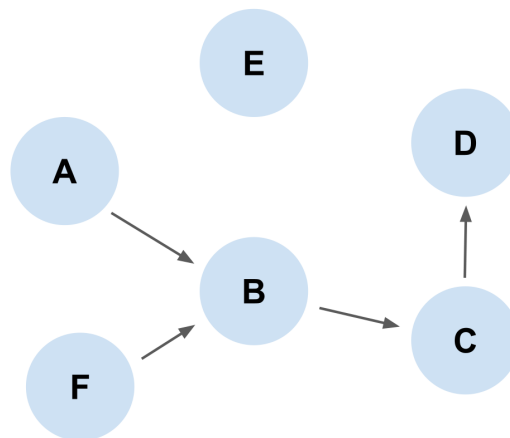


Homework 03

Exercise 1

1. Write the factorization of the joint distribution $P(A, B, C, D, E, F)$ corresponding to the Bayesian network:



2. Indicate whether the following statements on conditional indepenence are True or False and motivate your answer.

- a. $A \perp\!\!\!\perp B$
- b. $A \perp\!\!\!\perp F$
- c. $A \perp\!\!\!\perp C \mid \{B, E\}$
- d. $F \perp\!\!\!\perp D \mid B$
- e. $B \perp\!\!\!\perp D \mid C$

Exercise 2

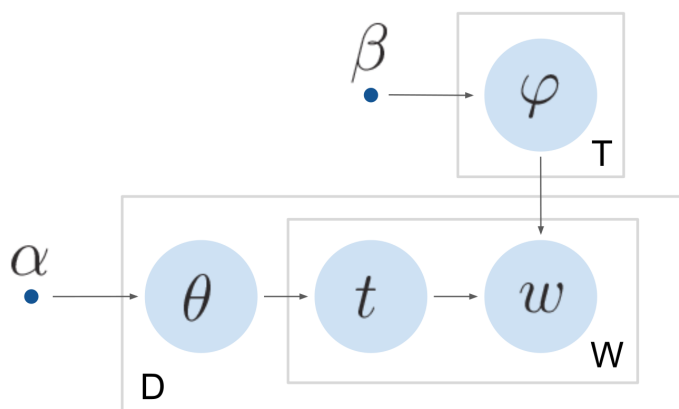
Topic models are statistical models that learn the distribution of the abstract topics occurring in a collection of documents. In this context, documents are collections of topics and topics are collections of words.

We consider a dictionary of N words, an ordered collection of D documents and an ordered collection of T possible topics appearing in each document. We build our model using the indexes representing each quantity in its collection (e.g. i is the i -th word in the dictionary). For simplicity, we assume that all documents contain the same number of words. Each document can contain multiple topics; specifically, we associate a topic to each word appearing in the document.

We make the following assumptions:

- D is the total number of documents
- T is the total number of topics
- $W < N$ is the number of words per document
- For the j -th word and the i -th document, we sample a topic $t_{i,j}$ from a Categorical distribution on θ_i
- The distribution θ_i of topics in the i -th document is a Dirichlet distribution with concentration parameters α (the length of α is T)
- The j -th word in the i -th document, namely $w_{i,j}$, is sampled from a Categorical distribution on $\varphi_{t_{i,j}}$
- The distribution φ_k of words per topic k is a Dirichlet distribution with concentration parameters β (the length of β is N)

1. Write the generative process of the following graphical model. Be aware of plate notation!



2. Set $T = 5$, $D = 10$, $W = 50$, $N = 100$ and use pyro to implement this graphical model as a function of the hyperparameters `model(alpha, beta)`, that outputs `theta`, `phi`, `t`, `w`.
3. Evaluate your function on the hyperparameters

```
alpha = torch.tensor([0.5, 0.3, 0.1, 0.4, 0.2])
beta = dist.Gamma(1./T, 1.).sample((W,))
```

and print the shape of the output tensors `theta`, `phi`, `t`, `w`.