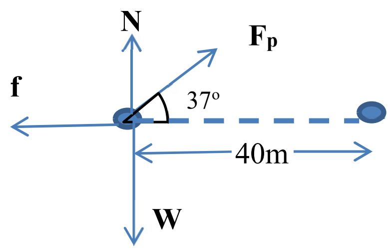
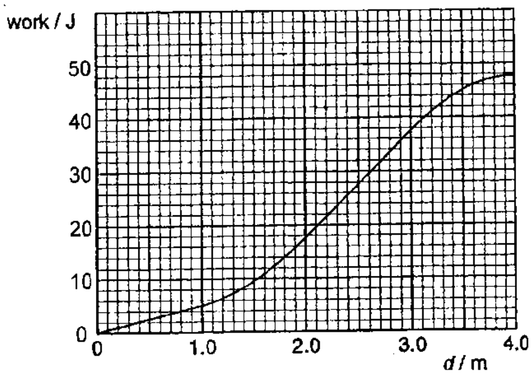
TUTORIAL 5: WORK ENERGY & POWER SOLUTIONS

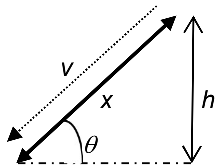
<u>Level 1 Solutions</u>

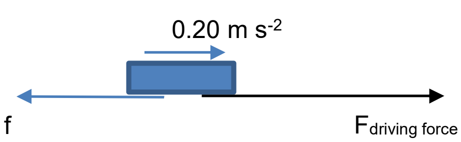
1.	C Work done on weight = gravitational force W × height q {No work is done to increase the KE of the weight, only the GPE.}	[1]
2(a)	Work done by F_p = 100 cos 37° x 40 = <u>3190 J</u> Work done by the friction force f = 50 x (-40) = <u>- 2000 J</u> Work done by weight W = 0 J Work done by normal contact force N= 0 J  	[1] [1] [1] [1]
(b)(i)	Net work done = ∑ Work done by each force = 3190 + (– 2000) +0 +0 = <u>1190 J</u>.	 [1] [1]
(b)(ii)	KE gained by the crate	[1]
3.	By Work Energy Theorem, Net Work done by the 10 N force over 2.5 m = change in KE W = 10 x 2.5 J = <u>25 J</u> *Alternatively*, solve using kinematics method, To determine the final K.E, we need to calculate the final velocity. Hence apply 3 equations of motion to determine final velocity. Given u = 4.0 m s^{-1}, a = 10/5 = 2 m s^{-2} , s = 2.5 m $v^2 = u^2 + 2as$ $v^2 = 4.0^2 + 2(2)(2.5)$ $= 26$ Increase in K.E = Final K.E - Initial K.E = ½ (5)(26) – ½ (5)(4)² = 65 – 40 = 25 J	 [1] [1]
4.	Work done by gas = P_{ext} x ΔV { NOT: p_{gas} x ΔV } = 1.0 x 10^5 x (4.0) = <u>4.0 x 10^5 J</u>	[1] [1]
7.	90% of stored EPE is converted to KE 0.9 (95) = KE_f - KE_i Since KE_i=0 $0.9 \times 95 = \frac{1}{2}\ 0.170\ v^2$ v = 31.7 m s^{-1} ≈ 32 m s^{-1} (shown)	

8.	(a) Gain in KE = Loss of GPE $\frac{1}{2}mv^2 - 0 = mgh$ $v = \sqrt{(2gh)} = \sqrt{(2 \times 9.81 \times 3.2)}$ = <u>7.9 m s^{-1}</u> **(b)** $\Delta GPE = GPE_f - GPE_i$ = 0 – 5.1(28)(9.81) = -1400 J Loss of GPE =1400 J **(c)** Work done by frictional force between B & C = change in GPE between B & C Therefore change of KE=0, so speed is unchanged.	[1] [1] [1] [1] [1] [1]
	<u>**Power & Efficiency**</u>	
14.	Ans: C Power is defined as the rate at which work is done. Although power is also power x velocity, it only applies when force is constant. Hence Option C's answer is more appropriate.	[1]
15.	Ans: C Accelerated from rest to a speed v → Gain in K.E. Hence we apply Power = $\frac{\text{Work done}}{\text{time}} = \frac{\text{Gain in kinetic energy}}{\text{time}}$ $P = \frac{1}{2} mv^2 / t$	[1]
16.	Ans: D Generates 1000 MW of electrical power → useful power output Since the efficiency is 40%, power input = $\frac{100}{40} \times 1000\text{MW}$ = <u>2500 MW</u> Wasted power = input power – useful output power = 2500 MW – 1000MW = <u>1500 MW</u>.	[1]

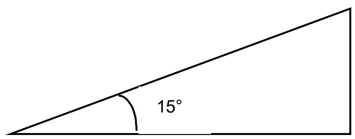
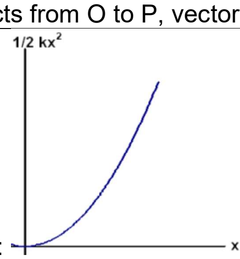
Level 2 Solutions

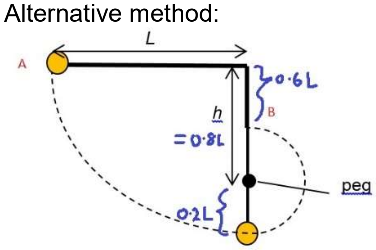
	Work	
5.	EPE = area of trapezium between x_0 to x_1 = ½ $(W_1x_1-W_ox_o)$	[1]
6.	 Straight line between 0-1.0 and 2.0-3.0. Note: Free-hand sketches of straight lines should be discouraged. Units are given on the axes thus the values must be calculated correctly and indicated on the axes.	[1] for each section
	Principle of conservation of energy	
9.	Ans: A Solve by applying using proportional relationship Gain in GPE = Loss in K.E (no work done against friction as the incline is frictionless) $mgh = \frac{1}{2} mv^2$ $2gh = v^2$ Thus h is proportional to v^2 (as 2g = constant) Therefore, $\frac{h_2}{h_1} = (\frac{v_2}{v_1})^2$ $\frac{h_2}{h} = (\frac{\frac{1}{2}v}{v})^2$ $\Rightarrow h_2 = \frac{1}{4} h$	[1]
10.	{Concept: Connected bodies have the same acceleration & speed} Loss in GPE_y = Gain in GPE_x + Gain in KE_x + Gain in KE_y (5.0)(9.81)(2.0) = (4.0)(9.81)(2.0 sin 30°) + Gain in KE_x + Gain in KE_y Gain in KE_x + Gain in KE_y = 58.86 ≈ <u>59 J</u> OR: $(TE_{X+Y})_{initial} = (TE_{X+Y})_{final}$ $0 = KE_{x+Y} + GPE_X + GPE_Y$ Assume initial position is where GPE = 0 for both X and Y GPE_x is +ve as the vertical height increases (more GPE) GPE_Y is -ve as the vertical height decreases (less GPE) $0 = KE_{x+Y}$ + (4.0)(9.81)(2.0 sin 30°) + (-(5.0)(9.81)(2.0)) KE_{x+Y} = Gain in KE of X + Y as initial KE of X + Y = 0 = 58.86 ≈ <u>59 J</u>	[1] [1]

11.	By the principle of conservation of energy, loss in GPE as space vehicle enters atmosphere = gain in thermal energy There is no change in KE in vehicle, since it is travelling at constant speed. So, Rate of loss in GPE = rate of thermal energy gained = rate of thermal energy dissipated by heat shield {Since the temperature of the vehicle does not change}  $m\,g\,(h/t) = P$ $m\,g\,(x \sin\theta / t) = P$ $m\,g\,(x/t) \sin\theta = P$ $m\,g\,v \sin\theta = P$	[1] [1]
12	Work Done on spring = Change in energy of the ball, $F_{average}\, y$ = GPE lost by ball + KE lost by ball $= mg\,y + \frac{1}{2}mv^2$ ..... $F_{av} = \frac{m}{2y}(v^2 + 2gy)$ OR: Gain in EPE of Spring = GPE lost by ball + KE lost by ball *during compression* $\frac{1}{2} k\, y^2 = mg\,y + \frac{1}{2}mv^2$ $\frac{1}{2} k\, y^2 = mg\,y + \frac{1}{2}mv^2$ $k = \frac{2}{y^2}(mg\,y + \frac{1}{2}mv^2)$ $F_{average} = \frac{1}{2} k\, y$ $= \frac{1}{2}[\frac{2}{y^2}(mg\,y + \frac{1}{2}mv^2)]y$ .... $= \frac{m}{2y}(v^2 + 2gy)$ Note: EPE = $\frac{1}{2}kx^2 = \frac{1}{2}Fx$. The F is not an average force, it is a varying force. In this case, it is the maximum force when the extension is x as the extension increases fom 0 to x. **Wrong method:** Work done by spring = F_{av} x amount of compression $= F_{av}\, y = \frac{1}{2} Fy$ $= \frac{1}{2} ky^2$ where $F_{ave} = \frac{1}{2} F$, and $F = ky$ (at max compression) assuming Hooke's law is obeyed. This method does not meet the requirement of the quest. So can't proceed with this method since *k* can't be expressed in terms of *m, v, y*.	[1] [1] [1] [1]

13(i)	From the graph, when W = 4 N, L_{eqm} = 9.2 cm { L_{eqm} = equilibrium length} Hence, total length = L_{eqm} + 0.80 cm = <u>10.0 cm</u>	[1]
(ii)	1. Change in GPE = mgh = 4.0 x 0.80 x 10^{-2} = <u>3.2×10^{-2} J</u> 2. Change in EPE = Area of trapezium formed by (0, 9.2), (0, 10.0), (4.0, 9.2) and (5.0, 10.0) = ½ (4.0 + 5.0) × (0.80 × 10^{-2}) = <u>3.6×10^{-2} J</u>	[2] [3]
(iii)	Work done to cause additional extension = Gain in EPE - Loss in GPE = 3.6×10^{-2} - 3.2×10^{-2} = <u>4.0×10^{-3} J</u> Net change in PE of system (this concept will be dealt with in greater detail in Topic 9: Oscillations)	[1]
	<u>Power & Efficiency</u>	
17	Output power of motor, $P = F v$. Linear speed at which wheel is turning, v = Circumference × number of revolutions per second = 0.5 × 20 = 10 m s^{-1} Force exerted by motor on wheel (via friction), F = force that resists the wheel = 50 – 20 = 30 N Thus output power of motor P = F × v = 30 N × 10 m s^{-1} = <u>300 W</u>	[1] [1]
18.	Ans: D  By N2, $F_{driving\ force} - f = m a$ $F_{driving\ force} - 160 = (1.2 \times 10^3)(0.20)$ $F_{driving\ force}$ = 400 N Output power of car, P = $F_{driving\ force}$ v = 400 × 10 = 4000 W	[1] [1]
19	Ans: C <u>By conservation of energy</u> Work done by motor (in 1 s) = Gain in KE of a column of water $W_{motor} = \frac{1}{2} m v^2$ $= \frac{1}{2}(density)(volume) v^2$ $= \frac{1}{2}(density)(area \times length) v^2$ $= \frac{1}{2}(1000)(\pi r^2 \times v) v^2$ $= \frac{1}{2}(1000)(\pi (0.10)^2 \times 10) 10^2$ = 1.57×10^4 W $P_{average}$ = W_{motor} /Time = 1.57×10^4 /1 = 15.7 <u>kW</u>	[1] [1]

20	Power output of motor = rate of GPE gain Efficiency = $\dfrac{\text{rate of GPE gain}}{\text{Power Supplied}}$ = $\dfrac{mgh/t}{\text{Power Supplied}}$ Power supplied = mg (h/t) ÷ efficiency = (2)(0.8 / 4) ÷ 0.20 = 2 W	 [1] [1]
21(a)	$P = F v$, where F = force that Bobby exerts in pedalling forward. {"Constant speed" implies that this force <u>F is in equilibrium with the total resistive force</u>, ie F is equal and opposite to the total resistive force.} Thus total resistive force, $F = \dfrac{P}{v} = \dfrac{220}{10}$ = <u>22 N</u>	 [1]
(b)	By Principle of Conservation of Energy, KE at A + GPE at A = KE at B $\frac{1}{2} m v_A^2 + m g h = \frac{1}{2} m v_B^2$ $(\frac{1}{2} \times 80 \times 10^2) + (80 \times 9.81 \times 4.0) = \frac{1}{2} \times 80 \times v_B^2$ v_B = <u>13.4 m s^{-1}</u>	 [1] [1]
(c)	At a higher speed, the total resistive force would be greater {recall Topic 4, air resistance depends on speed}; hence, Bobby needs to exert a <u>greater</u> force F to maintain a higher constant speed. Since power $P = Fv$, more power is required since <u>both</u> the force F and the speed v are increased.	[1] [1]
(d)	Since the cyclist is travelling at constant speed, his <u>kinetic energy is constant</u>, and is therefore not transforming any form of energy into kinetic energy. However, in order to maintain constant speed, the cyclist has to overcome various resistive forces. The energy required is transformed from the chemical energy stored in the cyclist's muscles. The chemical energy that he is using is being transformed into internal energy (or thermal energy) in, for example, the tyres, the road, the gearing systems and the surrounding air and into sound. *{It is insufficient to say the chemical energy is transformed into work done against resistive forces. This was a 5-mark question.}* Tutor's Comments: Incorrect to state that no chemical change is involved since no fuel is required, or that no work is being done because the resultant force on the cycle is zero.	[1] [1] [1] [1] [1]
22(i)	Gain in GPE per second = mgh ÷ t, where h = x sinθ = mg (x sinθ) ÷ t = (mg) v sinθ = (7000) 8 sin 15^0 = 14493.9 J s^{-1} = <u>1.45 x 10^4 W</u>	 [1] [1]

(ii)	Work done per second against friction = friction × displacement upslope per second = friction × velocity up the slope = 5 000 × 8 = 4.0 × 10^4 W {recall $P = F\,v$}	 [1] [1]
(iii)	Energy supplied by engine is converted into GPE gain + Work done against friction {no KE gain as speed is constant} Power supplied by engine = rate of GPE gain per second + Work done against friction per second = (i) + (ii) = 5.45 x 10^4 W Alternatively, engine power = applied force × velocity 	 [1] [1]
23	Ans:D V = kx^2, $F = -\frac{dV}{dx} = -2kx$. The negative sign indicates that the force vector acts in a direction opposite to that of the x vector. Since vector x acts from O to P, vector F must be in the PO direction.	 [1]
24	Quadratic shape:  F = -gradient of EPE-x graph	
25(a)	Loss in GPE = Gain in K.E $m\,g\,h = \frac{1}{2}\,m\,v_{bottom}^2$ $g\,L = \frac{1}{2}\,v^2$ $\Rightarrow v_{bottom} = \sqrt{2gL}$	 [1] [1]

(b)	Speed of the ball at the lowest point of its path, $v_{bottom} = \sqrt{2gL}$ Thus initial KE at lowest point = ½ $m\,v_{bottom}^2 = mgL$ As the peg moves up to the top of the circular path, loss in K.E = Gain in GPE $= m\,g\,(L - 0.60\,L)$ $= m\,g\,(0.4\,L)$ Final K.E = initial KE - loss in KE $= mg\,L - m\,g\,(0.4\,L)$ $\frac{1}{2}\,m\,v_B^2 = mg\,(0.6\,L)$ $\Rightarrow v_B = \sqrt{1.2gL}$ Alternative method:  Consider points A and B: Loss in GPE = Gain in KE mg (0.6L) = ½ m $v_B^{\ 2}$ $v_B = \sqrt{1.2gL}$	[1] [1]