Homework #1, AA2019-2020: Identification, deconvolution and order analysis for SISO and MIMO systems

Single Input Single Output (SISO) and Multiple Input Multiple Output (MIMO) are typical jargons that denotes the conventional and multichannel systems. The purpose of this Hw is to investigate the correct orders in estimation by MSE analysis, and some linear filtering manipulations by using Matlab as software. Notice that students can use primitives, and not evolved Matlab packages.

a) Consider the SISO system $y[n] = h[n] * x[n] + w[n]$, where the filter $h[n] \leftrightarrow H(z)$ is as follows:

- a.1) $H_1(z) = 1/(1 - 0.9z^{-1})$
- a.2) $H_2(z) = 1/A(z)$ with $A(z) = (1 - 0.8e^{-j\pi/4}z^{-1})(1 - 0.8e^{j\pi/4}z^{-1})$
- a.3) $H_3(z) = H_2^*(1/z) = 1/A^*(1/z)$ with $A(z) = (1 - 0.8e^{-j\pi/4}z^{-1})(1 - 0.8e^{j\pi/4}z^{-1})$

assume that the sequence $x[n]$ is a zero-mean white Gaussian random process with unitary power of length N (suggested values ranges from 10^2 to 10^4), and $w[n]$ is AR(1) with autocorrelation $r_{ww}[k] = \sigma_w^2(0.95)^{|k|}$. We now simulate the case where the filter to be estimated $\hat{h}[n]$ has lenght p that is not known and has to be derived. For the 3 filtering cases a.1,2,3 compute the filters' identification for parametric filter length p, and make one or more plot of MSE vs p for Signal to Noise Ratio $SNR = r_{hh}[0]/\sigma_w^2$=`[-20,-10,0,10,20,30]`dB;

Evaluate the optimum estimated filters' length and compare with the true impulse response (use stem for making the comparison).

a.4) Repeat the exercise above assuming that $H_4(z) = H_2(z) \cdot H_3(z)$.

a.5) Plot for all the exercises above the positions of the poles/zeros for all the filters $H(z)$ above and for $\hat{H}(z)$, can you justify the numerical results?

a.6) Plot the Power Spectral Density (PSD)of the random processes $h[n] * x[n]$ the the cases a.1,2,3,4 and superimpose the PSD of $w[n]$, comment.

a.7) Derive the Wiener filter that estimate $x[n]$ from $y[n]$ for all the cases a.1,2,3,4 above and plot superimposed to each plot in a.6), comment the result.

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b) Consider now a 2×2 MIMO system system

$$y_i[n] = h_{i1}[n] * x_1[n] + h_{i2}[n] * x_2[n] + w_i[n], \qquad \text{for } i = 1, 2$$

where the noise terms$w_i[n]$ have the same statistical properties of exercise a) (notice that the noises are independent and identically distributed). The input file are in file `Hw#1.mat` in this folder.

b.1) Assume that the filters are from exercise a), more specifically:$\mathcal{Z}\{h_{11}[n]\} = \mathcal{Z}\{h_{22}[n]\} = H_1(z)$ and $\mathcal{Z}\{h_{21}[n]\} = \mathcal{Z}\{h_{12}[n]\} = H_2(z)$, solve the MIMO identification problem (i.e., estimate the 4 filters' impulse responses) assuming that the estimated filters' $\hat{h}_{ij}[n]$ lenght p (same for all) is not known, and plot the MSE vs p for the same SNR as in a) where the SNR is defined by $SNR = r_{h_{11}h_{11}}[0]/\sigma_w^2$, comment.

b.2) Assume that filters are

$$h_{ij}[n] = \begin{cases} \alpha^{|i-j|}e^{-|n|/4} & |n| = 0, 1, 2, 3 \\ 0 & otherwise \end{cases}$$

estimate the filters responses using the same noise terms as in b.1), illustrate for each filter the MSE vs SNR (chooes an appropriate range) and and compare the MSE with the CRB.