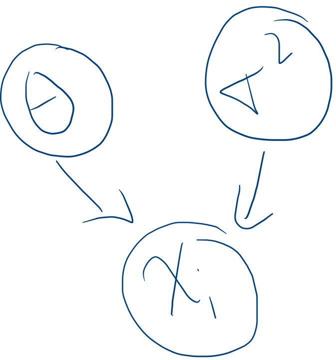
Primer modelo:



$$X_i \sim N(\theta, \sigma^2) \qquad i = 1, \dots, n$$

$$\theta \sim N(\theta_0, \tau^2)$$

$$\sigma^2 \sim IG(a, b) \qquad f(\sigma^2) = \frac{b^a}{\Gamma(a)} \left(\frac{1}{\sigma^2}\right)^{a+1} e^{-\frac{b}{\sigma^2}}$$

$$x = (x_1, \dots, x_n), \quad \theta \perp \sigma^2$$

$$f(\theta, \sigma^2 | x) \propto f(x | \theta, \sigma^2) f(\theta, \sigma^2)$$

$$\propto f(x | \theta, \sigma^2) f(\theta) f(\sigma^2)$$

$$\propto \left[\left(\frac{1}{\sigma^2}\right)^{n/2} e^{-\sum_{i=1}^{n} \frac{(x_i - \theta)^2}{2\sigma^2}} \right] \times \left[\frac{1}{\tau} e^{-\frac{(\theta - \theta_0)^2}{2\tau^2}} \right] \times \left[\frac{1}{(\sigma^2)^{a+1}} e^{-\frac{b}{\sigma^2}} \right]$$

$$f(\theta, \sigma^2 \mid x) \propto \left[\frac{1}{(\sigma^2)^{n/2}} e^{-\sum_{i=1}^{n} \frac{(x_i - \theta)^2}{2\sigma^2}} \right]$$

$$\times \left[\frac{1}{\tau} e^{-\frac{(\theta - \theta_0)^2}{2\tau^2}} \right]$$

$$\times \left[\frac{1}{(\sigma^2)^{a+1}} e^{-\frac{b}{\sigma^2}} \right]$$

$$\pi(\theta \mid x, \sigma^2) \propto e^{-\sum_{i=1}^{n} \frac{(x_i - \theta)^2}{2\sigma^2}} \times e^{-\frac{(\theta - \theta_0)^2}{2\tau^2}}$$

$$\sum_{i=1}^{n} \frac{(x_i-\theta)^2}{\sigma^2} + \frac{(\theta-\theta_0)^2}{\tau^2}$$

$$= \sum_{i=1}^{n} \frac{x_i^2 - 2x_i\theta + \theta^2}{\sigma^2} + \frac{\theta^2 - 2\theta\theta_0 + \theta_0^2}{\tau^2}$$

$$= C_0 + \frac{n\theta^2 - 2\theta\sum_{i=1}^{n} x_i}{\sigma^2} + \frac{\theta^2 - 2\theta\theta_0}{\tau^2}$$

$$= C_0 + \frac{n\theta^2 - 2\theta n\bar{x}}{\sigma^2} + \frac{\theta^2 - 2\theta\theta_0}{\tau^2}$$

$$= C_0 + \frac{n\tau^2\theta^2 - 2\theta n\tau^2\bar{x} + \sigma^2\theta^2 - 2\theta\sigma^2\theta_0}{\sigma^2\tau^2}$$

$$= C_0 + \frac{\theta^2(n\tau^2+\sigma^2) - 2\theta(n\tau^2\bar{x} + \sigma^2\theta_0)}{\sigma^2\tau^2}$$

$$= C_0 + \frac{(n\tau^2+\sigma^2)\left[\theta^2 - 2\theta\left(\frac{n\tau^2}{n\tau^2+\sigma^2} + \frac{\sigma^2}{n\tau^2+\sigma^2}\theta_0\right)\right]}{\sigma^2\tau^2}$$

$$\pi(\theta \mid x, \sigma^2) \propto e^{-\frac{(\theta - \mu_n)^2}{\frac{\sigma^2 \tau^2}{\beta_n}}}$$

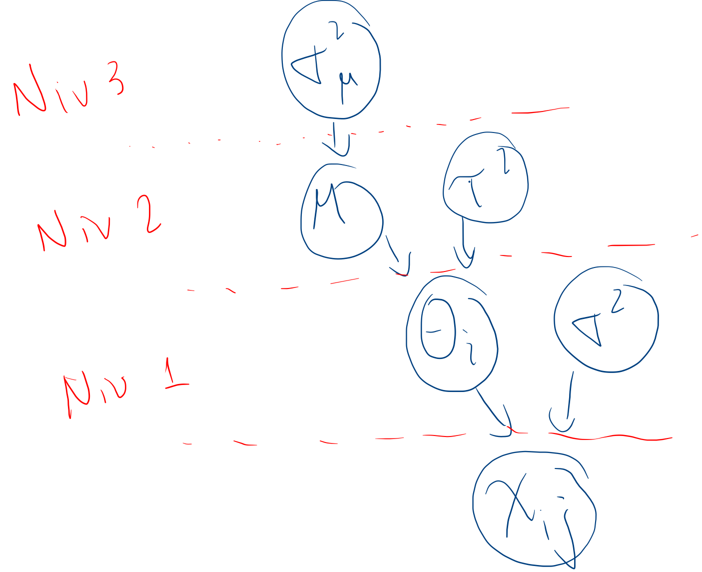
$$\mu_n = \left(1 - \frac{\sigma^2}{\beta_n}\right)\bar{x} + \frac{\sigma^2}{\beta_n}\theta_0$$

$$\beta_n = \sigma^2 + n\tau^2$$

$$\pi(\sigma^2 \mid x, \theta) \propto \frac{1}{(\sigma^2)^{\frac{n}{2}+a+1}} e^{-\sum_{i=1}^{n}\frac{(x_i-\theta)^2}{2\sigma^2}} e^{-\frac{b}{\sigma^2}}$$

$$= \frac{1}{(\sigma^2)^{\frac{n}{2}+a+1}} e^{-\frac{1}{\sigma^2}\left[\frac{1}{2}\sum_{i=1}^{n}(x_i-\theta)^2 + b\right]}$$

$$\sigma^2 \mid x, \theta \sim IG\left(\frac{n}{2}+a, \frac{1}{2}\sum_{i=1}^{n}(x_i-\theta)^2 + b\right)$$



$$
\begin{aligned}
X_{ij} &\sim \mathcal{N}(\theta_i, \sigma^2), \quad i = 1, \ldots, k, \quad j = 1, \ldots, n_i, \\
\theta_i &\sim \mathcal{N}(\mu, \tau^2), \quad i = 1, \ldots, k, \\
\mu &\sim \mathcal{N}(\mu_0, \sigma_\mu^2), \\
\sigma^2 &\sim \mathcal{IG}(a_1, b_1), \quad \tau^2 \sim \mathcal{IG}(a_2, b_2), \quad \sigma_\mu^2 \sim \mathcal{IG}(a_3, b_3).
\end{aligned}
$$

$$
\begin{aligned}
n &= \textstyle\sum_i n_i \\
\bar{\theta} &= \textstyle\sum_i n_i \theta_i / n.
\end{aligned}
$$

$$
\begin{aligned}
\theta_i \mid \ldots &\sim \mathcal{N}\left(\frac{\sigma^2}{\sigma^2 + n_i\tau^2}\mu + \frac{n_i\tau^2}{\sigma^2 + n_i\tau^2}\bar{X}_i, \ \frac{\sigma^2\tau^2}{\sigma^2 + n_i\tau^2}\right), \quad i = 1, \ldots, k, \\
\mu \mid \ldots &\sim \mathcal{N}\left(\frac{\tau^2}{\tau^2 + k\sigma_\mu^2}\mu_0 + \frac{k\sigma_\mu^2}{\tau^2 + k\sigma_\mu^2}\bar{\theta}, \ \frac{\sigma_\mu^2\tau^2}{\tau^2 + k\sigma_\mu^2}\right),
\end{aligned}
$$

$$
\begin{aligned}
\sigma^2 \mid \cdots \quad &\sim \mathcal{IG}\left(n/2 + a_1,\ (1/2)\sum_{ij}(X_{ij} - \theta_i)^2 + b_1\right), \\
\tau^2 \mid \cdots \quad &\sim \mathcal{IG}\left(k/2 + a_2,\ (1/2)\sum_{i}(\theta_i - \mu)^2 + b_2\right), \\
\sigma_\mu^2 \mid \cdots \quad &\sim \mathcal{IG}\left(1/2 + a_3,\ (1/2)(\mu - \mu_0)^2 + b_3\right),
\end{aligned}
$$