$$F^{\leftarrow}(u) = \inf\{x : F(x) \geq u\} = \min\{x : F(x) \geq u\}, \quad 0 < u < 1.$$

Proposición

Dada F una función de distribución cumulativa:

- a) $u \leq F(x) \iff F^{\leftarrow}(u) \leq x.$
- b) Si $U \sim \mathrm{Uniforme}(0,1) \implies F^{\leftarrow}(U)$ tiene función cumulativa de distribución F (es decir $\mathcal{L}(X) = \mathcal{L}(F^{\leftarrow}(U))$).
- c) Si F es continua $\implies F(X) \sim \mathrm{Uniforme}(0,1)$.

a) Sup. que $u \leq F(x)$, ent. $F^{\leftarrow}(u) \leq x$ por def

Ahora, sup. que $F^{\leftarrow}(u) \leq x$ ¿P.D $u \leq F(x)$.

Sea $(a_n)_{n \in \mathbb{N}} \subset \{x : F(x) \geq u\}$ t.

$\lim_{n \to \infty} a_n = F^{\leftarrow}(u)$. Como F es cont. por

la derecha, ent. $\lim_{n \to \infty} F(a_n) = F(F^{\leftarrow}(u))$

$u \leq F(a_n)$ para toda n

Fn-l. $u \leq \lim_{n \to \infty} F(a_n) = F\left(F^{\leftarrow}(u)\right)$

Fn-l. $u \leq F\left(F^{\leftarrow}(u)\right) \leq F(x)$

Fn-l. $u \leq F(x)$ $\blacksquare$

b) P.D. $\mathcal{L}(X) = \mathcal{L}(F^{\leftarrow}(U)),\ U \sim U(0,1)$

$$P(F^{\leftarrow}(U) \leq x) = P(U \leq F(x))$$

$$= F(x)$$

$$\therefore \mathcal{L}(F^{\leftarrow}(U)) = \mathcal{L}(X)$$

c) Sea F cont. P.D $\boxed{F(X) \sim U(0,1)}$.

$$\mathbb{P}(F(X) \leq x) = \mathbb{P}(X \leq F^{-1}(x))$$

$$= F(F^{-1}(x)) = x = \mathbb{P}(U \leq x)$$

$$a < b \Rightarrow F(a) < F(b)$$

Si $X \sim \exp(\lambda)$, ent.

$$F(x) = 1 - e^{-\lambda x}.$$

$$x = 1 - e^{-\lambda y} \iff 1 - x = e^{-\lambda y}$$

$$\iff -\frac{\ln(1-x)}{\lambda} = y$$

$$F^{\leftarrow}(u) \sim \exp(\lambda)$$

$$-\frac{\ln(1-u)}{\lambda} \sim \exp(\lambda)$$

$$-\frac{\ln(u)}{\lambda} \sim \exp(\lambda)$$

Sea $X \sim Beta\left(\frac{1}{2}, \frac{1}{2}\right)$

P.D. $\frac{2}{\pi} \arcsin\left(\sqrt{x}\right) \sim U(0,1)$

$$g(x) = \frac{1}{\pi \sqrt{x(1-x)}}$$

$$G(x) = \int_0^x \frac{dt}{\pi \sqrt{t(1-t)}} = \frac{2}{\pi} \arcsin\left(\sqrt{x}\right)$$

Por c), $G(x) \sim U(0,1)$

$$\frac{2}{\pi}\arcsin(\sqrt{x}) \sim U(0,1)$$