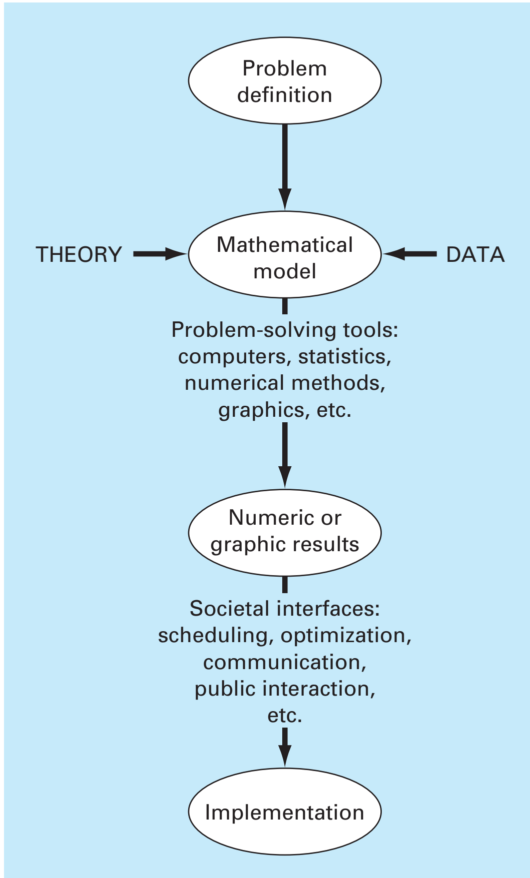


FIGURE 1.1
The engineering problem-solving process.

where the *dependent variable* is a characteristic that usually reflects the behavior or state of the system; the *independent variables* are usually dimensions, such as time and space, along which the system's behavior is being determined; the *parameters* are reflective of the system's properties or composition; and the *forcing functions* are external influences acting upon the system.

The actual mathematical expression of Eq. (1.1) can range from a simple algebraic relationship to large complicated sets of differential equations. For example, on the basis of his observations, Newton formulated his second law of motion, which states that the time rate of change of momentum of a body is equal to the resultant force acting on it. The mathematical expression, or model, of the second law is the well-known equation

$$F = ma \tag{1.2}$$

where F = net force acting on the body (N, or kg m/s^2), m = mass of the object (kg), and a = its acceleration (m/s^2).

FIGURE 1.2
Schematic diagram of the forces acting on a falling parachutist. F_D is the downward force due to gravity. F_U is the upward force due to air resistance.

The second law can be recast in the format of Eq. (1.1) by merely dividing both sides by m to give

$$a = \frac{F}{m} \tag{1.3}$$

where a = the dependent variable reflecting the system's behavior, F = the forcing function, and m = a parameter representing a property of the system. Note that for this simple case there is no independent variable because we are not yet predicting how acceleration varies in time or space.

Equation (1.3) has several characteristics that are typical of mathematical models of the physical world:

1. It describes a natural process or system in mathematical terms.
2. It represents an idealization and simplification of reality. That is, the model ignores negligible details of the natural process and focuses on its essential manifestations. Thus, the second law does not include the effects of relativity that are of minimal importance when applied to objects and forces that interact on or about the earth's surface at velocities and on scales visible to humans.
3. Finally, it yields reproducible results and, consequently, can be used for predictive purposes. For example, if the force on an object and the mass of an object are known, Eq. (1.3) can be used to compute acceleration.

Because of its simple algebraic form, the solution of Eq. (1.2) can be obtained easily. However, other mathematical models of physical phenomena may be much more complex, and either cannot be solved exactly or require more sophisticated mathematical techniques than simple algebra for their solution. To illustrate a more complex model of this kind, Newton's second law can be used to determine the terminal velocity of a free-falling body near the earth's surface. Our falling body will be a parachutist (Fig. 1.2). A model for this case can be derived by expressing the acceleration as the time rate of change of the velocity (dv/dt) and substituting it into Eq. (1.3) to yield

$$\frac{dv}{dt} = \frac{F}{m} \tag{1.4}$$

where v is velocity (m/s) and t is time (s). Thus, the mass multiplied by the rate of change of the velocity is equal to the net force acting on the body. If the net force is positive, the object will accelerate. If it is negative, the object will decelerate. If the net force is zero, the object's velocity will remain at a constant level.

Next, we will express the net force in terms of measurable variables and parameters. For a body falling within the vicinity of the earth (Fig. 1.2), the net force is composed of two opposing forces: the downward pull of gravity F_D and the upward force of air resistance F_U:

$$F = F_D + F_U \tag{1.5}$$

If the downward force is assigned a positive sign, the second law can be used to formulate the force due to gravity, as

$$F_D = mg \tag{1.6}$$

where g = the gravitational constant, or the acceleration due to gravity, which is approximately equal to 9.81 m/s^2.

Air resistance can be formulated in a variety of ways. A simple approach is to assume that it is linearly proportional to velocity[1] and acts in an upward direction, as in

$$F_U = -cv \tag{1.7}$$

where c = a proportionality constant called the *drag coefficient* (kg/s). Thus, the greater the fall velocity, the greater the upward force due to air resistance. The parameter c accounts for properties of the falling object, such as shape or surface roughness, that affect air resistance. For the present case, c might be a function of the type of jumpsuit or the orientation used by the parachutist during free-fall.

The net force is the difference between the downward and upward force. Therefore, Eqs. (1.4) through (1.7) can be combined to yield

$$\frac{dv}{dt} = \frac{mg - cv}{m} \tag{1.8}$$

or simplifying the right side,

$$\frac{dv}{dt} = g - \frac{c}{m}v \tag{1.9}$$

Equation (1.9) is a model that relates the acceleration of a falling object to the forces acting on it. It is a *differential equation* because it is written in terms of the differential rate of change (dv/dt) of the variable that we are interested in predicting. However, in contrast to the solution of Newton's second law in Eq. (1.3), the exact solution of Eq. (1.9) for the velocity of the falling parachutist cannot be obtained using simple algebraic manipulation. Rather, more advanced techniques, such as those of calculus, must be applied to obtain an exact or analytical solution. For example, if the parachutist is initially at rest ($v = 0$ at $t = 0$), calculus can be used to solve Eq. (1.9) for

$$v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right) \tag{1.10}$$

Note that Eq. (1.10) is cast in the general form of Eq. (1.1), where $v(t)$ = the dependent variable, t = the independent variable, c and m = parameters, and g = the forcing function.

EXAMPLE 1.1 Analytical Solution to the Falling Parachutist Problem

Problem Statement. A parachutist of mass 68.1 kg jumps out of a stationary hot air balloon. Use Eq. (1.10) to compute velocity prior to opening the chute. The drag coefficient is equal to 12.5 kg/s.

Solution. Inserting the parameters into Eq. (1.10) yields

$$v(t) = \frac{9.81(68.1)}{12.5}\left(1 - e^{-(12.5/68.1)t}\right) = 53.44\left(1 - e^{-0.18355t}\right)$$

which can be used to compute

[1]In fact, the relationship is actually nonlinear and might better be represented by a power relationship such as $F_U = -cv^2$. We will explore how such nonlinearities affect the model in problems at the end of this chapter.

t, s	v, m/s
0	0.00
2	16.42
4	27.80
6	35.68
8	41.14
10	44.92
12	47.54
∞	53.44

According to the model, the parachutist accelerates rapidly (Fig. 1.3). A velocity of 44.92 m/s is attained after 10 s. Note also that after a sufficiently long time, a constant velocity, called the *terminal velocity,* of 53.44 m/s is reached. This velocity is constant because, eventually, the force of gravity will be in balance with the air resistance. Thus, the net force is zero and acceleration has ceased.

Equation (1.10) is called an *analytical,* or *exact, solution* because it exactly satisfies the original differential equation. Unfortunately, there are many mathematical models that cannot be solved exactly. In many of these cases, the only alternative is to develop a numerical solution that approximates the exact solution.

As mentioned previously, *numerical methods* are those in which the mathematical problem is reformulated so it can be solved by arithmetic operations. This can be illustrated

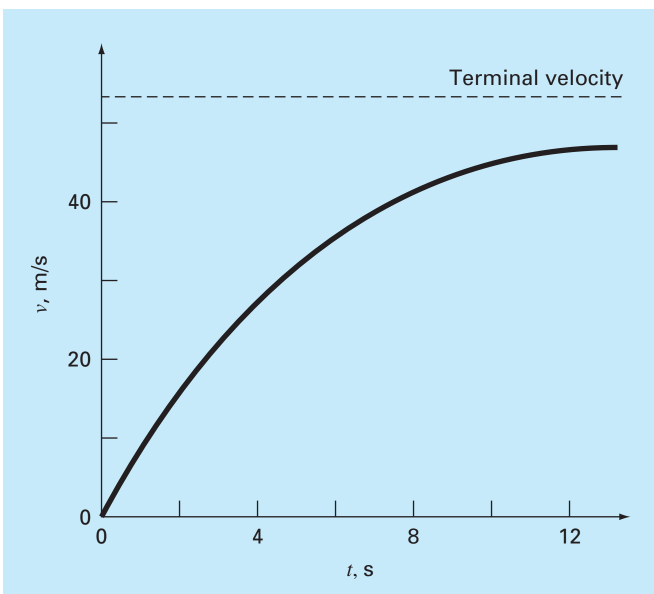


FIGURE 1.3
The analytical solution to the falling parachutist problem as computed in Example 1.1. Velocity increases with time and asymptotically approaches a terminal velocity.

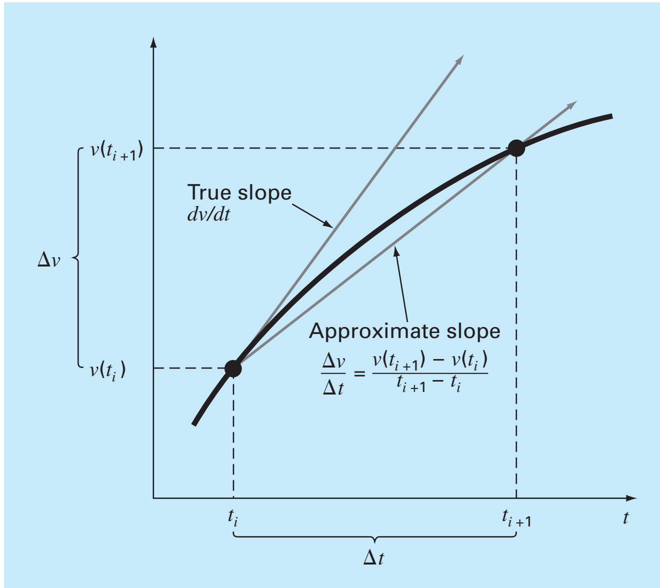


FIGURE 1.4
The use of a finite difference to approximate the first derivative of v with respect to t.

for Newton's second law by realizing that the time rate of change of velocity can be approximated by (Fig. 1.4):

$$\frac{dv}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} \tag{1.11}$$

where Δv and Δt = differences in velocity and time, respectively, computed over finite intervals, $v(t_i)$ = velocity at an initial time t_i, and $v(t_{i+1})$ = velocity at some later time t_{i+1}. Note that $dv/dt \cong \Delta v/\Delta t$ is approximate because Δt is finite. Remember from calculus that

$$\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Equation (1.11) represents the reverse process.

Equation (1.11) is called a *finite divided difference* approximation of the derivative at time t_i. It can be substituted into Eq. (1.9) to give

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m} v(t_i)$$

This equation can then be rearranged to yield

$$v(t_{i+1}) = v(t_i) + \left[g - \frac{c}{m} v(t_i) \right] (t_{i+1} - t_i) \tag{1.12}$$

Notice that the term in brackets is the right-hand side of the differential equation itself [Eq. (1.9)]. That is, it provides a means to compute the rate of change or slope of v. Thus, the differential equation has been transformed into an equation that can be used to determine the velocity algebraically at t_{i+1} using the slope and previous values of

v and t. If you are given an initial value for velocity at some time t_i, you can easily compute velocity at a later time t_{i+1}. This new value of velocity at t_{i+1} can in turn be employed to extend the computation to velocity at t_{i+2} and so on. Thus, at any time along the way,

$$\text{New value} = \text{old value} + \text{slope} \times \text{step size}$$

Note that this approach is formally called *Euler's method.*

EXAMPLE 1.2

Numerical Solution to the Falling Parachutist Problem

Problem Statement. Perform the same computation as in Example 1.1 but use Eq. (1.12) to compute the velocity. Employ a step size of 2 s for the calculation.

Solution. At the start of the computation ($t_i = 0$), the velocity of the parachutist is zero. Using this information and the parameter values from Example 1.1, Eq. (1.12) can be used to compute velocity at $t_{i+1} = 2$ s:

$$v = 0 + \left[9.81 - \frac{12.5}{68.1}(0)\right]2 = 19.62 \text{ m/s}$$

For the next interval (from $t = 2$ to 4 s), the computation is repeated, with the result

$$v = 19.62 + \left[9.81 - \frac{12.5}{68.1}(19.62)\right]2 = 32.04 \text{ m/s}$$

The calculation is continued in a similar fashion to obtain additional values:

t, s	*v*, m/s
0	0.00
2	19.62
4	32.04
6	39.90
8	44.87
10	48.02
12	50.01
∞	53.44

The results are plotted in Fig. 1.5 along with the exact solution. It can be seen that the numerical method captures the essential features of the exact solution. However, because we have employed straight-line segments to approximate a continuously curving function, there is some discrepancy between the two results. One way to minimize such discrepancies is to use a smaller step size. For example, applying Eq. (1.12) at l-s intervals results in a smaller error, as the straight-line segments track closer to the true solution. Using hand calculations, the effort associated with using smaller and smaller step sizes would make such numerical solutions impractical. However, with the aid of the computer, large numbers of calculations can be performed easily. Thus, you can accurately model the velocity of the falling parachutist without having to solve the differential equation exactly.

As in the previous example, a computational price must be paid for a more accurate numerical result. Each halving of the step size to attain more accuracy leads to a doubling

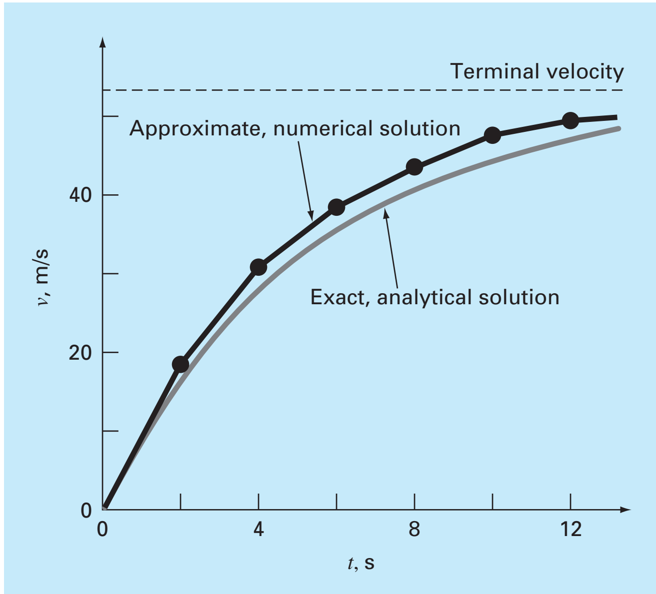


FIGURE 1.5
Comparison of the numerical and analytical solutions for the falling parachutist problem.

of the number of computations. Thus, we see that there is a trade-off between accuracy and computational effort. Such trade-offs figure prominently in numerical methods and constitute an important theme of this book. Consequently, we have devoted the Epilogue of Part One to an introduction to more of these trade-offs.

1.2 CONSERVATION LAWS AND ENGINEERING

Aside from Newton's second law, there are other major organizing principles in engineering. Among the most important of these are the conservation laws. Although they form the basis for a variety of complicated and powerful mathematical models, the great conservation laws of science and engineering are conceptually easy to understand. They all boil down to

$$\text{Change} = \text{increases} - \text{decreases} \tag{1.13}$$

This is precisely the format that we employed when using Newton's law to develop a force balance for the falling parachutist [Eq. (1.8)].

Although simple, Eq. (1.13) embodies one of the most fundamental ways in which conservation laws are used in engineering—that is, to predict changes with respect to time. We give Eq. (1.13) the special name *time-variable* (or *transient*) computation.

Aside from predicting changes, another way in which conservation laws are applied is for cases where change is nonexistent. If change is zero, Eq. (1.13) becomes

$$\text{Change} = 0 = \text{increases} - \text{decreases}$$

or

$$\text{Increases} = \text{decreases} \tag{1.14}$$

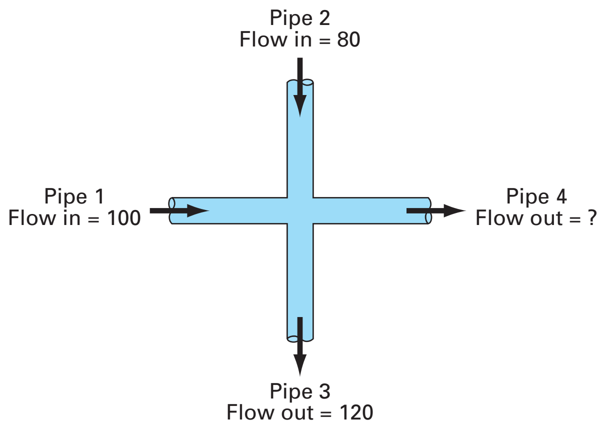


FIGURE 1.6
A flow balance for steady-state incompressible fluid flow at the junction of pipes.

Thus, if no change occurs, the increases and decreases must be in balance. This case, which is also given a special name—the *steady-state* computation—has many applications in engineering. For example, for steady-state incompressible fluid flow in pipes, the flow into a junction must be balanced by flow going out, as in

Flow in = flow out

For the junction in Fig. 1.6, the balance can be used to compute that the flow out of the fourth pipe must be 60.

For the falling parachutist, steady-state conditions would correspond to the case where the net force was zero, or [Eq. (1.8) with $dv/dt = 0$]

$$mg = cv \tag{1.15}$$

Thus, at steady state, the downward and upward forces are in balance, and Eq. (1.15) can be solved for the terminal velocity

$$v = \frac{mg}{c}$$

Although Eqs. (1.13) and (1.14) might appear trivially simple, they embody the two fundamental ways that conservation laws are employed in engineering. As such, they will form an important part of our efforts in subsequent chapters to illustrate the connection between numerical methods and engineering. Our primary vehicles for making this connection are the engineering applications that appear at the end of each part of this book.

Table 1.1 summarizes some of the simple engineering models and associated conservation laws that will form the basis for many of these engineering applications. Most of the chemical engineering applications will focus on mass balances for reactors. The mass balance is derived from the conservation of mass. It specifies that the change of mass of a chemical in the reactor depends on the amount of mass flowing in minus the mass flowing out.

Both the civil and mechanical engineering applications will focus on models developed from the conservation of momentum. For civil engineering, force balances are utilized to analyze structures such as the simple truss in Table 1.1. The same principles are employed for the mechanical engineering applications to analyze the transient up-and-down motion or vibrations of an automobile.

TABLE 1.1 Devices and types of balances that are commonly used in the four major areas of engineering. For each case, the conservation law upon which the balance is based is specified.

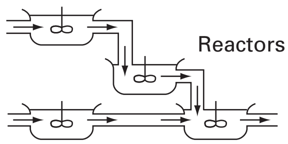
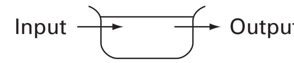
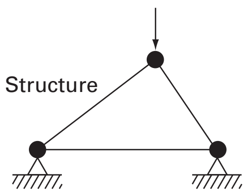
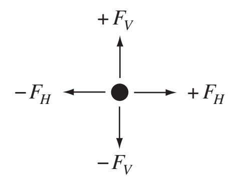
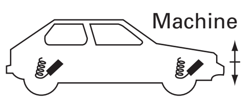
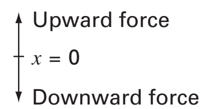
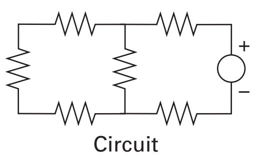
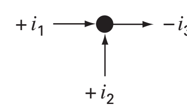
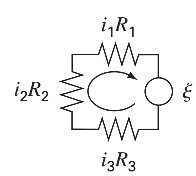
Field	Device	Organizing Principle	Mathematical Expression
Chemical engineering		Conservation of mass	Mass balance:  Over a unit of time period Δmass = inputs − outputs
Civil engineering		Conservation of momentum	Force balance:  At each node Σ horizontal forces (F_H) = 0 Σ vertical forces (F_V) = 0
Mechanical engineering		Conservation of momentum	Force balance:  $m \dfrac{d^2x}{dt^2}$ = downward force − upward force
Electrical engineering		Conservation of charge	Current balance:  For each node Σ current (i) = 0
		Conservation of energy	Voltage balance:  Around each loop Σ emf's − Σ voltage drops for resistors = 0 $\Sigma\, \xi - \Sigma\, iR = 0$

TABLE 1.2 Some practical issues that will be explored in the engineering applications at the end of each part of this book.

1. *Nonlinear versus linear.* Much of classical engineering depends on linearization to permit analytical solutions. Although this is often appropriate, expanded insight can often be gained if nonlinear problems are examined.
2. *Large versus small systems.* Without a computer, it is often not feasible to examine systems with over three interacting components. With computers and numerical methods, more realistic multicomponent systems can be examined.
3. *Nonideal versus ideal.* Idealized laws abound in engineering. Often there are nonidealized alternatives that are more realistic but more computationally demanding. Approximate numerical approaches can facilitate the application of these nonideal relationships.
4. *Sensitivity analysis.* Because they are so involved, many manual calculations require a great deal of time and effort for successful implementation. This sometimes discourages the analyst from implementing the multiple computations that are necessary to examine how a system responds under different conditions. Such sensitivity analyses are facilitated when numerical methods allow the computer to assume the computational burden.
5. *Design.* It is often a straightforward proposition to determine the performance of a system as a function of its parameters. It is usually more difficult to solve the inverse problem—that is, determining the parameters when the required performance is specified. Numerical methods and computers often permit this task to be implemented in an efficient manner.

Finally, the electrical engineering applications employ both current and energy balances to model electric circuits. The current balance, which results from the conservation of charge, is similar in spirit to the flow balance depicted in Fig. 1.6. Just as flow must balance at the junction of pipes, electric current must balance at the junction of electric wires. The energy balance specifies that the changes of voltage around any loop of the circuit must add up to zero. The engineering applications are designed to illustrate how numerical methods are actually employed in the engineering problem-solving process. As such, they will permit us to explore practical issues (Table 1.2) that arise in real-world applications. Making these connections between mathematical techniques such as numerical methods and engineering practice is a critical step in tapping their true potential. Careful examination of the engineering applications will help you to take this step.

PROBLEMS

1.1 Use calculus to solve Eq. (1.9) for the case where the initial velocity, $v(0)$ is nonzero.

1.2 Repeat Example 1.2. Compute the velocity to $t = 8$ s, with a step size of **(a)** 1 and **(b)** 0.5 s. Can you make any statement regarding the errors of the calculation based on the results?

1.3 Rather than the linear relationship of Eq. (1.7), you might choose to model the upward force on the parachutist as a second-order relationship,

$$F_U = -c'v^2$$

where c' = a bulk second-order drag coefficient (kg/m).

(a) Using calculus, obtain the closed-form solution for the case where the jumper is initially at rest ($v = 0$ at $t = 0$).

(b) Repeat the numerical calculation in Example 1.2 with the same initial condition and parameter values, but with second-order drag. Use a value of 0.22 kg/m for c'.

1.4 For the free-falling parachutist with linear drag, assume a first jumper is 70 kg and has a drag coefficient of 12 kg/s. If a second jumper has a drag coefficient of 15 kg/s and a mass of 80 kg, how long will it take him to reach the same velocity the first jumper reached in 9 s?

1.5 Compute the velocity of a free-falling parachutist using Euler's method for the case where $m = 80$ kg and $c = 10$ kg/s. Perform the calculation from $t = 0$ to 20 s with a step size of 1 s. Use an initial condition that the parachutist has an upward velocity of 20 m/s at $t = 0$. At $t = 10$ s, assume that the chute is instantaneously deployed so that the drag coefficient jumps to 60 kg/s.

1.6 The following information is available for a bank account:

Date	Deposits	Withdrawals	Interest	Balance
5/1				1522.33
	220.13	327.26		
6/1				
	216.80	378.51		
7/1				
	450.35	106.80		
8/1				
	127.31	350.61		
9/1				

Note that the money earns interest which is computed as

$$\text{Interest} = i\,B_i$$

where i = the interest rate expressed as a fraction per month, and B_i the initial balance at the beginning of the month.

(a) Use the conservation of cash to compute the balance on 6/1, 7/1, 8/1, and 9/1 if the interest rate is 1% per month (i = 0.01/month). Show each step in the computation.

(b) Write a differential equation for the cash balance in the form

$$\frac{dB}{dt} = f(D(t), W(t), i)$$

where t = time (months), $D(t)$ = deposits as a function of time (\$/month), $W(t)$ = withdrawals as a function of time (\$/month). For this case, assume that interest is compounded continuously; that is, interest = iB.

(c) Use Euler's method with a time step of 0.5 month to simulate the balance. Assume that the deposits and withdrawals are applied uniformly over the month.

(d) Develop a plot of balance versus time for **(a)** and **(c)**.

1.7 The amount of a uniformly distributed radioactive contaminant contained in a closed reactor is measured by its concentration c (becquerel/liter or Bq/L). The contaminant decreases at a decay rate proportional to its concentration—that is,

$$\text{decay rate} = -kc$$

where k is a constant with units of day^{-1}. Therefore, according to Eq. (1.13), a mass balance for the reactor can be written as

$$\frac{dc}{dt} \quad = \quad -kc$$

$$\begin{pmatrix}\text{change}\\ \text{in mass}\end{pmatrix} = \begin{pmatrix}\text{decrease}\\ \text{by decay}\end{pmatrix}$$

(a) Use Euler's method to solve this equation from t = 0 to 1 d with $k = 0.175\text{d}^{-1}$. Employ a step size of $\Delta t = 0.1$. The concentration at t = 0 is 100 Bq/L.

(b) Plot the solution on a semilog graph (i.e., ln c versus t) and determine the slope. Interpret your results.

1.8 A group of 35 students attend a class in a room that measures 11 m by 8 m by 3 m. Each student takes up about 0.075 m^3 and gives out about 80 W of heat (1 W = 1 J/s). Calculate the air temperature rise during the first 20 minutes of the class if the room is completely sealed and insulated. Assume the heat capacity, C_v, for air is 0.718 kJ/(kg K). Assume air is an ideal gas at 20°C and 101.325 kPa. Note that the heat absorbed by the air Q is related to the mass of the air m, the heat capacity, and the change in temperature by the following relationship:

$$Q = m\int_{T_1}^{T_2} C_v dT = mC_v\,(T_2 - T_1)$$

The mass of air can be obtained from the ideal gas law:

$$PV = \frac{m}{\text{MwT}} RT$$

where P is the gas pressure, V is the volume of the gas, Mwt is the molecular weight of the gas (for air, 28.97 kg/kmol), and R is the ideal gas constant [8.314 kPa m^3/(kmol K)].

1.9 A storage tank contains a liquid at depth y, where y = 0 when the tank is half full. Liquid is withdrawn at a constant flow rate Q to meet demands. The contents are resupplied at a sinusoidal rate $3Q\sin^2(t)$.

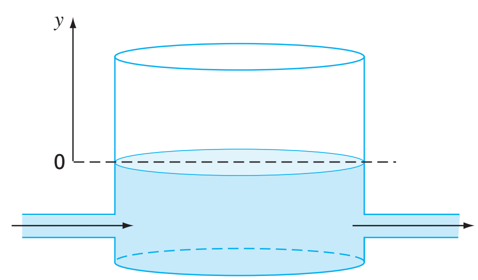


FIGURE P1.9

Equation (1.13) can be written for this system as

$$\frac{d(Ay)}{dt} \quad = 3Q\sin^2(t) - \quad Q$$

$$\begin{pmatrix}\text{change in}\\ \text{volume}\end{pmatrix} = (\text{inflow}) \quad - (\text{outflow})$$

or, since the surface area A is constant

$$\frac{dy}{dt} = 3\frac{Q}{A}\sin^2(t) - \frac{Q}{A}$$

Use Euler's method to solve for the depth y from $t = 0$ to 10 d with a step size of 0.5 d. The parameter values are $A = 1250\ \text{m}^2$ and $Q = 450\ \text{m}^3/\text{d}$. Assume that the initial condition is $y = 0$.

1.10 For the same storage tank described in Prob. 1.9, suppose that the outflow is not constant but rather depends on the depth. For this case, the differential equation for depth can be written as

$$\frac{dy}{dt} = 3\frac{Q}{A}\sin^2(t) - \frac{\alpha(1+y)^{1.5}}{A}$$

Use Euler's method to solve for the depth y from $t = 0$ to 10 d with a step size of 0.5 d. The parameter values are $A = 1250\ \text{m}^2$, $Q = 450\ \text{m}^3/\text{d}$, and $\alpha = 150$. Assume that the initial condition is $y = 0$.

1.11 Apply the conservation of volume (see Prob. 1.9) to simulate the level of liquid in a conical storage tank (Fig. P1.11). The liquid flows in at a sinusoidal rate of $Q_{\text{in}} = 3\sin^2(t)$ and flows out according to

$$Q_{\text{out}} = 3(y - y_{\text{out}})^{1.5} \qquad y > y_{\text{out}}$$

$$Q_{\text{out}} = 0 \qquad y \le y_{\text{out}}$$

where flow has units of m^3/d and y = the elevation of the water surface above the bottom of the tank (m). Use Euler's method to solve for the depth y from $t = 0$ to 10 d with a step size of 0.5 d. The parameter values are $r_{\text{top}} = 2.5$ m, $y_{\text{top}} = 4$ m, and $y_{\text{out}} = 1$ m. Assume that the level is initially below the outlet pipe with $y(0) = 0.8$ m.

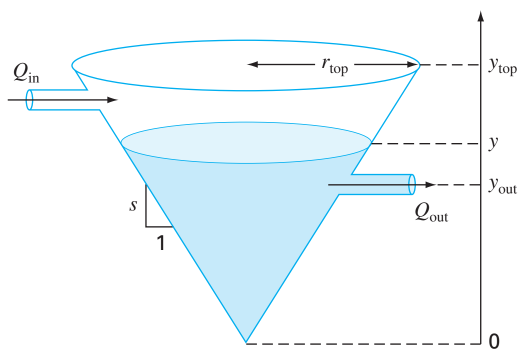


FIGURE P1.11

1.12 In our example of the free-falling parachutist, we assumed that the acceleration due to gravity was a constant value. Although this is a decent approximation when we are examining falling objects near the surface of the earth, the gravitational force decreases as we move above sea level. A more general representation based on *Newton's inverse square law* of gravitational attraction can be written as

$$g(x) = g(0)\frac{R^2}{(R+x)^2}$$

where $g(x)$ = gravitational acceleration at altitude x (in m) measured upward from the earth's surface (m/s^2), $g(0)$ = gravitational acceleration at the earth's surface ($\cong 9.81\ \text{m/s}^2$), and R = the earth's radius ($\cong 6.37 \times 10^6$ m).

(a) In a fashion similar to the derivation of Eq. (1.9) use a force balance to derive a differential equation for velocity as a function of time that utilizes this more complete representation of gravitation. However, for this derivation, assume that upward velocity is positive.

(b) For the case where drag is negligible, use the chain rule to express the differential equation as a function of altitude rather than time. Recall that the chain rule is

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}$$

(c) Use calculus to obtain the closed form solution where $v = v_0$ at $x = 0$.

(d) Use Euler's method to obtain a numerical solution from $x = 0$ to 100,000 m using a step of 10,000 m where the initial velocity is 1500 m/s upward. Compare your result with the analytical solution.

1.13 Suppose that a spherical droplet of liquid evaporates at a rate that is proportional to its surface area.

$$\frac{dV}{dt} = -kA$$

where V = volume (mm^3), t = time (min), k = the evaporation rate (mm/min), and A = surface area (mm^2). Use Euler's method to compute the volume of the droplet from $t = 0$ to 10 min using a step size of 0.25 min. Assume that $k = 0.08$ mm/min and that the droplet initially has a radius of 2.5 mm. Assess the validity of your results by determining the radius of your final computed volume and verifying that it is consistent with the evaporation rate.

1.14 *Newton's law of cooling* says that the temperature of a body changes at a rate proportional to the difference between its temperature and that of the surrounding medium (the ambient temperature),

$$\frac{dT}{dt} = -k(T - T_a)$$

where T = the temperature of the body (°C), t = time (min), k = the proportionality constant (per minute), and T_a = the ambient temperature (°C). Suppose that a cup of coffee originally has a temperature of 70°C. Use Euler's method to compute the temperature from $t = 0$ to 10 min using a step size of 2 min if $T_a = 20$°C and $k = 0.019$/min.

1.15 As depicted in Fig. P1.15, an *RLC circuit* consists of three elements: a resistor (R), and inductor (L) and a capacitor (C). The flow of current across each element induces a voltage drop.

Kirchhoff's second voltage law states that the algebraic sum of these voltage drops around a closed circuit is zero,

$$iR + L\frac{di}{dt} + \frac{q}{C} = 0$$

where i = current, R = resistance, L = inductance, t = time, q = charge, and C = capacitance. In addition, the current is related to charge as in

$$\frac{dq}{dt} = i$$

(a) If the initial values are $i(0) = 0$ and $q(0) = 1$ C, use Euler's method to solve this pair of differential equations from $t = 0$ to 0.1 s using a step size of $\Delta t = 0.01$ s. Employ the following parameters for your calculation: $R = 200\ \Omega$, $L = 5$ H, and $C = 10^{-4}$ F.
(b) Develop a plot of i and q versus t.

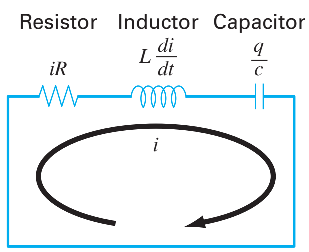


FIGURE P1.15

1.16 Cancer cells grow exponentially with a doubling time of 20 h when they have an unlimited nutrient supply. However, as the cells start to form a solid spherical tumor without a blood supply, growth at the center of the tumor becomes limited, and eventually cells start to die.
(a) Exponential growth of cell number N can be expressed as shown, where μ is the growth rate of the cells. For cancer cells, find the value of μ.

$$\frac{dN}{dt} = \mu N$$

(b) Write an equation that will describe the rate of change of tumor volume during exponential growth given that the diameter of an individual cell is 20 microns.
(c) After a particular type of tumor exceeds 500 microns in diameter, the cells at the center of the tumor die (but continue to take up space in the tumor). Determine how long it will take for the tumor to exceed this critical size.

1.17 A fluid is pumped into the network shown in Fig. P1.17. If $Q_2 = 0.6$, $Q_3 = 0.4$, $Q_7 = 0.2$, and $Q_8 = 0.3\ \text{m}^3/\text{s}$, determine the other flows.

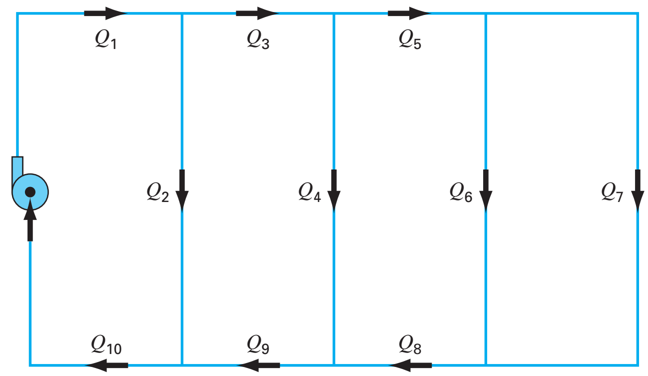


FIGURE P1.17

1.18 The velocity is equal to the rate of change of distance x (m),

$$\frac{dx}{dt} = v(t) \tag{P1.18}$$

(a) Substitute Eq. (1.10) and develop an analytical solution for distance as a function of time. Assume that $x(0) = 0$.
(b) Use Euler's method to numerically integrate Eqs. (P1.18) and (1.9) in order to determine both the velocity and distance fallen as a function of time for the first 10 s of free-fall using the same parameters as in Example 1.2.
(c) Develop a plot of your numerical results together with the analytical solution.

1.19 You are working as a crime-scene investigator and must predict the temperature of a homicide victim over a 5-hr period. You know that the room where the victim was found was at 10°C when the body was discovered.
(a) Use *Newton's law of cooling* (Prob. 1.14) and Euler's method to compute the victim's body temperature for the 5-hr period using values of $k = 0.12$/hr and $\Delta t = 0.5$ hr. Assume that the victim's body temperature at the time of death was 37°C, and that the room temperature was at a constant value of 10°C over the 5-hr period.
(b) Further investigation reveals that the room temperature had actually dropped linearly from 20 to 10°C over the 5-hr period. Repeat the same calculation as in **(a)** but incorporate this new information.
(c) Compare the results from **(a)** and **(b)** by plotting them on the same graph.

1.20 Suppose that a parachutist with linear drag ($m = 70$ kg, $c = 12.5$ kg/s) jumps from an airplane flying at an altitude of a kilometer with a horizontal velocity of 180 m/s relative to the ground.
(a) Write a system of four differential equations for x, y, $v_x = dx/dt$ and $v_y = dy/dt$.

(b) If the initial horizontal position is defined as $x = 0$, use Euler's methods with $\Delta t = 1$ s to compute the jumper's position over the first 10 s.

(c) Develop plots of y versus t and y versus x. Use the plot to graphically estimate when and where the jumper would hit the ground if the chute failed to open.

1.21 As noted in Prob. 1.3, drag is more accurately represented as depending on the square of velocity. A more fundamental representation of the drag force, which assumes turbulent conditions (i.e., a high Reynolds number), can be formulated as

$$F_d = -\frac{1}{2}\rho A C_d v|v|$$

where F_d = the drag force (N), ρ = fluid density (kg/m^3), A = the frontal area of the object on a plane perpendicular to the direction of motion (m^2), v = velocity (m/s), and C_d = a dimensionless drag coefficient.

(a) Write the pair of differential equations for velocity and position (see Prob. 1.18) to describe the vertical motion of a sphere with diameter d (m) and a density of ρ_s (kg/km^3). The differential equation for velocity should be written as a function of the sphere's diameter.

(b) Use Euler's method with a step size of $\Delta t = 2$ s to compute the position and velocity of a sphere over the first 14 s. Employ the following parameters in your calculation: $d = 120$ cm, $\rho = 1.3$ kg/m^3, $\rho_s = 2700$ kg/m^3, and $C_d = 0.47$. Assume that the sphere has the initial conditions: $x(0) = 100$ m and $v(0) = -40$ m/s.

(c) Develop a plot of your results (i.e., y and v versus t) and use it to graphically estimate when the sphere would hit the ground.

(d) Compute the value for the bulk second-order drag coefficient c_d' (kg/m). Note that, as described in Prob. 1.3, the bulk second-order drag coefficient is the term in the final differential equation for velocity that multiplies the term $v|v|$.

1.22 As depicted in Fig. P1.22, a spherical particle settling through a quiescent fluid is subject to three forces: the downward force of gravity (F_G), and the upward forces of buoyancy (F_B) and drag (F_D). Both the gravity and buoyancy forces can be computed with Newton's second law with the latter equal to the weight of the displaced fluid. For laminar flow, the drag force can be computed with *Stokes's law*,

$$F_D = 3\pi\mu d v$$

where μ = the dynamic viscosity of the fluid (N s/m^2), d = the particle diameter (m), and v = the particle's settling velocity (m/s). Note that the mass of the particle can be expressed as the product of the particle's volume and density ρ_s (kg/m^3) and the mass of the displaced fluid can be computed as the product of the particle's volume and the fluid's density ρ (kg/m^3). The volume of a sphere is $\pi d^3/6$. In addition, laminar flow corresponds to the case where the dimensionless Reynolds number, Re, is less than 1, where Re $= \rho d v/\mu$.

(a) Use a force balance for the particle to develop the differential equation for dv/dt as a function of d, ρ, ρ_s, and μ.

(b) At steady-state, use this equation to solve for the particle's terminal velocity.

(c) Employ the result of **(b)** to compute the particle's terminal velocity in m/s for a spherical silt particle settling in water: $d = 10\ \mu$m, $\rho = 1$ g/cm^3, $\rho_s = 2.65$ g/cm^3, and $\mu = 0.014$ g/(cm·s).

(d) Check whether flow is laminar.

(e) Use Euler's method to compute the velocity from $t = 0$ to 2^{-15} s with $\Delta t = 2^{-18}$ s given the parameters given previously along with the initial condition: $v(0) = 0$.

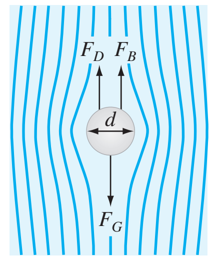


FIGURE P1.22

1.23 As described in Prob. 1.22, in addition to the downward force of gravity (weight) and drag, an object falling through a fluid is also subject to a buoyancy force that is proportional to the displaced volume. For example, for a sphere with diameter d (m), the sphere's volume is $V = \pi d^3/6$ and its projected area is $A = \pi d^2/4$. The buoyancy force can then be computed as $F_b = -\rho V g$. We neglected buoyancy in our derivation of Eq. (1.9) because it is relatively small for an object like a parachutist moving through air. However, for a more dense fluid like water, it becomes more prominent.

(a) Derive a differential equation in the same fashion as Eq. (1.9), but include the buoyancy force and represent the drag force as described in Prob. 1.21.

(b) Rewrite the differential equation from **(a)** for the special case of a sphere.

(c) Use the equation developed in **(b)** to compute the terminal velocity (i.e., for the steady-state case). Use the following parameter values for a sphere falling through water: sphere diameter = 1 cm, sphere density = 2700 kg/m^3, water density = 1000 kg/m^3, and $C_d = 0.47$.

(d) Use Euler's method with a step size of $\Delta t = 0.03125$ s to numerically solve for the velocity from $t = 0$ to 0.25 s with an initial velocity of zero.

1.24 As depicted in Fig. P1.24, the downward deflection y (m) of a cantilever beam with a uniform load w (kg/m) can be computed as

$$y = \frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$$

where x = distance (m), E = the modulus of elasticity = 2×10^{11} Pa, I = moment of inertia = 3.25×10^{-4} m^4, w = 10,000 N/m, and