Homework 01

Exercise 1

Let X be a continuous random variable with PDF

$$f(x) = \begin{cases} \frac{5}{32}x^4 & \text{for } 0 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

and let $Y = X^2$.

1. Find the CDF of Y.
2. Find the PDF of Y.
3. Find $\mathbb{E}[Y]$.

Exercise 2

Suppose that the joint PDF of X and Y is

$$f(x, y) = \begin{cases} \frac{15}{4}x^2 & \text{for } 0 \leq y \leq 1 - x^2 \text{ and } -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the marginal PDFs of X and Y.

Exercise 3

Let X and Y be continuous random variables with joint PDF

$$f(x, y) = \begin{cases} 6e^{-(2x+3y)} & \text{for } x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

1. Are X and Y independent?
2. Find $\mathbb{E}[Y|X > 2]$.
3. Find $P(X > Y)$.

Exercise 4

Let X and Y be two continuous random variables with joint PDF

$$\begin{cases} x + \frac{3}{2}y^2 & \text{for } 0 \leq x, y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the MAP and the ML estimates of X given $Y = y$.

Exercise 5

Find the VC-dimension of the set of the hyperplanes in a d-dimensional space.

Hint: consider the problem of binary classification in $\mathbb{R}^d$.