Dennis, in concordance with the third paragraph of your last email

> *For the mussel data, I suggest that you also do confidence intervals for each of the four beta coefficients and compare them with the usual linear regression confidence intervals. This should provide a nice contrast with the confidence intervals for $E(Y|X_i)$, i=1,...,10, in the dopamine data.*

we present here the four confidence intervals for $\widehat{\beta}_i$ for $i = 1, 2, 3, 4$ with both methods. I tried to print both results in the same plot but the scale was too different to be noticeable (I also compute some of these intervals using the estimations in section 4.4.4 from your book just to see if they were similar. The plots make sense because of the difference in $se(\widehat{\beta}_i)$ for both methods). Confidence intervals for Figure 2 were constructed following the Corollary in the main paper

$$\left[\widehat{\boldsymbol{\beta}}^T \boldsymbol{e}_i \pm z_{\alpha/2} V_{\boldsymbol{e}_i}^{1/2}\right]$$

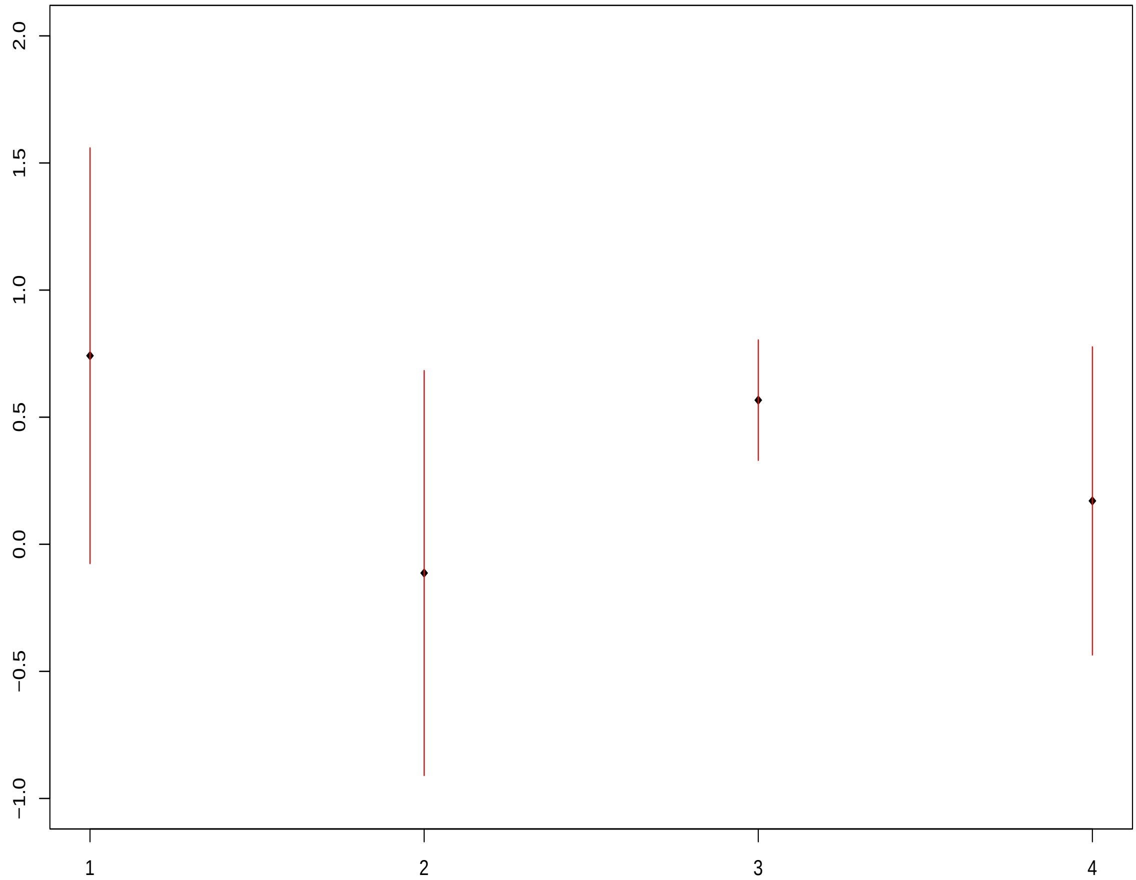


Figure 1: Confidence intervals for $\widehat{\beta}_i$ for $i = 1, 2, 3, 4$ using OLS regression.

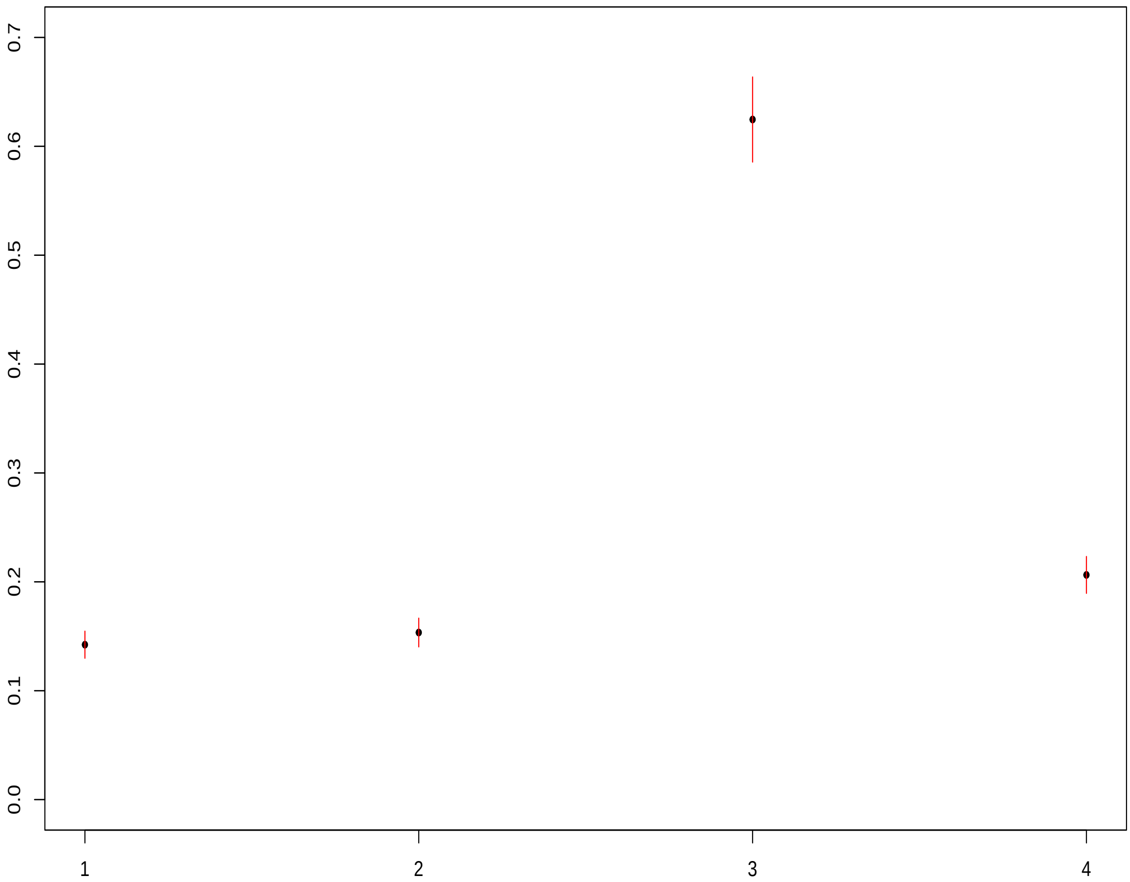


Figure 2: Confidence intervals for $\widehat{\beta}_i$ for $i = 1, 2, 3, 4$ using PLS regression with $u = 1$.

Table 1: Results for the OLS regression

| | Value | Std | Confidence Interval |
|---|---|---|---|
| $\widehat{\beta}_1$ | 0.741 | 0.410 | $(-0.0759, 1.559)$ |
| $\widehat{\beta}_2$ | -0.113 | 0.399 | $(-0.909, 0.683)$ |
| $\widehat{\beta}_3$ | 0.567 | 0.118 | $(0.330, 0.803)$ |
| $\widehat{\beta}_4$ | 0.170 | 0.304 | $(-0.435, 0.776)$ |

Table 2: Results for the PLS regression with $u = 1$

| | Value | Std | Confidence Interval |
|---|---|---|---|
| $\widehat{\beta}_1$ | 0.142 | 0.0063 | $(0.129, 0.154)$ |
| $\widehat{\beta}_2$ | 0.153 | 0.0067 | $(0.140, 0.166)$ |
| $\widehat{\beta}_3$ | 0.624 | 0.0199 | $(0.585, 0.663)$ |
| $\widehat{\beta}_4$ | 0.206 | 0.0086 | $(0.189, 0.223)$ |

Now, with respect to the other topic, I think there's a misconfusion because we were not too clear. What we are doing (in the case of the mussels data) is the prediction intervals using leave-one-out. In fact, we have to add the following Corollary that we must use in the examples

**Corollary 1.** *Under the hypothesis of Theorem 1 an approximate $1-\alpha$ level prediction interval is given by*

$$PI_\alpha = \left[\widehat{\boldsymbol{\beta}}_{PLS}^T \boldsymbol{X}_N \pm z_{\alpha/2} \left(V + \tau^2\right)^{1/2}\right].$$

More specifically, in Figure 3 we have the same plot than before but I changed the labels so now it should be correct. The way they were constructed was, for the $i$-th observation

- fit the $\text{PLS}_{u=1}$ model for $y_{(-i)}, \boldsymbol{X}_{(-i)}$
- define $r = y_{(-i)} - \boldsymbol{X}_{(-i)}^T \widehat{\boldsymbol{\beta}}_{(-i)}$
- $\widehat{\tau}^2 = \text{var}(r)$
- $y_n = \boldsymbol{X}_i^T \widehat{\boldsymbol{\beta}}_{(-i)} + \bar{y}_{(-i)}$
- construct the prediction interval with $\left[y_n \pm \left(V + \widehat{\tau}^2\right)^{1/2} z_{\alpha/2}\right]$

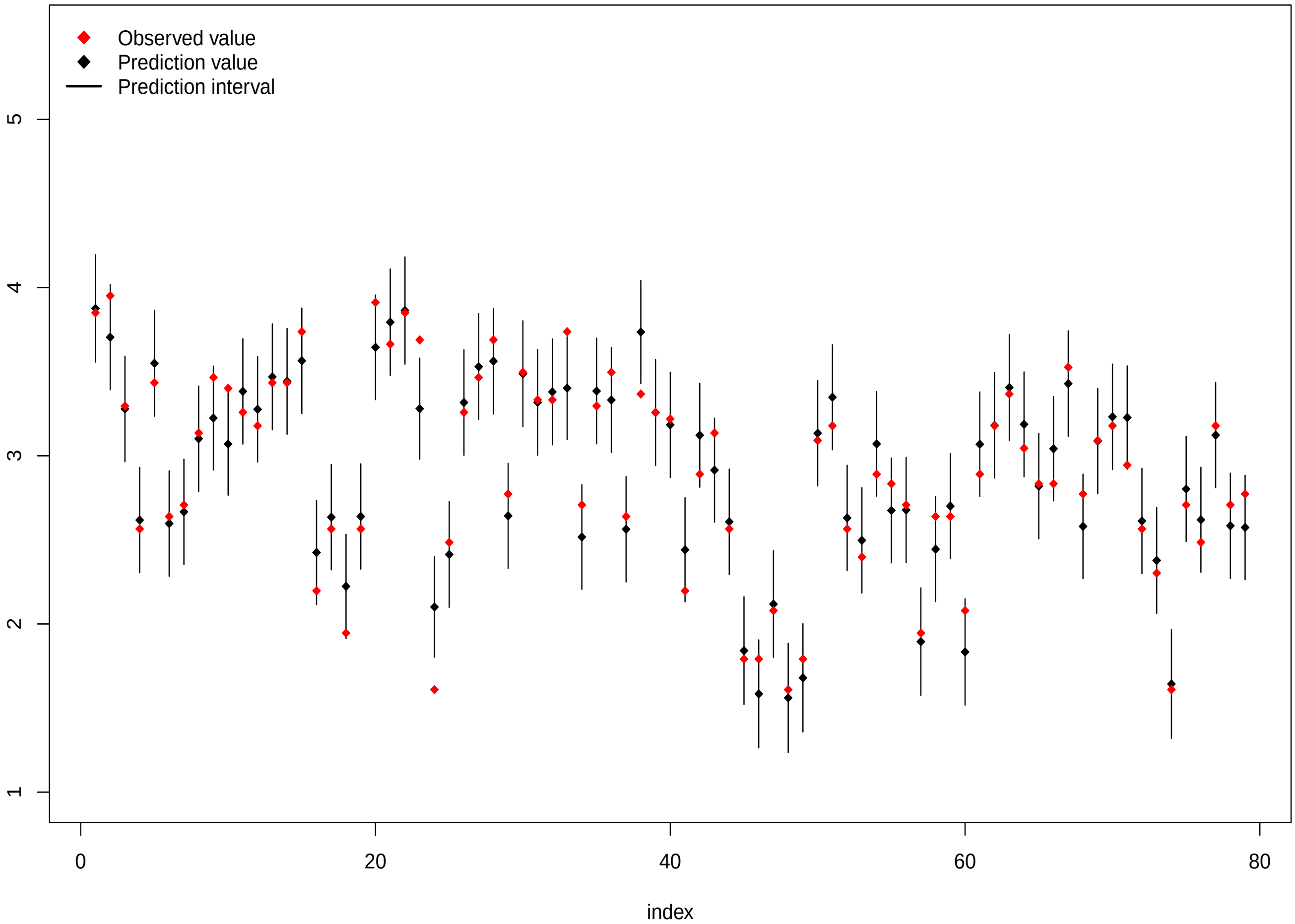


Figure 3: Intervals for the $n = 79$ observations in the mussels data. 93.67% of the intervals contain the observed value.