Quiz Se $T_n = x^2 \delta_{e^n} \in \mathcal{D}'(\mathbb{R})$ $\forall n \in \mathbb{N}$, allora:

(A) $T_n \to \delta_0$ in $\mathcal{D}'(\mathbb{R})$

(B) $T_n \to 1$ in $\mathcal{D}'(\mathbb{R})$

(C) T_n non ammette limite in $\mathcal{D}'(\mathbb{R})$

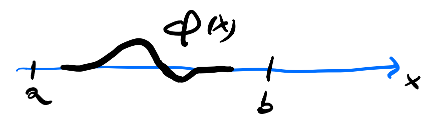
(D) $T_n \to 0$ in $\mathcal{D}'(\mathbb{R})$

Sol

$\forall n \in \mathbb{N}$ si ha $T_n = x^2 \delta_{e^n} = (e^n)^2 \delta_{e^n} = e^{2n} \delta_{e^{2n}}$

quindi se $\varphi \in \mathcal{D}(\mathbb{R})$, esistono $a, b \in \mathbb{R}$, $a < b$, tali che

$\varphi(x) = 0$ $\forall x \notin [a,b]$, per cui



$$\langle T_n, \varphi \rangle = \langle e^{2n} \delta_{e^{2n}}, \varphi \rangle = e^{2n} \langle \delta_{e^{2n}}, \varphi \rangle$$

$= e^{2n} \varphi(e^{2n}) = e^{2n} \cdot 0 = 0$ $\forall n \geq n_0$ per un opportuno $n_0 \in \mathbb{N}$

(per la precisione per $e^{2n_0} > b$, ma non è importante il valore di n_0)

$$\Rightarrow \lim_{n\to\infty} \langle T_n, \varphi \rangle = 0 = \langle 0, \varphi \rangle$$

$$\Rightarrow T_n \to 0 \text{ in } \mathcal{D}'(\mathbb{R}) \longrightarrow (D)$$