Tidal Deformation of Neutron Stars

Marco Zenari

Supervisor: Prof. Albino Perego

25 November 2022

Introduction

- GW150914: first detection of GWs;
- GWs are ripples in the curvature of space-time;
- GW170817: GWs from the inspiral phase of BNSs;
- Our aim: main aspects of NSs, BNSs, tidal effects and Love number k_2 calculation.

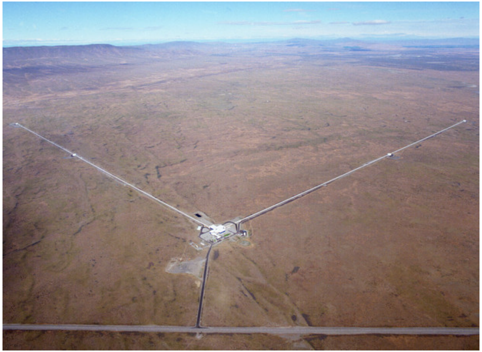


Figure: LIGO detector site at Hanford (Washington). Credits: LIGO.

Neutron stars

- Remnant left behind SNs can give birth to NSs;
- First detection of a pulsar: Cavendish Laboratory in 1967;
- Pulsars are highly magnetized, rotating NSs;
- Magnetic dipole model:
  - $\dot{T} \propto B^2$;
  - $B \propto \sqrt{T\dot{T}}$.

Tolman-Oppenheimer-Volkoff equations

- The structure of stars is determined by *hydrostatic equilibrium*;
- Gravity inside and at the surface of NSs is so strong that to describe their structure GR is needed;
- In GR all form of energy, including pressure, contribute to gravity.

TOV equations

$$\frac{dm}{dr} = \frac{4\pi r^2 \varepsilon}{c^2}$$

$$\frac{dP}{dr} = -\frac{G\varepsilon m(r)}{c^2 r^2}\left(1+\frac{P}{\varepsilon}\right)\left[1+\frac{4\pi r^3 P}{m(r)c^2}\right]\left[1-\frac{2Gm(r)}{c^2 r}\right]^{-1}$$

Equation of state of neutron stars

- The EOS is the information on the matter composition together with the relationships between:
  - matter pressure P;
  - energy density ε;
  - baryonic density n;
  - temperature T.

- Non-interacting fermions:

Polytrope EOS

$P = K\rho^{\Gamma}$

Polytrope exponent

Non-relativistic limit $\Gamma = 5/3$
Ultra-relativistic limit $\Gamma = 4/3$

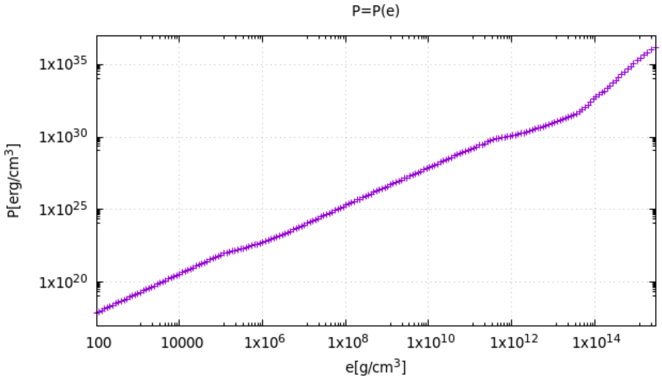
- Interactions in a NS cannot be ignored;

- Some neutrons will decay via $n \to p + e + \bar{\nu}_e$.

Bombaci-Logoteta EOS[1]

- The EOS can be used in tabulated form in numerical simulations.

- Plot in logarithmic scale of the pressure P in $\mathrm{erg}/\mathrm{cm}^3$ as function of the energy density ε in g/cm^3 for the BL equation of state.



[1] Astronomy & Astrophysics 646 (2021), A55

Compact binary systems and tidal deformation

- Binary systems in which at least one star is a compact object (WDs, NSs, BHs);
- Hulse-Taylor binary (PSR 1913+16), discovered in 1974;
- GR predicts the emission of GWs that carry away energy and momentum;
- The two objects become closer and closer, until they merge;
- In the final phase of the inspiral tidal forces between the NSs tend to stretch the compact objects.

Physics of tides: Earth-Moon system

- The force on an object due to his non-zero size is known as a tidal force;
- This phenomenon happens in every binary system;

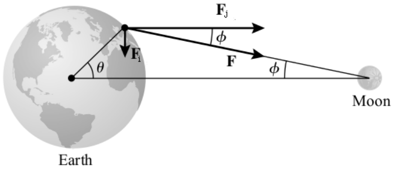


Figure: The geometry of tidal forces acting on Earth due to the Moon.

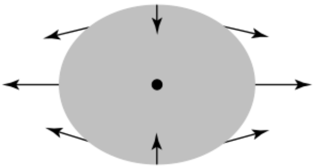


Figure: The differential gravitational force on Earth, relative to its center.

Differential gravitational force

$$\Delta \mathbf{F} \simeq \frac{GMmR}{r^3}\left(2\cos\theta\hat{\mathbf{i}} - \sin\theta\hat{\mathbf{i}}\right)$$

The result is that Earth is elongated.

General relativity tidal theory in Newtonian approximation

- The inspiral phase of BNSs can be studied with the PPN formalism;
- Tidal effects depend on the EOS of the NSs.

In PPN U_{ext} generates a quadrupolar tidal field $\mathscr{E}_{ij}$. In GR $\mathscr{E}_{ij}$ depends on the Riemann Tensor.

Quadrupolar tidal field

$$\mathscr{E}_{ij} = -\partial_i \partial_j U_{\text{ext}}$$

Quasi-static approximation and expansion at the linear order (for some constant λ):

Induced quadrupole

$$Q_{ij}(t) = -\lambda \mathscr{E}_{ij}(t).$$

We can define the dimensionless Love number k_2 as:

Love number k_2

$$k_2 = \frac{3}{2} \frac{G\lambda}{R^5}.$$

Love number k_2

- Typical value of k_2 for NSs is between 0.05 and 0.15;
- To calculate k_2 we need to integrate the following equations[2]:

Love number k_2

$$k_2 = \frac{8C^5}{5}(1-2C)^2[2+2C(y_R-1)-y_R]\Big\{2C[6-3y_R+3C(5y_R-8)]+4C^3[12-11y_R+C(3y_R-2)+2C^2(1+y_R)]+3(1-2C)^2[2-y_R+2C(y_R-1)]\times\ln(1-2C)\Big\}^{-1}$$

Compactness

$$C = \frac{GM}{c^2R},$$

y function

$$y_R = y(R)$$

[2]Physical Review C 98.3 (2018), p. 035804; Phys. Rev. D80 (2009), p. 084035; The Astrophysical Journal 677.2 (2008), p. 1216.

y function

Differential equation

$$r\frac{dy(r)}{dr}+y(r)^2+y(r)F(r)+r^2Q(r)=0$$

Initial condition

$$y(0)=2$$

F(r)

$$F(r)=\frac{r-\dfrac{4\pi G\ r^3}{c^4}[\varepsilon(r)-P(r)]}{r-\dfrac{2m(r)G}{c^2}}$$

Q(r)

$$Q(r)=\frac{\dfrac{4\pi\ G\ r}{c^4}\left[5\varepsilon(r)+9P(r)+\dfrac{\varepsilon(r)+P(r)}{\partial P(r)/\partial\varepsilon(r)}\right]-\dfrac{6}{4\pi r^2}}{r-\dfrac{2Gm(r)}{c^2}}-4\left[\frac{\dfrac{Gm(r)}{c^2}+4\pi\dfrac{Gr^3P(r)}{c^4}}{r^2\left[1-\dfrac{2Gm(r)}{c^2r}\right]}\right]^2$$

Overview of the code

- Code written in C;
- Dimensionless units in order to avoid numerical errors;
- We have integrated the TOV equations together with the equations for the Love number k_2 and the information from the BL EOS;
- Integrator: Runge-Kutta IV;
- Linear interpolation of the EOS.

Initial condition

$$P(0) = P_c$$
$$m(0) = 0$$
$$y(0) = 2$$

End of the integration

$$P < \delta$$

with δ arbitrarily chosen

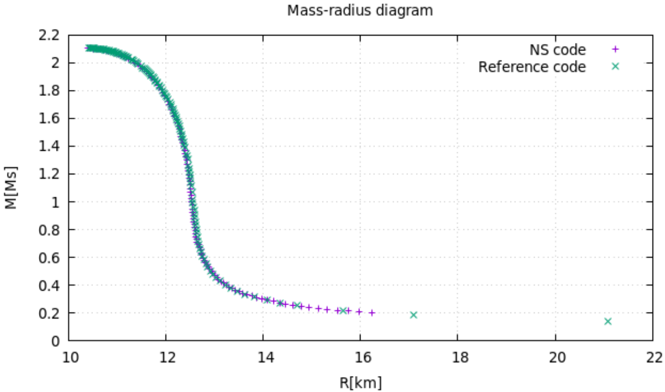
Results

- The output of our code consists in a sequence of possible value of mass, radius, compactness and Love number k_2 that correspond to a given central pressure, for a NS with BL EOS;
- The results are compatible with the ones obtained with a public Matlab code [3].

[3] Phys. Rev. D78 (2008), p. 024024; Phys. Rev. D80 (2009), p. 084035

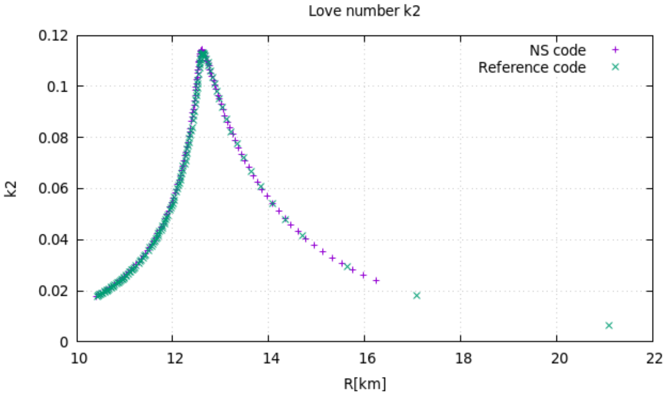
Results: mass-radius diagram

- The radius decrease with the increasing of the mass;
- Typical radius for a NS is between 12 and 13 km;
- The mass has a maximum value of about 2.102 $\mathrm{M}_\odot$.



Results: Love number k_2

- The value of the obtained k_2 are in the range expected $(0.05 - 0.15)$;
- There is a cusp for a radius that is between 12.5 and 13 km. This is a typical trend for low masses NSs ($M < 1\mathrm{M}_\odot$).



Conclusions

- The detection of GWs had a profound impact in our understanding of gravity and the physics of dense matter;
- GWs from the inspiral phase of BNSs can be used to measure the tidal deformability of the NSs, setting stringent constraints on their EOS;
- The derivation of an EOS suitable for their description requires a tremendous theoretical and computational effort;
- Continuing the detection of GWs due to inspiral of binary systems could provide a large amount of data that could help in the understanding of matter under such extreme conditions.