Introduction to Electromagnetism

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Introduction

I will introduce to you the theory of electromagnetism through this article, not in the order of history, but in the order of a self-contained logic that satisfies people. The reason I did this is that history is too complicated, confusing and sometimes even wrong. One would not fully understand the trajectory of any subject (geometry, Newton's law, General relativity, etc) without *experiencing* it. So I would NOT recommend you go through the tedious history of *any* subject but develop **your own** logic-complete explanation of it after reading tons of wikipedia and stackexchange, in other words, that explanation should make sense in the history in another parallel universe!

Physics World to Math World

Physics originates from observations in the real world. If you keep an eye on it, you could discover the following phenomena, which can be grouped into four:

1. Two parallel wires attract or repel each other when current is applied.
2. The force between two charges is proportional to their charge, and inversely proportional to their distance, squared.

Maxwell's First Equation

Inverse Square Law

According to observation 2, we could write[1]:

$$F \propto \frac{Qq}{r^2},$$

or in an equation:

$$\boxed{F = k\frac{Qq}{r^2},}$$

where k is a constant, Q and q are the amount of charge on two small objects, r is their distance.

[1] Non-bold-face letters (F) are scalars, bold-face letters ($\mathbf{F}$) are vectors.

Conservative Property

Let's consider the work done when we fix Q and gradually move q in a certain path. Since the force from Q to q is always radially from Q, it turns out that the work done only depends on the *initial* and *final* position of q regardless of the moving process in between! We say a force like this ***Conservative***[2]. We would like to think that there is a number U attached at every spatial point around Q such that the the work done from point A and B is just that $U(A) - U(B)$[3], i.e.,

$$W_{AB} =: U(A) - U(B).$$

$U(r)$ is called the ***potential energy*** at r. We claim[4] that

$$\boxed{U(r) = -k\frac{Qq}{r},}$$

because

$$W_{AB} = \int_{A\to B} \mathbf{F}\cdot \mathrm{d}\mathbf{r} = \int_{r_0}^{r_1} k\frac{Qq}{r^2}\mathrm{d}r = -k\frac{Qq}{r_1} - k\frac{Qq}{r_0} =: U(A) - U(B).$$

Therefore, the work done is just the difference of U with a negative sign:

$$W_{AB} = -\Delta U.$$

We also have the following relationship:

$$\boxed{\mathbf{F} = -\,U.} \qquad (1)$$

[2] **NOTE** It's worth noting that *conservativity* has nothing to do with the inverse square property. The potential energy of hypothetical "inverse force" and "inverse cubic force" are:

$$U_{\frac{1}{r}} = -kQq\ln r,$$

and

$$U_{\frac{1}{r^3}} = \frac{kQq}{2}\frac{1}{r^2}.$$

[4] Actually, should be $U(r) = -k\frac{Q}{r} +$ const. But when $r \to \infty$, we expect U to be 0 because q do not have the ability to do work.

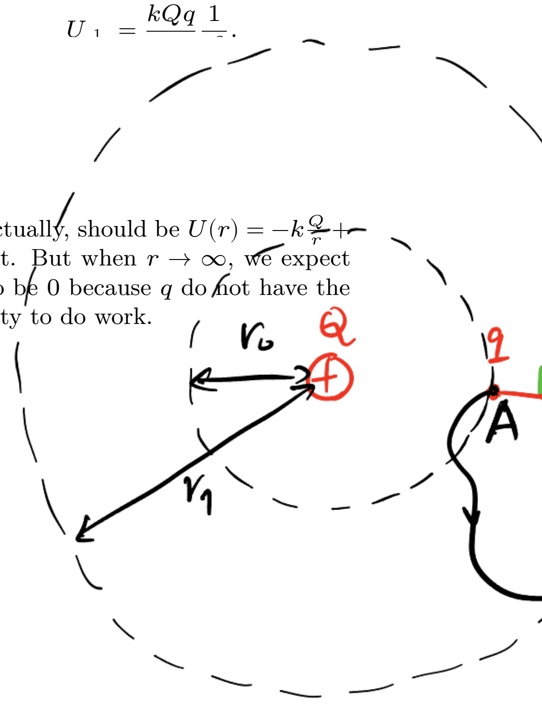

Figure 1: $U(A)$ is expected to be larger t

Get Rid of Test-charge

When q is far smaller than Q, it is called a test charge, which is used to "test" the effect of Q to its surroundings and minimize other interference. It is naturally to get rid of q and define a quantity E that only depends on Q, we expect that E satisfies:

$$\mathbf{F} =: \mathbf{E}q,$$

where E obviously equal to:

$$\boxed{E = k\frac{Q}{r^2}.}$$

We call E the **electric field** generated by Q. Since $\mathbf{F}$ is *conservative*, we also call $\mathbf{E}$ a ***conservative field***.

Therefore, Equation 1 could be written as:

$$\mathbf{E}q = -\ U. \tag{2}$$

We could also get rid of q in Equation 2 by defining a quantity V called the ***(Electric) potential*** generated by Q:

$$\boxed{U := Vq,}$$

so we have

$$\begin{aligned} \mathbf{E}q &= -\nabla V q \\ \implies \boxed{\mathbf{E} = -\ V.} \end{aligned} \tag{3}$$

You can compare @FU and @EV, the latter is test-charge-free version of the former!

?quarto-cite:FU

?quarto-cite:EV

Definition Chain of Basic Concepts

Based on the above observations, scientist in this parallel universe reached these consensus:

- **Current: Ampere** (I: A): The intensity of current on two ideal wires that are 1 metre apart and produces a force of 2×10^{-7} N between them (1N is defined to be the force that makes an object of mass 1kg moves at the acceleration of 1m/s^2)

- **Charge: Coulomb** (Q: C): The amount of charge that passes through a point with a current of 1A over 1 second[5].

[5] The reason *Ampere* are defined first is because *Ampere* is chosen to be one of the *base unit* of SI.
Just a quick review on SI units: These are the building blocks (7 of them) of any other physical units, they are defined in the first place independently, usually using quantities that do not change in our universe (such as the speed of light). They are **metre**, **kg**, **second**, **Ampere**, **kelvin**, **mole**, **candela**. Check wiki for details.