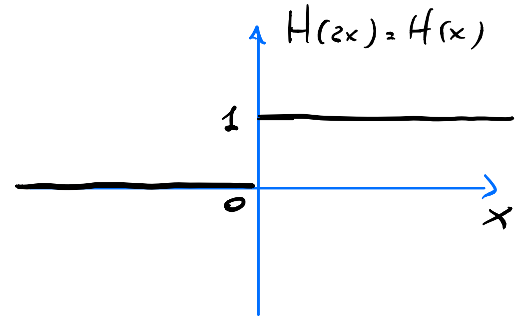
Es. $T = T_{H(2x)} + 5\delta_3(2x)$

$T' = ?$

Sol



$$T'_{H(2x)} = \delta_0$$

cos'è $\delta_3(2x)$? $\forall \varphi \in \mathcal{D}$ si ha

$$\langle \delta_3(2x), \varphi \rangle = \langle \delta_3, \tfrac{1}{2}\varphi\left(\tfrac{x}{2}\right) \rangle$$

$$= \tfrac{1}{2}\varphi\left(\tfrac{3}{2}\right) = \tfrac{1}{2}\langle \delta_{\frac{3}{2}}, \varphi \rangle$$

$$= \langle \tfrac{1}{2}\delta_{\frac{3}{2}}, \varphi \rangle$$

quindi $\delta_3(2x) = \frac{1}{2}\delta_{\frac{3}{2}}$ per cui $(\delta_3(2x))' = \frac{1}{2}\delta'_{\frac{3}{2}}$

perciò

$$T' = T'_{H(2x)} + 5(\delta_3(2x))' = \delta_0 + \tfrac{5}{2}\delta'_{\frac{3}{2}}$$

Altro modo di calcolare $(\delta_3(2x))'$:

$(\delta_3(2x))' = 2\delta_3'(2x)$ apparentemente diverso da $\frac{1}{2}\delta_{\frac{3}{2}}'$; in realtà sono uguali:

$\forall \varphi \in \mathcal{D}$ si ha infatti

$$\langle 2\delta_3'(2x), \varphi \rangle = 2\langle \delta_3'(2x), \varphi \rangle$$

$$= 2\langle \delta_3', \tfrac{1}{2}\varphi(\tfrac{x}{2}) \rangle$$

$$= -2\langle \delta_3, \tfrac{d}{dx}(\tfrac{1}{2}\varphi(\tfrac{x}{2})) \rangle$$

$$= -2\langle \delta_3, \tfrac{1}{4}\varphi'(\tfrac{x}{2}) \rangle$$

$$= -2\left(\tfrac{1}{4}\varphi'(\tfrac{3}{2})\right) = \tfrac{1}{2}\left(-\varphi'(\tfrac{3}{2})\right)$$

$$= \tfrac{1}{2}\langle \delta_{\frac{3}{2}}', \varphi \rangle = \langle \tfrac{1}{2}\delta_{\frac{3}{2}}', \varphi \rangle$$

per cui $2\delta_3'(2x) = \frac{1}{2}\delta_{\frac{3}{2}}'$