Es.

Trovare la serie di Laurent della funzione

$$f(z) = \frac{z+1}{z-1}$$

con centro $z_0 = 0$ nell'insieme $\{z \in \mathbb{C} : |z| > 1\}$

Sol

$$f(z) = (z+1)\frac{1}{z-1} = (z+1)\frac{1}{z\left(1-\frac{1}{z}\right)}$$

$$= \left(\frac{z+1}{z}\right)\frac{1}{1-\frac{1}{z}}$$

serie geometrica di ragione $\frac{1}{z}$

$$= \left(\frac{z+1}{z}\right)\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n$$

se e solo se $\left|\frac{1}{z}\right| < 1 \iff \frac{1}{|z|} < 1 \iff 1 < |z|$ OK

$$= \left(1+\frac{1}{z}\right)\sum_{n=0}^{\infty}\frac{1}{z^n}$$

$$= \sum_{n=0}^{\infty}\frac{1}{z^n} + \sum_{n=0}^{\infty}\frac{1}{z^{n+1}}$$

$$\left[= \left(1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right) + \left(\frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right) \right]$$

$$= \left(1 + \sum_{n=1}^{\infty} \frac{1}{z^n}\right) + \left(\sum_{n=0}^{\infty} \frac{1}{z^{n+1}}\right)$$

$$= \left(1 + \sum_{n=0}^{\infty} \frac{1}{z^{n+1}}\right) + \left(\sum_{n=0}^{\infty} \frac{1}{z^{n+1}}\right)$$

$$= 1 + \sum_{n=0}^{\infty} \left(\frac{1}{z^{n+1}} + \frac{1}{z^{n+1}}\right)$$

$$= 1 + \sum_{n=0}^{\infty} \frac{2}{z^{n+1}} \quad \forall z \in \{z \in \mathbb{C} : 1 < |z|\}$$