

Homework_3

April 9, 2021

0.1 Exercise 1

0.1.1 1.

$$P(A, B, C, D, E, F) = P(E)P(A)P(F)P(B|A, F)P(C|B)P(D|C) \quad (1)$$

2. 1. False: $P(A, B) = P(A)P(B|A) \neq P(A)P(B)$ 2. True: $P(A, F) = P(A)P(F)$ 3. True: $P(A, B, C, E) = P(A)P(E)P(B|A)P(C|B)$ implies $P(A, C|B, E) = \frac{P(A)P(E)P(B|A)P(C|B)}{P(E)P(B)} = P(A|B)P(C|B) = P(A|B, E)P(C|B, E)$ as they are both independent from E. 3. True: As for 3., it is a head to case tail where the intermediate node is given. 4. True: same as for 3 and 4.

0.2 Exercise 2

0.2.1 1.

The generative model is:

For each document i : - $\theta_i \sim \text{Dirichlet}_T(\alpha)$ - for each word j : - $t_{i,j}|\theta_i \sim \text{Categorical}(\theta_i)$ - $\varphi_{t_{i,j}} \sim \text{Dirichlet}_N(\beta)$ - $w_{i,j}|\varphi_{t_{i,j}} \sim \text{Categorical}(\varphi_{t_{i,j}})$

0.3 2-3.

```
[1]: import pyro
import torch
import pyro.distributions as dist

#set values
T = 5
D = 10
W = 50
N = 100

def model(alpha, beta):

    with pyro.plate('documents', T):
        phi = pyro.sample('theta', dist.Dirichlet(beta * torch.ones(N)))

    with pyro.plate('documents', D):
        theta = pyro.sample('theta', dist.Dirichlet(alpha * torch.ones(T)))
        with pyro.plate('words', W):
            t = pyro.sample('t', dist.Categorical(probs=theta))
```

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w = pyro.sample('w', dist.Categorical(probs=phi[t]))

print("theta =",theta.shape,"\nphi =",phi.shape,"\nt =", t.shape,"\nw =",w.
↪shape)

return theta, phi, t, w

alpha = torch.tensor([0.5, 0.3, 0.1, 0.4, 0.2])
beta = dist.Gamma(1./T, 1.).sample((N,))
model(alpha, beta)

```

```

theta = torch.Size([10, 5])
phi = torch.Size([5, 100])
t = torch.Size([50, 10])
w = torch.Size([50, 10])

```

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