Robotics HW2

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The aim of this homework is to test the operation of two controllers seen in the lecture, the PD with gravity compensation controller and the feedback linearisation controller, using two different types of reference signals, a step and a sinusoidal signal of different frequencies.

The tests were carried out considering a SCARA robot, whose dynamic model was obtained using the CasADi tool.

Simulations and related graphs were obtained by means of simulations in Simulink.

Performance was evaluated using the RMSE index.

1 PD with gravity compensation controller

Before analysing the simulation results, it is important to highlight the performance index used. The RMSE values, an index of controller performance, were obtained with respect to the difference between the desired end effector pose and the actual one. These two poses were calculated using the mex function f(x), which allows me to obtain the pose of the end effector from the pose of each individual link.

As for the tests performed, the reference signals were given in the so-called Joint Space, i.e. with respect to each individual joint. In particular, by means of the PD controller with gravity compensation with reference to Step, an initial condition of:

$$\boldsymbol{q} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \quad \dot{\boldsymbol{q}} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$$

where the initial condition for speed was required for the integrator block in Simulink.
The tracking reference considered was

$$\boldsymbol{q_d} = \begin{bmatrix} \pi/2 & \pi/3 & 0.1 & -\pi/4 \end{bmatrix}^T$$

In contrast, in the sinusoid tests, the initial condition was kept null as before, while the reference signal had a fixed amplitude and a variable frequency, specified by means of the formula:

$$\boldsymbol{q_d}(t) = \begin{bmatrix} \frac{\pi}{2}\sin(\omega t) & \frac{\pi}{2}\sin(\omega t) & 0.2\sin(\omega t) & \frac{\pi}{2}\sin(\omega t) \end{bmatrix}^T$$

Finally, the initial value of the gain considered for the derivative and proportional part of the PD was:

$$K_P = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix} \qquad K_D = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 1000 \end{bmatrix}$$

As we know from the theory seen in class, the PD controller with gravity compensation is designed for the stabilisability of a constant equilibrium posture. Indeed, we can see that it performs well in tracking the step signal. However, in subsequent tests with a sine wave signal, we can see that performance decreases.
In particular, the performance of the PD controller with gravity compensation depends on the frequency of the reference signal. This is because a sine wave signal is a typical example of a non-constant signal.
In particular, the lower the frequency of the signal, the more constant we can consider it to be, and in fact we can see from the table 3 how performance decreases with increasing frequency.

Subsequent tests have been performed by trying to modify the simulation sample time and the PD controller gain values considering the sinusoidal reference signal with pulse $w = 10rad/s$.

Possible improvements to the RMSE index were obtained by increasing the sampling time T_s, as shown in Table1.

More significant results, however, were obtained by modifying the gains of the proportional and derivative controller, as shown in table 2. Analysing the latter data, we can generally conclude, as we might have expected, that the higher the value of the controller gain, the higher the speed of the controller and thus the better the performance.

However, it is important to point out that in these simulations, saturation of the actuators was not taken into account. This in fact places a limit on the achievable controller, and therefore on the gain of the controllers.

2 Feedback linearization controller

The second controller tested was the feedback Linearisation Controller. The control in this case is achieved by a combination of the linearisation and decoupling of the system and a stabilisation part, again achieved by means of a proportional term a derivative term, but also the addition of a feedforward term, capable of ensuring perfect tracking of the trajetory under ideal conditions.

In this case, tests were carried out using only the sinusoidal reference of fixed amplitude and variable frequency

$$\boldsymbol{q_d}(t) = \begin{bmatrix} \frac{\pi}{2}\sin(\omega t) & \frac{\pi}{2}\sin(\omega t) & 0.2\sin(\omega t) & \frac{\pi}{2}\sin(\omega t) \end{bmatrix}^T$$

but with two different initial conditions

$$\boldsymbol{q} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \quad \text{and} \quad \boldsymbol{q} = \begin{bmatrix} \pi/2 & \pi/4 & 0 & 0 \end{bmatrix}^T$$

in both cases with zero initial conditions on the velocity of the joints.
The value of the gain considered for the derivative and proportional part of the PD was obtained by trial and error:

$$K_P = \begin{bmatrix} 400 & 0 & 0 & 0 \\ 0 & 400 & 0 & 0 \\ 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 400 \end{bmatrix} \qquad K_D = \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 20 \end{bmatrix}$$

In this case, this type of controller has been designed for tracking, thus generally non-constant references.
We note immediately with respect to the table 3 how the performance index RMSE is lower than that of the reference controller for each sinusoidal reference considered.

In this case, the test was also carried out with respect to a non-zero initial condition. This allows us to analyse the speed of convergence of the real trajectory with respect to the desired one. In fact, until now all our tests, starting from a null initial condition, the real trajectory and the desired one coincided.
As in the previous case, the performance index RMSE decreases with increasing frequency, also due to the much slower convergence of the two trajectories, as can be seen from the figures of the generalised coordinates for feedback linearization with non-null condition.

The same behaviour can also be observed in the central figures, where the trend of the trajectory error of each individual generalised coordinate with respect to time is shown.
The initial condition other than 0 implies a high error in the first instants, which however quickly converges to a sinusoidal type error that we can assume to be quite similar to that with respect to the corresponding graphs with null initial condition.

Simulation data performances

	PD with $sin\ w = 10$
Ts= 0.0001	0.8198
Ts= 0.00001	0.8180

Table 1: RMSE with different Ts, $K_P = 1000$ and $K_D = 1000$

	PD with $sin\ w = 10$
$K_P = 1000$, $K_D = 1000$	0.8198
$K_P = 100$, $K_D = 100$	1.3058
$K_P = 1000$, $K_D = 100$	1.4401
$K_P = 2000$, $K_D = 1000$	0.8193
$K_P = 10000$, $K_D = 1000$	0.8532

Table 2: RMSE with different K_P and K_D, Ts= 0.0001

	PD	**FL_IC_null**	**FL_IC_NON_null**
Step	0.0449		
sin $w = 0.5$	0.1150	0.0227	0.0716
sin $w = 3$	0.4008	0.1392	0.2138
sin $w = 6$	0.6148	0.3084	0.3990
sin $w = 10$	0.8198	0.7856	0.9263

Table 3: RMSE with Ts = 0.0001

Figure PD

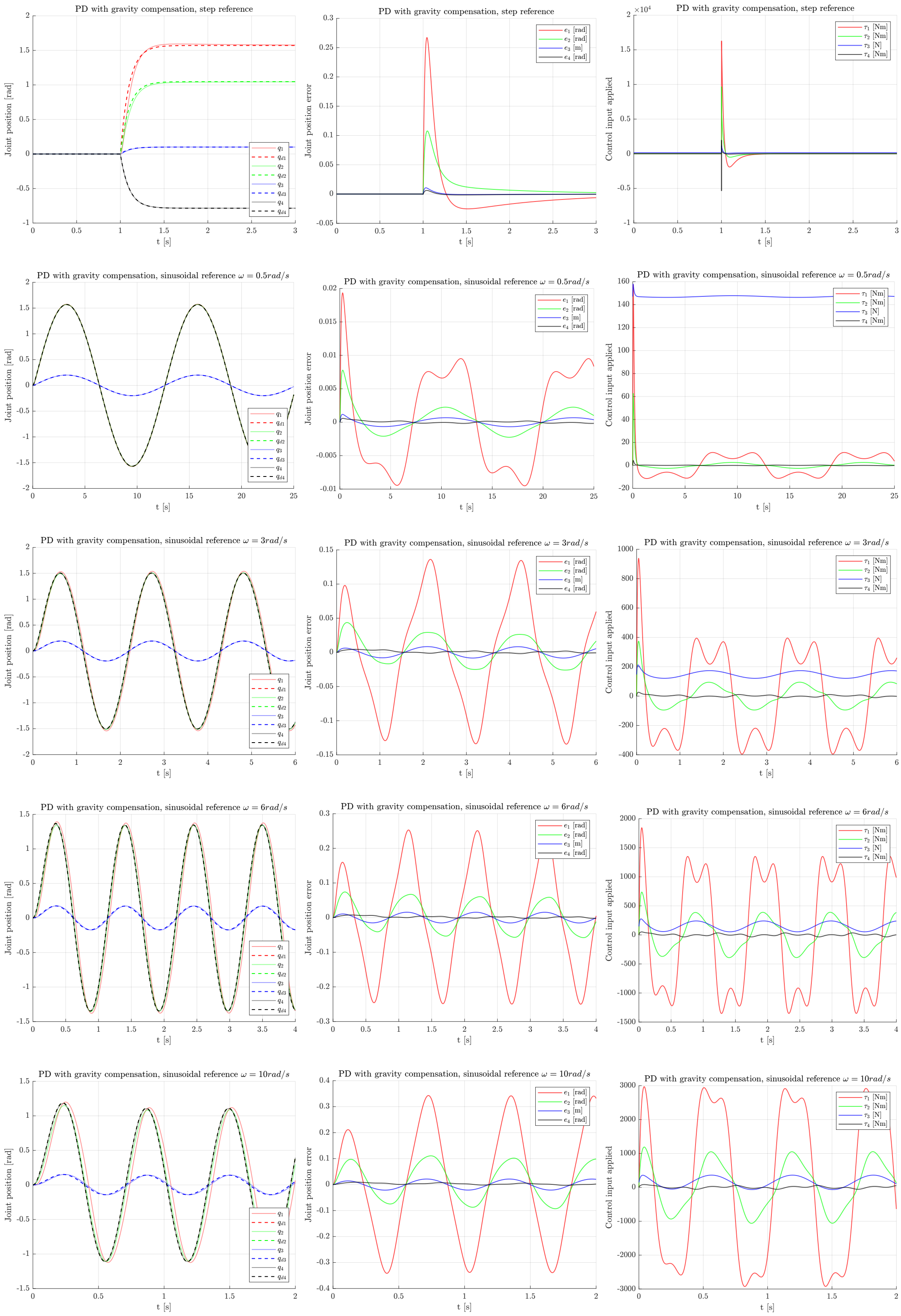
PD with gravity compensation, step reference
PD with gravity compensation, sinusoidal reference $\omega = 0.5rad/s$
PD with gravity compensation, sinusoidal reference $\omega = 3rad/s$
PD with gravity compensation, sinusoidal reference $\omega = 6rad/s$
PD with gravity compensation, sinusoidal reference $\omega = 10rad/s$
Joint position [rad]
Joint position error
Control input applied
t [s]
q_1 q_{d1} q_2 q_{d2} q_3 q_{d3} q_4 q_{d4}
e_1 [rad] e_2 [rad] e_3 [m] e_4 [rad]
τ_1 [Nm] τ_2 [Nm] τ_3 [N] τ_4 [Nm]
$\times 10^4$

Figure FL null initial condition

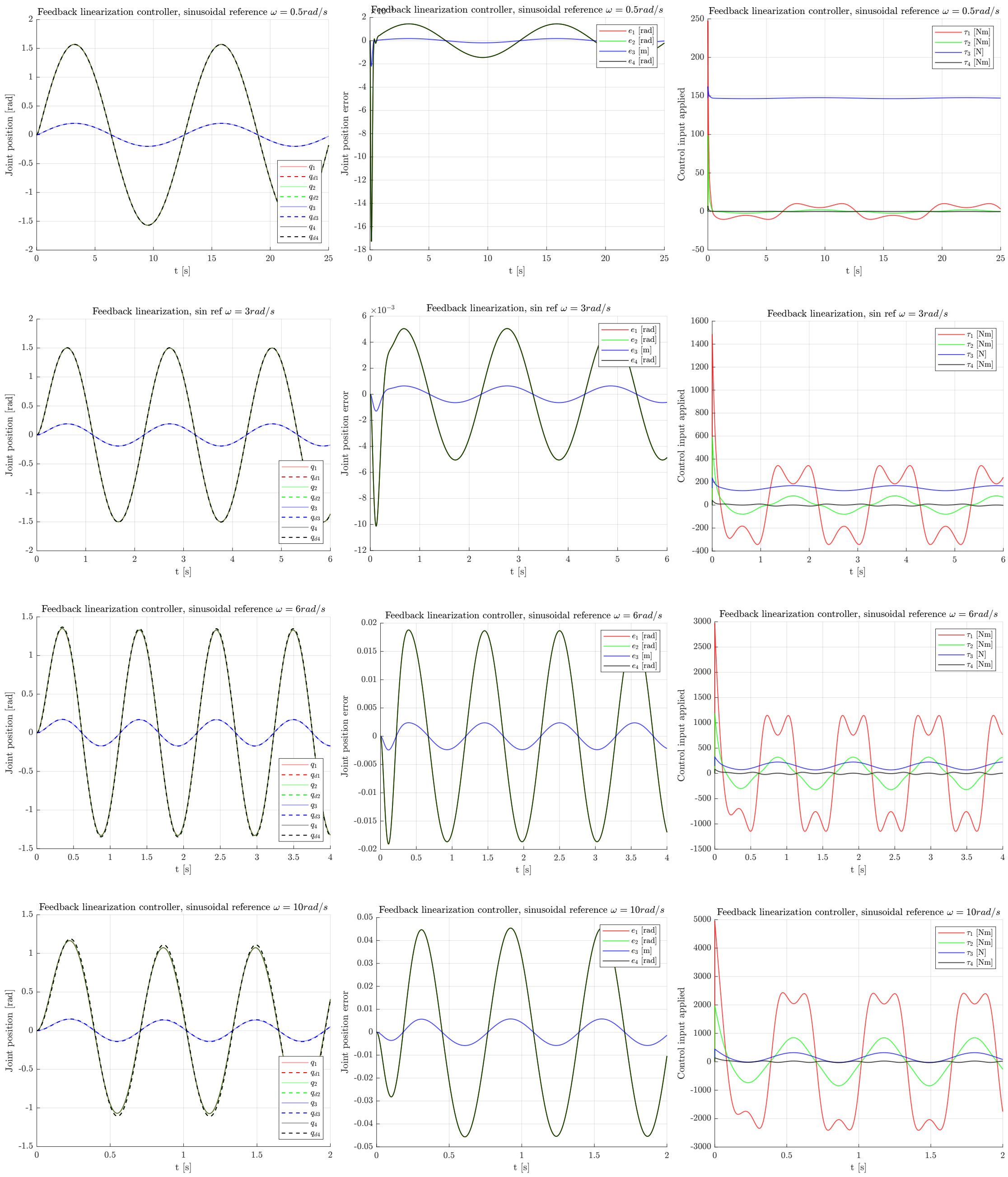

Figure FL NON null initial condition