TOPIC 13: Current of Electricity

Contents

13.1 Electric Current, Electric Charge and Coulomb 2
13.1.1 Number density and drift velocity of electrons 5
13.2 Potential Difference and Volt 6
13.3 Electromotive Force 8
13.4 Resistance and Ohm 9
13.5 I-V Characteristics Graphs 11
13.5.1 Resistance-Temperature Characteristics of an NTC Thermistor 13
13.6 Factors Affecting Resistance 14
13.7 Power 15
13.7.1 Kilowatt-hour 16
13.8 Internal Resistance 17
13.9 Efficiency 18
SUMMARY 19
TUTORIAL 13: CURRENT OF ELECTRICITY 21
Numerical Answers 32
ADDITIONAL QUESTIONS 33
TUTORIAL 13: CURRENT OF ELECTRICITY SOLUTIONS 42

Learning Outcomes: Candidates should be able to:

a.	Show an understanding that electric current is the rate of flow of charge.
b.	Derive and use the equation $I = nAvq$ for a current-carrying conductor, where n is the number density of charge carriers and v is the drift velocity
c.	Recall and solve problems using the equation $Q = I\,t$.
d.	Recall and solve problems by using $V = W / Q$.
e.	Recall and solve problems by using $P = V\,I$, $P = I^2 R$ and $P = V^2 / R$
f.	Define the resistance of a circuit component as the ratio of the potential difference across the component to the current passing through it, solve problems using the equation V=IR.
g.	Sketch and explain the I-V characteristics of various electrical components such as an ohmic resistor, a semiconductor diode, and a filament lamp and a negative temperature coefficient (NTC) thermistor.
h.	Sketch the resistance-temperature characteristics of an NTC thermistor.
i.	Recall and solve problems using the equation $R = \rho L / A$.
j.	Distinguish between e.m.f. and P.D. in terms of energy considerations.
k.	Show an understanding of the effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power.

Four broad areas:

- Electric current
- Potential difference
- Resistance and resistivity
- Electromotive force

Key questions:

- What is the difference between electric current, potential difference, electromotive force and resistance?
- What is resistivity?
- How and why is the I-V characteristic of ohmic resistor, semiconductor diode, filament lamp and a negative temperature coefficient thermistor different?
- How is potential difference and electromotive force related to energy?

13.1 Electric Current, Electric Charge and Coulomb

Many subatomic particles and ions possess a property called charge. When these charges are made to move, a current is produced. The ability to control the flow of these charges allow us to cook food, light our home, provide air-con, entertain us with video and music, mine bitcoin, defeat other Pokémons, etc.

Electric current (I):

Electric current is the rate of flow of <u>charge</u> with respect to time.

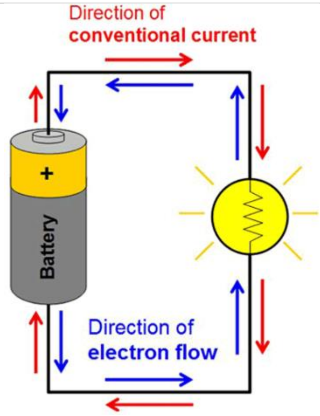
$$I = dQ/dt$$

{*NOT <u>charged particles</u>, since each particle may have <u>different amount of charges</u>*}

- For constant I, $I = Q/t$
- Scalar quantity.
- S.I. unit: Ampere (A) or Coulomb per second ($C\ s^{-1}$)
- A is the S.I. base unit, $C\ s^{-1}$ is the derived unit.

Common charge carriers are:

Material type	Charge carriers
Conductors (e.g. metals)	Free electrons (-)
Semiconductors	Free electrons (-) and holes (+) or vacancies for electrons.
Electrolytes	Negative (-) and Positive (+) ions
Ionized gases	Electrons (-) and positive ions (+)



Conventional current: defined as moving in the same direction as the <u>positive</u> charge flow. (Hence electrons in a circuit are moving opposite to conventional current)

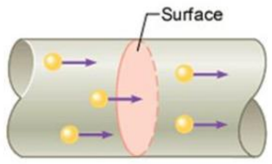
The <u>direction of the current in a circuit</u> is that of the conventional current. The 'direction' here is not a vector direction.

If the direction of charge flow is fixed, it is a **direct current**. If periodically reversed, it is an **alternating current** (Topic 18).

Electric charge (Q)

Electric charge passing a point is defined as the product of the (steady) current at that point and the time for which the current flows

Scalar quantity; S.I. unit: coulomb (C).
1 C = 1 A × 1 s

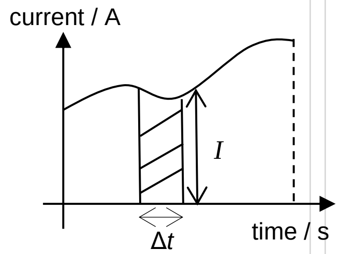


If <u>current is not constant</u>, $I = \mathrm{d}Q/\mathrm{d}t$

Total charge Q that flows in a time interval of Δt is given by

$$Q = \int I \,\mathrm{d}t$$
$$= \text{Area under the } I\text{-}t \text{ graph (i.e., the strip area} = I\,\Delta t)$$

Similarly, if a Q-t, charge-time graph is given, the instantaneous current I can be obtained from the gradient of the graph.



For <u>constant</u> current, $I = Q/t$

$$\boxed{Q = I\,t}$$

Where Q = total electric charge [C]
= **number of charged particles × charge per particle**
I = current [A]
t = time [s]

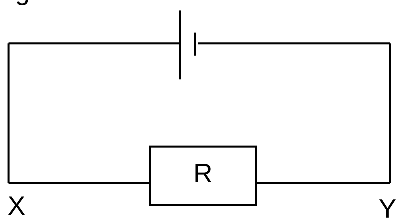
Note: one electron carries a charge of -1.6×10^{-19} C, which is also the smallest magnitude of charge called elementary charge. All other charges are whole number multiples of this elementary charge.

Example 1:

The current in the circuit is 4.8 A.

What is the rate of flow and the direction of flow of electrons through the resistor R?

A $3.0 \times 10^{19}\ \mathrm{s}^{-1}$ in direction X to Y
B $6.0 \times 10^{18}\ \mathrm{s}^{-1}$ in direction X to Y
C $3.0 \times 10^{19}\ \mathrm{s}^{-1}$ in direction Y to X
D $6.0 \times 10^{18}\ \mathrm{s}^{-1}$ in direction Y to X



Solution:
$Q = I\,t$
$N\,e = I\,t$,
where N = no of electrons and e = charge per electron = elementary charge = 1.6×10^{-19} C

Rate of flow of electrons = $= 3.0 \times 10^{19}$

Since conventional current flows from positive terminal to negative terminal (X to Y), flow of electrons will be in the opposite direction, i.e., <u>from Y to X</u>.

Ans: **C**

Worked Example 2:

An ion beam of singly charged Na^+ and K^+ ions is passing through vacuum. If the beam current is 20 μA, calculate the total number of ions passing any fixed point in the beam per second. (The charge on each ion is 1.6×10^{-19} C.)

Solution:

Current, $I = \frac{Q}{t} = \frac{Ne}{t}$ where N is the no. of ions and e is the charge on one ion.

No. of ions per second $= \frac{N}{t} = \frac{I}{e}$

$= \frac{20\times10^{-6}}{1.6\times10^{-19}}$

$= 1.25 \times 10^{14}$

Extension: If instead the ion beam is made up of Al^{3+} and Ga^{3+} ions, how would the above answer change?

Solution:

No. of ions per second $= \frac{N}{t} = \frac{I}{2e}$, where 2e is the charge on each ion

$= \frac{20\times10^{-6}}{2\times1.6\times10^{-19}}$

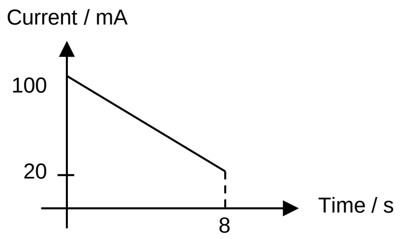
$= 6.25 \times 10^{13}$

Example 3:

The current in a component is reduced uniformly from 100 mA to 20 mA over a period of 8.0 s. Determine the charge that flows during this time interval. [480 mC]

Solution:

If the current is varying, total charge that flows is the area under the current-time graph.



Charge that flows during this time interval = Area under I-t graph

=

= 480 mC

Tutorial qn: Q1 to Q3

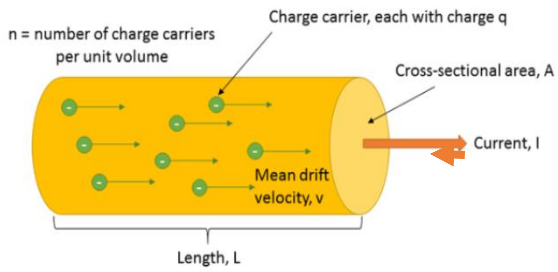
13.1.1 Number density and drift velocity of electrons

From $I = \dfrac{Q}{t} = \dfrac{Ne}{t}$,

$I = \dfrac{nVe}{t}$, where $N = nV$

(n = number density, V = Volume)

$= nAe \times \dfrac{l}{t}$, where V = area A x length l.



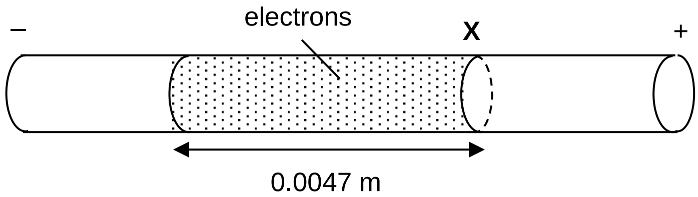
$\dfrac{l}{t}$ is the average speed of the electrons in a material due to an electric field. It is also known as the **drift velocity**[1], v.

Therefore, $\boxed{I = nAve}$ (**Derivation required**)

Watch this 4-minute video which illustrates what drift velocity of electrons is and how the above formula is derived. You may scan the QR code or go to tinyurl.com/ybqvhy32

Example 4: 2012 H2P1Q27

Copper has 8.5 x 10^{28} conduction electrons per cubic metre. A piece of copper wire has an area of cross-section 3.2 x 10^{-7} m^2.



When a potential difference is applied to the wire, the electrons within a cylinder of length 0.0047 m all pass point X in 60 s. What is the current in the wire?

Solution:

The average speed of electrons, or drift velocity, m s^{-1}

Hence, current, A

Note: Although the drift velocity of the electrons is low, it is not related to the actual transmission of the current (which is instantaneous!!). i.e., When a light bulb in a circuit is switched on, the current is flowing instantaneously in the circuit, and the bulb lights up almost immediately, despite the drift velocity of the electrons being low. All the electrons in the circuit respond to the electric field which propagates through the circuit at the speed of electromagnetic wave.

Watch this 3-minute video which illustrates why although drift velocity of electrons is extremely slow, its effect such as that of turning on a light bulb is instantaneous. You may scan the QR code or go to tinyurl.com/yasxtxow

[1] For metals, drift velocity ≈ 10^{-6} m s^{-1} and number density ≈ 10^{29} electrons per m^3;
for semiconductors, drift velocity ≈ 10 m s^{-1} and number density ≈ 10^{18} electrons per m^3

(2023 H2P3Q3(cii))

The mean speed of a conduction electron in the tungsten wire is very much greater than the answer obtained in **(c)(i)**.

Explain this observation.

....

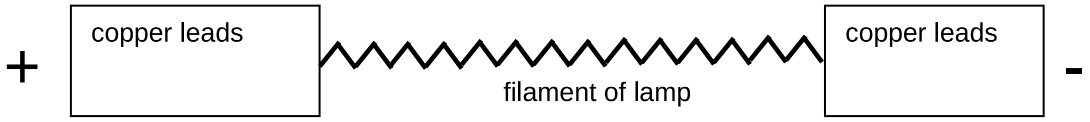
....

....

.... [2]

Worked Example 5 (2013 H1P2Q8(c))

Fig 13.1 illustrates the connection between the copper leads to the filament and the filament itself.



Describe, giving reasons where appropriate, and without using numerical values, the following features of the current in these **three** components.

(i) the direction of electron flow
Answer: Electrons flow from the negative terminal copper wire, to the positive terminal copper wire, via the filament.

(ii) the rate of electron flow.
Answer: From Current Law (Topic 14), where charge is conserved, therefore the rate of electron flow in and out of the wires is constant.

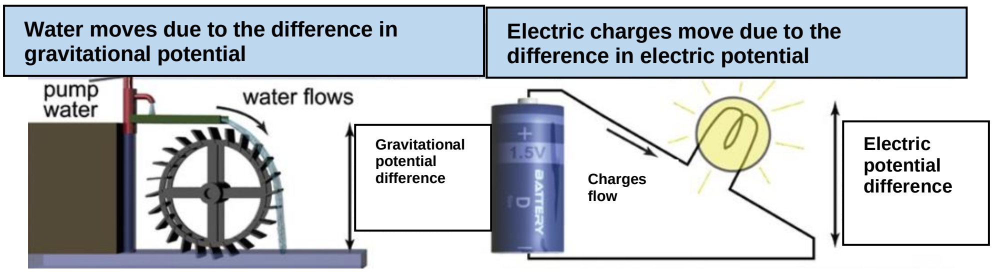
(iii) the average speed of electrons.
Answer: Since rate of electron flow is constant, the copper wire with larger cross-sectional area will have slower electrons (from $I = nAve$). Conversely, the filament of lamp will have faster electrons.

Tutorial qn: Q4

13.2 Potential Difference and Volt

In 'O' Level, we learnt that when current of 0.2 A passes through a resistor of 3 Ω, using $V = IR$, potential difference across the resistor is 0.6 V. But what exactly is potential difference?

We can use the following as an analogy:



- Electric charges is analogous to mass
- Electric current is analogous to the rate of flow of mass of water.
- Battery is analogous to water pump.
- Potential difference is analogous to change in gravitational potential, Φ.

Potential difference is defined as the energy transferred from electrical energy to other forms of energy when unit charge passes through an electrical device.

$$V = \frac{W}{Q}$$

where V: Potential difference
W: Energy transferred from electrical to other forms of energy
Q: Total Charge

Scalar quantity; S.I. unit: volt (V).
1 V = 1 J C^{-1} or 1 W A^{-1}

If W and Q in the above equation is divided by t, the time during which the charge is transferred,

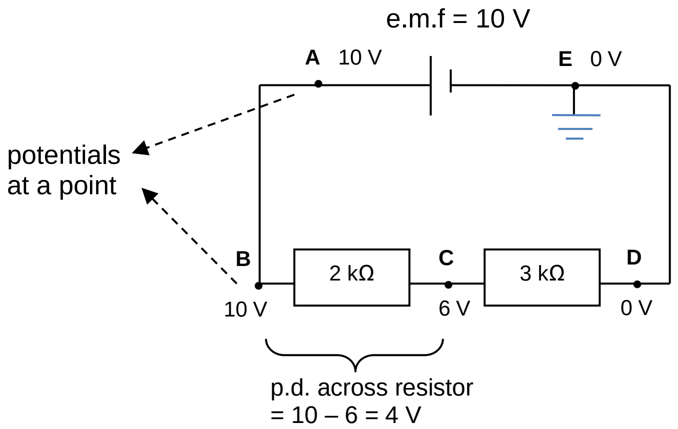
$$V = \frac{W/t}{Q/t} = \frac{P}{I}$$

where P: Power
I: Current

Thus, electric potential difference between 2 points could *alternatively* be defined as the electric **power** per unit current transferred to other forms of **power** when the current passes through an electrical component.

Potential difference is measured across a component.

Difference between Potential and Potential Difference:



Potential at a point	The amount of potential energy per unit of charge at a specified location. Potential drops when current passes through a load (energy used up by load). There is change in potential along the circuit when current passes through a battery or a load. There needs to be a reference potential (e.g. earthed at 0V) to define potential at all points.
Potential Difference (p.d.)	Difference in potential between any given two points. E.g., p.d. across BC, or V_{BC}, is 10 V – 6 V = 4 V. In addition, when there is no energy loss between two points of the circuit, the potential of these points is same. P.d. = 0 V. (e.g., V_{AB} = 0 V)

Conventional current flows from a point of higher potential to a point of lower potential across resistors of any forms (if they are joined by a conducting path). However, conventional current flows from a point of lower potential to a point of higher potential across a battery or any current generating devices.

Example 6

A current of 5 mA passes through a bulb for 1 minute. The potential difference across the bulb is 4 V. Calculate the work done to operate the bulb *for 1 minute*.

Solution:

Charge Q passing through bulb in a min $= I\,t$

$=$

$= 0$ C

V across bulb $= \dfrac{\text{Work Done}}{\text{Charge}}$

$4 = \dfrac{W}{0.3}$

Work done to operate bulb for 1 minute = = 1.2 J

13.3 Electromotive Force

The **electromotive force** (e.m.f.) is defined as the energy transferred per unit charge from other forms of energy into electrical energy (by a source) when charge is moved round a complete circuit

$$\text{e.m.f.} = \frac{\text{Work Done}}{\text{Charge}} = \frac{\text{Energy transferred}}{\text{Charge}}$$

Scalar quantity; S.I. unit: volt (V).

Note:

- e.m.f. is NOT a force and it does NOT push the charge around the circuit.
- e.m.f. of an electrical source exists even if $I = 0$ e.g. when battery is not connected in a closed circuit.

Distinguish between e.m.f. and PD in terms of energy considerations:
e.m.f. refers to the electrical energy generated from non-electrical energy forms.
p.d. refers to the non-electrical energy generated from electrical energy.

For example,

e.m.f. sources	**Energy Change (non-electrical energy to electrical energy)**
Chemical Cell	Chemical → Elec
Generator	Mechanical → Elec
Thermocouple	Thermal → Elec
Solar Cell	Solar → Elec

loads	**Energy Change (electrical to non-electrical energy)**
Bulb	Elec → Light
Fan	Elec → Mechanical
Doorbell	Elec → Sound
Heating element	Elec → Thermal

Example 7

A battery is connected in series with a resistor R. The battery transfers 2000 C of charge completely round the circuit. During the process, 2500 J of energy is dissipated in the resistor R and 1500 J is dissipated in the battery. The e.m.f of the battery is

A 0.500 V **B** 0.7500 V **C** 1.250 V **D** 2.000 V

Solution:

E = $\frac{W}{Q}$, where W = total energy transferred.

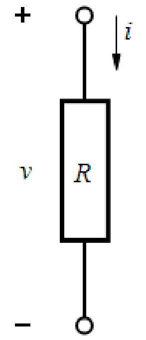
By conservation of energy,
Energy input = Energy output
Energy transferred in cell = Energy dissipated in the external resistor + energy dissipated in the battery due to internal resistance
= = 4000 J

Hence Emf = $\frac{\text{Energy transferred}}{\text{Total Charge}}$ = = 2.000 V

Ans: D

Tutorial qn: Q5, Q6

13.4 Resistance and Ohm



Resistance (R) is defined as the ratio of the potential difference across a component to the current flowing through it.

Resistance is NOT defined as the gradient of a V-I graph, i.e. NOT = dV/dI

$$\boxed{R = \frac{V}{I}}$$

Scalar quantity; S.I. unit: ohm (Ω).

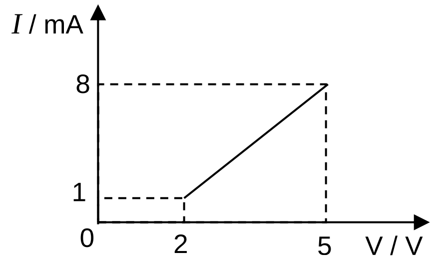
In general, the resistance of a component often varies with the voltage across and the current flowing through it.

However, resistance of certain components remains constant regardless of voltage across them. This happens if temperature is kept constant for these components. Such component is termed as ohmic and is said to obey the Ohm's Law (current through a conductor is proportional to the potential difference across it provided its temperature remains constant). An ohmic conductor will have a graph of current vs potential difference will be a linear graph, and passes through the origin as I $\propto$ V

The effect of temperature on resistance is discussed in the explanation for I-V characteristics graphs.

Example 8:

The graph shows the I ~ V characteristic of a certain conductor.



(a) Calculate the resistance of the conductor when the voltage is 5 V.
(b) State, with reason, whether the conductor obeys Ohm's law.
(c) Explain how to use the graph to compute the minimum and maximum resistance.

Solution:

(a) The resistance, $R = \frac{V}{I} =$ $= 630\ \Omega$ (2 sig. fig.)

(b)

(c) Resistance is the ratio of voltage and current.
Hence, resistance is computed from the ______________________ **<u>the line that joins the point to the origin</u>.**

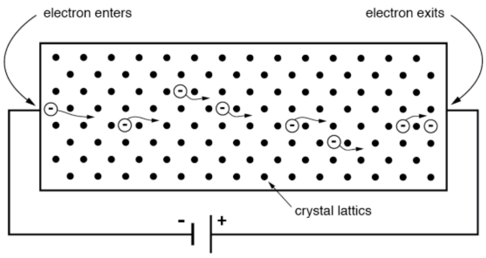
Therefore, at 1 mA, the resistance is maximum (lowest gradient for line from the origin) with resistance of $\frac{2}{0.001} = 2000\ \Omega$,
and at 8 mA, the resistance is the minimum (highest gradient for line from the origin) with 630 Ω.

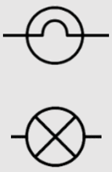
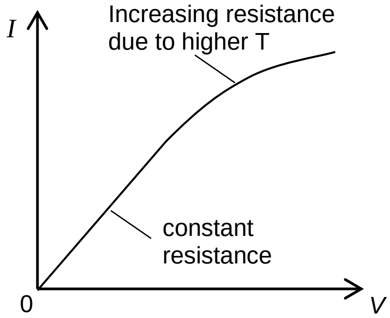
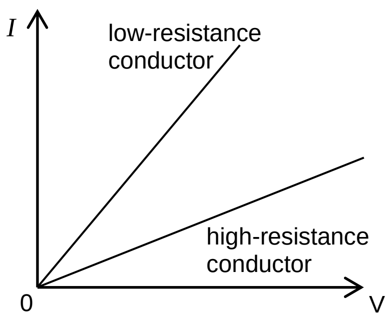
Note:
R is the ratio of *V* to the corresponding value of I.
Hence**,** <u>*R* is NOT in any way related to the gradient of the I -V graph.</u>
However, in the special case where a V-I graph be plotted for an <u>ohmic</u> component, its resistance <u>*equals*</u> the gradient of its V-I graph as the graph <u>passes through the origin</u>**.**

13.5 *I-V* Characteristics Graphs

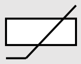
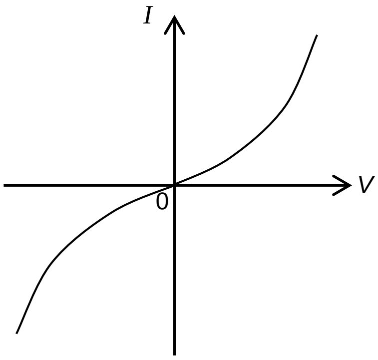
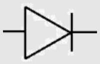
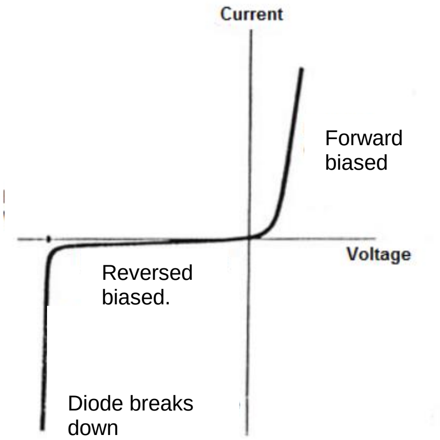
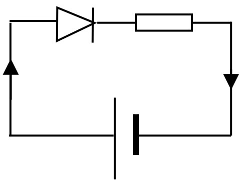
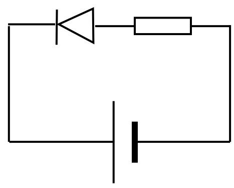
It is important to know how the current, I, passing through a component varies with the p.d., V across it. This is called the I-V characteristics[2] of a component. Resistance is affected by current flow which is dependent on

1. amplitude of the lattice vibration – the larger the vibration, the bigger the resistance as it is difficult for the charge carriers to move through the conductor thus collision frequency of electrons with lattice ions increases. The greater scattering of the electrons leads to a reduced rate of flow of charges (i.e. less current for the same potential difference)
2. number of charge carrier – the larger the number of charge carriers, the smaller the resistance as there is an increased rate of flow of charges (i.e. more current for the same potential difference)



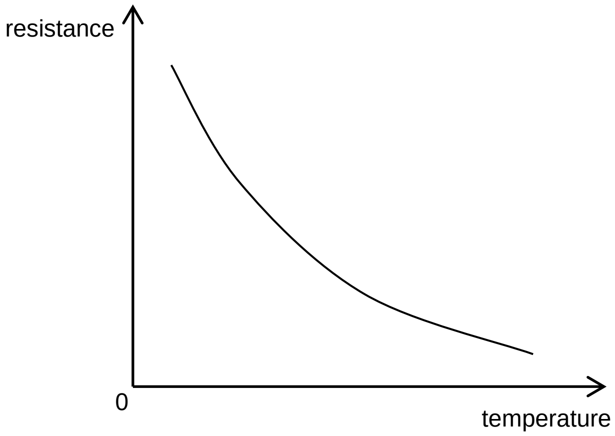
Component	*I-V* graph	Explanation
Filament lamp (*temperature changed* due to heating) 		Metallic conductors' resistance increases when temperature increases.[3] As V increases and hence, I increase, • filament gets hotter (temperature increases i.e. gains thermal energy) • Amplitude of vibration of lattice ions increase • Number of charge carriers (free electrons) is constant due to the atomic structure of metal atoms, and it does not increase significantly with temperature. • Hence resistance increases
Metallic conductor (maintained at *constant temperature*)	 Straight-line graph through origin	Metallic conductors' resistance remains constant when temperature is maintained at constant temperature. As V increases and hence, I increases, • Amplitude of vibration of lattice ions is constant • Number of charge carriers (free electrons) is constant • Hence resistance remains constant

[2] I is plotted against V. It is not a V-I graph.
[3] When a potential difference is put across a metal, it produces an electric field and exerts an electric force on the free electrons. Hence, the free electrons are accelerated.

Negative Temperature Coefficient (NTC) Thermistor  		A NTC thermistor's (made from semi-conductor material) resistance decreases when temperature increases.[4] As *V* increases and hence, *I* increase, • thermistor gets hotter (temperature increases i.e. gains thermal energy) • Amplitude of vibration of lattice ions increase • More bound electrons to (gain enough energy to) be released as negative charge carriers. • In addition, when electrons are freed from atoms, vacancies known as holes (which are positive charge carriers) are created. • The increased in number of charge carriers caused by the increasing temperature is more significant than the increased amplitude of lattice vibrations • Hence resistance decreases.
Semi-conductor diode 		A diode (made from semi-conductor material) has different resistances at different voltages. • In *forward biased* p.d. (> about 0.7 V for Silicon[5]), a diode's resistance is not infinitely high ⇒ current to flow  • In *reversed-biased* p.d. (before the diode breaks down), the diode has infinitely high resistance and behaves like an insulator ⇒ no current flows 

[4] When a potential difference is put across a metal, it produces an electric field and exerts an electric force on the free electrons. Hence, the free electrons are accelerated.
[5] Germanium is less widely used and offers a low turn on voltage of around 0.2 to 0.3 V

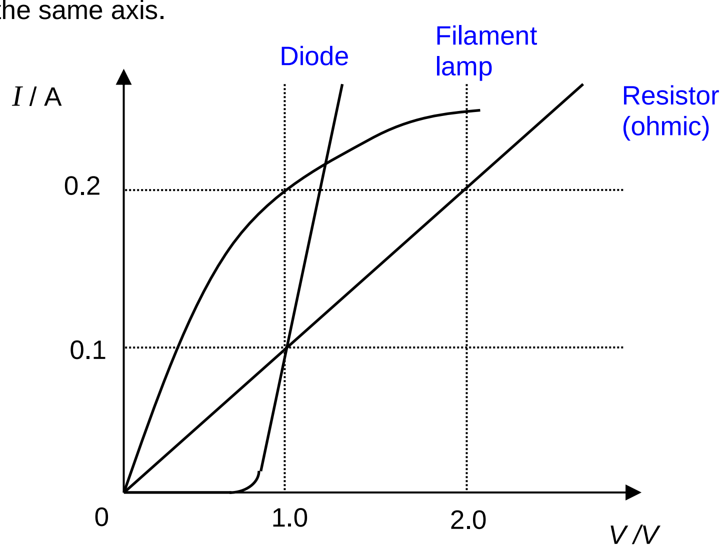
13.5.1 Resistance-Temperature Characteristics of an NTC Thermistor



NTC thermistors are resistors with a negative temperature coefficient, which means that the resistance decreases with increasing temperature.

Example 9: 2013 H1P1Q23

The graph shows the *I*-*V* characteristics of three electrical components, a diode, a filament lamp, and a resistor, plotted on the same axis.



Which statement is correct?

A The resistance of the diode equals that of the filament lamp at about 1.2 V.
B The resistance of the diode is constant above 0.8 V.
C The resistance of the filament is twice that of the resistor at 1.0 V.
D The resistance of the resistor equals that of the filament lamp when V = 0.8 V.

Solution:

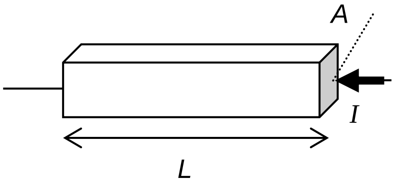
It is important that the students **memorize** the *I*-*V* characteristics of the filament lamp, diode and resistor (ohmic), as shown in the diagram.

Hence, when V = , the diode and filament lamp have the same current, so it will have the same resistance from $V = I R$.

Ans:

Tutorial qn: Q7 to Q10

13.6 Factors Affecting Resistance



Besides temperature, the resistance of a uniform conductor is proportional to its length L, its resistivity ρ and inversely proportional to its cross-sectional area A.

$$\boxed{R = \frac{\rho L}{A}}$$

where L : Length [m]
A : Cross sectional area [m^2]
ρ : Resistivity [Ω m]

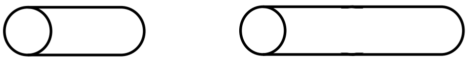
Resistivity of a material refers to the resistance per unit length and per unit cross-sectional area of the material. It is a property of the material. For example, gold has lower resistivity than copper. This means gold is a material that readily allows the movement of electrons.

$$\rho = \frac{RA}{l}$$

Scalar quantity; S.I. unit: ohm-metre (Ω m).

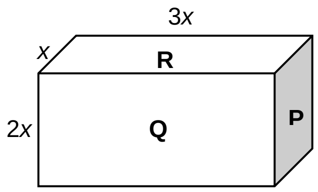
Note: Resistivity is an intrinsic property of a material. It is temperature-dependent but it is a constant for material under a given temperature. The temperature dependence of the resistivity is different for conductors and semiconductors.

Applications:

 Thinner wires have higher resistance than thicker wires (for similar lengths and material).	 Longer wires have higher resistance than shorter wires (for similar thickness and material)

Example 10:

The diagram shows a rectangular block with dimensions x, 2x and 3x. Electrical contacts can be made to the block between opposite pairs of faces (for example, between the face labelled P and its opposite face). Determine which two faces the maximum electrical resistance would be obtained.



A the face labelled P and its opposite face
B the face labelled Q and its opposite face
C the face labelled R and its opposite face
D the resistance is the same, whichever pair of faces is used.

Solution:
Maximum resistance is when the ratio of length (parallel to the current flow) to the cross sectional area (perpendicular to current flow) is largest.

Ans:

Example 11:

A wire of resistance *R* is stretched uniformly until it is twice its original length. What happens to its resistance?

Solution:
If the length *L* doubles, then the cross sectional area *A* is halved, since the volume ($V = AL$) of the wire remains the same.

Hence, the resistance will increase by a factor of

Tutorial qn: Q11 and Q12

13.7 Power

Electric Power (*P*) may be related to potential difference, current and/or resistance as follows:

$$\boxed{P = VI = I^2 R = \frac{V^2}{R}}$$

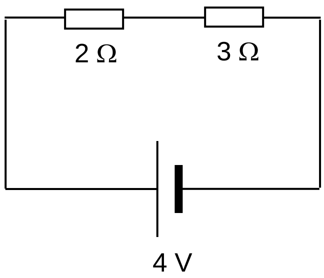
Note:

- All the three power formulas can be used to determine power loss. However, $P = I^2 R$ is more commonly used to calculate power loss in cables (instead of P = I V) as the current and resistance in the cables are usually known whereas the potential differences across the cables are difficult to determine.

- $P = IV$ is commonly used to calculate power supplied by a source because the current supplied and the e.m.f. of the power source is usually known.

- Brightness of bulb is determined by the power dissipated - not by I, V or R alone.

Example 12:



Referring to the circuit below, determine
(a) the power produced by the battery,
(b) the power dissipated in each resistor.

Solution:

(a) current in circuit $= \frac{V}{R} = \frac{4}{2+3} =$

power produced by battery =

(b) power dissipated in 2 Ω resistor =

power dissipated in 3 Ω resistor =

Worked Example 13:

A high-voltage transmission line with a resistance of 0.4 Ω km^{-1} carries a current of 500 A. There is a p.d. of 1200 kV across the power station and the line carries the current to a city located 160 km from the power station. Calculate

(a) the power loss in the lines.
(b) the fraction of the transmitted power that is lost.

Solution:

(a) power loss in line P $= I^2 R = 500^2 \times (0.4 \times 160) \times 2$ (to and fro)
$= 32$ MW

(b) total power transmitted $= I V = 500 \times 1200 \times 10^3 = 600$ MW

fraction of power loss $= \dfrac{32}{600} = 0.0533$

Example 14: (2009 SAJC Prelim P2 Q5 part)

An A.C generator delivers power to a resistive load in a remote location over a cable transmission line of resistance 6.0 kΩ. The potential difference across the load is 80 kV, and the power delivered by the generator is 250 kW.

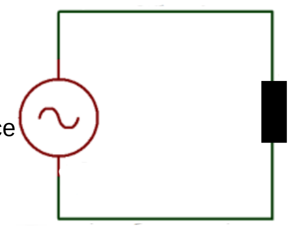
Show that the current flowing in the cables is 2.61 A.

Solution:

Given: R_c = 6 kΩ, V_{load} = 80 kW, Power delivered = 250 kW

By principle of conservation of energy,

$$P_{gen} = P_{load} + P_{loss\ in\ cables}$$



$\Rightarrow$ $I_c = 2.61$A

Tutorial qn: Q13 to Q14

13.7.1 Kilowatt-hour

Kilowatt-hours are a measurement of energy, describing the total amount of electricity used over time.

One kilowatt-hour (1 kWh) is the energy consumed by a power consumption of 1kW for 1 hour.

1 kWh = 1000 J s^{-1} x (60 x 60) s = 3.6 x 106 J

E.g., if you switched on a 100 W light bulb, it would take 10 hours to rack up 1 kWh of energy.
For December 2023, the electricity tariff in Singapore is 28.70 cents per kWh (excluding GST).

13.8 Internal Resistance

If you use a voltmeter to measure the open circuit e.m.f. of an AA-sized battery, you will find that the e.m.f. is about 1.5 V.

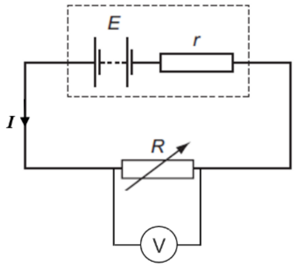
But if you are using a closed circuit to draw a current from the battery, you will find that the p.d. across the battery is now less than 1.5 V.

This is because the battery itself has an **resistance called internal resistance**.

One way to think of internal resistance is to imagine a real battery as being made up of an ideal battery of e.m.f. E, connected in series with a resistor r which represents the internal resistance.

In an open circuit with a voltmeter connected, no current is flowing through the battery and voltmeter, since we assume it is an 'ideal' voltmeter (infinite resistance) thus no potential drop across the internal resistance and the voltmeter measures the e.m.f. of 1.5 V.

In a closed circuit, current is flowing through the battery (even with an ideal voltmeter) thus there is a potential drop across the internal resistance and the voltmeter measures a p.d. of less than 1.5 V.



Note:

- E.m.f. of battery = p.d across load + p.d across internal resistor

$$\boxed{E = I R + I r}$$

- We can get the above equation by Conservation of Energy:
  Power supplied by cell = Power dissipated in the external & internal resistors
  $E I = I^2 R + I^2 r$
  Dividing the entire equation by I, we get back the same equation: $E = I R + I r$

- The reading on the voltmeter,
  V = I R = p.d. across the external resistor R = **terminal p.d. of battery = V**
  (This is because the voltmeter is connected **across R**, **and** to the **terminals** of the battery.)

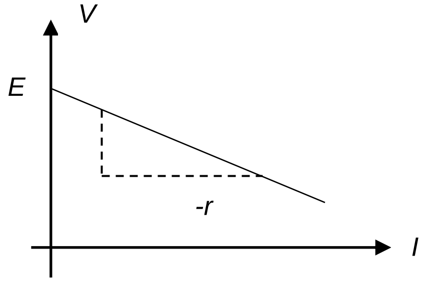
- Starting from, $E = I R + I r$
  $E = V + I r$

$$\boxed{\boldsymbol{V = E - Ir}}$$

- P.d. across internal resistance of the battery = $I\,r$. (However, this **cannot** be measured directly using a voltmeter, since the internal resistance is woven into the very existence of the battery.)

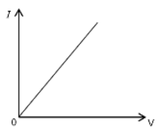
- If I = 0 A (open circuit) or if r = 0 Ω, *terminal p.d. of battery* = E

- Plotting V *vs* I, we get:



🡐 This graph is obtained by ***varying R***, and then plotting the values of V and I obtained.

Food for thought:
Why is this graph so different from the one that you learnt in secondary school? ➔



Example 15

A battery in a hearing aid supplies a current of 2.5 mA through a load-resistance of 400 Ω. When the wearer turns up the volume, the resistance is changed to 100 Ω and the current rises to 6.0 mA.

Determine
(a) the internal resistance of the battery.
(b) the e.m.f. of the battery,
(c) the PD across the terminals of the battery when the current is 2.5 mA.

Solution:

(a) & (b)
Applying the equation $E = I\,R + I\,r$ for both cases,

Solving (1) and (2) simultaneously, (a) r = 114 Ω, (b) E = 1.29 V

(c) When I = 2.5 mA, R = 400 Ω, E = 1.29 V

p.d. across the terminals, $V = I\,R$

=

= 1.00 V

Tutorial qn: Q15 to Q22

13.9 Efficiency

Efficiency = (Useful energy or power output/Total energy or power input) x 100%

$$\text{Efficiency} = \frac{\text{Energy supplied to Load}}{\text{Energy supplied by Battery}} \times 100\%$$

$$= \frac{\text{Energy supplied by Battery - Energy used to overcome Internal Resistance}}{\text{Energy supplied by Battery}} \times 100\%$$

Or

$$\text{Efficiency} = \frac{\text{Power supplied to Load}}{\text{Power supplied by Battery}} \times 100\%$$

$$= \frac{\text{Power supplied by Battery - Power dissipated by Internal resistance}}{\text{Power supplied by Battery}} \times 100\%$$

SUMMARY

1. **Electric current** (I)
rate of flow of charge with respect to time. Scalar quantity; S.I. unit: amperes (A) or coulomb per second ($C\ s^{-1}$)

2. Conventional current: defined as moving in the same direction as the positive charge flow. (Hence electrons in a circuit are moving opposite to conventional current)

3. **Electric charge** (Q) passing a point
defined as the product of the (steady) current at that point and the time for which the current flows. Scalar quantity; S.I. unit: coulomb (C).

For constant current, $Q = I\,t$
For varying current, $Q = \int I\,dt$ = Area under I-t graph

4. **Elementary charge** = 1.6×10^{-19} C

5. $I = nAve$ (Derivation required)

6. **Potential difference**
defined as the energy transferred from electrical energy to other forms of energy when unit charge passes through an electrical device.

$$V = \frac{W}{Q}$$

Or, it is the electric power per unit current transferred to other forms of power when the current passes through an electrical component.

$$V = \frac{W/t}{Q/t} = \frac{P}{I}$$

Potential difference is measured across a component.
Scalar quantity; S.I. unit: volt (V).

7. **Electromotive force** (e.m.f.)
energy transferred per unit charge from other forms of energy into electrical energy (by a source) when charge is moved round a complete circuit.

$$\text{E.m.f.} = \frac{\text{Work Done}}{\text{Charge}} = \frac{\text{Energy transferred}}{\text{Charge}}$$

Scalar quantity; S.I. unit: volt (V).

8. **Resistance** (R)
ratio of the potential difference across a component to the current flowing through it.

$$R = \frac{V}{I}$$

Scalar quantity; S.I. unit: ohm (Ω).
Resistance is NOT defined as the gradient of a V-I graph.

9. **I-V characteristics graphs**
- metallic conductor kept at constant temperature - resistance is constant.
- filament lamp - resistance *increases* as temperature increases.
- NTC thermistor - resistance *decreases* as temperature increases.
- Diode - a diode has a forward bias p.d (very low value of ~ 0.7 V for Silicon) where the resistance will drop as voltage increases and current flows. In reverse bias, the diode has infinitely high resistance and no current flows.

10. **Resistivity** of a material
refers to the resistance per unit length and per unit cross-sectional area of the material. It is a property of the material.

$$\rho = \frac{RA}{l}$$

Scalar quantity; S.I. unit: ohm-metre (Ω m).

12. **Electric Power**, $P = VI = I^2 R = \frac{V^2}{R}$

13. **Internal resistance**
resistance to current flow within the power source. It reduces the potential difference (not e.m.f.) across the terminal of the power supply when it is delivering a current.

$$E = IR + Ir$$

E.m.f. = p.d across load + p.d across internal resistor

14. **Efficiency** = $\frac{\text{Power supplied to Load}}{\text{Power supplied by Battery}} \times 100\%$

TUTORIAL 13: CURRENT OF ELECTRICITY

Concept on Q = I t

(L1) **1** A wire carries a steady current of 0.1 A over a period of 20 s. What total charge passes through the wire in this time interval? [1]

(L2) **2** Four small conductors on the edge of an insulating disc of radius r, are each given a charge of Q. The frequency of rotation of the disc is f. Express the equivalent electric current at the edge of the disc in terms of r, Q and/or f. [N98/1/14] [2]

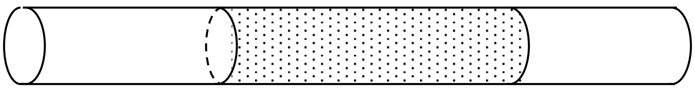
(L2) **3** When sufficiently high voltage is applied between two electrodes in a gas, the gas ionises. Electrons move towards the positive electrode, and positive ions move towards the negative electrode. (The charge on each ion is 1.6×10^{-19} C.)

(a) Determine the current in a hydrogen discharge tube if 4.4×10^{15} electrons and 1.5×10^{15} protons move in opposite directions past the tube's cross-section in one second. [2]

(b) State the direction of the current. [1]

Number density and drift velocity of electrons

(L2) **4** The electric current through a cylinder of semiconductor material is supplied by copper wires, as shown.



The charge carriers in the semiconductor material are electrons. The semiconductor material and the copper have the same cross-sectional area.
The table gives details of the number of free electrons per cubic metre in each material.

	Copper wire	semiconductor
Number of free electrons per cubic metre	8.6×10^{28}	4.3×10^{21}

The mean speed of electrons though the copper wire is 0.58 mm s^{-1}. What is the mean speed of electrons through the semiconductor material? [N2016/1/27]

A 2.0×10^{-6} m s^{-1} **B** 2.9×10^{-3} m s^{-1}
C 1.16×10^{4} m s^{-1} **D** 1.16×10^{7} m s^{-1}

Potential Difference / Electromotive Force

(L1) **5** Which statement describes the electric potential difference between two points in a wire that carries a current?

A the force required to move a unit positive charge between the points
B the ratio of the energy dissipated between the points to the current
C the ratio of the power dissipated between the points to the current
D the ratio of power dissipated between the points to the charge moved [1]

[J2000/1/16]

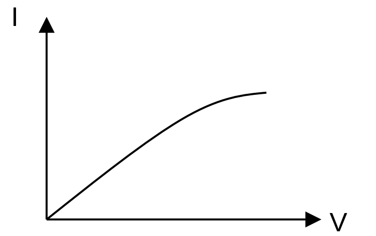
(L2) **6** A cell of e.m.f. E delivers a charge Q to an external circuit. Which statement is correct?

A The energy dissipation in the external circuit is E Q.
B The energy dissipation within the cell is E Q.
C The external resistance is E Q.
D The total energy dissipation in the cell and the external circuit is E Q. [1]

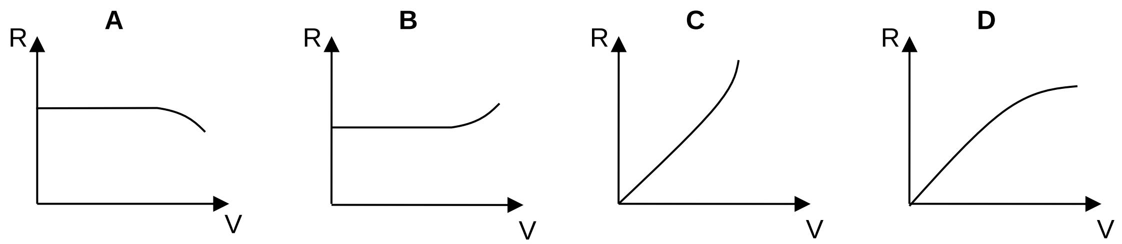
[N99/1/13]

Resistance, Ohm and Ohm's Law / I-V characteristic

(L2) **7** The current (I) flowing through a component varies with the potential difference V across it as shown.



Which graph best represents how the resistance R varies with V? [1]



(L1) **8** A torch bulb X is connected in series with a variable resistor and a 18 V battery of negligible internal resistance. The variable resistor is adjusted until the torch bulb operates at its normal rating, which is 12V and 24 W.

(a) Calculate the current in X and its resistance. [2]
(b) Calculate the p.d and the power dissipated in the variable resistor [2]

2012/2/Q3

(L2) 9 The variation with potential difference V of current I for a semiconductor diode is shown in Fig. 3.1.

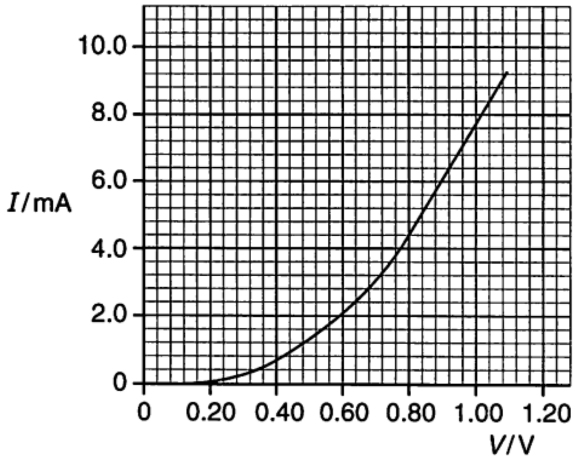


Fig. 3.1

(a) (i) Describe the main features of Fig. 3.1 that show the characteristics of the diode in terms of current, voltage and resistance.

....

....

....

....

....

....[4]

(ii) Use Fig. 3.1 to determine the resistance of the diode at 0.80V.

resistance = ... Ω [2]

(L2) 10 (a) A potential difference of 9.0 V causes electrons in a wire to flow such that 3.0×10^{20} electrons pass a point in the wire in 60 s. Calculate

(i) the charge which passes the point in 60 s; [1]
(ii) the electric current in the wire; [1]
(iii) the resistance of the wire. [1]

(b) Sketch graphs with labelled axes to show how the current through the wire will vary with the p.d. across if the temperature of the wire

(i) is kept constant; [1]
(ii) increases as the current increases. [1]

Factors affecting resistance

(L1) 11 Calculate the resistance of a nichrome wire of length 500 mm and diameter 1.0 mm, given that the resistivity of nichrome is $1.1 \times 10^{-6}\ \Omega$ m. [1]

(L2) 12 Given that the resistivity of copper at 25 ^{0}C is $1.72 \times 10^{-8}\ \Omega$ m,

(a) Calculate the resistance of a piece of copper wire whose length is 8.5 cm and whose diameter is 0.068 mm. [1]

(b) The wire is then stretched so that its length increases by 2% and its diameter reduces by 1%. Find the value of its resistance when under strain. [1]

Power

(L1) 13 A car headlamp is marked 12 V, 72 W. It is switched on for a 20 minutes journey.

(a) Calculate
(i) the current in the lamp. [1]
(ii) the energy supplied to the lamp during the journey, [1]

(b) State the assumption made. [1]

(L1) 14 A light-bulb is marked 240 V, 60 W. It is switched on for 3 Hrs. Assuming that the bulb used at the rated power, calculate

(a) the total energy converted by the bulb from electrical energy; [1]
(b) the current supplied to the bulb; [1]
(c) the total charge supplied to the bulb. [1]

Internal Resistance

(L1) **15** A 65.0 Ω resistor is connected to the terminals of the battery whose emf is 12.0 V and whose internal resistance is 0.5 Ω. Calculate

(a) the current in the circuit. [1]
(b) the terminal voltage of the battery. [1]

(L2) **16** An electrical source with internal resistance r is used to operate a lamp of resistance R. What fraction of the total power is delivered to the lamp? [1]

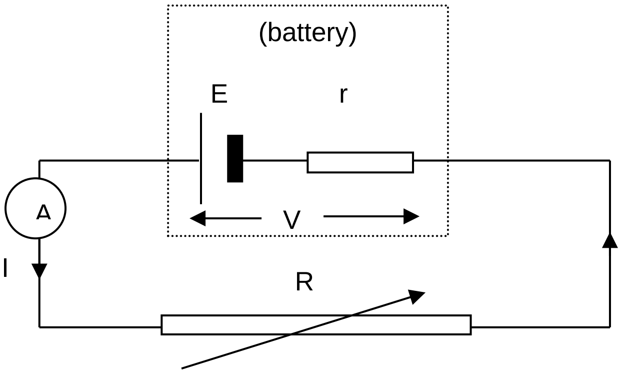
A $\frac{R+r}{R}$ **B** $\frac{R-r}{R}$ **C** $\frac{R}{R+r}$ **D** $\frac{R}{r}$

[J89/1/14]

(L2) **17** A battery is connected to a variable resistor and a voltmeter is connected across its terminals. When the variable resistor has a resistance of 6.0 Ω the voltmeter reading is 4.0 V. When the resistance is 10 Ω the voltmeter reading is 4.4 V.

Calculate the e.m.f. and the internal resistance of the battery. [3]

(L2) **18** A battery of e.m.f *E* and the internal resistance *r* delivers a current *I* through a variable *R*.



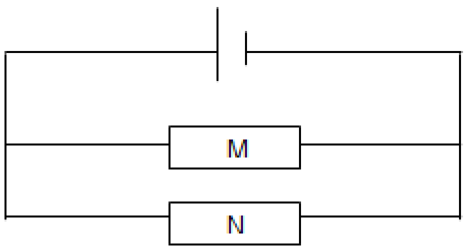
(a) *R* is set as two different values and the corresponding currents *I* are measured using an ammeter of negligible resistance. Compute the value of the e.m.f *E*. [3]

R / Ω	*I*/A
1.0	3.0
2.0	2.0

(b) Express *V*, the terminal potential difference of the battery, in terms of *E*,*R*, and *r*. [2]

[N04/1/23]

(L2) **19** In the circuit below, a battery supplies a current of 0.025 A for 80 s. During this time, it produces 18 J of electrical energy while the resistor M receives 11 J and resistor N receives 4.0 J. Calculate the e.m.f of the battery and its internal resistance. [3]



(L2) 20 A battery of e.m.f. E and internal resistance r is connected to a variable resistor, as shown in Fig. 22.1

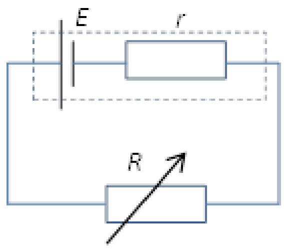


Fig 22.1

The total power produced in the battery is P_T. The power dissipated in the variable resistor is P_R. The variations of P_T, and of P_R with resistance R of the variable resistor are shown in Fig 22.2

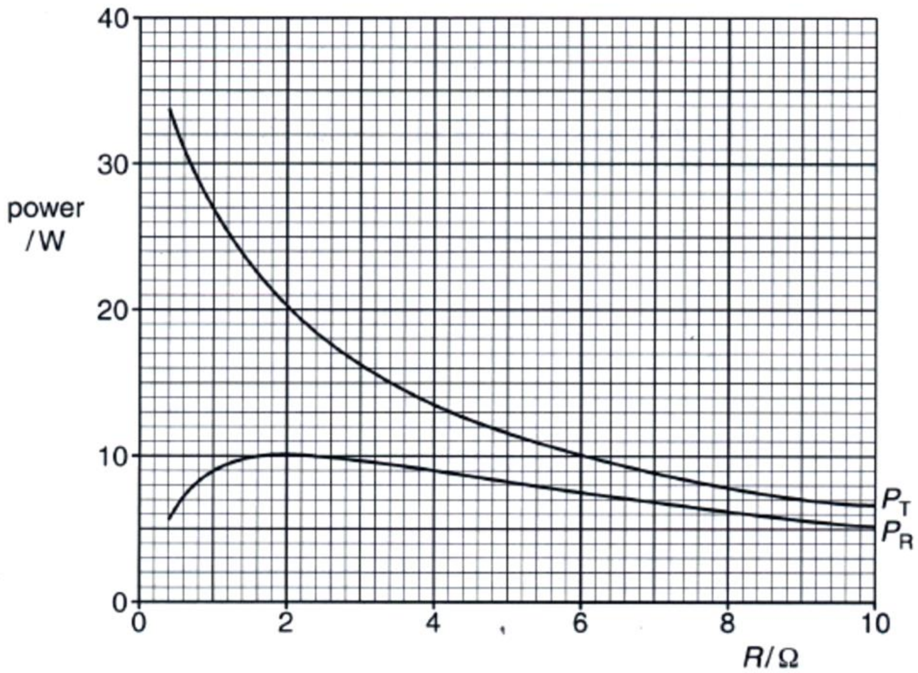


Fig 22.2

(a) For resistance R = 4.0 Ω, use Fig 22.1

(i) to show that the current in the circuit is 1.5 A, [2]

(ii) to determine the e.m.f. E of the battery [2]

(b) For any values of R, the values of P_T is greater than that of P_R.

(i) Suggest what is represented by the quantity (P_T - P_R). [1]

(ii) Use the values of P_T and P_R at R = 4.0 Ω and your answer to **(a)(i)** to determine the internal resistance r of the battery. [1]

(c) **(i)** Use Fig 22.1 to state the value of *R* at which P_R is maximum. [1]

(ii) For the values of *R* stated in **(i)**, determine the efficiency of power transfer from the battery to the variable resistor. [1]

(iii) State how the efficiency of power transfer changes for values of *R* between 4 Ω and 10 Ω. [1]

[2013P3/Q2]

(L2) 21 **(a)** An electrical component C has an *I-V* characteristic as shown in Fig. 23.1 below.

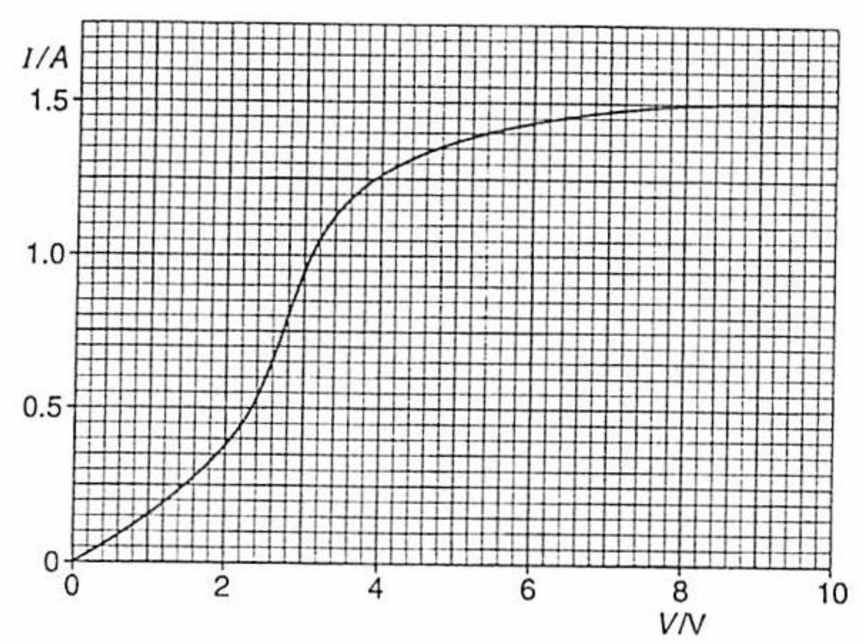


Fig. 23.1

(i) Calculate the resistance of component C when a p.d. of 6.0 V is applied across it. [1]

(ii) Deduce the minimum value of the resistance of component C over the range 0 – 10 V. [2]

(c) Component C is then connected into a circuit with a supply of internal resistance 0.80 Ω and a resistor of constant resistance 5.0 Ω, as shown in Fig. 23.2.

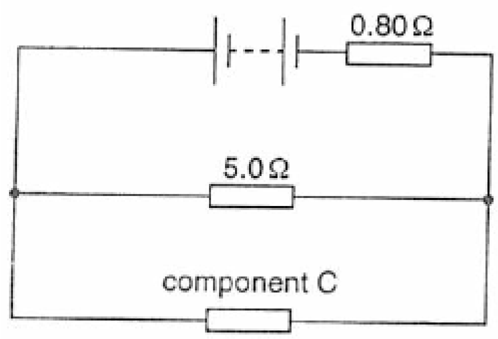


Fig. 23.2

The current through the 5.0 Ω resistor is found to be 0.85 A.
Calculate

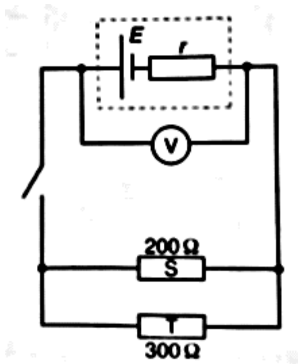
(i) The p.d. across component C, [2]

(ii) The total current from the supply, [2]

(iii) The e.m.f. of the supply, [2]

(iv) The energy supplied to component C in 20 minutes. [3]

(L2) **22** A power supply of e.m.f. E and internal resistance r is connected to two resistors S and T and a voltmeter as show
The voltmeter has infinite resistance.
Resistors S and T have resistances of 200 Ω and 300 Ω, respectively.



When the switch is open the voltmeter reading is 12.0 V.
When the switch is closed the voltmeter reading is 10.8 V.

(i) Determine the values of E and r. [5]

(ii) When the switch is closed, calculate the energy dissipated in the power supply in a time of 5.0 minutes. [2]

(iii) Resistor T is replaced by an NTC thermistor. Initially the resistance of the thermistor is 300 Ω. The switch is closed. The temperature of the thermistor is then increased.
State and explain the change, if any, to the voltmeter reading when the switch is closed. [3]

Numerical Answers

1 2 C

2 4Qf

3(a) 9.44×10^{-4} A

8(a) 2.0 A, 6.0 Ω **(b)** 6.0 V, 12 W

10 (a)(i) 48 C **(a)(ii)** 0.8 A **(a)(iii)** 11.3 Ω

11 0.70 Ω

12(a) 0.403 Ω **(b)** 0.419 Ω

13(a)(i) 6.0 A **(a)(ii)** 8.6×10^{4} J

14(a) 6.5×10^{5} J **(b)** 0.25 A **(c)** 2700 C

15(a) 0.183 A **(b)** 11.9 V

17 E = 5.17 V, r = 1.76 Ω

18(a) r = 1.0 Ω, E = 6.0 V **(b)** $\frac{ER}{r+R}$

19 E = 9 V, r = 60 Ω

20(a)(ii) E = 9.00 V **(b)(ii)** r = 2.0 Ω **(c)(i)** R = 2.0 Ω **(c)(ii)** 50%

(c)(iii) efficiency increases

21(a)(i) 4.2 Ω **(ii)** 3.0 Ω **(b)(i)** 4.25 V **(ii)** 2.15 A

(b)(iii) 5.97 V **(iv)** 6.6×10^{3} J

22(i) 12 V, 13.3 Ω **(ii)** 32.4 J

ADDITIONAL QUESTIONS

SAJC 2021 Prelims/2/Q5 & 2013/3/Q3(c)

1 **(a)** A battery of electromotive force (e.m.f.) 12.0 V and internal resistance *r* is connected to a filament lamp and a resistor, as shown in Fig. 1.1.

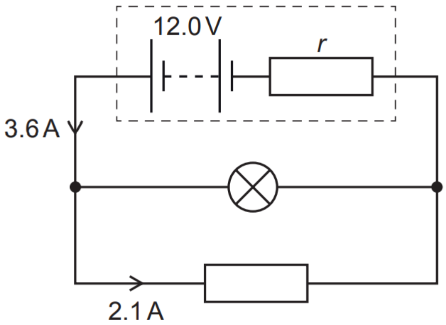


Fig. 1.1

The current in the battery is 3.6 A and the current in the resistor is 2.1 A. The I-V characteristic for the lamp is shown in Fig. 1.2.

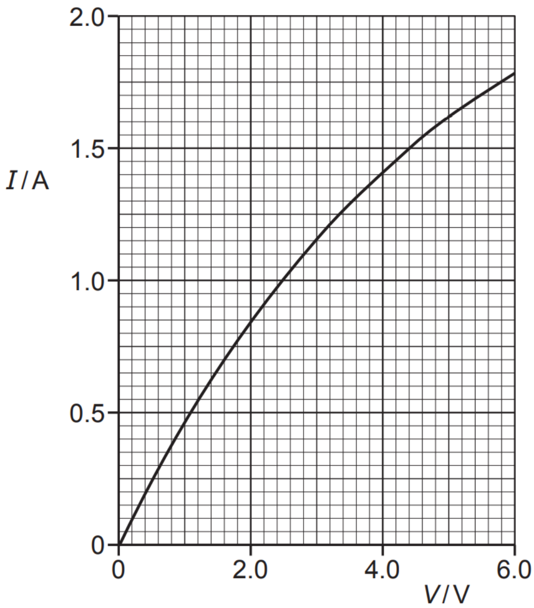


Fig. 1.2

(c) Explain, with reference to the graph, whether the resistance of filament lamp increases or decreases with increasing potential difference.

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…… [2]

(ii) Determine the internal resistance r of the cell in Fig. 1.1.

internal resistance = ……………………….. Ω [3]

(d) The filament wire of the lamp is connected in series with the adjacent copper connecting wire of the circuit, as illustrated in Fig. 1.3.

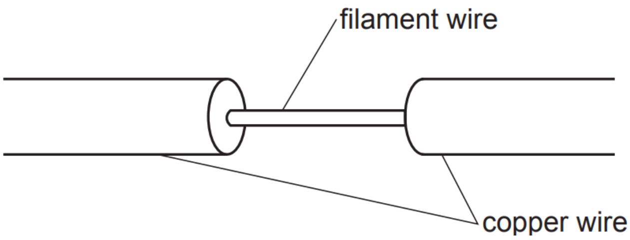

Fig. 1.3

Some data for the filament wire and the adjacent copper connecting wire are given in the table below.

	filament wire	copper wire
cross-sectional area	A	$360A$
number density of free electrons	n	$2.5n$

Calculate the ratio

$$\frac{\text{average drift speed of free electrons in filament wire}}{\text{average drift speed of free electrons in copper wire}}.$$

ratio = ……………………….. [1]

(b) Two identical filament lamps are connected first in series, and then in parallel, to a 12 V power supply that has negligible internal resistance.

The circuits are shown in Fig. 1.4 & Fig. 1.5 respectively.

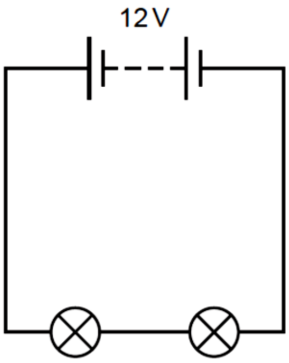

Fig. 1.4

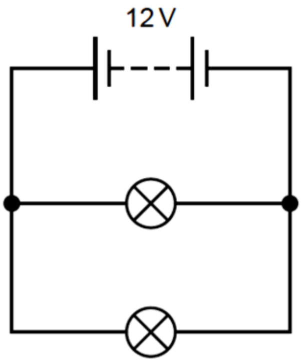

Fig. 1.5

Explain why, after some time, the resistance of each lamp when they are connected in series is different from the resistance of each lamp when they are connected in parallel.

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……... [3]

(c) A circuit contains a battery, a lamp and a switch, as shown in Fig. 1.6.

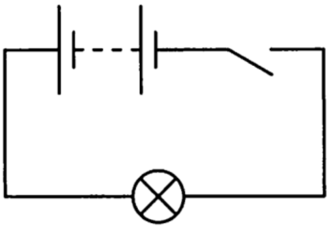

Fig. 1.6

Suggest why, when the switch is closed, the lamp is lit almost immediately and yet the free electrons in the wire are moving at a drift velocity of less than 1 mm s^{-1}.

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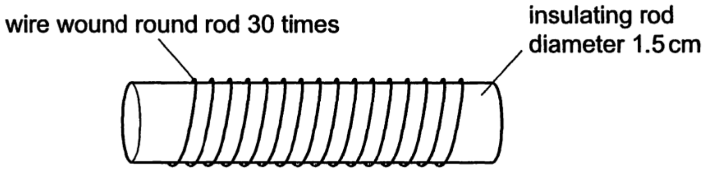
……..

…… [2]

2008/1/Q26

2 The material of a wire has resistivity $1.3 \times 10^{-8}\ \Omega$ m. The wire has diameter 0.50 mm and its length is just enough to enable it to be wound tightly round an insulating rod 30 times. The rod has diameter 1.5 cm.

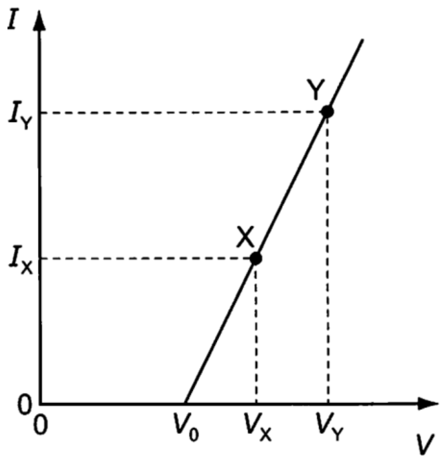


Calculate the resistance of the wire.

resistance of wire = …………………….. Ω

2009/1/Q27

3 The graph shows the variation with potential difference V of the current I in an electrical component.



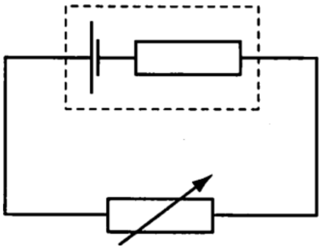
The resistance is measured for current I_Y and for current I_X.

Determine the change in resistance of the component from Y to X in terms of V_X, V_Y, I_X and I_Y.

change in resistance = …………………….. Ω

2010/1/Q26 modified

4 A variable resistor is connected across the terminals of a cell as shown.



The cell has constant internal resistance *r*.
The resistance of the variable resistor *R* is initially higher than *r* but is gradually reduced.

What happens to the terminal potential difference, the power wasted in the internal resistance *r* and the power delivered to the load *R*?

	terminal potential difference	power wasted in internal resistance *r*	power delivered to the load *R*
A	decreases	increases	increases
B	decreases	decreases	decreases
C	increases	increases	increases
D	increases	decreases	decreases

2022 RJC Prelims P2/Q6 modified

5 Fig 6.1 shows a miniature E10 filament light bulb with a rating of 6.0 V, 3.0 W.

Fig. 6.1

(a) **(i)** Calculate the resistance of the bulb.

resistance = ………………………… Ω [1]

(ii) The filament of the bulb is made of tungsten wire of length 2.0 cm and diameter 78 μm.

Calculate the resistivity of the tungsten filament.

resistivity = ………………………… Ω m [2]

(iii) The resistivity of tungsten from a table of constants is stated to be $5.6 \times 10^{-8}\ \Omega$ m, which was measured at room temperature.

Explain, in microscopic terms, the difference between this value and your answer in **(a)(ii)**.

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…………………………………………………………………………………………………… [2]

(b) Six identical E10 light bulbs are connected to a 6.0 V d.c. supply of negligible internal resistance in the arrangement shown in Fig. 6.2.

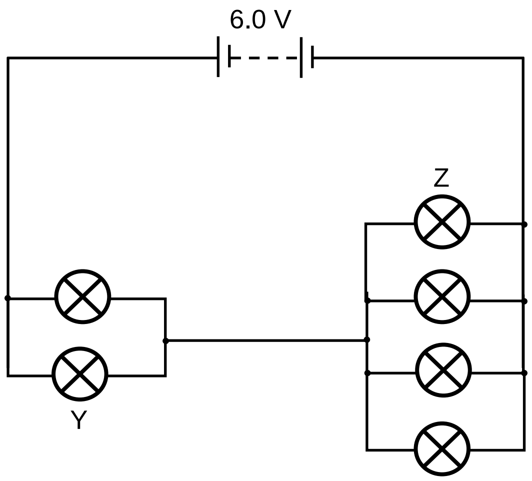

Fig. 6.2

Assume that the resistance of each bulb is as calculated in **(a)(i)**.

(i) Determine the amount of charge that passes through bulb Y in 2 minutes.

charge = ………………………… C [2]

(ii) Explain how the mean drift velocity of the electrons in the filament of bulb Y compare with that of the electrons in the filament of bulb Z.

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……………………………………………………………………………………………………..

……………………………………………………………………………………………………..

…………………………………………………………………………………………………… [2]

CIE 9702/S05/P2/Q7 modified

6 In the circuit of Fig. 6.1, the battery has an e.m.f. of 3.00 V and an internal resistance *r*. *R* is a variable resistor. The resistance of the ammeter is negligible and the voltmeter has an infinite resistance.

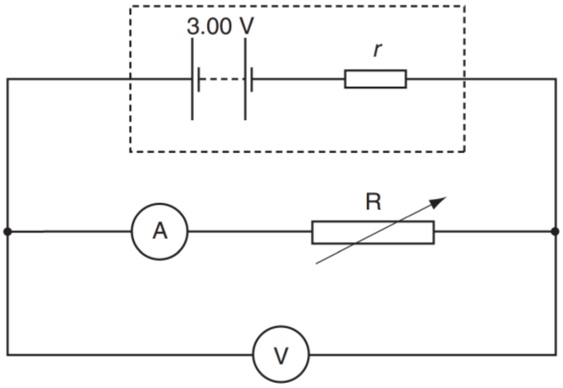

Fig. 6.1

The resistance of *R* is varied. Fig. 6.2 shows the variation of the power *P* dissipated in *R* with the potential difference *V* across *R*.

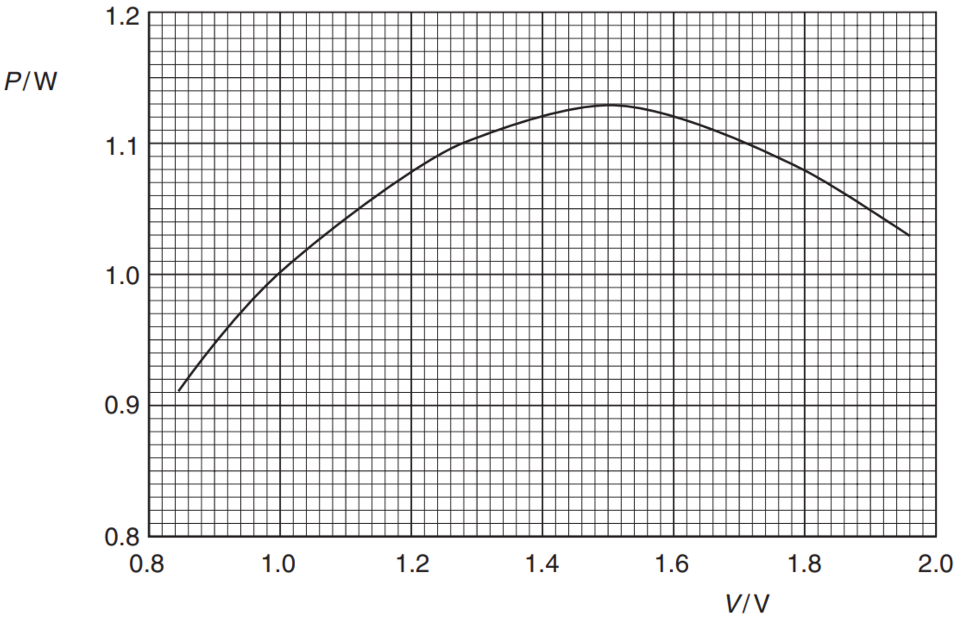

Fig. 6.2

(i) Determine the internal resistance *r* of the battery.

r = ………………………………. Ω [2]

(ii) By reference to Fig. 6.2, it can be seen that there are two values of potential difference *V* for which the power dissipation is 1.05 W.

State, with a reason, which value of *V* will result in less power being dissipated in the internal resistance.

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…………………………………………………………………………………………………….. [3]

CIE 9702/M16/P22/Q5

7 A power supply of electromotive force (e.m.f.) 8.7 V and negligible internal resistance is connected by two identical wires to three filament lamps, as shown in Fig. 7.1.

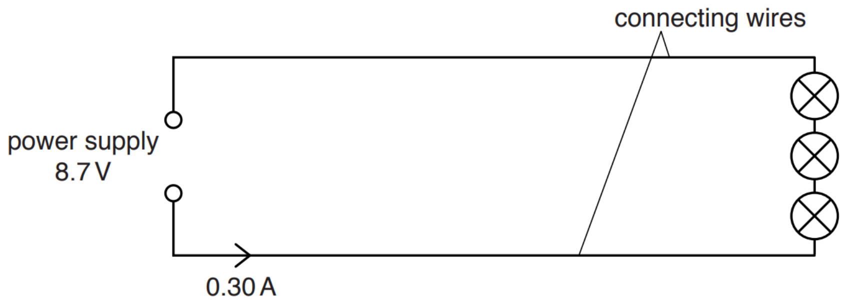


Fig. 7.1

The power supply provides a current of 0.30 A to the circuit. The filament lamps are identical. The I–V characteristic for one of the lamps is shown in Fig. 7.2.

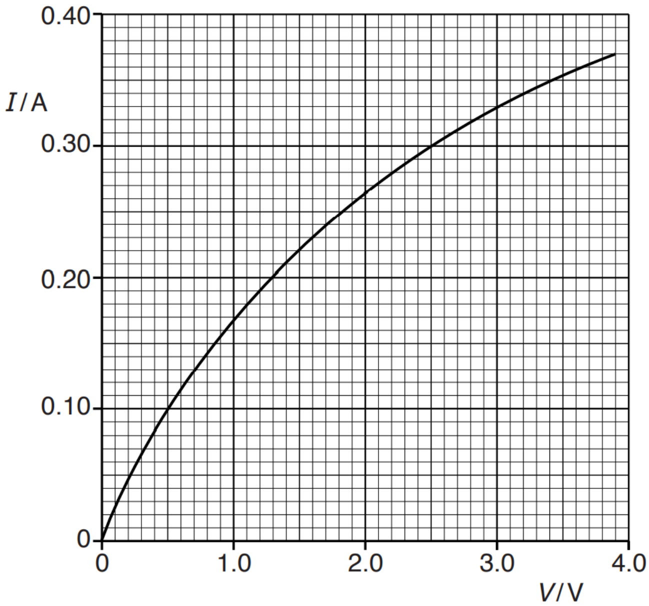


Fig. 7.2

(i) Show that the resistance of each connecting wire is 2.0 Ω.

[2]

(ii) The resistivity of the metal of the connecting wires does not vary with temperature.
On Fig. 7.2, sketch the I–V characteristic for one of the connecting wires. [2]

(iii) Calculate the power loss in one of the connecting wires.

power = ………………………….. W [2]

(iv) Some data for the connecting wires are given below.

cross-sectional area = 0.40 mm^2
resistivity = 1.7×10^{-8} Ω m
number density of free electrons = 8.5×10^{28} m^{-3}

Calculate

1. the length of one of the connecting wires,

length = ………………………….. m [2]

2. the drift speed of a free electron in the connecting wires.

drift speed = ………………………… m s^{-1} [2]

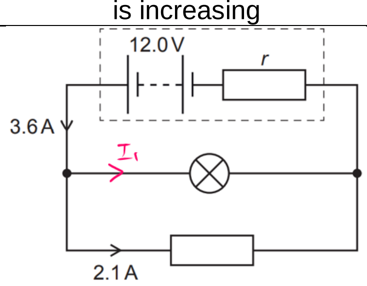
TUTORIAL 13: CURRENT OF ELECTRICITY SOLUTIONS

Level 1 Solutions

1	$Q = I\,t$ $= 0.1 \times 20$ $= 2$ C	[1]
5	**Ans: C** Potential difference = $\dfrac{\text{Energy transferred}}{\text{Charge}}$ Dividing the equation by time Potential difference ($\dfrac{\text{Charge}}{\text{time}}$) = Power Hence Power = VI	[1]
8(a)	$P = VI$ $24 = 12 \times I$ $I = 2.0$ A	[1]
	$V = R\,I$ $R = \dfrac{12}{2} = 6.0\ \Omega$	[1]
(b)	p.d across the variable resistor = 18 – 12 = 6.0 V	[1]
	Power dissipation = I V = 6.0 × 2.0 = 12 W	[1]
11	Resistance, R $= \dfrac{\rho L}{A}$ $= \dfrac{(1.1\times10^{\ 6})(500\times10^{\ 3})}{\pi\left(\dfrac{1.0\times10^{\ 3}}{2}\right)^2}$ $= 0.70\ \Omega$	[1]
13(a)(i)	$I = P / V$ $= 72 / 12$ $= 6.0$ A	[1]
(ii)	$W = P\,t$ $= 72 \times (20 \times 60)$ $= 8.6 \times 10^4$ J	[1]
(b)	The lamp is operating at 12 V constantly.	[1]
14(a)	Energy = $P\,t$ $= 60 \times (3 \times 60 \times 60)$ $= 6.5 \times 10^5$ J	[1]
(b)	At rated power, $P = V\,I$ $I = 60 / 240$ $= 0.25$ A	[1]
(c)	$Q = I\,t$ $= 0.25 \times (3 \times 60 \times 60)$ $= 2700$ C	[1]
15(a)	$E = I\,r + I\,R$ $I = \dfrac{E}{r + R}$	[1]

	$= \dfrac{12.0}{(65.0+0.5)}$ = 0.183 A	
(b)	$E = I\,r + I\,R$, where $I\,R$ is the terminal p.d. of battery. $V = I\,R = 0.183 \times 65$ = 11.9 V	[1]

Solutions to Additional Questions

1(a)(i)	Since resistance is the ratio of potential difference to current, and from the graph, From graph, this ratio is increasing and thus resistance increases. Accept: Resistance at any point is the reciprocal of the gradient of the line joining the point to the origin, and since reciprocal of this gradient is increasing, resistance is increasing	[M1] [A1]
(ii)	 Current across filament lamp, $I_1 = 3.6 - 2.1 = 1.5$ A Reading off Fig. 4.2, p.d across filament lamp V = 4.4 V p.d across internal r = 12.0 – 4.4 = 7.6 V Internal resistance, r = 7.6 / 3.6 = 2.11 Ω	[1] [1] [1]
(iii)	I = nAvq and current in series connection is constant $(nAvq)_{filament} = (nAvq)_{copper}$ $nAv_{filament}e = (2.5n)(360A)(v_{copper})(e)$ ratio = (2.5)(360) = 900	[1]
(b)	Potential difference across each lamp in series is 6 V, while potential difference across each lamp in parallel is 12 V. Since power = V^2 / R, each lamp in parallel receives more power than each lamp in series. Hence, temperature increases more for each lamp in parallel. Since resistance of filament lamp increases with temperature, resistance of each lamp in parallel will be greater.	[1] [1] [1]
(c)	When the switch is closed, the emf of the battery will set up an electric field almost immediately across the circuit. The electric field will cause all the free electrons to start moving. Free electrons are distributed throughout the wire, including those near and at the filament lamp. Hence, those electrons near and at the filament can power the lamp immediately even though the drift velocity is less than 1 mm s^{-1}.	[1] [1]

2	Entire length of the wire = 30 turns × circumference of each turns = 1.4137 m Cross-sectional area = πr^2 = $\pi(0.25 \times 10^{-3})^2$ = 1.9634×10^{-7} m^2 $R = \rho L / A = (1.3 \times 10^{-8})(1.4137) / (1.9634 \times 10^{-7})$ $= 9.4 \times 10^{-2}\ \Omega$	
3	Resistance at X = V_X / I_X, Resistance at Y = V_Y / I_Y Change in resistance = final – initial = $V_Y / I_Y – V_X / I_X$ Note: Resistance is NOT gradient.	
4	**Ans: A** When resistance of *R* decreases, current in the circuit increases. Hence, potential drop across *r* increases since $V_r = Ir$ Hence, terminal PD decreases (V_R = EMF – Ir) and power wasted in *r* increases ($P_r = I^2r$) Using maximum power theorem, P delivered to the load increases as *R* gets <u>closer</u> to *r*. Hence, power delievered to the load *R* will also increase.	
5(a)(i)	$P = \frac{V^2}{R}$ $R = \frac{V^2}{P} = \frac{(6.0)^2}{3.0} = 12\ \Omega$	[1]
(ii)	$R = \frac{\rho L}{A}$ $\rho = \frac{RA}{L}$ $= \frac{(12)\left(\pi \times \left(\frac{78 \times 10^{-6}}{2}\right)^2\right)}{0.020}$ $= 2.8670 \times 10^{-6} = 2.87 \times 10^{-6}\ \Omega$ m	[1] [1]
(iii)	The <u>table of constants states the resistivity of tungsten at room temperature</u> whereas the resistivity value in (a)(i) is the resistivity at a much <u>higher temperature when the light bulb is in use</u>. When <u>temperature increases, the lattice ions</u> in tungsten gain thermal energy and <u>vibrate with larger amplitudes</u>. This <u>increases the collisions</u> between the free electrons and the lattice ions which hinder the movement of the electrons. Hence, <u>resistivity increases with increasing temperature</u>.	[1] [1]
(b)(i)	p.d. across Y, $V_Y = \left(\frac{\frac{R}{2}}{\frac{R}{2} + \frac{R}{4}}\right)(6.0) = 4.0$ V current through Y, $I_Y = \frac{V_Y}{R} = \frac{4.0}{12}$ A $Q = I_Y t$ $= \left(\frac{4.0}{12}\right)(2 \times 60)$ $= 40$ C	

(ii)	Consider $I = Anvq = Anev$. Since both filaments are identical, *Ane* is constant. Hence the current I through the filament is directly proportional to the mean drift velocity *v* of the electrons in the filament i.e. $I \propto v$. The current through Y is twice the current through Z as the potential difference across Y is twice the potential difference across Z (OR the total current flowing through 2 bulbs and 4 bulbs in parallel are equal). Therefore, the mean drift velocity of the electrons in Y is twice that of the electrons in Z.	
6(i)	(Using maximum power theorem, r = R when P across R is maximum.) Hence, at max power, $P = V^2/R$ $1.13 = (1.50^2)/R$ $R = 1.99\ \Omega$ Hence, $r = 1.99\ \Omega$	 [1] [1]
(ii)	Larger p.d. across *R* means smaller p.d. across *r*, since e.m.f. is a constant. Smaller power dissipation across *r* at larger values of *V* since power = current × potential difference, and current is the same for *R* and *r*	[1] [1] [1]
7(i)	Since current = 0.30 A, p.d. across each lamp = 2.5 V (read off from graph) Hence, p.d. across each connecting wire = (8.7 – 7.5) / 2 = 0.60 V Therefore, resistance of each connecting wire = 0.60 / 0.30 = 2.0 Ω	[1] [1]
(ii)	straight line through origin with gradient of 0.5	[1] [1]
(iii)	power loss $= I^2R$ $= (0.30^2)(2.0)$ $= 0.18$ W	[1] [1]
(iv) 1.	$R = \rho L/A$ $2.0 = (1.7 \times 10^{-8})(L) / (0.40 \times 10^{-6})$ $L = 47$ m	[1] [1]
2.	$I = nAvq$ $v = 0.30 / (8.5 \times 10^{28} \times 0.40 \times 10^{-6} \times 1.6 \times 10^{-19})$ $= 5.5 \times 10^{-5}$ m s^{-1}	 [1] [1]