ESERCIZI (CONTINUAZIONE)

Es Sia $T_n = n\left(\delta_{\frac{1}{n}} - \delta_{-\frac{1}{n}}\right)$

$T_n \to ?$ in $\mathcal{D}'$

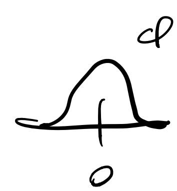
SOL

Sia $\varphi \in \mathcal{D}$

$$\langle T_n, \varphi \rangle = n\left(\langle \delta_{\frac{1}{n}}, \varphi \rangle - \langle \delta_{-\frac{1}{n}}, \varphi \rangle\right)$$

$$= \underbrace{n}_{\to\infty} \underbrace{\left(\varphi\left(\tfrac{1}{n}\right) - \varphi\left(-\tfrac{1}{n}\right)\right)}_{\to 0} \qquad \text{“}\infty \cdot 0\text{”}$$

$$= \frac{\varphi\left(\frac{1}{n}\right) - \varphi\left(-\frac{1}{n}\right)}{\frac{1}{n}} \qquad \text{“}\tfrac{0}{0}\text{”}$$



De L'Hopital con $\left(x = \frac{1}{n} : \quad \frac{\varphi(x) - \varphi(-x)}{x}\right)$

$$\sim \frac{\varphi'\left(\frac{1}{n}\right) + \varphi'\left(-\frac{1}{n}\right)}{1} \xrightarrow{n \to \infty} 2\varphi'(0)$$

$$= 2\langle \delta_0, \varphi' \rangle = 2\langle -\delta_0', \varphi \rangle = \langle -2\delta_0', \varphi \rangle$$

[RICORDIAMO CHE $\langle \delta_0', \varphi \rangle = -\langle \delta_0, \varphi' \rangle$]

Es (30/6/14) $f_n(x) = \mathbb{1}_{[1,n]}(x)$

$$T_n = \delta_0\left(x-3+\frac{1}{n}\right) + T_{f_n}$$

$T_n \to \,?$ in $\mathcal{D}'$

Sol $\forall \varphi \in \mathcal{D}$

(i)

$$\left\langle \delta_0\left(x-3+\frac{1}{n}\right), \varphi \right\rangle = \left\langle \delta_{3-\frac{1}{n}}, \varphi \right\rangle$$

$$= \varphi\left(3-\frac{1}{n}\right) \xrightarrow{n\to\infty} \varphi(3) = \langle \delta_3, \varphi \rangle$$

$$\Rightarrow \delta_0\left(x-3+\frac{1}{n}\right) \longrightarrow \delta_3 \quad \text{in } \mathcal{D}'$$

(ii)

$$\langle T_{f_n}, \varphi \rangle = \int_{-\infty}^{+\infty} \mathbb{1}_{[1,n]}(x)\,\varphi(x)\,dx$$

$$= \int_1^n \varphi(x)\,dx \xrightarrow[\substack{\uparrow \\ \text{oss. 2}}]{n\to\infty} \int_1^{+\infty} \varphi(x)\,dx = \int_{-\infty}^{+\infty} \mathbb{1}_{[1,\infty)}(x)\,\varphi(x)\,dx$$

$$= \left\langle T_{\mathbb{1}_{[1,\infty)}}, \varphi \right\rangle$$

$$\Rightarrow T_{f_n} \longrightarrow T_{\mathbb{1}_{[1,+\infty)}} = T_{H(x-1)} \quad \text{in } \mathcal{D}'$$

$$\Rightarrow T_n \rightarrow \delta_3 + T_{H(x-1)} \text{ in } \mathcal{D}'$$

VALE IL SEGUENTE

TH Se $f_n, f : \mathbb{R} \to \mathbb{R}$ loc. sommabili

e se

$f_n \longrightarrow f$ uniformemente su tutti gli intervalli limitati di $\mathbb{R}$

Allora $T_{f_n} \longrightarrow T_f$

NO DIM

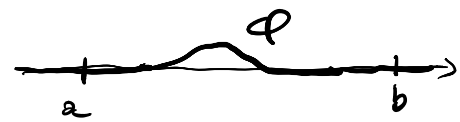
ES (9/7/15)

$$f_n(x) = \frac{1}{n} P_n\left(x+\frac{n}{2}\right)$$

$$T_n = \delta^{(n)}_{n^2} + T_{f_n} \longrightarrow \ ? \quad \text{in } \mathcal{D}'$$

SOL sia $\varphi \in \mathcal{D}$ $\left(\Rightarrow \exists\, a,b \text{ finiti} : \varphi(x)=0 \ \forall x \notin [a,b]\right)$

(i)



$$\langle \delta^{(n)}_{n^2}, \varphi \rangle = (-1)^n \langle \delta_{n^2}, \varphi^{(n)} \rangle$$

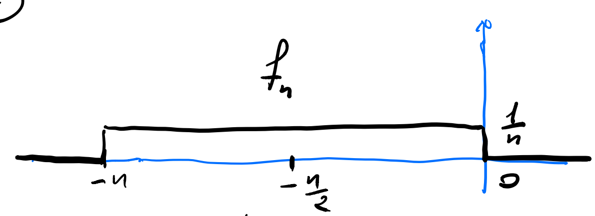
$$= (-1)^n \underbrace{\varphi^{(n)}(n^2)}_{=\,0} \xrightarrow{n\to\infty} 0 = \langle 0, \varphi \rangle$$

DA UN CERTO n_0 in poi

PERCHE' $\varphi^{(n)}(x) = 0 \ \forall x \notin [a,b] \ \forall n \in \mathbb{N}$

$$\Rightarrow \quad \delta^{(n)}_{n^2} \longrightarrow 0 \quad \text{in } \mathcal{D}'$$

(ii)

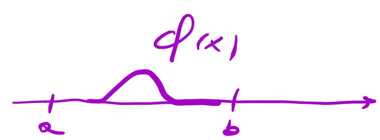


$$\langle T_{f_n}, \varphi \rangle = \int_{-\infty}^{+\infty} \frac{1}{n} P_n\left(x+\frac{n}{2}\right) \varphi(x)\, dx \ \longleftarrow$$

$$= \frac{1}{n} \boxed{\int_{-n}^{0} \varphi(x)\, dx} \xrightarrow{n\to\infty} 0 = \langle 0, \varphi \rangle$$

$\frac{1}{n} \xrightarrow{n\to\infty} 0$

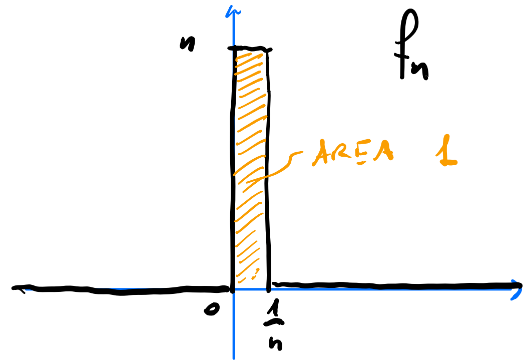
$\int_{-n}^{0} \varphi(x)\,dx \xrightarrow{n\to\infty} \int_{-\infty}^{0} \varphi(x)\, dx \in \mathbb{R}$



$$\Rightarrow \quad T_{f_n} \to 0 \quad \text{in } \mathcal{D}'$$

$$\Rightarrow \quad T_n = \delta_{n^2}^{(n)} + T_{f_n} \longrightarrow 0 + 0 = 0 \quad \text{in } \mathcal{D}'$$

Es $f_n(x) = n\, p_{\frac{1}{n}}\left(x-\frac{1}{2n}\right)$. $\qquad T_{f_n} \to ?$ in $\mathcal{D}'$



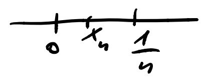
Sol

Sia $\varphi \in \mathcal{D}$

$$\langle T_{f_n}, \varphi \rangle = \int_0^{\frac{1}{n}} n\,\varphi(x)\,dx$$

$$= \underbrace{n\,\frac{1}{n}}_{=1} \underbrace{\left(\frac{1}{1/n}\int_0^{\frac{1}{n}} \varphi(x)\,dx\right)}_{\text{MEDIA INTEGRALE}} = \varphi(x_n) \xrightarrow{n\to\infty} \varphi(0) =$$

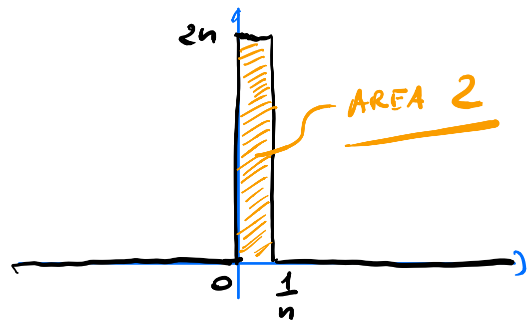
con $x_n \in \left(0, \frac{1}{n}\right)$, $\quad x_n \xrightarrow{n\to\infty} 0$



$$= \langle \delta_0, \varphi \rangle \quad \Rightarrow \quad T_{f_n} \to \delta_0 \text{ in } \mathcal{D}'$$

Es $f_n(x) = 2n\, P_{\frac{1}{n}}\left(x - \frac{1}{2n}\right)$; $T_{f_n} \to \ ?$ in $\mathcal{D}'$

<u>SOL</u> $\forall \varphi \in \mathcal{D}$ si ha



$$\langle T_{f_n}, \varphi \rangle =$$

$$= \int_0^{\frac{1}{n}} 2n\, \varphi(x)\, dx$$

con $x_n \in \left(0, \frac{1}{n}\right)$, $x_n \xrightarrow{n \to \infty} 0$

$$= 2n \, \frac{1}{n} \left(\frac{1}{\frac{1}{n}} \int_0^{\frac{1}{n}} \varphi(x)\, dx \right) = 2\, \varphi(x_n) \xrightarrow{n \to \infty}$$

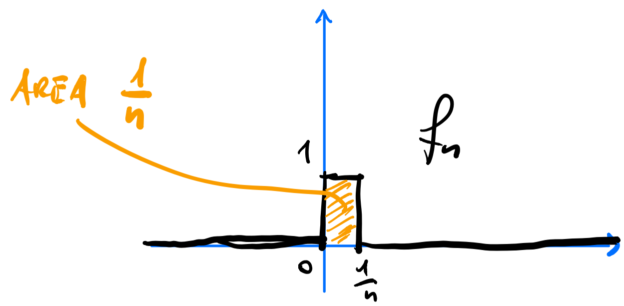
$$\longrightarrow 2\varphi(0) = 2 \langle \delta_0, \varphi \rangle = \langle 2\delta_0, \varphi \rangle$$

$$\Rightarrow \quad T_{f_n} \longrightarrow 2\delta_0 \quad \text{in } \mathcal{D}'$$

ES $\underline{5}$ $\quad f_n(x) = P_{\frac{1}{n}}\left(x - \frac{1}{2n}\right); \quad T_{f_n} \longrightarrow \, ? \text{ in } \mathcal{D}'$

<u>SOL</u>

Sia $\varphi \in \mathcal{D}$



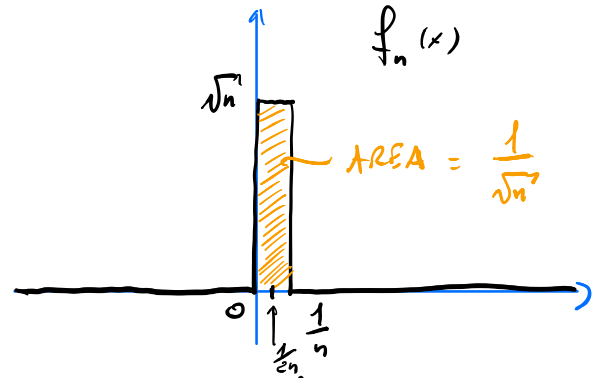
$$\langle T_{f_n}, \varphi \rangle = \int_0^{\frac{1}{n}} \varphi(x)\,dx \xrightarrow[n\to\infty]{} 0 = \langle 0, \varphi \rangle$$

OSS. 1

$\Rightarrow T_{f_n} \to 0 \text{ in } \mathcal{D}'$

Es

$$f_n(x) = \sqrt{n}\, p_{\frac{1}{n}}\left(x - \frac{1}{2n}\right) \; ; \quad T_{f_n} \to \; ? \quad \text{in } \mathcal{D}'$$



Sol

$\forall \varphi \in \mathcal{D}$

$$\langle T_{f_n}, \varphi \rangle = \int_0^{\frac{1}{n}} \sqrt{n}\, \varphi(x)\, dx = \left(\sqrt{n}\ \frac{1}{n}\right) \frac{1}{\frac{1}{n}} \int_0^{\frac{1}{n}} \varphi(x)\, dx$$

con $x_n \in \left(0, \frac{1}{n}\right) \Rightarrow x_n \to 0$

$$= \underbrace{\left(\frac{1}{\sqrt{n}}\right)}_{\downarrow\, 0} \underbrace{\varphi(x_n)}_{\downarrow\, \varphi(0)} \xrightarrow{n \to \infty} 0 = \langle 0, \varphi \rangle$$

$$\Rightarrow \quad T_{f_n} \to 0 \quad \text{in } \mathcal{D}'$$

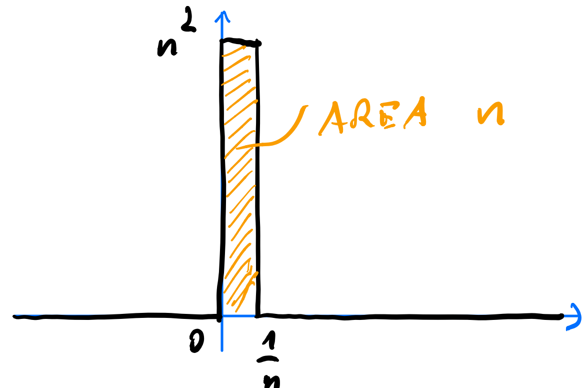
Es

$$f_n(x) = n^2 \, p_{\frac{1}{n}}\left(x - \frac{1}{2n}\right) ; \quad T_{f_n} \to \ ? \ \text{in } \mathcal{D}'$$

sol

$\forall \varphi \in \mathcal{D}$

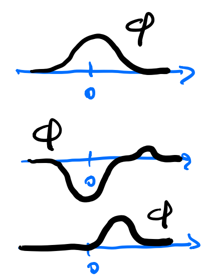
$$\langle T_{f_n}, \varphi \rangle =$$



$$= \int_0^{\frac{1}{n}} n^2 \varphi(x)\,dx = \left(n^2 \frac{1}{n}\right)\left(\frac{1}{1/n}\int_0^{\frac{1}{n}} \varphi(x)\,dx\right)$$

con $x_n \in (0, \frac{1}{n}) \Rightarrow x_n \to 0$

$$= \underbrace{n}_{\to +\infty} \; \underbrace{\varphi(x_n)}_{\to \varphi(0)} \longrightarrow \begin{cases} +\infty & \text{se } \varphi(0) > 0 \\ -\infty & \text{se } \varphi(0) < 0 \\ \text{FORMA IND.} & \text{se } \varphi(0) = 0 \end{cases}$$



quindi in generale $\exists \, \varphi$ t.c. il limite non esiste

$\Longrightarrow$ <u>NON</u> ESISTE IL LIMITE DI T_n in $\mathcal{D}'$