Formulario di Metodi Matematici - Analisi

Trasformata di Fourier

$\mathcal{F}(g)(\nu) = \hat{g}(\nu) := \int_{-\infty}^{+\infty} g(t)e^{-2\pi i\nu t}dt, \qquad \nu \in \mathbb{R}.$

Proprietà

(a) $\mathcal{F}(e^{2\pi i\nu_0 t}T(t))(\nu) = \mathcal{F}(T(t))(\nu - \nu_0) \qquad (\nu_0 \in \mathbb{R})$

(b) $\mathcal{F}(T(t - t_0))(\nu) = e^{-2\pi i t_0 \nu}\,\mathcal{F}(T(t))(\nu) \qquad (t_0 \in \mathbb{R})$

(c) $\mathcal{F}(T(at))(\nu) = \dfrac{1}{|a|}\,\mathcal{F}(T(t))\left(\dfrac{\nu}{a}\right) \qquad (a \in \mathbb{R}\setminus\{0\})$

(d) $\mathcal{F}(t^k T(t))(\nu) = \left(-\dfrac{1}{2\pi i}\right)^k (\mathcal{F}(T))^{(k)}(\nu) \qquad (k \in \mathbb{N})$

(e) $\mathcal{F}(T^{(k)})(\nu) = (2\pi i\nu)^k\,\mathcal{F}(T)(\nu) \qquad (k \in \mathbb{N})$

Tavola di trasformate ($a > 0$)

Distribuzione	Trasformata
$T(t)$	$\mathcal{F}(T(t))(\nu)$
$H(t)e^{-at}$	$\dfrac{1}{a + 2\pi i\nu}$
$e^{-a\lvert t\rvert}$	$\dfrac{2a}{a^2 + 4\pi^2\nu^2}$
$p_a(t)$	$\dfrac{\sin(a\pi\nu)}{\pi\nu}$
e^{-at^2}	$\sqrt{\dfrac{\pi}{a}}e^{-\pi^2\nu^2/a}$
$\dfrac{1}{a^2 + t^2}$	$\dfrac{\pi}{a}e^{-2\pi a\lvert\nu\rvert}$

Distribuzione	Trasformata
$\dfrac{\sin(at)}{t}$	$\pi p_{a/\pi}(\nu)$
δ_{x_0}	$e^{-2\pi i x_0 \nu}$
$e^{2\pi i x_0 t}$	δ_{x_0}
$\text{v.p.}\,\dfrac{1}{t}$	$-\pi i\,\text{sign}(\nu)$
$\text{sign}(t)$	$\dfrac{1}{\pi i}\,\text{v.p.}\,\dfrac{1}{\nu}$
$H(t)$	$\dfrac{1}{2\pi i}\,\text{v.p.}\,\dfrac{1}{\nu} + \dfrac{\delta_0}{2}$

Trasformata di Laplace

Proprietà per funzioni f con semipiano di convergenza Ω_f

(a) $\mathcal{L}(e^{s_0 t} f(t))(s) = \mathcal{L}(f(t))(s - s_0)$, $\quad s \in \Omega_f + s_0 \quad (s_0 \in \mathbb{C})$

(b) $\mathcal{L}(f(t - t_0)H(t - t_0))(s) = e^{-t_0 s}\, \mathcal{L}(f(t))(s)$, $\quad s \in \Omega_f \quad (t_0 > 0)$

(c) $\mathcal{L}(f(at))(s) = \dfrac{1}{a}\, \mathcal{L}(f(t)) \left(\dfrac{s}{a}\right)$, $\quad s \in a\Omega_f \quad (a > 0)$

(d) $[\mathcal{L}(f(t))]'(s) = -\,\mathcal{L}(tf(t))(s)$, $\quad s \in \Omega_f$

(e) $\mathcal{L}(f')(s) = s\, \mathcal{L}(f)(s) - f(0+)$, $\quad s \in \Omega_f \cap \Omega_{f'}$

(f) $\mathcal{L}(\int_0^t f(\tau)\, \mathrm{d}\tau)(s) = [\mathcal{L}(f(t))(s)]/s$, $\quad s \in \Omega_f, \quad \operatorname{Re} s > 0$

(g) $\mathcal{L}(f(t)/t)(s) = \int_s^{+\infty} \mathcal{L}(f)(\sigma)\, \mathrm{d}\sigma$, $\quad s \in \Omega_f \cap \mathbb{R}, \quad s > 0$

Proprietà per distribuzioni T con semipiano di convergenza Ω_T

(a) $\mathcal{L}(e^{s_0 t} T(t))(s) = \mathcal{L}(T(t))(s - s_0)$, $\quad s \in \Omega_T + s_0 \quad (s_0 \in \mathbb{C})$

(b) $\mathcal{L}(T(t - t_0))(s) = e^{-t_0 s}\, \mathcal{L}(T(t))(s)$, $\quad s \in \Omega_T \quad (t_0 > 0)$

(c) $\mathcal{L}(T(at))(s) = \dfrac{1}{a}\, \mathcal{L}(T(t)) \left(\dfrac{s}{a}\right)$, $\quad s \in a\Omega_T \quad (a > 0)$

(d) $[\mathcal{L}(T(t))]'(s) = -\,\mathcal{L}(tT(t))(s)$, $\quad s \in \Omega_T$

(e) $\mathcal{L}(T')(s) = s\, \mathcal{L}(T)(s)$, $\quad s \in \Omega_T$

Tavola di trasformate

Distribuzione $T(t)$	Trasformata $\mathcal{L}(T(t))(s)$		
$t^k H(t)$	$\dfrac{k!}{s^{k+1}}$	$\operatorname{Re} s > 0$	$(k \in \mathbb{N})$
$e^{s_0 t} H(t)$	$\dfrac{1}{s - s_0}$	$\operatorname{Re} s > \operatorname{Re} s_0$	$(s_0 \in \mathbb{C})$
$\cos(\omega t) H(t)$	$\dfrac{s}{s^2 + \omega^2}$	$\operatorname{Re} s > 0$	$(\omega \in \mathbb{R})$
$\sin(\omega t) H(t)$	$\dfrac{\omega}{s^2 + \omega^2}$	$\operatorname{Re} s > 0$	$(\omega \in \mathbb{R})$
$\cosh(\omega t) H(t)$	$\dfrac{s}{s^2 - \omega^2}$	$\operatorname{Re} s > \lvert\omega\rvert$	$(\omega \in \mathbb{R})$
$\sinh(\omega t) H(t)$	$\dfrac{\omega}{s^2 - \omega^2}$	$\operatorname{Re} s > \lvert\omega\rvert$	$(\omega \in \mathbb{R})$
$\delta_{x_0}^{(k)}$	$s^k e^{-x_0 s}$	$s \in \mathbb{C}$	$(x_0 \in \mathbb{R},\ k \in \mathbb{N})$