Esercizio #4, Cap. 6 Dispense

1° metodo (non troppo consigliato)

$$\mathcal{F}(\cos(2t+1))(\omega)$$

RISCALAMENTO

$$= \frac{1}{2}\mathcal{F}(\cos(t+1))\left(\frac{\omega}{2}\right)$$

TRASLAZIONE

$$= \frac{1}{2} e^{-2\pi i(-1)\frac{\omega}{2}} \mathcal{F}(\cos(t))\left(\frac{\omega}{2}\right)$$

EULERO

$$= \frac{1}{2} e^{\pi i\omega} \mathcal{F}\left(\frac{e^{it}+e^{-it}}{2}\right)\left(\frac{\omega}{2}\right)$$

$$= \frac{e^{\pi i\omega}}{4}\left[\mathcal{F}(e^{it})\left(\frac{\omega}{2}\right) + \mathcal{F}\left(\frac{e^{-it}}{2}\right)\left(\frac{\omega}{2}\right)\right]$$

$$= \frac{e^{\pi i\omega}}{4}\left[\mathcal{F}\left(e^{2\pi i\left(\frac{1}{2\pi}\right)t}\right)\left(\frac{\omega}{2}\right) + \mathcal{F}\left(e^{2\pi i\left(-\frac{1}{2\pi}\right)t}\right)\left(\frac{\omega}{2}\right)\right]$$

$$= \frac{e^{\pi i\omega}}{4}\left[\delta_{\frac{1}{2\pi}}\left(\frac{\omega}{2}\right) + \delta_{-\frac{1}{2\pi}}\left(\frac{\omega}{2}\right)\right]$$

RISCALAMENTO δ

$$= \frac{e^{\pi i\omega}}{4}\left[\frac{1}{1/2}\delta_{\frac{1}{2\pi}2} + \frac{1}{1/2}\delta_{\left(-\frac{1}{2\pi}\right)2}\right]$$

$$= \frac{e^{\pi i \omega}}{2} \left[\delta_{\frac{1}{\pi}} + \delta_{-\frac{1}{\pi}} \right]$$

$$\overset{(*)}{=} \frac{e^{\pi i \omega}}{2} \delta_{\frac{1}{\pi}} + \frac{e^{\pi i \omega}}{2} \delta_{-\frac{1}{\pi}}$$

(*) $h(t)\,\delta_{t_0} = h(t_0)\,\delta_{t_0}$

$$= \frac{e^{\pi i \frac{1}{\pi}}}{2} \delta_{\frac{1}{\pi}} + \frac{e^{\pi i \left(-\frac{1}{\pi}\right)}}{2} \delta_{-\frac{1}{\pi}}$$

$$= \frac{e^{i}}{2} \delta_{\frac{1}{\pi}} + \frac{e^{-i}}{2} \delta_{-\frac{1}{\pi}}$$

<u>2° metodo</u> (più consigliato)

$$\mathcal{F}\left(\cos(2t+1)\right)(\omega)$$

$$= \mathcal{F}\left(\frac{e^{(2t+1)i} + e^{-(2t+1)i}}{2} \right)(\omega)$$

$$= \frac{1}{2} e^{i} \mathcal{F}\left(e^{2ti}\right)(\omega) + \frac{1}{2} e^{-i} \mathcal{F}\left(e^{-2ti}\right)$$

$$= \frac{1}{2} e^{i} \mathcal{F}\left(e^{2\pi i\left(\frac{1}{\pi}\right)t}\right)(\omega) + \frac{1}{2} e^{-i} \mathcal{F}\left(e^{2\pi i\left(-\frac{1}{\pi}\right)t}\right)(\omega)$$

$$= \frac{e^{i}}{2} \delta_{\frac{1}{\pi}} + \frac{e^{-i}}{2} \delta_{-\frac{1}{\pi}}$$