TOPIC 5: WORK, ENERGY AND POWER

Learning Outcomes: Candidates should be able to:

a.	define and use work done by a force as the product of the force and displacement in the direction of the force.
b.	Calculate the work done in a number of situations including the work done by a gas which is expanding against a constant external pressure: $W = p\,\Delta V$.
c.	Give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation.
d.	Derive, from the equations for uniformly accelerated motion in a straight line, the equation $E_k = ½\,m\,v^2$.
e.	Recall and use the formula $E_k = ½\,m\,v^2$.
f.	Distinguish between gravitational potential energy, electric potential energy and elastic potential energy.
g.	Deduce that the elastic potential energy in a deformed material is related to the area under the force-extension graph
h.	Show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems.
i.	Derive, from the definition of work done by a force, the formula $E_p = m\,g\,h$ for gravitational potential energy changes near the Earth's surface.
j.	Recall and use the formula $E_p = m\,g\,h$ for gravitational potential energy changes near the Earth's surface.
k.	Show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems.
l.	Define power as work done per unit time and derive power as the product of force and velocity.

Three broad areas:
- Work done by a force, KE and GPE
- Conservation of energy
- Power

Key questions:
- In physics, how do we determine if there is work done?
- When the force applied and the displacement are not in the same direction, how do we calculate work done?
- When there is a force applied and there is displacement, what are the situations where there is no work done?
- What is the relationship between work done and energy?
- How can I apply conservation of energy to solve problems (without dissipative forces and with dissipative forces)?
- What is the relationship between a Force, and the Potential Energy (U) arising from the force?

Why is the study of Work and Energy important in everyday life?
Apart from force and momentum, energy and work are also important concepts in physics. Energy and work are common used by layman in daily life however these concepts in physics is different from layman's term.

As we tense our bodies and hold on to a yoga pose, we can feel the strain and effort required. We feel that we are "working" very hard.

In physics, we have a very different definition of work. A person holding perfectly still in a yoga pose is not considered to be doing any work, since the person is not gaining any Kinetic Energy or Gravitational Potential Energy. (This is akin to a student who spent 10 hours "studying", but still got 0/10 marks. This student is deemed to not have "learnt" anything).

To watch a Physics Professor put his faith in the Principle of Conservation of Energy, and risk catastrophic brain injury, scan this QR code or go to:
https://www.youtube.com/watch?t=1613&v=4a0FbQdH3dY&feature=youtu.be

5.1 Energy

Energy is the ability to do work. Energy exists in many different forms. The common ones in the are

Form of energy stored	due to
Kinetic energy (KE)	a mass in motion [Derivation in Sect. 5.3.1]
Gravitational potential energy (GPE)	a mass's position in the gravitational field of other masses [Derivation in Sect 5.5]
Elastic potential energy (EPE) or strain energy	compressed or stretched system (e.g. springs, strings, rods, rubber band) [Discussion in Sect 5.3.2]
Electric potential energy	a charge position's in the electric field of other charges. [Refer to Electric field for H2]

Typically KE and all the potential energies are termed as mechanical energy. The S.I. unit of energy is *joule* (J). It is a scalar quantity.

5.1.1 Energy conversion and conservation

Energy can neither be created nor destroyed but can transformed from one form to another, and from one body to another, but the total amount remains constant. This is termed as Principle of Conservation of Energy (PCE).

Note: Do not use short form PCE in examinations. PCE is one of the most fundamental laws of physics along with PCM.

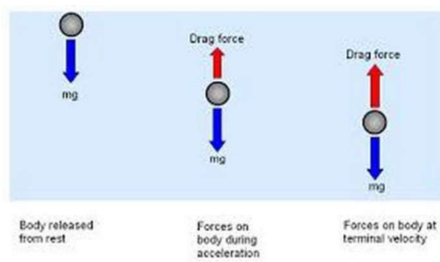
Examples of energy conversions:

(a) Burning of fuel (oil, coal or wood):
Chemical energy (of fuel) → Light, Sound & Thermal energies

(b) A body falling through a *viscous* fluid (recall Forces):

Before terminal velocity:
GPE → KE (of body) & Thermal energy (of body & fluid, while overcoming viscous forces)

During terminal velocity:
GPE → Thermal energy (of body & fluid)
{Why no KE change at terminal velocity?}



(c) A ball rolling on a *horizontal* surface comes to a stop:
KE → Thermal energy (of ball, ground) (or equivalently, w*ork done against friction)*

Food for thought: *Booming sounds from a sand dune*

When the sand on top layer of a steep dune is disturbed, it slides down the slope. Gravitational potential energy is transformed into kinetic energy & sound (booming noise)!

Scan this QR code for the Youtube video

5.2 Work done by a force

In our everyday language, work is related to expenditure of muscular effort, but this is *not* the case in the language of physics!

In physics, work is done when a force acting upon an object causes a displacement in direction of the force.

If you push hard against a ship and the ship is not moving, you are tired because of continuously repeated muscular contraction that are required, and you are, in common sense working. However, such effort does no work as the force you exert does not cause a displacement of the ship.

Food for thought:
BMT recruits during the 24 km route march, carrying a 10kg field pack do NOT do any work, if they move at constant speed, and the 10 kg field pack is at a constant height. There is no displacement of the fieldpack in direction of the force (upward contact force from your shoulder in supporting the load)

Work done by a force is the product of the force and the displacement in the direction of the force.

i.e. Work done by a force = Force x displacement in the direction of the force …. (5.1)

In physics, "doing work" is an act of transferring energy. It thus has the same S.I. unit of joule (J) as energy and it is also a scalar quantity.

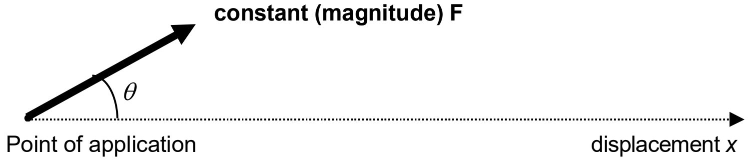
The (derived unit) *joule* is defined as the work done by a force of 1 N which produces a displacement of 1 m in the direction of the force.

Thus 1 J = 1 N $\times$ 1 m = 1 kg m^2 s^{-2}.

We shall now consider work done by a

- constant force (including work done by gas in expanding against a constant external pressure)
- variable force (including work done by applied force in extending or compressing a spring)

5.2.1 Work done by a constant force F



Consider the case of a constant force on an object that produces a displacement x at angle θ to F. Mathematically, equation (5.3) becomes

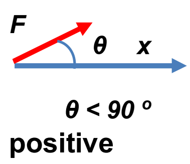
$$\boxed{W = (F)(x\cos\theta)}$$

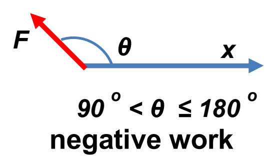
or resolving F into the direction of s:

$$\boxed{W = (F\cos\theta)(x)} \quad \text{…. (5.2)}$$

You can either resolve F into the dir of x, or resolve x into the dir of F, before taking the product.

Positive work is done by F if x, or its component, is parallel to F.
Negative work is done by F if x, or its component, is anti-parallel to F (i.e. x opposite to F).





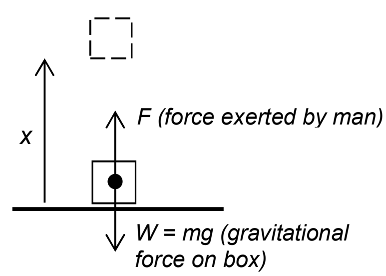
Pro Tip:
The 2 arrows must point "away" from each other in defining the angle
IMPORTANT

Note:
Positive and negative here for a scalar like work done does not refer to the direction. It is to keep track of the increase or decrease of energy of the object due to the force that is doing the work.

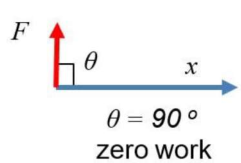
Illustration:

Consider a man lifting a box vertically upwards from the floor.

Positive work is done by the force man exerted on the box while negative work is done by the gravitational force by Earth on the box.



No work is done by F if
- $\theta = 90^0$, i.e. when F and x are perpendicular to each other, or
- $x = 0$, i.e. when the force does not cause any displacement of the body.



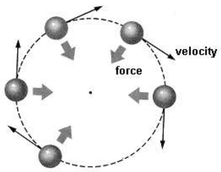
Concept check:
Is work done in the following scenarios?

(a) A barbell being held at certain height and the person becomes exhausted.

Answer:
No work is done by the person on the barbell because the barbell is not displaced.

(b) Motion of a satellite in a *uniform* circular motion (constant speed) around the Earth, where the centripetal force is perpendicular to the displacement at all times.

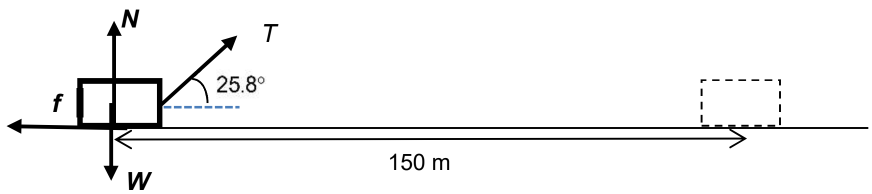
Answer:
No work is done by the gravitational force *F* on the satellite because the gravitational force is always perpendicular to the displacement *x* where x is in the same direction as the velocity.
Work done by F_G, gravitational force of Earth on satellite = $F_G(x \cos 90°) = 0$ J



Example 1
A man pulls a box horizontally for a distance of 150 m with a rope making an angle of 25.8° with the horizontal. A frictional force of 10 N acts between the box and the ground and the tension in the rope is 40 N. Determine the work done by each force acting on the box.

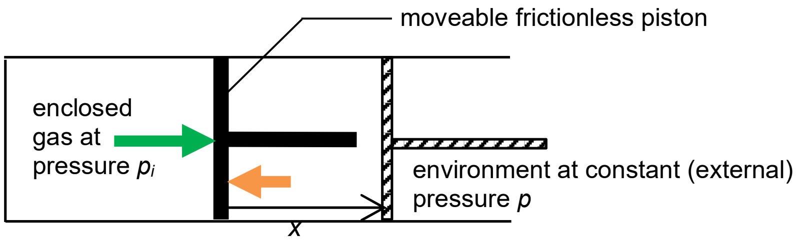
[5.4×10^3 J, -1.5×10^3 J, 0 J, 0 J]

Solution:



Tutorial qn: Q1, Q2, Q3

5.2.1.1 Constant force – Work done by a gas expanding against a constant external pressure



The gas expands as the piston moved out against a constant external pressure p, the change in volume of the gas ΔV

By definition, $W = Fx\cos\theta$ since gas is expanding against constant external pressure, force is constant

Thus work done by gas, W = force exerted by gas x distance moved by the piston

Since pressure $p = \dfrac{F}{A}$, $F = pA$ where A = cross sectional area of piston

Thus $W = (p_i A)x$ as p_i and x are in the same direction

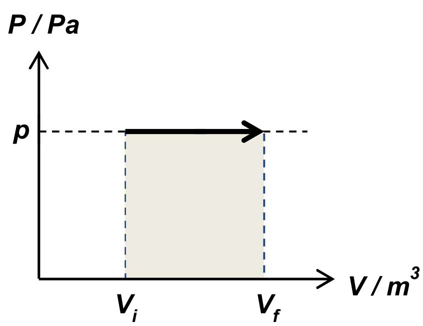
$W = p_i \Delta V$ where $\Delta V = V_f - V_i = Ax$

$\boxed{W = p_e \Delta V}$ ..... (5.3) $p_i = p_e$ as gas is expanding against that constant pressure

<u>Note:</u> The derivation of the equation is not required in the syllabus. More will be discussed about work done related to gas in a piston in the 1st law of Thermodynamics.

Example 2:

A gas is expanding against a constant external pressure of 1.5 x 10^4 Pa and its volume changes from 0.8 x 10^{-4} m^3 to 1.2 x 10^{-4} m^3. Determine the work done by the gas.



Solution:

Graph is as shown,

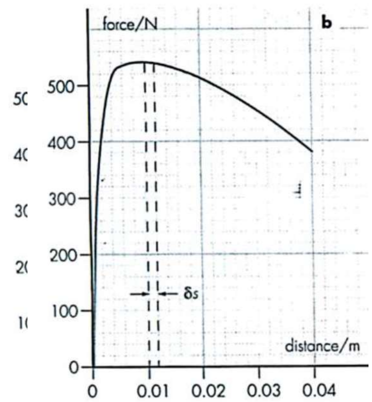
where p = 1.5 x 10^4 Pa

V_i = 0.8 x 10^{-4} m^3

V_f = 1.2 x 10^{-4} m^3.

Tutorial qn: Q4

5.2.2 Work done by a variable force F parallel to x



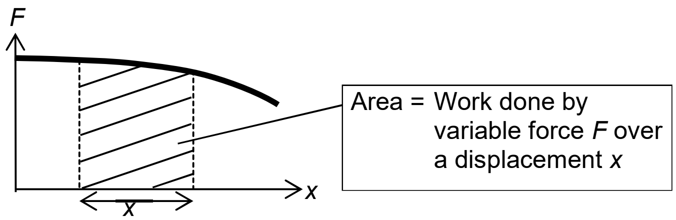
In many practical situations, the magnitude of the force being exerted on an object varies.

For example, the maximum tension in a muscle in the arm depends on its cross-sectional area and also on the length of the arm. A graph showing the way it which the force varies with the distance when the muscle contracts is shown.

To calculate the work done by this muscle in contracting by 0.040 m, it is necessary to find the sum of force × distance moved for all the small distances, *d*s.

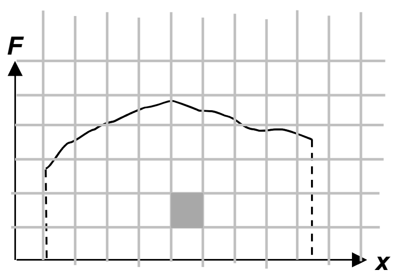
If a *variable* force *F*, produces a displacement x in the direction of *F*, then work done

$$\boxed{W = \int F\,dx = \text{Area under } F - x \text{ graph}} \quad \ldots.\ (5.4)$$

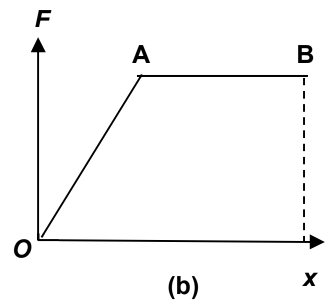


{This integral, in fact, is the definition of work done by a force. But for the purpose of the A-level Exam, the Syllabus only requires the simpler definition expressed by equation 5.2 and as stated in 5.1}

The area under the $F - x$ graph (approximately) can be determined by the method of "counting the squares" since the equation for *F* as a function of *x* is unknown. Determine the work done represented by 1 square. (In this case the graph of *F-x* will be printed on grid lines in the exam question paper.)



(a)



(b)

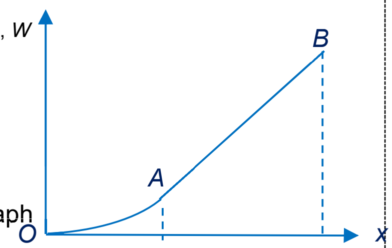
For graph (b), the work done can be accurately calculated using the formula for the area of a trapezium.

Note: In finding area and even gradient from a graph, one needs to note the units and scale on the axes.

Worked Example 3

On the axes shown below, sketch the corresponding work done-displacement graph for graph (b).

Solution:

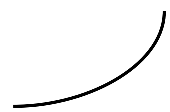
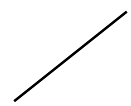


$W = \int F\,dx$

$|F| = \left|\frac{dW}{dx}\right|$

= gradient of W-x graph

The gradient of W-x graph should gives the values of F in F-x graph

Section	Value of F	Gradient of W-x graph
O to A	+ve and increasing	+ve and increasing 
A to B	+ve and constant	+ve and constant 

From O $\rightarrow$ A, increasing value of F in F-x graph means increasing gradient of W-x graph

From A $\rightarrow$ B, constant value of F in F-x graph means constant gradient of W-*x* graph

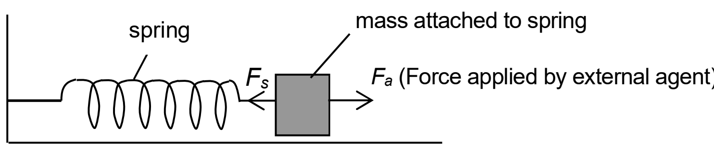
Note: Do NOT attempt to use area enclosed by F-x graph to plot W =x graph. Always think about gradient to draw graph.

Tutorial qn: Q5

5.2.2.1 Variable force – Elastic Potential Energy / Strain Energy due to Elastic Force

Recall in forces, during elastic deformation, an object (e.g. spring, string, rod) can recover to its original shape when the force is removed. The work done by the applied force on such an object is stored as elastic potential energy EPE in the object.

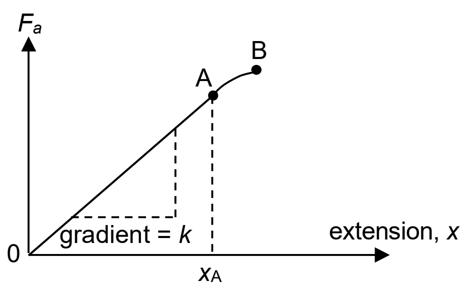
Work done by an external agent in stretching a spring horizontally.



Recall Hooke's law: the force exerted by a spring F_s = $k\,x$, where x is the *extension* and k = spring constant, provided it has not been stretched beyond its proportionality limit

Relationship between Elastic Potential Energy and Area under a Force-displacement graph

(a) For extension within the proportionality limit (between O & A):



A: proportionality limit
B: elastic limit

k = spring constant
= gradient of F_a – x graph

Assume stretching is achieved such that $\boldsymbol{F_a = F_s}$ i.e. external force is applied such that there is *no change in KE.*

Work done by F_a from O to A = $\int_0^{x_A} F_a\,dx = \int_0^{x_A} (kx)\,dx$ (since Hooke's Law is applicable)

$\boxed{\text{area under the F-x graph}} = [\frac{1}{2}kx^2]_0^{x_A}$

$\boxed{\frac{1}{2} F_a x_A} = \frac{1}{2}kx_A^{2}$ {which can be calculated} ….. (5.5)

In general, the EPE stored in a spring extended by x from its unstretched length is $\boxed{\frac{1}{2}kx^2}$. ….. (5.6)

Note:
F-x axes may be swapped in the A-level exam. (refer to Tut Q13) and x-axis may be the total length instead of extension.

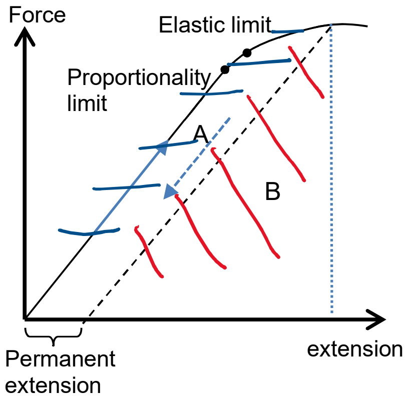
(b) For extension beyond the proportionality limit (between A to B):

Work done by F_a is no longer $= \frac{1}{2}kx^2$ (as $F_a \neq k\,x$ in this case)

Here, the work done can only be found by the approximate method of "counting the squares".

Tutorial qn: Q6

(c) For extension beyond elastic limit (Not so important!)



Within the elastic limit, we say that the spring is showing "elastic behaviour" and it'll go back to its original length when we remove the force. $F_s = F_a = kx$, $EPE = \frac{1}{2} kx^2$.

Beyond the proportionality limit but before the elastic limit, the spring will still return to its original length when unloaded.

Beyond the elastic limit, it *suffers a permanent deformation* when the force is removed (i.e. the material does not return to its original length).

Note: According to our syllabus, it is not required to state the differences between elastic and proportionality limit.

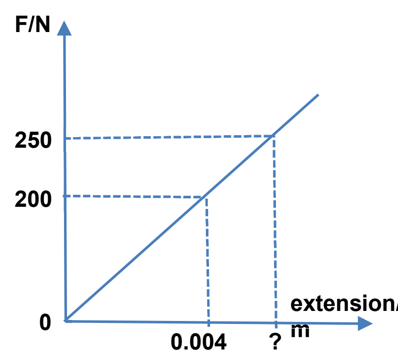
Looking at the diagram,

Area	What it represents
A	**Work done to cause permanent deformation in the spring which is not recoverable**
B	**Elastic potential energy recoverable when you remove the force**
A + B	Work done by the force on the spring

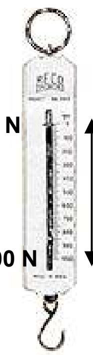
Example 4: [N11/1/10]
A wire is stretched elastically by a force of 200 N, causing an extension of 4.00 mm. The force is then steadily increased to 250 N. The wire still behaves elastically.

How much extra work is done in producing the additional extension?

Solution:
Assuming that that the wire still behaves elastically means the proportionality limit is still not reached and Hooke's Law still applies,



Worked Example 5:
The scale of a spring balance that reads from 0 to 100 N, spans a length of 0.15 m.



(a) Determine the strain energy in the spring when the balance shows a reading of 40 N. (Assume that the spring obeys Hooke's law) [1.2 J]
(b) Determine the increase in strain energy when the reading increase from 40 N to 80 N. [3.6 J]

Solution:
(a) Applying $F_o = k\,x_o$, $k = F_o / x_o = 100 / 0.15 = 666.7\ \text{N m}^{-1}$

When balance shows 40 N, $x = \frac{40}{100} \times 0.15 = 0.06$ m

Elastic Potential Energy = $\frac{1}{2}kx^2 = \frac{1}{2}(666.7)(0.06)^2 = 1.2$ J

(b) When balance shows 80 N, $x = \frac{80}{100} \times 0.15 = 0.12$ m

Increase in EPE = EPE when reading is 80 N - EPE when reading is 40 N
= $\frac{1}{2}(666.7)(0.12)^2 - \frac{1}{2}(666.7)(0.06)^2$
= 3.6 J

Problem Solving:
1. Many students made the mistake of calculating change in EPE as $\frac{1}{2}k(\Delta x)^2$, i.e. change in EPE = $\frac{1}{2}(666.7)(0.12 - 0.06)^2$
2. That is absolutely WRONG.
3. Change in EPE = $\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$
4. Mathematically, $\frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = \frac{1}{2}k\left(x_1^2 - x_2^2\right) \neq \frac{1}{2}k\left(x_1 - x_2\right)^2$

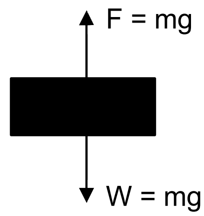
5.3 Relationship between Work Done and Change in Energy

Work done is not a form of energy but a process by which the energy is transferred.

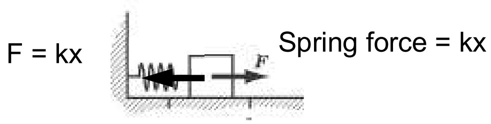
Work done by an applied force F on a system WD = KE gain/loss + PE gain/loss + Thermal energy ….. (5.7)

Thus if applied force, F does work against

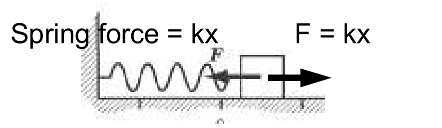
- the **weight, W** only, then the **WD = Gain in GPE = mgh** [Derivation in Sect. 5.3.2]



- a **spring force only**, then the **WD = Gain in EPE = ½ kx**^{2} [Derivation in Sect. 5.2.2.1]

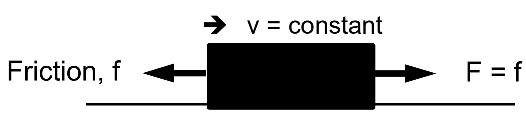


Spring in tension being compressed



Spring in tension being extended

- **a resistive force e.g. friction, drag force only**, then the **WD = Gain in Thermal Energy**



If **F > (weight + spring force + resistive force) ➔ F$_{net}$ = F– (weight + spring force + resistive force)**

In such a case, the work done by the net force results in the gain of KE= ½ mv^2 [Derivation in Sect. 5.3.1]

In general,

Net work done = Work done by net force = Change in KE

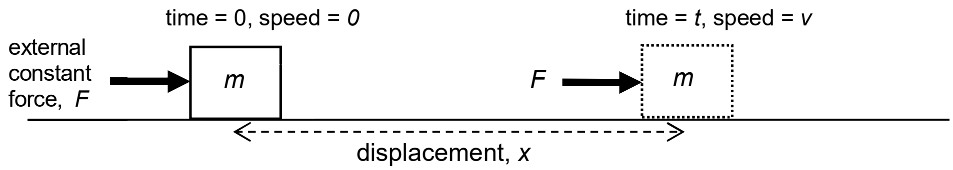
….. (5.8) (refer to Example 6 and Tut Q3,)

Note:

- WD by force, F against friction, f (= Fd) is numerically equal to WD by friction, f (= fd).
  Similarly, WD by force, F against weight, W (= Fh) is numerically equal to WD by weight, W (=Wh = mgh)
- Another way to transfer energy is by heating which is not discussed in this topic
  [Refer to 1st law of Thermodynamics for H2]

5.3.1 Derivation of KE = ½ mv2 from equations for uniformly accelerated motion

Consider an external constant force *F* acting on an object of mass *m* on a level surface. During the application of *F*, its speed changes from *u* to *v* over a displacement *x*. We conclude that work done by the force *F* results in a change in Kinetic Energy *KE*. For simplicity, we assume initial speed $u = 0$ m s^{-1}.



Since the force is constant, by Newton's 2nd law that there will be uniform / constant acceleration *a*.

As the object displaces by *x* during the action of the force,

Work done by force, *F* = Force x distance moved in direction of force

$W = F x = (m a) x$ [by Newton's 2nd Law]

Using the kinematics equation $v^2 = u^2 + 2ax$ and substituting u=0

$$ax = \frac{v^2}{2}$$

Multiply the equation by *m* and recalling that W = *m a x* from above

$$W = m\,\frac{v^2}{2}$$

$$W = \tfrac{1}{2} m\, v^2$$

For an object initially at rest, *u* = 0, hence $\boxed{KE = \tfrac{1}{2} mv^2}$ …. (5.9)

<u>Note:</u> The F here is also a net force as $F_{net} = ma$

Example 6:
During take-off, an aeroplane's speed changes from zero to 400 m s^{-1}, while travelling 1 000 m along a horizontal runway. If its mass is 2.0×10^4 kg, find the minimum driving force which needs to be applied to it. Assume that friction and air resistance are negligible. (What is the significance of the *horizontal* runway?) [1.6 x 10^6 N]

Solution :

Let *F* = minimum driving force

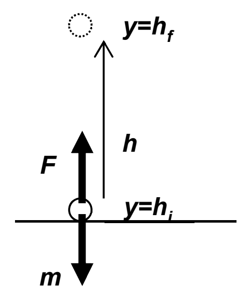
Work done by *F on plane* = ΔE (of plane)

F s cos θ = ΔKE

F s cos 0° = ΔKE

5.3.2 Derivation of GPE = mgh from definition of work done by a force

Consider a mass m, being raised by a vertical displacement h, by an external constant force F at constant speed (no acceleration) and where $h <<$ Earth's radius.



work done W by the external force F, = $F \times h$

Since the mass is moving at constant speed,
net force acting on the mass is zero. So $F = mg$

$$= (mg)\,(h_f - h_i)$$
$$= mgh$$

The gravitational potential energy of a body is dependent on the vertical height above some reference level and is given by

$$\boxed{\text{GPE} = m\,g\,h} \quad \ldots.\ (5.10)$$

In the above derivation:

- work done by F is transformed to GPE only, none to KE, as speed is a constant.
- value of g is constant over the displacement h, as $h <<$ Earth's radius [Refer to Gravitational Field for H2]

Note:

(a) The work done by the external force against weight has resulted in a change in GPE of the mass when it is raised by vertical height h. The change in GPE that occurs when a body is displaced is mostly used in problem solving rather than the absolute value of GPE. (Similar situation as in "pressure difference = $h\rho g$").

(b) Equation is valid only where g is constant throughout the distance moved. For displacement near the earth's surface, g is fairly constant - so equation can be used. But if a mass such as a spaceship is sent to an orbit high above the earth's surface, then g varies appreciably- so the equation cannot be applied. As a rule of thumb, it is not applicable if h exceeds 1 km.

(c) The change in GPE of the mass is independent of the path taken. It is dependent only on its vertical displacement.

(d) Level at which GPE is zero is arbitrarily assigned. For example, to determine the GPE of a student standing at level 2 of the classroom block, we can assign GPE to be zero at level 1 (lowest point). GPE of the student will be positive. Alternatively, we can select level 3 as the level of zero GPE. GPE of the student will be negative.

Food for Thought: What happens if the external force F applied is > mg?

The work done by the external agent will result not just an increase in GPE but in KE as well as the work done after working against weight to increase GPE will be used to increase KE.

5.4 Problems solving involving conservation of energy and work done

Three approaches to solve problem involving energy and work done using conservation:

1. Total initial energy = Total final energy
2. Loss in GPE/KE = Gain in KE/GPE
3. Work done by external force = Total gain in mechnical energy + Total gain in thermal energy (Work done against dissipative forces)

Worked Example 7:

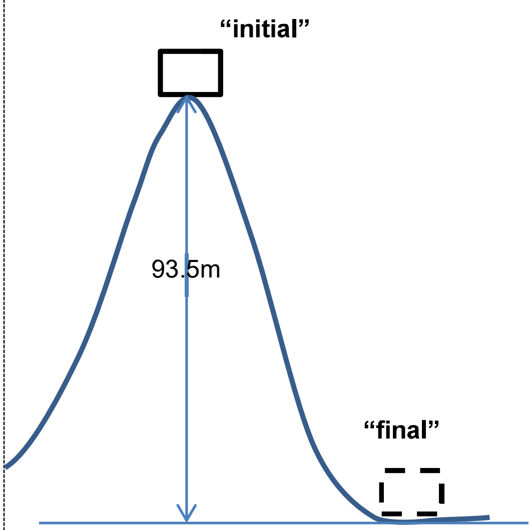
One of the tallest and fastest roller coasters in the world is the Steel Dragon in Japan. The ride includes a vertical drop of 93.5 m.
The coaster has a speed of 3.0 m s^{-1} at the top of the drop.

Mie,

(a) **Neglecting friction,** calculate the speed of the riders at the bottom.

(b) **Taking friction into consideration** and that the actual speed of the riders at the bottom is 41 m s^{-1}, determine the work done by friction on a 55.0 kg rider during the descent of 93.5 m. [42.9 m s^{-1}, 4470 J]

Solution:
(a)



Step 1: Draw a sketch of the problem.

Step 2: Choose an "initial" point (the top) and a "final" point (the bottom).

<u>**Step 3:** Method 1: "Loss = Gain"</u>
By Principle of conservation of energy,
Loss in GPE = Gain in K.E

$$m\,g\,\Delta h = \tfrac{1}{2}\,m\,(v^2 - u^2)$$
$$m\,(9.81)(93.5) = \tfrac{1}{2}\,m\,(v^2 - 3^2)$$
$$v = \underline{42.9\text{ m s}^{-1}}$$

<u>**Step 3**: Method 2: "$TE_i = TE_f$"</u>
By Principle of conservation of energy,
TE_i at the top = TE_f at bottom
$KE_i + GPE_i = KE_f + GPE_f$

Set h = 0 at the bottom of coaster

$$\tfrac{1}{2}mu^2 + mgh = \tfrac{1}{2}mv^2 + mg(0)$$
$$\tfrac{1}{2}m(3)^2 + m(9.81)(93.5) = \tfrac{1}{2}mv^2 + 0$$

Solving, speed of the riders at the bottom v, is <u>42.9 m s^{-1}</u>

(b)

<u>Method 1: "Loss = Gain"</u>
By Principle of conservation of energy,
Loss in GPE = Gain in K.E + Work done against friction

$$m\,g\Delta h = \tfrac{1}{2}\,m\,(v^2 - u^2) + W$$
$$(55.0)\,(9.81)(93.5) = \tfrac{1}{2}\,(55.0)\,(41^2 - 3^2) + W$$
$$W = 4467.925$$
$$= 4470\text{ J (to 3 sf)}$$

<u>Method 2: "Total Initial = Total Final Energy"</u>
By Principle of conservation of energy,
$KE_i + GPE_i = KE_f + GPE_f$ + Work done against friction

$$\tfrac{1}{2}mu^2 + mgh = \tfrac{1}{2}mv^2 + mg(0) + W$$

$$\tfrac{1}{2}(55.0)(3)^2 + (55.0)(9.81)(93.5) = \tfrac{1}{2}(55.0)(41)^2 + 0 + W$$

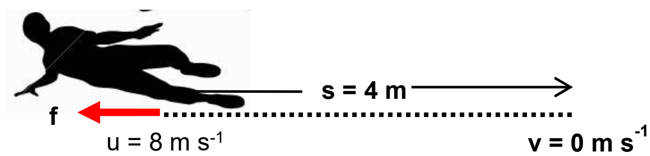
W = 4470 J (to 3 s.f.)

Pro Tip:
Method 1 should be used in most questions which usually only involve GPE and KE. Method 2 would be useful when we are dealing with 3 forms of energy (GPE, KE and EPE), like in Worked Example 10.

Worked Example 8:
An 80 kg soccer player pretends to fall down and slides to a stop from a speed of 8 m s^{-1} in a distance of 4 m.

What is the average horizontal frictional force exerted on him by the ground? Is the work done by the frictional force on the man *positive* or *negative* work? [640 N]

Solution:



Let f be the average horizontal frictional force.

Applying the principle of conservation of energy on the man

Loss in KE = Work done against friction

KE = W

$\frac{1}{2}mu^2 = fs$

$\frac{1}{2}(80)(8)^2 = f(4)$

$f = 640$ N

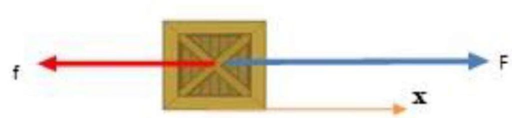
Negative work done by frictional force on the man, since f and displacement, s are in opposite directions.

(**θ = 180°**, so
Work done by friction on the man **= fs cosθ** = (640)(4) cos **180°** = -ve)

Worked Example 9: (N90P1Q7)
A crate initially at rest is pushed 10 m along a horizontal surface by a horizontal force of 80 N. The frictional force opposing the motion is 60 N.

How much of the work done is converted into thermal energy and how much into the KE of the crate? [600 J, 200 J]

Solution:



"final"

Thermal energy generated = work done against **frictional force**
= |f s cos θ |
= | 60 x 10 cos 180° | = 600 J

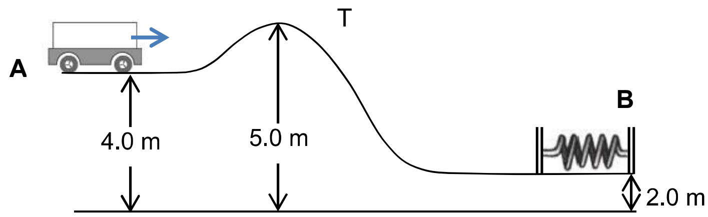
Force to do work against friction is in direction opposite to friction but has the same numerical values as f (F = f numerically). Thus WD against friction = - WD by friction but numerically they are the same.

Applying the principle of conservation of energy,

Work done by F = Gain in KE + Work done against f
80(10) = Gain in KE + 600 J
Gain in KE = 200 J

Worked Example 10: Max compression of a spring
A 3.2 kg trolley initially moving at 5.0 m s^{-1} at a height of 4.0 m encounters a hill of height 5.0 m as shown in the figure. At a later point there is a horizontal spring (force constant k = 120 N m^{-1}) at a height of 2.0 m.

Show using numerical calculations whether the trolley reaches the spring. If so, calculate the maximum compression of the spring. [1.31 m]



Solution:
In order for the trolley to reach the spring, it must have sufficient KE to reach the top of the hill. Applying the principle of conservation of energy on the trolley,

$$\text{Total Energy at A} = \text{Total Energy at T}$$
$$KE_i + GPE_i = KE_f + GPE_f$$
$$\tfrac{1}{2}(3.2)(5^2) + (3.2)(9.81)(4) = KE_f + (3.2)(9.81)(5)$$
$$KE_f = 8.61 \text{ J}$$

Since $KE_f > 0$, the trolley can reach the top of the hill, and then reach the spring.

To calculate the maximum compression of the spring:

Applying the principle of conservation of energy

$$\text{Total Energy at A} = \text{Total Energy at B}$$
$$KE_i + GPE_i + EPE_i = KE_f + GPE_f + EPE_f$$
$$\tfrac{1}{2}(3.2)(5^2) + (3.2)(9.81)(4) + 0 = 0 + (3.2)(9.81)(2) + \tfrac{1}{2}kx^2$$
$$\tfrac{1}{2}(120)x^2 = 102.72$$
$$x = 1.31 \text{ m}$$

Alternative method
From A to B,

$$\text{Loss of KE} + \text{Loss of GPE} = \text{Gain in EPE}$$
$$\tfrac{1}{2}(3.2)(5^2) + (3.2)(9.81)(4-2) = \tfrac{1}{2}(120)x^2$$
$$x = 1.31 \text{ m}$$

Pro Tip:
The "Total Initial Energy = Total Final Energy" method is slightly preferable in this case, if we are unsure which energy is a "gain" or a "loss".

Tutorial qn: Q7, Q8, Q9, Q10, Q11, Q12, Q13

5.5 Relationship between force F & potential energy U

Whenever a force F experienced by an object is a function of its position, its potential energy U is related to the force by:

$$\boxed{F = -\frac{dU}{dx}} \quad \text{.... (5.11)}$$

U = potential energy of a body, F = force it experiences, x = displacement.

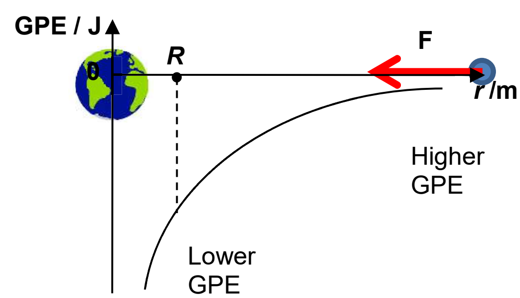
Note:
- negative sign indicates that the potential energy decreases in the direction of F.
- it is valid for 3 types of forces (elastic force, gravitational force, electric force) and their associated U. This equation will be important in the topics of Gravitational Field and Electric Field for H2.

Pro Tip:
$F = -\frac{dU}{dx}$ is the mathematical expression of this: "Everything wants to go to the lowest possible energy". E.g. A ball released from rest "wants" to go down, because its U (GPE) will be lower and its F (weight) is making it go down.

Worked Example 11:
A body is at a distance R from the centre of a planet. State how the force on the body may be obtained from its GPE – r graph (will be taught in Topic 7).

Solution:
We need to draw a **tangent** to the curve at R, and determine the gradient since F = - dU/dx

Force at R = negative gradient of the graph



Tutorial qn: Q23, Q24

5.6 Power

Work is only an indication of the amount of *energy* expended. The term *power* can indicate how fast work is being done or energy is being expended from one form to another when the *time* taken is known.

Who has greater power if the rock of the same mass is rolled to the same height? How can you tell?

Power is the work done per unit time.

Common error:
"work done per *second*". "Work" is a Physical Quantity, while "second" is a Unit.
Both terms should be Physical Quantities, since we are defining a Physical Quantity (Power).
Both terms should be in units if we are defining a Unit, e.g. 1 *Horse Power* is the power necessary to lift a total mass of 33,000 *pounds* one foot in one *minute*.

Alternative definition: **Rate at which energy (*E*) is transferred with respect to time (*t*).**

Hence, $$\boxed{P = \frac{W}{t} = \frac{E}{t}} \quad \text{.... (5.12)}$$

The S.I. unit of power is *watt* (W), i.e. 1 W = 1 J s^{-1}. It is a scalar quantity.

If the work done on a system is NOT at a constant rate, the *average* power <*P*> over a certain time interval, *t*, is defined as:

$$\boxed{\text{Average Power} < P > = \frac{W}{t} = \frac{E}{t}} \quad \text{.... (5.13)}$$

where W = work done during time interval t,
E = energy converted during interval t.

Similarly, the instantaneous power P is:

$$\boxed{\text{Instantaneous Power, } P = \frac{dW}{dt} = \frac{dE}{dt}} \quad \text{.... (5.14)}$$

5.6.1 Derivation of P = Fv

First Method: (without calculus)

Definition gives P = work done per unit time
= force × displacement per unit time
= force × velocity (shown)

3 definitions involved in this derivation

i.e. $\boxed{P = F\,v}$ .... (5.15)

Note: F is **NOT the net force**; it is the **applied force/ driving force!**

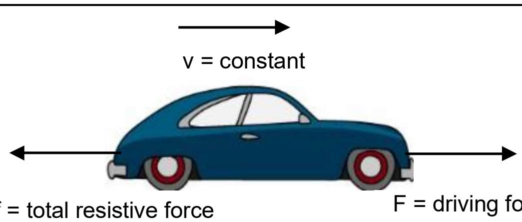
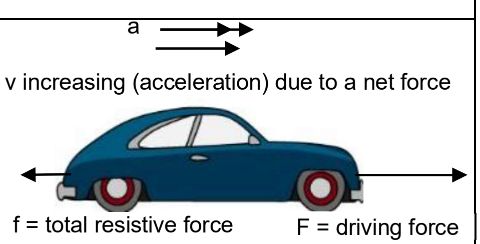
Second Method: (using calculus, not in Syllabus)

Definition (5.13) gives $P = \frac{dW}{dt}$

Since $W = F\,x$, $dW = F\,dx$. Thus, $P = F\,\frac{dx}{dt}$. Since $v = \frac{dx}{dt}$, $\Rightarrow P = F\,v$

Application of *P* = *Fv*:

This expression can be applied to determine the driving power of a vehicle, where F is the driving force and v is the velocity of the vehicle.

For object moving at constant speed	For object accelerating
F = force to proving the driving power (e.g. engine thrust). Since it is moving at constant speed, Magnitude of this applied force, F = total resistive force, f (In general, Total resistive force, f = friction + air resistance)	F = force to proving the driving power (e.g. engine thrust). F = total resistive force + m a (Derived from $F_{net} = ma$)
	

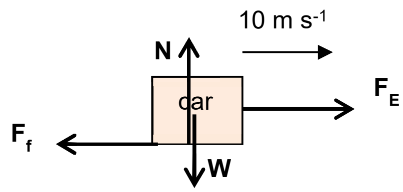
Worked Example 12:
A car of mass 1 000 kg moving on a horizontal road with a *steady* speed of 10 m s^{-1} has a total frictional force on it of 400 N.

(a) Determine the power due to the engine. [4 kW]

(b) The car now climbs a hill at an angle of $8°$ to the horizontal. Assuming that the frictional force stays constant at 400 N, what engine power is now needed to keep the car moving at 10 m s^{-1}? [17.7 kW]

Solution:

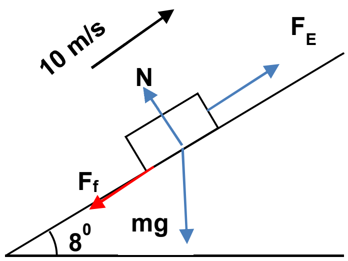
(a) Draw a simple free-body diagram of the forces acting on the car



To move at steady speed of 10 m/s, by Newton's 2nd law,
the net force, $F_{net} = 0$
$F_E = F_f$ (the car engine must exert a force, F_E equal to that of the resistive force, F_f).

Power supplied by engine, $P_E = F_E \times$ velocity
$= F_f\ v$
$=$ **400 (10)**
$=$ **4 kW**

(b)



Since velocity is constant at 10 m s^{-1}, by Newton's 2nd Law,
the net force along the slope, $F_{net} = 0$,
$F_E - F_f - mg \sin\theta = 0$
$F_E = F_f + mg \sin\theta$
$=$ **400 + 1000 (9.81) sin 8°** $=$ 1765 N

$P_E = F_E \times v$

$=$ **1765 × 10**

$=$ **1.77×10^4 W**

Worked Example 13: (N09P1Q13)

A speedboat with two engines each of power output 36 W, can travel at a maximum speed of 12 m s^{-1}. The total drag D on the boat is related to the speed v of the boat by the equation shown: $D \propto v^2$

What is the maximum speed of the boat when only one engine is working?

A 3.0 m s^{-1} B 6.0 m s^{-1} C 8.5 m s^{-1} D 9.5 m s^{-1}

Solution:

$D \propto v^2$

$D = k v^2$

With 2 engines,

At maximum speed, acceleration is 0, so

$2F_E = D$

$2F_E v = Dv$

$2P = Dv$

$2P = (kv^2)v$

$2P = kv^3$

$2(36) = k(12)^3$ ------------ (1)

With 1 engine,

At maximum speed, acceleration is 0, so

$F_E = D'$

$P = F_E v'$

$P = D' v'$

$P = (kv'^2)v'$

$P = kv'^3$

$36 = kv'^3$ ------------ (2)

$\frac{(2)}{(1)}$: $\frac{36}{2(36)} = \frac{kv'^3}{k(12)^3}$

$\frac{1}{2} = \frac{v'^3}{(12)^3}$

$(\frac{1}{2})^{1/3} = \frac{v'}{(12)}$

$v' = 9.52$ m s^{-1}

Answer is D.

Tutorial qn: Q14, Q15, Q17, Q18, Q19, Q21, Q22

5.7 Energy losses in practical devices & Efficiency

In an *ideal* situation, the (total) *energy output* of a practical device (e.g. a machine) is equal to the *energy input*. However in practice, the former is always less than the latter; i.e. (useful) energy output < energy input. In other words, efficiency of the device is <100%.

This is because some energy is inevitably used to overcome *dissipative* forces (like friction and air resistance).

The energy conservation equation is thus actually:

$$\boxed{\text{Energy input} = \text{energy output} + \text{work done against dissipative forces}} \quad \text{.... (5.16)}$$

The work done against dissipative forces is transformed into thermal energy.

Definition:

$$\text{Efficiency of a practical device} = \frac{\text{Useful energy output}}{\text{Total energy input}} \times 100\%$$
$$= \frac{\text{Useful work done}}{\text{Total energy input}} \times 100\%$$
$$= \frac{\text{Power output}}{\text{Power input}} \times 100\% \quad \text{.... (5.17)}$$

Examples

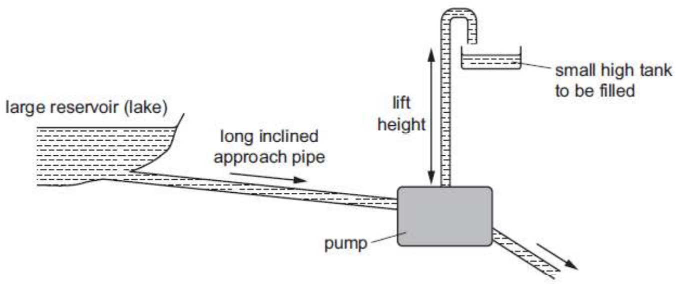
Activity/ Machine	Approximate Efficiency /% {Topic 1, Learning Outcome (e)}
Cycling	20
Swimming	3
Steam engine	20
Petrol engine	30- 40

Worked example 14: J15 P1Q18
The diagram shows a pump called a hydraulic ram.

In one such pump the long approach pipe holds 500 kg of water. A valve shuts when the speed of this water reaches 2.0 m s^{-1} and the kinetic energy of this water is used to lift a small quantity of water by a height of 15 m.

The efficiency of the pump is 10%.

What is the maximum mass of water that can be lifted 15 m? [0.68 kg]



Solution:

KE of the water flowing in the long approach pipe = $\frac{1}{2}$m v^2

$= \frac{1}{2}(500)2.0^2 = 1000$ J

Using efficiency = $\frac{\text{Power Output}}{\text{Power Input}} \times 100\%$, and since pump is 10% efficient, only 10% of KE is converted to PE or 100 J

PE of the mass *m* of water that rises by 15 m = *mgh*

$m\,(9.81)(15) = 100$

$m = 0.68$ kg

Tutorial qn: Q16, Q20

SUMMARY

1. Energy: ability to do work. S.I unit: Joule (J); scalar quantity.

- Various forms of energy: eg. mechanical, chemical, thermal, etc.
- Principle of Conservation of Energy (PCE): energy is never created nor destroyed; may be transferred / converted from one form to another. Total energy of remains constant.
- KE = $\frac{1}{2} m v^2$
- GPE = $m g h$
- EPE = $\frac{1}{2} k x^2$

2. Work done

- process / means by which energy is transferred; amount of energy converted
- Definition: Work by a force = Force x Displacement in the direction of the force
- S.I. unit: Joules (J); scalar quantity
- 1 J = 1 N $\times$ 1 m = 1 kg m^2 s^{-2}
- $\boldsymbol{W} = \boldsymbol{Fs}\cos\theta$ for a constant force
- $\boldsymbol{W} = \int \boldsymbol{F}\, \boldsymbol{dx}$ = Area under F – x graph for a variable force
- $W = p\Delta V$ for work done by expanding gas against constant external pressure
- $\boldsymbol{W} = \frac{1}{2}\boldsymbol{kx^2}$ for extension within the proportionality limit
- Work done by an applied force F on a system WD = KE gain/loss + PE gain/loss + Thermal energy
- Work done by a net force = Change in KE

3. Force and PE: $F = -\dfrac{dU}{dx}$

- negative sign indicates that the potential energy decreases in the direction of F

4. Power

- Definition: <u>rate</u> at which work is done (W) (i.e. work done per unit time), or, rate at which energy (E) is transferred with respect to time (t).
- $\boldsymbol{P} = \frac{W}{t} = \frac{E}{t}$; Average Power $\langle \boldsymbol{P} \rangle = \frac{\Delta W}{\Delta t} = \frac{\Delta E}{\Delta t}$; $P = F v$
- S.I. unit: watt (W), i.e. 1 W = 1 J s^{-1}; scalar quantity

5. Efficiency

- Efficiency $= \dfrac{\text{Useful energy output}}{\text{Total energy input}} \times 100\%$

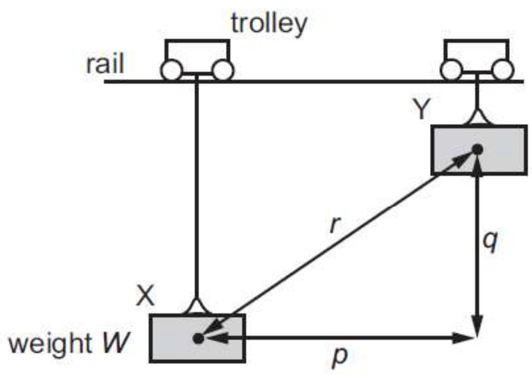
$$= \frac{\text{Useful work done}}{\text{Total energy input}} \times 100\%$$

$$= \frac{\text{Power output}}{\text{Power input}} \times 100\%$$

TUTORIAL 5: WORK, ENERGY AND POWER

Work

(L1)1. (J03/1/17)
A weight *W* hangs from a trolley that runs along a rail. The trolley moves horizontally through a distance *p* and simultaneously raises its weight through a height *q*. As a result, the weight moves through a distance *r* from X to Y. It starts and finishes at rest. How much work is done on the weight during this process.



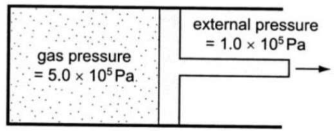
A *Wp* **B** *W* (*p*+*q*) **C** *Wq* **D** *Wr*

(L1)2. (Giancoli, sixth edition, pg 138)
A person pulls a 50 kg crate 40 m along a horizontal floor by a constant force F_p = 100 N, which acts at a 37° angle above the horizontal. The floor is rough and exerts a frictional force, *f* = 50 N. Determine

(a) the work done by each force acting on the crate, [4]
(b)(i) the net work done on the crate, [2]
(ii) suggest what form of energy is gained by the crate due to (b)(i). [1]

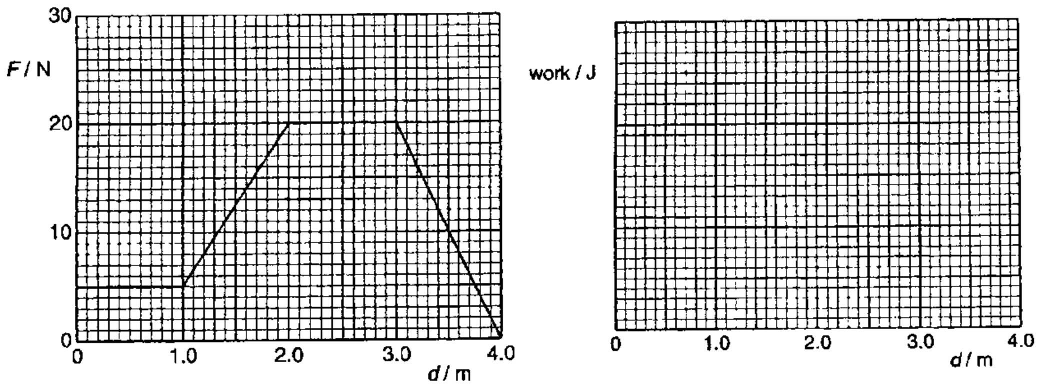
(L1)3. (H1/08/1/13)
A mass of 5.0 kg is moving in a straight line with a constant speed of 4.0 m s^{-1}. A force of 10 N is then applied to the mass along the direction of motion and removed after the mass has travelled a distance of 2.5 m.
What is the increase in the kinetic energy of the mass due to the action of the force? [2]

(L1)4. (H1/09/1/13) (Work done by a gas at constant pressure)
A gas at a pressure of 5.0×10^5 Pa is enclosed in a cylinder fitted with a piston.



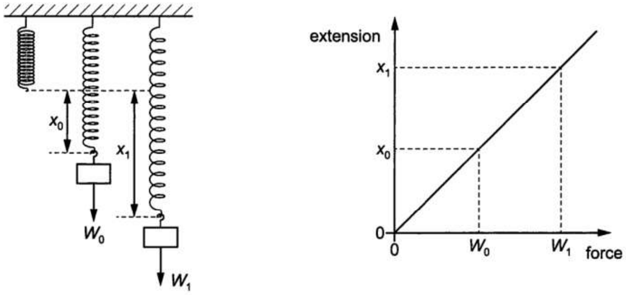
The gas expands by 4.0 m^3 against a constant external pressure of 1.0×10^5 Pa .
How much work does the gas do against the external pressure? [2]

(L2)5. (N08/3/1b)
The figure below shows the variation with displacement *d* of the force *F* applied to an object. *F* and *d* are in the same direction.



On the adjacent figure, draw a graph showing the variation with *d* of the work done. [4]

(L2)6. (N21/1/7) AN object of weight W_o is suspended from a spring so that the spring extends by x_o, as shown. The extension-force graph for the spring is shown.

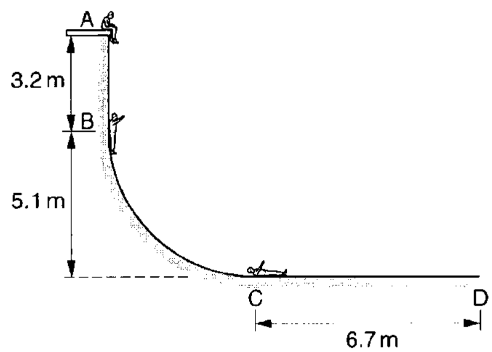


How much extra potential energy is stored in the spring when its extension increases from x_o to x_1?

Principle of conservation of energy

(L1)7. (N02/2/2 part)
The strain energy stored in a bow just before release of an arrow is 95 J. When the arrow of mass 170 g is fired, 90% of the strain energy is transferred to the arrow.
Show that the speed of the arrow as it leaves the bow is 32 m s^{-1}.

(L1)8. (H1/N10/2/1)
A particular type of slide for children in a theme park is called a 'drop slide'. This is a slide in which the first part of the fall is vertical. Figure below shows a child of mass 28 kg on a drop slide.



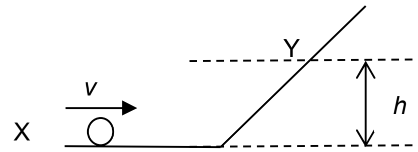
The child drops from A to B, a distance of 3.2 m, before reaching the surface of the slide and then travels around a bend from B to C, while falling a further vertical distance of 5.1 m. At C the child's speed is the same as it was at B. After this the child travels 6.7 m horizontally before stopping at D.

(a) Assuming air resistance to be negligible between A and B, calculate the child's speed at B. [2]

(b) Determine the loss of gravitational potential energy of the child between B and C. [2]

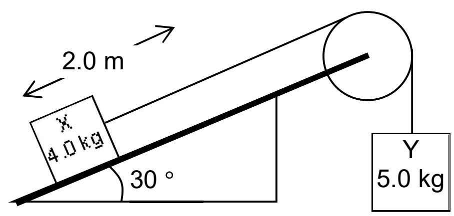
(c) Explain how it is possible for the child's speed to be the same at B and C. [2]

(L2)9. (J87/1/5)
An object of mass m passes a point X with a velocity v and slides up a frictionless incline to stop at point Y which is at a height h above X. A second object of mass ½ m passes X with a velocity of ½ v. To what height will it rise to?



A. ¼ h **B.** ½ h **C.** $\frac{1}{\sqrt{2}}h$ **D.** h **E.** h $\sqrt{2}$

(L2)10. (J90/1/7)
The diagram shows two bodies X and Y connected by a light cord passing over a light, free-running pulley. X starts from rest and moves on a smooth plane inclined at 30° to the horizontal. Calculate the total kinetic energy of the system when X has travelled 2.0 m along the plane. [2]



Extension of Question 9:
In the A level exam, this question will not be classified as a "WEP" question.
Would it be faster or slower to solve this question using "dynamics/kinematics" methods?
What is the clue in the question that we can solve this using "WEP" methods?
Can you solve "dynamics style" questions using WEP methods instead?

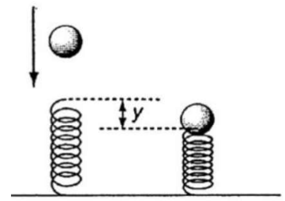
(L2)11. (N95/I/6)
A space vehicle of mass m re-enters the Earth's atmosphere at an angle θ to the horizontal. Because of air resistance, the vehicle travels at a constant speed v. The heat shield of the vehicle dissipates heat at a rate P, so that the mean temperature of the vehicle remains constant.

Taking g as the relevant value of the acceleration of free fall, express P in terms of m, g, v and θ. [2]

Extension of Question 10:
Why can't the angle be too gentle or too steep?
Too gentle: take too long to re-enter, or may "skip"/"bounce" off Earth's atmosphere back into space.
Too steep: rate of heat generated is too large, and the shield may be destroyed.

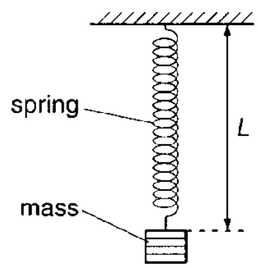
(L2)12. (N06/1/7)

A ball of mass m falls freely from rest. When it has reached a speed v, it strikes a vertical spring. The spring is compressed by a distance y before the ball moves upwards again. Assume that all the energy the ball loses becomes elastic potential energy in the spring. Determine an expression for the average force exerted by the spring during its compression in terms of m, v, g, and y. *[2]*



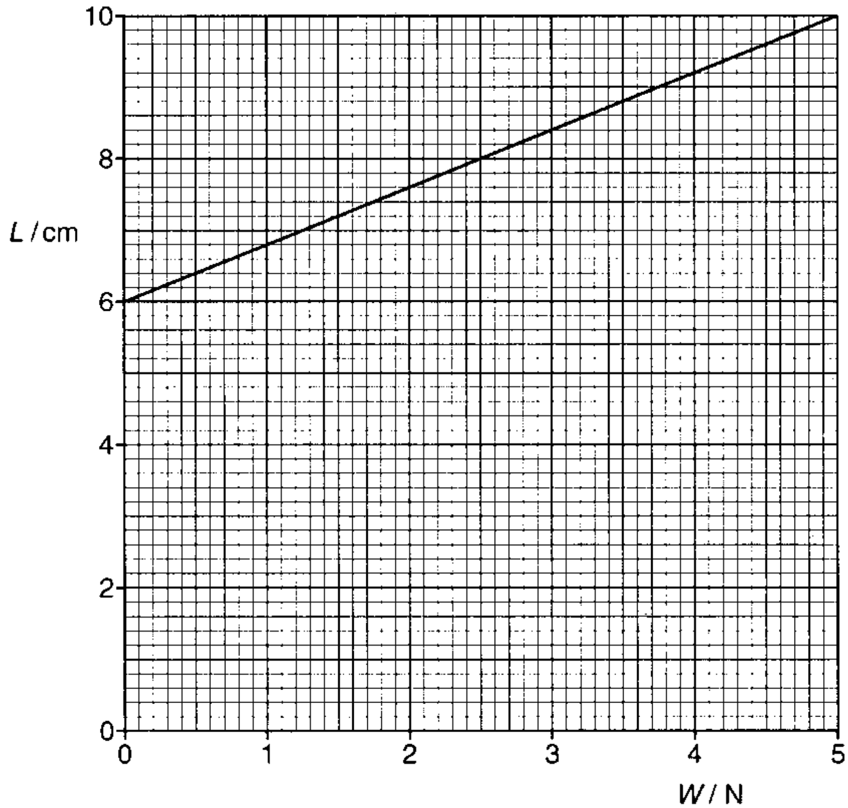
(L2)13. (N10/3/6c)

A light helical spring is suspended vertically from a fixed point, as shown in the figure below.



Different masses are suspended from the spring. The weight W of the mass and the length L of the spring are noted.

The variation with weight W of the length L is shown in the figure below.



A mass of weight 4.0 N is suspended from the spring. When the mass is stationary, it is then pulled downwards a distance of 0.80 cm and held stationary.

(i) Determine the total length of the spring. [1]

(ii) For the increase in extension of 0.80 cm, determine the magnitude of the change in

1. the gravitational potential energy of the mass, [2]

2. the elastic potential energy of the spring. [3]

(iii) Use your answers in (ii) to show that the work done to cause the additional extension of 0.80 cm is 4.0×10^{-3} J. [1]

Power & Efficiency

(L1)14. (J03/1/16)

Which of the following expression defines power?

A force × distance moved in the direction of the force
B force × velocity
C work done ÷ time taken
D work done × time taken

(L1)15. (N86/1/4)

A car of mass *m* has an engine which can deliver power *P*. What is the minimum time in which the car can be accelerated from rest to a speed *v*?

A. $\frac{mv}{P}$ **B**. $\frac{P}{mv}$ **C**. $\frac{mv^2}{2P}$ **D**. $\frac{2P}{mv^2}$ **E**. $\frac{mv^2}{4P}$

(L1)16. (N94/1/6)

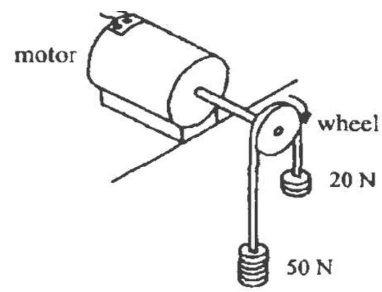
A power station has an efficiency of 40% and generates 1000 MW of electrical power. What is the input power and the wasted power?

	Input power /MW	Wasted power/MW
A	1000	400
B	1000	600
C	1400	400
D	2500	1500

(L2)17. (J96/1/6, N02/1/8, N98/1/6)

The diagram shows an arrangement used to find the output power of an electric motor. The wheel attached to the motor's axle has a circumference of 0.5 m and the belt which passes over it is stationary when the weights have the values shown. If the wheel is making 20 revolutions per second, calculate the output power.

[2]



(L2)18. (N07/1/10)

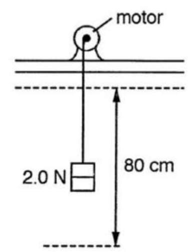
A car of mass 1.2×10^3 kg travels along a horizontal road at a speed of 10 m s^{-1}. It then accelerates at 0.20 m s^{-2}. At the time it begins to accelerate, the total resistive force acting on the car is 160 N. What total output power is developed by the car *as it begins the acceleration*?

A 0.80 kW **B** 1.6 kW **C** 2.4 kW **D** 4.0 kW

(L2)19. The outboard motor of a small boat has a propeller of radius 0.10 m. If the boat is at rest, the propeller sends back a stream of water at a speed of 10 m s^{-1}. What is the *average* power of the motor? (take the density of water = 1000 kg m^{-3})

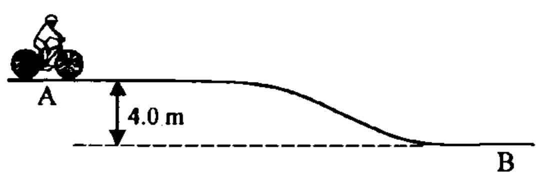
A 1.25 kW **B** 6.50 kW **C** 15.7 kW **D** 31.4 kW

(L2)20. (N04/1/9)
A small electric motor is used to raise a weight of 2.0 N at constant speed through a vertical height of 80 cm in 4.0 s.



The efficiency of the motor is 20%.
Determine the electrical power supplied to the motor. [2]

(L2)21. (N99/3/1 modified)
Bobby together with his bicycle has a total mass of 80 kg and is travelling with a constant speed of 10 m s^{-1} on a flat road at A, as illustrated below. Bobby exerts a power of 220 W at A to overcome resistive forces. He then goes down a small slope to B by descending 4.0 m.



(a) Calculate the total resistive force acting on Bobby at A [1]

(b) Calculate the speed at B, assuming that all the lost potential energy at A is transformed into kinetic energy of Bobby and his bicycle at B. [2]

(c) At B, Bobby travels at a higher constant speed. Explain why he needs to provide a power greater than 220 W. [2]

(d) It is often stated that many forms of transport transform chemical energy into kinetic energy. Explain why a cyclist travelling at constant speed is not making this transformation. Explain what transformations of energy are taking place. [5]

(L2)22. A car of weight 7000 N travels at a steady speed of 8 m s^{-1} up a steady incline at 15° above the horizontal. The car's motion is opposed by a constant frictional force of 5 000 N. Calculate

(i) the gain in gravitational PE per second, [2]

(ii) the work done *per second* against friction, [2]

(iii) the car's engine (driving) power (2 methods to solve this part are possible) [2]

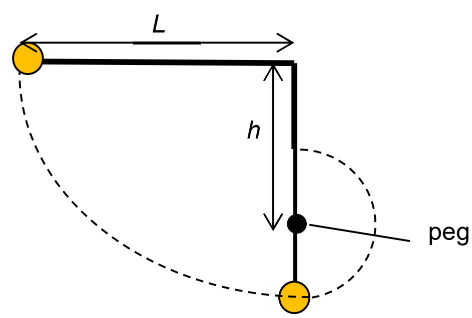
Relationship between force F & potential energy U

(L2)23. (J84/2/2)
The potential energy of a body when it is at a point P a distance x from a reference point O is given by $V = kx^2$, where k is a constant. What is the force acting on the body when it is at P?

A $2kx$ in the direction OP
B kx in the direction OP
C kx in the direction PO
D $2kx$ in the direction PO

(L2)24. Sketch a graph to show the variation with extension of the elastic potential energy of an object that obeys Hooke's law. Explan how you can determine the load, F, from the graph.

(L2)25. (Giancoli, sixth edition, pg 165)
A ball is attached to a horizontal cord of length L whose other end is fixed as shown below.



(a) If the ball is released, what will be its speed at the lowest point of its path? [2]

(b) A peg is located a distance h directly below the point of attachment of the cord. If $h = 0.80\,L$, what will be the speed of the ball when it reaches the top of its circular path about the peg? [2]