Homework_2_solutions_Marco_Sciorilli

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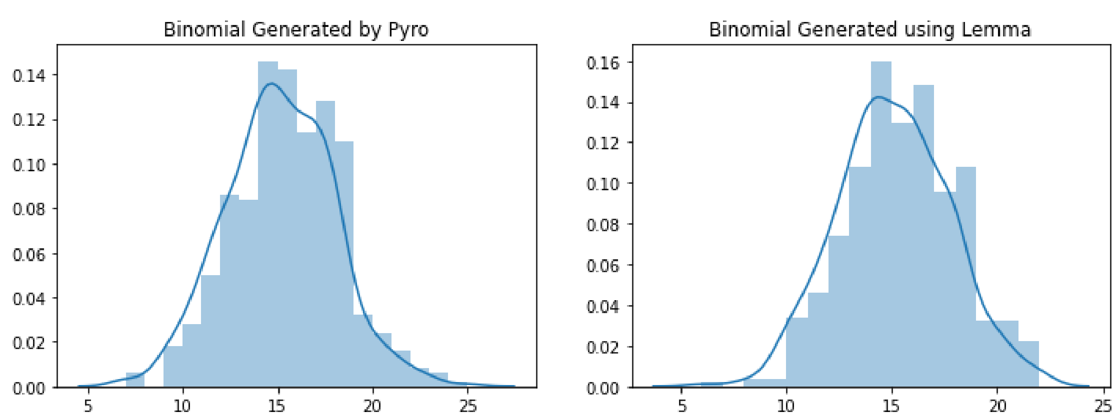
0.1 Exercise 1

[84]:
```
import torch
import pyro
import seaborn as sns
import matplotlib.pyplot as plt

#my binomial function
def binomial(a, b):
    bernoulli = dist.Bernoulli(b)
    bernoulli.sample = [pyro.sample("n", bernoulli).item() for i in range(a)]
    bernoulli.sample = sum(bernoulli.sample)
    return bernoulli.sample

#samples
binomial_auto = dist.Binomial(30, 0.5)
binomial_auto_samples = [pyro.sample("n", binomial_auto ) for i in range(500)]
binomial_samples = [binomial(30, 0.5) for i in range(500)]

#plot
fig, axes = plt.subplots(1, 2, figsize=(12,4))
sns.distplot(binomial_auto_samples, ax=axes[0])
sns.distplot(binomial_samples, ax=axes[1])
axes[0].set_title('Binomial Generated by Pyro')
axes[1].set_title('Binomial Generated using Lemma')
plt.show()
```



0.2 Exercise 2

0.2.1 1.

If the posterior distribution $p(\theta|x)$ belongs to the same family as the prior distribution $p(\theta)$, then the prior is said to be a **conjugate prior** for the likelihood function $p(x|\theta)$. In our case:

$$\lambda \sim \text{Gamma}(\alpha, \beta)$$

$$x \sim \text{Poisson}(\lambda)$$

Where the probability distribution for the Poisson likelihood is

$$P(x_i|\lambda) = \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

And the prior distribution

$$P(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$

And the Bayes Theorem states for a vector of n observations $\vec{x}$

$$P(\lambda|\vec{x}) = \frac{P(\vec{x}|\lambda)P(\lambda)}{P(\vec{x})}$$

The denominator doesn't contain λ, so

$$P(\lambda|\vec{x}) \propto P(\vec{x}|\lambda)P(\lambda)$$

If every x is independent

$$P(\vec{x}|\lambda) = \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} = \frac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{\prod_{i=1}^{n} x_i!} \propto \lambda^{n\overline{x}} e^{-n\lambda}$$

So once again the posterior is, dropping all the terms not related to λ as before

$$P(\lambda|\vec{x}) \propto P(\vec{x}|\lambda)P(\lambda) = \lambda^{n\overline{x}} e^{-n\lambda} \lambda^{\alpha-1} e^{-\beta\lambda} = \lambda^{n\overline{x}+\alpha-1} e^{-(\beta+n)\lambda} \sim Gamma(n\overline{x} + \alpha, \beta + n)$$

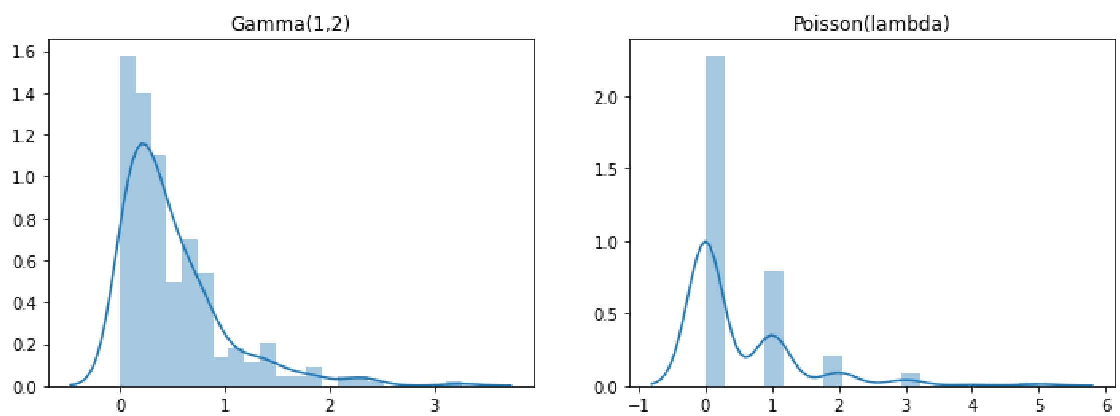
So we have proved that Gamma distribution is a conjugate prior for the Poisson likelihood.

0.2.2 2.

[98]:
```
import torch
import pyro
import seaborn as sns
import matplotlib.pyplot as plt


#samples
gamma_auto = dist.Gamma(1, 2)
#get 300 gamma samples
gamma_auto_samples = [pyro.sample("n", gamma_auto ).item() for i in range(300)]
#get 300 x for a poisson distibution with the found-before gammas
poisson_samples = [pyro.sample("n", dist.Poisson(gamma_auto_samples[i]) ) for i␣
↪in range(300)]

#plot
fig, axes = plt.subplots(1, 2, figsize=(12,4))
sns.distplot(gamma_auto_samples, ax=axes[0])
sns.distplot(poisson_samples, ax=axes[1])
axes[0].set_title('Gamma(1,2)')
axes[1].set_title('Poisson(lambda)')
plt.show()
```



0.2.3 3.

[126]:
```
import torch
import pyro
import seaborn as sns
import matplotlib.pyplot as plt

prior_list = [1, 2]
x = [3, 10]
```

```
lamb_posterior_samples = [pyro.sample("lambda_posterior", dist.
 ↪Gamma(prior_list[0] + sum(x), prior_list[1] + len(x)))for i in range(300)]
x1 = [3, 10, 2, 5]
lamb_posterior_samples1 = [pyro.sample("lambda_posterior", dist.
 ↪Gamma(prior_list[0] + sum(x1), prior_list[1] + len(x1)))for i in range(300)]
x2 = [3, 10, 2, 5, 6, 7]
lamb_posterior_samples2 = [pyro.sample("lambda_posterior", dist.
 ↪Gamma(prior_list[0] + sum(x2), prior_list[1] + len(x2)))for i in range(300)]
prior_samples = [pyro.sample("Prior", dist.Gamma(prior_list[0],␣
 ↪prior_list[1]))for i in range(300)]


#plot
plt.figure("Lambda distribustions",figsize=(12,8))
sns.distplot(lamb_posterior_samples, hist=False, label = 'Posterior after first␣
 ↪x')
sns.distplot(lamb_posterior_samples1, hist=False, label = 'Posterior after␣
 ↪second x')
sns.distplot(lamb_posterior_samples2, hist=False, label = 'Posterior after␣
 ↪third x')
sns.distplot(prior_samples, hist=False, label = 'Prior distribution')
plt.legend()
plt.show()
```