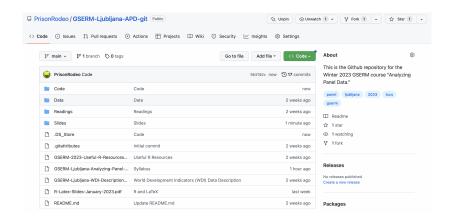
GSERM -Ljubljana 2023 Analyzing Panel Data

January 9, 2023

Preliminaries

- Instructor: Prof. Christopher Zorn (zorn@psu.edu).
- Class: January 9-13, 2023, 13:00-18:00 CET, at the University of Ljubljana (via Zoom).
- The course outline / syllabus is here.
- More important: The syllabus, slides, readings, code, data, etc. are all available on the course github repo (viewable at https://github.com/PrisonRodeo/GSERM-Ljubljana-APD-git).
- Class sessions will be recorded; links to those recordings will sent out via email to class participants.



Software

R

- All examples, plots, etc.
- Current version is 4.2.2
- Also be sure to get the RStudio IDE...
- The Github repo contains a bit of introductory code for people who may never have used R, and a list of R resources.
- A few of the primary packages we'll use include:
 - · plm
 - · 1me4
 - · gee

See the econometrics task view for more.

Stata

- Current version is 17
- Mostly use the -xt- series of commands (for "cross-sectional time series")



R version 4.2.2 (2022-10-31) -- "Innocent and Trusting" Copyright (C) 2022 The R Foundation for Statistical Computing Platform: aarch64-apple-darwin20 (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY. You are welcome to redistribute it under certain conditions. Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors. Type 'contributors()' for more information and 'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or 'help.start()' for an HTML browser interface to help.

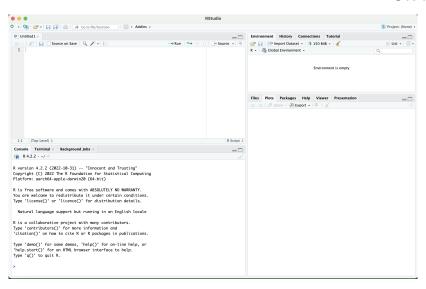
Type 'q()' to quit R.

[R.app GUI 1.79 (8160) aarch64-apple-darwin20]

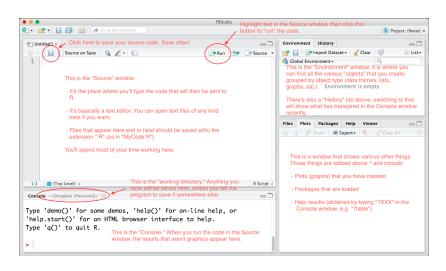
[History restored from /Users/cuz10/.Rapp.history]

>

RStudio



RStudio (annotated)



Starting Points

- <u>Panel</u> data: Data comprising repeated observations over time on a set of cross-sectional units.
- Terminology:
 - "Unit" / "Units" / "Units of observation" / "Panels" = Things we observe repeatedly
 - "Observations" = Each (one) measurement of a unit
 - "Time points" = When each observation on a unit is made
 - $i \in \{1...N\}$ indexes units
 - $t \in \{1...T\}$ or $\{1...T_i\}$ indexes observations / time points
 - If $T_i = T \ \forall i$ then we have "balanced" panels / units
 - Balanced panels also imply $N_t = N \ \forall t$
 - NT = Total number of observations (if balanced)
- "Panel" ≠ "Time Series"
- "Panel" \neq "Multilevel" / "Hierarchical" / etc.

More Terminology

 $N >> T \rightarrow$ "panel" data...

- (American) National Election Study panel studies $(N \approx 2000, T = 3)$
- Swiss Household Panel (FORS) (N = large, T = 21)
- Often:
 - · Cross-sectional units are a sample from a population
 - · T is (relatively) fixed

T>>N or $T\approx N \rightarrow$ "time-series cross-sectional" ("TSCS") data

- National OECD data (N=20 original members, $T\approx 70$)
- Often:
 - · N is an entire population, and is (relatively) fixed
 - · Asymptotics are in T

 $N=1 \rightarrow$ "time series" data

 $T=1 \rightarrow$ "cross-sectional" data

${\sf Panel\ Data\ Structure}\ +\ {\sf Organization}$

id	t	Y	X_1	
1	1	250	3.4	
1	2	290	3.3	
:	:	:	:	
2	1	160	4.7	
2	2	150	4.9	
:	:	:	:	
	•	•	•	•••

Introduction to Panel Variation: A Tiny (Fake) Example

	ID	Year	Female	Pres	GOP	Approve
1	1	2014	1	obama	0	4
2	1	2016	1	obama	0	5
3	1	2018	1	trump	0	2
4	1	2020	1	trump	0	1
5	2	2014	0	obama	1	2
6	2	2016	0	obama	1	1
7	2	2018	0	trump	1	4
8	2	2020	0	trump	1	3
9	3	2014	0	obama	1	2
10	3	2016	0	obama	1	2
11	3	2018	0	trump	1	4
12	3	2020	0	trump	0	1

Aggregation (means)

Cross-Sectional:

	ID	Year	${\tt Female}$	Pres	GOP	Approve
1	1	2017	1	?	0.00	3.00
2	2	2017	0	?	1.00	2.50
3	3	2017	0	?	0.75	2.25

Temporal:

	Year	${\tt Female}$	Pres	GOP	Approve
1	2014	0.333	obama	0.667	2.67
2	2016	0.333	obama	0.667	2.67
3	2018	0.333	trump	0.667	3.33
4	2020	0.333	trump	0.333	1.67

The Point

Aggregation:

- Always loses information
- Sometimes distorts relationships
- Occasionally forces arbitrary decisions

If you have variation in multiple dimensions, use it.

Two-Way Variation

Two "dimensions" of variation:

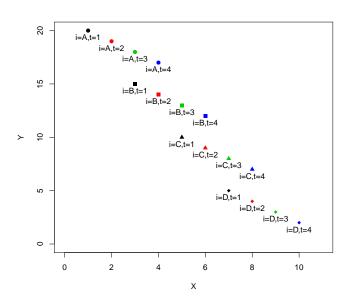
- <u>Unit-Level</u> Variation (how each unit is on average different from other units on average) a/k/a **between-unit** variation
- <u>Time-Level</u> Variation (how each measurement / time point is on average different from other time points on average) a/k/a/within-unit variation

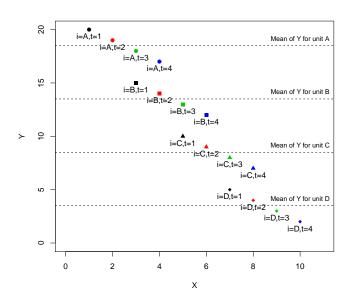
A random variable may:

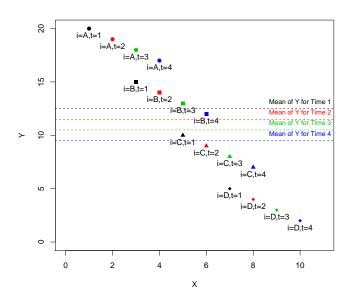
- Have only between-unit variation (i.e., lack temporal variation)
- Have only within-unit variation (i.e., lack cross-sectional variation)
- Have both within- and between-unit variation

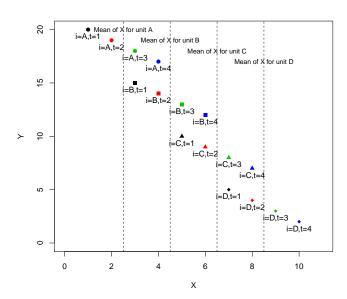
For Y_{it} , a variable that varies over both units and time:

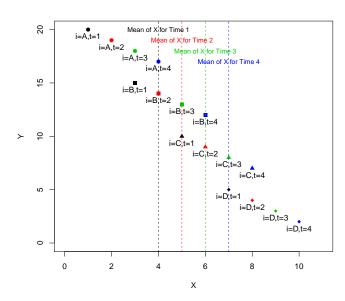
- $\bar{Y}_i = \frac{1}{T_i} \sum_{t=1}^{T} Y_{it}$ is the over-time mean of Y for unit i,
- $\bar{Y}_t = \frac{1}{N} \sum_{i=1}^{N} Y_{it}$ is the across-unit mean of Y at time t, and
- $\bar{Y} = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} Y_{it}$ is the grand mean of Y.











Within- and Between-Unit Variation

The within-unit mean of Y is:

$$\bar{Y}_i = \frac{1}{T_i} \sum_{t=1}^{T_i} Y_{it}$$

That means that we can write:

$$Y_{it} = \bar{Y}_i + (Y_{it} - \bar{Y}_i).$$

That is, the *total* variation in Y_{it} can be decomposed into:

- The between-unit variation in the \bar{Y}_i s, and
- The within-unit variation around \bar{Y}_i (that is, $Y_{it} \bar{Y}_i$).

Within- and Between-Unit Variation (continued)

Note that (while unusual) one could do a similar decomposition vis-à-vis time:

$$Y_{it} = \bar{Y}_t + (Y_{it} - \bar{Y}_t).$$

That is, the *total* variation in Y_{it} can be decomposed into:

- The temporal variation in the \bar{Y}_t s, and
- The within-time-point variation around \bar{Y}_t (that is, $Y_{it} \bar{Y}_t$).

In a similar fashion, we can also calculate the within- and between-unit variability (e.g., the standard deviations) of the constituent variables \bar{Y}_i and $(Y_{it} - \bar{Y}_i)$...

Variation ("Toy" Data from Above)

"Total" Variation:

```
> with(toy, describe(Y))
  vars n mean sd median trimmed mad min max range skew kurtosis se
X1 1 16 11 5.9 11 11 7.4 2 20 18 0 -1.5 1.5
```

"Between" Variation:

"Within" Variation:

Regression!

Model:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

...makes all the usual OLS assumptions, plus

- $\beta_{0i} = \beta_0 \forall is$
- $\beta_{1i} = \beta_1 \ \forall is$

For the model:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

...the same is true.

Variable Intercepts

Unit-specific intercepts:

$$Y_{it} = \beta_{0i} + \beta_1 X_{it} + u_{it} \tag{1}$$

Time-point-specific intercepts:

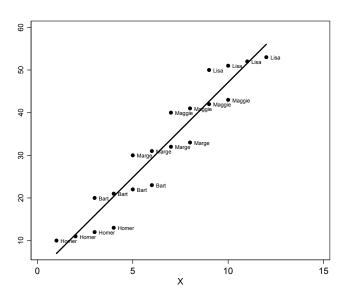
$$Y_{it} = \beta_{0t} + \beta_1 X_{it} + u_{it} \tag{2}$$

Both:

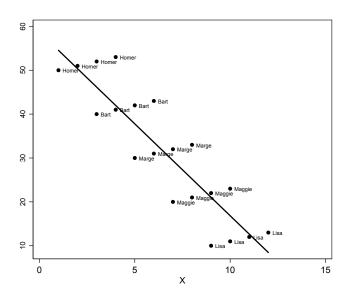
$$Y_{it} = \beta_{0it} + \beta_1 X_{it} + u_{it} \tag{3}$$

Note: Equation 3 is not identified (as written)!

Varying Intercepts



Varying Intercepts



Varying Slopes (+ Intercepts)

Unit-specific slopes:

$$Y_{it} = \beta_0 + \beta_{1i} X_{it} + u_{it} \tag{4}$$

(...one can also have time-point specific slopes, or both – again, the last of those is not identified as written.)

Unit-specific slopes + intercepts:

$$Y_{it} = \beta_{0i} + \beta_{1i} X_{it} + u_{it} \tag{5}$$

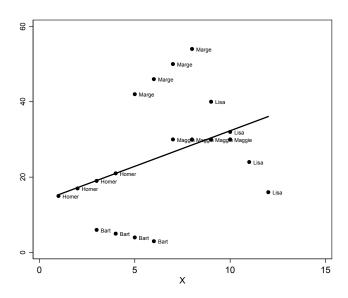
Time-point-specific slopes + intercepts:

$$Y_{it} = \beta_{0t} + \beta_{1t} X_{it} + u_{it} \tag{6}$$

Both...:

$$Y_{it} = \beta_{0it} + \beta_{1it} X_{it} + u_{it} \tag{7}$$

${\sf Varying\ Slopes}\,+\,{\sf Intercepts}$



The Error Term...

Usual OLS assumption:

$$u_{it} \sim \text{i.i.d.} N(0, \sigma^2) \ \forall \ i, t$$

or, equivalently:

$$\mathbf{u}\mathbf{u}'\sim\sigma^2\mathbf{I}$$

implies:

$$Var(u_{it}) = Var(u_{jt}) \ \forall \ i \neq j \ (i.e., no cross-unit heteroscedasticity)$$

 $Var(u_{it}) = Var(u_{is}) \ \forall \ t \neq s \ (i.e., no temporal heteroscedasticity)$
 $Cov(u_{it}, u_{is}) = 0 \ \forall \ i \neq j, \ \forall \ t \neq s \ (i.e., no auto- or spatial correlation)$

Pooling

Pooling pros:

- Adds data (→ precision)
- Enhances generalizability

BUT: fitting the model:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

Implies

- that the process governing the relationship between *X* and *Y* is exactly the same for each *i*,
- that the process governing the relationship between *X* and *Y* is the same for all *t*.
- that the process governing the us is the same $\forall i$ and t as well.

Q: When can we "pool" data on different units?

"Partial" Pooling (Bartels 1996)

Two regimes:

$$Y_A = \beta_A' \mathbf{X}_A + u_A$$

$$Y_B = \beta_B' \mathbf{X}_B + u_B$$

with $\sigma_A^2 = \sigma_B^2$, and $Cov(u_A, u_B) = 0$.

Estimators:

$$\hat{eta}_{A,B} = (\mathbf{X}_{A,B}^{\prime}\mathbf{X}_{A,B})^{-1}\mathbf{X}_{A,B}^{\prime}Y_{A,B}$$

and

$$\widehat{\mathsf{Var}(eta_{A,B})} = \hat{\sigma}_{A,B}^2(\mathbf{X}_{A,B}'\mathbf{X}_{A,B})^{-1},$$

A Pooled Estimator

Pooling As and Bs gives:

$$\hat{\beta}_{P} = (\mathbf{X}'_{A}\mathbf{X}_{A} + \mathbf{X}'_{B}\mathbf{X}_{B})^{-1}(\mathbf{X}'_{A}Y_{A} + \mathbf{X}'_{B}Y_{B})
= (\mathbf{X}'_{A}\mathbf{X}_{A} + \mathbf{X}'_{B}\mathbf{X}_{B})^{-1}[\beta_{A}(\mathbf{X}'_{A}\mathbf{X}_{A}) + \beta_{B}(\mathbf{X}'_{B}\mathbf{X}_{B})],$$

What is the expectation?

$$E(\hat{\beta}_P) = \beta_A + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_B'\mathbf{X}_B(\beta_B - \beta_A)$$
$$= \beta_B + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_A'\mathbf{X}_A(\beta_A - \beta_B)$$

Pooling: A Test

We can assess whether $\hat{\beta}_A = \hat{\beta}_B$ via:

$$F = \frac{\frac{\hat{\mathbf{u}}_{P}'\hat{\mathbf{u}}_{P} - (\hat{\mathbf{u}}_{A}'\hat{\mathbf{u}}_{A} + \hat{\mathbf{u}}_{B}'\hat{\mathbf{u}}_{B})}{K}}{\frac{(\hat{\mathbf{u}}_{A}'\hat{\mathbf{u}}_{A} + \hat{\mathbf{u}}_{B}'\hat{\mathbf{u}}_{B})}{(N_{A} + N_{B} - 2K)}} \sim F_{[K,(N_{A} + N_{B} - 2K)]}$$

Fractional Pooling

Bartels suggests:

$$\hat{\beta}_{\lambda} = (\lambda^2 \mathbf{X}_A' \mathbf{X}_A + \mathbf{X}_B' \mathbf{X}_B)^{-1} (\lambda^2 \mathbf{X}_A' Y_A + \mathbf{X}_B' Y_B)$$

with $\lambda \in [0, 1]$:

- $\lambda=0$ ightarrow separate estimators for \hat{eta}_{A} and \hat{eta}_{B} ,
- $\lambda = 1 \rightarrow$ "fully pooled" estimator $\hat{\beta}_P$,
- $0 < \lambda < 1 \rightarrow$ a regression where data in regime A are given some "partial" weighting in their contribution towards an estimate of β .

Pooling, Summarized

"(R)oughly speaking, it makes sense to pool disparate observations if the underlying parameters governing those observations are sufficiently similar, but not otherwise."

- Bartels (1996)

Exploring Variation: A Running Example

The U.S. Supreme Court, 1946-2020

- Court has nine "justices" at any time
- Appointed by the President, confirmed by the Senate (simple majority vote)
- Serve for life / good behavior
- One is appointed as the "Chief Justice" (a sitting justice may be elevated to that position)
- Sit in annual "terms" (October through June/July); decide 80-150 cases per term
- Cases are appealed from lower federal and state supreme courts
- Simple majority decision rule ("five")
- Nearly all decisions have a ideological (left / right) valence ("liberal" vs. "conservative")

The Supreme Court Database



http://scdb.wustl.edu/

Supreme Court Panel Data

Structure: One observation per justice (i) per term (t)

Important variables:

- justice: A justice (unit) ID variable [range: 78-116]
- term: A term (time) variable [range: 1946-2019]
- LiberalPct: The percentage of cases in that term in which that justice voted in a politically left / "liberal" direction
- MajPct: The percentage of cases in that term in which that justice voted with the majority in a case
- Ideology: A variable measuring the justice's (pre-confirmation) political ideology [range: 0 (most conservative) - 1 (most liberal)]*
- Qualifications: A measure of the justice's qualifications prior to his/her appointment [range: 0 (least qualified) - 1 (most qualified)]*
- President: The name of the president who appointed that justice*
- YearApptd: The year that justice was appointed*
- NCases: The number of cases the Court decided during that term**
- ChiefJustice: The identity of the Chief Justice during that term**

 $^{^{}st}$ indicates variables that are non-time-varying (that is, that have only between-unit variation)

 $^{^{**}}$ indicates variables that are non-unit-varying (that is, that have only within-unit variation)

Summary Statistics

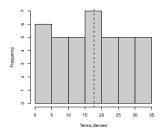
> summary(SCDat	a)			
term	justice	justiceName	LiberalPct	MajPct
Min. :1946	Min. : 78.0	Length:672	Min. :16.7	Min. : 46.7
1st Qu.:1964	1st Qu.: 91.0	Class : character	1st Qu.:38.4	1st Qu.: 76.2
Median:1982	Median :100.0	Mode :character	Median:49.5	Median: 82.8
Mean :1982	Mean : 98.1		Mean :51.9	Mean : 81.7
3rd Qu.:2001	3rd Qu.:106.0		3rd Qu.:65.7	3rd Qu.: 88.0
Max. :2019	Max. :116.0		Max. :87.7	Max. :100.0
Order	Nominee	SenateVote	Ideology	Qualifications
Min. : 1.0	Length: 672	Length:672	Min. :0.0	000 Min. :0.125
1st Qu.:15.0	Class : characte	r Class :charac	ter 1st Qu.:0.1	.60 1st Qu.:0.750
Median:27.0	Mode :characte	r Mode :charac	ter Median:0.4	88 Median :0.885
Mean :24.5			Mean :0.4	88 Mean :0.802
3rd Qu.:36.0			3rd Qu.:0.7	50 3rd Qu.:0.978
Max. :47.0			Max. :1.0	000 Max. :1.000
President	YearApptd	NCases	ChiefJustice	
Length:672	Min. :193	7 Min. : 76	Length:672	
Class :charact	er 1st Qu.:195	5 1st Qu.: 96	Class :character	•
Mode :charact	er Median:197	0 Median :141	Mode :character	
	Mean :197	0 Mean :142		
	3rd Qu.:198	8 3rd Qu.:182		
	Max. :201	8 Max. :258		

Some Basics

How many justices are in the data?

```
> length(unique(SCData$justice))
[1] 38
```

How many terms do justices typically serve?

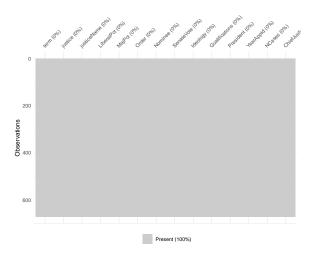


So we have:

- N = 38 units (justices)
- $\overline{T}=17.7$ time periods (terms) of data per justice, on average [range: 2-35], for a total of
- NT = 672 justice-terms in the data

Missing Data

- > library(naniar)
- > vis_miss(SCData)



Variation: LiberalPct

```
> # Total variation:
>
> with(SCData, describe(LiberalPct))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 672 51.9 16.2 49.5 51.5 19 16.7 87.7 71 0.19
> # Between-Justice variation:
> LibMeans <- ddply(SCData,.(justice),summarise,
                MeanLibPct=mean(LiberalPct))
> with(LibMeans, describe(MeanLibPct))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 38 52.5 14.8 48.3 52.2 16.5 29.9 77.2 47.2 0.31 -1.29 2.39
> # Within-Justice variation:
>
> SCData <- ddply(SCData,.(justice), mutate,
              LibMean=mean(LiberalPct))
> SCData$LibWithin <- with(SCData, LiberalPct-LibMean)
> with(SCData, describe(LibWithin))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 672 0 7.36 -0.16 -0.02 7.05 -30.8 32.8 63.6 0.04
```

Variation: Ideology

```
> # Total variation:
>
> with(SCData, describe(Ideology))
         n mean sd median trimmed mad min max range skew kurtosis
X 1
     1 672 0.49 0.32 0.49
                              0.48 0.39 0 1 1 0.09
> # Between-Justice variation:
> IdeoMeans <- ddply(SCData,.(justice),summarise,
                  MeanIdeo=mean(Ideology))
> with(IdeoMeans, describe(MeanIdeo))
                sd median trimmed mad min max range skew kurtosis
  vars n mean
X1 1 38 0.54 0.33 0.58 0.54 0.43 0
                                           1
                                                 1 -0.11
                                                            -1.460.05
> # Within-Justice variation (hint - there is none):
>
> SCData <- ddply(SCData,.(justice), mutate,
                 IdeoMean=mean(Ideology))
> SCData$IdeoWithin <- with(SCData, Ideology-IdeoMean)
> with(SCData, describe(IdeoWithin))
  vars n mean sd median trimmed mad min max range skew kurtosis se
X 1
     1 672
             0 0
                       0
                               0
                                   0
                                      0
                                          0
                                                0 NaN
```

Variation: NCases

```
> # Total variation:
>
> with(SCData, describe(NCases))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 672 142 44.7 141
                              141 60.8 76 258 182 0.18 -1.06 1.73
> # Between-Term variation:
> NCMeans <- ddply(SCData,.(term),summarise,
                   MeanNCases=mean(NCases))
> with(NCMeans, describe(MeanNCases))
  vars n mean sd median trimmed mad min max range skew kurtosis
                             141 60.8 76 258 182 0.17 -1.11 5.24
X1 1 74 142 45.1 142
> # Within-Term variation (none):
>
> SCData <- ddply(SCData,.(term), mutate,
                NCMean=mean(NCases))
> SCData$NCWithin <- with(SCData, NCases-NCMean)
> with(SCData, describe(NCWithin))
  vars n mean sd median trimmed mad min max range skew kurtosis se
X1 1 672 0 0
                       0
                              0
                                  0
                                      0
                                          Ω
                                               0 NaN
```

Visualization: ExPanDaR

An interactive tool for exploring panel data...

- Creator: Joachim Gassen (Department of Accounting, Humboldt-Universität zu Berlin)
- Built upon / consistent with ggplot / tidyverse
- Requires installing the ExPanDaR package
- Calling
 - > ExPanD()

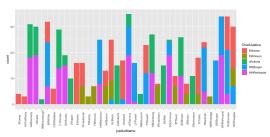
...opens the Shiny app, and asks for a (pre-formatted) data frame (typically in CSV format)

 More information is here: https://joachim-gassen.github.io/ExPanDaR/

Some examples...

ExPanDaR: Summaries

Counts by factors:

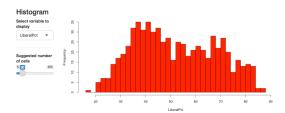


Summary statistics:

Descriptive Statistics	Analysis Set	Base Set							
	Variable	N	Mean	Std. dev.	Min.	25 %	Median	75 %	Max.
Hover over variable names with mouse to see variable definitions	V1	672	351.475	208.836	1.000	168.750	336.500	538.250	707.00
	LiberalPct	672	51.856	16.213	16.667	38.446	49.473	65.713	87.662
Select Tab to choose the analysis set of variables or the base set of variables (to define new variables).	MajPct	672	81.714	8.827	46.667	76.193	82.828	87.984	100.000
	Order	672	24.531	12.784	1.000	15.000	27.000	36.000	47.000
	Ideology	672	0.488	0.320	0.000	0.160	0.487	0.750	1.00
	Qualifications	672	0.802	0.245	0.125	0.750	0.885	0.978	1.000
Click here to delete selected variables from the analysis sample.	YearApptd	672	1,970.324	20.975	1,937.000	1,955.000	1,970.000	1,988.000	2,018.00
Delete Variables									

ExPanDaR: Distributions

Histograms:

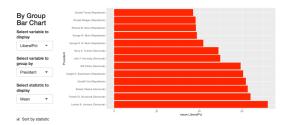


Outlier Detection:

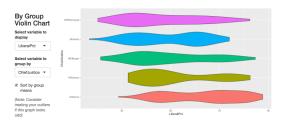
Extreme	justice	term	LiberalPct
Observations	81	1958	87.7
Select variable to	81	1956	87.0
sort data by LiberalPct ▼	81	1971	84.7
	90	1963	84.1
Select period to subset to	81	1955	83.7
All 💌			
	102	1979	21.3
	108	2003	21.3
	102	1998	20.2
	108	1998	20.2
	115	2016	16.7

ExPanDaR: More Distributions

Bar Charts of Means (by factors):

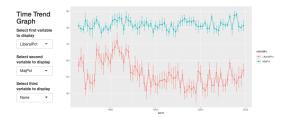


Violin Plots:

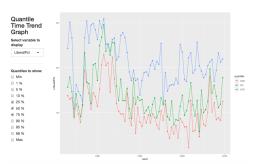


ExPanDaR: Trends

General Trends + Variation:

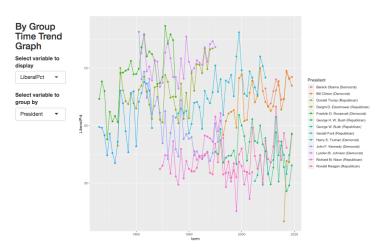


Trends in Quantiles:



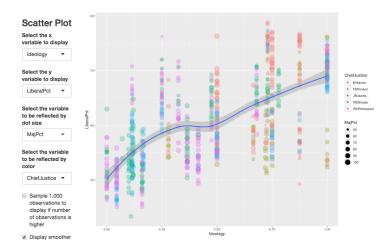
ExPanDaR: More Trends

Trends By Group:



ExPanDaR: Scatterplots

Fancy Scatterplots:

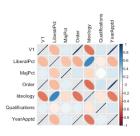


ExPanDaR: Correlations / Regression

Bivariate Correlations:

Correlation Plot This plot visualizes

sample correlations (Pearson above, Spearman below diagonal). Reports correlations for all continuous variables. Hover over ellipse to get rho, P-Value and n.



Regression (OLS) Analysis:





The Plan

- Tuesday, 10 January: One- and Two-Way "Unit Effects" Models (fixed, "random," etc.)
- Wednesday, 11 January: Dynamics in Panel Data
- Thursday, 12 January: Panel Data and Causal Inference
- Friday, 13 January: Models for Discrete Responses