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Multi Objective Differential Evolution approach for voltage stability constrained reactive power planning problem

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ABSTRACT

This paper presents the application of Multi Objective Differential Evolution (MODE) algorithm to solve the Voltage Stability Constrained Reactive Power Planning (VSCRPP) problem. Minimization of total cost of energy loss and reactive power production cost of capacitors and maximization of voltage stability margin are taken as the objectives in the Reactive Power Planning (RPP) problem. The *L*-index of the load buses is taken as the indicator of voltage stability. In the proposed approach, generator bus voltage magnitudes, transformer tap settings and reactive power generation of capacitor bank are taken as the control variables and are represented as the combination of floating point numbers and integers. The MODE emphasizes the non dominated solutions and simultaneously maintains diversity in the non dominated solutions. DE/randSF/1/bin strategy scheme of Differential Evolution with self tuned parameter which employs binomial crossover and difference vector based mutation is used for the VSCRPP problem. A fuzzy based mechanism is employed to get the best compromise solution from the pareto front to aid the decision maker. The proposed reactive power planning model is implemented on two test systems, IEEE 30 bus and IEEE 57 bus test systems. The simulation results of the proposed optimization approach show that MODE is better in maintaining diversity and optimality of solutions.

1. Introduction

Reactive power planning is one of the most challenging problems in power systems. It deals with the decisions of finding location and amount of reactive power resources in normal and stressed system conditions. It involves the simultaneous minimization of two objective functions; the first objective deals with the minimization of operation cost by reducing real power loss, the second objective minimizes the allocation cost of additional reactive power sources. The VAR planning aims at reduced VAR support to maintain feasible operation within acceptable voltage profile. When the transmission system is stressed due to various reasons, voltage instability limit the operation of the system and hence should be included in the VAR planning process. Ajjarapu et al. [1] proposed a method of determining the minimum amount of shunt reactive power support which indirectly maximizes the real power transfer before voltage collapse is encountered. A sequential quadratic programming algorithm is adopted to solve the optimal solution. Vaahedi et al. [2] proposed an algorithm for optimal VAR planning which takes into account voltage profile and voltage stability margins simultaneously. Wang et al. [3] proposed a flexible compensation method based on multi scenario and reactive power divisions to adapt the changes in future environment. The conventional optimization methods [4–6] may lead to local minimum and sometimes result in divergence in solving complex RPP problems.

Recently, evolutionary computation techniques like Genetic Algorithm (GA) [7] and Evolutionary Programming (EP) [8] have received greater attention to obtain global optimum for RPP problem. Lai and Ma [5] has proposed an evolutionary programming approach to RPP problem. The test results are compared with conventional gradient based optimization method. In [9], an integer-coded multi objective genetic algorithm is applied to reactive power planning problem considering both intact and contingent operating states. A modified Non Dominated Sorting Genetic Algorithm II (NSGA II) for multi objective RPP problem by incorporating dynamic crowding distance has been discussed in [10].

In this work, *L*-index proposed in [11] is used as the indicator of voltage stability. In this paper, VSCRPP problem is treated as multi objective optimization problem with minimization of cost of energy loss, reactive power production cost of capacitors and *L*-index (voltage stability index) as the objectives. Due to the presence of conflicting objectives, a multi objective optimization problem

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Nomenclature

G_{ij}, B_{ij}	conductance and susceptance of transmission line connected between *i*th and *j*th bus
P_i, Q_i	real and reactive power injection of *i*th bus
P_s	real power generation of slack bus
Q_{ci}	reactive power generation of *i*th VAR source installment
V_{gi}	generator voltage magnitude at bus *i*
t_{ki}	tap setting of transformer at branch *k*
N_l	number of transmission lines
N_C	number of reactive power source installation buses
N_T	number of tap-setting transformer branches
N_{PV}	number of voltage buses
N_B	total number of buses
N_{PQ}	number of load buses
N_{B-1}	total number of buses excluding slack bus
N_d	number of load level durations
d_l	duration of load level (h)
h	per unit energy cost
e_i	fixed VAR source installment cost at bus *i*
C_{ci}	per unit VAR source purchase cost at bus *i*
V_i	voltage magnitude of *i*th generator
V_j	voltage magnitude of *j*th generator
θ_{ji}	phase angle of the term F_{ji}
δ_i	voltage phase angle of *i*th generator unit
δ_j	voltage phase angle of *j*th generator unit
N_g	number of generating units
$L_{\max}$	maximum value of *L*-index in load buses

results in a number of optimal solutions known as pareto optimal solutions [12]. In multi objective optimization, effort must be made in finding the set of trade off pareto solutions by considering all objectives to be important. The ability of evolutionary techniques like Differential Evolution (DE) to find multiple solutions in one single simulation run makes them unique in solving multi objective optimizations. This paper proposes a Multi Objective Differential Evolution (MODE) with self tuned parameters for VSC-RPP problem. DE/randSF/1/bin scheme [13] is used for the RPP problem in which mutation scheme uses a randomly selected vector and only one weighted difference vector is used to perturb it. The mutation scheme is combined with binomial type crossover and with random scale vector. Due to the convergence speed, simplicity and robustness by MODE to reach the optimal solutions makes it suitable for large scale optimization problem like VSCRPP. The effectiveness of the proposed approach to solve the multi objective voltage stability constrained reactive power planning problem has been demonstrated in IEEE 30 bus and IEEE practical 57 bus test systems.

2. Problem formulation

Generally, the RPP problem is formulated as an optimization problem in which cost of energy loss and cost of reactive power production of capacitors are minimized satisfying a number of equality and inequality constraints. In this work, in addition to the above objective, minimization of $L_{\max}$ in the contingency state is included as additional objective of the RPP problem. The control variables of the problem are generator bus voltage magnitudes, tap settings of transformers and reactive power generation of capacitor banks. The mathematical formulation of the multi-objective RPP problem is given below:

2.1. Minimization of cost of energy loss and cost of reactive power production of capacitors

The objective function in RPP problem comprises of two terms, namely, the total cost of energy loss, W_C and the cost of reactive power production, I_C which is given by:

$$\text{Minimize } F_C = W_C + I_C \tag{1}$$

The first term W_C represents the total cost of energy loss as follows:

$$W_C = h\sum_{l\varepsilon N_l} d_l P_{loss,l} \tag{2}$$

where $P_{loss,l}$ is the network real power loss during the period of load level d_l and is given by equation:

$$P_{loss,l} = \sum_{\substack{k\in N_l \\ k=(i,j)}} g_k\left(V_i^2 + V_j^2 - 2V_iV_jCos\theta_{ij}\right) \tag{3}$$

The second term I_C represents the cost of VAR production of capacitors which has two components namely a fixed installation cost, e_i and variable cost, C_{ci}.

$$I_C = \sum_{i\in N_C} e_i + C_{ci}|Q_{ci}| \tag{4}$$

where Q_{ci} is reactive power source installation at bus *i* and Q_{ci} can be either positive or negative, depending on whether the installation is capacitive or reactive. Therefore, absolute values are used to compute the cost. The above two equations are put in one equation to obtain a comprehensive one.

2.2. Minimization of L-index

Static voltage stability analysis involves determination of an index called voltage stability index. This index is an appropriate measure of closeness of the system to voltage collapse. There are various methods of determining voltage stability index. One such method is *L*-index proposed in [11] which is based on load flow analysis. The bus with the highest *L* index value will be the most vulnerable bus in the system. The *L*-indices for a given load condition are computed for all the load buses and the maximum of the *L*-indices ($L_{\max}$) gives the proximity of the system to voltage collapse. The *L*-index has an advantage of indicating voltage instability proximity of current operating point without calculation of the information about the maximum loading point. Hence the minimization of *L*-index makes the system less prone to voltage collapse. The *L*-index of the *j*th node is given by the expression,

$$L_j = \left|1 - \sum_{i=1}^{N_g} F_{ji}\frac{V_i}{V_j}\angle(\theta_{ji} + \delta_i - \delta_j)\right| \tag{5}$$

The detailed calculation of *L* index is given in Appendix A.1.

2.3. System constraints

The RPP problem is subjected to the following equality and inequality constraints:

(i) Real power balance equation:

$$P_i - V_i\sum_{j=1}^{N_B} V_j[G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}] = 0; \quad i = 1,2\ldots,N_{B-1} \quad (6)$$

(ii) Reactive power balance equation:

$$Q_i - V_i\sum_{j=1}^{N_B} V_j[G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}] = 0; \quad i = 1,2\ldots,N_{PQ} \quad (7)$$

(iii) Slack bus real power generation limit:

$$P_s^{\min} \leqslant P_s \leqslant P_s^{\max} \quad (8)$$

(iv) Generator reactive power generation limit:

$$Q_{gi}^{\min} \leqslant Q_{gi} \leqslant Q_{gi}^{\max} \quad i \in N_{PV} \quad (9)$$

(v) Generator bus voltage limit:

$$V_{gi}^{\min} \leqslant V_{gi} \leqslant V_{gi}^{\max} \quad i \in N_B \quad (10)$$

(vi) Capacitor bank reactive power generation limit:

$$Q_{ci}^{\min} \leqslant Q_{ci} \leqslant Q_{ci}^{\max} \quad i \in N_C \quad (11)$$

(vii) Transformer tap setting limit:

$$t_{ki}^{\min} \leqslant t_{ki} \leqslant t_{ki}^{\max} \quad i \in N_T \quad (12)$$

(viii) Line flow limit:

$$S_l \leqslant S_l^{\max} \quad l \in N_l \quad (13)$$

3. Multi objective optimization

3.1. Overview of multi objective optimization

An optimization problem in which more than one objectives is involved is called as multi-objective optimization problem. A Multiobjective Optimization Problem (MOP) can be mathematically formulated as,

$$\text{Min} \quad F(x) = [f_1(x), \ldots, f_m(x)] \quad (14)$$

$$\text{S.t.}: \quad g_j(x) = 0 \quad j = 1, \ldots, M$$
$$h_k(x) \leqslant 0 \quad k = 1, \ldots, K$$

where $F(x)$ consists of m conflicting objective functions, x is the decision vector, g_j is the jth equality constraint and h_k is the kth inequality constraint.

In multi objective optimization, the improvement of one objective may lead to deteriotion of another. Thus, a single solution which can optimize all objective functions does not exist. The best trade-offs solutions called the pareto optimal solutions in one single simulation run can best be approximated by Multi Objective Evolutionary Algorithms (MOEAs). The above multi-objective optimization problem is solved using Multi Objective Differential Evolution algorithm to obtain the pareto optimal solutions.

3.2. Multi Objective Differential Evolution (MODE)

Differential Evolution (DE) [14] is a population-based stochastic search algorithm that works in the general framework of evolutionary algorithms. The optimization variables are represented as floating point numbers in the DE population. It starts to explore the search space by randomly choosing the initial candidate solutions within the boundary. Differential Evolution creates new off springs by generating a noisy replica of each individual of the population. The DE algorithm is extended in this work to solve

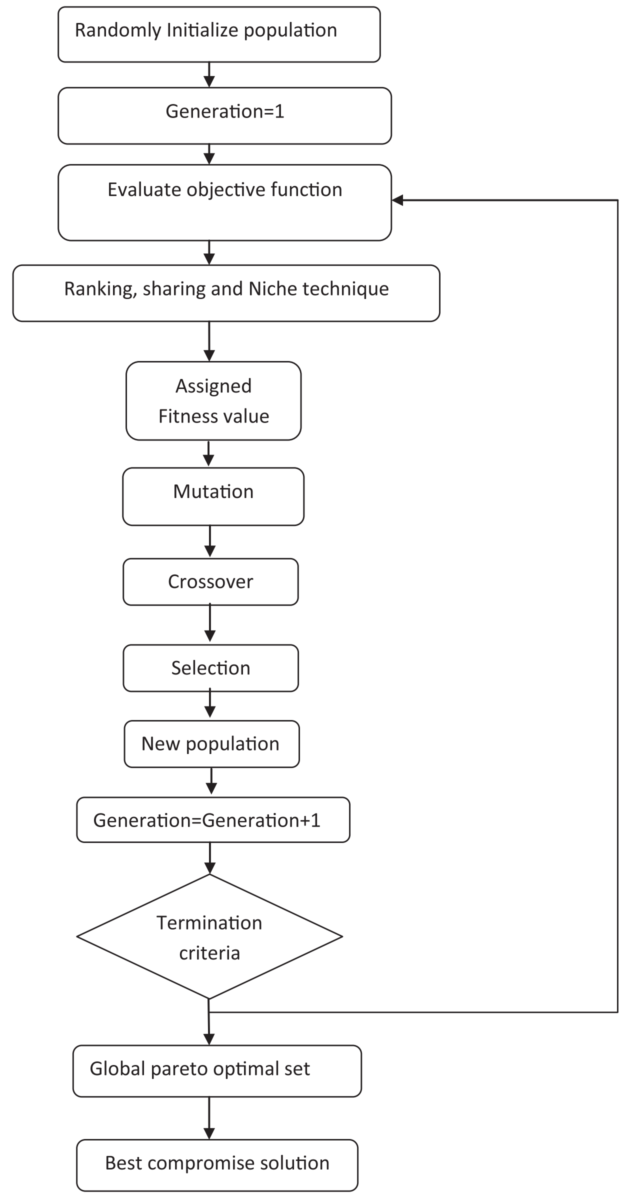

Fig. 1. Flowchart of Multi Objective Differential Evolution.

the multi-objective optimization problem. The algorithm of MODE is given in Fig. 1 and can be described as given below:

DE begins with a randomly initiated population, NP known as genomes/chromosomes which forms a candidate solution to multidimensional optimization problem and is expressed as:

$$X_{i,G} = x_{1,i,G}, x_{2,i,G}, x_{3,i,G}, \ldots, x_{D,i,G}$$

where G is the generation number and D is the problem's dimension. For each parameter of the problem, there will be minimum and maximum value within which the parameter should be restricted.

Hence the jth component of ith vector is initialized as follows:

$$x_{j,i,0} = x_{j,\min} + rand_{i,j}[0,1] \times (x_{j,\max} - x_{j,\min}) \quad (15)$$

where $rand_{i,j}[0, 1]$ is a uniformly distributed random number lying between 0 and 1.

Each solution is checked for its dominance in the population. Two solutions ($x(1)$ and $x(2)$) are compared on the basis of whether one dominates the other solution or not. A solution $x(1)$ is said to dominate the other solution $x(2)$, if the following conditions are satisfied:

(a) The solution $x(1)$ is no worse than $x(2)$ in all objectives, or $f_i(x^{(1)}) < f_i(x^{(2)})$ for all $i = 1, 2, M$, where M is the number of objective functions.
(b) The solutions $x(1)$ is strictly better than $x(2)$ in at least one objective, or $f_i(x^{(1)}) < f_i(x^{(2)})$ for at least one j ($j \varepsilon \{1, 2, \ldots, M\}$).

If any of the above condition is violated, the solution $x(1)$ does not dominate the solution $x(2)$ (or mathematically $x(1) \leqslant x(2)$). To a solution i, a rank equal to one plus the number of solutions η_i that dominate solution i is assigned:

$$r_i = 1 + \eta_i \tag{16}$$

The non dominated solutions are assigned a rank equal to 1, since no solution would dominate a non dominated solution in a population. After ranking, raw fitness is assigned to each solution based on its rank by sorting the ranks in ascending order of magnitude. The raw fitness is assigned to each solution by linear mapping function which is chosen to assign fitness between N (for best rank solution) and 1 (for worst rank solution). Thereafter, solutions of each rank are considered at a time and their averaged raw fitnesses are called assigned fitness. Thus the mapping and averaging procedure ensures that the better ranked solutions have a higher assigned fitness. To maintain diversity in the population, niching is introduced among solutions of each rank. The niche count is calculated by summing the sharing function value as below:

$$nc_i = \sum_{j=1}^{\mu(r_i)} Sh(d_{ij}) \tag{17}$$

where $\mu(r_i)$ is the number of solutions in a rank and $Sh(d_{ij})$ is the sharing function value of two solution i and j.

The sharing function is calculated by using objective function as distance metric as:

$$Sh(d_{ij}) = \left\{ \begin{array}{cc} 1 - \left(\frac{d_{ij}}{\sigma_{share}}\right)^{\alpha} & \text{if } d \leqslant \sigma_{share} \\ 0 & \text{otherwise} \end{array} \right\} \tag{18}$$

The parameter d is the distance between any two solutions in the population and σ_{share} is the sharing parameter which signifies the maximum distance between any two solutions before they can be considered to be in the same niche. The above function takes a value in [0,1] depending on the values of d and σ_{share}. If $\alpha = 1$ is used, the effect linearly reduces from one to zero.

The normalized distance between any two solutions can be calculated as follows:

$$d_{ij} = \sqrt{\sum_{k=1}^{M} \left(\frac{f_k^{(i)} - f_k^{(j)}}{f_k^{\max} - f_k^{\min}} \right)^2} \tag{19}$$

where $f_k^{\max}$ and $f_k^{\min}$ are the maximum and minimum objective function value of the kth objective.

The shared fitness is calculated by dividing the fitness of a solution by its niche count. Although all solutions of any particular rank have the identical fitness, the shared fitness value of each solution residing in less crowded region has a better shared fitness which produces a large selection pressure for poorly represented solutions in any rank. Dividing the assigned fitness by the niche count reduces the fitness of each solution. To keep the average fitness of the solutions in a rank same as before sharing, the fitness values are scaled. This procedure is continued until all ranks are processed. After evaluating the individuals in the population, the mutation operator is applied. The mutation operator creates mutant vectors by perturbing a randomly selected vector (X_{r1}) with the difference of two other randomly selected vectors (X_{r2} and X_{r3}). All of these vectors must be different from each other, requiring the population to be of at least four individuals to satisfy this condition. To control the perturbation and improve convergence, the difference vector is scaled by a user defined constant. The process can be expressed as follows:

$$V_{i,G} = X_{r1,G} + F(X_{r2,G} - X_{r3,G}) \tag{20}$$

where F is scaling constant.

In this work, DERANDSF (DE with Random Scale Factor) is used in which the scaled parameter F is varied in a random manner in the range (0.5–1) by using the relation:

$$F = 0.5 \times (1 + rand[0, 1]) \tag{21}$$

where $rand[0,1]$ is a uniformly distributed random number within the range [0,1]. This allows for the stochastic variations in the amplification of the difference vector and thus helps retain population diversity as the search progresses. The difference vector based mutation is believed to be the strength of DE because of the automatic adaptation in improving the convergence of the algorithm which comes from the idea of difference based recombination operator, i.e. Blend crossover operator [15,16].

The crossover operator creates the trial vectors which are used in the selection process. A trial vector is a combination of mutant vector and a parent vector based on different distributions like uniform distribution, binomial distribution and exponential distribution is generated in the range [0,1] and compared against a user defined constant referred to as the crossover constant. In this work, binomial crossover is performed on each of the D variables. If the value of the random number is less or equal to the value of the crossover constant, the parameter will come from the mutant vector, otherwise the parameter comes from the parent vector. The crossover operation maintains diversity in the population preventing local minima convergence. The crossover constant must be in the range from 0 to 1. If the value of crossover constant is one then the trial vector will be composed of entirely mutant vector parameters. If the value of crossover constant is zero then the trial vector will be composed of entirely parent vector. Trial vector gets at least one parameter from the mutant vector even if the crossover constant is set to zero.

The scheme may be outlined as follows:

$$U_{i,j}(G) = \begin{cases} V_{i,j}(G) & \text{if } rand(0,1) \leqslant CR \text{ or } j = q \\ X_{i,j}(G) & \text{otherwise} \end{cases} \tag{22}$$

where q is randomly chosen index in the D dimensional space. CR is crossover constant, $X_{i,j}(G)$ is parent vector, and $V_{i,j}(G)$ is mutant vector.

To keep the population size constant over subsequent generations, the selection process determines which one of the target vector and trial vector will survive in the next generation and is outlined as follows:

$$\begin{aligned} X_i(G+1) &= U_i(G) \text{ if } f(U_i(G)) \leqslant f(X_i(G)), \\ &= X_i(G) \text{ if } f(X_i(G)) < f(U_i(G)) \end{aligned} \tag{23}$$

where $f(X)$ is the objective function to be minimized. So if the new trial vector yields a better value of the fitness function, it replaces its target in the next generation; otherwise the target vector is retained in the population. In the proposed work, DE/randSF/1/bin strategy with self tuned parameters is used. Here rand denotes randomly selected vector to be perturbed, 1 denotes the number of difference vectors considered for perturbation and bin stands for binomial type of crossover operator. This process is continued until the convergence criterion is satisfied.

3.3. Best compromise solution

It is often practical to choose one solution from all solutions that satisfy different goals after the pareto optimal set is obtained. Due to imprecise nature of the Decision Maker's (DM) judgement, it is natural to assume that the DM may have fuzzy or imprecise nature of goals of each objective function. Hence in this work, a technique based on fuzzy set theory [17] was applied to extract the best compromise solution. Using linear membership function, a membership function is assigned for each objective functions and is defined as follows:

$$\mu_i = \begin{cases} 1, & F_i \geqslant F_i^{\max} \\ \frac{F_i^{\max} - F_i}{F_i^{\max} - F_i^{\min}}, & F_i^{\min} < F_i < F_i^{\max} \\ 0, & F_i \leqslant F_i^{\min} \end{cases} \quad (24)$$

where $F_i^{\max}$ and $F_i^{\min}$ are the maximum and minimum value of the ith objective function among all non-dominated solutions. The above equation gives a degree of satisfaction for each objective function for a particular solution and map the objectives in the range of 0 to 1.

The membership function for the non dominated solutions in a fuzzy set is calculated as follows:

$$\mu^k = \frac{\sum_{i=1}^{N_{obj}} \mu_i^k}{\sum_{k=1}^{M} \sum_{i=1}^{N_{obj}} \mu_i^k} \quad (25)$$

where M is the number of non dominated solutions and N_{obj} is the number of objectives. Finally, the best compromise solution is the one achieving the maximum membership function.

3.4. Performance measures

The performance of proposed MODE is studied by comparing the reference pareto-front which is obtained with multiple runs. In this paper, the proposed algorithm with conventional weighted sum approach is used as reference pareto front. The reference pareto-front is generated by considering the problem as single objective optimization problem by linear combination of the objective functions as given below:

$$\text{Minimize } f = w_1 \times L_{\max} + w_2 \times Total \text{ VAR } \cos t, F_C \quad (26)$$

where w_1 and w_2 are the weighting factors and the sum of weighting factor should be 1. The non dominated solutions are obtained by applying the algorithm 20 times with different weighting factors between 0 and 1.

The performance of proposed MODE is studied under various performance measures such as convergence metric, generational distance and spacing [12]. The details of the performance metrices are discussed below:

(a) *Convergence metric (γ):* This metric finds an average distance between non dominated solutions found and the actual pareto optimal front as follows:

$$\gamma = \frac{\sum_{i=1}^{N} d_i}{N} \quad (27)$$

where d_i is the distance between non dominated solutions and actual pareto front and N is the number of solutions in the front.

(b) *Generational Distance (GD):* This reports how far on average pareto front known from reference pareto front and is given as follows:

$$GD = \frac{\left(\sum_{i=1}^{n} d_i^p\right)^{1/p}}{|PF_{known}|} \quad (28)$$

where PF_{known} is number of vectors in known pareto front, $p = 2$ and d_i is the Euclidean phenotypic distance between each member, i of PF_{known} and closest member in reference pareto front to that member.

(c) *Spacing (D_i):* This describes the spread of the vectors in known pareto front.

$$D_i = \min_j \left(\left| f_1^i(x) - f_1^j(x) \right| + \left| f_2^i(x) - f_2^j(x) \right| \right) \quad (29)$$

4. Implementation of Differential Evolution algorithm

The following issues are addressed in the implementation of MODE in VSC-RPP problem:

4.1. Problem representation

The control variables namely, generator bus voltages (V_{gi}) are represented as floating point numbers, transformer tap positions (t_{ki}) and reactive power generation of VAR sources (Q_{ci}) are represented as integers for the VSC-RPP problem. The transformer tap setting with tapping ranges of ±10% and a tapping step of 0.025 pu is represented from the alphabet (0, 1, ..., 8) and the VAR sources with limits of 1 and 5 MVAR and step size of 1 MVAR is represented from the alphabet (0, 1, ..., 5).

4.2. Evaluation function

In the reactive power optimization problem under consideration, the objective function comprises of minimization of total cost of energy loss, VAR production cost and minimization of L-index satisfying a number of equality and inequality constraints (6–13). For each individual, the equality constraints are satisfied by running the Newton Raphson power flow algorithm. The inequality constraints on the control variables are taken into account in the problem representation itself, and the constraints on the state variables are taken into consideration by adding a quadratic penalty function to the objective function. With the inclusion of penalty function the new objective function becomes,

$$\text{Min } f = f_c + SP + \sum_{j=1}^{N_{PQ}} VP_j + \sum_{j=1}^{N_g} \mathbf{QP}_j + \sum_{j=1}^{N_l} \mathbf{LP}_j \quad (30)$$

Here, SP, VP_j, QP_j and LP_j are the penalty terms for the reference bus generator active power limit violation, load bus voltage limit violation, reactive power generation limit violation and line flow limit violation respectively. These quantities are defined by the following equations:

$$SP = \begin{cases} K_s\left(P_s - P_s^{\max}\right)^2 & \text{if } P_s > P_s^{\max} \\ K_s\left(P_s - P_s^{\min}\right)^2 & \text{if } P_s < P_s^{\min} \\ 0 & \text{otherwise} \end{cases} \quad (31)$$

$$VP_j = \begin{cases} K_v\left(V_j - V_j^{\max}\right)^2 & \text{if } V_j > V_j^{\max} \\ K_v\left(V_j - V_j^{\min}\right)^2 & \text{if } V_j < V_j^{\min} \\ 0 & \text{otherwise} \end{cases} \quad (32)$$

$$QP_j = \begin{cases} K_q\left(Q_j - Q_j^{\max}\right)^2 & \text{if } Q_j > Q_j^{\max} \\ K_q\left(Q_j - Q_j^{\min}\right)^2 & \text{if } Q_j < Q_j^{\min} \\ 0 & \text{otherwise} \end{cases} \quad (33)$$

$$LP_j = \begin{cases} K_l(S_l - S_l^{\max})^2 & \text{if } S_l > S_l^{\max} \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

where K_s, K_v, K_q and K_l are the penalty factors. The success of the penalty function approach lies in the proper choice of these penalty parameters. Using the above penalty function approach, one has to experiment to find a correct combination of penalty parameters K_s, K_v, K_q and K_l. Since DE maximizes the fitness function, the minimization objective function f is transformed to a fitness function to be maximized as,

$$\text{Fitness} = \frac{k}{f} \quad (35)$$

where k is a large constant.

5. Simulation results

The proposed single objective DE and MODE-based approaches for contingency constrained VAR planning model incorporating voltage stability is applied in IEEE 30-bus and IEEE 57-bus test systems. The real and reactive loads are scaled up according to predetermined weighting factors to analyze the system under stressed condition. Generator voltage magnitudes are treated as continuous variables whereas transformer tap-settings and shunt capacitor banks are treated as discrete variables with 9 levels and 6 levels respectively. The program was written in MATLAB 7.1 and executed on a PC with 2.4 GHz Intel Pentium IV processor. To demonstrate the effectiveness of the proposed approach, three different cases have been considered:

Case 1: Single objective VSCRPP in IEEE 30 bus system by single objective DE.
Case 2: Multi objective VSCRPP in IEEE 30 bus system by MODE.
Case 3: Multi objective VSCRPP in IEEE 57 bus test system by MODE.

5.1. Case 1:Single objective VSCRPP in IEEE 30 bus system

The network data for IEEE 30 bus system are taken from [18]. Buses 30, 29, 26, 25 and 24 are identified for reactive power injection based on maximum L-indices of load buses. The single objective RPP problem is formulated with the minimization of total cost of energy loss and VAR investment cost as the objective with

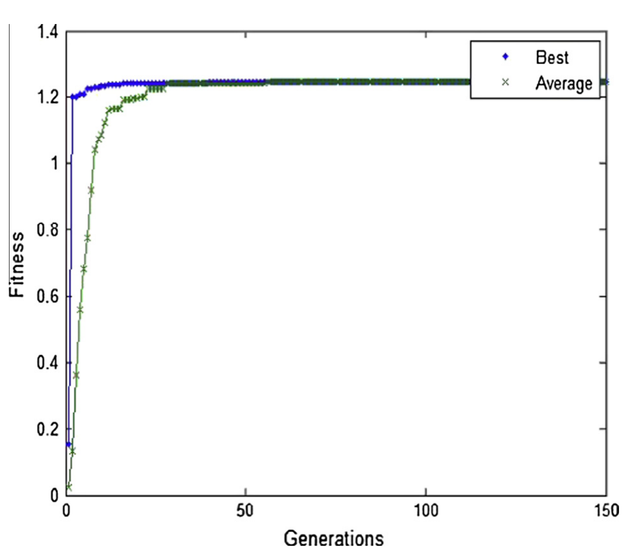

Fig. 2. Convergence of single objective DE-RPP algorithm for IEEE 30 bus system.

Table 1
Controller settings under base case for IEEE 30 bus system.

Control variables	Initial setting	Optimal setting
Voltage magnitudes (pu)		
V_1	1.074	1.0472
V_2	1.065	1.0324
V_5	1.043	1.0450
V_8	1.042	1.0432
V_{11}	1.069	1.0312
V_{13}	1.058	0.9761
Transformer tap settings (pu)		
T_{6-9}	0.981	0.956
T_{6-10}	1.042	1.034
T_{4-12}	1.029	1.046
T_{28-27}	1.037	1.023
VAR source installments (MVAR)		
Q_{C30}	0	0
Q_{C29}	0	0
Q_{C26}	0	0
Q_{C25}	0	0
Q_{C24}	0	0
Var installation cost, I_C	0	0
Cost of energy loss, W_C ($/yr)	2.6085×10^6	2.4216×10^6
Total VAR cost, F_C ($/yr)	2.6085×10^6	2.4216×10^6
Transmission loss, T_L (MW)	4.963	4.545
$L_{\max}$	0.1978	0.1645

$L_{\max}$ as additional constraint along with system constraints. The improved DE-based algorithm with DE/randSF/1/bin scheme was applied to solve the RPP problem with different parameter settings and the best results are obtained with the following setting with the population size of 30, crossover rate of 0.7, scaled parameter of variable random tuned value and number of generations to be 150.

The proposed approach took 184.75 s to reach the optimal solution. Fig. 2 shows the variation of fitness during the DE run for the best case. The optimal values of the control variables from the proposed algorithm along with initial control variable setting are given in Table 1. The algorithm reached a minimum loss of 4.545 MW and the total cost is 2.4216×10^6 $/yr. As indicated in

Table 2
Comparison of total VAR cost.

Method	Minimum total VAR cost, F_C
Conventional method [5]	4.0133×10^6 $/yr
Evolutionary programming [5]	2.6085×10^6 $/yr
Modified genetic algorithm	2.4314×10^6 $/yr
Proposed differential evolution	2.4216×10^6 $/yr

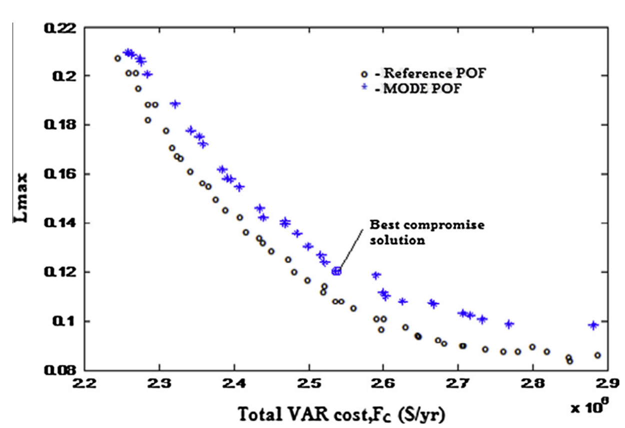

Fig. 3. Pareto optimal front of MODE in VSCRPP problem under base case in IEEE 30 bus system.

Table 3
Pareto optimal solutions by MODE in IEEE 30 bus system under 100% loaded condition.

Control variables		Best VAR cost	Best L_{max}	Best compromise solution
Generator voltage magnitudes, Vvar (pu)	V_1	1.0498	1.0498	1.05
	V_2	1.0426	1.0416	1.0393
	V_5	1.0284	1.0098	1.0142
	V_8	1.0277	1.016	1.0179
	V_{11}	1.0388	1.0973	1.0999
	V_{13}	0.9675	1.0999	1.0838
Transformer tap settings, Tvar (pu)	T_{6-9}	1.1	0.975	0.975
	T_{6-10}	1.1	0.9	0.95
	T_{4-12}	1.1	0.975	1
	T_{28-27}	0.9	1.025	0.9
Settings of shunt capacitors, Cvar (MVAR)	Q_{C30}	0	0	0
	Q_{C29}	0	0	0
	Q_{C26}	0	0	0
	Q_{C25}	0	0	0
	Q_{C24}	0	0	0
Objective values				
Total VAR cost, F_C ($/yr)		2.2583×10^6	2.8810×10^6	2.5387×10^6
L_{max}		0.21	0.0981	0.1204
Transmission loss, T_L (MW)		4.2966	5.4814	4.83

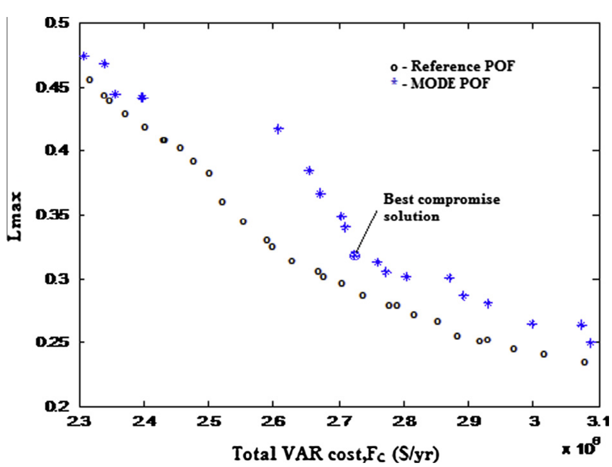

Fig. 4. Pareto optimal front of MODE – VSCRPP for line outage (28–27) under 125% loaded condition in IEEE 30 bus system.

Table 2, this total cost is less than the value reported in the literature for IEEE 30 bus system under similar operating condition. The real power savings and annual energy cost savings are calculated as follows:

$$P_{save(\%)} = \frac{P_{loss}^{init} - P_{loss}^{opt}}{P_{loss}^{init}} \times 100 \tag{36}$$

$$W_C^{save}(\$) = h \cdot d_l \left(P_{loss}^{init} - P_{loss}^{opt}\right) \times 1000 \tag{37}$$

The values of the constants and the limits of the control variables are given in Appendix A.2. The real power savings from proposed DE is 8.42% and annual energy cost savings is $ 219700.8 of those from evolutionary programming [5] in normal operating conditions. For comparison, the RPP problem was solved using modified genetic algorithm [19] and it took 196.30 s to reach the optimal solution and hence it is evident that the proposed DE algorithm is more effective in reaching the optimal solution than the other evolutionary approaches.

Table 4
Pareto optimal solutions by MODE in IEEE 30 bus system in line outage (28–27) under 125% loaded condition.

Control variables		Best VAR cost	Best L_{max}	Best compromise solution
Generator voltage magnitudes, Vvar (pu)	V_1	1.049	1.05	1.05
	V_2	1.046	1.045	1.04
	V_5	1.028	1.032	1.035
	V_8	1.03	1.026	1.019
	V_{11}	1.017	1.074	1.08
	V_{13}	0.962	1.097	1.098
Transformer tap settings, Tvar (pu)	T_{6-9}	1.1	1	1
	T_{6-10}	1.1	0.95	0.9
	T_{4-12}	1.1	1	0.9
	T_{28-27}	0.9	0.9	0.9
Settings of shunt capacitors, Cvar (MVAR)	Q_{C30}	3	2	2
	Q_{C29}	4	4	4
	Q_{C26}	1	4	4
	Q_{C25}	2	5	3
	Q_{C24}	1	3	2
Objective values				
Total VAR cost, F_C ($/yr)		2.3×10^6	3.1×10^6	2.72×10^6
L_{max}		0.47	0.24	0.32
Transmission loss, T_L (MW)		6.1	5.54	5.58

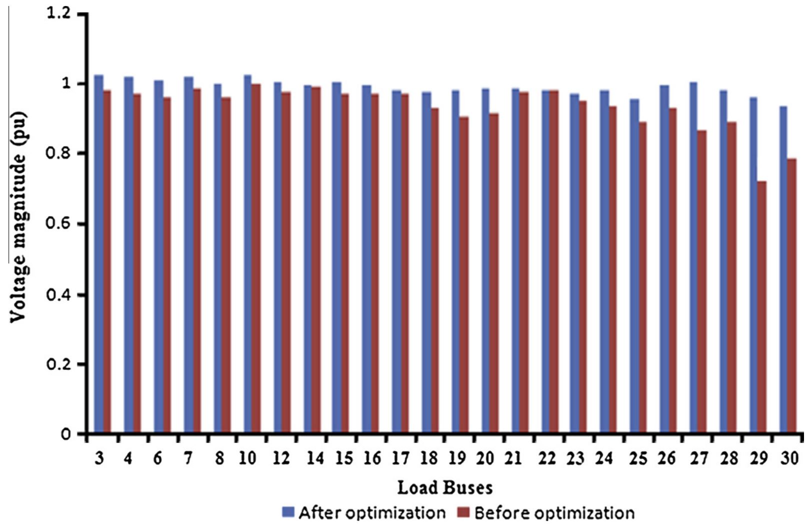

Fig. 5. Voltage profile improvement for line outage (28–27) under 125% loaded condition in IEEE 30 bus system.

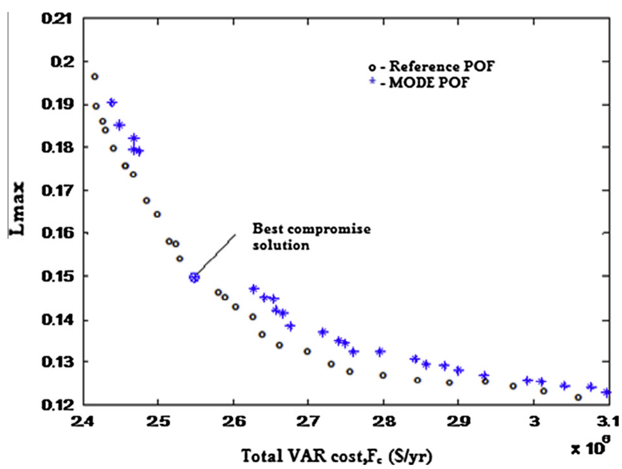

Fig. 6. Pareto optimal front for line outage (27–30) under 125% loaded condition in IEEE 30 bus system.

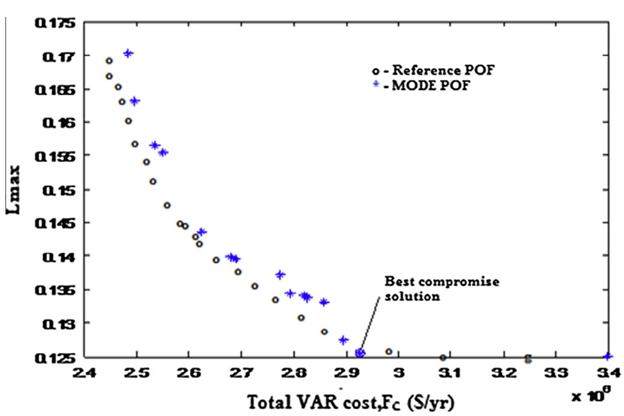

Fig. 7. Pareto optimal front for line outage (27–29) under 125% loaded condition in IEEE 30 bus system.

5.2. Case 2: Multi objective VSCRPP in IEEE 30 bus system

The multi objective VSCRPP problem by MODE is carried out in IEEE 30 bus system by considering two conflicting objectives: minimization of total cost of energy loss, VAR investment cost and also improving the voltage stability by minimization of L_{max} simultaneously. Firstly, the problem is formulated under base case operating conditions and the best pareto optimal front among 20 different simulations is displayed in Fig. 3. The MODE has generated 33 pareto solutions with the following given parameters: Number of generations: 150, population size: 50, crossover rate: 0.7 and scaled parameter of variable random tuned value. From the graph, it is clear that MODE is better in diversity and finding optimal solutions in search space. The two extreme optimal solutions of MODE that represents the best total VAR cost and the best L_{max} is given in Table 3. From this table, it is seen that the extreme points of pareto optimal front and best solution of each functions are identical. Hence it is verified that the proposed method is capable of exploring more efficient search space. Further, from the decision making strategy, the best compromise solution in the overall non dominated solutions are computed and are also given in Table 3.

Next, the contingency analysis was conducted on the same system under 125% loaded condition. From the contingency analysis, line outages 28–27, 27–30 and 27–29 have been identified as severe cases with the L_{max} values of 0.4165, 0.2352 and 0.2146 respectively. The multi objective optimization problem is carried out with minimization of cost of energy loss and VAR production cost in base case and minimization of L_{max} under contingency states in stressed system conditions. The simulation runs for the Multi objective VSCRPP problem under contingency state (28–27) are displayed in Fig. 4 and the statistical results are given in Table 4. From this figure, it is worth mentioning that proposed MODE is capable of exploring more efficient non inferior solutions. From the best compromise solution in Table 4, the maximum L-index

Table 5
Results of MODE optimization for the two severe contingencies in IEEE 30 bus system.

	Line outage 27–30 (125% loaded condition)			Line outage 27–29 (125% loaded condition)		
	Best VAR cost, F_C	Best L_{max}	Best solution	Best VAR cost, F_C	Best L_{max}	Best solution
F_C ($/yr)	2.4398×10^6	3.0979×10^6	2.5497×10^6	2.484×10^6	3.39×10^6	2.92×10^6
L_{max}	0.1907	0.1229	0.1498	0.1703	0.1251	0.1255
T_L (MW)	4.48	5.60	4.85	4.68	6.12	5.39

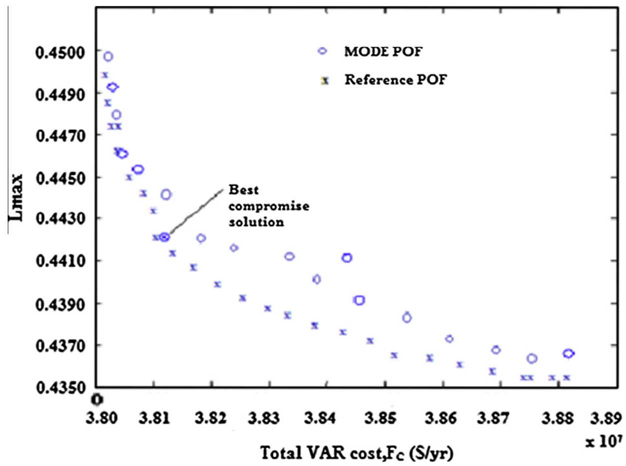

Fig. 8. Pareto optimal front for line outage (46–47) under 125% loaded condition in IEEE 57 bus system.

has decreased from 0.4165 to 0.32, a reduction of about 10% and the minimum voltage of the system in the 29th load bus have been increased from 0.6721 to 0.989, an improvement of about 31.7% has been obtained by MODE. The real power savings from MODE in the best total VAR cost is 13.42% and annual energy cost savings is $ 350259.8 when compared to initial settings. The voltage profile improvement for the severe contingency (28–27) after the application of the proposed optimization approach is displayed in Fig. 5. From this figure, it is proved that by using the proposed method voltage stability is improved under most critical contingency in stressed system conditions in the test system.

In addition, the VSCRPP problem is carried out under the next two severe contingencies (27–30) and (27–29) under stressed operating conditions. The pareto optimal front obtained for the two severe contingencies are plotted in Figs. 6 and 7. The two extreme solutions along with best compromise solution for the two contingencies using proposed MODE are summarized in Table 5. From this table, it is clear that the proposed MODE is better in finding the optimality of solutions due to span over the entire pareto optimal front.

Table 6
Pareto optimal solutions of MODE in IEEE 57 bus test system for line outage (46–47) under 150% loaded condition.

Control variables	Best VAR cost	Best L_{max}	Best compromise solution
Generator voltage magnitudes, Vvar (pu)	1.0577, 1.0299, 0.9957, 0.9921, 1.0233, 0.9897, 1.0109	1.0577, 1.0299, 0.9957, 0.9922, 1.0332, 0.9906, 1.0103	1.0577, 1.0299, 0.9957, 0.9939, 1.0269, 0.9919, 1.0099
Transformer tap settings, Tvar (pu)	1.0, 1.025, 1.1, 0.95, 1.0, 0.975, 1.075, 0.9, 0.975, 1.075, 0.9, 0.975, 0.9, 1.0, 1.05, 1.075	1.0, 1.025, 1.1, 0.95, 1, 0.975, 1.0, 0.975, 1.1, 0.9, 0.975, 1.075, 0.9, 0.975, 0.9, 1.0, 1.05, 1.075	1.0, 1.025, 1.1, 0.95, 1.0, 0.975, 1.075, 0.9, 0.975, 1.075, 0.9, 0.975, 0.9, 1.0, 1.05, 1.075
Settings of shunt capacitors, Cvar (MVAR)	3, 5, 5, 3, 4	3, 5, 5, 3, 4	3, 5, 5, 3, 4
Objective values			
Total VAR cost, F_C ($/yr)	3.8514×10^7	3.8862×10^7	3.8523×10^7
L_{max}	0.4485	0.4399	0.4469
Transmission loss, T_L (MW)	65.18	72.4	68.9

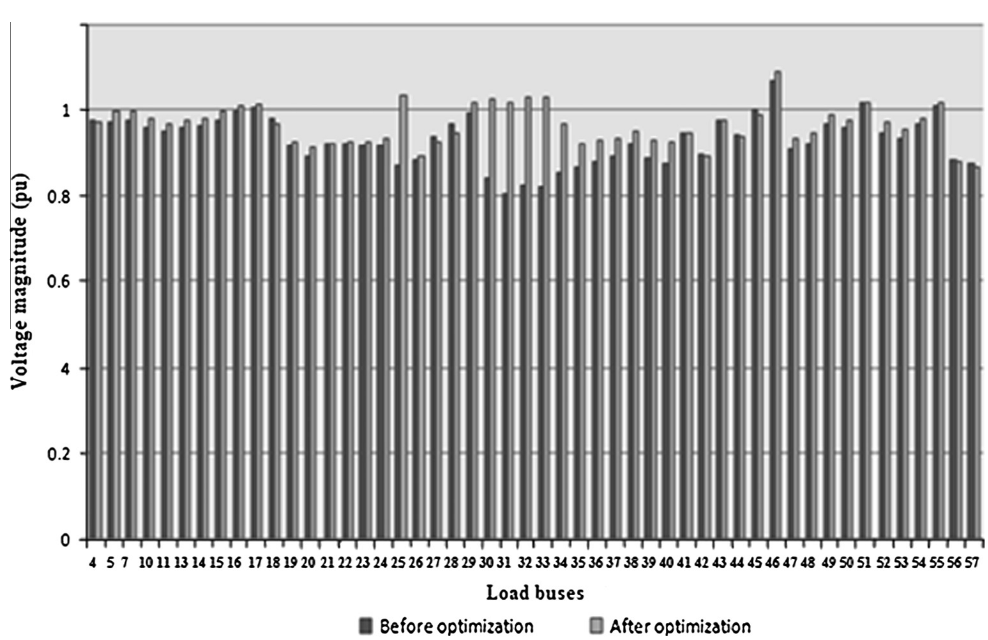

Fig. 9. Voltage profile improvement for line outage (46–47) under 150% loaded condition in IEEE 57 bus system.

Table 7
Performance metrics of MODE in IEEE 30 and IEEE 57 bus test systems.

Performance metrics	Solution	IEEE 30 bus system	IEEE 57 bus system
Convergence metric	Mean	0.1735	0.2671
	Best	0.1589	0.2345
	Worst	0.2123	0.3231
Generational distance	Mean	0.367	0.5173
	Best	0.2819	0.4129
	Worst	0.5219	0.6713
Spacing	Mean	0.0271	0.0357
	Best	0.0273	0.0646
	Worst	0.0963	0.1478

5.3. Case 3: Multi objective VSCRPP in IEEE 57 bus test system

The IEEE 57-bus system was chosen as the second test system to demonstrate the method's usefulness on a large system. IEEE 57-bus system has 4 generators, 3 synchronous condensers, 50 load buses, 80 transmission lines and 16 tap changing transformers. The proposed MODE was applied to solve the VSCRPP problem under base load condition and has brought the total VAR cost to 1.005×10^7 \$/yr. From the contingency analysis, line outage (46–47) in 150% loaded condition is found to be the most severe case with the $L_{\max}$ value of 0.4578. From the weak bus ranking, buses 30, 32, 31, 33 and 34 were selected for reactive power injection. The pareto optimal front along with reference front for the critical contingency (46–47) is displayed in Fig. 8. It can be seen that the obtained solutions are well distributed on trade-off surface, except some discontinuity, caused by the discrete decision variables. The optimal control variable settings of the best total VAR cost and best $L_{\max}$ along with best compromise solution after the application of the MODE for the severe contingency under 150% stressed system condition are summarized in Table 6. Corresponding to these control variable settings, there is no limit violations in any of the state variables in the base case and contingency states. The real power savings from MODE is 16.67% and annual energy cost savings is \$ 6,853,824 in critical contingency state (46–47) when compared to initial settings which is given in Appendix A.3. The improvement of voltage profile in the system after the application of the algorithm under contingency (46–47) are displayed in Fig. 9. The minimum voltage in the 31st load bus of the system has been increased by 22% by proposed DE approach. Hence the improvement in voltage profile of the system after the application of the proposed algorithm is evident from this result. The performance measures namely, convergence metric, generational distance and spacing for the two test systems are computed for the best, mean and worst solutions for 20 simulation runs and are tabulated in Table 7. From this it is observed that all the three statistical performance measures have minimum mean values in the two test systems using proposed MODE. This shows the efficiency of the proposed algorithm in diversity and convergence characteristics in solving the VSCRPP problem.

6. Conclusion

In this paper, the voltage stability enhancement is achieved by the formulation of voltage stability constrained reactive power planning problem by the implementation of Multi Objective Differential Evolution. The improved Differential Evolution algorithm with self tuned parameter has been applied to solve reactive power planning in power systems. To improve the efficiency of the Differential Evolution algorithm in the search process, the optimization variables were represented in natural form. Further, of variable random scale vector to find the true global optimum in addition to binomial crossover and differential mutation vector is used. A multi objective formulation of RPP problem has been developed in which candidate solutions are selected to reduce the reactive power installation cost and transmission loss while improving the voltage profile of the system. Two conflicting objectives such as reactive power production cost and voltage stability index are considered. The proposed approach has been tested on IEEE 30 bus system and IEEE 57 bus test system under base case and contingency states. The simulation results show that the proposed MODE algorithm is effective in reducing the VAR installation cost and improving the voltage security of the system simultaneously. The MODE is better in characterizing the pareto optimal front in solving the multi objective reactive power planning problem.

Appendix A

A.1. Appendix

The L-index calculation for a power system is briefly discussed below:

Consider a N-bus system in which there are N_g generators. The relationship between voltage and current can be expressed by the following expression:

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix} \tag{A1}$$

where I_G, I_L and V_G, V_L represent currents and voltages at the generator buses and load buses.

Rearranging the above equation we get,

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \tag{A2}$$

where

$$F_{LG} = -[Y_{LL}]^{-1}[Y_{LG}] \tag{A3}$$

The L-index of the jth node is given by the expression,

$$L_j = \left| 1 - \sum_{i=1}^{N_g} F_{ji} \frac{V_i}{V_j} \angle(\theta_{ji} + \delta_i - \delta_j) \right| \tag{A4}$$

The values of F_{ji} are obtained from the matrix F_{LG}.

A.2. Appendix

Per unit energy cost, h	0.06 \$/h
Duration of load level, d_l	8760 h for 1 yr
Fixed VAR source installment cost at bus i, e_i	Capacitor settings × 1000
VAR source purchase cost at bus I, C_{ci}	Capacitor settings × 30 × 1000
$P_s^{\min}$ (MW)	50
$P_s^{\max}$ (MW)	200
$Q_{gi}^{\min}$ (MVAR)	−20, −20, −15, −15, −10, −15
$Q_{gi}^{\max}$ (MVAR)	150, 60, 62.5, 48.73, 40, 44.72
$V_{gi}^{\min}$ (pu)	0.95
$V_{gi}^{\max}$ (pu)	1.05
$t_k^{\min}$ (pu)	0.95
$t_k^{\max}$ (pu)	1.05

A.3. Appendix

Control variables	Initial setting
Generator voltage magnitudes, Vvar (pu)	1.0045; 0.9865; 0.9817; 0.9776; 0.9679; 0.9996
Transformer tap settings, Tvar (pu)	0.925; 1; 0.975; 1.025; 1; 0.925; 1.075; 1; 0.925; 0.925; 0.925; 0.95; 0.9; 1.025; 1.1; 1.05
Settings of shunt capacitors, Cvar (MVAR)	4, 2, 2, 4, 3
Total VAR cost, F_C ($/yr)	4.1571×10^7
Transmission loss, T_L (MW)	78.22
$L_{\max}$	0.4578

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