Principles of Electronics and Electronic Systems - 01MCJHX

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Exercises on Operational Amplifiers

1 Exercises on Ideal Opamps

Assuming that the operational amplifier is ideal, evaluate the expression of the voltage v_{OUT} in the circuit of Fig.1 in terms of the independent sources v_1, v_2 and v_3.

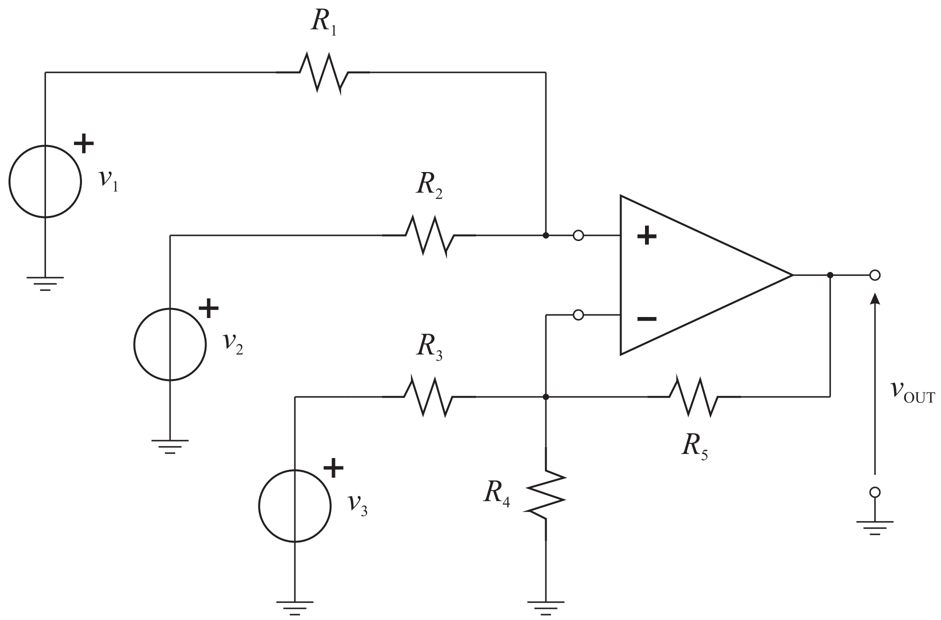

Figure 1: Circuits to be analyzed.

1.1 Direct Analysis Method

In any negative feedback opamp-based circuit, the opamp output voltage can be expressed by KVL as

$$v_{\mathrm{OUT}} = v^{+} - v_{\mathrm{D}} + v_{\mathrm{RF}} \tag{1}$$

where v^{+} is the non-inverting terminal voltage, v_{D} is the differential input voltage of the opamp and v_{RF} is the voltage across the feedback resistor R_{F}, as depicted in Fig.2. Moreover, since an ideal opamp is considered, it follows that

$$\begin{cases} v_{\mathrm{D}} = v^{+} - v^{-} = 0 \\ i^{+} = i^{-} = 0 \end{cases} \tag{2}$$

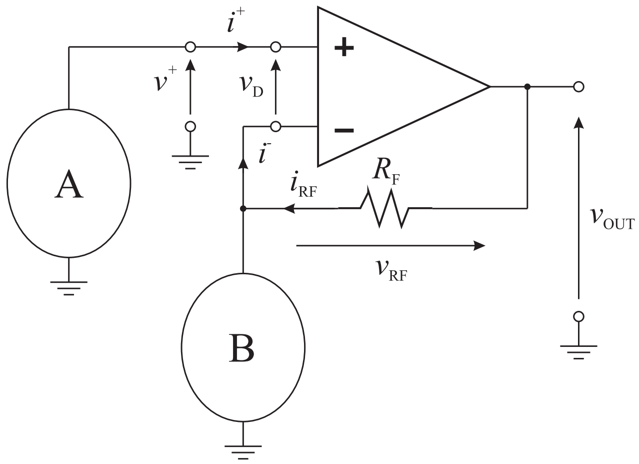

Figure 2: Analysis of negative-feedback opamp circuits.

and Eqn.(1) gives

$$\boxed{v_{OUT} = v^+ + v_{RF}} \tag{3}$$

On the basis of Eqn.(3), the output voltage v_{OUT} can be evaluated in three steps:

- evaluation of v^+
- evaluation of v_{RF}
- calculation of v_{OUT} by Eqn.(3).

Evaluation of v^+

In order to evaluate v^+, it can be observed that, from Eqn.(2), the current i^+ is zero, so the non-inverting input port of the opamp is equivalent to an open circuit and does not affect the part of the circuit connected to it. As a consequence, the voltage v^+ can be evaluated without considering the connection to the opamp circuit, as depicted in Fig.3. It can be observed that such a voltage depends only on the network A in Fig.3, which is directly or indirectly connected to the opamp non-inverting input.

With reference to the circuit in Fig.11, the network that is obtained at this first step is depicted in Fig.4 and includes the resistors R_1 and R_2 and the independent sources v_1 and v_2. The voltage v^+ can be evaluated, either by superposition or by the Millman theorem, as

$$v^+ = v_1 \frac{R_2}{R_1 + R_2} + v_2 \frac{R_1}{R_1 + R_2} \tag{4}$$

Evaluation of v_{RF}

In order to evaluate the voltage v_{RF}, it is convenient to evaluate the current i_{RF} flowing through R_{RF}. Once i_{RF} is known, v_{RF} is immediately evaluated as

$$v_{RF} = R_F i_{RF}.$$

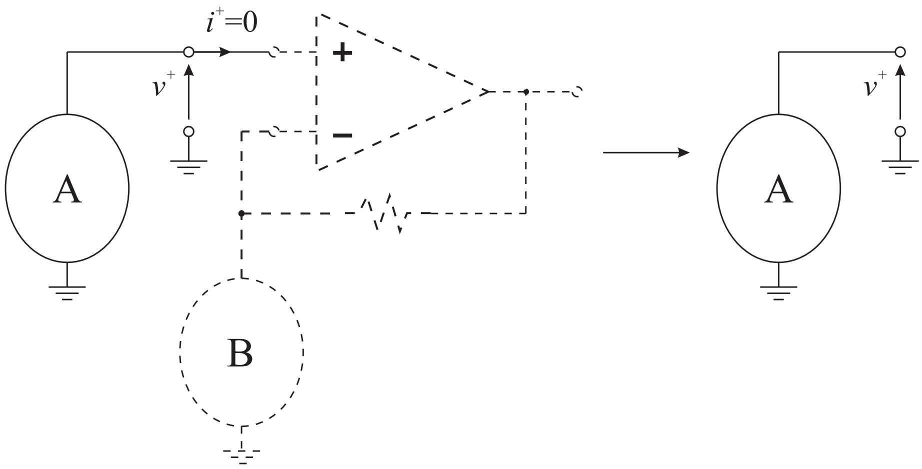

Figure 3: Analysis of negative-feedback opamp circuits: evaluation of v^+.

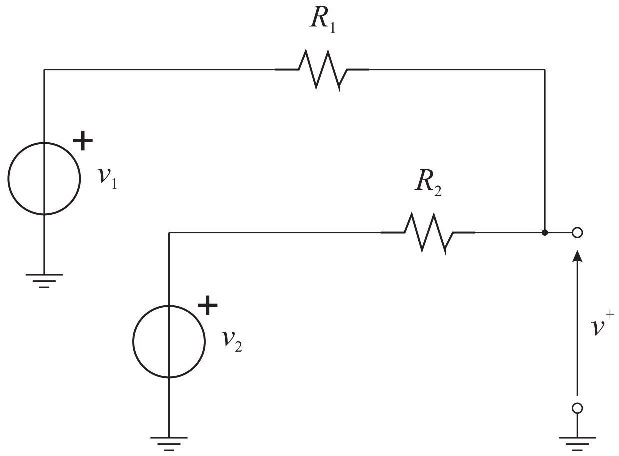

Figure 4: Network to be considered for the evaluation of v^+ in the circuit of Fig.1.

Since the current i^- entering the inverting input of the opamp is zero from Eqn.(2), the current i_{RF} is equal to the current i_{B} entering the network B connected to the opamp inverting input. In order to evaluate such a current, it should be recalled that, again from Eqn.(2), the voltage v^- at the inverting input of the opamp is forced to the voltage v^+ at the opamp non-inverting input (that has been evaluated above) because of negative feedback. As a consequence, the current i_{RF} can be evaluated considering the network B connected to the opamp inverting input and considering the inverting input as an ideal voltage source which forces a voltage v^+, as depicted in Fig.5.

With reference to the circuit in Fig.11, the circuit to be considered to evaluate i_{RF} is reported in Fig.6. Either by superposition or by considering KCL, which gives that

$$i_{\mathrm{RF}} = i_{\mathrm{R3}} + i_{\mathrm{R4}}$$

where

$$i_{\mathrm{R3}} = \frac{v^+ - v_3}{R_3}$$

and

$$i_{\mathrm{R4}} = \frac{v^+}{R_4},$$

the expression of i_{RF} in terms of v_3 and the equivalent source v^+ is

$$i_{\mathrm{RF}} = \frac{v^+}{R_3 \parallel R_4} - \frac{v_3}{R_3} \tag{5}$$

as a consequence, taking into account that $R_{\mathrm{F}} = R_5$

$$v_{\mathrm{RF}} = R_5 i_{\mathrm{RF}} = v^+ \frac{R_5}{R_3 \parallel R_4} - v_3 \frac{R_5}{R_3} \tag{6}$$

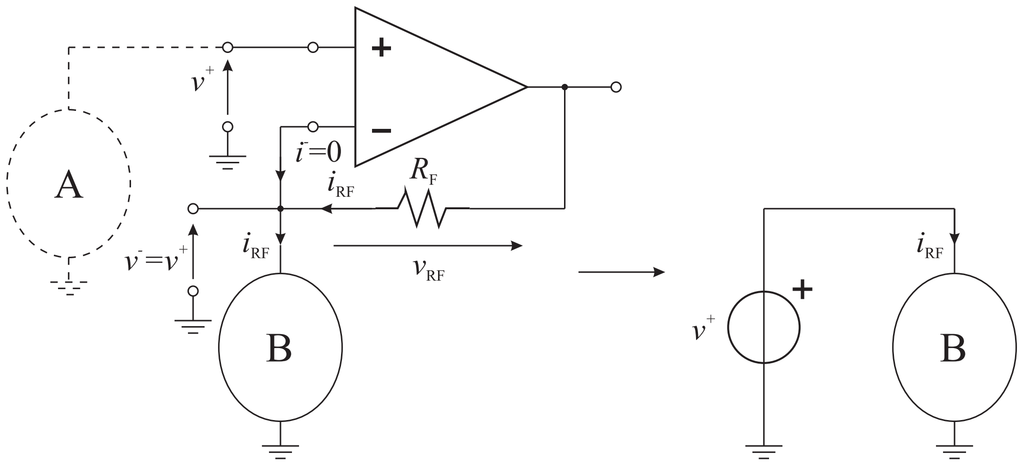

Figure 5: Analysis of negative-feedback opamp circuits: evaluation of v_{RF}.

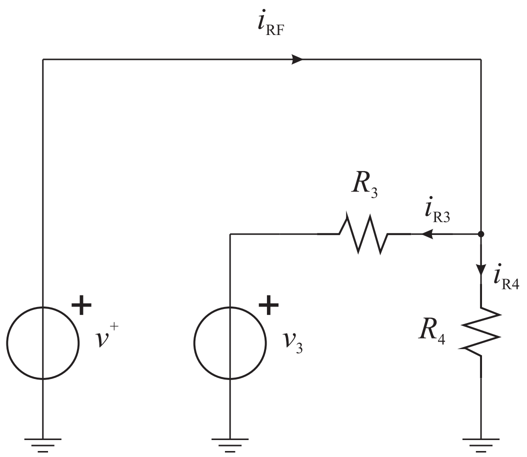

Figure 6: Network to be considered for the evaluation of i_{RF} in the circuit of Fig.11.

Evaluation of v_{OUT}

Since both v^+ and i_{RF} have been calculated, the opamp output voltage v_{OUT} can be evaluated by Eqn.(3). With reference to the circuit in Fig.11, one finally gets

$$\begin{aligned} v_{\mathrm{OUT}} &= v^+ + v_{\mathrm{RF}} \\ &= v^+ + v^+ \frac{R_5}{R_3 \parallel R_4} - v_3 \frac{R_5}{R_3} \\ &= \left(v_1 \frac{R_2}{R_1 + R_2} + v_2 \frac{R_1}{R_1 + R_2} \right) \left(1 + \frac{R_5}{R_3 \parallel R_4} \right) - v_3 \frac{R_5}{R_3} \end{aligned} \tag{7}$$

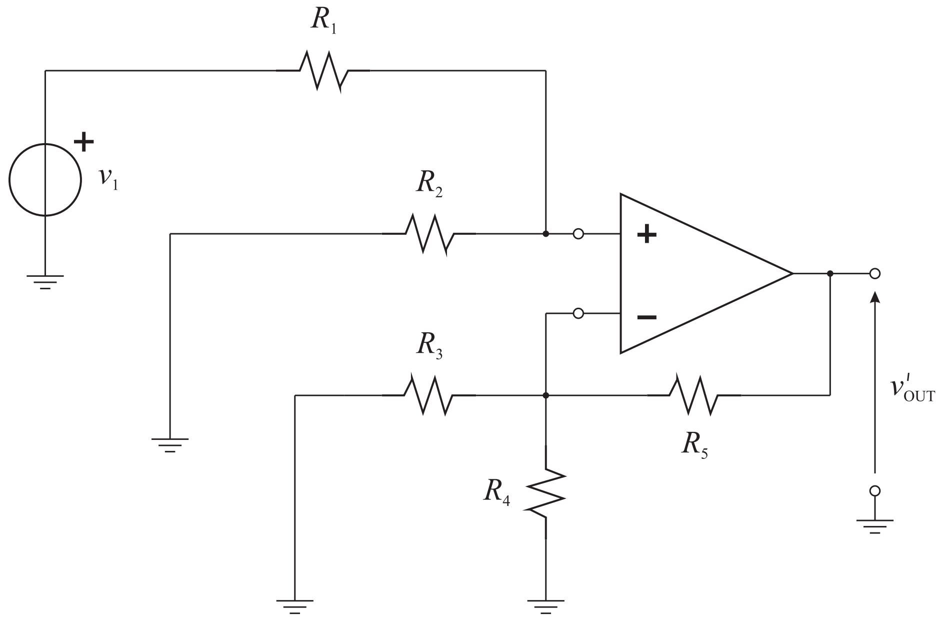

Figure 7: Network to be considered for the evaluation of the contribution $v'_{\rm OUT}$ of v_1 in the circuit of Fig.11.

1.2 Reduction to Basic Configurations

The analysis of the circuit in Fig.11 can be also performed by superposition recalling the properties of basic amplifier configurations. According to this method, the output voltage $v_{\rm OUT}$ can be evaluated by superposition as

$$v_{\rm OUT} = v'_{\rm OUT} + v''_{\rm OUT} + v'''_{\rm OUT} \tag{8}$$

where $v'_{\rm OUT}$ is the contribution of v_1, $v''_{\rm OUT}$ is the contribution of v_2 and $v'''_{\rm OUT}$ is the contribution of v_3.

The contribution of v_1 can be evaluated with reference to the circuit in Fig.7. In such a circuit, the voltage v^+ at the opamp non-inverting input is

$$v^+ = v_1 \frac{R_2}{R_1 + R_2}.$$

and resistors R_3 and R_4 are in parallel and can be replaced by their equivalent resistance $R_3 \parallel R_4$. Taking into account of the above considerations, the circuit in Fig.7 can be reduced to the circuit in Fig.8, that is analogous to the voltage amplifier basic topology. Recalling such a configuration, the output voltage $v'_{\rm OUT}$ is immediately evaluated as

$$v'_{\rm OUT} = v_1 \frac{R_2}{R_1 + R_2}\left(1 + \frac{R_5}{R_3 \parallel R_4}\right) \tag{9}$$

By a similar reasoning, the contribution of the voltage source v_2 can be evaluated as

$$v''_{\rm OUT} = v_2 \frac{R_1}{R_1 + R_2}\left(1 + \frac{R_5}{R_3 \parallel R_4}\right). \tag{10}$$

In order to evaluate the contribution of v_3, the circuit in Fig.9 is to be considered. In such a circuit, the current flowing through resistors R_1 and R_2 is zero since the current entering the

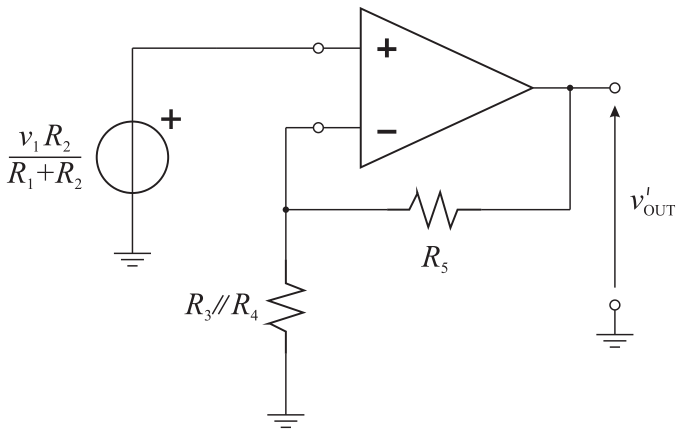

Figure 8: Equivalent non-inverting voltage amplifier for the evaluation of the contribution v'_{OUT} of v_1 in the circuit of Fig.11.

opamp non-inverting input is zero. As a consequence, the voltage drop across them is zero and can be replaced by a short circuit connection of the non-inverting input to the reference voltage. Moreover, since the voltage v^+ is zero, the voltage v^- at the opamp inverting input is zero as well. As a consequence, the voltage drop across the resistor R_4 is zero, no current flows through it and it gives no contribution in the evaluation of the output voltage v'''_{OUT}. Taking into account of the above considerations, the circuit in Fig.9 can be reduced to the circuit in Fig.10, that is analogous to the so-called inverting amplifier topology. Recalling such a configuration, the output voltage v'''_{OUT} is immediately evaluated as

$$v'''_{\mathrm{OUT}} = -v_3 \frac{R_5}{R_3}. \tag{11}$$

By merging the results of Eqn.(9), Eqn.(10) and Eqn.(11), one finally gets

$$\begin{aligned} v_{\mathrm{OUT}} &= v'_{\mathrm{OUT}} + v''_{\mathrm{OUT}} + v'''_{\mathrm{OUT}} \\ &= v_1 \frac{R_2}{R_1+R_2}\left(1+\frac{R_5}{R_3 \parallel R_4}\right) + v_2 \frac{R_1}{R_1+R_2}\left(1+\frac{R_5}{R_3 \parallel R_4}\right) - v_3 \frac{R_5}{R_3} \\ &= \left(v_1 \frac{R_2}{R_1+R_2} + v_2 \frac{R_1}{R_1+R_2}\right)\left(1+\frac{R_5}{R_3 \parallel R_4}\right) - v_3 \frac{R_5}{R_3} \end{aligned} \tag{12}$$

as obtained in Eqn.(7).

The same method could be employed by first evaluating the overall voltage at the opamp non-inverting terminal and then considering the voltage at the non-inverting terminal as an ideal voltage source. By so doing, the superposition principle can be applied considering this equivalent source v^+ and the source v_3 as input sources.

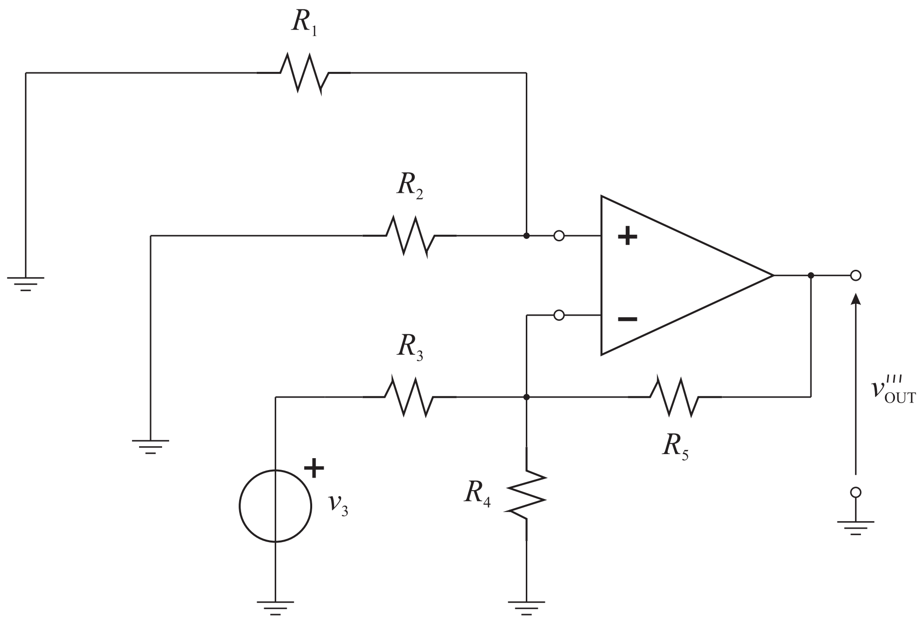

Figure 9: Network to be considered for the evaluation of the contribution v'''_{OUT} of v_3 in the circuit of Fig.11.

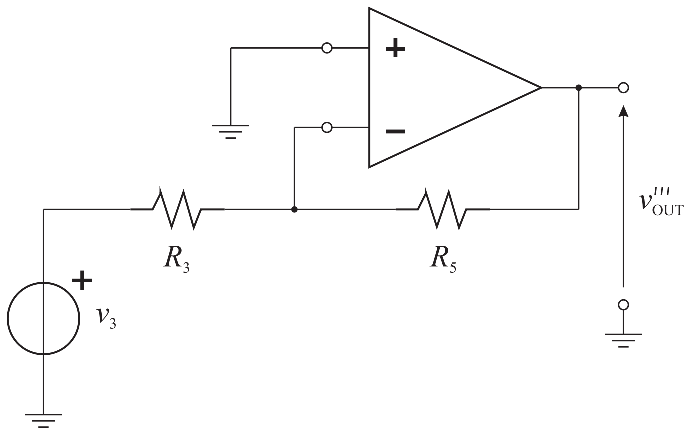

Figure 10: Equivalent inverting voltage amplifier for the evaluation of the contribution v'''_{OUT} of v_3 in the circuit of Fig.11.

Further considerations on circuits with ideal opamps

Some hints that can be useful when solving circuits including opamp circuits are now given.

- Circuits including more than one opamp circuit can be analyzed by considering first the opamp circuits whose inputs are only connected to independent sources and evaluating their output voltages using the techniques considered before. Then, since the output of an ideal opamp can be considered as an ideal voltage source, such amplifiers can be replaced by ideal voltage sources with the previously calculated values and the rest of the circuit can be solved.
- Ideal opamp circuits are linear elements, so circuits including ideal opamps and other linear elements can be analyzed in the frequency domain and transfer functions relating their input and output quantities can be defined. To this purpose, passive components are described in terms of their impedances/admittances.
- When a quantity different from the opamp output voltage is requested, the above technique should be adapted to the requested output (e.g. Eqn.(3) should be replaced with another relation). Nonetheless, the first two steps considered in the direct solution method are useful to solve almost all ideal opamp-based circuits.

Exercises

Exercise 1. Evaluate the expression of the voltage v_{OUT} of the circuit of Fig.11 in terms of the independent sources v_1, i_2 and v_3 for $R_1 = R_2 = R_3 = R_4 = R_5 = 10\mathrm{k}\Omega$.

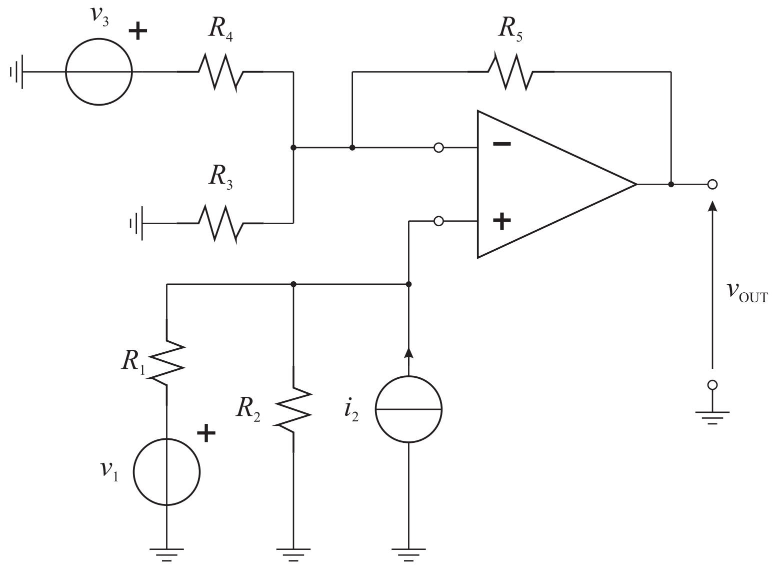

Figure 11: Circuit for Ex.1.

Exercise 2. Evaluate the expression of the voltage v_{OUT} of the circuit of Fig.12 in terms of the independent sources v_1, i_2 and v_3 for $R_1 = R_2 = R_3 = R_4 = 10\mathrm{k}\Omega$ and $R_5 = 50\mathrm{k}\Omega$.

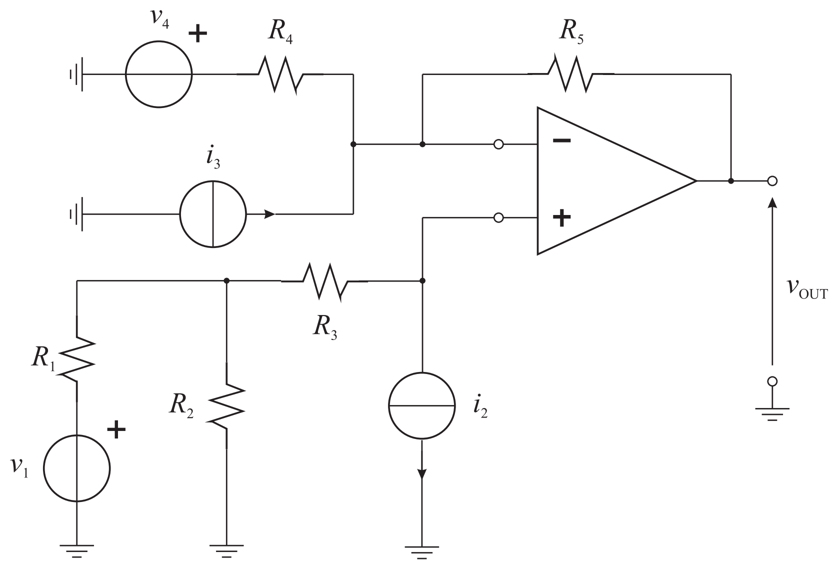

Figure 12: Circuit for Ex.2.

Exercise 3. Evaluate the expression of the voltage v_{OUT} of the circuit of Fig.13 in terms of the independent sources v_1, i_2 and v_3.

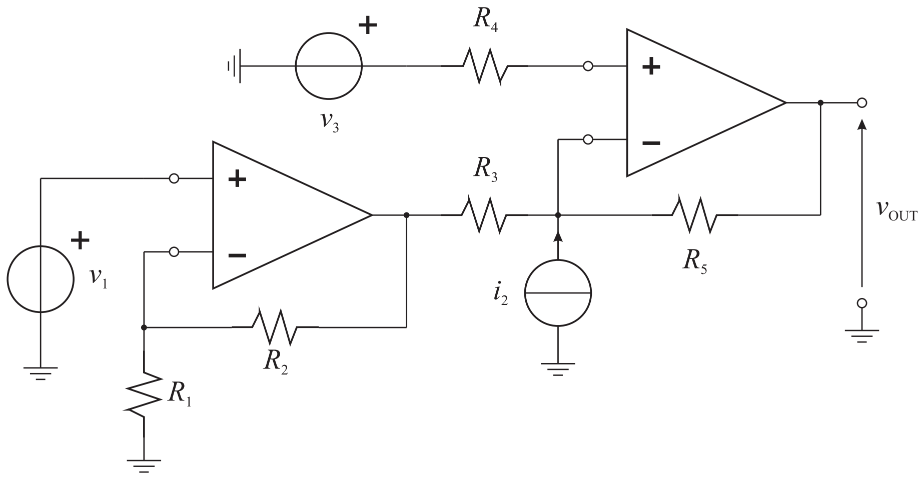

Figure 13: Circuit for Ex.3.

Exercise 4. Evaluate the expression of the voltage v_{OUT} of the circuit of Fig.14 in terms of the independent sources v_1, i_2 and v_3.

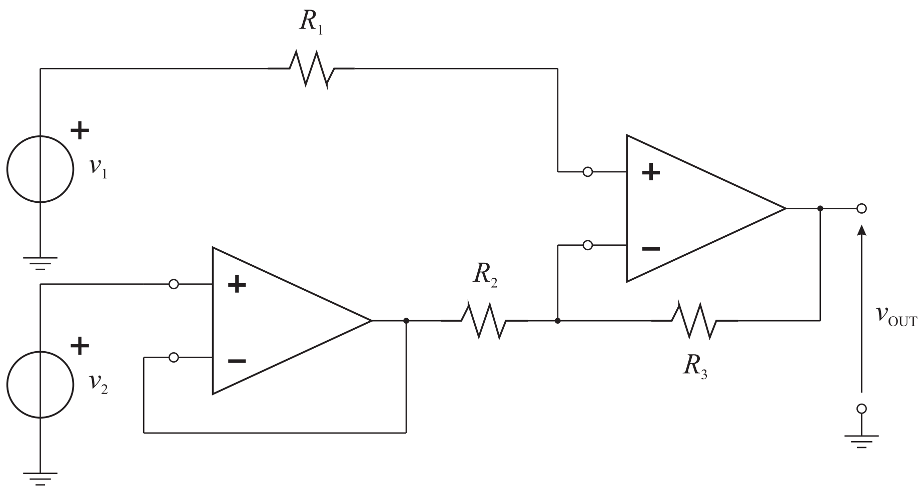

Figure 14: Circuit for Ex.4.

Exercise 5. Evaluate the expression of the current i_{OUT} in the circuit of Fig.15 in terms of the independent sources v_1 and v_2.

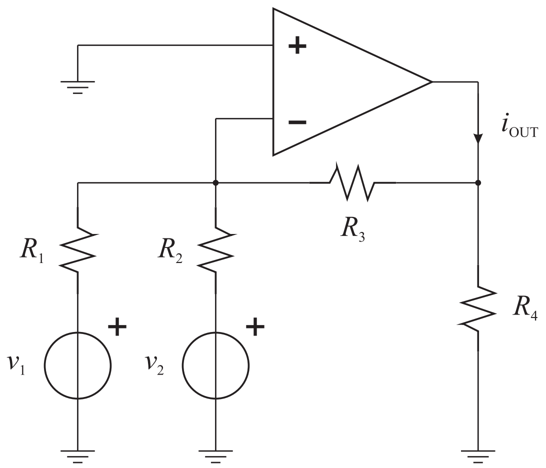

Figure 15: Circuit for Ex.5.

Exercise 6. With reference to the circuit reported in Fig.16, in which $R_1 = 10\text{k}\Omega$, $R_2 = 22\text{k}\Omega$ and $C = 10\text{nF}$, evaluate the expression of the transfer function $H(f) = \frac{V_{\text{out}}(f)}{V_{\text{in}}(f)}$ and plot the magnitude and phase Bode diagrams of $H(f)$.

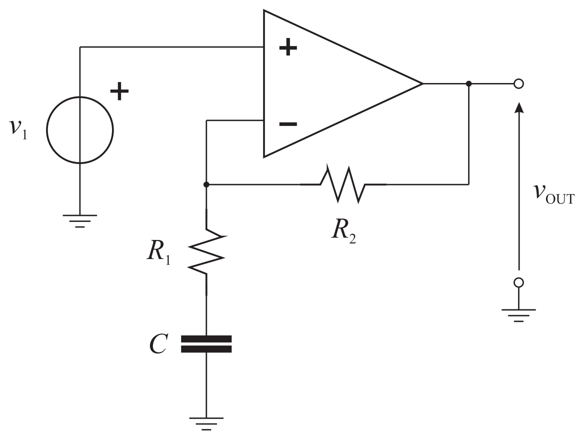

Figure 16: Circuit for Ex.6.

Exercise 7. With reference to the circuit in Fig.17, in which $R_1 = 10\text{k}\Omega$, $R_2 = 100\text{k}\Omega$, $C_1 = \frac{10}{\pi}\text{nF}$ and $C_2 = \frac{100}{\pi}\text{nF}$, evaluate $H(f) = \frac{V_{\text{out}}(f)}{V_{\text{in}}(f)}$ and plot the Bode diagrams of $H(f)$.

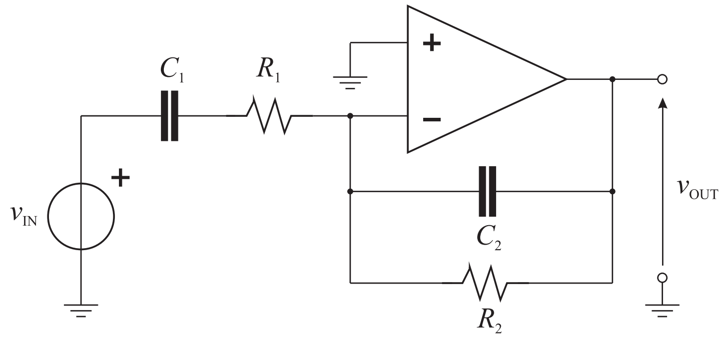

Figure 17: Circuit for Ex.7.

Exercise 8. With reference to the circuit in Fig.18, in which $R = 100\text{k}\Omega$ and $C = 2.2\text{nF}$, evaluate $H(f) = \frac{V_{\text{out}}(f)}{V_{\text{in}}(f)}$ and plot the Bode diagrams of $H(f)$.

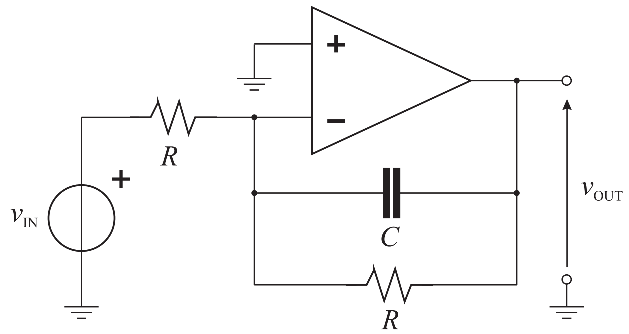

Figure 18: Circuit for Ex.8.

Exercise 9. With reference to the circuit in Fig.19, for $R = 100\text{k}\Omega$ and $C = \frac{10}{\pi}\text{nF}$, evaluate $H(f) = \frac{V_{\text{out}}(f)}{V_{\text{in}}(f)}$ and plot the Bode diagrams of $H(f)$.

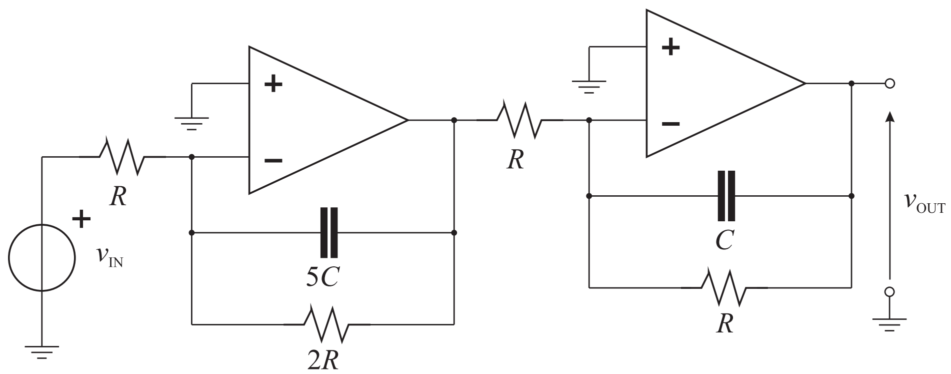

Figure 19: Circuit for Ex.9.

2 Exercises on Real Opamp Limitations

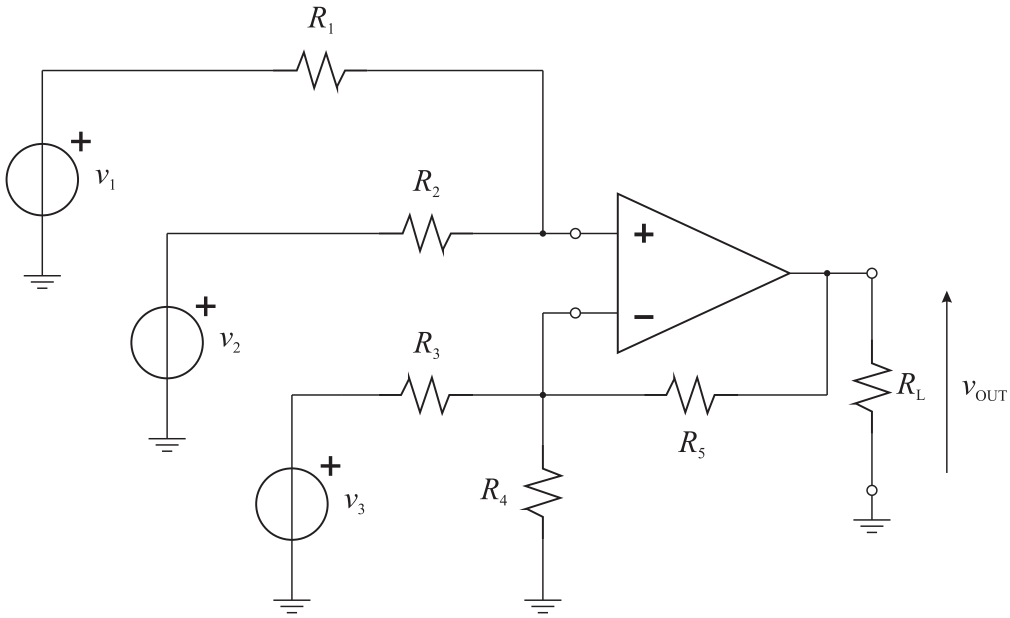

Figure 20: Circuit considered in the example.

2.1 Voltage/Current Swing

Consider the circuit in Fig.20, in which $R_1 = R_2 = R_3 = R_4 = 100\text{k}\Omega$ and the output port of the circuit drives a resistor $R_L = 1\text{k}\Omega$. The swings of the input signals v_1, v_2 and v_3 are reported in Tab.1.

Table 1: Swing of the input signals in Fig.11

Source	V_{MIN}	V_{MAX}
v_1	-2V	1V
v_2	0	1V
v_3	0	500mV

Evaluate

- the minimum output voltage swing;
- the minimum output current swing;
- the minimum common-mode input range

requested for the opamp in Fig.11 in order to operate properly with the specified input signals.

Solution

Output Voltage Swing The expression of the output voltage under the assumption that the opamp is ideal is reported in Eqn.(7). By replacing the numerical values, one gets

$$\begin{aligned} v_{\text{OUT}} &= \left(v_1 \frac{R_2}{R_1+R_2} + v_2 \frac{R_1}{R_1+R_2}\right)\left(1+\frac{R_5}{R_3 \parallel R_4}\right) - v_3 \frac{R_5}{R_3} \\ &= \frac{3}{2}v_1 + \frac{3}{2}v_2 - v_3 \end{aligned} \tag{13}$$

Taking into account of the swing of the input signals, the maximum value of the opamp output should be

$$\begin{aligned} v_{\text{OUT,MAX}} &= \max\left(\frac{3}{2}v_1 + \frac{3}{2}v_2 - v_3\right) \\ &= \frac{3}{2}\max v_1 + \frac{3}{2}\max v_2 - \min v_3 \\ &= \frac{3}{2}\cdot V_{1,\text{MAX}} + \frac{3}{2}\cdot V_{2,\text{MAX}} - V_{3,\text{MIN}} \\ &= \frac{3}{2}\cdot 1\text{V} + \frac{3}{2}\cdot 1\text{V} - 0\text{V} \\ &= 3\text{V}. \end{aligned} \tag{14}$$

Similarly, the minimum value of the opamp output should be

$$\begin{aligned} v_{\text{OUT,MIN}} &= \min\left(\frac{3}{2}v_1 + \frac{3}{2}v_2 - v_3\right) \\ &= \frac{3}{2}\min v_1 + \frac{3}{2}\min v_2 - \max v_3 \\ &= \frac{3}{2}\cdot V_{1,\text{MIN}} + \frac{3}{2}\cdot V_{2,\text{MIN}} - V_{3,\text{MAX}} \\ &= \frac{3}{2}\cdot(-2\text{V}) + \frac{3}{2}\cdot 0\text{V} - 0.5\text{V} \\ &= -3.5\text{V} \end{aligned} \tag{15}$$

Taking into account of Eqns.(14) and (15), the opamp should have a minimum output swing of (-3.5V, 3V) in order to process the input signals properly. An opamp with an output swing $(V_{\text{OUT,MIN}}, V_{\text{OUT,MAX}})$ with $V_{\text{OUT,MIN}} < -3.5\text{V}$ and $V_{\text{OUT,MAX}} > 3\text{V}$ could be employed in the circuit of Fig.11.

Output Current Swing Provided that the circuit in Fig.11 is working properly, the current flowing in the load resistor R_{L} is proportional to the output voltage so, for a given load, the opamp should source a maximum output current when the maximum output voltage is requested and should sink a maximum output current when the minimum (negative) output voltage is requested. As a consequence

$$i_{\text{OUT,MAX,sourced}} = \frac{v_{\text{OUT,MAX}}}{R_{\text{L}}} = \frac{3\text{V}}{1\text{k}\Omega} = 3\text{mA} \tag{16}$$

and

$$i_{\text{OUT,MAX,sunk}} = \frac{v_{\text{OUT,MIN}}}{R_{\text{L}}} = \frac{-3.5\text{V}}{1\text{k}\Omega} = -3.5\text{mA} \tag{17}$$

Common-Mode Input Voltage Swing Provided that the opamp circuit works properly, we have that $v^+ \simeq v^-$, as a consequence,

$$v_{\mathrm{CM}} = \frac{v^+ + v^-}{2} \simeq v^+.$$

In order to evaluate the minimum common-mode input range of the opamp in Fig.11, the minimum and the maximum values of the non-inverting input voltage, which is expressed as

$$\begin{aligned} v^+ &= v_1 \frac{R_2}{R_1+R_2} + v_2 \frac{R_1}{R_1+R_2} \\ &= \frac{1}{2} v_1 + \frac{1}{2} v_2, \end{aligned} \tag{18}$$

should be evaluated taking into account of the swing of the input signals. By so doing, one gets

$$\begin{aligned} v_{\mathrm{CM,MAX}} = v^+_{\mathrm{MAX}} &= \max\left(\frac{1}{2} v_1 + \frac{1}{2} v_2\right) \\ &= \frac{1}{2} \max v_1 + \frac{1}{2} \max v_2 \qquad (19) \\ &= \frac{1}{2} \cdot 1\mathrm{V} + \frac{1}{2} \cdot 1\mathrm{V} \\ &= 1\mathrm{V}. \qquad (20) \end{aligned}$$

and

$$\begin{aligned} v_{\mathrm{CM,MIN}} = v^+_{\mathrm{MIN}} &= \min\left(\frac{1}{2} v_1 + \frac{1}{2} v_2\right) \\ &= \frac{1}{2} \min v_1 + \frac{1}{2} \min v_2 \qquad (21) \\ &= \frac{1}{2} \cdot (-2\mathrm{V}) + \frac{1}{2} \cdot 0\mathrm{V} \\ &= -1\mathrm{V} \qquad (22) \end{aligned}$$

Taking into account of Eqns.(20) and (22), the opamp should have a minimum common-mode input swing of (-1V, 1V) in order to process the input signals properly. An opamp with an output swing $(V_{\mathrm{CM,MIN}}, V_{\mathrm{CM,MAX}})$ with $V_{\mathrm{CM,MIN}} < -1\mathrm{V}$ and $V_{\mathrm{CM,MAX}} > 1\mathrm{V}$ could be employed in the circuit of Fig.11.

2.2 Offset Voltage

Suppose that an integrated opamp circuit TLC271, which has a declared maximum input offset voltage of 5mV, is employed in the circuit of Fig.20 with the value of the components considered at the previous point. Evaluate the maximum DC offset on the output voltage v_{OUT} of the circuit in Fig.20.

Solution

In order to evaluate the output offset voltage in the circuit of Fig.20, all signal sources should be switched off and the opamp model including the input offset voltage source V_{OFF} should be considered, as depicted in Fig.21.

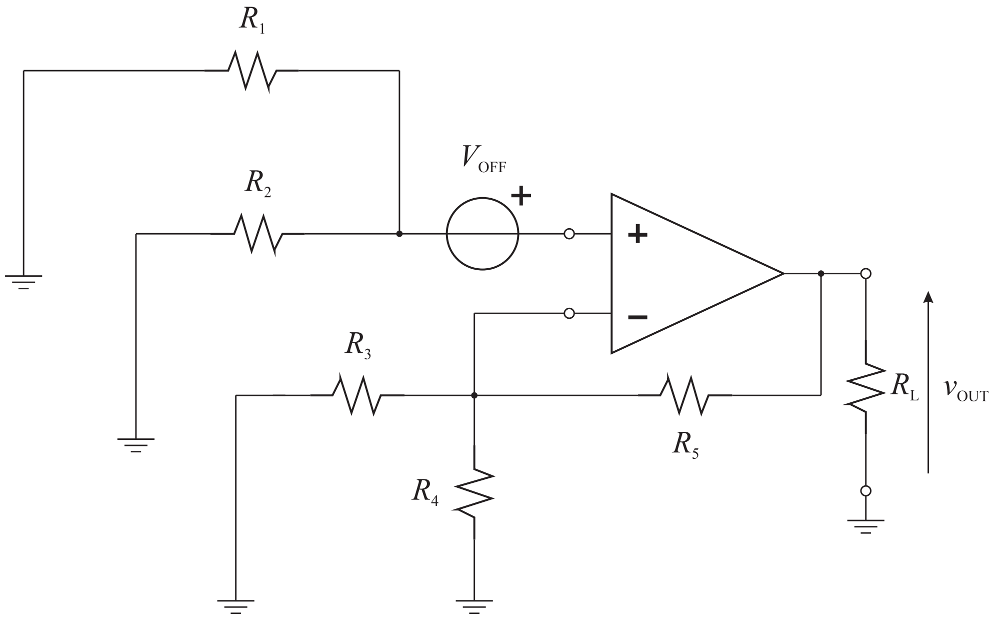

Figure 21: Circuit to be considered for the evaluation of the maximum DC offset voltage on v_{OUT} due to the input offset voltage V_{OFF} of the opamp in the circuit of Fig.20.

As a consequence, from the analysis of the circuit in Fig.21, assuming the opamp ideal, one gets that the contribution $V_{\mathrm{OFF,OUT}}$ of the opamp offset voltage source V_{OFF} to the output voltage v_{OUT} is given by:

$$V_{\mathrm{OFF,OUT}} = V_{\mathrm{OFF}} \left(1 + \frac{R_5}{R_3 \parallel R_4} \right) = 3V_{\mathrm{OFF}}. \tag{23}$$

The worst-case, maximum, output offset voltage is therefore given in magnitude by

$$\max |V_{\mathrm{OFF,OUT}}| = 3 \max |V_{\mathrm{OFF}}| = 15\mathrm{mV}. \tag{24}$$

As a consequence, the output voltage v_{OUT} of the circuit in Fig.21 can be affected by a DC error $-15\mathrm{mV} < V_{\mathrm{OFF,OUT}} < 15\mathrm{mV}$.

2.3 Effects of Finite Differential Amplification

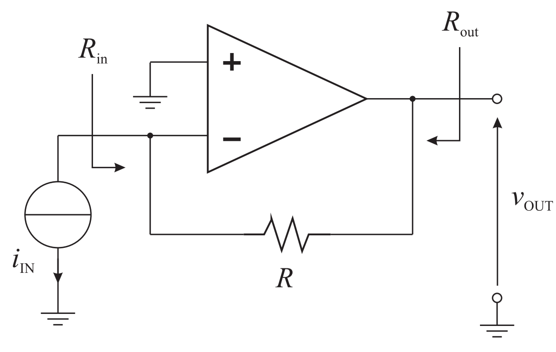

Figure 22: Circuit considered in the Exercise

Consider the transresistance amplifier circuit in Fig.22, in which $R = 10\mathrm{k}\Omega$. With reference to this circuit, evaluate

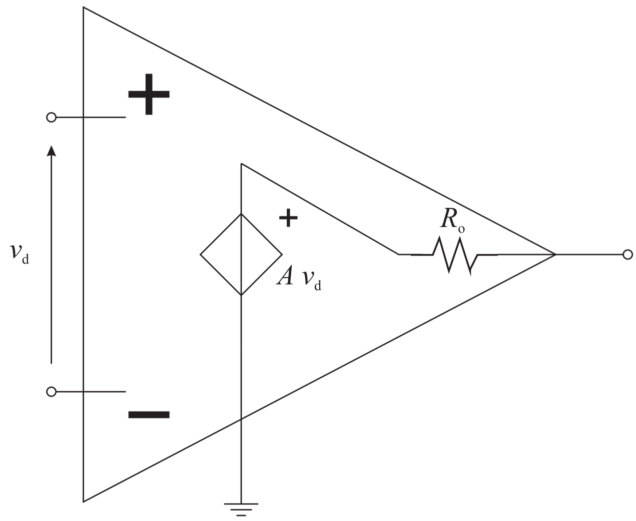

Figure 23: Equivalent circuit of an opamp with finite amplification and finite output resistance.

- the transresistance $R_{\rm m} = \frac{v_{\rm OUT}}{i_{\rm IN}}$;
- the input resistance $R_{\rm in}$ indicated in Fig.22;
- the output resistance $R_{\rm out}$ indicated in Fig.22

in the case of an ideal operational amplifier (i.e. for a differential amplification $A \to \infty$) and for a real operational amplifier with the equivalent circuit in Fig.23, with $R_{\rm o} = 10\mathrm{k}\Omega$ and $A_{\rm d} = 10^4$, $A_{\rm d} = 10^2$, $A_{\rm d} = 10$ and $A_{\rm d} = 1$.

Solution

The transresistance $R_{\rm m}$ and the input and output resistances, in the case that the opamp is assumed ideal are immediately evaluated as follows:

Transresistance If the opamp is assumed ideal, $v^- = v^+ = 0$ and the output voltage $v_{\rm OUT}$ is equal to the voltage across the feedback resistor R by the KVL. Moreover, since the current i^- flowing through the opamp inverting input is zero, by the KCL it follows that $i_{\rm R} = i_{\rm IN}$.

As a consequence,

$$v_{\rm OUT} = v_{\rm R} = R i_{\rm IN}$$

and the transresistance of the amplifier is therefore

$$R_{\rm m} = R = 10\mathrm{k}\Omega \tag{25}$$

Input Resistance If the opamp in Fig.22 is assumed ideal, $v^- = v^+ = 0$. As a consequence, if all the independent sources are switched off and a test current $i_{\rm T}$ is applied to the input port, the test voltage across this source is zero. As a consequence, the input resistance, which is defined as the ratio between the test voltage and the test current, is given by

$$R_{\rm in} = \frac{v_{\rm T}}{i_{\rm T}} = 0,$$

as expected in an ideal transresistance amplifier.

Output Resistance In order to evaluate the output resistance, all independent sources should be switched off and a test current i_{T} should be applied to the output port. Then, the output resistance is evaluated by definition as

$$R_{\mathrm{out}} = \frac{v_{\mathrm{T}}}{i_{\mathrm{T}}}.$$

With reference to the circuit in Fig.22, if the input source is switched off, the current flowing through R is zero since $i^- = 0$. As a consequence, since $v_{\mathrm{T}} = v_{\mathrm{OUT}} = v_{\mathrm{R}}$, it follows that the test voltage v_{T} is zero and

$$R_{\mathrm{out}} = \frac{v_{\mathrm{T}}}{i_{\mathrm{T}}} = 0,$$

as expected in an ideal transresistance amplifier.

The transresistance R_{m} and the input and output resistances, in the case that the opamp is assumed represented by the model reported in Fig.23 can be evaluated by analyzing the circuit in Fig.24.

Transresistance In order to evaluate the transresistance $R_{\mathrm{m}} = \frac{v_{\mathrm{OUT}}}{i_{\mathrm{IN}}}$, we apply the usual analysis method for circuits including controlled sources. By so doing, the control quantity is first evaluated in terms of i_{IN} and of $\hat{e} = Av_{\mathrm{D}}$ as

$$v_{\mathrm{D}} = (R + R_{\mathrm{out}})\, i_{\mathrm{IN}} - \hat{e}$$

hence

$$v_{\mathrm{D}} + Av_{\mathrm{D}} = (R + R_{\mathrm{o}})\, i_{\mathrm{IN}}$$

and finally

$$v_{\mathrm{D}} = \frac{R + R_{\mathrm{o}}}{1 + A} i_{\mathrm{IN}} \tag{26}$$

The output voltage can be therefore expressed as

$$\begin{aligned} v_{\mathrm{OUT}} &= -R_{\mathrm{o}} i_{\mathrm{IN}} + A\frac{R + R_{\mathrm{o}}}{1 + A} i_{\mathrm{IN}} \\ &= +\frac{AR + AR_{\mathrm{o}} - AR_{\mathrm{o}} - R_{\mathrm{o}}}{1 + A} i_{\mathrm{IN}} \\ &= \frac{AR - R_{\mathrm{o}}}{1 + A} i_{\mathrm{IN}} \end{aligned} \tag{27}$$

As a consequence, the transresistance of the stage is given by

$$R_{\mathrm{m}} = \frac{AR - R_{\mathrm{o}}}{1 + A} \tag{28}$$

For the given numerical values, and for the three values of the opamp differential amplification A one gets the transresistance values reported in Tab.2.

It can be observed that the transresistance of the stage is close to the one obtained considering an ideal opamp if the differential amplification is higher that 10^3. For $A = 10$ an error or about 20% can be noticed, whereas for $A = 1$, the transresistance is completely different with respect to the ideal opamp case.

Table 2: Transresistance of the amplifier in Fig.22 for different values of A

A	R_m
10^4	9.998kΩ
10^3	9.98kΩ
10	8.18kΩ
1	0Ω

Input Resistance In order to evaluate the input resistance of the circuit in Fig.24, we switch off the independent source and apply a test current source at the input port. Dependent sources (i.e. Av_D) should be kept ON. The circuit is then analyzed by the usual method for circuits including controlled sources. The control quantity is first evaluated in terms of the test current i_T and of $\hat{e} = Av_\mathrm{D}$ as

$$v_\mathrm{D} = (R + R_\mathrm{o})\, i_\mathrm{T} - \hat{e}$$

hence

$$v_\mathrm{D} = \frac{R + R_\mathrm{o}}{1 + A} i_\mathrm{T} \tag{29}$$

Since $v_\mathrm{T} = v_\mathrm{D}$ in this circuit, one immediately gets that

$$R_\mathrm{in} = \frac{v_\mathrm{T}}{i_\mathrm{T}} = \frac{R + R_\mathrm{o}}{1 + A}. \tag{30}$$

For the given numerical values, and for the three values of the opamp differential amplification A one gets the input resistance values reported in Tab.3.

Table 3: Input resistance of the amplifier in Fig.22 for different values of A

A	R_in
10^4	2Ω
10^3	19.8Ω
10	1.8kΩ
1	10kΩ

It can be observed that the input resistance of the amplifier is close to the ideal value (i.e. zero) if the differential amplification is higher that 10^3.

Output Resistance In order to evaluate the output resistance of the circuit in Fig.24, we switch off the independent source and apply a test current source at the output port. Dependent sources (i.e. Av_D) should be kept ON. The circuit is then analyzed by the usual method for circuits including controlled sources. The control quantity is first evaluated in terms of the test current i_T and of $\hat{e} = Av_\mathrm{D}$ as

$$v_\mathrm{D} = -R_\mathrm{o} i_\mathrm{T} - \hat{e}$$

hence

$$v_\mathrm{D} = -\frac{R_\mathrm{o}}{1 + A} i_\mathrm{T} \tag{31}$$

Since $v_\mathrm{T} = -v_\mathrm{D}$ in this circuit, one immediately gets that

$$R_\mathrm{out} = \frac{v_\mathrm{T}}{i_\mathrm{T}} = \frac{R_\mathrm{o}}{1+A}. \tag{32}$$

For the given numerical values, and for the three values of the opamp differential amplification A one gets the output resistance values reported in Tab.4.

Table 4: Output resistance of the amplifier in Fig.22 for different values of A

A	R_out
10^4	1 Ω
10^3	9.99Ω
10	909Ω
1	5kΩ

It can be observed that the output resistance of the stage is close to the ideal value (i.e. zero) if the differential amplification is higher that 10^3.

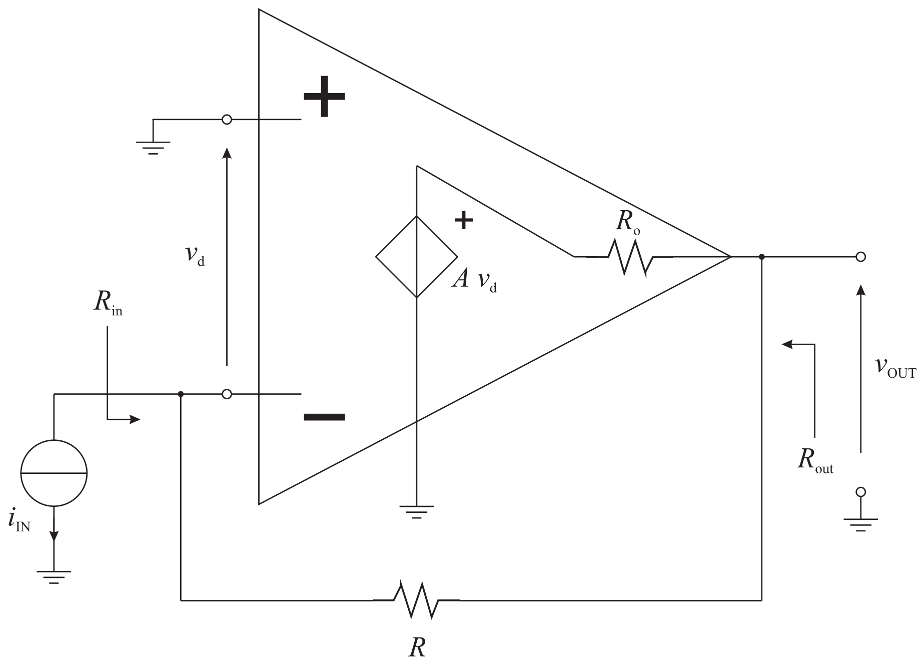

Figure 24: Circuit to be analyzed taking into account of the finite opamp amplification A_d

2.4 Opamp Bandwidth Limitation

Evaluate the transimpedance $Z_\mathrm{m}(f) = \frac{V_\mathrm{out}(f)}{I_\mathrm{out}(f)}$ of the amplifier in Fig.22 where $R = 10\mathrm{k}\Omega$ and where the operational amplifier is represented by the equivalent circuit in Fig.23, in which $R_\mathrm{o} = 100\mathrm{k}\Omega$ and the differential voltage amplification transfer function of the opamp is

$$A(f) = \frac{A_0}{1 + j\frac{f}{f_\mathrm{p}}} \tag{33}$$

where $A_0 = 10^4$ and $f_\mathrm{p} = 100\mathrm{Hz}$. Plot the Bode diagrams of $Z_\mathrm{m}(f)$ in magnitude and phase.

Solution

The expression of the transimpedance Z_m, taking into account of the finite amplification $A(f)$ can be calculated as in the previous exercise (Eqn.(28)):

$$Z_\mathrm{m}(f) = \frac{A(f)R - R_\mathrm{o}}{1 + A(f)} \tag{34}$$

By replacing the expression of $A(f)$ one gets

$$\begin{aligned}
Z_\mathrm{m}(f) &= \frac{\frac{A_0}{1+j\frac{f}{f_\mathrm{p}}}R - R_\mathrm{o}}{1 + \frac{A_0}{1+j\frac{f}{f_\mathrm{p}}}} \\
&= \frac{A_0 R - R_\mathrm{o}\left(1 + j\frac{f}{f_\mathrm{p}}\right)}{A_0 + 1 + j\frac{f}{f_\mathrm{p}}} \\
&= \frac{A_0 R - R_\mathrm{o} - R_\mathrm{o} j\frac{f}{f_\mathrm{p}}}{A_0 + 1 + j\frac{f}{f_\mathrm{p}}} \\
&= \frac{A_0 R - R_\mathrm{o}}{A_0 + 1}\frac{1 - j\frac{f}{\left(A_0\frac{R}{R_\mathrm{o}} - 1\right)f_\mathrm{p}}}{1 + j\frac{f}{(A_0+1)f_\mathrm{p}}} \\
&= Z_\mathrm{m}(0)\frac{1 - j\frac{f}{f_\mathrm{zc}}}{1 + j\frac{f}{f_\mathrm{pc}}}
\end{aligned} \tag{35}$$

where, considering the numerical values, $Z_\mathrm{m}(0) \simeq 10\mathrm{k}\Omega$, $f_\mathrm{pc} \simeq 1\mathrm{MHz}$ and $f_\mathrm{zc} \simeq 100\mathrm{kHz}$. The Bode plot of Z_m in Fig.25 is therefore obtained.

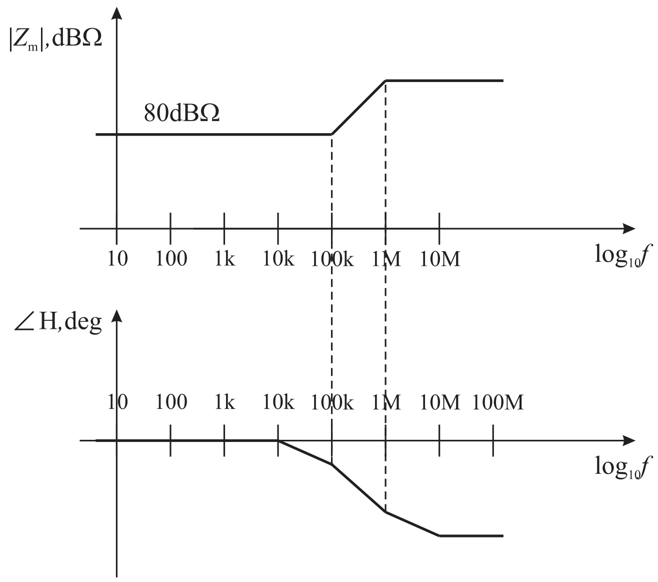

Figure 25: Bode diagrams of the transimpedance Z_m in the circuit of Fig.22.

2.5 Exercises

Exercise 1. The current amplifier in Fig.26, where $R_1 = 10\text{k}\Omega$, $R_2 = 1\text{k}\Omega$ and $R_L = 10\text{k}\Omega$ includes an opamp circuit with an output voltage swing (-10V,+10V) and with an output current swing of (-3mA,5mA). Evaluate the minimum and the maximum value of the input signal i_{IN} that can be properly amplified.

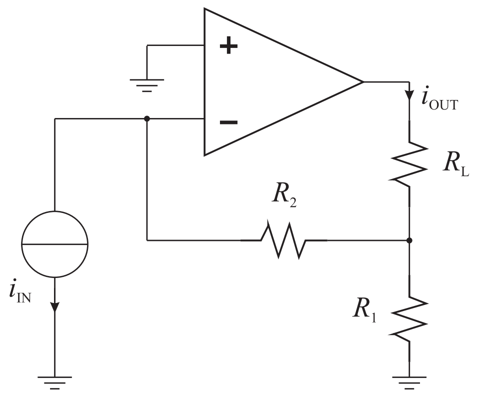

Figure 26: Circuit for Ex.1.

Exercise 2. The opamp circuit in Fig.26, where $R_1 = 10\text{k}\Omega$, $R_2 = 1\text{k}\Omega$ and $R_L = 10\text{k}\Omega$, shows a maximum input offset voltage of 5mV and a maximum input bias current of 100nA. The input offset current is negligible. Evaluate the maximum offset in the output current i_{OUT}.

Exercise 3. The circuit in Fig.26, where $R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$ and $R_3 = 10\text{k}\Omega$ has two input signals: $v_1(t)$ with a swing $(V_{1,\text{MIN}}, V_{1,\text{MAX}})$=(10mV, 500mV) and $v_2(t)$ with a swing $(V_{2,\text{MIN}}, V_{2,\text{MAX}})$=(-250mV, 1V).

- Calculate the minimum opamp output voltage swing that is requested for proper operation;
- supposing that the output current swing of the opamp is (-10mA, +10mA), evaluate the minimum resistive load that can be driven by the circuit.

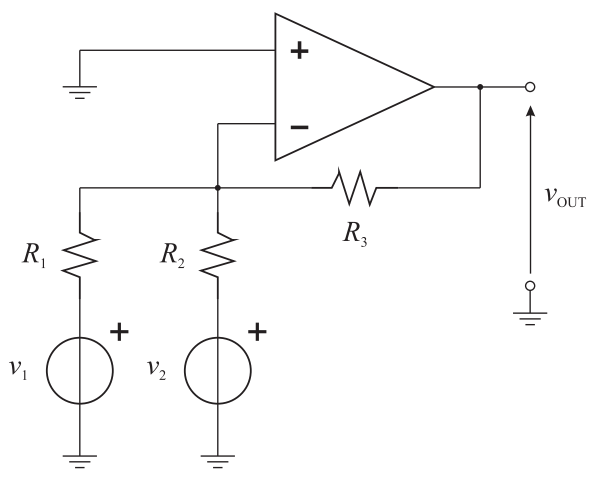

Figure 27: Circuit for Ex.3.

Exercise 4. Both the opamp circuits in Fig.28 show a maximum input offset voltage of 4mV. Evaluate the offset on the output voltage v_{OUT}, assuming $R_1 = R_2 = R_3 = R_4 = R_5 = 10\mathrm{k}\Omega$.

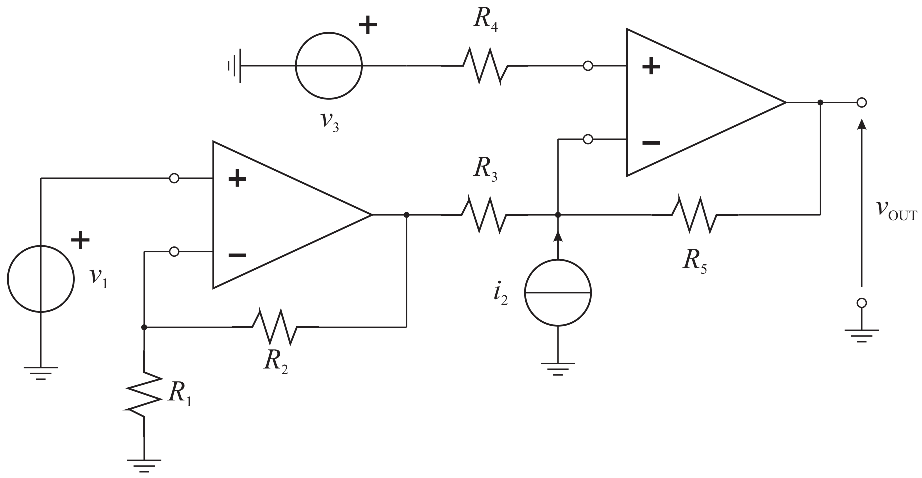

Figure 28: Circuit for Ex.4.

Exercise 5. With reference to the circuit in Fig.29, in which $R = 10\mathrm{k}\Omega$, evaluate the transconductance amplification $g_{\mathrm{m}} = \frac{i_{\mathrm{OUT}}}{v_{\mathrm{IN}}}$, the input resistance R_{in} and the output resistance R_{out} considering the equivalent circuit of the operational amplifier shown in Fig.23, where $R_{\mathrm{o}} = 5\mathrm{k}\Omega$

- assuming that the differential amplification A of the opamp is infinite (ideal opamp);
- assuming $A = 10^3$;
- assuming $A = 10$.

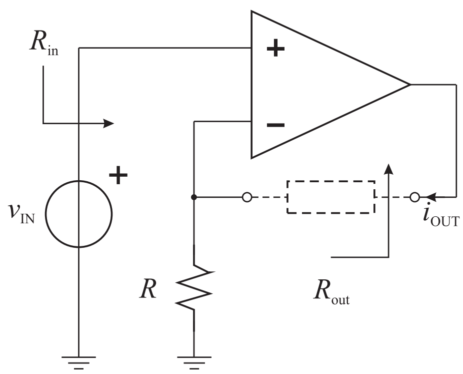

Figure 29: Circuit for Ex.5.

Exercise 6. With reference to the circuit in Fig.30, in which $R_1 = 10\text{k}\Omega$ and $R_2 = 100\text{k}\Omega$, evaluate the voltage amplification $A_\text{v} = \frac{v_\text{OUT}}{v_\text{IN}}$, the input resistance R_in and the output resistance R_out considering the equivalent circuit of the operational amplifier shown in Fig.23, where $R_\text{o} = 10\text{k}\Omega$

- assuming that the differential amplification A of the opamp is infinite (ideal opamp);
- assuming $A = 10^3$;
- assuming $A = 10$.

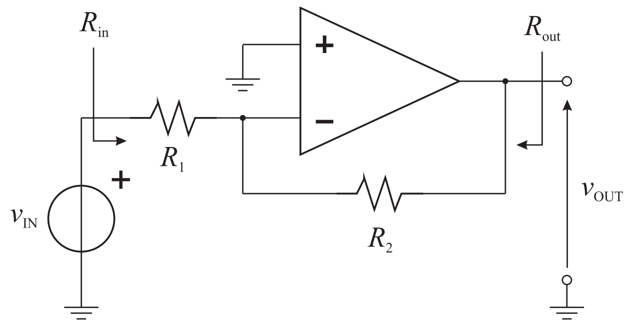

Figure 30: Circuit for Ex.6.

Exercise 7. With reference to the circuit in Fig.30, in which $R_1 = 10\text{k}\Omega$ and $R_2 = 100\text{k}\Omega$, evaluate the voltage amplification $A_\text{v}(f) = \frac{V_\text{out}(f)}{V_\text{in}(f)}$, the input impedance Z_in and the output impedance Z_out considering the equivalent circuit of the operational amplifier shown in Fig.23, where $R_\text{o} = 10\text{k}\Omega$ and $A(f) = \frac{A_0}{1+j\frac{f}{f_\text{p}}}$, $A_0 = 10^4$ and $f_\text{p} = 50\text{Hz}$.

Exercise 8. An opamp circuit in the voltage follower configuration is employed to drive a resistive load $R_\text{L} = 1\text{k}\Omega$ with a signal provided by a source with an internal resistance $R_\text{S} = 100\text{k}\Omega$. The opamp circuit has the equivalent circuit in Fig.32, where $R_\text{i} = 10\text{k}\Omega$, $R_\text{o} = 100\text{k}\Omega$ and $A(f) = \frac{A_0}{1+j\frac{f}{f_\text{p}}}$, $A_0 = 10^6$ and $f_\text{p} = 1\text{Hz}$.

- evaluate the transfer function $\frac{v_\text{OUT}}{v_\text{IN}}$, supposing that the load is directly connected to the signal source without the voltage follower circuit;
- evaluate the transfer function $\frac{v_\text{OUT}}{v_\text{IN}}$, assuming that the opamp is ideal (i.e. $A \to \infty$);
- evaluate the transfer function $\frac{v_\text{OUT}}{v_\text{IN}}$ at frequency $f = 0$;
- evaluate the expression of the transfer function $\frac{V_\text{out}(f)}{V_\text{in}(f)}$ over frequency and plot its Bode diagrams in magnitude and phase.

Exercise 9. With reference to the circuit in Fig.16, evaluate the frequency domain transfer function $H(f) = \frac{V_\text{out}(f)}{V_\text{in}(f)}$, taking into account of the frequency dependent voltage amplification $A(f) = \frac{A_0}{1+j\frac{f}{f_\text{p}}}$, with $A_0 = 10^4$ and $f_\text{p} = 100\text{Hz}$, of the operational amplifier. Assume that the opamp input impedance is infinite and that the opamp output impedance is zero.

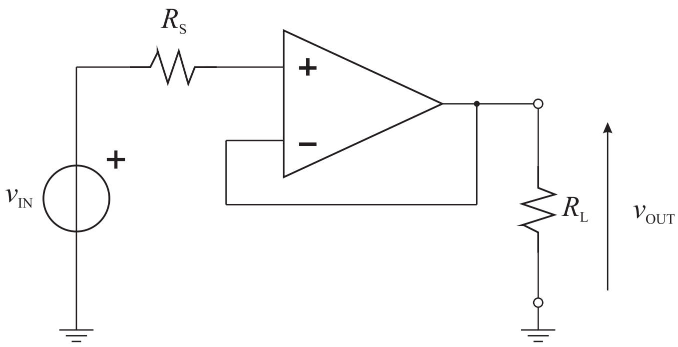

Figure 31: Circuit for Ex.8.

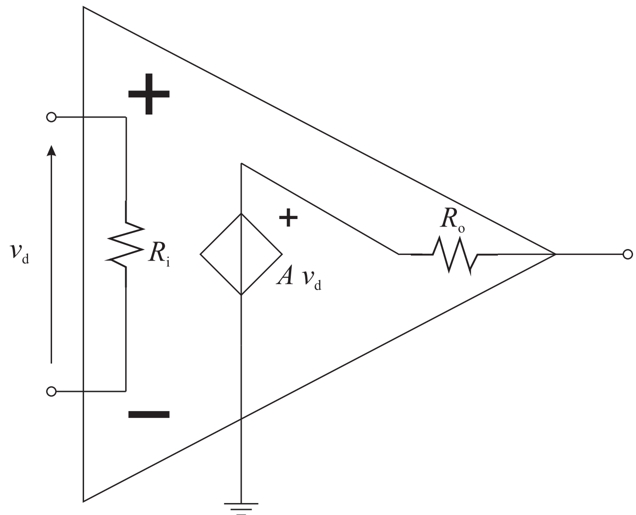

Figure 32: Equivalent circuit of the opamp for Ex.8.