Robotics and Control 1
Homework 1

May 9, 2022

1 Robotic Arm

Consider the two-link planar arm with a prismatic joint and a revolute joint given in Figure 1 for which the vector of generalized coordinates is $q = [d_1 \quad \theta_2]^T$.

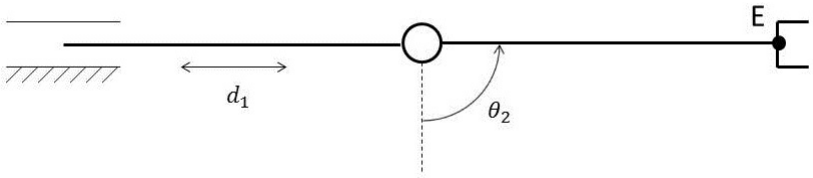

Figure 1: Two-link planar arm with a prismatic joint and a revolute joint. Initial configuration.

Let a_1, a_2, be the lenghts of link 1 and link 2, respectively. Let m_{ℓ_1}, m_{ℓ_2} be the masses of the two links and m_{m_1}, m_{m_2}, the masses of the rotors of the two joints motors. Assume the centres of mass of the links coincide with the middle points of the links. It is assumed that $p_{m_i} = p_{i-1}$ and $z_{m_i} = z_{i-1}$, for $i = 1, 2$, i.e., the motors are located on the joints axes with centres of mass located at the origins of the respective frames. Let I_{m_1}, I_{m_2} be the moments of inertia with respect to the axes of the two rotors, and I_{ℓ_1}, I_{ℓ_2} the moments of inertia relative to the centres of mass of the two links, respectively. Finally, let k_{r1} and k_{r2} be the gear reduction of Motor 1 and Motor 2, respectively. Tasks:

1. Find the dynamic model with the Lagrangian formulation (define the C matrix using the Christoffel symbols and verify the validity of the notable property analyzed during the lectures).

2. Design a control law that leads the end-effector from the initial position given in Figure 1 to the final position given in Figure 2. The trajectory depicted by the point E has to satisfy the following requirements:

   - it has to be a straight line;
   - it has to last 5 seconds;
   - the initial and the final velocities have to be 0.

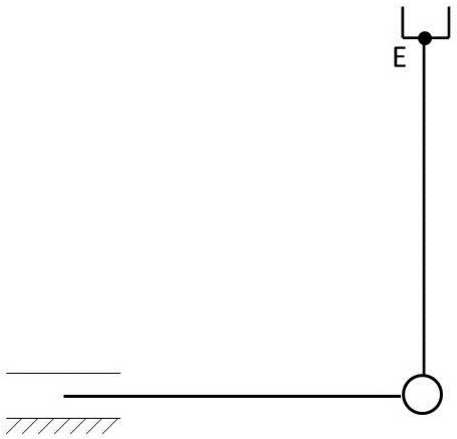

Figure 2: Two-link planar arm with a prismatic joint and a revolute joint. Final configuration.

Assume all the frictions are negligible. Use the following numerical values for the parameters introduced above:

$a_1 = a_2 = 1$; $m_{\ell_1} = m_{\ell_2} = 50$; $m_{m_1} = m_{m_2} = 5$; $I_{\ell_1} = I_{\ell_2} = 10$; $I_{m_1} = I_{m_2} = 0,01$; $k_{r1} = k_{r2} = 100$