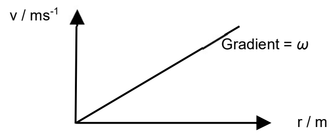
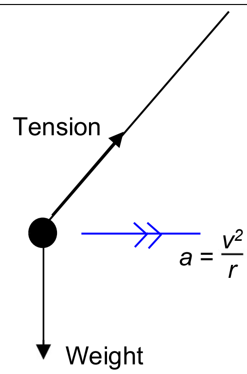
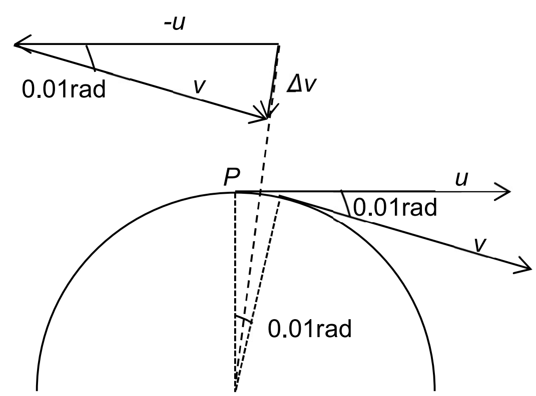
TUTORIAL 6: MOTION IN A CIRCLE SOLUTIONS

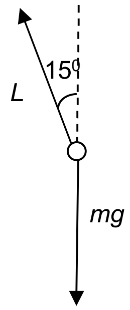
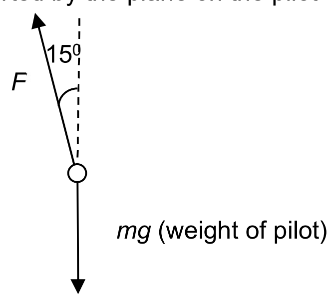
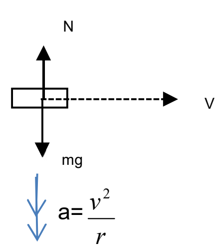
Level 1 Solutions

1(a)	300 rpm = 5rps = 5 x 2π s^{-1} = 10π s^{-1} = 31.42 s^{-1} = 31.4 rad s^{-1}	[1]
(b)	Since $v = r\omega$, and ω is constant, v α r. Therefore, it is a linear graph. 	[1]
(c)	$v_1 = r_1\omega$ = (0.030)(31.42) = 0.943 m s^{-1} $v_2 = r_2\omega$ = (0.050)(31.42) = 1.57 m s^{-1}	[1] [1]
2	 Note: A common mistake is including the centripetal force. Students must remember that centripetal force is a resultant force that does not exist by itself but is the resultant of the forces acting on the object. In this case, the horizontal component of the tension. {Tension [1], Weight [1] }	
3(a)	$r = \frac{0.2}{2}$ = 0.1 m ω = 600 rpm = 10 rps = 62.831 rad s^{-1} F_c = $mr\omega^2$ = 10^{-4} (0.1) $(62.831)^2$ = 3.9478 x 10^{-2} N = 3.95 x 10^{-2} N (3.s.f)	[1]
(b)(i)	F_{max} = $mr\omega_{max}^2$ 0.1 = 10^{-4} (0.1) ω^2 ω^2 = 10^4 ω = 100 rad s^{-1}	[1] [1]
(ii)	v = $r\omega$ = 0.1 (100) = 10 m s^{-1} The moment the pea leaves the wheel, its linear velocity is the same as the linear velocity of the wheel.	[1]

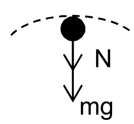
Level 2 Solutions

4	$a = \frac{v^2}{r}$ --- (1) KE = ½ m v^2 $v^2 = \frac{2\,KE}{m}$ --- (2) Sub (2) in (1): $a = \frac{\frac{2\,KE}{m}}{r} = \frac{\frac{2\,(5.0\times10^{-13})}{1.67\times10^{-27}}}{0.6} = 9.98\times10^{14}$ m s^{-2}	
5(a)(i)	$v = r\omega = 0.7 \times 500 = 350$ m s^{-1}	[2]
(ii)	$a = \frac{v^2}{r}$ or $r\omega^2 = 0.7 \times 500^2 = 1.75 \times 10^5$ m s^{-2}	[2]
(b)	As the blade moves in a circle, it experiences a **continuous change in the direction of velocity.** A change in the direction of velocity is a change in velocity. Hence, the blade experiences an acceleration and by Newton's 2nd Law, a force is required to produce this acceleration. The molecular force holding the blade to the wheel provides the centripetal force. At higher angular velocities, the centripetal force increases and may exceed the molecular force.	[1] [1] [1] [1]
(c)	Minimum centripetal force required F_c = ma = 0.8 x 175 000 = 1.40 x 10^5 N (3.s.f.)	[2]
6(a)	The vehicle is constantly changing its direction of travel. As velocity is a vector with direction and magnitude, thus its velocity is changing even though the magnitude of velocity remains constant. Since acceleration is rate of change of velocity, the vehicle is accelerating.	[1] [1]
(b)	Period, angular velocity, and kinetic energy. Note: Angular velocity is a vector, but it has constant magnitude and direction (clockwise or anticlockwise) Linear momentum, linear velocity, Centripetal acceleration, and Centripetal force are constantly changing direction.	[2]
(c)	The centripetal force required to keep the vehicle moving in a circle is mv^2/r and is provided by the friction between the tyres and the road. A more massive vehicle requires a bigger centripetal force which the tyres cannot fully provide, and hence, it must either travel slower or risk its wheels slipping. Note: Sliding friction is less than static friction. Once a car slips, it is likely to keep slipping.	[1] [1]

7	When the car is rounding an unbanked corner, the centripetal force is provided by the lateral friction between the wheels and the road surface. Given v_d = 20 m s^{-1} When the road is dry, the lateral friction, $F_d = \frac{mv_d^2}{r}$ --- (1) When the road is wet, the lateral friction, $F_w = ½\, F_d = \frac{mv_w^2}{r}$ --- (2) Divide (2) by (1), $½ = \frac{v_w^2}{v_d^2}$ $v_w^2 = ½\, v_d^2$ $v_w = \frac{v_d}{\sqrt{2}} = \frac{20}{\sqrt{2}}$ = 14.1 m s^{-1} Thus, the maximum safe speed when the road is wet is 14.1 m s^{-1}.	
8(a)		[1 for accurate diagram]
	Given that, Acceleration = $\frac{\Delta v}{t}$ and $\Delta v = v - u$ By drawing the vector diagram of Δv, even if there is no change in magnitude of v and u, there is a finite value for Δv. Hence, there is acceleration and it is pointing towards the centre because Δv is perpendicular to v and u when the angle subtended is small.	[1, explanation]
(b) (i)	Refer to previous diagram, By Geometry, it can be proved that the angle between u and v is also 0.01 rad. With small angle approximation, $\|\Delta v\|$ is equal to the arc length of the arc subtend by u and v. $s = r\,\theta$ Therefore, $\|\Delta v\| = \|u\|\,\theta = 0.01\|v\|$	[1]
(ii)	Acceleration of P is towards centre.	[1]
(iii)	Distance travelled is the arc length of radius r of angle 0.01 rad ⇨ $s = 0.01\, r$ Hence time, $t = s/v = 0.01\, r/v$	[1] [1]
(iv)	Acceleration $a = \Delta v/t$ $= (0.01v) / (0.01\, r/v)$ $= v^2 / r$ Note: Clear workings must be presented in presented for all "show" questions. No mark awarded for writing down $a = v^2 / r$.	[1]

9 (a)	 	900 km h^{-1} = 250 m s^{-1} $L \sin 15^0 = \frac{mv^2}{r}$ $L \cos 15^0 = mg$ Thus, $\tan 15^0 = v^2/(rg)$ $r = 250^2 / (9.81 \tan 15^0)$ $= 2.377 \times 10^4$ (m) $= 2.38 \times 10^4$ m to 3.s.f. (or 23.8 km) Note: Horizontally, there is acceleration (because it is moving in a circle), the horizontal component of L is providing the net force. Vertically, the body is in equilibrium, so the vertical component of L is equal to the weight.	[1] [1]
(b)	Ratio = $\frac{L\ sin15^o}{L\ cos15^o}$ = tan 15° = 0.26794 = 0.268 (3.s.f.)		[1]
(c)	Let F be the force exerted by the plane on the pilot  Considering vertical forces acting on the pilot $F \cos 15^o = W$ $\frac{W}{F} = \cos 15^o$ $F = \frac{W}{cos15^o}$ = 711 N (3.s.f.)		[1]
(d)	For the plane, $L \cos 15^0 = mg$ (from part a) (Mass of 560 tons of the SIA Airbus 380 includes the pilot). $\frac{mg}{L} = \cos15^o$ $L = (5.6 \times 10^5 \times g)/\cos 15^o$ $= 5.687393 \times 10^6$ N $= 5.69 \times 10^6$ N (3.s.f.)		[1]
10 (a)	At the top of the bridge, the resultant of the normal force of road on car and weight provides the centripetal force for circular motion over the humpback bridge. Taking the downwards direction to be positive, $F_{net} = mg - N = \frac{mv^2}{r}$ $N = mg - \frac{mv^2}{r}$  		[1] [1]

(b)	When the vehicle just remains in contact with the road, the normal reaction force N just reduces to zero. Thus, the equation $N = mg - \frac{mv^2}{r}$ $0 = mg - \frac{mv^2}{r}$ $mg = \frac{mv^2}{r}$ $g = \frac{v^2}{r}$ $r = \frac{v^2}{g} = \frac{20^2}{9.81}$ r = 40.8 m (3.s.f.)	[1] [1]
11 (a)	Let the net force acting on the carriage at the top of the arc = ΣF_y $\Sigma F_y = N + mg = \frac{mv^2}{r}$ (downwards as positive)	
(i)	At the lowest possible speed, v_{min}, N=0 $\frac{mv^2}{r} = mg$ $v_{min}^2 = rg$ $v_{min} = \sqrt{(rg)}$ $= \sqrt{(8.6 \times 9.81)}$ = 9.1850 m s^{-1} = 9.19 m s^{-1} (3.s.f.)	[1] [1]
(ii)	$N + mg = \frac{mv^2}{r}$ $N = \frac{mv^2}{r} - mg$ $= \frac{800(17)^2}{8.6} - 800(9.81)$ = 19035 N = 19.0 x 10^3 N downwards (3.s.f.)	[1] [1] [1]
(b)	The carriage has a kinetic energy of ½ mv^2 at the top of the loop. To gain that much kinetic energy, it must have converted an equal amount of gravitational potential energy. Assuming the carriage started at zero velocity at the top of the slope, and that no energy is lost through friction or air resistance, ½ $mv^2 = mg\Delta h$ Δh = x – diameter of loop = x – 17.2 ½ $mv^2 = mg\ \Delta h$ ½ $v^2 = g\ (x - 17.2)$ v^2 / (2 g) = x -17.2 289 / (19.62) = x – 17.2 x = 14.73 + 17.2 = 31.9 m (3.s.f.)	[1] [1] [1] [1]
12	$a = r\,\omega^2 = r\,(\frac{2\pi}{T})^2 = (4.00)\,(\frac{2\pi}{3.70})^2$ = 11.535 m s^{-2} When the passenger is at the top, acceleration is downwards (towards the centre of the circle). (Taking downwards to be positive, and applying N2) $W + N_{top} = ma$	

	$77(9.81) + N_{top} = 77(11.535)$ $N_{top} = 133$ N acting downwards When the passenger is at the bottom, acceleration is upwards (towards the centre of the circle). (Taking upwards to be positive, and applying N2) $- W + N_{bottom} = ma$ $-77(9.81) + N_{bottom} = 77(11.535)$ $N_{bottom} = 1640$ N acting upwards	
13(a)(i)	$v = r\,\omega$	[1]
(ii)	With ω kept constant, vary the length (r) of the cord .	[1]
(iii)	Since the weight acts perpendicular (downwards), it has no component in the radial direction. Hence, the tension is the only force that constitutes the centripetal force that must be non-zero to continuously change the direction of the velocity for circular motion to be possible.	[1] [1]
(iv)	Acceleration = $\frac{v^2}{r}$ $T = m\frac{v^2}{r} = m\,v\,(\frac{v}{r}) = m\,v\,\omega$	[1] [1]
(b)(i)(1)	$a = \frac{v^2}{r} = \frac{12^2}{7.0} = 20.6 \approx 21$ m s^{-2}	[1]
(2)	 ↓+ve, $F = m\,a$ $N + mg = m\frac{v^2}{r}$ $N = m\frac{v^2}{r} - mg$ $= 60\,(20.6) - 60\,(9.81)$ $= 650$ N	[1] [1] [1]
(ii)(1)	$\Delta E_p = m\,g\,\Delta h = (60)\,(9.81)\,(-14) = -8240$ J	[1]
(2)	v_b = velocity at the bottom of loop v_t = velocity at the top of loop= 12 m s^{-1} KE at bottom of loop = KE at top of loop + GPE change $\frac{1}{2}m\,v_b^2 = \frac{1}{2}m\,v_t^2 + \Delta E_p$ $\frac{1}{2}(60)\,v_b^2 = \frac{1}{2}(60)\,(12)^2 + 8240$ $v_b = 20$ m s^{-1}	[1] [1] [1]
(iii)	The cart must enter the bottom of the loop with a minimum KE that is required to increase its GPE to reach the top at a minimum speed. This minimum speed is the speed required to enable the cart to just remain in contact with the track. If the speed of the cart is lesser than this minimum speed, the cart and its passengers will lose contact with the track and fall.	[1] [1]