

$$\theta \sim \text{Beta}(\alpha, \beta)$$

$$y|\theta \sim \text{Bernoulli}(\theta)$$

$$f_{\theta}(\theta) = \frac{\theta^{\alpha-1} (1-\theta)^{\beta-1}}{B(\alpha, \beta)} \quad B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$f(y|\theta) = \theta^y (1-\theta)^{1-y}$$

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \propto p(y|\theta)p(\theta)$$

$$= \theta^y (1-\theta)^{1-y} \frac{\theta^{\alpha-1} (1-\theta)^{\beta-1}}{B(\alpha, \beta)}$$

$$\propto \theta^{y+\alpha-1} (1-\theta)^{1-y+\beta-1}$$

$$\theta|y \sim \text{Beta}(y+\alpha, 1-y+\beta)$$

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \propto p(y|\theta)p(\theta)$$

$$= \prod_{i=1}^n p(y_i|\theta) p(\theta)$$

$$= \theta^{\sum_{i=1}^n y_i} (1-\theta)^{n-\sum_{i=1}^n y_i} \theta^{\alpha-1} (1-\theta)^{\beta-1}$$

$$y = (y_1, \dots, y_n)$$

$$\theta|y \sim \text{Beta}(\alpha', \beta')$$

$$\alpha' = \sum_{i=1}^n y_i + \alpha$$

$$\beta' = n - \sum_{i=1}^n y_i + \beta$$

$$y|\theta \sim \text{Bernoulli}(\theta)$$

Nivel 1  $\rightarrow \theta \sim \text{Beta}(\alpha, \beta)$

$$\alpha \sim \text{Gamma}(a, b)$$

