2

$$L = \frac{m_1}{2}(l_1 \dot{\varphi}_1)^2 + \frac{m_2}{2}\left[(l_1\dot{\varphi}_1)^2 + (l_2\dot{\varphi}_2)^2 + 2l_1l_2\dot{\varphi}_1\dot{\varphi}_2\cos(\varphi_1-\varphi_2)\right]$$

$$-m_1 g l_1(1-\cos\varphi_1) - m_2 g\left[l_1(1-\cos\varphi_1) + l_2(1-\cos\varphi_2)\right)$$

(a) $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$$

first φ_1:

$$q_1 \equiv \frac{\partial L}{\partial \dot{\varphi}_1} = m_1 l_1^2 \dot{\varphi}_1 + m_2 l_1^2 \dot{\varphi}_1 + l_1 l_2 m_2 \dot{\varphi}_2 \cos(\varphi_1-\varphi_2)$$

$$= \dot{\varphi}_1 l_1^2(m_1 + m_2) + m_2 l_1 l_2 \dot{\varphi}_2 \cos(\varphi_1-\varphi_2)$$

Euler lagrange!

$$\frac{d}{dt}(q_1) = \frac{\partial L}{\partial \varphi_1}$$

$$\frac{\partial L}{\partial \varphi_1} = \frac{dq_1}{dt} = -m_2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 \sin(\varphi_1-\varphi_2) - m_1 g l_1 \sin\varphi_1$$

$$- m_2 g l_1 \sin\varphi_1 = -m_2 l_1 l_2 \dot{\varphi}_1 \dot{\varphi}_2 \sin(\varphi_1-\varphi_2)$$

$$- g l_1 \sin\varphi_1 (m_1 + m_2)$$

then φ_2:

$$p_2 \equiv \frac{\partial L}{\partial \dot\varphi_2} = m_2 l_2^2 \dot\varphi_2 + l_1 l_2 \dot\varphi_1 \cos(\varphi_1 - \varphi_2) m_2$$

$$\frac{d}{dt}(p_2) = \frac{\partial L}{\partial \varphi_2} = + m_2 l_1 l_2 \dot\varphi_1 \dot\varphi_2 \sin(\varphi_1 - \varphi_2)$$
$$- m_2 2 g l_2 \sin(\varphi_2)$$

$$p_1 = \dot\varphi_1 l_1^2 (m_1 + m_2) + m_2 l_1 l_2 \dot\varphi_2 \cos(\varphi_1 - \varphi_2)$$

$$p_2 = m_2 l_2^2 \dot\varphi_2 + l_1 l_2 \dot\varphi_1 \cos(\varphi_1 - \varphi_2) m_2$$

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \underbrace{\begin{pmatrix} l_1^2 (m_1 + m_2) & m_2 l_1 l_2 \cos(\varphi_1 - \varphi_2) \\ m_2 l_1 l_2 \cos(\varphi_1 - \varphi_2) & m_2 l_2^2 \end{pmatrix}} \begin{pmatrix} \dot\varphi_1 \\ \dot\varphi_2 \end{pmatrix}$$

$$\vec p = M \frac{d\vec\varphi}{dt} \Rightarrow M^{-1} \vec p = \frac{d\vec\varphi}{dt}$$

$$M^{-1} = \frac{1}{\det(M)} \begin{pmatrix} m_2 l_2^2 & -m_2 l_1 l_2 \cos(\varphi_1-\varphi_2) \\ -m_2 l_1 l_2 \cos(\varphi_1-\varphi_2) & l_1^2(m_1+m_2) \end{pmatrix}$$

$$\det(M) = \left(l_1^2 l_2^2 m_2(m_1+m_2)\right) - \left(m_2 l_1 l_2 \cos(\varphi_1-\varphi_2)\right)^2$$

$$= \left(l_1 l_2 \sqrt{m_2(m_1+m_2)} - m l_1 l_2 \cos(\varphi_1-\varphi_2)\right)\left(l_1 l_2 \sqrt{m_2(m_1+m_2)} + m l_1 l_2 \cos(\varphi_1-\varphi_2)\right)$$

b)

$$\begin{cases} \dot\varphi_1 = \dfrac{q_1 m_2 l_2^2 - q_2 m_2 l_1 l_2 \cos(\varphi_1-\varphi_2)}{\det(M)} \\[2ex] \dot\varphi_2 = \dfrac{-q_1 m_2 l_1 l_2 \cos(\varphi_1-\varphi_2) + q_2 l_1^2(m_1+m_2)}{\det(M)} \\[2ex] \dot q_1 = -m_2 l_1 l_2 \dot\varphi_1 \dot\varphi_2 \sin(\varphi_1-\varphi_2) - g l_1 \sin\varphi_1 (m_1+m_2) \\[2ex] \dot q_2 = m_2 l_1 l_2 \dot\varphi_1 \dot\varphi_2 \sin(\varphi_1-\varphi_2) - 2 g l_2 \sin(\varphi_2) m_2 \end{cases}$$