2}(\hat{\Phi}_i^\dagger\Phi_i - \Phi_i^\dagger\hat{\Phi}_i)\right]^2, \tag{15}$$

where σ^a, $a = 1, 2, 3$, denote the Pauli matrices, and ξ_Q^W, ξ_Q^B are parameters associated with the gauge fixing of the quantum fields. The background Higgs field $\hat{\Phi}$ has the usual non-vanishing vacuum expectation value v, while the one of the quantum Higgs field Φ is zero:

$$\hat{\Phi}(x) = \begin{pmatrix} \hat{\phi}^+(x) \\ \frac{1}{\sqrt{2}}(v + \hat{H}(x) + i\hat{\chi}(x)) \end{pmatrix}, \qquad \Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(H(x) + i\chi(x)) \end{pmatrix}. \tag{16}$$

Here $\hat{H}$ and H denote the physical background and quantum Higgs field, respectively, and $\hat{\phi}^+, \hat{\chi}, \phi^+, \chi$ are unphysical degrees of freedom. The gauge-fixing term (15) translates to the conventional one upon replacing the background Higgs field by its vacuum expectation value and omitting the background $\mathrm{SU}(2)_\mathrm{W}$ triplet field $\hat{W}_\mu^a$. Background-field gauge invariance restricts the number of quantum gauge parameters to two, one for $\mathrm{SU}(2)_\mathrm{W}$ and one for $\mathrm{U}(1)_\mathrm{Y}$.

In the spirit of the BFM, one should also split the fermion fields into background and quantum fields. However, for all fields that do not enter the gauge-fixing term, quantization in the BFM is equivalent to the conventional formalism. The Feynman rules for background and quantum fields are identical for these fields and there is no need to distinguish them. We therefore use a common symbol for the fermion fields, i.e. we do not write a caret for the fermion background fields.

Next, we express the gauge-fixing term (15) by physical fields. In order to avoid tree-level mixing between the photon and the Z boson one has to chose $\xi_Q = \xi_Q^W = \xi_Q^B$. This yields

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi_Q}\left[(G^A)^2 + (G^Z)^2 + 2G^+G^-\right], \tag{17}$$

where

$$G^A = \partial^\mu A_\mu + ie(\hat{W}_\mu^+ W^{-\mu} - W_\mu^+ \hat{W}^{-\mu}) + ie\xi_Q(\hat{\phi}^-\phi^+ - \hat{\phi}^+\phi^-),$$
$$G^Z = \partial^\mu Z_\mu - ie\frac{c_\mathrm{W}}{s_\mathrm{W}}(\hat{W}_\mu^+ W^{-\mu} - W_\mu^+ \hat{W}^{-\mu}) - ie\xi_Q\frac{c_\mathrm{W}^2 - s_\mathrm{W}^2}{2c_\mathrm{W}s_\mathrm{W}}(\hat{\phi}^-\phi^+ - \hat{\phi}^+\phi^-)$$
$$+ e\xi_Q\frac{1}{2c_\mathrm{W}s_\mathrm{W}}(\hat{\chi}H - \hat{H}\chi - v\chi),$$
$$G^\pm = \partial^\mu W_\mu^\pm \pm ie(\hat{A}^\mu - \frac{c_\mathrm{W}}{s_\mathrm{W}}\hat{Z}^\mu)W_\mu^\pm \mp ie(A^\mu - \frac{c_\mathrm{W}}{s_\mathrm{W}}Z^\mu)\hat{W}_\mu^\pm$$
$$\mp ie\xi_Q\frac{1}{2s_\mathrm{W}}\left[(v + \hat{H} \mp i\hat{\chi})\phi^\pm - (H \mp i\chi)\hat{\phi}^\pm\right]. \tag{18}$$

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Finally, we add a Faddeev–Popov part to the Lagrangian

$$\mathcal{L}_{\mathrm{FP}} = -\bar{u}^{\alpha}\frac{\delta G^{\alpha}}{\delta\theta^{\beta}}u^{\beta}, \tag{19}$$

where $\alpha = A, Z, \pm$, and $\delta G^{\alpha}/\delta\theta^{\beta}$ is the variation of the gauge-fixing terms G^{α} under the infinitesimal quantum gauge transformations of the quantum fields

$$\delta W_{\mu}^{\pm} = \partial_{\mu}\delta\theta^{\pm} \mp ie(W_{\mu}^{\pm} + \hat{W}_{\mu}^{\pm})(\delta\theta^A - \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\delta\theta^Z) \pm ie\left[(A_{\mu} + \hat{A}_{\mu}) - \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}(Z_{\mu} + \hat{Z}_{\mu})\right]\delta\theta^{\pm},$$

$$\delta Z_{\mu} = \partial_{\mu}\delta\theta^Z - ie\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\left[(W_{\mu}^+ + \hat{W}_{\mu}^+)\delta\theta^- - (W_{\mu}^- + \hat{W}_{\mu}^-)\delta\theta^+\right],$$

$$\delta A_{\mu} = \partial_{\mu}\delta\theta^A + ie\left[(W_{\mu}^+ + \hat{W}_{\mu}^+)\delta\theta^- - (W_{\mu}^- + \hat{W}_{\mu}^-)\delta\theta^+\right],$$

$$\delta\phi^{\pm} = \pm\frac{ie}{2s_{\mathrm{W}}}\left[H + \hat{H} + v \pm i(\chi + \hat{\chi})\right]\delta\theta^{\pm} \mp ie(\phi^{\pm} + \hat{\phi}^{\pm})(\delta\theta^A - \frac{c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2}{2c_{\mathrm{W}}s_{\mathrm{W}}}\delta\theta^Z),$$

$$\delta H = \frac{ie}{2s_{\mathrm{W}}}\left[(\phi^+ + \hat{\phi}^+)\delta\theta^- - (\phi^- + \hat{\phi}^-)\delta\theta^+\right] + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}(\chi + \hat{\chi})\delta\theta^Z,$$

$$\delta\chi = \frac{e}{2s_{\mathrm{W}}}\left[(\phi^+ + \hat{\phi}^+)\delta\theta^- + (\phi^- + \hat{\phi}^-)\delta\theta^+\right] - \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}(H + \hat{H} + v)\delta\theta^Z. \tag{20}$$

Using the Lagrangian specified above, an effective action $\Gamma[\hat{V}, \hat{S}, F, \bar{F}]$ is constructed following Ref. [2], where $\hat{V}$ collectively denotes the background gauge fields, $\hat{S}$ the background Higgs fields and F the fermion fields. $\Gamma[\hat{V}, \hat{S}, F, \bar{F}]$ is invariant under the background gauge transformations of the background fields

$$\delta\hat{W}_{\mu}^{\pm} = \partial_{\mu}\delta\hat{\theta}^{\pm} \mp ie\hat{W}_{\mu}^{\pm}(\delta\hat{\theta}^A - \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\delta\hat{\theta}^Z) \pm ie(\hat{A}_{\mu} - \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\hat{Z}_{\mu})\delta\hat{\theta}^{\pm},$$

$$\delta\hat{Z}_{\mu} = \partial_{\mu}\delta\hat{\theta}^Z - ie\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}(\hat{W}_{\mu}^+\delta\hat{\theta}^- - \hat{W}_{\mu}^-\delta\hat{\theta}^+),$$

$$\delta\hat{A}_{\mu} = \partial_{\mu}\delta\hat{\theta}^A + ie(\hat{W}_{\mu}^+\delta\hat{\theta}^- - \hat{W}_{\mu}^-\delta\hat{\theta}^+),$$

$$\delta\hat{\phi}^{\pm} = \pm\frac{ie}{2s_{\mathrm{W}}}(\hat{H} + v \pm i\hat{\chi})\delta\hat{\theta}^{\pm} \mp ie\hat{\phi}^{\pm}(\delta\hat{\theta}^A - \frac{c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2}{2c_{\mathrm{W}}s_{\mathrm{W}}}\delta\hat{\theta}^Z),$$

$$\delta\hat{H} = \frac{ie}{2s_{\mathrm{W}}}(\hat{\phi}^+\delta\hat{\theta}^- - \hat{\phi}^-\delta\hat{\theta}^+) + \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}\hat{\chi}\delta\hat{\theta}^Z,$$

$$\delta\hat{\chi} = \frac{e}{2s_{\mathrm{W}}}(\hat{\phi}^+\delta\hat{\theta}^- + \hat{\phi}^-\delta\hat{\theta}^+) - \frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}(\hat{H} + v)\delta\hat{\theta}^Z, \tag{21}$$

and the corresponding transformations of the fermion fields

$$\delta f_u^{\mathrm{L}} = \frac{ie}{\sqrt{2}s_{\mathrm{W}}}f_d^{\mathrm{L}}\delta\hat{\theta}^+ - ie\left[Q_{f_u}\delta\hat{\theta}^A - \left(\frac{1}{2c_{\mathrm{W}}s_{\mathrm{W}}} - Q_{f_u}\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}\right)\delta\hat{\theta}^Z\right]f_u^{\mathrm{L}},$$

$$\delta f_d^{\mathrm{L}} = \frac{ie}{\sqrt{2}s_{\mathrm{W}}}f_u^{\mathrm{L}}\delta\hat{\theta}^- - ie\left[Q_{f_d}\delta\hat{\theta}^A + \left(\frac{1}{2c_{\mathrm{W}}s_{\mathrm{W}}} + Q_{f_d}\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}\right)\delta\hat{\theta}^Z\right]f_d^{\mathrm{L}},$$

$$\delta f^{\mathrm{R}} = -ieQ_f(\delta\hat{\theta}^A + \frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}\delta\hat{\theta}^Z)f^{\mathrm{R}}, \tag{22}$$

where f_u^{L} stands for all left-handed up-type quarks and neutrinos of (8), f_d^{L} denotes their isospin partners, and f^{R} represents the right-handed singlets of (9).

The effective action $\Gamma[\hat{V}, \hat{S}, F, \bar{F}]$ is the generating functional of the vertex functions which are obtained by differentiating $\Gamma[\hat{V}, \hat{S}, F, \bar{F}]$ with respect to its arguments. The vertex functions can be calculated from Feynman rules that distinguish between quantum and background fields. Whereas the quantum fields appear only inside loops, the background fields are associated with the external lines. Apart from doubling of the gauge and Higgs fields, the BFM Feynman rules differ from the conventional ones only owing to the gauge-fixing and ghost terms, which affect only vertices that involve both background and quantum fields. Since the gauge-fixing term is non-linear in the fields, the gauge parameter enters also the gauge-boson vertices. As mentioned above, the lowest-order Feynman rules involving fermion fields are the same as in the conventional formalism.

The S matrix is constructed in the usual way by forming trees with vertices from $\Gamma[\hat{V}, \hat{S}, F, \bar{F}]$ which are connected by lowest-order background-field propagators [5]. As a simple example, we calculated the one-loop process $Z \to b\bar{b}$ for arbitrary values of ξ_Q. We verified that the resulting S-matrix element is in fact independent of ξ_Q and equal to the one obtained in the conventional formalism.

We have evaluated the complete set of BFM Feynman rules in the electroweak SM for arbitrary values of the quantum gauge parameter ξ_Q. They are listed in the appendix. Despite the distinction between background and quantum fields, calculations in the BFM become in general simpler than in the conventional formalism. This is in particular the case in the 't Hooft-Feynman gauge ($\xi_Q = 1$) for the quantum fields where many vertices simplify considerably (see appendix). Moreover, the gauge fixing of the background fields is totally unrelated to the gauge fixing of the quantum fields. This freedom can be used to choose a particularly suitable background gauge, e.g. the unitary gauge or a non-linear gauge [18]. In this way the number of Feynman diagrams can be reduced drastically. The gauge fixing of the background fields does not affect $\Gamma[\hat{V}, \hat{S}, F, \bar{F}]$. It is only relevant for the construction of connected Green functions and S-matrix elements. In particular, in linear background gauges only the tree-level propagators are concerned.

Since the background gauge parameters enter only tree-level quantities, their cancellation in S-matrix elements is a direct consequence of the BFM Ward identities. As an example, this can easily be checked for background R_ξ gauges in four-fermion processes. In this case, the BFM Ward identities imply the cancellation of the background gauge parameters separately for self-energy and vertex contributions.

In Ref. [7], the BFM was applied to the process $Z \to 3\gamma$ at one-loop order. However, the gauge-fixing term used there breaks background-field gauge invariance since no background Higgs field has been introduced. This influences the vertex functions with external Higgs fields. Since for the specific process treated in Ref. [7] no such vertex function contributes, the results obtained there are nevertheless unaffected. In Ref. [7], the Feynman rules for vertices involving exactly two quantum fields and no background Higgs fields were given for the special case $\xi_Q = 1$. Putting $\xi_Q = 1$ in the corresponding rules given in appendix A we find agreement except for the ones in (A49b) which differ by a factor 2.

3 WARD IDENTITIES

The invariance of the effective action under the background gauge transformations specified in (21) and (22),

$$\delta\Gamma = \sum_i \frac{\delta\Gamma[\hat{V},\hat{S},F,\bar{F}]}{\delta\hat{V}_i}\delta\hat{V}_i + \sum_j \frac{\delta\Gamma[\hat{V},\hat{S},F,\bar{F}]}{\delta\hat{S}_j}\delta\hat{S}_j$$

$$+ \sum_k \left(\delta\bar{F}_k \frac{\delta\Gamma[\hat{V},\hat{S},F,\bar{F}]}{\delta\bar{F}_k} - \frac{\delta\Gamma[\hat{V},\hat{S},F,\bar{F}]}{\delta F_k}\delta F_k\right) = 0, \tag{23}$$

where i,j,k run over all background gauge fields, background Higgs fields and fermion fields, respectively, gives rise to simple Ward identities. Since the gauge invariance has been retained in the background-field formulation, these are precisely the Ward identities related to the classical Lagrangian. This is in contrast to the conventional formalism where owing to the gauge-fixing procedure the explicit gauge invariance is lost and the Ward identities are obtained from the invariance under BRS transformations. These Slavnov–Taylor identities have a more complicated structure and in general involve ghost contributions (see e.g. Ref. [19]).

The BFM Ward identities follow from differentiating (23) with respect to the fields and are valid in all orders of perturbation theory. Note that the identities hold for arbitrary values of the quantum gauge parameter.

We first list some identities for self-energies. These are related to the two-point vertex functions as follows

$$\Gamma_{\mu\nu}^{\hat{V}\hat{V}'}(k,-k) = i(-g_{\mu\nu}k^2 + k_\mu k_\nu + g_{\mu\nu}M_{\hat{V}}^2)\delta_{\hat{V}\hat{V}'}$$

$$+ i\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right)\Sigma_{\mathrm{T}}^{\hat{V}\hat{V}'}(k^2) - i\frac{k_\mu k_\nu}{k^2}\Sigma_{\mathrm{L}}^{\hat{V}\hat{V}'}(k^2),$$

$$\Gamma_\mu^{\hat{W}^\pm\hat{\phi}^\mp}(k,-k) = ik_\mu\left[\pm M_{\mathrm{W}} + \Sigma^{\hat{W}^\pm\hat{\phi}^\mp}(k^2)\right],$$

$$\Gamma_\mu^{\hat{Z}\hat{\chi}}(k,-k) = ik_\mu\left[iM_{\mathrm{Z}} + \Sigma^{\hat{Z}\hat{\chi}}(k^2)\right],$$

$$\Gamma_\mu^{\hat{A}\hat{\chi}}(k,-k) = ik_\mu\Sigma^{\hat{A}\hat{\chi}}(k^2),$$

$$\Gamma_\mu^{\hat{A}\hat{H}}(k,-k) = ik_\mu\Sigma^{\hat{A}\hat{H}}(k^2),$$

$$\Gamma^{\hat{H}\hat{H}}(k,-k) = i(k^2 - M_{\mathrm{H}}^2) + i\Sigma^{\hat{H}\hat{H}}(k^2),$$

$$\Gamma^{\hat{\chi}\hat{\chi}}(k,-k) = ik^2 + i\Sigma^{\hat{\chi}\hat{\chi}}(k^2),$$

$$\Gamma^{\hat{\phi}^+\hat{\phi}^-}(k,-k) = ik^2 + i\Sigma^{\hat{\phi}^+\hat{\phi}^-}(k^2),$$

$$\Gamma^{\hat{H}} = iT^{\hat{H}},$$

$$\Gamma^{\bar{f}f}(p,-p) = -i(\slashed{p} + m_f) - i\slashed{p}\,\omega_-\Sigma_{\mathrm{L}}^{\bar{f}f}(p^2) - i\slashed{p}\,\omega_+\Sigma_{\mathrm{R}}^{\bar{f}f}(p^2) + im_f\Sigma_{\mathrm{S}}^{\bar{f}f}(p^2) \tag{24}$$

where $\hat{V},\hat{V}'$ indicate vector fields, and, as throughout this paper, all momenta and fields in the vertex functions are incoming. In the following we omit the second argument of the two-point vertex functions which is fixed by momentum conservation. Note that no gauge-fixing terms for the background fields are included in the vertex functions, i.e. the

lowest-order contributions to the vertex functions follow directly from $\mathcal{L}_{\mathrm{C}}$. The self-energies contain no tadpole contributions; these appear explicitly as $T^{\hat{H}}$. We obtain the following Ward identities for the self-energies

$$\Sigma_{\mathrm{L}}^{\hat{A}\hat{A}}(k^2) = 0, \tag{25}$$

$$\Sigma_{\mathrm{L}}^{\hat{A}\hat{Z}}(k^2) = 0, \tag{26}$$

$$\Sigma^{\hat{A}\hat{\chi}}(k^2) = 0, \tag{27}$$

$$\Sigma^{\hat{A}\hat{H}}(k^2) = 0, \tag{28}$$

$$\Sigma_{\mathrm{L}}^{\hat{Z}\hat{Z}}(k^2) - iM_{\mathrm{Z}}\Sigma^{\hat{Z}\hat{\chi}}(k^2) = 0, \tag{29}$$

$$k^2\Sigma^{\hat{Z}\hat{\chi}}(k^2) - iM_{\mathrm{Z}}\Sigma^{\hat{\chi}\hat{\chi}}(k^2) + i\frac{e}{2c_{\mathrm{W}}s_{\mathrm{W}}}T^{\hat{H}} = 0, \tag{30}$$

$$\Sigma_{\mathrm{L}}^{\hat{W}^{\pm}\hat{W}^{\mp}}(k^2) \mp M_{\mathrm{W}}\Sigma^{\hat{W}^{\mp}\hat{\phi}^{\pm}}(k^2) = 0, \tag{31}$$

$$k^2\Sigma^{\hat{W}^{\pm}\hat{\phi}^{\mp}} \mp M_{\mathrm{W}}\Sigma^{\hat{\phi}^{\pm}\hat{\phi}^{\mp}} \pm \frac{e}{2s_{\mathrm{W}}}T^{\hat{H}} = 0. \tag{32}$$

As a direct consequence of the Ward identities (25) and (26) and of the analyticity of $\Gamma_{\mu\nu}^{\hat{A}\hat{A}}(k)$ and $\Gamma_{\mu\nu}^{\hat{A}\hat{Z}}(k)$ at $k^2 = 0$ their transverse parts vanish at zero momentum, i.e.

$$\Sigma_{\mathrm{T}}^{\hat{A}\hat{A}}(0) = 0, \tag{33}$$

and

$$\Sigma_{\mathrm{T}}^{\hat{A}\hat{Z}}(0) = 0. \tag{34}$$

Whereas the QED relations (25) and (33) are valid in the BFM to all orders, they only hold at one-loop order in the conventional formalism. The identities (26), (27) and (34) have no conventional counterpart. Note that the vanishing of the photon–Z-boson mixing at zero momentum is explicitly enforced through a renormalization condition in the usual on-shell scheme (see e.g. Ref. [16]), while in the BFM it is automatically fulfilled as a consequence of gauge invariance. The impact of the BFM on the renormalization program will be discussed in more detail in the next section. Equation (27) shows that in contrast to the R_{ξ} gauges of the conventional formalism the photon does not mix with the unphysical scalar χ in the BFM. For the Z-boson self-energy one has in the usual formalism at one-loop order[2]

$$k^2\left[\Sigma_{\mathrm{L}}^{ZZ}(k^2) - 2iM_{\mathrm{Z}}\Sigma^{Z\chi}(k^2)\right] - M_{\mathrm{Z}}^2\Sigma^{\chi\chi}(k^2) + \frac{eM_{\mathrm{Z}}}{2c_{\mathrm{W}}s_{\mathrm{W}}}T^H = 0. \tag{35}$$

In the BFM, this relation decouples into two simpler Ward identities, (29) and (30), which are valid to all orders.

The three-point function $\Gamma_{\mu}^{\hat{A}\bar{f}f}$ obeys

$$k^{\mu}\Gamma_{\mu}^{\hat{A}\bar{f}f}(k,\bar{p},p) = -eQ_f[\Gamma^{\bar{f}f}(\bar{p}) - \Gamma^{\bar{f}f}(-p)], \tag{36}$$

---

[2]For the relations corresponding to (25) and (35) in the conventional formalism at two-loop order see Ref. [20].

i.e. just the QED Ward identity. Note that despite the $U(1)_{\text{em}}$ gauge invariance of the classical Lagrangian the conventional formalism does not yield the QED-type Ward identity in the SM. In the BFM, the Ward identities for the $\hat{Z}\bar{f}f$ and $\hat{W}\bar{f}f$ vertices read

$$k^\mu \Gamma_\mu^{\hat{Z}\bar{f}f}(k,\bar{p},p) - iM_Z\Gamma^{\hat{\chi}\bar{f}f}(k,\bar{p},p) = e[\Gamma^{\bar{f}f}(\bar{p})(v_f - a_f\gamma_5) - (v_f + a_f\gamma_5)\Gamma^{\bar{f}f}(-p)], \tag{37}$$

$$k^\mu \Gamma_\mu^{\hat{W}^+\bar{f}_u f_d}(k,\bar{p},p) - M_W\Gamma^{\hat{\phi}^+\bar{f}_u f_d}(k,\bar{p},p) = \frac{e}{\sqrt{2}s_W}[\Gamma^{\bar{f}_u f_u}(\bar{p})\omega_- - \omega_+\Gamma^{\bar{f}_d f_d}(-p)],$$

$$k^\mu \Gamma_\mu^{\hat{W}^-\bar{f}_d f_u}(k,\bar{p},p) + M_W\Gamma^{\hat{\phi}^-\bar{f}_d f_u}(k,\bar{p},p) = \frac{e}{\sqrt{2}s_W}[\Gamma^{\bar{f}_d f_d}(\bar{p})\omega_- - \omega_+\Gamma^{\bar{f}_u f_u}(-p)], \tag{38}$$

where $v_f = (I_{W,f}^3 - 2s_W^2 Q_f)/(2s_W c_W)$ and $a_f = I_{W,f}^3/(2s_W c_W)$. Also the triple gauge-boson vertex fulfills a QED-like Ward identity

$$k^\mu \Gamma_{\mu\rho\sigma}^{\hat{A}\hat{W}^+\hat{W}^-}(k,k_+,k_-) = e[\Gamma_{\rho\sigma}^{\hat{W}^+\hat{W}^-}(k_+) - \Gamma_{\rho\sigma}^{\hat{W}^+\hat{W}^-}(-k_-)]. \tag{39}$$

An example involving vertex functions with Higgs bosons is:

$$k_Z^\mu \Gamma_\mu^{\hat{Z}\hat{\chi}\hat{H}}(k_Z,k_\chi,k_H) - iM_Z\Gamma^{\hat{\chi}\hat{\chi}\hat{H}}(k_Z,k_\chi,k_H) = -i\frac{e}{2s_W c_W}[\Gamma^{\hat{H}\hat{H}}(k_H) - \Gamma^{\hat{\chi}\hat{\chi}}(k_\chi)]. \tag{40}$$

Further Ward identities are listed in Refs. [8, 9].

4   RENORMALIZATION OF THE STANDARD MODEL

As we will show in this section, the BFM gauge invariance has important consequences for the structure of the renormalization constants necessary to render Green functions and S-matrix elements finite. The arguments which we give in the following are made explicit for the one-loop level. It is easy, however, to extend them by induction to arbitrary orders in perturbation theory.

Following the QCD treatment of Ref. [2], we introduce field renormalization only for the background fields. We start with the following set of renormalization constants for the parameters

$$\begin{aligned}
e_0 &= Z_e e = (1 + \delta Z_e)e, \\
M_{W,0}^2 &= M_W^2 + \delta M_W^2, \\
M_{Z,0}^2 &= M_Z^2 + \delta M_Z^2, \\
M_{H,0}^2 &= M_H^2 + \delta M_H^2, \\
m_{f,0} &= m_f + \delta m_f, \\
t_0 &= t + \delta t,
\end{aligned} \tag{41}$$

and fields

$$\hat{W}_0^\pm = Z_{\hat{W}}^{1/2}\hat{W}^\pm = (1 + \frac{1}{2}\delta Z_{\hat{W}})\hat{W}^\pm,$$

$$\begin{pmatrix} \hat{Z}_0 \\ \hat{A}_0 \end{pmatrix} = \begin{pmatrix} Z_{\hat{Z}\hat{Z}}^{1/2} & Z_{\hat{Z}\hat{A}}^{1/2} \\ Z_{\hat{A}\hat{Z}}^{1/2} & Z_{\hat{A}\hat{A}}^{1/2} \end{pmatrix} \begin{pmatrix} \hat{Z} \\ \hat{A} \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{\hat{Z}\hat{Z}} & \frac{1}{2}\delta Z_{\hat{Z}\hat{A}} \\ \frac{1}{2}\delta Z_{\hat{A}\hat{Z}} & 1 + \frac{1}{2}\delta Z_{\hat{A}\hat{A}} \end{pmatrix} \begin{pmatrix} \hat{Z} \\ \hat{A} \end{pmatrix},$$

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$$\hat{H}_0 = Z_{\hat{H}}^{1/2}\hat{H} = (1 + \tfrac{1}{2}\delta Z_{\hat{H}})\hat{H},$$

$$\hat{\chi}_0 = Z_{\hat{\chi}}^{1/2}\hat{\chi} = (1 + \tfrac{1}{2}\delta Z_{\hat{\chi}})\hat{\chi},$$

$$\hat{\phi}_0^{\pm} = Z_{\hat{\phi}}^{1/2}\hat{\phi}^{\pm} = (1 + \tfrac{1}{2}\delta Z_{\hat{\phi}})\hat{\phi}^{\pm},$$

$$f_0^{\mathrm{L}} = \left(Z_f^{\mathrm{L}}\right)^{1/2} f^{\mathrm{L}} = (1 + \tfrac{1}{2}\delta Z_f^{\mathrm{L}})f^{\mathrm{L}},$$

$$f_0^{\mathrm{R}} = \left(Z_f^{\mathrm{R}}\right)^{1/2} f^{\mathrm{R}} = (1 + \tfrac{1}{2}\delta Z_f^{\mathrm{R}})f^{\mathrm{R}}. \tag{42}$$

The tadpole counterterm δt renormalizes the term in the Lagrangian linear in the Higgs field $\hat{H}$ which we denote by $t\hat{H}(x)$ with $t = v(\mu^2 - \lambda v^2/4)$. It corrects for the shift in the minimum of the Higgs potential due to radiative corrections. Choosing v as the correct vacuum expectation value of the Higgs field $\hat{\Phi}$ is equivalent to the vanishing of t.

In order to preserve the background-field gauge invariance when renormalizing the theory it is necessary to require that the renormalized vertex functions fulfill Ward identities of the same form as the unrenormalized ones. As a consequence, also the counterterms have to fulfill these Ward identities. This yields relations between the counterterms.

For example, from (26) one obtains immediately

$$0 = \Sigma_{\mathrm{L}}^{\hat{A}\hat{Z},\mathrm{ren}}(k^2) = \Sigma_{\mathrm{L}}^{\hat{A}\hat{Z}}(k^2) - M_{\mathrm{Z}}^2\tfrac{1}{2}\delta Z_{\hat{Z}\hat{A}} = -M_{\mathrm{Z}}^2\tfrac{1}{2}\delta Z_{\hat{Z}\hat{A}}, \tag{43}$$

i.e.

$$\delta Z_{\hat{Z}\hat{A}} = 0. \tag{44}$$

Expressing bare quantities in the QED Ward identity (36) through renormalized ones and counterterms yields

$$k^{\mu}\Gamma_{\mu}^{\hat{A}\bar{f}f}(k,\bar{p},p) = k^{\mu}\Gamma_{\mu}^{\hat{A}\bar{f}f,\mathrm{ren}}(k,\bar{p},p) + ieQ_f\slashed{k}\left(\delta Z_e + \tfrac{1}{2}\delta Z_{\hat{A}\hat{A}} + \delta Z_e^{\mathrm{R}}\omega_+ + \delta Z_e^{\mathrm{L}}\omega_-\right),$$

where (44) was used, and

$$-eQ_f\left[\Gamma^{\bar{f}f}(\bar{p}) - \Gamma^{\bar{f}f}(-p)\right] = -eQ_f\left[\Gamma^{\bar{f}f,\mathrm{ren}}(\bar{p}) - \Gamma^{\bar{f}f,\mathrm{ren}}(-p)\right] + ieQ_f\slashed{k}\left(\delta Z_e^{\mathrm{R}}\omega_+ + \delta Z_e^{\mathrm{L}}\omega_-\right).$$

Using the Ward identity both for bare and renormalized quantities implies

$$\delta Z_{\hat{A}\hat{A}} = -2\delta Z_e. \tag{45}$$

This is just the famous relation between the renormalizations of field and coupling known from QED. In contrast to the conventional formalism, the BFM yields this relation also for the electroweak SM. Note that after fixing the charge renormalization there is no more freedom to impose an extra condition for the field renormalization. Just as in QED, the on-shell definition of the electric charge together with gauge invariance automatically fixes the residue of the photon propagator to unity. This can be derived using the Ward identities (34) and (36). Instead of considering (36), the relation (45) can equivalently be obtained from the Ward identity (39) for the non-Abelian coupling.

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From the Ward identities (37), (38) and (40) one derives in a similiar way the following relations between the renormalization constants

$$\delta Z_{\hat{A}\hat{Z}} = 2\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2},$$

$$\delta Z_{\hat{Z}\hat{Z}} = -2\delta Z_e - \frac{c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2},$$

$$\delta Z_{\hat{W}} = -2\delta Z_e - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2},$$

$$\delta Z_{\hat{H}} = \delta Z_{\hat{\chi}} = \delta Z_{\hat{\phi}} = -2\delta Z_e - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2} + \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2}, \tag{46}$$

where

$$\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2} = \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2} - \frac{\delta M_{\mathrm{Z}}^2}{M_{\mathrm{Z}}^2}.$$

Finally, we get for the field renormalizations of the fermions

$$\delta Z_f^{\mathrm{L}} = \delta Z_{f_u}^{\mathrm{L}} = \delta Z_{f_d}^{\mathrm{L}}, \tag{47}$$

i.e. the field renormalization constants for the two left-handed fermions in a doublet must be equal.

The relations (44) – (46) express the field renormalization constants of all gauge bosons and scalars completely in terms of the renormalization constants of the electric charge and the particle masses. If the renormalized parameters are identified with the physical electron charge and the physical particle masses, they are manifestly gauge-independent. Moreover, the bare quantities $g_{1,0}$, $g_{2,0}$, λ_0, μ_0 and $G_{i,0}^f$ in the Lagrangian obviously are also gauge-independent, as they represent free parameters of the theory. According to (12) and (13), the same is true for the bare charge and the bare weak mixing angle. Consequently, the counterterms δZ_e and δc_{W}^2 for the gauge couplings are gauge-independent. The relations (45) and (46) therefore imply that the field renormalizations of all gauge-boson fields are gauge-independent. This is in contrast to the conventional formalism where the field renormalizations in the on-shell scheme are gauge-dependent.

It should be recalled at this point that in contrast to δZ_e and δc_{W}^2 the counterterms for the masses are not gauge-independent. This can be traced back to the mechanism of spontaneous symmetry breaking. The non-vanishing vacuum expectation value of the Higgs field, which generates the mass terms, is clearly not invariant under gauge transformations. Whereas the renormalized value $v = 2s_{\mathrm{W}}M_{\mathrm{W}}/e$ is gauge-independent, the bare quantity v_0 and the corresponding counterterm δv are not [21]. As a consequence, the bare masses which depend on v_0 are gauge-dependent. Thus, the counterterms δM_{W}^2, δM_{Z}^2, δM_{H}^2, δm_f and δt are also gauge-dependent. The physical masses, however, are determined by the pole positions of the propagators, i.e. the zeros of $k^2 - M^2 - \delta M^2 + C\delta t/M_{\mathrm{H}}^2 + \Sigma(k^2) + CT^{\hat{H}}/M_{\mathrm{H}}^2$, where C denotes the coupling of the fields to the Higgs field. The linear combination $\delta M^2 - C\delta t/M_{\mathrm{H}}^2$ of the mass and tadpole counterterm is independent of δv and thus gauge-independent.

The relations (44) – (46) reduce the number of independent renormalization constants considerably. One is left with the parameter renormalizations appearing in (41) and the

fermion field renormalization constants δZ_f^L, $\delta Z_{f_u}^R$ and $\delta Z_{f_d}^R$. We choose on-shell renormalization conditions for the parameters as in Ref. [16][3] and express the renormalization constants in terms of unrenormalized self-energies and the tadpole

$$\delta Z_e = \frac{1}{2} \frac{\partial \Sigma_T^{\hat{A}\hat{A}}(k^2)}{\partial k^2}\bigg|_{k^2=0},$$

$$\delta M_W^2 = \text{Re}\left(\Sigma_T^{\hat{W}\hat{W}}(M_W^2)\right),$$

$$\delta M_Z^2 = \text{Re}\left(\Sigma_T^{\hat{Z}\hat{Z}}(M_Z^2)\right),$$

$$\delta M_H^2 = \text{Re}\left(\Sigma_T^{\hat{H}\hat{H}}(M_H^2)\right),$$

$$\delta m_f = \frac{1}{2} m_f \text{Re}\left[\Sigma_L^{\bar{f}f}(m_f^2) + \Sigma_R^{\bar{f}f}(m_f^2) + 2\Sigma_S^{\bar{f}f}(m_f^2)\right],$$

$$\delta t = -T^{\hat{H}}. \tag{48}$$

The fermion field renormalization constants can be fixed as follows

$$\delta Z_f^L = -\text{Re}\,\Sigma_L^{\bar{f}_d f_d}(m_{f_d}^2) - m_{f_d}^2 \frac{\partial}{\partial k^2} \text{Re}\left(\Sigma_L^{\bar{f}_d f_d}(k^2) + \Sigma_R^{\bar{f}_d f_d}(k^2) + 2\Sigma_S^{\bar{f}_d f_d}(k^2)\right)\bigg|_{k^2=m_{f_d}^2},$$

$$\delta Z_{f_u}^R = -\text{Re}\,\Sigma_R^{\bar{f}_u f_u}(m_{f_u}^2) - m_{f_u}^2 \frac{\partial}{\partial k^2} \text{Re}\left(\Sigma_L^{\bar{f}_u f_u}(k^2) + \Sigma_R^{\bar{f}_u f_u}(k^2) + 2\Sigma_S^{\bar{f}_u f_u}(k^2)\right)\bigg|_{k^2=m_{f_u}^2},$$

$$\delta Z_{f_d}^R = -\text{Re}\,\Sigma_R^{\bar{f}_d f_d}(m_{f_d}^2) - m_{f_d}^2 \frac{\partial}{\partial k^2} \text{Re}\left(\Sigma_L^{\bar{f}_d f_d}(k^2) + \Sigma_R^{\bar{f}_d f_d}(k^2) + 2\Sigma_S^{\bar{f}_d f_d}(k^2)\right)\bigg|_{k^2=m_{f_d}^2}. \tag{49}$$

Although there is no freedom to choose the field renormalizations of the gauge bosons, scalars and left-handed up-type fermions in the BFM, the specified set of renormalization constants is still sufficient to render all background-field vertex functions finite[4]. This is evident since the divergences of the vertex functions are subject to the same restriction as the counterterms. In order to illustrate this fact at one-loop order we list the divergent part of the self-energies in the BFM using dimensional regularization and writing the dimension as $D = 4 - \epsilon$,

$$\left(\Sigma_T^{\hat{A}\hat{A}}(k^2)\right)^{\text{div}} = \frac{e^2}{16\pi^2} k^2 \left(\frac{32}{9}n - 7\right) \frac{2}{\epsilon},$$

$$\left(\Sigma_T^{\hat{A}\hat{Z}}(k^2)\right)^{\text{div}} = \frac{e^2}{16\pi^2} k^2 \left(\frac{32s_W^2 - 12}{9c_W s_W}n + \frac{42c_W^2 + 1}{6c_W s_W}\right) \frac{2}{\epsilon},$$

$$\left(\Sigma_T^{\hat{Z}\hat{Z}}(k^2)\right)^{\text{div}} = \frac{e^2}{16\pi^2} \left[k^2 \left(\frac{32s_W^4 - 12(2s_W^2 - 1)}{9c_W^2 s_W^2}n - \frac{42c_W^4 + 2c_W^2 - 1}{6c_W^2 s_W^2}\right)\right.$$

---

[3]The charge renormalization condition formulated in Ref. [16] assumes that the residue of the renormalized photon propagator equals unity and that the photon–Z-boson mixing vanishes for on-shell photons. Owing to the Ward identities, these conditions are fulfilled and we can use the same condition in the BFM.

[4]Beyond one-loop order one needs in addition a renormalization of the quantum gauge parameters [2]. At the one-loop level these counterterms do not enter the background-field vertex functions because ξ_Q does not appear in pure background-field vertices. Clearly, the renormalization of gauge parameters is irrelevant for S-matrix elements at any order as these are gauge-independent.

$$
-\sum_f \frac{m_f^2}{2c_{\mathrm{W}}^2 s_{\mathrm{W}}^2} + \frac{2M_{\mathrm{W}}^2 + M_{\mathrm{Z}}^2}{4c_{\mathrm{W}}^2 s_{\mathrm{W}}^2}(\xi_Q + 3)\Bigg] \frac{2}{\epsilon} \, ,
$$

$$
\left(\Sigma_{\mathrm{T}}^{\hat{W}\hat{W}}(k^2)\right)^{\mathrm{div}} = \frac{e^2}{16\pi^2}\left[k^2\left(\frac{4}{3s_{\mathrm{W}}^2}n - \frac{43}{6s_{\mathrm{W}}^2}\right) - \sum_f \frac{m_f^2}{2s_{\mathrm{W}}^2} + \frac{2M_{\mathrm{W}}^2 + M_{\mathrm{Z}}^2}{4s_{\mathrm{W}}^2}(\xi_Q + 3)\right]\frac{2}{\epsilon} \, ,
$$

$$
\left(\Sigma^{\hat{H}\hat{H}}(k^2)\right)^{\mathrm{div}} = \frac{e^2}{16\pi^2}\Bigg[k^2\left(\sum_f \frac{m_f^2}{2M_{\mathrm{W}}^2 s_{\mathrm{W}}^2} - \frac{2c_{\mathrm{W}}^2 + 1}{4c_{\mathrm{W}}^2 s_{\mathrm{W}}^2}(\xi_Q + 3)\right) - \sum_f \frac{3m_f^4}{M_{\mathrm{W}}^2 s_{\mathrm{W}}^2}
$$
$$
+ \frac{3(5M_{\mathrm{H}}^4 + 12M_{\mathrm{W}}^4 + 6M_{\mathrm{Z}}^4) + 5M_{\mathrm{H}}^2(2M_{\mathrm{W}}^2 + M_{\mathrm{Z}}^2)\xi_Q}{8M_{\mathrm{W}}^2 s_{\mathrm{W}}^2}\Bigg]\frac{2}{\epsilon} \, ,
$$

$$
\left(\Sigma^{\hat{\chi}\hat{\chi}}(k^2)\right)^{\mathrm{div}} = \left(\Sigma^{\hat{\phi}\hat{\phi}}(k^2)\right)^{\mathrm{div}}
$$
$$
= \frac{e^2}{16\pi^2}\left[k^2\left(\sum_f \frac{m_f^2}{2M_{\mathrm{W}}^2 s_{\mathrm{W}}^2} - \frac{2c_{\mathrm{W}}^2 + 1}{4c_{\mathrm{W}}^2 s_{\mathrm{W}}^2}(\xi_Q + 3)\right)\right]\frac{2}{\epsilon} + \frac{e}{2M_{\mathrm{W}}s_{\mathrm{W}}}\left(T^{\hat{H}}\right)^{\mathrm{div}} \, ,
$$

$$
\left(\Sigma_{\mathrm{L}}^{\bar{f}f}(k^2)\right)^{\mathrm{div}} = \frac{e^2}{16\pi^2}\left(\frac{m_f^2 + m_{f'}^2}{4M_{\mathrm{W}}^2 s_{\mathrm{W}}^2} + \frac{4s_{\mathrm{W}}^2 Q_f^2 - 8I_{\mathrm{W},f}^3 Q_f s_{\mathrm{W}}^2 + 2c_{\mathrm{W}}^2 + 1}{4c_{\mathrm{W}}^2 s_{\mathrm{W}}^2}\xi_Q\right)\frac{2}{\epsilon} \, ,
$$

$$
\left(\Sigma_{\mathrm{R}}^{\bar{f}f}(k^2)\right)^{\mathrm{div}} = \frac{e^2}{16\pi^2}\left(\frac{m_f^2}{2M_{\mathrm{W}}^2 s_{\mathrm{W}}^2} + \frac{Q_f^2}{c_{\mathrm{W}}^2}\xi_Q\right)\frac{2}{\epsilon} \, ,
$$

$$
\left(\Sigma_{\mathrm{S}}^{\bar{f}f}(k^2)\right)^{\mathrm{div}} = -\frac{e^2}{16\pi^2}\left(\frac{m_{f'}^2}{2M_{\mathrm{W}}^2 s_{\mathrm{W}}^2} + \frac{(Q_f - I_{\mathrm{W},f}^3)Q_f}{c_{\mathrm{W}}^2}(\xi_Q + 3)\right)\frac{2}{\epsilon} \, ,
$$

$$
\left(T^{\hat{H}}\right)^{\mathrm{div}} = \frac{e^2}{16\pi^2}\Bigg(-\sum_f \frac{2m_f^4}{eM_{\mathrm{W}}s_{\mathrm{W}}}
$$
$$
+ \frac{3(M_{\mathrm{H}}^4 + 4M_{\mathrm{W}}^4 + 2M_{\mathrm{Z}}^4) + M_{\mathrm{H}}^2(2M_{\mathrm{W}}^2 + M_{\mathrm{Z}}^2)\xi_Q}{4eM_{\mathrm{W}}s_{\mathrm{W}}}\Bigg)\frac{2}{\epsilon} \, , \tag{50}
$$

where f' is the isospin partner of fermion f, n denotes the number of fermion generations and the summations run over all fermion flavors and colors. The fermion self-energies and the fermionic contributions to the gauge-boson and scalar self-energies are included for completeness. They have the same form as in the conventional formalism.

Using (48) and (49) we obtain the divergent parts of the renormalization constants

$$
(\delta Z_e)^{\mathrm{div}} = \frac{e^2}{16\pi^2}\frac{1}{2}\left(\frac{32}{9}n - 7\right)\frac{2}{\epsilon} \, ,
$$

$$
\left(\frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2}\right)^{\mathrm{div}} = \frac{e^2}{16\pi^2}\left(\frac{4}{3s_{\mathrm{W}}^2}n - \frac{43}{6s_{\mathrm{W}}^2} - \sum_f \frac{m_f^2}{2M_{\mathrm{W}}^2 s_{\mathrm{W}}^2} + \frac{2c_{\mathrm{W}}^2 + 1}{4c_{\mathrm{W}}^2 s_{\mathrm{W}}^2}(\xi_Q + 3)\right)\frac{2}{\epsilon} \, ,
$$

$$
\left(\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}\right)^{\mathrm{div}} = \frac{e^2}{16\pi^2}\left(\frac{-32s_{\mathrm{W}}^2 + 12}{9c_{\mathrm{W}}^2}n - 7 - \frac{1}{6c_{\mathrm{W}}^2}\right)\frac{2}{\epsilon} \, ,
$$

$$
\left(\delta M_{\mathrm{H}}^2\right)^{\mathrm{div}} = \frac{e^2}{16\pi^2}\Bigg(\sum_f \frac{m_f^2(M_{\mathrm{H}}^2 - 6m_f^2)}{2M_{\mathrm{W}}^2 s_{\mathrm{W}}^2}
$$
$$
+ \frac{3[5M_{\mathrm{H}}^4 + 12M_{\mathrm{W}}^4 + 6M_{\mathrm{Z}}^4 + M_{\mathrm{H}}^2(2M_{\mathrm{W}}^2 + M_{\mathrm{Z}}^2)(\xi_Q - 2)]}{8M_{\mathrm{W}}^2 s_{\mathrm{W}}^2}\Bigg)\frac{2}{\epsilon} \, ,
$$

$$(\delta m_f)^{\text{div}} = \frac{e^2}{16\pi^2} m_f \left(\frac{3(m_f^2 - m_{f'}^2)}{8M_{\text{W}}^2 s_{\text{W}}^2} - \frac{3Q_f(Q_f - I_{\text{W},3}^f)}{c_{\text{W}}^2} + \frac{(2c_{\text{W}}^2 + 1)\xi_Q}{8c_{\text{W}}^2 s_{\text{W}}^2} \right) \frac{2}{\epsilon},$$

$$(\delta t)^{\text{div}} = -\left(T^{\hat{H}} \right)^{\text{div}},$$

$$\left(\delta Z_f^{\text{L}} \right)^{\text{div}} = -\left(\Sigma_{\text{L}}^{\bar{f}_u f_u}(k^2) \right)^{\text{div}} = -\left(\Sigma_{\text{L}}^{\bar{f}_d f_d}(k^2) \right)^{\text{div}},$$

$$\left(\delta Z_{f_u}^{\text{R}} \right)^{\text{div}} = -\left(\Sigma_{\text{R}}^{\bar{f}_u f_u}(k^2) \right)^{\text{div}},$$

$$\left(\delta Z_{f_d}^{\text{R}} \right)^{\text{div}} = -\left(\Sigma_{\text{R}}^{\bar{f}_d f_d}(k^2) \right)^{\text{div}}. \tag{51}$$

According to (45) and (46), this also fixes the divergent parts of the gauge-boson and scalar field renormalization constants yielding

$$(\delta Z_{\hat{A}\hat{A}})^{\text{div}} = -\frac{e^2}{16\pi^2} \left(\frac{32}{9}n - 7 \right) \frac{2}{\epsilon},$$

$$(\delta Z_{\hat{A}\hat{Z}})^{\text{div}} = -\frac{e^2}{16\pi^2} 2 \left(\frac{32s_{\text{W}}^2 - 12}{9c_{\text{W}}s_{\text{W}}}n + \frac{42c_{\text{W}}^2 + 1}{6c_{\text{W}}s_{\text{W}}} \right) \frac{2}{\epsilon},$$

$$(\delta Z_{\hat{Z}\hat{Z}})^{\text{div}} = -\frac{e^2}{16\pi^2} \left(\frac{32s_{\text{W}}^4 - 12(2s_{\text{W}}^2 - 1)}{9c_{\text{W}}^2 s_{\text{W}}^2}n - \frac{42c_{\text{W}}^4 + 2c_{\text{W}}^2 - 1}{6c_{\text{W}}^2 s_{\text{W}}^2} \right) \frac{2}{\epsilon},$$

$$(\delta Z_{\hat{W}})^{\text{div}} = -\frac{e^2}{16\pi^2} \left(\frac{4}{3s_{\text{W}}^2}n - \frac{43}{6s_{\text{W}}^2} \right) \frac{2}{\epsilon},$$

$$(\delta Z_{\hat{H}})^{\text{div}} = (\delta Z_{\hat{\chi}})^{\text{div}} = \left(\delta Z_{\hat{\phi}} \right)^{\text{div}}$$

$$= -\frac{e^2}{16\pi^2} \left(\sum_f \frac{m_f^2}{2M_{\text{W}}^2 s_{\text{W}}^2} - \frac{2c_{\text{W}}^2 + 1}{4c_{\text{W}}^2 s_{\text{W}}^2}(\xi_Q + 3) \right) \frac{2}{\epsilon}. \tag{52}$$

The divergent parts of the gauge-boson field renormalization constants are independent of ξ_Q in accordance with the general discussion given above.

The renormalized self-energies are obtained by adding the counterterms specified in (A3) – (A10) to the unrenormalized self-energies. It is evident from (50) – (52) that although the field renormalization constants cannot be chosen freely in the BFM, all renormalized self-energies are nevertheless finite. Whereas in the conventional formalism the field renormalization constants are adjusted in order to obtain finite self-energies, this happens automatically in the BFM as a consequence of the Ward identities. The finiteness of the longitudinal parts of the gauge-boson self-energies and of the gauge-boson–scalar mixing energies follows directly from the finiteness of the renormalized tadpole and scalar self-energies and the Ward identities (27) – (32).

A renormalization based on the on-shell definition of all parameters can therefore consistently be used in the BFM. It renders all vertex functions finite while respecting the full gauge symmetry of the BFM.

Since the divergent parts of the unrenormalized self-energies fulfill the Ward identities by themselves, it is obvious that renormalization in the minimal-subtraction scheme also preserves the symmetry of the BFM.

As mentioned above, the on-shell renormalization in the BFM fixes the residue of the photon propagator to unity. The propagators of the other gauge bosons, scalars and

left-handed up-type fermions acquire residues different from unity. This is similar to the minimal on-shell scheme of the conventional formalism and has to be corrected in the S-matrix elements by a UV-finite wave-function renormalization.

The renormalization constants introduced in (41) and (42) correspond to the physical fields, i.e. the mass and electric charge eigenstates $\hat{A}, \hat{Z}, \hat{W}^\pm$. Alternatively, one can introduce renormalization constants in the symmetric formulation (see e.g. Ref. [19]) resulting in the minimal on-shell scheme. In the bosonic sector these renormalization constants are given by

$$
\begin{aligned}
\hat{W}_0^a &= (Z_2^{\hat{W}})^{1/2}\hat{W}^a, \\
\hat{B}_0 &= (Z_2^{\hat{B}})^{1/2}\hat{B}, \\
\hat{\Phi}_0 &= (Z^{\hat{\Phi}})^{1/2}\hat{\Phi}, \\
g_{2,0} &= Z_1^{\hat{W}}(Z_2^{\hat{W}})^{-3/2}g_2 = Z_{g_2}g_2, \\
g_{1,0} &= Z_1^{\hat{B}}(Z_2^{\hat{B}})^{-3/2}g_1 = Z_{g_1}g_1, \\
v_0 &= (Z^{\hat{\Phi}})^{1/2}(v - \delta v), \\
\mu_0^2 &= (Z^{\hat{\Phi}})^{-1}(\mu^2 - \delta\mu^2), \\
\lambda_0 &= Z^\lambda (Z^{\hat{\Phi}})^{-2}\lambda.
\end{aligned}
\tag{53}
$$

In this formulation, the gauge symmetry of the BFM implies in addition to $Z_1^{\hat{B}} = Z_2^{\hat{B}}$

$$
\begin{aligned}
Z_1^{\hat{W}} &= Z_2^{\hat{W}}, \\
\delta v &= 0.
\end{aligned}
\tag{54}
$$

Thus, for both the isotriplet fields of $SU(2)_W$ and the isosinglet field of $U(1)_Y$ a QED-like relation between coupling constant and field renormalization holds, and there is no renormalization of the vacuum expectation value other than the one owing to the Higgs-field renormalization. The other restrictions following from (44) – (47) are already taken into account in the ansatz (53) for the field renormalization. It is clear that also in this on-shell scheme the field renormalizations of the gauge bosons are gauge-independent. With the restrictions imposed by the BFM, the two renormalization schemes become in fact equivalent, i.e. both schemes yield identical renormalized Green functions.

We have derived the relations between the renormalization constants from the background-field Ward identities given in the last section. As the gauge invariance of the effective action is directly related to the gauge invariance of the classical Lagrangian [2], those relations can also be inferred directly from the Lagrangian. One can check that the relations listed above are precisely those required to render the renormalized classical Lagrangian $\mathcal{L}_C$ gauge-invariant.

As a consequence of the relations (44) – (47), the counterterm vertices of the background fields have a much simpler structure than the ones in the conventional formalism (see e.g. Ref. [16]). Their explicit form is given in the appendix. Moreover, all vertices resulting from an irreducible gauge-invariant part of the Lagrangian and in particular all realizations of a generic vertex, e.g. $\hat{V}\hat{V}\hat{V}\hat{V}$, are renormalized in the same way.

In the appendix we have listed the counterterms for all vertices involving only background fields. These are sufficient for the renormalization of all one-loop processes.

Through the parameter renormalizations and the renormalizations of the background fields also the vertices containing both quantum and background fields and the pure quantum-field vertices acquire counterterms. These become relevant in higher orders. Their explicit form can easily be obtained using (41), (42) and the Feynman rules given in the appendix.

5 PROPERTIES OF BFM VERTEX FUNCTIONS

As mentioned above, the BFM vertex functions possess improved theoretical properties compared to their conventional counterparts. In previous treatments, such properties were either explicitly enforced by construction [10, 11] or could only be verified for specific examples [12, 13]. Since the properties could not be derived from the theory, their theoretical understanding remained unclear. Moreover, the new "vertex functions" were obtained by rearranging contributions between different conventional Green functions. The field-theoretical meaning of these objects is obscure. In the BFM, the background-field vertex functions themselves exhibit the improved properties. As will be illustrated in this section, these properties can be directly deduced from the Ward identities discussed in section 3. The Ward identities are a direct consequence of the background-field gauge invariance and are valid independent of the value of the quantum gauge parameter ξ_Q. Consequently, the properties of the BFM vertex functions following from these identities also hold for arbitrary ξ_Q.

We first consider the fermion–gauge-boson vertex functions. In Ref. [13] it was found by explicit calculation that in the pinch technique the one-loop fermion–gauge-boson vertex functions are UV-finite when the fermion field renormalization has been added. In the BFM, this fact is an obvious consequence of the relations between the renormalization constants derived in the last section. As follows from (A25), the counterterm for the $\hat{V}\bar{F}F$-vertex is solely given by the fermion field renormalization. Adding it to the vertex function evidently cancels the UV divergence. Obviously, this fact holds for all values of the quantum gauge parameter ξ_Q. From the counterterm structure given in the appendix similar conclusions can be drawn for other vertex functions. In particular, the $\hat{V}\hat{W}\hat{W}$ and $\hat{V}\hat{V}'\hat{W}\hat{W}$ vertices become UV-finite after adding the field renormalization of two $\hat{W}$ fields as can be read from (A11) and (A13). In Ref. [13] it was also noted that the one-loop fermion–photon vertex functions including fermion field renormalization vanish at zero momentum transfer of the photon. In the BFM, the inclusion of the fermion field renormalization amounts to the complete renormalization of this vertex. But the renormalized vertex correction vanishes owing to the renormalization condition for the electric charge.

Next, we investigate the asymptotic behavior of the gauge-boson self-energies in the BFM for $|q^2| \to \infty$. In Ref. [9], the explicit one-loop result for the leading logarithms of the bosonic contributions to the gauge-boson self-energies in the BFM has been given showing that their coefficients are independent of ξ_Q. However, this feature can also be deduced from the Ward identities as follows. In dimensional regularization the unrenormalized one-loop self-energies obey ($\hat{V}, \hat{V}' = \hat{A}, \hat{Z}, \hat{W}$)

$$\Pi^{\hat{V}\hat{V}'}(q^2) = \frac{\Sigma_{\mathrm{T}}^{\hat{V}\hat{V}'}(q^2) - \Sigma_{\mathrm{T}}^{\hat{V}\hat{V}'}(0)}{q^2} = g_{\hat{V}\hat{V}'}^2 \, \mu^\epsilon \left(-c_{\hat{V}\hat{V}'} \frac{2}{\epsilon} + \text{UV-finite terms} \right), \quad (55)$$

17

where $c_{\hat{V}\hat{V}'}$ is a q^2-independent coefficient, which can be read off from (50), $g_{\hat{A}\hat{A}} = e$, $g_{\hat{W}\hat{W}} = g_2 = e/s_W$, $g_{\hat{Z}\hat{Z}} = e/(c_W s_W)$, $g_{\hat{A}\hat{Z}} = e/\sqrt{c_W s_W}$, and μ is a mass parameter necessary to keep $g^2_{\hat{V}\hat{V}'}$ dimensionless. In the limit $|q^2| \to \infty$ all masses can be neglected and on dimensional grounds the self-energies behave as

$$\Pi^{\hat{V}\hat{V}'}(q^2) \underset{|q^2|\to\infty}{\widetilde{}} g^2_{\hat{V}\hat{V}'} \left(-c_{\hat{V}\hat{V}'}\frac{2}{\epsilon} + c_{\hat{V}\hat{V}'} \log\frac{|q^2|}{\mu^2} + \text{UV-finite constant} \right). \qquad (56)$$

Using the identities

$$\left(\delta Z_{\hat{V}\hat{V}}\right)^{\text{div}} = -\left(\Pi^{\hat{V}\hat{V}}(q^2)\right)^{\text{div}}, \quad \left(\delta Z_{\hat{A}\hat{Z}}\right)^{\text{div}} = -2\left(\Pi^{\hat{A}\hat{Z}}(q^2)\right)^{\text{div}} \qquad (57)$$

we can identify the divergent parts of $\delta Z_{\hat{V}\hat{V}}$ and $\delta Z_{\hat{A}\hat{Z}}$ as

$$\left(\delta Z_{\hat{V}\hat{V}}\right)^{\text{div}} = g^2_{\hat{V}\hat{V}}c_{\hat{V}\hat{V}}\frac{2}{\epsilon}, \quad \left(\delta Z_{\hat{A}\hat{Z}}\right)^{\text{div}} = 2g^2_{\hat{A}\hat{Z}}c_{\hat{A}\hat{Z}}\frac{2}{\epsilon}. \qquad (58)$$

We found in section 4 that the field renormalization constants for the gauge bosons and thus $(\delta Z_{\hat{V}\hat{V}'})^{\text{div}}$ are gauge-independent. As a consequence, also the coefficients $c_{\hat{V}\hat{V}'}$ of the leading logarithms of $\Sigma_{\text{T}}^{\hat{V}\hat{V}'}(q^2)$ are independent of ξ_Q.

In Ref. [2] it has been shown for QCD that in the BFM the β-function of the gauge coupling is related to the anomalous dimension and thus to the field renormalization constant of the gauge boson. The same applies to the SM as well. The relation $Z_e = Z_{\hat{A}\hat{A}}^{-1/2}$ implies for the β-function associated with the electromagnetic coupling in the minimal-subtraction scheme

$$\beta_e(e) = c_{\hat{A}\hat{A}}e^3 + \mathcal{O}(e^5), \qquad (59)$$

i.e. in analogy to QED, the coefficient of the leading logarithm of the photon self-energy in the BFM equals the coefficient of the one-loop β-function. Analogously, the relation $Z_{g_2} = (Z_2^{\hat{W}})^{-1/2} = Z_{\hat{W}}^{-1/2}$, which can be inferred from (46) and (54), yields for the charged-current coupling

$$\beta_{g_2}(g_2) = c_{\hat{W}\hat{W}}g_2^3 + \mathcal{O}(g_2^5). \qquad (60)$$

The fact that the coefficients of the leading logarithms of the self-energies equal the coefficients of the β-functions implies that the asymptotic behavior of effective coupling constants $e^2(q^2)$ and $g_2^2(q^2)$ defined via Dyson summation of self-energies (see e.g. Refs. [10, 11, 13]) is governed by the renormalization group. As a consequence, we can introduce running couplings as follows

$$e^2(q^2) = \frac{e_0^2}{1 + \text{Re}\,\Pi^{\hat{A}\hat{A}}(q^2)} = \frac{e^2}{1 + \text{Re}\,\Pi^{\hat{A}\hat{A},\text{ren}}(q^2)},$$

$$g_2^2(q^2) = \frac{g_{2,0}^2}{1 + \text{Re}\,\Pi^{\hat{W}\hat{W}}(q^2)} = \frac{g_2^2}{1 + \text{Re}\,\Pi^{\hat{W}\hat{W},\text{ren}}(q^2)}, \qquad (61)$$

where the quantities on the right-hand side are the renormalized ones and the second equality holds because of $Z_e = Z_{\hat{A}\hat{A}}^{-1/2}$ and $Z_{g_2} = Z_{\hat{W}}^{-1/2}$, respectively. As these running couplings can be expressed in terms of bare quantities, they are manifestly renormalization-scheme independent in the BFM . Asymptotically these couplings are equivalent to the

ones defined in Refs. [10, 11, 13], but for finite values of q^2 there are differences.[5] More-over, the running couplings (61) depend on ξ_Q in the non-asymptotic region. This indicates that any definition of running couplings via Dyson summation of self-energies that take into account mass effects is not unique but a matter of convention. This arbitrariness is made transparent in the BFM and has to be taken into account when considering applications.

We can define a running $s_{\mathrm{W}}(q^2)$ as the ratio of the electromagnetic and charged-current running coupling constants

$$s_{\mathrm{W}}^2(q^2) = \frac{e^2(q^2)}{g_2^2(q^2)} = \frac{e^2}{g_2^2} \frac{1 + \mathrm{Re}\,\Pi^{\hat{W}\hat{W},\mathrm{ren}}(q^2)}{1 + \mathrm{Re}\,\Pi^{\hat{A}\hat{A},\mathrm{ren}}(q^2)}. \tag{62}$$

In the leading-logarithmic approximation this can be written as

$$s_{\mathrm{W}}^2(q^2) \underset{|q^2| \to \infty}{\widetilde{}} s_{\mathrm{W}}^2 \left(1 - \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}} \mathrm{Re}\,\Pi^{\hat{A}\hat{Z},\mathrm{ren}}(q^2) \right) + \mathcal{O}(e^4). \tag{63}$$

This resembles the definition of a running $s_{\mathrm{W}}^2(q^2)$ used for example in Ref. [22].

6 CONCLUSION

In this paper we have studied the application of the BFM to the electroweak SM. We have given the full Lagrangian for the SM and indicated how the gauge-invariant effective action of the BFM and the S matrix are constructed. A complete set of Feynman rules for arbitrary values of a quantum gauge parameter has been listed including all counterterms necessary for one-loop calculations.

We have shown that the gauge invariance of the BFM implies simple QED-like Ward identities. They have been discussed in comparison with the Slavnov–Taylor identities of the conventional formalism. As a consequence of the Ward identities, the vertex functions in the BFM possess improved theoretical properties compared to their conventional counterparts. In particular, this has been worked out for the example of running couplings directly defined via Dyson summation. In contrast to the conventional formalism, their asymptotic behavior is automatically governed by the renormalization group and independent of the quantum gauge parameter. In comparison to former treatments like the pinch technique, where desirable properties of Green functions could only be verified by explicit computation, the BFM offers a well-suited framework for studying the properties of off-shell Green functions by relating them to the gauge invariance of the effective action.

Moreover, practical calculations of S-matrix elements simplify considerably in the BFM. The freedom to choose an appropriate gauge, e.g. the unitary gauge, for the background fields independently of the quantum gauge fixing allows to reduce the number of contributing Feynman diagrams drastically. In addition, also the evaluation of loop diagrams simplifies. This holds in particular in the 't Hooft–Feynman gauge for the quantum fields.

When considering applications of the BFM in the SM it is particularly important to establish a consistent renormalization which does not violate the explicit gauge invariance,

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[5]Those differences also exist between the different formulations of the previous treatments.

i.e. which does not alter the form of the Ward identities. This has been done starting from two different renormalization schemes, a complete and a minimal on-shell scheme. We have shown that the gauge symmetry imposes relations between field renormalization constants and the renormalization constants of the SM parameters, i.e. electric charge and particle masses. It was pointed out that even with this reduced set of independent renormalization constants all Green functions of the SM become finite. This has been verified explicitly at one-loop order by calculating the relevant quantities. The renormalization constants of the physical parameters are still independent of each other so that all on-shell parameter renormalization conditions can be maintained. Thus, the on-shell scheme is compatible with the symmetries of the BFM. Furthermore, it is obvious that the same holds for the minimal-subtraction scheme.

As a consequence of gauge invariance, the renormalization in the BFM drastically simplifies compared to the conventional formalism both technically and conceptually. In the BFM, much less independent renormalization constants are needed and the counterterms have a much simpler structure. All realizations of a generic vertex have one single universal counterterm. If charge and particle masses are identified with their physical values, the field renormalizations of all gauge bosons become gauge-independent.

Acknowledgement

Note added

Shortly before completion of this paper we became aware of a simultaneous work [23] focussing on the renormalization of the electroweak SM (omitting fermions) in the BFM. As in this reference the residue of the Higgs field is required to be unity, in contrast to our result (54) a nonzero (but nevertheless finite) correction δv to the vacuum expectation value of the Higgs field is needed. This violates the naïve Ward identities and is cured in Ref. [23] by including δv into the renormalized Ward identities. Since the renormalization in the BFM necessarily involves fields whose residues differ from unity we find it preferable to carry it out in such a way that the explicit gauge invariance and correspondingly the form of the Ward identities is retained. Furthermore, we disagree with the conclusion of Ref. [23] that the Landau gauge would be enforced for the background fields. In fact, we do not find any reason that would require this restriction.

A FEYNMAN RULES IN THE BACKGROUND-FIELD METHOD

In this appendix we list the Feynman rules of the SM in the BFM for an arbitrary quantum gauge parameter $\xi_Q = \xi_Q^W = \xi_Q^B$. We write down generic Feynman rules for all vertices and give the possible actual insertions. We use here the shorthand notation

$$c = c_\mathrm{W}, \qquad s = s_\mathrm{W}. \tag{A1}$$

From the Feynman rules given here, the vertex functions corresponding to the gauge-invariant effective action of the BFM can be calculated. No gauge-fixing term is included

for the background fields. Such a term is only relevant for the construction of connected Green functions and S-matrix elements from the vertex functions. It can be chosen independently from the gauge-fixing of the quantum fields. If a linear gauge is used, only the propagators of the background fields are affected. In a background R_ξ gauge, the background-field propagators take the same form as the quantum-field propagators given below with ξ_Q replaced by the background gauge parameter ξ_B. Note, however, that it is preferable to use a more convenient gauge for the background fields like the unitary gauge.

We first list the vertices containing only background fields including counterterms. In lowest order, these vertices are identical to the ones in the conventional formalism (see e.g. Ref. [16]). Their counterterms, however, have a much simpler structure. Note that in the BFM apart from the two-point functions each generic vertex has a universal counterterm. As mentioned above, these counterterms are sufficient for the renormalization of all one-loop processes.

In the vertices all momenta and fields are considered as incoming.

- tadpole:

$$
\boxed{\quad \times\!-\!-\!-\underline{\hat{H}} \quad} = i\delta t. \tag{A2}
$$

- $\hat{V}\hat{V}$ counterterm:

$$
\boxed{\hat{V}_{1,\mu}, k \;\wwww\!\times\!\wwww\; \hat{V}_{2,\nu}} = i\Big[(-g_{\mu\nu}k^2 + k_\mu k_\nu)C_1 + g_{\mu\nu}C_2\Big] \tag{A3}
$$

with the actual values of $\hat{V}_1$, $\hat{V}_2$ and C_1, C_2

$\hat{V}_1\hat{V}_2$	$\hat{W}^+\hat{W}^-$	$\hat{Z}\hat{Z}$	$\hat{A}\hat{Z}$	$\hat{A}\hat{A}$
C_1	$\delta Z_{\hat{W}}$	$\delta Z_{\hat{Z}\hat{Z}}$	$\frac{1}{2}\delta Z_{\hat{A}\hat{Z}}$	$\delta Z_{\hat{A}\hat{A}}$
C_2	$M_W^2\delta Z_{\hat{W}} + \delta M_W^2$	$M_Z^2\delta Z_{\hat{Z}\hat{Z}} + \delta M_Z^2$	0	0

(A4)

- $\hat{V}\hat{S}$ counterterm:

$$
\boxed{\hat{V}_\mu, k \;\wwww\!\times\!-\!-\!-\underline{\hat{S}}} = ik_\mu C\delta Z_{\hat{H}} \tag{A5}
$$

with the actual values of $\hat{V}$, $\hat{S}$ and C

$\hat{V}\hat{S}$	$\hat{W}^\pm\hat{\phi}^\mp$	$\hat{Z}\hat{\chi}$
C	$\pm M_W$	iM_Z

(A6)

- $\hat{S}\hat{S}$ counterterm:

$$\boxed{\underset{\hat{S}_1, k \qquad\qquad \times \qquad\qquad \hat{S}_2}{}} = i\Big[\delta Z_{\hat{H}} k^2 - C\Big] \qquad (A7)$$

with the actual values of $\hat{S}_1$, $\hat{S}_2$ and C_1, C_2

$\hat{S}_1\hat{S}_2$	$\hat{H}\hat{H}$	$\hat{\chi}\hat{\chi},\hat{\phi}\hat{\phi}$
C	$M_{\mathrm{H}}^2 \delta Z_{\hat{H}} + \delta M_{\mathrm{H}}^2$	$-\frac{e}{2s}\frac{\delta t}{M_{\mathrm{W}}}$

$$(A8)$$

- $F\bar{F}$ counterterm:

$$\boxed{\underset{F_1, p \qquad\qquad \times \qquad\qquad \bar{F}_2}{}} = i\Big[C_{\mathrm{L}}\not{p}\omega_- + C_{\mathrm{R}}\not{p}\omega_+ - C_{\mathrm{S}}\Big] \qquad (A9)$$

with the actual values of F_1, $\bar{F}_2$ and C_{L}, C_{R}, C_{S}

$F_1\bar{F}_2$	$f\bar{f}$
C_{L}	δZ_f^{L}
C_{R}	δZ_f^{R}
C_{S}	$m_f \frac{1}{2}\left(\delta Z_f^{\mathrm{L}} + \delta Z_f^{\mathrm{R}}\right) + \delta m_f$

$$(A10)$$

- $\hat{V}\hat{V}\hat{V}\hat{V}$ coupling:

$$\boxed{\begin{array}{cc} \hat{V}_{1,\mu} & \hat{V}_{3,\rho} \\ & \\ \hat{V}_{2,\nu} & \hat{V}_{4,\sigma} \end{array}} = ie^2 C\Big[2g_{\mu\nu}g_{\sigma\rho} - g_{\nu\rho}g_{\mu\sigma} - g_{\rho\mu}g_{\nu\sigma}\Big](1 + \delta Z_{\hat{W}}) \qquad (A11)$$

with the actual values of $\hat{V}_1$, $\hat{V}_2$, $\hat{V}_3$, $\hat{V}_4$ and C

$\hat{V}_1\hat{V}_2\hat{V}_3\hat{V}_4$	$\hat{W}^+\hat{W}^+\hat{W}^-\hat{W}^-$	$\hat{W}^+\hat{W}^-\hat{Z}\hat{Z}$	$\hat{W}^+\hat{W}^-\hat{A}\hat{Z}$	$\hat{W}^+\hat{W}^-\hat{A}\hat{A}$
C	$\frac{1}{s^2}$	$-\frac{c^2}{s^2}$	$\frac{c}{s}$	-1

$$(A12)$$

- $\hat{V}\hat{V}\hat{V}$ coupling:

$$= -ieC\Big[g_{\mu\nu}(k_2 - k_1)_\rho + g_{\nu\rho}(k_3 - k_2)_\mu \\ + g_{\rho\mu}(k_1 - k_3)_\nu\Big](1 + \delta Z_{\hat{W}}) \tag{A13}$$

with the actual values of $\hat{V}_1$, $\hat{V}_2$, $\hat{V}_3$ and C

$\hat{V}_1\hat{V}_2\hat{V}_3$	$\hat{A}\hat{W}^+\hat{W}^-$	$\hat{Z}\hat{W}^+\hat{W}^-$
C	1	$-\frac{c}{s}$

$$\tag{A14}$$

- $\hat{S}\hat{S}\hat{S}\hat{S}$ coupling:

$$= ie^2 C\Big[1 + \frac{\delta M_{\mathrm{H}}^2}{M_{\mathrm{H}}^2} + \frac{e}{2s}\frac{\delta t}{M_{\mathrm{W}}M_{\mathrm{H}}^2} + \delta Z_{\hat{H}}\Big] \tag{A15}$$

with the actual values of $\hat{S}_1$, $\hat{S}_2$, $\hat{S}_3$, $\hat{S}_4$ and C

$\hat{S}_1\hat{S}_2\hat{S}_3\hat{S}_4$	$\hat{H}\hat{H}\hat{H}\hat{H}, \hat{\chi}\hat{\chi}\hat{\chi}\hat{\chi}$	$\hat{H}\hat{H}\hat{\chi}\hat{\chi}, \hat{H}\hat{H}\hat{\phi}^+\hat{\phi}^-, \hat{\chi}\hat{\chi}\hat{\phi}^+\hat{\phi}^-$	$\hat{\phi}^+\hat{\phi}^-\hat{\phi}^+\hat{\phi}^-$
C	$-\frac{3}{4s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}$	$-\frac{1}{4s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}$	$-\frac{1}{2s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}$

$$\tag{A16}$$

- $\hat{S}\hat{S}\hat{S}$ coupling:
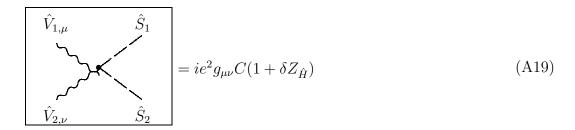
$$= ieC\Big[1 + \frac{\delta M_{\mathrm{H}}^2}{M_{\mathrm{H}}^2} + \frac{e}{2s}\frac{\delta t}{M_{\mathrm{W}}M_{\mathrm{H}}^2} + \delta Z_{\hat{H}}\Big] \tag{A17}$$

with the actual values of $\hat{S}_1$, $\hat{S}_2$, $\hat{S}_3$ and C

$\hat{S}_1\hat{S}_2\hat{S}_3$	$\hat{H}\hat{H}\hat{H}$	$\hat{H}\hat{\chi}\hat{\chi}, \hat{H}\hat{\phi}^+\hat{\phi}^-$
C	$-\frac{3}{2s}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}}$	$-\frac{1}{2s}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}}$

$$\tag{A18}$$

- $\hat{V}\hat{V}\hat{S}\hat{S}$ coupling:

$$= ie^2 g_{\mu\nu} C(1 + \delta Z_{\hat{H}}) \qquad (A19)$$

with the actual values of $\hat{V}_1$, $\hat{V}_2$, $\hat{S}_1$, $\hat{S}_2$ and C

$\hat{V}_1\hat{V}_2\hat{S}_1\hat{S}_2$	$\hat{Z}\hat{Z}\hat{H}\hat{H}$ $\hat{Z}\hat{Z}\hat{\chi}\hat{\chi}$	$\hat{W}^+\hat{W}^-\hat{H}\hat{H}, \hat{W}^+\hat{W}^-\hat{\phi}^+\hat{\phi}^-$ $\hat{W}^+\hat{W}^-\hat{\chi}\hat{\chi}$	$\hat{A}\hat{A}\hat{\phi}^+\hat{\phi}^-$	$\hat{Z}\hat{A}\hat{\phi}^+\hat{\phi}^-$	$\hat{Z}\hat{Z}\hat{\phi}^+\hat{\phi}^-$
C	$\frac{1}{2c^2 s^2}$	$\frac{1}{2s^2}$	2	$-\frac{c^2-s^2}{cs}$	$\frac{(c^2-s^2)^2}{2c^2 s^2}$

and

$\hat{V}_1\hat{V}_2\hat{S}_1\hat{S}_2$	$\hat{W}^\pm\hat{A}\hat{\phi}^\mp\hat{H}$	$\hat{W}^\pm\hat{A}\hat{\phi}^\mp\hat{\chi}$	$\hat{W}^\pm\hat{Z}\hat{\phi}^\mp\hat{H}$	$\hat{W}^\pm\hat{Z}\hat{\phi}^\mp\hat{\chi}$
C	$-\frac{1}{2s}$	$\mp\frac{i}{2s}$	$-\frac{1}{2c}$	$\mp\frac{i}{2c}$

$$(A20)$$

- $\hat{V}\hat{S}\hat{S}$ coupling:

$$= ieC(k_1 - k_2)_\mu (1 + \delta Z_{\hat{H}}) \qquad (A21)$$

with the actual values of $\hat{V}$, $\hat{S}_1$, $\hat{S}_2$ and C

$\hat{V}\hat{S}_1\hat{S}_2$	$\hat{Z}\hat{\chi}\hat{H}$	$\hat{A}\hat{\phi}^+\hat{\phi}^-$	$\hat{Z}\hat{\phi}^+\hat{\phi}^-$	$\hat{W}^\pm\hat{\phi}^\mp\hat{H}$	$\hat{W}^\pm\hat{\phi}^\mp\hat{\chi}$
C	$-\frac{i}{2cs}$	-1	$\frac{c^2-s^2}{2cs}$	$\mp\frac{1}{2s}$	$-\frac{i}{2s}$

$$(A22)$$

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- $\hat{S}\hat{V}\hat{V}$ coupling:

$$= ieg_{\mu\nu}C(1+\delta Z_{\hat{H}}) \tag{A23}$$

with the actual values of $\hat{S}$, $\hat{V}_1$, $\hat{V}_2$ and C

$\hat{S}\hat{V}_1\hat{V}_2$	$\hat{H}\hat{Z}\hat{Z}$	$\hat{H}\hat{W}^+\hat{W}^-$	$\hat{\phi}^\pm\hat{W}^\mp\hat{A}$	$\hat{\phi}^\pm\hat{W}^\mp\hat{Z}$
C	$\frac{1}{c^2 s}M_{\mathrm{W}}$	$\frac{1}{s}M_{\mathrm{W}}$	$-M_{\mathrm{W}}$	$-\frac{s}{c}M_{\mathrm{W}}$

(A24)

- $\hat{V}\bar{F}F$ coupling:

$$= ie\gamma_\mu \Big[C_{\mathrm{L}}\omega_-(1+\delta Z_{F_1}^{\mathrm{L}}) + C_{\mathrm{R}}\omega_+\Big(1+\tfrac{1}{2}(\delta Z_{F_1}^{\mathrm{R}}+\delta Z_{F_2}^{\mathrm{R}})\Big)\Big] \tag{A25}$$

with the actual values of $\hat{V}$, $\bar{F}_1$, F_2 and C_{R}, C_{L}

$\hat{V}\bar{F}_1F_2$	$\hat{A}\bar{f}f$	$\hat{Z}\bar{f}f$	$\hat{W}^+\bar{f}_uf_d,\hat{W}^-\bar{f}_df_u$
C_{L}	$-Q_f$	$\frac{I_{\mathrm{W},f}^3-s^2Q_f}{cs}$	$\frac{1}{\sqrt{2}s}$
C_{R}	$-Q_f$	$-\frac{s}{c}Q_f$	0

(A26)

- $\hat{S}\bar{F}F$ coupling:

$$= ie\Big[C_{\mathrm{L}}\omega_-\Big(1+\frac{\delta m_{F_1}}{m_{F_1}}+\frac{1}{2}\delta Z_{F_1}^{\mathrm{L}}+\frac{1}{2}\delta Z_{F_1}^{\mathrm{R}}\Big) + C_{\mathrm{R}}\omega_+\Big(1+\frac{\delta m_{F_2}}{m_{F_2}}+\frac{1}{2}\delta Z_{F_1}^{\mathrm{L}}+\frac{1}{2}\delta Z_{F_2}^{\mathrm{R}}\Big)\Big] \tag{A27}$$

with the actual values of $\hat{S}$, $\bar{F}_1$, F_2 and C_{R}, C_{L}

$\hat{S}\bar{F}_1F_2$	$\hat{H}\bar{f}f$	$\hat{\chi}\bar{f}f$	$\hat{\phi}^+\bar{f}_uf_d$	$\hat{\phi}^-\bar{f}_df_u$
C_{L}	$-\frac{1}{2s}\frac{m_f}{M_{\mathrm{W}}}$	$-i\frac{1}{2s}2I_{\mathrm{W},f}^3\frac{m_f}{M_{\mathrm{W}}}$	$+\frac{1}{\sqrt{2}s}\frac{m_{f_u}}{M_{\mathrm{W}}}$	$-\frac{1}{\sqrt{2}s}\frac{m_{f_d}}{M_{\mathrm{W}}}$
C_{R}	$-\frac{1}{2s}\frac{m_f}{M_{\mathrm{W}}}$	$i\frac{1}{2s}2I_{\mathrm{W},f}^3\frac{m_f}{M_{\mathrm{W}}}$	$-\frac{1}{\sqrt{2}s}\frac{m_{f_d}}{M_{\mathrm{W}}}$	$+\frac{1}{\sqrt{2}s}\frac{m_{f_u}}{M_{\mathrm{W}}}$

(A28)

Note that in contrast to the conventional formalism no counterterms are needed for the $\hat{Z}\hat{A}\hat{H}\hat{H}$, $\hat{Z}\hat{A}\hat{\chi}\hat{\chi}$, $\hat{A}\hat{\chi}\hat{H}$ and $\hat{H}\hat{Z}\hat{A}$ couplings.

We now consider the Feynman rules for vertices containing quantum fields. We treat these vertices in lowest order, i.e. we do not list the counterterms explicitly. As mentioned above, all lowest-order vertices involving fermions have the usual form. Since the gauge-fixing term is quadratic in the quantum fields, apart from vertices involving ghost fields only vertices containing exactly two quantum fields differ from the conventional ones. Thus, the other vertices involving quantum fields have in lowest order the same form as the pure background-field vertices given above. Their insertions can be obtained from the ones listed for the pure background-field vertices by forming all possible combinations of quantum and background fields, e.g. one infers $\hat{W}^+W^-AZ, W^+\hat{W}^-AZ, W^+W^-\hat{A}Z$ and $W^+W^-A\hat{Z}$ as the possible insertions for the $\hat{V}VVV$ coupling corresponding to $\hat{W}^+\hat{W}^-\hat{A}\hat{Z}$.

Some of the vertices containing two quantum fields also have the usual Feynman rules. These are $\hat{V}\hat{V}SS$, $\hat{S}\hat{S}VV$, $\hat{V}SS$ and $\hat{S}VV$. In the following we list those couplings for which the generic form or actual insertion differs from the ones in the conventional formalism. Note that some of the insertions appearing in the conventional couplings have no counterpart here. We list only the non-vanishing insertions.

- $\hat{V}\hat{V}VV$ coupling:
  The $\hat{V}\hat{V}VV$ coupling has two generic forms depending on the actual insertions,

$$\hat{V}_{1,\mu}\hat{V}_{2,\nu}V_{3,\rho}V_{4,\sigma} : ie^2 C\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}(1 - \frac{1}{\xi_Q}) - g_{\mu\rho}g_{\nu\sigma}(1 - \frac{1}{\xi_Q})\Big] \qquad (A29)$$

for the insertions

$\hat{V}_1\hat{V}_2V_3V_4$	$\hat{W}^\pm\hat{W}^\pm W^\mp W^\mp$	$\hat{Z}\hat{Z}W^+W^-$ $\hat{W}^+\hat{W}^-ZZ$	$\hat{A}\hat{Z}W^+W^-$ $\hat{W}^+\hat{W}^-AZ$	$\hat{A}\hat{A}W^+W^-$ $\hat{W}^+\hat{W}^-AA$
C	$\frac{1}{s^2}$	$-\frac{c^2}{s^2}$	$\frac{c}{s}$	-1

$\qquad (A30)$

and

$$\hat{V}_{1,\mu}\hat{V}_{2,\nu}V_{3,\rho}V_{4,\sigma} : ie^2 C\Big[2g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}(1 + \frac{1}{\xi_Q})\Big] \qquad (A31)$$

for the insertions

$\hat{V}_1\hat{V}_2V_3V_4$	$\hat{W}^+\hat{W}^-W^+W^-$	$\hat{W}^\pm\hat{Z}W^\mp Z$	$\hat{W}^\pm\hat{A}W^\mp Z$ $\hat{W}^\pm\hat{Z}W^\mp A$	$\hat{W}^\pm\hat{A}W^\mp A$
C	$\frac{1}{s^2}$	$-\frac{c^2}{s^2}$	$\frac{c}{s}$	-1

$\qquad (A32)$

- $\hat{V}VV$ coupling:

$$\hat{V}_{1,\mu}(k_1)V_{2,\nu}(k_2)V_{3,\rho}(k_3) :$$
$$-ieC\Big[g_{\nu\rho}(k_3 - k_2)_\mu + g_{\mu\nu}(k_2 - k_1 + \frac{k_3}{\xi_Q})_\rho + g_{\rho\mu}(k_1 - k_3 - \frac{k_2}{\xi_Q})_\nu\Big] \tag{A33}$$

with the actual values of $\hat{V}_1$, V_2, V_3 and C

$\hat{V}_1V_2V_3$	$\hat{A}W^+W^-, \hat{W}^+W^-A, \hat{W}^-AW^+$	$\hat{Z}W^+W^-, \hat{W}^+W^-Z, \hat{W}^-ZW^+$
C	1	$-\frac{c}{s}$

$$\tag{A34}$$

- $\hat{S}\hat{S}SS$ coupling:

$$\hat{S}_1\hat{S}_2S_3S_4 : ie^2C \tag{A35}$$

with the actual values of $\hat{S}_1$, $\hat{S}_2$, S_3, S_4 and C

$\hat{S}_1\hat{S}_2S_3S_4$	$\hat{H}\hat{H}HH$ $\hat{\chi}\hat{\chi}\chi\chi$	$\hat{H}\hat{H}\chi\chi$ $\hat{\chi}\hat{\chi}HH$	$\hat{H}\hat{\chi}H\chi$	$\hat{\phi}^+\hat{\phi}^-HH, \hat{H}\hat{H}\phi^+\phi^-$ $\hat{\phi}^+\hat{\phi}^-\chi\chi, \hat{\chi}\hat{\chi}\phi^+\phi^-$
C	$-\frac{3}{4s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}$	$-\frac{1}{4s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2} - \frac{\xi_Q}{2c^2s^2}$	$-\frac{1}{4s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2} + \frac{\xi_Q}{4c^2s^2}$	$-\frac{1}{4s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2} - \frac{\xi_Q}{2s^2}$

and

$\hat{S}_1\hat{S}_2S_3S_4$	$\hat{\phi}^\pm\hat{H}\phi^\mp H$ $\hat{\phi}^\pm\hat{\chi}\phi^\mp\chi$	$\hat{\phi}^+\hat{\phi}^-\phi^+\phi^-$	$\hat{\phi}^\pm\hat{\phi}^\pm\phi^\mp\phi^\mp$	$\hat{\phi}^\pm\hat{H}\phi^\mp\chi$ $\hat{\phi}^\mp\hat{\chi}\phi^\pm H$
C	$-\frac{1}{4s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2} + \frac{\xi_Q}{4s^2}$	$-\frac{1}{2s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2} - \frac{\xi_Q}{4c^2s^2}$	$-\frac{1}{2s^2}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2} + \frac{\xi_Q}{2c^2s^2}$	$\mp\frac{i\xi_Q}{4c^2}$

$$\tag{A36}$$

- $\hat{S}SS$ coupling:

$$\hat{S}_1S_2S_3 : ieC \tag{A37}$$

with the actual values of $\hat{S}_1$, S_2, S_3 and C

$\hat{S}_1S_2S_3$	$\hat{H}HH$	$\hat{H}\chi\chi$	$\hat{\chi}H\chi$	$\hat{H}\phi^+\phi^-$
C	$-\frac{3}{2s}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}}$	$-\frac{1}{2s}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}} - \xi_Q\frac{M_{\mathrm{W}}}{c^2s}$	$-\frac{1}{2s}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}} + \xi_Q\frac{M_{\mathrm{W}}}{2c^2s}$	$-\frac{1}{2s}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}} - \xi_Q\frac{M_{\mathrm{W}}}{s}$

and

$\hat{S}_1S_2S_3$	$\hat{\phi}^\pm\phi^\mp H$	$\hat{\phi}^\pm\phi^\mp\chi$
C	$-\frac{1}{2s}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}} + \xi_Q\frac{M_{\mathrm{W}}}{2s}$	$\mp i\xi_Q M_{\mathrm{W}}\frac{s}{2c^2}$

$$\tag{A38}$$

- $\hat{V}V\hat{S}S$ coupling:

$$\hat{V}_{1,\mu}V_{2,\nu}\hat{S}_1S_2 : ie^2 g_{\mu\nu}C \tag{A39}$$

with the actual values of $\hat{V}_1$, V_2, $\hat{S}_1$, S_2 and C

$\hat{V}_1V_2\hat{S}_1S_2$	$\hat{Z}Z\hat{H}H$ $\hat{Z}Z\hat{\chi}\chi$	$\hat{W}^\pm W^\mp\hat{H}H$ $\hat{W}^\pm W^\mp\hat{\chi}\chi$	$\hat{W}^\pm W^\mp\hat{\phi}^\mp\phi^\pm$	$\hat{A}A\hat{\phi}^\pm\phi^\mp$	$\hat{Z}A\hat{\phi}^\pm\phi^\mp$ $\hat{A}Z\hat{\phi}^\pm\phi^\mp$	$\hat{Z}Z\hat{\phi}^\pm\phi^\mp$
C	$\frac{1}{2c^2s^2}$	$\frac{1}{2s^2}$	$\frac{1}{s^2}$	2	$-\frac{c^2-s^2}{cs}$	$\frac{(c^2-s^2)^2}{2c^2s^2}$

and

$\hat{V}_1V_2\hat{S}_1S_2$	$\hat{W}^\pm A\hat{H}\phi^\mp$ $\hat{A}W^\pm\hat{\phi}^\mp H$	$\hat{W}^\pm A\hat{\chi}\phi^\mp$ $\hat{A}W^\pm\hat{\phi}^\mp\chi$	$\hat{W}^\pm Z\hat{\phi}^\mp H$ $\hat{Z}W^\pm\hat{H}\phi^\mp$	$\hat{W}^\pm Z\hat{\phi}^\mp\chi$ $\hat{Z}W^\pm\hat{\chi}\phi^\mp$	$\hat{W}^\pm Z\hat{H}\phi^\mp$ $\hat{Z}W^\pm\hat{\phi}^\mp H$	$\hat{W}^\pm Z\hat{\chi}\phi^\mp$ $\hat{Z}W^\pm\hat{\phi}^\mp\chi$
C	$-\frac{1}{s}$	$\mp\frac{i}{s}$	$-\frac{1}{2cs^2}$	$\mp\frac{i}{2cs^2}$	$\frac{c^2-s^2}{2cs^2}$	$\pm i\frac{c^2-s^2}{2cs^2}$

and

$\hat{V}_1V_2\hat{S}_1S_2$	$\hat{W}^\pm W^\mp\hat{\chi}H$ $\hat{W}^\mp W^\pm\hat{H}\chi$
C	$\pm\frac{i}{2s^2}$

$$\tag{A40}$$

- $V\hat{S}S$ coupling:

$$V_\mu\hat{S}_1(k_1)S_2(k_2) : ieC2k_{1,\mu} \tag{A41}$$

with the actual values of V, $\hat{S}_1$, S_2 and C

$V\hat{S}_1S_2$	$Z\hat{\chi}H$	$Z\hat{H}\chi$	$A\hat{\phi}^\pm\phi^\mp$	$Z\hat{\phi}^\pm\phi^\mp$	$W^\pm\hat{\phi}^\mp H, W^\mp\hat{H}\phi^\pm$	$W^\pm\hat{\phi}^\mp\chi$	$W^\pm\hat{\chi}\phi^\mp$
C	$-\frac{i}{2cs}$	$\frac{i}{2cs}$	∓ 1	$\pm\frac{c^2-s^2}{2cs}$	$\mp\frac{1}{2s}$	$-\frac{i}{2s}$	$\frac{i}{2s}$

$$\tag{A42}$$

- $S\hat{V}V$ coupling:

$$S\hat{V}_{1,\mu}V_{2,\nu} : ieg_{\mu\nu}C \tag{A43}$$

with the actual values of S, $\hat{V}_1$, V_2 and C

$S\hat{V}_1V_2$	$H\hat{Z}Z$	$H\hat{W}^\pm W^\mp$	$\chi\hat{W}^\pm W^\mp$	$\phi^\pm\hat{W}^\mp A$	$\phi^\pm\hat{W}^\mp Z$	$\phi^\pm\hat{Z}W^\mp$
C	$\frac{1}{c^2s}M_{\rm W}$	$\frac{1}{s}M_{\rm W}$	$\mp\frac{i}{s}M_{\rm W}$	$-2M_{\rm W}$	$\frac{c^2-s^2}{cs}M_{\rm W}$	$-\frac{1}{cs}M_{\rm W}$

$$\tag{A44}$$

Next, we list the Feynman rules for couplings involving ghost fields. As above, pure quantum-field vertices have the usual Feynman rules.

- $\hat{V}\bar{G}G$ coupling:

$$= ie(k_1 - k_2)_\mu C \tag{A45}$$

with the actual values of $\hat{V}$, $\bar{G}_1$, G_2 and C

$\hat{V}\bar{G}_1 G_2$	$\hat{A}\bar{u}^\pm u^\pm, \hat{W}^\pm\bar{u}^A u^\mp, \hat{W}^\mp\bar{u}^\mp u^A$	$\hat{Z}\bar{u}^\pm u^\pm, \hat{W}^\pm\bar{u}^Z u^\mp, \hat{W}^\mp\bar{u}^\mp u^Z$
C	± 1	$\mp\frac{c}{s}$

$$\tag{A46}$$

- $V\bar{G}G$ coupling:

$$V_\mu \bar{G}_1 G_2 : iek_{1,\mu}C \tag{A47}$$

with the actual values of V, $\bar{G}_1$, G_2 and C as given in (A46).

- $\hat{V}\hat{V}\bar{G}G$ coupling:

$$= ie^2 g_{\mu\nu}C \tag{A48}$$

with the actual values of $\hat{V}_1$, $\hat{V}_2$, $\bar{G}_1$, G_2 and C

$\hat{V}_1\hat{V}_2\bar{G}_1 G_2$	$\hat{W}^\pm\hat{W}^\pm\bar{u}^\pm u^\mp$	$\hat{W}^+\hat{W}^-\bar{u}^A u^A$ $\hat{A}\hat{A}\bar{u}^\pm u^\pm$	$\hat{W}^+\hat{W}^-\bar{u}^A u^Z, \hat{A}\hat{Z}\bar{u}^\pm u^\pm$ $\hat{W}^+\hat{W}^-\bar{u}^Z u^A$	$\hat{W}^+\hat{W}^-\bar{u}^Z u^Z$ $\hat{Z}\hat{Z}\bar{u}^\pm u^\pm$
C	$-\frac{2}{s^2}$	2	$-2\frac{c}{s}$	$2\frac{c^2}{s^2}$

$$\tag{A49a}$$

and

$\hat{V}_1\hat{V}_2\bar{G}_1 G_2$	$\hat{W}^+\hat{W}^-\bar{u}^\pm u^\pm$	$\hat{A}\hat{W}^\pm\bar{u}^\pm u^A$ $\hat{A}\hat{W}^\pm\bar{u}^A u^\mp$	$\hat{Z}\hat{W}^\pm\bar{u}^\pm u^A, \hat{A}\hat{W}^\pm\bar{u}^\pm u^Z$ $\hat{Z}\hat{W}^\pm\bar{u}^A u^\mp, \hat{A}\hat{W}^\pm\bar{u}^Z u^\mp$	$\hat{Z}\hat{W}^\pm\bar{u}^\pm u^Z$ $\hat{Z}\hat{W}^\pm\bar{u}^Z u^\mp$
C	$\frac{1}{s^2}$	-1	$\frac{c}{s}$	$-\frac{c^2}{s^2}$

$$\tag{A49b}$$

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- $\hat{V}V\bar{G}G$ coupling:

$$\hat{V}_{1,\mu}V_{2,\nu}\bar{G}_1G_2 : ie^2 g_{\mu\nu}C \tag{A50}$$

with the actual values of $\hat{V}_1$, $\hat{V}_2$, $\bar{G}_1$, G_2 and C

$\hat{V}_1V_2\bar{G}_1G_2$	$\hat{W}^\pm W^\pm \bar{u}^\pm u^\mp$	$\hat{W}^\pm W^\mp \bar{u}^A u^A$, $\hat{A}A\bar{u}^\pm u^\pm$	$\hat{W}^\pm W^\mp \bar{u}^A u^Z$, $\hat{A}Z\bar{u}^\pm u^\pm$, $\hat{W}^\pm \hat{W}^\mp \bar{u}^Z u^A$, $\hat{Z}A\bar{u}^\pm u^\pm$	$\hat{W}^\pm W^\mp \bar{u}^Z u^Z$, $\hat{Z}Z\bar{u}^\pm u^\pm$
C	$-\frac{1}{s^2}$	1	$-\frac{c}{s}$	$\frac{c^2}{s^2}$

and

$\hat{V}_1V_2\bar{G}_1G_2$	$\hat{W}^\pm W^\mp \bar{u}^\pm u^\pm$	$\hat{A}W^\pm \bar{u}^\pm u^A$, $\hat{W}^\pm A\bar{u}^A u^\mp$	$\hat{Z}W^\pm \bar{u}^\pm u^A$, $\hat{A}W^\pm \bar{u}^\pm u^Z$, $\hat{W}^\pm Z\bar{u}^A u^\mp$, $\hat{W}^\pm A\bar{u}^Z u^\mp$	$\hat{Z}W^\pm \bar{u}^\pm u^Z$, $\hat{W}^\pm Z\bar{u}^Z u^\mp$
C	$\frac{1}{s^2}$	-1	$\frac{c}{s}$	$-\frac{c^2}{s^2}$

(A51)

- $\hat{S}\bar{G}G$ coupling:

 $= ieC\xi_Q$ (A52)

with the actual values of $\hat{S}$, $\bar{G}_1$, G_2 and C

$\hat{S}\bar{G}_1 G_2$	$\hat{H}\bar{u}^Z u^Z$	$\hat{H}\bar{u}^\pm u^\pm$	$\hat{\phi}^\pm \bar{u}^\pm u^A$, $\hat{\phi}^\pm \bar{u}^A u^\mp$	$\hat{\phi}^\pm \bar{u}^\pm u^Z$, $\hat{\phi}^\pm \bar{u}^Z u^\mp$
C	$-\frac{1}{c^2 s}M_{\mathrm{W}}$	$-\frac{1}{s}M_{\mathrm{W}}$	M_{W}	$\frac{s}{c}M_{\mathrm{W}}$

(A53)

- $S\bar{G}G$ coupling:

$$S\bar{G}_1G_2 : ieC\xi_Q \tag{A54}$$

with the actual values of S, $\bar{G}_1$, G_2 and C

$S\bar{G}_1 G_2$	$H\bar{u}^Z u^Z$	$H\bar{u}^\pm u^\pm$	$\chi\bar{u}^\pm u^\pm$	$\phi^\pm \bar{u}^\pm u^A$	$\phi^\pm \bar{u}^\pm u^Z$	$\phi^\pm \bar{u}^Z u^\mp$
C	$-\frac{1}{2c^2 s}M_{\mathrm{W}}$	$-\frac{1}{2s}M_{\mathrm{W}}$	$\mp\frac{i}{2s}M_{\mathrm{W}}$	M_{W}	$-\frac{c^2-s^2}{2cs}M_{\mathrm{W}}$	$\frac{1}{2cs}M_{\mathrm{W}}$

(A55)

- $\hat{S}\hat{S}\bar{G}G$ coupling:

$$= ie^2C\xi_Q \tag{A56}$$

with the actual values of $\hat{S}_1$, $\hat{S}_2$, $\bar{G}_1$, G_2 and C

$\hat{S}_1\hat{S}_2\bar{G}_1G_2$	$\hat{H}\hat{H}\bar{u}^Zu^Z$ $\hat{\chi}\hat{\chi}\bar{u}^Zu^Z$	$\hat{H}\hat{H}\bar{u}^\pm u^\pm, \hat{\phi}^+\hat{\phi}^-\bar{u}^\pm u^\pm$ $\hat{\chi}\hat{\chi}\bar{u}^\pm u^\pm$	$\hat{\phi}^+\hat{\phi}^-\bar{u}^Au^A$	$\hat{\phi}^+\hat{\phi}^-\bar{u}^Au^Z$ $\hat{\phi}^+\hat{\phi}^-\bar{u}^Zu^A$	$\hat{\phi}^+\hat{\phi}^-\bar{u}^Zu^Z$
C	$-\frac{1}{2c^2s^2}$	$-\frac{1}{2s^2}$	-2	$\frac{c^2-s^2}{cs}$	$-\frac{(c^2-s^2)^2}{2c^2s^2}$

and

$\hat{S}_1\hat{S}_2\bar{G}_1G_2$	$\hat{H}\hat{\phi}^\pm\bar{u}^\pm u^A$ $\hat{\phi}^\pm\hat{H}\bar{u}^Au^\mp$	$\hat{\chi}\hat{\phi}^\pm\bar{u}^\pm u^A$ $\hat{\phi}^\pm\hat{\chi}\bar{u}^Au^\mp$	$\hat{H}\hat{\phi}^\pm\bar{u}^\pm u^Z$ $\hat{\phi}^\pm\hat{H}\bar{u}^Zu^\mp$	$\hat{\chi}\hat{\phi}^\pm\bar{u}^\pm u^Z$ $\hat{\phi}^\pm\hat{\chi}\bar{u}^Zu^\mp$
C	$\frac{1}{2s}$	$\mp\frac{i}{2s}$	$\frac{1}{2c}$	$\mp i\frac{1}{2c}$

$$\tag{A57}$$

- $\hat{S}S\bar{G}G$ coupling:

$$\hat{S}_1S_2\bar{G}_1G_2 : ie^2C\xi_Q \tag{A58}$$

with the actual values of $\hat{S}_1$, S_2, $\bar{G}_1$, G_2 and C

$\hat{S}_1S_2\bar{G}_1G_2$	$\hat{H}H\bar{u}^Zu^Z$ $\hat{\chi}\chi\bar{u}^Zu^Z$	$\hat{H}H\bar{u}^\pm u^\pm$ $\hat{\chi}\chi\bar{u}^\pm u^\pm$	$\hat{\phi}^\pm\phi^\mp\bar{u}^\pm u^\pm$	$\hat{\phi}^\pm\phi^\mp\bar{u}^Au^A$	$\hat{\phi}^\pm\phi^\mp\bar{u}^Au^Z$ $\hat{\phi}^\pm\phi^\mp\bar{u}^Zu^A$	$\hat{\phi}^\pm\phi^\mp\bar{u}^Zu^Z$
C	$-\frac{1}{4c^2s^2}$	$-\frac{1}{4s^2}$	$-\frac{1}{2s^2}$	-1	$\frac{c^2-s^2}{2cs}$	$-\frac{(c^2-s^2)^2}{4c^2s^2}$

and

$\hat{S}_1S_2\bar{G}_1G_2$	$\hat{H}\phi^\pm\bar{u}^\pm u^A$ $\hat{\phi}^\pm H\bar{u}^Au^\mp$	$\hat{\chi}\phi^\pm\bar{u}^\pm u^A$ $\hat{\phi}^\pm\chi\bar{u}^Au^\mp$	$\hat{H}\phi^\pm\bar{u}^\pm u^Z$ $\hat{\phi}^\pm H\bar{u}^Zu^\mp$	$\hat{H}\phi^\pm\bar{u}^Zu^\mp$ $\hat{\phi}^\pm H\bar{u}^\pm u^Z$	$\hat{\chi}\phi^\pm\bar{u}^\pm u^Z$ $\hat{\phi}^\pm\chi\bar{u}^Zu^\mp$	$\hat{\chi}\phi^\pm\bar{u}^Zu^\mp$ $\hat{\phi}^\pm\chi\bar{u}^\pm u^Z$
C	$\frac{1}{2s}$	$\mp\frac{i}{2s}$	$-\frac{c^2-s^2}{4cs^2}$	$\frac{1}{4cs^2}$	$\pm i\frac{c^2-s^2}{4cs^2}$	$\mp\frac{i}{4cs^2}$

and

$\hat{S}_1S_2\bar{G}_1G_2$	$\hat{H}\chi\bar{u}^\pm u^\pm$ $\hat{\chi}H\bar{u}^\mp u^\mp$
C	$\mp\frac{i}{4s^2}$

$$\tag{A59}$$

Finally, we give the quantum-field propagators:

- gauge bosons $V = A$, Z, W ($M_A = 0$)

$$\boxed{V_\mu \sim\!\!\!\sim\!\!\!\sim^{\;k}\!\!\!\sim V_\nu} = -i\left[\frac{g_{\mu\nu}}{k^2 - M_V^2} - \frac{(1 - \xi_Q)k_\mu k_\nu}{(k^2 - M_V^2)(k^2 - \xi_Q M_V^2)}\right] \qquad (A60)$$

- Faddeev–Popov ghosts $G = u^A$, u^Z, $u^\pm$ ($M_{u^A} = 0$, $M_{u^Z} = \xi_Q M_Z$, $M_{u^\pm} = \xi_Q M_W$)

$$\boxed{G \bullet\cdots\cdots^{\;k}\cdots\cdots\bullet \bar{G}} = \frac{i}{k^2 - M_G^2} \qquad (A61)$$

- scalar fields $S = H$, χ, ϕ ($M_\chi = \xi_Q M_Z$, $M_\phi = \xi_Q M_W$)

$$\boxed{S \bullet\!-\!-^{\;k}\!-\!-\!\bullet S} = \frac{i}{k^2 - M_S^2} \qquad (A62)$$

- fermion fields $F = f$

$$\boxed{F \bullet\!\!-\!\!-^{\;p}\!\!-\!\!-\!\!\bullet \bar{F}} = \frac{i(\not{p} + m_F)}{p^2 - m_F^2} \qquad (A63)$$

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