CAP 1, ES 10

Scrivere $f(x+iy) = x^2 - y^2 - 2y + 2ix(1-y)$

come una funzione di $z = x+iy$ $\quad (x, y \in \mathbb{R})$

SOL

$$
\begin{aligned}
f(x+iy) &= x^2 - y^2 - 2y + 2ix(1-y) \\
&= x^2 - y^2 - 2y + 2ix - 2ixy \\
&= (x-iy)^2 - 2(y-ix) \\
&= (x-iy)^2 + 2i(x+iy) \\
&= \bar{z}^2 + 2iz
\end{aligned}
$$

Oppure si sostituiscono x con $\frac{z+\bar{z}}{2}$ e y con $\frac{z-\bar{z}}{2i}$:

$$
\begin{aligned}
f(x+iy) = &\left(\frac{z+\bar{z}}{2}\right)^2 - \left(\frac{z-\bar{z}}{2i}\right)^2 - 2\left(\frac{z-\bar{z}}{2i}\right) + 2i\left(\frac{z+\bar{z}}{2}\right) \\
&- 2i\left(\frac{z+\bar{z}}{2}\right)\left(\frac{z-\bar{z}}{2i}\right)
\end{aligned}
$$

$$= \frac{z^2}{4} + \frac{z\bar{z}}{2} + \frac{\bar{z}^2}{4} + \left(\frac{z^2 - 2z\bar{z} + \bar{z}^2}{4} \right) + iz \cancel{- i\bar{z}}$$

$$+ iz \cancel{+ i\bar{z}} - \frac{1}{2}\left(z^2 - \bar{z}^2\right)$$

$$= \cancel{\frac{z^2}{4}} + \cancel{\frac{z\bar{z}}{2}} + \frac{\bar{z}^2}{4} + \cancel{\frac{z^2}{4}} - \cancel{\frac{z\bar{z}}{2}} + \frac{\bar{z}^2}{4} + 2iz$$

$$\cancel{- \frac{z^2}{2}} + \frac{\bar{z}^2}{2}$$

$$= \bar{z}^2 + 2iz$$