Formulario di Teoria ed Elaborazione dei Segnali

AA. 2015-2016

Formule trigonometriche

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$	$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$
$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$	$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$
$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$	$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$
$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$	

Serie e integrali

$\sum_{k=0}^{N} r^k = \frac{1-r^{N+1}}{1-r}$	$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$
$\sum_{k=0}^{+\infty} r^k = \frac{1}{1-r}$	

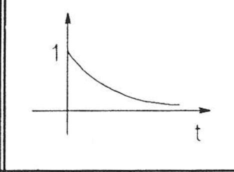
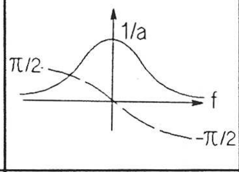
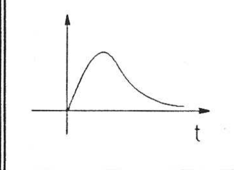
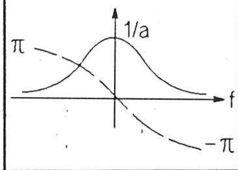
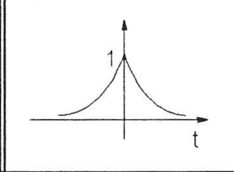
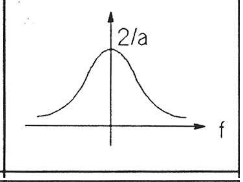
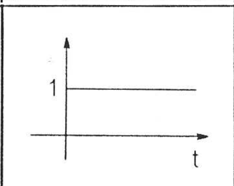
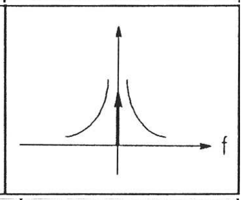
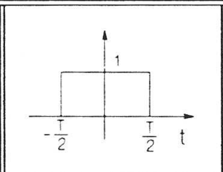
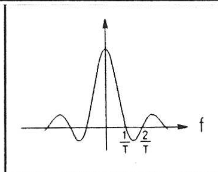
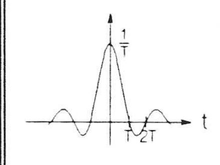
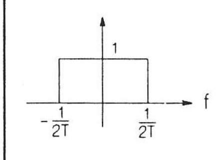
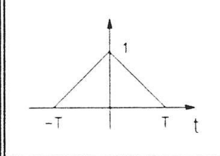
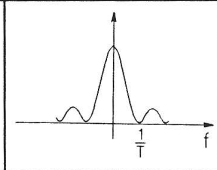
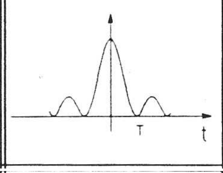
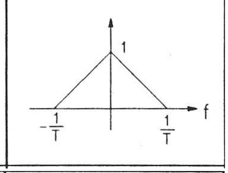
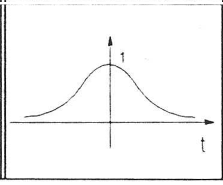
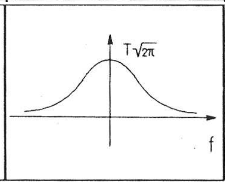
La funzione Gaussiana o distribuzione normale

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\nu^2}d\nu$	$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_x^{+\infty} e^{-\nu^2}d\nu$
$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-\nu^2}d\nu$	$Q(x) = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$

Trasformate di Fourier

inversione assi	$x(-t)$	$X(-f)$
coniugazione	$x^*(t)$	$X^*(-f)$
anticipo o ritardo	$x(t\pm\theta)$	$X(f)e^{\pm j2\pi f\theta}$
scalamento in t	$x(kt)$	$\frac{1}{\lvert k\rvert}X(\frac{f}{k})$
scalamento in f	$\frac{1}{\lvert k\rvert}x(\frac{t}{k})$	$X(kf)$
traslazione in f	$x(t)e^{\pm j2\pi f_0 t}$	$X(f\mp f_0)$
derivazione	$\dot{x}(t)$	$j2\pi f X(f)$
integrazione	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{2}X(0)\delta(f) + X(f)/j2\pi f$
dualità	$X(t)$	$x(-f)$

	Funzione del tempo $x(t)$		Funzione della frequenza $X(f)$	
1		$e^{-at}u(t)$ $(a>0)$	$\dfrac{1}{a+j2\pi f}$	
2		$ate^{-at}u(t)$ $(a>0)$	$\dfrac{a}{(a+j2\pi f)^2}$	
3		$e^{-a\|t\|}$ $(a>0)$	$\dfrac{2a}{a^2+4\pi^2 f^2}$	
7		$u(t)=\begin{cases}1, & t>0\\ 0, & t<0\end{cases}$	$\dfrac{1}{j2\pi f}+\dfrac{\delta(f)}{2}$	
23		$p_T(t)=\begin{cases}1, & \|t\|<T/2\\ 0, & \|t\|>T/2\end{cases}$	$T\,\mathrm{Sinc}(fT)$ $=\dfrac{\sin(\pi fT)}{\pi f}$	
24		$\dfrac{1}{T}\,\mathrm{Sinc}(t/T)=$ $\dfrac{\sin(\pi t/T)}{\pi t}$	$p_{1/T}(f)=\begin{cases}1, & \|f\|<1/2T\\ 0, & \|f\|>1/2T\end{cases}$	
25		$\mathrm{tri}(t/T)$ $=\begin{cases}1-\|t\|/T, & \|t\|<T\\ 0, & \|t\|>T\end{cases}$	$T\,\mathrm{Sinc}^2(fT)$ $=T\dfrac{\sin^2(\pi fT)}{(\pi fT)^2}$	
26		$\dfrac{1}{T}\mathrm{Sinc}^2(t/T)$ $=T\left[\dfrac{\sin(\pi t/T)}{\pi t}\right]^2$	$\mathrm{tri}(fT)$ $=\begin{cases}1-\|f\|T, & \|f\|<1/T\\ 0, & \|f\|>1/T\end{cases}$	
28		$e^{-t^2/2T^2}$	$T\sqrt{2\pi}e^{-2\pi^2 f^2 T^2}$	

Trasformata zeta

Sequenza $x(n),\ y(n)$	$X(z),\ Y(z)$	ROC $R_x,\ R_y$
$x(n-N)$	$z^{-N}X(z)$	se $N>0 \rightarrow R_x\backslash\{z=0\}$ se $N<0 \rightarrow R_x\backslash\{z=\infty\}$
$\alpha_1 x(n)+\alpha_2 y(n),\ \alpha_1,\alpha_2$ costanti	$\alpha_1 X(z)+\alpha_2 Y(z)$	contiene $R_x \cap R_y$
$x(-n)$	$X(z^{-1})$	$\frac{1}{R_x}$
$x^*(n)$	$X^*(z^*)$	R_x
$x^*(-n)$	$X^*(\frac{1}{z^*})$	$\frac{1}{R_x}$
$\Re(x(n))$	$\frac{1}{2}\left[X(z)+X^*(z^*)\right]$	contiene R_x
$\Im(x(n))$	$\frac{1}{2j}\left[X(z)-X^*(z^*)\right]$	contiene R_x
$x(-n)u(-n-1)$	$X(z^{-1})-x(0)$, $x(n)$ causali	$-$
$\alpha^n x(n)$	$X(z/\alpha)$	$\lvert\alpha\rvert \cdot R_x$
$nx(n)$	$-z\frac{d}{dz}X(z)$	R_x meno $z=\infty$ o $z=0$
$nx(-n)$	$-z\frac{d}{dz}X(z^{-1})$	contiene $\frac{1}{R_x}$
$n\alpha^n x(n)$	$-z\frac{d}{dz}X(z/\alpha)$	$\lvert\alpha\rvert \cdot R_x$ meno $z=\infty$ o $z=0$
$\cos(2\pi fn)x(n)$	$\frac{1}{2}\left[X(ze^{j2\pi f})+X(ze^{-j2\pi f})\right]$	$-$
$\sin(2\pi fn)x(n)$	$\frac{j}{2}\left[X(ze^{j2\pi f})-X(ze^{-j2\pi f})\right]$	$-$
$x(n)\star y(n)$	$X(z)Y(z)$	contiene $R_x \cap R_y$

$u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u(-n-1)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\alpha^n u(n)$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-\alpha^n u(-n-1)$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
$n\alpha^n u(n)$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$(n+1)\alpha^n u(n)$	$\frac{1}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$n\alpha^{n-1}u(n)$	$\frac{z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$(n-1)\alpha^n u(n)$	$\frac{2\alpha z^{-1}-1}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$n^2\alpha^n u(n)$	$\frac{\alpha z^{-1}(1+\alpha z^{-1})}{(1-\alpha z^{-1})^3}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-n\alpha^n u(-n-1)$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$

DTFT

segnale	DTFT
$\mathrm{sgn}(n)$	$\frac{1+\exp(-j2\pi f)}{1-\exp(-j2\pi f)}$
$u(n)$	$\frac{1}{2}\delta(f) + \frac{1}{1-\exp(-j2\pi f)}$
$\exp(j2\pi f_0 n)$	$\delta(f - f_0)$
$\mathrm{sinc}(n/N)$	$NP_{1/N}(f)$
$p_{2K+1}(n)$	$\frac{\sin(\pi f(2K+1))}{\sin(\pi f)}$