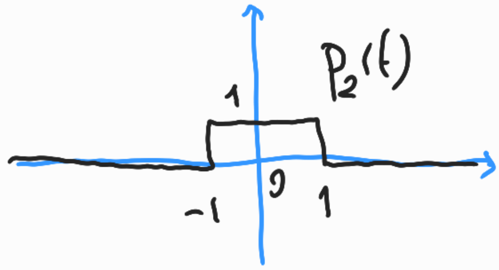
Es (3/7/17)

$\mathcal{F}\mathcal{F}(p_2) = ?$

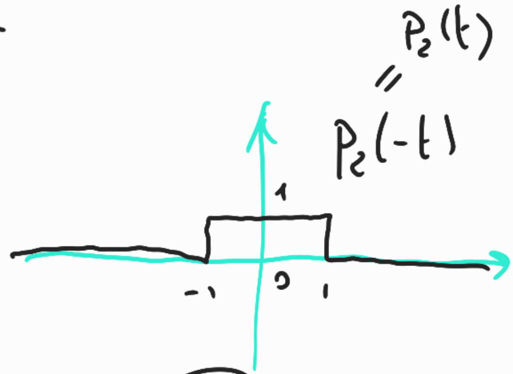


Ⓐ $p_2(t)$

Ⓑ $\dfrac{\sin(2\pi t)}{\pi t}$

Ⓒ $-\dfrac{\sin(2\pi t)}{\pi t}$

Ⓓ $-p_2(t)$



SOL

$$\mathcal{F}\mathcal{F}(p_2(t))(t) = p_2(-t) \overset{p_2 \text{ PARI}}{=} p_2(t) \longrightarrow \text{A}$$

Es(20/7/18) $T(t) = t^2 \delta_4(t+1)$, $\mathcal{F}(T') = ?$

$\delta_{x_0}(t-t_0) = \delta_{x_0+t_0}$

$h(t)\delta_{t_0} = h(t_0)\delta_{t_0}$

SOL

N.B. !! $\underbrace{t^2\delta_4(t+1)}_{= \, t^2[\delta_4(t+1)]} = t^2\delta_3 = (3)^2\delta_3 = 9\delta_3$

$t^2 f(t+1)$

$\Rightarrow T' = (9\delta_3)' = 9\delta_3'$

$\Rightarrow \mathcal{F}(T')(\omega) = (2\pi i\omega)^1 \mathcal{F}(T)(\omega)$

$= 2\pi i\omega \, \mathcal{F}(9\delta_3) = 2\pi i\omega \, 9 \, e^{-2\pi i 3\omega}$

$= 18\pi i\omega \, e^{-6\pi i\omega}$

Es (17-7-17) Se

$$g(t) = \frac{2t e^{\pi i t}}{(1+t^2)^2} \quad , \quad \mathcal{F}(g) = ?$$

SOL

$$\mathcal{F}\left(\frac{2t e^{\pi i t}}{(1+t^2)^2}\right)(\omega) = \mathcal{F}\left(\frac{2t e^{2\pi i \frac{1}{2} t}}{(1+t^2)^2}\right)(\omega)$$

$$\overset{(*)}{=} \mathcal{F}\left(\frac{2t}{(1+t^2)^2}\right)\left(\omega - \frac{1}{2}\right) = \mathcal{F}\left(\frac{d}{dt}\left(-\frac{1}{1+t^2}\right)\right)\left(\omega - \frac{1}{2}\right)$$

$$= \left(2\pi i\left(\omega - \frac{1}{2}\right)\right)^1 \mathcal{F}\left(-\frac{1}{1+t^2}\right)\left(\omega - \frac{1}{2}\right)$$

$$= -\left(2\pi i\left(\omega - \frac{1}{2}\right)\right) \frac{\pi}{1} e^{-2\pi 1 \left|\omega - \frac{1}{2}\right|}$$

$$= -2\pi^2 i\left(\omega - \frac{1}{2}\right) e^{-2\pi\left|\omega - \frac{1}{2}\right|}$$

(*) $\mathcal{F}\left(e^{2\pi i \omega_0 t} T(t)\right)(\omega)$

$\shortparallel$

$\mathcal{F}(T(t))(\omega - \omega_0)$

con $\omega_0 = \frac{1}{2}$

Es (3-7-17) Sia $g(t) = H(t+2)e^{-t}$ e $T(t) = (t-1)\delta_3(t+1) + T_g$

Verificare che $T \in \mathcal{S}'$ e calcolare $\mathcal{F}(T)$

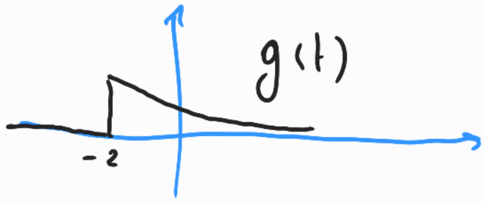
SOL

$$T(t) = (t-1)\,\delta_3(t+1) + T_g = (t-1)\,\delta_2 + T_g = (2-1)\,\delta_2 + T_g$$
$$= \delta_2 + T_g\ ;$$

δ_2 ha supporto compatto $\Rightarrow \delta_2 \in \mathcal{S}'$ (temperata);

g limitata $\Rightarrow T_g \in \mathcal{S}'$

$\uparrow$ oppure perché g è sommabile



$$\Longrightarrow T = \delta_2 + T_g \in \mathcal{S}'$$

$$\mathcal{F}(T) = \mathcal{F}(\delta_2)(\omega) + \mathcal{F}(T_g)(\omega)$$
$$= \mathcal{F}(\delta_2)(\omega) + \mathcal{F}(g)(\omega)$$
$$= e^{-2\pi i 2\omega} + \mathcal{F}(H(t+2)e^{-t})(\omega)$$
$$= e^{-4\pi i\omega} + \mathcal{F}(H(t+2)e^{-(t+2)}e^{2})(\omega)$$
$$= e^{-4\pi i\omega} + e^{2}e^{-2\pi i(-2)\omega}\mathcal{F}(H(t)e^{-t})(\omega)$$
$$= e^{-4\pi i\omega} + e^{2}e^{4\pi i\omega}\frac{1}{1+2\pi i\omega}$$

Es (8-9-16) $S = \mathcal{F}\left(\mathbb{1}_{[2,4]}(t)\right) + \mathcal{F}^{-1}\left(t^3 \delta_5(t+3)\right)$. Allora in $\mathcal{S}'$:

(A) $\mathcal{F}(S)$ non è temperata (B) $\mathcal{F}(S) = \mathbb{1}_{[-4,-2]} + 8\delta_{-2}$

(C) $\mathcal{F}(S) = p_2(t+3) - 8\delta_{-2}$ (D) $\mathcal{F}(S) = p_2(t+3) + 8\delta_2$

SOL (A) FALSA!

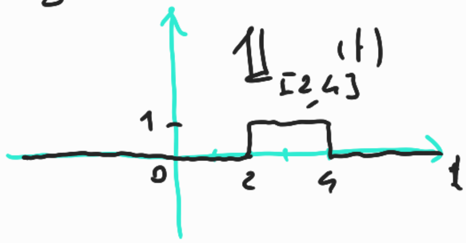
$$\mathcal{F}(S)(t) = \mathcal{F}\mathcal{F}\left(\mathbb{1}_{[2,4]}(t)\right)(t) + \mathcal{F}\mathcal{F}^{-1}\left(t^3\delta_5(t+3)\right)(t)$$

$$= \mathbb{1}_{[2,4]}(-t) + t^3\delta_5(t+3)$$

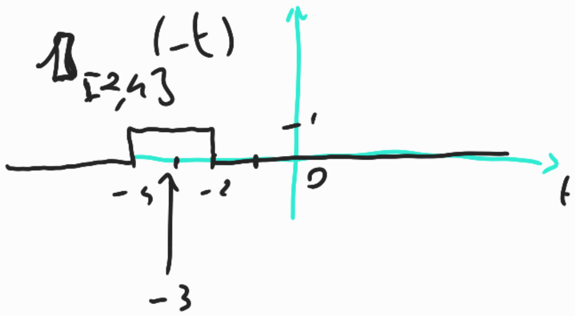
$$= \mathbb{1}_{[-4,-2]}(t) + t^3\delta_2$$

$$= \mathbb{1}_{[-4,-2]}(t) + 8\delta_2$$

$$= p_2(t+3) + 8\delta_2 \longrightarrow \text{(D)}$$



$\mathbb{1}_{[-4,-2]}(t) = \mathbb{1}_{[2,4]}(-t)$



Es (20/1/20)

$$T = (t+1)\delta_7 - T_g \quad \text{dove } g(t) = \cos t$$

$$\mathcal{F}(T) = ?$$

Sol

$$T = (t+1)\delta_7 - \cos t = 8\delta_7 - \cos t$$

$$\Rightarrow \mathcal{F}(T)(\omega) = 8\mathcal{F}(\delta_7)(\omega) - \mathcal{F}(\cos t)(\omega)$$

$$= 8e^{-2\pi i 7\omega} - \mathcal{F}\left(\frac{e^{it} + e^{-it}}{2}\right)(\omega)$$

$$= 8e^{-14\pi i\omega} - \frac{1}{2}\mathcal{F}\left(e^{2\pi i \frac{1}{2\pi} t}\right)(\omega) - \frac{1}{2}\mathcal{F}\left(e^{2\pi i\left(-\frac{1}{2\pi}\right)t}\right)(\omega)$$

$$= 8e^{-14\pi i\omega} - \frac{1}{2}\delta_{\frac{1}{2\pi}} - \frac{1}{2}\delta_{-\frac{1}{2\pi}}$$

— · —

Def Se $T \in \mathcal{D}'$,

(i) T si dice PARI se $T(-t) = T(t)$

(ii) T si dice DISPARI se $T(-t) = -T(t)$

Es δ_0 è PARI: infatti $\delta_0(-t) = \delta_{-0}(t) = \delta_0(t)$

Es

$$\boxed{\begin{array}{l} T \text{ PARI} \Rightarrow \mathcal{F}(T) \text{ PARI};\\ T \text{ DISPARI} \Rightarrow \mathcal{F}(T) \text{ DISPARI}\end{array}}$$

PROP

(a) T PARI $\Rightarrow$ T' DISPARI

(b) T DISPARI $\Rightarrow$ T' PARI

Dim

(a) T PARI (cioè $T(-t) = T(t)$)

$$\langle T'(-t), \varphi \rangle = \langle T', \varphi(-t) \rangle = -\langle T, -\varphi'(-t) \rangle$$

$$= +\langle T, \varphi'(-t) \rangle \underset{T \text{ PARI}}{=} \langle T(-t), \varphi'(-t) \rangle$$

$$= \langle T(t), \varphi'(t) \rangle = -\langle T'(t), \varphi(t) \rangle$$

$$\Rightarrow T'(-t) = -T'(t) \quad \text{OK.}$$

(b) ESERCIZIO

— · —

È possibile dimostrare che se $f(t) = \log|t|$ allora f è localmente sommabile (Analisi 1)

e $T'_{\log|t|}$ è una

distribuzione non regolare chiamata VALORE PRINCIPALE DI $\frac{1}{t}$, denotata col simbolo $v.p. \frac{1}{t} \in \mathcal{D}'$ che agisce in questo modo: $\forall \varphi \in \mathcal{D}$

$$\left\langle v.p.\frac{1}{t}, \varphi \right\rangle = \lim_{\varepsilon \to 0^+} \int_{-\infty}^{-\varepsilon} \frac{\varphi(t)}{t}\, dt + \int_{\varepsilon}^{+\infty} \frac{\varphi(t)}{t}\, dt$$

QUINDI

$$T'_{\log|t|} = v.p.\frac{1}{t}$$

(SI DIMOSTRA)

[N.B. LA DERIVATA DELL'ANALISI 1

$$\frac{d}{dt}(\log|t|) = \frac{1}{t}$$ CHE NON È LOC. SOMMABILE

PERCHÉ $\int_0^1 \frac{1}{t}\, dt = +\infty$

$\Rightarrow$ $T_{\frac{1}{t}}$ NON HA SENSO]

Inoltre

$\log|t|$ PARI $\Rightarrow$ $T_{\log|t|}$ PARI $\Rightarrow$ $\overbrace{\text{v.p.}\,\frac{1}{t}}^{T'_{\log|t|}}$ DISPARI

— o —

È POSSIBILE VERIFICARE CHE

$$\boxed{\mathcal{F}(H(t))(\omega) = \frac{1}{2\pi i}\,\text{v.p.}\,\frac{1}{\omega} + \frac{\delta_0}{2}}$$

E ANCHE (NON FACILE)

$$\mathcal{F}\left(\sum_{n=-\infty}^{+\infty} \delta_n\right) = \sum_{n=-\infty}^{+\infty} \delta_n$$