UNIVERSITY OF TRIESTE

DEPARTMENT OF ENGINEERING AND ARCHITECTURE

Course of

Mechanical Vibrations

SIMULATION OF FUZZY AND PID CONTROLLERS FOR ACTIVE SUSPENSIONS

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Contents

Abstract

The aim of this project is to analyze the behaviour of various solutions for vehicles's suspensions, with a focus on active suspensions. Active suspensions allow to overcome the trade-off between performance and comfort, which is typical of passive suspensions. With active suspensions it is possible to modify the behaviour of the system, and thus to obtain the desired dynamic response, through the calibration of the logic controller.

Many controllers are described in literature; hereby, the fuzzy and PID controllers are developed and analyzed. In order to evaluate the performance, passive and skyhook suspensions have also been modelled.

The fuzzy and PID controllers have been modelled on Python and Simulink, respectively, once that the same behaviour of the passive suspension has been verified in both development environments.

The results of the simulations show that both active controllers guarantee an increment in performance and comfort with respect to the passive suspension. However, it is not possible to define a better system without considering the objective of the suspension system itself.

Chapter 1

Models of suspensions

The first task is to develop a simple model which can realistically describe the vehicles' behaviour. Thus, the one-quarter vehicle models have been chosen. Moreover, in the literature it is possible to retrieve much information about these models. The drawback of the one-quarter vehicle models is that it is not possible to analyse the roll, pitch and yawn motions, which highly affect the driving experience and the performance of a vehicle.

The one-quarter model will be a 2DOF model, in which both the body of the car and the wheel are represented. This choice is supported by a large number of articles in the literature.

1.1 2DOF passive suspension

The classic 2DOF passive suspension is represented in Figure 1.1; the body m_b is linked to the wheel m_w through a spring k_s and a dumper c_s. The wheel is linked to the road surface through a different spring k_t. The dumper of the tire can be neglected, according to [Ro, 1993]. This allows to obtain a simplified model.

Following the nomenclature defined in the figure, the equations of motions can be written in the state-space notation:

$$\dot{\overrightarrow{X}} = \left\{ \begin{array}{c} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{array} \right\} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_b} & \frac{k_s}{m_b} & -\frac{c_s}{m_b} & \frac{c_s}{m_b} \\ \frac{k_s}{m_w} & -\frac{k_s+k_t}{m_w} & \frac{c_s}{m_w} & -\frac{c_s}{m_w} \end{bmatrix} \left\{ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_w} \end{array} \right\} r$$

where the state $\overrightarrow{X}$ is defined as:

$$\overrightarrow{X} = \left\{ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right\} = \left\{ \begin{array}{c} x_b \\ x_w \\ \dot{x_b} \\ \dot{x_w} \end{array} \right\}$$

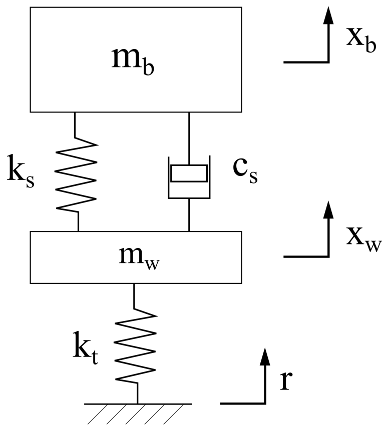

FIGURE 1.1: Model of the 2DOF passive suspension.

The values of the parameters are chosen accordingly to [Barr, 1996]; those are described in the Table 1.1.

m_b	m_w	k_s	k_t	c_s
kg	kg	N/m	N/m	$N/(m/s)$
250	30	15000	150000	1000

TABLE 1.1: Model parameters.

1.1.1 Natural frequencies

The model is characterized by its natural frequencies; in order to find them, the equations of motion should be rearranged in the canonical representation:

$$m_b\ddot{x_b} + c_s(\dot{x_b} - \dot{x_w}) + k_s(x_b - x_w) = 0$$
$$m_w\ddot{x_w} - c_s(\dot{x_b} - \dot{x_w}) - k_s(x_b - x_w) + k_t(x_w - r) = 0$$

The characteristic equation of the system is given by:

$$det \begin{bmatrix} -\omega^2 m_b + k_s & -k_s \\ -k_s & -\omega^2 m_w + k_s + k_t \end{bmatrix} = 0$$

The natural frequencies are the roots of the characteristic equations:

$$f_1 = 1.2\ Hz$$
$$f_2 = 11.8\ Hz$$

It is straightforward to calculate the eigenvectors, which represent the modes of the system:

$$\overrightarrow{X}^{(1)} = \begin{Bmatrix} 1 \\ 0.09 \end{Bmatrix} \qquad \overrightarrow{X}^{(2)} = \begin{Bmatrix} 1 \\ -90.8 \end{Bmatrix}$$

These vectors could be interpreted as follows: at low frequencies, the dynamic of the system affect almost exclusively the suspended body; at high frequencies, the wheel displacement is preponderant. In other words, the motion of the wheel will be characterized by a high frequency dynamic. Since, as will be shown later, the driving comfort depends on the body displacement, and the drivability depends on the wheel displacement, it is clear that the control system, in order to ensure high comfort and performance, must operate in a wide frequency range.

1.2 2DOF active suspension

The 2DOF active suspension differs from the 2DOF passive one for the addition of an actuator between the body and the wheel, as shown in Figure 1.2. The entity of the force depends on the logic of the controller, and it is a function of the parameters chosen as inputs of the system - suspension deflection, velocity and acceleration of the body.

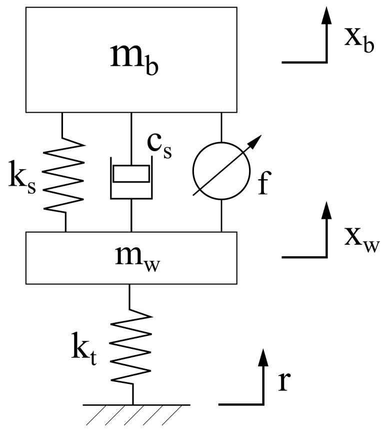

FIGURE 1.2: Model of the 2DOF active suspension.

The EOM in the state-space notation are, in this case:

$$\dot{\vec{X}} = \begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_b} & \frac{k_s}{m_b} & -\frac{c_s}{m_b} & \frac{c_s}{m_b} \\ \frac{k_s}{m_w} & -\frac{k_s+k_t}{m_w} & \frac{c_s}{m_w} & -\frac{c_s}{m_w} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{m_b} \\ \frac{k_t}{m_w} & \frac{1}{m_w} \end{bmatrix} \begin{Bmatrix} r \\ f \end{Bmatrix}$$

1.3 Skyhook 2DOF suspension

In the *skyhook* model the body is linked to the sky with a dumper; this way, a force is generated which mitigates the body displacement. The aim of this model is to isolate the body dynamics from the rest of the model. This results in a highly reduced body displacement (with respect to the classic 2DOF passive suspension), and thus it can be a valid metric to assess the performance of the active suspension developed in this work.

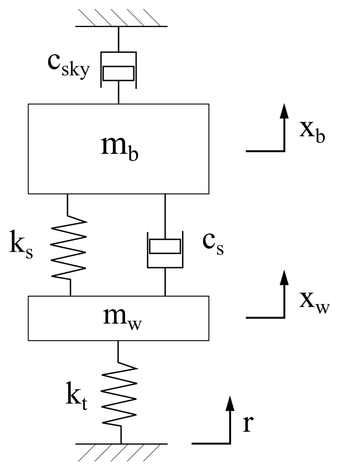

FIGURE 1.3: Model of the 2DOF Skyhook suspension.

The problem of the skyhook suspension is that a practical realization is impossible - it is impossible to hook a dumper in the sky. Some constructors - Maserati and Ducati - have developed skyhook systems: these are to be intended as systems where a low body displacement is achieved, as in the classical skyhook model. However, these systems adopt a variable dumper, controlled with a sophisticated logic (Figure 1.4).

The EOM for the skyhook model are:

$$\dot{\vec{X}} = \begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_b} & \frac{k_s}{m_b} & -\frac{c_s+c_{sky}}{m_b} & \frac{c_s}{m_b} \\ \frac{k_s}{m_w} & -\frac{k_s+k_t}{m_w} & \frac{c_s}{m_w} & -\frac{c_s}{m_w} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_w} \end{Bmatrix} r$$

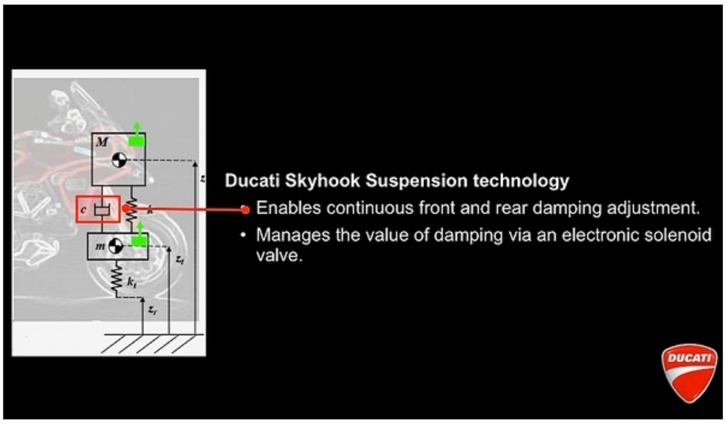

FIGURE 1.4: Skyhook suspension.

Chapter 2

Simulations

2.1 Controllers

The controllers described in the following sections are aimed to determine the entity of the force developed by the actuator, in order to increase the level of comfort and drivability. It is not possible to assign a numerical values to these quantities; however, they are related to: [Mulla, 2014]

- *body displacement* and *acceleration*: a high value of these determine a low drive comfort;
- *wheel displacement*: a high value indicates a low adherence with the road, and therefore a low drivability.

The evaluation of the force will depend on the values retrieved from - some or all - the sensors of position, velocity and acceleration installed on the suspension system.

2.1.1 Fuzzy controller

The fuzzy logic allows to deal with a decision problem in a way which is rather similar to the human decision-making process. In the fuzzy logic, the truth values of an expression could assume values in the $[0, 1]$ interval (whereas the crisp logic allows either 0 - false - or 1 - true - values). This permits a more realistic representation of the reality.

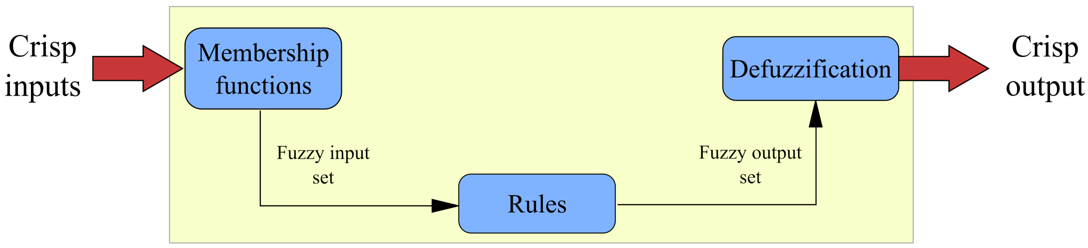

FIGURE 2.1: Fuzzy logic controller.

The fuzzy inference mechanism is made up of these steps:

1. inputs fuzzification: the membership degree of each input is computed for each fuzzy set;
2. fire strength: the degree of activation for each rule is computed;
3. effect on consequent sets, with minimum or product correlation;
4. superposition of consequent sets, with union or sum;
5. defuzzifucation with the centroid method, in order to obtain a scalar value as output.

The strength of a fuzzy logic controller, based on a set of rules, is that each rule could be variously satisfied.

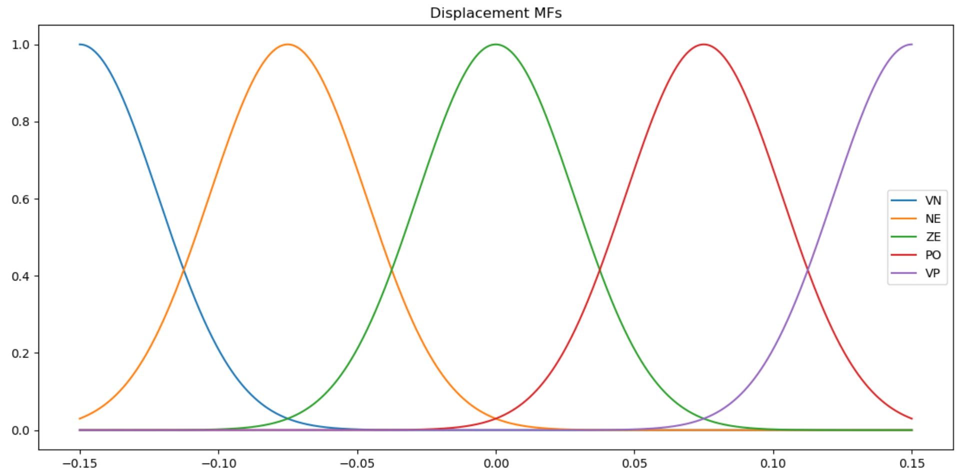

FIGURE 2.2: Membership functions for displacement.

The choice of the inputs is critical: in literature several solutions are found; [Ro, 1993] and [Barr, 1996] uses velocity and acceleration of the body. [Senthilkumar, 2016] chooses the spring elongation and the body velocity. [Changizi, 2011] uses three inputs (spring elongation, velocity and acceleration of the body). In order to obtain a simple fuzzy model, it has been chosen to use as inputs the spring elongation $(x_b - x_w)$ and the body velocity. The output of the model is trivial: it is the actuator force[1].

The membership functions have been defined for each quantity. The domain limits for actuator force and body velocity are taken from [Barr, 1996], whereas for the body displacement are deduced from the passive suspension model; then, the domains are divided with 5 membership functions. Therefore, there will be 25 fuzzy rules.

[1]Note that, in the case of semi-active suspensions, the output of the system will be the dumping coefficient.

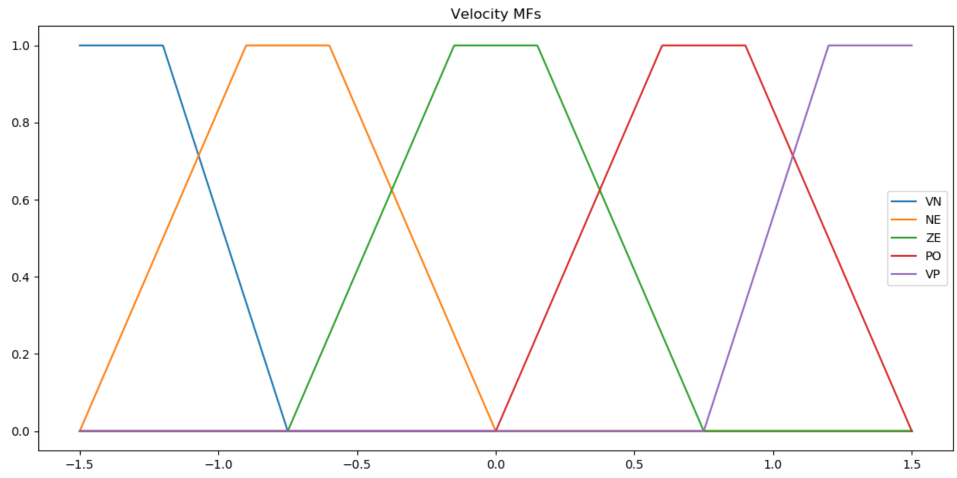

FIGURE 2.3: Membership functions for body speed.

The shape of the MFs could be gaussian, triangular or trapezoidal - the choice described here is the one which allows to obtain better overall results. Each membership is described by a linguistic label: *POsitive*, *ZEro*, *NEgative*, eventually combined with *Strong*, *Weak* e *Very*.

The key of the fuzz controller is its rulebase; this contains the IF/THEN rules which allows to assign a value the output based on the values of the inputs.The rules, shown in Table 2.1, has been developed by refining a common-sense set of rules with a trial-and-error approach.

		Velocity				
		VN	NE	ZE	PO	VP
Deflection	VN	PS	PS	PS	PW	PW
	NE	PS	PS	PW	NS	NS
	ZE	PS	PW	ZE	NW	NS
	PO	PS	PS	NW	NS	NS
	VP	NW	NW	NS	NS	NS

TABLE 2.1: Rulebase.

This rulebase has a rather aggressive behaviour, which allows to minimize the oscillation of the body, which leads to benefits in the comfort.

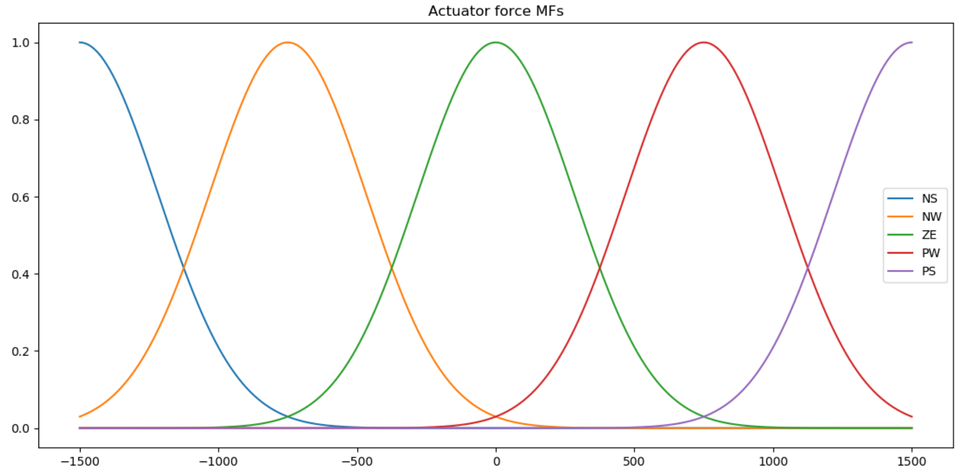

FIGURE 2.4: Membership functions for the actuator force.

2.1.2 PID controller

The *Proportional-Integral-Derivative* controller, commonly shortened as PID, represent the most used logic in the industry controllers. The intuition of the PID controller is to compare the signal with a reference signal: the difference between these defines the error, which is the signal to minimize with an adequate variation of the output variable.

A PID controller is therefore made up of three separate actions:

$$u(t) = k_p \cdot e(t) + k_i \int_0^t e(\tau)d\tau + k_d \cdot \frac{de(t)}{dt}$$

where $u(t)$ is the output variable, and $e(t) = r(t) - y(t)$ is the error; the coefficients k_p, k_i, k_d define, respectively, the proportional, integral and derivative behaviour of the controller. These can be described as follows:

- *Proportional*: it is proportional to the error signal at the present time. A controller with only a proportional term cannot guarantee the convergence of the error: in fact, if the signal error is null, the output variable will be null as well.

- *Integral*: it proportional to the integral of the error signal between the start time and the present time. Thus, this term account for previous values of the error. By doing this, the residual error is minimized, and convergence will be reached. The integral term will be not zero even if the error will be null at a certain time.

- *Derivative*: it is the proportional to the time derivative of the error signal. It represents the future trend of the error. This term allows to achieve a fast compensation of the error.

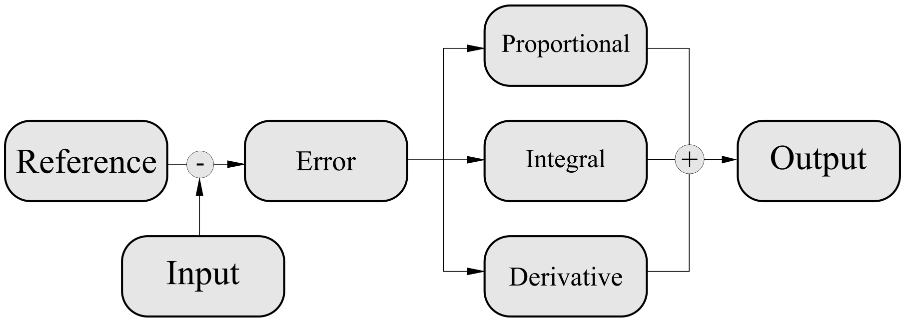

FIGURE 2.5: PID controller.

In the case of the active suspension here studied, the reference signal is evaluated on the suspension elongation. The effect of the single actions of the PID controller on the system are described in Table 2.2.

Parameter	Controller		
	P	PI	PID
Overshoot	Increase	Increase	Decrease
Steady-state error	Decrease	Substantial decrement	-
Settling time	Small variation	Increase	Decrease

TABLE 2.2: Effect of single actions on the whole system. [Smriti Rao, 2014]

The tuning of the coefficient is based on trial-and-error approach, following the behaviour described in the Table 2.2. The experimental tests has shown that a high k_d reduced the settling time of both wheel and body displacement, while causing an instability in the acceleration of the suspended mass acceleration.

2.2 Test conditions

Given the test conditions found in literature, four types of test signals have been chosen: [Ro, 1993, Barr, 1996, Senthilkumar, 2016]:

1. step;

2. impulse;

3. bump;

4. road profile ISO 8608;

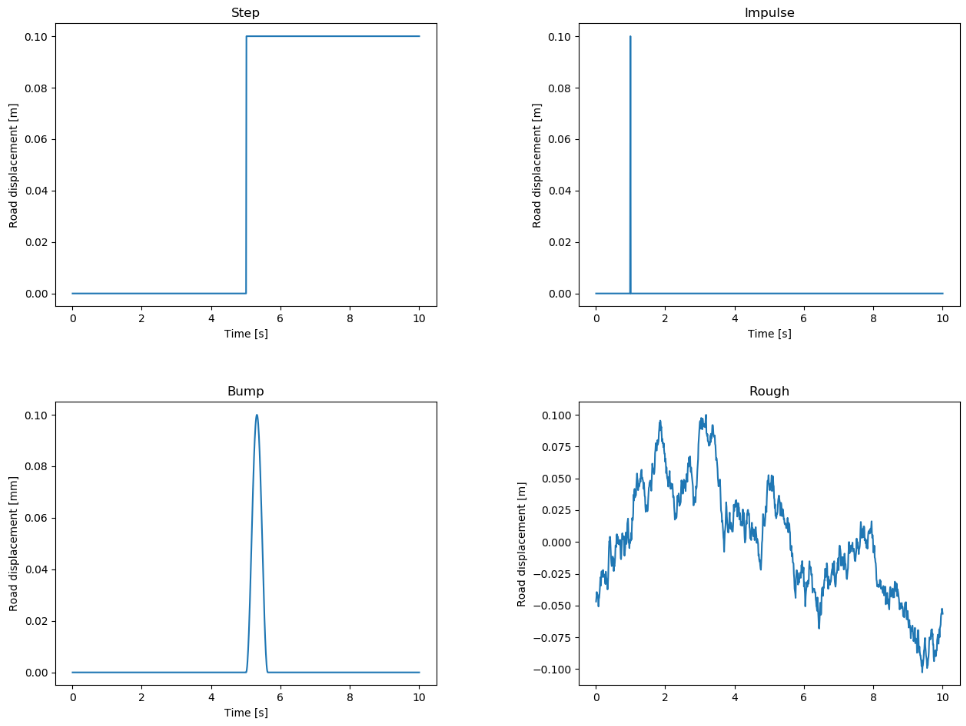

FIGURE 2.6: Road profiles used in the simulations.

The first two signal are useful to characterize the dynamic behaviour of the system; the others try to emulate a more realistic scenario, where the variation of the road profile are less sudden. Each profile has a maximum amplitude of 0.1 m, where this is supposed to be the feasible limit for a real suspension. The chosen time step is 0.01 s, for a total simulation time of 10 s.

2.3 Operative scheme

The operative scheme of the simulation is independent on the controller used: in both cases, the controller is placed in a closed-loop, as shown in Figure 2.7.

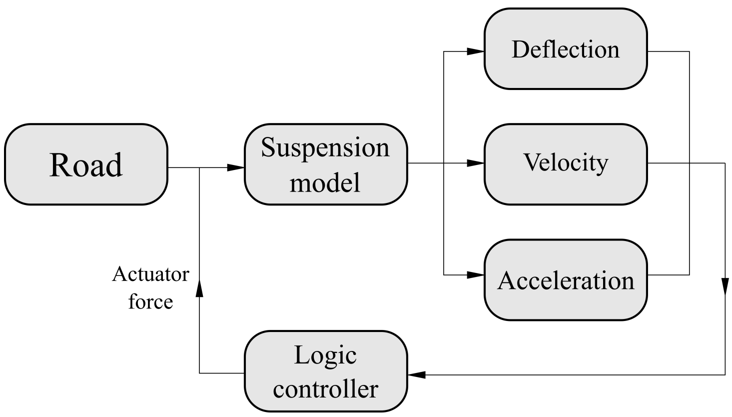

FIGURE 2.7: Simulations scheme.

2.4 Implementation

2.4.1 Python

Python is a general purpose interpreted programming language; this means that it is possible to personalize the working environment via the installation of specific packages. In the case of this work, in order to manipulate arrays, solve differential equations and plot the results, the package installed are *numpy*, *scipy* and *matplotlib*. The main problem in the development of the model has been the process of solving the EOM: in the case of the active suspension, the coefficients of the differential equations are not constant, and therefore a package like *ode45* is not suitable.

The implementation of the fuzzy logic has been made using some functions implemented in the *scikit-fuzzy*[Fuzzy Toolbox] package.

2.4.2 Simulink

Simulink is an environment to model and simulate dynamic systems, which uses a user interface where the system is represented and modelled with a block diagram. Each Simulink model can be linked with Matlab. In the following, the model of the suspension is described.

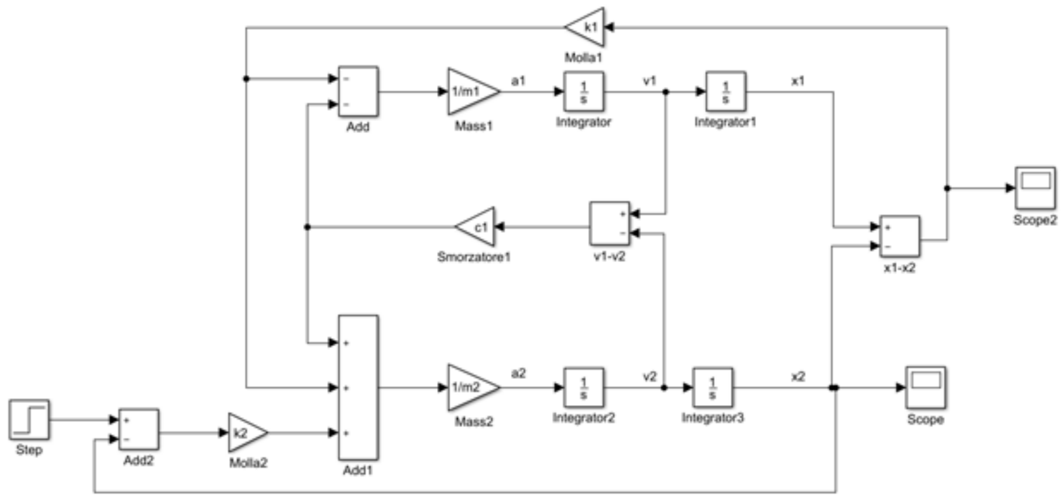

FIGURE 2.8: Simulink model of the passive suspension.

The passive suspension model is shown in Figure 2.8 (the indices 1 and 2 represent the body and the wheel, respectively).

The active model is obtain by simply adding a signal generator in the scheme, as shown in Figure2.9.

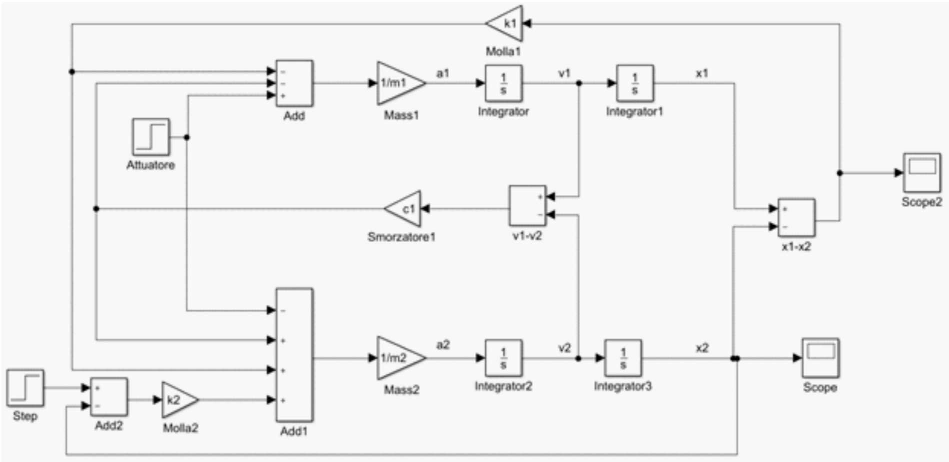

FIGURE 2.9: Simulink model of the active suspension.

This model will be linked with the controllers, which evaluates the force of the actuator. The final scheme of the PID controlled active suspension is shown in Figure 2.10.

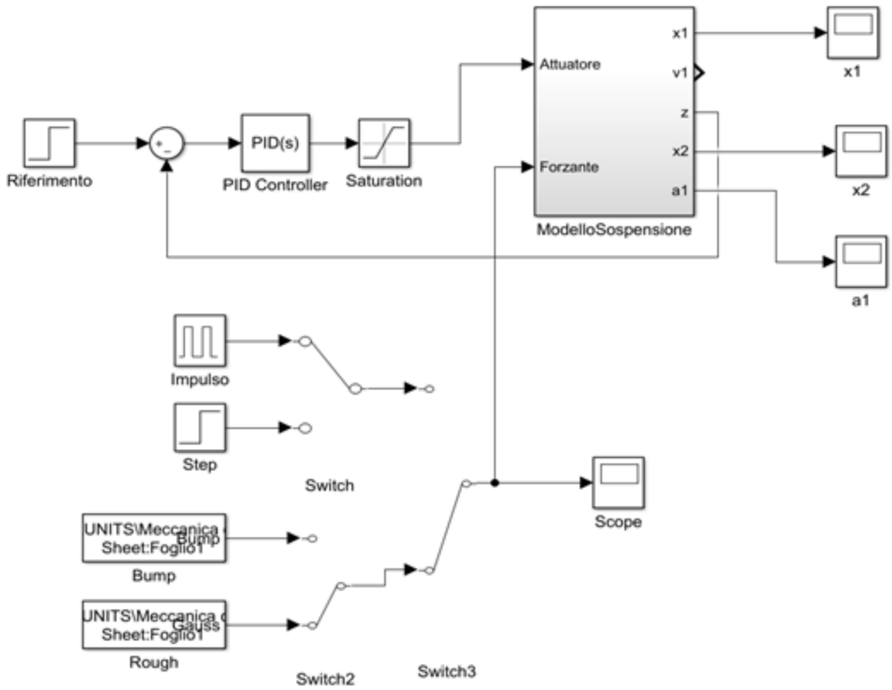

FIGURE 2.10: Simulink model with PID controller.

In the figure, the block "*ModelloSospensione*" represents the sub-model of the active suspension (the one shown in Figure 2.10).

Chapter 3

Results

In this chapter the results of the simulations are reported and analysed, both in numerical and graphical representations. The responses will be compared with the passive suspension (where a significant improvement is expected), and with the skyhook suspension.

3.1 Comparison of the suspension models and controllers

3.1.1 Peak values and settling time

The first evaluation metrics are the values of the overshoot amplitudes and the settling time; the less these values, the better the controller has performed.

The overshoot values are defined as the maximum variation with respect to the steady-state response; the settling time are evaluated as the time at which the difference between the actual and the steady-state responses are constrained into a range of:

- $\pm 1\ mm$ for displacements;
- $\pm 0.1\ m/s^2$ for accelerations.

3.1.1.1 Body displacement

In Figure 3.1-3.4 the responses of passive, fuzzy, skyhook and PID suspensions are shown, for each test signal. Qualitatively, an improvement is observed both in overshoot amplitude and settling time for the active models and the skyhook. The analytical values are reported in Table 3.1. Note that in this table the rough road profile is not considered, because the evaluation metrics previously defined do not make sense; in this case, the RMS values will be studied.

Control system	Input signal	Overshoot [mm]		Settling time [s]
		1st	2nd	
Passive	Step	55.2	27.3	3.0
	Impulse	6.8	3.0	1.1
	Bump	130.2	65.6	2.7
Fuzzy	Step	26.4	1.9	1.0
	Impulse	6.7	0	0.4
	Bump	108.6	23.6	0.8
Skyhook	Step	19.7	3.2	1.1
	Impulse	6.2	0.8	0.3
	Bump	98.7	17.1	1.0
PID	Step	18.6	0.4	2.4
	Impulse	8.7	1.2	0.4
	Bump	125	35.6	0.6

TABLE 3.1: Overshoot and settling time for bady displacement.

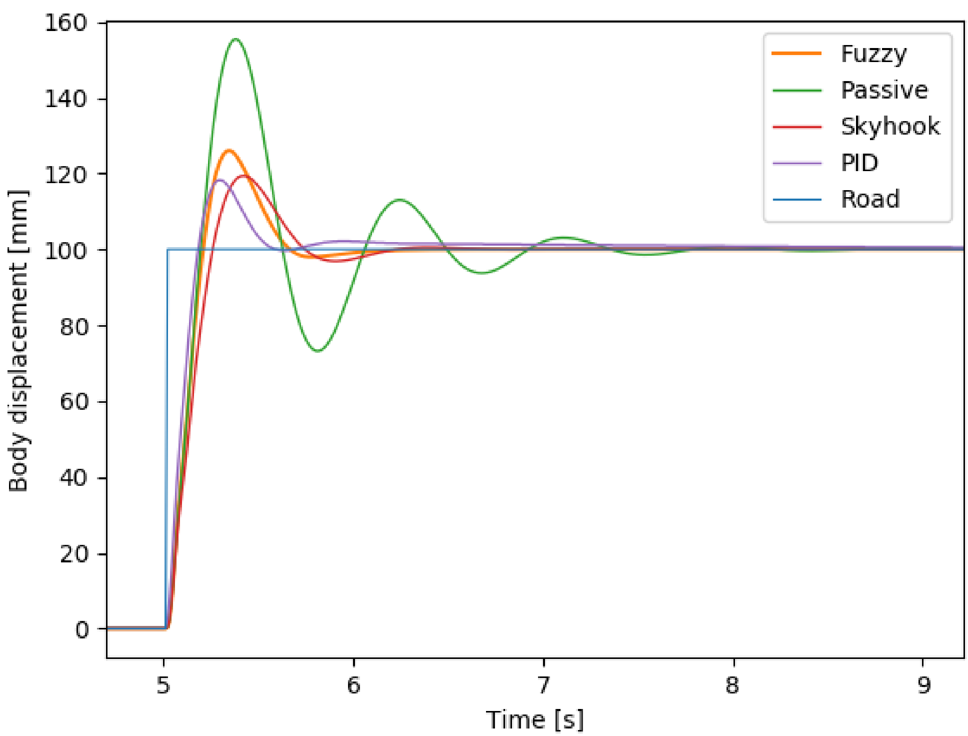

FIGURE 3.1: Body displacement: step profile.

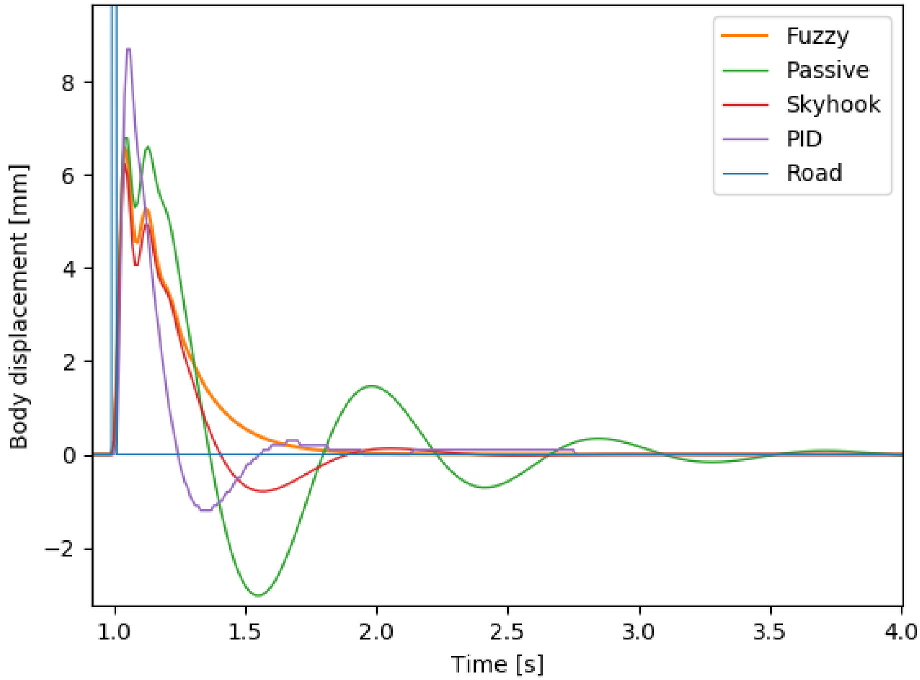

FIGURE 3.2: Body displacement: impulse profile.

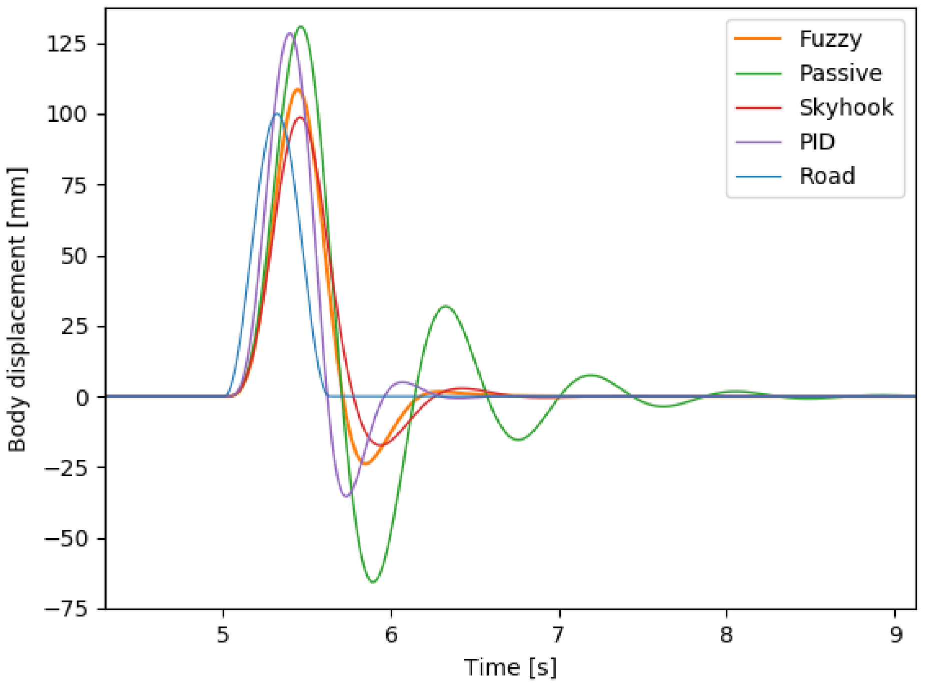

FIGURE 3.3: Body displacement: bump profile.

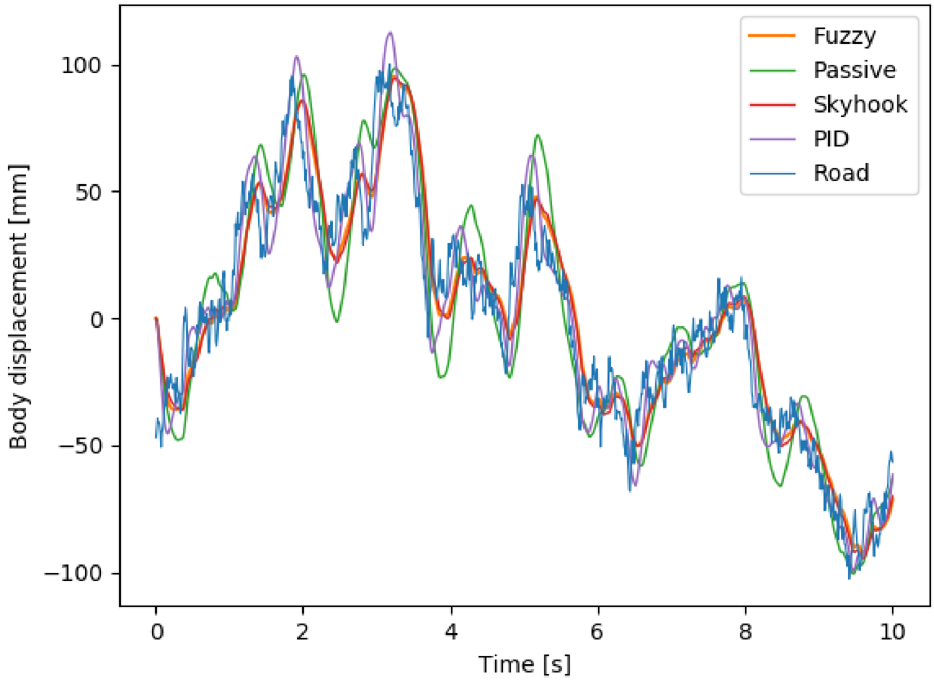

FIGURE 3.4: Body displacement: ISO 8608 road profile.

The fuzzy logic controller is the best solution for settling time, because it is the most robust with all test signals; however, also PID and skyhook systems guarantee an evident improvement on the settling time with respect to the passive suspension.

On the overshoot metric, the overall best solution in not clear: the skyhook suspension reduces the most the first peak, but the second peak is better handled by the active systems.

3.1.1.2 Body acceleration

The same analysis can be made on the body acceleration (Figure 3.5-3.8).

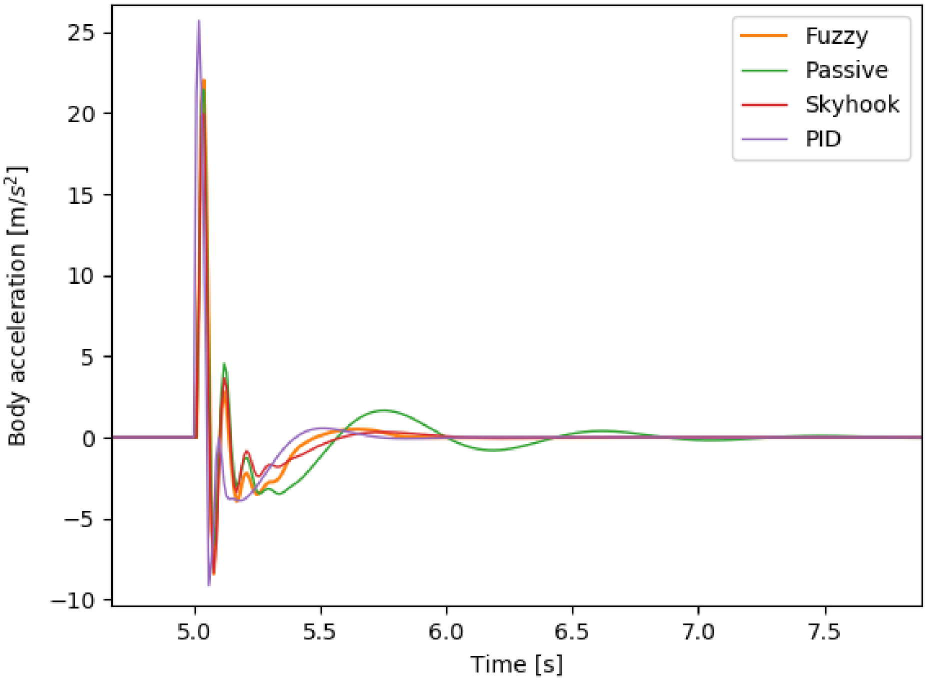

FIGURE 3.5: Body acceleration: step profile.

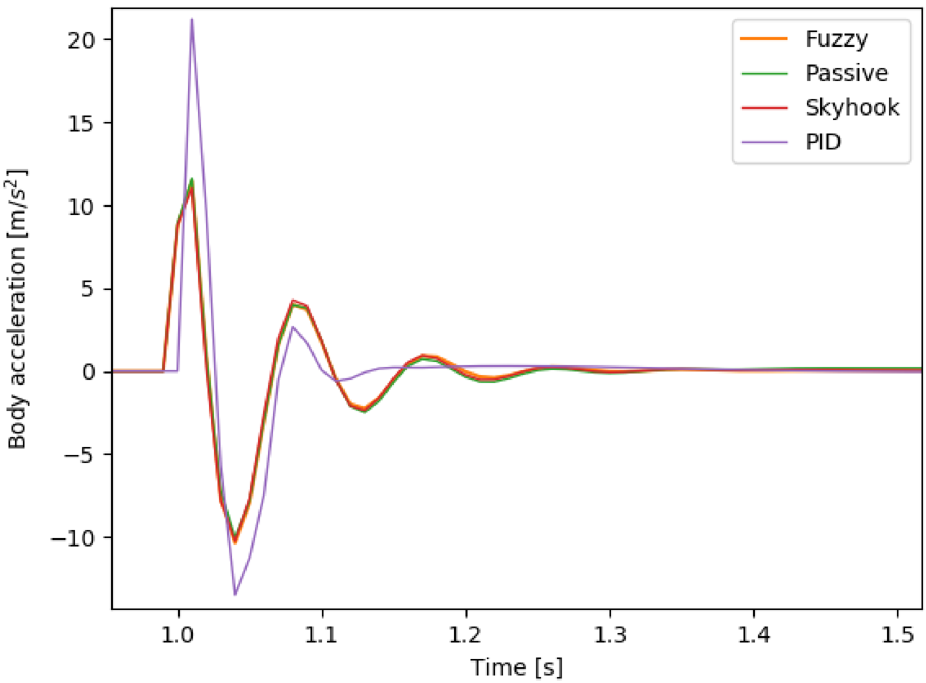

FIGURE 3.6: Body acceleration: impulse profile.

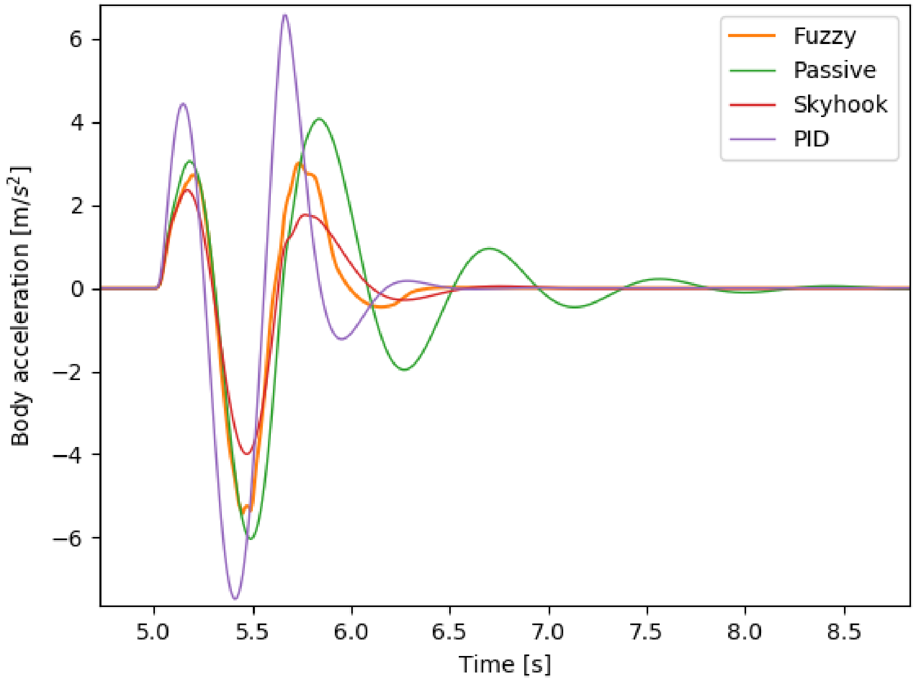

FIGURE 3.7: Body acceleration: bump profile.

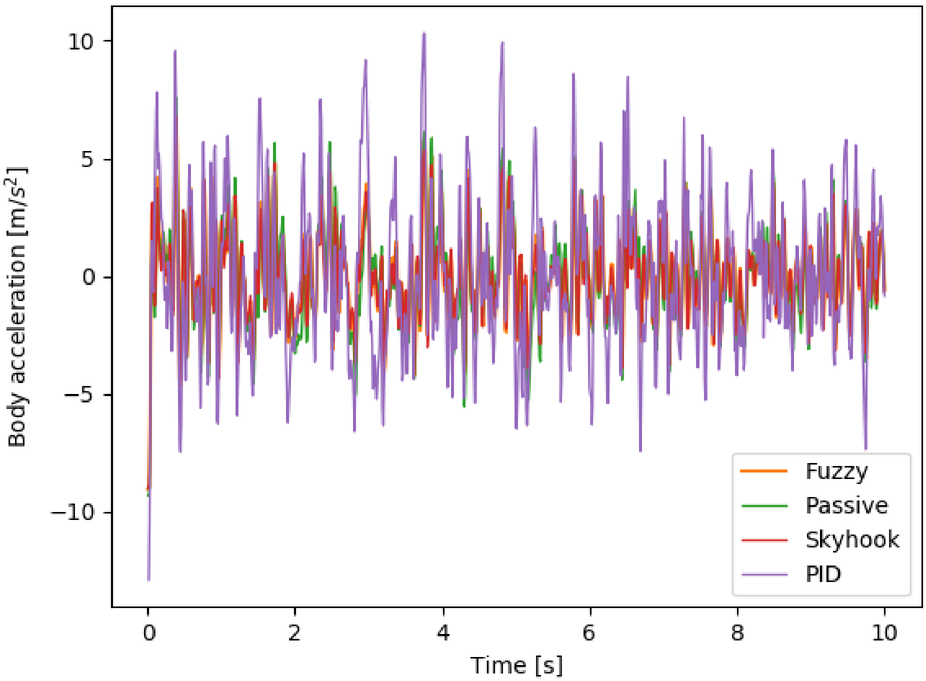

FIGURE 3.8: Body acceleration: ISO 8608 road profile.

Control System	Input signal	**Overshoot** $[m/s^2]$	**Settling time** $[s]$
Passive	Step	21.3	2.5
	Impulse	11.5	0.9
	Bump	6.0	2.4
Fuzzy	Step	22.0	0.8
	Impulse	11.3	0.3
	Bump	5.4	0.7
Skyhook	Step	19.9	1.0
	Impulse	10.9	0.3
	Bump	4.0	0.9
PID	Step	25.7	0.7
	Impulse	21.3	0.3
	Bump	7.4	0.8

TABLE 3.2: Settling time for body acceleration.

In this analysis it is worth to be noted an increment in the values of overshoot for the PID controlled suspension.

3.1.1.3 Wheel hop

The wheel hop determines the drivability of the vehicle: a lower displacement allows a better contact between the tire and the road surface. In the following, the responses of each suspension are presented, for each test signal (Figure 3.9-3.12).

Again, a decrement of the settling time is observed for the active suspensions. The PID controller, with step and impulse test signals, exhibits the lowest overshoot among all suspensions.

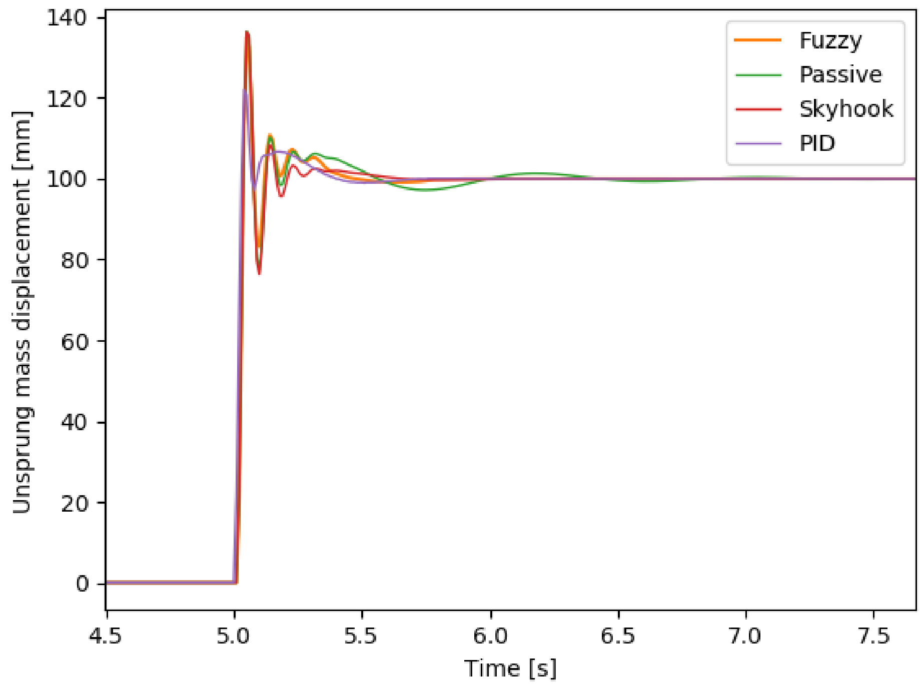

FIGURE 3.9: Wheel hop: step profile.

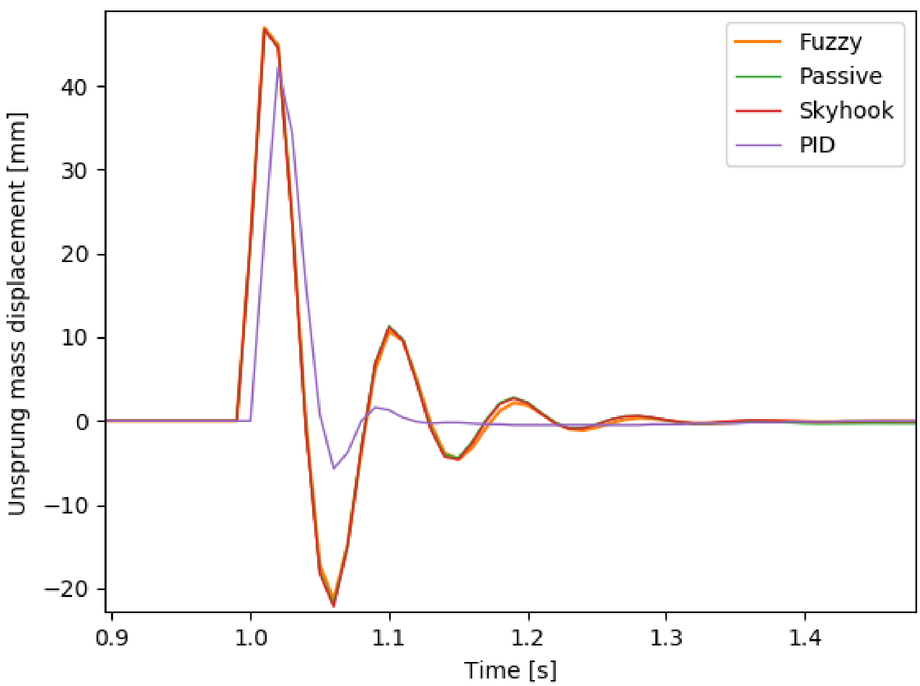

FIGURE 3.10: Wheel hop: impulse profile.

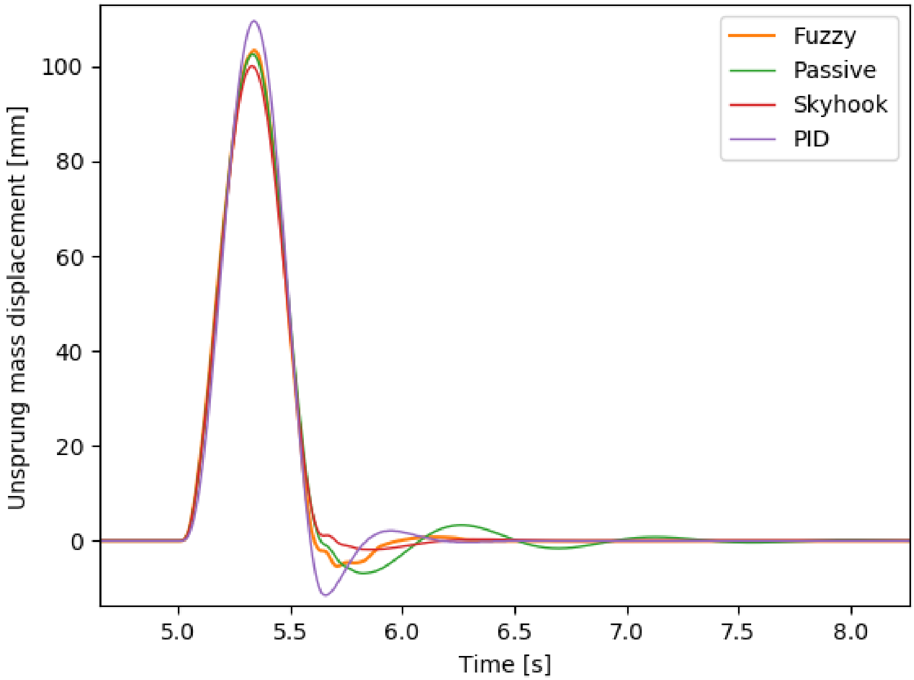

FIGURE 3.11: Wheel hop: bump profile.

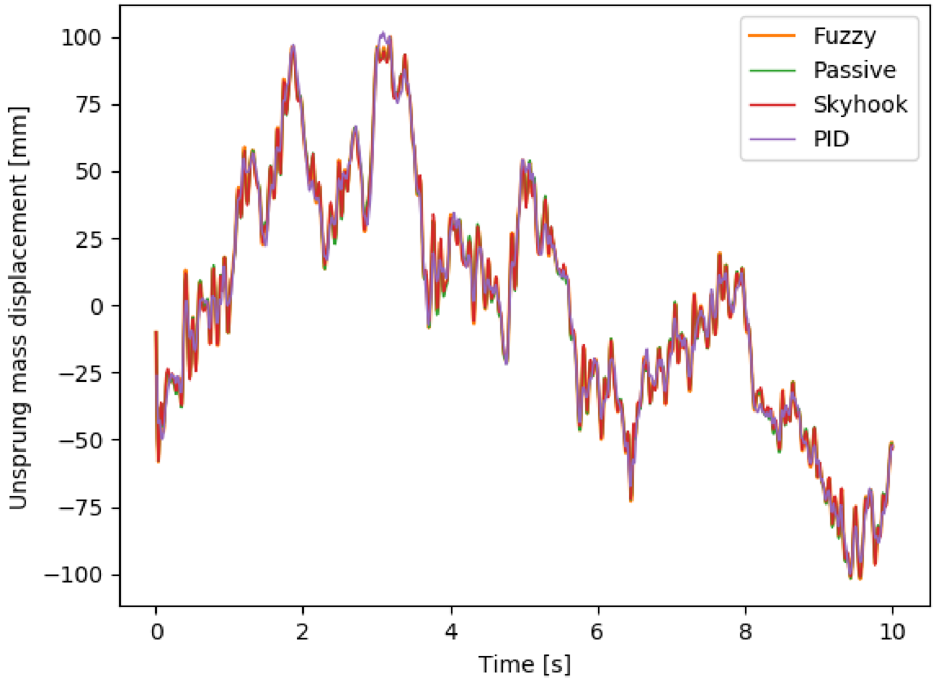

FIGURE 3.12: Wheel hop: ISO 8608 road profile.

Control System	Input signal	**Overshoot** $[mm]$	**Settling time** $[s]$
Passive	Step	36.5	1.6
	Impulse	46.6	0.3
	Bump	102	1.2
Fuzzy	Step	35.1	0.4
	Impulse	47.0	0.2
	Bump	103	0.3
Skyhook	Step	36.5	0.5
	Impulse	46.5	0.3
	Bump	100	0.4
PID	Step	22.1	0.5
	Impulse	42.1	0.1
	Bump	109	0.4

TABLE 3.3: Overshoot and settling time for body displacement.

3.1.2 RMS values

For the road profile ISO 8608 test signal, the performance of the controller are evaluated via the comparison of the RMS values of selected quantities [Barr, 1996]. The *root mean squared* quantity is defined as:

$$RMS = \sqrt{\frac{1}{N}\sum_{i=1}^{N} x^2}$$

The actual values of the RMS are not important; the variation of the RMS with respect to the passive case are the interesting metric.

Control system	**Rough**		
	Body displacement	Body acceleration	Wheel hop
Fuzzy	−9.1%	−8.6%	−1.7%
Skyhook	−8.2%	−11.4%	−3.1%
PID	−4.2%	54.1%	−24.1%

TABLE 3.4: Variation of RMS values for fuzzy and skyhook model with respect to passive model, with road profile ISO 8608.

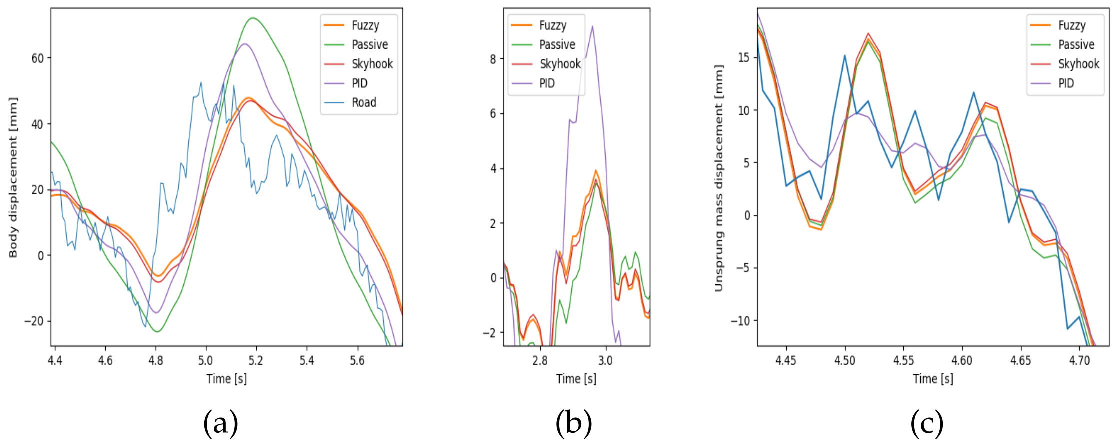

FIGURE 3.13: System responses in case of road profile ISO 8608. (a) Body displacement (b) Acceleration (c) Wheel hop.

The high increment in the RMS values of the body acceleration for the PID controller is given by the high peak values which the system shows (Figure 3.13b). This causes a high decrement in the wheel hop RMS values, as shown in Figure 3.13c.

3.1.3 Frequency analysis

It could be interesting to analyse the frequency response of the body acceleration, as suggested by [Yoshimura, 1999]. An FFT has been applied, and the results are shown in Figure 3.14 and 3.15.

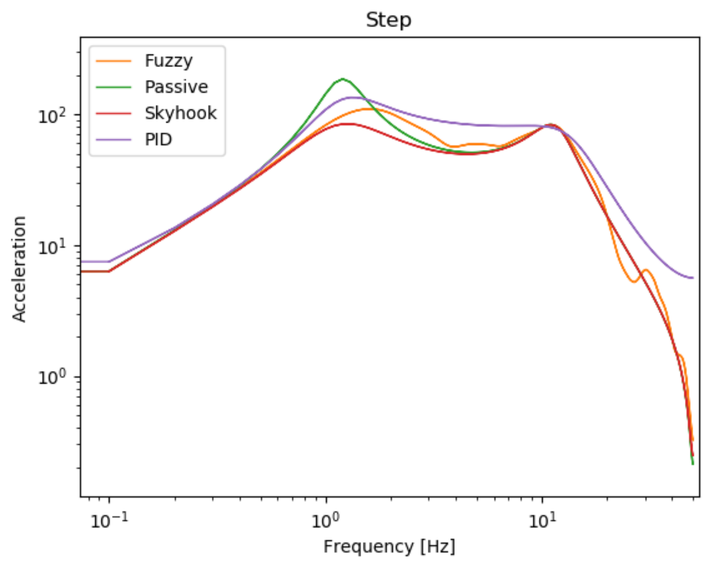

FIGURE 3.14: Frequency analysis of body acceleration response to step profile.

Note that, as expected there are peak values in correspondence with the natural frequencies of the system. Moreover, the passive suspension's first peak value has higher values than the others, showing that the active and skyhook systems actually better control the dynamic of the system.

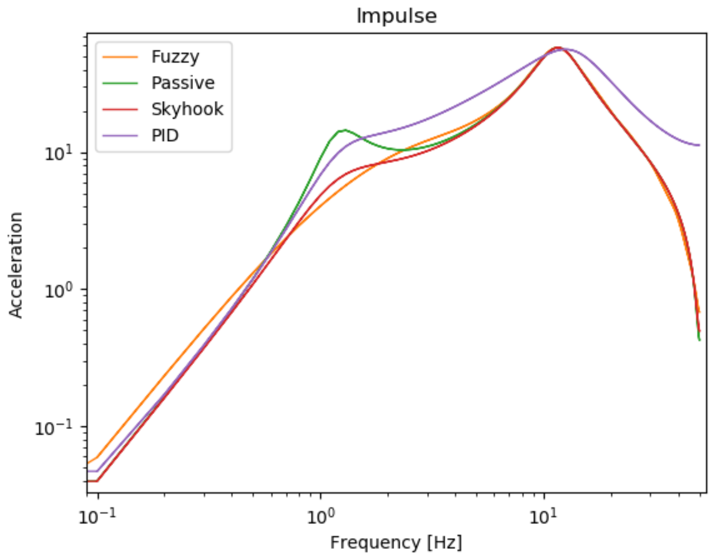

FIGURE 3.15: Frequency analysis of body acceleration response to impulse profile.

3.2 Conclusions

In this section the results of the simulations are interpreted, for each controller.

The PID controller offers the advantage of simple development and research of the optimal values. In general, an improvement of the settling time and overshoot values are observed for the displacements, at the expense of an increment of the acceleration peaks. This is explained by the structure of the controller, which operates on a single signal. This because only the spring deflection has been chosen as the reference signal, and therefore only the displacements will be controlled. On the other hand, if the acceleration would have been chosen as the reference signal, better results would have been reached for the body acceleration, while worse results in the case of the displacements.

The fuzzy controller exhibits a higher robustness of improvement for all the evaluation metrics; the reason for this is that the controller operates with two input variables. However, this reflects into a less aggressive optimization on single metrics.

The skyhook model is an analytical model which offers several advantages, but given its impracticability it could not directly implemented in a real-world scenario. The solutions named as skyhook are a derivation of this model.

It is worth to be noted that the step and impulse test signals are useful to characterize the model, but are unlike to evaluate real-life performance, given theirs discontinuous nature.

Concluding, given the results of the simulations conducted in this work, it is not possible to assess which model is the better one. Each solution offers its advantages and disadvantages: the choice must be made considering the objective to be reached. For scenarios where only the drivability of the vehicle is important (e.g. race cars), the best solution seems to be the PID controller. Otherwise, if a compromise between performance and comfort is needed, the fuzzy controller is preferable.

3.3 Future developments

The suspension model studied in this work are rather simple and simplistic; a deep study should be made with half-vehicle and full vehicle models, where roll, yawn and pitch motions play a role in the driving performance. Such implementation, mathematically more complex, puts new problems on the choice of the objective to optimized - a multi-objective optimization framework would be more reasonable.

The fuzzy logic controller could be improved if three inputs would be considered (displacement, velocity and acceleration); the drawback of this would be that a far less intuitive rulebase needs to be developed.

Lastly, a semi-active suspension could be modelled, where the controller acts on the dumping effect.

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