Def: Sea $X=(X_1,\dots,X_d)$, F_i continuas

$$F_i(x_i) \sim U(0,1)$$

$$U=(U_1,\dots,U_d)=(F_1(X_1),\dots,F_d(X_d))$$

$u\in[0,1]^d$

$$(x_1,\dots,x_d)=(F_1^{-1}(u_1),\dots,F_d^{-1}(u_d))$$

$$C(u_1,\dots,u_d)=\mathbb{P}(U_1\le u_1,\dots,U_d\le u_d)$$

$$=\mathbb{P}(X_1\le F_1^{-1}(u_1),\dots,X_d\le F_d^{-1}(u_d))$$

$$f(x_1, \ldots, x_d) = c(u_1, \ldots, u_d)\, f_1(x_1) \cdots f_d(x_d)$$

Para el ejercicio de Alan,

$$f(x_1, x_2) = c(u_1, u_2) \cdot f_1(x_1) \cdot f_2(x_2)$$

Vamos a considerar una cópula de Gumbel de parámetro $\theta > 1$

$$C(u,v) = \exp\left(-\left((-\ln u)^{\theta} + (-\ln v)^{\theta}\right)^{\frac{1}{\theta}}\right)$$

$$h(u,v) = -\left((-\ln u)^{\theta} + (-\ln v)^{\theta}\right)^{\frac{1}{\theta}}$$

Así, tenemos que $C(u,v) = \exp(f(u,v))$

$$\frac{\partial h(u,v)}{\partial u} = -\frac{1}{\theta} h(u,v)^{1-\theta}\, \theta\, (-\ln u)^{\theta-1} \left(-\frac{1}{u}\right)$$

$$= h(u,v)^{1-\theta} \frac{(-\ln u)^{\theta-1}}{u}$$

$$\frac{\partial h(u,v)}{\partial v} = -\frac{1}{\theta} h(u,v)^{1-\theta} \theta (-\ln v)^{\theta-1} \left(-\frac{1}{v}\right)$$

$$= h(u,v)^{1-\theta} \frac{(-\ln v)^{\theta-1}}{v}$$

$$\frac{\partial C(u,v)}{\partial u} = \frac{\partial}{\partial u} \exp(h(u,v))$$

$$= \exp(h(u,v))\ h(u,v)^{1-\theta} \frac{(-\ln u)^{\theta-1}}{u}$$

$$\frac{\partial C(u,v)}{\partial v} = \frac{\partial}{\partial u} \exp(h(u,v))$$

$$= \exp(h(u,v))\, h(u,v)^{1-\theta} \frac{(-\ln v)^{\theta-1}}{v}$$

$$c(u,v) = \frac{\partial^2}{\partial u \partial v} C(u,v) = \frac{\partial^2}{\partial v \partial u} C(u,v)$$

$$= \frac{\partial}{\partial v}\left[C(u,v)\, h(u,v)^{1-\theta} \frac{(-\ln u)^{\theta-1}}{u} \right]$$

$$= \frac{(-\ln u)^{\theta-1}}{u} \frac{\partial}{\partial v}\left[C(u,v)\, h(u,v)^{1-\theta} \right]$$

$$= \frac{(-\ln u)^{\theta-1}}{u} \frac{\partial}{\partial v}\left[C(u,v)\, h(u,v)^{1-\theta} \right]$$

$$\vdots$$

$$c(u,v) = \frac{(\ln u \cdot \ln v)^{\theta-1}}{uv} C(u,v)\, h(u,v)^{1-2\theta} \left[h(u,v) + (1-\theta) \right]$$

Ahora, si $X_1 \sim \log Norm(0, 0.25)$

$X_2 \sim exp(1, 10)$

$$Sop((X_1, X_2)) = (0, \infty) \times (10, \infty)$$

$$g: \mathbb{R}^2 \longrightarrow I \times J$$

$$(z_1, z_2) \longmapsto (g_1(z_1), g_2(z_2))$$

$$g_1(z_1) = exp(z_1)$$

$$g_2(z_2) = exp(x) + 10$$

$$\tilde{q}_K(z \mid w) = \text{Normal Multivariada}(z \mid w, \sigma^2 I_2)$$

$$q_K(y \mid X) = \tilde{q}_K(\overset{=z}{g^{-1}(y)} \mid g^{-1}(X))\,|J|$$

$$|J| = \begin{vmatrix} \dfrac{\partial g_1^{-1}(y_1)}{\partial y_1} & \boxed{\cancel{\dfrac{\partial g_1^{-1}(y_2)}{\partial y_2}}}^{\to 0} \\ \boxed{\cancel{\dfrac{\partial g_2^{-1}(y_1)}{\partial y_1}}}^{\to 0} & \dfrac{\partial g_2^{-1}(y_2)}{\partial y_2} \end{vmatrix} = \frac{g_1^{-1}(y)}{\partial y_1} \cdot \frac{g_2^{-1}(y)}{\partial y_2}$$

$$g^{-1}(y) = z$$

$$y = g(z)$$