TUTORIAL 14: DIRECT CURRENT CIRCUITS

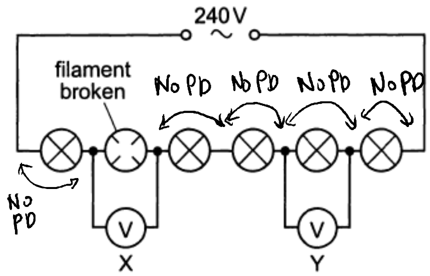
Level 1 Solutions

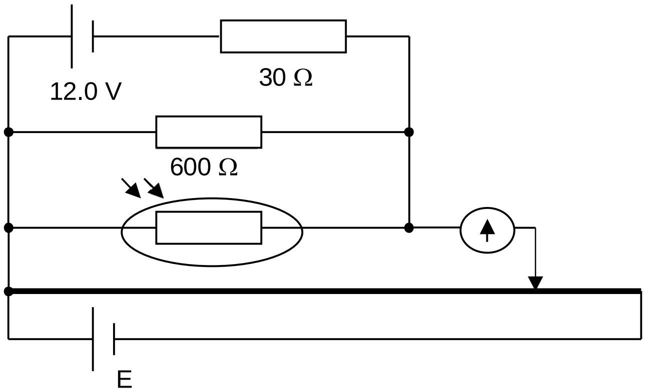
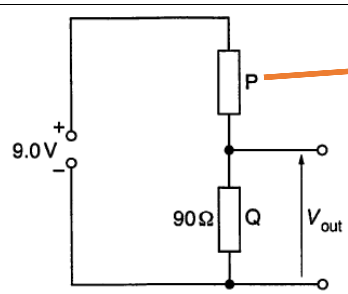
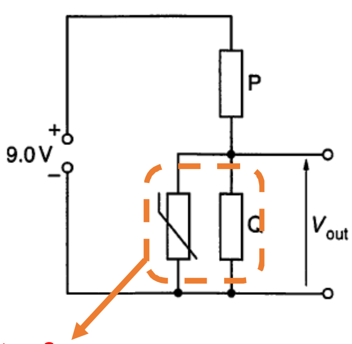
1	Resistance for X = $(\frac{1}{2}+\frac{1}{4+3})^{-1}$ = 1.56 Ω Resistance for Y = $2+(\frac{1}{4}+\frac{1}{3})^{-1}$ = 3.71 Ω or $\frac{4x3}{4+3}$ +2 Resistance for Z = $(\frac{1}{3}+\frac{1}{2}+\frac{1}{4})^{-1}$ = 0.923 Ω Therefore, in order of increasing resistance is **<u>Z X Y</u>**.	
4	**Method 1:** Total R = 3 + $(\frac{1}{6}+\frac{1}{6})^{-1}$ = 6.0 Ω I = V / R = 12 / 6.0 = 2.0 A Hence the ammeter reading is 1 A. **Method 2:** 12 – I (3.0) = 0.5 (I) (6.0) → I = 2 A Hence the ammeter reading is 1 A. Note: The total current supplied by the battery will be split equally at the junction.	
7	**Ans: B** Let current flowing through R_1, R_2 and R_3 be *i*. $(R_1 + R_2 + R_3)\ i.$ = 5 V (1) $(R_2 + R_3)\ i.$ = 3 V (2) $(R_3)\ i.$ = 2 V (3) From (2) / (3), $\frac{R_2+R_3}{R_3}=\frac{3}{2}$ → $\frac{R_2}{R_3}=\frac{1}{2}$ Subst (2) into (1), $(R_1)\ i.$ = 2 V → $R_1 = R_3$	
8	Total R = R + $\left(\frac{1}{R}+\frac{1}{R}\right)^{-1}$ = R + 0.5 R = 1.5 R V = $\frac{0.5R}{1.5R}\times 6$ = 2 V Hence the voltmeter reading is 2 V.	[1] [1]
9	**Ans: D** For Option A: VJ = $\frac{2\,k\Omega}{2\,k\Omega+4\,k\Omega}$ x 12 V = 4 V For Option B: Diode is in reverse direction, therefore no current. For Option C: Diode is in reverse direction, therefore no current. For Option D: VJ = $\frac{4\,k\Omega}{2\,k\Omega+4\,k\Omega}$ x 12 V = 8 V For Option E: Diode is in reverse direction, therefore no current.	

14	$E_A = \frac{l_A}{l_S} E_S = \frac{66}{90}(15) = 11\text{ V}$	[2]

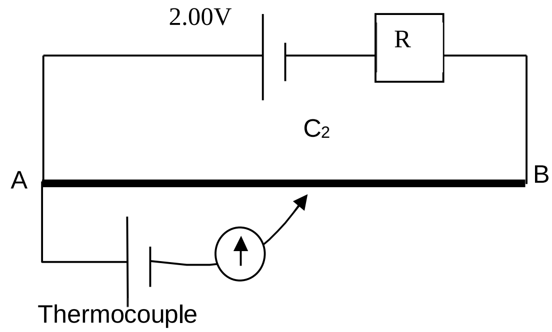
Level 2 Solutions

2	When variable resistor is 0 kΩ, Rc = 1.0 kΩ. Voltmeter reading = 12 V (maximum) When variable resistor is 1.0 kΩ, Voltmeter reading = $\frac{1}{1+(1+1)^{-1}} \times 12$ = 8 V (minimum)	[1] [1] [1]
3(a)	It is equivalent to the three branches in parallel (12 Ω, 6 Ω, 12 Ω), total resistance = $\left(\frac{1}{12}+\frac{1}{6}+\frac{1}{12}\right)^{-1}$ = 3 Ω	[1]
(b)	Consider resistor between AB to be in series to resistor between BC, resistance = 6 +6 = 12 Ω. This will then be parallel the diagonal 6 Ω resistor, resistance = $\left(\frac{1}{12}+\frac{1}{6}\right)^{-1}$ = 4 Ω. This will then be in series to the 6 Ω resistor between CD, resistance will be 4 + 6 = 10 Ω. Finally, this is in parallel to the 6 Ω resistor between AD, resistance = $\left(\frac{1}{10}+\frac{1}{6}\right)^{-1}$ = 3.75 Ω.	 [1] [1] [1]
5	Total R = 20 + 50 + $\left(\frac{1}{800}+\frac{1}{12000}\right)^{-1}$ = 820 Ω Ammeter reading, I = $\frac{6}{820}$ = 7.32 mA Voltmeter reading = 6 – (0.00732 × 20) - (0.00732 × 50) = 5.49 V	[1] [1] [1]
6	**Ans: D** When one or more filaments are broken, the circuit will be an open circuit and hence current is zero. PD across each of the <u>unbroken</u> filaments = $V = I R$ = (0)(R) = 0 V, where R is the resistance of each filament. Voltmeter Y thus reads <u>0 V</u>. For Voltmeter X, its left end is at the same potential as the left end of the 240 V supply (since there is no PD across the lamp on its left). The right end of Volmeter X is at the same potential as the right end of the 240 V supply (since there is no PD across all the 4 lamps on its right!) Voltmeter X thus measures the PD across the <u>240 V supply</u>.	

		
10	$V_{\text{at Y}} = \dfrac{R}{R+R_{LDR}} \times 6$ When illuminated, $2 = \dfrac{R}{R+1200} \times 6$ R = 600 Ω Note: When in the dark, resistance of LDR (10 MΩ) is much bigger than resistance of R (600 Ω). Therefore potential at Y will be approximately 0.	
11 (a)(i)(1)	<u>Method 1: Potential Divider method</u> Voltage across LDR = $\dfrac{500}{30+500}$ x 12 = 11.32 V Therefore, Current through LDR = $\dfrac{11.32}{3000}$ = 3.77 mA <u>Method 2:</u> Current supplied by battery = $\dfrac{12}{30+500}$ = 0.02264 A Current supplied by battery = $\dfrac{12}{30+500}$ = 0.02264 A Voltage across LDR = 12 – (0.02264 × 30) = 11.32 V Therefore, Current through LDR = $\dfrac{11.32}{3000}$ = 3.77 mA	[1] [1] [1] [1]
(2)	Power dissipated in LDR = 0.00377 × 11.32 = 0.0427 W	[1]
(a)(ii)	When R_{LDR} drops to 100 Ω, Current supplied by battery = $\dfrac{12}{30+\left(\dfrac{1}{600}+\dfrac{1}{100}\right)^{-1}}$ = 0.1037 A Voltage across LDR = 12 – (0.1037 × 30) = 8.889 V Power dissipated in LDR = $\dfrac{8.889^2}{100}$ = 0.790 W > 0.5 W	 [1] [1]

	Hence, LDR will be overheated and damaged.	[1]
(b)	 Note: Ensure that the polarity of E is correct as shown.	[2]
12 (a)(i)	$V_{out} = I\,R_Q = 0.027 \times 90 = 2.43$ V	[1]
(ii)	 Step 1: $R_P = \frac{(9.0-2.43)}{0.027} = 243.33\ \Omega$	[1]
	 Step 2: Combined parallel resistance = $(\frac{1}{120} + \frac{1}{90})^{-1}$ = 51.43 Ω.	[1]
	Step 3: By potential divider rule, $V_{out} = \frac{R_{//}}{R_{//}+R_P} \times 9.0\text{V}$ $= \frac{51.43}{51.43+243.33} \times 9 = 1.57$ V	[1]
(iii)	When temp of thermistor increases, thermistor's resistance decreases. Thus the lower section's parallel combined resistance <u>decreases,</u> resulting in a <u>decrease</u> in the total circuit's resistance.	[1]

	Since the emf remains the same, current delivered by battery increases. Thus the p.d. across resistor P increases, and by (Principle 2) V_{out} = emf – V_P, V_{out} thus decreases.	[1] [1]
13 (b)(i)	The variation of the resistance of the thermistor is much greater and more linear for temperature between 273 K to 293 than for temperature between 313 K to 333 K. Hence, the thermometer is more sensitive in the range 273 K to 293 K.	[1] [1]
(ii)	T = 292 K (approx.)	[1]
(c)(i)	$V_A = \frac{600}{600+400}\times 6 = 3.6$ V For voltmeter to read 0 V, V_B has to be at 3.6 V as well. Hence, $V_B = \frac{1200}{1200+R_T}\times 6$ $3.6 = \frac{1200}{1200+R_T}\times 6$, $R_T = 800\ \Omega$ Therefore, T = 305 K (from the graph)	[1] [1] [1]
(ii)(1)	When voltmeter read 1.2 V, it suggests that the voltage at point B could be either 1.2 V higher or lower than the voltage at A. Higher voltage at B will mean that the resistance of the thermistor is lower and hence the temperature is higher. Lower voltage will mean lower temperature. Hence the thermistor can be at 2 different temperatures.	[1] [1]
(2)	$V_B = \frac{1200}{1200+R_T}\times 6$ $3.6 \pm 1.2 = \frac{1200}{1200+R_T}\times 6$ $4.8 = \frac{1200}{1200+R_T}\times 6$ OR $2.4 = \frac{1200}{1200+R_T}\times 6$ $R_T = 300\ \Omega$ OR $R_T = 1800\ \Omega$ T = 333 K OR T = 289 K Hence, the lower temperature will be 289 K.	[1] [1] [1]

15	 V_{AC} = 4.00 mV, therefore V_{AB} = 8.00 mV V_R = 2.00 – 0.008 = 1.992V $V_R = \frac{R}{R+2.00}$ x 2 = 1.992 R = 498 Ω If R' = 498 + 10 = 508 Ω $V_{AB} = \frac{2}{2+508}$ x 2 = 0.00784 V $V_{AC} = \frac{L_{AC}}{L_{AB}}$ x 7.84 mV = 4.00 mV Therefore L_{AC} = 0.51 cm The slider has to move 1 cm towards B.	[1] [1] [1] [1]
16 (a)	Potentiometer does not draw any current from the unknown emf when the balanced length is measured. whereas for a voltmeter, V = E – I r and I is finite, hence, V will always show a reading lower than the true value.	[1] [1]
(b) i	By potential divider rule to lower /secondary circuit, $V_{XZ} = \frac{100}{100+30} \times 2$ V = 1.54 V	[1] [1]
ii	Using V is proportional to L, $\frac{L}{100cm} \times 4V = V_{XZ} = 1.54$ V (ans in (i)) L = 38.5 cm	[1] [1]
iii	Since there is no current in the secondary circuit when S_2 is closed and S_1 open, V_{XZ} = emf of secondary cell, 2V $\frac{L}{100cm} \times 4V = V_{xz} = 2V$ L = 50.0 cm	[1] [1]
iv	I = 0 at balance point (same as (iii), no current in the secondary circuit when S2 is closed and S1 open)	[1]
v	The fractional uncertainty is reduced when the balance length (XZ) measured is larger.	[1]

	This can be obtained when the potential across XY is lower by having another resistor connected in series with wire XY. Note that there is no internal resistance in driver cell! (as compared to lecture example 18.	[1]
17(a)(i)	Indicate on (1.5, 0.2) [draw a straight line from origin with gradient 1/(7.5) =0.133. The intersection with the graph is the answer]	[1]
(ii)	Max power = Max p.d. x max current = 4.5 x 0.36 = 1.6 W	[1]
(b)	As *V* increase, *I* increases and hence, temperature increases (as more electrical energy (*IVt*) is converted to thermal energy.) Amplitude of vibration of lattice ions increases and thus collision frequency of electrons with lattice ions increases. Since the number of free electrons is fixed, there is no increase in charge carriers. Hence, resistance increases with temperature.	[1] [1] [1]
(c) (i)	From the graph, when current is 0.36 A, p.d. across the lamp is 4.5 V (accept 4.45 to 4.55) ∴ p.d. across 14 Ω resistor = (5.0 x 0.36) + 4.5 = 6.3 V (accept 6.25 to 6.35) Current in the 14 Ω resistor = 6.3 / 14 = 0.45 A (allow ECF)	[1] [1]
(ii)	Current through battery = 0.36 + 0.45 = 0.81 A (allow ECF of 0.45A and/or 6.3V) Internal resistance $r = \frac{p.d.\ across\ r}{I} = \frac{7.5-6.3}{0.81}$ = 1.5 Ω	[1] [1]
(d)(i)	V_{AB} = (12/15) x 15 = 12 V V_{AX} = (0.5/1.2) x 12 = 5 V E = V_{AX} = 5 V	[1] [1]
(ii)	The fractional error could be reduced by: - using a resistance wire with smaller resistance per unit length/ smaller resistivity/ thicker wire. - add (an appropriate) resistor in series to the driver cell - decrease e.m.f. of driver cell {1m for any one shown above} By potential divider rule, p.d. across resistance wire AB will decrease, hence length AX will increase and fractional error will be reduced. **Do not accept increase of length of wire.**	[1] [1]

(iii)	V_{AY} = p.d across 4Ω resistor = (4/5) x 5 V = 4 V	 [1]
	AY = (4/12) x 1.2 m = 0.4 m	 [1]

- End of tutorial solutions -

Solutions to Additional Questions

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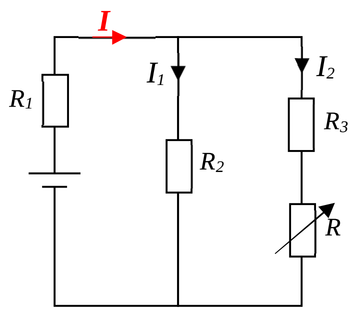
Answer: B

Since *R* decreases, overall resistance of the circuit decrease. With the same EMF, the total current ***I*** will increase.



Therefore, the potential drop across R_1 increases.
Therefore, the potential drop across R_2 (EMF – V_{R1}) decreases.
Therefore, the current I_1 decreases.

Since total current ***I*** will increase as discussed earlier,
<u>therefore, I_2 must increase.</u>

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Answer: C

Since 1 filament breaks, therefore the parallel circuit is reduced by 1 branch. Hence, overall resistance increases (for parallel circuit in general, the more branches, the lower the resistance)

Therefore, overall current decreases.
Therefore, the potential drop across the internal resistor decreases.
Therefore, the terminal p.d. (which is equal to the p.d. across each lamp) increases.

Since, power = V^2/R, power delivered to each bulb has increased, brightness increases.

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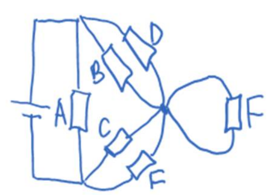
Answer: B

Effective resistance of B and D
= effective resistance of C and E
= (10 x 10)/(10 +10) = 5.00 Ω



Total effective resistance
= $(\frac{1}{10} + \frac{1}{10})^{-1}$
= 5.00 Ω

Current through cell = V/I = 9.0 /5.00 = 1.80 A

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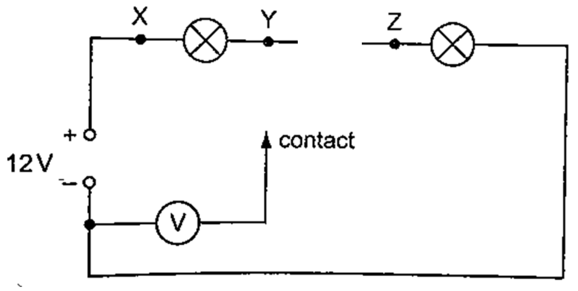
Answer: C



Since bulb L has a broken filament, the circuit is broken and no current flows. Hence,

- No pd across X and Y. Potential at Y = X = potential at +ve terminal.
- Therefore, p.d. reading at X = 12 V = p.d. reading at Y
- Since no current flows, potential Z = potential at -ve terminal. Therefore, p.d. reading at Z = 0 V.

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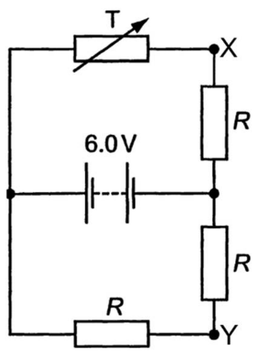
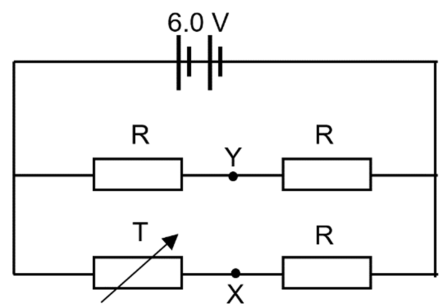
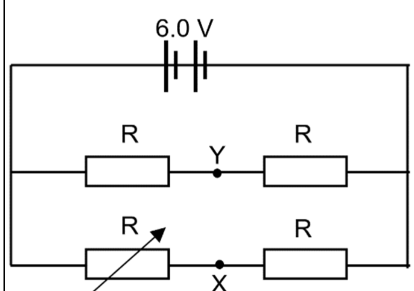
<tr><td>5</td><td>

Answer: B

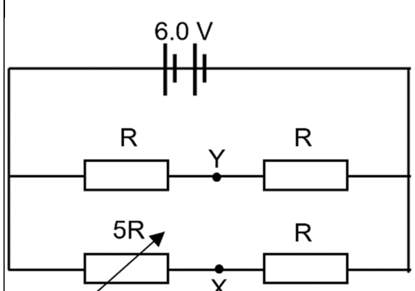




It is easier to solve the question by removing the voltmeter and just keep in mind that we are looking at p.d. between X and Y. After that, we can redraw the circuit such that it make it easier for us to understand the circuit (on the right).



When T = R, p.d. across X and Y = 0 (since p.d. between +ve terminal to Y = p.d between +ve terminal to X = 3.0 V)



When T = 5R,

- p.d. across +ve to Y = 3.0 V
- p.d. across +ve to X = 5/6 *6 = 5.0 V

Hence, p.d. across X and Y when T varies from R to 5 R is 5.0 – 3.0 = 2.0 V

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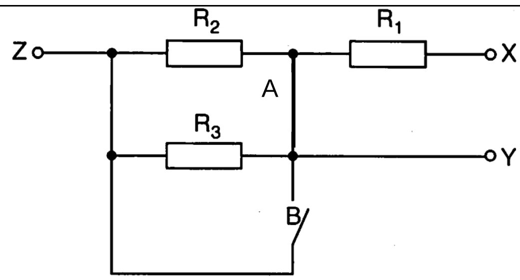
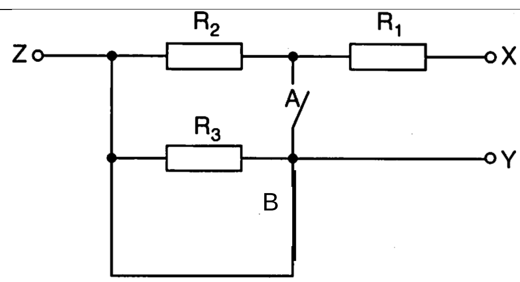
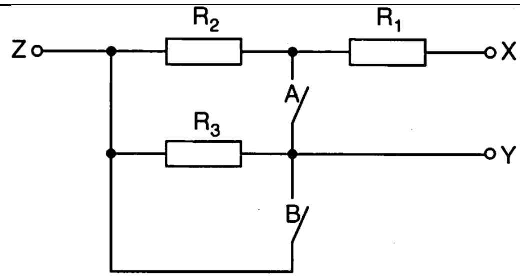
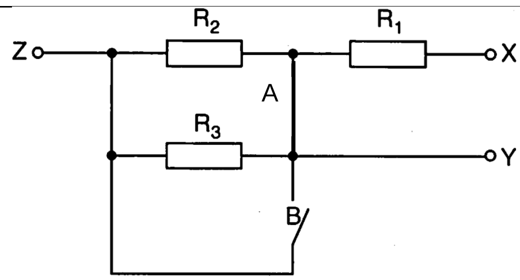
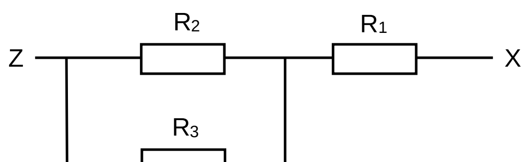
6	**Answer: C** Resistance of 1 branch of *N* resistors in series = *NR* Resistance of *N* branch in parallel $= (1/NR + \ldots.. + 1/NR)^{-1}$ $= (N / NR)^{-1}$ $= R$	
7	From the first circuit, $V_{BC} = 1.5$ V and $V_{BD} = \dfrac{R_{BD}}{R_{BD} + 1.0} \times 2.0$ At balance length, $0.9375 \times V_{BD} = 1.5$ $0.9375 \times \dfrac{R_{BD}}{R_{BD} + 1.0} \times 2.0 = 1.5$ $R_{BD} = 4.0\ \Omega$ From the second circuit, to balance at point D, then $V_{BD} = 1.0 / 1.5 \times 1.5$ $\dfrac{4.0}{4.0 + 1.0 + R} \times 2.0 = 1.0$ Solving, $R = 3.0\ \Omega$	[1] [1] [1] [1]
8(ai)	 When A is closed and B is open, effective resistance between X & Y = $R_1 = 6\ \Omega$ since there is a short circuit that allows current to bypass R_2 and R_3 and goes straight to Y. Therefore, $R_1 = 6\ \Omega$	
(ii)	 When A is open and B is closed, effective resistance between X & Y = $R_1 + R_2 = 10\ \Omega$ since there is a short circuit that allows current to bypass R_3. Therefore, $R_2 = 4\ \Omega$	

(iii)	 When both A and B are open, effective resistance between X & Y = $R_1 + R_2 + R_3 = 12\ \Omega$ since the current has to flow through all 3 resistors to reach Y. Therefore, $R_3 = 2\ \Omega$	
(b)	 When A is closed and B is open, the above is the actual circuit. The circuit can be redrawn as below:  R_2 is parallel to R_3, and their effective resistance is in series with R_1. Hence, resistance between Z and X = $(1/4 + 1/2)^{-1} + 6 = 7.33\ \Omega$	