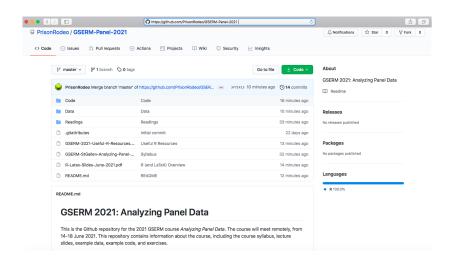
GSERM 2021Analyzing Panel Data

June 14, 2021

Preliminaries

- Instructor: Prof. Christopher Zorn (zorn@psu.edu).
- Class: June 14-18, 2021, 13:00-18:00 CET, at the University of St. Gallen (via Zoom).
- The course outline / syllabus is here.
- More important: The syllabus, slides, readings, code, data, etc. are all available on the course github repo (viewable at https://github.com/PrisonRodeo/GSERM-Panel-2021), and on the HSG CANVAS page.
- Class sessions will be recorded; links to those recordings will be available on CANVAS only.



Software

R

- All examples, plots, etc.
- Current version is 4.1.0
- Packages you'll use (see the econometrics task view for more):
 - · plm
 - \cdot lme4
 - · gee

Stata

- Current version is 17
- Mostly use the -xt- series of commands (for "cross-sectional time series")

Starting Points

- <u>Panel</u> data: Data comprising repeated observations over time on a set of cross-sectional units.
- Terminology:
 - "Unit" / "Units" / "Units of observation" / "Panels" = Things we observe repeatedly
 - "Observations" = Each (one) measurement of a unit
 - "Time points" = When each observation on a unit is made
 - $i \in \{1...N\}$ indexes units
 - $t \in \{1...T\}$ or $\{1...T_i\}$ indexes observations / time points
 - If $T_i = T \ \forall i$ then we have "balanced" panels / units
 - Balanced panels also imply $N_t = N \ \forall t$
 - NT = Total number of observations (if balanced)
- "Panel" \neq "Time Series"
- "Panel" ≠ "Multilevel" / "Hierarchical" / etc.

More Terminology

 $N >> T \rightarrow$ "panel" data...

- (American) National Election Study panel studies ($N \approx 2000, T = 3$)
- Swiss Household Panel (FORS) (N = large, T = 21)
- Often:
 - · Cross-sectional units are a sample from a population
 - · T is (relatively) fixed

T>>N or $T \approx N \rightarrow$ "time-series cross-sectional" ("TSCS") data

- National OECD data (N=20 original members, $T\approx70$)
- Often:
 - · N is an entire population, and is (relatively) fixed
 - · Asymptotics are in T

 $N=1 \rightarrow$ "time series" data

 $T=1 \rightarrow$ "cross-sectional" data

${\sf Panel\ Data\ Structure}\,+\,{\sf Organization}$

id	t	Y	X_1	
1	1	250	3.4	
1	2	290	3.3	
:	:	:	:	
2	1	160	4.7	
2	2	150	4.9	
:	:	:	:	
_ •	•	•	•	•••

Introduction to Panel Variation: A Tiny (Fake) Example

id	year	gender	pres	pid	approve
1	2014	female	obama	dem	3
1	2016	female	obama	dem	3
1	2018	female	trump	dem	5
1	2020	female	trump	dem	3
2 2 2 2 2	2014 2016 2018 2020	male male male male	obama obama trump trump	gop gop gop gop	2 1 4 3
3 3 3 3	2014 2016 2018 2020	male male male male	obama obama trump trump	gop gop gop dem	2 2 4 1

Aggregation: Cross-Sectional

1 female ? dem 3.50 2 male ? gop 2.50 3 male ? ? 2.25	id	gender	pres	pid	approve
	_	male	?	gop	2.50

Aggregation: Temporal

2014 0.33 obama 0.66(?) 2.33 2016 0.33 obama 0.66(?) 2.00 2018 0.33 trump 0.66(?) 4.33 2020 0.33 trump 0.33(?) 2.33	year	female	pres	pid	approve
	2016 2018	0.33	obama trump	0.66(?) 0.66(?)	2.00 4.33

The Point

Aggregation:

- Loses information
- Distorts relationships
- Forces arbitrary decisions

If you have variation in multiple dimensions, use it.

Two-Way Variation

Two "dimensions" of variation:

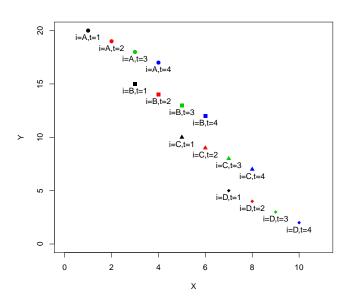
- <u>Unit-Level</u> Variation (how each unit is on average different from other units on average) a/k/a **between-unit** variation
- <u>Time-Level</u> Variation (how each measurement / time point is on average different from other time points on average) a/k/a/within-unit variation

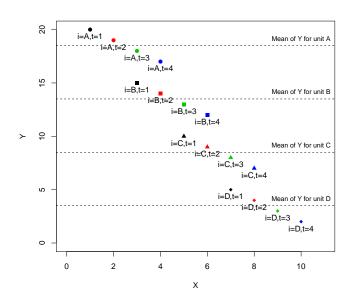
A random variable may:

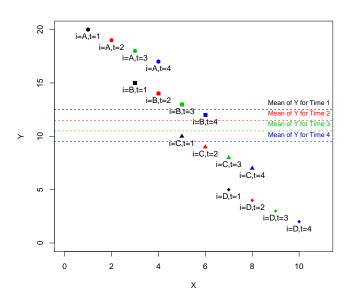
- Have only between-unit variation (i.e., lack temporal variation)
- Have only within-unit variation (i.e., lack cross-sectional variation)
- Have both within- and between-unit variation

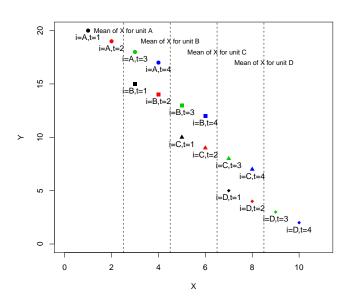
For Y_{it} , a variable that varies over both units and time:

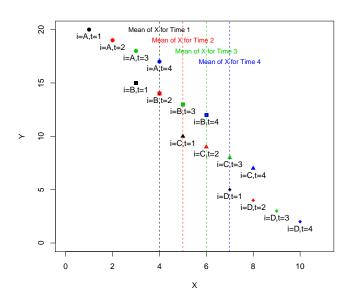
- $\bar{Y}_i = \frac{1}{T_i} \sum_{t=1}^{T} Y_{it}$ is the over-time mean of Y for unit i,
- $\bar{Y}_t = \frac{1}{N_t} \sum_{i=1}^N Y_{it}$ is the across-unit mean of Y at time t, and
- $\bar{Y} = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} Y_{it}$ is the grand mean of Y.











Within- and Between-Unit Variation

The within-unit mean of Y is:

$$\bar{Y}_i = \frac{1}{T_i} \sum_{t=1}^{T_i} Y_{it}$$

That means that we can write:

$$Y_{it} = \bar{Y}_i + (Y_{it} - \bar{Y}_i).$$

That is, the *total* variation in Y_{it} can be decomposed into:

- The between-unit variation in the \bar{Y}_i s, and
- The within-unit variation around \bar{Y}_i (that is, $Y_{it} \bar{Y}_i$).

Within- and Between-Unit Variation (continued)

Note that (while unusual) one could do a similar decomposition vis-à-vis time:

$$Y_{it} = \bar{Y}_t + (Y_{it} - \bar{Y}_t).$$

That is, the *total* variation in Y_{it} can be decomposed into:

- The temporal variation in the \bar{Y}_t s, and
- The within-time-point variation around \bar{Y}_t (that is, $Y_{it} \bar{Y}_t$).

In a similar fashion, we can also calculate the within- and between-unit variability (e.g., the standard deviations) of the constituent variables \bar{Y}_i and $(Y_{it} - \bar{Y}_i)$...

Variation ("Toy" Data from Above)

"Total" Variation:

```
> with(toy, describe(Y)) vars n mean sd median trimmed mad min max range skew kurtosis se X1 1 16 11 5.9 11 11 7.4 2 20 18 0 -1.5 1.5
```

"Between" Variation:

"Within" Variation:

Regression!

Model:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

...makes all the usual OLS assumptions, plus

- $\beta_{0i} = \beta_0 \forall is$
- $\beta_{1i}=\beta_1 \ \forall \ \textit{is}$

For the model:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

...the same is true.

Variable Intercepts

Unit-specific intercepts:

$$Y_{it} = \beta_{0i} + \beta_1 X_{it} + u_{it} \tag{1}$$

Time-point-specific intercepts:

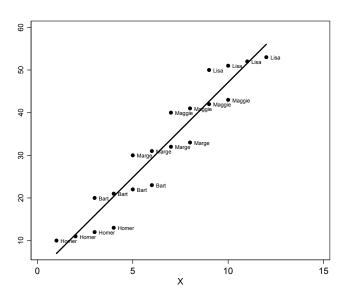
$$Y_{it} = \beta_{0t} + \beta_1 X_{it} + u_{it} \tag{2}$$

Both:

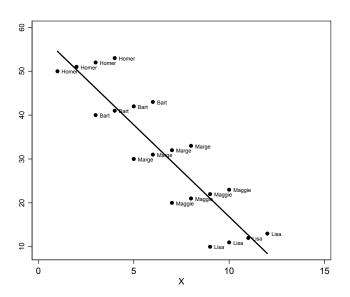
$$Y_{it} = \beta_{0it} + \beta_1 X_{it} + u_{it} \tag{3}$$

Note: Equation 3 is not identified (as written)!

Varying Intercepts



Varying Intercepts



Varying Slopes (+ Intercepts)

Unit-specific slopes:

$$Y_{it} = \beta_0 + \beta_{1i} X_{it} + u_{it} \tag{4}$$

(...one can also have time-point specific slopes, or both – again, the last of those is not identified as written.)

Unit-specific slopes + intercepts:

$$Y_{it} = \beta_{0i} + \beta_{1i} X_{it} + u_{it} \tag{5}$$

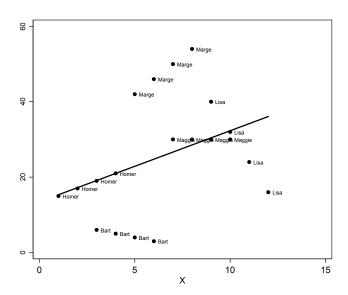
Time-point-specific-specific slopes + intercepts:

$$Y_{it} = \beta_{0t} + \beta_{1t} X_{it} + u_{it} \tag{6}$$

Both...:

$$Y_{it} = \beta_{0it} + \beta_{1it} X_{it} + u_{it} \tag{7}$$

${\sf Varying\ Slopes}\,+\,{\sf Intercepts}$



The Error Term...

Usual OLS assumption:

$$u_{it} \sim \text{i.i.d.} N(0, \sigma^2) \ \forall \ i, t$$

or, equivalently:

$$\mathbf{u}\mathbf{u}'\sim\sigma^2\mathbf{I}$$

implies:

$$Var(u_{it}) = Var(u_{jt}) \ \forall \ i \neq j \ (i.e., no cross-unit heteroscedasticity)$$

 $Var(u_{it}) = Var(u_{is}) \ \forall \ t \neq s \ (i.e., no temporal heteroscedasticity)$
 $Cov(u_{it}, u_{is}) = 0 \ \forall \ i \neq j, \ \forall \ t \neq s \ (i.e., no auto- or spatial correlation)$

Pooling

Pooling pros:

- Adds data (→ precision)
- Enhances generalizability

BUT: fitting the model:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

Implies

- that the process governing the relationship between *X* and *Y* is exactly the same for each *i*,
- that the process governing the relationship between X and Y is the same for all t,
- that the process governing the us is the same $\forall i$ and t as well.

Q: When can we "pool" data on different units?

"Partial" Pooling (Bartels 1996)

Two regimes:

$$Y_A = \beta_A' \mathbf{X}_A + u_A$$

$$Y_B = \beta_B' \mathbf{X}_B + u_B$$

with $\sigma_A^2 = \sigma_B^2$, and $Cov(u_A, u_B) = 0$.

Estimators:

$$\hat{\beta}_{A,B} = (\mathbf{X}_{A,B}^{\prime}\mathbf{X}_{A,B})^{-1}\mathbf{X}_{A,B}^{\prime}Y_{A,B}$$

and

$$\widehat{\mathsf{Var}(eta_{A,B})} = \hat{\sigma}_{A,B}^2(\mathbf{X}_{A,B}'\mathbf{X}_{A,B})^{-1},$$

A Pooled Estimator

$$\hat{\beta}_{P} = (\mathbf{X}'_{A}\mathbf{X}_{A} + \mathbf{X}'_{B}\mathbf{X}_{B})^{-1}(\mathbf{X}'_{A}Y_{A} + \mathbf{X}'_{B}Y_{B})
= (\mathbf{X}'_{A}\mathbf{X}_{A} + \mathbf{X}'_{B}\mathbf{X}_{B})^{-1}[\beta_{A}(\mathbf{X}'_{A}\mathbf{X}_{A}) + \beta_{B}(\mathbf{X}'_{B}\mathbf{X}_{B})],$$

$$E(\hat{\beta}_P) = \beta_A + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_B'\mathbf{X}_B(\beta_B - \beta_A)$$
$$= \beta_B + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_A'\mathbf{X}_A(\beta_A - \beta_B)$$

Pooling: A Test

$$F = \frac{\frac{\hat{\mathbf{u}}_{P}'\hat{\mathbf{u}}_{P} - (\hat{\mathbf{u}}_{A}'\hat{\mathbf{u}}_{A} + \hat{\mathbf{u}}_{B}'\hat{\mathbf{u}}_{B})}{K}}{\frac{(\hat{\mathbf{u}}_{A}'\hat{\mathbf{u}}_{A} + \hat{\mathbf{u}}_{B}'\hat{\mathbf{u}}_{B})}{(N_{A} + N_{B} - 2K)}} \sim F_{[K,(N_{A} + N_{B} - 2K)]}$$

Fractional Pooling

Bartels suggests:

$$\hat{\beta}_{\lambda} = (\lambda^2 \mathbf{X}_A' \mathbf{X}_A + \mathbf{X}_B' \mathbf{X}_B)^{-1} (\lambda^2 \mathbf{X}_A' Y_A + \mathbf{X}_B' Y_B)$$

with $\lambda \in [0,1]$:

- $\lambda=0$ ightarrow separate estimators for \hat{eta}_{A} and \hat{eta}_{B} ,
- $\lambda=1$ \rightarrow "fully pooled" estimator $\hat{\beta}_P$,
- $0 < \lambda < 1 \rightarrow$ a regression where data in regime A are given some "partial" weighting in their contribution towards an estimate of β .

Pooling, Summarized

"(R)oughly speaking, it makes sense to pool disparate observations if the underlying parameters governing those observations are sufficiently similar, but not otherwise."

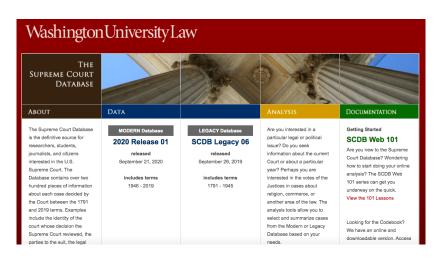
- Bartels (1996)

Exploring Variation: A Running Example

The U.S. Supreme Court, 1946-2020

- Court has nine "justices" at any time
- Appointed by the President, confirmed by the Senate (simple majority vote)
- Serve for life / good behavior
- One is appointed as the "Chief Justice" (a sitting justice may be elevated to that position)
- Sit in annual "terms" (October through June/July); decide 80-150 cases per term
- Cases are appealed from lower federal and state supreme courts
- Simple majority decision rule ("five")
- Nearly all decisions have a ideological (left / right) valence ("liberal" vs. "conservative")

The Supreme Court Database



http://scdb.wustl.edu/

Supreme Court Panel Data

Structure: One observation per justice (i) per term (t)

Important variables:

- justice: A justice (unit) ID variable [range: 78-116]
- term: A term (time) variable [range: 1946-2019]
- LiberalPct: The percentage of cases in that term in which that justice voted in a politically left / "liberal" direction
- MajPct: The percentage of cases in that term in which that justice voted with the majority in a case
- Ideology: A variable measuring the justice's (pre-confirmation) political ideology [range: 0 (most conservative) - 1 (most liberal)]*
- Qualifications: A measure of the justice's qualifications prior to his/her appointment [range: 0 (least qualified) - 1 (most qualified)]*
- President: The name of the president who appointed that justice*
- YearApptd: The year that justice was appointed*
- NCases: The number of cases the Court decided during that term**
- ChiefJustice: The identity of the Chief Justice during that term**

^{*} indicates variables that are non-time-varying (that is, that have only between-unit variation)

^{**} indicates variables that are non-unit-varying (that is, that have only within-unit variation)

Summary Statistics

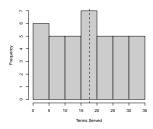
```
> summary(SCData)
                                justiceName
                                                      LiberalPct
      term
                   iustice
                                                                       MaiPct
 Min.
        :1946
                Min.
                       : 78.0
                                Length:672
                                                    Min.
                                                           :16.7
                                                                   Min.
                                                                          : 46.7
 1st Qu.:1964
                1st Qu.: 91.0
                                Class :character
                                                    1st Qu.:38.4
                                                                   1st Qu.: 76.2
 Median:1982
                Median :100.0
                                Mode
                                     :character
                                                    Median:49.5
                                                                   Median: 82.8
 Mean
        :1982
                Mean
                       : 98.1
                                                    Mean
                                                           :51.9
                                                                   Mean
                                                                          : 81.7
 3rd Qu.:2001
                3rd Qu.:106.0
                                                    3rd Qu.:65.7
                                                                   3rd Qu.: 88.0
                                                           :87.7
 Max.
        :2019
                Max.
                       :116.0
                                                    Max.
                                                                   Max.
                                                                          :100.0
     Order
                                    SenateVote
                                                          Ideology
                                                                       Qualifications
                  Nominee
                                                              :0.000
                                                                              :0.125
 Min.
        : 1.0
                Length: 672
                                   Length:672
                                                       Min.
                                                                       Min.
 1st Qu.:15.0
                Class : character
                                   Class : character
                                                       1st Qu.:0.160
                                                                       1st Qu.:0.750
 Median :27.0
                Mode :character
                                   Mode :character
                                                       Median :0.488
                                                                       Median :0.885
        :24.5
                                                       Mean
                                                              :0.488
                                                                       Mean
                                                                              :0.802
 Mean
 3rd Qu.:36.0
                                                       3rd Qu.:0.750
                                                                       3rd Qu.:0.978
        :47.0
                                                       Max.
                                                              :1.000
                                                                       Max.
                                                                              :1.000
 Max.
  President
                      YearApptd
                                       NCases
                                                  Chief.Justice
                           :1937
                                   Min.
                                           : 76
 Length:672
                    Min.
                                                 Length:672
 Class :character
                    1st Qu.:1955
                                   1st Qu.: 96
                                                 Class :character
 Mode :character
                    Median:1970
                                   Median:141
                                                 Mode :character
                    Mean
                           :1970
                                   Mean
                                           :142
                    3rd Qu.:1988
                                   3rd Qu.:182
                    Max.
                           :2018
                                   Max.
                                           :258
```

Some Basics

How many justices are in the data?

> length(unique(SCData\$justice))
[1] 38

How many terms do justices typically serve?

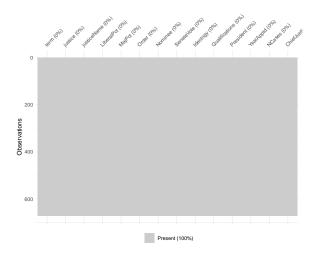


So we have:

- N = 38 units (justices)
- $\bar{T}=17.7$ time periods (terms) of data per justice, on average [range: 2-35], for a total of
- NT = 672 justice-terms in the data

Missing Data

- > library(naniar)
- > vis_miss(SCData)



Variation: LiberalPct

```
> # Total variation:
> with(SCData, describe(LiberalPct))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 672 51.9 16.2 49.5 51.5 19 16.7 87.7 71 0.19
> # Between-Justice variation:
>
> LibMeans <- ddply(SCData,.(justice),summarise,
                MeanLibPct=mean(LiberalPct))
> with(LibMeans, describe(MeanLibPct))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 38 52.5 14.8 48.3 52.2 16.5 29.9 77.2 47.2 0.31 -1.29 2.39
> # Within-Justice variation:
>
> SCData <- ddply(SCData,.(justice), mutate,
             LibMean=mean(LiberalPct))
> SCData$LibWithin <- with(SCData, LiberalPct-LibMean)
> with(SCData, describe(LibWithin))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 672 0 7.36 -0.16 -0.02 7.05 -30.8 32.8 63.6 0.04
```

Variation: Ideology

```
> # Total variation:
> with(SCData, describe(Ideology))
         n mean sd median trimmed mad min max range skew kurtosis
X1
  1 672 0.49 0.32 0.49 0.48 0.39 0 1 1 0.09
> # Between-Justice variation:
>
> IdeoMeans <- ddply(SCData,.(justice),summarise,
                  MeanIdeo=mean(Ideology))
> with(IdeoMeans, describe(MeanIdeo))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 38 0.54 0.33 0.58 0.54 0.43 0
                                          1
                                              1 -0.11 -1.46 0.05
> # Within-Justice variation (hint - there is none):
>
> SCData <- ddply(SCData,.(justice), mutate,
                IdeoMean=mean(Ideology))
> SCData$IdeoWithin <- with(SCData, Ideology-IdeoMean)
> with(SCData, describe(IdeoWithin))
         n mean sd median trimmed mad min max range skew kurtosis se
X1
     1 672
             0 0
                       0
                              0
                                  0
                                      0
                                         0
                                               0 NaN
                                                          NaN 0
```

Variation: NCases

```
> # Total variation:
> with(SCData, describe(NCases))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 672 142 44.7 141 141 60.8 76 258 182 0.18 -1.06 1.73
> # Between-Term variation:
>
> NCMeans <- ddply(SCData,.(term),summarise,
                   MeanNCases=mean(NCases))
> with(NCMeans, describe(MeanNCases))
  vars n mean sd median trimmed mad min max range skew kurtosis
                             141 60.8 76 258 182 0.17 -1.11 5.24
X1 1 74 142 45.1 142
> # Within-Term variation (none):
>
> SCData <- ddply(SCData,.(term), mutate,
                NCMean=mean(NCases))
> SCData$NCWithin <- with(SCData, NCases-NCMean)
> with(SCData, describe(NCWithin))
  vars n mean sd median trimmed mad min max range skew kurtosis se
X1 1 672
             0 0
                       0
                              0
                                  0
                                      0
                                         0
                                               0 NaN
                                                          NaN 0
```

Visualization: ExPanDaR

An interactive tool for exploring panel data...

- Creator: Joachim Gassen (Department of Accounting, Humboldt-Universität zu Berlin)
- Built upon / consistent with ggplot / tidyverse
- Requires installing the ExPanDaR package
- Calling
 - > ExPanD()

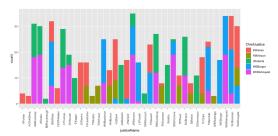
...opens the Shiny app, and asks for a (pre-formatted) data frame (typically in CSV format)

 More information is here: https://joachim-gassen.github.io/ExPanDaR/

Some examples...

ExPanDaR: Summaries

Counts by factors:

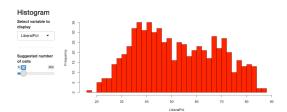


Summary statistics:

iariable peralPct	N 672 672	Mean 351.475 51.856	Std. dev. 208.836 16.213	Min. 1.000 16.667	25 % 168.750 38.446	Median 336.500	75 % 538.250	Max. 707.000
	672							
		51.856	16.213	16.667	20 446			
iPet					J3.440	49.473	65.713	87.662
apr or	672	81.714	8.827	46.667	76.193	82.828	87.964	100.000
der	672	24.531	12.784	1.000	15.000	27.000	36.000	47.000
ology	672	0.488	0.320	0.000	0.160	0.487	0.750	1.000
alifications	672	0.802	0.245	0.125	0.750	0.885	0.978	1.000
arApptd	672	1,970.324	20.975	1,937.000	1,955.000	1,970.000	1,988.000	2,018.000
	ology alifications	ology 672 alifications 672	ology 672 0.488 allifications 672 0.802	ology 672 0.488 0.320 allifications 672 0.802 0.245	ology 672 0.488 0.320 0.000 allifications 672 0.802 0.245 0.125	ology 672 0.488 0.320 0.000 0.160 allifications 672 0.802 0.245 0.125 0.750	ology 672 0.488 0.320 0.000 0.160 0.487 allfications 672 0.802 0.245 0.125 0.750 0.885	ology 672 0.488 0.320 0.000 0.160 0.487 0.750 allfications 672 0.802 0.245 0.125 0.750 0.885 0.978

ExPanDaR: Distributions

Histograms:

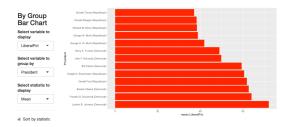


Outlier Detection:

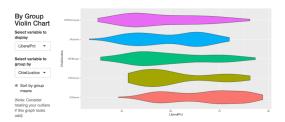
Extreme	justice	term	LiberalPct
Observations	81	1958	87.7
Select variable to	81	1956	87.0
sort data by	81	1971	84.7
LiberaiPct *	90	1963	84.1
Select period to subset to	81	1955	83.7
All -			***
	102	1979	21.3
	108	2003	21.3
	102	1998	20.2
	108	1998	20.2
	115	2016	16.7

ExPanDaR: More Distributions

Bar Charts of Means (by factors):

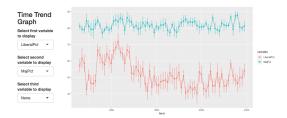


Violin Plots:

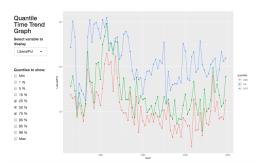


ExPanDaR: Trends

General Trends + Variation:

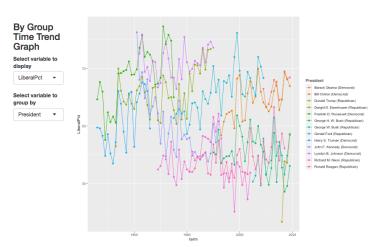


Trends in Quantiles:



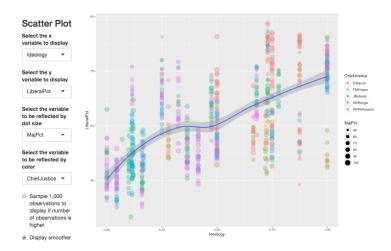
ExPanDaR: More Trends

Trends By Group:



ExPanDaR: Scatterplots

Fancy Scatterplots:

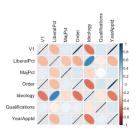


ExPanDaR: Correlations / Regression

Bivariate Correlations:



sample correlations (Pearson above, Spearman below diagonal). Reports correlations for all continuous variables. Hover over ellipse to get rho, P-Value and n.



Regression (OLS) Analysis:



Select a categorial variable as the first fixed effect

None

	Dependent variable:
	LiberalPct
deology	31.200***
	(1.490)
MajPct	-0.287
	(0.054)
Constant	60.100
	(4.650)
stimator	ols
ixed effects	None
Std. errors clustered	No
Observations	672
R ²	0.445
Adjusted R ²	0.443
Vote:	p<0.1; "p<0.05; "p<0

The Plan

- Tuesday, 15 June: One- and Two-Way "Unit Effects" Models (fixed, "random," etc.)
- Wednesday, 16 June: Dynamics in Panel Data
- Thursday, 17 June: Panel Data and Causal Inference
- Friday, 18 June: Models for Discrete Responses