TUTORIAL 1: MEASUREMENT SOLUTIONS

Quantities and Units

(L1)1	**A** (There are only 7 base units: only kg, m, s, A, K, mol. and cd)	[1]
(L1)2	**prefix** / **decimal equivalent** pico — 10^{-12} micro — 10^{-6} centi — 10^{-2} deci — 10^{-1} giga — 10^{9} tera — 10^{12}	[5]
(L2)3	**B** Work = *Fs* Joule = unit of Fx s = kg m s^{-2} m = kg m^2 s^{-2}	[1]

Estimation (to 1 sf)

(L2)4(a)	Volume of Olympic-sized pool – 50 x 25 x 2 = 2500 m^3 Density of water = 1000 kg m^{-3} Mass = density x volume = 1000 kg m^{-3} x 2500 m^3 ≈ 3 x 10^6 kg	[1] [1]
(b)	A good estimate of a basketball, Diameter = 24 cm, Mass = 0.5 kg Hence volume = $\frac{4}{3}\pi\left(\frac{0.24}{2}\right)^3 = 0.00724$ Density = $\frac{0.5}{0.00724}$ = 70 kg m^{-3}	[1] [1]
(c)	The maximum speed of the SMRT train is 90 kmh^{-1} = 25 ms^{-1}, and it roughly took 10s to reach the maximum speed from rest Acceleration = (25 – 0) / 10 = 2.5 ms^{-2} ≈ 3 ms^{-2}	[1] [1]
(d)	car reaches 30 m s^{-1} about 100 km h^{-1}in 20 s, so if its mass is 1000 kg, it gains kinetic energy of 450 000 J in 20 s, a power around 20 kW. OR Average cruising speed = 80 km h^{-1} Average drag force on car at 80 km h^{-1} = 1000 N Power = *F v* = 1000 x (80000/ 3600) = 22.2 kW = 20kW (1 sf) *{Tutors' Comments: The expected answers for this question allowed for a huge range of actual values. Credit will be awarded for justified reasoning. Unit given should be W instead of kg m^2s^{-3}.}*	[1] [1]

(L2)5	**C** The mass of the graphic calculator is quite similar to that of a smartphone. The mass of a graphic calculator is approximately 150 grams, hence, an approximate weight of 1.5 N or 150 cN. $1.5 = 150 \times 10^{-2}$ where $c = 10^{-2}$.	[1]

Errors and Uncertainties

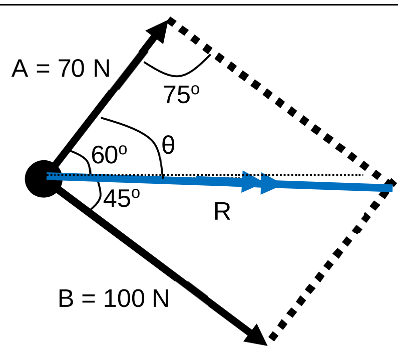
(L2)6	**B** Accuracy refers to how close a reading is to the true value. Since the voltmeter has a zero error, the reading is 0.08 V greater than the true value, which should be 2.08 V, thus rendering it not accurate, as the student did not correct for the zero error (since he was unaware of it). On the other hand, the uncertainty in the reading is half of the smallest division (±0.01 V), and since the reading is recorded to the nearest 0.01 V as well, it is precise.	[1]
(L2)7	**C** as per definition for systematic error (zero error is a common systematic error)	[1]
(L2)8	**B** Poor accuracy refers to the mean being far from the true value. Precise measurements mean a small spread.	[1]
(L2)9	**B** For option **B**, the mean is close to the true value, hence small systematic error. There is a significant spread amongst the other values, hence 'not very precise'.	[1]
(L2)10	**B** The graph shows a systematic error in the time measured because it does not cut through the origin, as should be the case for the given equation $\sqrt{h} = \left(\sqrt{\tfrac{1}{2}a}\right)t$.	[1]

Error Computation

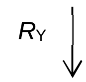
(L1)11	Perimeter, P = 2 x lengths + 2 x width = 178.2 m. Absolute error, ΔP = 2 x Δlength + 2 x Δwidth = 0.4 m. Hence, answer = (178.2 ± 0.4) m	 [1] [1]
(L2)12	**E** Let $d = d_1 - d_2$ $\Delta d = \Delta d_1 + \Delta d_2 = 2 + 1 = 3$ mm $\frac{\Delta d}{d} = \frac{3}{64 - 47} = 0.176 \approx 18\%$	[1]
(L2)13	**C** 3% of 327.66 = 9.83. Therefore, the error (Δ v) is in the whole numbers and the answer cannot be more accurate than the tens; i.e. 330 m s^{-1}.	[1]
(L2)14	**B** Let $A = \frac{xy^2}{z}$ => $\frac{\Delta A}{A} = \frac{\Delta x}{x} + \frac{2\Delta y}{y} + \frac{\Delta z}{z}$ $0.06 = (0.02) + 2(0.01) + (0.02)$ Thus, the set of values in option B would yield an uncertainty of 0.06 or 6%.	[1]

(L2)15	$P = 8.515$ Let $3X + 2Y = A$, $\Delta A = 3(0.02) + 2\,(0.01) = 0.08$ $\dfrac{\Delta P}{P} = \dfrac{\Delta A}{A} + \dfrac{\Delta Z}{Z}$ $\dfrac{\Delta A}{A} = \dfrac{0.08}{13.54} = 0.0059, \quad \Rightarrow \dfrac{\Delta P}{P} = 0.0059 + \dfrac{0.05}{1.59} = 0.037$ $\Delta P = 0.318 = 0.3$ (to 1.sf.) $\Rightarrow P \pm \Delta P = 8.5 \pm 0.3$ (note that the d.p. of P and ΔP are the same!)	[1] [1] [1]
(L2)16	$s = ut + ½ at^2$ $= 4.0 \times 3.32 + ½ \times 5.8 \times 3.32^2$ $= 45.244$ m Max $s = 4.1 \times 3.33 + ½ \times 6.0 \times 3.33^2 = 46.9197$ Min $s = 3.9 \times 3.31 + ½ \times 5.6 \times 3.31^2 = 43.5861$ Therefore error in s = ½(46.9197-43.5861) =1.66 = 2 m (1 sf) $\Rightarrow s = (45 \pm 2)$ m	[1] [1] [1]

Vectors and Scalars

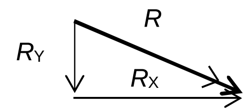
(L1)17	**B** force is a vector, kinetic energy is scalar	[1]
(L1)18	**C** Options A, B and D has a resultant of 1 unit (arrow) each. The resultant for option C is $\sqrt{2^2 + 1^2} = \sqrt{5}$ units	[1]
(L2)19(a)	 Using cosine rule, $R^2 = 70^2 + 100^2 - 2(70)(100)\cos 75°$ $R = 106.19\text{ N} = 106\text{ N}$ Using sine rule, $\dfrac{\sin\theta}{100} = \dfrac{\sin 75°}{106.19}$ $\theta = 65.5°$ from Force A or 5.5° below the horizontal.	[1] [1]
(b)	Sum of x-components: $R_X = A_x + B_x$ $= 70\cos 60° + 100\cos 45°$ $= 105.7$ N (i.e. rightwards) $\xrightarrow{R_X}$	

<table>
<tr>
<td></td>
<td>
Sum of y-components:
$R_Y = A_{y} + B_{y}$
$= 70 \sin 60° - 100 \sin 45°$
$= -10.09$ N (i.e. in the downwards direction)

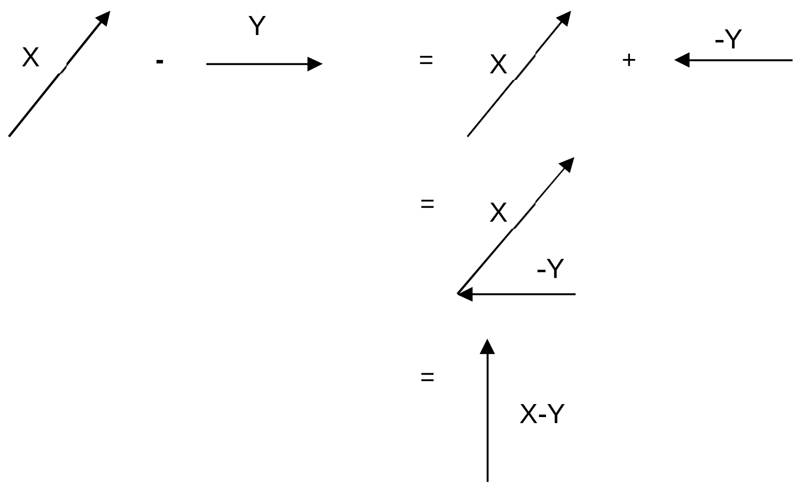


|Resultant|, I*R*I= $\sqrt{105.7^2 + 10.09^2} = \sqrt{11274}$ = 106.2 N = 106 N

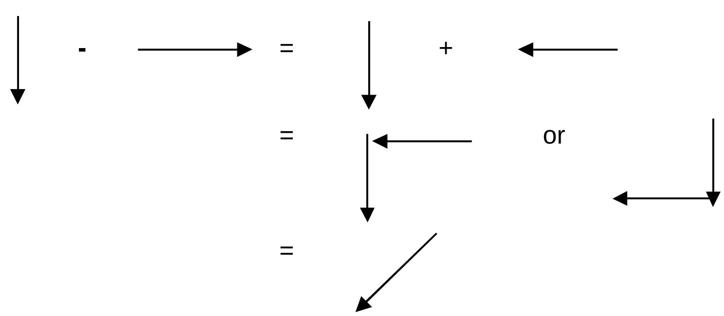
Direction (or argument) = $\tan^{-1}(-10.09/105.7) = -5.5°$,
i.e. 5.5° below the horizontal.



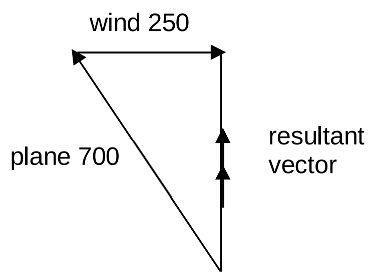
</td>
<td>[1]
[1]</td>
</tr>
<tr>
<td>(L2)20</td>
<td>
B



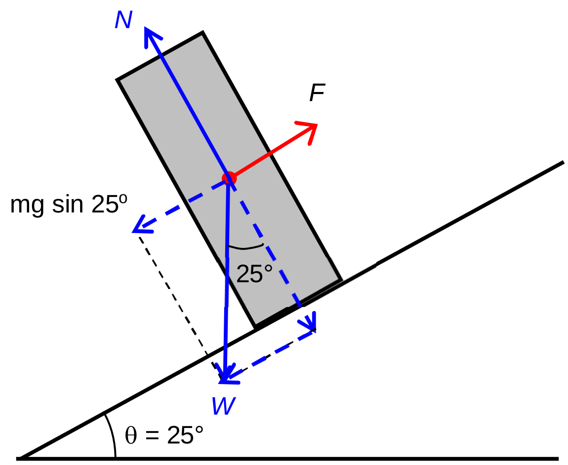
</td>
<td>[1]</td>
</tr>
<tr>
<td>(L2)21</td>
<td>
B



</td>
<td>[1]</td>
</tr>
<tr>
<td>(L2) 22</td>
<td>
The resultant vector of the aircraft is directly north, and it is due to the addition of the eastward wind vector and the northwest 700 km/h vector of the plane.



Magnitude of resultant vector = $\sqrt{700^2 - 250^2}$ = 654 km/h
</td>
<td>[1]</td>
</tr>
</table>

(L2)23	 To push the box up the slope, the force required must be **at least equal to** the component of the weight of the box down the slope. Min. force, $F = mg \sin \theta$ $= mg \sin 25°$ $= 200\ (9.81) \sin 25°$ $= 829$ N Since $F = mg \sin \theta$ Longer inclined plane $\Rightarrow$ smaller θ $\Rightarrow$ Less force is needed.	 [1] [1]