Es RIEPILOGO

Es (26-6-12) Risolvere $e^{2z-\bar{z}} = 1$

Ⓐ $\left\{ z = \frac{2k\pi}{3i} : k \in \mathbb{Z} \right\}$ Ⓑ $\{ z : \operatorname{Re} z = 0 \}$

Ⓒ $\{0\}$ Ⓓ $\left\{ z = \frac{4k\pi}{3i} : k \in \mathbb{Z} \right.$

Sol

$z = x + iy$ con $x, y \in \mathbb{R}$

$$e^{2z-\bar{z}} = 1 \iff e^{2x+2iy-x+iy} = 1 \iff e^{x+i3y} = 1$$

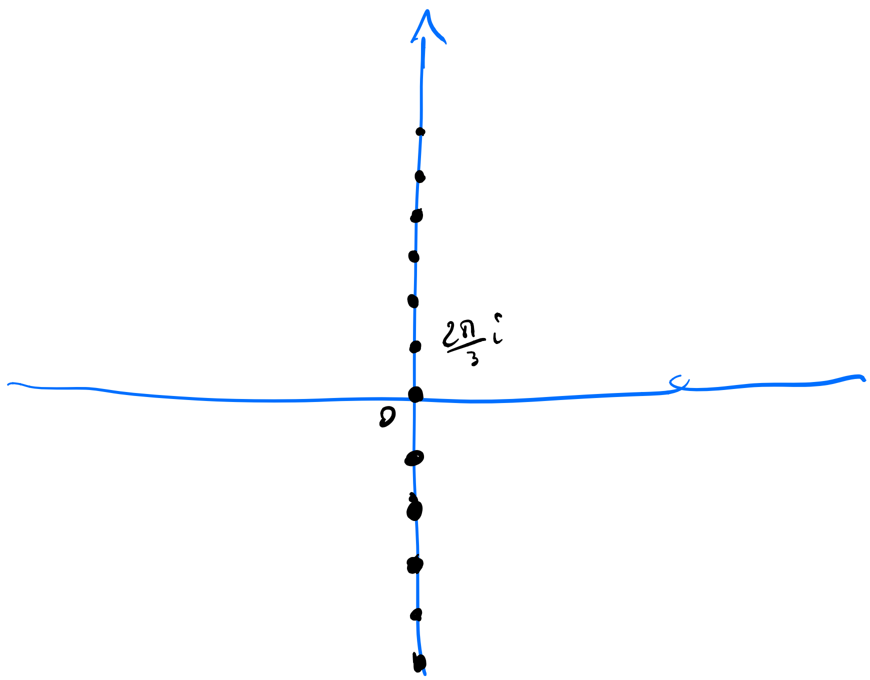
$$\iff \underbrace{e^{x}}_{\overset{\vee}{0}} e^{\overbrace{3iy}^{3\theta}} = 1\, e^{i0} \iff \begin{cases} e^{x} = 1 \\ 3y = 0 + 2k\pi, \quad k \in \mathbb{Z} \end{cases}$$

$$\iff \begin{cases} x = 0 \\ y = \frac{2}{3} k\pi, \quad k \in \mathbb{Z} \end{cases} \iff z = 0 + i \frac{2}{3} k\pi, \quad k \in \mathbb{Z}$$

$$\iff z = \frac{2k\pi}{3} i \quad k \in \mathbb{Z} \longrightarrow \text{Ⓐ}$$

perché $i = -\frac{1}{i}$ e $k \in \mathbb{Z}$

$$\left[\left\{ \frac{2k\pi}{3i} : k \in \mathbb{Z} \right\} = \left\{ -\frac{2k\pi}{3i} : k \in \mathbb{Z} \right\} \right] !!$$



Oss Volendo si poteva utilizzare la formula

$$e^{w} = 1 \iff w = 2k\pi i, \quad k \in \mathbb{Z}$$

Es (15/7/14) $\bar{z}^2 = z^2$

(A) $\{z \in \mathbb{C} : \operatorname{Re} z = 0\}$ (B) $\{z \in \mathbb{C} : (\operatorname{Re} z)(\operatorname{Im} z) = 0\}$

(C) $\{z \in \mathbb{C} : |\operatorname{Re} z| = |\operatorname{Im} z|\}$ (D) $\{0\}$

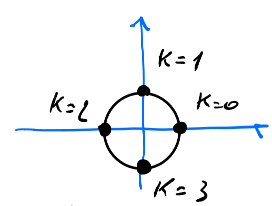
SOL coord. polari : $z = \rho e^{i\theta}$ con $\rho \geq 0$, $\theta \in \mathbb{R}$

$$\bar{z}^2 = z^2 \iff (\rho e^{-i\theta})^2 = (\rho e^{i\theta})^2 \iff$$

$$\iff \underbrace{\rho^2 e^{-2i\theta}}_{\text{forme polari}} = \underbrace{\rho^2 e^{2i\theta}}_{\text{forme polari}} \iff$$

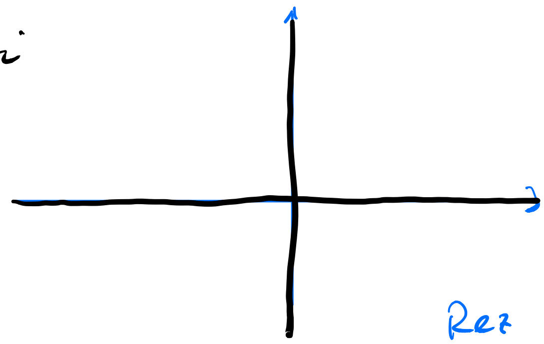
$$\iff \begin{cases} \rho^2 = \rho^2 \\ -2\theta = 2\theta + 2k\pi, \quad k \in \mathbb{Z} \end{cases} \iff \begin{cases} \forall \rho \geq 0 \\ \theta = \frac{k\pi}{2}, \quad k \in \mathbb{Z} \end{cases}$$

$$\iff \begin{cases} \forall \rho \geq 0 \\ \theta = 0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi \end{cases}$$



basta $k = 0, 1, 2, 3$

ho i 2 assi cartesiani



$\longrightarrow$ B $\quad \left\{ z = x + iy \ : \ \overbrace{x}^{\mathrm{Re}\,z}\overbrace{y}^{\mathrm{Im}\,z} = 0 \right\}$

Alternativamente $z = x + iy$, $\quad x, y \in \mathbb{R}$

$$\bar{z}^2 = z^2 \iff (x - iy)^2 = (x + iy)^2 \iff$$

$$\iff x^2 - 2ixy - y^2 = x^2 + 2ixy - y^2 \iff$$

$$\iff -2ixy = 2ixy \iff$$

$$\iff -xy = xy \iff xy = 0 \longrightarrow \text{B}$$

Es (20/6/16) Risolvere $|z| = 2i$

Sol

NON CI SONO SOLUZIONI.

$|z| \in \mathbb{R} \quad \forall z \in \mathbb{C} \Rightarrow |z| \neq 2i \ \forall z \in \mathbb{C}$

ES (22-9-17) Per quali $\alpha \in \mathbb{R}$ la funzione

$f(z) = iz + \alpha \bar{z}$ è intera?

(A) $\alpha = 0$ (B) $\alpha = i$

(C) nessun $\alpha \in \mathbb{R}$ (D) $\forall \alpha \in \mathbb{R}$

Sol N.B. "intera" significa "olomorfa su tutto $\mathbb{C}$"

Pongo $z = x + iy$, $x, y \in \mathbb{R}$ $\Rightarrow$

$$f(z) = iz + \alpha \bar{z} = i(x+iy) + \alpha(x-iy) \qquad (\underline{\alpha \in \mathbb{R}})$$

$$= \underbrace{(\alpha x - y)}_{u(x,y)} + i \underbrace{(x - \alpha y)}_{v(x,y)} \qquad \begin{matrix} u(x,y) \in \mathbb{R} \\ v(x,y) \in \mathbb{R} \end{matrix}$$

$$\text{C-R} \begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \end{cases} \Leftrightarrow \begin{cases} \alpha = -\alpha \\ -1 = -1 \end{cases} \Leftrightarrow \begin{cases} \alpha = 0 \\ \forall (x,y) \in \mathbb{R}^2 \end{cases}$$

$\Leftrightarrow \alpha = 0 \longrightarrow$ (A) per $\alpha = 0$ OK, $f(z) = iz$ olomorfa;

se $\alpha \neq 0$ $f(z)$ **NON** è olomorfa, perché se lo fosse avrei che

$$\alpha \bar{z} = \underbrace{f(z)}_{\text{OLOMORFA}} - \underbrace{iz}_{\text{OLOMORFA}} \quad \text{olomorfa, ASSURDO}$$

Es (8/9/16) Se $f: \mathbb{C} \to \mathbb{C}$ olomorfa

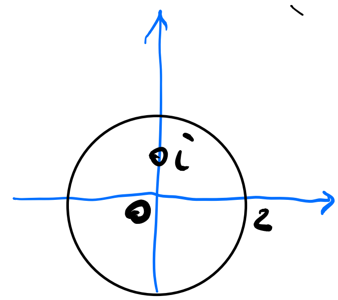
e se $f'(i) = \pi i$ allora

$$I = \int_{\partial B_2(0)} \frac{f(z)}{(z-i)^2}\, dz \qquad \text{vale}$$

(A) $-2\pi^2$ (B) $-\frac{\pi}{i}$

(C) $\frac{1}{2}$ (D) 0

Sol Per il th. dei residui



$$I = \int_{\partial B_2(0)} \frac{f(z)}{(z-i)^2}\, dz = 2\pi i \, \underset{\frac{f(z)}{(z-i)^2}}{\mathrm{Res}} (i)$$

$m=2$
f olomorfa intorno a i

$$= 2\pi i \frac{1}{1!} \frac{d}{dz}\left(f(z)\right)\Big|_{z=i}$$

IPOTESI

$$= 2\pi i \, f'(i) = (2\pi i)(\pi i) = -2\pi^2 \to \text{(A)}$$

ALTERNATIVA formula di Cauchy per le derivate:

$$\boxed{\underbrace{f'(i)}_{=\pi i} = \underbrace{\frac{1!}{2\pi i}\int_{\partial B_2(0)} \frac{f(z)}{(z-i)^2}\,dz}_{=\frac{I}{2\pi i}}}$$

$$\Longrightarrow \quad I = (\pi i)(2\pi i) = -2\pi^2$$