Es Calcolare $\mathcal{F}\left(\frac{t\sin(2t)}{(t^2+4)^2}\right)$.

Sol

$g(t) = \frac{t\sin(2t)}{(t^2+4)^2}$ è limitata (anche sommabile).

$\mathcal{F}\left(\frac{t}{(t^2+4)^2}\right)$ è facilmente calcolabile con una tecnica vista a lezione:

$$\mathcal{F}\left(\frac{t}{(t^2+4)^2}\right)(\omega) = \mathcal{F}\left(\frac{d}{dt}\left(-\frac{1}{2(t^2+4)}\right)\right)(\omega)$$

$$= (2\pi i\omega)\,\mathcal{F}\left(-\frac{1}{2(t^2+4)}\right)(\omega) = -\pi i\omega\,\mathcal{F}\left(\frac{1}{4+t^2}\right)(\omega)$$

$$= -\pi i\omega\,\frac{\pi}{2}\,e^{-2\pi 2|\omega|} = -\frac{\pi^2}{2} i\omega\, e^{-4\pi|\omega|}$$

Quindi

$$\mathcal{F}\left(\frac{t\sin(2t)}{(t^2+4)^2}\right)(\omega) = \mathcal{F}\left(\frac{t}{(t^2+4)^2}\cdot\frac{e^{2it}-e^{-2it}}{2i}\right)(\omega)$$

$$= \frac{1}{2i}\left[\mathcal{F}\left(e^{2it}\frac{t}{(t^2+4)^2}\right)(\omega) - \mathcal{F}\left(e^{-2it}\frac{t}{(t^2+4)^2}\right)(\omega)\right]$$

$$= \frac{1}{2i}\left[\mathcal{F}\left(e^{2\pi i\left(\frac{1}{\pi}\right)t}\frac{t}{(t^2+4)^2}\right)(\omega) - \mathcal{F}\left(e^{2\pi i\left(-\frac{1}{\pi}\right)t}\frac{t^2}{(t^2+4)^2}\right)(\omega)\right]$$

$$= \frac{1}{2i}\left[\mathcal{F}\left(\frac{t}{(t^2+4)^2}\right)\left(\omega-\frac{1}{\pi}\right) - \mathcal{F}\left(\frac{t}{(t^2+4)^2}\right)\left(\omega+\frac{1}{\pi}\right)\right]$$

$$= \frac{1}{2i}\left[-\frac{\pi^2}{2}i\left(\omega-\frac{1}{\pi}\right)e^{-4\pi\left|\omega-\frac{1}{\pi}\right|} + \frac{\pi^2}{2}i\left(\omega+\frac{1}{\pi}\right)e^{-4\pi\left|\omega+\frac{1}{\pi}\right|}\right]$$

"volendo"

$$\downarrow$$

$$= \frac{\pi^2}{4}\left[-\left(\frac{\pi\omega-1}{\pi}\right)e^{-4\pi\left|\omega-\frac{1}{\pi}\right|} + \left(\frac{\pi\omega+1}{\pi}\right)e^{-4\pi\left|\omega+\frac{1}{\pi}\right|}\right]$$

$$= \frac{\pi}{4}\left[(1-\pi\omega)e^{-4\pi\left|\omega-\frac{1}{\pi}\right|} + (1+\pi\omega)e^{-4\pi\left|\omega+\frac{1}{\pi}\right|}\right]$$