Homework 02

Exercise 1

Use the following lemma to generate 500 samples from a $Binomial(30, 0.5)$ distribution in pyro. Plot the resulting distribution using `sns.distplot` function.

Lemma Let $X_1, \ldots, X_n$ be independent $Bernoulli(p)$ random variables, then $X = X_1 + \ldots + X_n$ is a random variable with $Binomial(n, p)$ distribution.

Exercise 2

1. (**theory**) Consider the Gamma distribution[1] $\text{Gamma}(\alpha, \beta)$ with p.d.f.
$$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$
and the Poisson distribution $\text{Poisson}(\lambda)$ with p.m.f.
$$\frac{\lambda^k e^{-\lambda}}{k!}.$$
Given the generative model
$$\begin{aligned} \lambda &\sim \text{Gamma}(\alpha, \beta) \\ x &\sim \text{Poisson}(\lambda) \end{aligned}$$
prove that the Gamma distribution is a conjugate prior[2] for the Poisson likelihood.
2. (**code**) Set the parameters of the Gamma distribution to $\alpha = 1$ and $\beta = 2$. Write the pyro code to sample λ and x, extract 300 samples from the distributions of both random variables and plot their histograms.
3. (**code**) Suppose you observe some data $x = [3, 10, 2, 5, 6, 7]$. Plot 300 samples from the posterior distribution of λ as the number of observed data points from x increases:
$$\begin{aligned} x &= [3, 10] \\ x &= [3, 10, 2, 5] \\ x &= [3, 10, 2, 5, 6, 7]. \end{aligned}$$

[1] Section "Probability distributions" in notebook 01.

[2] Section "Conjugate priors" in notebook 02.