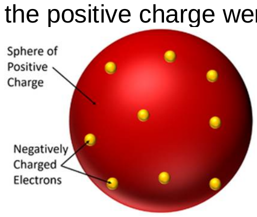
TOPIC 20: NUCLEAR PHYSICS

Learning Outcomes: Candidates should be able to:

	(I) Structure of the Atom
a.	Infer from the results of the Rutherford α-particle scattering experiment the existence and small size of the atomic nucleus.
b.	Distinguish between nucleon number (mass number) and proton number (atomic number).
c.	Show an understanding that an element can exist in various isotopic forms each with a different number of neutrons.
	(II) Energy-Mass Equivalence in Nuclear Processes
d.	Use the usual notation for the representation of nuclides and represent simple nuclear reactions by nuclear equations of the form $^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$.
e.	State and apply to problem solving the concept that nucleon number, charge and mass-energy are all conserved in nuclear processes.
f.	Show an understanding of the concept of mass defect.
g.	Recall and apply the equivalence between energy and mass as represented by $E = m c^2$ to solve problems.
h.	Show an understanding of the concept of nuclear binding energy and its relation to mass defect.
i.	Sketch the (graph of) variation of binding energy per nucleon with nucleon number.
j.	Explain the relevance of binding energy *per nucleon* to nuclear fusion and to nuclear fission.
	(III) Radioactive Decay
k.	Show an understanding of the spontaneous and random nature of nuclear decay.
l.	Infer the random nature of radioactive decay from the fluctuations in count rate.
m.	Show an understanding of the origin and significance of background radiation.
n.	Show an understanding of the nature of α, β and γ radiations (knowledge of positron emission is not required)
o.	Show an understanding of how the conservation laws for energy and momentum in β decay were used to predict the existence of the neutrino (knowledge of antineutrino and antiparticles is not required)
p.	Define the terms activity and decay constant and recall and solve problems using the equation $A = \lambda N$.
q.	Infer and sketch the exponential nature of radioactive decay and solve problems using the relationship $x = x_o e^{-\lambda t}$, where x could represent activity, number of undecayed particles or received count rate.
r.	Define and use half-life as the time taken for a quantity *x* to reduce to half its initial value.
s.	Solve problems by using the relation $\lambda = \dfrac{\ln 2}{t_{1/2}}$.
t.	Discuss qualitatively the effects, both direct and indirect, of ionising radiation on living tissues and cells.

Introduction

In the early years of the 20th century, not much was known about the structure of atoms beyond the fact that they contained electrons. This particle had been discovered (by JJ Thomson) only in 1897, and its mass was unknown in those early days. Atoms are electrically neutral, so they must also contain some positive charge, but <u>at that time nobody knew what form this compensating positive charge took</u>. How the electrons moved within the atom and how the mass of the atom was divided between the electrons and the positive charge were also open questions.



JJ Thomson suggested a **model of the atom** as a sphere of positive charge with electrons spread out the entire volume, much like the seeds in a watermelon. ("Plum-pudding" Model)

In 1911, Ernest Rutherford while interpreting some experiments carried out in his laboratory was led to propose a very different model for the structure of the atom.

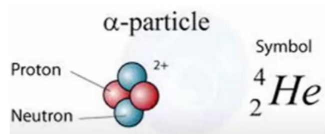
Part I

20.1 Rutherford's alpha–scattering experiment

a.	Infer from the results of the Rutherford α-particle scattering experiment the existence and small size of the atomic nucleus.

20.1.1 Experimental set-up

Ernest Rutherford's alpha-scattering experiment involved firing energetic alpha-particles through a thin gold foil and measuring the extent to which the alpha-particles were deflected as they passed through the foil. The set-up is as shown in Figure 20.1a & b (No need to recall set-up).



By 1908, Rutherford had established that alpha particles were positively charged particles identical to helium atoms which have lost 2 electrons (i.e. helium nucleus).

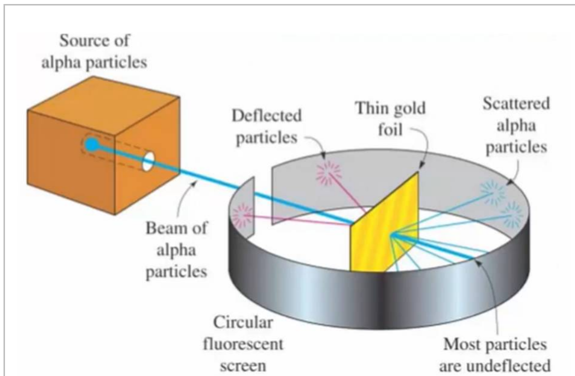


Fig. 20.1a:
Setup with a circular fluorescent screen.

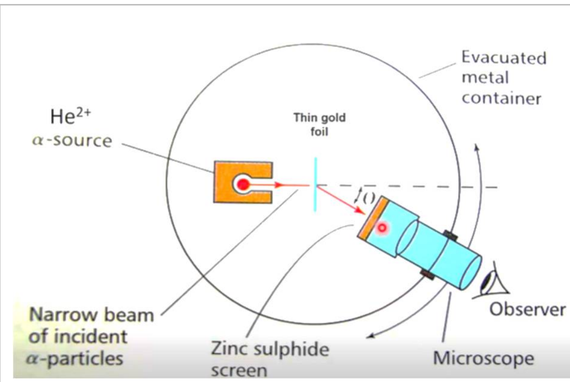


Fig. 20.1b:
Setup with a microscope as alpha-particle detector.

20.1.2 Experimental results

The results of the experiment are as shown in Figure 20.1.2:

1. Most of the α-particles passed through the metal foil undeflected (i.e. went straight through, or with very minimal deflection),

2. A very small proportion (about 1 in 8000) was deflected at large angles (by more than 90°, some of these approaching 180°!)

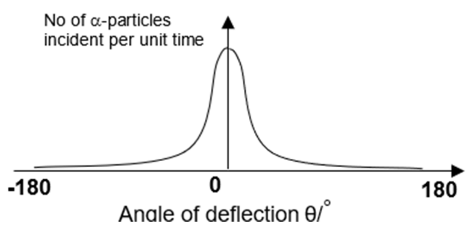


Fig. 20.2

In Rutherford's words:
"*It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you*."

20.1.3 Discovery of a Small Core of Positive Charge

Large angle deflections were not expected on the basis of Thomson's model (with uniformly distributed positive and negative charges) because the maximum deflecting force acting on the α-particle as it passes through such a positive sphere of charge would be too small to deflect the α-particle by even one degree.

Rutherford explained that to produce such a large deflection (180 degrees), there must be a large force, which could only be provided if the positive charge in an atom was concentrated in a relatively small volume of the atom. (He called this concentration of positive charge the nucleus of the atom.) Any electrons belonging to the atom were assumed to be in the relatively large volume outside the nucleus.

Figure 20.1.3 shows the path taken by typical alpha-particles as they pass through the atoms of the gold foil. The angle through which an alpha-particle is scattered depends on how close its path lies to the nucleus of the gold atom. Large angle deflections occur when alpha-particles were incident almost head-on to a nucleus.

In addition, since a very small proportion of the particles were scattered through large angles (i.e. the probability of a head-on approach is small), Rutherford concluded that the positive core/nucleus occupies only a small proportion of the atom, i.e. the atom is mostly empty space.

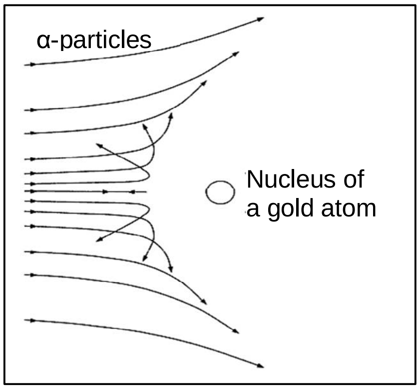


Fig. 20.3

20.1.4 Observations & Inferences

Observations/Results	Inferences
1. Most of the α-particles passed through the metal foil undeflected (i.e. went straight through, or with very minimal deflection)	1. The size of the nucleus is very small relative to size of the whole atom (means the atom consists of mainly empty space).
2. A very small proportion (about 1 in 8000) was deflected at large angles (by more than 90°, some of these approaching 180°)	2. There exists a positively charged core (called the nucleus).

20.1.5 Size of the nucleus deduced from Distance of Closest Approach

Considering an alpha-particle on a head-on collision course toward a nucleus, the kinetic energy of the incoming alpha-particle must be converted completely to electrical potential energy when the particle stops momentarily at the point of closest approach (before turning around).

Using the principle of conservation of energy, the initial kinetic energy of the alpha-particle is equal to the maximum electrical potential energy of the system (alpha-particle and target nucleus),
when the alpha-particle is at the **distance of closest approach**.
The distance of closest approach is thus an indication of the upper limit to the size of the nucleus.

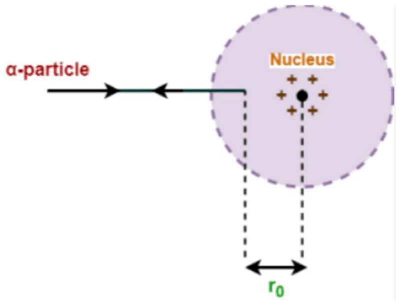


Fig. 20.4

Candidates are expected to recall the order of magnitude of an atomic radius & the radius of a nucleus.
The nuclear radius is of the order of **10^{-15} m** while that of an atom (including the orbiting electrons) is about **10^{-10} m**.
(If you imagine the nuclear diameter to be 1 mm, then the nearest electron is 10 m away.)

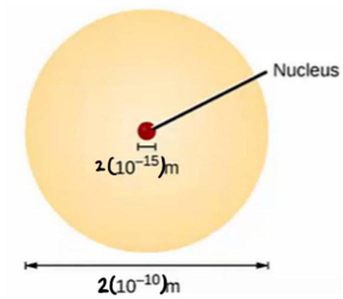
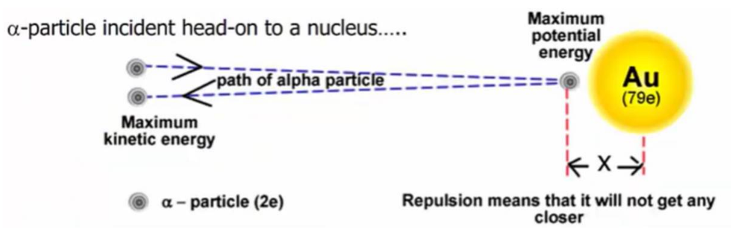


Fig. 20.5

Worked Example 1: Estimating the size of the nucleus from the distance of closest approach

In a Rutherford scattering experiment, an alpha particle (charge = +2e) heads directly toward a gold nucleus (charge = +79e). The alpha particle has a kinetic energy of 5.0 MeV *when very far from the nucleus.* Assuming the gold nucleus to be fixed in space, determine the distance of closest approach. Hence estimate the size of the nucleus.



Solution:
Let x = distance of closest approach

	Kinetic Energy of system	Electric Potential Energy
At initial position (when α-particle is very far from nucleus)	KE_i = 5 MeV	EPE_i = 0
At distance of closest approach (when α-particle is momentarily at rest)	KE_f = 0	$EPE = q\,V_f = (2e)(\frac{Q_{gold}}{4\pi\varepsilon_0 x})$

By Principle of Conservation of Energy,

$$KE_i + EPE_i = KE_f + EPE_f$$

$$5\text{ MeV} + 0 = 0 + (2e)\left(\frac{Q_{gold}}{4\pi\varepsilon_0 x}\right),$$ where 1 MeV = 10^6 eV = 10^6 x (1.6 x 10^{-19} J) = 1.6 x 10^{-13} J

$$5\text{ MeV} + 0 = 0 + (2e)\left(\frac{79e}{4\pi\varepsilon_0 x}\right),$$

⇒ x (distance of closest approach) = 4.55 x 10^{-14} m

Hence, the (upper limit of) estimated size of the nucleus is 5 x 10^{-14} m.

20.2 Properties of Constituents of a Nucleus

All nuclei are made up of two types of particles: protons and neutrons. (The only exception is the ordinary hydrogen nucleus, which is a single proton.) Protons and neutrons are classified together as nucleons.

The proton and the neutron are very similar particles, aside from the difference in their electric charges (q = +e for the proton, q = 0 for the neutron). They have nearly equal masses and experience identical nuclear forces inside the nuclei.

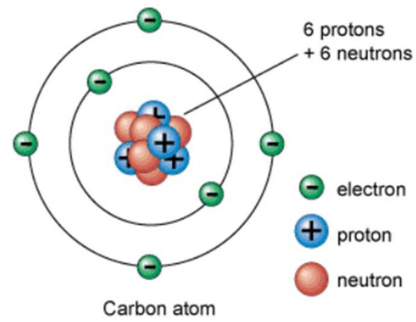


Fig. 20.6

20.2.1 Notation to represent specific nuclides

b.	Distinguish between nucleon number (mass number) and proton number (atomic number).

The term nuclide is used to specify an atom with a particular number of protons, Z and a particular number of neutrons. For example $^{6}_{3}Li$, $^{7}_{3}Li$, $^{16}_{8}O$, $^{18}_{8}O$ are 4 different nuclides.

The usual notation to represent a specific nuclide is $^{A}_{Z}X$, where:

Nucleon number (mass number): **no. of protons + no. of neutrons**

$^{A}_{Z}X$ — X: symbol of the element

Proton number (atomic number): **no. of protons** in the nucleus

Fig. 20.7

Note:
- The proton number, *Z* is sometimes omitted because it gives the same information as the chemical symbol. For example, all lithium (Li) atoms have 3 protons and all oxygen (O) atoms have 8 protons.
- **A – Z** = number of neutrons

20.2.2 Isotopes

c.	Show an understanding that an element can exist in various isotopic forms each with a different number of neutrons.

Isotopes are atoms of the same element but having different number of neutrons**,** i.e. isotopes are atoms with the same proton number (Z) but a different nucleon number (A).

Eg: $^{16}_{8}O$, $^{17}_{8}O$, $^{18}_{8}O$ are 3 isotopes of Oxygen

$^{1}_{1}H\ (hydrogen)$, $^{2}_{1}H\ (deuterium)$, $^{3}_{1}H\ (tritium)$ are 3 isotopes of Hydrogen

Isotopes have identical chemical properties since they have an identical configuration of electrons but different nuclear and physical properties.

20.3 The Unified Atomic Mass Constant

Atomic masses are measured in unified atomic mass, u.
1 u is defined such that the atomic mass of ${}^{12}_{6}C$ is exactly 12 u.
1 u = 1.66 x 10^{-27} kg (provided in the List of Data)

The mass of a proton and the mass of a neutron are considered as equal (unless otherwise stated), and equal to 1 u.

Thus, for example, the mass of 1 nucleus of the nuclide ${}^{17}_{8}O$ is 17 u, i.e. = 17 x (1.66 x 10^{-27} kg)
= 2.82 x 10^{-26} kg

Alternatively, we can find its nuclear mass in the following way:

Mass of 1 mole of any substance = Relative Atomic Mass (expressed in grams)

1 mole of ${}^{17}_{8}O$ atoms/nuclei has a mass of 17 g (17 x 10^{-3} kg).

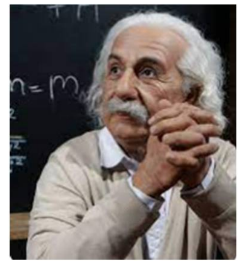
Thus mass of one nucleus of ${}^{17}_{8}O$ = $\dfrac{17 \times 10^{-3}\ kg}{N_A}$ = $\dfrac{17 \times 10^{-3}\ kg}{6.02x10^{23}}$ = 2.82 x 10^{-26} kg

Part II

20.4 Equivalence between Mass and Energy

g.	Recall and apply the equivalence between energy and mass as represented by $E = m\,c^2$ to solve problems.

20.4.1 Einstein's Equation


Albert Einstein!

According to Albert Einstein (1905), mass and energy are equivalent; expressed by the famous equation

$$\boxed{E = m\,c^2} \quad \ldots\ldots\ldots\ldots(1.1)$$

where c = velocity of light in vacuum = 3.00×10^8 m s^{-1}

In other words, mass is a form of energy. He also showed that energy and mass are inter-changeable (interconvertible), i.e. for any change in the energy of an object, ΔE, there is a corresponding change in its mass, Δm, and vice-versa.

$$\boxed{\Delta E = (\Delta m)\,c^2} \quad \ldots\ldots\ldots\ldots(1.2)$$

Thus even a small change in mass corresponds to an enormous amount of energy release because c^2 is a very large number.

Worked Example 2: Determining the increase in mass when water gets hotter

When 1.0 kg of water at 25°C gains thermal energy Q, such that its temp rises to 100°C, what is the amount of heat gain? What is the increase in mass?

[3.5×10^{-12} kg]

Solution:

$$Q = m\,c\,\Delta\theta = 1 \times 4200 \times (100 - 25)\text{ J} = 315\text{ kJ}$$

According to equation 1.2, the increase in the mass of the water,

$$\Delta m = \frac{\Delta E}{c^2} = \frac{Q}{c^2} = 3.5 \times 10^{-12}\ kg$$

This is a negligible (and thus "impossible" to detect by most instruments) increase in mass.

Example 3: Determining energy equivalence in MeV

Calculate the energy equivalent of 1 u of hydrogen in MeV.

Solution:

$E = m\,c^2$

20.4.2 Rest-Mass Energy

Einstein was able to show that the mass of an object *varies* with its speed. At rest, the object has a mass m_o, its so-called **rest mass**.

The *energy equivalent* that it possesses, since mass and energy are equivalent, is called its **rest-mass energy**.

Thus by equation (1.1) $\boxed{\text{Rest-mass energy} = m_o c^2}$ m_o: mass when object is at rest.

Thus, for example, the total energy of a car moving along a level road = $m_o c^2$ + KE

20.5 Mass Defect and Binding Energy

f.	Show an understanding of the concept of mass defect.

20.5.1 Mass Defect

The mass of a nucleus is always less than the sum of the masses of the constituent particles (i.e. the nucleons) taken *separately.* This discrepancy is called the mass defect.

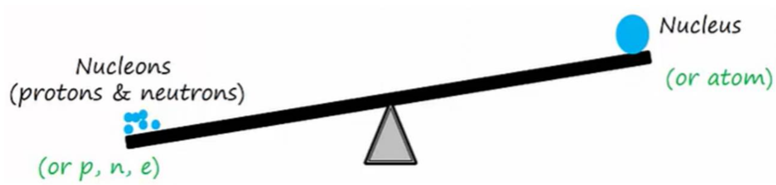


Fig. 20.8

For nucleus:	For whole atom:
Mass Defect = Σ Mass of nucleons when separated – Nuclear Mass = (Z) m_p + (A – Z) m_n – Nuclear Mass ⇒ Σ Mass of nucleons when separated = Nuclear mass + **Mass defect**	Mass Defect = Σ Mass of protons, neutrons & electrons when separated – Atomic Mass Σ Mass of neutrons, protons and electrons = Atomic mass + Mass defect

Example 4: Basic Application of Mass Defect Formula

The atomic mass of calcium is 39.96259 u . Given that the
mass of a proton = 1.00728 u ,
mass of a neutron = 1.00867 u ,
mass of an electron = 0.00055 u ,
and that there are 20 protons, 20 neutrons and 20 electrons in 1 Ca atom,
determine the mass defect of calcium.

[0.36741 u]

Solution:

Note: *The atomic mass (NOT nuclear mass) is being considered. Thus electrons must be considered as part of the constituent particles.*

Mass defect = (Mass of neutrons, protons and electrons) – Atomic mass

=

The discrepancy between the mass of a nucleus and the sum of the masses of its constituent particles is a startling result, as it appears that the Principle of Conservation of Mass is broken.

(This discrepancy is due to the strong nuclear force. Since the strong nuclear force is an attractive force the potential energy due to this force is negative, which means that the total energy of a nucleus is lower than the case when the same number of protons and neutrons are completely separated. Lower total energy means a lower mass.)

20.5.2 Nuclear Binding energy

h.	Show an understanding of the concept of nuclear binding energy and its relation to mass defect.

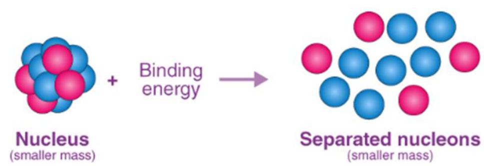
Since $E = m\,c^2$, any mass or mass difference has an energy equivalent. Therefore, when the total mass of a nucleus is less than the sum of the masses of its nucleons, the total energy of the bound system (the nucleus) is less than the combined energy of the separated nucleons.

This difference in energy, is the energy equivalent of the mass defect, and is called the **binding energy** of the nucleus. Hence,

$$\boxed{\text{Binding energy} = (\text{Mass defect}) \times c^2}$$

Concept of Binding Energy:

The **binding energy** of the nucleus is the energy that must be supplied to a nucleus to *completely* separate (unbind) the nucleus into its individual nucleons (neutrons and protons).



Note:

1. It is necessary to supply energy to unbind the nucleus because the nucleons are held close together by the attractive strong nuclear force between the nucleons.

2. Nuclear binding energy can also be thought of as the energy *released* when a nucleus is *formed* from its individual nucleons.

Worked Example 5 (J84P1Q12): Determining Binding Energy

Deuterium is represented by the symbol $^{2}_{1}H$. What nucleons make up its nucleus?

Use the data below to calculate the binding energy of the deuteron (the deuterium nucleus):

Nuclear mass of deuterium,	2.01355 u,
rest mass of proton, m_p,	1.00728 u,
rest mass of neutron, m_n,	1.00867 u.

[3.59 x 10^{-13} J]

Solution:

Nucleons in deuterium nucleus: 1 proton and 1 neutron

Mass defect = (Mass of neutron + mass of proton) – Nuclear mass
= (1.00867 u + 1.00728 u) - 2.01355 u
= 0.002388 u

Binding energy = Mass defect x c^2
= 0.002388 u x c^2
= (0.0024 x 1.66 x 10^{-27} kg) (3 x 10^8 m s^{-1})^2 *Must convert u to kg.*
= 3.5856 x 10^{-13} **J**

{If question above requires the ans in MeV: $= \dfrac{3.5856 \times 10^{-13}}{10^6 \times 1.6 \times 10^{-19}}$ MeV = 2.24 MeV}

Note**:**

- This result tells us that to completely separate a deuterium nucleus into a proton and a neutron, it is necessary to supply 2.24 MeV of energy to overcome the attractive nuclear force between the proton and the neutron.
- One way of supplying the deuterium with this energy is by bombarding it with energetic particles.

20.5.3 Graph of Binding energy per nucleon & Stability of Nucleus

i.	Sketch the (graph of) variation of binding energy per nucleon with nucleon number.
j.	Explain the relevance of binding energy *per nucleon* to nuclear fusion and to nuclear fission.

Binding energy per nucleon of a nuclide = $\dfrac{\text{Binding energy of the nucleus}}{\text{Nucleon no of the nucleus}}$

From 20.5.2, it can be deduced that the binding energy per nucleon of a nucleus is a measure of how difficult it is to break up the nucleus – the higher the value, the more difficult it is to break up (unbind) the nucleus, since more energy must be supplied.

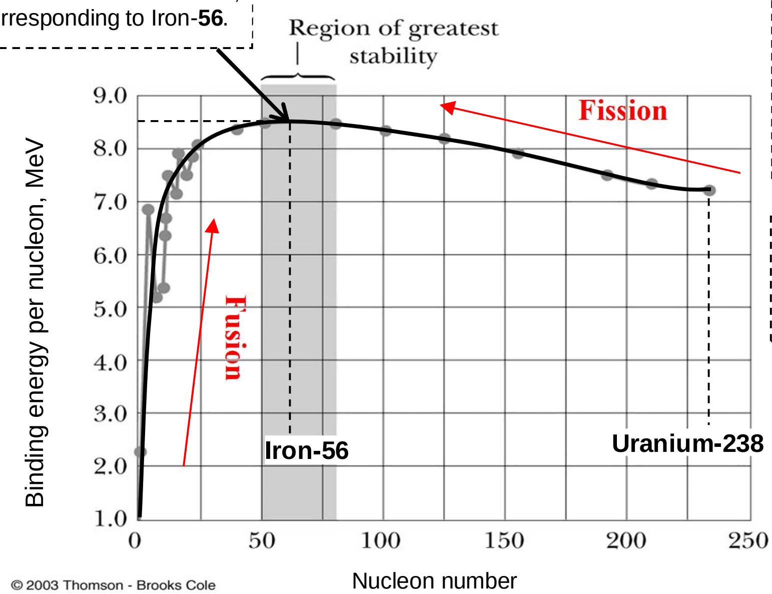
In other words, binding energy per nucleon is a measure of the stability of the nucleus. The higher the value, the more stable (difficult to unbind) is the nucleus.

Common mistake: Binding energy is the energy required to bind the nucleons.

20.5.3.1 Graph of binding energy per nucleon

The variation of binding energy per nucleon with nucleon number is shown in Fig. 20.9 below. [J01P3Q6]

Max. binding energy per nucleon of about **8.8 MeV**, corresponding to Iron-**56**.



Need to know how to sketch this graph, & recall the approximate value of the maximum binding energy per nucleon (8.8 MeV) and the corresponding nucleon no (Iron-56).

Recall:
1 MeV = 10^6 eV
= 10^6 x (1.6×10^{-19} C) x 1 V
= 1.6×10^{-13} J

Fig. 20.9

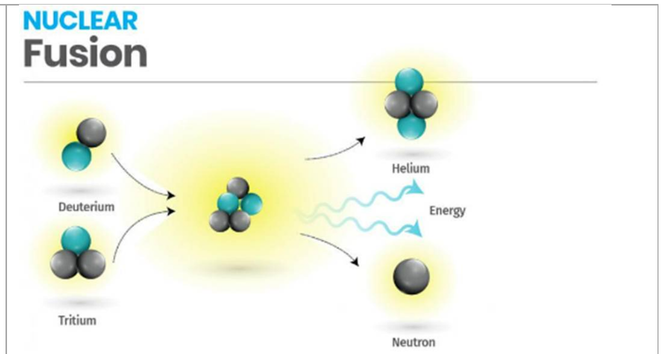
- The graph is a maximum when the nucleon no. is approximately 56; i.e. Iron – 56 is the most stable nuclide.
- The graph is NOT symmetrical.

20.5.3.2 Nuclear Fusion & Fission

The release of the Oppenheimer movie, the story of the director of the Manhattan Project, has prompted many people to go online and search for an explanation of the difference between fission and fusion, two fundamental scientific concepts.

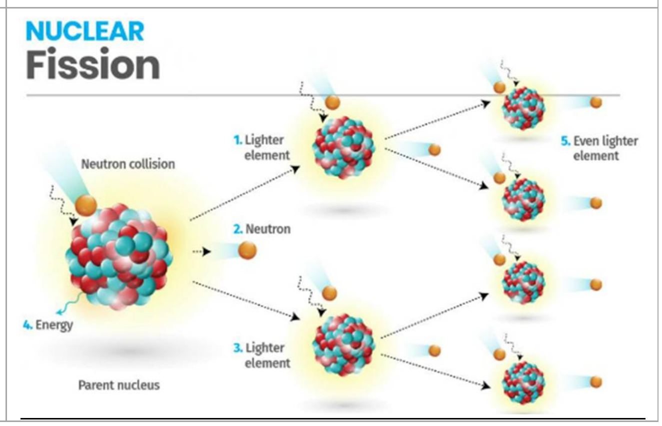
Nuclear Fusion
Nuclear fusion is the process where two light nuclei are combined to produce a heavier nucleus with the release of energy.

The Sun, along with all other stars, is powered by this reaction.



Nuclear Fission
Nuclear fission is a reaction where a heavy nucleus disintegrates into two lighter nuclei with the release of energy.

For instance, when hit by a neutron, the nucleus of an atom of uranium-235 splits into two smaller nuclei, for example, a barium nucleus and a krypton nucleus and two or three neutrons. These extra neutrons will hit other surrounding uranium-235 atoms, which will also split and generate additional neutrons in a multiplying effect generating a chain reaction in a fraction of a second.



Referring to the graph of binding energy per nucleon (Fig. 20.9):

- [*By reference to the graph in Fig. 20.9, explain how the process of nuclear fusion may result in the release of energy. [J01P3Q9]*]
  When a nuclide with a low binding energy per nucleon {i.e. one on the left of the peak} fuses with another nuclide also of low binding energy per nucleon, the end product of the **nuclear fusion** may be a nuclide whose nucleon number gets closer to the peak of the graph above.

  Since a nuclear reaction *resulting* in nuclei with a higher binding energy per nucleon is associated with the release of energy, fusion results in the release of energy.

- The variation of binding energy per nucleon at high nucleon numbers tells us that nucleons are more tightly bound (stable) when they are assembled into two middle-mass nuclei rather than into a high-mass nucleus. In other words, energy will be released in the **nuclear fission** of a single massive nucleus into two smaller fragments.

In summary, any nuclear reaction which **results in moving towards the max of the graph** has the effect of
- ✓ releasing energy and hence,
- ✓ increasing the stability of the resulting products.

Worked Example 6: Implication of graph of binding energy per nucleon

(i) **With reference to the graph in Figure 20.9, explain the implications for the stability of a given nuclide of a *low* value of the BE per nucleon. (N85P2Q11)**

Understanding of question: "low value of BE per nucleon" is to be interpreted as having a nucleon no which is to the left of the maximum point (Refer to Fusion arrow on Figure 20.5.3).

Solution:
A nuclide with a *low* value of BE per nucleon will tend to undergo fusion with another nuclide of low BE per nucleon because the end product of the fusion will then be one of a higher nucleon no and thus greater stability.

Eg. ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$ + Energy release

(ii) **Why is the fusion of nuclei having *high* nucleon numbers *not* associated with the release of energy? (J92P3Q6)**

Understanding of question: "high nucleon number" is to be interpreted as having a nucleon number which is to the right side of the maximum point (Refer to Fission arrow on Figure 20.5.3)

Solution:
If two such nuclei were to undergo fusion, the resulting nucleus would have an even larger nucleon number & thus a lower value of binding energy per nucleon. This would mean that it becomes less stable; hence energy could ***not*** have been released.

On the other hand, nuclei having high nucleon numbers will tend to undergo nuclear **fission** instead, to produce 2 nuclei (each of lower nucleon no) which will be more stable as they would possess a higher BE per nucleon.

Eg. ${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{148}_{57}La + {}^{85}_{35}Br + 3\,{}^{1}_{0}n$ + Energy release

- In both nuclear fission and fusion associated with the release of energy, the total mass of the products after the reaction is less than the total mass of the reactants before. The amount of energy released is the energy equivalent of the "missing mass", which can be calculated by $E = m c^2$.

- In both cases of fusion and fission, the energy released is mostly in the form of kinetic energy of the products.

E.g. ${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{148}_{57}La + {}^{85}_{35}Br + 3{}^{1}_{0}n + \underbrace{E_{release}}_{\text{(KE of products)}}$

E.g. $\underbrace{{}^{2}_{1}H + {}^{2}_{1}H}_{\text{(deuterium)}} \rightarrow \underbrace{{}^{3}_{2}He + {}^{1}_{0}n}_{\textbf{Smaller}\text{ mass}} + \underbrace{E_{release}}_{\text{(KE of products)}}$

20.5.4 Principle of Conservation of Energy-Mass and Binding Energy

It can be deduced from the principle of conservation of energy-mass (Sect. 20.6.1) and the definition of binding energy that in a nuclear reaction:

Energy released in nuclear reaction	=	Total BE of **Products** – Total BE of **Reactants**

We can use the following to visualize the BEs involved when nuclei A and B reacts to form C, releasing a certain amount of energy (5 J).

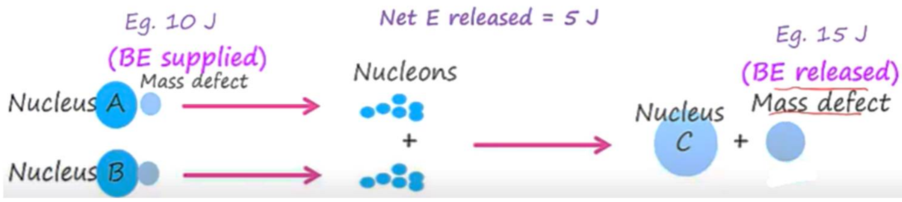


Fig. 20.10

Example 7 (N99P1Q29): Calculating Energy Release using Binding Energies

Two deuterium nuclei fuse together to form a Helium-3 nucleus, with the release of a neutron. The reaction is represented by

$${}^{2}_{1}H + {}^{2}_{1}H \longrightarrow {}^{3}_{2}He + {}^{1}_{0}n + \text{energy}$$

The binding energies per nucleon are: for ${}^{2}_{1}H$ 1.09 MeV, for ${}^{3}_{2}He$ 2.54 MeV.

How much energy is released in this reaction?

[3.26 MeV]

Solution:

Does ${}^{1}_{0}n$ contribute to the term "Total Binding Energy of Products"? Why?

Unlike the neutrons that are found in a nucleus, the neutron here exists as a free neutron, i.e. one which does not experience any nuclear force. Hence there is no binding energy associated with it.

Energy released in nuclear process = Total BE of **products** – Total BE of **reactants**

=

=

=

20.6 Conservation of Mass and Energy in Nuclear Processes

d.	Use the usual notation for the representation of nuclides and represent simple nuclear reactions by nuclear equations of the form ${}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$.
e.	State and apply to problem solving the concept that nucleon number, charge and mass-energy are all conserved in nuclear processes.

20.6.1 Balancing Nuclear Equations

In any nuclear process, the **proton no. and the nucleon no**. are **conserved**, as illustrated in the following examples:

(i) $\,{}^{7}_{3}Li + {}^{1}_{1}H \rightarrow {}^{4}_{2}He + {}^{4}_{2}He$

(ii) $\,{}^{14}_{7}N + {}^{1}_{0}n \rightarrow {}^{12}_{6}C + {}^{3}_{1}H$

(iii) $\,{}^{211}_{83}Bi \rightarrow {}^{211}_{84}Po + {}^{0}_{-1}\beta$

However as mass and energy are equivalent, having the same number of nucleons on both sides of the equation does not necessarily mean that the total mass (and hence total energy) is conserved,

Since mass has an energy equivalent, mass can be considered as a form of energy; it would be appropriate to combine the 2 classical conservation principles of mass and of energy into one – to be called the **Principle of Conservation of Energy-Mass.**

The term "principle of conservation of *energy-mass*" may not be used in the A-Level exams. Instead, principle of conservation of *energy* may be used interchangeably.

The other important conservation principle: Principle of Conservation of Momentum is also valid for nuclear reactions and may be used in calculations for some problems where relevant information is provided.

20.6.2 Application of the Principle of Conservation of Energy-Mass

Total energy-mass before the process = Total energy-mass after process
In addition to kinetic energies of the reactants and products, rest-mass energy must also be considered as one of the energy forms.

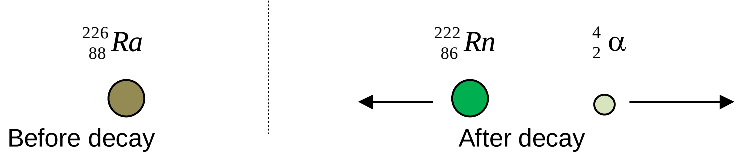
If a γ-ray is emitted in the process, its photon energy, *hf*, is the other energy form that needs to be considered.

Thus, the **Principle of Conservation of Energy-Mass** can be expressed as:

$\sum$ (rest mass energy & KE of reactants) = $\sum$ (rest mass energy & KE of products + *hf* of emitted γ-ray, if any)

Consider an isotope ${}^{226}_{88}Ra$ that decays into ${}^{222}_{86}Rn$ with the emission of an α-particle and a γ-ray photon of frequency f.

The nuclear equation for the decay is: ${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}\alpha + \gamma\text{-}ray$



Applying the principle of conservation **of energy-mass** to this decay:

Total energy-mass *before* decay = Total energy-mass *after* decay

Total rest mass energy + kinetic energy of *reactants* = Total rest mass energy + kinetic energy of *products* + photon energy, hf (if γ-ray is emitted)

Thus,

$$m_{Ra}c^2 + \tfrac{1}{2} m_{Ra}u_{Ra}^2 = (m_{Rn}c^2 + \tfrac{1}{2} m_{Rn}u_{Rn}^2) + (m_{\alpha}c^2 + \tfrac{1}{2} m_{\alpha}u_{\alpha}^2) + hf$$

Assume that KE of parent nucleus (i.e. ${}^{226}_{88}Ra$) = zero (i.e. assume that it is initially stationary)

$$m_{Ra}c^2 + \mathbf{0} = (m_{Rn}c^2 + \tfrac{1}{2} m_{Rn}u_{Rn}^2) + (m_{\alpha}c^2 + \tfrac{1}{2} m_{\alpha}u_{\alpha}^2) + hf$$

$$[m_{Ra} - (m_{Rn} + m_{\alpha})]c^2 = \tfrac{1}{2} m_{\alpha}u_{\alpha}^2 + \tfrac{1}{2} m_{Rn}u_{Rn}^2 + hf$$

i.e.

(Total rest mass of reactants – total rest mass of products) x c^2 = KE of products and γ-photon

Energy equivalent of mass lost = Energy released (in the form of KE of products and γ-photon)

What can be implied from the above:

1. Total rest mass of products is less that the total rest mass of reactants. i.e. there is a loss of mass. The "mass difference" between reactants and products is a positive value.
2. The energy equivalent of the mass loss is released in the form KE of products & γ-photon
3. Since energy is released in the reaction, it is a spontaneous reaction.

Example 8 (N83P1Q13 (part)) – Spontaneous Reaction

A stationary radon atom may decay spontaneously into a polonium atom and a helium atom, as shown below:

$${}^{222}_{86}Rn \rightarrow {}^{218}_{84}Po + {}^{4}_{2}He$$

The rest masses of these atoms are:

${}^{222}_{86}Rn$, 222.0176u, ${}^{218}_{84}Po$, 218.0090u, ${}^{4}_{2}He$, 4.0026u

(i) Explain how energy is conserved in this decay.

(ii) Assuming that no γ-ray is emitted, calculate the total kinetic energy of the decay products.

Solution:

(i) By the Principle of Conservation of Energy-Mass,
Total energy-mass before decay = Total energy-mass after decay,
$m_{Rn}c^2 + KE_{Rn} = m_{Po}c^2 + KE_{Po} + m_{\alpha}c^2 + KE_{\alpha}$ (No γ-ray emitted in this case),
where KE_{Rn} = zero (stationary radon atom initially)

(ii) Total KE of decay products = $KE_{Po} + KE_{\alpha}$ (= energy released)

Worked Example 9 (N82 P1Q11) – Induced Reaction (First Artificial Transmutation of a Nucleus, 1919 in Rutherford's Lab)

A nitrogen nucleus ${}^{14}_{7}N$, bombarded with an alpha particle of a certain energy, transmutes to an oxygen nucleus ${}^{17}_{8}O$ and a proton.

(a) Write an equation for this nuclear reaction, showing the nucleon numbers and the proton numbers of the particles involved.

(b) Find the **minimum** energy of the alpha particle to make this reaction occur.

	Mass/kg
${}^{14}_{7}N$	2.32530×10^{-26}
${}^{17}_{8}O$	2.82282×10^{-26}
proton mass m_p	0.16735×10^{-26}
α	0.66466×10^{-26}

Solution:

(a) ${}^{14}_{7}N + {}^{4}_{2}\alpha \longrightarrow {}^{17}_{8}O + {}^{1}_{1}p$

(b) By Principle of Conservation of Energy-Mass,

$(mc^2)_N + KE_N + (mc^2)_\alpha + KE_\alpha = (mc^2)_O + KE_O + (mc^2)_P + KE_P$

Assume KE_N = zero.

Note: Always assume target nucleus is stationary unless otherwise stated

$\therefore KE_\alpha = \{(mc^2)_O + KE_O + (mc^2)_p + KE_p\} - \{(mc^2)_N + (mc^2)_\alpha\}$

The rest mass energies of Oxygen, proton, Nitrogen and α-particle each has a definite value independent of the kinetic energy of the bombarding α-particle

However the kinetic energies of Oxygen and proton depend on it, i.e. if the kinetic energy of the α-particle is reduced, so will the kinetic energies of the products.

Thus, the **minimum** ke of α-particle corresponds to the case when KE_O and KE_p = **0**

Thus for reaction to occur, **minimum** $\mathbf{KE_\alpha} = \{(mc^2)_O + (mc^2)_p\} - \{(mc^2)_N + (mc^2)_\alpha\}$

Note:

1. What happens if $KE_\alpha <$ this minimum value?
   The reaction as described in (a) will not occur
   since the Principle of Conservation of Energy-Mass will be violated.

2. Induced vs Spontaneous Nuclear Reactions/ Processes
   In Example 9, we note that *energy must be supplied* to the alpha particle (in the form of kinetic energy) to ***induce*** the reaction to occur. A nuclear reaction such as this is said to be an **induced** nuclear reaction i.e. it cannot take place spontaneously. The reaction will not occur spontaneously as the reactants would have less energy than the products.

The next example deals with the question: *Will a given nuclear reaction occur **spontaneously**?*
In this topic, such a question will be decided by considering **only the energy point of view** – even though there may be other factors which play a part in determining whether or not the reaction will occur spontaneously.

Worked Example 10 (J87P2Q6) – Spontaneous Reaction (Origin of beta radioactive decay)

The rest masses of the neutron, proton and electron are 1.0087 u, 1.0073 u and 0.0005 u respectively.

Explain why it is energetically possible for a neutron to spontaneously emit a beta-particle.
Using conventional symbols, write down an equation for this decay.

Solution:

$${}^{1}_{0}n \longrightarrow {}^{0}_{-1}\beta + {}^{1}_{1}p$$

Before decay	After decay	
•	←•	•→
neutron	proton	β

By Principle of Conservation of Energy-Mass,

$$(mc^2)_n + KE_n = (mc^2)_\beta + KE_\beta + (mc^2)_P + KE_P$$

Which KE term may be zero?

KE_n: In all questions involving radioactive decay, you can always assume that the parent nucleus is initially *stationary* (Recall Example 9). Thus $KE_n = 0$.

KE_β: can't be zero since it is ***emitted***

KE_p: **can't be zero**, otherwise the principle of conservation of momentum will be violated.

Thus, $(mc^2)_n + 0 = (mc^2)_\beta + KE_\beta + (mc^2)_p + KE_p$

And $(mc^2)_n$ must > $(mc^2_p + (mc^2)_\beta$ (by the amt $KE_\beta + KE_p$) for decay to be energetically possible.

{The "mass lost" during reaction is the source of the Energy Released.

Rest mass energy of reactants = $(mc^2)_n$ = 1.0087 uc^2

Rest mass energy of products = $(mc^2)_p + (mc^2)_\beta$ = [1.0073 u + 0.0005 u]c^2 = 1.0078 uc^2

Since rest mass energy of reactant > rest mass energy of products, the reaction is energetically possible.

If on the other hand, rest mass energy of reactant < rest mass energy of product, then the decay is NOT energetically possible.

Summary

1. $E = mc^2 \rightarrow \Delta E = (\Delta m)c^2$

2. In nuclear processes, 4 quantities must be conserved:

   (i) nucleon no. (ii) proton no. (iii) energy & mass (**PC E-M**)

   (iv) Momentum [Eg 10, where the vector sum of the momentum of the proton and electron = initial momentum (of the neutron)]

3. $\boxed{\text{Binding Energy} = (\text{Mass Defect}) \times c^2}$

4. Recall <u>asymmetrical</u> shape of BE per nucleon vs nucleon no graph.
   In both nuclear fusion and fission, resulting nuclides have a <u>higher</u> BE per nucleon and they become <u>more stable</u>.
   The **mass "loss"** during these processes is the **source of the energy released**.

5.

Energy released in nuclear process	=	Total BE of **products** – Total BE of **reactants**

   In general, this energy released appears in the form of

   (i) KE (mostly) (ii) gamma-ray (energy = *hf*) {not in all cases}

6. By Principle of Conservation of Energy-Mass,

   $\boxed{\text{Total energy-mass before reaction = Total energy-mass after reaction,}}$

Total rest mass energy & kinetic energy of reactants	=	Total rest mass energy & kinetic energy of products + photon energy, hf (if γ-ray is emitted in the process)

   <u>Notes</u>:

   a) If the reaction/process is a radioactive decay (which is a spontaneous process), assume that the KE of the radioactive parent nucleus to be initially stationary, i.e. its KE = 0 unless otherwise stated.

   b) For an induced nuclear reaction: assume KE of target nucleus (nucleus being bombarded) = 0 unless otherwise stated.

   c) Rest-mass energy = $m_o c^2$, m_o = rest mass

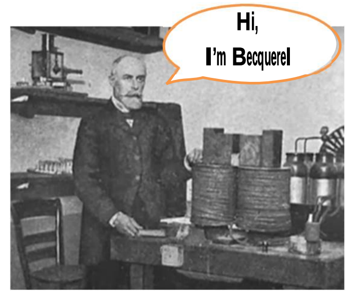
Energy released in nuclear process	=	Energy equivalent of mass loss
	=	(Total rest mass of **reactants** – total rest mass of **products**) x c^2

Part III

20.7 Radioactive decay

k.	Show an understanding of the spontaneous and random nature of nuclear decay.

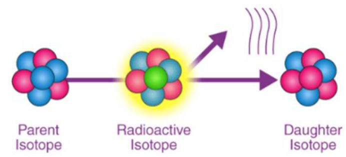
20.7.1 Nature of radioactive decay



(General Knowledge: In 1896, Henri Becquerel accidentally discovered that uranium salt crystals emit an invisible radiation that can darken a photographic plate even if the plate is covered to exclude light. This spontaneous emission of radiation was soon called radioactivity.)

- Radioactivity is a process which involves the particles of the nucleus - not the extra-nuclear electrons as in chemical reactions. An unstable radioactive nucleus emits radiation in an attempt to become more stable.

- Radioactive decay is a **spontaneous** process. This means that it cannot be controlled or affected by external factors- factors outside (external to) the nucleus, like pressure, temperature.



- Radioactive decay is also a **random** process. This means that it is impossible to know when the next disintegration will occur.
  Nevertheless, for a sufficiently large number of nuclei in a sample, it decays exponentially with time.

  'Random' here thus means:
  - it is impossible to know when the next disintegration will occur, even though
  - the probability of decay per unit time of a nucleus is constant.

Radioactive decay is the spontaneous and random disintegration of an unstable nucleus by emitting one or more of 3 types of radiations: alpha-particles (α-particles), beta-particles *(β-particles) and gamma rays (γ-rays)*.

Worked Example 11 (J96P3Q6a)

Distinguish between the radioactive decay and the fission of a nucleus. [5]

Solution:

Radioactive decay:

1. Spontaneous
2. Random
3. Disintegration of an unstable nucleus by emitting one or more of 3 types of radiations: alpha-particles (α-particles), beta-particles *(β-particles) and gamma rays (γ-rays)*.

Nuclear Fission:

1. Usually induced, or occurs when nucleus is bombarded with neutrons
2. Results in 2 fragments of about equal mass typically

20.7.2 Nature of α-particles, β-particles and γ-rays

n.	Show an understanding of the nature of α, β and γ radiations (knowledge of positron emission is not required)

- **Alpha-particles** are helium nuclei; they consist of 2 protons and 2 neutrons and have a charge of +2e.
- **Beta-particles** are electrons emitted from the nucleus of a radioactive substance when a neutron decays.
  (These electrons are not the electrons that orbit around the nucleus.
  Recall Example 10: ${}^{1}_{0}n \rightarrow {}^{0}_{-1}\beta + {}^{1}_{1}p$, i.e. an electron is produced and then emitted when a neutron "transforms" itself into a proton within the nucleus.)
- **Gamma rays** are electromagnetic radiations of very short wavelengths, i.e. high energy photons.
  (After α or β decay, the nuclei of some atoms are left in an excited state. They return to the ground state by emission of a γ-ray.)

20.7.2.1 Nature & Characteristics of α-particles, β-particles and γ-rays

	Alpha	Beta	Gamma
Notation	${}^{4}_{2}\alpha$ or ${}^{4}_{2}He$	${}^{0}_{-1}\beta$ or β^{-} or ${}^{0}_{-1}e$	γ
Charge	+2e	-e	No charge
Mass	4 u	≈1/2000 u	Massless
Nature	Particle	Particle	EM radiation
Penetrating ability *(Important for Planning in Pract Exam)*	Stopped by a few cm of air at stp or a thin sheet of paper	Stopped by a few mm of aluminium or ≈ 1 m of air at stp	Stopped by a few cm of lead or 1 m of concrete
Relative ionising ability (least impt pt)	Strong	Weak	Very weak

To distinguish the three forms of radiations:
the set-up as shown in Figure 20.11 can be used.

The radiation from a radioactive sample is directed into a region with a magnetic field. The beam splits into three components, two bending in opposite directions and the third not changing direction.

It can be concluded using Fleming's Left Hand Rule, that:

1. the radiation of the undeflected beam (γ-ray) carries no charge,
2. the component deflected upward contains positively-charged particles (α-particles) and
3. the component deflected downward contains negatively-charged particles (β-particles).

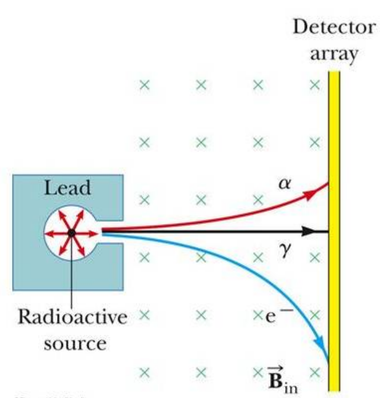
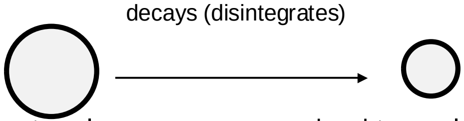


Fig. 20.11

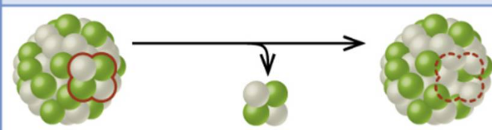
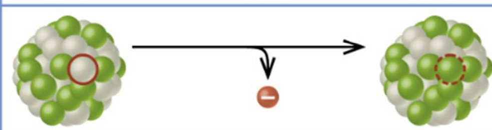
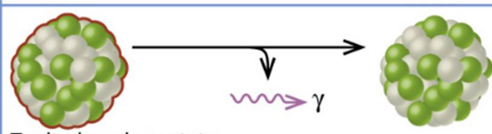
20.7.3 Effect of Radioactive Decay on the Nucleon & Proton Nos

As stated in the earlier section, a radioactive nucleus decays spontaneously via alpha, beta and gamma decay. The original nucleus that undergoes the radioactive decay is called the <u>parent nucleus</u> and the new nucleus formed after the decay is called the <u>daughter nucleus</u>.



The daughter nucleus may still be unstable and so undergoes further decay. It is thus possible for a nucleus to undergo a series of changes, resulting in a radioactive decay series until a stable end-product is formed. For example,

$$^{232}_{90}\text{Th} \xrightarrow{\alpha} {}^{228}_{88}\text{Ra} \xrightarrow{\beta} {}^{228}_{89}\text{Ac} \xrightarrow{\beta} {}^{228}_{90}\text{Th} \xrightarrow{\alpha} {}^{224}_{88}\text{Ra} \xrightarrow{\alpha} {}^{220}_{86}\text{Rn} \xrightarrow{\alpha} {}^{216}_{84}\text{Po}$$

Type	Nuclear equation	Representation	Change in mass/atomic numbers
Alpha decay	$^{A}_{Z}\text{X} \rightarrow {}^{4}_{2}\text{He} + {}^{A-4}_{Z-2}\text{Y}$		A: decrease by 4 Z: decrease by 2
Beta decay	$^{A}_{Z}\text{X} \rightarrow {}^{0}_{-1}\text{e} + {}_{Z+1}^{A}\text{Y}$		A: unchanged Z: increase by 1
Gamma decay	$^{A}_{Z}\text{X} \rightarrow {}^{0}_{0}\gamma + {}^{A}_{Z}\text{Y}$	 γ Excited nuclear state	A: unchanged Z: unchanged

Note: γ-rays are EM waves

Fig. 20.12

20.7.4 What happens in a Beta Decay

During a beta decay, the emission of electrons ***from a nucleus*** is surprising because the nucleus is composed of protons and neutrons only.

This apparent discrepancy can be explained by noting that the emitted electron is created in the nucleus by a process in which a neutron is transformed into a proton. This process can be represented by

$$^{1}_{0}\boldsymbol{n} \rightarrow {}^{1}_{1}\boldsymbol{p} + \boldsymbol{e}^{-}$$

The next example illustrates how to calculate the amount of energy released in the beta decay of $^{14}_{6}C$ and also how experimental results led to the prediction of the existence of neutrino.

Worked Example 12- The Beta Decay

Calculate the energy released in the beta decay of a stationary $^{14}_{6}C$ to $^{14}_{7}N$, given that the mass of a neutral $^{14}_{6}C$ **atom** is 14.003242u. The SUM of the mass of the daughter nuclide & the mass of the beta particle is 14.003074 u.

Solution:

$^{14}_{6}\boldsymbol{C} \rightarrow {}^{14}_{7}\boldsymbol{N} + \boldsymbol{e^-}$ (if neutrino is <u>NOT</u> emitted)

Mass difference between the reactant and products,
Δm = 14.003242 u – 14.003074 u = 0.000168 u

This corresponds to an energy release of E = $\Delta m\, c^2$ = ………….. = 0.156 MeV.

20.7.4.1 Prediction of Existence of Neutrino

o.	Show an understanding of how the conservation laws for energy and momentum in β decay were used to predict the existence of the neutrino (knowledge of antineutrino and antiparticles is not required)

- Energy released in Eg 12 is in the form of kinetic energy of products and photon energy (if any). In this beta decay, no photons were detected or produced.
- By the **<u>Law of Conservation of Energy</u>**,
  it was thus expected that $KE_{\text{daugher nucleus}} + KE_{\text{beta particle}}$ = 0.156 MeV …………………………..…. (A)
- By the **<u>Law of Conservation of Momentum</u>**, since the initial momentum of $^{14}_{6}C$ was zero, the total final momentum of the daughter nucleus and the beta particle was also expected to be zero, ie

$$p_{\text{daugher nucleus}} + p_{\text{beta particle}} = 0 \text{ (expected)}$$

$$\Rightarrow |\, p_{\text{daugher nucleus}} \,| = |\, p_{\text{beta particle}} \,| \text{ (expected)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(B)}$$

- Since $\boldsymbol{KE} = \frac{1}{2}\boldsymbol{mv^2} = \frac{(\boldsymbol{mv})^2}{2\boldsymbol{m}}$
  $= \frac{\boldsymbol{p^2}}{2\boldsymbol{m}}$, $\rightarrow$ KE $\propto$ 1/m

  and since the beta particle is much lighter than the daugher nucleus and they have equal momenta,

  it was expected that: $KE_{\text{beta particle}} >> KE_{\text{daugher nucleus}}$ (ie $KE_{\text{daugher nucleus}}$ is negligible) ……….(C)
- From (A) & (C), it was expected**:** $KE_{\text{beta particle}} \approx$ 0.156 MeV …………………………………….. (D)
- However experimental results showed:
  1. the vast majority of emitted beta particles (electons) had energy <u>lower</u> than that (0.156 MeV),
  2. the **KE of the emitted electrons had a range** of values from zero up to its respective calculated maximum value (of 0.156 MeV) for any beta decay.

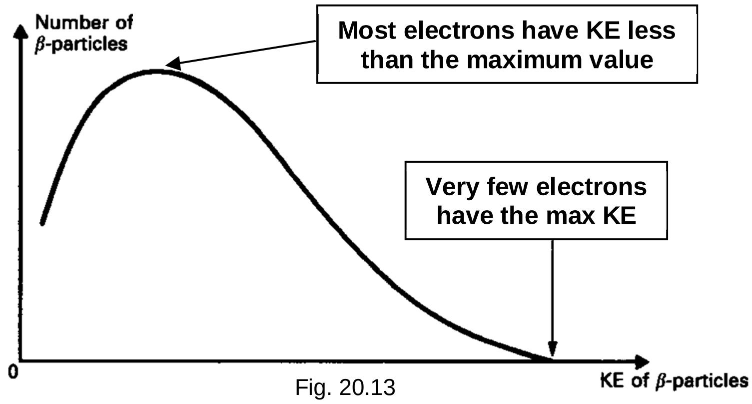


Fig. 20.13

- Violation of the Laws of Conservation of Energy & Momentum?

  If the daughter nucleus and the electron were not carrying away all the energy released, how to account for the missing energy? And why was there a range of energies (instead of just **one** value as predicted in (D)?

  It was as if the Laws of Conservation of Energy and Momentum were violated!

- Prediction of the Neutrino: (years & names are not required)
  In 1930, Wolfgang Pauli predicted that a third particle must be present to carry away the "missing energy" and to conserve momentum.

  In 1932, Enrico Fermi proposed that in beta decay the neutron could decay to a proton, electron and the neutrino (now called an **electron antineutrino**) as shown: ${}^1_0\boldsymbol{n} \rightarrow {}^1_1\boldsymbol{p} + \boldsymbol{e}^- + \bar{\boldsymbol{\nu}}$

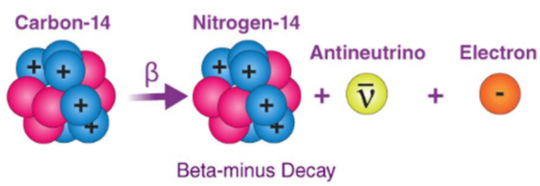


Fig. 20.14

- Discovery of the Neutrino:

  In 1938 the first evidence of the reality of neutrinos/electron antineutrinos came via simultaneous cloud-chamber measurements of the electron and the recoil of the nucleus where the tracks show that the beta particle and daughter nucleus do not move off in a straight line. See figure below.

  If there were no antineutrino, the beta particle and the remaining nucleus were expected to move in a straight line, according to the principle of conservation of linear momentum.

  In 1956 the electron antineutrino was discovered by Cowan and Reines. With that direct detection, the apparent violation of the Laws of Conservation of Energy & Momentum was resolved, a result that was rewarded almost forty years later with the 1995 Nobel Prize.

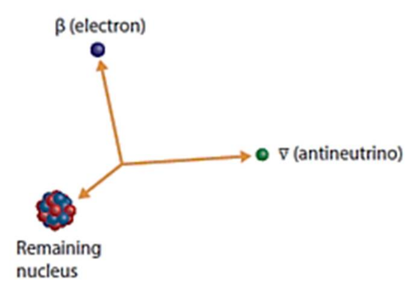


Fig. 20.15

Video on Why Neutrinos Matter https://www.youtube.com/watch?v=nkydJXigkRE	Video on The giant science experiment hunting for the 'ghost particle' https://www.youtube.com/watch?v=aE6vRfCp4E4

20.8 Radioactive decay law

p.	Define the terms activity and decay constant and recall and solve problems using the equation $A = \lambda N$.

20.8.1 The Decay Constant

The **rate of disintegration,** $\frac{dN}{dt}$**,** of a radioactive sample is proportional to the number of undecayed nuclei N, remaining in the sample at that instant
(since radioactive decay is a random process. See Appendix.)

i.e. $\frac{dN}{dt} \propto N$ or $\boxed{\frac{dN}{dt} = -\lambda N}$ ....(8.1)

where λ is the proportionality constant called the **decay constant.**

- The negative sign signifies that N decreases with time; ie dN is negative.
- It is omitted when solving numerical problems.
- From equation (8.1), $\lambda = \frac{(\frac{dN}{N})}{dt}$.

Thus,

The **decay constant** λ is defined as <u>the fraction of the total no. of undecayed nuclei which will decay per unit time,</u>

or, the **probability** of a nucleus decaying **per unit time.**

- Unit of λ: $\mathbf{s^{-1}}$
- The value of λ is characteristic of the radioactive material; it determines the rate at which the nuclei will decay. The larger the value of λ, the more rapidly the nuclei decay (i.e. the higher the activity for a given N).

20.8.2 Activity

The ***activity*, *A*** of a radioactive <u>source</u> is defined as the <u>rate at which the nuclei are disintegrating,</u>

i.e. $A = \frac{dN}{dt}$

- Hence $A = \lambda N$

- S.I. unit of A: *second*$^{-1}$, s^{-1} or the *becquerel* (Bq)
  where 1 Bq = 1 disintegration per second

20.8.3 Radioactive decay law

l.	Infer the random nature of radioactive decay from the fluctuations in count rate.
q.	Infer and sketch the exponential nature of radioactive decay and solve problems using the relationship x = xo e-λt, where x could represent activity, number of undecayed particles or received count rate.

We can rewrite equation (8.1) as $\frac{dN}{N} = -\lambda\, dt$, which can be integrated to give

$$\boxed{N = N_0 e^{-\lambda t}}$$ …….(8.2) **Radioactive Decay Law**

where N = number of undecayed (**active**) nuclei at time t,
N_0 = number of undecayed nuclei at t = 0 (i.e. at the start of the observation)

- The number of undecayed nuclei of a radioactive substance decays **exponentially** with time.

- Since the *activity* A ∝ number of undecayed nuclei at that instant, it can be deduced that the exponential decay law will also apply to it.

$$A = \lambda N \qquad \text{where } A = \text{Activity at time } t$$
$$= \lambda N_0 e^{-\lambda t}$$
$$= A_0 e^{-\lambda t} \qquad \text{where } A_0 = \text{Activity at } t = 0$$
$$A_0 = \lambda N_0$$

Thus, $$\boxed{A = A_0 e^{-\lambda t}}$$ ……(8.3)

- The ***count-rate***, ***C***, refers to the number of counts per unit time recorded by a radiation detector (eg a GM tube connected to a ratemeter).

- Since count-rate C, ∝ activity A, we expect

$$\boxed{C = C_0 e^{-\lambda t}}$$ ……(8.4) where C = count-rate at time t,
C_0 = count-rate at t = 0

- Summarising equations (8.2) ,(8.3) and (8.4), we have

$$\boxed{x = x_0 e^{-\lambda t}}$$ ……(8.5) **Decay law** {Given in List of Formulae}

where x may be the number of undecayed nuclei N, activity A, or (received) count-rate C.

- For equations (8.2) to (8.4), only recall and use is required. Derivation (see Appendix) is not required.

- The random nature of radioactive decay can be inferred from **fluctuations** in the count-rate

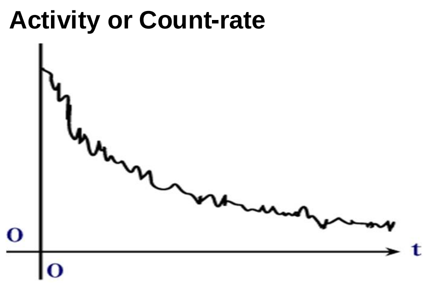


Fig. 20.16a: Actual experimental result

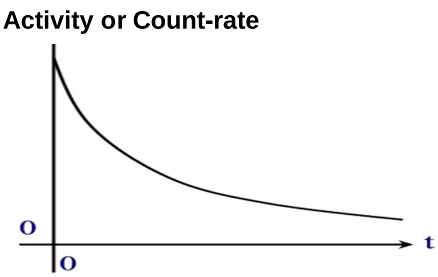


Fig. 20.16b: Best Fit Line is exponential

The problem of the random nature of the **count-rate** in a radioactivity experiment can be dealt with by ignoring the **fluctuations** and drawing the line of best fit.
(See Appendix, pt 2: Random and yet with an Exponential Pattern)

20.8.4 Half-life

r.	Define and use half-life as the time taken for a quantity x to reduce to half its initial value.
s.	Solve problems by using the relation $\lambda = \dfrac{\ln 2}{t_{1/2}}$.

The **half-life** of a radioactive source is defined as the *average* time taken for half the **number of undecayed nuclei** present in the sample to disintegrate.

Alternatively, it can be defined as the average time for the *activity* to be halved.
Common Mistake: It is NOT the average time for half the mass (or amount) to decay.

Using the concept of half-life, when N decreases to $\dfrac{N_0}{2}$, time taken $t = t_{1/2}$. Substituting the above values into $N = N_0\, e^{-\lambda t}$, gives: $\dfrac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$

Solving for $t_{1/2}$,

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$ ……..(8.6) (Derivation is required)

Like λ, half-life is *characteristic* of each nuclide and is an important alternative (and more convenient) means of identifying the nuclide.

Substituting equation (8.6) into $A = \lambda N$ gives:

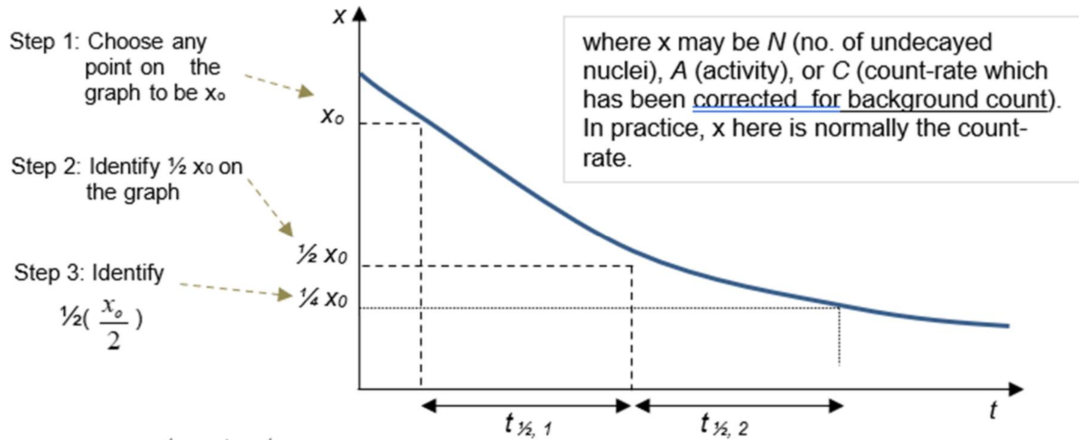
$$\boxed{A = \frac{\ln 2}{t_{1/2}} \, x \, N}$$ ………..(8.7)

Thus, 2 factors which determine Activity, A of a given sample are:

1. half-life (or λ)
2. number of undecayed nuclei N

Determination of $t_{1/2}$ by graphical method

1st Method



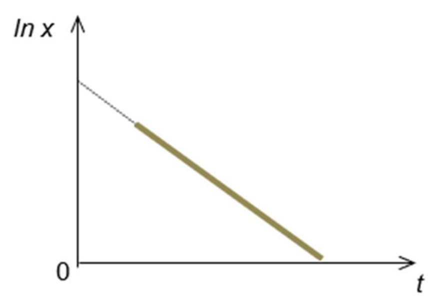
Step 4: $t_{1/2} = \dfrac{t_{1/2,1} + t_{1/2,2}}{2}$ {take average}

2nd Method

Using $x = x_0 e^{-\lambda t}$, $\ln x = \ln x_0 - \lambda t$.

Since λ is a constant,
a plot of $\ln x$ against t will give a linear graph,
where **gradient = $-\lambda$,**
vertical intercept = $\ln x_o$

Half-life can be found by: $t_{1/2} = \dfrac{\ln 2}{\lambda}$



Worked Example 13: Using half-life to determine fraction of remaining sample
Radon-220 is a naturally occurring radioactive gas with a half-life of 54 s. What fraction of a sample of this gas remains after (a) 54 s (b) 108 s.

[½, ¼]

Solution:

(a) Fraction of undecayed nuclei at time t is $\frac{N}{N_0}$.

Using $N = N_o e^{-\lambda t}$,

$\frac{N}{N_0} = e^{-\lambda t}$, where $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{54}$

$\rightarrow \frac{N}{N_0} = ½$

Alternatively: since a time of 54 s corresponds to ***one*** half-life, by definition, the fraction that remains after 54 s = ½

(b) 108 s $\Rightarrow$ 2 half-lives have elapsed, $\Rightarrow N = (1/2)(1/2)N_o = (1/2)^2 N_o$

$$\frac{N}{N_0} = 1/4$$

In general, the number of nuclei that remains undecayed,

$$\boxed{N = \left(\frac{1}{2}\right)^n N_o} \quad \text{........ (8.8)}$$

where n = number of half-lives that have passed = $\frac{t}{t_{1/2}}$

Similarly,

$$\boxed{A = \left(\frac{1}{2}\right)^n A_o} \quad \& \quad \boxed{C = \left(\frac{1}{2}\right)^n C_o} \quad \text{........ (8.9)}$$

Summary of useful formulae:

$$\frac{N}{N_o} = \left(\frac{1}{2}\right)^n \qquad \frac{A}{A_o} = \left(\frac{1}{2}\right)^n \qquad \frac{C}{C_o} = \left(\frac{1}{2}\right)^n$$

$$N = N_o e^{-\lambda t} \qquad A = A_o e^{-\lambda t} \qquad C = C_o e^{-\lambda t}$$

$$\frac{dN}{dt} \propto N \qquad \frac{dN}{dt} = -\lambda N \qquad A = \frac{dN}{dt} = \lambda N \qquad t_{1/2} = \frac{\ln 2}{\lambda}$$

$$A = \frac{\ln 2}{t_{1/2}} \times N \qquad \text{No. of nuclei in sample} = \frac{\text{sample mass}}{\text{mass of 1 mol}} \times N_A$$

Example 14: Application of Decay Law

Antimony-124 has a half-life of 60 days. If a sample of antimony-124 has an initial activity of 6.5×10^6 Bq, what will its activity be after 1 year?

[9.6×10^4 Bq]

Solution:

Using $A = A_0 e^{-\lambda t}$ and $t_{1/2} = \dfrac{\ln 2}{\lambda}$

Note: Neither t nor $t_{1/2}$ needs to be substituted in the unit of seconds as long as they are in the same units. Here it's more convenient to use the unit of "days".

Example 15 (J85P2Q7) Relationship betw Number of Nuclei in a Sample & the Mass of Sample

${}^{32}_{15}P$ is a beta-emitter with a decay constant of $5.6 \times 10^{-7}\ s^{-1}$. For a particular application the initial rate of disintegration must yield 4.0×10^7 beta particles every second.

(i) How many nuclei (or atoms, to a good approximation) does this mass correspond to?

(ii) What mass of pure ${}^{32}_{15}P$ will give this decay rate?

[7.14×10^{13}; 3.8×10^{-12} kg]

Solution:

Given: $\lambda = 5.6 \times 10^{-7}\ s^{-1}$, Initial $\dfrac{dN}{dt}$, i.e. $A_0 = 4.0 \times 10^7\ s^{-1}$

(i) Using,

$A_o = \lambda N_o$,

Required N_o =

(ii) mass of 1 mol of ${}^{32}_{15}P$ = 32 g = 32×10^{-3} kg

Recall: Mass of 1 mol of nuclide= nucleon number of nuclide (or relative atomic mass) expressed in grams, NOT in kg

Number of nuclei in sample = $\dfrac{\text{Sample Mass}}{\text{Mass of 1 mol}} \times N_A$

Since 1 mol of nuclei, N_A $\propto$ mass of 1 mol of nuclei,

and no. of nuclei in sample $\propto$ mass of sample

Sample mass =

=

Worked Example 16 (2014 P3Q8b): Spacecraft powered by Radioactivity – a 6 mark calculation

In a Voyager spacecraft, electrical power is provided using plutonium-238 (${}^{238}_{94}\boldsymbol{Pu}$).
Plutonium-238 nuclei emit α-particles of energy 5.48 MeV.
The half-life of plutonium-238 is 86.4 years.
Some of the energy of the emitted α-particles is converted into thermal energy and then into electrical energy.
Calculate (i) the probability per second of the decay of a plutonium-238 nucleus. [3]
(ii) the mass of plutonium-238 required for the energy per unit time of the emitted α-particles to be 2400 W. Explain your working. [6]

Solution:

(i) probability per second of decay = decay const λ

$= \frac{\boldsymbol{ln2}}{\boldsymbol{t_{\frac{1}{2}}}}$ where $\boldsymbol{t_{\frac{1}{2}}}$= 86.4 years = 86.4 x(365 x 24 x 60 x 60) s

= 2.54 x 10^{-10} s^{-1}

(ii) Let M = mass of plutonium required.
Each α-particle provides 5.48 MeV (given) = 5.48 x (10^6) x (1.6 x 10^{-19}) Joules…..(A)

Since activity A = λN,
number of α-particles emitted per sec = λN, where N = Number of ${}^{238}_{94}\boldsymbol{Pu\ nuclei\ required}$
(Each decay of ${}^{238}_{94}\boldsymbol{Pu}$ emits one α-particle)

From (i) = 2.54 x 10^{-10} x N ……….. (B)

where number of ${}^{238}_{94}\boldsymbol{Pu\ nuclei\ required}, \boldsymbol{N} = \frac{\boldsymbol{M}}{\boldsymbol{Molar\ Mass}}\boldsymbol{x\ Avogadro\ Const}$

$= \frac{\boldsymbol{M}}{\boldsymbol{238x10^{-3}}}$ x 6.02 x 10^{23} …………(C)

Thus, (C) into (B) gives: number of α emitted per sec = (2.54 x 10^{-10}) x $\frac{\boldsymbol{M}}{\boldsymbol{238x10^{-3}}}$ x 6.02 x 10^{23}..(D)

Deduce that (A) x (D) = the required power of 2400 W

ie, 5.48 x (10^6) x (1.6 x 10^{-19}) x (2.54 x 10^{-10}) x $\frac{\boldsymbol{M}}{\boldsymbol{238x10^{-3}}}$ x 6.02 x 10^{23} = 2400 W

Solving for M, M = 4.26 kg

(Alternatively, to det number of ${}^{238}_{94}\boldsymbol{Pu\ nuclei\ required}, \boldsymbol{N}$ in (C):

1 ${}^{238}_{94}\boldsymbol{Pu\ nucleus\ has\ 238\ nucleons, each\ of\ mass\ 1\ u}$.
Thus number of ${}^{238}_{94}\boldsymbol{Pu\ nuclei\ required}, \boldsymbol{N} = \frac{\boldsymbol{M}}{\boldsymbol{238\ u}}$, where 1 u = 1.66 x 10^{-27} kg)

Worked Example 17 (2021 P3 Q9(c) (i)) Probability of decay in a certain Time Interval

The isotope beryllium-7 (${}^{7}_{4}\mathbf{Be}$) is radioactive with a half-life of 53 days. A beryllium-7 nucleus decays by the emission of a γ-ray photon of energy 0.48 MeV.
(i) Determine the probability of decay of a beryllium-7 nucleus **in a time of 1.0 day**.

Soln:($\frac{\boldsymbol{dN}}{\boldsymbol{N}}$ = λdt is the probability of decay in a time dt)

Hence for 1.0 day, the probability = λ x 1 day
= λ x (24 x 3600 s) (A)

where $\lambda = \frac{\boldsymbol{ln2}}{\boldsymbol{t_{\frac{1}{2}}}} = \frac{\boldsymbol{ln2}}{\boldsymbol{53\ x\ (24x3600)}}$ =1.5 x 10^{-7}s (B)

Thus in (A), probability = 0.013.
(Note: OK to work with the time in BOTH eqns in 'day' instead of converting to 'second'.)

20.9 Background Radiation

m.	Show an understanding of the origin and significance of background radiation.

Background radiation refers to **radiation from sources other than the source being considered.**

We are all unavoidably exposed to background radiation, no matter where we live. Primary contributions of background radiation from natural radiation sources include: (no need to recall)

1. cosmic rays (from outer space),
2. radon in the air (Radon gas is part of the natural decay series of uranium found in rocks of the earth's crust & it seeps into houses through cracks in the foundations and become attached to airborne dust. It is often the single largest contributor to an individual's background radiation dose.)
3. food & water (particularly an isotope of potassium which are then incorporated in our body).

A very small fraction of background radiation comes from man-made sources such as:

1. radioactive contamination due to nuclear weapons tests; the two nuclear attacks against Japan; & the tsunami-flooded, earthquake-stricken nuclear plant in Japan in 2011
2. medical X-rays (especially through CAT scans) & radiation therapy

Significance of Background Radiation:
In a laboratory, background radiation refers to the measured value from any sources that affect an instrument when a radiation source sample is not being measured. Background radiation thus makes it necessary to correct any experiment in which radioactivity is measured.

Two precautionary steps:

(i) Carry out a background count using the *same* detector to be used in the experiment and *in the location where the experiment is to be done* (in absence of radioactive source)

(ii) The background count rate must then be deducted from subsequent measurements made so as to obtain the true (or sometimes called 'corrected') count rate.

True count rate = Measured count rate – Background count rate

However, in experiments where the count rate is much greater than the background count rate, this correction for background count will be unnecessary. Background radiation from location to location. There are many factors that influence exposure levels, including naturally-occurring radionuclides in the soil and altitude. The typical average value can be taken as about order of 10 count per minute e.g 20 counts per minute (Take note of this value given in question).

Example 18 (J94P1Q30): Taking into account the background count

A radioactive source contains the nuclide $^{187}_{74}W$ which has a half-life of 24 hours.
In the absence of this source, a constant *average* count-rate of 10 s^{-1} is recorded.
Immediately after the source is placed in a *fixed position* near the counter, the *average* count-rate rises to 90 s^{-1}.

What *average* count-rate is expected with the source still in place 24 hours later?

A 30 s^{-1} B 40 s^{-1} C 45 s^{-1} D 50 s^{-1}

Solution:
Note: Question is asking for the measured count rate, NOT the true count rate.

Deduce: background count-rate = 10 s^{-1}

Thus, at t = 0, corrected count-rate C_0 = Use $C = C_0 e^{-\lambda t}$, where t = 24 h & $t_{1/2} = \frac{\ln 2}{\lambda}$

C =

Therefore, measured count rate = i.e. Option …….

Example 19 (J91P1Q30): Radiocarbon Dating (C-14 Dating)

Video on How Does Radiocarbon Dating Works
https://www.youtube.com/watch?v=phZeE7Att_s

Concept:

- Ratio of C-14 to C-12 nuclei in all living matter is a constant value (equal to the ratio in the atmosphere)
- When the organism dies, the activity of the C-14 in the dead organism starts to decrease with time since no fresh carbon is taken in.
- By comparing the residual activity of a dead specimen with that of a living one of the *same mass*, the age (actually, how long it has been dead) of any carbon-containing material may be calculated.

Radioactive ^{14}C dating was used to find the age of a wooden archaeological specimen. Measurements were taken in three situations for which the following count-rates were obtained:

Specimen	count-rate
1 g sample of living wood	80 counts per minute
1 g sample of archaeological specimen	35 counts per minute
no sample	20 counts per minute

If the half-life of ^{14}C is known to be 5 700 years, what was the approximate age of the archaeological specimen? [1.1 x 10^4 years]

Solution:

Background count-rate = 20 per min

Using $C = C_0 e^{-\lambda t}$ and $t_{1/2} = \dfrac{\ln 2}{\lambda}$

C_0 = corrected count-rate of specimen when it just died (t = 0)
= corrected count-rate of equal mass of any living plant
= 80 – background count-rate
= 80 – 20 = 60 per min

C = corrected count-rate of specimen now
=

Substituting,
$\Rightarrow$ t =

Worked Example 20: Radioactive dating of rocks containing U-238

In a particular sample of rock, it is found that the ratio of ${}^{238}U$ to ${}^{206}Pb$ is 3:5. The decay process of U-238 (uranium decay series) eventually converts a ${}^{238}U$ atom to ${}^{206}Pb$ (**stable** end-product). Taking the half-life of U-238 as 4.5 x 10^9 years, estimate the age of the rock.

State any assumption you make.

[6.4 x 10^9 years]

Solution:

For U-238, $N = N_0 e^{-\lambda t}$ where N = number of undecayed U-238 nuclei remaining in the rock,
N_0 = original number of U-238 nuclei in the rock

Let N_0 be the number of U nuclei present when the rock was formed.

Assumption 1: No addition or removal of uranium by whatever means occurred during the lifetime of the rock.

Rewriting, $\frac{N}{N_0} = e^{-\lambda t}$ (1)

Assumption 2: Every Pb atom found in the rock came only from the decay of U-238 and not from any other source and none was subsequently removed,

it can thus be deduced that for U-238, $\frac{N}{N_0} = \frac{3}{5+3} = \frac{3}{8}$

Sub into (1), $\frac{3}{8} = e^{-\lambda t}$ where $t_{1/2} = \frac{\ln 2}{\lambda}$, $t_{1/2}$ = 4.5 x 10^9 years

$\rightarrow$ t = 6.4 x 10^9 years

Alternatively,

Since every Pb atom is assumed to have come only from the decay of a U-238 atom,

N_0 (of U-238) = $N_u + N_{Pb}$ where N_u = present number of U-238 atoms,
N_{Pb} = present no of Pb atoms.

Thus $N = N_0 e^{-\lambda t}$ becomes $N_u = (N_u + N_{Pb})\, e^{-\lambda t}$

i.e. $\frac{N_u + N_{Pb}}{N_u} = e^{+\lambda t}$

$1 + \frac{N_{Pb}}{N_u} = e^{+\lambda t}$ where $\frac{N_{Pb}}{N_u} = \mathbf{5/3}$

Solving, $\rightarrow$ t = 6.4 x 10^9 years

20.10 Direct & Indirect Effects of Ionising Radiation on Living Tissues & Cells

t.	Discuss qualitatively the effects, both direct and indirect, of ionising radiation on living tissues and cells.

Ionising radiation may damage or destroy biologically important molecules directly or indirectly.

Direct effect	Indirect effect
This is when radiation interacts with the atoms of the DNA molecule directly, or some other cellular component critical to the survival of the cell. Such an interaction may damage or destroy the cell by "direct" interference with its life-sustaining system.	This occurs through chemical changes to the surrounding **medium** of the cells, which is mainly water. The ionization of water molecules produces H and OH free radicals which could combine to form toxic substances, such as hydrogen peroxide (H_2O_2), which can then attack the chromosomes in the nucleus of each cell.

Low doses	High doses
▪ tend to cause harmful changes in the DNA (i.e. mutations) ▪ increase the risk of longer term effects such as cancer.	▪ can kill so many cells that tissues and organs are damaged.

Example 21 (H1 N23P2Q7): Effects of Ionising Radiation

Discuss the effect of **different amounts** of ionizing radiation on living cells [4]

Appendix: Derivation of N = N_o exp ($-\lambda$t) (Enrichment)

Radioactive decay is a completely random process in which nuclei disintegrate independently. Since there is always a very large number of active nuclei (i.e. undecayed nuclei) in a given amount of radioactive material, we can apply the *method of statistics,* i.e. *the law of chance,* to the process and obtain an expression for the fraction of the nuclei originally present that will have decayed on average in a given time interval.

If we have *N* undecayed nuclei (or atoms) at a particular time *t*, we can write the rate of disintegration (decay) of the number of undecayed atoms with time as $\frac{dN}{dt}$.

This rate of disintegration is thus proportional to the number of undecayed atoms *N* present at that time - by the law of chance or the method of statistics, so may write:

$$\boxed{\frac{dN}{dt} = -\lambda N} \quad \text{.......... (1)}$$

where λ is the proportionality constant, called the decay constant. (The negative sign is included because *N* is a decreasing quantity and therefore $\frac{dN}{dt}$ is negative while *N* itself is +ve and so, a negative sign is required to ensure the equality of equation (1).)

If there are N_o undecayed nuclei at some time *t* = 0 and a smaller number *N* at a later time *t*, then integrating (1),

$$\int_{N_o}^{N} \frac{dN}{N} = -\lambda \int_o^t dt$$

Thus, $$[\ln N]_{N_o}^{N} = -\lambda t$$

$$\ln N - \ln N_o = -\lambda t$$

$$\ln \frac{N}{N_o} = -\lambda t$$

$$e^{-\lambda t} = \frac{N}{N_o}$$

$$\boxed{N = N_o e^{-\lambda t}} \quad \text{........(2)}$$ Derivation is not required.

Note

1. This is the **radioactive decay law** and it states that a radioactive substance (i.e. the number of undecayed nuclei NOT the *mass* or the *amount*) decays exponentially with time.

2. Random and yet with an Exponential Pattern
   - The law is a statistical one; it does not tell us when a particular nucleus will decay but only that after a certain time, a certain fraction will have decayed.
   - At the level of the *individual nucleus*, the process is purely random as is evident from the fluctuations which occur when radiations from a source are counted.
   - At the macroscopic level, i.e. when the behaviour of the *group* of radioactive nuclei is considered *as a whole*, there is a definite pattern to the behaviour - the exponential decrease.
     (Analogy:

if you throw a dice say , 500 times, you can be quite certain that 1/6 of the times, you will obtain a given number, say 3; but you can never be certain at which throw, the dice will give the number 3.)

TUTORIAL 20: NUCLEAR PHYSICS

Rutherford's scattering experiment

(L1) 1. (J95P3Q6)

(a) In the alpha-scattering experiment, alpha-particles, travelling in a vacuum, are incident on a gold foil. Draw on the same diagram, 3 paths of an alpha-particle which

(i) is directly towards the nucleus of a gold atom,
(ii) passes close to the nucleus of a gold atom,
(iii) passes some distance from the nucleus. [3]

(b) Describe and explain how the alpha-particle scattering experiment which you have illustrated in part (a) gives evidence for the existence and small size of the nucleus. [4]

(L2) 2. (N2021P2Q7)

In the original Rutherford alpha particle scattering experiment, alpha particles were fired at a thin gold ($^{197}_{79}Au$) foil. As a result of the experiment, the model of the atom was changed.

(a) State and explain the evidence from this experiment that changed the model of the atom in terms of its charge distribution and its mass distribution.

charge distribution ....

....

mass distribution ....

....

....

....
[4]

(b) The alpha particles had an initial energy of 5.59 MeV when a long distance from a gold ($^{197}_{79}Au$) nucleus.

Calculate the minimum possible separation between an alpha particle and the gold nucleus.

[3]

Balancing Nuclear Equations

(L1) 3. (J80P2Q34)

The decay of $^{238}_{92}U$ to $^{239}_{93}Np$ by β-emission is not possible because

A β-decay only occurs in isotopes of low mass

B $^{239}_{93}Np$ is not a stable isotope

C nucleon number cannot increase in a decay process

D proton number cannot increase in a decay process

E nucleon and proton numbers must both decrease in a decay process

Mass Defect & Binding Energy

(L1) 4. (N98P1Q29)

The nucleus of the nuclide $^{A}_{Z}X$ has a mass M. In terms of the rest mass of the proton m_p, the rest mass of the neutron m_n and the velocity of light c, write down an expression for the binding energy per nucleon of this nucleus.

(L1) 5. (N90P3Q6b)

An atom of magnesium consists of:

12 electrons each of mass 0.00055 u
12 protons each of mass 1.00728 u
13 neutrons each of mass 1.00866 u

(i) Which particles are in the nucleus and what is the nucleon number of this nucleus?
(ii) Which no. determines that the atom is an atom of magnesium rather than of any other element?

(iii) The mass of the magnesium atom is 24.98584 u. Calculate the total mass of the constituent particles. Explain this difference.

(iv) Calculate, in joules, the binding energy of the atom.

(L2) 6 (i) Sketch a graph to show the variation with nucleon number of the binding energy per nucleon of nuclei. [2]

(ii) By reference to your graph, explain how the process of nuclear fusion may result in the release of energy. [2]

(L2) 7. (N2021P2Q7c)

(c) Alpha particles may be produced from the radioactive decay of an isotope of radon ($^{222}_{86}Rn$) into an isotope of polonium (Po).

A stationary nucleus of radon emits an alpha particle.
The alpha particle has a binding energy per nucleon of 7.08 MeV.
The products of the radioactive decay have a total kinetic energy of 6.62 MeV.

The binding energy per nucleon of radon is 7.69 MeV.

Determine the binding energy per nucleon of polonium. Show your working.

[4]

Principle of Conservation of Energy-Mass

(L2) 8. (N97P3Q6part)
When Uranium-235 nuclei undergo fission by slow-moving neutrons, 2 possible reactions are:

Reaction 1: $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{139}_{54}Xe + ^{95}_{38}Sr + 2\,^{1}_{0}n$ + energy, Reaction 2: $^{235}_{92}U + ^{1}_{0}n \rightarrow 2\ ^{116}_{46}Pd + xc +$ energy,

(a) For reaction 2, identify the particle c and state the number x of such particles produced in the reaction. [2]

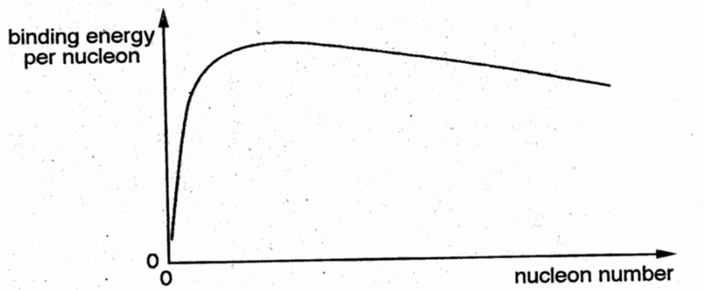
(b) The binding energy per nucleon E for a number of nuclides is given.

Nuclide	E / Mev
$^{95}_{38}Sr$	8.74
$^{139}_{54}Xe$	8.39
$^{235}_{92}U$	7.60

(i) Show that the energy released in reaction 1 is 210 MeV.

(ii) The energy released in reaction 2 is 163 MeV. Suggest, with a reason, which one of the two reactions is more likely to occur. [6]

(L2) 9. (J04P4Q8)
The figure shows the variation with nucleon number of the binding energy per nucleon of a nucleus.



(a) On the figure, mark with the letter S the position of the nucleus with the greatest stability

(b) One possible fission reaction is:

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{144}_{56}\mathrm{Ba} + {}^{90}_{36}\mathrm{Kr} + 2\,{}^{1}_{0}\mathrm{n}.$$

(i) On the figure, mark possible positions for

(1) the Uranium-235 ($^{235}_{92}$U) nucleus (label this position U), [1]

(2) the Krypton-90 ($^{90}_{36}$Kr) nucleus (label this position Kr). [1]

(ii) The binding energy per nucleon of each nucleus is as follows:

$^{235}_{92}$U	1.2191×10^{-12} J
$^{144}_{56}$Ba	1.3341×10^{-12} J
$^{90}_{36}$Kr	1.3864×10^{-12} J

Use this data to calculate

(1) the energy released in this fission reaction (give your answer to three significant figures), [3]

(2) the mass equivalent of this energy. [2]

(iii) Suggest why the neutrons were not included in your calculation in (ii). [1]

(L2) 10. Annihilation (J08 P4Q8)

An electron and a positron (a particle of equal mass to an electron but with a positive charge) will interact with each other to form two γ-ray photons.

$$^{0}_{+1}e + ^{0}_{-1}e \rightarrow 2\gamma$$

Assuming that the kinetic energy of the positron and the electron is negligible when they interact,

(a) suggest why the 2 photons will move off in opposite directions with equal energies, [3]

(b) calculate the energy of one of the γ-ray photons. [2]

(L2) 11. (2008P3Q7b)

A stationary Uranium-235 nucleus absorbs a slow neutron and undergoes fission. Occasionally, a fission can take place in which no neutrons are emitted. One such fission is shown by the following nuclear equation.

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{x}_{52}Te + ^{98}_{y}Zr$$

The masses of these particles are

uranium	$^{235}_{92}U$	235.0439 u
tellurium	$^{x}_{52}Te$	137.9603 u
zirconium	$^{98}_{y}Zr$	97.9197 u
neutron	$^{1}_{0}n$	1.0087 u

(i) State the values of x and y in the equation. [2]

(ii) Deduce the energy released in the fission reaction. [4]

(iii) Of the energy released, 2.3×10^{-11} J becomes kinetic energy of the tellurium and zirconium nuclei. State one way in which the remaining energy may be released. [1]

(iv) Calculate

1. the ratio $\dfrac{\text{speed of zirconium nucleus}}{\text{speed of tellurium nucleus}}$, [2]

2. the ratio $\dfrac{\text{kinetic energy of zirconium nucleus}}{\text{kinetic energy of tellurium nucleus}}$. [2]

(v) Deduce the speed of the zirconium nucleus. [3]

(vi) State the two assumptions that you made in your calculations in (iv) part 1. [2]

(L2) 12. (N2019/P3Q9b)

(b) When an α-particle bombards a stationary nitrogen nucleus, a nuclear reaction that can take place is given by the equation

$$^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{H}.$$

Data for the nuclei in the reaction are given in Fig. 9.1.

nucleus	mass/u
α-particle $^{4}_{2}\text{He}$	4.002604
hydrogen $^{1}_{1}\text{H}$	1.007825
nitrogen $^{14}_{7}\text{N}$	14.003074
oxygen $^{17}_{8}\text{O}$	16.999130

Fig. 9.1

(i) Use data from Fig. 9.1 to determine, to three significant figures, the energy associated with the change in mass in this reaction.

[4]

(ii) The incident α-particle has a kinetic energy of 1.76×10^{-13} J.

State and explain whether the reaction will take place.

....

....

.... **[2]**

Nature of α, β, γ-rays & Balancing Nuclear Equations

(L1) 13. (N92P1Q28)
As a result of successive decays in a radioactive series, the nucleon number (mass number) of an isotope decreases by 4 while its proton number (atomic number) is unchanged. How many α-particles and β-particles are emitted?

number of particles emitted

	α	β
A	1	1
B	1	2
C	1	4
D	2	1
E	2	2

(L2) 14. Ionisation Current (J2000 P1Q30)
A radioactive source produces 10^6 α-particles per second. When all the ions produced in air by these α-particles are collected, the ionization current is about 0.01 μA. If the charge on an ion is about 10^{-19} C, estimate the average number of ions produced by each α-particle.

(L2) 15. Discovery of Neutrino (9749 Specimen Paper 3)
Strontium-90 undergoes beta decay to form yttrium.
The emitted beta particles have a range of energies up to a maximum of 0.55 MeV.
Use conservation laws to explain why this range of energies leads to the suggestion that another particle is emitted by the decaying strontium-90 nucleus together with the beta particle. [4]

Radioactive decay law

(L1) 16. {N93P1Q29)
A radioactive isotope has a decay constant λ and a molar mass *M*. Taking the Avogadro constant to be *L,* what is the activity of a sample of mass *m* of this isotope?

(L1) 17. (N95P2Q7)
The activity of a piece of radioactive material is 4.3×10^5 Bq at time t = 0.
The number of undecayed atoms in the material at time t = 0 is 7.9×10^{15}.
Calculate
(a) (i) the activity after 4.0 half-lives have elapsed,
(ii) the number of undecayed atoms after 4.0 half-lives, [4]

(b) the decay constant λ, [3]

(c) the half-life [2]

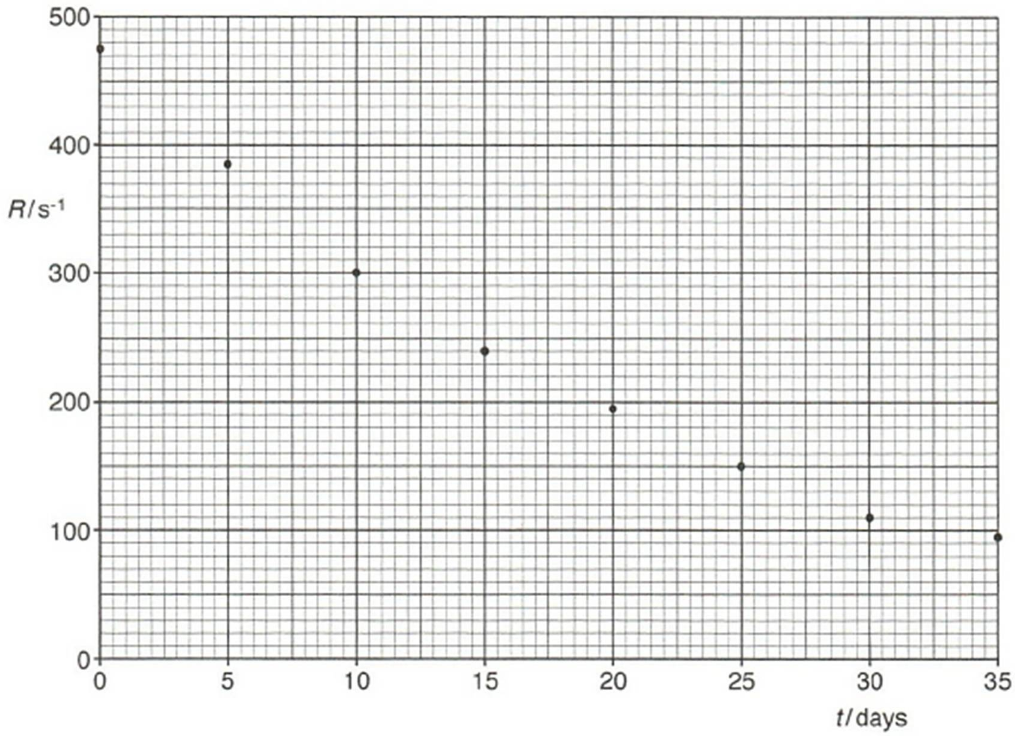
(L2) 18. (N94P2Q6)
A student stated that "radioactive materials with a short half-life always have a high activity".
(a) What is meant by (i) half-life, (ii) activity? [3]
(b) Discuss whether the student's statement is valid. [3]

(L2) 19. (N94P1Q30)
Samples of 2 radioactive nuclides, X and **Y**, each have equal activity A_0 at time $t = 0$.
X has a half-life of 24 years and **Y** a half-life of 16 years. The samples are mixed together.
What will be the total activity of the mixture at $t = 48$ years? [2]

(L2) 20. (N2016P3Q6)

A student determines the half-life of the radioactive isotope phosphorus-32.
Phosphorus-32 decays by beta emission (β-emission) to form sulfur-32 which is stable.

The student measures the average count rate R from a sample of phosphorus-32 at various times t.
The results are shown in Fig. 6.1.



(a) Suggest why

(i) the determination of the half-life of phosphorus-32 by this method requires that the product of the decay is stable,

....

....[1]

(ii) the student did not need to make an allowance for background radiation.

....

....

.... [2]

(b) Use Fig. 6.1 to determine a value for the half-life, in days, of phosphorus-32.

half-life = ... days [3]

(c) Explain why, although the count rates are too low for the radiation to cause immediate symptoms in the student, careful shielding of the source is necessary.

....

....

.... [2]

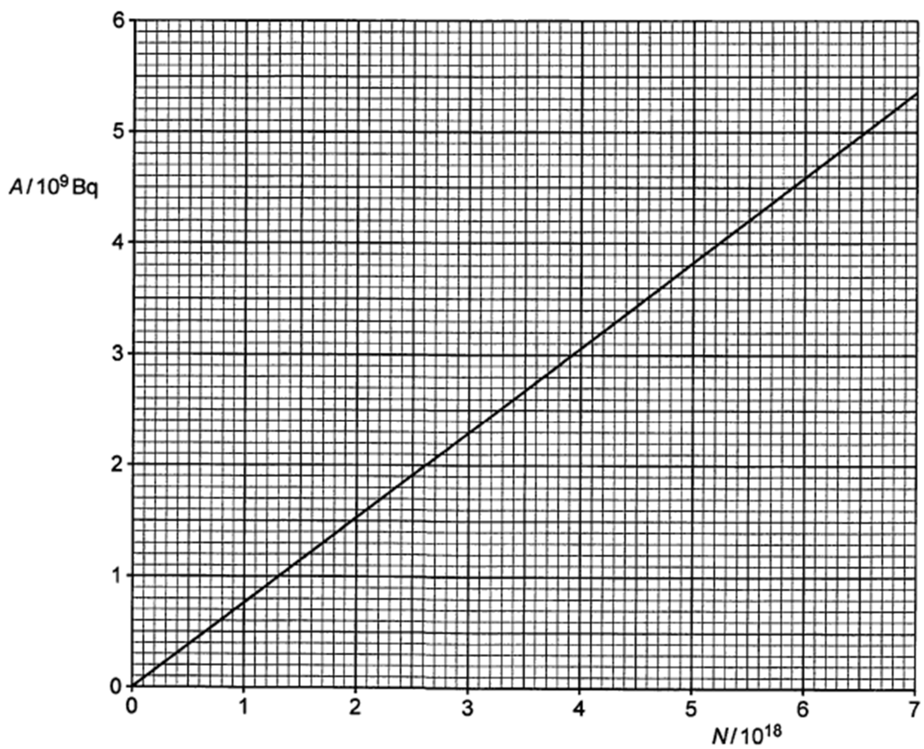
(L2) 21. (N2022P2Q6d)

(d) Strontium-90 ($^{90}_{38}Sr$) emits a beta particle to become yttrium (Y).

(i) Complete the nuclear decay equation, including all the decay products.

$^{90}_{38}Sr \rightarrow$ **[3]**

(ii) The activity A of a sample of unstable strontium-90 varies with the number N of its nuclei, as shown in Fig. 6.2.



Determine the half-life of strontium-90.

[2]

(e) An unstable isotope J decays into a stable isotope K.

Determine the ratio:

$$\frac{\text{number of atoms of J}}{\text{number of atoms of K}}$$

for an initially pure sample of isotope J after a time equal to 3.5 half-lives.

[3]

(L2) 22. C-14 Dating: (N77P2Q35)

The $^{14}C :\ ^{12}C$ ratio of **living** material has a constant value during life but the ratio decreases after death because the ^{14}C is not replaced. The half-life of ^{14}C is 5 600 years.

The ^{14}C content of a 5 g sample of **living** wood has a radioactive count rate of about 100 per minute. If the count rate of a 10 g sample of **ancient wood** is 50 per minute, estimate the age of the sample.

(L2) 23. (N99P2Q7)

One isotope of potassium, $^{40}_{19}K$, has a half-life of 1.4 x 10^9 years and decays to form argon, $^{40}_{18}Ar$, which is stable. A sample of rock taken from the Sea of Tranquility on the Moon contains both potassium and argon in the ratio:

$$\frac{\text{number of Potassium 40 atoms}}{\text{number of Argon 40 atoms}} = \frac{1}{7}$$

(a) The decaying potassium nucleus emits a particle X.

(i) Write down the nuclear equation representing this decay. [1]

(ii) Suggest an identity of X. [1]

(b) Assume that when the rock was formed, there was no Argon-40 present in the sample and that none has escaped subsequently.

(i) Estimate the age of the rock. [3]

(ii) State, with a reason, whether your answer in (i) is an overestimate or an underestimate of the age of the rock if some escape of argon has occurred. [2]

(L2) 24. (N87P2Q12 part)

The equation $^{238}_{92}U \rightarrow {}^{234}_{90}Th + \quad \alpha$

represents the decay, by alpha-emission, of a uranium nucleus.

The half-life of $^{238}_{92}U$ is 1.42 x 10^{19} s. What mass of this nuclide would give an emission of one α-particle per second?

Numerical Answers

2 (b) 4.07×10^{-14} m
5 (iii) 25.20654 u (iv) 3.3×10^{-11} J
7 (c) 7.73 MeV
8 (a) 4 (b)(i) 210 MeV (ii) reaction 1
9 (b) (ii)(1) 3.04×10^{-11} J; (2) 3.38×10^{-28} kg
10 (b) 8.2×10^{-14} J
11 (i) 138, 40 (ii) 2.58×10^{-11}J (iv) 1.41 (v) 1.29×10^{7} m s^{-1}
12 (b)(i) 1.91×10^{-13} J (ii) will not
14 10^{5}
16 $A = \frac{\lambda m L}{M}$

17 (a)(i) 2.69×10^{4} Bq, (ii) 4.94×10^{14}, (b) 5.44×10^{-11} s^{-1}, (c) 1.27×10^{10} s
19 3/8 A_0
20 (b) 15.1 days

21 (d)(ii) 9.04×10^{8} s (e) 0.097
22 11200 years
23 (b)(i) 4.2×10^{9} yrs
24 8.10×10^{-6} kg

ADDITIONAL QUESTIONS

2017 CJC P3

1 (a) Before Rutherford's alpha scattering experiment, the plum-pudding model (Fig. 6.1) was a proposed model of the atom. In this model, the atom is a sphere (the "pudding"), the positive charges are uniformly distributed throughout the sphere, and the negative charges (the "plums") were found throughout the positive sphere.

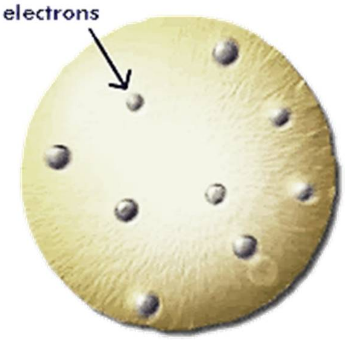


Fig. 6.1

Explain how the results in Rutherford's alpha scattering experiment showed that the atom was made up of a small nucleus instead of the plum-pudding model.

....

....

....

....

....

.... [3]

(b) An oxygen fusion reaction that occurs in red supergiants is given below. The energy released in the reaction is 9.594 MeV.

$$ {}^{16}_{8}O + {}^{16}_{8}O \rightarrow {}^{28}_{14}Si + {}^{4}_{2}He $$

Mass of ${}^{16}_{8}O$ = 15.9905 u
Mass of ${}^{4}_{2}He$ = 4.0015 u
Mass of neutron = 1.0087 u
Mass of proton = 1.0073 u

(i) Calculate the binding energy per nucleon of a ${}^{16}_{8}O$ nucleus.

binding energy per nucleon = ... J [3]

(ii) Calculate the mass of the $^{28}_{14}Si$ nucleus to 6 significant figures.

mass of the $^{28}_{14}Si$ = ... kg [3]

2017 CJC P3

2 **(a)** The thickness of paper is measured using a beta radiation source and detector as shown in Fig. 8.1.

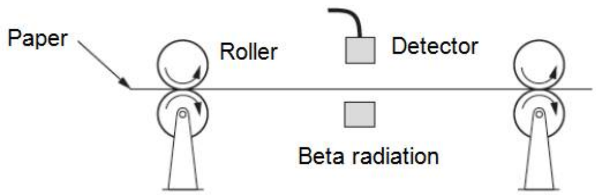


Fig. 8.1

State why it would be inappropriate to use either alpha radiation or gamma radiation for this task.

....

....

.... [2]

(b) Fig. 8.2 shows three different beta radiation sources.

Beta radiation source	**Half-life (years)**	**Remarks**
Nickel-66	0.0063	Decays to Copper-66, a pure beta emitter with a 29 seconds half-life.
Sulfur-35	0.240	
Strontium-90	28.8	Decays to Yttrium-90, a beta emitter with a 2.7 days half-life.

Fig. 8.2

(i) Define *half-life*.

....

....

.... [2]

(ii) Explain why Strontium-90 would be most appropriate for this task.

....

....

....

....

.... [2]

(iii) Calculate the decay constant of Strontium-90.

decay constant = ... year $^{-1}$ [1]

(iv) If the initial activity of the Strontium-90 source is 140 GBq, calculate its activity after 10 years.

activity = ... GBq [2]

(v) In Fig. 8.3, sketch a graph to show how, starting with an initial activity of 140 GBq, the activity of strontium-90 changes with time. Label the axes with suitable values.

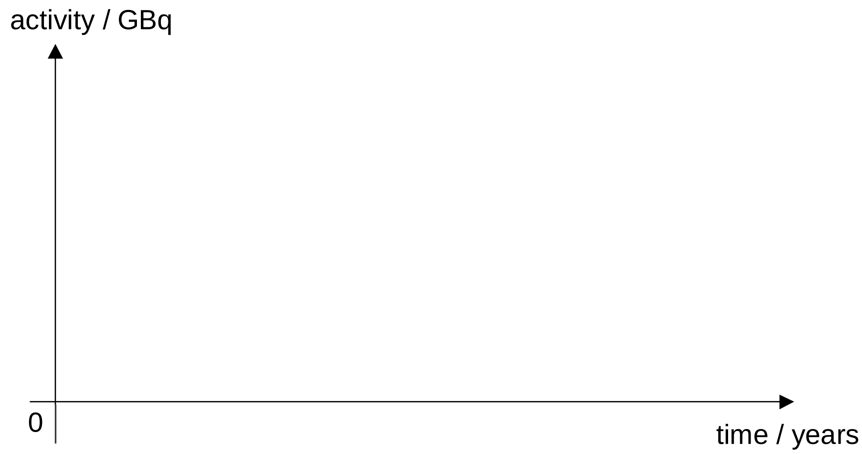


Fig. 8.3

[2]

(vi) Suggest why it is almost impossible to sketch the activity of Yttrium-90 on the same axes of Fig. 8.3 if the scale is not a logarithm scale.

....

....

....

....

.... [2]

(c) Suggest one more use for radiation in our daily life.

....

.... [1]

....

(d) To illustrate the effect of a uniform electric field on the paths of emissions from a radioactive source when travelling in a vacuum, a student draws a diagram similar to Fig. 8.4.

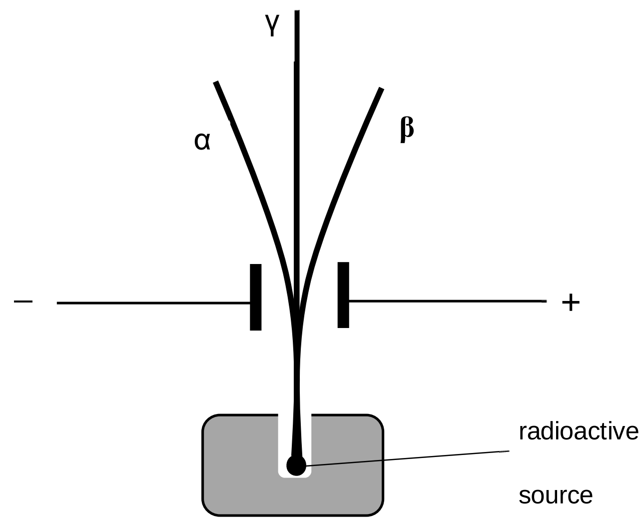


Fig.8.4

(i) Explain why there is no need to consider the effect of gravitational field on the paths of the emissions.

....

....

....

.... [2]

(ii) Explain why the paths of the alpha (α) particles and beta (β) particles curve when in the electric field.

....

....

....

.... [2]

(iii) Suggest two reasons why a pattern of tracks, such as that shown in Fig.8.4, is impossible to observe in practice.

....

....

....

.... [2]

2017 HCI P2

3 **(a)** Isotope X undergoes radioactive decay to form isotope Y. The half-life of isotope X is 2.0×10^5 years. The activity of a pure sample of isotope X extracted from an ore is measured to be 1.1×10^7 Bq.

(i) State what is meant by isotopes.

……………………………………………………………………………………………………

…………………………………………………………………………………………………...[1]

(ii) Explain why the measured activity of the sample X is relatively constant.

……………………………………………………………………………………………………

…………………………………………………………………………………………………...[1]

(iii) It is discovered that isotope Y undergoes radioactive decay to form isotope Z. The half-life of isotope Y is 1.5 hours.

1. Calculate the decay constant of Y.

decay constant of Y = ……………………………….. s^{-1} [1]

2. The number of isotope Y in the sample is found to be constant. Explain how this is possible.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

…………………………………………………………………………………………………...[1]

3. Hence, determine this amount of Y.

amount of Y = …………..………..…….. atoms [2]

(b) Th-232 decays by alpha-emission with a decay constant of 1.57×10^{-18} s^{-1}. This is the beginning of a decay chain which eventually ends in Pb-208. A sample of rock is found to contain both Th-232 and Pb-208 in the ratio of 5:1.

(i) When the rock was formed, there was no Pb-208 present in the sample. Estimate the age of the rock.

age of rock = …………..………..…….. years [2]

(ii) State the assumption made in **(b)(i)** regarding the intermediate product nuclei.

……………………………………………………………………………………..………………

…………………………………………………………………………………………………..…[1]

(iii) State with a reason, whether your answer in **(b)(i)** is an overestimate or an underestimate of the age of the rock if the assumption in **(b)(ii)** is not valid.

……………………………………………………………………………………..………………

……………………………………………………………………………………..………………

……………………………………………………………………………………..………………

…………………………………………………………………………………………………..…[1]

2017 TJC P3

4 A uranium ($^{238}_{92}U$) nucleus, originally at rest, spontaneously decays to form a thorium (Th) nucleus and an α-particle. A γ-ray is not emitted.

(a) Write down the nuclear equation which describes this disintegration.

.... [1]

(b) The α-particle emitted in this disintegration travelled 25 mm in a cloud chamber, ionising the fluid within. Given that, on average, an α-particle creates 5.0×10^3 ion pairs per mm of track in the cloud chamber and that the energy required to produce an ion pair is 5.2×10^{-18} J, show that the speed with which the α-particle was emitted is 1.4×10^7 m s^{-1}.

[2]

(c) **(i)** Deduce the speed of the thorium nucleus after the disintegration.

speed of thorium nucleus = ... m s^{-1} [2]

(ii) Describe the motion of the thorium nucleus relative to the α-particle.

....

.... [1]

(iii) Briefly describe how your answer to **(ii)** would differ if the uranium nucleus is originally moving in one direction and the α-particle is emitted in a perpendicular direction.

....

.... [1]

(d) Calculate the difference between the rest mass of the original uranium nucleus and the sum of the rest masses of the products of the disintegration.

difference = ... kg [3]

(e) Uranium-238 has a half-life of 4.5×10^9 years.

(i) Define *half-life* and *decay constant* for a radioactive substance.

half-life ....

....

decay constant ....

.... [2]

(ii) Calculate the decay constant for uranium-238.

decay constant = ... s^{-1} [2]

(f) A certain coal-fired power station burns 80 megatonnes of coal a year. The coal contains a 0.0002% impurity of uranium-238 of which 10% is discharged into the atmosphere as fly ash during combustion.

(i) Assuming that, on average, the fly ash takes one year to fall to earth, show that the mass of uranium-238 in the air at any one time is 1.6×10^4 kg.

[1]

(ii) Hence calculate the number of α-particles produced per unit time from the uranium-238 in the air.

number per unit time = ... s^{-1} [3]

(iii) Suggest a reason why fly ash poses a radiation hazard.

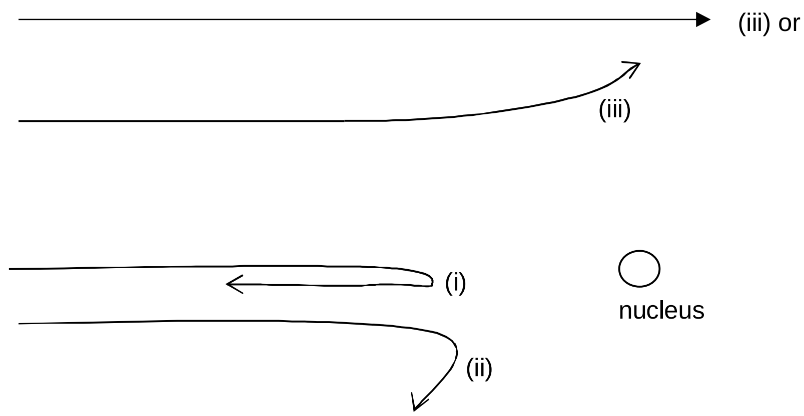
....

....

.... [2]

TUTORIAL 20: NUCLEAR PHYSICS SOLUTIONS

Level 1 Solutions

1.	(a)		[3]
	(b)	Most of the α-particles passed through the metal foil undeflected, and a very small proportion (about 1 in 8000) was deflected at large angles thus the size of the nucleus is very small relative to size of the whole atom, and the atom has a positively-charged core (nucleus)	[2] [2]
3.		Option C A is a false statement. D & E are false because they are not valid for a beta decay. B is irrelevant to the decay process.	
4.		Binding energy = $\Delta m c^2$ = $\{[Z m_p + (A - Z) m_n] - M\} \times c^2$ Therefore binding energy per nucleon = $\left(\frac{Zm_p + (A-Z)m_n - M}{A}\right)c^2$	
5.	(i)	The protons & the neutrons. Nucleon number is 25 (12 protons + 13 neutrons)	
	(ii)	Proton number is the "signature" of an element. {Nucleon number, however is not characteristic of an element due to such a thing called an isotope.}	
	(iii)	Total mass of constituent particles = 12(0.00055 u) + 12(1.00728 u) + 13(1.00866 u) = 25.20654 u The difference is the mass defect which is the mass-equivalent of the atomic binding energy (the energy required to completely separate the atom into its constituent particles).	
	(iv)	Binding Energy = Δmc^2 = $(25.20654 - 24.98584)(1.66 \times 10^{-27})(3 \times 10^8)^2$ = 3.3×10^{-11} J {Remember to convert any mass expressed in u to kg}	

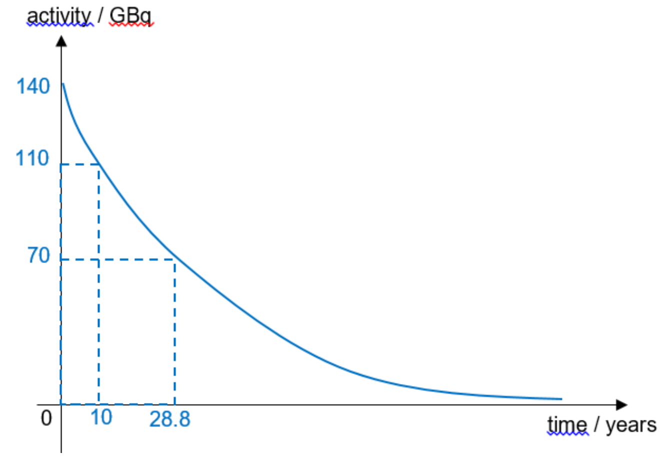
13.		Option: B One alpha decay means A – 4 and Z – 2, but Z is then restored by 2 beta emissions since each beta decay leads to an <u>increase</u> of 1 for the proton no Z.	
16.		$A = \lambda N$ N (number of active nuclei) = $\frac{\text{Sample Mass}}{\text{Mass of 1 mol}} \times N_A = \frac{mL}{M}$ Thus $A = \frac{\lambda mL}{M}$	
17.	(a)(i)	Activity after 4.0 half-lives have elapsed : $A = (½)^4 A_o = \frac{4.3 \times 10^5}{16} = 2.69 \times 10^4$ Bq	
	(ii)	No. of undecayed atoms after 4.0 half-lives: $N = (½)^4 N_o = \frac{7.9 \times 10^{15}}{16} = 4.94 \times 10^{14}$	
	(b)	$A = \lambda N$; $\lambda = \frac{A_o}{N_o} = \frac{4.3 \times 10^5}{7.9 \times 10^{15}} = 5.44 \times 10^{-10}\ s^{-1}$	
	(c)	$t_{½} = \frac{\ln 2}{\lambda} = 1.27 \times 10^{10}$ s	

Solutions to Additional Questions

1	(a)		There was a very small proportion of alpha particles deflected by very large angles (over 90°). This would not be possible for the plum-pudding model as the positive charges were spread all over the sphere as the force would not be strong enough to deflect the alpha particles with such big angle.	**B1** **B1** **B1**
	(b)	(i)		
		L2	Binding energy per nucleon = (mass defect) c^2 / (number of nucleons) = (8 $m_{neutron}$ + 8 m_{proton} – mass of $^{16}_{8}\boldsymbol{O}$) c^2 / 16 = [8 (1.0087 + 1.0073) – 15.9905] x 1.66 x 10^{-27} x (3 x 10^8)^2 / 16 = 1.28 x 10^{-12} J	**B2** **A1**
		(ii)		
		L2	(Mass of reactants – mass of products) c^2 = 9.594 MeV (2 x 15.9905 – 4.0015 – mass of Si) x 1.66 x 10^{-27} x (3 x 10^8)^2 = 9.594 x 10^6 x 1.6 x 10^{-19} Mass of Si = 27.9692 u = 4.64289 x 10^{-26} kg	**B2** **A1**

2	(a)			
	L2		All alpha radiation will be absorbed/ stopped by the paper. Nearly all gamma radiation will pass through the paper.	B1 B1
	(b)	(i)		
		L1	The half-life of a radioactive sample is defined as either: The <u>average</u> <u>time taken</u> for <u>the number of undecayed nuclei to be halved</u>.	B2
		(ii)		
		L2	Strontium-90 has the longest half-life (hence the activity will decrease at a low rate) In order to measure the thickness of the paper, the activity of the source should not vary too much. Otherwise the result will be different even when the thickness of the paper is the same.	B1 B1
		(iii)		
		L1	$\boldsymbol{\lambda} = \frac{\ln 2}{t_{1/2}}$ = ln 2/ 28.8 = 0.024068 = 0.0241 yr^{-1}	B1

		(iv)		
		L2	A = $A_0\ e^{-\lambda t}$ = 140 $\boldsymbol{e}^{-(\frac{\ln 2}{28.8}\times 10)}$ = 110 GBq	M1 A1
		(v)	Solution:	

			 *Award one mark for correct shape* *Award one mark for all labels*	
		(vi)		
		L3	The half-life for yttrium is almost 4000 times smaller than that of strontium, that means that the time scale for yttrium is much smaller than that of strontium. Also, the moment when strontium decay to form yttrium, yttrium might decay to something else. The number of yttrium nucleus is very insignificant as compared to that of strontium. Since activity is directly proportional to the number of undecay nucleus, the activity of yttrium is very insignificant as compared to that of strontium as well.	B1 B1
	(c)			
	L1	*Any relevant application:* • Tracing in medical field, e.g. to evaluate the performance of the thyroid by measuring the radiation intensity around the neck determined the amount of iodine in the thyroid gland and thus the performance of the thyroid. • Radiation therapy: radiation can damage rapidly dividing cells and thus it is useful in cancer treatment. • Food preservation: radiation can destroy or incapacitate bacteria and mold spores.		
	(d)	(i)		
		L2	As masses of the particles are very small, the gravitational force on them is almost negligible <u>as compared to</u> the electric force on them. **Comparison is needed for the 2nd mark.*	B1 B1
		(ii)		
		L2	While there is no change in their vertical velocities, there is a constant horizontal acceleration (in opposite directions) on both types of particles as they pass throught the electric field. This causes the particles to travel in parabolic paths.	B1 B1
		(iii)		
		L3	• In vacuum, no tracks can be observed as tracks are created by the ionised gas particles in the medium. • The vastly different charge to mass ratio of beta particles and alpha particles should made the alpha path almost no curvature and beta particles will curve and hit the plates.	B2

			• As the speeds of beta particles vary over a wide range, beta particles will have more than one track. *Any two reasons for two marks.*	

3	(a)	(i)	Isotopes are atoms **{**not elements**}** that have the same number of protons but different number of neutrons **{**in the nuclei**}**.	[1]
		(ii)	As the half-life of X (in years) is very long, the measured activity of sample X is thus relatively constant.	[1]
		(iii) 1	Decay constant of Y, $\lambda_Y = \dfrac{\ln 2}{t_{1/2}} = \dfrac{0.693}{1.5 \times 60 \times 60}$ = 1.283 x 10^{-4} s^{-1}	[1]
		2	Equilibrium is reached when the rate of production of Y (from the decay of X) is equal to its rate of decay. Hence, the number of isotope Y in the sample will stabilise at a constant value.	[1]
		3	Thus, the activity of Y is about 1.1 x 10^7 Bq. Amount of Y, $N_Y = \dfrac{A_Y}{\lambda_Y} = \dfrac{1.1 \times 10^7}{1.283 \times 10^{-4}} =$ 8.6 x 10^{10} atoms	[1] [1]
	(b)	(i)	By $N = N_o e^{-\lambda t}$: $5N = 6Ne^{-\lambda t}$ Take ln on both sides, $\ln 5 = \ln 6 - \lambda t$ $t = \dfrac{\ln 6 - \ln 5}{1.570 \times 10^{-18}} = 1.161 \times 10^{17}$ s = 3.682 x 10^9 years = 3.7 x 10^9 years	[1] [1]
		(ii)	Decay of Th-232 will give rise to a radioactive series where there will be a number of radioactive daughter products before ending up as the stable Pb-208. It is assume that these intermediate radioactive daughter products have very short half-life (much shorter than that of Th-232) so the number of intermediate daughter products are insignificant compared to Th-232 and Pb-208.	[1]
		(iii)	If the assumption is not valid, the current amount of decay products will be more than 1N. The fraction of undecayed Th-232 is actually less than $\frac{5}{6}$, thus answer for **(b)(i)** will be an under-estimate.	[1]

4	(a)		${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$
	(b)		Energy of the emitted α-particle = $(5.0 \times 10^{3}) \times 25 \times (5.2 \times 10^{-18}) = 6.5 \times 10^{-13}$ J $\frac{1}{2} mv^2 = 6.5 \times 10^{-13}$ $\frac{1}{2} \times 4 \times 1.67 \times 10^{-27} v^2 = 6.5 \times 10^{-13}$ $\therefore v = 1.4 \times 10^{7}$ m s^{-1}
	(c)	(i)	By conservation of momentum, $m_{Th}v_{Th} = m_{\alpha}v_{\alpha}$ $v_{Th} = \frac{4}{234} \times 1.4 \times 10^{7}$ $= 2.4 \times 10^{5}$ m s^{-1}
		(ii)	The thorium nucleus travels in the opposite direction relative to the α-particle.
		(iii)	The thorium nucleus and the α-particle no longer travel in opposite directions/ move off in a fork. (total momentum is conserved.)
	(d)		K.E. of thorium nucleus = $\frac{1}{2} \times 234 \times 1.67 \times 10^{-27} \times (2.4 \times 10^{5})^2 = 1.125 \times 10^{-14}$ J $E = \triangle m c^2$ $1.125 \times 10^{-14} + 6.5 \times 10^{-13} = \triangle m \times (3.00 \times 10^{8})^2$ $\triangle m = 7.3 \times 10^{-30}$ kg
	(e)	(i)	Half-life is the average time taken for half of the number of radioactive nuclei to decay. Decay constant is the probability per unit time that a radioactive nucleus will decay.
		(ii)	$\lambda = \ln 2 / t_{1/2}$ $= 4.9 \times 10^{-18}$ s^{-1}
	(f)	(i)	Mass of uranium-238 = $0.000002 \times 80 \times 10^{9} \times 0.1 = 1.6 \times 10^{4}$ kg
		(ii)	number of α-particles produced per unit time = $A = \lambda N$ $= \lambda \times \frac{M}{M_m} N_A$ $= 4.9 \times 10^{-18} \times \frac{1.6 \times 10^{4}}{238 \times 10^{-3}} \times 6.02 \times 10^{23}$ $= 2.0 \times 10^{11}$ s^{-1}
		(iii)	When fly ash settles on landfills, mines or quarries, people living next to these areas are exposed to higher levels of radiation from uranium. OR When fly ash dissolves in water/seeps into soil, uranium contaminates rivers/streams/lakes/vegetation, making drinking water unsafe/ affecting crops and making food unsafe for consumption. OR When fly ash is inhaled, uranium accumulates in the lungs causing destruction of DNA/mutation of cells leading to cancer.