TOPIC 1: MEASUREMENT

Learning Outcomes: Candidates should be able to:

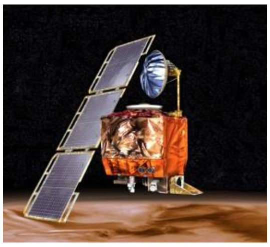
a.	recall the following base quantities and their SI units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).
b.	express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate.
c.	use SI base units to check the homogeneity of physical equations.
d.	show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *Signs, Symbols and Systematics: The ASE Companion to 16–19 Science, 2000.*
e.	use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).
f.	make reasonable estimates of physical quantities included within the syllabus.
g.	distinguish between scalar and vector quantities, and give examples of each.
h.	add and subtract coplanar vectors.
i.	represent a vector as two perpendicular components.
j.	show an understanding of the distinction between systematic errors (including zero error) and random errors.
k.	show an understanding of the distinction between precision and accuracy.
l.	assess the uncertainty in a derived quantity by addition of actual, fractional, percentage uncertainties or by numerical substitution (a rigorous statistical treatment is not required).

Three broad areas:
- Physical quantities and SI units
- Errors and uncertainties
- Scalars and vectors

Key questions:
- What are base/derived quantities and base/derived units?
- How to check the homogeneity of physical equations?
- What are prefixes?
- How to make reasonable estimates of physical quantities?
- What are systematic errors and random errors? Precision and accuracy?
- How to determine actual, fractional, percentage uncertainties?
- What are scalar and vector quantities?
- How to add/subtract vectors? (How to determine relative velocity?)
- How to represent a vector as two perpendicular components?

Why are units important in everyday life?
Nov. 10, 1999: Metric Math Mistake Muffed Mars Meteorology Mission.

A disaster investigation board reports that NASA's Mars Climate Orbiter burned up in the Martian atmosphere because engineers failed to convert units from British Imperial to metric.



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http://www.wired.com/thisdayintech/2010/11/1110mars-climate-observer-report/

1.1 Physical Quantities and SI Units

Physics can be summarised as a collection of mathematical relationships between physical phenomena.

Each scientifically measurable physical quantity has a numerical magnitude and a unit!

Side story:
There are two major systems of units used in the world: **SI units** (metric system) and **English units** (imperial system). English units were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard. The acronym "SI" is derived from the French *Système International*. (Read more about them at https://www.bipm.org/en/about-us/)

E.g.

$$F = m\,a = 1000\ N$$

Physical quantities (force, mass, acceleration) — F, m, a

numerical magnitude — 1000

unit — N

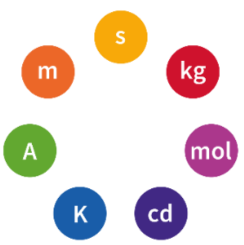
Base quantities & Units

Base quantities are physical quantities that are the most *fundamental* and they are independent of each other.

The SI Units consists of **7 base units**:

Base Quantities	SI Base Units	
	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Amount of substance	mole	mol
Temperature	kelvin	K
Current	ampere	A
*Luminous intensity	candela	cd

* Luminous intensity is not required in the syllabus.



Read more about history of units at https://pressbooks.bccampus.ca/practicalphysicsphys1104/chapter/1-2-physical-quantities-and-units/

Derived Quantities & Units

Derived quantities are obtained from one or more of the base quantities through a defining equation. A derived unit can be expressed in terms of products or quotients of base units.

Derived Quantities	Equation	Derived Unit		Derived Units in terms of base units
		Common Name	Symbol	
Force	$\Delta p / t$	Newton	N	kg m s^{-2}
Pressure	F / A	Pascal	Pa	kg m s^{-2} / m^2 = kg m^{-1} s^{-2}
Work or Energy	$F \times d$	Joule	J	
Power	E / t	Watt	W	kg m^2 s^{-2} / s = kg m^2 s^{-3}
Frequency	$1 / t$	Hertz	Hz	1 / s = s^{-1}
Charge	$I \times t$	Coulomb	C	A s
Potential Difference	E / Q	Volt	V	kg m^2 s^{-3} A^{-1}
Resistance	V / I	Ohm	Ω	kg m^2 s^{-3} A^{-2}

Pro Tip:
- To derive the SI base units of a quantity, think of an equation that helps express that quantity in terms of other base quantities.
- Use " Let [ ] denote 'unit of' " in your working.

Note: Unitless quantities

- Trigonometrical functions, e.g. sine, cosine, tangent (of an angle)
- Logarithmic functions, e.g. $\log_x$, $\ln$ (quantity)
- Powers, e.g. in $10^{a/b}$, the ratio a/b is unit-less (but a & b may have the same units)
- Certain physical constants, e.g. refractive index of materials; angle θ (= arc length / radius) is a dimensionless quantity but also given a 'practical' unit --- radian or degree

Pro Tip: For all calculations in this syllabus

All physics equations should be written in terms of SI units.
e.g. in $F = ma$, SI units for F, m, a are N, kg and ms^{-2}.
If we use units of g (gram) instead of kg, the equation would have to be modified to "$F = \underline{0.001}\, ma$" where F is in N, m in g and a in ms^{-2}!

Tutorial qn: Q1, Q3

1.1.1 Homogeneous Equations (Dimensionally Consistent)

An equation is homogeneous if <u>every term</u> on both sides of the equation have the <u>same SI base units</u>.

E.g. $v = u + at$
Let [ ] denote 'unit of'
$[v] = \text{m s}^{-1}$; $[u] = \text{m s}^{-1}$; $[at] = \text{m s}^{-2}\ \text{s} = \text{m s}^{-1}$

Note:
A <u>physically correct</u> equation must be homogeneous!

But some homogeneous equations can be physically incorrect (i.e. not a physics eqn!).

E.g.

Error	Example
1. Incorrect unit-less coefficient	$v = \mathbf{5}u + at$
2. Extra term	$v = u + at + \sqrt{as}$
3. Incorrect sign	$v = \mathbf{-}u + at$

Worked Example 1:
Determine the SI base units of acceleration.

Solution:
[*Since a correct physics equation must be homogeneous, and acceleration can be related to other quantities velocity & time*]
Let [ ] denote 'unit of'
$[a] =$

Example 2:
Express the ohm in SI base units [H1 N20 P2 Q4].

Solution: Let [ ] denote 'unit of'

$[R] =$

Example 3:

The flow of fluid can be described by the Bernoulli equation $P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$,

where P = pressure, ρ = density, v = velocity, g = gravitational acceleration, h = elevation.

Determine if the equation is a correct one.

Solution: Let [ ] denote 'unit of'
If the equation is homogeneous, ……....

Term 1:

Term 2:

Term 3:

Since all 3 terms have the ……..... SI base units, equation is ……………………..……..... (but we cannot comment on the correctness)

1.1.2 Prefixes for SI Units

Prefixes are useful for expressing units of physical quantities that are either very big or very small.

10^n	Prefix	Symbol	Name	Decimal equivalent
10^{12}	tera	T	Trillion	1 000 000 000 000
10^{9}	giga	G	Billion	1 000 000 000
10^{6}	mega	M	Million	1 000 000
10^{3}	kilo	k	Thousand	1 000
10^{0}	(none)	(none)	One	1
10^{-1}	deci	d	Tenth	0.1
10^{-2}	centi	c	Hundredth	0.01
10^{-3}	milli	m	Thousandth	0.001
10^{-6}	micro	μ	Millionth	0.000 001
10^{-9}	nano	n	Billionth	0.000 000 001
10^{-12}	pico	p	Trillionth	0.000 000 000 001

Powers of Ten™ (1977 video by IBM)
Starting at a picnic by the lakeside in Chicago, this famous film transports us to the outer edges of the universe.

Scan this QR code or go to
https://youtu.be/0fKBhvDjuy0

Example 4:
Convert the following to suitable units with prefix: (a) 600000 W; (b) 0.0000790 m

Solution:
(a)

Example 5:
Determine the equivalent value of 5.0×10^2 g cm^{-3} in kg m^{-3}.

Solution:

Tutorial qn: Q2

1.1.3 Estimation of Physical Quantities

When making an estimate, it is only reasonable to give the figure to **1 significant figure** since an estimate is not very precise.

Often, when making an estimate, we need to express a more complicated quantity in terms of other simpler quantities using a formula first.

Some examples:

Physical Quantity	Reasonable Estimate
Height of a 10 storey building	40 m
Mass of a small car	1000 kg
Speed of car on expressway	80 km h^{-1}
Current in an electric kettle	8 A
Length of a football field	100 m
Reaction time of a young person	0.2 s
*Separation between 2 atoms in a solid	10^{-10} m
*Radius of a nucleus	10^{-15} m

*****_essential for recall in later chapters_

What are significant figures?

3 rules to follow:
- All non-zero digits are significant digits
- Zeros that occur between significant digits are significant digits
- Zeros to the right of the decimal point and to the right of a non-zero digit are significant

E.g.
- 40.001 has …. significant digits
- 0.0010 has …. significant digits (the last two)
- 2.01 x 10^7 has ….. significant digits
- 100 has ….. significant digit

Pro Tip:
Give your final answers in structured qns to 2 or 3 s.f., unless it's an estimation question. (Final answers cannot be more precise than the data given in question. A-level questions usually gives data to 2 s.f.)

Common useful formulae for estimation questions:
- Volume of cylinder = $\pi r^2 h$
- Volume of sphere = $\frac{4}{3}\pi r^3$
- Curved surface area of sphere = $4\pi r^2$

Example 6:
Give a reasoned estimate of the area of the island of Singapore, giving your answer in SI unit. [2]
(N09 P2 Q1 (H1))

Solution:
Singapore is about ………. km north-south and ………. km east-west.
Area of such a rectangle is ………. km^2
By estimating that Singapore is roughly ………. ………. of the rectangle,
the area of the island of Singapore is about ………. km^2. (Actual area is 728.6 km^2)

Example 7:
Give reasoned estimate of the acceleration of a train on the Singapore rapid transit system, giving your answer in an SI unit.

Solution:
Top speed of train = ………. $km\ h^{-1}$; Time to reach top speed from rest = ………. s

Acceleration =

Tutorial qn: Q4, Q5

Note:
The phrase 'correct to an **order-of-magnitude**' means that the value quoted is reliable to within a factor of ten or so.

Example 8: [N13/P1/Q2]
What is the order-of-magnitude of the mass of a single sheet of A4 paper?

A 0.01 g B 0.3 g C 1 g D 30 g

Solution:
A4 paper usually has a mass per unit area of **……….** g m^{-2}. The area of A4 paper is **…….…………. ,**
giving it a mass of about **……….** g.
The order of magnitude is thus **……….** g.
Ans:

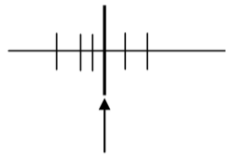
1.2 Errors and Uncertainties

An error is the <u>difference between the measured value and the 'true value'.</u>

Sources of errors could be man, machine, material, experimental technique or environment.
Errors can be classified as either **random or systematic** errors. The total error can be a combination of both systematic error and/or random error.

1.2.1 Random Error

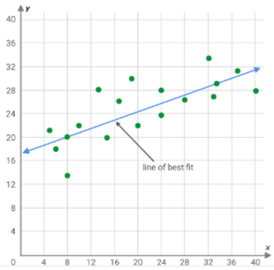
> Random error is an error which *causes measurements* to be <u>sometimes larger</u> than the true value and <u>sometimes smaller</u> than the true value.



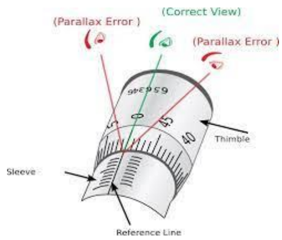
true value

Random errors are equally likely to be <u>positive or negative</u> with respect to the true value. They can have <u>different magnitudes</u>.

We can <u>reduce random errors</u> by
- repeating the measurements to obtain an average value. (Contrast: Systematic errors cannot be reduced by averaging repeated readings)
- Plotting a graph to obtain the line or curve of best fit (Note: presence of random errors is represented by the scattering of points about the best-fit line).

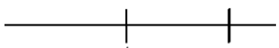


Examples of random errors:
- Random variations in external experimental conditions. E.g. random disturbances caused by mechanical vibrations, tremors produced by the wind, fluctuations in temperature etc.
- Non-constancy of experimental specimen. E.g. non-uniform diameter of wire.
- Random human error. E.g. parallax error (not placing the eye directly over the pointer, misjudgment in the interpolation of the smallest division of the scale of measuring instruments), inability of an experimenter to repeat his action precisely: e.g. human judgement in starting and stopping a stopwatch.



1.2.2 Systematic Error

> Systematic error is an error which *causes measurements* to be <u>either</u>, <u>always larger</u> than the true value, or <u>always smaller</u> than the true value.



true value

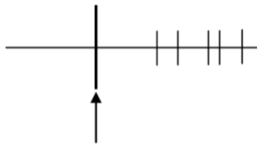
(without random errors)

Common Mistake: "always" must be stated twice. If the above is written as "always larger or smaller", it may imply that it is "always (either) larger or smaller" which basically refers to random error.

Systematic errors are of the <u>same magnitude</u> with the <u>same sign</u>.

Systematic errors are difficult to detect.

Taking the average of repeated readings does <u>not reduce</u> systematic errors. However if a faulty instrument is replaced, or the experimental technique/procedure is improved, or a different experimental approach is used, systematic errors <u>can be eliminated</u>.

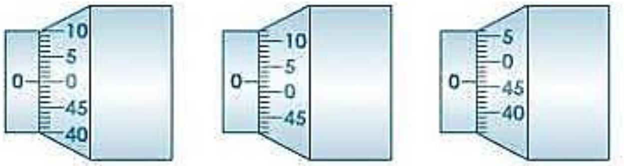


true value

(with random errors)

Examples of systematic errors:

- Zero error of an instrument. E.g. micrometer screw gauge (see diagram below).
- Incorrectly calibrated instrument e.g. a slow-running stop-watch.
- Using the wrong constant e.g. taking g to be 10 m s^{-2} when it should be 9.81 m s^{-2}.
- Incorrect experimental procedure e.g. not subtracting background count rate when determining the count rate from a radioactive source.



(a) No zero error | (b) Positive zero error | (c) Negative zero error

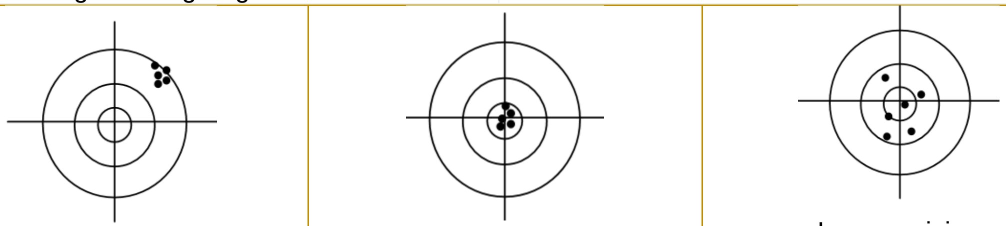
Tutorial qn: Q7

1.2.3 Precision & Accuracy

Precision refers to the degree of agreement (scatter, spread) of <u>repeated measurements</u> of the same quantity.

Accuracy refers to the degree of agreement between the <u>result of measurement(s) and the true value</u> of the quantity.

Example using shooting target board:



High precision Small random error Low accuracy Large systematic error	High precision Small random error High accuracy Small systematic error	Low precision Large random error High accuracy Small systematic error

Note:

- Precision of a set of measurements (above) is **not** the same thing as <u>precision of an instrument</u> (related to the smallest division of the instrument)(See **Notes for Phy Practical Exam**)
- If several readings are taken, we take the <u>average</u> of the readings and compare it with the true value to see if it is **accurate**.

Example 9 (N96 P1 Q3):
Four students each made a series of measurements of the acceleration of free fall g. The table shows the results obtained. Which student obtained a set of results that could be best described as *precise but not accurate*?

Student	results, g/ m s^{-2}			
A	9.81	9.79	9.84	9.83
B	9.81	10.12	9.89	8.94
C	9.45	9.21	8.99	8.76
D	8.45	8.46	8.50	8.41

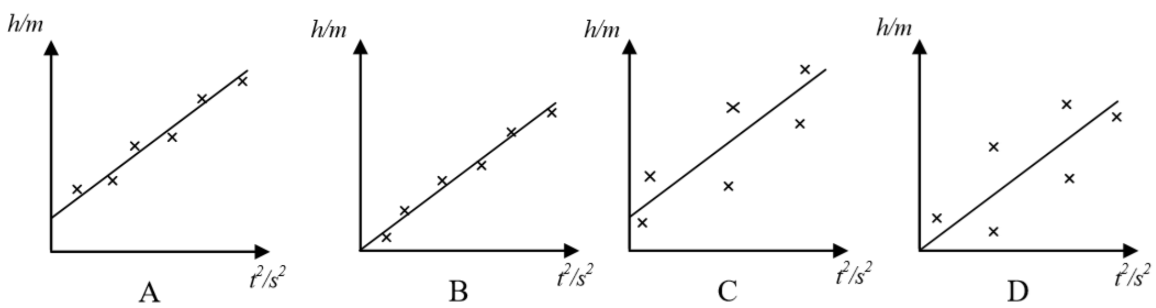
Solution:
For precise results, the spread (difference between largest and smallest values) should be ……
For inaccurate results, the average should be ………………………..

For example for Student B,
Spread = largest value – smallest value = …..
Average = …..

Ans:

Example 10:

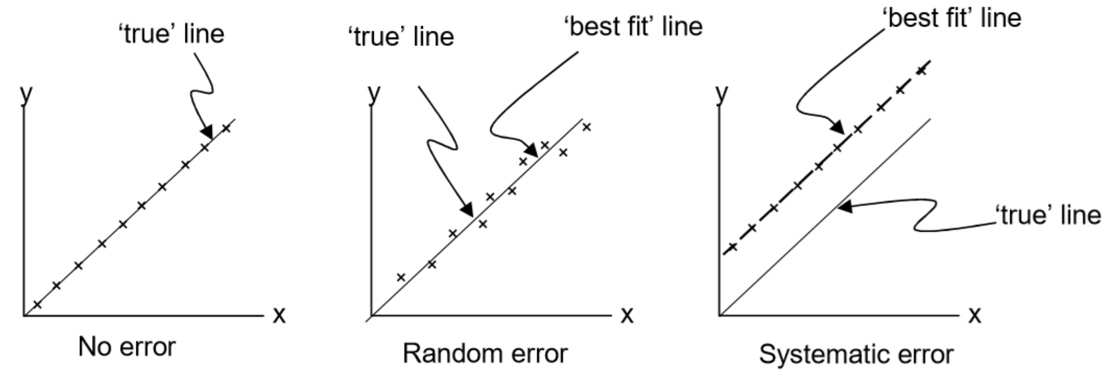
A student measures the time t for a ball to fall from rest through a vertical height, h. He repeats the measurement for various different values of h. Knowing that the equation $h = \frac{1}{2} g t^2$ applies, he plots a graph of h against t^2. Comment on the accuracy and precision of the given data in the boxes below. Assume that the gradient of the graphs below have the value of ½ g.
(Question: how do you know these graphs are accurate or precise?)



Solution:

Data A:	Data B:	Data C:	Data D:
Not accurate (graph not thru origin) but more precise than C (smaller scatter of plots)			

Note: to start off, we need to linearise the equation $h = \frac{1}{2} g t^2$.
We should see a straight line graph passing through the origin (with h in y-axis & t^2 in x-axis)



Accuracy depends on how far the best-fit graph is wrt true line (that passes through the origin); Precision depends on whether the graphs have a small scatter or plots about the best-fit line.

Tutorial qn: Q6, Q8, Q9, Q10

1.2.4 Actual (Absolute) Uncertainty in a Reading (*also for H2 Practical):

A **scale reading** is the single determination of a value of an unknown quantity at one point on a measuring scale.

A **measurement** is the final result of the analysis of a series of readings.

As all measuring instruments have an inherent uncertainty when reading off their scales, scientists reporting their results usually specify a range of values that they expect this "true value" to fall within.

E.g. The scale reading taken at the tip might be stated as (15.80 ± 0.05) cm.

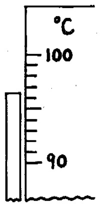
The most common way to show the range of values is: **($X \pm \Delta X$),** where ΔX is the **actual/ absolute uncertainty** or maximum uncertainty in the scale reading.

For ***X***:
- Reading/measurement must always be recorded to the <u>same degree of precision (same d.p.)</u> as ΔX

For **ΔX**:
- always be rounded off to <u>1 significant figure</u> only.
- based on the finite precision in the instrument used. ΔX is normally estimated to be <u>half of the smallest division</u> on a scale (of the instrument).

- Both should be given in standard form (where possible) with the <u>same power of ten</u>. E.g., NOT $t = (1.43 \times 10^6 \pm 2 \times 10^4)$ s, but as $t = (1.43 \pm 0.02) \times 10^6$ s instead.

E.g. 1 Thermometer scale (below).(*Also for measuring cylinder & analogue meters)



Smallest division = 1˚C;
Actual uncertainty = 0.5 ˚C;

Only one scale reading is required.
Hence, temperature = (96.5 ± 0.5)˚C
(Note: both to 1 d.p. precision)

E.g. 2 Measurement of length using 2 scale readings (below).

Smallest division = 1 mm;
Actual uncertainty (of *each* scale reading) = 0.5 mm;
Hence, Max uncertainty = 2(½ smallest div) = 1 smallest division = 1 mm.
Length = (158 ± 1) mm or (15.8 ± 0.1) cm
(Note: both to 1 d.p. precision in cm)

Note:

- In Practical, you will learn more about the precision of different measurement instruments (eg. protractor, mass balance, voltmeter etc).
- For measurements where the actual uncertainty arising from the procedure/ technique (in Practical) is insignificant (henceforth referred to as **IDEAL CONDITIONS**), students must record the data to the precision of the instrument. (Refer to Table 1 of Practical Exam Notes) (similar to Eg. 1 and 2 above)
- **CHALLENGING CONDITIONS** occur when the actual uncertainty becomes significantly larger (than the precision of the instrument) due to difficulties encountered during the measurement process arising from the procedure/technique (e.g. rapidly fluctuating temperatures using a thermometer in Eg. 1).
- In theory papers (Papers 1 to 3), the CHALLENGING CONDITIONS are generally not tested.

Example 11:
Are these measurement recordings correct?

Measurement	Is it correct?	Why?
A = (929.345 ± 0.012) Pa	No	The error should be recorded to 1 significant figure. Should be (929.35 ± 0.01).
B = (929.345 ± 0.01) Pa		
C = (929 ± 10) Pa		
D = (9.29 ± 0.01) x 10^2 Pa		

1.2.5 Fractional and Percentage Uncertainty

E.g. mass of an object measured is $(\underbrace{14.50}_{m} \pm \underbrace{0.01}_{\Delta m})$ kg.

Actual uncertainty	Fractional uncertainty	Percentage uncertainty
$\Delta m = 0.01$ kg	$\dfrac{\Delta m}{m} = \dfrac{0.01}{14.50} = 0.00069$	$\dfrac{\Delta m}{m} \times 100\% = \dfrac{0.01}{14.50} \times 100\%$ $= 0.069\%$
Always expressed in 1 s.f.	Need not be in 1 s.f. Can be up to 2.s.f.	

1.2.6 Consequential Uncertainties

The result of an experiment is seldom obtained by a single measurement; very often it is obtained by measuring a few related quantities. When a quantity Y is calculated using two measured quantities A and B (having uncertainties ΔA and ΔB respectively), the overall uncertainty ΔY is called the consequential uncertainty.

Pro Tip (for problem-solving involving all 3 Rules below):
- Make the required quantity the <u>subject of the equation</u> (if required) (Sometimes the equation is deduced, if not already given)
- Calculate the value of the unknown quantity (leave in many s.f. first)
- Apply 1 or more of the Rules 1, 2 or 3 (below)
- Calculate and express the uncertainty (e.g. ΔY) to <u>1 s.f.</u>
- Write the calculated value (Y) to the <u>same place value</u> as the uncertainty (ΔY).
- Express answer to standard form (not always compulsory but good practice)

IMPORTANT

Rule 1: For Addition & Subtraction of Quantities

If $Y = nA \pm mB$, (n & m are constants),
$\Rightarrow$ $\boldsymbol{\Delta Y = |n|\Delta A + |m|\Delta B}$

Note:
- Add all absolute uncertainties! (since finding the max possible error)
- Retain the coefficients (i.e. each measurement's uncertainty is "weighted" by their coefficient)

However, if ΔB is to be found from $Y = nA \pm mB$, we need to <u>make B the subject</u> first!!

i.e. $B = \dfrac{1}{m}(Y - nA)$

$\Rightarrow$ $\Delta B = \left|\dfrac{1}{m}\right| \Delta Y + \left|\dfrac{n}{m}\right| \Delta A$

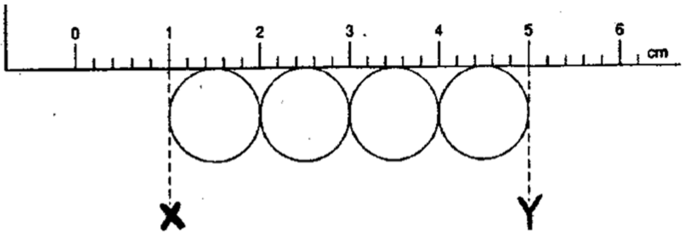
Example 12:

Quantity	Absolute uncertainty
$P = a + 2b$	$\Delta P = \Delta a + 2\Delta b$
$Q = 3c - 4\pi d$	
$R = \frac{1}{3}p - \frac{2}{3}q$	

Example 13:
A student uses a stopwatch, which has an uncertainty of ± 0.2 s, to time 20 oscillations of a pendulum bob. The time recorded t is 38.5 s. Determine the period T and its uncertainty.

Solution:

Example 14 [J94 P1 Q2 modified]:
A student attempts to measure the diameter of a steel ball by using a metre rule to measure four similar balls in a row.



What is the diameter of a steel ball together with its associated uncertainty?

Solution:

[**Special Note**: the absolute uncertainty of length for this ruler is 0.1, which is half of the smallest division of 0.2 cm!]
The diameter of the steel ball, together with its associated uncertainty, is (1.00 ± 0.05) cm.

Tutorial qn: Q11, Q12, Q13

Rule 2: for Multiplication & Division of Quantities

If $Y = aX^nZ^m$, (a, n & m are constants)

$$\Rightarrow \frac{\Delta Y}{Y} = |n|\frac{\Delta X}{X} + |m|\frac{\Delta Z}{Z}$$

Note:
- Add all fractional (or percentage) errors.
- Coefficient *a*, is ignored as it is considered error-free.
- Power *n (or m):* each measurement's fractional/percentage uncertainty is "weighted" by its power.
- See derivation in **Appendix**.

Example 15:

Quantity	Fractional uncertainty
$V = 3AB$ (where A and B are variables)	
$D = \dfrac{M}{4\pi V}$	
Resistivity, $\rho = \dfrac{\pi R d^2}{4L}$	

Worked Example 16:

Given that: $Q = \dfrac{3ab^2}{\sqrt{c}}$

The percentage errors of *a*, *b* and *c* are 1 %, 3 % and 2 % respectively. Calculate the percentage error in *Q*.

Solution:

$$\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c}$$

Hence $\dfrac{\Delta Q}{Q} \times 100 = \left(\dfrac{\Delta a}{a} \times 100\right) + 2\left(\dfrac{\Delta b}{b} \times 100\right) + \dfrac{1}{2}\left(\dfrac{\Delta c}{c} \times 100\right)$ = 1 + 2 (3) + ½ (2) = 8 %

Example 17:

The acceleration of free fall, *g* is $g = \dfrac{4\pi^2 L}{T^2}$, where *L* is the length of the pendulum, measured as (20.0 ± 0.1) cm and the value of g is found to be 9.81 ± 2%. Calculate the percentage uncertainty in the calculated value of T.

Solution:

Important Note:
It is a common error for students to state fractions and ratios in fractional form. The answer should always be in decimal form with the appropriate number of significant figures.

Worked Example 18 (N12 P1 Q2) :
The equation connecting object distance *u*, image distance *v* and focal length *f* for a lens is

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}.$$

A student measures values of *u* and *v*, with their associated uncertainties.
These are *u* = 50 mm ± 3 mm, v = 200 mm ± 5 mm. He calculates the value of *f* as 40 mm.
What is the uncertainty in this value?

A ± 2.1 mm **B** ± 3.4 mm **C** ± 4.5 mm **D** ± 6.8 mm

Solution:
Rearranging and using Rules 1 & 2,

$$f = \frac{uv}{u+v}$$

$$\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta(u+v)}{u+v}$$

$$\frac{\Delta f}{40} = \frac{3}{50} + \frac{5}{200} + \frac{3+5}{50+200}$$

$$\Delta f = 4.68 \text{ mm}$$

Answer must be WRONG as it does not match any of the options.

Rules 1 & 2 cannot be used as the terms (u + v) and uv are **mutually dependent**. (i.e. variables are repeated). Use Rule 3 instead.

Tutorial qn: Q14, 15

Rule 3: for Equations with Dependent Terms / Other Mathematical Functions (Trigo/ lg/ ln)

$$\boxed{\Delta Y = \tfrac{1}{2}\,(Y_{max} - Y_{min})}$$

Note:
Calculate the maximum and minimum of the quantity Y. Then determine the absolute error ΔY.

Worked Example 19: (cont'd from previous example)

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$f = \left(\frac{1}{u} + \frac{1}{v}\right)^{-1}$$

$$f_{max} = \left(\frac{1}{u_{max}} + \frac{1}{v_{max}}\right)^{-1}$$

$$= \left(\frac{1}{50+3} + \frac{1}{200+5}\right)^{-1} = 42.11$$

$$f_{min} = \left(\frac{1}{50-3} + \frac{1}{200-5}\right)^{-1} = 37.87$$

$$\Delta f = \frac{42.11 - 37.87}{2} = 2.1$$ Ans: A

Example 20:
Let z = sin θ where θ = (48 ± 3)°. Express z together with its associated uncertainty.

Solution:
z = sin 48° =

Using the smallest and largest possible values of θ,
z_{min} =
z_{max} =

Δz =

Thus, z ± Δz =

Tutorial qn: Q16

1.3 Scalars & Vectors

Scalars	Vectors
A **scalar** quantity has a <u>magnitude only</u>. It does not have a direction. It is completely described by a numerical value and a unit.	A **vector** quantity has both <u>magnitude and direction</u>.
Examples: distance, speed, mass, time, temperature, work done, kinetic energy, pressure, power, electric charge etc.	Examples: displacement, velocity, moment (or torque), momentum, force, electric field, gravitational field strength etc.
	It can be described by an arrow whose length represents the magnitude of the vector and the arrow-head represents the direction of the vector. In print, a vector is often denoted by a letter in bold type, e.g. force ***F***. In written form, it is $\vec{F}$.
For scalars with negative values, it means that the quantity has a value that is *less than zero.*	For vectors with *negative* values, it means that the quantity has the same magnitude but pointing in the *opposite* direction. 

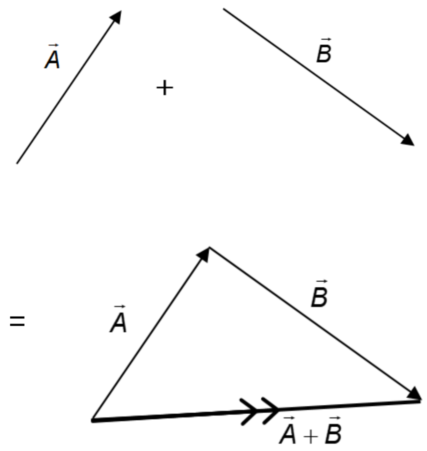
Common Mistake:
Students tend to associate work done (or energy) and pressure with vectors because of the vector quantity (force) involved. For example, Work Done = Force x Displacement in the direction of Force. This type of multiplication of 2 vectors to yield a scalar is called the scalar (dot) product of two vectors and is in your H2 Mathematics syllabus.

Tutorial qn: Q17

1.3.1 Vector Addition (& Subtraction)

Unlike scalar quantities, vector quantities cannot be added or subtracted using algebra. Instead a <u>vector diagram</u> is needed.

Triangle Law method:



Note:
Head of one to Tail of the other and the Resultant is from the tail of the first to the head of the second.

Works for addition of more than two vectors (polygon addition).

For 3 or more vectors, we can add them using polygon additon instead. This is just an extension of the triangular addition above (seldom tested).

Vector addition obeys:

- commutative law, i.e. ***A*** + ***B*** = ***B*** + ***A*** (i.e. order of addition is not important)
- associative law, i.e. (***A*** + ***B***) + ***C*** = ***A*** + (***B*** + ***C***)

Example 21 (N85 P1 Q1): (Vector Subtraction)
A particle moves eastwards with an initial velocity of 15 m s^{-1}, as shown in Fig.1. At a later time, its velocity is 15 m s^{-1} at an angle of 60° North of East (Fig. 2)

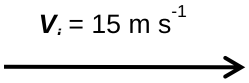


Fig.1

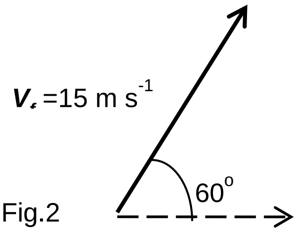


Fig.2

What is the *change in velocity* that has taken place in this interval?

Solution:
Change in velocity = Final velocity – Initial velocity Note: not initial – final !!

= –

= +

=

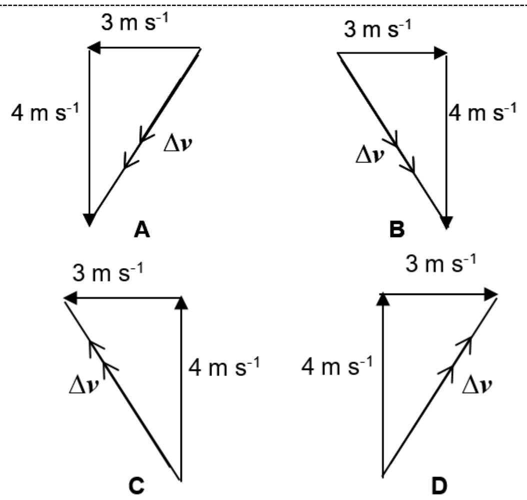
Example 22: (Vector Subtraction)

A ship travelling 3 m s^{-1} due East changes its course to 4 m s^{-1} due South.

What is the change in velocity $\Delta \boldsymbol{v}$ of the ship?

Solution:
change in velocity, $\Delta \boldsymbol{v}$ =

Ans:



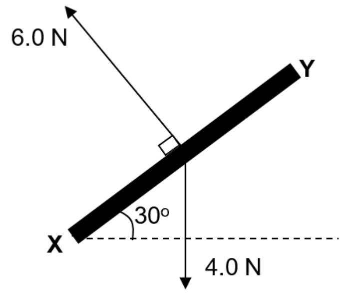
Tutorial qn: Q18, Q20, Q21

1.3.2 Calculation of Resultant Vector

Four methods for computing resultant vectors are:

(i) Accurate scale drawing (obsolete in A Level)
(ii) Pythagoras' theorem (only for 2 perpendicular vectors)
(iii) Sine and cosine rule (for non-perpendicular vectors)
(iv) Resolution and addition of component vectors (for non-perpendicular vectors)

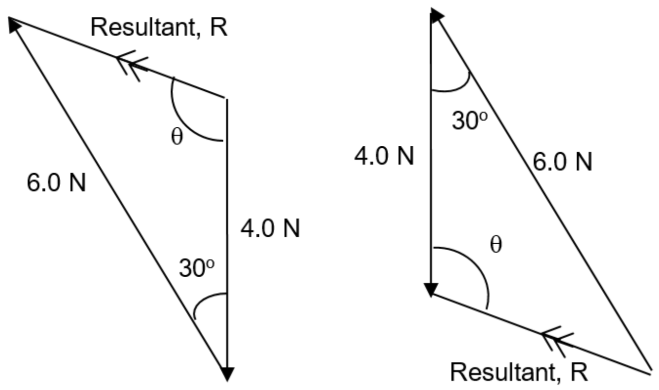
Worked Example 23: (Accurate scale drawing)
In the diagram below, XY represents a flat kite of weight 4.0 N. At a certain instant, XY is inclined at 30^o to the horizontal and the wind exerts a steady force of 6.0 N at right angles to XY so that the kite flies freely.



Draw a scale diagram to find the magnitude and direction of the resultant force acting on the kite.

Solution:

Scale: 1 cm ≡ 1.0 N



The 2 diagrams above illustrate the commutative law of addition of vectors.

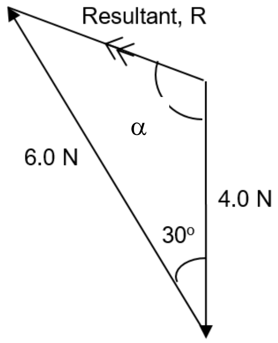
R = 3.2 N (≡ 3.2 cm) at θ = 112° to the 4 N vector.

Note: In general do not use scale drawing in problem-solving, unless specifically asked to do so by the question.

Example 24: (sine and cosine rule)
Solve the previous Example again, this time using sine/cosine rule.

Solution:
Using cosine rule,
$a^2 = b^2 + c^2 - 2bc \cos A$
$\boldsymbol{R}^2 = 4^2 + 6^2 - 2(4)(6)(\cos 30°)$
$\boldsymbol{R} = 3.2$ N

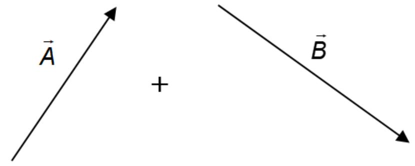
Using sine rule,
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{6}{\sin \alpha} = \frac{3.23}{\sin 30°}$$
$\alpha = 68°$ or $112°$
$= 112°$ to the 4 N vector



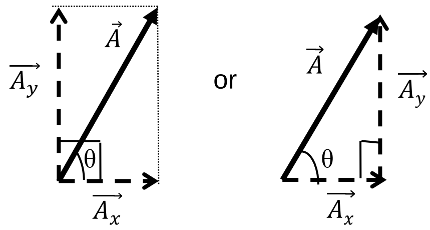
Tutorial qn: Q19(a), Q22

Method (iv) - Resolution and Addition of Component Vectors (Components Method)

Let's say we wish to sum 2 non-perpendicular vectors ***A*** and ***B.***



Step 1 (**resolution of vectors**) :
Since two vectors can be added to give a resultant vector, any vector can be broken up (or resolved) into two vectors or 'components'. E.g. vector ***A*** can be resolved into two <u>perpendicular components of $\boldsymbol{A_x}$ and $\boldsymbol{A_y}$</u> (see dotted arrows)



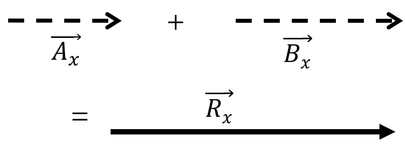
Here, $\boldsymbol{A_x} = \boldsymbol{A} \cos \theta$ and $\boldsymbol{A_y} = \boldsymbol{A} \sin \theta$

Similarly, vector ***B*** is resolved into two perpendicular components of $\boldsymbol{B_x}$ and $\boldsymbol{B_y}$.

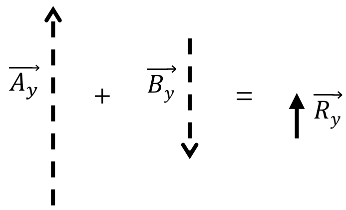
components exist at any one time.

(Note: vectors may sometimes be resolved into directions that are not along x- or y-directions. The choice of directions depends on the problem at hand. See future Example of object on a slope.)

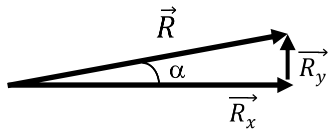
Step 2:
Sum all the x-components ($\boldsymbol{A_x}$ & $\boldsymbol{B_x}$) to obtain vector $\boldsymbol{R_x}$



Similarly, sum all the y-components ($\boldsymbol{A_y}$ & $\boldsymbol{B_y}$) to obtain vector $\boldsymbol{R_y}$.



Step 3:
$\boldsymbol{R_x}$ and $\boldsymbol{R_y}$ are then added "vectorially" using Pythagoras theorem to obtain the <u>magnitude</u> of resultant $\boldsymbol{R}$,



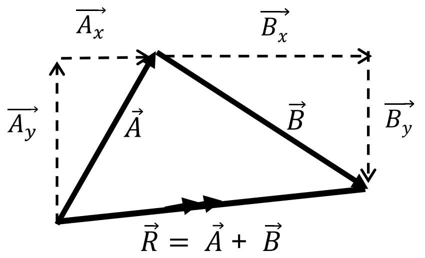
i.e. $\boxed{|\boldsymbol{R}| = \sqrt{R_x^2 + R_y^2}}$

The <u>direction</u>, angle α of $\boldsymbol{R}$ (with respect to the horizontal) is obtained using the formula,

$$\boxed{\tan\alpha = \frac{|R_y|}{|R_x|}}$$

Note: Direction of $\boldsymbol{R}$ may also be described with respect to the vertical (depending on question).

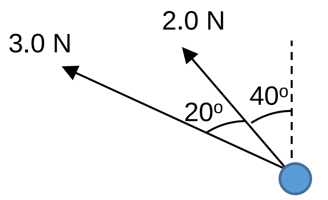
Putting all vectors into one diagram:



PhET 'Explore 2D' vector simulation:
Try adding vectors and seeing the components using this simulation!

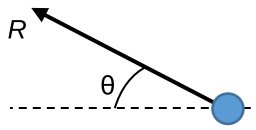
Scan this QR code or go to:
https://phet.colorado.edu/sims/html/vector-addition/latest/vector-addition_en.html

Example 25:
Determine the magnitude and direction of the resultant force acting on the body.



Solution:
R_x =

Note:
Indicate positive directions for R_x (ie. leftwards or rightwards) & R_y (upwards or downwards).

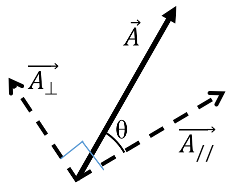


Tutorial qn: Q19(b)

1.3.3 Resolution of vectors into other directions (Slope problems)

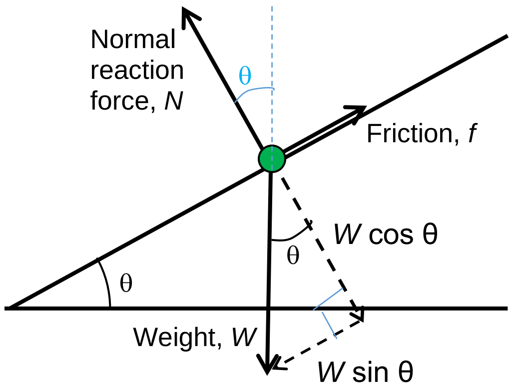
Note that a vector may be resolved into 2 perpendicular directions that may not include the horizontal and vertical directions (see right diagram):

$A_{//} = A\cos\theta$ and $A_{\perp} = A\sin\theta$



Thus for objects on a slope, the weight vector ***W*** may be resolved into 2 perpendicular components:

- one <u>along the slope</u> ($W\sin\theta$), (usually the acceleration direction)
- one <u>perpendicular to the slope</u> ($W\cos\theta$), as follows:



Pro-Tip 1:
how do we know how to draw this vector triangle with W, $W\cos\theta$, and $W\sin\theta$?

Answer:
The vector triangle must be right-angled, where W is the hypotenuse, and the 2 resolved components of W are at 90º to one another!

Pro-Tip 2:
How do we know where the angle of the slope, θ, is within the right-angled triangle drawn?

Answer:
Visualise the "slope floor" inclining at a bigger and bigger angle from the horizontal, then the Normal Reaction will also incline at the same angle θ from the vertical direction. Using 'opposite angles', the angle θ is thus found.

If the above object is *resting* in *equilibrium* on the slope (i.e. resultant forces is zero), since N is perpendicular to the slope and f is parallel to the slope, by finding the components of W parallel and perpendicular to the slope, we can find N and f :

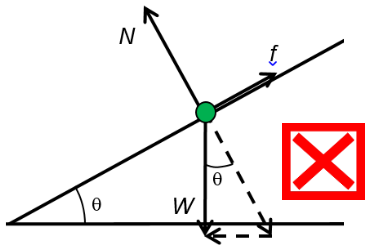
$N = W\cos\theta$ and $f = W\sin\theta$

Pro Tip 3:
Since we are "breaking" up the W force,

- it must always be the **hypotenuse** of the right-angled triangle.
- the W force (hypotenuse) must always be larger than the resolved components, ie. $W > W\sin\theta$, and $W > W\cos\theta$.

You must also check that the two components are perpendicular to each other.

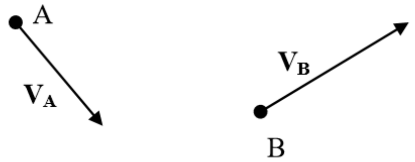
Common mistake!!!



Tutorial qn: Q23

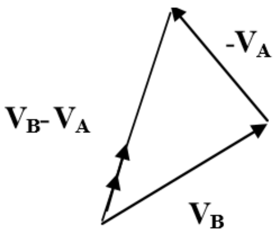
1.3.4 Relative Velocity

If A and B are 2 moving objects,

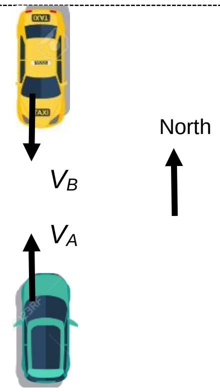


then the apparent velocity of B when observed from A is called the **velocity of B relative to A**.

i.e. $\boxed{\boldsymbol{V}_{BA} = \boldsymbol{V}_B - \boldsymbol{V}_A}$

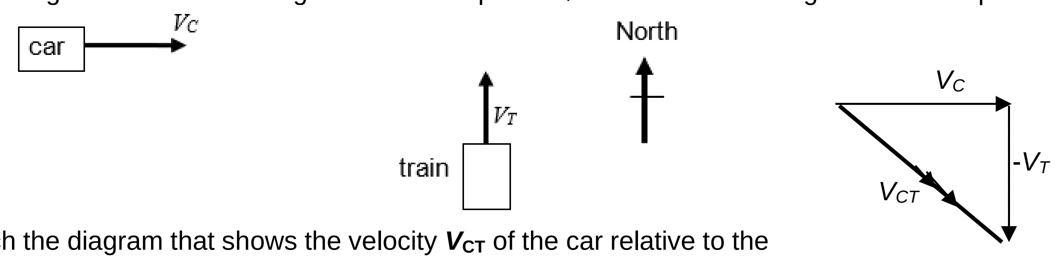


Example 26:
A car A is travelling north at a constant speed of 80 km h^{-1} and another car B is approaching car A in opposite direction at a constant speed of 100 km h^{-1}. What is B's velocity relative to A?



Solution:
Taking southward as positive direction,
Velocity of car B relative to car A
=

Example 27 (H1 N16/1/5 (modified)):
A passenger in a train travelling due north at speed V_T sees a car travelling due east at speed V_C.



Sketch the diagram that shows the velocity $\boldsymbol{V}_{CT}$ of the car relative to the passenger in the train.

Solution:

Hence we are looking for $\boldsymbol{V}_{CT}$, relative velocity of car with respect to train.
Using vector subtraction,
$\boldsymbol{V}_{CT}$ =

Tutorial qn: Q22

SUMMARY

A physical quantity has a numerical magnitude and a unit.

Base units: m, kg, s, mol, K, A, cd.
Derived unit: a unit expressed as a product and/or quotient of the (7) base units.

Problem solving method: To find the unit of an unknown quantity, state the defining equation (if not given), make the unknown quantity the subject, then derive the unit of the unknown quantity in terms of the base units of the known quantities.

Homogeneous Equation: every term on both sides of the equation have the same resultant units.
A physically correct equation must be homogeneous!

Prefixes for units: tera (10^{12}), giga (10^{9}), mega (10^{6}), kilo (10^{3}), deci (10^{-1}), centi (10^{-2}), milli (10^{-3}), micro (10^{-6}), nano (10^{-9}), pico (10^{-12}).

Errors and Uncertainties

Random error	Systematic error
An error which causes measurements to be <u>sometimes larger</u> than the <u>true value</u> <u>and</u> <u>sometimes smaller</u> than the true value	An error which causes measurements to be <u>either</u>, <u>always</u> larger than the <u>true value</u>, <u>or</u> <u>always</u> smaller than the true value.
Examples: Random variations in external experimental conditions, Non-constancy of experimental specimen, Random human error.	Examples: Zero error of an instrument. E.g. micrometer screw gauge, incorrectly calibrated instrument.
Can be <u>reduced.</u> • E.g. by taking the average of repeated readings; • Plotting a graph to obtain the line or curve of best fit (Note: presence of random errors is represented by the scattering of points about the best-fit line).	Averaging repeated measurements cannot reduce the errors, but can be <u>eliminated</u> • E.g. by replacing faulty instrument; • Improving the experimental technique/procedure, or • Using a different experimental approach

Precision	Accuracy
Refers to the degree of agreement (scatter, spread) of repeated measurements of the same quantity.	Refers to the degree of agreement between the result of a measurement and the true value of the quantity.
A measure of the magnitude of the random errors present; high precision means small random error	A measure of the magnitude of the systematic error present; high accuracy means small systematic error
Precision of a set of measurements (above) is <u>not</u> the same thing as <u>precision of an instrument</u> (related to the smallest division of the instrument)!	If several readings are taken, we take the average of the readings and compare it with the true value to see if it is accurate.
Precision depends on whether the graphs have a small scatter or plots about the best-fit line.	Accuracy depends on how far the best-fit graph is wrt true line (that passes through the origin)

Actual uncertainty	Fractional uncertainty	Percentage uncertainty
Δm	$\dfrac{\Delta m}{m}$	$\dfrac{\Delta m}{m}$ x 100%

Always expressed in 1 s.f.	Need not be in 1 s.f. Can be up to 2.s.f.	
Normally estimated to be <u>*half* of the smallest division</u> on a scale (of the instrument).		

The reading/measurement must always be recorded to the same degree of precision (same d.p.) as the actual uncertainty (which is one s.f.)

Consequential uncertainties:

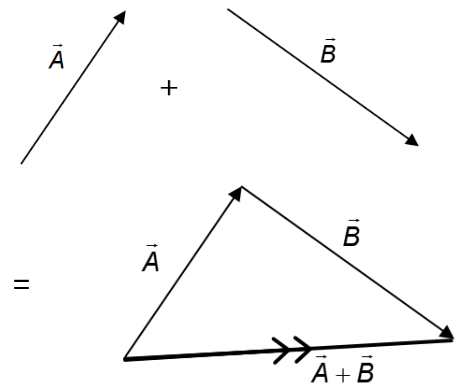
- Make the required quantity the <u>subject</u> of the equation (if required) (Sometimes the equation is deduced, if not already given)
- Calculate the value of the unknown quantity (leave in many d.p. first)
- Apply 1 or more of the Rules 1, 2 or 3 (below)
- Calculate and express the uncertainty (e.g. ΔY) to <u>1 s.f.</u>
- Write the calculated value (Y) to the <u>same place value</u> as the uncertainty (ΔY).
- Express answer to standard form (not always compulsory but good practice)

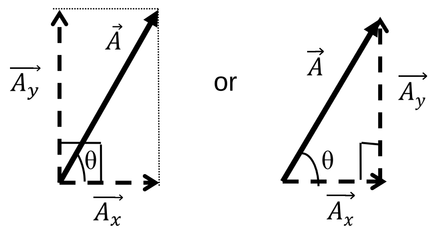
Addition and Subtraction (Rule 1)	**Product and Division (Rule 2)**	**Dependent terms / other Mathematical Functions (Trigo/ lg/ ln) (Rule 3)**
If $Y = nA \pm mB$, then $\Delta Y = \lvert n\rvert\Delta A + \lvert m\rvert\Delta B$	If $Y = aX^n Z^m$, then $\frac{\Delta Y}{Y} = \lvert n\rvert \frac{\Delta X}{X} + \lvert m\rvert \frac{\Delta Z}{Z}$	□$\Delta Y = \frac{1}{2}\,(Y_{max} - Y_{min})$
Constants are included in computation of error.	Constants are not included in computation of error.	

Scalars & Vectors:

Scalar	**Vector**
A **scalar** quantity has a <u>magnitude only</u>. It does not have a direction. It is completely described by a numerical value and a unit.	A **vector** quantity has both <u>magnitude and direction</u>.
Examples: distance, speed, mass, time, temperature, work done, kinetic energy, pressure, power, electric charge etc.	Examples: displacement, velocity, moment (or torque), momentum, force, electric field, gravitational field strength etc.
	It can be described by an arrow whose length represents the magnitude of the vector and the arrow-head represents the direction of the vector. In print, a vector is often denoted by a letter in bold type, e.g. force ***F***. In written form, it is $\vec{\boldsymbol{F}}$.
For scalars with negative values, it means that the quantity has a value that is less than zero.	For vectors with negative values, it means that the quantity has the same magnitude but pointing in the opposite direction. 

Vector Addition (& Subtraction)

Triangle Law method:	
	Note: Head of one to Tail of the other and the Resultant is from the tail of the first to the head of the second. Vector addition obeys: • commutative law, i.e. ***A*** + ***B*** = ***B*** + ***A*** (i.e. order of addition is not important) • associative law, i.e. (***A*** + ***B***) + ***C*** = ***A*** + (***B*** + ***C***)

Resolution of vector	
A vector ***A*** can be resolved into two <u>perpendicular components of $\boldsymbol{A_x}$ and $\boldsymbol{A_y}$</u> (see dotted arrows) Here, $\boldsymbol{A_x}$ = ***A*** cos θ and $\boldsymbol{A_y}$ = ***A*** sin θ *(Note: vectors may sometimes be resolved into directions that are not along x- or y-directions. The choice of directions depends on the problem at hand. See future Example of object on a slope.)*	 components exist at any one time.

Calculation of Resultant Vector:

Method (iv) - Resolution and addition of component vectors

Step 1 (**resolution of vectors**) :
Resolve vectors into two perpendicular components

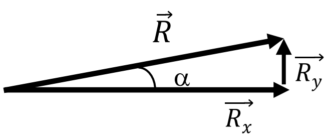
Step 2:
Sum all the x-components ($\boldsymbol{A_x}$ & $\boldsymbol{B_x}$) to obtain vector $\boldsymbol{R_x}$; and sum all the y-components ($\boldsymbol{A_y}$ & $\boldsymbol{B_y}$) to obtain vector $\boldsymbol{R_y}$.

Step 3:
$\boldsymbol{R_x}$ and $\boldsymbol{R_y}$ are then added "vectorially" using Pythagoras theorem to obtain the magnitude of resultant ***R***,

i.e. $\boxed{|\boldsymbol{R}| = \sqrt{R_x^2 + R_y^2}}$

The direction, angle α of ***R*** (with respect to the horizontal) is obtained using the formula,

$$\boxed{\tan\alpha = \frac{|R_y|}{|R_x|}}$$



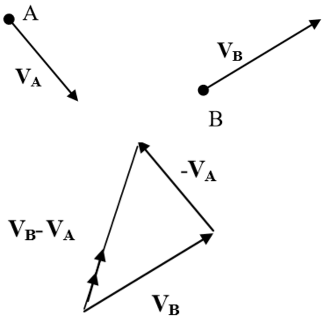
Note: Direction of ***R*** may also be described with respect to the vertical (depending on question).

Relative Velocity

If A and B are 2 moving objects,

then the apparent velocity of B when observed from A is called the **velocity of B relative to A**.

i.e. $\boxed{\boldsymbol{V}_{BA} = \boldsymbol{V}_B - \boldsymbol{V}_A}$



APPENDIX

Number of significant figures for calculated quantities in general calculations

Expression	Rule	Example
multiplication, division and trigonometric functions	number of **significant figures** in the answer should equal the least number of significant figures in any of the quantities involved	I = 1.7 A, P = 2.005 W, $V = P / I$ = 1.2 V (to 2 sig. fig.)
addition and subtraction	number of **decimal places** in the answer should equal the least number of decimal places in any of the quantities involved	17.2911 J + 2.03 J = 19.32 J (to 2 d.p.)

Note:
Leaving the answer with 1 more significant figure (eg. V = 1.18 V) is tolerable for A-Level Exam. However leaving the answer with 1 less significant figure (eg. V = 1 V) or too many significant figures (eg. V = 1.17941 V) is not acceptable!

Derivation of fractional error for product and quotient: (Not in syllabus)

Given an equation:

$$Y = aX^nZ^m \quad \text{...(1)}$$

Where X, Y and Z are variables with a given amount of uncertainty,
And a, n and m are numerical constants.
Differentiating (1) by partial fraction yields,

$$dY = (m)aX^nZ^{m-1}dZ + (n)aZ^mX^{n-1}dX \quad \text{........(2)}$$

$$\frac{(2)}{(1)} \Rightarrow \qquad \frac{dY}{Y} = \frac{(m)aX^nZ^{m-1}}{aX^nZ^m}dZ + \frac{(n)aX^{n-1}Z^m}{aX^nZ^m}dX$$

$$\frac{dY}{Y} = n\frac{dX}{X} + m\frac{dZ}{Z} \qquad \text{(Shown)}$$

For a quotient like, $Y = a\dfrac{X^n}{Z^m}$, you get a negative ,- $m\dfrac{dZ}{Z}$, for the fractional error.

We need to understand that in the computation of the fractional error, we need to consider the "worst case" scenario to find the highest possible error, so we need to <u>**add**</u> up all the fractional errors.

Thus for, $Y = a\dfrac{X^n}{Z^m}$

The fractional error, $\dfrac{dY}{Y} = n\dfrac{dX}{X} + m\dfrac{dZ}{Z}$

Therefore a more generic equation for the fraction error for product or quotient is

$$\frac{dY}{Y} = |n|\frac{dX}{X} + |m|\frac{dZ}{Z}$$

SUMMARY OF KEY QUANTITIES, SYMBOLS & UNITS

The following list illustrates the symbols and units that will be used in question papers.

Quantity	Usual symbols	Usual units	Quantity	Usual symbols	Usual units
Base Quantities					
mass	m	kg	electric current	I	A
length	l	m	thermodynamic temperature	T	K
Time	t	s	amount of substance	n	mol
Other Quantities					
distance	d	m	elementary charge	e	C
displacement	s, x	m	electric potential	V	V
area	A	m^2	electric potential difference	V	V
volume	V, v	m^3	electromotive force	E	V
density	ρ	kg m^{-3}	resistance	R	Ω
speed	u, v, w, c	m s^{-1}	resistivity	ρ	Ω m
velocity	u, v, w, c	m s^{-1}	electric field strength	E	N C^{-1}, V m^{-1}
acceleration	a	m s^{-2}	permittivity of free space	ε_o	F m^{-1}
acceleration of free fall	g	m s^{-2}	magnetic flux	Φ	Wb
force	F	N	magnetic flux density	B	T
weight	W	N	permeability of free space	μ_o	H m^{-1}
momentum	p	N s	force constant	k	N m^{-1}
work	w, W	J	Celsius temperature	θ	°C
energy	E,U,W	J	specific heat capacity	c	J K^{-1} kg^{-1}
potential energy	E_p	J	molar gas constant	R	J K^{-1} mol^{-1}
kinetic energy	E_k	J	Boltzmann constant	k	J K^{-1}
heating	Q	J	Avogadro constant	N_A	mol^{-1}
change of internal energy	ΔU	J	number	N, n, m	
power	P	W	number density (number per unit volume)	n	m^{-3}
pressure	p	Pa	Planck constant	h	J s
torque	T	N m	work function energy	Φ	J
gravitational constant	G	N kg^{-2} m^2	activity of radioactive source	A	Bq
gravitational field strength	g	N kg^{-1}	decay constant	λ	s^{-1}
gravitational potential	φ	J kg^{-1}	half-life	$t_{1/2}$	s
angle	θ	°, rad	relative atomic mass	A_r	
angular displacement	θ	°, rad	relative molecular mass	M_r	
angular speed	ω	rad s^{-1}	atomic mass	m_a	kg, u
angular velocity	ω	rad s^{-1}	electron mass	m_e	kg, u
period	T	s	neutron mass	m_n	kg, u
frequency	f	Hz	proton mass	m_p	kg, u
angular frequency	ω	rad s^{-1}	molar mass	M	kg
wavelength	λ	m	proton number	Z	
speed of electromagnetic waves	c	m s^{-1}	nucleon number	A	
electric charge	Q	C	neutron number	N	

TUTORIAL 1: MEASUREMENT

Quantities and Units

(L1)1. (N17/P1/Q1)
Which list of SI units contains only **base** units? [1]

A kelvin, metre, mole, ampere, kilogram
B kilogram, metre, second, ohm, mole
C kilogram, newton, metre, ampere, mole
D newton, kelvin, second, volt, mole

(L1)2. (N01/P2/Q3 modified)
The unit of a physical quantity may be shown with a prefix. For example, the prefix *micro* (μ) has the decimal equivalent 10^{-6}, so that 1 microampere (1 μA) can be written as 10^{-6} A. Complete the table below to show each prefix with its corresponding decimal equivalent. [5]

prefix	decimal equivalent
pico	……………….
micro	10^{-6}
centi	………………..
deci	………………..
giga	………………..
…………	10^{12}

(L2)3. (J03/P1/Q2)
The unit of work, the joule, may be defined as the work done when the point of application of a force of 1 newton is moved a distance of 1 metre in the direction of the force.

Express the joule in terms of the base units of mass, length and time, the kg, m and s. [1]

A kg m^{-1} s^2 **B** kg m^2 s^{-2} **C** kg m^2 s^{-1} **D** kg s^{-2}

Estimation

(L2)4. Make a reasoned estimate (to 1 sf) for the following quantities, with an appropriate SI unit:
(a) Mass of water in an Olympic-size swimming pool [2]
(Hint: Use Density = Mass/Volume)
(b) Volume of a standard basketball, and hence the average density of a basketball. [2]
(c) The acceleration of a train on the Singapore rapid transit system. (H1 N09/2/1) [2]
(Acceleration = change in Velocity/change in Time)
(d) The power of a car travelling along an expressway. (H1 N09/2/1) [2]
(Power = Force x Velocity)

(L2)5. (N21/P1/Q1 modified)
What is the best estimate of the weight of a graphic calculator? [1]

A 1.5 cN **B** 1.5 dN **C** 150 cN **D** 150 dN

Errors and Uncertainties

(L2)6. (N12/P1/Q1)
A student uses an analogue voltmeter to measure the potential difference across a lamp. The voltmeter is marked 0.02 V but has a zero error of 0.08 V. The student is not aware of this zero error and writes down a reading of 2.16 V. Is the reading accurate and is it precise? [1]

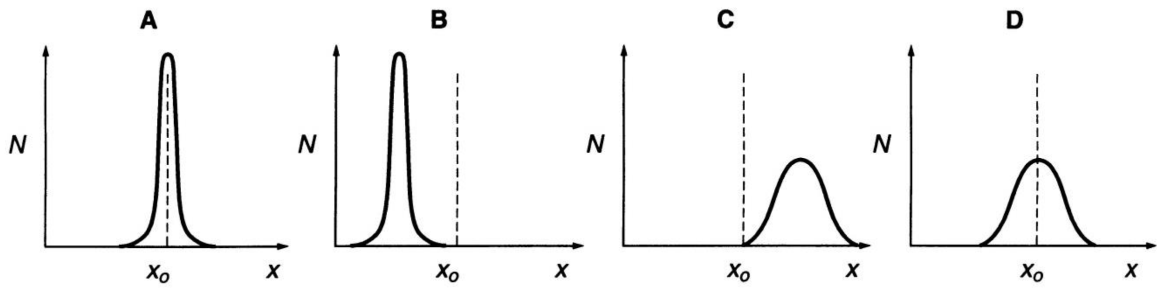
	accurate	precise
A	no	no
B	no	yes
C	yes	no
D	yes	yes

(L2)7. (J92/P1/Q1)
Which of the following experimental techniques reduces the systematic error of the quantity being investigated? [1]

A timing a large number of oscillations to find a period
B measuring the diameter of a wire repeatedly and calculating the average
C adjusting an ammeter to remove its zero error before measuring a current
D plotting a series of voltage and current readings for an ohmic device on a graph and using its gradient to find resistance

(L2)8. (N97/P1/Q2)
A quantity x is measured many times and the number N of measurements giving a value x is plotted against x. The true value of the quantity is x_o.

Which graph best represents precise measurements with poor accuracy?



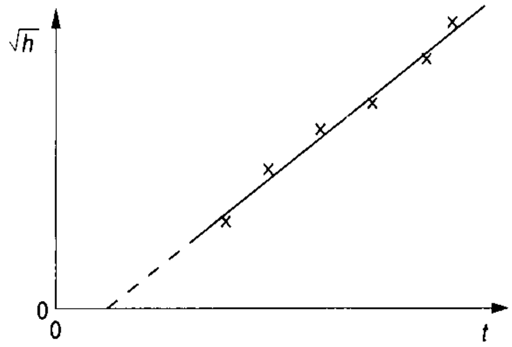
[1]

(L2)9. (N02/P1/Q2)
An object of mass 1.000 kg is placed on four different balances. For each balance the reading is taken five times. The table shows the values obtained together with the means. Which balance has the smallest systematic error but is not very precise? [1]

balance	reading/kg					mean/kg
	1	2	3	4	5	
A	1.000	1.000	1.002	1.001	1.002	1.001
B	1.011	0.999	1.001	0.989	0.995	0.999
C	1.012	1.013	1.012	1.014	1.014	1.013
D	0.993	0.987	1.002	1.000	0.983	0.993

Extension Question
Label all the options with accurate/precise, accurate/ not precise, not accurate/precise, neither

(L2)10. (N99/P3/Q1)
A student measures the time t for a ball to fall from rest through a vertical distance h. Knowing that the equation $h = ½at^2$ applies, the student plots the graph shown.



Which of the following is a possible explanation for the intercept? [1]

A Air resistance has not been taken into account for larger values of h.
B There is a constant delay between starting the timer and releasing the ball.
C There is an error in the timer that consistently makes it run fast.
D The student should have plotted h against t^2.

Error Computation

(L1)11. A rectangular plot of land has length (56.8 ± 0.1) m and width (32.3 ± 0.1) m. Calculate its perimeter and its uncertainty. [2]

(L2)12. (N81/P2/Q5)
In an experiment, the external diameter d_1 and internal diameter d_2 of a tube are found to be (64 ± 2) mm and (47 ± 1) mm respectively. The percentage error in $(d_1 – d_2)$ expected from these readings is at most [1]

A 0.3% **B** 1% **C** 5% **D** 6% **E** 18%

(L2)13. (J03/P1/Q5)

A student makes measurements from which she calculates the speed of sound as 327.66 m s^{-1}. She estimates that her result is accurate to ±3%.

Which of the following gives her result expressed to the appropriate number of significant figures? [1]

A 327.7 m s^{-1} **B** 328 m s^{-1} **C** 330 m s^{-1} **D** 300 m s^{-1}

(L2)14. (N17/P1/Q2)

After taking measurements of the quantities in the expression $\frac{xy^2}{z}$, the total uncertainty is calculated as 6%.

Which individual percentage uncertainties of *x*, *y* and *z* when combined give this total of 6%?

	X / %	*Y* / %	*Z* / %
A	1	1	4
B	2	1	2
C	3	2	2
D	4	1	1

[1]

(L2)15. Given the following measurements:

X = (2.48 ± 0.02) m, Y = (3.05 ± 0.01) m, Z = (1.59 ± 0.05) m

Express in terms of $P \pm \Delta P$, where $P = \dfrac{3X + 2Y}{Z}$. [3]

(L2) 16. Use Rule 3 for error computation to determine the error in s from s = ut + ½ at^2, given that u = 4.0 ± 0.1 m s^{-1}, t = 3.32 ± 0.01 s and a = 5.8 ± 0.2 m s^{-2}. Express your answer in s ± Δs. [3]

Extension Question

Use Rule 1 and 2 to compute error in s and compare the answers

Vectors and Scalars

(L1)17. (J00/P1/Q1)

Which pair includes a **vector** quantity and a **scalar** quantity? [1]

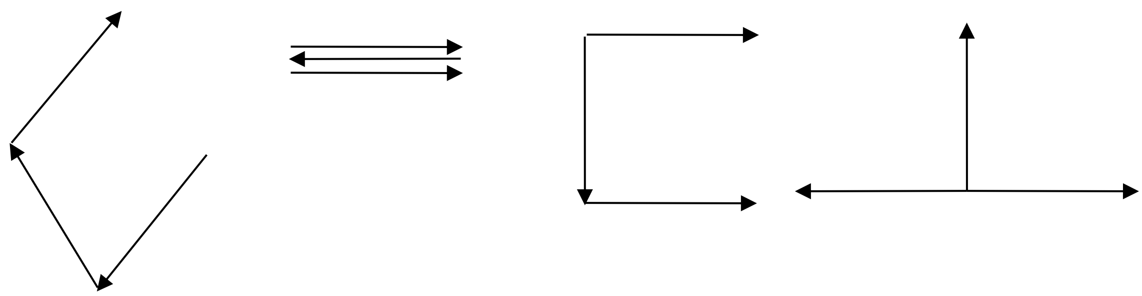
A displacement; acceleration
B force; kinetic energy
C power; speed
D work; potential energy

(L1)18. (N05/P1/Q1)

Each diagram shows three vectors of equal magnitude.

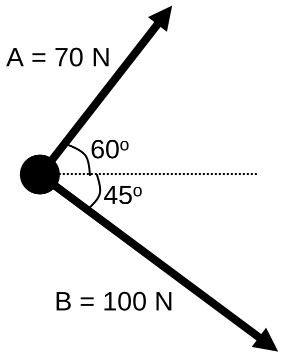
In which diagram is the magnitude of the resultant vector different from the other three? [1]

A **B** **C** **D**



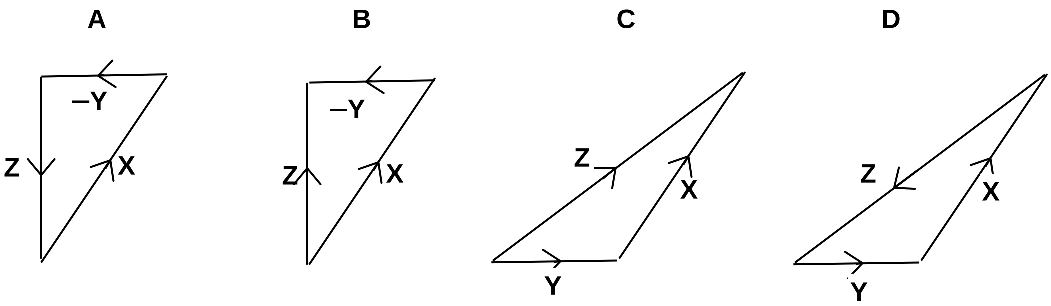
(L2)19. Determine the magnitude and direction of the resultant of the two forces shown in the figure below using

(a) sine and cosine rule method; [2]
(b) components method. [2]



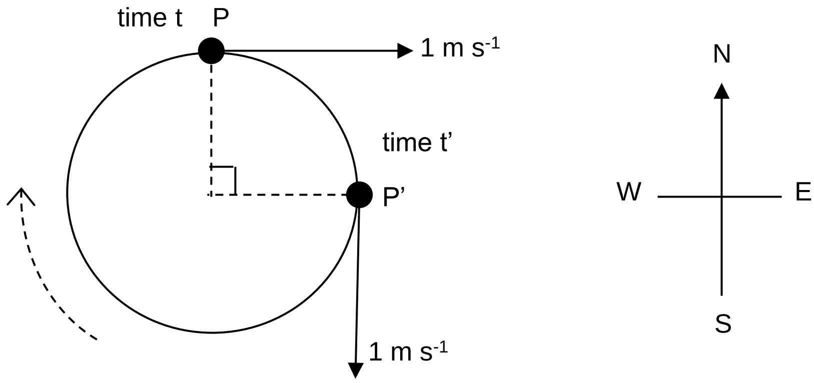
(L2)20. (J02/P1/Q2)
The diagram shows two vectors **X** and **Y**.

In which vector triangle does the vector **Z** show the magnitude and direction of vector **X** – **Y**?



[1]

(L2)21. A body rotates steadily clockwise at a constant speed of 1 m s^{-1}. At time t the body is at point P as shown. At time t', the body has rotated through 90° and it is now at point P'.



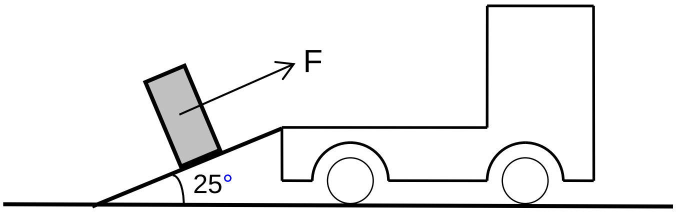
What is the change in the velocity of the body between t and t'?

A	Zero	
B	1.4 m s^{-1}	south west
C	1.4 m s^{-1}	north east
D	2.0 m s^{-1}	south east

[1]

(L2)22. (N16/P1/Q3**)**
An aircraft flies with an airspeed of 700 km h^{-1} through a 250 km h^{-1} jet-stream wind from the west. The pilot wishes to fly directly north from Australia towards Changi airport in Singapore. To achieve this, the pilot points the air craft away from the north direction. Determine the speed of the aircraft in the direction of the north relative to the ground. [2]

(L2)23. With the aid of an inclined plane, a 200 kg box is pushed up a flat-bed lorry. Assuming that the frictional force is negligible, find the minimum force needed to push the box up the inclined plane.



If a longer inclined plane is used, state whether more, equal or less force is needed to push the box up the incline. Give your reason. [2]

Numerical Answers

3	B	**5**	C	**9**	B
11	(178.2 ± 0.4) m				
12	E	**13**	C	**14**	B
15	8.5 ± 0.3				
16	(45 ± 2) m				
19	106 N, 5.5° below horizontal				
21	B				
22	654 km h^{-1}				
23	829 N				

ASSIGNMENT

1 [H1 N07 P1 Q2 modified]
When a beam of light is incident on a surface, it delivers energy to the surface. The intensity of the beam is defined as the energy delivered per unit area per unit time. What is the unit of intensity, expressed as SI base units? [2]

2 [2017/CT/Q21]

(a) In an experiment to measure the viscosity η of a liquid, the following equation was used.

$$\eta = \frac{kr^2}{v}$$

where:

r = (0.83 ± 0.01) mm,

v = (0.065 ± 0.002) m s^{-1}, and

k is a constant of value 93.7 N m^{-3}.

Using the equation above,

(i) determine the base unit of η.

base unit for η = ……………………… [2]

(ii) calculate the magnitude of η.

η = ……………………… [1]

(iii) determine the magnitude of the associated uncertainty for the viscosity of water $\Delta\eta$, and hence express the magnitude of the viscosity of water with its associated uncertainty.

η = ……………… ± …………… [3]

(b) A student measures the time taken t for a small metal ball to fall a vertical distance h from rest. The measurements are as shown below:

$$h = 266 \pm 1 \text{ cm}$$
$$t = 0.740 \pm 0.005 \text{ s}$$

Suggest two reasons why, in this experiment, although the value of t is precise, it may not be accurate.

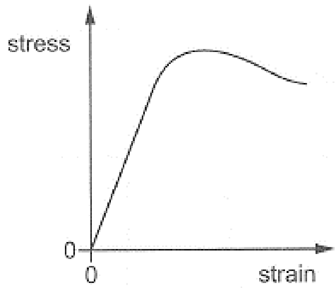
………………………………………………………………………………………………………

………………………………………………………………………………………………………

……………………………………………………………………………………………………… [2]

ADDITIONAL QUESTIONS

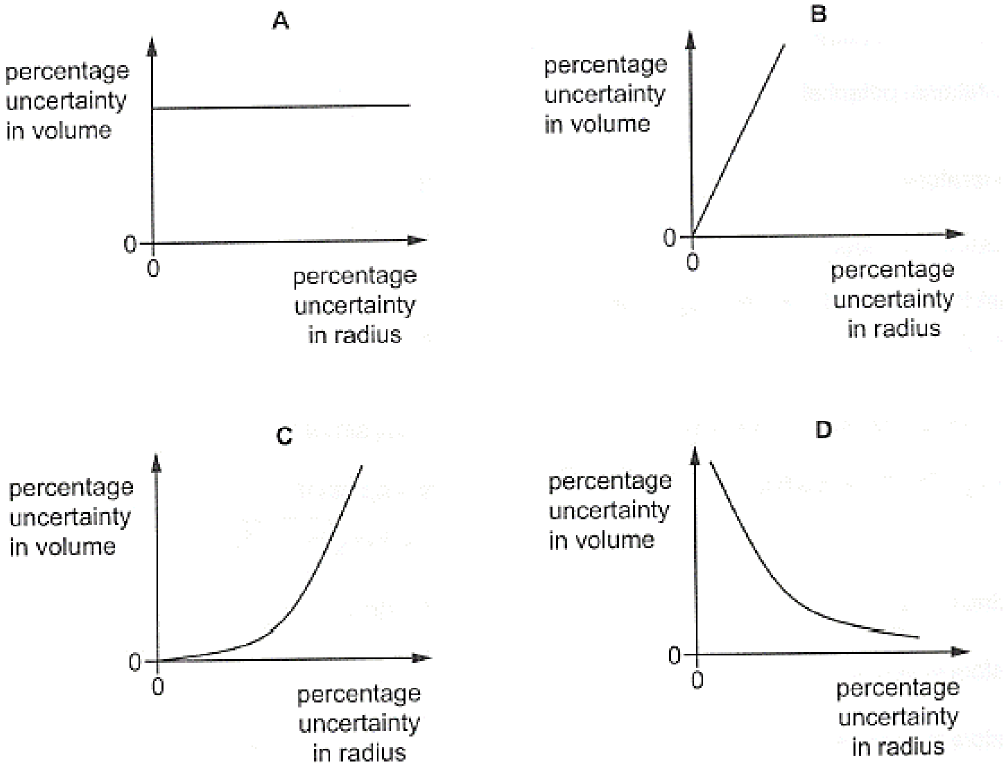
1 [N2020 P1 Q3]
When a metal wire is stretched, the elastic potential energy stored in the wire increases. A quantity called stress can be calculated by dividing the stretching force by the cross-sectional area of the wire. A quantity called strain can be calculated by dividing the extension of the wire by the original length of the wire. A graph of stress-strain for a metal wire is shown.



What is the unit of the quantity that represents the area under the stress-strain graph?

A J **B** J m^{-1} **C** J m^{-2} **D** J m^{-3}

2 [N2020 P1 Q1]
A spherical balloon is gradually inflated. The radius of the balloon Is measured at regular intervals and the uncertainty of the measurement is estimated. The radius of the balloon is used to calculate the volume of the balloon at each stage. Which graph shows how the percentage uncertainty in the volume of the balloon varies with the percentage uncertainty in the radius of the balloon?



*3 (H1 2010/P1/Q3)
A student times the oscillations of a simple pendulum but accidentally starts by counting "1", instead of "0", when the bob is released and finishes at a count of "10".

What is the percentage error in the student's calculation of the period due to this mistake?

A 10% high **B** 10% low **C** 11% high **D** 11% low

4 (a) The drag force F experienced by a steel sphere of radius r dropping at speed v through a liquid is given by

$$F = arv$$

where a is a constant.
Express the unit for a in terms of SI base units.

unit for a = ……………………………… [2]

(b) A cylindrical thermos flask is used to store hot water. The internal diameter and depth of the thermos flask are measured to be (8.50 ± 0.01) cm and (17.0 ± 0.1) cm respectively.

(i) State the instrument used to measure its diameter and a systematic error that can occur with the use of this instrument.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………[2]

(ii) Calculate the volume of the thermos flask and its associated uncertainty.

volume = ……………………………… cm^3 [3]

(c) A lorry changes its velocity from 50 m s^{-1} due East to 30 m s^{-1} due South.

(i) Draw a vector diagram to show the initial and final velocities and the change in velocity. [2]

(ii) Calculate the magnitude in the change in velocity.

change in velocity = …………………..……….m s^{-1} [1]

TUTORIAL 1: MEASUREMENT SOLUTIONS

Quantities and Units

(L1)1	**A** (There are only 7 base units: only kg, m, s, A, K, mol. and cd)	[1]
(L1)2	<table><tr><th>prefix</th><th>decimal equivalent</th></tr><tr><td>pico</td><td><u>10^{-12}</u></td></tr><tr><td>micro</td><td>10^{-6}</td></tr><tr><td>centi</td><td><u>10^{-2}</u></td></tr><tr><td>deci</td><td><u>10^{-1}</u></td></tr><tr><td>giga</td><td><u>10^{9}</u></td></tr><tr><td><u>tera</u></td><td>10^{12}</td></tr></table>	[5]

Error Computation

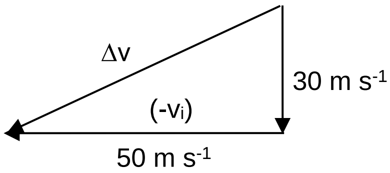
(L1)11	Perimeter, P = 2 x lengths + 2 x width = 178.2 m. Absolute error, ΔP = 2 x Δlength + 2 x Δwidth = 0.4 m. Hence, answer = (178.2 ± 0.4) m	 [1] [1]

Vectors and Scalars

(L1)17	**B** force is a vector, kinetic energy is scalar	[1]
(L1)18	**C** Options A, B and D has a resultant of 1 unit (arrow) each. The resultant for option C is $\sqrt{2^2 + 1^2} = \sqrt{5}$ units	[1]

Solutions to Additional Questions

1	**D** Stress has units of N m^{-2} (since it is force divided cross-sectional area). Strain is dimensionless (since it is extension divided by original length of wire). The area under the graph can be gotten by multiplying the units of both axes: which is just N m^{-2}. Multiplying both the numerator and denominator by metre turns it into (N m) / (m^2 m) = J m^{-3}.	[1]
2	**B** Volume of sphere = $\frac{4}{3}\pi r^3$ Percentage uncertainty in volume, $\frac{\Delta V}{V} \times 100\% = 3\frac{\Delta r}{r} \times 100\%$ A graph of percentage uncertainty in volume versus percentage uncertainty in radius is of the form Y = mX + C, and gradient = 3.	[1]
*3	**B** There are 10 oscillations counted by the student, whereas there are 9 oscillations in reality. Thus student's calculated period = 0.9 T, where T is the actual period. Percentage error in student's calculation = $\frac{0.9\,T - T}{T} \times 100\%$ = -10 % (hence 10% lower) {Note: If it is difficult to visualize why the "student's calculated period = 0.9 T", Try the following thought process: Say, the actual period of oscillation is supposed to be 1 s (which is T). Total time taken by 9 oscillations (in reality) was 9 s only. But the student THOUGHT he was measuring 10 oscillations, so, his calculated period would be 9 s/ 10 oscillations = 0.9 s = 0.9 T. }	[1]
4(a)	Let [ ] denote the unit of [*F*] = [m] [a] = kg m s^{-2} [*arv*] = [*a*] x (m)(m s^{-1}) Equating both sides of eqn (since equation is homogeneous) kg m s^{-2} = [*a*] x m^2 s^{-1} $\Rightarrow$ [*a*] = <u>kg m^{-1} s^{-1}</u>	[1] [1]
(b)(i)	vernier calipers (deduced from statement that internal diameter is measured) zero error (do not accept parallax)	[1] [1]
(ii)	$V = \pi\left(\frac{d^2}{4}\right)h$ $= 964.665\ \text{cm}^3$ $\frac{\Delta V}{V} = \frac{2\Delta d}{d} + \frac{\Delta h}{h}$ $\Delta V = \left(\frac{2\times 0.01}{8.50} + \frac{0.1}{17.0}\right)\times 964.665$ $= 8\ \text{cm}^3$ $V = (965 \pm 8)\ \text{cm}^3$	[1] [1] [1]

(c)(i)		[2]
(ii)	$\Delta v^2 = 50^2 + 30^2 =$ <u>58.3 m s^{-1}</u> Examiners do not require the direction in this case (arbitrary decision)	[1]

- End of tutorial solutions -