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On the Dipole Moments of Fermions at Two Loops

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Abstract

Complete two-loop electroweak corrections to leptonic anomalous magnetic moments have been calculated recently using asymptotic expansions of Feynman diagrams. Techniques developed in that context can also be applied to the electric dipole moments (EDM) of fermions. In the Standard Model the EDM was proved to vanish at two loops. In this talk we discuss a simplification of that proof.

1 Introduction

In a recent calculation of two-loop electroweak corrections to $g-2$ of leptons [1] we have applied computing techniques based on asymptotic expansion of the Feynman diagrams [2]. Similar tools have many other important applications, especially in low energy physics. In the present talk we focus on the two-loop contributions to the electric dipole moments (EDM) of fermions.

Precise measurements of the electric dipole moment of the neutron and the electron impose severe constraints on extensions of the Standard Model (SM) and even on the SM itself (θ-term in the QCD Lagrangian) [3]. If a non-vanishing EDM of an elementary particle were found, it would be the first manifestation of CP-violation outside the $K^0-\bar{K}^0$ system and could provide essential information about the nature of CP-violation. For this reason two decades ago several groups started the theoretical investigation of EDMs in the SM [4]. They found that EDMs vanish trivially at the one-loop level in the SM because only the absolute values of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements enter the relevant amplitudes, not permitting for a CP-violating complex phase. However, at the two-loop level (with two virtual W bosons) they found a quark flavor structure rich enough to generate a non-vanishing EDM of the neutron.

In 1978 a more detailed analysis of Shabalin [5] and Donoghue [6] showed that the EDM of both the neutron and the electron must vanish identically at the two-loop level. This result is somewhat surprising since it could not be traced back to any symmetry which would enforce zero EDM. Subsequent studies [7, 8, 9] found that QCD corrections may generate a non-vanishing EDM of the order of $G_F^2\alpha_s$. These papers, however, differ strongly in the predictions of the size of the three-loop contributions.

The proofs of the vanishing of EDM are based on cancellations among certain groups of diagrams. We noticed that these cancellations are stronger than previously believed, that is they occur among smaller subsets of diagrams. Before presenting our results we summarize in the next section the original argument of Shabalin's proof.

We believe the simplifications we found contribute to a better understanding of the perturbative contributions to EDM and may eventually help in the clarification of the three-loop value of the quark EDM.

2 Shabalin's Argument [8]

In the unitary gauge the diagrams contributing to EDM at the two-loop level are obtained by attaching an external photon to every internal line of the diagram depicted in fig. 1.

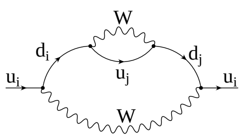


Fig. (1)

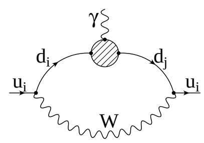


Fig. (2a)

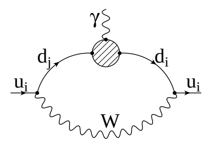


Fig. (2b)

These diagrams possess a complex CKM phase, necessary to generate EDM (a T violating observable) if all quarks u_i, d_i are different.

We now divide the diagrams generated from fig. 1 into two sets: the first set consists of only one diagram, where the photon couples to the outer W line; one can check by an explicit calculation that the part of this diagram which contains γ_5 vanishes, and therefore it does not contribute to EDM. The sum of vertex and selfenergy counterterms for diagrams obtained from fig. 1 gives no contribution to EDM either.

In the rest of this paper we will focus on the remaining set of diagrams. It can be viewed as a one-loop diagram with an insertion of a flavor-changing effective vertex $F_1F_2\gamma$ with off-shell fermions $F_{1,2}$ (see fig. 2a). This effective vertex is given by a sum of one-loop diagrams in fig. 3.

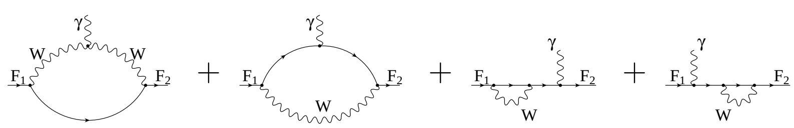
$\Gamma^{F_1F_2\gamma} =$ 

Fig. (3)

An explicit calculation [10] of $\Gamma^{F_1 F_2 \gamma}$ shows that it is proportional to Δ, the external photon momentum. Therefore, in the calculation of d_e we can neglect Δ in the remainder of the diagram in fig. 2a. Also, we have $\Gamma^{F_1 F_2 \gamma} = \Gamma^{F_2 F_1 \gamma *}$. These two observations are enough to show that the diagram (2a) is complex conjugate of the diagram (2b). The imaginary part of their sum vanishes and no EDM is generated. This finishes the original proof [8].

We would like to remark that interesting arguments in favor of vanishing of the two-loop contributions to EDM were also made in [9, 11].

3 Explicit Two-Loop Calculation

Our proof which we present in this section can be summarized by saying: the diagram in fig. (2a) vanishes. In other words, for the vanishing of EDM we need not examine the sum (2a)+(2b) because they are zero independently.

The contribution of the diagram (2a) to EDM is obtained from the sum of four diagrams. Its vanishing can be seen from the two equalities displayed in fig. 4, which we established by an explicit calculation of the two-loop diagrams (the insertions of the external photon are indicated by $\otimes$.)

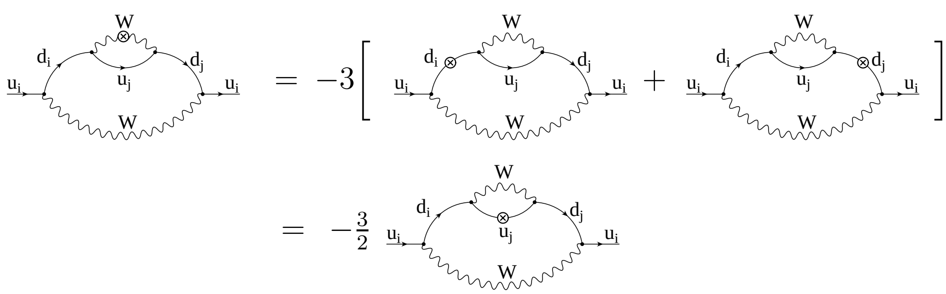


Fig. (4)

The above equalities are true only in the unitary gauge. In a different gauge, e.g. the linear 't Hooft–Feynman gauge, there are additional diagrams with Goldstone bosons; in that situation only a sum over all possible photon insertions *and* all flavors of the fermion in the internal loop vanishes. (Still, the vanishing occurs for a fixed order of the quarks d_i and d_j — no summation over the mirror reflections is necessary.)

There is a simple relation between the contributions to the EDM form factor of a diagram and its mirror reflection: for a given photon insertion the reflection gives a minus sign and complex conjugation of the CKM factors (this is different from the contributions to the *magnetic* moment — there is no sign change there.) As a result the sum of a pair of mirror diagrams has *twice* the imaginary part of an individual diagram (only the real part vanishes).

Finally, we notice that the original argument summarized in section 2 did not say anything about the vanishing of the real part of the sum of the mirror diagrams; a non-vanishing result would contribute to the imaginary part of d_e, forbidden by the hermiticity of the Hamiltonian. From the equalities we have displayed above it is clear that both real and imaginary parts vanish.

4 Projection Operator for EDM Form Factor

In calculations of the anomalous magnetic moment the standard technique is to project out the relevant Lorentz structure and average over the directions of the external photon momentum. The problem is then reduced to a calculation of scalar propagator integrals. Such calculations are most easily performed using algebraic manipulation programs.

In this section we derive a projector also for the electric dipole moment operator; the derivation follows the case of the magnetic moment [12, 13]. We consider the most general matrix element of a current between spin 1/2 fermions

$$\begin{aligned}\langle \alpha_f | M_\mu | \alpha_i \rangle &= \bar{u}_f(p_2) \Big[F_1(t)\gamma_\mu - \frac{i}{2m} F_2(t)\sigma_{\mu\nu}\Delta^\nu + \frac{1}{m} F_3(t)\Delta_\mu \\ &+ \gamma_5 \Big(G_1(t)\gamma_\mu - \frac{i}{2m} G_2(t)\sigma_{\mu\nu}\Delta^\nu + \frac{1}{m} G_3(t)\Delta_\mu \Big) \Big] u_i(p_1) \end{aligned} \tag{1}$$

with $\Delta = p_1 - p_2$ and $t = \Delta^2$. For on-shell external fermions we have

$$p_1^2 = p_2^2 = m^2 \tag{2}$$

We introduce $p = \frac{1}{2}(p_1 + p_2)$ for which we find

$$p^2 = \frac{1}{4}(4m^2 - t)\,, \qquad p \cdot \Delta = 0\,. \tag{3}$$

Conservation of the electromagnetic current requires $F_3(t) = 0$. $F_1(t)$ is the charge form factor, $F_2(t)$ the magnetic moment form factor, $G_2(t)$ the electric moment form factor. The electric dipole moment d_e of the fermion is given by

$$d_e = G_2(0) \tag{4}$$

In order to extract the electric dipole moment form factor one can introduce a projection operator

$$L_\mu = (\not{p}_1 + m)\gamma_5 \left[g_1\gamma_\mu - \frac{1}{m} g_2 p_\mu - \frac{1}{m} g_3 \Delta_\mu \right] (\not{p}_2 + m)\,. \tag{5}$$

In order to determine the coefficients of g_i we take the trace of $L_\mu M^\mu$ (we work in d dimensions):

$$\begin{aligned}\mathrm{Tr}(L_\mu M^\mu) &= \Big\{ - \Big[8m^2(d-1) + 2t(2-d) \Big] g_1 - 4t g_3 \Big\} G_1(t) \\ &+ \left[g_2 t \left(2 - \frac{t}{2m^2} \right) \right] G_2(t) \\ &+ \left[-4 g_1 t - 2\frac{t^2}{m^2} g_3 \right] G_3(t) \end{aligned} \tag{6}$$

The resulting system of equations for $G_2(t)$ can be solved by choosing $g_1 = g_3 = 0$ and

$$g_2 = \frac{2m^2}{t(4m^2 - t)}\,. \tag{7}$$

We can write the EDM form factor $G_2(t)$ as

$$G_2(t) = \frac{2mp^\mu}{t(t-4m^2)} \mathrm{Tr}\left[(\not p_1 + m)\gamma_5(\not p_2 + m)M_\mu\right] \tag{8}$$

Since we are only interested in the special case $t = 0$ we can make further simplifications. As a first step the general amplitude M_μ can then be expanded to first order in Δ_μ:

$$M_\mu(p,\Delta) \approx M_\mu(p,0) + \Delta_\nu \frac{\partial}{\partial \Delta_\nu} M_\mu(p,\Delta)\bigg|_{\Delta=0} \equiv V_\mu(p) + \Delta^\nu T_{\nu\mu}(p)\,. \tag{9}$$

The next step is to average over the spatial directions of Δ with the formulas

$$\begin{aligned} \langle \Delta_\mu \Delta_\nu \rangle &= \frac{1}{d-1}\Delta^2 \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right), \\ \langle \Delta_\mu \rangle &= 0\,; \end{aligned} \tag{10}$$

after that, the limit $t \to 0$ can be taken. The result is

$$d_e = -\frac{p_\mu}{4m} \mathrm{Tr}\left\{ \gamma_5 V^\mu + \frac{1}{2m^2(d-1)} (\not p + m)\gamma_5 [\not p, \gamma_\nu](\not p + m) T^{\nu\mu} \right\} \tag{11}$$

5 Acknowledgement

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