
THE OUT-OF-SAMPLE FAILURE OF EMPIRICAL EXCHANGE RATE MODELS – A COINTEGRATION PERSPECTIVE.

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ABSTRACT

This study reports an estimation of the out-of-sample performance of univariate and multivariate monetary models contributing to the analysis of the Meese Rogoff paradox for the empirical CHF/USD, GBP/USD and YEN/USD series. 20 years of monthly observations and five models are analysed. In line with the academic literature we find a random walk model to be the best univariate model to capture the unit root behaviour of FX-rates and to forecast the latter out-of-sample. Nonetheless, in contrast with the general academic literature we do not find multivariate models to underperform a random walk model when forecasting FX-rates out of sample. Finally, we find that a vector error correction model representing the cointegration evidence among macroeconomics fundamentals and FX-rates outperforms a random walk model for all the country series and estimation lags.

Keywords FX-rates · Monetary Models · Meese- Rogoff Puzzle · Cointegration · VECM

1 Introduction

Many approaches tried to model FX-rates since the start of a free-floating system after Bretton Woods in the 1970s. Monetary models, linking macroeconomics variables to FX-rates were the first to emerge and due to their compelling theoretical intuition soon became the reference in the field. Nonetheless, the enthusiasm for such elegant models was diminished by two seminal works. Meese and Rogoff (1983a, [1]) and Meese and Rogoff (1983b [2]) demonstrated how none of the different monetary models could beat a random walk without a drift in modeling FX-rates out of sample. After the evidence was presented the opinion divided into four distinct diverging groups. Some argued for a sample issue and confined the problem to the specific historical period, others argued for the limit of the linear models used by the authors to approach monetary models, others argued for a fail of monetary models and their underlying assumptions

and, finally, others started to approach the issue in an innovative way turning towards micro-based models that aim to capture the complexity of market information asymmetries and investors heterogeneity ¹.

While all the above-mentioned different reactions contributed to a further understanding of FX-rates properties we question whether the runaway from linear monetary models is justified given the methodological approach presented in the studies of Meese and Rogoff, the small sample of use and the lack of a cointegration analysis later formalized by Granger and Engle (1987, [4]). This methodological study stands therefore as a revision of the out of sample failure of foreign exchange rates testing whether macroeconomics variables are indeed incapable of explaining FX-rates movements before exploring more nuanced fields such as nonlinear and micro-based modeling. The hope is therefore to avoid over-engineering and over-complication of the issue going back to the fundamentals.

Specifically, our study aims to explore the above at a monthly lag frequency on a self-developed data set for the recent period ranging from 1986 to 2006. This period allows for a consistent estimation of all of the necessary parameters given the chosen models. Moreover, it avoids modeling the most recent financial crises where structural breaks are present in the series.

As in Meese and Rogoff (1983a and 1983b, [1, 2]) we explore the out of sample forecasting performance of foreign exchange rates for three major monetary models. Firstly, the monetary models presented by Frenkel (1976, [5]) and Mussa (1976, [6]) claiming a link among monetary mass differentials, interest rates differentials, output gap and FX-rates. Secondly the sticky price model where prices adjust sluggishly to macroeconomics shocks, exchange rates overshoot and inflation rates differential determine the long run behaviour of FX-rates (Dornbush 1976, [7]). Finally, a portfolio balance model claiming for the importance of monetary flows and net current account differentials as a key determinant for FX-rates changes as in Hooper and Morton (1982, [8]).

Based on such monetary models five different stochastic models are analyzed in order to forecast FX-rates. Three of them are presented in the seminal work of Meese and Rogoff (1983a, [1]) and represent the benchmark to verify whether the results of the paper hold for the time frame of interest. These models are a simple random walk forecast, an OLS estimation without inclusion of lagged terms and a vector autoregressive model. On the top of these models we analyze the performance of transfer models discussed in Montgomery and Weatherby (1980, [9]) and the vector error correction model representation of the cointegration relationship between macroeconomic variables and FX-rates proposed by Granger and Engle (1987, [4]).

If our results are in line with the seminal papers of Meese and Rogoff in the case of univariate models, the picture resulting from a multivariate fit contradicts the results of the reference papers. We do not find enough evidence to support the claim of an underperformance of multivariate linear models in comparison to a simple random walk model. Moreover, we find a strong evidence for cointegration among macroeconomic series and FX-rates for all of the analyzed country series. This suggests a long term stable relation between the fundamentals and the FX-rates rejecting the claim of Meese and Rogoff about a weak relation between the two. Finally, we do find that a vector error correction model representation of the cointegration relation between macroeconomic series and FX-rates outperforms the out-of-sample forecast of FX-rates at all lags and for all of the country series. This proof further strengthen the hypothesis that a link between monetary models and FX-rates does indeed exist at monthly lag frequency given a long enough sample period that allows for a reliable multivariate model estimation.

¹See Balliu and King (2005, [3]) for a general survey.

The paper continues as follows. Section 2 outlines the methodological approach and describes the techniques used for the empirical analysis and the out-of-sample forecast. Section 3 continues by introducing the dataset used and the chosen proxies to capture the macroeconomic fundamentals of interest, Section 4 reports the main results of the empirical analysis and Section 5 concludes the findings.

2 Methodology

2.1 Structural Model

The basic structural model encompassing all of the different monetary models discussed in the introductory session is of the following form:

$$s = \beta_0 + \beta_1(m - m^*) + \beta_2(y - y^*) + \beta_3(r - r^*) + \beta_4(\pi - \pi^*) + \beta_5(TB - TB^*) + \varepsilon \quad (1)$$

where s represents the logarithm of the indirect quote of FX-rates, i.e. the foreign exchange value of a dollar unit, $m - m^*$ represents the logarithm of the U.S. and foreign country money mass supply ratio, $y - y^*$ represents the logarithm of the U.S. and foreign GDP level ratio, $r - r^*$ represents the short term U.S./foreign country interest rate differential, $\pi - \pi^*$ represents the U.S./foreign country inflation rate differential, $TB - TB^*$ represents the U.S. foreign country current account differential. Finally, ε represents the zero expectation error term capturing all of the other factors not expressed in the model.

The model above encompasses all of the monetary models previously discussed. Specifically, the most basic monetary model, the Frenkel-Bilson model logically inferred from the relative PPP proposition, assumes $\beta_4 = \beta_5 = 0$. The Dornbusch model that allows for a sluggish price adjustment behaviour and predicts FX-overshooting assumes $\beta_5 = 0$. Finally the Hooper-Morton model poses no restrictions on the coefficients of equation 1. Such a structural model is consequently estimated on four different parametric models of interest, which will be discussed next.

2.2 Univariate Models

The most basic model that is tested is an OLS model as in the benchmark papers of Meese and Rogoff (1983a, 1983b, [1, 2]). In this model the coefficients of equation 1 are computed without taking into account any lagged effect. On the other hand, the second more flexible model is modeled to capture possible lagged effects of the macroeconomics fundamentals. Opposed to the Meese and Rogoff papers where the authors decided to capture the possibility of lagged terms by incorporating autoregressive models giving a higher importance on more recent observations we decided to apply the transfer function models widely spread in the fields of engineering such as control systems and electronic circuits. These models have been poorly discussed in the field of economics with the exception of Tustin (1957, [10]) that tried to make the point for applying the models in the economic modeling field.

Transfer function models relates a given set of inputs to an output variable through the following general formula

$$Y_t = \mu + \frac{(\omega_0 + \omega_1 B^1 + \dots + \omega_s B^s)}{1 - \delta_1 B^1 - \dots - \delta_r B^r} X_{t-b} + \varepsilon_t \quad (2)$$

where X represent a matrix of exogenous terms, μ the optional term modeling the mean of the series and ε the non-captured variation in the series. Moreover the above ratio represents the transform function of the regressors matrix and is especially characterized by the order of the denominator and nominator terms.

r , the order of the denominator term, expresses the rate of the decay pattern, where a higher term indicates a slower decay. s , the order of the nominator term, expresses the persistence of so called unpatterned spikes, that is the persistence of effects that are not captured in the decay pattern. Finally, the b term in the matrix of input represents the dead time, that is the time it takes for the dependent variable to react to some changes in the input matrix. This is of primary importance as it might very well be that some shift in macroeconomic fundamentals just begins to display effects after a certain amount of time when the economic agents begin to perceive the change.

Due to the flexible nature of the latter model, we believe that it is better suited and preferable to capture the true nature of distributed lags present in the structural model of equation 1 in comparison with the smoothed autoregressive model, applied by Meese and Rogoff, with the arbitrarily chosen smoothing term of 0.95 (See Meese and Rogoff (1983a pp. 7, [1])).

2.3 Multivariate Models

The above discussed models rely on the exogeneity of the independent variables. If the assumption fails, the model are biased and lead to misleading conclusion as the independence of the sample distribution is not guaranteed. In practice this poses an issue for the estimation of the structural model of equation 1.

While some variables as the monetary mass and the output gap are commonly treated as exogenous variables in the underlying monetary models the practice suggests that such macroeconomic variables might very well be influenced by the movements of FX-rates. On the top of it other variables such as short-term interest rates differentials are treated as endogenous even in the underlying monetary models and therefore require a different estimation compared to the one outlined in the previous section.

To obviate the above issues Meese and Rogoff (1983a and 1983b, [1, 2]) analyzed the structural model of interest through a vector autoregressive model firstly introduced by Sims (1980, [11]). In the general case the model consists of a system of equations in the form

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \cdots + \Phi_n y_{t-n} + \varepsilon_t \quad (3)$$

where y is a vector containing the endogenous variables of interest, Φ_0 is a vector of constants, Φ_1, \dots, Φ_n are matrices describing the effect of lagged endogenous variables on the levels of the current variables and ε captures the equations specific error term. The resulting model will capture the endogeneity present in the monetary structural model allowing for a consistent OLS estimation as far as the error terms in the equations are uncorrelated.

Despite the described vector autoregressive model manages well to model the endogeneity of macroeconomic variables we question whether the restricted form of it could yield more efficient and more reliable estimates capturing the FX-rates and the monetary models relation. This is especially motivated by the cointegration theory developed by Engle and Granger (1987, [4]) and the well known evidence of non stationary macroeconomics times series ². Moreover,

²See for instance Gil-Alana and Robinson (1997, [12])

the importance of such method is underlined by the evidence of Phillips (1986, [13]) that set down the theoretical fundamentals showing how parameter estimates of cointegrated series do not converge in probability and do not converge to any non-degenerate distribution in the asymptotic case if the case of a misspecified OLS estimate as potentially is equation 3.

We propose therefore a test for cointegration among the macroeconomic and the FX-rates series based on Johansen (1991, [14]) and we consequently estimate a vector error correction model of the form

$$\begin{aligned}\Delta y_t &= \Pi y_{t-1} + \sum_{i=1}^{p-1} \Phi^* \Delta y_{t-i} + \varepsilon_t \\ \Phi^* &= - \sum_{i=j+1}^p \Phi_i, \quad j = 1, \dots, p-1 \\ \Pi &= - (I - \Phi_1 - \dots - \Phi_p)\end{aligned}\tag{4}$$

where Πy_{t-1} of equation 4 represent the error correction term and y_i and Φ refer to the variables described in the unrestricted vector autoregressive model of 3.

2.4 Forecasting Approach

The four described models and a random walk model without drift are validated by looking at their ability to forecast FX-rates out of sample. In this sense the 234 observations sample is split in a training and a validation sample. Three fourth of the total observations are training the four models while the remaining fourth of the observations is used for the out of sample validation.

In comparison to Meese and Rogoff (1983b, [2]) we do not try any restricted estimation based on the theoretical monetary models literature. We rather estimate unrestricted versions for all of the models outlined with the exception of the vector error correction model described in 4, given the by product VAR model restriction imposed by the latter.

To select the best performing monetary model we use a two step approach. Firstly, we estimate the four parametric models according to the most general structural model described in 1. Secondly we perform Wald tests to check whether there is in sample evidence for using restricted models. Based on positive evidence we proceed by running restricted models out of sample and look for an increased performance.

The out of sample performance of the different models is measured by applying a rolling out-of-sample forecast in analogy to the benchmark papers. This consists of a re-estimation of the four outlined models for each new forecasting point. Given the decision to estimate the out of sample model performance at one, three, six and twelve months lags the rolling forecast technique involves a model reestimation at respective frequencies. Important is to underline how the results of the Wald tests are extended for each of the subsequent model estimation in the rolling forecast. This means that given the statistically significant evidence for the Null of a restricted model in the first three fourth of the sample the same restricted model is used for the subsequent model estimations.

Explicitly, our approach intends to fit a structural model containing all of the macroeconomics series in the first 176 observations - from September 1986 to April 2001 - and perform Wald tests to verify whether a restricted monetary

model is supported in the sample. Based on such results the rolling forecast method intends to estimate the next point forecast at 1, 3, 6 and 12 month lags and shift the data sample of one period such that a new model is estimated for the October 1986 to May 2001 series. According to the new parametric fit, point forecast at 1, 3, 6 and 12 months are estimated and the method iterated until the last out of sample observation for February 2006 is reached.

Finally, it is important to underline that as in the benchmark papers, we allow the univariate models described in 2.2 a richer set of information compared to the random walk. Specifically, the random walk and the multivariate model use the $\mathcal{F}_{t-1} = F_0, \dots, F_{t-1}$ information set, where F_i represents a set containing all of the available information at timepoint i . By contrast the univariate models dispose of $\mathcal{F}_t \setminus s_t$, where s_t represents the FX-rate at time point t . In simple terms this means that we are going to give the univariate models the advantage of using the actual realizations of macroeconomics variables without the need to estimate them, therefore outlying the possibility of poor out-of-sample fit due to poor macroeconomic fundamentals estimation.

Based on the obtained rolling forecasts three statistics are computed to compare the model fit. These are the root mean squared error (RMSE), the mean absolute error (MAE) and the mean directional accuracy (MDA). A particular importance is assigned to the MAE results given Westerfield (1977, [15]) that analyzed the empirical exchange rates distribution finding evidence for FX-rates non-normal stable-Paretian distributions with infinite variance.

Opposed to Meese and Rogoff (1983a and 1983b, [1, 2]) we decided to further explore the point estimators of the above mentioned statistics by computing a model confidence set (MCS) as described in Hansen et al. (2011, [16]). The idea of the latter consists of a procedure yielding a model set, \mathcal{M}^* , built to contain the best model with a chosen level of confidence. The exact procedure is based on an equivalence test δ_m and an elimination rule ϵ_m , consistent with the chosen confidence level. In a first step the competing models are compared with each other. If δ_m does not support evidence for the equal performance of the models, ϵ_m is applied, the poorly performing models are discarded, and the general problem reiterated until δ_m is accepted for all of the surviving models.

In this paper we decided to make use of superior predictive ability test of Hansen (2005, [17]) and to obtain p-values for the equal predictive hypothesis of models according to a bootstrap implementation outlined by Hansen et al. (2011, [16]).

3 Dataset

The dataset, comprehensively discussed in Appendix A 6 consists of monthly observations over 21 years ranging from January 1986 to January 2006. All the time series of use are selected in accordance with the underlying monetary structural model 1 and are consistent with the series first utilized by Meese and Rogoff (1983a, [1]). Three different FX-rates series are analyzed, namely the JPN/USD rate, the GBP/USD rate and the CHF/USD rate. With respect to the independent variables of the structural model, we decided to work with the 3-months treasury bills to capture the short term interest rates differentials. The only exception were of CH series where we worked with the 3-month US-CH LIBOR spread due to missing reported publicly available data for the seasonally unadjusted short term treasury rates at monthly lag.

For measuring money mass differentials, we worked with the M1 and M3 measures testing the both given the important difference among the two and the lack of theoretical models to favour one measure above the other. We obtained stronger results for the M3 series and therefore proceeded to report the results obtained through such measure in the

paper.

For measuring the inflation rate differentials important for the Dornbusch model we worked with consumer price indices and for measuring the trade balance we worked with the net trade amount for goods as a proxy for the current account balance. This trade balance is indeed just a rough approximation given the fraction of traded goods in comparison to the total amount of trade that includes services and monetary transfers. Nonetheless we preferred such measure from an interpolation of the quarterly published current account balances.

Finally, we worked with the unemployment rate to capture the output differential of the economies as we believe that such variable reflects well the fluctuations in output levels, i.e. the quantity of interest, leaving the constant term in equation 1 to capture the other information present in the output measure such as the size of the economies.

As in our reference papers we decided not to work with seasonal time series in order to avoid the possible bias introduced by different seasonal adjustments to the structural parameters documented in Sims (1973a and 1973b, [18, 19]). Henceforth we decided to approach our model validation by fitting the models on seasonal adjusted and detrended time series.

With respect to the time series adjustments we proceeded by detrending the series exploring three different possibilities. Firstly, detrending via differentiation, secondly detrending through a linear time trend and thirdly detrending through a moving average filter. Detrending through differencing proved to be the most effective due to the quadratic behaviour displayed by the series. This approach was therefore applied to all of the series. The obtained mean stationary series were consequently inspected for the presence of seasonality and appropriately differentiated to the point where no significance was found for seasonal unit roots according to the methods proposed in Canova and Hansen (1995, [20]) and in Wang and Smith (2006, [21]).

4 Empirical Results

4.1 Univariate Models

The results for the univariate model fit are presented in table 1. For all of the univariate models described in section 2.2 a Wald test assessing the empirical evidence for a restricted structural model is computed. We find that the joint Null of the Frenkel-Bilson model cannot be rejected on a 1% confidence level for the simple OLS modeling of FX-rates. We proceeded therefore to estimate the out of sample performance for the OLS applying both the most general structural model and the Frenkel-Bilson model.

For the transfer function model described in 2 the rate of decay, the persistence term and the dead lag were estimated at each model estimation in the rolling forecast. The optimal lags for the terms above were selected minimizing the Akaike information criteria (See Akaike (1998, [22])). We found a compelling evidence for modeling the differentiated series with one term in the nominator capturing the effect of unexpected shifts in the independent variables and two terms in the denominator capturing the general high persistency of FX-rates movements through time.

The results of table 1 confirm the one of the general literature where an OLS model vastly underperforms a random walk in the out of sample forecast of FX-rate at all tested lags.

Table 1: Mean Absolute Error – Univariate Models

Lag	Univariate Models			
	Model	Switzerland	United Kingdom	Japan
MAE 1	Random Walk	0.0193 (0.503)	0.0168 (0.506)	0.0162 (0.506)
	Transfer Function Model	0.0205 (0.119)	0.0176 (0.128)	0.0173 (0.228)
	OLS - unrestricted	0.0234 (0.031)	0.0211 (0.053)	0.0194 (0.192)
	OLS - restricted	0.1435 (0.000)	0.1352 (0.002)	0.0534 (0.000)
MAE 3	Random Walk	0.0390 (0.307)	0.0330 (0.093)	0.0291 (0.534)
	Transfer Function Model	0.0384 (0.499)	0.0341 (0.094)	0.0280 (0.111)
	OLS - unrestricted	0.0603 (0.133)	0.0443 (0.143)	0.0497 (0.000)
	OLS - restricted	0.1551 (0.000)	0.1389 (0.007)	0.0583 (0.000)
MAE 6	Random Walk	0.0580 (0.483)	0.0495 (0.785)	0.0439 (0.484)
	Transfer Function Model	0.0581 (0.468)	0.0498 (0.526)	0.0442 (0.396)
	OLS - unrestricted	0.0789 (0.003)	0.0531 (0.486)	0.0669 (0.000)
	OLS - restricted	0.1716 (0.000)	0.1446 (0.041)	0.0669 (0.000)
MAE 12	Random Walk	0.0964 (0.171)	0.0584 (0.726)	0.0753 (0.505)
	Transfer Function Model	0.0954 (0.489)	0.0598 (0.678)	0.0752 (0.147)
	OLS - unrestricted	0.1210 (0.000)	0.0744 (0.523)	0.1001 (0.000)
	OLS - restricted	0.2100 (0.000)	0.1660 (0.459)	0.0776 (0.000)

In contrast, transfer function models performed much better in modeling FX-rates out of sample. While the model still underperforms a random walk in forecasting the FX-rates of the countries of interest at 1 month lag, the evidence at higher lags is rather mixed and it seems the two models performs equally in forecasting FX-rates out of sample. This is confirmed by the superior predictive test of Hansen (2005, [17]) and the p-value reported in parenthesis in table 1 obtained by the bootstrap method of Hansen (2011, [16]). For all of the models the equal predictability hypothesis of the unrestricted and restricted OLS model is rejected with 5% confidence.

Two possible causes for such observations might exist. While on one hand the increased forecast performance at higher lags might be explained by the different information set provided to the two models as described in 2.4, on the other hand, the fact might be caused by an increasing importance of macroeconomic variables for explaining the long run behaviour of FX-rates movements.

Finally, the Null of equal predictability of the random walk model and the transfer function models in the superior predictability test could not be rejected with 10% confidence. This suggests the random walk model as the best

univariate model to fit FX-rates out of sample given its parsimonious computation power combined with a restricted information set compared to the transfer function model as discussed in 2.4.

We observe no systematic difference when looking at RMSE. The picture of the latter looks similar to the results obtained looking at the MAE at all lags and for all models. No particularly important outliers in forecast errors seem to be of significant importance in the analysis. The final word is given to the directional accuracy measure defined as the average number of times the sign of the realized FX-rate difference is matched by an equivalent sign in the FX-rate forecast difference. The values for such statistics lie between 0.5-0.7 for all the frequencies and there does not seem to be systematic differences among the models. Such realizations for the random walk model suggest that it is important to include certain degree of momentum for FX-rates in the models.

4.2 Multivariate Models

The evidence from the previous section together with the general empirical literature suggests the presence of a unit root in the FX-rates. This was further confirmed by augmented Dickey-Fuller tests (Dickey and Fuller (1979, [23])) applied to the FX-rates and macroeconomics variables suggesting the statistically significant presence of unit roots. Dickey-Fuller tests were estimated once again after differentiating the series. All of the series but the US-JP interest rates differential and the US-UK unemployment rate differential proved to be stationary after such adjustment, which supports an evidence for an integration order of 1 for most of the series.

While in the previous section the FX-rate unit root was proved to be best modeled by a simple random walk model this section further expands the analysis by looking at the performance of multivariate models. Those may capture and efficiently estimate the relation among macroeconomics variables and FX-rates in the case of endogeneity among the series.

While the reference paper of Meese and Rogoff (1983a and 1983b, [1, 2]) attempted to use a multivariate model through a vector autocorrelation model we will analyze the extent of cointegration among the series using the results of the previous analysis showing a higher order integration for the macroeconomic and FX-rates series.

In order to do that we tested the hypothesis of cointegration for the macroeconomics variables and the FX-rates taking into account the trend displayed by the times series. The results of this analysis are based on the trace statistic for the computed eigenvector discussed in Johansen (1991, [14]). Running the Johansen cointegration test we obtained the results presented in table 2. All of the series display a profound evidence for the presence of three cointegration vectors at 5% percent confidence level.

Given the five series of macroeconomic fundamentals it is possible that the three cointegration relations might exist just among such variables and that a long term relation between the macroeconomic fundamentals and the FX-rate for the three different series does not exists. In order to test such a hypothesis we decided to first run Johansen tests verifying for cointegration between the FX-rate and a single macroeconomic variable. We then iterated the process by gradually adding macroeconomic variables until the estimation of a model included all of the variables present in the structural model as in 1. The results in this case confirm the hypothesis of cointegration between FX-rates and the macroeconomic variables. For the period analyzed we found strong evidence for cointegration among the FX-rate and macroeconomic fundamentals for the UK-US series. In such a case the Null of no cointegration could be rejected with 5% confidence

for the FX-M3, FX-CPI, FX-Current Account, FX-interest rate series. The picture looks more fragmented in the case of the other series. In the case of the CH-US series evidence for cointegration is displayed between {M3, M1, Interest Rate} and the FX-rate respectively, while for the JP-USA series the Null of no cointegration can be rejected only for the FX-M3 series.

Table 2: Johansen Cointegration Test – Trace Statistics

		Trace Score	Quantiles Test Statistics		
Series			90%	95%	99%
Structural JP-US	Cointegrated Series ≤ 3	26.46	28.71	31.52	37.22
	Cointegrated Series ≤ 2	48.79	45.23	48.28	55.43
	Cointegrated Series ≤ 1	87.80	66.49	70.60	78.87
	Cointegrated Series $= 0$	138.18	85.18	90.39	104.20
Structural CH-US	Cointegrated Series ≤ 3	25.97	28.71	31.52	37.22
	Cointegrated Series ≤ 2	53.22	45.23	48.28	55.43
	Cointegrated Series ≤ 1	85.88	66.49	70.60	78.87
	Cointegrated Series $= 0$	145.12	85.18	90.39	104.20
Structural UK-US	Cointegrated Series ≤ 3	26.94	28.71	31.52	37.22
	Cointegrated Series ≤ 2	51.26	45.23	48.28	55.43
	Cointegrated Series ≤ 1	88.47	66.49	70.60	78.87
	Cointegrated Series $= 0$	178.10	85.18	90.39	104.20

Given the evidence of cointegration relations among FX-rates and macroeconomic fundamentals the Granger's theorem postulates the existence of a vector error correction model representation as in 4. We selected the optimal lagged terms of it by estimating three different information criteria for the model fit. Specifically, we computed the Akaike, Quinn-Hannan (See Hannan and Quinn (1979, [24]) and Schwarz (See Schwarz (1978, [25]) information criteria for the models with diverse lag terms and from parsimony reasons selected the minimum lag number identified by any of the three models above. An analogous approach was used for determining the optimal lag terms of the benchmark multivariate model – a vector autoregressive model in difference. Based on this approach multivariate models of term one, and with a smaller frequency also of term two, resulted in the model estimates of the rolling forecast method. This is an important result given the restricted sample size and the exponential increase of parameters in the number of lagged terms.

The results of the rolling forecast are presented in table 3. As in the case of the univariate fit and in line with the expectations the error increases in the estimation lag. In contrast to Meese and Rogoff (1983a and 1983b, [1, 2]) we do not find evidence for a general underperformance of the vector autoregressive models in predicting out of sample movements of foreign exchange rates in comparison to the simple random walk model. Especially in the short term when looking at the one month out of sample performance of vector autoregressive models we find that the this

marginally outperformed a random walk model. For higher term lags the evidence is rather mixed and the superiority of random walk models cannot be claimed. Moreover, important is to underline that in comparison to univariate models of 2.2, the multivariate models make use of the same information set \mathcal{F}_{t-1} as the random walk models.

One possible explanation for the striking difference between the results reported in this paper and the one presented in Meese and Rogoff (1983a and 1983b, [1, 2]) may be resulting from the different sample size used for the reported study compared to the one used by Meese and Rogoff. In this sense, Meese and Rogoff worked with a sample size of 87 observations, using as few as 37 observations for fitting their models (See Meese and Rogoff (1983a Section 3 and 1983b Section 3.2, [1, 2])). It is therefore highly likely that the multivariate results of their study suffer from overfitting issues leading the vector autoregressive model to perform poorly out of sample.

Table 3: Mean Absolute Error – Multivariate Models

Multivariate Models				
Lag	Model	Switzerland	United Kingdom	Japan
MAE 1	Random Walk	0.0193 (0.125)	0.0168 (0.177)	0.0162 (0.625)
	VAR	0.0183 (0.207)	0.0186 (0.178)	0.0160 (0.170)
	VECM	0.0176 (0.491)	0.0174 (0.476)	0.0153 (0.355)
MAE 3	Random Walk	0.0390 (0.392)	0.0330 (0.495)	0.0291 (0.529)
	VAR	0.0394 (0.359)	0.0333 (0.486)	0.0292 (0.488)
	VECM	0.0376 (0.624)	0.0328 (0.016)	0.0320 (0.770)
MAE 6	Random Walk	0.0580 (0.675)	0.0495 (0.081)	0.0439 (0.449)
	VAR	0.0578 (0.666)	0.0488 (0.504)	0.0418 (0.575)
	VECM	0.0580 (0.535)	0.0483 (0.355)	0.0434 (0.647)
MAE 12	Random Walk	0.0964 (0.258)	0.0584 (0.232)	0.0753 (0.458)
	VAR	0.0974 (0.165)	0.0603 (0.092)	0.0763 (0.267)
	VECM	0.0881 (0.507)	0.0542 (0.676)	0.0731 (0.573)

Turning to the vector error correction model of equation 4 that models the cointegration relation previously described we can see from table 3 that the rolling forecast of such models beats the random walk model fit for all of the countries and at all lags when measured in terms of mean absolute error.

While the vector error correction model successfully outperformed the simple random walk model when looking at the MAE statistics, no statistically significant evidence for the difference among the two models is found when modeling a model confidence set as in Hansen (2011, [16]). Looking at the p-value of the Null Hypothesis of equal predictability, that results from a bootstrapped superior ability tests reported in parenthesis of table 3, the results are clear. All of the three models are indistinguishable in terms of their out of sample performance across all of the estimation lags and series.

5 Conclusion

The reported results generally confirmed the conclusions presented by Meese and Rogoff (1983a and 1983b, [1, 2]). None of the four models analyzed could improve with statistical significance the out of sample performance of a simple random walk model without drift when forecasting foreign exchange rates. Moreover, we found evidence for a statistically significant underperformance of OLS models compared to a random walk model.

When looking at the point estimation of out of sample error statistics we observed a mixed evidence for the vector autoregressive models and the transfer function models. Both models could sporadically yield more accurate results compared to the random walk model but generally underperformed the latter.

Given the evidence for cointegration among macroeconomic variables and FX-rates we estimated a vector error correction model which displayed promising results, outperforming a random walk model when looking at out of sample forecasting error statistics. These results were consistent among all estimation lags and country series. Nonetheless they did not display enough evidence for rejecting the Null of equal predictability with a random walk model based on the interval estimation through the model confidence test of Hansen (2011, [16]).

Therefore, we can conclude that our study does support the strength of a random walk model in comparison to univariate models for FX-rates estimation. It is also generally sound model for capturing the unit root behaviour observed in the FX-rates movements. Nonetheless, our study vastly contrasted with the results of Meese and Rogoff papers when analyzing multivariate models. The latter models were able to well capture the movements of FX-rates through a parametric model based on macroeconomic fundamentals suggesting an estimation failure due to a small sample bias in the results reported by Meese and Rogoff (1983a and 1983b, [1, 2]). Moreover the cointegration between macroeconomic fundamentals and FX-rates suggests that a stable long run relation between the two exists.

Based on our results we reject the claim of weak relation among macroeconomic variables and foreign exchange rates and we claim that the macroeconomic fundamentals should be considered as a primary determining factor driving the foreign exchange rates movements at monthly lag.

Finally, we note how the successful performance of the random walk models is explained by the long memory displayed by foreign exchange series that is visible by analyzing autocorrelation plots for the series in virtually all time periods. In contrast to it, the comparable performance of macroeconomic models is explained by the long run relation and correlation among foreign exchange and macroeconomic variables and FX-rates. This is a remarkable achievement that leads us to claim for an underlying relation between fundamentals and FX-rates rejecting the exchange rate disconnect puzzle for the studied period.

6 Appendix A

Series: Unemployment rate.

Description: Unemployment Rate: Aged 15-64: All Persons.

Source: Main Economic Indicators - complete database, Main Economic Indicators (database), <http://dx.doi.org/10.1787/data-00052-en>, April 13, 2019.

Series: 3-month Treasury Bills USA, percent annum.

Description: 3-Month Treasury Bill - Secondary Market Rate

Source Board of Governors of the Federal Reserve System (US), 3-Month Treasury Bill: Secondary Market Rate [TB3MS], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/TB3MS>, April 13, 2019.

Series: Treasury Bills UK.

Description: Treasury Bills discount rate, percent per annum.

Source: Bank of England, Treasury Bill Discount Rate in the United Kingdom [TBDRUKM], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/TBDRUKM>, April 13, 2019.

Series: Treasury Bills JP.

Description: Interest Rates, Government Securities, Treasury Bills for Japan.

Source: International Monetary Fund, Interest Rates, Government Securities, Treasury Bills for Japan [INTGSTJPM193N], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/INTGSTJPM193N>, April 13, 2019.

Series: 3-month LIBOR.

Description: 3-Month or 90-day Rates and Yields: Interbank Rates

Source: Organization for Economic Co-operation and Development, Main Economic Indicators (database), <http://dx.doi.org/10.1787/data-00052-en>, April 13, 2019.

Series: M3 USA & JP.

Description: M3 comprises notes and coins in circulation outside banking corporations; demand and savings deposits, fixed and installment savings deposits, time deposits, and certificates of deposit of households, nonfinancial corporations, local governments, securities companies, Tanshi companies, and some other financial corporations such as securities finance companies with depository corporations in national and foreign currency; and nonresident deposits with banking corporations in national currency.

Source: International Monetary Fund, M3 for Japan, and US retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/MYAGM3JPM189N> & <https://fred.stlouisfed.org/series/MYAGM3USM052N>, April 13, 2019.

Series: M3 CHF & UK.

Description: M3 comprises notes and coins in circulation outside banking corporations; demand and savings deposits, fixed and installment savings deposits, time deposits, and certificates of deposit of households, nonfinancial corporations, local governments, securities companies, Tanshi companies, and some other financial corporations such as securities finance companies with depository corporations in national and foreign currency; and nonresident deposits with banking corporations in national currency.

Source: Organization for Economic Co-operation and Development, Main Economic Indicators (database), <http://dx.doi.org/10.1787/data-00052-en>, April 13, 2019.

Series: M1 UK & JP & CH.

Description: M1 consists of: (1) currency outside the U.S. Treasury, Federal Reserve Banks, and the vaults of depository institutions; (2) traveler's checks of nonbank issuers; (3) demand deposits; and (4) other checkable deposits (OCDs), which consist primarily of negotiable order of withdrawal (NOW) accounts at depository institutions and credit union share draft accounts.

Source: Organization for Economic Co-operation and Development, Main Economic Indicators (database), <http://dx.doi.org/10.1787/data-00052-en>, April 13, 2019.

Series: CPI.

Description: Consumer Price Index: All Items

Source: Organization for Economic Co-operation and Development, Main Economic Indicators (database), <http://dx.doi.org/10.1787/data-00052-en>, April 13, 2019.

Series: Trade.

Description: Net Trade. Value of Goods.

Source: Organization for Economic Co-operation and Development, Main Economic Indicators (database), <http://dx.doi.org/10.1787/data-00052-en>, April 13, 2019.

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