Intro2Astro: Exoplanet Atmospheres Problem Set
July 29, 2025

Section I: Life & Habitability

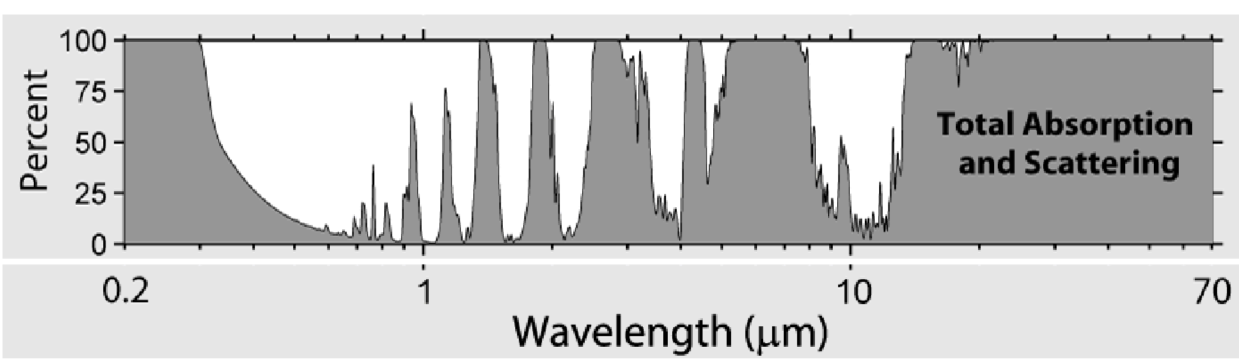
1. In Kelvin, the equilibrium temperature (T_{eq}) of a planet is the calculated bare-rocky surface temperature from stellar insolation for a perfect blackbody, given by the equation below. Assume *A,* the bond albedo, to be 0.3, the stellar luminosity to be solar-like ($L_\bigstar$ = 3.827×10^{26} watts), and *a* to be the average radial distance of the planet from its star. σ is the Stefan-Boltzmann constant given as 5.670×10^{-8} [W m^{-2} K^{-4}].
    a. Calculate the bounds of the radial extent of the habitable zone where water is liquid in astronomical units (1 au = 1.496×10^{11} m). *Hint: water freezes at 273 K, and boils at 373 K. Watch your units!*

$$T_{\text{eq}} = \left(\frac{(1 - A)L_\star}{16\pi\sigma a^2} \right)^{\frac{1}{4}}$$

    b. Is our habitable zone toy model an underestimate or overestimate. *(Hint: Earth orbits the Sun at 1 au)*. If so, why? Name at least one reason.

Section II: Interpreting Atmospheric Absorption Spectra

2. Observing the Sun from the ground, sunlight passes through Earth's atmosphere before reaching our ground-based detector. The figure below shows the total absorption of stellar radiation at different wavelengths for Earth's atmosphere as seen on the planet's surface. Label the major absorption features on the figure, including H_2O, CO_2, O_2 and O_3, and CH_4.



Section III: Characterizing Atmospheric Loss

3. The Cosmic Shoreline model from Zahnle & Katling (2017) shows that an exoplanet's atmosphere can be predicted to be lost or retained depending on the unique escape velocity of that planet and the extreme UV (XUV) radiation that it receives from its star. Super-Earth exoplanet "Terra II" orbits a Sun-like star at a very close orbit and receives 100x the same amount of XUV stellar irradiation as on Earth ($I_{XUV,\text{Terra II}} = 100 \times I_{XUV,\oplus}$).
   a. Calculate the escape velocity on Terra II. Assume the planet's mass to be 4x Earth, and the planet's radius to be 2x that of Earth. Where one Earth mass is equal to ($M\oplus = 5.972 \times 10^{24}$ kg), one Earth radii is equal to ($R\oplus = 6.378 \times 10^{6}$ m), and G is the gravitational constant ($6.6743 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$). The equation for the escape velocity v_e in [m/s] of a planet is given below, where G is the gravitational constant, M is the planetary mass, and R is the planetary radius.

$$v_e = \sqrt{\frac{2GM}{r}}$$

   b. Predict if an atmosphere on Terra II is likely to exist based on your calculated escape velocity and label Terra II on the Cosmic Shoreline diagram. The Cosmic Shoreline diagram from Pass, Charbonneau, & Vanderburg (2025) is given below for you to make your prediction. *(Hint: The XUV influence on Terra II is stated previously and read the escape velocity in units of km/s).*