TOPIC 14: Direct Current Circuits

Learning Outcomes: Candidates should be able to:

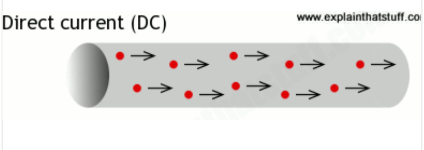
a.	Recall and use appropriate circuit symbols as set out in the ASE publication *Signs, Symbols and Systematics* (*The ASE Companion to 16-19 Science, 2000*).
b.	Draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, and/or any other type of component referred to in the syllabus.
c.	Solve problems using the formula for the combined resistance of two or more resistors in series.
d.	Solve problems using the formula for the combined resistance of two or more resistors in parallel.
e.	Solve problems involving series and parallel circuits for one source of e.m.f.
f.	Show an understanding of the use of a potential divider circuit as a source of variable p.d.
g.	Explain the use of thermistors and light-dependent resistors in potential dividers to provide a potential difference which is dependent on temperature and illumination respectively.
h.	Recall and solve problems using the principle of the potentiometer as a means of comparing potential differences.

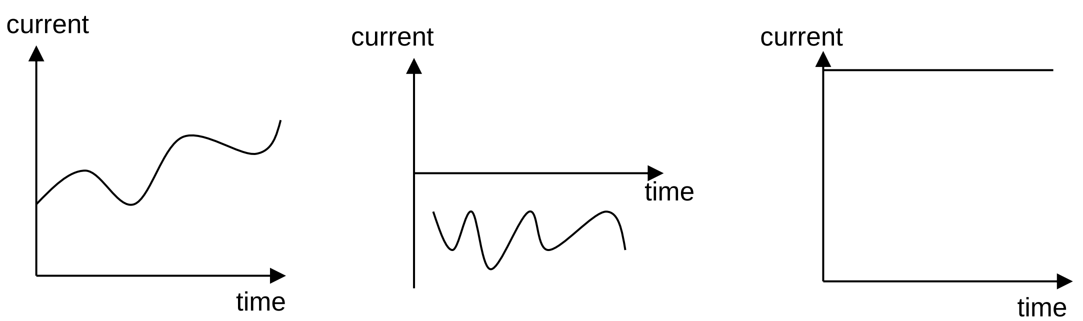
Four broad areas:
- Circuit symbols and diagrams
- Series and parallel arrangements
- Potential divider
- Balance potentials

Key questions:
- What is a direct current?
- How to use circuit symbols in a diagram to represent a real circuit?
- What are the differences between series and parallel circuit arrangement in terms of current, potential difference and resistance?
- How to simplify a complex/unfamiliar circuit to solve a problem?
- How can a potential divider be used as a source of variable potential difference?
- How can the principle of potentiometer be used to solve problems?

14.1 What is a D.C.?

D.C. stands for direct current. As the term implies, it means current flowing only in <u>one direction</u> in a closed circuit. The magnitude of the current, however, may vary with time. The following are some examples of graphs representing the variation of direct current with time.





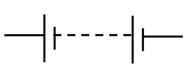
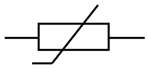
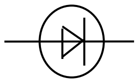
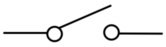
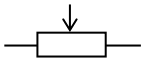
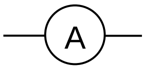
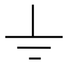
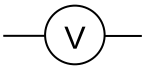
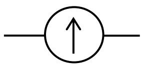
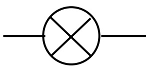
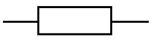
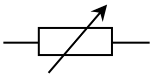
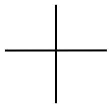
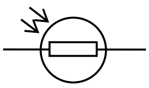
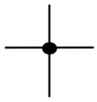
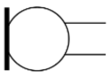
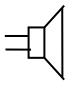
(a) (b) (c)

In (a) and (b), the magnitudes of the current changes with time. In (a) the current flows in the *opposite* direction as that in (b). In (c), the current flows with *constant* magnitude and direction. We say, the current is a *steady* D.C. Your syllabus focuses mainly on the type of current shown in (c) – steady D.C (usually just called D.C.).

14.2 The Electrical Circuit Symbols

A simple D.C. circuit typically consist of:
a) source(s) of electrical energy, e.g. cell / battery / power supply
b) resistor / LDR / thermistor / filament lamp / diode
c) switches
d) simple measuring instruments such as voltmeter, ammeter, etc.

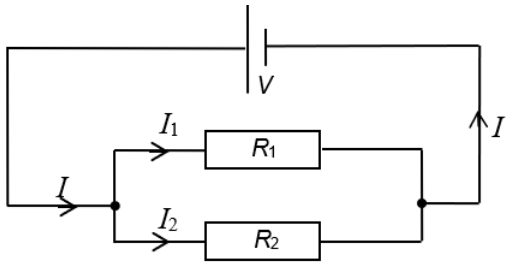
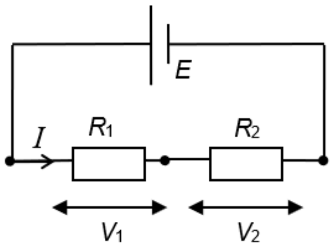
Standard circuit symbols are used to simplify circuit diagrams. Each circuit symbol represents an electrical device.

Symbol	Meaning	Symbol	Meaning
	cell / battery		thermistor
	power supply		diode
	switch		potential divider
	ammeter		earth
	voltmeter		
	galvanometer		
	filament lamp		
	resistor		
	variable resistor		wires crossing with no connection
	light dependent resistor		wires crossing with connection
	microphone		loudspeaker

*aerial/antenna, capacitor and inductor are no longer in the A-Level syllabus.

14.3 Principles Involved in Solving Circuit Problems

The two principles below were first described in 1845 by the German physicist Gustav Kirchhoff.

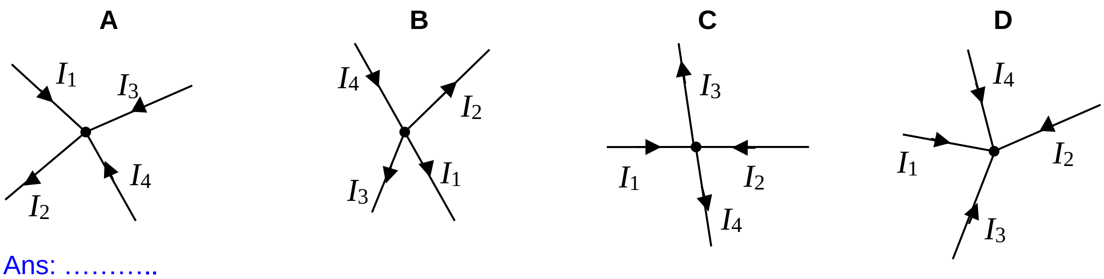
Principle 1 (Kirchhoff's Current Law)	Principle 2 (Kirchhoff's Voltage Law)
Based on: Principle of conservation of electric charge.	Based on: Principle of conservation of energy. See explanation below.
At any circuit junction, the sum of the currents flowing into and out of that junction are equal.	The net electromotive force around a closed circuit loop is equal to the sum of potential drops around the loop.
Illustration:  $I = I_1 + I_2$	Illustration:  $E = V_1 + V_2$

Explanation for Principle 2:
By Principle of Conservation of Energy, energy cannot be created nor destroyed. Therefore, the electrical energy supplied by the source is equal to the sum of electrical energy dissipated by all the components. Consider a closed circuit of e.m.f. E and resistance R_1 and R_2.

Power supplied by the source		=	Power dissipated by the resistors
	$I E$	=	$I^2 R_1 + I^2 R_2$
	E	=	$I R_1 + I R_2$
i.e.	E	=	$V_1 + V_2$

Example 1
The diagrams show connected wires which carry currents I_1, I_2, I_3 and I_4. The currents are related by the equation $I_1 + I_2 = I_3 + I_4$.

To which diagram does this equation apply? [N98/P1/Q16]

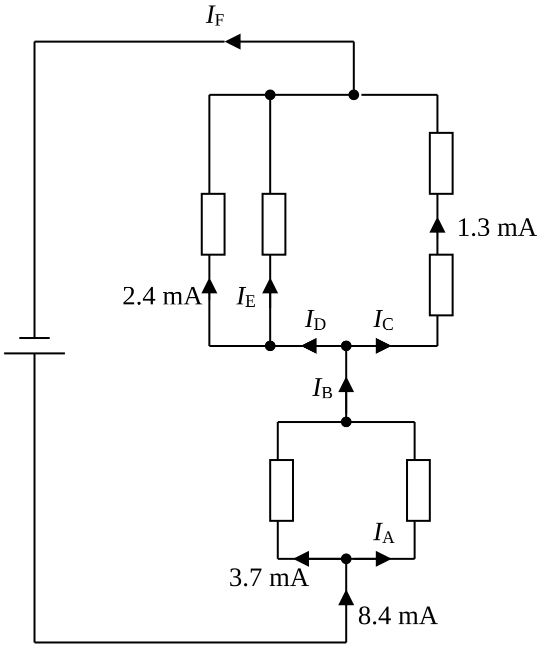


Ans: ………..

Worked Example 2

Calculate the currents I_A, I_B, I_C, I_D, I_E, and I_F as shown in the diagram below.

[I_A = 4.7 mA, I_B = 8.4 mA, I_C = 1.3 mA,
I_D = 7.1 mA, I_E = 4.7 mA, I_F =8.4 mA]



$I_A = 8.4 - 3.7 = 4.7$ mA

$I_B = 8.4$ mA

$I_C = 1.3$ mA

$I_D = 8.4 - 1.3 = 7.1$ mA

$I_E = 7.1 - 2.4 = 4.7$ mA

$I_F = 8.4$ mA

Example 3A

Referring to the circuit drawn, determine the value of the current I and the potential difference across each resistor.

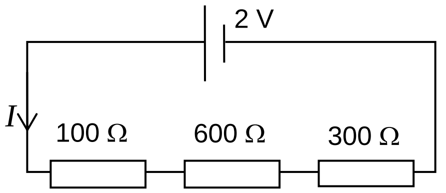


Fig. 3A

E =

I =

Applying Ohm's law,
Potential difference across the 100 Ω resistor = 0.002 x 100 = 0.2 V
Potential difference across the 600 Ω resistor = 0.002 x 600 = 1.2 V
Potential difference across the 300 Ω resistor = 0.002 x 300 = 0.6 V

Food for thought -
Earthing a point in a circuit:

Sometimes a specific point in the circuit is earthed, which means the electrical potential at that point is zero (0 V). The potential of the other points can be obtained by calculating the potential difference between the earth point and the other points.

Potential of a point can only be defined if there is a reference potential. This reference is given as 0 V (earthen) in example 3B below.

Example 3B
Considering the same circuit (Fig. 3A) with the earth point O as shown in Fig. 3B.

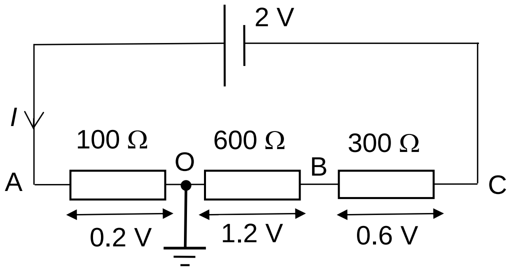


Fig. 3B

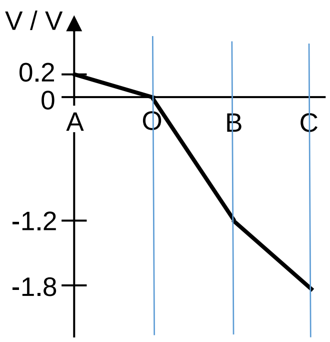
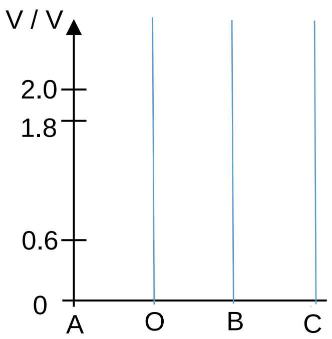
Find the potential at A, B and C.

Current flows from higher to lower potential, which means
$V_A > V_O > V_B > V_C$,   and $V_O = 0$ V (a reference point) since it is an earth point.

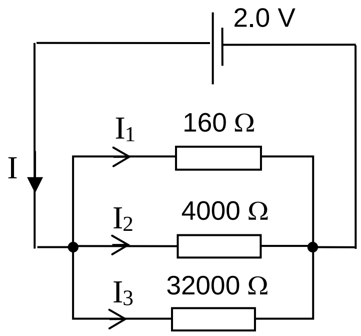
The potential at A, V_A, the potential at B, V_B, and the potential at C, V_C can be obtained by calculating the potential difference between the earth point and each point (A, B and C).
Hence,
$V_A = 0 + 0.2 = +0.2$ V
$V_B = 0 – 1.2 = -1.2$ V
$V_C = 0 – (1.2) – (0.6) = -1.8$ V

By Principle 2, total potential difference between A & C = $V_A - V_C = 0.2 - (-1.8) = 2.0$ V = Emf of cell.

Note that "earthing" a certain point in a circuit does not change the pd across each resistor.

With point O as earth point.	**With point C as earth point *instead*.**
	

Example 4
Referring to the circuit drawn, determine the value of I_1, I and R, the combined resistance in the circuit.



E =

$\Rightarrow$ I_1 =

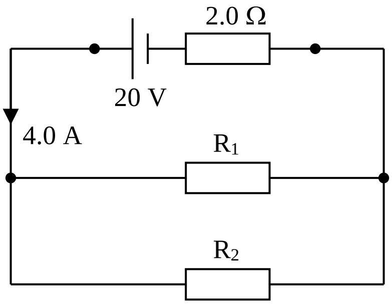
Similarly, I_2 =

and I_3 =

Since I = $\Rightarrow$ I =

Applying Ohm's Law, R =

Example 5
A battery with an e.m.f. of 20 V and an internal resistance of 2.0 Ω is connected to resistors R_1 and R_2 as shown in the diagram. A total current of 4.0 A is supplied by the battery and R_2 has a resistance of 12 Ω. Calculate the resistance of R_1 and the power supplied to each circuit component.

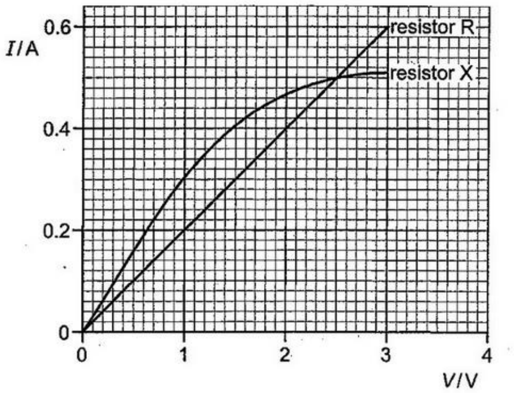


Solution:

Power supplied to R_1 =

Power supplied to R_2 =

Example 6 Graph problem (N2015/ P1/Q9)
The graph shows the I-V characteristics of two resistors R and X.



The resistors R and X are connected in series with a cell of negligible internal resistance. The current in the circuit is 0.3 A.

The resistors R and X are then connected in parallel with the same cell.

What is the e.m.f. of the cell and the current in the cell when the resistors are connected in parallel?

Solution:
In series: current is the same for X and R. Draw a horizontal line at 0.3 A that cuts the two lines at 1.0 V and 1.5 V.
Therefore, total e.m.f. =

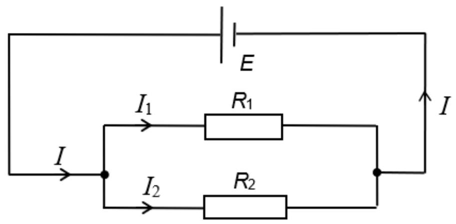
In parallel: voltage is the same for X and R. Draw a vertical line at the point of intersection of the 2 characteristics i.e. 2.5 V

Therefore, total current =

14.4 Determining the Combined Resistance of Two or More Resistors

Combined Resistance of Resistors in Parallel

From Principle 1, we know that $I = I_1 + I_2$



Since the e.m.f. E of the battery (of negligible internal resistance) is the same across each of the resistors, the above equation implies that:

$$\frac{E}{R_c} = \frac{E}{R_1} + \frac{E}{R_2}$$

where R_c is the combined resistance in the circuit.

Thus, $$\frac{1}{R_c} = \frac{1}{R_1} + \frac{1}{R_2}.$$

For N resistors in parallel, the formula becomes: $$\boxed{\frac{1}{R_c} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}}$$

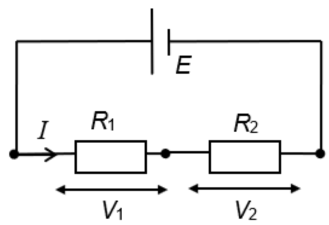
Special case: For two resistors in parallel, it can be shown that $R_c = \dfrac{R_1 R_2}{R_1 + R_2}$.

This 'product divided by the sum' rule enables quick calculation of combined resistance.

Note: The combined resistance, R_c of resistors in parallel is always **smaller** than the **smallest** resistance in the parallel network. If the 2 resistors are **identical**, then the resultant resistance is **half** of the value of one of them.

Combined Resistance of Resistors in Series

From Principle 2, we know that $E = V_1 + V_2$



Since the current in the circuit does not change, the above equation implies that:

$$I R_c = I R_1 + I R_2$$

where R_c is the combined resistance in the circuit.

Thus $$R_c = R_1 + R_2$$

For N resistors in series, the formula becomes: $$\boxed{R_c = R_1 + R_2 + \ldots + R_N}$$

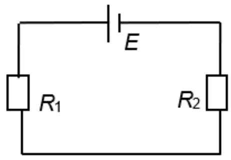
Note: Resultant resistance of resistors in series is always **larger** than the **largest** resistance in the series.

In general, the resistors are

- in *series* if they are connected in a branch with the *same current* flowing through the resistors;
- in *parallel* if they have the *same potential difference* across the resistors.

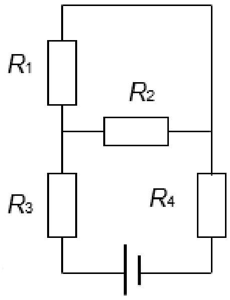
Exercises:

1. Are R_1 and R_2 in series or parallel?



Ans: They are in ……….. because

2. Are R_1 and R_2 in series or parallel?

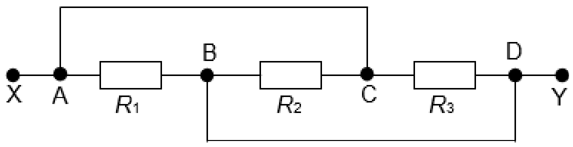


Ans: They are in …………. because

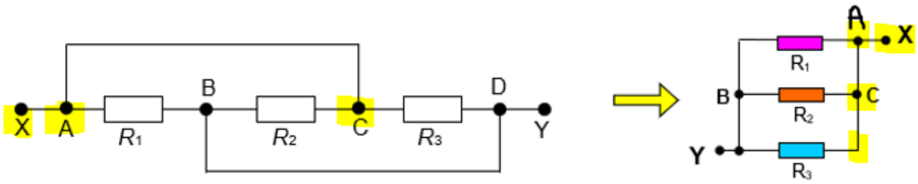
Are R_2 and R_3 in series or parallel?

Ans: They are ………………………………. because …….

3. Are R_1, R_2 and R_3 in series or parallel?



Ans:
Pro tip: You can use 'potential analysis' by colour-coding points in the circuit that are at the same potential (since there is no potential drop across any resistor along that path).
E.g. X, A, C in the above circuit are at the same potential; B, D, Y are at the same potential too (but different from that of X).



Hence, it is now obvious that R_1, R_2 and R_3 are in parallel.

Alternatively we can understand the circuit better by physically shifting the resistors around without changing the connections.

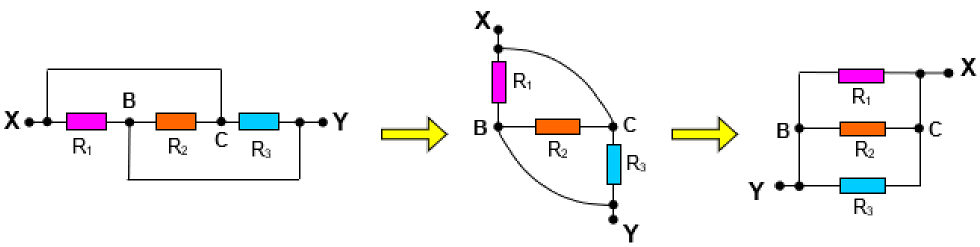
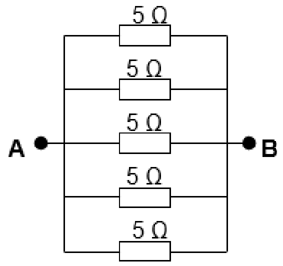


Fig. 1 **Fig. 2** **Fig. 3**

From Fig. 1 to Fig. 2, keep the position of R_2 fixed, rotate R_1 clockwise about B and rotate R_3 clockwise about C. From Fig. 2 to Fig. 3, again keep the position of R_2 fixed and continue to rotate R_1 and R_3 clockwise about B and C respectively.

4. How do you determine the equivalent resistance across **AB** by inspection?

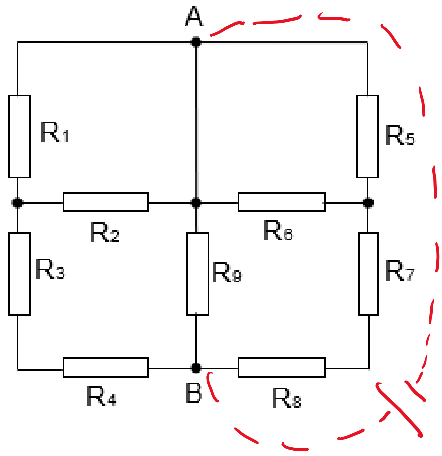


Surely you are not going to calculate it the long way >>

$\frac{1}{R_c} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{5}{5}$, hence, $R_c = 1\ \Omega$

If there are n parallel branches each with the same resistance R, then the equivalent resistance, $R_c = \frac{R}{n}$

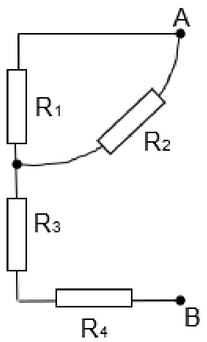
5. So how do you determine the following equivalent resistance across **AB** by inspection, taking the value of each resistor to be 2 Ω? [1.11 Ω]



For such questions, never seek to map current flow as it is too complex to be useful.

Instead, consider which resistors are in series or parallel. Consider whether there are identical branches.

Tip: Imagine a battery being connected to the 2 points of interest in the questions i.e. A and B before analysing if the branch is series or parallel



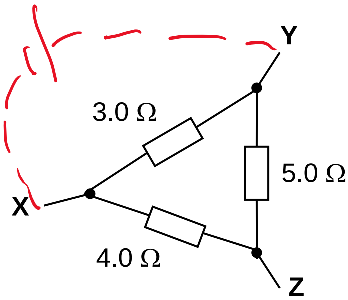
After redrawing the left part of the above circuit (comprising R_1, R_2, R_3, and R_4), we get the diagram on the left. i.e. this is the 'left branch' of the given circuit, where the combined resistance of this left branch is 5 Ω. The "right branch" is symmetrical to the "left branch".

Overall, we have 3 parallel branches between points A and B.

So, given circuit is basically: 5 Ω (left branch)// 2 Ω (centre) // 5 Ω (right branch)

which is equal to 2.5 Ω // 2 Ω = $\frac{(2.5)(2)}{2.5 + 2}$ = 1.11 Ω.

Example 7 [J91/P1/Q14 modified]
The diagram shows a network of three resistors of 3.0 Ω, 4.0 Ω and 5.0 Ω as shown.
Determine the resistance between **X** and **Y**. [2.25 Ω]



Consider again which resistors are in series or parallel.

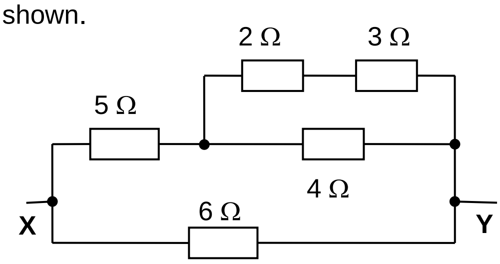
The 4.0 Ω and 5.0 Ω resistors are in …………, combined resistance is ………. Ω.

This 9.0 Ω combined resistance is in ………… to the 3.0 Ω resistor as they have the same pd (ie. V_{XY})

Hence, the combined resistance

Tip: Imagine a battery being connected to the 2 points of interest in the questions i.e. X and Y before analysing if the branch is series or parallel

Worked Example 8
Calculate the resistance across **XY** for the network shown.



$$\text{Resistance} = \left\{\left[\left(\frac{1}{2+3}+\frac{1}{4}\right)^{-1}+5\right]^{-1}+\frac{1}{6}\right\}^{-1}$$

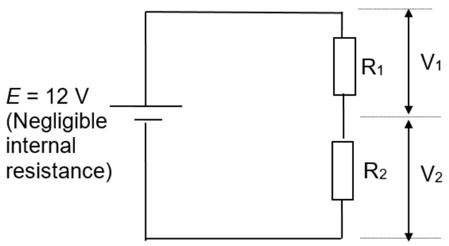
$= \underline{\mathbf{3.277\ \Omega}}$

Tutorial qn: Q1 to Q6

14.5 The Potential Divider

A potential divider is an arrangement of resistors in which two (or more) resistors, connected in series with a supply voltage (e.m.f.), "divide" the e.m.f. proportionally according to their resistance.

The potential difference, V across each resistor is a fraction of the e.m.f. of the cell and can be calculated as shown below.



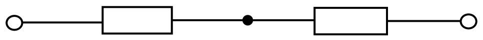
$$V_1 = \frac{R_1}{R_1+R_2}\times E \qquad \text{Similarly, } V_2 = \frac{R_2}{R_1+R_2}\times E$$

Derive the above by using $E = V_1 + V_2$. Since I is the same through the two resistors in series, $E = I\,(R_1 + R_2)$ thus $I = E / (R_1 + R_2)$. Substitute this I into $V_1 = IR_1$ and $V_2 = IR_2$.

The general formula for the p.d. V_R across a resistor of resistance R can be deduced to be:

$$\boxed{V_R = \frac{R}{R_{eff}} \times total\ p.d. across\ all\ resistors}$$

Example 9
Two resistors, of resistance 300 kΩ and 500 kΩ respectively, form a potential divider with outer junctions maintained at potentials of +3 V and –15 V.



Determine the potential at the junction **X** between the resistors. [-3.75 V]

Solution:
The potential difference across the 300 kΩ resistor =

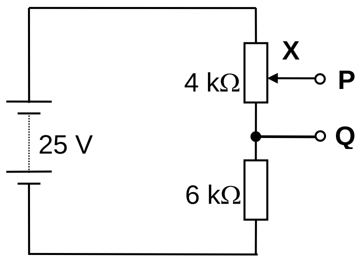
The potential at X =

Or consider the pd across the 500 kΩ resistor =

The potential at X =

Tip: Calculate potential difference first before the potential (with respect to the reference point)

Worked Example 10
The diagram below shows a potential divider circuit which by adjustment of the position of the movable contact **X**, can be used to provide a variable potential difference between the terminals **P** and **Q**.



Determine the limits of this potential difference.

Solution:
When **X** is at the upper end of the resistor, the potential difference across **PQ**
$= (\frac{4}{4+6}) \times 25 = 10$ V

When **X** is at the lower end of the resistor, the potential difference across **PQ**
$= (\frac{0}{4+6}) \times 25 = 0$ V

or **PQ** = 0 V as there is no potential drop when current did not go through a resistor between **PQ** when **X** is at the lower end.

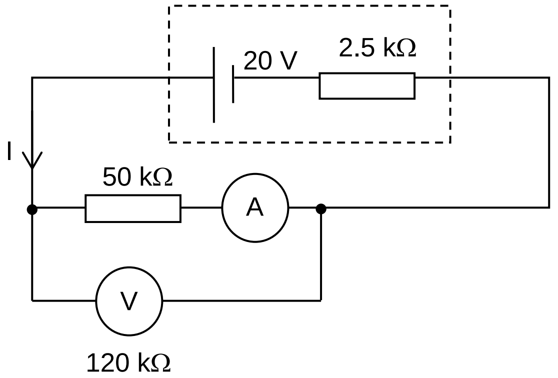
Example 11
A 20 V battery with an internal resistance of 2.5 kΩ is connected to a 50 kΩ device. A voltmeter with an internal resistance of 120 kΩ and an ammeter with negligible internal resistance are used to measure the potential difference across the device and the current passing through it respectively.

(a) Sketch the circuit diagram for this setup.

(b) Determine the reading on the voltmeter and the ammeter.

Solution:

(a)



(b) Combined resistance parallel to the voltmeter, R =

=

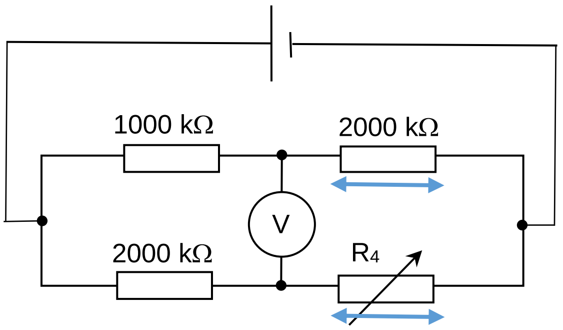
Voltmeter reading, V =

Ammeter Reading =

Tip: Ignore ammeter and voltmeter if they are ideal. Image them as a resistor in the circuit and solve the problem if they are not ideal.

Worked Example 12

What is the value of resistor R_4 if the voltmeter has no reading?



Solution:

The voltmeter has no reading when its upper terminal has the same potential as the lower terminal.

Using potential divider rule, this happens when

$$\frac{2000}{1000 + 2000} E = \frac{R_4}{2000 + R_4} E$$

Solving the equation gives $R_4 = 4000\ \Omega$.

Alternatively, we can use the resistor ratio of the two parallel branches. Let the current on the top branch be I_1 and the lower branch be I_2

$$\frac{I_1 \times 1000}{I_1 \times 2000} = \frac{I_2 \times 2000}{I_2 \times R_4}$$

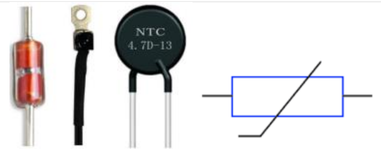
$$\frac{1000}{2000} = \frac{2000}{R_4}$$

$R_4 = 4000\ \Omega$.

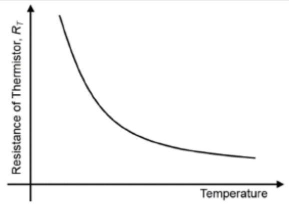
14.5.1 Use of a Thermistor in a Potential Divider

A thermistor is a resistor whose resistance varies with temperature.

It can be used in potential divider circuits to monitor and control temperatures, such as a heat sensor.



For a NTC or Negative temperature coefficient thermistor, its resistance varies inversely with the temperature. NTC thermistors are made of oxides of nickel, cobalt, copper, iron and titanium.

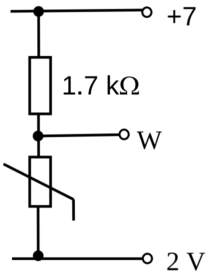


Here are some of the applications of NTC thermistors:

- Digital Thermometer
- Temperature Monitoring and Control
- Fire Alarm

Example 13

In the figure below, the thermistor has a resistance of 800 Ω when hot, and a resistance of 5000 Ω when cold. Determine the potential at W when the temperature is hot.



Solution:

When thermistor is hot, potential difference across it

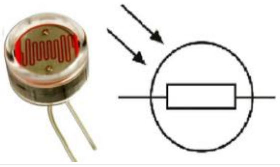
=

=

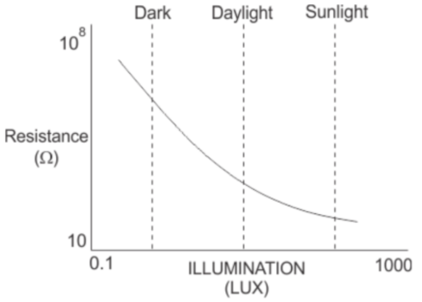
The potential at W =

14.5.2 Use of a Light Dependent Resistor (LDR) in a Potential Divider

An LDR is a resistor whose resistance varies with the intensity of light falling on it. Its resistance decreases with increasing light intensity. It can be used in a potential divider circuit to monitor light intensity, such as a light sensor.

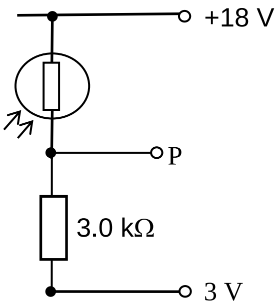


Photoresistors work based off of the principle of photoconductivity. Photoconductivity is an optical phenomenon in which the material's conductivity is increased when light is absorbed by the material.



Worked Example 14

In the figure below, the resistance of the LDR is 6.0 MΩ in the dark but then drops to 2.0 kΩ in the light. Determine the potential at point P when the LDR is in the light.



Solution:

In the light the potential difference across the LDR

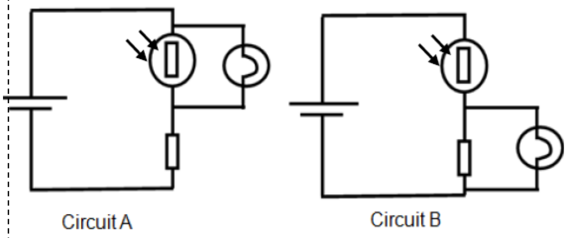
$= \left(\frac{2.0\text{ k}}{3.0\text{ k} + 2.0\text{ k}}\right) \times (18 - 3)$

= 6 V

The potential at P = 18 – 6

= **12 V**

Food for thought:



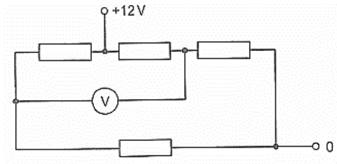
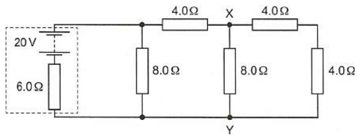
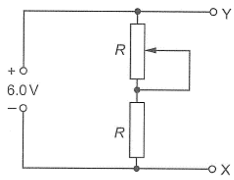
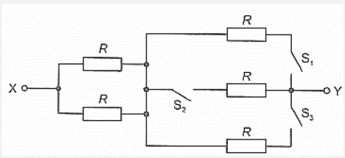
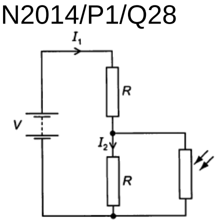
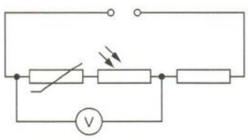
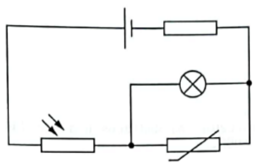
In which circuit would the bulb shine more brightly at night? (Assuming the light from the blub does not affect the LDR)

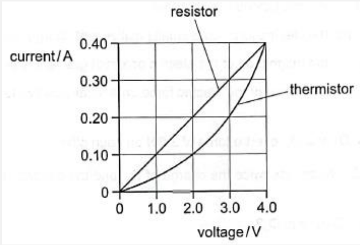
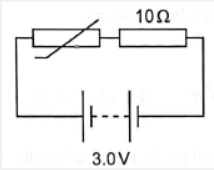
Ans:

Tutorial qn: Q7 to Q13

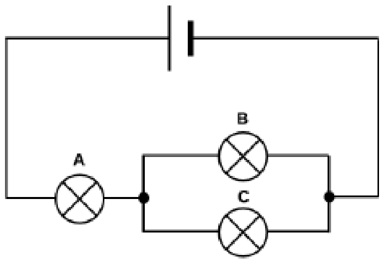
14.6 Circuit Analysis

Many practical circuits may seem difficult to solve due to one of the following reasons.

	Type of Question	Examples	Pro tip for solving
1	Complex layouts with multiple current paths	• Ex. Q3 & Q5 on pg 14-8 • Lecture e.g. 7 • N2012/P1/Q30  • N2016/P1/Q29  • N2020/P1/Q24  (one of the 4 options given)	Redraw given circuit by: • visualising common points in the circuit with the same potential, or • visualising the current paths in circuit diagram, or • reducing the circuit to a number of identifiable pure series and pure parallel circuits, after shifting the components around * N2020/P1/Q24 What is the p.d. across XY?
2	Multiple switches to evaluate (with different on/off combinations asked for in the qn)	• N2012/P1/Q31 	Redraw given circuit by: • visualising the current paths in circuit diagram.
3	Components with variable resistance (e.g. variable resistor, thermistor, LDR)	• N2014/P1/Q28  • N2016/P1/Q28  • N2018/P2/Q8 • N2021/P1/Q23 	Redraw given circuit by: • ignoring ideal ammeter & voltmeter • drawing components as simple resistors • applying Kirchhoff's laws, Ohm's law and/or potential divider rule accurately • recalling that brightness of a bulb depends on its power ($P = V^2/R$) *N2021/P1/Q23 (hint: for bulb to glow brighter, p.d. across bulb/thermistor parallel combination must ……?)

4	Graphs (e.g. I-V) to analyse for more than 1 component	• Lecture e.g. 6 • N2010/P1/Q9 • N2021/P1/Q22   	• Trial and error method with knowledge/application of Kirchhoff's laws, Ohm's law and/or potential divider rule, using info from the graphs * N2021/P1/Q22 What is the current in the circuit?

Example 15
3 identical bulbs, A, B and C are connected in a circuit as shown. What will happen to the brightness of the remaining bulb when bulb C blows?



	Brightness of bulb A	Brightness of bulb B
A	decrease	increase
B	increase	decrease
C	decrease	decrease
D	increase	increase

Solution:

When C blows, effective resistance across B and C will now …………. (from R/2 previously to R), so p.d. across B will ………….., while p.d. across A ………………. (by potential divider rule).

Since brightness of a bulb is dependent on the …………………………… (given by $P = V^2/R$), and R remains constant for *each* bulb, the Power in A will …………., while power in B ………………….

14.7 The Potentiometer

14.7.1 Introduction & Comparison Method

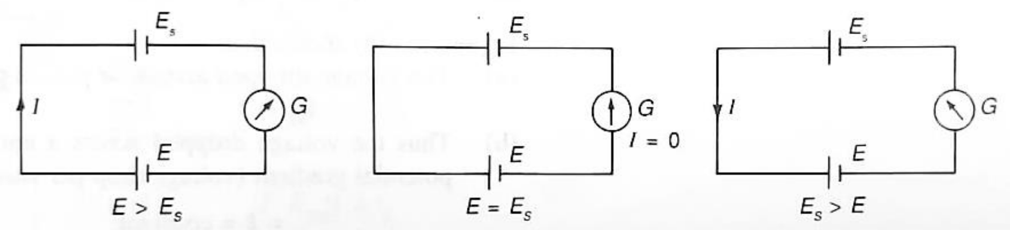
The voltmeter and ammeter are used to read voltages and currents in real circuits. In practice however, these meters affect the circuit when connected due to their own resistances.

	Ideal	Practical	Connections
Voltmeter	- Infinite resistance - Does not draw current from circuit	- Has a large but finite resistance - Draws a small current from circuit	- Always connected in parallel, across the component whose p.d. is to be measured
Ammeter	- Zero resistance - Zero p.d. across its ends	- Has a small resistance - Has a non-zero p.d. across its ends	- Always connected in series along the path where the current is to be measured

Pro Tip: In A-level questions where the resistance of the ammeter or voltmeter is not given, we assume that they are ideal.

To measure p.d. or e.m.f, there are two methods to do so:
1. With the voltmeter – fast and efficient, but note that its resistance will affect the circuit current and hence p.d./e.m.f under investigation.
2. With the *potentiometer* – a comparison method (explained below).

To measure the p.d./e.m.f. (E) of an unknown source (e.g. test cell), one can use the following set-up. A set of standard e.m.f. sources (E_s) of different values is needed.



By trial and error, connect different standard e.m.f. sources into the circuit. Once the chosen standard e.m.f. source causes null (zero) deflection in the galvanometer G (a current detector), we can conclude that the unknown e.m.f. has the same value as the standard e.m.f. source.

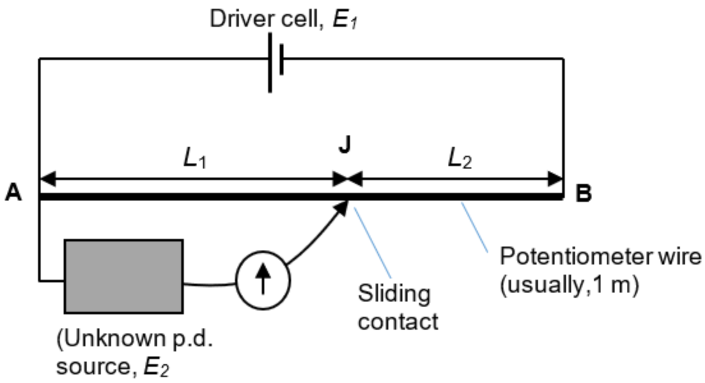
However, a more convenient arrangement is to have a set-up that yields a continuous (instead of discrete as above), variable p.d., by using a *potential divider* circuit in a *potentiometer* circuit.

14.7.2 Components and Working Principle

1. What is it

The potentiometer is a device used to compare p.d. or e.m.f. It can measure an unknown e.m.f. by comparing this e.m.f. with a known standard e.m.f., thus, the potentiometer set-up can be used as an improvised voltmeter.

2. Components



The circuit consists of a bare resistance wire (AB) of uniform resistance R and length L, connected in series with a driver cell (of known e.m.f.), which maintains a steady current in the wire AB.

It also consists of a galvanometer (an analogue ammeter whose pointer can indicate magnitude and direction of current flow) and an unknown p.d. source (or test cell), E_2 as shown.

The sliding contact (or jockey) at J can be moved along the bare resistance wire, to tap out a continuous range of values of p.d. (as mentioned in pg 17).

3. Working Principle

a) Potential Divider Rule.

The above potentiometer circuit is likened to a potential divider with 2 resistances R_{AJ} and R_{JB} in series.

b) Balance point & balance length.

The sliding contact **J** will move along wire AB until it finds a point along the wire such that the galvanometer shows a zero reading/ null deflection (when there is no current flowing through the branch containing the unknown p.d. source).

Hence the potentiometer is said to be "*balanced*". The balance length is L_1.

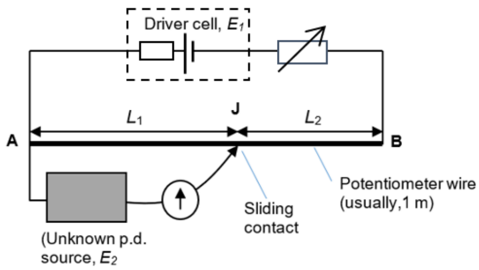
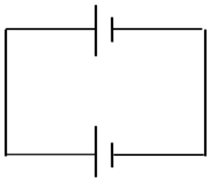
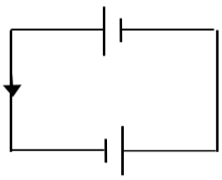
There is no current in the secondary circuit (**A**E_2**J**) while there is a constant current in the primary/main circuit (E_1**AJB**).

c) The **potential difference along the wire AB is proportional to the length** of the wire. Unlike a typical connecting wires made of copper, the resistance wire has significant resistance and its resistance cannot be taken to be negligible.

Proof: since the resistance of wire AB with a uniform cross-section area A and uniform resistivity ρ, is $R = \rho L/A$, resistance is proportional to the length of wire L. And since the same current (from driver cell) I flows through wire AB, the potential difference across a section of wire AB is proportional to the resistance, and hence length of that section.

If the driver cell has negligible internal resistance, and if the potentiometer is *balanced*,

e.m.f. or p.d. of the unknown source, $E_2 = \dfrac{R1}{R1 + R2} \times E_1 = \dfrac{L_1}{L_1 + L_2} \times E_1$

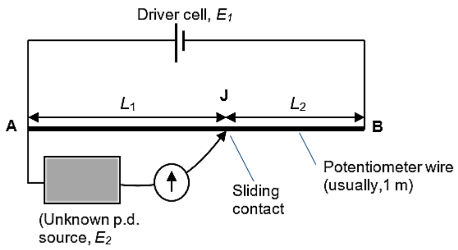
	Note: some questions might define the **potential gradient *k*** across the wire as $\frac{V_{AJ}}{L_{AJ}}$. [see N2022/P1/Q22]
4.	**Additional note on using a potentiometer**
a)	From the equation above, we see that the accuracy of E_2 depends on the accuracy of L_1 and E_1. It is desirable to have a <u>large balance length</u> (L_1), to ensure the percentage uncertainty in the reading of balance length is small. Since p.d. across balance length, V_{AJ} = (p.d. per unit length of pot wire) x (balance length) = (potential gradient, k) x (balance length), by <u>decreasing the potential gradient k</u>, balance length will be longer. A **smaller potential gradient *k* achieved by** the following (assuming wire kept constant at 1 m): • smaller driver cell e.m.f. E_1 • larger resistance of variable resistor (in series with driver cell) • smaller resistivity of bare resistance wire AB (i.e. another type of wire) • larger diameter of potentiometer wire AB (Not acceptable to say: decrease resistance of potentiometer wire.) This is because potential gradient is determined by applying the potential divider rule to: E_1, driver cell's internal resistance, driver cell's variable resistor (if any), resistance of potentiometer wire (which is in turn determined by its resistivity, length (kept constant) and diameter/thickness since R = ρL/A) 
b)	A balance point cannot be achieved if both cells (driver cell and test cell) are driving current in the same direction (see (b) below). They must therefore have both positive (or both negative) terminals connected together. Also, in order for the circuit to have a balance point, E_1 must be greater than E_2.  (a) No current if $E_1 = E_2$  (b) current flows

14.7.3 Types of Potentiometer Circuit Questions

Type A

When the potentiometer is balanced,

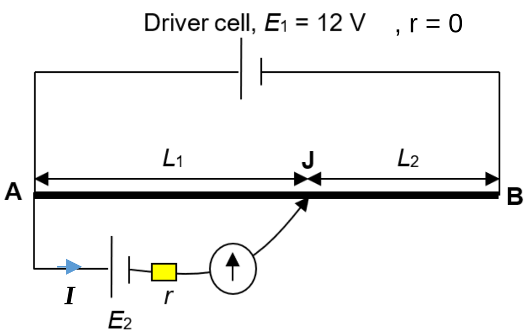
p.d. across **AJ** (balance length) = e.m.f. of unknown p.d. source/ test cell



Examples of some type A circuits:

a) Test cell with internal resistance *r*.

Illustration:



Note:
- $I = 0$ at balance
- Potential gradient $k = \frac{12\ V}{1\ m} = 12$ V m^{-1}

$E_2 - I\,r = V_{AJ}$

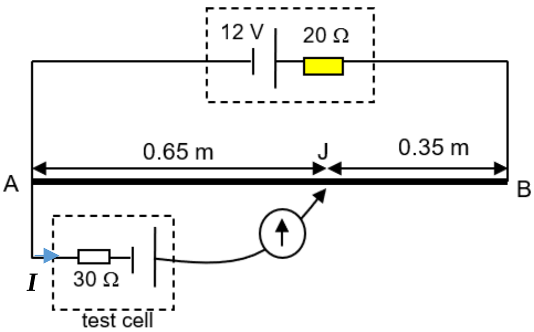
When the potentiometer is balanced, I in secondary cct = 0,

thus $E_2 = V_{AJ}$ = p.d. across AJ

Note: presence of internal resistance r does not affect the balance length L_1.

b) Driver cell with internal resistance *r*.

Illustration:



Note:
- $I = 0$
- Potential gradient $k \neq 12$ V m^{-1}

In the circuit shown, the potentiometer wire has a resistance of 60 Ω. Determine the e.m.f. of the test cell if the balance point is at J.

Solution:

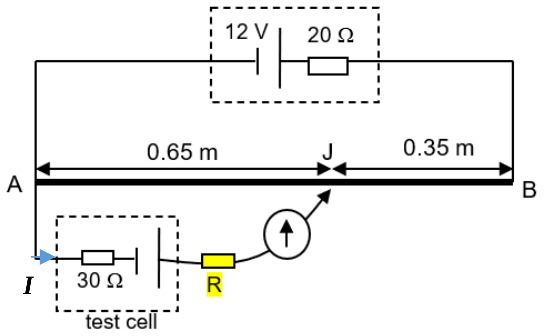
Resistance of wire AJ = $\frac{0.65}{0.65 + 0.35} \times 60 = 39\ \Omega$

$\Rightarrow$ E.m.f. of test cell = $V_{AJ} = \frac{39}{60 + 20} \times 12 = 5.85$ V (using potential divider rule)

Note this r !

c) Test cell with resistor R in series.

Illustration:



Note:
- I = ...??
- Potential gradient $k \neq 12$ V m^{-1}

Compared with circuit (b) above, will the balance length change if a resistor of R is connected in series with the test cell? Why?

Solution:

Balance length will …………….

…….. current flows through the resistor R at balance point. Hence, there is no p.d. drop across R (and 30 Ω) .

E.m.f. of test cell is ……………..

Type B

For this type, the test cell has a 'complete circuit' of its own. Hence, there is always a current through its internal resistance, even at balance point.

Note that it is now the terminal p.d. of the test cell (not its e.m.f. since part of the e.m.f. is dropped across its internal resistance) that is being balanced.

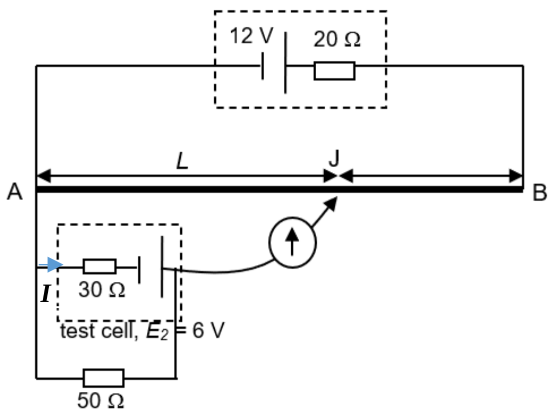
i.e. p.d. across **AJ** (balance length) = terminal p.d. of unknown p.d. source

$= E_2 - I\,r$

Examples of some type B circuits:

a) Test cell with its own circuit

Illustration:



In the circuit above, the potentiometer wire has a resistance of 60 Ω. Determine the balance length *L*.

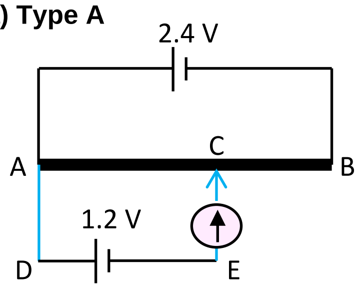
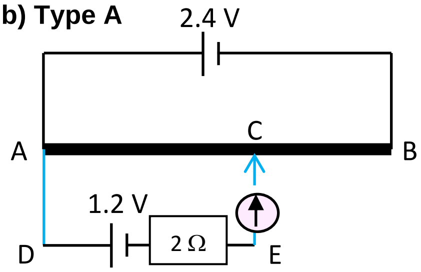
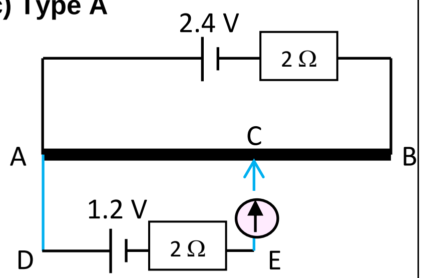
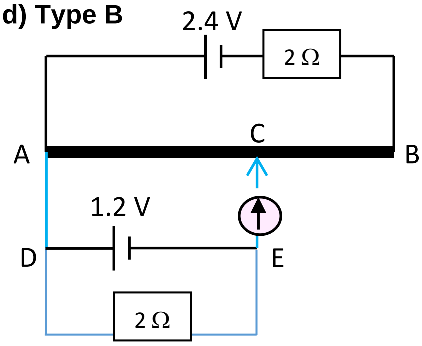
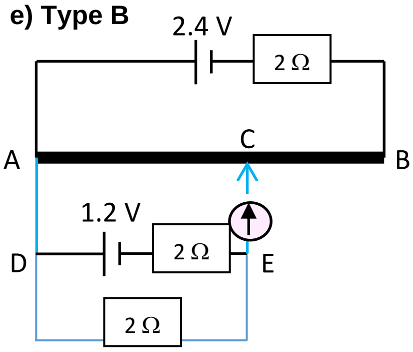
Solution:

Resistance of wire AJ =

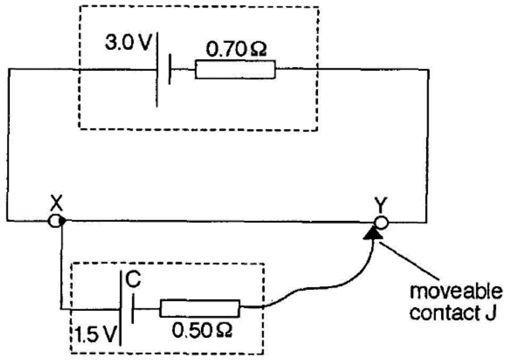
	V_{AJ} = terminal p.d. across test cell = = = $\Rightarrow L =$ Note: The upper 'main circuit/loop' is effectively independent of the lower 'secondary circuit/loop' as there is no current flowing between them.

Watch this 9-minute video to for illustrations of potential dividers and potentiometer. You may scan the QR code or go to https://tinyurl.com/y8g8ydrz

Additional Exercises on potentiometer questions:

a) Type A  	AB has a resistance of 10 Ω and length of 100 cm. The galvanometer is undeflected. Determine the length AC. Can you solve this mentally? [Ans: 50 cm]
b) Type A  	AB has a resistance of 10 Ω and length of 100 cm. The galvanometer is undeflected. Determine the length AC. Can you solve this mentally? [Ans: 50 cm]
c) Type A  	AB has a resistance of 10 Ω and length of 100 cm. The galvanometer is undeflected. Predict how the balanced length AC would be compared to that in part (b)? (Solve mentally). Determine the length AC. [Ans: 60 cm]
d) Type B  	AB has a resistance of 10 Ω and length of 100 cm. The galvanometer is undeflected. Predict how the balanced length AC would be compared to that in part (c)? (Solve mentally). Determine the length AC. [Ans: 60 cm]
e) Type B  	AB has a resistance of 10 Ω and length of 100 cm. The galvanometer is undeflected. Predict how the balanced length AC would be compared to that in part (d)? Determine the length AC. [Ans: 30 cm]

Worked Example 16
The diagram below shows a simple potentiometer circuit. The length and resistance of the wire XY is 1.2 m and 1.4 Ω respectively. [N02/P2/Q6]



The moveable contact J can be connected to any point along the wire XY.

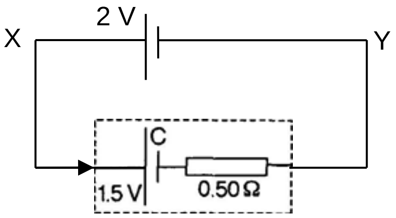
(i) Initially, the contact J is connected to end Y. The P.D. across the wire XY is then equal to the P.D. applied across cell C. On the figure above, mark with an arrow the direction of the current through the cell C.

(ii) Determine the position of the contact J on XY such that there is no current through the cell C.

(iii) To improve accuracy, the position found in **(ii)** should be nearer to end Y. Suggest one way in which the circuit above may be modified so that this is achieved.

Solution:

(i) Pd across **XY** = $\left(\frac{1.4}{1.4 + 0.7}\right)$(3 V) = 2 V, hence an equivalent circuit would be :



As seen from diagram, V_{XY} which is 2 V is larger than the emf of cell C, which is 1.5 V. Hence, the current will flow from the **XY** to cell C in an anti-clockwise manner, and the arrow drawn is from left to right entering cell C.

(ii) If there is no current through cell C, the potential difference **XJ** should be equal to the pd across cell C, which is 1.5 V. Hence, the balance point is the length **XJ** such that V_{XJ} = 1.5 V.
Since length **XY** = 1.2 m and corresponds to 2.0 V, length **XJ** would correspond to 1.5 V when it is of length 0.9 m.

(iii) Remember that this question wants the action suggested, hence, an answer such as 'increase the resistance of **XY**' would not be a practical answer. An answer must indicate the exact action to be carried out. In theory, to increase the balance length, we would want to reduce V_{XY} so that a larger length is required to get the voltage required.

1. Change the material of the wire to one with a lower resistivity.
2. Add a resistor to the main circuit so that the potential difference XY is smaller.

For 2., if this question carries high marks, you should go further to indicate what is the limit of the resistance that could be added in series to the main circuit. In this case, the limit is 0.70 Ω because any more than that will make it impossible to get a balance point as V_{XY} would have been lower than the emf of cell C.

Tutorial qn: Q14 to Q17

SUMMARY

1. **Principle 1 (Kirchoff's current law):** The sum of currents entering any junction in an electric circuit is always equal to the sum of currents leaving that junction.

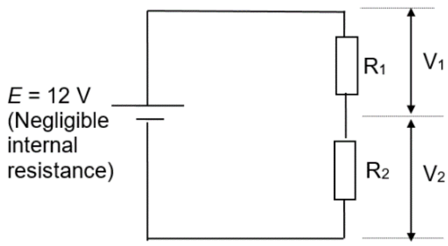
   Principle 2 (Kirchoff's voltage law) : In any closed loop in a circuit, the net electromotive force equals to sum of potential drops around the loop.

2. Formula for the combined resistance of two or more resistors in:

   - **Series** - $R_c = R_1 + R_2 + \ldots + R_N$
   - **Parallel** - $\frac{1}{R_c} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$

   or $R_c = \frac{R_1 R_2}{R_1 + R_2}$ i.e. 'product over the sum' [ONLY for 2 in parallel]

3. A **potential divider** is a circuit in which two (or more) resistors, connected in series with a supply voltage (e.m.f.), "divide" the e.m.f. proportionally according to their resistance.



   General formula for the p.d. V_R across a resistor of resistance R can be deduced to be:

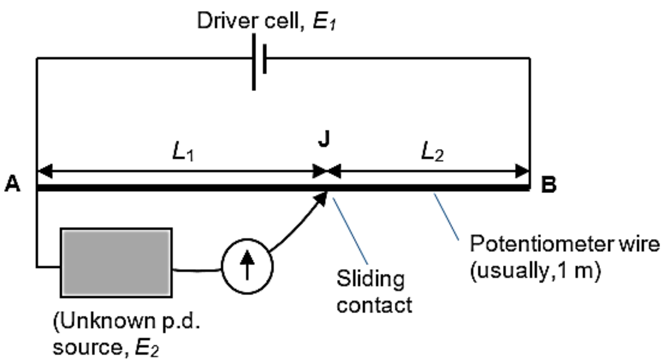
   $$V_R = \frac{R}{R_{eff}} \times total\ p.d. across\ all\ resistors$$

4. A **thermistor** is a resistor whose resistance varies with temperature. For a NTC or Negative temperature coefficient thermistor, its resistance varies inversely with the temperature.

   An **LDR** (Light dependent resistor) is a resistor whose resistance varies with the intensity of light falling on it. Its resistance decreases with increasing light intensity.

5. The **potentiometer** is a device used to compare p.d. or e.m.f. where $E_2 = \frac{L_1}{L_1 + L_2} \times E_1$.

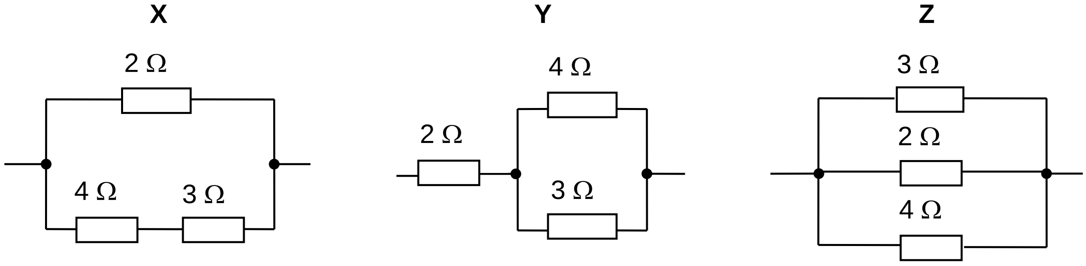
   Refer to lecture for type A and type B circuits.



TUTORIAL 14: DIRECT CURRENT CIRCUITS

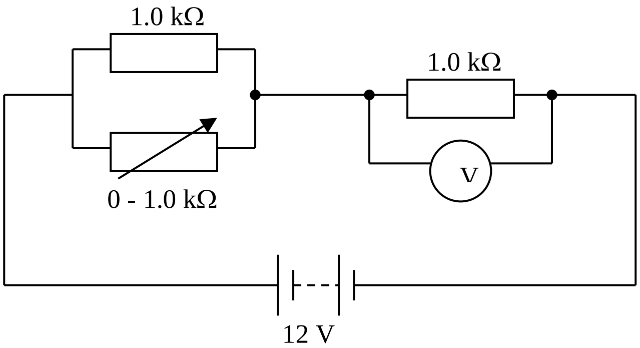
Combined resistances

(L1) 1 Three resistors of resistance 2 Ω, 3 Ω and 4 Ω respectively are used to make the combinations **X, Y** and **Z** shown in the diagrams. List the combinations in order of increasing resistance.



[N07/P1/29]

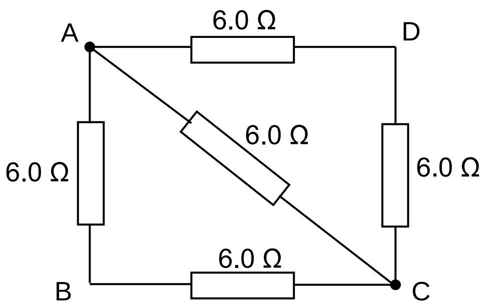
(**L2) 2** The diagram shows a resistor network connected to a 12 V battery of negligible internal resistance. The variable resistor has the range indicated, and the voltmeter has infinite resistance.



Calculate the maximum and minimum possible values of the voltmeter reading as the variable resistor is altered.

[Modified N01/P2/4]

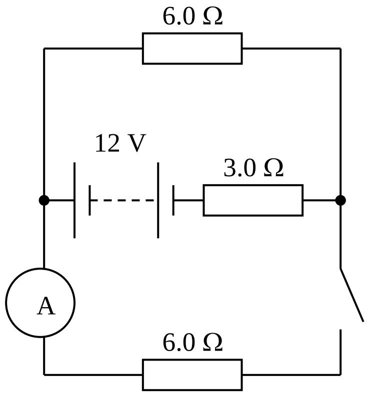
(L2) 3 A network of resistors, each of resistance 6.0 Ω, is constructed as shown below. Determine the total resistance of the network between **(a)** terminals A and C, and **(b)** terminals A and D. [4]



Kirchhoff's Laws

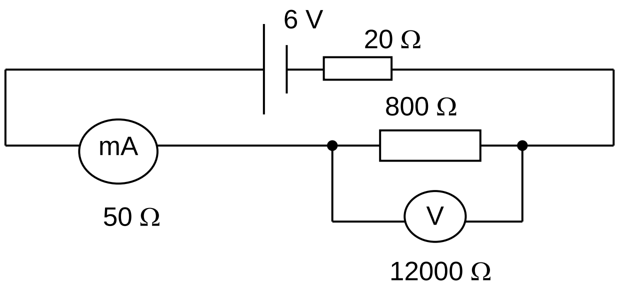
[N07/P1/28]

(L1) 4 In the circuit, the battery has an e.m.f. of 12 V and an internal resistance of 3.0 Ω. The ammeter has negligible resistance.

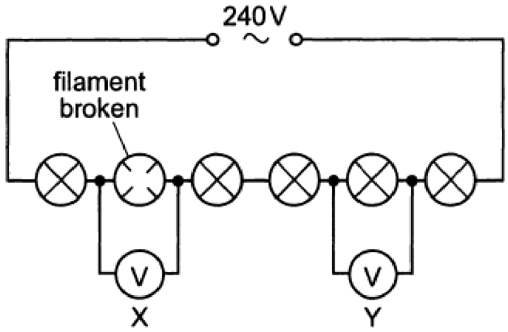


The switch is closed.
Calculate the reading on the ammeter.

(L2) 5 A circuit is set up as shown to measure the PD across and current through an 800 Ω device. The battery, ammeter and voltmeter have an resistance of 20 Ω, 50 Ω and 12000 Ω respectively. Determine the reading of the ammeter and the voltmeter. [3]



(L2) 6 [H1 N09/1/24]
A mains circuit contains six similar lamps connected in series. One of the lamps has a broken filament. Voltmeters X and Y of infinite resistance are placed in the circuit as shown.

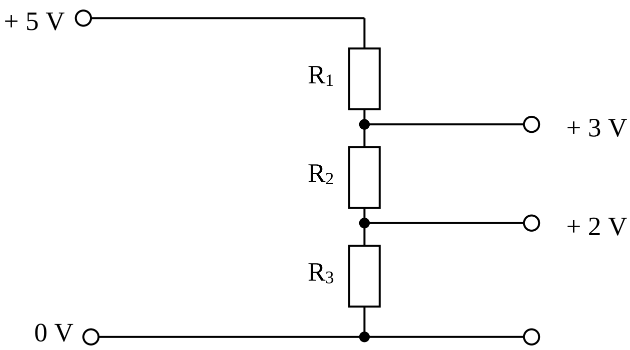


What are the voltmeter readings?

	X reading / V	Y reading / V
A	0	0
B	0	240
C	40	40
D	240	0

Potential Divider

[J96/P1/15]
(L1) 7 A potential divider is used to give outputs of 2 V and 3 V from a 5 V source, as shown.

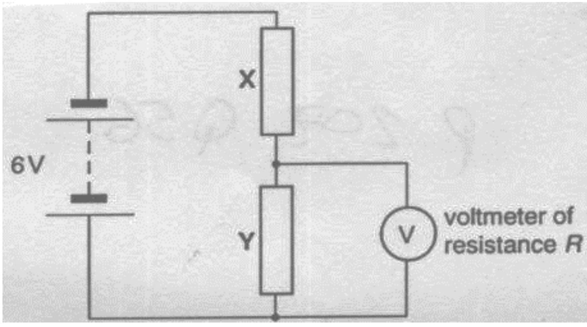


Which combination of resistances R_1, R_2 and R_3 gives the correct voltages?

	R_1 / kΩ	R_2 / kΩ	R_3 / kΩ
A	1	1	2
B	2	1	2
C	3	2	2
D	3	2	3

[N96/P1/14]

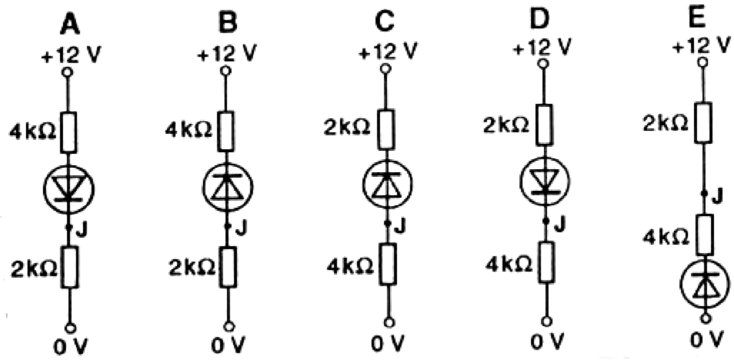
(L1) 8 In the circuit shown, resistors X and Y, each of resistance R, are connected to a 6 V battery of negligible internal resistance. A voltmeter, also of resistance R, is connected across Y. Determine the reading of the voltmeter.



[N92/P1/13]

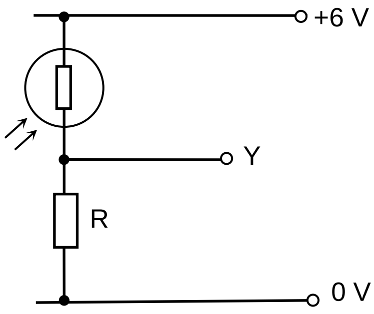
(L1)9 Assume the diodes shown in the diagrams are ideal (i.e. they have zero resistance in the forward direction and infinite resistance in the reverse direction).

In which of the component arrangements is the potential at J equal to 8 V?



As an exercise, work out the potential at **J** for the other four options.

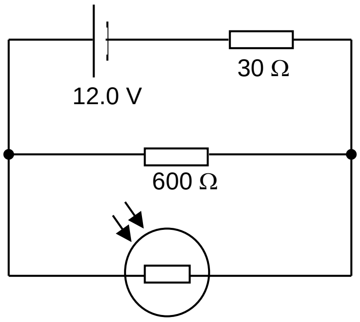
(L2)10 The light dependent resistor (LDR) in the circuit shown below has a resistance of 1200 Ω when illuminated, and a resistance of 10 MΩ when in the dark.



Determine the value R of a suitable resistor which can be placed in series with it so that the potential at Y changes from zero to 2 V when the LDR is illuminated. [2]

[N06/III/6 mod]

(L2)11 **(a)** A light dependent resistor (LDR) is placed in parallel with a 600 Ω resistor and connected to a 12.0 V battery of internal resistance 30 Ω as shown below.



(i) In conditions of low intensity light, the resistance of the LDR is 3000 Ω. Calculate

(1) the current through the LDR.

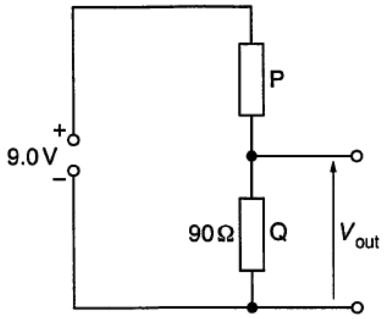
(2) the power dissipated in the LDR. [3]

(ii) Accidentally, the LDR is exposed to sunlight and its resistance falls to 100 Ω. Discuss whether the LDR, which is marked 0.5 W, will be damaged. [3]

(b) Draw a labelled diagram showing how a potentiometer could be connected to the circuit in the figure above to compare the potential differences across the LDR in (c) for different levels of illumination. [2]

[N08/P2/Q6a]

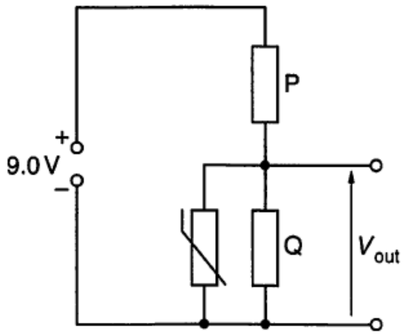
(L2)12 (a) Two resistors P and Q are connected in series to a power supply of e.m.f. 9.0 V and negligible internal resistance to form a potential divider, as shown below.



The resistance of Q is 90 Ω. The current in P is 27 mA. The potential difference across Q is V_{out}.

(i) Calculate V_{out}. [1]

(ii) An NTC thermistor is now connected in parallel with Q, as shown below. The resistance of the thermistor is 120 Ω.

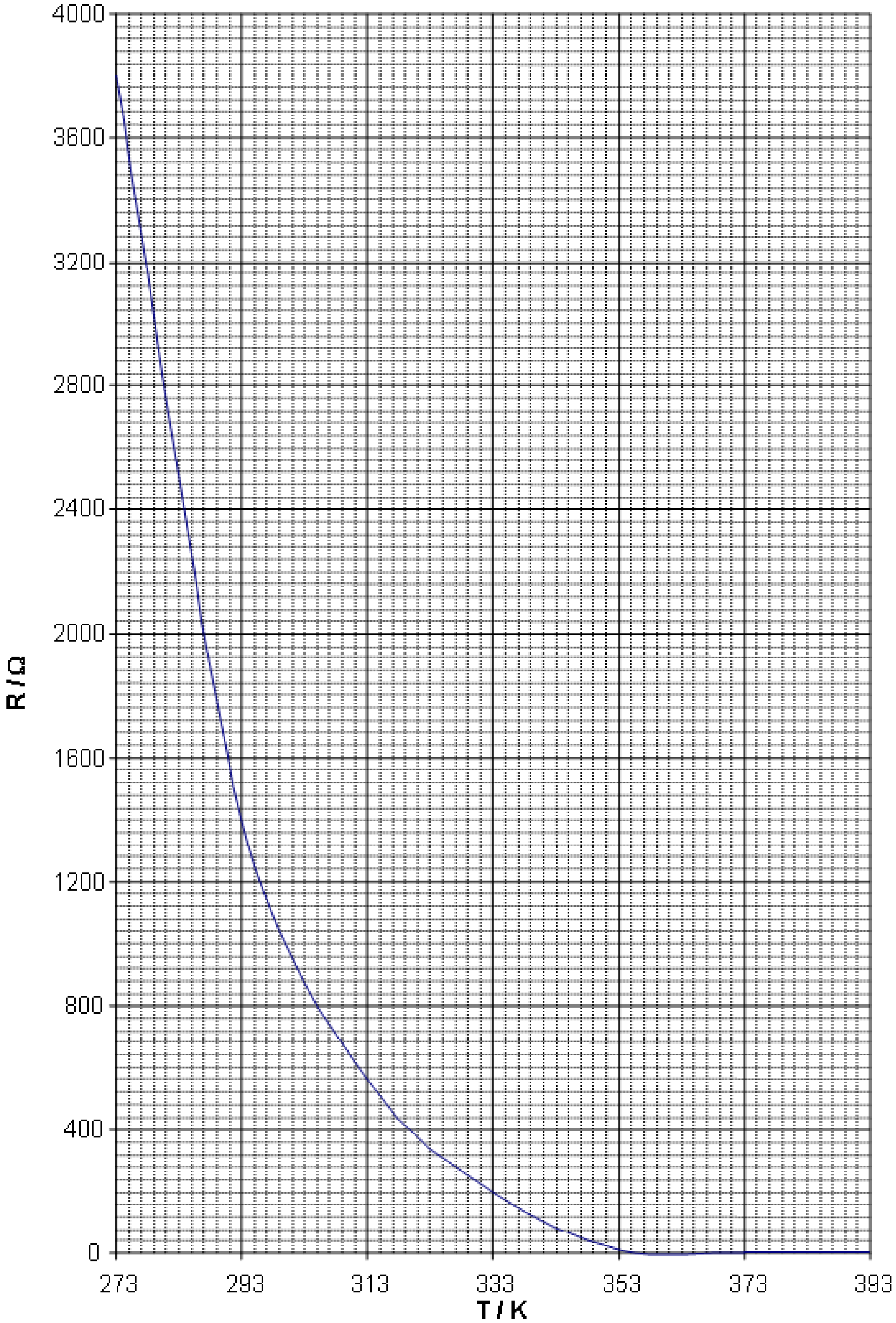


Calculate the new value of V_{out}. [3]

(iii) The temperature of the thermistor is increased. State and explain the change, if any, to the value of V_{out}. [3]

[N06/II/4 mod]

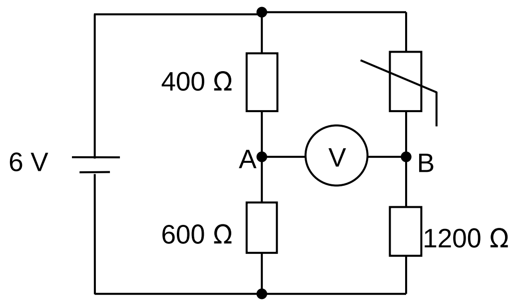
(L2) 13 (b) The variation with thermodynamic temperature T of the resistance R of a thermistor is shown below.



(i) Suggest, with a reason, why the thermistor, used as a thermometer, is more appropriate for measuring temperature in the range 273 K to 293 K than in the range 313 K to 333 K. [2]

(ii) Use the graph to determine the temperature corresponding to a thermistor resistance of 1500 Ω when measured on the thermodynamic scale of temperature. [1]

(c) The thermistor is connected into the circuit below.



The voltmeter connected between A and B has infinite resistance. The battery has e.m.f. 6.00 V and negligible internal resistance.

(i) Determine the thermodynamic temperature at which the voltmeter reads zero. Explain your working. [3]

(ii) The temperature of the thermistor is now changed and the voltmeter reads 1.20 V.

(1) Suggest why the thermistor could be at one of two different thermodynamic temperatures. [2]

(2) Calculate the lower of these two thermodynamic temperatures. [3]

Potentiometer

(L1) 14 A potentiometer is used to measure the e.m.f of a Cell A. The potentiometer consists of a driver cell of e.m.f. of 20 V and negligible internal resistance and a 1-metre resistance wire.
The balance length is 66 cm when the Cell A is connected to the lower circuit.
The balance length is 90 cm when a standard cell of e.m.f. 15 V is connected to the lower circuit.
Calculate the e.m.f. of Cell A. [2]

(L2)15 A potentiometer circuit is made up of a 2.00V cell in series with a resistance R and a uniform wire AB of length 1.000 m and resistance 2.00 Ω.

(a) A thermocouple of emf 4.00mV is connected through a galvanometer when the slider is exactly at the mid-point of AB. What is the value of R? [2]

(b) If now the resistance R is increased by 10 Ω, how far would the slider have to be moved to obtain zero current in the galvanometer again? [2]

[SAJC 2012 BT1 Question 25]

(L2) 16 **(a)** Explain why potentiometer can give a more accurate reading of the emf of a cell compared to using a voltmeter. [2]

(b) Fig. 12.1 below shows a potentiometer which shows a balance point at **Z** when switches S_1 and S_2 are *closed*.

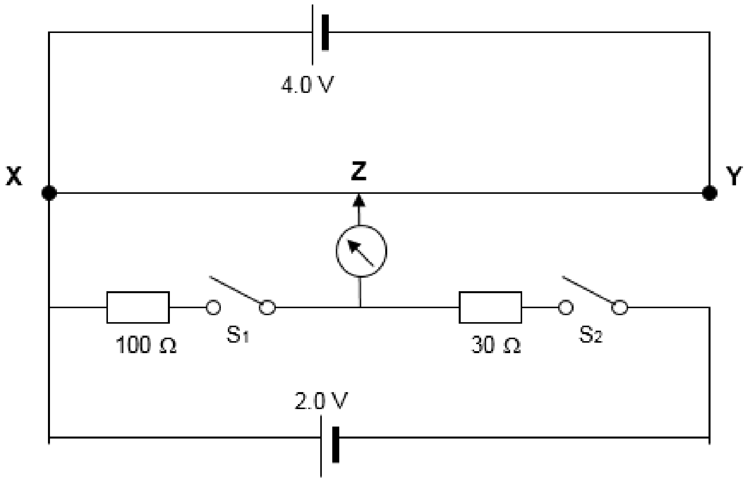


Fig. 12.1

(i) Assuming that both cells have negligible resistance, determine the potential difference across **XZ**. [2]

(ii) Hence, find the balance length **XZ**, if wire **XY** is 100 cm long. [2]

(iii) Determine the new balance length **XZ'** when switch S_2 is closed and S_1 is open. [2]

(iv) Determine, for the new balance length **XZ'**, the current through the 30 Ω resistor when the switch S_2 is closed and S_1 is open. [1]

(v) State and explain one way to improve the potentiometer measurements above. [2]

[SAJC 2021 BT1 Q23]

17 The variation with potential difference V of the current I in a filament lamp is shown in Fig. 17.1.

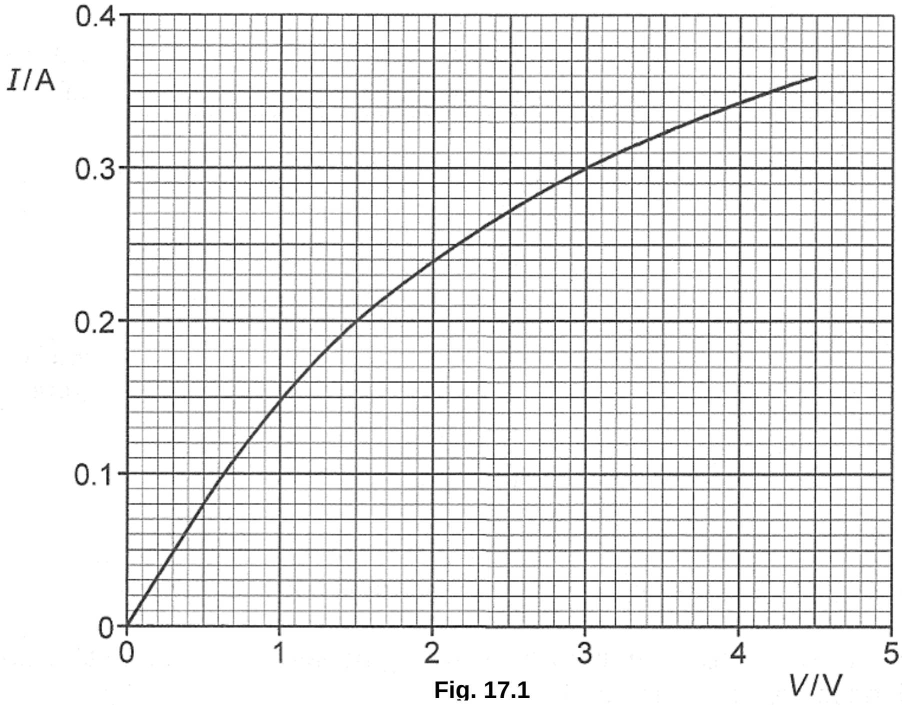


Fig. 17.1

(a) **(i)** Indicate with a cross (x) on Fig. 17.1 where the resistance of the lamp is 7.5 Ω. [1]

(ii) Calculate the maximum power dissipated by the lamp based on the data given in Fig. 17.1.

(b) Explain in terms of the movement of the charged particles how the resistance of the lamp varies with potential difference V.

[3]

(c) The lamp is connected into the circuit shown in Fig. 17.2.

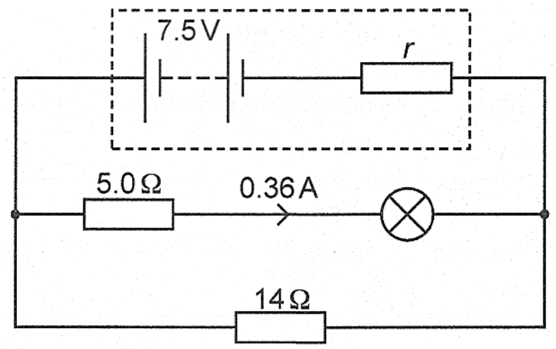


Fig. 17.2

The battery in the circuit has an electromotive force *E* of 7.5 V and internal resistance *r*. The current in the resistor of resistance 5.0 Ω is 0.36 A.

Using the graph or otherwise, determine:

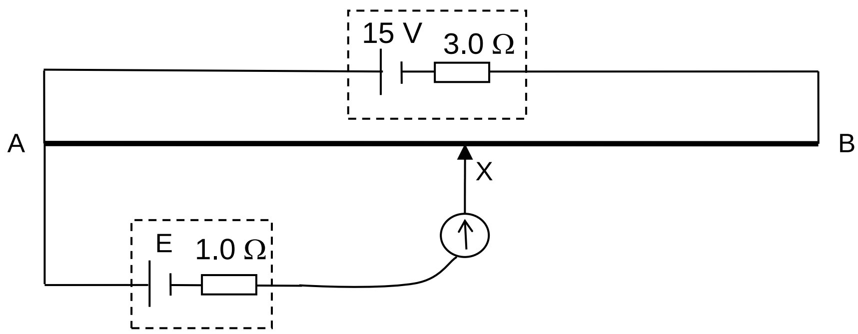
(i) the current in the resistor of resistance 14 Ω

[2]

(ii) the internal resistance r of the battery.

[2]

(d) A potentiometer circuit is set up as shown in Fig. 17.3. The resistance wire AB is 1.2 m long and has a total resistance of 12 Ω. The 15 V cell has an internal resistance of 3.0 Ω.



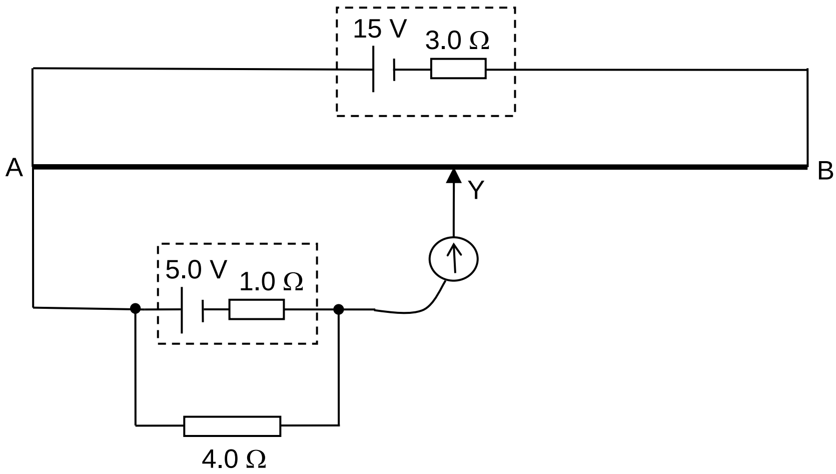
(i) Determine the e.m.f. of cell, *E*, if the balance length AX is 0.50 m.

[2]

(ii) Describe and explain one way in which the accuracy of the experiment to determine the e.m.f. of the cell, *E*, can be improved.

[2]

(iii) The potentiometer circuit using the same resistance wire AB is set up as shown in Fig 23.4. Determine the balance length AY.



[2]

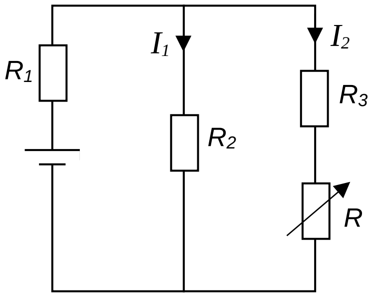
Numerical Answers

1) Z X Y
2) 12 V (max), 8 V (min)
3) (a) 3.0 Ω, (b) 3.75 Ω
4) 1 A
5) 7.32 mA, 5.49 V
8) 2 V
10) 600 Ω
11 (a)(i) 1) 3.77 mA 2) 0.0427 W
12(a)(i) 2.43 V (ii) 1.57 V
13 (b)(ii) T = 292 K (c)(i) 305 K (ii) 2) 289 K
14) 11 V
15) 498 Ω, 1.0 cm towards B
16) (b) (i) 1.54 V (ii) 38.5 cm (iii) 50.0 cm (iv) 0
17) (a) (ii) 1.6 W (c) (i) 0.45 A (ii) 1.5 Ω (d) (i) 5 V (iii) 0.4 m

ADDITIONAL QUESTIONS

SAJC 2017 BT2

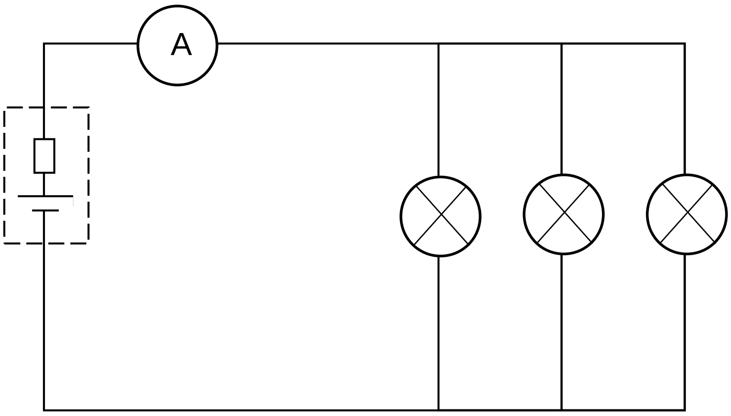
1 In the circuit below, R_1, R_2 and R_3 are fixed resistors and R is a variable resistor.



As R decreases,

A I_1 remains unchanged, I_2 increases
B I_1 decreases, I_2 increases
C I_1 increases, I_2 decreases
D I_1 increases, I_2 increases

2 Three identical light bulbs are connected to a constant voltage d.c. supply with internal resistance as shown in the diagram. Each bulb operates at normal brightness and the ammeter (of negligible resistance) registers a steady current.



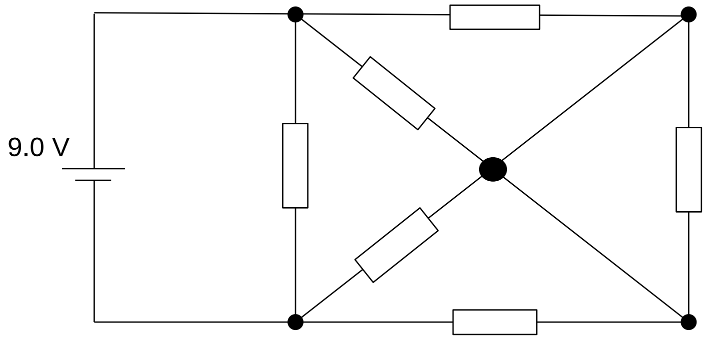
The filament of one of the bulb breaks.

What happens to the ammeter reading and to the brightness of the remaining bulbs?

	ammeter reading	bulb brightness
A	increases	decreases
B	increases	increases
C	decreases	increases
D	decreases	unchanged

SAJC 2019 Prelim

3 A cell of e.m.f. 9.0 V with negligible internal resistance is connected to 6 identical resistors, each of 10.0 Ω as shown below.

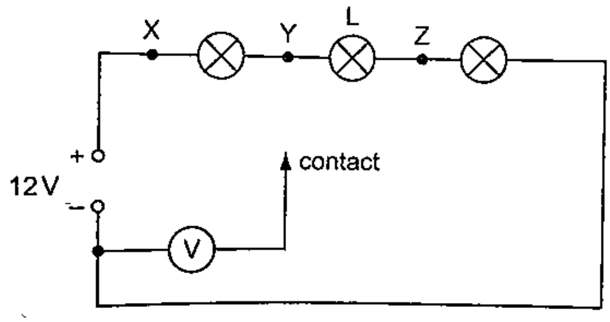


What is the current flowing through the cell in diagram above?

A 1.35 A **B** 1.80 A **C** 4.00 A **D** 5.45 A

2005/1/Q23

4 The diagram shows three lamps in series with a 12 V supply.



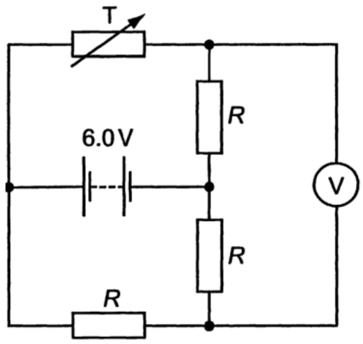
To test the circuits, the contact is connected in turn to points X, Y and Z. The lamps **do not light** because lamp L has a broken filament.

Which line of the table below shows the readings of the voltmeter?

	reading at X	reading at Y	reading at Z
A	12 V	8 V	4 V
B	8 V	8 V	0 V
C	12 V	12 V	0 V
D	8 V	12 V	4 V

2008/1/Q29

5 A battery of e.m.f. 6.0 V and negligible internal resistance is connected to three resistors, each of resistance *R*, and a variable resistor *T*, as shown.



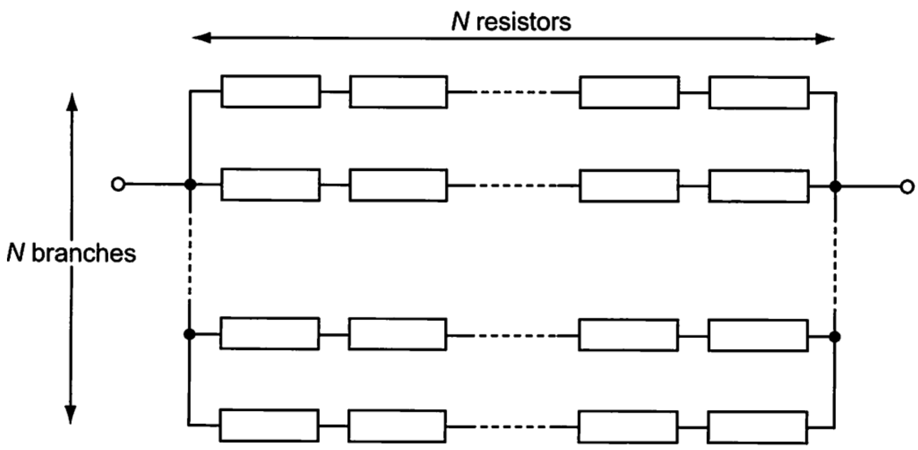
The resistance of T changes from *R* to 5*R*.

What is the change in the reading of the high resistance voltmeter?

A zero **B** 2 V **C** 4 V **D** 5 V

2009/1/Q28

6 An array of resistors, each of resistance *R*, consists of *N* parallel branchers. Each branch contains *N* resistors connected in series.



What is the total resistance of the array?

A $\dfrac{1}{RN}$ **B** $\dfrac{R}{N}$ **C** R **D** NR

SAJC Prelim 2021/2/Q5(c)

7 A potentiometer is setup as shown in Fig. 1.1. A resistive wire of 1.0 m is connected between point B and point D.

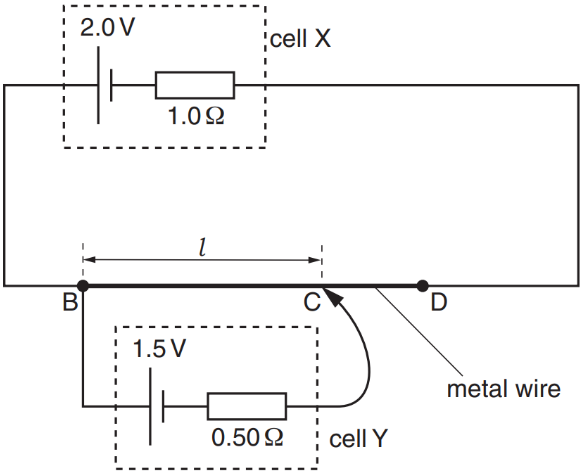


Fig. 1.1

When the length l is set at 93.75 cm, the current in cell Y is zero.
Two resistors are added to the potentiometer circuit, as shown in Fig. 1.2.

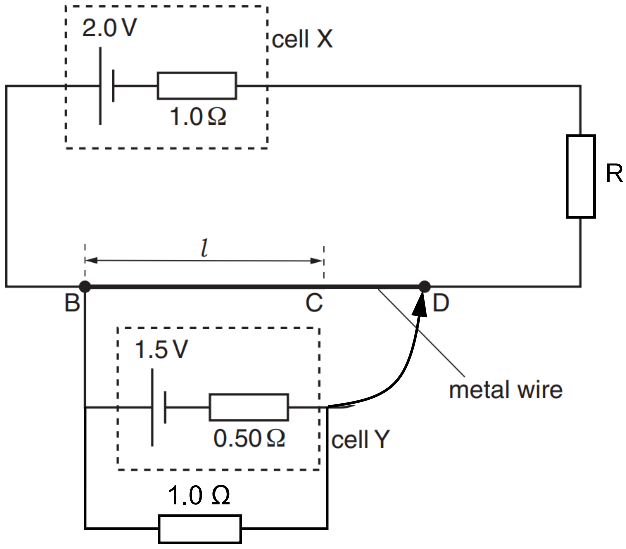


Fig. 1.2

Calculate the value of R such that the balance point of the circuit will be at point D.

R = …………………………….. Ω [4]

2010/3/Q4

8 A circuit consists of three resistors R_1, R_2 and R_3, and two switches A and B, as shown in Fig. 8.1.

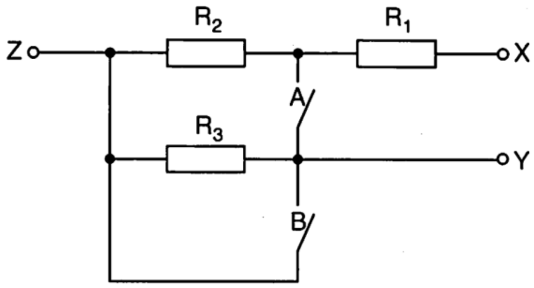


Fig. 8.1

The resistance between terminals X and Y is measured for different settings of the switches A and B. The results are shown in Fig. 8.2.

switch A	switch B	resistance between X and Y / Ω
open	open	12
open	closed	10
closed	open	6
closed	closed	6

Fig. 8.2

(a) Determine the resistance of

(i) resistor R_1,

resistance = ………………….. Ω [1]

(ii) resistor R_2,

resistance = ………………….. Ω [1]

(iii) resistor R_3,

resistance = ………………….. Ω [1]

(b) Switch A is now closed and switch B is open.

Calculate the resistance between terminals X and Z.

resistance = ………………….. Ω [2]

TUTORIAL 14: DIRECT CURRENT CIRCUITS

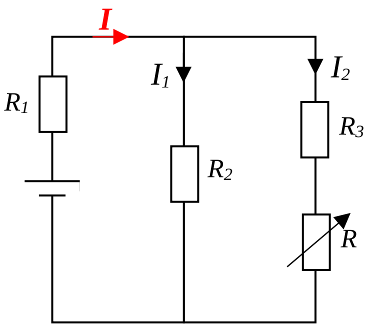
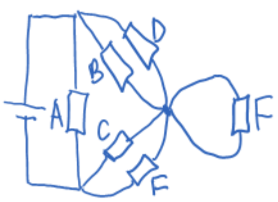
Level 1 Solutions

1	Resistance for X = $(\frac{1}{2}+\frac{1}{4+3})^{-1}$ = 1.56 Ω Resistance for Y = $2+(\frac{1}{4}+\frac{1}{3})^{-1}$ = 3.71 Ω or $\frac{4x3}{4+3}$ +2 Resistance for Z = $(\frac{1}{3}+\frac{1}{2}+\frac{1}{4})^{-1}$ = 0.923 Ω Therefore, in order of increasing resistance is **<u>Z X Y</u>**.	
4	**Method 1:** Total R = 3 + $(\frac{1}{6}+\frac{1}{6})^{-1}$ = 6.0 Ω I = V / R = 12 / 6.0 = 2.0 A Hence the ammeter reading is 1 A. **Method 2:** 12 – I (3.0) = 0.5 (I) (6.0) → I = 2 A Hence the ammeter reading is 1 A. Note: The total current supplied by the battery will be split equally at the junction.	
7	**Ans: B** Let current flowing through R_1, R_2 and R_3 be *i*. $(R_1 + R_2 + R_3)$ *i*. = 5 V (1) $(R_2 + R_3)$ *i*. = 3 V (2) (R_3) *i*. = 2 V (3) From (2) / (3), $\frac{R_2+R_3}{R_3}=\frac{3}{2}$ → $\frac{R_2}{R_3}=\frac{1}{2}$ Subst (2) into (1), (R_1) *i*. = 2 V → $R_1 = R_3$	
8	Total R = R + $\left(\frac{1}{R}+\frac{1}{R}\right)^{-1}$ = R + 0.5 R = 1.5 R V = $\frac{0.5R}{1.5R}\times 6$ = 2 V Hence the voltmeter reading is 2 V.	[1] [1]
9	**Ans: D** For Option A: VJ = $\frac{2\,k\Omega}{2\,k\Omega+4\,k\Omega}$ x 12 V = 4 V For Option B: Diode is in reverse direction, therefore no current. For Option C: Diode is in reverse direction, therefore no current. For Option D: VJ = $\frac{4\,k\Omega}{2\,k\Omega+4\,k\Omega}$ x 12 V = 8 V For Option E: Diode is in reverse direction, therefore no current.	

14	$E_A = \frac{l_A}{l_S} E_S = \frac{66}{90}(15) = 11\text{ V}$	[2]

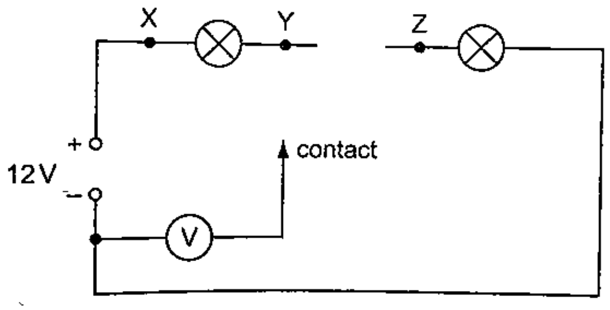
- End of tutorial solutions -

Solutions to Additional Questions

1	**Answer: B** Since *R* decreases, overall resistance of the circuit decrease. With the same EMF, the total current ***I*** will increase.  Therefore, the potential drop across R_1 increases. Therefore, the potential drop across R_2 (EMF – V_{R1}) decreases. Therefore, the current I_1 decreases. Since total current ***I*** will increase as discussed earlier, therefore, I_2 must increase.	
2	**Answer: C** Since 1 filament breaks, therefore the parallel circuit is reduced by 1 branch. Hence, overall resistance increases (for parallel circuit in general, the more branches, the lower the resistance) Therefore, overall current decreases. Therefore, the potential drop across the internal resistor decreases. Therefore, the terminal p.d. (which is equal to the p.d. across each lamp) increases. Since, power = V^2/R, power delivered to each bulb has increased, brightness increases.	
3	**Answer: B** Effective resistance of B and D = effective resistance of C and E = (10 x 10)/(10 +10) = 5.00 Ω  Total effective resistance = $(\frac{1}{10} + \frac{1}{10})^{-1}$ = 5.00 Ω Current through cell = V/I = 9.0 /5.00 = 1.80 A	

4

Answer: C

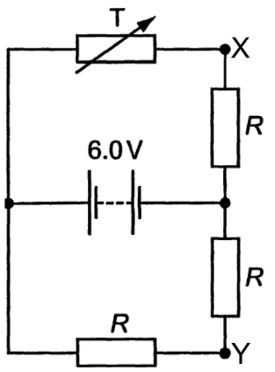


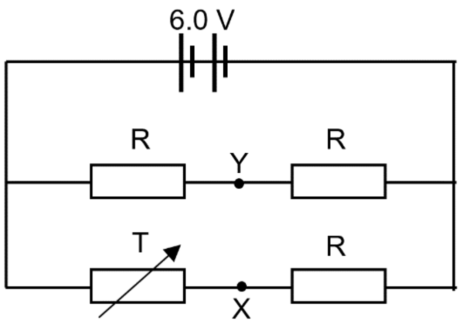
Since bulb L has a broken filament, the circuit is broken and no current flows. Hence,

- No pd across X and Y. Potential at Y = X = potential at +ve terminal.
- Therefore, p.d. reading at X = 12 V = p.d. reading at Y
- Since no current flows, potential Z = potential at -ve terminal. Therefore, p.d. reading at Z = 0 V.

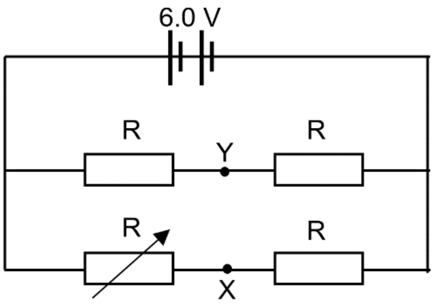
5

Answer: B

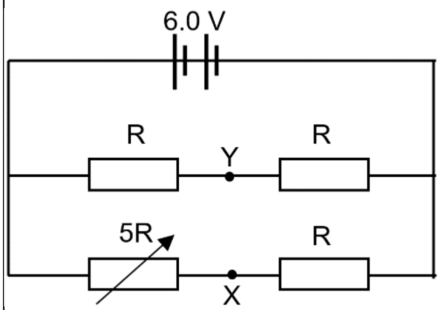




It is easier to solve the question by removing the voltmeter and just keep in mind that we are looking at p.d. between X and Y. After that, we can redraw the circuit such that it make it easier for us to understand the circuit (on the right).



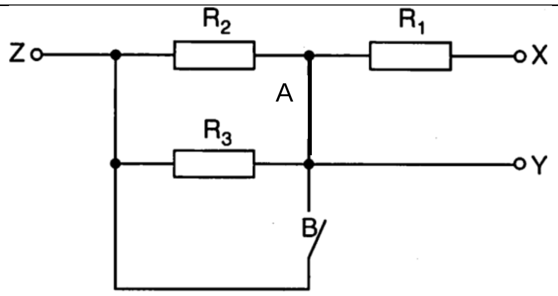
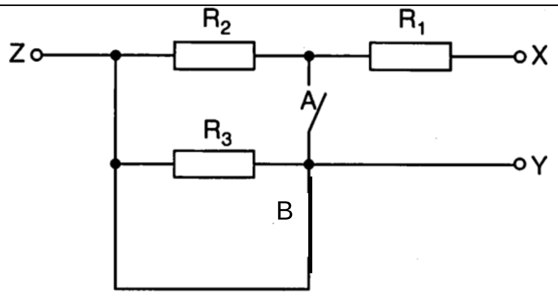
When T = R, p.d. across X and Y = 0 (since p.d. between +ve terminal to Y = p.d between +ve terminal to X = 3.0 V)

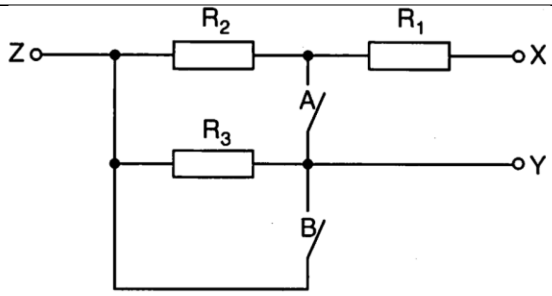


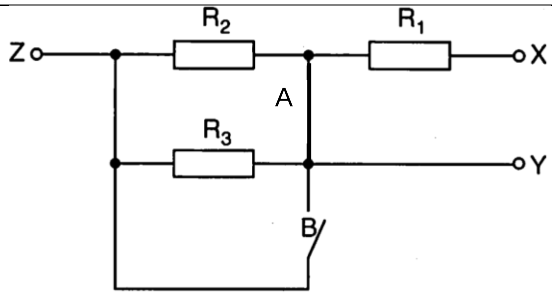
When T = 5R,

- p.d. across +ve to Y = 3.0 V
- p.d. across +ve to X = 5/6 *6 = 5.0 V

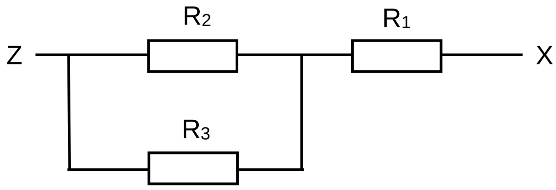
Hence, p.d. across X and Y when T varies from R to 5 R is 5.0 – 3.0 = 2.0 V

6	**Answer: C** Resistance of 1 branch of *N* resistors in series = *NR* Resistance of *N* branch in parallel $= (1/NR + \ldots + 1/NR)^{-1}$ $= (N / NR)^{-1}$ $= R$	
7	From the first circuit, $V_{BC} = 1.5$ V and $V_{BD} = \dfrac{R_{BD}}{R_{BD} + 1.0} \times 2.0$ At balance length, $0.9375 \times V_{BD} = 1.5$ $0.9375 \times \dfrac{R_{BD}}{R_{BD} + 1.0} \times 2.0 = 1.5$ $R_{BD} = 4.0\ \Omega$ From the second circuit, to balance at point D, then $V_{BD} = 1.0 / 1.5 \times 1.5$ $\dfrac{4.0}{4.0 + 1.0 + R} \times 2.0 = 1.0$ Solving, $R = 3.0\ \Omega$	[1] [1] [1] [1]
8(ai)	 When A is closed and B is open, effective resistance between X & Y = $R_1 = 6\ \Omega$ since there is a short circuit that allows current to bypass R_2 and R_3 and goes straight to Y. Therefore, $R_1 = 6\ \Omega$	
(ii)	 When A is open and B is closed, effective resistance between X & Y = $R_1 + R_2 = 10\ \Omega$ since there is a short circuit that allows current to bypass R_3. Therefore, $R_2 = 4\ \Omega$	

<table>
<tr><td>(iii)</td><td>

When both A and B are open,
effective resistance between X & Y = $R_1 + R_2 + R_3 = 12\ \Omega$ since the current has to flow through all 3 resistors to reach Y.
Therefore, $R_3 = 2\ \Omega$</td><td></td></tr>
<tr><td>(b)</td><td>

When A is closed and B is open, the above is the actual circuit.

The circuit can be redrawn as below:


R_2 is parallel to R_3, and their effective resistance is in series with R_1.
Hence, resistance between Z and X = $(1/4 + 1/2)^{-1} + 6 = 7.33\ \Omega$</td><td></td></tr>
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