TOPIC 3: DYNAMICS

Learning Outcomes: Candidates should be able to:

a.	State and apply each of **Newton's laws of motion**.
b.	Show an understanding that mass is the property of a body which resists change in motion (**inertia**).
c.	Describe and use the concept of weight as the force experienced by a mass in a gravitational field.
d.	Define and use **linear momentum** as the product of mass and velocity.
e.	Define and use **impulse** as the product of force and time (duration) of impact.
f.	Relate **resultant force** to the **rate of change of momentum**.
g.	Recall and solve problems using the relationship **F = ma**, appreciating that **resultant force** and **acceleration** are always in the **same direction**.
h.	State the **principle of conservation of momentum**.
i.	Apply the principle of conservation of momentum to solve simple problems including **inelastic** and (perfectly) **elastic interactions** between two bodies in one dimension. (Knowledge of the concept of coefficient of restitution is not required.)
j.	Show an understanding that, for a (perfectly) **elastic** collision between two bodies, the **relative speed of approach is equal to the relative speed of separation.**
k.	Show an understanding that, whilst the **momentum** of a **closed system** is always **conserved** in interaction between bodies, some change in kinetic energy usually takes place.

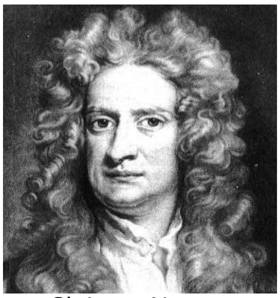
Two broad areas:
- Newton's laws of motion
- Linear momentum and its conservation

Key questions:
- What scenario is each Newton's law being used? E.g. Newton's 1st law is used for stationary objects or objects moving with constant velocity (not same as constant speed)
- What is the condition for using conservation of momentum? Why is it important to know this?
- What are the 3 types of collision and what are the equations (if any) related to each of them?

Dynamics, we look at the causes of the motion namely, the forces. The foundation of Dynamics are the Newton's laws of motion beyond the description of motion in kinematics. Derived from Newton's 3rd law, conservation of momentum will provide a new perspective for looking at problems of force and motion.

Why is the study of dynamics important?
Newton's laws are very important because they govern many experiences everyday life. These laws can explain how things can start moving, stop moving, speed up, slow down or change direction, why you don't float out of bed or fall through the floor of your house. It enables space travel and ensure safety design in cars. Newton's laws give an accurate account of nature, except for very small bodies such as electrons or for bodies moving close to the speed of light which requires quantum mechanics and relativity to understand.

Newton's Laws of Motion



Sir Isaac Newton
(1642 -1727)

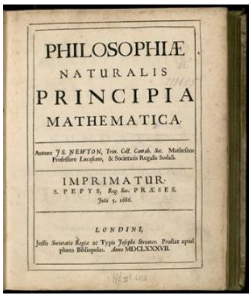


Fig. 3.1

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. Newton's monumental work, Philosophiae Naturalis Principia Mathematica, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects.
It was not until the advent of modern physics early in the 20th century that it was discovered that Newton's laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the size of most molecules (about 10^{-9}m in diameter). These constraints define the realm of classical mechanics.

3.1 Newton's First Law of Motion

Every object continues in its state of **rest** or **constant speed in a straight line** unless a **net** force acts on it to change that state.

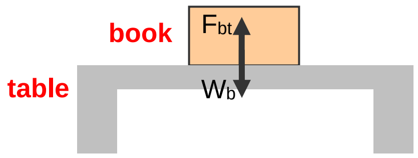
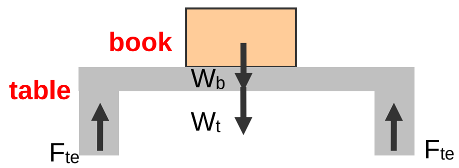
(1) **Cause for change in velocity:** The first law of motion states that there must be a cause (which is the net force) for there to be any change in velocity (either a change in magnitude OR direction, or both). E.g. An object sliding across the floor slows down due to the net force of friction acting on the object. Note that **no net force is required to maintain a constant velocity** (i.e. constant speed in a straight line or zero speed).

(2) **Inertia and mass:** A force is required because matter has a "natural reluctance" to have its state changed - a property called **inertia**. (Hence, this law is also called the *Law of Inertia).* Mass which is the amount of matter in a body, will affect inertia, hence **mass** is the **property of a body which resists change in motion**; the larger the mass, the greater its inertia.

(3) **Weight and gravitational field:** **Weight** is the **force experienced by a mass in a gravitational field.** Weight $W = m g$

(4) **External and Internal forces:** Only **external forces** can **change the state of motion** of a body – "**Internal" forces** cannot.

Whether a force is internal or external depends on the **system** defined. **Only external forces are drawn in a free body diagram (FBD)** as only external forces can change the state of motion of a body.

External forces	Internal forces
The forces acting on objects of a system **from outside the system** Consider a book at rest on a table  **System:** book **External forces:** Weight, W_b of book exerted on book by the Earth, Normal contact force, F_{bt} exerted on book by table	The forces acting among the objects **within the system** Consider a book at rest on a table  **System:** book and table **External forces:** Weight W_b of book exerted on book by the Earth, Weight W_t of book exerted on table by the Earth, Normal contact force F_{te} external on table by the Earth. **Internal forces (not drawn in the FBD):** Normal contact force, F_{bt} exerted on book by table and normal contact force F_{tb} exerted on table by book.

More about "**internal" forces**

Internal forces are forces acting on "objects of the **same** system". For every internal force that acts on a particular part of the body/system, there will always be another internal force acting on another part of the body/system. These 2 internal forces, by the third law of motion, are of equal magnitude and opposite in direction; hence the body/system as a whole experiences no net force.

Concept Check:

Explain why internal forces can't cause a change in motion in the examples below.

Try pulling yourself up by your shoelaces

Blowing at the sails of your own sailboat

Fig. 3.2

Tutorial qn: Q1

3.1.1 Illustration of Newton's First Law

Why does an object (e.g. tissue box) on a car dashboard slide to the *<u>left</u>* *as the car turns right? Assume lateral friction between box and dashboard is negligible.*

By Newton's First Law, the box will tend to **<u>continue moving in a straight line</u>** even while the car is turning right, unless a net force acts on the box. Viewed by an **<u>observer in the car</u>**, the box appears to be sliding to the left although in reality the box is moving straight ahead (viewed by an **<u>observer outside the car</u>**) when the car is turning right.

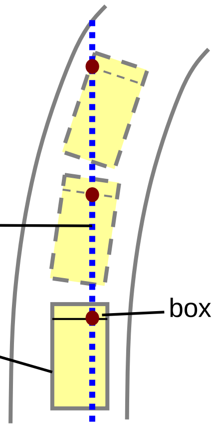


Fig. 3.3

CAUTION! Many people think that there is a "centrifugal" force pulling the box radially outwards of the circular path. This is a misconception as there is no such force pulling the box outwards.

Concept Check:
Why does an *<u>unstrapped</u>* *passenger fall forward when a car brakes suddenly?*

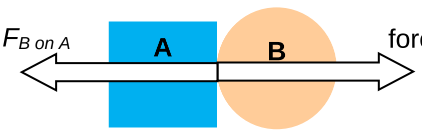
The brakes reduce the car speed but the passenger (who was previously moving with the speed of the car before the car brakes) <u>continues to move forward according to Newton's first law</u> since there is no net force to change his state of motion (until he hits something).

3.2 Newton's Third Law of Motion

When body A exerts a force on body B, body B exerts a force of the <u>same type</u> that is <u>equal in magnitude</u> and <u>opposite in direction</u> on body A.

CAUTION! In A-level MCQ, Newton's third law is sometimes given as "for every force (or action), there is an equal and opposite force (or reaction)." However, <u>this is not an acceptable definition</u>, because it is unclear that the two forces are acting on different bodies.

force of **B** on **A**, $F_{B\ on\ A}$



force of **A** on **B**, $F_{A\ on\ B}$

Fig. 3.4

(1) **Forces exists in pairs with equal magnitude and opposite in direction:** Third law implies that it's impossible to have a single force acting at any one time; forces always exist *in pairs*; called an **action-reaction pair**. They are **equal in magnitude** and **opposite in direction.**

(2) **Action-reaction pair acting on different bodies:** The 2 forces of an action-reaction pair always act on different bodies, never on the same body. Thus, an action-reaction pair of forces cannot possibly cancel each other.

(3) **Action-reaction pair are of the same type:** The two forces of an action-reaction pair are of the **same type**. What does this mean? (N2005 P3Q1a)(i))

Forces are classified into 4 fundamental types:
1. Gravitational (e.g. weight)
2. Electromagnetic[1] (e.g. contact force)
3. Strong nuclear (e.g. force within a nucleus of an atom)
4. Weak nuclear (which is responsible for radioactive decay at subatomic level)

The statement means that the 2 forces of any action-reaction pair are of the **same fundamental type.**

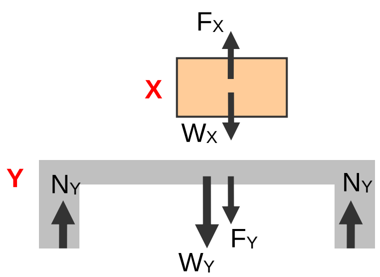
Concept Check:
*Explain how one can deduce the weight of an apple resting on a table, and the contact force by the table on the apple (also known as the normal reaction), do **not** form an action-reaction pair.*

The two forces
- are not of the same (fundamental) type; weight is a gravitational force, contact force is electromagnetic.
- act on same body, hence they cannot be an action-reaction pair

Worked Example 1 (N05P3Q1a, part)

(a) Object X is a book ***resting in equilibrium*** on a table (object Y). Draw labelled force diagrams to show the forces on X and on Y. Make it clear which forces are equal in magnitude and opposite in direction and which forces are action-reaction pair(s). [3]

Solution:



W_X : Weight of book, X (Earth pulls book)

W_Y : Weight of table, Y (Earth pulls table)

F_X : Contact force on X by Y

F_Y : Contact force on Y by X

N_Y : Reaction by ground on Y/Contact force on Y by Earth

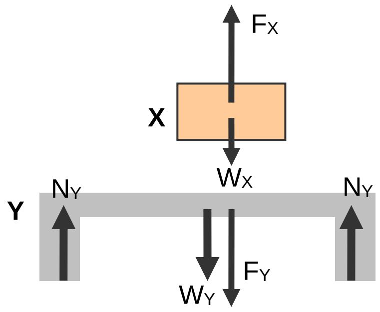
Drawing F_X and W_X acting on X and F_y, W_Y and N_Y acting on Y. [1]
F_X and F_Y are equal in magnitude and opposite in direction (action-reaction pair). [1]
W_X and F_X are also equal in magnitude and opposite in direction. (But **not action-reaction pair** as they act on the same body and they are not of the same type of force. Equal in magnitude because X is in equilibrium where $F_{net} = 0$.) [1]

Exam skills:
- Draw arrow to represent a force starting from the object it is acting on
- The length of the arrow should represent its magnitude as far as possible e.g. length of W_x and F_x should be the same as $F_{net} = 0$.
- Name all the forces when marks are awarded to FBD e.g. a label of W without explanation will not be awarded a mark

Worked Example 2 (N05P3Q1a, part)

(b) Now, suppose object X is a book that has fallen onto the table (object Y) and has just landed on it. ***At the instant of arrival***, draw labelled force diagrams to show the forces on X and on Y. Make it clear which forces are equal in magnitude and opposite in direction, and which forces are different in magnitude from those in (a). [3]

Solution:



W_X	: Weight of book X (Earth pulls book)
W_Y	: Weight of table Y (Earth pulls table)
F_X	: Contact force on X by Y
F_Y	: Contact force on Y by X
N_Y	: Contact force by ground on Y

Drawing F_X and W_X acting on X and F_Y, W_Y and N_Y acting on Y, with $\mathbf{F_x > W_x}$. [1]
F_X and F_Y are equal in magnitude and opposite in direction (action-reaction pair) [1]
Magnitude of F_X and F_Y are bigger compared to their previous values in (a). [1]
This is because as the box hits the table, the **net** force on box is **upwards**,
causing it to **decelerate** to rest while in (a), acceleration = 0 i.e. net force is zero ($W_X = F_X$)

Elaboration:
Unlike in (a), here in (b), W_X and F_X are **NOT** equal in magnitude ($F_X > W_X$);
the book experiences a net upward force from the instant where it just touches the table till it comes to rest. (ie book decelerates when it hits the table.

↑: $F_{net} = ma$

$F_X - W_X = ma$, where a = instantaneous deceleration, NOT = 0.

$F_X = W + ma$,

$\Rightarrow F_X > W_X$

Tutorial qn: Q2

3.3 Newton's Second Law of Motion

The **rate of change of momentum** of a body is proportional to the **net force** acting on it, and this (rate of) change of momentum takes place in the direction of the net force.

- To define force[1], or to answer a question like "State the relation between force and momentum" (N2012P3Q6, N2020P3Q6a 2m), use:

Force on a body is defined as the **rate of change of momentum** of the body, and it acts in the direction of the change in momentum.

[1] At A-levels, a force should not be defined simply as a *push or a pull (lacks essential details)*.

3.3.1 Linear Momentum

The linear **momentum** of a body *p* is defined as the **product of its mass and its velocity**.

i.e. $\boxed{p = mv}$ --------- eqn 3.1

- Direction of the momentum vector is given by the direction of the velocity vector.
- Unit: kg m s^{-1}

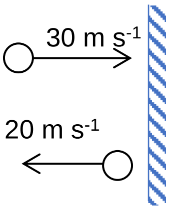
Worked Example 3 - to find change in momentum along a line

A ball of mass 58 g travels at a velocity of 30 m s^{-1}. It hits a wall and rebounds with a velocity of 20 m s^{-1}. Calculate the change in momentum of the ball.

Solution:

initially: 30 m s^{-1}

finally: 20 m s^{-1}



Problem Solving:

Step 1: *Draw arrows to represent the initial and final velocities. Either label the arrows appropriately or write down the known values for easy mathematical manipulation.* It is a good habit to draw length of arrow to be approximately proportional to its values.

Step 2: *Define a direction to be positive. It does not matter which direction is taken to be positive. In this case, rightward is taken as positive*

Step 3: *Apply formula for change in momentum. Must take into account direction of velocity vectors (which is the direction of the momentum vectors).*

Change in momentum = **Final – Initial** momentum (by **definition**)
= *mv – mu {vector subtraction}*
= (0.058)(**–**20) – (0.058)(**+**30)
= –2.9 kg m s^{-1}

Note: the solution involves vector subtraction (where the sign of the direction is important), instead of a simple scalar subtraction.
{The + & -ve signs serve to indicate the direction of the vectors.}

A final answer with -ve sign means the change in momentum (also a vector) is in the leftward direction as rightward is taken to be positive. If leftward was taken to be positive instead, the final answer would have a +ve sign.
The direction of the change in momentum is not important in this case as the question only ask for a calculation of its magnitude.

Concepts:

- Momentum is a vector with direction and is the same direction as the velocity vector
- Change in a vector = final vector – initial vector

Worked Example 4 (N2010P1Q2 modified) - to find change in momentum NOT along a line

A toy boat of mass 100 g changes its velocity from 8 m s^{-1} due north to 6 m s^{-1} due east. Calculate the change in momentum and state its direction.

Solution:

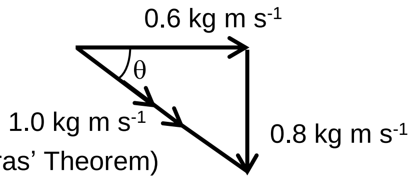
Change in momentum = Final momentum – Initial momentum

= $\longrightarrow$ – $\uparrow$

= $\longrightarrow$ + $\downarrow$

(Indicate direction of the change in momentum (eg with θ as shown)

=



(By Pythagoras' Theorem)

$\tan\theta = \frac{0.8}{0.6}$

$\rightarrow \theta = 53^o$

Hence, Δp = 1.0 kg m s^{-1} at an angle of 53^o (or 53° South of East, or ,37^o East of South).

Note: Here there is no need to assign a direction to be positive like in Worked Example 3 as the direction would be taken into account in the vector addition/subtraction. Direction of the vector Δp can be described as an angle. The best is to draw the angle in the diagram.

3.3.2 Mathematical Statement of Newton's Second Law

Mathematically, net force, $F_{net} \propto \frac{d(mv)}{dt} = k\frac{d(mv)}{dt}$,

where the value of the proportionality constant *k* would depend on the (chosen) definition of the *unit of force*.

Note: This is the reason why "proportional" instead of "equal" is used in Newton's second law of motion.

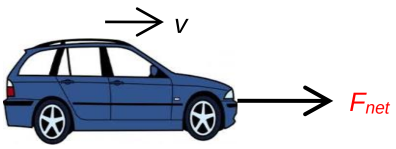
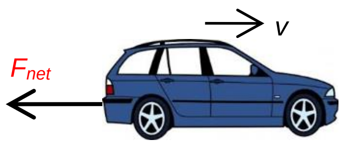
Since the definition of the *newton* was chosen as "the force which causes a mass of 1 kg to have an acceleration of 1 m s^{-2}, thus,

$$1\text{ N} = k \times 1\text{ kg} \times 1\text{ m s}^{-2}$$
$$k = 1$$

Thus, $$F_{net} = \frac{d(mv)}{dt} \quad \text{------- eqn 3.2}$$

Note: If mass is in g instead with force in newton and acceleration in 1 m s^{-2}, k would be 1/1000.

3.3.3 Illustration of Newton's Second Law

 **Fig. 3.5**	 **Fig 3.6**
If the net force F_{net} acts in the same direction as the velocity *v*, the momentum of the body increases. We say that the body **accelerates in the direction of its motion**, ie its speed **increases**.	However, if the net force F acts **opposite** to the velocity *v* of the body, its momentum decreases. We say that the body decelerates, or, it **accelerates in the direction opposite to its motion**; its speed **decreases**.

3.3.4 For a System of Constant Mass with Varying Velocity

From definition of force, $F_{net} = \dfrac{d(mv)}{dt}$.

If m = constant, $F_{net} = m\dfrac{dv}{dt}$

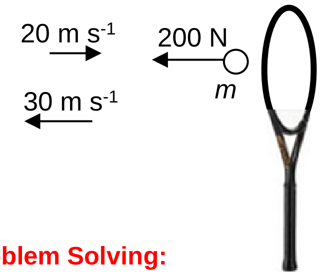
Since $\dfrac{dv}{dt} = a$, $\boxed{\boldsymbol{F_{net} = ma}}$ --------- eqn 3.3

F_{net} is the net or resultant of all the external forces on the system

Example 5

A tennis player hits a ball of mass 58 g which reaches her with a velocity of 20 m s^{-1}. If the maximum force she can exert on the ball is 200 N, what is the minimum duration the ball must be in contact with the racket in order for the player to return it to her opponent with a speed of 30 m s^{-1}? [t = 0.0145 s]

Solution:



Taking leftward as positive,

Problem Solving:

- Draw the diagram
- Vector subtraction considering the direction
- Max force corresponds to a min time.

Concepts:

- a = Δv/Δt = (v – u)/t where a = constant with a constant force during duration of time t
- F_{net} = ma

3.3.5 Applying F_{net} = *ma*: Problem-Solving Strategy

The second law can be applied to a complete system or to any part of a system. You must decide clearly which part or system to consider. If forces on only a **part** of a **system** are to be considered, then the **mass** of **only that part** must be used when applying F_{net} = ma. (See Examples 8 & 9)

This technique of isolating a part of the system is called the **free-body method** whereby all forces and masses outside the selected part are ignored.

Procedure:

1. Draw a **free-body diagram (FBD)**.

   i.e. a diagram showing **ALL** the forces acting **ON** the selected system (complete or part), ignoring those forces exerted by the system on other bodies.
   - Use a point to represent the system
   - Indicate forces acting on this system with arrows (magnitude of force is proportional to the length of the arrows) starting from the system
2. Indicate the direction of the acceleration with an arrow. (If it's unknown, indicate an ***assumed* direction.** If the final answer of the acceleration turns out to be negative, it indicates that the actual direction is opposite to your assumed direction.)

3. Resolve all the forces in along the line of **direction** of the **acceleration** and perpendicular to it.

4. Substitute into F_{net} = *ma*, where $\boldsymbol{F_{net}}$ = **vector sum** of all forces in (3). Consider dir of acc to be positive.
   - LHS: *all forces*. Ensure their directions with appropriate sign are correct.
   - RHS: *ma*

3.3.5.1 Single Body Problems

In problem solving, we often represent a single body system e.g. a human, a car as a dot or a box. The weight of this single body will be acting from the centre of this dot or box. Normal reaction (i.e. normal contact force between the body and the surface) is acting at the contact surface.

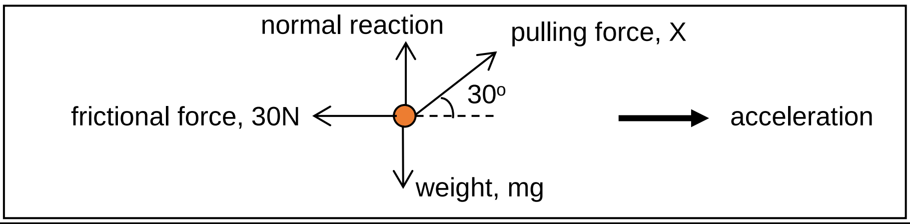
Worked Example 6 – resolving forces in direction of a to apply F_{net} = ma

A 20 kg wagon is pulled along a level ground by a rope inclined at 30° above the horizontal. A frictional force of 30 N opposes the motion. How large is the pulling force *X* if the wagon is moving rightward along the ground with

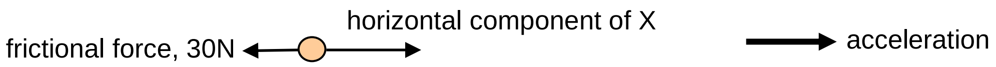
(a) an acceleration of 0.40 m s^{-2} (b) constant speed of 25 m s^{-1}? [44 N; 35 N]

Solution:

(a)



- Resolving all forces in the direction of acceleration (i.e. horizontally in this case):



(The 2 forces perpendicular to direction of acceleration, normal reaction & *mg*, have a zero component in the direction of acceleration.)

- *When acceleration is to the right*

Consider horizontal direction and taking rightward to be positive, {Good practice to write this}

$\rightarrow$: $F_{net} = ma$

$(+X \cos 30^0) + (-30) = 20 \times (+0.40)$

$\Rightarrow X = 44$ N

Note: The vertical forces here are not useful in solving this problem. Why? They may be useful in other problems.

(b) When speed = constant , $\Rightarrow$ acceleration a = 0.

Consider horizontal direction and taking rightward to be positive,

$\rightarrow$: $F_{net} = ma$

$(+X \cos 30^0) + (-30) = 0$

$\Rightarrow$ X = 35 N

Tutorial qn: Q4, Q5, Q6, Q7, Q8

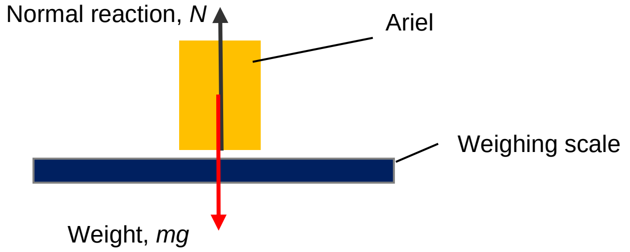
Example 7: Apparent Weight in a Lift – understanding apparent weight is the normal reaction

Ariel, who has a mass of 50 kg, is standing on a weighing scale in a lift. Calculate her "apparent weight" if the lift is

(a) moving upwards at a constant speed of 4.0 m s^{-1},
(b) accelerating upwards at 2.0 m s^{-2},
(c) accelerating downwards at 2.0 m s^{-2},
(d) falling after the lift cable snaps. [491 N; 591 N; 391 N; 0 N]

Concept:
Apparent weight refers to the **normal reaction** *N* that the surface i.e. weighing scale, exerts on Ariel. This is the "weight" Ariel will read from the weighing scale. This may or may not be the true weight which is due to gravitational pull of Earth. Thus it is called the apparent weight. By the 3rd law, it is **numerically equal to the force that she exerts on the surface (weighing scale)** as they form an action-reaction pair.

Solution:



Forces acting on Ariel:

- Her weight *mg* (Earth pulls Ariel, often referred to as the "true weight")
- Normal reaction *N* (scale pushes Ariel upwards)

By Newton's third law, the force that Ariel pushes downwards on the scale (i.e. her **apparent weight) is always equal and opposite to that of *N*.**

Hence **apparent weight (given by scale reading) = normal reaction *N* (in magnitude**)

(a) If the lift is moving upwards at const speed
Taking upwards as positive,
Consider vertical direction:

$\uparrow$: $F_{net} = ma$
$(+N) + (-mg) = ma$ where a =0
$\Rightarrow N = mg$

Note that weight mg is downward thus negative as upward is taken as positive.
Thus g is simply a numerical value here in the mathematical manipulation.

Apparent weight $= N = 50 \times 9.81 = 491$ N (i.e. equal to the true weight)

(b) If the lift is accelerating upwards,
Taking upwards as positive,

$\uparrow$: $F_{net} = ma$
$(+N) + (-mg) = ma$
$\Rightarrow \quad N = mg + ma = (50 \times 9.81) + [50 \times (+2.0)] = 591$ N

Apparent weight = N = 591 N (i.e. larger than the true weight)

{Note that <u>decelerating</u> downwards at 2 m s^{-2} would cause the same apparent weight as (b).
Taking downwards as positive, mg – N = m(–a) $\Rightarrow$ N = mg + ma }

(c) If the lift is accelerating downwards,

(d) If the lift is falling with acceleration g,

Note: Question may ask the condition for apparent weight to be 0. This occurs when the body is falling at acceleration of free fall. This is also known as apparent weightlessness.

Tutorial qn: Q3

3.3.5.2 Connected-Bodies Problems - Masses Connected by Rod, Rope, or Physical Contact

There are times when the system is not a single body but connected bodies e.g. a train consisting of many cabins. Each cabin is connected to the next. In such cases, we could consider each cabin or the whole train as a single body, depending what needs to be determined. When the bodies are connected, they should have same velocity and acceleration.

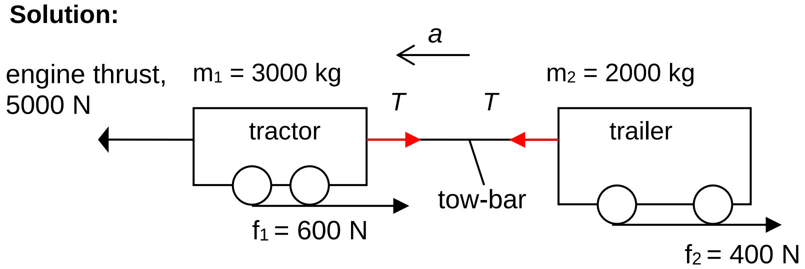
Worked Example 8: Masses Connected Horizontally by tow-bar

A tractor of mass 3000 kg pulls a trailer of mass 2000 kg. The tractor generates a thrust of 5000 N. Friction exerts a backward drag of 600 N on the tractor and 400 N on the trailer.

Determine:
(a) the acceleration of the tractor-trailer system,
(b) the tension in the tow-bar between the tractor and the trailer,

[0.80 m s^{-2}; 2000 N]

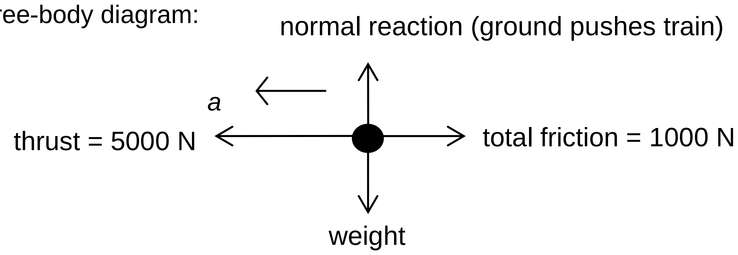
Solution:



(a) To find acceleration *a* of the tractor-trailer system:

Consider the <u>tractor and trailer</u> as one system:

Free-body diagram:



(Note: **tensions T acting on the tractor and trailer are not to be included because they are *internal* forces when the tractor and trailer are considered as a system**)

Applying F_{net} = ma, *in the direction of a* (i.e. horizontally):

$\leftarrow$: thrust – total friction = (5000) a

5000 – 1000 = (5000) a

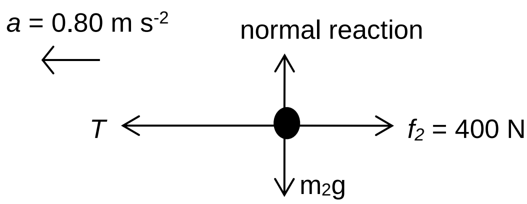
a = <u>0.80 m s^{-2}</u>

(Note: normal reaction and weight each has a zero component horizontally)

(b) To determine *T* (tension in the tow-bar):

Consider <u>only the trailer</u> as the system.

Free-body diagram:



(Note: **Engine thrust of 5000 N does not act on m_2. It only acts on m_1**)

Applying $F_{net} = ma$ to m_2 alone (horizontally),

$\leftarrow$: T - 400 = $m_2 a$,

T = 400 + [2000 x (+0.80)]

= <u>2000 N</u>

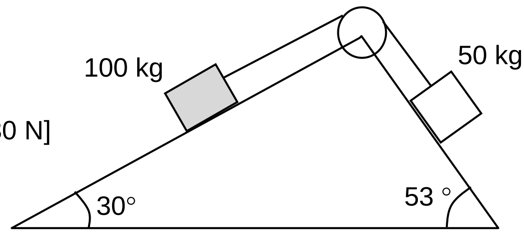
Worked Example 9: Connected-Bodies Problem on a Slope

Two blocks, connected by an inextensible string passing over a small, frictionless pulley, rest on frictionless planes as shown.

(a) Calculate the acceleration of the blocks.

(b) Calculate the tension in the cord.

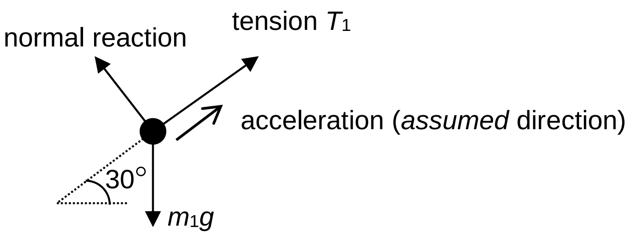
[0.66 m s^{-2}; 430 N]



Solution:

Apply F_{net} = ma to each of the connected bodies separately (i.e. one at a time).

Consider the <u>100 kg alone</u>

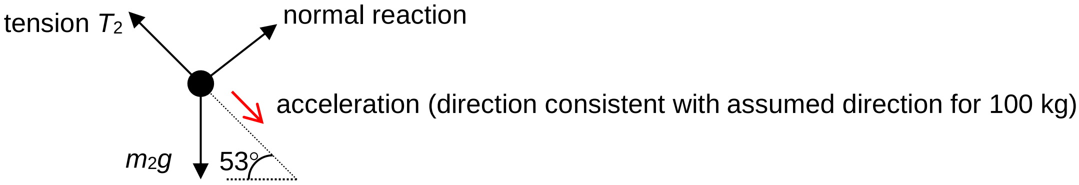


- Applying $F_{net} = ma$ to 100 kg mass.
  Since direction of acceleration is assumed to be up the slope, we should resolve <u>all forces along the slope</u> taking direction of acceleration to be positive.

  ↗ : $T_1 - m_1 g \sin 30° = m_1 a$ { NOT: $m_1 g \sin 30° - T_1 = m_1 a$ }

  $T - 100g \sin 30° = 100\,a$ ------- eqn (1) {Let $T_1 = T_2 = T$ since tensions form an action-reaction pair. Tensions in the same string are always equal}

Consider the <u>50 kg mass alone</u>



- Applying F_{net} = ma to 50 kg mass along the slope taking direction of acceleration to be positive:

  ↘ : $m_2 g \sin 53° - T_2 = m_2 a$ {Similarly, N_2 has a zero component here}

  $50\,g \sin 53^0 - T = 50\,a$ -------- eqn (2)

- Solving for T and a simultaneously gives:

  a = – 0.66 m s^{-2} & T = 425 N = 430 N (2sf)

  Since value of a is **negative**, the blocks accelerate in direction ***opposite to that assumed***.

Tutorial qn: Q9, Q10, Q11

3.3.6 For a System moving at Constant Velocity and Varying Mass (Less common)

From definition of force, $F = \dfrac{d(mv)}{dt}$.

When mass of the body changes during the motion as $\frac{dm}{dt}$ *and velocity v is constant,*

$$\boxed{F = v\frac{dm}{dt}}$$

Although there are many cases for which the above equation is applicable, one of obvious example is rockets where a significant fraction of the mass of a rocket is the fuel, which is expelled during flight at a high velocity. The propulsive force (known as thrust) exerted on the rocket by the expelled fuel is in the opposite direction (due to Newton's 3rd law) to the motion of the expelled fuel.

Worked Example (N2022P3Q1c – modified)

1(c) A rocket of initial mass 4000 kg is situated on the surface of the planet with a gravitational field strength of 1.62 N kg^{-1}.

The rocket fires its engines. The exhaust gases from the engines are all ejected vertically downwards, and the mass of the rockets reduces at a rate of 70.0 kg s^{-1}. A constant thrust of 10.0 kN is generated on the rocket, and it accelerates vertically away from the surface of the planet.

(i) Calculate the speed of the exhaust gases at the instant when rocket lifts off.

Solution:

Thrust by the exhaust gas F = v dm/dt

$$10.0 \times 10^3 = v\,(70)$$
$$v = 142.8 = 143 \text{ m s}^{-1}$$

(ii) Calculate the acceleration of the rocket after the engines have fired for 15.0 s.

You may assume that the gravitational field strength of the planet acting on the rocket is constant.

Solution:

After 15.0 s, W of rocket = [4 000 - 70.0 (15.0)] (1.62) = 4779 N

$$\uparrow: F_{net} = ma$$
$$10.0 \times 10^3 - 4779 = (4\,000 - 70.0(15.0))a$$
$$a = 1.77 \text{ m s}^{-2}$$

Common error:
1. Taking F = v dm/dt to be the net force = thrust - weight
2. Taking weight as W = mg = 4000(9.81) N

3.4 Impulse of a Force

1. The impulse of a force $\boldsymbol{I}$ is defined as the product of the force and the time Δt during which it acts.

Case 1: Constant Force

i.e. $\boxed{I = F\Delta t}$ --------- eqn 3.4

- When using this formula, we are assuming the (average) net force ***F* is constant** over the duration Δt.
- Impulse is a *vector* which takes the direction of the force. Unit: *N s* (newton second)

2. Mathematically,

$$\begin{aligned} I &= F\Delta t \\ &= \frac{\Delta(mv)}{\Delta t}\Delta t \\ &= \Delta(mv) \end{aligned}$$

$\Rightarrow$ Impulse = Change in momentum

Thus the **impulse of a force acting on a body is equal in magnitude to the change in momentum of the body.**

- **CAUTION!** It is **incorrect** to **define** impulse as the change in momentum.

3. **Case 2: Variable Force**

$\boxed{I = \int F\,dt = \text{Area under } F\text{-}t \text{ graph}}$ ---------- eqn 3.5

- Thus**, Area under net *F - t* graph = change in momentum of body**
  (Use "count-the-squares" method if area is not of a regular shape)
- Typical variation of an "impulsive force" with time

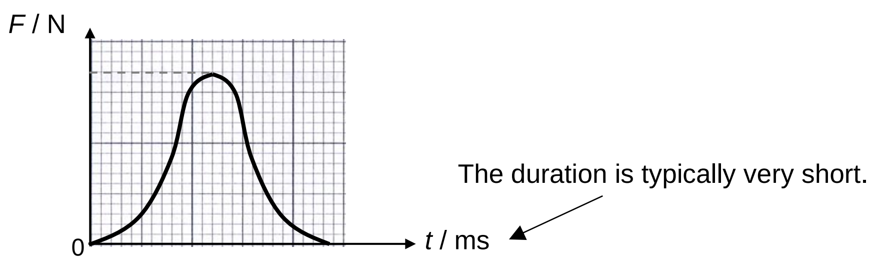
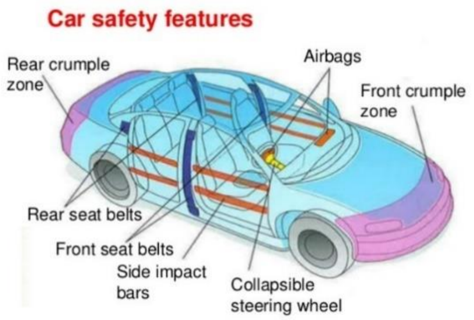


Fig. 3.7

3.4.1 Application: How to Reduce an Impulsive Force

There are many occasions when one wants to reduce the impulsive force. An important application would be to save the driver's and passengers' lives in car accidents. There are various safety features in a car to increase the time to reduce the impact to the occupants as the force is allowed to act over a longer time when there is a sudden stop.

Features targeting the driver and passengers	Features targeting the car
Dashboard padding Airbag	Crumple zone



The **<u>momentum change is the same</u>** (from an initial velocity to come to a stop) for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time.

In eqn 3.4, impulse, $I = F\Delta t$. If the **<u>time taken Δt</u>** for the momentum change is **<u>increased</u>** (in other words, the rate of momentum change is decreased), the net force acting on the system would be reduced. Hence, the **<u>force experienced can be reduced if the collision is prolonged</u>**.

Some examples of increasing impact duration to reduce injuries or damage:

- airbag in cars/ car seatbelt
- car bonnet made of collapsible material ('crumple zone')
- material of package used to contain eggs, refrigerator, TV, etc
- withdrawal of hand when catching a fast ball
- bending of knees when landing on floor from a height

What other examples can you think of?

Worked Example 10 (SAJC 2014 CT Q22c)

Using Newton's Laws of Motion, explain why an egg falling from the same height will be more likely to crack when knocked against a hard floor than against a soft cushion. [2]

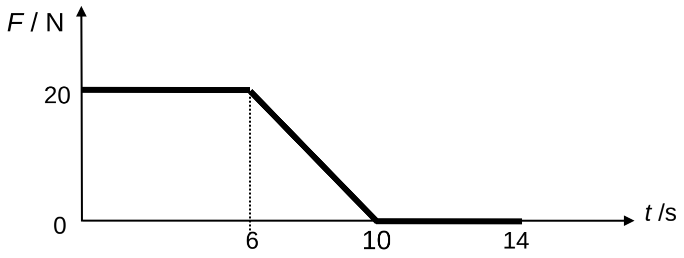
Solution:

The cushion <u>increases the time taken</u> for the change (decrease) in momentum of the egg. {Context} [1]
By Newton's second law, Force = change in momentum ÷ time taken,
with the <u>same change in momentum</u>,
the <u>increase in time</u> will result in a <u>smaller average net force</u> exerted on the egg. {Concept} [1]

Unacceptable solution: The cushion absorbs the impact of the fall.

Example 11

A body of mass 3.2 kg initially at rest is acted on by a resultant force which varies with time as shown below.



Calculate
(a) the momentum change of the body from 0 to 10 s,
(b) its velocity after 12 s [160 kg m s^{-1}; 50 m s^{-1}]

Solution:

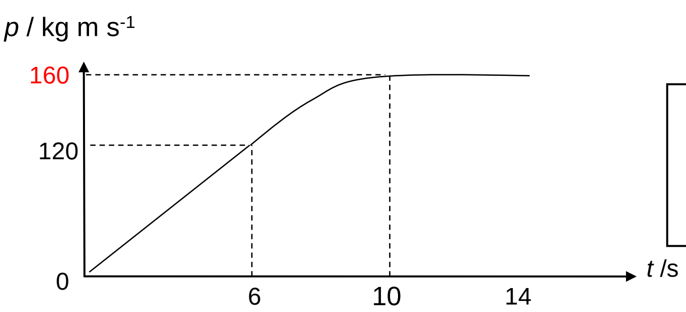
(a) Change in momentum after 10 s =

Using area of trapezium formula

(b)

Concept:
The velocity has increased as the force accelerate the body.
A question can be in a context where there is an initial velocity and a braking force. H1 (N2018P2Q1)

(c) Sketch the p – t graph. Include appropriate values of p on the y-axis



Change in momentum after 6 s
= Area under F-t graph from 0 to 6 s
= 20 x 6
= 120 kg m s^{-1}

Concept:
- F = dp/dt
- The object starts from rest and is acted on by a force thus velocity and thus momentum is increasing.

Problem solving:
- F is the gradient of d-t graph.
- Do not use area under F-t graph to draw p-t graph. Instead look at gradient of p-t to get the F values.
- Include appropriate values on the graph

Tutorial qn: Q12, Q13, Q14

3.5 Principle of Conservation of Linear Momentum (PCM)

For a **<u>system</u>** of interacting bodies, the **<u>total momentum</u>** of the bodies (i.e. momentum of the system) **remains constant**, provided **<u>no net force</u>** acts on the system.

i.e. $\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad \text{if net force} = 0}$ {for all interactions} ----- eqn 3.6

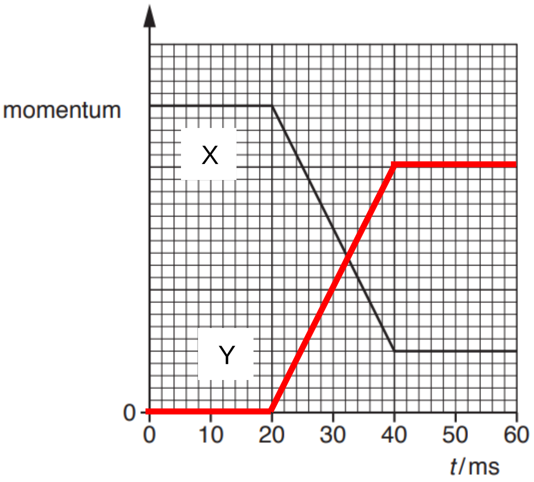
Note:

- Total momentum refers to the <u>vector sum</u> of the momentum of the interacting bodies; when adding momentum, the <u>direction</u> of the momentum vectors must be taken into account.

- Equivalent expression for "a system with no resultant force acting on it": a **closed system**.

- Eqn 3.6 $\Rightarrow$Total momentum of a closed system (JUST) **before** collision = (JUST) **after** collision.

- Similarly, the total momentum of a closed system **DURING** interaction is conserved.



A block X of mass m slides in a straight line along a horizontal frictionless surface collide with a stationary block Y of mass m. Block X makes contact with block Y at time t = 20 ms.

No net force – velocity and thus momentum (just) before and (just) after collision is constant.

Conservation of momentum - total momentum of X and Y before, after and during (from 20 to 40 ms) collision is constant (this takes into account the sign as momentum is a vector)

3.5.1 Verification of Principle of Conservation of Linear Momentum from Newton's Second & Third Laws:

We can deduce the principle of conservation of momentum by applying Newton's second and third laws to two colliding bodies as illustrated below:

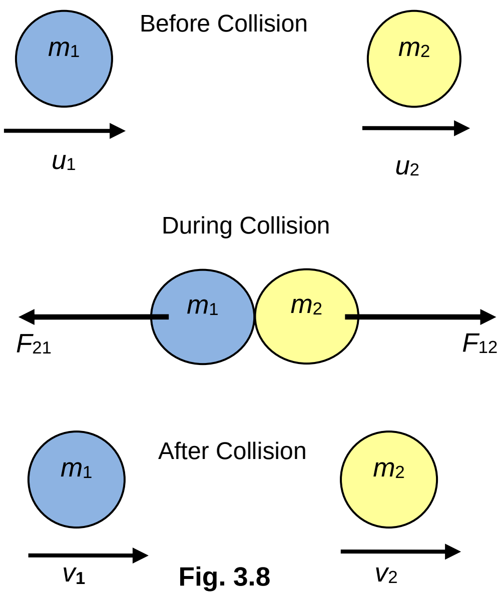


Fig. 3.8

During the collision, suppose m_1 exerts an average force of F_{12} on m_2, while m_2 exerts an average force of F_{21} on m_1 over a duration Δt.

By Newton's 3rd law,

$F_{12} = -F_{21}$

{**Negative sign** denotes the 2 forces are opposite in direction}

By Newton's 2nd law,

$$F_{12} = \frac{m_2(v_2 - u_2)}{\Delta t} \quad \& \quad F_{21} = \frac{m_1(v_1 - u_1)}{\Delta t}$$

Thus, $\dfrac{m_2(v_2 - u_2)}{\Delta t} = -\dfrac{m_1(v_1 - u_1)}{\Delta t}$

Rearranging, $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

i.e., total momentum before collision = total momentum after collision.
Thus the PCM is consistent with Newton's laws of motion.

3.5.2 Applying Principle of Conservation of Linear Momentum: Problem Solving Strategy

1. Identify all objects that constitute the closed system. Draw a diagram with the masses and velocities before/after/during the collision.
2. Choose a sign convention for the velocity (momentum) vectors
3. Equate total momentum before interaction and total momentum after interaction

Note:
Head-on collision means that they move along the same line joining their centres of masses before and after the collision (i.e. velocities of the objects before and after collision are collinear) as shown in Fig. 3.9. Generally all A -level qns are head-on collisions. (It does not mean colliding objects must move in opposite direction.)

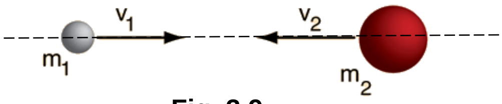
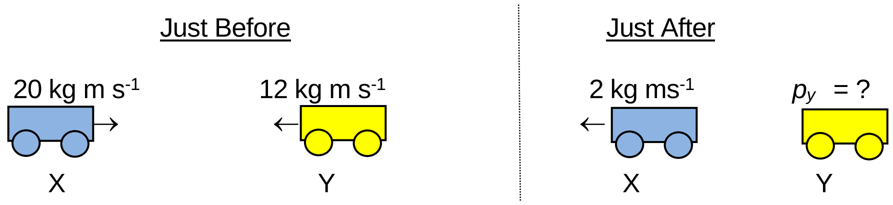


Fig. 3.9

Worked Example 12 (N88P1Q4)

The diagram shows 2 trolleys X and Y about to have a *head-on* collision. The momentum of each trolley before the impact is given.



After the collision, the trolley X reverses in direction and the momentum of X is 2 kg m s^{-1}.
What is the momentum of trolley Y? State its direction. [10 kg m s^{-1}]

Solution:

Consider the system as X and Y as a whole.

No net force acts on the system if we assume that friction between trolleys and Earth is negligible. {The weights are cancelled out by the normal reactions}

By the principle of conservation of momentum:
Total momentum of system before collision = Total momentum of system after collision

$\rightarrow$: $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$(+20) + (-12) = (-2) + p_y$

$+8$ kg m s^{-1} = $-2 + p_y$

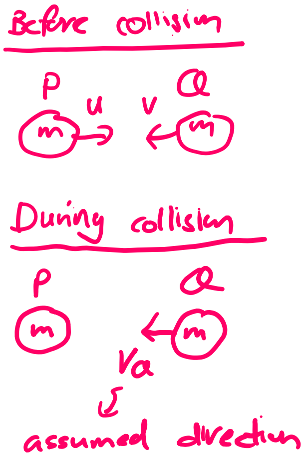
p_y = +10 kg m s^{-1} i.e. in the original direction of X (since p_y is +)

Example 13 **(J85P1Q5, J88P1Q4)**

Two bodies P and Q of equal mass, travelling towards one another on a level frictionless track at speeds *u* and *v* respectively, collide into each other. At some instant <u>during the collision</u> P is brought momentarily to rest. What is the speed of Q at that instant?

A zero **B** v-u **C** 2(v-u) **D** ½ (v-u) **E** $\sqrt{uv}$

Solution:



Let v_Q be the velocity of Q at that instant.

By Principle of Conservation of Momentum,

Note:

- **u_p must be substituted as negative u** because mu is a momentum vector and in opposite direction to the direction taken to be positive.
- We deduce from "level frictionless track" that P and Q maintain their speeds *u* and *v* respectively till they collide.

Additional Notes:

- Momentum, being a vector quantity, takes on either a positive or a negative value, depending on its direction. Thus, a mental note of a <u>sign convention for the vectors</u> is necessary.
- "<u>Interaction</u>" refers not only to a collision as is commonly understood, but to any situation after which, <u>the velocity of a body is changed</u>. Examples: an explosion, <u>radioactive emission, or a "collision" which does not involve physical contact</u> as illustrated in the following situation:

Fig. 3.10 magnet A, held fixed magnet B approaching

As B approaches A, B slows down - its velocity and momentum change.

- It is vital that the "closed system" be identified correctly because that will determine whether a given **force is an internal or an external force**; and <u>internal forces are not relevant</u> when deciding, whether or not, momentum of a system is conserved.
- For Principle of Conservation of Momentum to be applicable <u>no net</u> force must be acting on the system; otherwise by Newton's 2nd law, this <u>net force will cause a change in momentum in the system.</u>
- Whilst the (total) momentum of a system is always conserved (provided no net force acts), the <u>momenta of the individual bodies may change.</u>

Concept Check:

Suppose you are initially standing at rest and then, you jump upward, leaving the ground with a speed v. Obviously, your momentum is not conserved because your momentum before the jump was zero and became mv as you begin to rise.

Is the Principle of Conservation of Momentum applicable in this case? Discuss.

Solution: Both (A) and (B) below are acceptable answers.

(A) If the system being considered consists of only you**:**

the resultant of the external forces acting on this system (you) is not zero.
(since there is the gravitational force acting on you by Earth).

Hence, Principle of Conservation of Momentum is not applicable to this system as it is NOT a closed system).

(B) If the system consists of you AND the Earth:

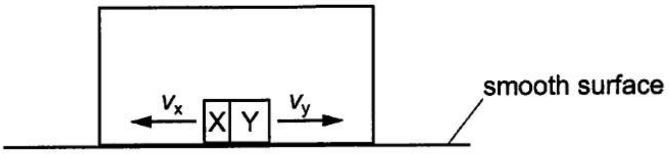
the resultant of the external forces acting on this system is zero.
Hence for this closed system, the Principle of Conservation of Momentum can be applied.
As you jump upward, the other body of this system (planet Earth) moves in the *opposite direction* with a momentum of *equal magnitude.*
{Of course the speed of the Earth would not be perceptible due to its large mass}

Note**:**
If the exam question does not specify the system, the candidate is allowed to choose what bodies/body constitute the system. Then discuss whether PCM is applicable or not to the chosen system.

Extension: Describe the motion of the **centre of gravity of the system** in (B).

5 A container placed on a smooth surface contains an object at its centre.
The object explodes into two pieces X and Y.
Y has a greater mass than X.

X moves at speed v_x to the left, hitting the end of the container at time t_1.
Y moves to the right at speed v_y, hitting the other end at time t_2.



Which statement describes the motion of the centre of gravity of the system (X, Y and the container)?

A It does not move.

B It moves to the left after t_1 and slows at t_2.

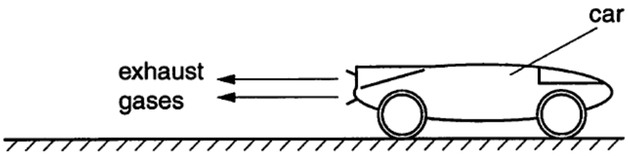
C It moves to the left at t_1 and stops at t_2.

D It moves to the right after t_2.

(N2019P1Q5)

Worked Example 14 with Visible Thinking (N2011P3Q1)

A toy car with a rocket engine moves along a horizontal track, as shown in the figure below.



The rocket engine produces a constant forward force of 4.6 N. The car loses mass continuously as exhaust gases are produced by the rocket.

Use momentum considerations to explain why the rocket produces a forward force on the car.

[3]

Thought process:

1. ***Trigger Keywords:*** *Use momentum considerations…*
   The first thing that comes to mind should be the PCM.

2. ***Trigger:*** *Diagram indicated that exhaust gases are moving backward*
   Exhaust gases are gaining momentum backward.

3. ***Trigger Keywords:*** *… forward force on the car...*
   The focus is on the car. The exhaust gases & the car are the 2 bodies of a closed system .

4. *The question requires us to link momentum and force. This should trigger you to think about Newton's 2nd Law (resultant force proportional to rate of change of momentum).*

5. *Considering the points above, the line of thought should be*
   a. *As a combined system, total momentum of the exhaust gases and the car should be conserved, since no net force acts on the system (exhaust gases and car)*
   b. *If the gases are gaining momentum backward, then to conserve total momentum, the car must gain momentum forward. (so that vector sum can be constant)*
   c. *Since car (only) is gaining momentum, there must be a force (N2L)*

Solution:

By PCM, the total momentum of the system (comprising the toy car and exhaust gases) should remain unchanged at any instant since no net force acts.

Since momentum is a vector quantity, the change in the exhaust gases' backward momentum at any instant must be matched by an equal change in the toy car's forward momentum, in order that total momentum remains unchanged.

The rate of change of the toy car's forward momentum, according to Newton's second law of motion, is the resultant force it experiences.

3.5.3 Elastic, Inelastic, & Completely Inelastic Interactions

- In any interactions between 2 bodies of a closed system, the total momentum and total energy of are always conserved.

- However, the total KINETIC energy of the system may or may not be conserved, depending on the type of interactions.

<table>
<tr><th rowspan="2">Type</th><th colspan="3">Total momentum conserved?</th><th colspan="3">Total KE conserved?</th><th rowspan="2">Other information</th></tr>
<tr><th>Before</th><th>During</th><th>After</th><th>Before</th><th>During</th><th>After</th></tr>
<tr><td>(Perfectly) Elastic</td><td colspan="3" rowspan="3">Yes</td><td>Yes</td><td>No</td><td>Yes</td><td>During collision, some KE is converted into other forms of energy
An example could be two rubber balls colliding. DURING the collision, the rubber balls will be deformed, and some KE become EPE. This KE will be recovered from the EPE when the rubber balls regain their former shape.</td></tr>
<tr><td>Inelastic</td><td colspan="3" rowspan="2">No, ie some KE is converted into other forms</td><td>After collision, particles do not stick together.</td></tr>
<tr><td>Perfectly Inelastic</td><td>After collision, particles coalesce ie stick together {& move with the same velocity}.</td></tr>
</table>

Fig. 3.11

3.5.4 Inelastic Collisions

This is defined as a collision in which total KE of system is NOT conserved

For such interactions, some KE has been converted to other forms: mostly used in deforming the colliding bodies - we say that some KE has been transformed into *internal energy*, which is the **work done to deform bodies permanently – KE cannot be recovered**. Some KE may also be converted into thermal energy as the environment heats up, (and to a certain extent, sound energy).

3.5.5 Perfectly (or Completely) Inelastic Collisions

This is defined as a collision in which

- total KE of system is NOT conserved, ***AND***

- the colliding bodies stick together (coalesce) to move as one body (i.e. with a common final velocity) after the collision.

The consequence is a simplification of PCM equation,

i.e. $$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

becomes $$\boxed{m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, v}\ ,$$ ----- eqn 3.7

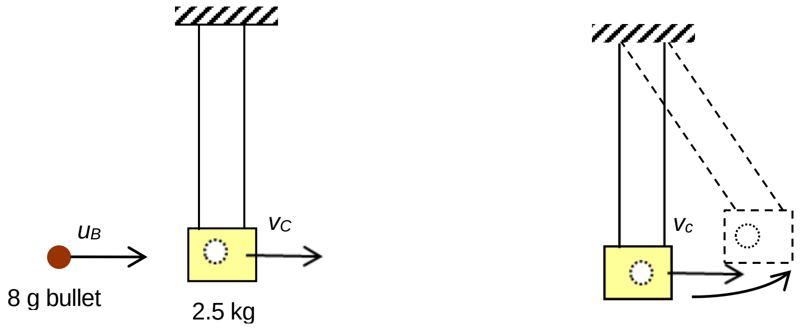
where v is the common final velocity

Worked Example 15: A Method to determine Speed of a Bullet – a perfectly inelastic collision

An 8 g bullet is fired into a 2.5 kg ballistic pendulum and becomes embedded in it. If the pendulum rises a vertical distance of 6 cm, calculate the initial speed of the bullet.

Stage 1: When bullet gets embedded in the ballistic pendulum

Stage 2: When the bullet and ballistic pendulum rises



Solution:

Stage 1 (to find u_B in terms of v_c using PCM):

("embedded" implies colliding bodies coalesce after collision)

Let u_B = initial speed of bullet (to find), and
v_c = velocity of "combined bodies" ***JUST*** after impact.

Total momentum before impact $= m_B u_B$ + zero for pendulum $= 0.008\ u_B$

Total momentum just after impact $= (m_B + M_P)\ v_c = 2.508\ v_c$

By PCM, $0.008\ u_B = 2.508\ v_c$ (1)

Stage 2 (to find v_c using COE):

Assuming air resistance is negligible,
Gain in gravitational PE = KE loss of combined bodies {after rising to max ht}

$(m_B + M_P)\ g\ \Delta h = ½\ (m_B + M_P)\ (v_c)^2$ (2)

Solving (1) and (2) simultaneously, initial speed of bullet, $u_B = 340$ m s^{-1}

Common Error:

Students frequently equate the initial KE of bullet with the GPE gained by bullet and ballistic pendulum. This would imply there is no KE converted to internal energy (when deformation occurs) or to thermal energy during the penetration of the bullet.

3.5.6 (Perfectly) Elastic Collisions

This is defined as a collision in which the total kinetic energy is conserved before and after the collision ie total KE BEFORE = total KE AFTER the collision.

- Total kinetic energy refers to the SUM of the KE of ALL bodies in the system.
- Individual KE of each of the 2 bodies may change, but the, sum of KE will NOT change.
- **DURING** an elastic collision, total KE may **not** be conserved as some KE may be converted into elastic PE or electric PE (for charged particles) *but* the PE is eventually reconverted back to KE (as in Fig. 3.10) so that total KE BEFORE = total KE AFTER collision.

(Truly elastic collisions are very rare (since it must be perfectly silent!), and are found to often occur between atomic and subatomic particles, such as electrons striking nuclei, or a collision between an air molecule and the wall of the container.
Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as *heat transfer due to friction and sound*. Examples of macroscopic collisions that are nearly elastic are that of two steel blocks on ice, two carts with spring bumpers on an air track, and billiard ball collisions.)

3.5.6.1 Problem-Solving in Elastic Collisions in 1-Dimension (Method A – using COKE)

The typical question on elastic collisions is solved by writing down 2 eqns:

Since it is **elastic**, total KE is conserved:

$$\boxed{\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2}$$ -------- eqn 3.8 (COKE – conservation of KE)

By Principle of Conservation of Momentum, (typically, we can assume no net force acts on system)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Worked Example 16

Two balls of masses 200 g and 500 g moving (in the same direction) along the line joining their centres with initial speeds 8.0 m s^{-1} and 2.0 m s^{-1} respectively undergo an elastic collision. Calculate their respective speeds after the collision. [0.57 m s^{-1}, 5.43 m s^{-1}]

Solution:

Since momentum is conserved, $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$$(0.20 \times 8.0) + (0.50 \times 2.0) = 0.20\, v_1 + 0.50\, v_2 \quad \text{--------------------(1)}$$

Since it's elastic: $\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$

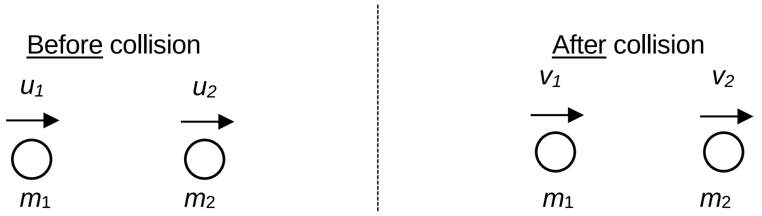
$$\tfrac{1}{2} \times 0.20 \times 8.0^2 + \tfrac{1}{2} \times 0.50 \times 2.0^2 = \tfrac{1}{2}\, 0.20 \times v_1^2 + \tfrac{1}{2}\, 0.50 \times v_2^2 \quad \text{------ (2)}$$

Solving simultaneously (using Method A) gives: $v_1 = -0.57$ m s^{-1}, $v_2 = 5.43$ m s^{-1}

Thus speed of 500 g and 200 g balls are 0.57 m s^{-1} and 5.43 m s^{-1} respectively.

Note: It can be tedious (do use the graphing calculator!) to solve simultaneous equations involving quadratic equation thus Method A is not used when Method B can be used where only linear equations are involved. However, one needs to be careful with sign convention while using Method B while Method A only deals with scalars, In addition, you do need to note there are two solutions when solving quadratic equation and the correct one has to be chosen according to the conditions given in the context.

3.5.6.2 Problem-Solving in Elastic Collisions in 1-Dimension (Method B – using relative speeds)



For any **elastic** collision,

Relative Speed of Approach = Relative Speed of Separation (Proof: see Appendix)

$$\boxed{\boldsymbol{u_1 - u_2 = v_2 - v_1}} \quad \text{-------- eqn 3.9}$$

- Eqn 3.9 can be used in place of eqn 3.8. This will avoid solving the problem involving quadratic equation which can be more tedious and time consuming.
- Eqn 3.9 does NOT apply to *inelastic* collisions.
- u_1, u_2, v_1, v_2 are <u>all vectors</u>. <u>You must substitute the sign of each vector into the equation</u>. Eqn 3.9 assumed all velocities are in the same direction.

Common error: Writing eqn 3.9 as: $u_1 - u_2 = \boldsymbol{v_1 - v_2}$

Exercise: You should try solving Example 16 using Method B.

Worked Example 17

In an elastic head-on collision, a ball of mass 2.0 kg moving at 3.0 m s^{-1} collides with a ball of mass 4.0 kg moving at 5.0 m s^{-1} in the opposite direction. Calculate the velocities of the balls after the collision.

Solution:

Let the directions of v_1 & v_2 be as shown.

Since this is an elastic collision & taking direction to the right to be +ve,

$$u_1 - u_2 = v_2 - v_1$$

becomes $(+3.0) - (-5.0) = v_2 - v_1$

$\Rightarrow \quad v_2 = 8.0 + v_1$ (1)

Note:
Do not forget to account for all the signs for all velocity vectors.

By PCM,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

i.e. $2.0\,(+3.0) + 4.0\,(-5.0) = 2.0\,(v_1) + 4.0(v_2)$ (2)

Substituting (1) into (2),

$-14 = 2.0\, v_1 + 4.0\,(8.0 + v_1)$

$v_1 = -7.7$ m s^{-1}

$\Rightarrow \quad v_2 = 0.33$ m s^{-1}

Tutorial qn: Q15, Q16, Q17, Q18, Q18, Q20, Q21

<u>Special Case 1:</u> Elastic Collision between <u>identical</u> masses, with one initially stationary
i.e. $m_1 = m_2$ & $u_2 = 0$

Worked Example 18

In an elastic collision, two spheres A and B of equal mass collide head-on. Before the collision, sphere B is stationary and sphere A is moving with speed 6.0 ms^{-1} directly towards sphere B.
Determine the velocity of sphere B after the collision.

Solution:

For an elastic collision,
→: $u_A - u_B = v_B - v_A$

$(+6.0) - (0) = v_B - v_A$

$v_B = v_A + 6.0$ (1)

By PCM
→: $m\,u_A + m\,u_B = m\,v_A + m\,v_B$

$(+6.0) + (0) = v_A + v_B$ (2)

Substituting (1) into (2),

$6.0 = v_A + (v_A + 6.0)$

$\Rightarrow v_A = 0$ m s^{-1} , $v_B = 6.0$ m s^{-1}

Conclusion:
Spheres A and B <u>exchange velocities</u> after the collision! You may observe this interaction between billiard balls.

This 3-min video demonstrates the <u>elastic collision</u> of 2 objects of <u>equal masses</u>. The objects **exchange velocities** after the collision.
https://www.youtube.com/watch?v=51IFubnEAsU&t=2s

Special Case 2: Elastic Collision between a "light" moving mass and a "heavy" stationary mass
i.e. $m_1 << m_2$ & $u_2 = 0$

Worked Example 19

A tennis ball of mass 100 g strikes normally at a flat surface of a stationary boulder of mass 10000 kg. The tennis ball is then reflected at a speed of 2.0 ms^{-1}. Assume that the collision is elastic, determine the speed at which the tennis ball strikes the boulder.

Solution:

Let u_t and v_t be the velocity of the tennis ball just before and just after it hits the boulder respectively.
Let u_B and v_B be the velocity of the boulder just before and just after the tennis ball hit it respectively.

For an elastic collision,
Taking direction towards the boulder as positive,

$\rightarrow$: $u_t - u_B = v_B - v_t$

$u_t - 0 = v_B - (-2.0)$

$u_t = v_B + 2.0$ (1)

By Principle of Conservation of Momentum,

$\rightarrow$: $m\,u_t + m\,u_B = m\,v_t + m\,v_B$

$0.10\,u_t + 10000\,(0) = 0.10(-2.0) + 10000\,v_B$

$0.10\,u_t = -0.20 + 10000\,v_B$ (2)

Substituting (1) into (2),

$0.10(v_B + 2.0) = -0.20 + 10000\,v_B$

$9999.9v_B = 0.40$

$v_B = 4.0 \times 10^{-5}$ m s^{-1} (2 sf)

$u_t \approx 2.0$ m s^{-1} (2 sf)

Conclusion:
1. Smaller mass (m_1) rebounds in opposite direction with almost the same speed after collision.
2. Much heavier mass (m_2) remains approximately at rest.

i.e. there is (almost) no transfer of KE between m_1 and m_2. You may observe this when a light object e.g. ping pong ball hits a massive stationary object e.g. a wall

Special cases 1 & 2 represent the 2 extremes of the transfer of KE from one object to another in a head-on elastic collision where one of the objects is initially stationary.

The results from special cases 1 and 2 can be generalised and proven. The results are **useful for A-level MCQs** and are summarised in the table below.

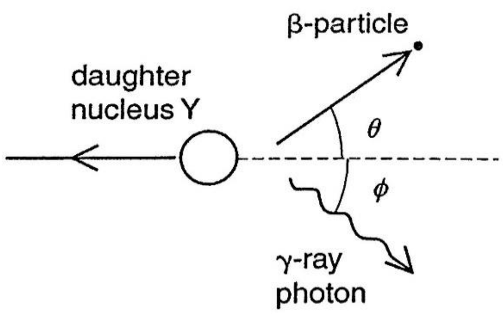
(Perfectly) Elastic Collisions (Total KE and Total momentum conserved)
Special Case 1: Elastic Collision between identical masses, with one initially stationary (i.e. $m_1 = m_2$ & $u_2 = 0$) • $v_1 = 0$ • $v_2 = u_1$ • In other words, the masses **exchange velocities**
Special Case 2: Elastic Collision between a "light" moving mass and a "heavy" stationary mass (i.e. $m_1 << m_2$ & $u_2 = 0$) • $v_1 \approx -u_1$ • $v_2 \approx 0$ • In other words, the incoming **light mass rebounds** in opposite direction with **no change in speed** • **Stationary heavy mass remains stationary**

3.5.7 Use of PCM in 2-dimensional Interactions:

- All the collisions discussed so far are **1-dimensional** interactions where the motions of the objects, before and after the interaction, occur along the same straight line.
- **A 2-dimensional** interaction is one where the motions of the objects before and after the interaction, are NOT along the same straight line but in a plane.
- Syllabus Learning Outcome (i) seems to suggest that 2 dim collisions are out of the syllabus but is contradicted in N2012 P3 Q6 (ii) & N2015 P3 Q5 (c) – in Topic 19 Quantum Physics

Example 20 (2012 P3 Q6 (ii))

In a student's model for radioactive decay, a stationary nucleus X decays by simultaneous emission of a β- particle (beta particle) and a γ-ray photon (gamma ray). A daughter nucleus Y is formed. Nucleus Y may move after the decay. Possible directions for the β- particle, the γ-ray photon and the nucleus Y are shown below.



Momentum is a vector quantity and can be resolved into perpendicular components.
Suggest and explain the relation between the angle θ and the angle ϕ. [3]

(Theory:
For a 2-dim interaction, since total momentum is conserved, **2 independent eqns** can be written, one to express the conservation of momentum in the "x-direction" & the other, in a perpendicular direction.)

Solution:

Let p_Y: momentum of nucleus Y (in x-dir)

Tutorial qn: Q22

SUMMARY

1. **Newton's First Law:**
Every object continues in its state of rest or constant speed in a straight line (or constant velocity) unless a net force acts on it to change that state.

2. **Newton's Second Law**:
The rate of change of momentum of a body is proportional to the net force acting on it, and this (rate of) change of momentum takes place in the direction of the net force.

3. **Force** on a body is defined as the rate of change of momentum of the body and it acts in the direction of the change in momentum.

{i.e. $F_{net} = \dfrac{d(mv)}{dt}$}

System	Equation	Example
Constant mass with varying velocty	*Net force,* $F = \dfrac{m(v-u)}{t} = ma$ *acting over a finite time interval t*	Forces acting on a point mass Forces acting on connected bodies
Constant velocity with varying mass	Thrust, $F = v\frac{dm}{dt}$ with mass varying at dm/dt and moving at velocity v	Expelled fuel from a rocket providing a forward thrust (in opp dir to the dir of expelled fuel) on the rocket due to Newton's 3rd law as the mass of the rocket decreases as dm/dt when the fuel is expelled at velocity v

4. **Newton's Third Law:**
When body A exerts a force on body B, body B exerts a force of the same type that is equal in magnitude and opposite in direction on body A.

- These two forces form an action-reaction pair, always act on different bodies; hence, they cannot cancel each other out
- They are of the same type of force. Must know how to identify forces which form an action-reaction pair; *weight and the normal reaction are not an action-reaction pair as they are different type of force!*

5. **Linear momentum** of a body is defined as the product of its mass and velocity. i.e. $\boxed{p = m\,v}$
Unit: kg m s^{-1}

6. **Impulse of a force** I is defined as the product of the force and the time Δt during which it acts.

Impulse is a vector.

Case 1: For a constant net force F over duration Δt - $\boxed{I = F \times \Delta t}$

Case 2: For a variable force - $\boxed{\text{Impulse = area under the } F\text{-}t \text{ graph} = \int F\,dt}$

- It is equal in magnitude to the change in momentum of the body.
{Incorrect to define impulse as a change in momentum}
- Hence the change in momentum of the body is equal in magnitude to the area under a (net) force-time graph. {For F-t graph of irregular shape, use "count-the-squares" method to determine the area}

7. Mass: a measure of the amount of matter in a body, hence, a measure of the inertia of the body

- **Inertia** is the property of a body which **resists change in its motion** (2022 P3Q1(a))

Weight: weight W is the force experienced by a mass in a gravitational field g. $W = mg$.

Apparent weight refers to the Normal reaction N that the surface/ weighing scale exerts on a body.

8. Principle of Conservation of Linear Momentum (PCM):
For a system of interacting bodies, the total momentum of the bodies (i.e. momentum of the system) remains constant, provided no net force acts on the system. {incorrect to say "no external force"}

or, the total momentum of an closed system is constant,

i.e. $\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$, if net $F = 0$ {applicable for all interactions}

Note: Total momentum *during* the interaction is also conserved. {See N08P1Q6}

9. Types of collisions:

<table>
<tr><th rowspan="2">Type</th><th colspan="3">Total momentum conserved?</th><th colspan="3">Total KE conserved?</th><th rowspan="2">Other information</th></tr>
<tr><th>Before</th><th>During</th><th>After</th><th>Before</th><th>During</th><th>After</th></tr>
<tr><td>(Perfectly) Elastic</td><td colspan="3" rowspan="3">Yes</td><td>Yes</td><td>No</td><td>Yes</td><td>During collision, KE is converted into other forms of energy but can be recovered such that KE before and after collision are the same.

Eg. Deformation of two rubber balls during the collision where KE is converted into EPE before converting back to KE after the balls regain their original shape.</td></tr>
<tr><td>Inelastic</td><td colspan="3" rowspan="2">No, ie some KE is converted into other forms and cannot be recovered.</td><td>After collision, particles do not stick together.</td></tr>
<tr><td>Perfectly Inelastic</td><td>After collision, particles stick together / coalesce {move with the same velocity}.</td></tr>
</table>

For all **elastic collisions**, $\boxed{u_1 - u_2 = v_2 - v_1}$ {this is a vector equation where u_1, u_2, v_1 and v_2 are all vectors and the signs for the four vectors must be substituted correctly}

i.e. relative speed of approach = relative speed of separation**.**

or, $\boxed{\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2}$ {this is a scalar quadratic equation and there will be two soln. The correct coln must be chosen based on context given}

10. Problem Solving:

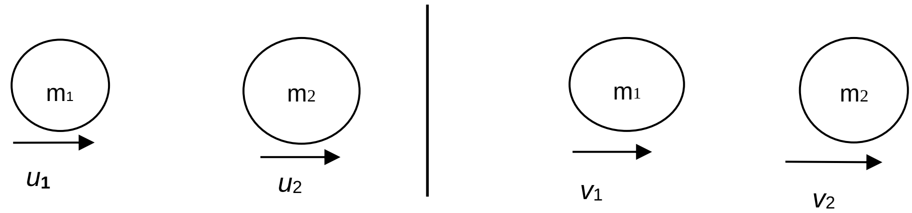
For inelastic collision: Use PCM
For elastic collision: Use PCM and the "relative speed equation" or "COKE".

Note:

- PCM applies to all interacting systems not subjected to a net force including explosion not just collision.
- Momentum and velocity are vectors thus direction must be taken into account in applying PCM and relative speed equation
- Always draw diagram of before and after collision/explosion to help keep track of the sign convention.

APPENDIX

Derivation of Relative Speed Formula (not required for A level)



Relative speed of approach refers to the **speed** of m_1 as observed by m_2.

- Thus, as m_1 approaches m_2, relative speed of approach is $\boldsymbol{u_1 - u_2}$ (m_2 is the observer).

Relative speed of separation refers to the speed of m_2 as observed by m_1.

- Thus as m_2 separates from m_1, relative speed of separation is $\boldsymbol{v_2 - v_1}$ (m_1 is the observer).

Now, consider an *elastic head-on* collision between two masses,

By **Principle of Conservation of Momentum**,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \qquad (1)$$

Since it's **elastic**, $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \qquad (2)$

From equation (1), $m_2 (u_2 - v_2) = m_1 (v_1 - u_1) \qquad (3)$

From equation (2), $m_2 (u_2^2 - v_2^2) = m_1 (v_1^2 - u_1^2) \qquad (4)$

Dividing (4) by (3), $u_2 + v_2 = v_1 + u_1$

i.e. $\boxed{u_1 - u_2 = v_2 - v_1}$

TUTORIAL 3: DYNAMICS

(Take *g* as 9.81 m s^{-2} unless otherwise stated.)

Newton's First Law: Law of Inertia

(L1)1. A parachutist of mass 60.0 kg, including his pack and parachute, is falling with a steady speed. Deduce using Newton's first law, the magnitude of

(a) the net force acting on him, [1]

(b) the air resistance acting on him and the parachute. [1]

Free Body Diagram of parachutist and the parachute

Newton's Third Law: Action-Reaction Pair

(L2)2. Imagine that you are holding a book weighing 4 N at rest on the palm of your hand. Complete the following sentences:

a) A downward force of magnitude 4 N is exerted on the book by ____________.

b) An upward force of magnitude ______ is exerted on ____________ by the hand.

c) Is the upward force in part (b) the reaction to the downward force in part (a)?

d) The reaction (force) to the force in part (a) is a force of magnitude _________, exerted on ___________ by ____________. Its direction is _____________.

e) The reaction to the force in part (b) is a force of magnitude _____________, exerted on ___________ by _________. Its direction is ____________.

f) A student thinks that the reason why the force described in (a) and that in (b) are equal is because they are an action-reaction pair.

Explain how one can deduce that these 2 forces ***cannot*** be an action-reaction pair.

Free Body Diagram of book

Newton's Second Law for Constant-Mass Systems: F = ma

(L1)3. A lift of mass 480 kg is designed to carry a maximum load of 3 000 N.
Calculate the tension in the lift cable at maximum load when

(a) the lift moves downwards at steady speed, [2]
(b) the lift moves downwards, accelerating at 1.5 m s^{-2}, [2]
(c) the lift moves downwards, decelerating at 1.5 m $^{-2}$, [2]
(d) the lift moves upward, accelerating at 1.5 m s^{-2}, [2]
(e) the lift moves upward, decelerating at 1.5 m s^{-2}. [2]

Free Body Diagram of lift

(L2)4. A 70.0-kg man jumps 1.00 m down onto a concrete walkway. As he forgets to bend his knees, his downward motion comes to a complete stop in 0.10 s. Determine the **average** force transmitted to his leg bones. [3]

Free Body Diagram of man

(L2)5. In a test of a car seat belt system, a dummy of mass 55 kg in a car seat is accelerated to a speed of 35 m s^{-1} before it hits a brick wall. The seat belt allows the dummy to move forward relative to the seat by a distance of 0.60 m. Calculate
(a) the deceleration of the dummy, [2]
(b) the force on the dummy due to the seat belt. [1]

(L2)6. A stream of water from a hose travels horizontally at speed v. The stream strikes a brick wall and falls vertically down it without splashing.
The stream of water is a cylinder of cross-sectional area A. The water has density ρ.
Which expression is the force exerted on the wall by the water? [N13 P1 Q6]

A $A\rho v$ **B** $A\rho v^2$ **C** $\frac{1}{2}A\rho v^3$ **D** $A\rho v^3$

(L2)7. Kimi Raikkonen (mass 74 kg) accelerates horizontally from rest to a speed of 160 km per hr in 4 s in his racing car. Calculate the ***total*** force exerted ***by the car seat*** on Kimi. [3]

Free Body Diagram of Kimi

Motion on an Inclined Plane

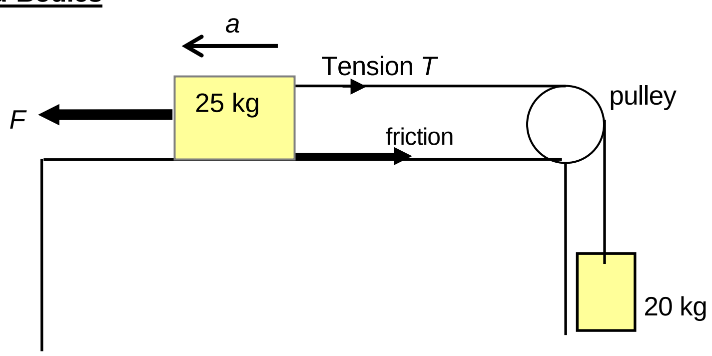
(L2)8. An inclined plane makes an angle of 30° with the horizontal. Neglecting friction, find the constant force X, applied parallel to the plane, required to cause a 15 kg box to slide

(a) up the plane with acceleration 1.2 m s^{-2}, [2]
(b) down the incline with acceleration 1.2 m s^{-2}. [2]
(c) What would be the magnitude and direction of the acceleration if the force X, is *not* applied to the box? [2]

Free Body Diagram of box

Connected-Bodies

(L2)9.

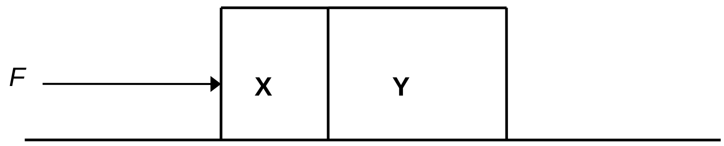


Determine the constant force F required to give the 20 kg block an upward acceleration of 0.60 m s^{-2} given that the friction on the 25 kg block is 49 N. What will be the tension T in the cord? [3]

Free Body Diagram of 25 kg

Free Body Diagram of 20 kg

(L2)10. Two blocks, X and Y, of masses m and $2m$ respectively, are accelerated along a smooth horizontal surface by a force F applied to block X, as shown.



What is the magnitude of the force exerted by block Y on block X during this acceleration? [1]

A 0 **B** F / 3 **C** F / 2 **D** 2F / 3

[J90 P1 Q2 & N99 P1 Q4]

Free Body Diagram of X and Y

Free Body Diagram of X or Y

(L2)11. A loaded wagon of mass 900 kg is pulled on a level road by a horse of mass 350 kg. A force of friction of 1300 N acts on the wagon.

Calculate

(i) the force the horse push (backward) on the ground in order to give the wagon a forward velocity of 6 m s^{-1} in 5 s from rest, [2]

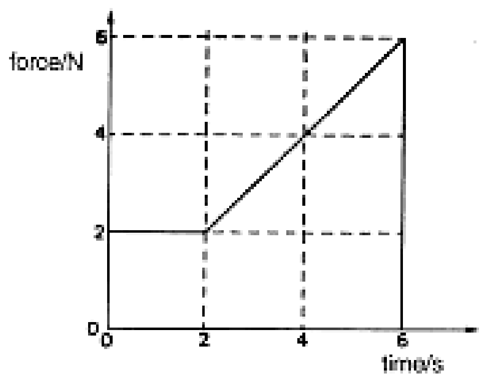
(ii) the tension in the rope. [2]

Free Body Diagram of wagon

Free Body Diagram of horse

Momentum Change & Force-time Graph

(L1)12. The graph shows how the force acting on a body varies with time. Assuming that the body is moving in a straight line, by how much does its momentum change? [2]
[N90 P1 Q5]



(L2)13. A model rocket of initial mass 1.3 kg is fired vertically into the air. Its mass decreases at a constant rate of 0.23 kg s^{-1} as the fuel burns. The final mass of the rocket is 0.38 kg. The rocket rises to a height such that, during the flight, the gravitational field strength of the Earth may be considered to have the constant value of 9.8 N kg^{-1}. [N96 P2 Q1]

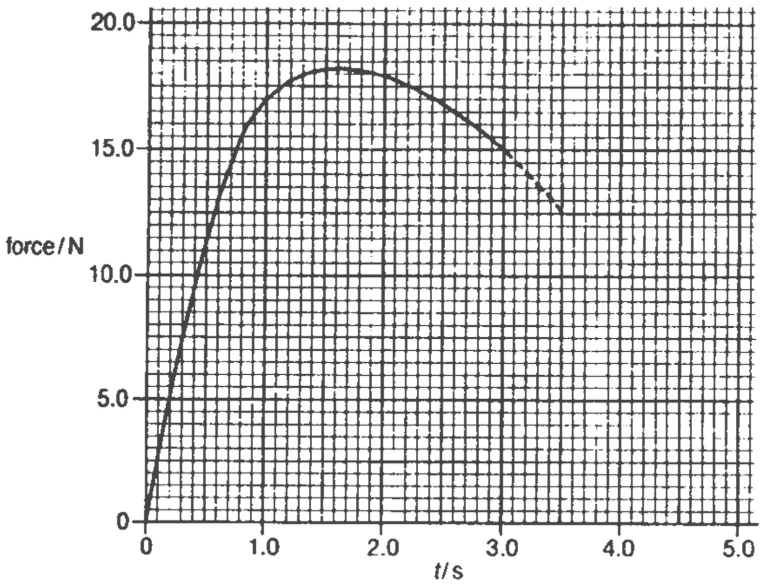
(a) Calculate

(i) the initial weight of the rocket,

(ii) the final weight of the rocket,

(iii) the time taken for the fuel to be burned. [3]

(b) The variation with time t of the upward force on the rocket during the first 3 seconds after firing is shown below.



(i) On the graph above, use the same scales to draw a line to represent the variation with time *t* of the total weight of the rocket during the first 5 seconds after firing. [2]

(ii) Hence read off from the graph, the time delay between firing the rocket and lift-off. [1]

(c) (i) Write down an equation to represent the relation between the resultant force F on a body, the time t for which the force acts and the change in momentum Δp of the body.

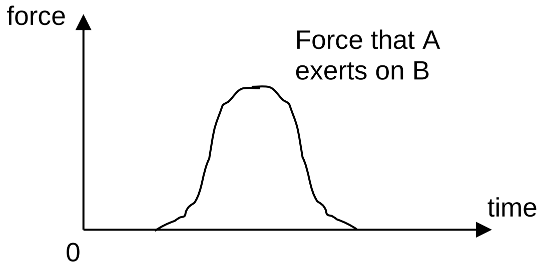
(ii) On the graph above, shade the area of the graph which represents the change in momentum of the rocket during the first 3 seconds after the rocket is fired. [3]

(d) The energy stored in the fuel is converted partly into kinetic energy of the rocket and thermal energy of the rocket. State two further forms of energy into which the energy of the fuel is converted. [2]

(L2)14. (a) (i) Define *linear momentum*. [1]

(ii) State the relationship between the change in linear momentum of an object, the constant force acting on the object, and the time for which the force acts. [1]

(b) In a collision between two bodies A and B, the force that A exerts on B varies with time in the way shown below.



(i) Sketch a graph on the axes below, to show the variation of the force that B exerts on A. [1]

(ii) Explain your answer to (i). [1]

(iii) Explain how your answer to (i) is consistent with the principle of conservation of momentum. [3]

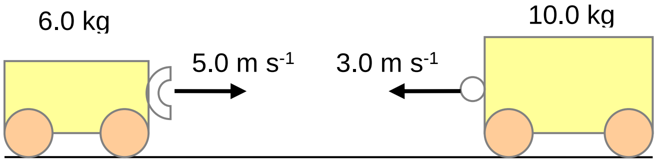
(c) In a collision, when a truck of mass 12 000 kg runs into the back of a car of mass 1200 kg, a constant force of 72 000 N acts for 0.25 s.
Calculate the change in the velocity of
(i) the car, [3]
(ii) the truck, [2]

(d) Suggest one way in which the conditions in (c) are unrealistic. [1]

(e) Discuss how seat belts and air bags in a car ensure greater safety. [2]
[N04 P3 Q2]

Application of PCM in 1-D Interactions

(L1)15. A trolley of mass 6.0 kg travelling at a speed of 5.0 m s^{-1} collides head on and locks together with another trolley of mass 10 kg which is travelling in the opposite direction at a speed of 3.0 m s^{-1}. The collision lasts for 0.20 s.



What is the total momentum of the two trolleys before the collision and the average force acting on each trolley during this collision?

	Total momentum Before collision / kg ms^{-1}	average force on each trolley / N
A	0	300
B	60	150
C	0	150
D	60	300

[N08 P1 Q6]

(L2)16. A 70 kg man stands in the middle of a frozen pond of radius 5.0 m. He is unable to get to the other side because of a lack of friction between his shoes and the ice. To overcome his difficulty, he throws his 1.2 kg physics textbook horizontally at a speed of 5.0 m s^{-1} towards the north shore. How long does it take him to reach the south shore? [2]

Assignment

(L2)17 (a) A piece of plasticine of mass 0.20 kg falls to the ground and hits the ground with a velocity of 8.0 m s^{-1} vertically downward. It does not bounce but sticks to the ground.

(i) Calculate the momentum of the plasticine just before it hits the ground. [1]

(ii) State the transfers of momentum and of kinetic energy of the plasticine which occur as a result of the collision. [2]

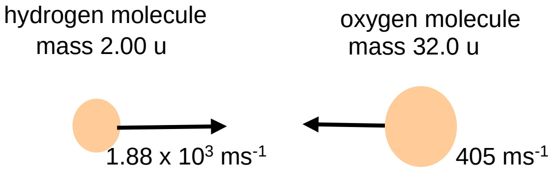
(b) A neutron of mass 1.00 u travelling with velocity 6.50 x 10^6 m s^{-1} collides head on with a stationary carbon atom of mass 12.00 u. The carbon atom moves off in the same direction with velocity 1.00 x 10^6 m s^{-1}. (1 u = 1.66 x 10^{-27} kg).

(i) Calculate the velocity of the neutron after the collision. [2]

(ii) State what happens to the total kinetic energy as a result of this collision. [2]

(c) When two strong magnets are held stationary with the north pole of one pushed against the north pole of the other. On letting go, the magnets spring apart. It is apparent that the kinetic energy of the magnets has increased.
Explain how the law of conservation of momentum applies in this case. [2]
[J94 P2 Q1]

(L2)18. (a) In a gas a hydrogen molecule of mass 2.00 u and velocity 1.88×10^3 m s^{-1}, collides head-on with an oxygen molecule of mass 32.0 u and velocity 405 m s^{-1}, as illustrated below. 1 u = 1.66×10^{-27} kg.



In qualitative terms, what can be stated about the subsequent motion as a result of knowing that

(i) the collision is elastic,

(ii) the collision is head-on? [2]

(b) Using your answers to (a),

(i) determine the relative velocity of separation of the 2 molecules after the collision,

(ii) determine the velocity of both molecules after the collision. [4]

[N2000 P3 Q1]

(L2)19. A stone is dropped from a point a few metres above the Earth's surface. Considering the system of stone and Earth, discuss briefly how the principle of conservation of momentum applies *before* the impact of the stone with the Earth. [3]

[J85 P2 Q2]

(L2)20. A tritium nucleus moves towards a deuterium nucleus as illustrated in Fig. 21.1.

Fig.21.1

The nuclei initially have the same speed v. The tritium nucleus consists of two neutrons and a proton. The deuterium nucleus consists of a neutron and a proton. The proton and the neutron have the same mass m.

(a) At one instant during the interaction between the nuclei, they are both travelling in the **same direction** with the same speed. Calculate this speed, in terms of v. [2]

(b) Fig. 21.2 is a velocity-time sketch graph showing how the velocity of each nucleus varies. The interaction between the nuclei is elastic.

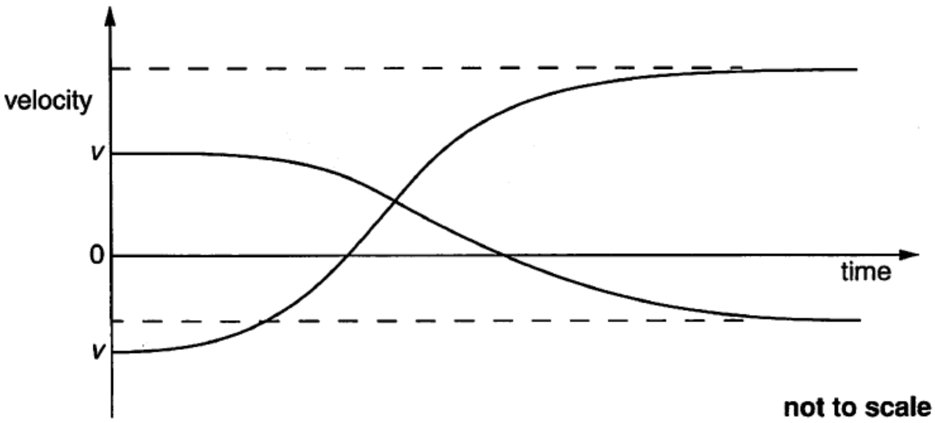


Fig. 21.2

(i) Label the graph to show

1. which curve is for the tritium nucleus,
2. the times at which each nucleus stops,
3. the time at which they are at their distance of closest approach. [3]

(ii) Determine the final speed of each nucleus in terms of v. [4]

[N2007 P3 Q1]

(L2)21. (a) A car of mass 750 kg is travelling at 25 ms^{-1} along a horizontal road. The brakes are applied and the car is brought to rest by an average resistive force *F*. The car has an average deceleration of 4.8 ms^{-2}.

(i) Show that the resistive force acting on the car is 3600 N.

[1]

(ii) Calculate the distance travelled by the car during this deceleration.

[2]

(iii) Describe, in terms of Newton's third law, the horizontal forces acting on the tyres of the car and on the road. [2]

(b) The car in (a) now travels at 25 ms^{-1} down a **slope** where the angle to the horizontal is 10°. The car is brought to rest by applying the brakes. The same resistive force of 3600 N acts on the car.

(i) Explain why the distance the car travels before coming to rest is greater than in (a).

[1]

(ii) Calculate the deceleration of the car.

[2]

[N2010 P2 Q1 part]

Application of PCM in 2-D Interactions

(L2)22. A car of mass 750 kg was travelling due North when it collided at an intersection with a bus of mass 2000 kg which was travelling due East. From witnesses, the police found out that the bus was travelling at a speed of 36 km h^{-1} and that, after collision, the car and bus entangled together, sliding freely on an oil patch produced in the accident in the direction N 53.1° E.

(a) Did the car exceed the speed limit of 70 km h^{-1}? [3]

(b) What was the speed of the wreckage just after the collision? [2]

Numerical Answers

1 (b) 5.9×10^2 N

3 (a) 7710N (b) 6530N (c) 8890N (d) 8890N (e) 6530N

4. 3800 N

5. (a) 1020 m s^{-2} (b) 56.1 kN

7. 1100 N

8. (a) 91.6 N (b) 55.6 N (c) 4.91 m s^{-2}

9. T = 208 N ; F = 272 N

10. $F_{Y\ on\ X} = 2/3\ F$

11. (a)(i) 2.80 kN (ii) 2.38 kN

12. 20 kg m s^{-1}

13. (a)(i) 12.7N (ii) 3.72 N (iii) 4 s (b)(ii) 0.5 s

14. (c)(i) 15 ms^{-1} (ii) 1.5 ms^{-1}

16. 58.3 s

17. (a)(i) 1.6 kg m s^{-1} (b)(i) $- 5.5 \times 10^6$ m s^{-1}

18. (b) (i) 2290 m s^{-1} (ii) 2420 m s^{-1} & 136 m s^{-1}; both to left

20. (a) v / 5 (bii) deuterium = 1.4v, tritium = 0.6v

22. (b) 32.7 km h^{-1}

ADDITIONAL QUESTIONS

ACJC Prelims 2020 P2 Q2

1 **(a)** An object of mass 1.5 kg is released vertically downwards from a stationary hot air balloon in mid-air. Fig. 2.1 shows how the velocity of the object varies with time.

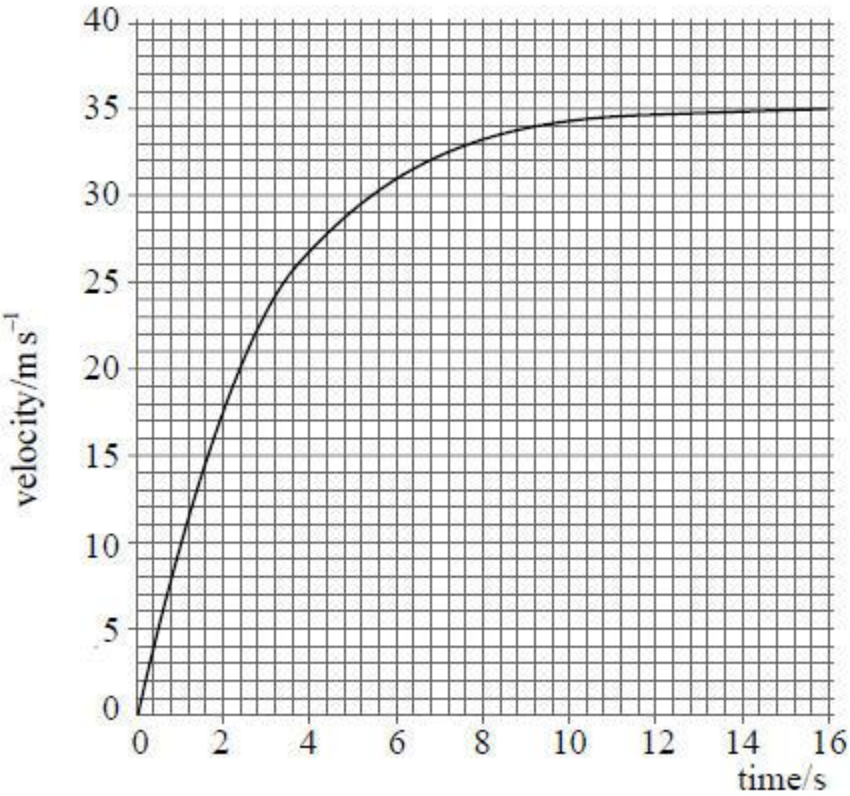


Fig. 2.1

(i) Define *acceleration*.

………………………………………………………………………………………………………

……………………………………………………………………………………………… [1]

(ii) Using Fig. 2.1, explain how the acceleration of the object changes with time.

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

……………………………………………………………………………………………… [2]

(iii) Determine the magnitude of the viscous force acting on the object at time = 5.0 s.

viscous force = ……………………… N [3]

(b) Fig. 2.2 shows two blocks A and B of mass 3.0 kg and 4.0 kg respectively connected by a light inextensible cord passing over a light frictionless pulley. When both blocks are released, block A starts to move from rest along a smooth plane inclined at 40° to the horizontal while block B starts to move from rest along a smooth plane inclined at 20° to the horizontal.

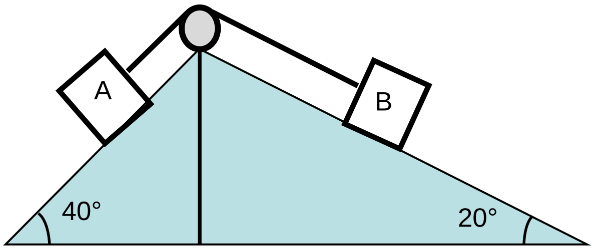


Fig. 2.2 (not to scale)

Calculate the magnitude of the acceleration of the two blocks when the blocks are released and the tension in the cord.

acceleration = ……………………… m s^{-2} [2]
tension = …..……………………… N [1]

ASRJC Prelims 2021 P3Q2

2 **(a)** A student suggests that Newton's third law implies that the weight of a book resting on a table is equal to the support force that the table exerts on the book.

Explain why

(i) the student is wrong,

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

…………………………………………………………………………………………………… [2]

(ii) the two forces are equal and opposite.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

…………………………………………………………………………………………………… [1]

(b) Use Newton's laws to deduce the principle of conservation of momentum.

[3]

(c) In space, an object of mass 28 kg travelling with velocity 88 m s^{-1} collides with a second object of mass 17 kg travelling in the same direction with a velocity of 53 m s^{-1}. The collision is inelastic.

After the collision, the 28 kg object continues to move in the original direction but with a velocity of 67 m s^{-1}.

Calculate the loss in kinetic energy in the collision.

loss in kinetic energy = …………………………….J [3]

(d) In **(c)**, the force exerted by the 28 kg object on the 17 kg object will not have a constant value during the time they are in contact with one another.

Sketch two graphs on the axes shown in Fig. 2.1 to show how the force varies with time if the collision in **(c)** is between

(i) two steel objects (label this line S),

(ii) two rubber objects (label this line R).

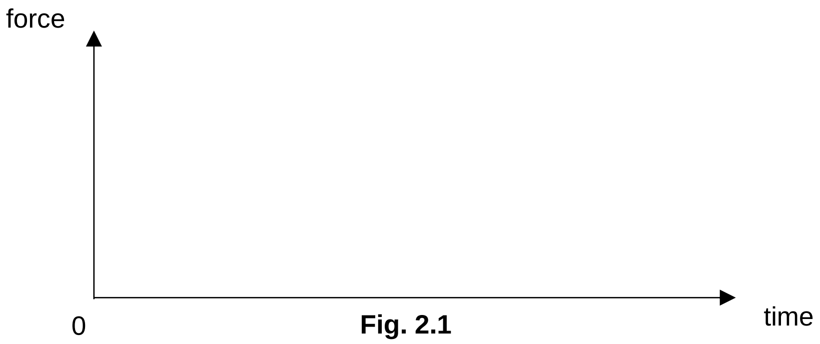


Fig. 2.1

[2]

TUTORIAL 3: DYNAMICS SOLUTIONS

Level 1 Solutions

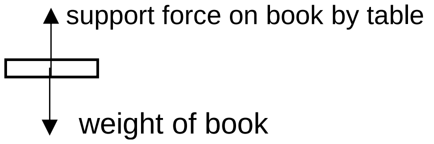
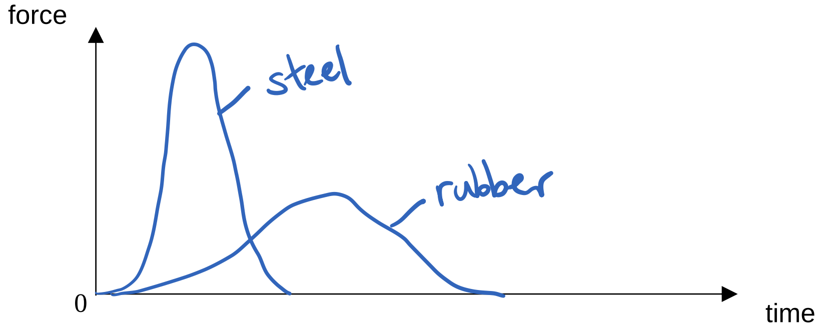
1 (a)	(Newton's First Law states that every body continues in a state of rest or uniform motion in a straight line unless a net external force acts on it.) Since the parachutist is falling with a <u>steady</u> speed, the net force acting on him is <u>zero</u>.	[1]
(b)	Since the net force acting on him is zero, total upward force (air resistance) is equal to total downward force (his total weight). Hence the air resistance acting on him and the parachute is equal in magnitude to his total weight = mg = (60.0)(9.81) = <u>5.9×10^2 N</u>	[1]
3	The general eqn of motion is either Case 1: if lift speeds up (i.e. accelerates) on the way up, or slows down downwards, then: ↑: T – mg = ma Case 2: if lift speeds up on the way down, or slows down (decelerates) upwards, then: ↓: mg – T = ma Case 3: for constant speed in either direction, T = mg since a = 0. a) 7710N b) 6530N c) 8890N d) 8890N e) 6530N	[1] [1]
12	Change in momentum = area under F-t graph = 4 + ½ (2 + 6) (4) = 20 kg m s^{-1}	[1] [1]
15	Ans: C Total momentum of system before collision = (6.0)(5.0) – (10)(3.0) = 30 Ns – 30 Ns = 0 Ns By Principle of conservation of momentum, total final momentum is thus 0 (since no net force acts on the system). Hence **both** trolleys will be brought to rest **during** collision. By Newton's 2nd Law, net force on each trolley is the rate of change of linear momentum. i.e. $F = \frac{30}{0.20}$ = 150 N.	

Solutions to Additional Questions

ACJC Prelims 2020 P2Q2

1(a)(i)	Acceleration is the rate of change of velocity.	1
(a)(ii)	The gradient of the tangent to the *v-t* graph gives the acceleration. Acceleration decreases continuously to zero (or near zero) at 16.0 s.	1 1
(a)(iii)	Finding gradient of tangent drawn at 5.0 s (accept reasonable tangent) $F = ma$ = weight – F_v $1.5(a) = 1.5(9.81) - F_v$ *Acceptable range of* F_v is 11.2 - 11.9 N	1 1 1
(b)(i)	Assuming block A moves down the plane, *Note: You can also solve by assuming A moves upwards and if the assumption is wrong, the acceleration you get will be negative but still the correct magnitude*. F = ma for A: $(3.0)(9.81)\sin 40° - T = (3.0)a$ ---- (1) F = ma for B: $T - (4.0)(9.81)\sin 20° = (4.0)a$ ----- (2) Solving both equations (1) and (2), a = 0.785 m s^{-2} (3sf) Substitute a = 0.785 in (1) $(3.0)(9.81)\sin 40° - T = (3.0)(0.785)$ T = 16.6 N (3sf)	 1 1 1
(b)(ii)	Consider both blocks as one system since friction is for both blocks and cannot be split for each block $m_A g \sin 40° - m_B g \sin 20° - f = (m_A + m_B)\, a$ $(3.0)(9.81)\sin 40° - (4.0)(9.81)\sin 20° - 4.0 = (7.0)a$ $a = 0.214$ m s^{-1} $v^2 = u^2 + 2as$ $v^2 = 0 + 2(0.214)(2.0)$ v = 0.925 m s^{-1} (3sf)	 1 1 1
	Alternative solution using energy method, Loss in GPE_A = Gain in GPE_B + Gain in KE_{A+B} + Work done against friction $m_A g h_A = m_B g h_B + \frac{1}{2}(m_A + m_b)v^2 + fs$ $(3.0)(9.81)(2.0\sin 40°) = (4.0)(9.81)(2.0\sin 20°) + \frac{1}{2}(3.0 + 4.0)v^2 + (4.0)(2.0)$ v = 0.925 m s^{-1} (3sf) Note: The cord remains taut thus A and B moves with the same speed.	 *1* *1* *1*

ASRJC Prelims 2021 P3Q2

2i	 The support force on book by table and the weight of book are both acting on the book. (N3L states that forces act on different bodies.) They are different types of forces, gravitational and electromagnetic/contact forces. (N3L states that forces must be of the same type.) (So, they are not a pair of action-reaction forces.) Alternate solution Reaction force of contact force by table on book is contact force that book exerts on table. Reaction force of weight of book is gravitational force that book exerts on Earth. (So, weight of book and contact force by table on book are not a pair of action-reaction forces.)	1 1 1 1
aii	Since book is resting on table, there is no net force on the book, so the two forces are equal and opposite.	1
b	Suppose two colliding bodies A and B (where A and B is an isolated system), By Newton's third law, force A exert on B, F_{AB} is equal in magnitude and opposite in direction to force B exert on A, F_{BA}. $F_{AB} = -F_{BA}$ Duration of collision is the same for A and B. By Newton's second law, net force on A, F_{BA} is equal to rate of change of momentum of A. Net force on B, F_{AB} is equal to rate of change of momentum of B. Hence, total (rate of) change of momentum is 0.	1 1 1
c	By Conservation of Linear Momentum Sum of initial momentum = Sum of final momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ (28)(88) + (17)(53) = (28)(67)+ (17)v_2 v_2 = 87.6 m s^{-1} Loss in kinetic energy = total initial kinetic energy – total final kinetic energy = ½ (28)(88)^2 + ½ (17)(53)^2 – (½ (28)(67)^2 + ½ (17)(87.6)^2) = 4200 J	1 1 1
d	 Correct shape – smooth curves for both lines. Line for steel objects has larger peak and smaller duration than line for rubber, with approximately equal area under the two lines.	1 1