TOPIC 4: FORCES

Learning Outcomes: Candidates should be able to:

a.	Recall and apply Hooke's Law to new situations or to solve related problems.
b.	Describe the forces on mass, charge and current in *gravitational, *electric and *****magnetic fields, as appropriate. *(ie.* $F_g = mg$, $F_E = qE$, $F_B = Bqv \sin\theta$*)*
c.	Show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance. (No treatment of coefficients of friction and viscosity is required.)
d.	Show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity.
e.	Define and apply the moment of a force and the torque of a couple.
f.	Show an understanding that a couple is a pair of forces which tends to produce rotation only.
g.	Apply the principle of moments to new situations or to solve related problems.
h.	Show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium.
i.	Use a vector triangle to represent forces in equilibrium
j.	Derive, from the definitions of pressure and density, the equation $p = h\rho g$.
k.	Solve problems using the equation $p = h\rho g$.
l.	show an understanding of the origin of the force of upthrust acting on a body in a fluid.
m.	state that upthrust is equal in magnitude and opposite in direction to the weight of the fluid displaced by a submerged or floating object.
n.	Calculate the upthrust in terms of weight of displaced fluid. *(i.e.: Archimedes' Principle)*
o.	recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object to new situations or to solve related problems.

*****To be covered in greater detail in a later topic.

Five broad areas:
- Types of force
- Centre of gravity
- Turning effects of forces
- Equilibrium of forces
- Upthrust

Key questions:
- What are the different type of forces?
- What determines whether a simple object such as a balance beam will rotate?
- What are the conditions for an object in equilibrium?

4.1 Introduction

In chapter 3, you learnt that a *net force* is needed to accelerate an object (Newton's 2nd law). What are *the kind of forces* that can constitute that net force?

In this chapter we will learn more about the following forces:
- normal contact (normal reaction) force,
- frictional force,
- viscous force,
- tension,
- compression
- and upthrust.

We will also learn how forces can act to cause an *equilibrium* of an object (i.e. acceleration being zero).

4.1.1 The Fundamental Forces

There are four basic types of forces that govern the Universe. Those that exist outside of the nucleus are the electromagnetic and gravitational forces.

Force	Effects	Relative strength
Gravitational*	Acts on all masses	much weaker
Weak nuclear	Causes radioactive decay in some atoms {Not in Syllabus}	↓
Electromagnetic*	Acts on electric charges	
Strong nuclear	Binds protons and neutrons in nucleus {Not in Syllabus}	much stronger

*Only the electromagnetic and gravitational forces will be covered in greater detail in the A-Level syllabus.

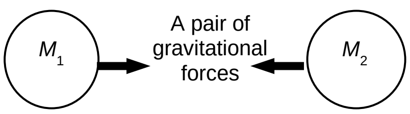
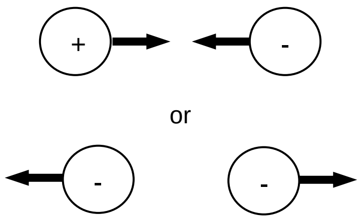
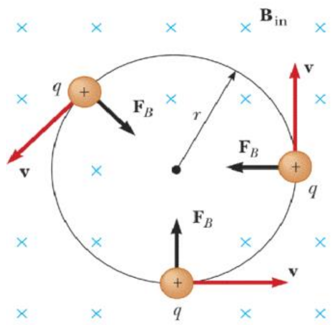
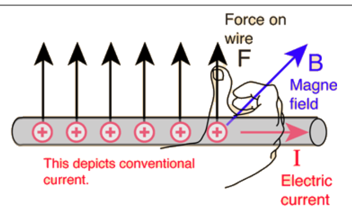
Recall that for action-reaction pair of forces, the two forces must be of the same fundamental force.

The normal contact (normal reaction) force, frictional force, viscous force, tension, compression and upthrust are actually all examples of the electromagnetic force, which arises from interactions between electric charges of the objects involved!

4.1.2 Fields of Force

A field of force refers to a <u>region of space</u> within which a body experiences a force without being in physical contact with any other body. {N16 P3 Q6}

Note: This concept helps us account for the forces between 2 bodies which are <u>not in physical contact</u>. Greater details for these forces would be discussed in later topics.

(1) Gravitational field	(2) Electric field	(3) Magnetic field
What it is…		
A region of space in which a <u>mass</u> experiences an *attractive* force due to the presence of <u>another mass</u>. 	A region of space where a <u>charge</u> experiences an *attractive or repulsive* force due to the presence of <u>another charge</u>. 	A region of space within which a force is experienced by a <u>moving charge</u> or a <u>current-carrying</u> conductor. 
On Earth, this force acting on us is known as our weight. Near the Earth's surface (up to about 1 km from its surface), $W = mg$, where $g = 9.81$ m s^{-2}. (Greater details would be discussed in the topic of Gravitational Field for H2 Physics.)	Whether the force is attractive or repulsive depends on the *polarity* of the charges. Like charges repel and unlike charges attract. (Greater details would be discussed in the topic of Electric Field for H2 Physics.)	 (Greater details would be discussed in the topic of Electromagnetism for both H1 and H2 Physics.)
Direction of force is…		
Gravitational forces always act <u>in the direction</u> of the gravitational field.	Forces due to an electric field always act in a direction <u>parallel</u> to the electric field.	Magnetic force will always be <u>perpendicular</u> to the magnetic field.

4.2 Drawing a Free body Diagram (FBD)

A FBD is a diagram showing <u>all external forces</u> acting <u>ON an isolated body</u>. This body is <u>free</u> because the diagram will show the body without its surroundings; i.e. the body is 'free' of its environment. This eliminates unnecessary information which might be given in a problem.

FBDs are not trivial. Most often, the first step to solve questions relating to forces is to draw an accurate FBD. Hence, the solution of a question depends on your ability to draw a correct FBD(s).

Steps to drawing a FBD: Let's illustrate the steps using the scenario below:

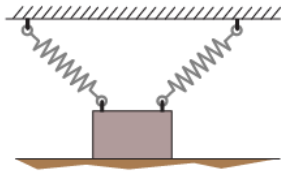
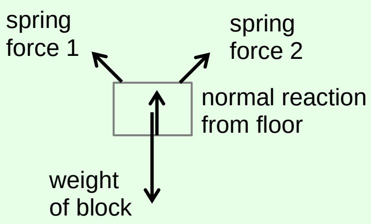
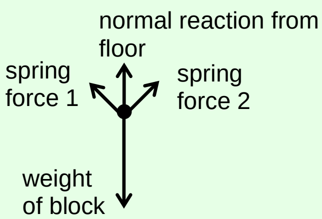


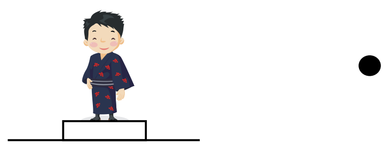
Fig. 1: A block of mass m sits on a floor partially suspended by two springs.

Outline method	Point-mass method
	
• Draw an outline of the object (block). • Draw forces at the location where they act. (This is so that we can represent moments acting on the body)	• Draw object represented as a point.

Note:
- The length of the arrow represents the magnitude of the force.
- Draw all external forces.
- Label the forces in full with meaningful names, not just by abbreviation. Part of the marks is awarded for the correct labelling of the type of forces acting on the body.

Key Learning pt: (1) Always draw FBD, even if not asked to do so in the question!
(2) Draw the magnitude accurately. Notice how magnitudes of the vertical components of the upward forces summed to be equal to the downward forces when object is in equilibrium.

Example 1
Draw the FBD for the following situations.

(a) man who has jumped off the ground	(b) man on a weighing balance
	

Note:
A common mistake for (a) (or any object being thrown off) is to draw in the force exerted by the ground/ hand that throws the object!

4.3 Types of Contact Forces

4.3.1 Normal Contact Force

Consider a stone resting on the ground. As the stone is in contact and is pressing on the ground, there will be a force acting on the stone by the ground. This force is known as **normal contact force/ normal reaction** as it will always be *perpendicular* to the surface upon which it is resting.

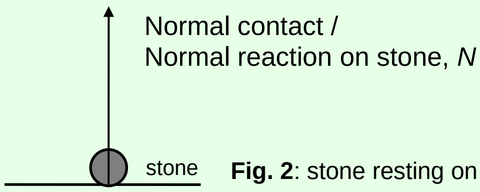


Fig. 2: stone resting on the ground.

In another scenario where a box is resting on a rough slope, normal reaction and friction both exist, and thus the resultant for these two forces is known as **contact** force.

Fig. 3: forces on a box resting on rough slope.

Note: The above is NOT a FBD. It is drawn to illustrate the relationship between contact force and its components (friction & normal reaction). External forces exerting on the box such as weight has been excluded. Unlike normal reaction, contact force in this case is not perpendicular to the surface of the slope.

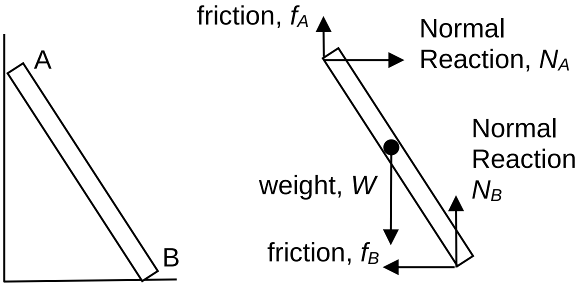
In general, the contact force acting on a ladder at the wall and at the ground has 2 perpendicular components: friction and the normal reaction, where

$$\boxed{\text{Contact force, } C = \sqrt{(\text{friction})^2 + (\text{normal reaction})^2}}$$

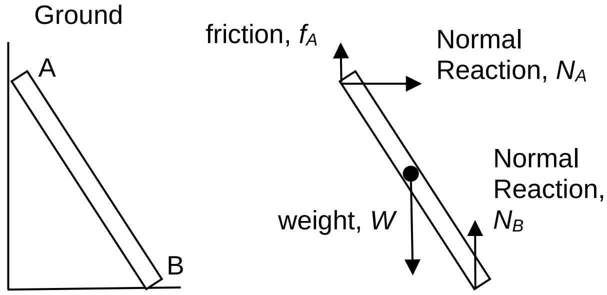
Example 2

Draw the FBD for the ladders for different situations. Resolve the force from the wall and ground into friction and normal reaction. Which of these situations will definitely not be in equilibrium?

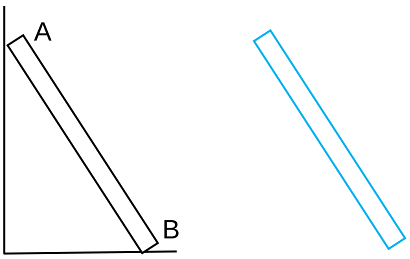
(a) Ladder resting on rough wall and rough ground



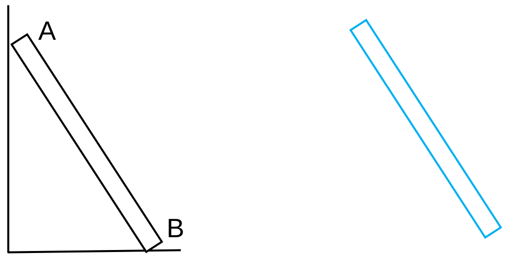
(b) Ladder resting on rough wall and smooth Ground



(c) Ladder resting on smooth wall and smooth ground



(d) Ladder resting on smooth wall and rough ground



Ans: ladders are definitely not in equilibrium; absence of leftward force(s) to cancel the rightward component forces on the ladder.

Common mistake:
Some students think that the contact force in (a) needs to pass through the body of the ladder. The direction of the contact force depends on the magnitude of friction (its horizontal component) and the normal reaction (its vertical component). Therefore, contact force might not pass through the body of the ladder.

The direction of the contact force can be found by, $\tan\phi = \frac{|N|}{|f|}$

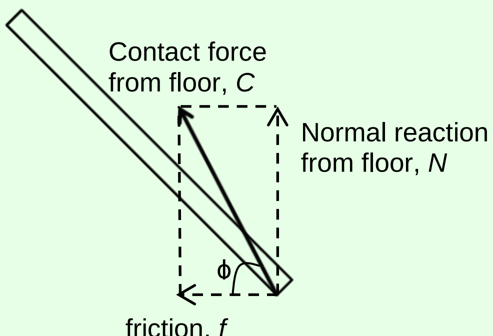


Fig. 4: forces on ladder.

To summarise,

1) For a *smooth* surface, contact force is made up of only the normal reaction.

2) For a *rough* surface, contact force is made up of both the normal reaction and friction.

Key Learning pt: (1) Direction of the contact force should be calculated using $\tan\phi = \frac{|N|}{|f|}$

(2) Do not assume contact force is along the ladder! **Contact** force is UNLIKELY to be along the ladder.

4.3.2 Frictional Force

A frictional force is a type of resistive force that arises when two surfaces move relative to each other or are in attempted motion. Friction always opposes relative motion.

Frictional forces are dissipative in nature, that is, the work done by frictional forces always leads to the production of thermal energy.

Watch this 5 min video to see what the world would be like if friction stopped working.
https://tinyurl.com/y6wgx5q6

4.3.2.1 Newton's 3rd Law and Friction

When you are walking along the pavement, the sole of your shoe pushes backwards on the pavement, and friction from the pavement opposes this backward push in accordance with Newton's third law. It is this frictional push (reaction) from the pavement that enables you to move forward. Walking would be impossible without this force of friction!

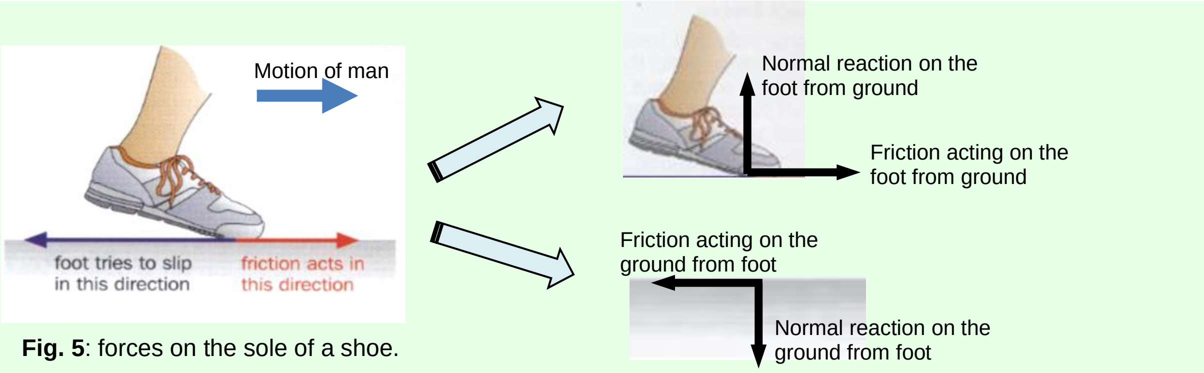


Fig. 5: forces on the sole of a shoe.

Similarly, when a car *accelerates*, the application of torque to the driving wheels' axle causes the tyre to push on the road backwards as shown in the diagram. The reaction force (Newton's 3rd law) acting on the tyre is the frictional force between the tyre and the road. This friction pushes the tyre forward, enabling the car to accelerate.

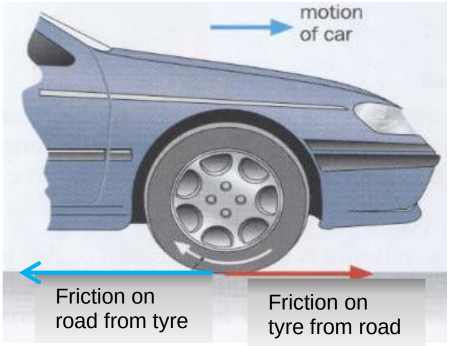


Fig. 6: forces on tyre and road.

For a car *braking*, the forces in the above figure will be *reversed*.

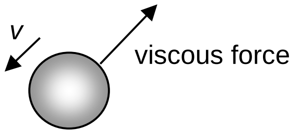
Food for thought:
If there was no friction present (e.g. car on ice), do you think the car can move forward?

Ans: The car wheel would spin but there would be no forward motion because of slippage of the wheel against the slippery ice surface as shown in this video:
https://tinyurl.com/y8coc9ht

Note: The term '**total resistive force**' used in some questions refers to the total force that opposes the motion of the entire body. It comprises the frictional force and air resistance. (Contrast this with Fig. 6 where friction opposes the motion of the *car tyre* only and not the entire car).

4.3.2.2 Viscous Forces

Viscous force (or drag force) is another kind of a resistive force that opposes relative motion. It is present when an object moves in a fluid (i.e. liquid or gas). Air resistance is a common example of a viscous force.



Viscous forces are dissipative in nature. They increase with speed (unlike friction). If the object is stationary, the viscous force is zero. For example, the drag of the air resistance against a car increases with the speed of the car.

Worked Example 3: [Specimen paper 07/P3/Q2(a)]

Explain the origin of air resistance. [1]

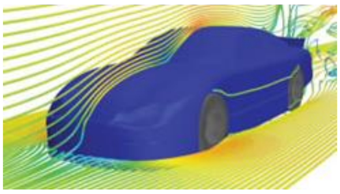
Ans: Air resistance is the force exerted on a moving body due to the collisions of the body and the air molecules in its path.

Factors affecting viscous forces:

(i) **Speed of Object**
The viscous force increases with the speed of the moving object or fluid (need NOT be proportional)

(ii) **Viscosity of Fluid**
Viscosity of a fluid is a measure of how difficult it will flow. For an object moving at a given speed, it will encounter a greater viscous force when travelling through fluid of higher viscosity (e.g. water compared to air). Common Error: Viscosity is not equal to density of fluid. It is independent of density.

(iii) **Shape and size of the object**
For example, an object which is more aerodynamic/ streamline will encounter less viscous force.



Aerodynamics in car racing

Key Learning pt: (1) Air resistance (a type of viscous force) is NOT proportional to speed. It increases with speed.
(2) Friction is not the same as viscous force.

Friction	Viscous force
Between 2 solid surfaces	Between solid/fluid and fluid surfaces
Can exists without motion	Only exists when there is motion
Constant with speed	Increaes with speed

Worked Example 4:
Show how forces acting on the sphere varies at different stages of a fall with air resistance.

Ans:
Consider a sphere (of weight *W*) falling through a fluid e.g. air. Initially, $W > F$, where *F*: air resistance.

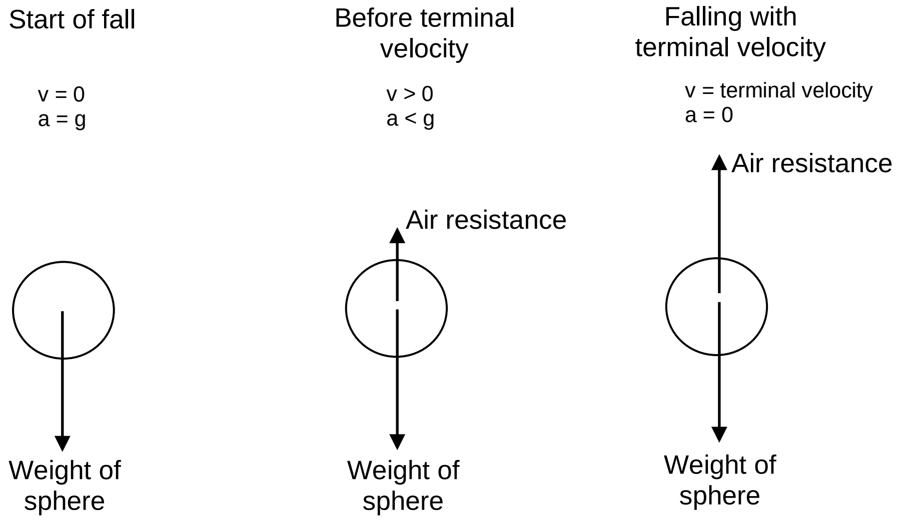


Fig. 7: forces on a falling sphere.

Note: the length of the arrow represents the magnitude of the force.

As the velocity of the sphere increases over time, the viscous force acting on it increases to a maximum constant value when $F = W$, and thus the object is at equilibrium. The velocity where this occurs is called the terminal velocity. [Refer to Chapter 2, Section 2.4]

When the sphere is moving through a fluid which has low density such as air, the upthrust acting on the sphere may be considered as negligible. When the sphere moves in fluids such as oil, upthrust has to be taken in account. Upthrust will be discussed in greater detail later in Section 4.3.4.

Key Learning pt:	(1)	When **dropping** from rest, air resistance (AR) can only increase to be equal to weight (not more than weight).

Watch this 4-min video to see how velocity changes until terminal velocity when the sky diver jumps out with a parachute:
https://tinyurl.com/yay8btkc

Example 5:
An object of mass 0.50 kg is falling under gravity with velocity *v* through a viscous fluid such that the viscous force it experiences is given by $F = k\,v$, where k = 26 N s m^{-1}.
(a) Calculate the terminal velocity of the object.
(b) Calculate the acceleration of the object when it is falling with velocity 0.14 m s^{-1}.

Ans:
(a) (b)

4.3.3 Tension and Compression

Tension: It refers to the 'pulling' force exerted by a string, cable, rope etc on another object. For example

- a taut rope attached to a load is *under tension* so the tension force pulls *on the rope* (Fig. 8(a)).
- As a result, the rope pulls back *on the load* (Newton's 3rd Law)(Fig. 8(b)).
- Note that the direction of tension is always *away* from the body and *along* the rope at the point of attachment.

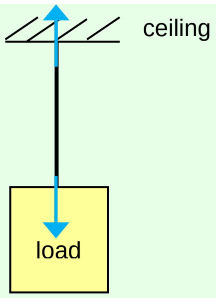


Fig. 8(a):
Forces on the rope under tension.

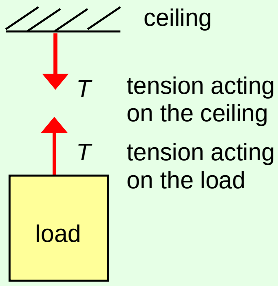


Fig. 8(b)

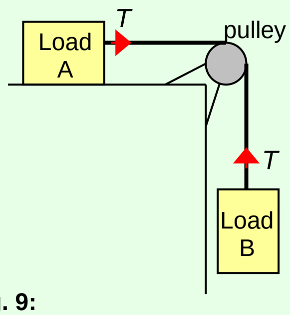


Fig. 9:
Tensions on loads A & B.

Compression: It refers to the 'compressing' force that acts on objects. For example

- a column used to support a heavy load is being compressed by the weight of the load.
- The compressive force acts on the column in both directions (Fig. 10(a)).
- By Newton's 3rd Law, the column pushes back on the load (Fig. 10(b)) and the ground simultaneously (Fig. 10(c)).

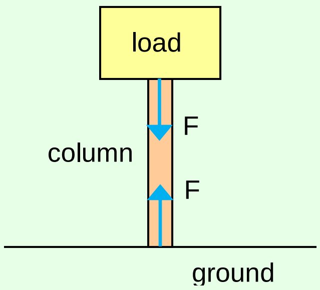


Fig. 10(a):
Forces on the vertical column supporting a heavy load.

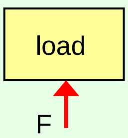


Fig. 10(b):
Force acting on the load.

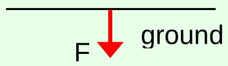


Fig. 10(c):
Force acting on the ground.

Key Learning pt:	(1)	Tension always acts away from the object, along the rope / spring / column.
	(2)	Compression always acts towards the object, along the spring / column.

4.3.4 Hooke's Law

Hooke's Law states that**:**

> If the limit of proportionality is not exceeded, the extension is directly proportional to the force/ load applied.

i.e. $\boxed{F = k\,x}$

F is the force applied to the material,
x is the extension (or compression) of the material,
k is constant of proportionality or *force constant*. (For a *spring*, *k* is known as the *spring constant*). The unit of *k* is Newton per metre.

*Common Mistake***:** Students often mistake the *x* to be the natural length or final length. *x* is the change in length of the material, i.e. *x* = final length – natural (unstretched) length

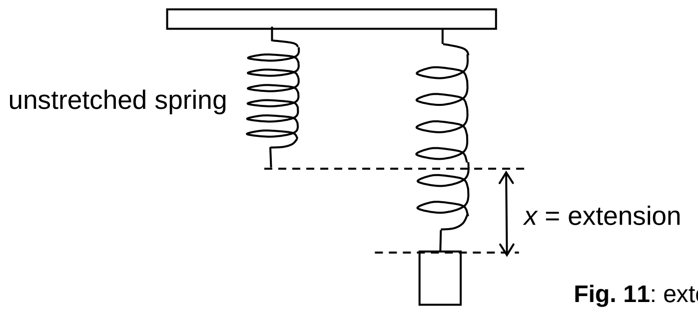


Fig. 11: extension in a stretched spring.

Refer to **Appendix** for questions involving 2 or more springs in *series / parallel* combination.

Try Tutorial Qn 1.

4.3.5 Pressure and Upthrust

Pressure is the *force per unit area* acting on an object. i.e. $p = \frac{F}{A}$, where *p* = pressure, *F* = normal force, *A* = cross-sectional area over which the force acts. It is a scalar and has SI units of pascal (Pa).

Upthrust is an upward force exerted by the fluid on a submerged or floating object due to the ***difference in pressure*** between the upper and lower surfaces of the object.

4.3.5.1 Pressure in a Fluid (Derivation Required)

Consider a *volume of fluid* of height *h,* cross-sectional area *A* and density ρ,

volume of liquid column, V = Ah,
mass of liquid column = Vρ = $Ah\rho$,
weight of liquid column = mg = $Ah\rho g$.

Pressure acting on the surface of the liquid = atmospheric pressure, P_{atm}
Therefore,
Pressure at the base of the liquid column
= pressure at the surface + pressure due to the weight of liquid
= p_{atm} + normal force / area
= $p_{atm} + Ah\rho g/ A$
= $p_{atm} + h\rho g$

Hence the pressure difference *due to liquid column* only, Δp
= pressure at the base – pressure at the surface
= $p_{atm} + h\rho g - p_{atm}$
= $h\rho g$

Hence, $\boxed{\Delta p = h\rho g}$ (Derivation Required)

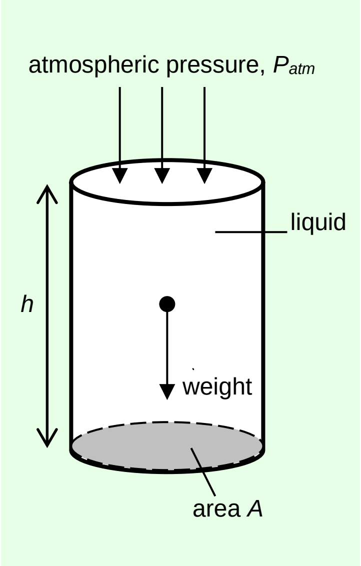


Fig. 14: pressure at base of liquid column.

Food for thought: Are the pressures at points A and B below the same?

Ans:
Since $p = h\rho g + p_{atm}$ and the atmospheric pressure and the vertical distance below the liquid surface (h) are the same, the pressures at A and B are the same.

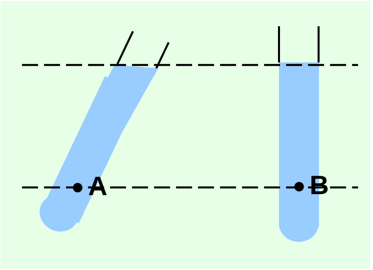
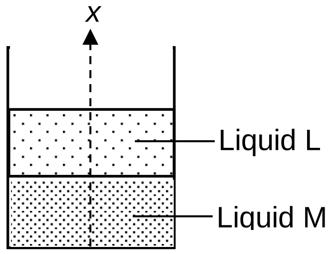


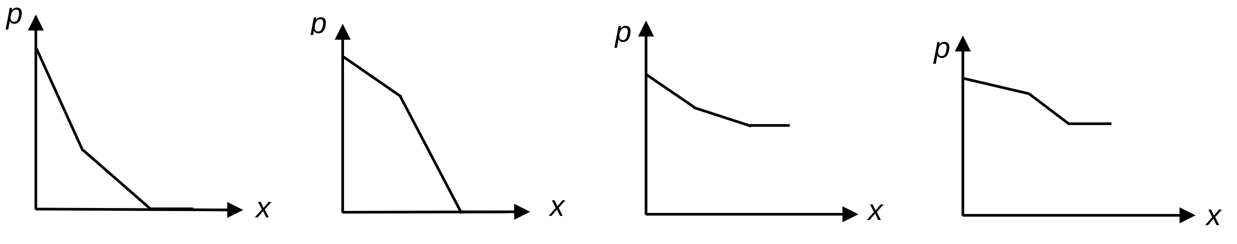
Fig. 15: pressures at the same depth.

Example 6: [SAJC JC1 Common Test 2010]

A small container which is open to the atmosphere contains a layer of liquid L, floating on liquid M. Liquid M has a density which is twice as great as that of liquid L.



Which graph shows how the pressure, p, at a point varies with its height, x above the base of the container?



A **B** **C** **D**

Ans:

- From $\Delta p = h\rho g$, where h is the height from base of container.
- So ρg gives the gradient of the graph.
- From the base of the beaker, the pressure is the highest.
- As M is of higher density, the gradient is steeper.
- The pressure deceases till the surface where its pressure is the atmospheric pressure.

4.3.5.2 Origin of Upthrust

As already mentioned, upthrust is an upward force that arises due due to the ***difference in pressure*** between the upper and lower surfaces of the object.

Consider a spring balance hooked to an object under water versus supporting it in air. The object in water seems less heavy. The spring balance reading (also equals to the tension in the string/ tension on object) decreases from T_1 to T_2 (We can also call T_2 the *apparent weight* of the object in water)
This effect is due to the buoyancy force or *upthrust* acting upwards on the object! Note that the upthrust U_2 is larger when the object is immersed in water than in air.

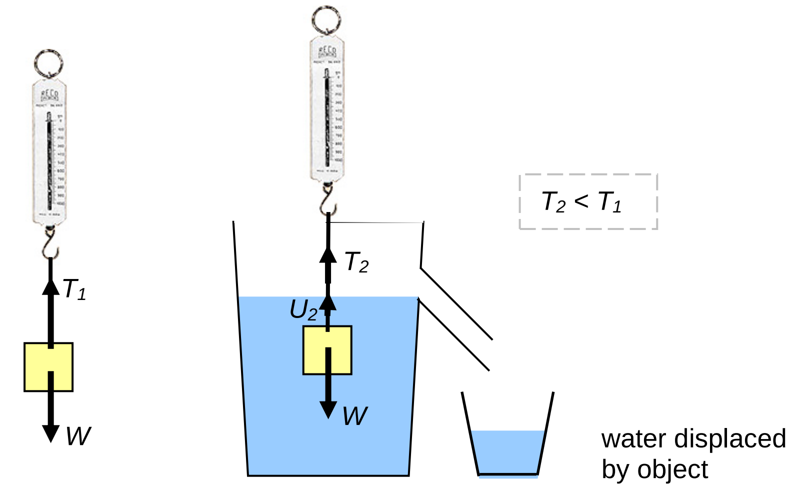


Fig. 16: FBD of an object hooked to a spring in two scenarios.

The upthrust is due to the difference in pressure acting on the upper and lower surfaces of the object when it is in a fluid (liquid or gas). The forces acting horizontally against the sides of the object cancel out one another. Hence,

To calculate the upthrust, consider an object of uniform cross-sectional area A and length L in a liquid of density ρ_f. .

F_1 is the force exerted on the upper surface of the cylinder due to hydrostatic pressure.
F_2 is the force exerted on the lower surface of the cylinder due to hydrostatic pressure.

i.e. Upthrust $= F_2 - F_1$
$= p_2 A - p_1 A$ {Label & define p_2 & p_1}
$= \rho_f\, g\, h_2\, A - \rho_f\, g\, h_1\, A$
$= \rho_f\, g\, (h_2 - h_1)\, A$
$= \rho_f\, g\, L\, A$ (where $h_2 - h_1 = L$)

Since $L \times A$ = volume of object, V

$$\boxed{\text{Upthrust} = V \rho_f\, g}$$

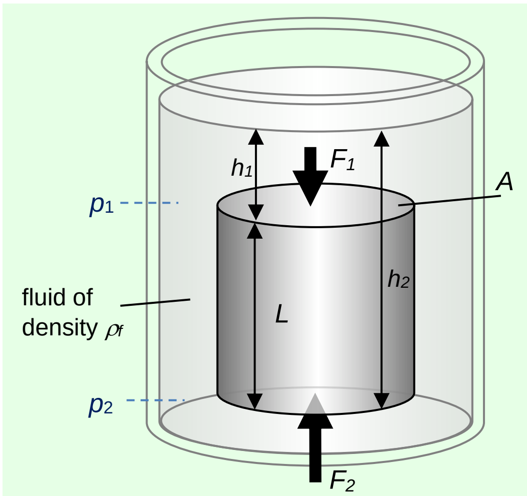


Fig. 17: forces on submerged object at different depths.

Try Tutorial Qn. 2

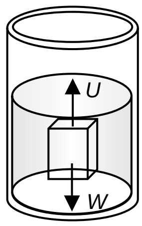
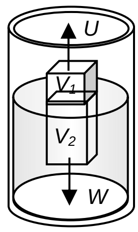
Upthrust is equal in magnitude and opposite in direction to the weight of the fluid displaced by a submerged or floating object. This principle is known as **Archimedes' Principle.**

Important Points:
- ρ_f is the density of the fluid (not of the object).
- V is the volume of object that is submerged in the fluid {not necessarily the entire volume of object unless it's completely submerged}.
- ($V\rho_f\, g$) is the weight of the fluid that has been displaced by the object.

4.3.5.3 Conditions for an object to float

When an object **floats**, the *upthrust* acting on it must be equal in magnitude and opposite in direction to the *weight of the object* since it is in vertical equilibrium.

- An object will float as long as the upthrust = weight of the object.
- This principle of flotation applies to both partially immersed bodies (i.e. ships) and totally immersed bodies (i.e. submarines).
- Archimedes principle states that the upthrust is the weight of the fluid displaced. So, for a floating object on a liquid, the weight of the displaced liquid is the weight of the object!

Object is **completely submerged** but suspended	Object is **partially submerged**
Upthrust = Weight of object $V \rho_f g = V \rho_{obj} g$ $\rho_f = \rho_{obj}$  	Upthrust = Weight of object $V_2 \rho_f g = (V_1 + V_2) \rho_{obj} g$ $\rho_f > \rho_{obj}$ 

It is possible that (max) upthrust < weight of object.

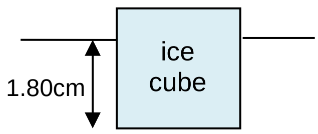
- Since upthrust = volume submerged × density of fluid × g, upthrust is maximum when the entire object is submerged in the water.
- If max upthrust is still less than weight, then the object will sink.
- This happens when $\rho_{object} > \rho_{fluid}$.

Key Learning pt:
(1) U = volume submerged × density of fluid × g
= (volume of fluid displaced × density of fluid) × g
= mass of fluid displaced × g
= weight of fluid displaced
(2) Upthrust = $V_{submerged} \rho_f g$ = weight of fluid displaced is always true!
(3) Upthrust = weight of object is only true IF the object floats!
(4) In Secondary School, we explain that things float when $\rho_{object} < \rho_{fluid}$.and that things sink when $\rho_{object} > \rho_{fluid}$. In JC, the explanation must include the concept of Upthrust.

Worked Example 7:

An ice cube of sides 2.0 cm floats in a cup of soda. One of its faces is 0.20 cm above the surface of the soda in the cup. Calculate the density of the soda if the density of ice is 920 kg m^{-3}.

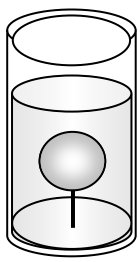
Ans:
By Archimedes' & Flotation Principles, when an object floats on a liquid,
weight of object = upthrust = weight of liquid displaced

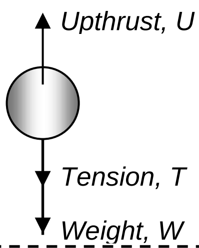


$$V_{ice} \rho_{ice} g = V_{displaced} \rho_{soda} g$$
$$\rho_{soda} = (0.02 \times 0.02 \times 0.02) / (0.02 \times 0.02 \times 0.018) \times 920$$
$$= 1020 \text{ kg m}^{-3}$$

Example 8:

A cylinder contains a liquid of density 1.2 x 10^3 kg m^{-3}. A body of volume 5.0 x 10^{-3} m^3 and density 9.0 x 10^2 kg m^{-3} is totally immersed in the liquid and is attached by a thread to the bottom of the cylinder. Calculate the tension in the thread.





Ans:
Consider all forces acting on the body {in vertical equilibrium}:

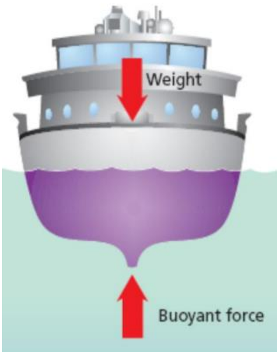
Example 9:

(a) Steel has a greater density than water. Explain how then a steel ship floats on water.

(b) 24 m of a super tanker is below the water surface when it is in sea water of density 1030 kg m^{-3}. What depth will be below the water surface when it enters fresh water of density 1000 kg m^{-3} ? Assume that the super tanker has vertical sides. [24.7 m]
{*i.e. will an object sink more or sink less in a denser fluid?*}

Ans:

(a) A steel ship is not entirely made of steel, it has a lot of empty space filled with air that has a much smaller density than water. Thus the effective density of the ship is smaller than that of water allowing it to float. Because of its shape, the volume of water a ship displaces is also very much larger than the volume of steel. Thus the upthrust created is sufficient to balance its weight.

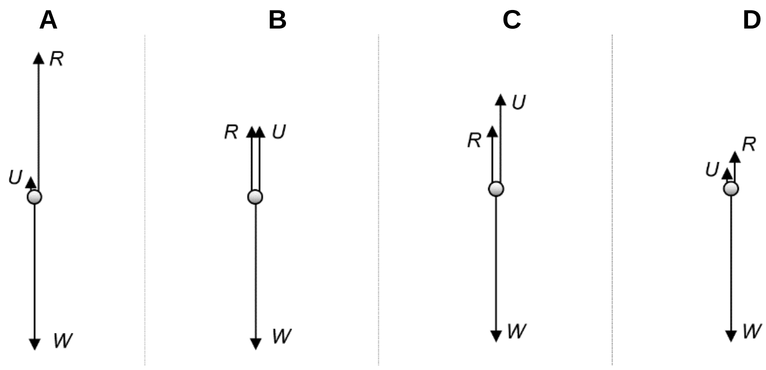


(b) For the ship to float, since its weight is constant,

Worked Example 10: [N10/01/08]

A water droplet in a cloud is falling through air and is *in equilibrium*. Three forces act on it, its weight *W*, upthrust *U* and air resistance *R*.

Which diagram, showing these three forces to scale, is correct?



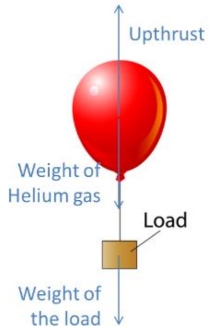
Ans: A
As the water droplet is in equilibrium, downwards *W* equates (in length) with upwards *U* and *R*. *U* is very small as weight of air displaced is small (due to low density of air).

Note: In this case, upthrust is not negligible although density of air is small. Read the question carefully to determine whether upthrust should be neglected.

Good Practice: In all force diagrams, try to draw to scale to show the relative magnitudes of the vectors.

Try Tutorial Qns 3 – 8.

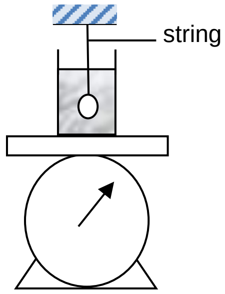
When do we include Upthrust in our Free Body Diagram?



There are many cases where upthrust is considered negligible and hence omitted in the FBD and calculation. Upthrust of an object is negligible when the density of the *fluid* in which the object is immersed is *relatively low* compared to that of the object AND the *displaced volume* of the fluid is very small.

In the case of a helium balloon in air, upthrust is NOT negligible since the density of helium is lower than the air AND volume of the balloon is NOT small, so upthrust due to weight of air displaced is significant, comparable to the weight of helium. In fact because the upthrust is bigger than the weight, the balloon rises up.

Worked Example 11 with Visible Thinking:



The weight of a beaker of water is 30 N. If a solid ball of weight 25 N in air displaces 10 N of water when immersed as shown, determine the total weight of the ball and the beaker of water as recorded on the weighing scale. [3]

Thought process:

1. ***Trigger Keywords:*** *total weight of ball and beaker as recorded on the weighing scale…*
   - ***<u>Realise that the weighing scale reading is the result of a normal reaction force acting downwards by the base of beaker on the scale's top surface.</u>***
   - *How do we then determine the magnitude of this normal reaction force?*
2. ***Trigger Keywords:*** *…ball…displaces 10 N of water when immersed..*
   - *This refers to the upthrust on the ball! i.e. U = 10 N*
3. ***Key consideration:*** *how many FBDs should we draw?*
   - *There are 3 objects: ball, beaker of water and weighing scale.*
   - *Drawing FBDs for each object will help you answer this question.*

Ans:

Drawing the FBD for each object as follows:

Weighing scale:	Beaker of water:	Ball:
 normal reaction on scale by beaker, N' (weight of scale & normal reaction by table has been ignored for clarity)	 normal reaction on beaker by scale, N reaction of upthrust acting on water, $U' = 10$ N weight of beaker + water, $W_w = 30$ N	 tension, T upthrust on ball, U = 10 N weight of ball, $W_b = 25$ N

By Archimedes' Principle,
upthrust = weight of the fluid displaced by the object = 10 N [1]

Since upthrust is the upward force that the water exerts on the object, hence by Newton's 3rd law, the downward reaction force that object exerts on water, U' = 10 N

Since the beaker of water is stationary (i.e. in vertical equilibrium),
the net force acting on the beaker of water is 0 N.
Hence, normal reaction force on beaker due to weighing scale, $N = W_w + U'$ [1]
$= 30 + 10 = 40$ N

Hence, magnitude of (normal reaction) force on weighing scale, N' = magnitude of N *{N3L}*
= **40 N** [1]

4.4 Centre of gravity (CG)

Whilst the weight of a mass is in fact distributed throughout the mass, it may be considered to be acting at a single point called the *centre of gravity*.

> The **centre of gravity** of an object is the point through which the entire weight of a body may be considered to act.

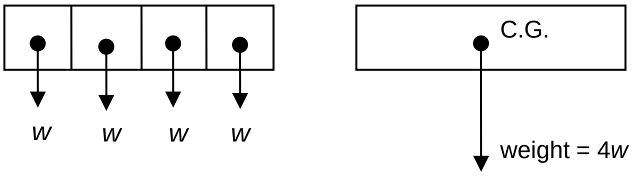


Fig. 19: weight of a mass.

The position of the centre of gravity affects the stability of an object. In general, a lower centre of gravity favours stability.

The centre of gravity of an object of uniform thickness and density is at its geometrical centre.

4.5 Moment of a force

> The **moment of a force** is defined as the product of the force and the perpendicular distance of its line of action from the pivot or axis of rotation.

> moment of a force, τ = force × perpendicular distance from pivot

i.e. it is the turning effect of the force about an the axis of rotation.

Note: the word torque has the same definition as moment

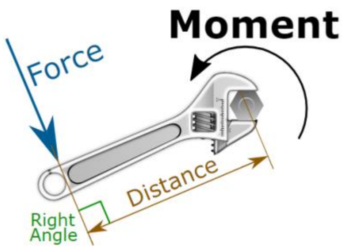
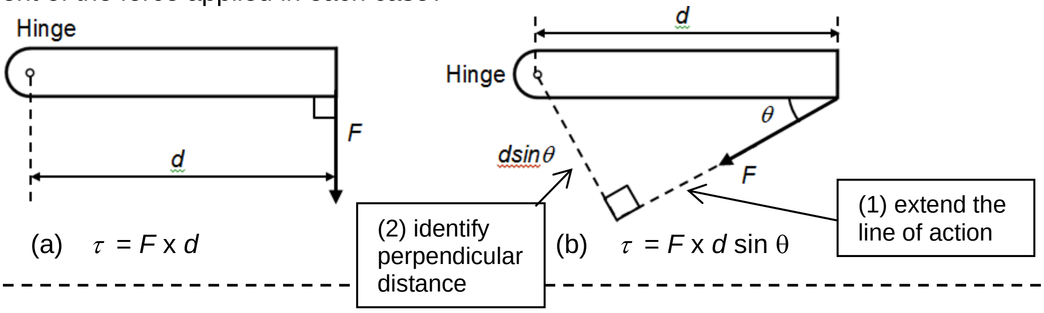


Fig. 20: moment of a force on a spanner.

Worked Example 12:

What is the moment of the force applied in each case?



(a) $\tau = F \times d$

(b) $\tau = F \times d \sin\theta$

4.5.1 Couple

A **couple** is a pair of <u>equal and opposite</u> forces whose <u>lines of action do not coincide</u> and which tends to produce rotation only. A couple cannot give rise to a resultant force since the 2 forces are equal and opposite. Instead, a couple creates only a turning effect known as *torque*.
Note: A couple is NOT an action-reaction pair because they act on the same body.

> The **torque of a couple** is defined as the product of <u>one</u> of the forces of the couple and the perpendicular distance between the lines of action of the forces.

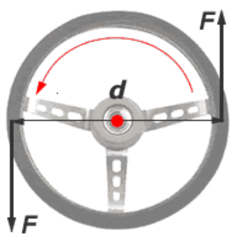


Fig. 21: moment of a couple on a steering wheel.

$\boxed{\tau = F \times d}$

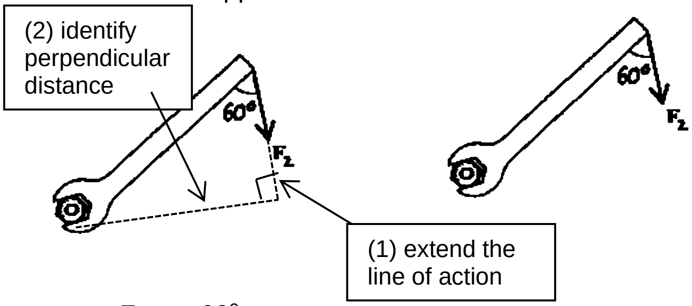
Both moment and torque are *vectors* and their S.I unit are Newton-metre (N m).

Example 13:

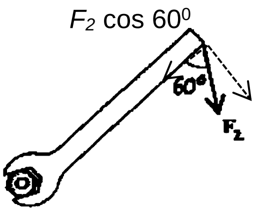
A nut requires a torque of 40 Nm to be loosened. If a spanner is 0.20 m long, calculate the force that must be applied to the end of the spanner in order to loosen it if it is applied at 60° to the handle.

Ans:

Method 1: Finding perpendicular distance

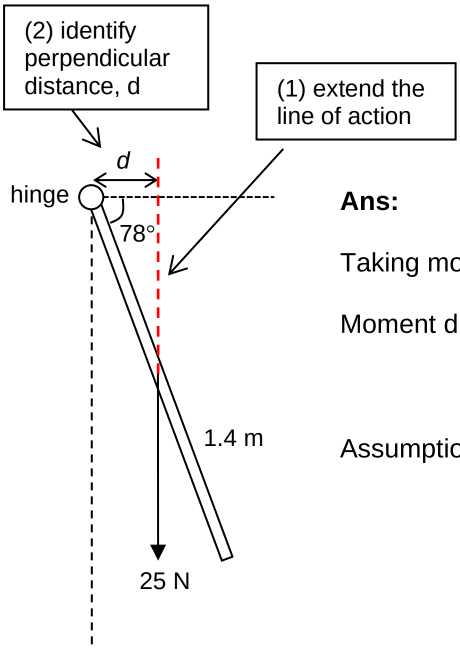


Method 2: Resolving force



Worked Example 14:

Determine the moment exerted on a rusty hinge on a pub sign, if it has a length of 1.4 m, a weight of 25 N and is tilted at an angle of 78° to the horizontal as shown. State the assumption made.



Ans:

Taking moments about the hinge,

Moment due to weight = 25 × d
= 25 × 0.7 cos 78°
= 3.64 Nm (clockwise direction)

Assumption: the sign is a uniform bar thus the weight acts at its middle.

Try Tutorial Qns 9 - 12.

4.6 Equilibrium of a Rigid Body

4.6.1 Conditions for Equilibrium

A rigid body is one whose shape would not change when acted upon by a force. A rigid body under the action of a number of forces is in *equilibrium* if

(1) The resultant force acting on it equals zero, (This ensures translational equilibrium)
AND
(2) The resultant moment about any point equals zero. (This ensures rotational equilibrium)

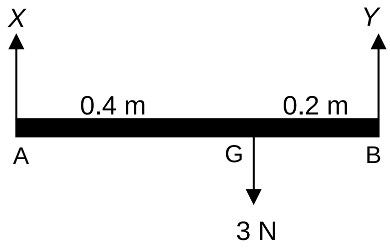
Note that for point (2) above, we can also say that the sum of clockwise moments about any point must be equal to the sum of anti-clockwise moments about that point when a body is in rotational equilibrium. This is otherwise known as the **Principle of Moments**.

The **Principle of Moments** states that for a body to be in rotational equilibrium, the sum of all the anticlockwise moments about any point must be equal to the sum of all the clockwise moments about that same point.

Note: Students are to explicitly state the point or pivot about which moments are taken from in problem-solving.

Worked Example 15: (Rotational and translational equilibrium for extended body)

A light horizontal rod AB is suspended at its ends by two vertical strings. The rod is 0.6 m long and its weight of 3 N acts at point G where AG is 0.4 m and BG is 0.2 m. Calculate the tensions X and Y in the strings.



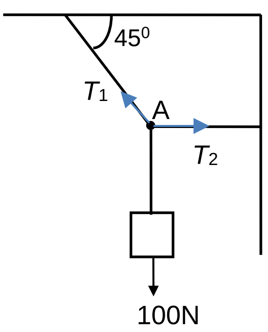
Ans:
Since rod AB is in equilibrium, by Principle of Moments,
Taking moment about point A,
Sum of clockwise moments = Sum of anti-clockwise moments
3 N x 0.4 m = Y x 0.6 m
Y = 2 N

Taking moments about B
Sum of clockwise moments = Sum of anti-clockwise moments
X x 0.6 m = 3 N x 0.2 m
X = 1 N

OR
By translational equilibrium
X + Y = 3 N
X = 3 – 2 = 1 N

Example 16: (Only Translational equilibrium for point body mass)

Calculate the tensions T_1 and T_2, in the cords supporting the object of weight 100 N.



Considering point A,
For the three forces to be in equilibrium:

How to solve quantitative equilibrium questions for extended body with 2 unknowns:

Step 1: **Use Rotational Equilibrium to find first unknown force**

Take moments at the point where the other unknown force passes through and find the first unknown force required.

Rationale: the second unknown force is not necessary (yet) and will not appear in the equation

(If required)
Step 2: **METHOD 1 (Recommended)**

Use Translational Equilibrium to find second unknown force

May need resolving into components to determine the second unknown force

Rationale: no need to use rotational equilibrium already as left only 1 unknown.

METHOD 2

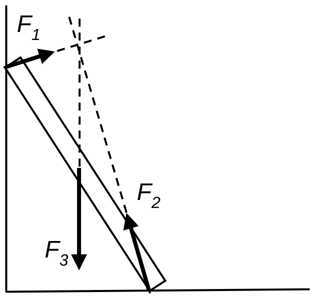
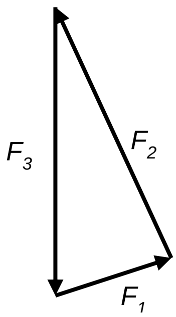
Use Rotational Equilibrium to find second unknown force

Take moments at a convenient point and find the second unknown force required.

Rationale: not recommended to use as we try to avoid using rotational equilibrium if possible as we need to determine perpendicular distances of the forces.

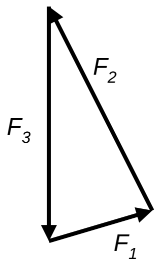
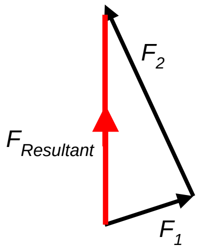
4.6.2 Triangle of Forces

If a body is acted upon by 3 coplanar forces (only) and remains in equilibrium, then

(i) The lines of action of the forces must pass through a single (common) point. (i.e. forces are ***concurrent***), and	(ii) When a vector diagram of the 3 forces is drawn, the forces will form a **closed triangle**, with the vectors pointing in the same orientation around the triangle.
	 Resultant force = 0 $\Sigma F_x = 0$ & $\Sigma F_y = 0$

Note: If more than 3 forces are in equilibrium, a closed vector diagram is formed, (also with the vectors pointing in the same orientation around the polygon.)

Caution: Do not confuse an equilibrium vector triangle with a resultant force vector triangle (see table below)

Equilibrium Vector Diagram	Resultant Vector Diagram
 For an equilibrium vector triangle, the resultant force = $F_1 + F_2 + F_3 = 0$. Note that F_3 is not a resultant force, and the 3 force vectors are pointing "head-to-tail" around the triangle.	 For a resultant force vector triangle, the resultant force, $F_{Resultant} = F_1 + F_2$. In this case, $F_{Resultant}$ is pointing from the starting point of F_1 to the ending point of F_2.

Example 17:

In the following two scenarios, two of the forces acting on the static object (the person in (a), and the beam in (b)) are already drawn for you. Identify the third force and draw it into the diagrams below.

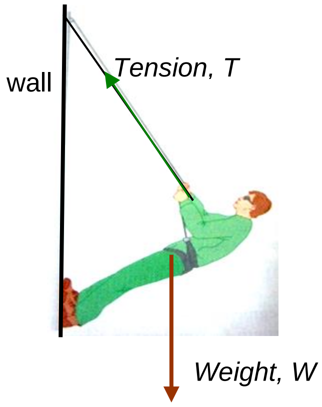


Fig. 23(a): A person lowering himself down the face of a cliff via a rope.

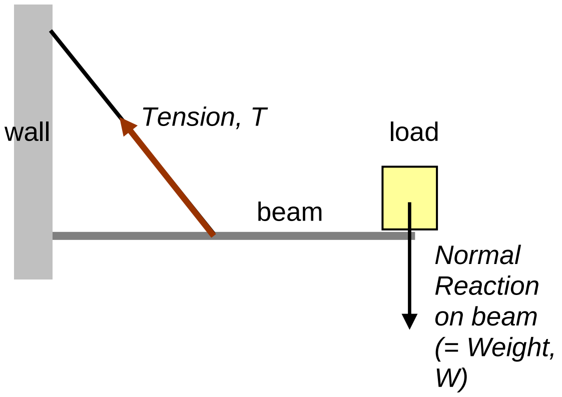
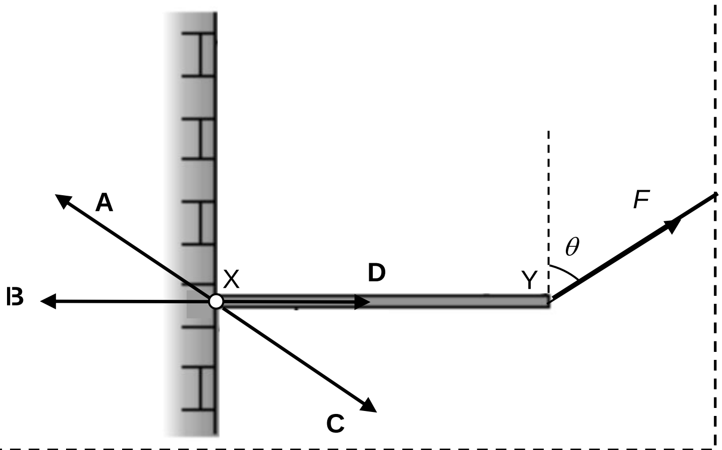


Fig. 23 (b): A light beam with a load placed on one end and secured by a cable in the middle

Example 18: [SAJC JC1 Common Test 2010]

A uniform rod XY is freely hinged to the wall at X. It is held horizontal by a force *F* acting from **Y** at an angle θ to the vertical as shown in the diagram. Which arrow shows the direction of the reaction force exerted by the hinge on the rod?



Ans:

There are 3 forces acting on the rod: , force *F* and the reaction force by the hinge.

They must all pass through a common point. Thus options and are eliminated.

is eliminated because there will be a resultant force to the right thus the rod will not be in equilibrium OR all 3 forces must form a closed triangle.

Key Learning pt: For an object in equilibrium

(1) Conditions: (i) Net moment = 0 about <u>any</u> pivot.
(ii) Net horizontal force = 0
(iii) Net vertical force = 0

(2) Vector diagram: (i) All forces must form a closed triangle/polygon in the same orientation.

(3) For object under <u>3 forces only</u>: (i) Lines of action of the forces must pass through a single point (i.e. concurrent)

(4) To solve equilibrium question: (i) 3 equations to solve
(ii) CW moment = ACW moment
(iii) $\sum F_x = 0$
(iv) $\sum F_y = 0$

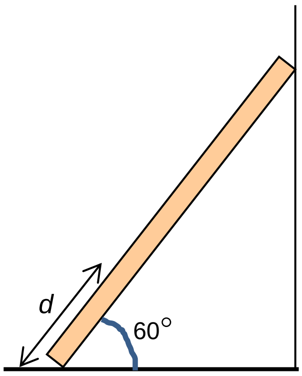
(5) Quite frequently, we need to use CW moment = ACW moment as a first step to solve equilibrium questions.

Try Tutorial Qns 13 – 16 and 18.

4.6.3 Problems involving Ladders against a Wall

Worked Example 19 with Visible Thinking: [SAJC Prelim 2008/P2/Q2]

A uniform ladder of length 3 m and mass 20 kg, leans with its upper end against a *smooth* vertical wall and its lower end on a *rough* ground. The angle of elevation of the ladder is 60° above the horizontal. A technician of mass 60 kg tries to climb up the ladder.
Given that the maximum friction between the rough ground and the ladder is 300 N, calculate the maximum distance, *d***,** in the diagram where the technician could climb just before the ladder slips.



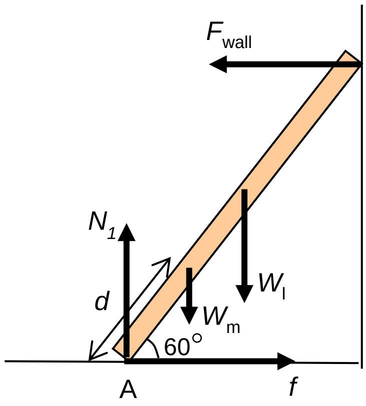
Thought process:

1. ***Trigger Keywords:*** *smooth vertical wall…*
   - *No friction at the wall. Only the normal reaction force (F_{wall}) exists.*

2. ***Trigger Keywords:*** *rough ground...*
   - *Friction (f) exists between the ladder and ground*

3. ***Key consideration:*** *what is the max d before the ladder starts to lose its rotational equilibrium? (Because when it 'slips', it rotates)*
   - *We need to use principle of moments for the equilibrium of ladder <u>just before</u> it starts to slip.. i.e. man is standing at distance d.*
   - *Where should the pivot be?*
   - *Drawing a FBD (outline method) for the ladder will help you answer this question.*
   - *Do we need to also consider translational equilibrium of the ladder? (Yes, since there is more than 1 unknown in this question)*
   -

Solution:



1. FBD (outline method): draw the forces acting <u>on the ladder</u>.
2. Given that the ladder is in equilibrium, ensure that forces drawn are possible to lead to translational equilibrium. i.e. leftward forces are countered by rightward forces.
3. Since the wall is smooth (no friction), only normal reaction, F_{wall} acts on the ladder.

Let's look at both the following:

Translational Equilibrium	Rotational Equilibrium
Resolve all forces on ladder into the vertical and horizontal components. Then write an equation for the equilibrium of the ladder in each direction: For horizontal equilibrium, (take rightward as +ve) $f - F_{wall} = 0$ $F_{wall} = f = 300$ N For vertical equilibrium, (take upward as +ve) $N_1 - W_l - W_m = 0$ $N_1 = W_l + W_m$ $= 20 \times 9.81 + 60 \times 9.81$ $= 80 \times 9.81$ $= 784.8$ N (although not needed, this is done to show how N_1 can be calculated if qn calls for it)	Which point should be picked as the pivot? In this case, <u>point A</u> (base of ladder) will be more ideal as there are more forces, N_1 and f passing through this point. The unknown force(s) (N_1 in this case) will be eliminated in the equation as the moment by this force is zero. Take moments about <u>point A</u>, Sum of clockwise moments = Sum of anti-clockwise moments $(W_l \times 1.5 \cos 60^o) + (W_m \times d \cos 60^o) = F_{wall} \times 3 \sin 60^o$ $(20g \times 1.5 \cos 60^o) + (60g \times d \cos 60^o) = 300 \times 3 \sin 60^o$ Hence d = <u>2.15 m</u>

Tip: In many other cases, where unknown forces passes through a particular point, they are often chosen as the pivot in problem-solving so that we need not consider the moments caused by these <u>unknown forces</u>.

Try Tutorial Qn 19.

Worked Example 20: [SAJC CT 2016 modified]

A technician attaches a rod PQ hinged at P to a vertical wall and supports it using a wire, as shown in Fig. 22.1

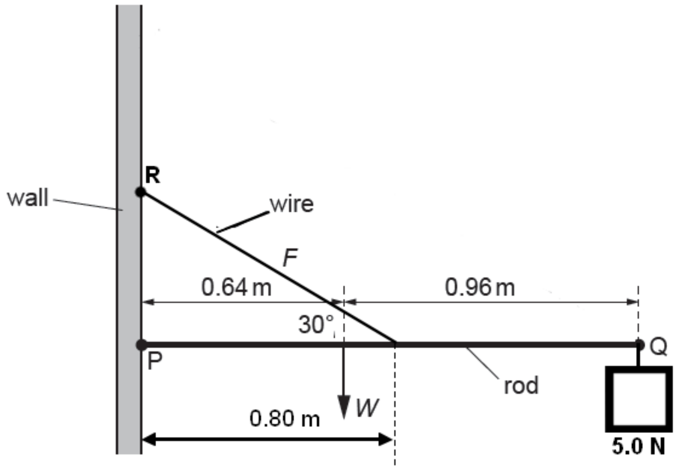


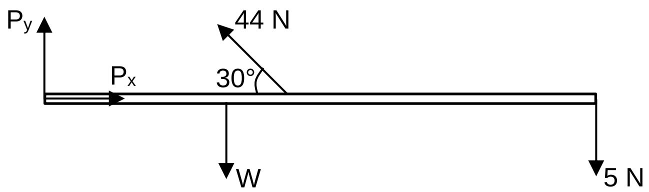
Fig. 22.1

The length of the rod is 1.60 m. The weight W of the rod acts 0.64 m from P and a load of 5.0 N is suspended at the end Q. The rod is kept horizontal and in equilibrium by a wire attached to the mid-point of the rod and to the wall at R. The wire provides a force F on the rod of 44 N at 30° to the horizontal.

(i) Calculate the weight W of the rod.

(ii) Calculate the force hinge exerts on the rod at P. State its direction.

Solution:

1. Always draw FBD	 ▪ *Recall that contact force at P is unlikely to be along rod.* ▪ *Since we do not know its actual direction, it suffices to assume a general vertical and horizontal component (you can assume in totally opposite direction)*
2. Use the 3 equations to solve. 3. More often than not, we start by using principle of moments (i.e. ACW = CW moment) 4. A good pivot to choose is P, since there are 2 unknown forces there.	Taking moment about P, $44 \sin(30°) \times 0.8 = [\,W \times 0.64\,] + [1.6 \times 5\,]$ $W = 15$ N
5. To solve for P, use remaining 2 equations to solve (i.e. $\sum F_x = 0$ and $F_y = 0$)	Assume rightward as positive, $P_x + (-44 \cos 30°) = 0$ $P_x = 38.1$ N Assume upwards as positive, $P_y + 44 \sin 30° + (-15) + (-5) = 0$ $P_y = -2.0$ N (*negative sign indicates P_y is downward*) Therefore, $P = \sqrt{38.1^2 + 2.0^2} = 38.2$ N Direction = $\tan^{-1}(2/38.1) = 3.0°$ below the horizontal

SUMMARY

1. A **field of force** refers to a region of space within which a body experiences a force without being in physical contact with any other body.
Note the characteristics of gravitational, electric and magnetic fields (pg 2).

2. **Free body diagram** – usually drawn for problem-solving. It shows all <u>external forces</u> acting <u>on an isolated body.</u>

3. Types of forces – contact /reaction force, frictional force, viscous/ drag force, tension & compression, upthrust.

4. In general, the reaction force acting on a ladder at the wall and at the ground has 2 perpendicular components: friction and the normal reaction, where

$$\boxed{\text{Reaction force, } R = \sqrt{(\text{friction})^2 + (\text{normal reaction})^2}}$$

5. **Hooke's Law** states if the limit of proportionality is not exceeded, the extension is directly proportional to the force/load applied.

i.e. $\boxed{F = k\,x}$ where F is the force applied to the material,
x is the extension (or compression) of the material,
k is constant of proportionality or *force constant*. (For a *spring*, k is known as the *spring constant*). The unit of k is Newton per metre.

6. **Pressure** difference due to a liquid column, $\Delta p = h\rho g$

7. **Upthrust** (U) is an upward force exerted by the fluid on a submerged or floating object due to the difference in pressure between the upper and lower surfaces of the object.

$\boxed{U = V\rho g}$ *ρ is the density of the fluid*
V is the volume of object that is submerged in the fluid

8. **Archimedes' Principle** states that any object immersed in a fluid will experience an upthrust which is equal in magnitude and opposite in direction to the weight of the fluid displaced by the object.

9. **Principle of Flotation** states the upthrust acting on a floating object must be equal in magnitude and opposite in direction to the weight of the object since it is in vertical equilibrium.

10. For an object that is *completely submerged* but suspended, $\rho_f = \rho_{obj}$
For an object that is *partially submerged* and suspended, $\rho_f > \rho_{obj}$

11. **Centre of gravity** of an object is the point through which the entire weight of a body may be considered to act.

12. **Moment of a force** is defined as the product of the force and the perpendicular distance of its line of action from the pivot or axis of rotation.

$$\boxed{\text{Moment of a force}, \tau = \text{Force} \times \text{perpendicular distance from pivot}}$$

13. A **couple** is a pair of <u>equal and opposite</u> forces whose <u>lines of action do not coincide</u> and which tends to produce rotation only.

14. **Torque of a couple** is defined as the product of one of the forces of the couple and the perpendicular distance between the lines of action of the forces.

$$\boxed{\tau = F \times d}$$

15. **Conditions for equilibrium:** a rigid body under the action of a number of forces is in equilibrium if
    (1) The resultant force acting on it equals zero. (ensures translational equilibrium)
    (2) The resultant moment about any point equals zero. (ensures rotational equilibrium)

16. **Principle of Moments** states that for a body to be in rotational equilibrium, the sum of all the anticlockwise moments about any point must be equal to the sum of all the clockwise moments about that same point.

17. If a body is acted upon by 3 forces (only) and remains in equilibrium, then
    (i) The lines of action of the forces must pass through a single (common) point. (i.e. forces are ***concurrent***), AND
    (ii) When a vector diagram of the 3 forces is drawn, the forces will form a **closed triangle**, with the vectors pointing head-to-tail around the triangle.
    (Caution: Do not confuse an equilibrium vector triangle with a resultant force vector triangle.)

APPENDIX

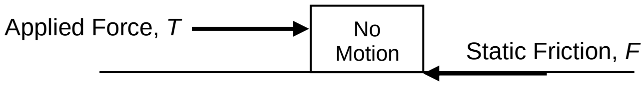
There are 2 kinds of friction: static friction and kinetic friction.

(a) Static friction

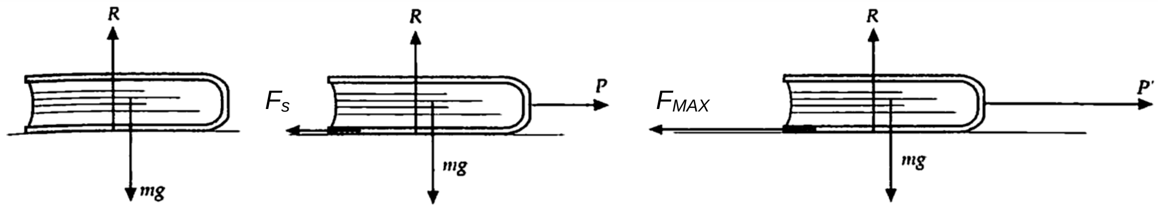
Static friction is a force existing at the interface between two <u>stationary</u> surfaces and it prevents the surfaces from sliding over each other (i.e. no relative motion).

The magnitude of static friction <u>varies</u> (up to a certain maximum value), such that it is just sufficient to prevent motion (of the object).

Hence, the static frictional force is always self-adjusting ("friction on demand"), constantly adjusting itself to be equal to the applied force and hence maintaining static equilibrium, as long as the maximum static friction is not breached.



Example: A stationary book is pulled along a table with an increasing force to the right. The book remains stationary in all three circumstances (i), (ii) and (iii) below. Hence the frictional force acting on the book must have been equal to the varying applied / pulling forces.



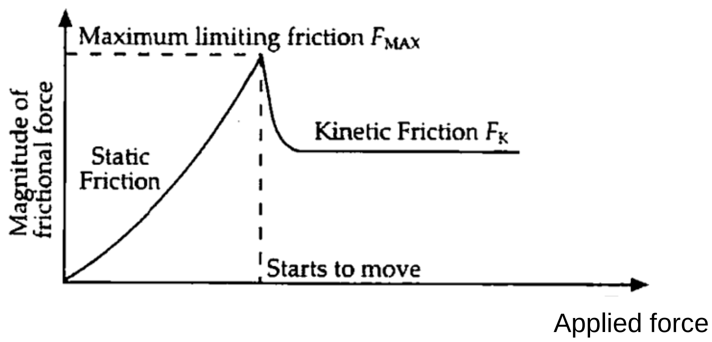
(i) no force applied (ii) book pulled with force P (iii) book pulled with larger force P' and book just about to move. (friction reaches its max value F_{max})

(b) Kinetic Friction (a.k.a Dynamic Friction)

If the applied force > maximum static friction, there will be a resultant force acting on the object. Object thus accelerates from rest and the frictional force <u>decreases to a constant</u> *value*. This new frictional force is known as **kinetic (or dynamic)** *friction*. Once the object starts to move, the frictional force will change from static friction to kinetic friction.

For the object to move at constant velocity, the applied force must then decrease to the same magnitude as the kinetic frictional force.

The graph below shows the variation of friction with applied force



………

Coefficient of Friction, μ

Two coefficients of friction are sometimes quoted for a given pair of surfaces - a *coefficient of static friction* μ_s and a *coefficient of kinetic friction* μ_k. The coefficient of friction μ is a dimensionless scalar value which describes the ratio of the force of friction (F) between two bodies and the force pressing them together, or the normal force (N). ($F = \mu N$)

The coefficient of friction depends on the materials used; for example, ice on steel has a low coefficient of friction, while rubber on pavement has a high coefficient of friction. The coefficient of friction is an empirical measurement – it has to be measured experimentally.

Did you know?

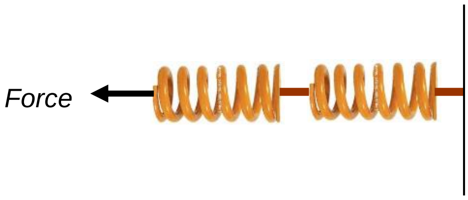
The most slippery solid known, discovered in 1999, dubbed BAM (for the elements boron, aluminium, and magnesium), has an approximate coefficient of friction of 0.02, about half that of Teflon (0.05)!

……

Springs in Series and Parallel Combinations

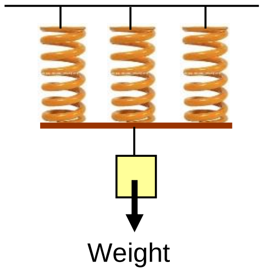
For springs arranged **in series**,

$$\frac{1}{k_T} = \frac{1}{k_A} + \frac{1}{k_B} + \ldots$$



For springs arranged **in parallel**,

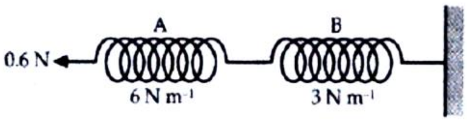
$$k_T = k_A + k_B + k_C + \ldots$$



Springs In Series - Example

Spring A with a force constant of 6 N m^{-1} is connected in series with spring B of a force constant 3 N m^{-1}. A force of 0.6 N is applied at one end while the other end is securely attached to a wall as shown.

(a) What is the force exerted on spring B?
(b) What is the extension of each spring?
(c) Deduce the effective force constant of the set-up.



Ans:

(a) Force on spring B = 0.6 N
{Why? The force, F, experienced by each spring in the series network is the same.}

(b) Let extension of a spring be e.
$e_A = F / k_A$ = 0.6 / 6 = 0.1 m

$e_B = F / k_B$ = 0.6 / 3 = 0.2 m

(c) The force, F, experienced by each spring in the series network is the same.

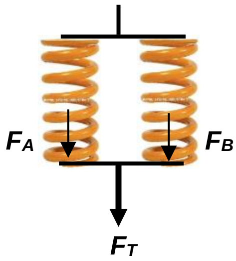
Since the springs are connected *in series*, total extension $e_T = e_A + e_B$

$$F / k = F / k_A + F / k_B \quad \text{(where } k = \text{force constant of the system)}$$
$$1 / k = 1 / k_A + 1 / k_B$$
$$k = (k_A . k_B) / (k_A + k_B)$$
$$= (6 \times 3) / (6 + 3)$$
$$= 2 \text{ N m}^{-1}$$

Alternatively, $k = \dfrac{\text{force}}{\text{total extension}} = \dfrac{0.6\text{N}}{0.3\text{m}} = 2 \text{ N m}^{-1}$

Springs In Parallel - Example

What if the springs were connected in parallel and have the same extension? What would be the force constant of this parallel setup?



Ans:

The force applied to the parallel system of springs (F_T = 0.6 N) is equal to the force pulling on both springs (i.e. F_A and F_B respectively),

Thus $F_T = F_A + F_B$

$k_T . x_T = k_A . x_A + k_B . x_B$

$k_T = k_A + k_B$ since $x_T = x_A = x_B$

$= 6 + 3$

$= 9 \text{ N m}^{-1}$

Tutorial 4: Forces

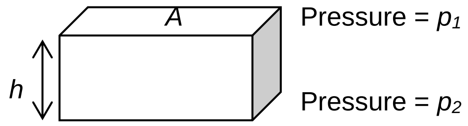
Hooke's Law

(L1) 1. A wire is stretched elastically by a force of 300 N, causing an extension of 6.0 mm. The force is then steadily reduced to 200 N. What is the new extension? [2]

Upthrust

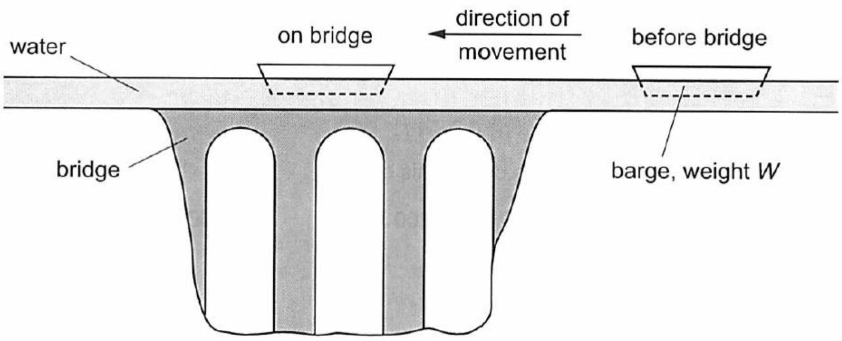
(L1) 2. A solid block of material of density ρ, height h and horizontal surface area A is immersed in a liquid. The pressures of the liquid at the upper and lower surfaces are p_1 and p_2 respectively. Which of the following is an expression for the upthrust on the block?

[N03/I/6]



A $Ah\rho g$ **B** $Ah\rho g + p_1A$ **C** p_2A **D** $p_2A - p_1A$

(L2) 3. A bridge (an aqueduct) carries a canal across a valley, as shown.



A barge of total weight W in air approaches and crosses on to the bridge. What is the *extra weight supported by the bridge* when the barge is on it? [N05/1/6 & N18/1/6]

A 0 **B** ½ W **C** W **D** $2W$

(L2) 4. A block of ice of density 0.9 g cm^{-3} is held below the surface of water, of density 1.0 g cm^{-3}. The block of ice is then released and floats to the surface.

Determine is the ratio $\dfrac{\text{upthrust when fully submerged}}{\text{upthrust when floating}}$?

[N15/P1/Q7]

A 0.1 **B** 0.9 **C** 1.0 **D** 1.1

[Extension Qn]: What happens to the ratio if there were impurities in the ice?
Ans: with impurities, density of ice block increases (>0.9), thus ratio decreases.
What happens to the ratio if Coca-Cola is used instead of water?
Ans: with Coca Cola, density of liquid increases (>1.0), thus ratio increases.

(L2) 5. A large concrete block has dimensions 3.00 m × 4.00 m × 5.00 m. It is completely submerged in sea water of density 1020 kg m^{-3} and needs to be lifted out of the water.

What is the range of minimum forces required to lift the block from when it is completely submerged to when it is clear of the water? (Density of concrete = 2300 kg m^{-3}.)

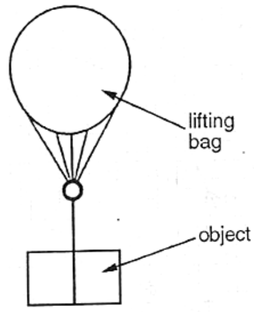
[2016 P1 Q8]

A 6.12×10^4 N to 1.38×10^5 N
B 7.68×10^4 N to 1.38×10^5 N
C 6.00×10^5 N to 1.35×10^6 N
D 7.53×10^5 N to 1.35×10^6 N

(L2) 6. Tom, a 60.0 kg lifeguard of a swimming pool, is inspecting the pool on a large rubber dinghy. On board the dinghy, there is a rectangular styrofoam slab of thickness 10.0 cm and density 300 kg m^{-3}, which is used as a float.

Tom threw the styrofoam slab into the pool, jumps into the water and sits on the styrofoam slab as a float. The slab is just completely submerged below the water surface. Calculate the area of the base of the slab. (Density of water = 1000 kg m^{-3}) [3]

(L2) 7. (a) In order to raise a heavy object from the sea-bed, a "lifting bag" may be attached to the object and then partially inflated with air, as shown in the figure.



Explain why air has to be released continuously from the lifting bag as the object rises to the surface so that a constant speed of ascent is maintained. {Hint: as pressure decreases, volume increases} [3]

(b) A submerged iron cannon of mass 800 kg and density 8000 kg m^{-3} is attached to a lifting bag of negligible volume and mass. Estimate the initial acceleration of the cannon when 0.70 m^3 of air is suddenly released into the bag. (The density of the water is 1050 kg m^{-3}.) [3]
[J90/III/17 part]

(L2) 8. A solid vertical cylinder has an area of cross-section 0.45 m^2 and is submerged in water of density 1000 kg m^{-3}. It is held in a fixed position by a vertical rod. The top of the cylinder is 1.4 m from the surface of the water and the bottom of the cylinder is 3.2 m from the surface of the water, as shown in Fig. 8.4. The cylinder is made from a material that has a density of 2400 kg m^{-3}.

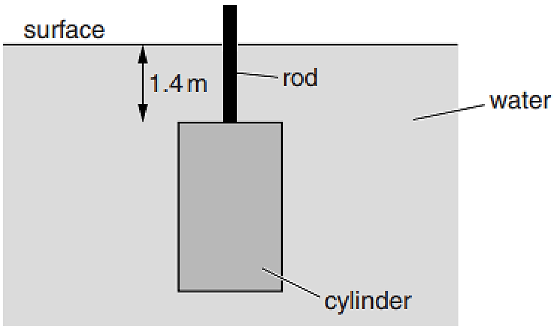


Fig. 8.4

(i) Explain why the water exerts an upthrust on the cylinder.

………………………………………………………………………………………………………

………………………………………………………………………………………………………

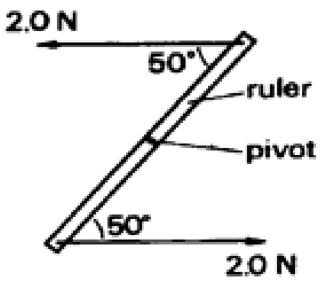
………………………………………………………………………………………………………

……………………………………………………………………………………………………… [3]

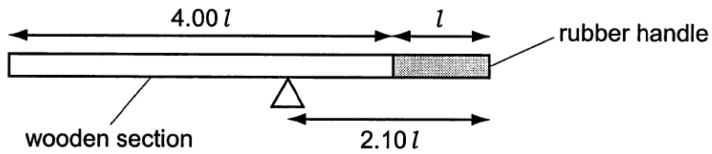
(ii) Determine the magnitude of the force exerted on the cylinder by the rod. [3]
[SAJC BT2 2021]

Moment of a Force & Couple

(L1) 9. A ruler of length 0.3 m is pivoted at its centre. Equal and opposite forces of magnitude 2 N are applied to the ends of the ruler, creating a couple as shown. Determine the magnitude of the torque of the couple on the ruler when it is in the position shown. [2]
[N99/I/5]



(L2) 10. A rod of uniform cross-sectional area has a wooden section and a solid rubber handle, as shown.

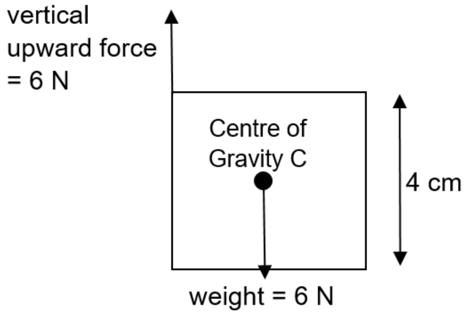


The length of the handle is *l* and the length of the wooden section is 4.00 *l*. The rod balances when the pivot is at a distance 2.10 from the rubber end of the rod.

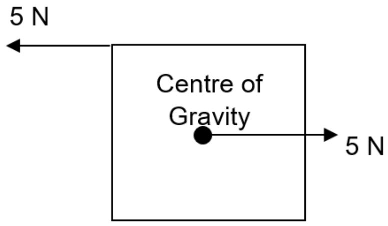
What is the ratio $\frac{\text{density of rubber}}{\text{density of wood}}$? [H1 N14/1/12]

A 1.71 **B** 2.25 **C** 2.50 **D** 3.27

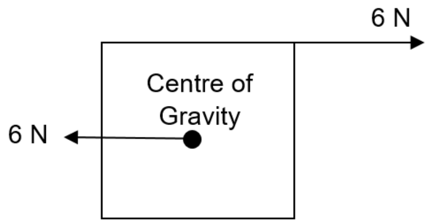
(L2) 11. The diagram below shows two forces acting on a uniform square plate of metal.



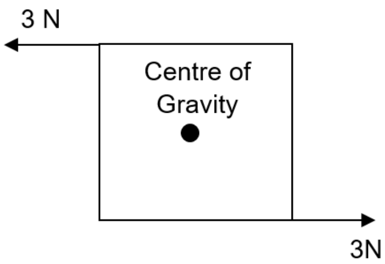
Which of the following force(s) would ensure equilibrium when added to the setup above?



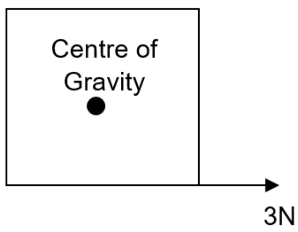
A



B



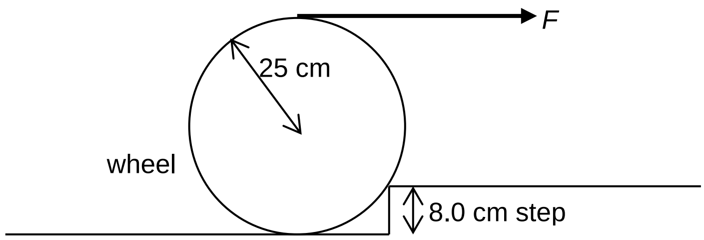
C



D

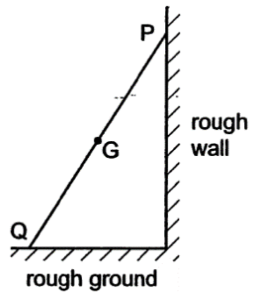
(L2) 12. The figure below shows a uniform wheel of mass 18 kg and radius 25 cm pulled by a horizontal force *F* against a step of height 8.0 cm. Determine the magnitude of force *F* so that the wheel *just* begins to turn over the step. [3]

[N07/II/6 part (H1)]

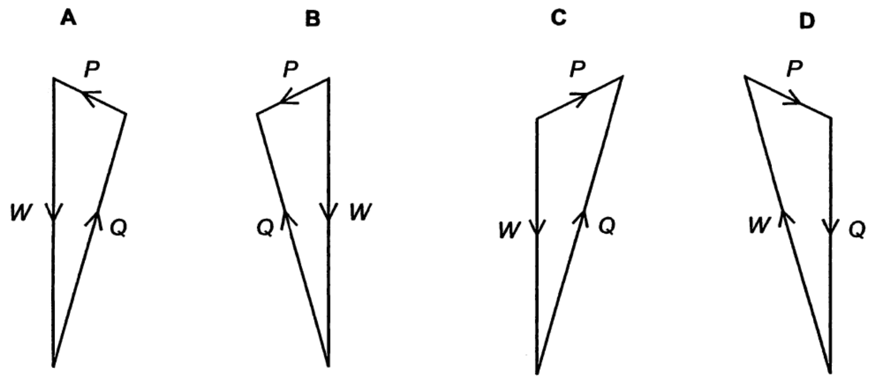


Equilibrium

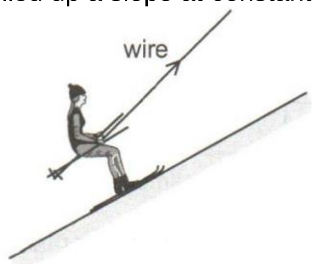
(L1) 13. A ladder rests on rough ground and leans against a rough wall. Its weight *W* acts through the centre of gravity G. Forces also act on the ladder at P and Q. These forces are P and Q respectively. Which vector triangle represents the forces on the ladder?



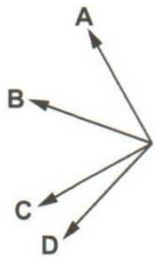
[N07/I/11 (H1)]



(L1) 14. The diagram shows a skier being pulled up a slope at constant speed.

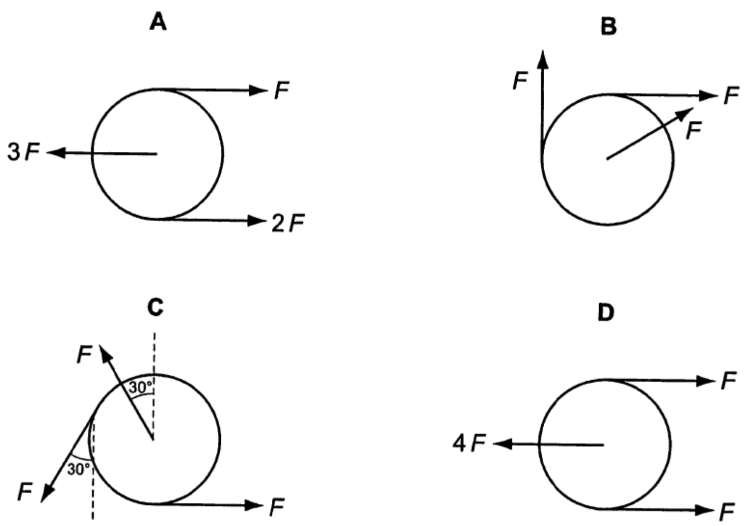


In which direction is the force from the slope on the skier? [2016 P1 Q9]



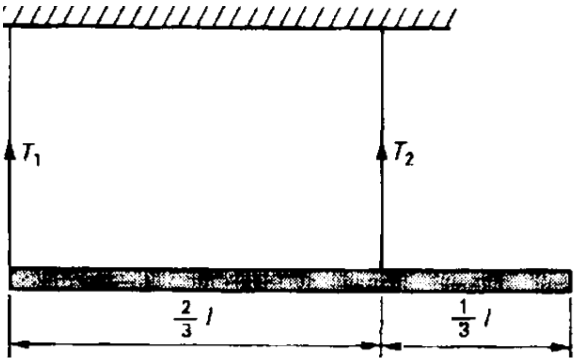
(L2) 15. An isolated disc is subjected to three forces, each given in terms of units of magnitude F. In which situation will the disc experience both a resultant force and a resultant torque?

[N08/I/9 (H1)]



(L2) 16. A heavy uniform beam of length l is supported by two vertical cords as shown in the diagram. Determine the ratio of the tensions in these cords. [2]

[J95/I/5]



[ASSIGNMENT]

(L2) 17. A metal sign of uniform thickness and of width 1.00 m at its upper edge hangs from two vertical, rigid supports, as shown in Fig. 17.1.

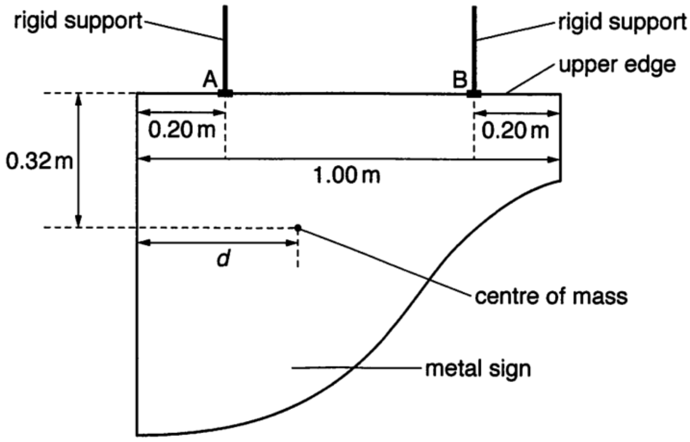


Fig. 17.1

The sign is hinged and swings freely from the rigid supports at points A and B.
The supports are 0.20 m from each edge of the sign. The mass of the sign is 4.5 kg. The centre of mass of the sign is 0.32 m below its upper edge.

(i) The ratio of the tensions in the two supports is $\frac{3}{7}$.
Calculate the magnitude of each tension at A and B respectively. [2]

(ii) Using your answers in (i), determine the horizontal distance d of the centre of mass from the left edge of the metal sign. [2]

(iii) A horizontal wind now blows on the face of the sign so it hangs at an angle θ to the vertical as shown in Fig. 17.2.

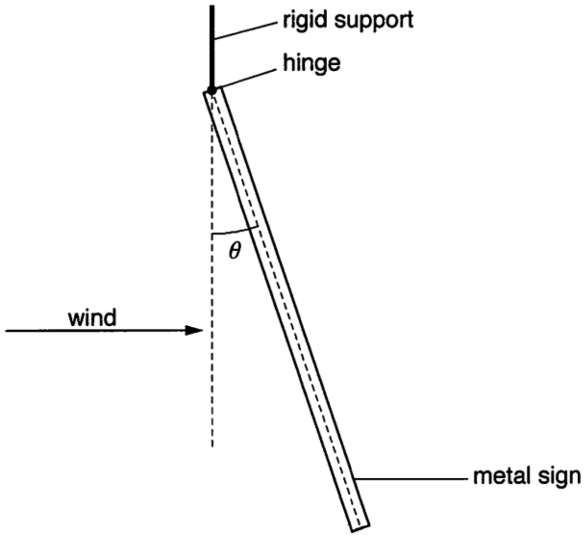


Fig. 17.2

Explain why the force exerted by each support on the sign now has a horizontal component.

……

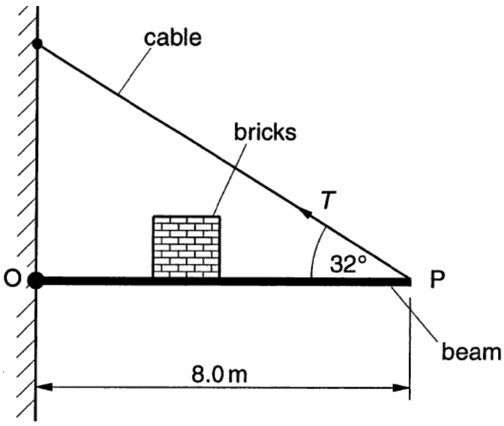
……

……… [1]

[2018 P2 Q1]

(L2) 18. The figure below shows a uniform metal beam pivoted at one end and held in equilibrium in a horizontal plane by a cable attached to its other end. The beam has mass 210.0 kg and length 8.0 m. The cable makes an angle $\theta = 32°$ to the beam and has a tension *T*. A pile of bricks of total mass 150 kg is placed on the beam.

[H1 N08/2/3 (modified)]



(a) Draw labelled arrows to show the **three** other forces acting on the beam. [1]

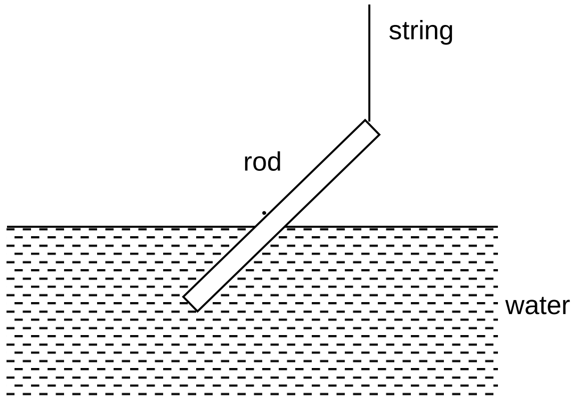
(b) Describe and explain what would happen to the tension *T* in the cable as the pile of bricks is slowly moved towards the end P of the beam [2]

(c) The pile of bricks is placed 5.3 m from the end P .Calculate the tension *T* in the supporting wire. [3]

(d) Compute the magnitude of the force that the wall exerts on the beam at the hinge. [2]

(L2) 19. A ladder of length 10 m and weight 200 N leans against a smooth wall such that it is at an angle of 60° to the horizontal. A boy of weight 500 N stands on the ladder 1/4 of the way from its lower end. Calculate the normal reaction at the wall and the magnitude and direction of the resultant force acting on the lower end of the ladder. [5]

(L2) 20. [Nov 2004/ Special Paper /7c]
A uniform cylindrical wooden rod of weight 60 N, attached at one end to a light string, is slowly lowered into water. It is found that, when the system is in equilibrium, the string is vertical and exactly half of the rod is underwater, as shown in figure below.



(i) Draw a labelled diagram showing the forces acting on the rod when it is in equilibrium. [2]

(ii) The density of water is 1.0×10^3 kg m^{-3}. Calculate the density of the wood from which the rod is made. [3]

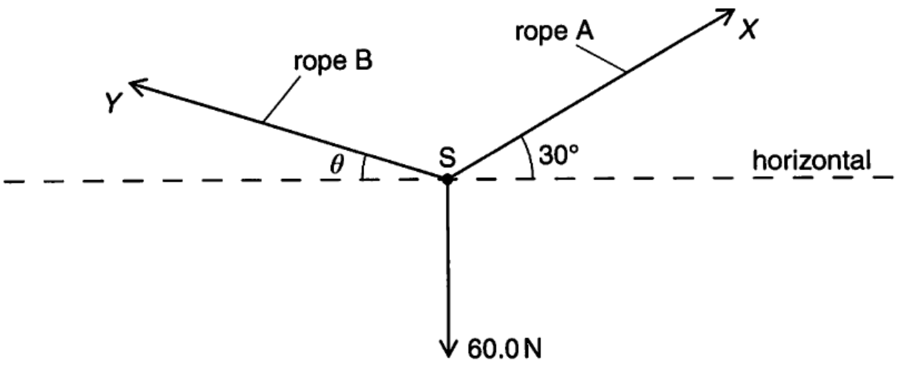
(iii) Calculate the tension in the string. [2]

Numerical Answers

1. 4.0 mm	6. 0.857 m^2
7. (b) 0.49 m s^{-2}	8. (ii) 11100 N
9. 0.46 N m	12. 77 N
16. 0.333	17(i). T_A = 30.9 N, T_B = 13.2 N (ii) d = 0.38 m
18.(c) 2880 N (d) 3160 N	19. θ = 79.5 ° R = 712 N
20.(ii) 750 kg m^{-3}, (iii) 20 N	

ADDITIONAL QUESTIONS

1 An object S of weight 60.0 N is supported by two ropes A and B, as shown in the figure.



Rope A is at 30° to the horizontal and exerts force X on S. Rope B is at an angle θ to the horizontal and exerts force Y on S. The magnitude of force X is varied from 0 to 200 N. Rope A is always kept at 30°to the horizontal. The force Y is varied in magnitude and direction to keep S in equilibrium

(a) Determine the magnitude and direction of force Y for the magnitude of force X equal to

(i) zero [2]

(ii) 200 N [3]

(b) By reference to figure, explain why the rope B cannot be parallel to the weight of S no matter how large the magnitude of X.

………

………

………

……. [2]

[N2015 P2Q1]

2 A boom can be used to assist a person to move heavy loads. A typical arrangement is shown below.

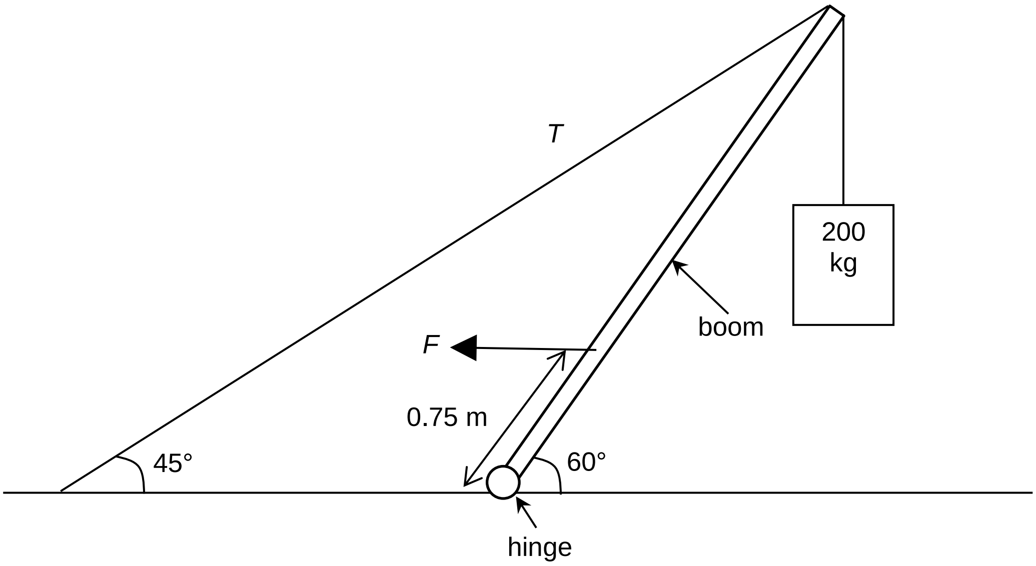


Fig. 1.1

The boom is angled at 60° to the horizontal, and a steel cable is attached to the top of the boom and the floor such that the cable makes an angle of 45° to the horizontal, as shown. The uniform boom has a mass of 45 kg and length 3 m.

A human operator exerts a force F = 120 N horizontally at a distance 0.75 m away from the hinge as measured along the boom. The system is in equilibrium.

(a) State the conditions for a body to be in static equilibrium.

………

……… [2]

(b) Show that the tension T in the cable connecting from the top of the boom to the floor is 4.1 kN.

[3]

(c) Determine the magnitude and direction of the force exerted by the hinge on the boom.

magnitude of force = ………………………….. N

direction of the force = ……………………………………………………… [4]

[SAJC 2019 Prelim]

3 A spherical ball, with one quarter of its volume removed, is allowed to float in water as shown in Fig. 23.2. Point **O** is the centre of the spherical ball, and it is in line with the water level when it is partially submerged. The density of the water is 1000 kg m^{-3}.

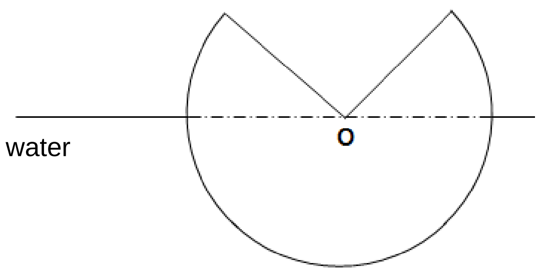


Fig. 23.2

(i) Determine the density of spherical ball.

density = ……..…………….. kg m^{-3} [2]

(ii) A tension of 300 N is required to pull the ball downwards to keep the ball completely submerged and stationary. Determine the radius of the ball.

radius of ball = …………..………….. m [3]

[SAJC 2015 FE]

4 A small air bubble in some water is rising to the surface with constant velocity.

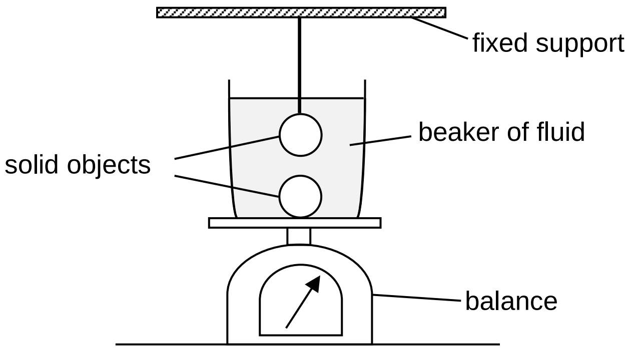
The volume of the bubble is 2.370 x 10^{-8} m^{-3}.
The density of water is 1000 kg m^{-3}.
The density of air is 1.290 kg m^{-3}.

What is the magnitude of the viscous force on the bubble?

A 2.367 x 10^{-5} N **B** 2.373 x 10^{-5} N
C 2.322 x 10^{-4} N **D** 2.328 x 10^{-4} N

[SAJC 2017 FE]

5 A beaker of fluid has weight *Z*. A solid object of weight *X* in air displaces weight *Y* of the fluid when it is fully immersed in the fluid. An identical solid object is placed at the bottom of the beaker as shown

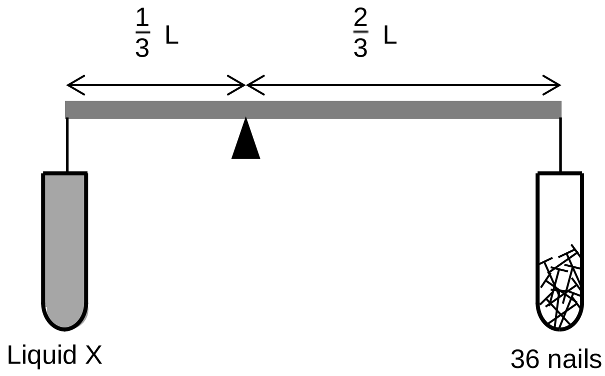


: is the balance reading?

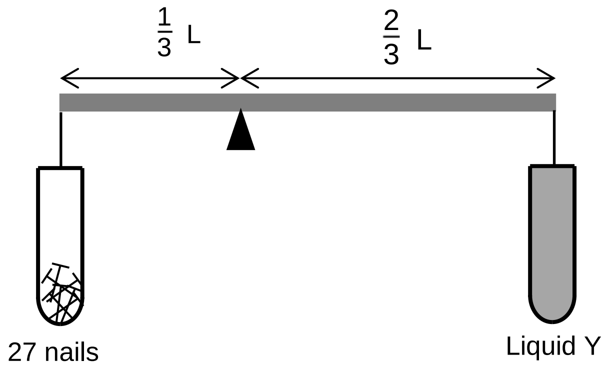
A $X + Z$ **B** $2Y + Z$ **C** $X + Y + Z$ **D** $2X - 2Y + Z$

[SAJC 2019 FE]

6 The diagram shows two light plastic tubes hanging from the ends of a massless beam which is pivoted at one-third of its length *L*. When the tube on the left is completely filled with liquid X of density ρ_X, it takes 36 nails in the tube on the right to make the beam horizontal.



Both tubes are then emptied. The tube on the right is now completely filled with liquid Y of density ρ_Y and it takes 27 nails in the tube on the left to make the beam horizontal again.

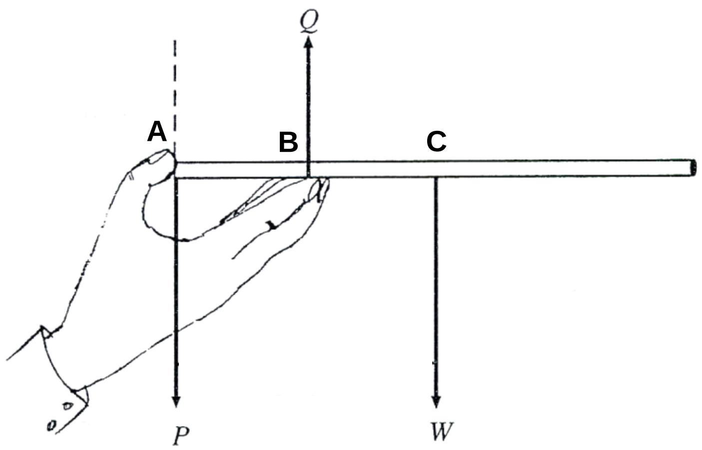


What is the ratio $\frac{\rho_X}{\rho_Y}$?

A $\frac{1}{3}$ **B** $\frac{4}{3}$ **C** $\frac{8}{3}$ **D** $\frac{16}{3}$

[SAJC 2016 BT2]

7 A waiter holds a tray horizontally in one hand between fingers and thumb as shown below.



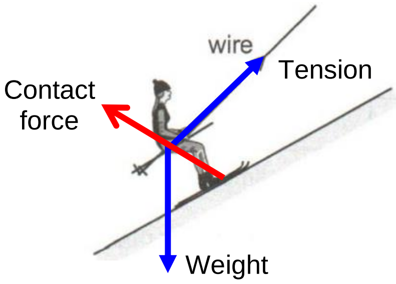
P, *Q* and *W* are the three forces acting on the tray. *P* and *Q* are the contact forces the fingers are exerting on the tray while *W* is the weight of the tray. The waiter is about to place a glass of water on the tray.

At which point should the glass be positioned on the tray such that the **magnitude of P** remains the **same** with or without the glass?

A Point A **B** Point B **C** Point C **D** None of the above

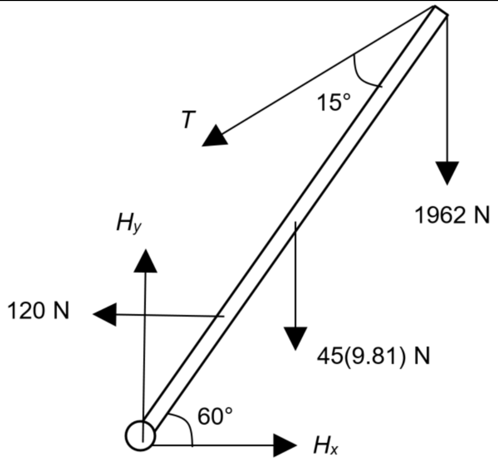
Tutorial 4: FORCES SOLUTIONS

Level 1 Solutions

1	Using Hooke's Law, $F = kx$ $300 = k(6.0 \times 10^{-3})$ $k = 50000 \text{ N m}^{-1}$ When force = 200 N, $200 = (50000)(x)$ $x = 4.0 \times 10^{-3}$ m or 4.0 mm	[1] [1]
2	Ans: D Upthrust = upward force – downward force $= p_2 A - p_1 A$	[1]
9	Torque of couple = Force x Perpendicular distance between line of action of forces $= 2 \times 0.3 \sin 50° = 0.46 Nm$	[1]
13	Ans: A Since the rough wall is used, force from P would have been upward and leftward. Since the rough ground is used, force from Q would have been upward and rightward. Finally, at equilibrium, a closed vector diagram is expected.	[1]
14	Ans: B Since there are only 3 forces acting on the person, the forces must meet at a common point. 	

Solutions to Additional Questions

1(a)	If X = 0 N, then Y has to be 60.0 N upwards. {to balance the downward force of the weight} Thus magnitude of Y = 60.0 N, Angle = 90°	[1] [1]
(b)	For horizontal eqm, Y_x = 200 cos 30° = 173.21 N leftwards For vertical eqm, Y_y = 200 sin 30° - 60.0 = - 40.0 N downwards Thus magnitude of Y = 178 N Angle = $\tan^{-1}$(40.0 / 173.21) = 13.0° Following the defined angle in the diagram, angle = -13.0° {Note: angle of Y is negative since Y acts below the horizontal.}	[1] [1] [1]
(c)	There will always be a non-zero horizontal component of X (X cos 30°). For equilibrium, the resultant of the horizontal component of Y and weight must equal X cos 30°. Since weight has no horizontal component, Y must have a horizontal component and thus rope B cannot be parallel to weight of S (vertical).	[1] [1]
2(a)	<u>Net</u> force is zero (in all direction). <u>Net</u> moment is zero about <u>any</u> point. **DO NOT ACCEPT: …a point OR a given point**	[1] [1]

(b)	 Evidence of angle between *T* and boom (in working) = 15° Taking moments about the hinge, (Tsin 15°)(3) + (120sin 60°)(0.75) = (45)(9.81)(1.5cos 60°) + (1962)(3cos 60°) T = 4116 = 4.1 kN	[1] [2]
(c)	(Let H_y and H_x be the vertical and horizontal components of the force hinge acts on the boom) H_y = 1962 + 45(9.81) + Tcos 45° = 5314 N H_x = 120 + Tsin 45° = 3030 N Therefore, magnitude of force hinge acts on boom = $\sqrt{5314^2+3030^2}$ = 6120 N Direction of the force = $\tan^{-1}$ (5314 / 3030) = 60.3° above horizontal	[1] [1] [1] [1]
3(i)	Given that it is in equilibrium, Upthrust = Weight of ball (Note: ½ of the ball is submerged, while ¾ of ball remains!) $\rho_W \times \frac{1}{2} V \times g = \rho_{Ball} \times \frac{3}{4} V \times g$ $1000 \times \frac{1}{2} V \times g = \rho_{Ball} \times \frac{3}{4} V \times g$ $\Rightarrow$ ρ_{Ball} = **667** kg m^{-3}	[1] [1]
(ii)	Given that it is in equilibrium, Upthrust = Weight of ball + tension $\rho_W \times \frac{3}{4} V \times g = \rho_{Ball} \times \frac{3}{4} V \times g + 300$ $1000 \times \frac{3}{4} V \times g = 667 \times \frac{3}{4} V \times g + 300$ V = 0.122 m^3 = $\frac{4}{3} \pi r^3$ $\Rightarrow$ r = **0.308 m**	[1] [1] [1]
4	**Ans: C** Upthrust = air bubble weight + viscous force $\rho_{water}Vg = \rho_{air}Vg$ + viscous force Viscous force = $V(\rho_{water} - \rho_{air})g = (2.370 \times 10^{-8})(1000 - 1.290)(9.81) = 2.322 \times 10^{-4}$ N	
5	**Ans: C** Total weight is Z (water) + X (sphere resting at bottom) + Y (downward reaction of suspended sphere)	
6	**Ans: D** With liquid X, $\rho_X V g \frac{1}{3} L = 36 W_{Nails} \frac{2}{3} L$ With liquid Y, $\rho_Y V g \frac{2}{3} L = 27 W_{Nails} \frac{1}{3} L$ Dividing the two equations yields $\frac{\rho_X}{\rho_Y} = \frac{16}{3}$	
7	**Ans: B** If the glass is placed at point B, only Q will increase in magnitude to maintain equilibrium. Since W will not change, P will remain the same magnitude to maintain rotational equilibrium about B.	