TOPIC 2: KINEMATICS

Learning Outcomes: Candidates should be able to:

a.	Show an understanding of and use the terms distance, displacement, speed, velocity and acceleration.
b.	Use graphical methods to represent distance, displacement, speed, velocity and acceleration.
c.	Identify and use the physical quantities from the gradients of displacement-time graphs and areas under and gradients of velocity-time graphs, including cases of non-uniform acceleration.
d.	Derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line.
e.	Solve problems using equations which represent uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance.
f.	Describe qualitatively the motion of bodies falling in a uniform gravitational field with air resistance.
g.	Describe and explain motion due to a uniform velocity in one direction and a uniform acceleration in a perpendicular direction.

Two broad areas:
- Rectilinear motion (in a straight line)
- Non-linear motion (not in a straight line)

Kinematics is the branch of classical mechanics that describes the motion of objects (how - kinematics) without consideration of the forces that cause the motion (why - dynamics). We will be analyzing one dimensional motion along a straight line (rectilinear motion) as well as two-dimensional motion along a curved path (non-linear motion).

Key questions:
- How do you describe and represent motion precisely using terms like position, distance, displacement, speed, velocity and acceleration and in graph?
- How do you apply mathematical analysis to quantify motion?
- How do objects move under the influence of gravity with and without air resistance?

Why is the study of kinematics important?
Objects are in motion all around us. Everything from molecules and atoms vibrating, your blood flowing to a moving bus to the rotation of the Earth around the Sun.

2.1 Definitions of Displacement, Speed, Velocity and Acceleration

2.1.1 Distance and Displacement

Fig. 2.1 below shows two possible paths between two positions A and B.

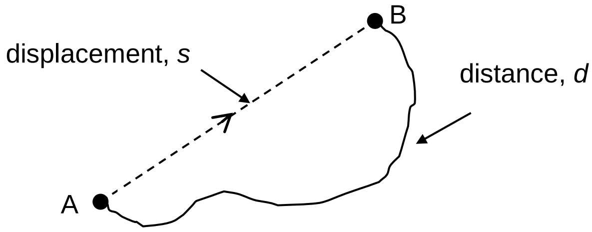


Fig 2.1

If a body moves from position A to position B along the curved path, the total length AB covered by the body irrespective of the direction of motion is called the distance, *d*.

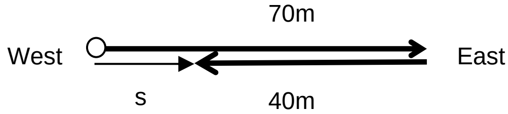
In contrast, the displacement, *s* of the body is the linear/shortest distance between position A and position B, AB with the direction shown along the dotted path.

	Distance	**Displacement**
Definition	**Distance** travelled is the total length of the actual path travelled by an object irrespective of the direction of motion	**Displacement** is defined as the distance moved **in a specific direction**
Scalar or Vector	Scalar	Vector Directed from the start position to the end position Use arrow to represent a vector
SI unit	metre (m)	metre (m)

Example 1 (Physics, Giancoli, 6th ed) : Distance vs Displacement – rectilinear motion

A person walks 70 m east and then turns around and walk 40 m to the west.

(a) What is the total distance travelled?
(b) What is his displacement?



Solution:

(a) Total distance travelled is

(b) Displacement, s =
=

Example 2: Distance vs Displacement

A person walks 4 m east, then walks 3 m north. Determine distance and displacement.

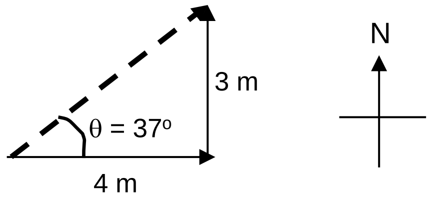
Solution:

Total distance travelled is = 4 + 3 = 7 m

Displacement, s = $\sqrt{4^2 + 3^2}$

= 5 m

$\theta = \tan^{-1}\frac{3}{4} = 37°$



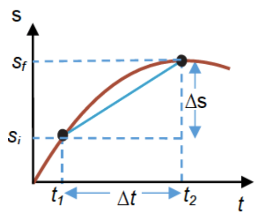
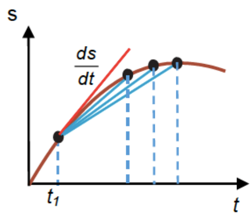
2.1.2 Speed and Velocity

	Speed (Instantaneous)	**Velocity (Instantaneous)**
Definition	**Speed** is the **rate of change of distance travelled**	**Velocity** is the **rate of change of displacement**
Scalar or Vector	Scalar	Vector Tangential to the path Use arrow to represent a vector
SI unit	m s^{-1}	m s^{-1}

Note: "rate of change" refers to "change with respect to time"

Common Error: "*Velocity is the rate of change of displacement per unit* time." This is incorrect because rate of change of displacement already define velocity. This statement would be refer to velocity per unit time.

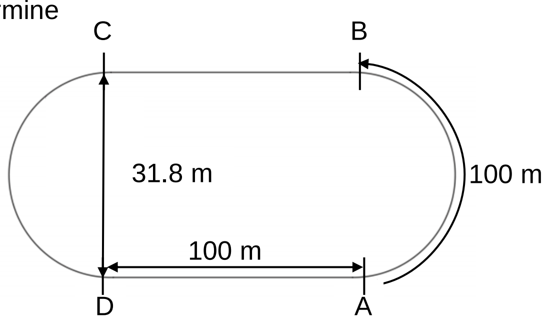
	Average Speed	**Average Velocity**
Definition	$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$	$\text{Average velocity} = \frac{\text{Displacement}}{\text{Total time taken}}$

Comparing average velocity and velocity (instantaneous)	
Average velocity	**Velocity (Instantaneous)**
Over a period of time	At a specific time
 $< v > = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_2 - t_1}$ where s is displacement and t is time	 $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$ where s is displacement and t is time

Note: Instantaneous velocity is equal to average velocity ONLY when the velocity is constant.

Example 3: Average vs Instantaneous, speed vs velocity

A student runs one round in an anticlockwise direction around a 400 m running track, starting from position A, in 120 seconds with constant speed. Determine



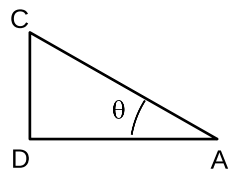
(a) his average speed for the whole journey,
(b) his average velocity from A to B,
(c) his average velocity from A to C,
(d) his speed at D,
(e) his velocity at D.
(f) his average velocity for the whole journey.

Solution:

(a) Average speed for the whole journey = d/t = 400/120 = 3.3 m s^{-1}

(b) Average velocity from A to B =

(c) Displacement from A to C =



Time from A to C =

Average velocity from A to C =

or tan θ =
θ =

(d) Speed at D =

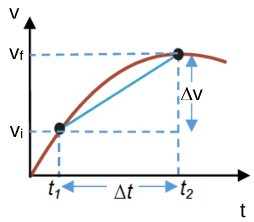
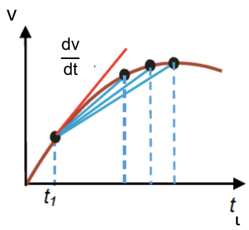
(e) Velocity at D =

(f) Average velocity for the whole journey =

2.1.3 Acceleration

	Acceleration (Instantaneous)
Definition	**Acceleration** is defined as the **rate of *change* of velocity**
Scalar or Vector	Vector Same direction as the resultant force. (More details in dynamic topic - Newton's 2nd law for fixed mass, $F_{net} = ma$) Use arrow to represent an acceleration.
SI unit	m s^{-2}

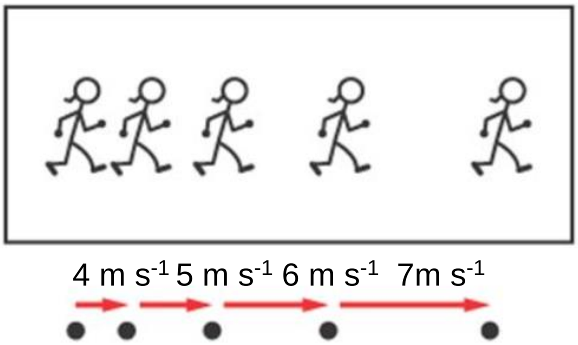
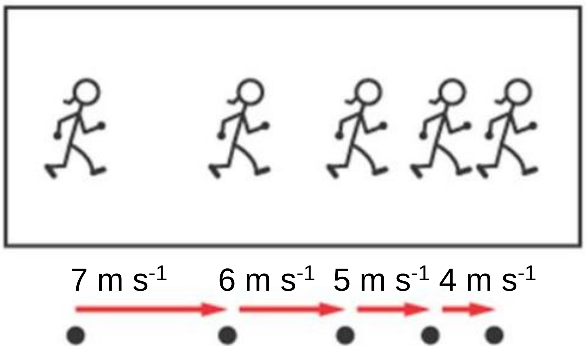
	Average acceleration
Definition	Average acceleration = $\frac{\text{Change in velocity}}{\text{Total time taken}}$

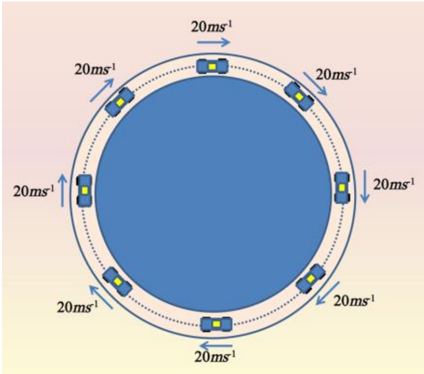
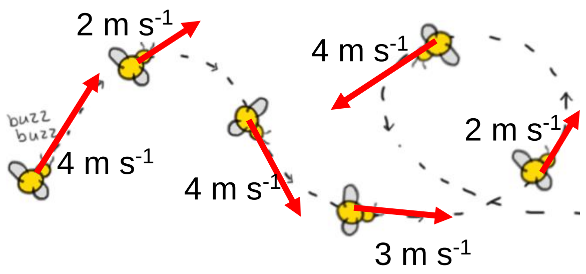
Comparing average acceleration and acceleration (instantaneous)	
Average acceleration	**Acceleration (Instantaneous)**
 $\langle a \rangle = \frac{v_f - v_i}{\Delta t}$ where v is velocity and t is time	 $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$ where v is velocity and t is time

Note: Instantaneous acceleration is equal to average acceleration ONLY when the acceleration is constant.

Since velocity is a vector, it changes when
- its magnitude changes (speed up or slow down) or/and
- its direction changes

A body is accelerating when its velocity is changing (with time) either
- in magnitude or
- in direction or
- in both magnitude and direction

Case	Situation	Acceleration direction
Change in magnitude - Speeding up		Same direction as the change in velocity - in the same direction as velocity as velocity is increasing.
Change in magnitude - Slowing down Slowing down is also known as deceleration		Same direction as the change in velocity - in the opposite direction to velocity as the velocity is decreasing.

<table>
<tr><td>Change in direction only</td><td>
</td><td>Same direction as the change in velocity – draw vector diagram, towards center of circular motion (more details in circular motion topic)</td></tr>
<tr><td>Change in both magnitude and direction</td><td>
</td><td>Same direction as the change in velocity.</td></tr>
</table>

Sign convention for velocity and acceleration

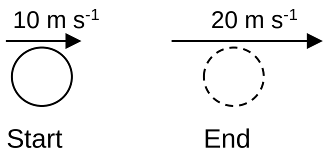
Acceleration can be positive or negative. Positive acceleration does not always mean "speeding up". Similarly, negative acceleration does not always mean 'slowing down".

The following cases will illustrate this.

- Using acceleration is the rate of change of velocity, $a = \frac{\text{final velocity - initial velocity}}{\text{time}}$, and
- Taking rightwards as the positive direction: ⟶ *+ve*

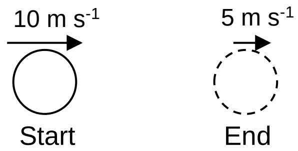
Note: Velocity and acceleration are vectors. If you define right as the positive direction, any velocity or acceleration to the left are considered as negative.

<table>
<tr><td>Case 1</td><td>A particle is speeding up (acceleration) in the positive direction</td></tr>
<tr><td colspan="2">→+ve



$a = \frac{(+20)-(+10)}{\text{time}}$ = a positive value

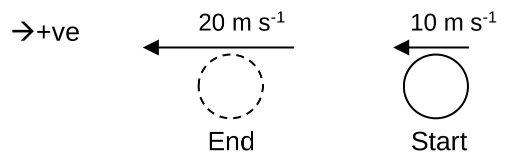
$v_f > v_i$ ➔ speeds up ➔ positive acceleration</td></tr>
<tr><td>Case 2</td><td>A particle is slowing down (deceleration) in the positive direction</td></tr>
<tr><td colspan="2">→+ve



$a = \frac{(+5)-(+10)}{\text{time}}$ = a negative value

$v_f < v_i$ ➔ slows down ➔ negative acceleration</td></tr>
</table>

Case 3 | A particle is *speeding up (acceleration)* in the *negative* direction

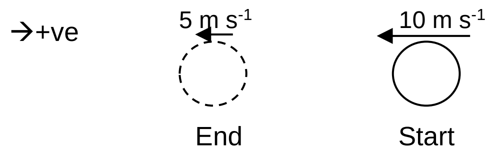


$$a = \frac{(-20)-(-10)}{\text{time}} = \text{a negative value}$$

Why are the velocities and acceleration negative? These vectors point to the left and right is taken to be the positive direction.

$v_f > v_i$ ➔ speeds up ➔ negative acceleration

Case 4 | A particle is *slowing down (deceleration)* in the *negative* direction



$$a = \frac{(-5)-(-10)}{\text{time}} = \text{a positive value}$$

$v_f < v_i$ ➔ slows down ➔ positive acceleration

From the cases, it can be seen that any positive value must be a vector that points in the same direction as the convention taken. The converse is true for any negative value.

To know if acceleration is positive or negative, the change in velocity is calculated taking into account that velocity is a vector and a specific direction is set to be positive.

Note that if acceleration and the velocity are in the same direction (i.e. resultant force also in same direction as the velocity) ➔ the particle will speed up.

Conversely, if acceleration and the velocity are in opposite direction (i.e. resultant force also in the opposite direction as velocity) ➔ the particle will slow down.

This makes sense as you can imagine an applied force in the direction of the velocity should make the particle speed up. An opposite force slows it down. You will learn more about this in dynamics.

Concept check:

For each of the following cases, explain and give an example if the answer to the question is a 'yes'.

No	Case	Yes/No	Example
1	Can a body have zero velocity and still be accelerating?	Yes	*A body that is momentarily at rest like one making a U-turn. There is a change in velocity thus an acceleration at that moment.*
2	Can a body undergo acceleration when its speed remains constant?	Yes	*A body undergoing circular motion has a constant speed (but changing velocity due to changing direction, with the acceleration pointing perpendicularly to the velocity vector at all times).*

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>Can a body be increasing in speed as its acceleration decreases?</td><td>Yes</td><td>A body can be increasing in speed but the increase decreases as time passes.

Example:
<table>
<tr><td>Time /s</td><td>Velocity /m s⁻¹</td><td>Acceleration/ m s⁻²</td></tr>
<tr><td>0</td><td>0</td><td rowspan="5">40 m s⁻²
30 m s⁻²
20 m s⁻²
10 m s⁻²</td></tr>
<tr><td>1</td><td>40</td></tr>
<tr><td>2</td><td>70</td></tr>
<tr><td>3</td><td>90</td></tr>
<tr><td>4</td><td>100</td></tr>
</table></td></tr>
<tr><td>4</td><td>Is acceleration always in the same direction as velocity?</td><td>No</td><td>Acceleration is defined as the rate of CHANGE of velocity. Hence acceleration must necessarily be in the same direction as the change in velocity and NOT the velocity.</td></tr>
</table>

Tutorial qn: Q1

2.2 Graphical Representations of Motion

The typical graphs are

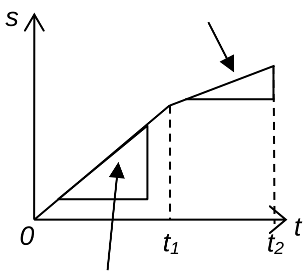
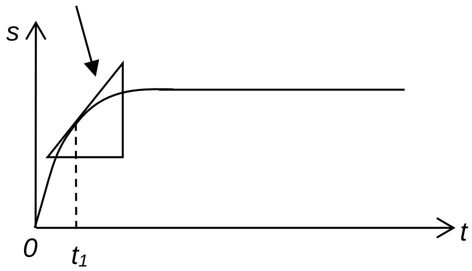
- Displacement-time (*s*-*t*) graph
- Velocity-time (*v*-*t*) graph
- Acceleration-time (*a*-*t*) graph

The useful information interpreted from these graphs are summarised below:

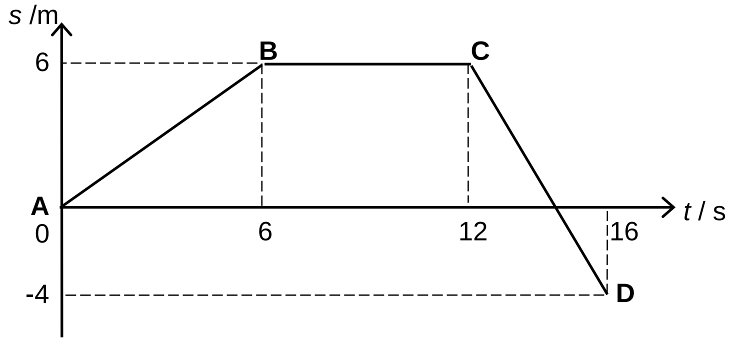
Graph	Gradient	Area enclosed by graph and x-axis
Displacement-time (*s* - *t*)	Velocity = $\frac{ds}{dt} = v$	-
Velocity-time (*v* - *t*)	Acceleration = $\frac{dv}{dt} = a$	Change in displacement = $\int v\,dt = \Delta s$
Acceleration-time (*a* - *t*)	-	Change in velocity = $\int a\,dt = \Delta v$

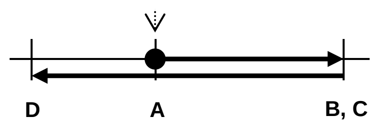
2.2.1 Displacement against time (*s* - *t*) graph

The **gradient of the displacement-time graph** at any instant is the **instantaneous velocity** of the body (at that instant), since velocity is defined as the rate of change of displacement.

Straight-line	Curve
Gradient of the tangent = constant velocity from t_1 to t_2.  Gradient of the tangent = constant velocity from *0* to t_1.	Gradient of the tangent at t_1 = instantaneous velocity at t_1. 

The s - t graph below describes the position of a body from a fixed-point **A**, over a period of time.





Stage/ Time	What is observed from s-t graph	Description of displacement	Description of velocity
A → B **0 to 6 s**	Value of s: increases from 0 to 6 m. Gradient: A straight line starting from s = 0 m with positive gradient	The body moves from 0 to 6 m.	The body moves towards **B** with a constant velocity of $\frac{6 - 0}{6}$ = 1 m s^{-1} **Note:** The gradient is positive meaning the velocity is in the same direction as the positive sign convention assumed
B → C **6 to 12 s**	Value of s: stays constant at 6 m Gradient: A horizontal line	The body remains at 6 m from start point	The body is stationary for 6 seconds
C → D **12 to 16 s**	Value of s: decreases from 6 m to - 4 m Gradient: A straight line with negative gradient passing through s = 0 m	The body moves from 6 m , passing by the start point at 0m and moves to 4 m before the start point.	The body makes a U-turn at **C** to move towards **D** with a constant velocity of $\frac{- 4 - 6}{16 - 12}$ = - 2.5 m s^{-1} **Note:** The gradient is negative meaning the velocity is in the direction opposite to the positive sign convention assumed)

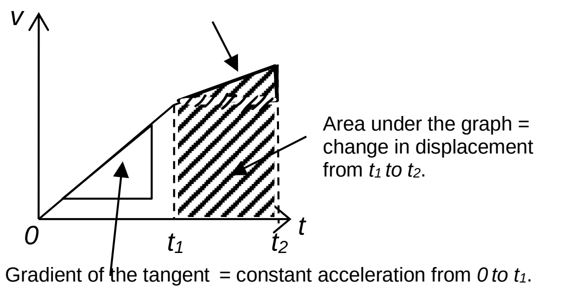
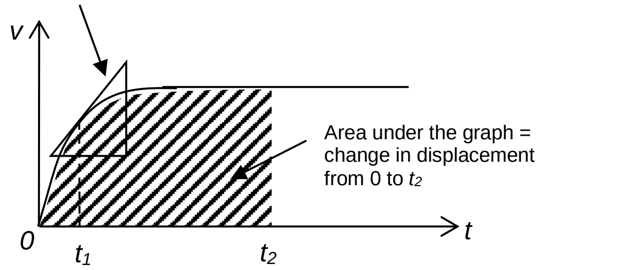
Other information:

- The distance travelled from **A** to **D** = 6m + 6m + 4m = 16 m
- Average speed between **A** & **D** = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$ = 16 / 16 = 1 m s^{-1}

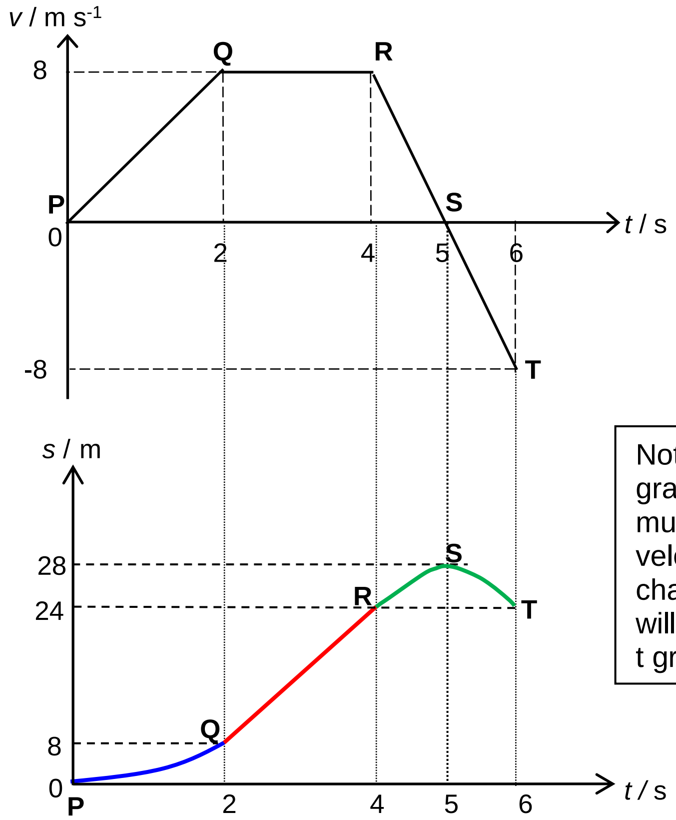
2.2.2 Velocity against Time (v - t) Graph

The **gradient of the *v-t* graph** at any instant is the **instantaneous acceleration** of the body (at that instant), i.e. $a = \frac{dv}{dt}$ by definition.

The **area under the *v-t* graph** is the **<u>change</u> in displacement** of the body within the time interval, since $\Delta s = \int v\, dt$.

Straight-line	Curve
Gradient of the tangent = constant acceleration from t_1 to t_2.  Gradient of the tangent = constant acceleration from 0 to t_1.	Gradient of the tangent at t_1 = instantaneous acceleration at t_1. 

The v - t graph below describes the motion of a body.
Draw the corresponding s - t graph.



Note: The change in the gradient of s-t graph must be smooth as velocity cannot be changing abruptly. Mark will be deducted if the s-t graph is not smooth

<u>How to draw s-t graph given *v-t* graph</u>

Physics concept linking *s* and *v*: Velocity is the gradient of the *s*-*t* graph. i.e. $v = \frac{ds}{dt}$.

At each stage:

PQ: Velocity is positive and increasing → Gradient of *s*-*t* graph is positive and increasing → upward sloping and gets steeper

QR: Velocity is constant. Gradient of *s*- *t* graph is constant → linear line

RS: Velocity is positive and decreasing → Gradient of *s*-*t* graph is positive and decreasing → upward sloping and gets less steep

ST: Velocity is negative and increasing → Gradient of *s*-*t* graph is negative and increasing → downward sloping and gets steeper

Prior knowledge:

Sign of gradient

- Positive gradient → slope upwards
- Negative gradient → slope downwards

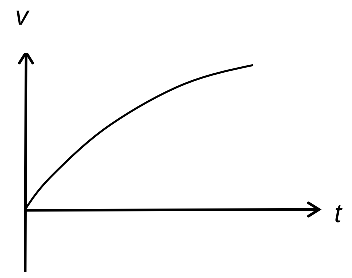
Change in gradient

- Increasing gradient→ slope becomes steeper
- Decreasing gradient→ slope becomes gentler

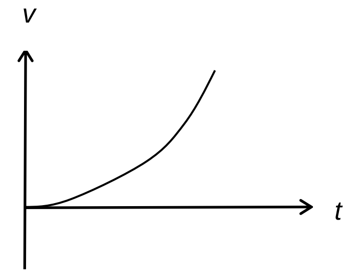
Note: It is easier to deduce shape of graph from gradient rather than from area under the graph. Thus if you have to draw s- t from v, look at the value of v and deduce the corresponding gradient of s rather than look at area under v- t graph (change in s).

Stage/ Time	What is observed from v- t graph	Description of displacement, velocity and acceleration
P → Q **0 to 2 s**	Value of v: increases from 0 to 8 m s^{-1}. Gradient: A straight line starting from v = 0 m with positive gradient	The body accelerates constantly/uniformly from rest to 8 m s^{-1} at $\frac{8}{2} = 4$ m s^{-2} (from gradient). The displacement from **P** is $\frac{1}{2}$ (8 x 2) = 8 m (from area under the graph)
Q → R **2 to 4 s**	Value of v: stays constant at 8 m s^{-1}. Gradient: A horizontal line	The body moves with constant velocity of 8 m s^{-1}; thus acceleration is zero. The displacement from **Q** is (4-2) x 8 = 16 m.
R → S → T **4 to 5 s to 6 s**	Value of v: decreases from 8 m s^{-1} to - 8 m s^{-1}. Gradient: A straight line with negative gradient passing through v = 0 m s^{-1} at. 5 s.	The body accelerate constantly/uniformly from 8 m s^{-1} to - 8 m s^{-1} at $\frac{-8-(-8)}{6-4} = -8$ m s^{-2}. <u>From 4 to 5 s:</u> Acceleration is negative while the velocity is positive in the opposite direction ➔ Slowing down thus decelerating at 8 m s^{-2} <u>From 5 to 6 s:</u> Acceleration is negative and the velocity is negative in the same direction ➔ Speeding up at 8 m s^{-2} **Note:** A negative acceleration does NOT means a deceleration. The body stops at *t* = 5 s. The displacement from R to S is $\frac{1}{2}$ (8)(5-4) = 4 m (area above the x-axis) The displacement from S to T is $\frac{1}{2}$ (-8)(5-4) = - 4 m (area below the x-axis)

Note:



Decreasing acceleration
i.e. velocity is *increasing at a decreasing rate*.

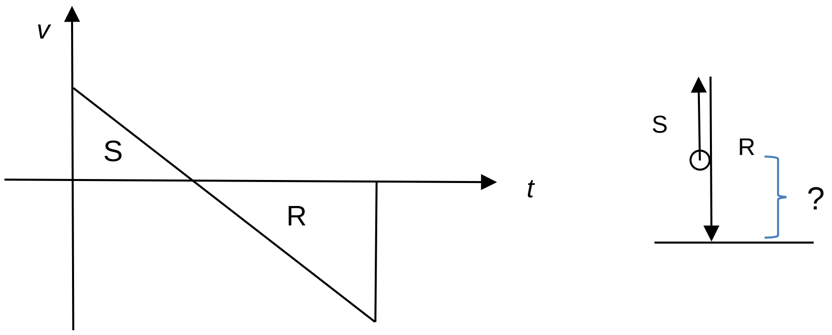


Increasing acceleration
i.e. velocity is *increasing at an increasing rate.*

Note that in both figures above, the magnitude of the velocity is increasing, thus both cases are NOT considered to be "deceleration".

Example 5 (N98/I/3): Area under v- t graph

A stone is thrown upwards from the top of the cliff. After reaching its maximum height, it falls past the cliff-top and into the sea. The graph shows how the vertical velocity *v* of the stone varies with time *t* after being thrown upwards. R and S are the magnitudes of the areas of the two triangles.

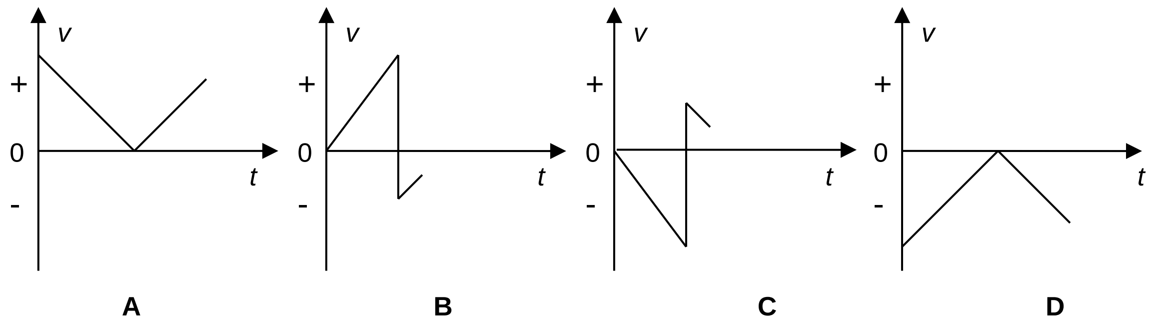


What is the height of the cliff-top above the sea?

A R **B** S **C** R + S **D** R – S

Worked Example 6 (J79/2/5): v-t graph of a free-falling object bouncing on the floor

A tennis ball is released so that it falls vertically to the floor and bounces back again. Taking velocity upwards as positive, which one of the following graphs best represents the variation of velocity ***v*** with time ***t***? Is there energy loss after the bounce? How do you know?



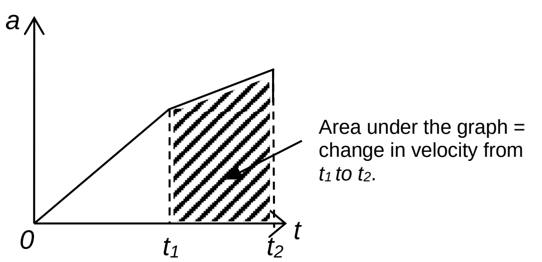
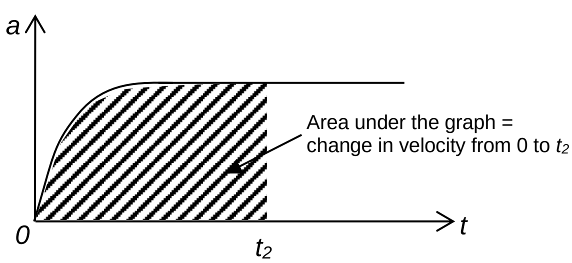
Solution:

This question states that the velocity upwards is taken as positive. Ball is released from rest. Hence velocity is negative as it falls downward. When the ball strikes the ground, it decelerates (slows down) and accelerates (speeds up) upwards within a very short period of time with a steep positive gradient. The velocity is positive as it bounces upwards.

Yes there is energy loss as the area enclosed represent the displacement. The displacement for falling > displacement for bouncing up as seen by the area enclosed.

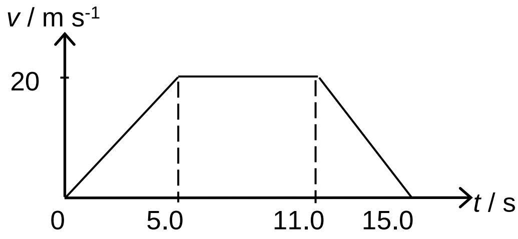
2.2.3 Acceleration against Time (*a* - *t*) Graph

Area under the *a*-*t* graph is the **change in velocity** of the body.

Straight-line	Curve
	

The v - t graph below describes the motion of a body.

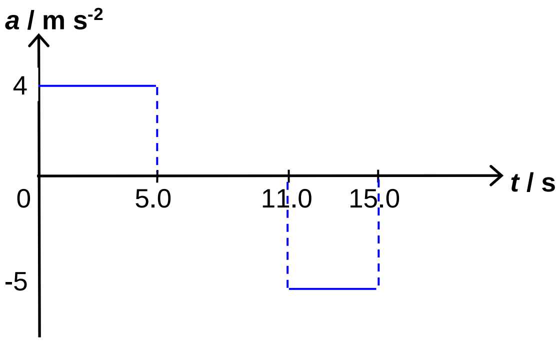
Draw the corresponding a - t graph.



How to draw *a-t* graph given *v-t* graph

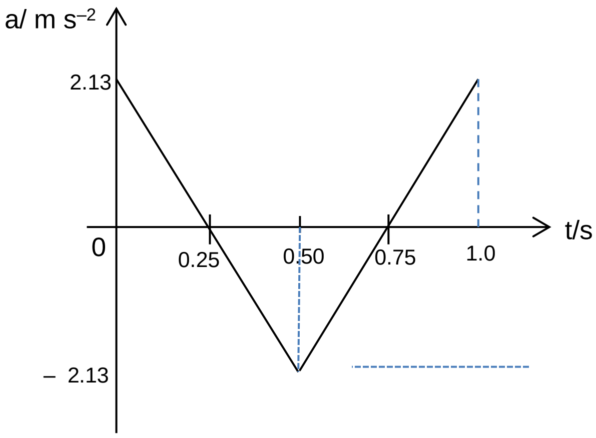
Physics concept linking *a* and *v*:

Acceleration is the gradient of the v-t graph. i.e. $a = \frac{dv}{dt}$. Hence



Worked Example 7 with Visible Thinking (N13/2/2b modified): Interpret velocity from a-t graph

The variation with time *t* of acceleration *a* of a mass is shown below. At time = 0, the mass is at rest.



Describe the change in velocity of the mass.

(a) from t = 0 to t = 0.25 s

> ***Thought process:***
> *Information: At t = 0, v = 0. Acceleration is positive. Acceleration decreases in magnitude.*
>
> *Concept: Since acceleration is a vector quantity, sign of a-t graph indicates the direction (direction of change in velocity) and magnitude of a-t indicates the extent of acceleration (rate of change of velocity).*
>
> *Application: Since acceleration is positive, velocity increases in the positive direction. Since magnitude of acceleration is decreasing, velocity changes at decreasing rate.*

Velocity increases at a decreasing rate {because acceleration is positive and decreasing}.

(b) from t = 0.25 s to t = 0.50 s

> ***Thought process:***
> *Information: Velocity is positive at t = 0.25 (from earlier part). Acceleration is negative. Acceleration increases in magnitude. Area under graph for (a) is equal in magnitude as area under graph for (b) but opposite sign.*
>
> *Concept: Sign of a-t graph indicates the direction (direction of change in velocity) and magnitude of a-t indicates the extent of acceleration (rate of change of velocity). Area of a-t graph represent change in velocity.*
>
> *Application: Since acceleration is opposite in sign to velocity, the mass is slowing down. Since magnitude of acceleration is increasing, velocity changes at increasing rate. Since the areas for (a) and (b) are equal and opposite, the amount of increase in velocity for (a) is equal to the amount of decrease for (b).*

Magnitude of its velocity is decreasing (mass is slowing down). Velocity decreases at an increasing rate {because acceleration is negative and increasing} and becomes zero at t = 0.50 s.

(c) from t = 0.50 s to t = 0.75 s

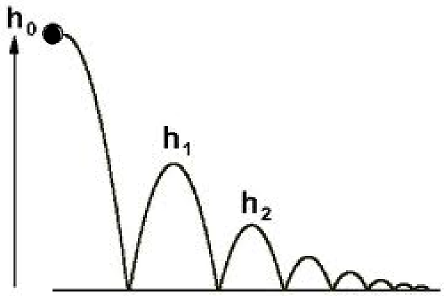
The velocity starts from zero. Mass moves in the opposite direction. Magnitude of velocity increases at a decreasing rate {because acceleration is negative and decreasing}.

(d) from t = 0.75 s to t = 1.0 s

Mass continues to move in the opposite direction. Magnitude of velocity decreases (mass is slowing down) at increasing rate and becomes zero at t = 1.0 s.

Worked Example 8: Graphs representing motion of a bouncing ball

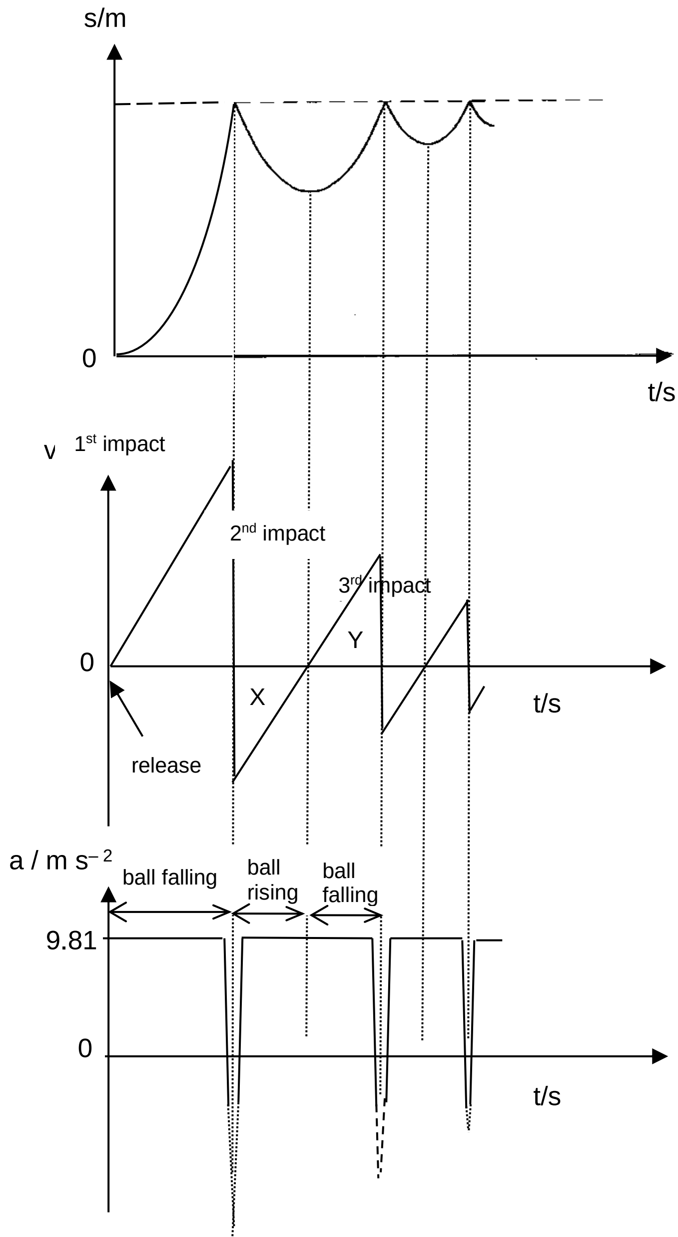
A steel ball is released from rest a distance above a rigid horizontal surface and is allowed to bounce, as shown in the diagram below.



https://www.youtube.com/watch?v=hEWJNrf8I

Taking air resistance as negligible and *downwards as positive*, sketch on the axes provided its corresponding graph representing the variation with time *t* of

(a) its displacement, *s* (b) its velocity, *v* (c) its acceleration, *a*.



Displacement-time graph

- Subsequent maximum height after each bounce gets lower due to energy loss to the ground on impact.

Velocity-time graph

- Gradient to remain the same for every bounce. It is because acceleration of free fall is always 9.81 m s^{-2} downwards ($\because$ F_{net} = ma and F_{net} = weight) regardless whether the ball is rising or falling.
- Velocities just before and just after impact are of opposite directions.
- Max velocity decreases because energy is lost when ball hits the ground.
- Area X = Area Y (same distance when ball rises and when ball falls)

Acceleration time graph

- Acceleration remains constant when ball is in the air regardless whether the ball is rising or falling. This is the acceleration of free fall.
- Time at which the ball is in the air decreases. (the horizontal line of the graph shortens)
- Dotted line to represent the steep negative gradient in a very short period of time. This is due to the upward reaction force acting on ball during impact. As energy is lost with each impact, this dotted line gest shorter.

Complete understanding of this part requires the understanding of dynamics.

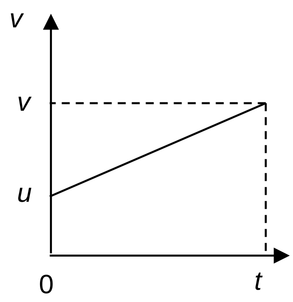
Tutorial qn: Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9

2.3 Derivation of Equations for Linear (Straight Line) Motion with Constant Acceleration

Suppose a body moving in linear motion with a constant acceleration *a*, increases its velocity from an initial value *u* to a final value *v* in a time interval *t*. Its *v*-*t* graph will be as shown below.

By definition, acceleration, $a = \dfrac{v-u}{t}$,

$$\Rightarrow \boxed{v = u + at} \quad \text{-------- (1)}$$



Since displacement *s* = area under v-t graph (trapezium)

$$\boxed{s = \left(\frac{v+u}{2}\right)t} \quad \text{-------- (2)}$$

From equations (1) & (2), eliminating *t*, we have:

$$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \left(\frac{v^2-u^2}{2a}\right)$$

$$\Rightarrow \boxed{v^2 = u^2 + 2as} \quad \text{-------- (3)}$$

Equation	Missing variable
$v = u + at$	s
$s = (\frac{v+u}{2})t$	a
$v^2 = u^2 + 2as$	t
$s = ut + \frac{1}{2}at^2$	v

s – displacement
u – initial velocity
v – final velocity
a – acceleration (must be constant)
t – time

Also from equations (1) & (2), eliminating *v*, we have

$$s = \left[\frac{(u+at)+u}{2}\right]t$$

$$s = \left[\frac{2u+at}{2}\right]t$$

$$\Rightarrow \boxed{s = ut + \tfrac{1}{2}at^2} \quad \text{-------- (4)}$$

Conditions to use these equations:

- the motion is linear and,
- the acceleration is constant (which means for eg, that *air resistance must be negligible* for a body falling in Earth's gravitational field). Unless questions mention, air resistance is ignored.

Note: All free-falling body has a constant downward acceleration of g = 9.81 m s^{-2}. Thus the kinematic equations can be used in all free fall questions. Note: g ≠ 10 m s^{-2} anymore!

Tutorial qn: Q10

Suggested strategy for solving kinematics problems:

1. Sketch an informative diagram using arrows to indicate the various information given in the question.

2. Set a sign convention and indicate clearly in your workings and answers the positive direction.
   - \+ direction is usually taken as the direction of the initial velocity *u* or in the direction where most vectors are in
   - any vector (v, s or a) which then has an opposite direction must thus have a negative sign

3. Choose appropriate start and end point.

4. Identify both the known and unknown quantities. Each kinematic equation requires 3 known quantities to solve for an unknown. Two kinematic equation will be needed to solve for two unknowns.

5. Identify the appropriate equation to determine the unknown from given information.

6. Substitute known values into the kinematics equation (include appropriate positive and negative values based on sign convention) and solve for the unknown quantity.

7. Check that the answer is sensible.

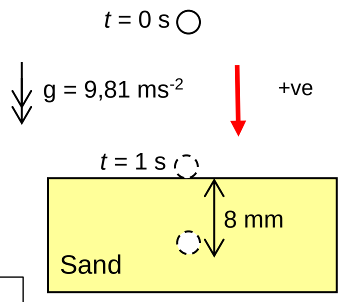
Common Error: Displacement, *s* is measured with respect to a reference point, which is usually the start point. Displacement may be + ve or - ve. It is positive if it is in the direction of the sign convention when measured from the reference point. Substitute the correct sign for *s* in the equations.

Worked Example 9 (N08/1/4): Free falling object decelerated by sand

A metal ball is dropped from rest over a bed of sand. It hits the sand bed one second later and makes an impression of maximum depth 8.0 mm in the sand. Air resistance is negligible. On hitting the sand, what is the average deceleration of the ball?

Solution:

Step 1: Sketch a diagram indicating start and end with information included



(a) Determine the velocity of the ball when it hits the sand bed.

Step 2: Define direction. Take downwards as positive
Step 3: Choose appropriate start and end point.
Step 4: List unknown and known quantities.

s	distance fallen after 1 s - ?
u	0 – start point (ball dropped from rest) – start
v	velocity when it hits the sand bed, v - ? - end
a	9.81 m s $^{-2}$ (acceleration of free fall in air)
t	1.0 s

Step 5: Select appropriate equation (s is not known or required)

: $v = u + a t$

Step 6: Substitute in values with appropriate +/- sign

$v = 0 + (9.81)(1.0)$

: $v = 9.81$ m s $^{-1}$ (upon hitting sand bed)

Step 7: v should be + as it is downwards. Value looks reasonable.

(b) Calculate the average deceleration of the ball in the sand.

Consider the start of the motion at top of sand bed, end at ball stopping within sand

: $v^2 = u^2 + 2\,a\,s$

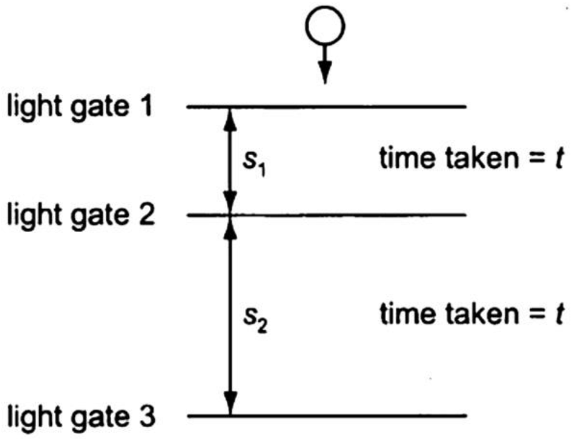
$0 = (9.81^2) + 2\,(a)(8.0 \times 10^{-3})$

$a = -\,6020$ m s $^{-2}$

Hence deceleration is 6.02×10^3 m s $^{-2}$ (Can ignore -ve sign as mentioned deceleration)

Worked Example 10 (N10/1/5): Free falling object

An object falls freely with constant acceleration *a* from above three light gates. It is found that it takes a time *t* to fall between the first two light gates a distance of s_1 apart. It then takes an additional time , also *t*, to fall between the second and third gates a distance of s_2 apart.



What is the acceleration in terms of s_1, s_2 and t ?

A. $\dfrac{(s_2 - s_1)}{t^2}$ B. $\dfrac{(s_2 - s_1)}{2t^2}$ C. $\dfrac{2(s_2 - s_1)}{3t^2}$ D. $\dfrac{2(s_2 - s_1)}{t^2}$

Solution: A

All the four options include $(s_2 – s_1)$ and t. Hence the equation of motion chosen will be $s = ut + ½ at^2$

Between gates 1 and 2:

$\downarrow$: $s = ut + \frac{1}{2}at^2$

$s_1 = ut + ½ at^2$ ----(1), where u = initial velocity of the ball at light gate 1

Between gates 1 and 3:

$\downarrow$: $s = ut + \frac{1}{2}at^2$

$s_1 + s_2 = u(2t) + ½ a(2t)^2$ ---(2)

From (2), $s_2 = u(2t) + ½ a(2t)^2 - s_1$

$$\begin{aligned} s_2 - s_1 &= u(2t) + ½ a(2t)^2 - 2s_1 \\ &= u(2t) + ½ a(2t)^2 - 2(ut + ½ at^2) \\ &= at^2 \end{aligned}$$

$$a = \frac{(s_2 - s_1)}{t^2}$$

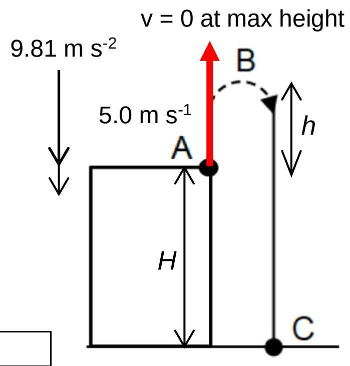
Example 11: Free falling object moving up and down

A stone is thrown vertically upward from cliff with velocity 5.0 m s^{-1}. It strikes the pond near the base of cliff after 4.0 s. Neglect air resistance.

Equation	Missing variable
$v = u + at$	*s*
$v^2 = u^2 + 2as$	*t*
$s = ut + \frac{1}{2}at^2$	*v*

Calculate
(a) time taken to reach the maximum height,
(b) max height reached as measured from the starting position,
(c) height of cliff,
(d) velocity just before hitting the pond.
(e) Sketch the *a-t, v-t* and *s-t* graphs of the stone with key values labelled.

Solution:



(a) From A to B

s	max height, h - ?
u	5.0 m s^{-1} – start
v	0 at max height – end (It is a turning point)
a	- 9.81 m s^{-1}
t	time to h - ?

↑: $v = u + at$
$0 = (\quad) + (\quad)t$
$t = 0.51$ s

(b) From A to B

↑: $v^2 = u^2 + 2as$
$0 = (\quad)^2 + 2(\quad)h$
$h = 1.27$ m

(c) From A to C

s	height of cliff, *H*
u	5.0 m s^{-1} – start
v	velocity upon reaching the pond – end - ?
a	- 9.81 m s^{-1}
t	4.0 s

↑: $s = ut + \frac{1}{2}at^2$
H = ( )( ) + ½ ( )($)^2$
= - 58 m [Negative as displacement is below A and upwards is taken to be positive]

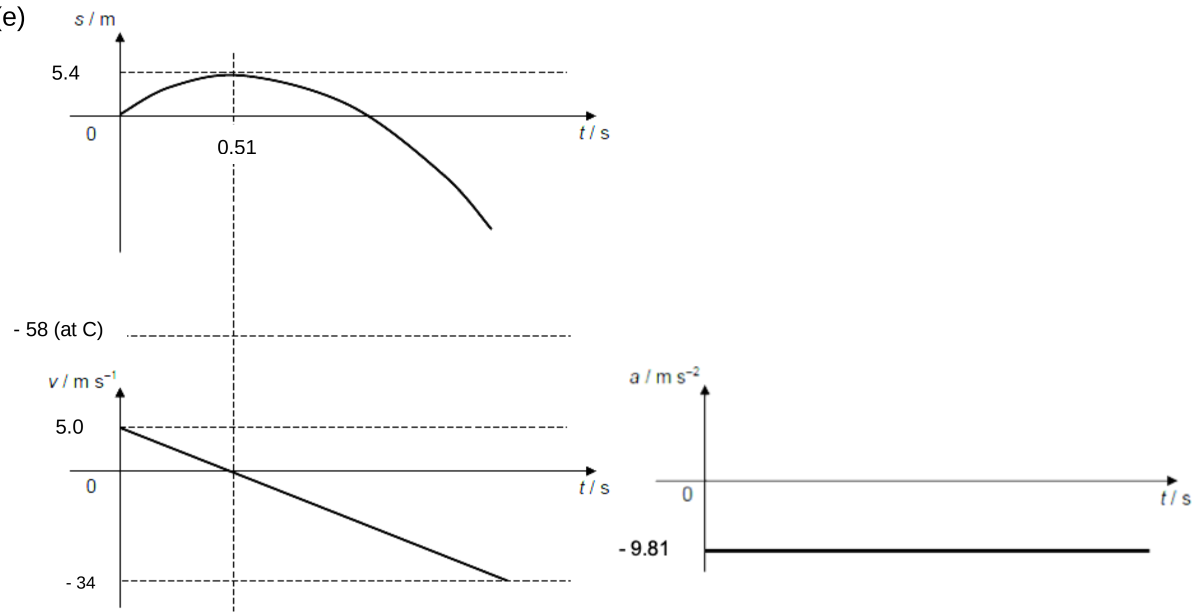
Height of cliff = 58 m

(d) From A to C

↑: $v = u + at$
$v = (\quad) + (\quad)(\quad)$
= - 34 m s^{-1} [Negative as final velocity is downwards and upwards is taken to be positive]

Velocity = 34 ms^{-1} downwards

(e)



Worked Example 12 (N13/1/3): Two unknowns solved using two kinematics equations

A car accelerates uniformly from rest along a level road. The effects of air resistance on the car are negligible. The car travels 12 m in the time between 1 s and 2 s after starting. How far does it travel in the time between 3 s and 4 s after starting.

A 28 m **B** 35 m **C** 48 m **D** 64 m

Solution: A

Method 1: Calculations

→ : $s = ut + \frac{1}{2}at^2$

s	?	?
u	0 (from rest)	0 (from rest)
v	?	?
a	?	?
t	1 s	2 s

Between t = 0 and t = 1s, u = 0 and t= 1 ➔ $s_1 = \frac{1}{2}a(1)^2 = \frac{1}{2}a$

Between t = 0 and t = 2 s, u = 0 and t = 2 ➔ $s_2 = \frac{1}{2}a(2)^2 = 2a$

Hence displacement between t = 1s and t = 2s

$2a - \frac{1}{2}a = 12$

Hence $a = 8 \text{ m s}^{-2}$

s	?	?
u	0 (from rest)	0 (from rest)
v	?	?
a	8 m s^{-2}	8 m s^{-2}
t	3 s	4 s

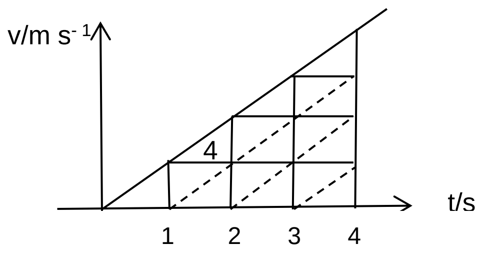
Displacement between t = 3 s to t = 4 s
Displacement between t = 0 to t = 4 s - Displacement between t = 0 to t = 3 s
$= \frac{1}{2}a(4)^2 - \frac{1}{2}a(3)^2$
= 28 m

Method 2: Graphical

Since the car accelerates uniformly from rest, the velocity-time graph is a straight line passing through the origin. The areas under the graph for 1, 2, 3 and 4 s are similar triangles.

As shown in the diagram below, distance travelled between 1 and 2 s = 3 small triangles = 12 m. Thus each small triangle represents 4 m.

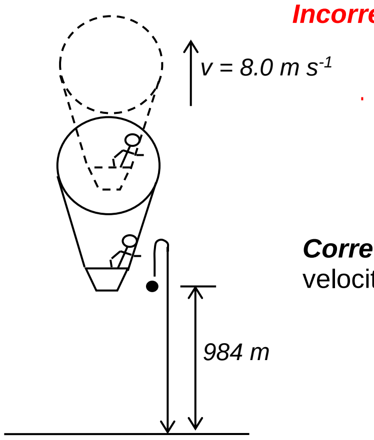
Required distance is 7 small triangles = 28 m.



Tutorial qn: Q11, Q12, Q13, Q14

Worked Example 13: An object released from another moving object

A hot air balloon is ascending with a constant vertical velocity of 8.0 m s^{-1}. At a height of 984 m above the ground, a sandbag is dropped. Determine the time taken for the sandbag to reach the ground.



Incorrect Solution: Deduce what is wrong with the following answer:

$\uparrow$: $s = ut + \frac{1}{2}at^2$, (since sandbag is dropped, $u = 0$)

$-984 = 0 + \frac{1}{2}(-9.81)\,t^2$

$t = 14.2$ s

Correct Solution: Instead of u = 0, the sandbag was moving at the velocity of the balloon at 8.0 m s^{-1} upwards when it was released

$\uparrow$: $s = ut + \frac{1}{2}at^2$

$-984 = 8.0\,t + \frac{1}{2}(-9.81)\,t^2$

$4.905\,t^2 - 8.0\,t - 984 = 0$

$$t = \frac{-(-8.0) \pm \sqrt{(-8.0)^2 - 4(4.905)(-984)}}{2(4.905)}$$

$t = 15.0$ s

s	-984 m
u	8.0 m s^{-1}
v	?
a	-9.81 m s^{-2}
t	?

2.4 Motion of a Body Falling in a Uniform Gravitational Field with Air Resistance

Strobe of a falling ball
https://www.youtube.com/watch?v=xQ4znShlK5A
A ball is dropped in front of a metre rule and lit by a strobe light. A long exposure photograph captures the position of the ball in certain time interval. The acceleration of the ball can then be measured from the photo.

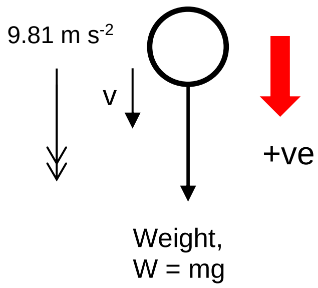
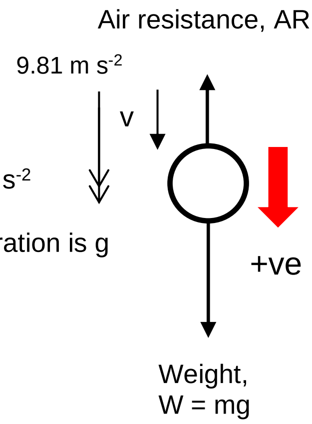
Vertical throw with and without air resistance
https://www.youtube.com/watch?v=Lhb87cf67cQ
Watch this video to see how the software Tracker was used to analyse the displacement-time graph and velocity-time graph of a tennis ball and beach ball.

Compare the fall of a bowling ball and feather in air and in vacuum
https://www.youtube.com/watch?v=E43-CfukEgs&feature=emb_logo
The giant vacuum chamber at the NASA's Space Power Facility is used to replicate Galileo's famous experiment where both heavy and light objects are dropped at the same time to see which will hit the ground faster. Modern technology allows us to see the results clearly by use of high-speed cameras to capture motion for every fraction of a second.

Terminal velocity of rain drop
https://www.sciencenewsforstudents.org/article/raindrops-break-speed-limit
Terminal velocity is not just observed in parachute but also in raindrops. This article introduces why scientists are interested in knowing the terminal velocity of raindrops and how data collected seemed to contradict current knowledge. The article shows how the real science is often complex.

Modelling of air resistance
https://www.wired.com/2015/01/air-resistance-force-make-difference/
When does air resistance makes a difference? When you drop a ball, air resistance is often ignored. How high do you have to drop something so that air resistance is significant?

Consider a body released from rest at height, *h*, above the ground in a uniform gravitational field. Take downwards as positive. The motions with and without air resistance are compared below.

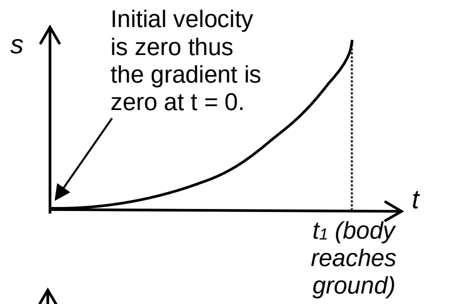
	<u>**No air resistance**</u> An object falling under the influence of gravity only is said to be free falling	<u>**With air resistance**</u>
Conditions	• Air resistance is absent in vacuum • Air resistance insignificant compared to object's weight	• Air resistance is comparable to the object's weight
Forces	Only weight is present or significant	Both weight and air resistance are significant
Motion	Constant downward acceleration of free fall at g = 9.81 m s^{-2} ➔ Velocity increases at a consent rate	Start with downward acceleration g = 9.81 m s^{-2}. The acceleration deceases ➔ Velocity increases at a decreasing rate
Link between force and motion	Taking downward to be positive, Newton's 2nd law, $F_{net} = ma$ Weight, W = mg. <u>Throughout the fall</u> $F_{net} = W$ $ma = mg$ $a = g = 9.81$ m s^{-2} • acceleration is constant at g 	Taking downward to be positive, Newton's 2nd law, $F_{net} = ma$ Weight, W = mg Air resistance, AR generally increases with velocity and is in direction to opposite velocity.  <u>Start of fall</u> u = 0, AR = 0 $F_{net} = W$ $ma = mg$ $a = g = 9.81$ m s^{-2} • initial acceleration is g <u>During the fall</u> $F_{net} = W - AR$ $W - AR = ma$ $mg - AR = ma$ $a = g - AR/m$ • velocity increases, air resistance which points upwards, increases. • resultant force F_{net} (weight – air resistance) decreases, acceleration decreases. • as velocity continues to increase due to a downward acceleration, upward air resistance increases till it balances the weight of the body. The acceleration is zero and the body falls at constant (terminal) velocity.

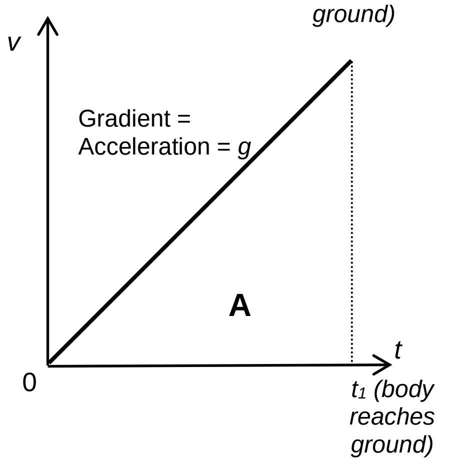
<table>
<tr>
<td>Equations and Graphs</td>
<td>
If object falls from rest (u = 0) and taking downwards to be positive

$s = ut + \frac{1}{2}at^2$
$s = \frac{1}{2}at^2$

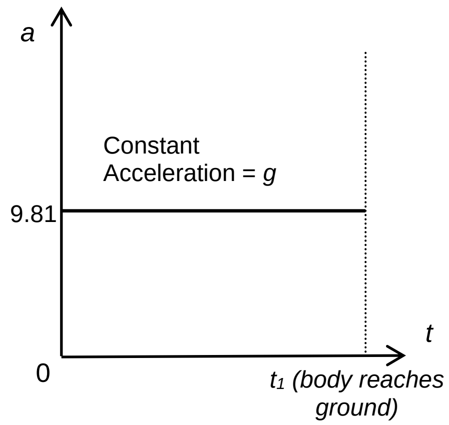
$v = u + at$
$v = at$

$a = 9.81 \text{ m s}^{-2}$



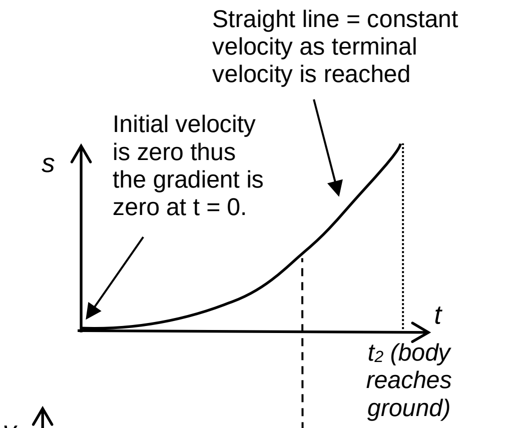


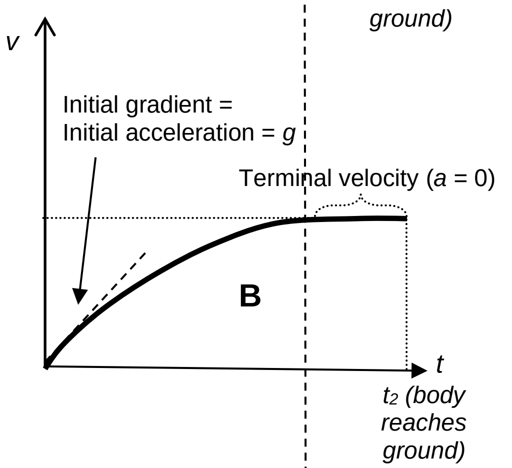
A = B = h as distance fallen is the same ➔ $t_1 < t_2$



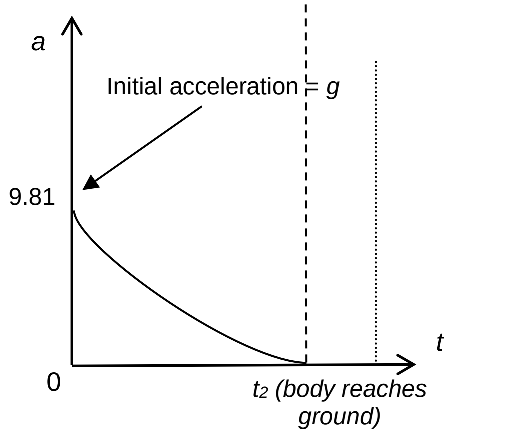
</td>
<td>
If object falls from rest (u = 0) and taking downwards to be positive

Kinematics equations cannot be used as acceleration is not constant but you can still sketch the graphs with known information.





A = B = h as distance fallen is the same ➔ $t_1 < t_2$



Assumption: The height of fall, has to be large enough for a body to reach terminal velocity, e.g. sky-diving from a plane. It is possible for a body to reach the ground before reaching terminal velocity, eg. Raindrops from the sky.
</td>
</tr>
</table>

Consider a body thrown upwards at initial velocity, *u*, in a uniform gravitational field, and subsequently caught by hand at the same position when thrown.

	No air resistance	**With air resistance**
	An object falling under the influence of gravity only is said to be free falling	

Link between force and motion

No air resistance

Taking downward to be positive,

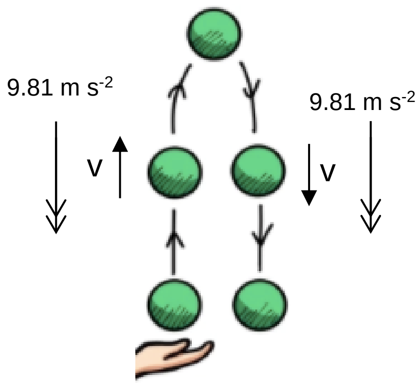
Newton's 2^{nd} law, $F_{net} = ma$

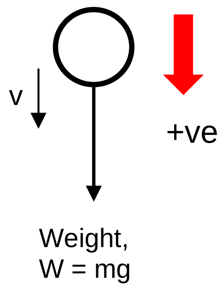
Weight, $W = mg$.

On the way up, highest point and on the way down

$F_{net} = W$
$ma = mg$
$a = g = 9.81$ m s^{-2}

- acceleration is constant at g





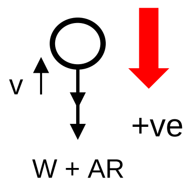
With air resistance

Taking downward to be positive,

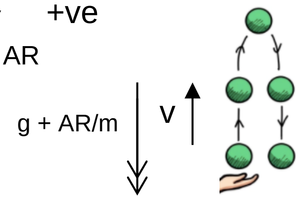
Newton's 2^{nd} law, $F_{net} = ma$

Weight, $W = mg$
Air resistance, AR generally increases with velocity and is in direction to opposite velocity.

C: On the way up



$F_{net} = W + AR$
$W + AR = ma$
$mg + AR = ma$
$a = g + AR/m$

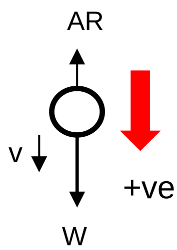


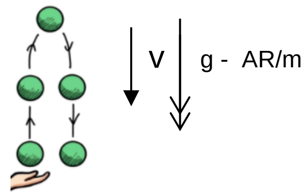
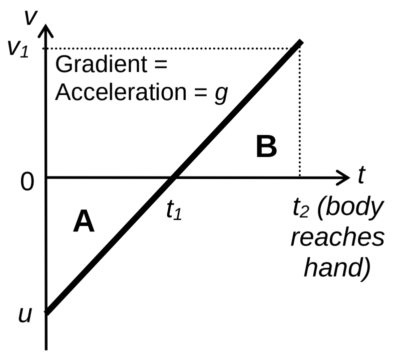
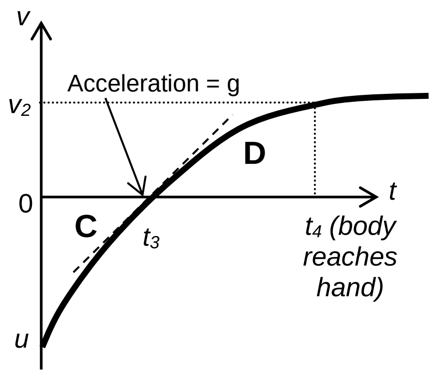
- $F_{net} > W$ ie. $ma = mg + AR$. This results in a deceleration greater than *g*.
- As velocity decreases, air resistance decreases too.
- Downward resultant force and hence deceleration decreases.

Highest point

- The body is momentarily at rest
- Air resistance becomes zero and hence, the only force acting on it is the weight. ie.ma = mg. The acceleration is thus *g* at this point.

D: On the way down



		$F_{net} = W - AR$ $W - AR = ma$ $mg - AR = ma$ $a = g - AR/m$  • Air resistance acts upwards (opposite to velocity) but weight acting downwards. ie. ma = mg – AR. This results in an <u>acceleration smaller than g</u> • As velocity increases, air resistance increases. • Downward resultant force and hence the acceleration decreases.
Equations and Graphs	If object is thrown upwards and caught again and taking downwards to be positive $v = u + at$ $a = 9.81$ m s^{-2}  A = B as distance fallen is the same. Time taken to move up and come down is the same ➔ $t_2 = 2\ t_1$	If object is thrown upwards and caught again and taking downwards to be positive Kinematics equations cannot be used as acceleration is not constant but you can still sketch the graphs with known information.  C = D as distance fallen is the same. But due to air resistance, the maximum height reached is lower ➔ areas C & D are smaller than areas A and B. Air resistance will result in the object slowing down to 0 m s^{-1} at a faster rate on its way up. Air resistance will result in the object speeding up at a slower rate on its way down. Since the deceleration on its way up is more than the acceleration on its way down and areas C = D ➔ $t_3 < (t_4 - t_3)$ v_2 is smaller than u due to air resistance (lose of energy) Assumption: The distance moved by the body is small so terminal velocity is not reached. https://www.youtube.com/watch?v=Sx7kyhaJuuU

Tutorial qn: Q15

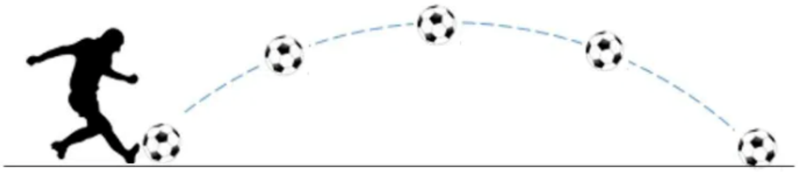
2.5 Projectile motion

https://www.stem.org.uk/elibrary/resource/27021/monkey-and-hunter

https://www.youtube.com/watch?v=TbWiMsfr_DQ

Watch the 2 videos above to learn about the classic Monkey and Hunter experiment. In this experiment, a "hunter" aims directly at a "monkey" in a tree, the monkey releases its grip to drop vertically at the same moment the gun is fired, will the bullet hit the monkey?

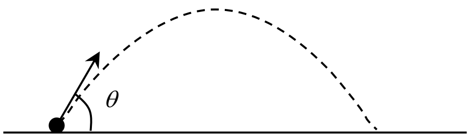
Beside objects being dropped or thrown up vertically as discussed earlier where the motion is only along a straight line (one-dimension motion) under the influence of gravity, it is common in our daily life that object moves in a plane (two-dimension motion) under the influence of gravity. The bullet in the Monkey and Hunter experiment follows such a path. Examples of such projectile motion include kicking a ball at an angle and water coming out of a hose at an angle.



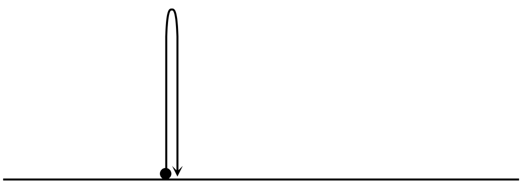
Projectile motion of a soccer ball

Projectile motion of water

If a body is projected at an angle θ (not = 90º) to the horizontal, the body will move in a parabolic path. If angle is 90˚, it becomes a linear path.



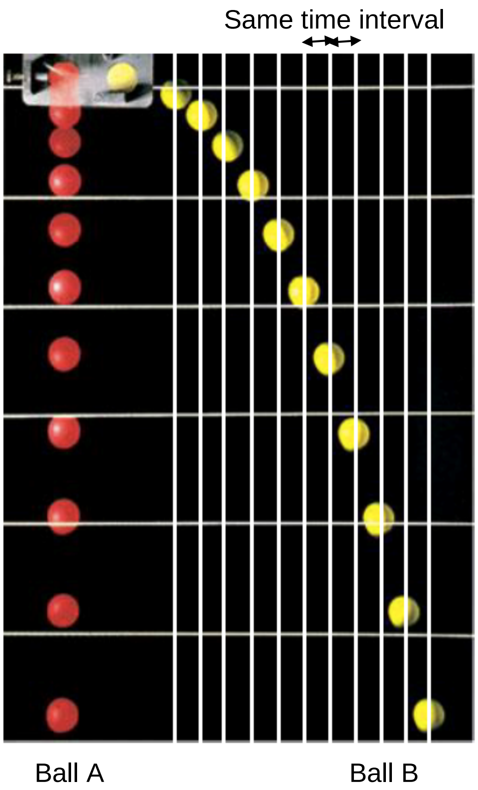
Projectile Motion
(two-dimensional)



Linear Motion
(one-dimensional)

Parabolic paths occur when motion is due

- <u>uniform acceleration</u> in a <u>perpendicular direction</u> e.g. acceleration of free fall g in downward vertical direction and
- <u>uniform velocity</u> in <u>one direction</u> e.g. a constant horizontal velocity



The diagram on the left show the image of ball A and ball B at equal interval of time.

Ball A is dropped simultaneously while ball B is projected horizontally.

The horizontal and vertical components of the motion can be investigated separately as the perpendicular components are "independent" of each other.

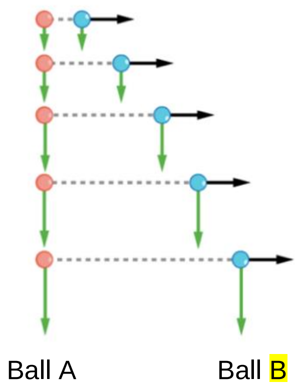
Vertical motion:

- Both balls are at the same state of motion in the same time interval throughout (in sync).
- This shows that ball A's motion is same as the vertical component of ball B motion.
- Since ball A is free falling with constant downward acceleration g, kinematics equations can be used. The analysis of ball B vertical motion would be exactly the same. Ball B experiences **free fall acceleration g = 9.81 m s^{-2} vertically downward.**

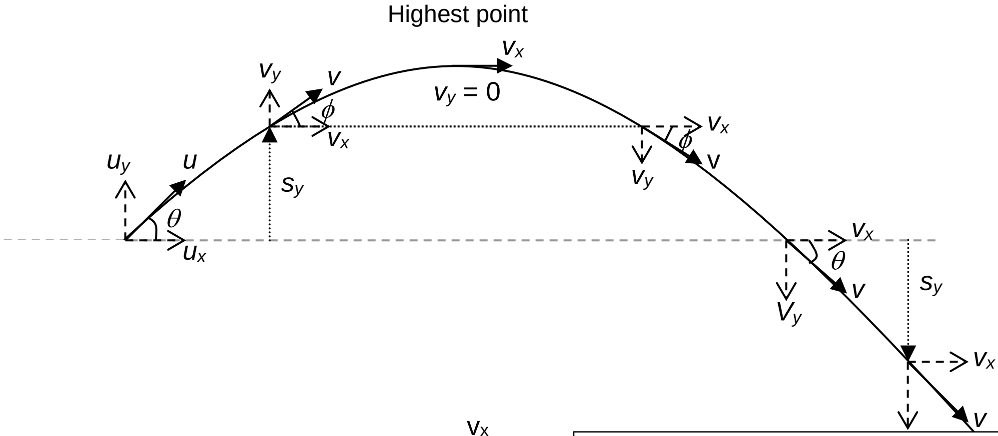
Horizontal motion:

- Ball A has no horizontal motion
- The horizontal displacement moved by ball B is the same at each equal time interval. This means **velocity is constant and acceleration is zero.**
- Since the acceleration is constant at zero, kinematics equations can also be applied

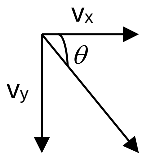
Vector arrows representation of the horizontal and vertical components of the velocities of ball A and ball B:



The velocity, horizontal component of the velocity and vertical component of the velocity of a projectile motion would look like this:



where $v = \sqrt{v_x^2 + v_y^2}$, $\theta = \tan^{-1}\dfrac{v_y}{v_x}$



Sometimes question may ask you to find other angle to describe the direction of the velocity. Always draw the vector diagram to show the angle and to use the correct trigo eqn to find the angle

Applying kinematics equations independently to the two perpendicular directions:

Horizontal	**Vertical**
$u_x = u \cos \theta$ (θ is w.r.t horizontal)	$u_y = u \sin \theta$ (θ is w.r.t horizontal)
$v_x = u_x + a_x t \Rightarrow v_x = u_x$ (since $a_x = 0$)	$v_y = u_y + a_y t$
$s_x = u_x t + \frac{1}{2} a_x t^2 \Rightarrow s_x = u_x t$ (since $a_x = 0$)	$s_y = u_y t + \frac{1}{2} a_y t^2$
	$v_y^2 = u_y^2 + 2a_y s$

Some common conditions to take note for projectile motion:

Condition	**Implication**
Projected horizontally	$u_y = 0$
At highest point of motion/ Maximum height	$v_y = 0$ (v_x is not zero thus $v = v_x$)
Retuned to the same horizontal level as starting point	$s_y = 0$

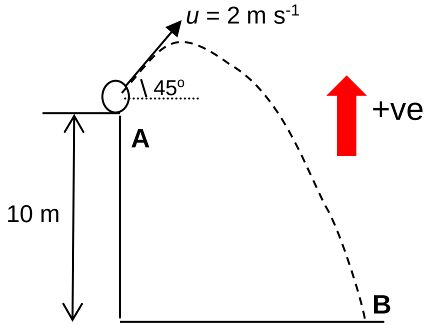
Suggested strategy for solving projectile motion problems:

1. Motion should have <u>acceleration in one direction</u> and <u>constant velocity in the perpendicular direction</u>.
2. <u>Sketch an informative diagram</u> using arrows to indicate the various information given in the question.
3. <u>Set a sign convention</u> in the horizontal and vertical directions.
4. <u>Choose appropriate start and end point.</u>
5. <u>Identify both the known and unknown quantities</u> using symbols u_x, v_x, s_x, u_y, v_y, s_y, a_y, t
6. <u>Identify the appropriate equation</u> to determine the unknown from given information.
7. <u>Substitute</u> known values into the kinematics equation (include appropriate <u>positive and negative</u> values based on sign convention) and solve for the unknown quantity.

Worked Example 14: Projectile motion with different launch angles

Determine the time of flight t in each situation for the object to land on ground.

(a) Object projected with velocity u at angle θ ***above the horizontal***



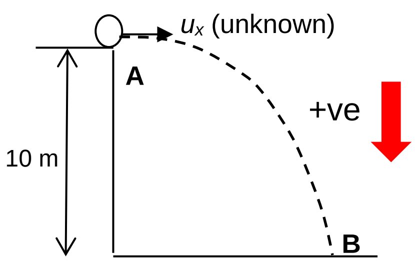
Solution:
Start at *A; end at B*

$$\uparrow: \; s_y = u_y.t + \frac{1}{2}a_y t^2$$

$$-10 = (2\sin45^o)t + \tfrac{1}{2}(-9.81)t^2$$

t = 1.58 s or -1.29 s (NA)

(b) Object ***projected horizontally*** off a surface



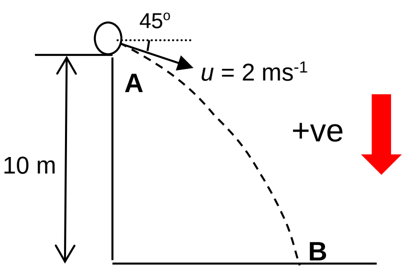
Solution:
Start at A; end at B

$$\downarrow: \; s_y = u_y.t + \frac{1}{2}a_y t^2$$

$$10 = (0)t + \tfrac{1}{2}(9.81)t^2$$

t = 1.43 s

(c) Object projected at velocity u at angle θ ***below the horizontal***



Solution:
Start at A; end at B

$$\downarrow: \; s_y = u_y.t + \frac{1}{2}a_y t^2$$

$$10 = (2\sin45^o)t + \tfrac{1}{2}(9.81)t^2$$

$$t = \frac{-1.41 \pm \sqrt{1.41^2 - 4(4.91)(-10)}}{2(4.91)}$$

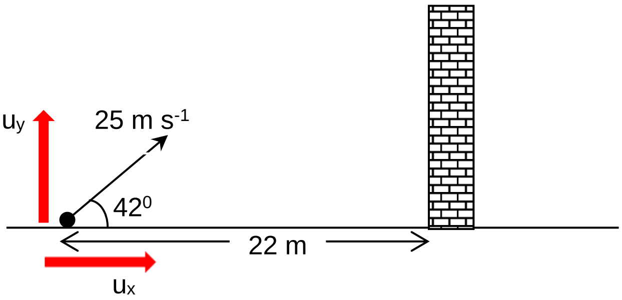
t = 1.29 s or -1.58 s (NA)

Note:

- Start each problem by by resolving the initial velocity *u* into its horizontal and vertical component, u_x and u_y.
- The sign convention in (a) is upwards +ve as vertical component of u is upwards. The sign convention in (b) is downwards as there is no vertical component of u and both s and a are downwards. Depending on sign convention, a_y= - 9.81m s^{-2} in (a) but a_y= 9.81m s^{-2} in (b)
- Setting a convenient sign convention and being consistent when you substitute values is absolutely necessary!

Example 15: Typical projectile motion problem

A ball is thrown with a speed of 25 m s^{-1} at an angle of 42° above the horizontal directly toward a wall as shown below. The wall is 22 m from the release point of the ball. Assume air resistance is negligible.



Preparing to ans qn:
- Include all quantities given in a diagram.
- Draw in vector arrow for u_x and u_y

(a) How long is the ball in the air before it hits the wall?

$u_x = u\cos\theta$ = 25 cos 42° = 18.6 m s^{-1}

Analysis of qn:
The ball hits the wall after travelling 22 m <u>horizontally</u>.

(b) How far above the release point does the ball hit the wall?

$u_y = u\sin\theta$ = 25 sin 42° = 16.7 m s^{-1}

↑:

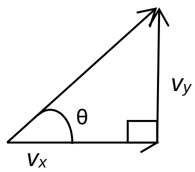
Analysis of qn:
- The qn needs to calculate the <u>vertical</u> displacement measured from the release point.
- Take upwards as +ve following u_y direction to find s_y.

(c) Determine the angle (with respect to the horizontal) at which the ball hits the wall.

→: $v_x = u_x$ = 18.6 m s^{-1}

↑:

Analysis of qn:
- To determine the angle, we need to calculate v_x and v_y
- Take upwards as +ve following u_y direction to find v_y.



$\tan\theta = \dfrac{v_y}{v_x} =$

$\theta =$

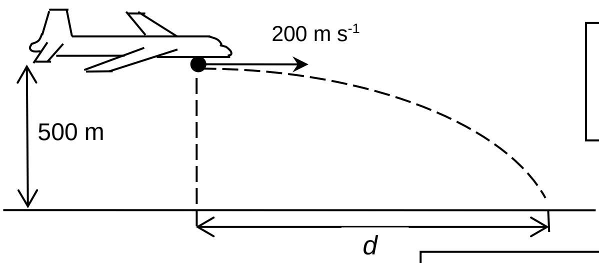
(d) Has it passed the highest point on its trajectory when it hits the wall?

As the ball hits the wall, since the _________ component of the velocity is ________ at the wall from part (c), the ball is still on the way ___. Hence the ball has not passed the highest point.

Worked Example 16 (J82/P2/Q3): Projectile motion when an object is dropped from another moving object

An aeroplane flying in a straight line at a constant height of 500 m with a speed of 200 m s^{-1} drops an object. The object takes a time *t* to reach the ground and travels a horizontal distance *d* in doing so. Taking *g* as 9.81 m s^{-2} and ignoring air resistance,

(a) find the values of *t* and *d*.
(b) determine how far from the plane is the object when it hits the ground.
(c) find the velocity of the object when it hits the ground.



Preparing to ans qn:
Include all quantities given in a diagram.

Solution:

(a) $\downarrow$: $s_y = u_y t + \frac{1}{2} a_y t^2$
$500 = 0 + \frac{1}{2}(+9.81)\, t^2$
$t = 10.1$ s

$\rightarrow$: $d = s_x = u_x t = (200)(10.1) = 2.02 \times 10^3$ m

Analysis of qn:
- The initial velocity of the object must be the velocity of the plane it is on as it is moving together with the plane. (IMPT!!)
- Use the vertical displacement 500 m to find t as there are insufficient information in the horizontal direction.
- Take downward as +ve as $u_y = 0$ and a_y and s_y are both downward.
- As t is common to both x and y dir at the same point. Find s_x using t found.

(b) 500 m

Analysis of qn:
The horizontal velocity of both plane and object stay constant at 200 m s^{-1}.

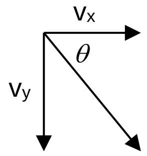
(c) $\rightarrow$: $u_x = v_x = 200$ m s^{-1}

$\downarrow$: $v_y^2 = u_y^2 + 2\, a_y\, s_y$
$v_y^2 = 2\,(+9.81)(500)$
$v_y = 99.0$ m s^{-1}

$v = \sqrt{v_x^2 + v_y^2}$
$= \sqrt{200^2 + 99.0^2}$
$= 223$ m s^{-1}

Analysis of qn:
- To determine the velocity and angle, we need to calculate v_x and v_y
- Take downward as +ve as $u_y = 0$ and a_y and s_y are both downward.

Direction of velocity, $\theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1}(99.0/200) = 26.3^\circ$ below the horizontal

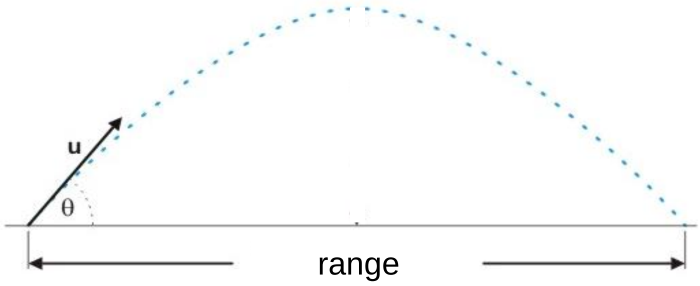


Tutorial qn: Q16, Q17, Q18, Q19, Q20

2.6 Maximum Range and Alternative Angles of Projection

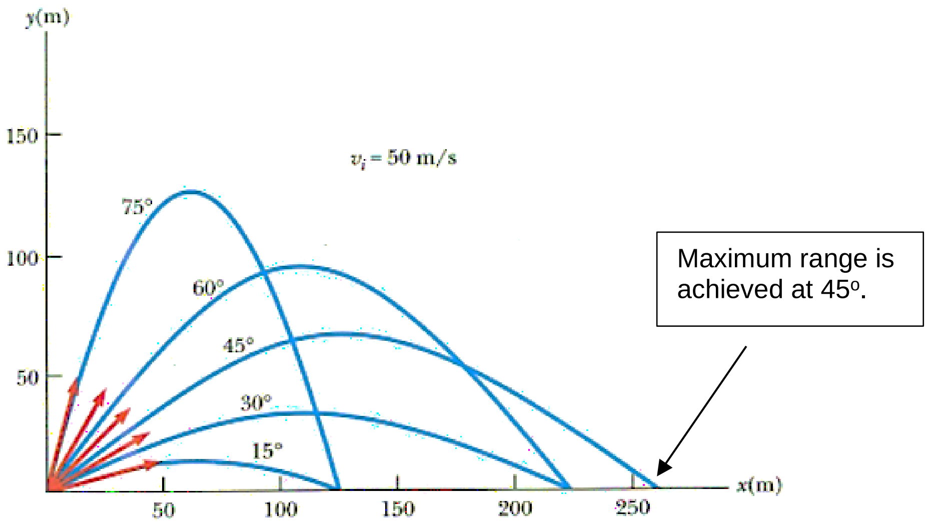
Maximum range

It can be shown (see appendix for derivation), that the maximum range is achieved when the body is projected at an angle of **45°** to the horizontal. The range of the projectile is the displacement in the horizontal direction before reaching the same vertical height as the starting point.



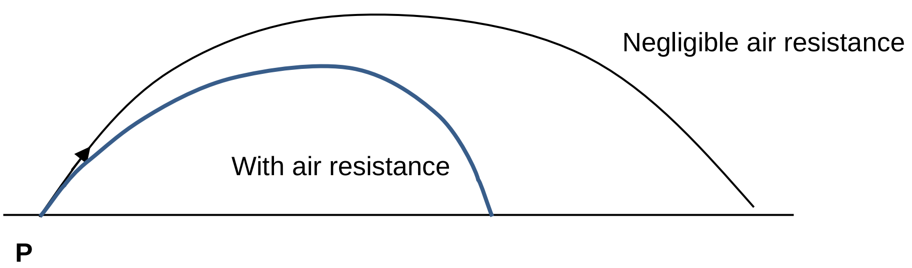
Alternative Angles of Projection

For a body to achieve a particular range on flat ground, it can be projected at two possible angles with the *same initial speed, u*. If one of the angles is θ, then the alternative angle of projection is always **(90° - θ).**



2.7 Effect of air resistance on Parabolic Motion

The following is the parabolic path of a body fired from point **P** at an angle to the horizontal in when there is negligible air resistance.



Comparing the path for projectile with air resistance to the path with negligible air resistance.

- Lower maximum height
- Smaller range
- Not symmetric (much steeper on the way down)

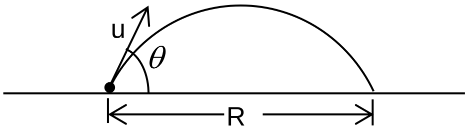
Suggest an explanation for any difference between the two paths.

Air resistance opposes the motion of the body and acts against <u>both</u> the horizontal and vertical motions. Hence the body's maximum height is reduced and the maximum range is also reduced.

As the body horizontal velocity keep decreasing. The horizontal velocity keeps decreasing. So the path is asymmetric.

APPENDIX

To find the max possible range, i.e. the max s_x:



$s_x = u_x T$ (where T = total time of flight)

$s_x = (u \cos \theta)\, T$ --------------- (1)

To find total time of flight, T :

↑ : $s_y = u_y T + \frac{1}{2} a_y t^2$,

$0 = (u \sin \theta)\, T + \frac{1}{2}(-g)\, T^2$ (where $s_y = 0$! Definition of "range")

$T = \dfrac{2u \sin\theta}{g}$ ----------------- (2)

Sub (2) into (1), $\boldsymbol{s_x} = (u \cos\theta)\, T = (u \cos\theta)\dfrac{2u \sin\theta}{g} = \boldsymbol{\dfrac{u^2 \sin 2\theta}{g}}$

The maximum range $R = s_x$ is achieved when $\sin 2\theta = 1$. This means that a body must be projected at an angle of 45^0 to the horizontal.

SUMMARY

1. **Distance** travelled is the total length of the actual path travelled by an object irrespective of the direction of motion

2. **Displacement** is defined as the distance moved in a specific direction.

3. **Speed** is defined as the distance travelled per unit time OR the rate of change of distance travelled.

4. **Velocity** is defined as the displacement per unit time OR the rate of change of displacement.

5. **Acceleration** is defined as the rate of change of velocity.

6. $v = \frac{ds}{dt}$. This means gradient of the *s-t* graph is the instantaneous velocity.

7. $a = \frac{dv}{dt}$. This means gradient of the *v-t* graph is the instantaneous acceleration.

8. Area under the *v-t* graph is the change in displacement of the body within the time interval, since $\Delta s = \int v \, dt$.

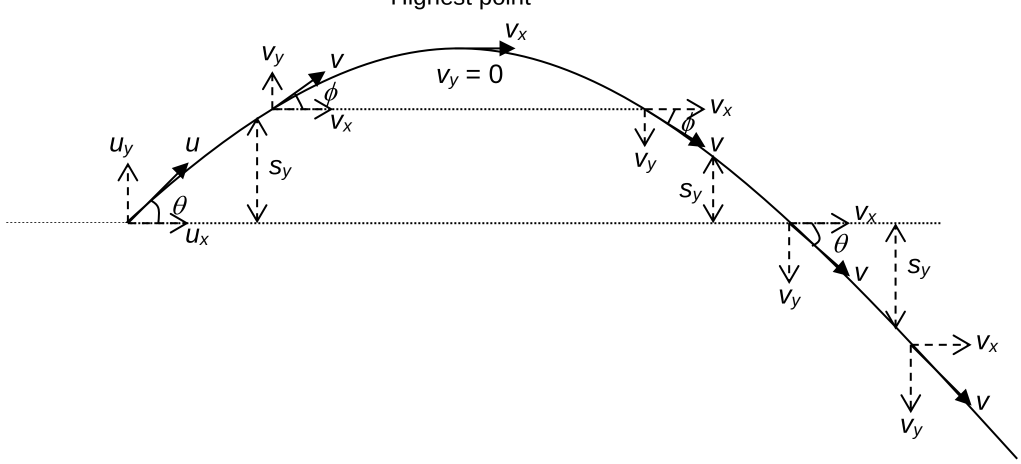
9. Area under the *a-t* graph is the change in velocity of the body, since $\Delta v = \int a \, dt$.

10. When a body is accelerating, it could mean its velocity is either changing in magnitude, in direction, or both magnitude and direction.

11. For a body **accelerating constantly** in a **straight line** from an initial velocity *u* to a final velocity *v* in a time interval *t*,

    - $v = u + at$
    - $s = \frac{1}{2}(u + v)\,t$
    - $v^2 = u^2 + 2as$
    - $s = ut + \frac{1}{2}at^2$

12. Projectile Motion:



At any instant: $v = \sqrt{v_x^2 + v_y^2}$; direction of the velocity, ϕ, at any instant: $\tan\phi = \frac{v_y}{v_x}$

13. Air resistance acts against both the horizontal and vertical motions. Hence the body's maximum height is reduced and the maximum range is also reduced. The path is also assymmetrical.

14. A body must be projected at an angle of 45^o to the horizontal if it were to achieve the maximum range on flat ground. If one of the angles is θ, then the alternative angle of projection is always $(90^o - \theta)$.

Most common difficulties in A-levels for kinematics

1. Unable to distinguish between the sign of the slope (+/- or upward/downward sloping) and value of the slope (getting steeper or more gentle).

2. Fail to notice variables and units on the x and y axes of graphs.

3. Not considering the sign convention and thus assigning appropriate and consistent +ve and -ve values to *s, u, v* and *a*. Draw a diagram and arrow that reminds you of the convention will solve this problem!

4. Unsure of the kinematics equation to use in both linear and projectile motion. Write down the 3 kinematics equations, identify the known variables and the unknown ones you are trying to find as well as the direction (whether vertical or horizontal) for projectile will help you make the right choice. You can draw this table.

s	
u	
v	
a	
t	

5. Getting confused in analysing *s,u,v* and *a* as vectors independently in the horizontal and vertical directions for selected start and end point in solving projectile motion question. Draw a diagram and show the horizontal and vertical component of velocity as well as use appropriate subscript e.g. u_y or u_x instead of u will minimise the confusion.

TUTORIAL 2: KINEMATICS

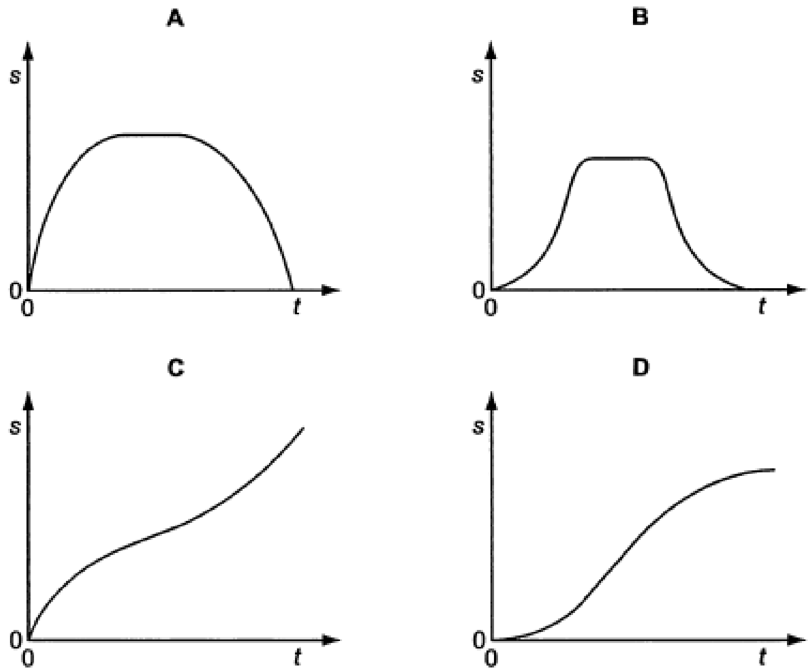
Definition of displacement, speed , velocity and acceleration

(L1)1. Which of the following statements is correct?

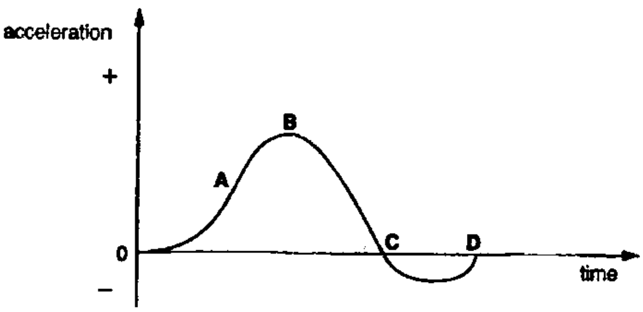
A When an object is in motion, its velocity and acceleration are always in the same direction.
B When an object is thrown upwards, its acceleration at the highest point is zero.
C When velocity of an object is zero, its acceleration can be non-zero.
D When acceleration of an object is zero, its velocity is zero.

Graphical representations of motion

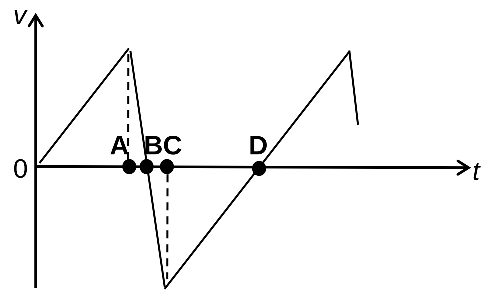
(L1)2. A cyclist accelerates down a hill and then travels at constant speed before decelerating as he climbs back up another hill. Which graph shows the variation with time *t* of the distance *s* moved by the cyclist? [N08/1/5]



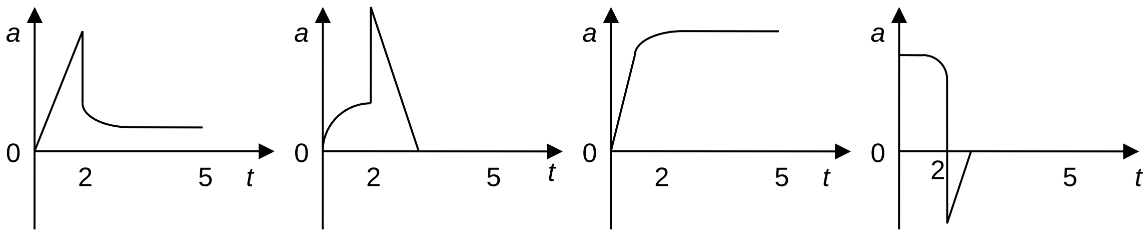
(L2)3. A car is travelling along a straight road. The graph below shows the variation with time of its acceleration during part of the journey. At which point on the graph does the car have its greatest velocity? [N96/1/4]



(L2)4. The graph shows the variation with time *t* of the velocity *v* of a bouncing ball, released from rest. Downward velocities are taken as positive. At which time does the ball reach its maximum height after bouncing? [J99/1/3]



(L2)5. A parachutist steps from an aircraft, falls without air resistance for 2 s and then opens his parachute. Which graph best represents how *a*, his vertical acceleration, varies with time *t* during the first 5 s of his descent? [J95/1/3]



A **B** **C** **D**

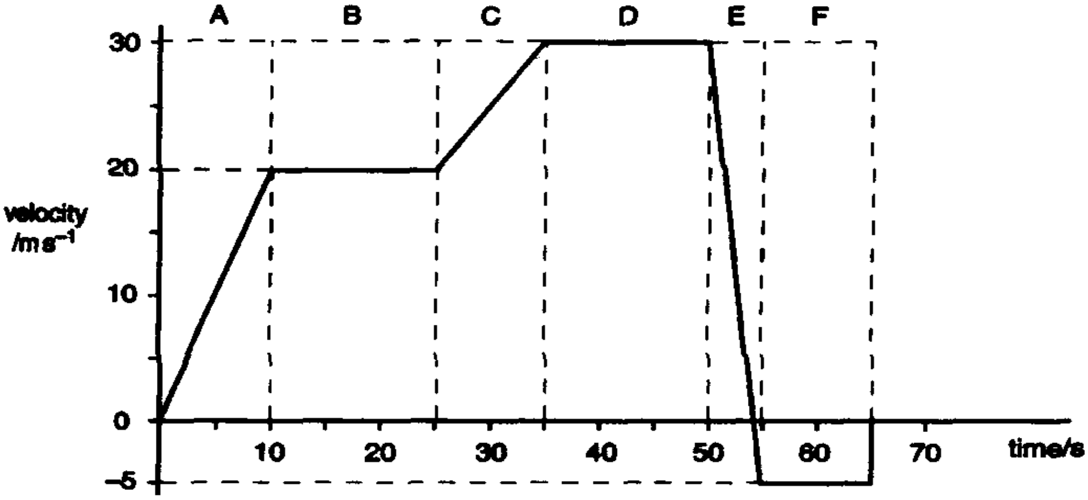
(L2)6. A stone falls freely from rest to the ground. The effect of air resistance on the stone is negligible.

The stone travels 0.75 of the total distance to the ground in the last 1.00s of its fall.

What is the time of the fall? [N19/1/3]

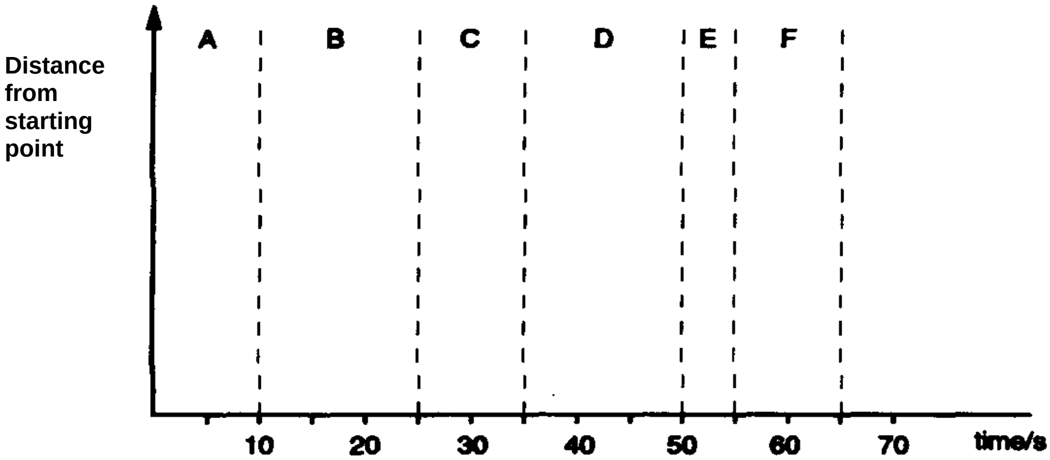
A 1.25 s **B** 1.50 s **C** 1.67 s **D** 2.00 s

(L2)7. Figure below shows a velocity-time graph for a journey lasting 65 s. It has been divided up into six sections for ease of reference.



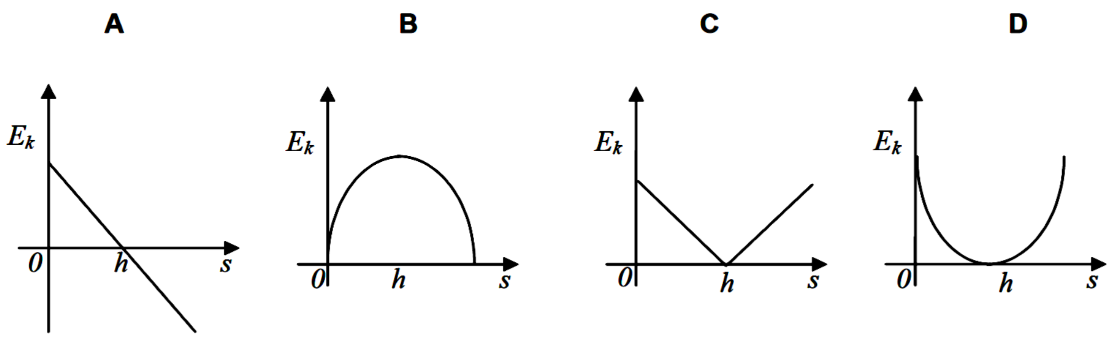
(a) Describe qualitatively in words what happens in sections E and F of the journey. [4]

(b) On figure below, sketch the shape of the corresponding distance-time graph. You are not expected to make detailed calculations of the distance travelled. [3]



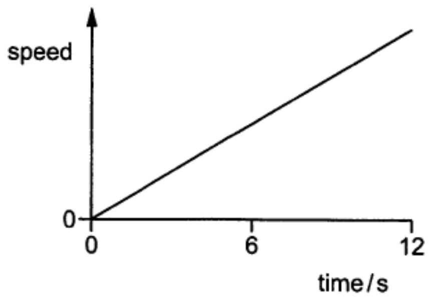
(L2)8. A stone, thrown vertically upwards from ground level, rises to a height *h* and then falls back to its starting point. Assuming that air resistance is negligible, which of the following graphs best shows how kinetic energy E_k of the stone, varies with *s*, the distance s travelled?

[N89/1/2 modified]



{Hint: Consider the equations of motion and energy. Express them in terms of energy}

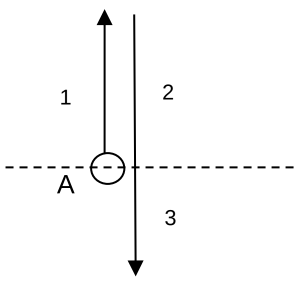
(L2)9. A car accelerates from rest for 12 s along a straight track. The variation with time of the speed of the car is shown below. [N17/1/3]



Determine the ratio $\dfrac{\text{distance travelled between 6s and 12s}}{\text{distance travelled between 0s and 6s}}$.

Equations for linear motion with constant acceleration

(L1)10. A ball is projected upwards from point A. Identify the sign of the displacement, velocity, acceleration at each stage during its flight, taking the upward direction as positive.



	s	*v*	*a*
1			
2			
3			

(L1)11. A man stands on the edge of a cliff. He throws a stone upwards with a velocity of 19.6 m s^{-1} at time t = 0. The stone reaches the top of its trajectory after 2.00 s and then falls towards the bottom of the cliff. Air resistance is negligible. Which row shows the correct velocity v and acceleration a of the stone at different times? [N09/1/4]

	t / s	v / m s^{-1}	a / m s^{-2}
A	1.00	9.81	9.81
B	2.00	0	0
C	3.00	9.81	- 9.81
D	5.00	- 29.4	- 9.81

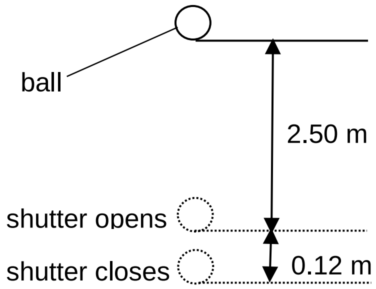
(L1)12. (a) A car, travelling along a long straight road at 30 km h^{-1}, accelerates at 2 m s^{-2} until it reaches a speed of 90 km h^{-1}. Calculate the time taken to accelerate.

(b) The same car then decelerates at 5 m s^{-2} until it comes to rest. Determine the time taken to decelerate and hence, calculate the distance travelled while decelerating.

(c) If the car in (b) subsequently accelerates from rest to 100 km h^{-1} while travelling only 50 m, determine the acceleration needed.

(L2)13. A motorist travelling at 13 m s^{-1} approaches the traffic lights, which turn red when he is 25 m away from the stop line. His reaction time is 0.7 s and the condition of the road and his tyres is such that the car cannot slow down at a rate of more than 4.5 m s^{-2}. If he brakes fully, how far from the stop line will he stop, and on which side of it? [J82/1/1]

(L2)14. A photographer wishes to check the time for which the shutter on a camera stays open when a photograph is being taken. To do this, a metal ball is photographed as it falls from rest. It is found that before the shutter opens, the ball falls 2.50 m from rest and, during the time that the shutter remains open, the ball falls a further 0.12 m, as illustrated in the figure below.



Assuming that air resistance is negligible, calculate

(i) the speed of the ball after falling 2.50 m,

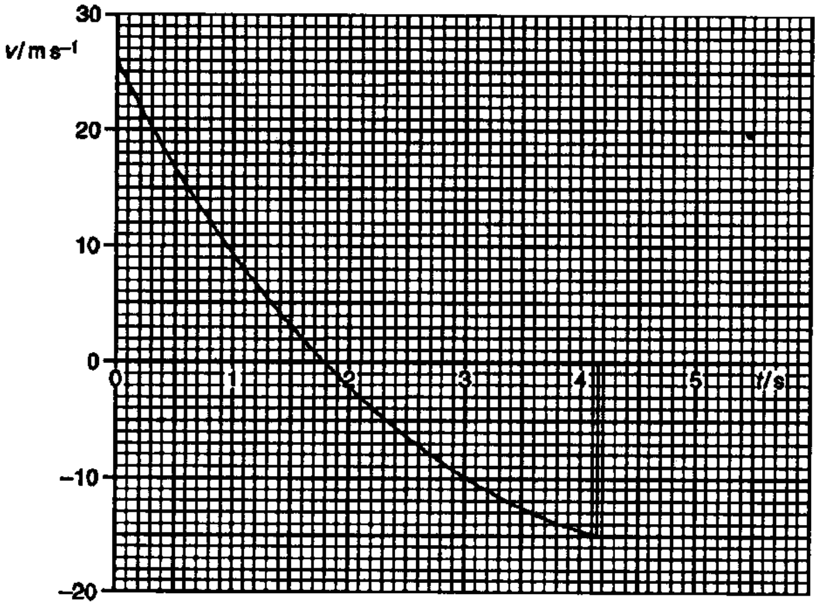
(ii) the time to fall the further 0.12 m,

(iii) the time for which the shutter stays open is marked on the camera as 1/60 s. Comment on whether the test confirms this time. [6]

[N99/2/2]

Motion of a body falling in a uniform gravitational field

(L2)15 (a) The graph shows the variation with time t of the velocity v of a ball from the moment it is thrown with a velocity of 26 m s^{-1} vertically upwards.



(i) State the time at which the ball reaches its maximum height. [1]

(ii) State the feature of a velocity-time graph that enables the acceleration to be determined. [1]

(iii) Just after the ball leaves the thrower's hand, it has a downward acceleration of approximately 20 m s^{-2}. Explain how this is possible. [2]

(iv) State the time at which the acceleration is g. Explain why the acceleration has this value only at this particular time. [2]

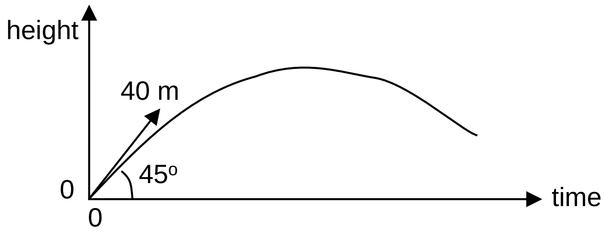
(v) Sketch an acceleration-time graph for the motion. Show the value of g on the acceleration axis. [3]

(b) Explain why, for all real vertical throws, the time taken to reach maximum height must be shorter than the time taken to return to the starting point. [2]

[N03/3/1 (part)]

Projectile motion

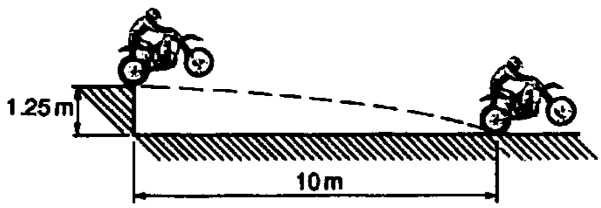
(L1)16. An object is projected with velocity 40 m s^{-1} at an angle of 45^{o} to the horizontal. Air resistance is negligible. [N09/1/5]



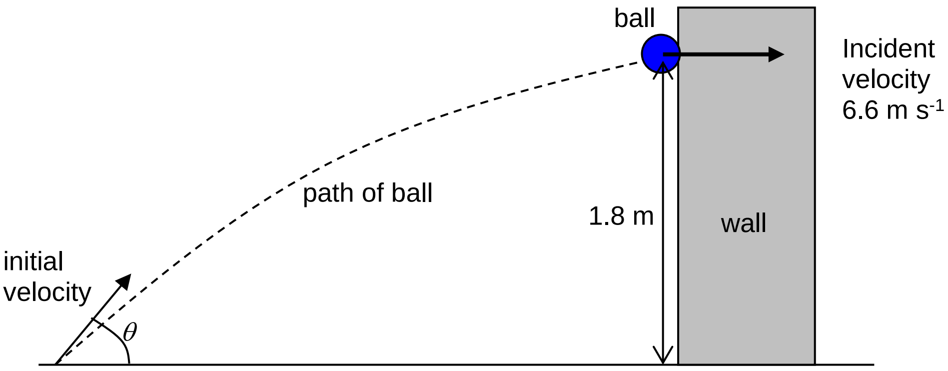
What is the speed of the object after 5.0 s?

A 21 m s^{-1} **B** 28 m s^{-1} **C** 35 m s^{-1} **D** 49 m s^{-1}

(L1)17. A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown in the diagram. What was the speed at take-off? [J93/I/3]



(L2)18. Figure shows the path of a ball that is kicked off the ground towards a vertical wall.



The ball hits the wall when it reaches it maximum height of 1.8 m. The ball is incident with a horizontal velocity of 6.6 m s^{-1} and rebounds in a horizontal direction with a velocity of 5.2 m s^{-1}. Assume air resistance is negligible.

(a) Calculate the initial vertical component of the ball's velocity. [2]

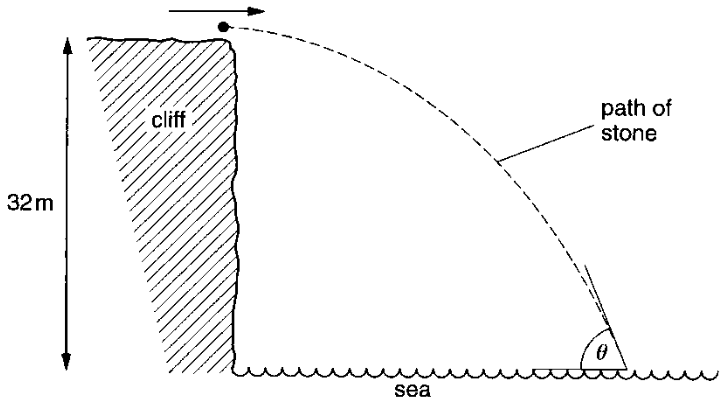
(b) Hence determine the angle of projection of the ball from the ground. [2]

(c) Explain why the ball does not rebound to the point on the ground from where it is kicked. [2]

[N09/2/2 (part)]

Assignment

(L2)19. A stone of mass 130 g is thrown horizontally from the top of a cliff of height 32 m, as illustrated below.



Air resistance is negligible. The stone enters the sea with a speed of 34 m s^{-1}.

(a) Determine, for the stone as it hits the sea,

(i) the vertical component of the velocity of the stone, [2]

(ii) the angle θ to the horizontal of the stone's path. [2]

(b) Use energy considerations to suggest why, if the stone causes a large splash on hitting the sea, it will be slowed down in a shorter distance than when no splash is produced. [2]

[N08/2/1 (part)]

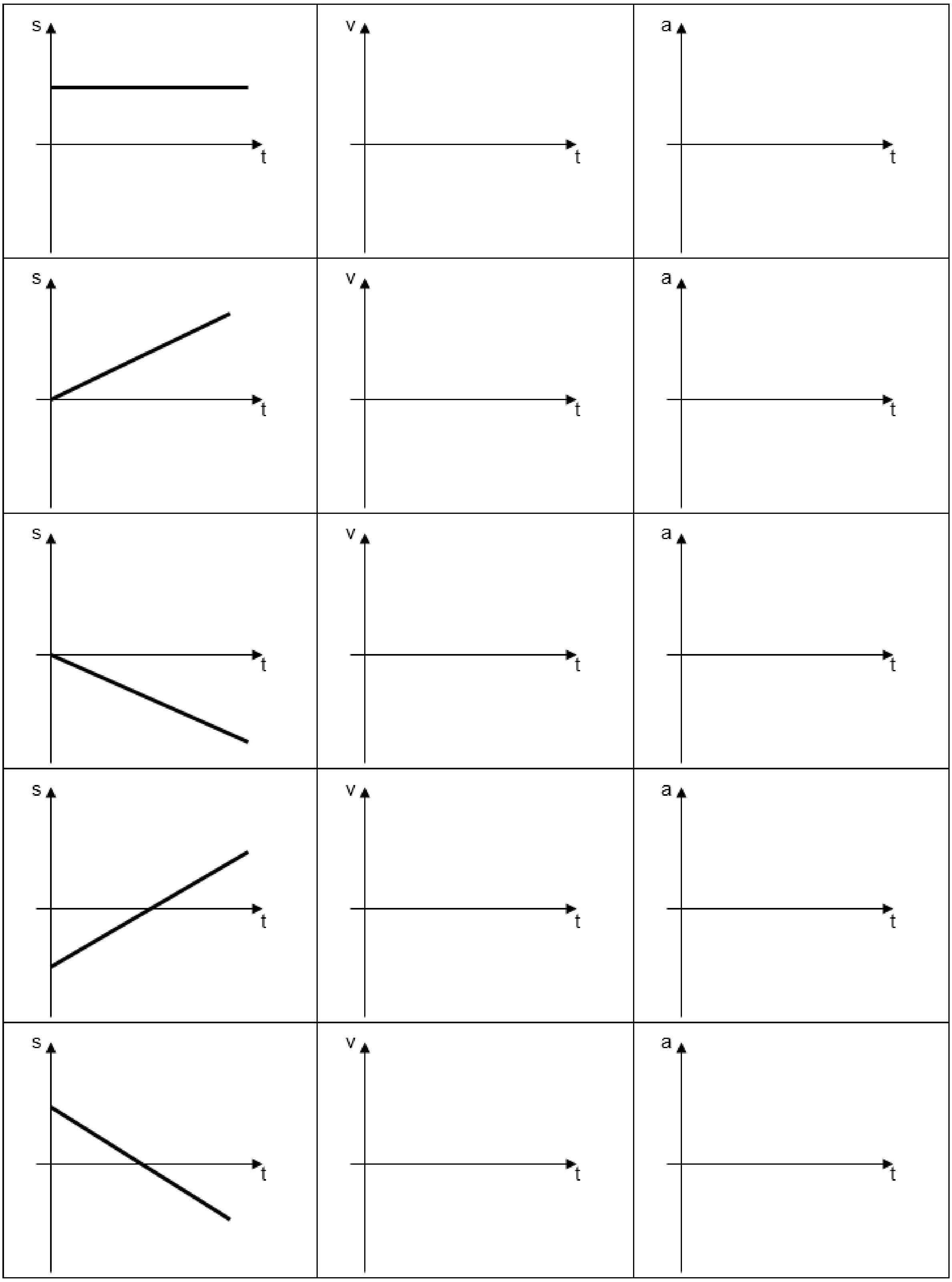
(L2)20. A golfer hits a ball on a flat golf course. The ball travels a horizontal distance of 500 m before bouncing. It reaches a maximum height of 210 m. What is the approximate angle to the horizontal at which the ball leaves the golf club? Air resistance may be assumed to be negligible. [N16/1/5]

Numerical / MCQ Answers

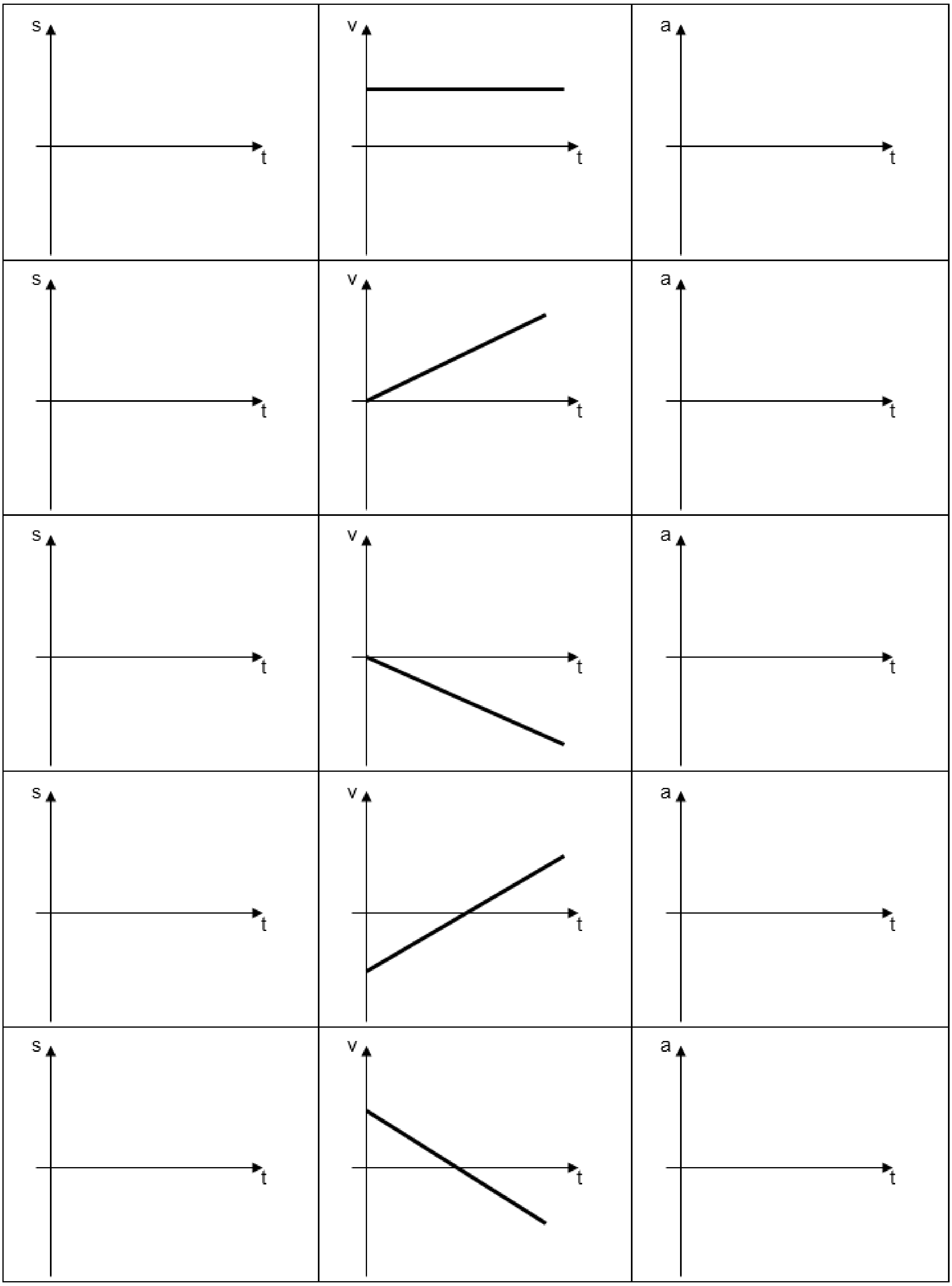
6 D
9 3
11 D
12 (a) 8.3 s (b) 62.5 m (c) 7.72 m s^{-2}
13 2.9 m after the stop line.
14 (i) 7.00 m s^{-1} (ii) 0.0169 s
15 (a)(i) 1.8 s (iv) 1.8 s
16 C
17 19.8 m s^{-1}
18 (a) 5.94 m s^{-1} (b) 42°
19 (a)(i) 25.1 m s^{-1} (a)(ii) 47.5°
20 59.2°

ADDITIONAL QUESTIONS

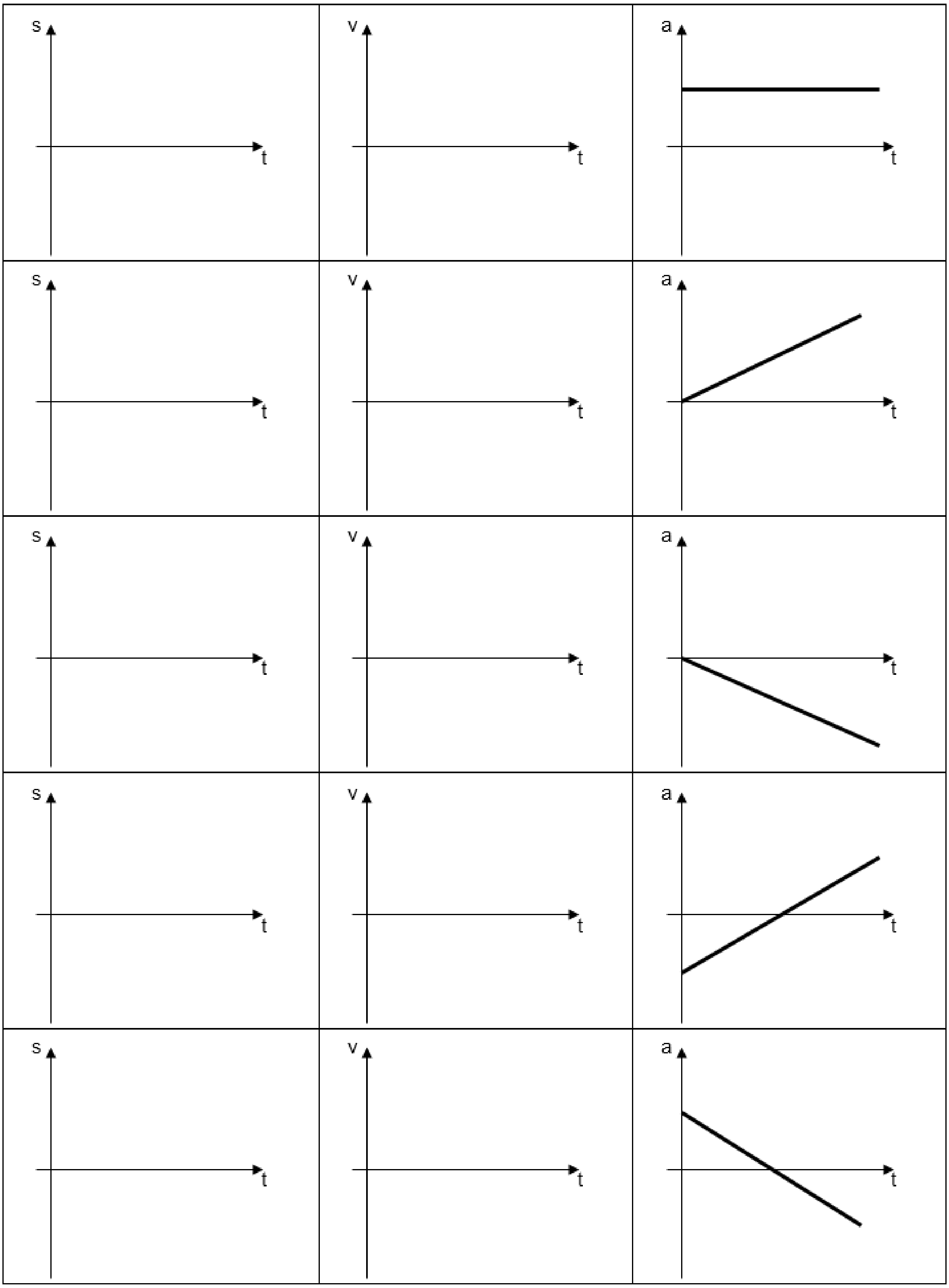
Starting from s-t graph Use *pencil* to sketch the graphs ahead of the lectures



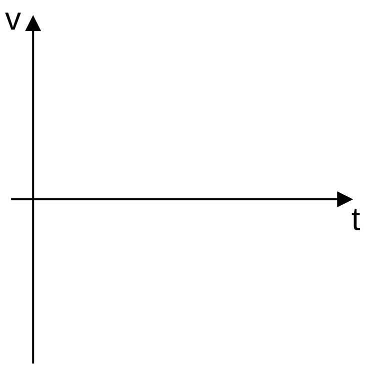
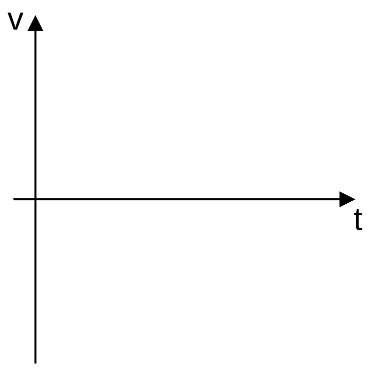
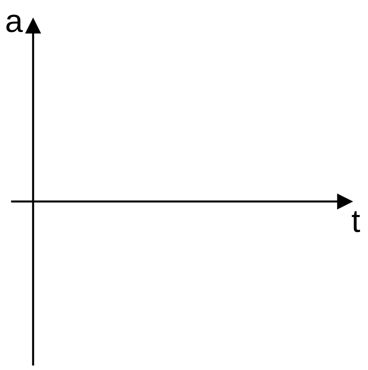
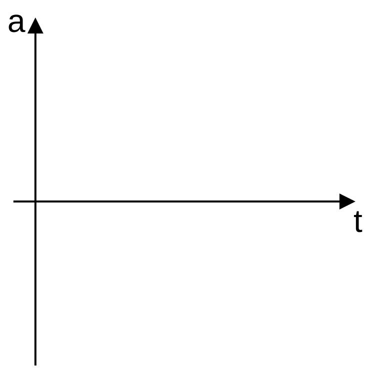
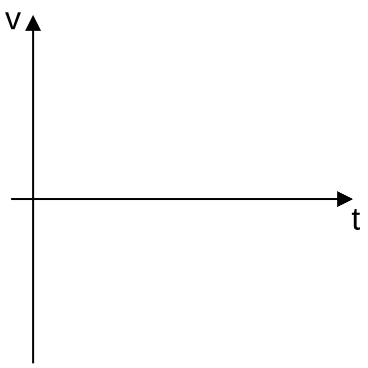
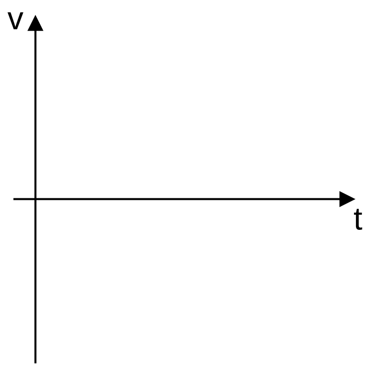
Starting from v-t graph



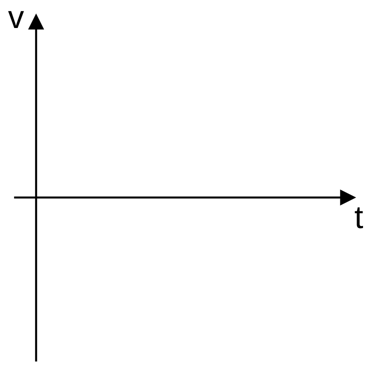
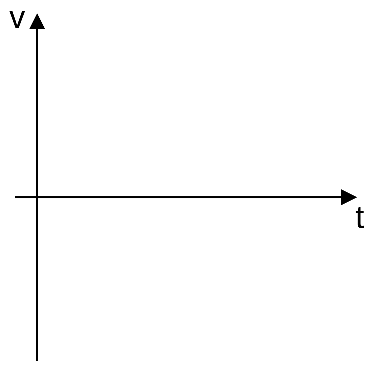
Starting from a-t graph



Case 1: Consider a ball released from rest from a height of h.

	Without air resistance	**With air resistance**
Draw the forces acting on the ball during the motion.		
What is the direction and magnitude of the acceleration of the ball? How does it change?		Hint: As speed increases, air resistance increases.
Draw the v-t graph. What is your positive direction? What can you say about the areas under the 2 graphs? What can you say about the time taken without and with air resistance?		
Draw the a-t graph.		
What if the ball was thrown downwards instead of being dropped? Draw the v-t graph.		

Case 2: Consider a ball thrown upwards with velocity *u* and landed at the same level.

	Without air resistance	**With air resistance**
Draw the forces acting on the ball during the motion.	During upward motion, During downward motion,	During upward motion, During downward motion,
What is the direction and magnitude of the acceleration of the ball? How does it change?	During upward motion, During downward motion,	Hint: As speed increases, air resistance increases. During upward motion, During downward motion,
Draw the v-t graph. What is your positive direction? What can you say about the areas under the 2 graphs? What can you say about the time taken for the upward motion and the downward motion?	 	

Paths of 3 soccer balls: The figure below shows three paths for a kicked football:

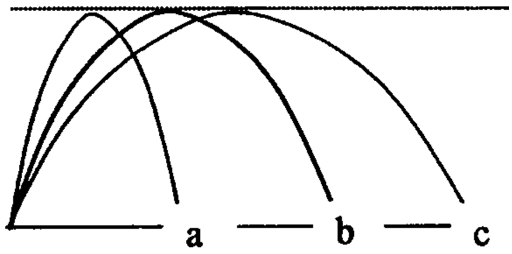


Fig. 20

Rank the paths according to

(a) initial vertical velocity component
(b) time of flight
(c) initial horizontal velocity component
(d) initial speed

TUTORIAL 2: KINEMATICS SOLUTIONS

<u>Level 1 Solutions</u>

<table>
<tr><td>1</td><td>[C]
Acceleration is defined as the rate of CHANGE of velocity & By Newton's 2nd Law, Resultant force = ma. Hence the acceleration, a, is always in the same direction as the change in velocity and net force.

Option A is wrong as acceleration is not related to velocity at that instant. Should be change in velocity.
Option C is correct as this can occur at the highest point of the projectile motion.
Option B is wrong as at the highest point, weight is acting on the object. Hence since weight is the resultant force, acceleration of the object is downwards.
Option D is wrong. When acceleration is zero, the object can also move at constant velocity.</td><td></td></tr>
<tr><td>2</td><td>[D]
The gradient of the s-t graph of option D shows that the velocity increases (accelerating down the hill), becomes constant, then decreases.</td><td></td></tr>
<tr><td>10</td><td><table><tr><td></td><td>s</td><td>v</td><td>a</td></tr><tr><td>1</td><td>+</td><td>+</td><td>-</td></tr><tr><td>2</td><td>+</td><td>-</td><td>-</td></tr><tr><td>3</td><td>-</td><td>-</td><td>-</td></tr></table></td><td></td></tr>
<tr><td>11</td><td>[D]
Option A is wrong because acceleration should be – 9.81 m s^{-2}
Option B is wrong because acceleration is not zero but – 9.81 m s^{-2}
Option C is wrong because velocity should be – 9.81 m s^{-1} since ball is travelling downwards after reaching its maximum height at t = 2.0 s .
Using v = u + a t (taking upwards as positive),
v = (19.6) + (-9.81)(5.0)
= - 29.45 m s^{-1}</td><td></td></tr>
<tr><td>12(a)</td><td>Using $v = u + at$,
Note: 1 km / 1 hour = 1000 m / 3600 s
i.e. 90 km hr^{-1} = 90000 m / 3600 s
(90 000 / 3 600) = (30 000 / 3 600) + 2 t
t = 8.3 s</td><td></td></tr>
<tr><td>(b)</td><td>Using $v = u + at$,
0 = (90 000 / 3 600) + (-5) t
Note: a = -5 as it is in the opposite direction of u
t = 5.00 s

Using $s = ut + ½ at^2$,
s = 25 (5) + ½ (-5)$(5.00)^2$
s = 62.5 m</td><td></td></tr>
<tr><td>(c)</td><td>Using $v^2 = u^2 + 2as$,
$(100\ 000 / 3600)^2$ = 0 + 2 (a)(50)
a = 7.72 m s^{-2}</td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

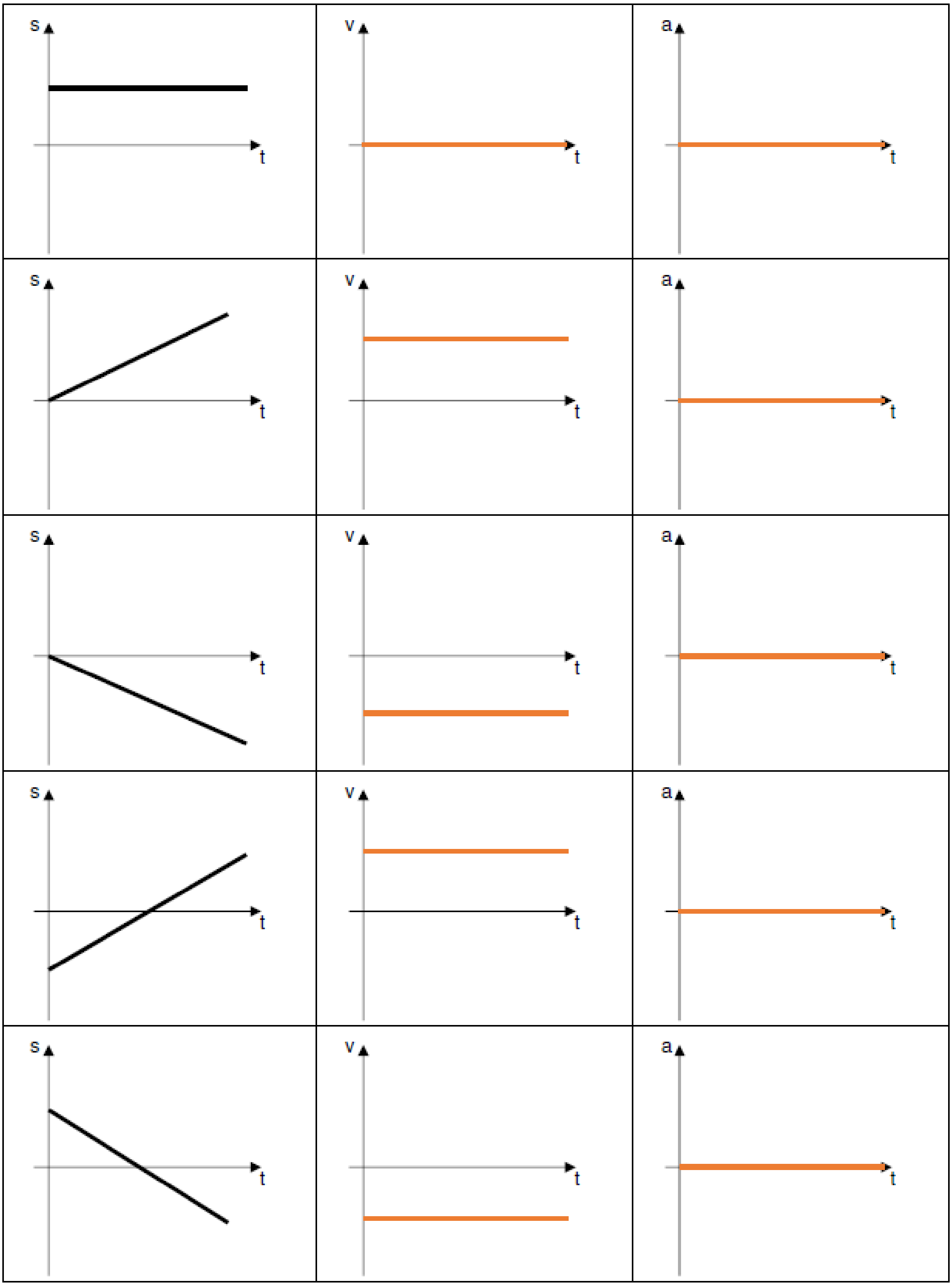
16	[C] Using $v_y = u_y + a_y t$, (taking upwards as positive), v_y = (40 sin 45°) + (-9.81)(5) = -20.77 m s^{-1} Hence $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(40\cos 45°)^2 + (-20.77)^2}$ **Note:** v_x is constant = 35.1 m s^{-1}	
17	Let speed at take-off = u_x. Taking downwards as +ve, Using $s_y = u_y t + ½ g t^2$ 1.25 = 0 + ½ (9.81) t^2 → t = 0.5 s Using $s_x = u_x t$ 10 = u_x (0.5) → u_x = 19.8 m s^{-1}	

- End of Level 1 tutorial solutions -

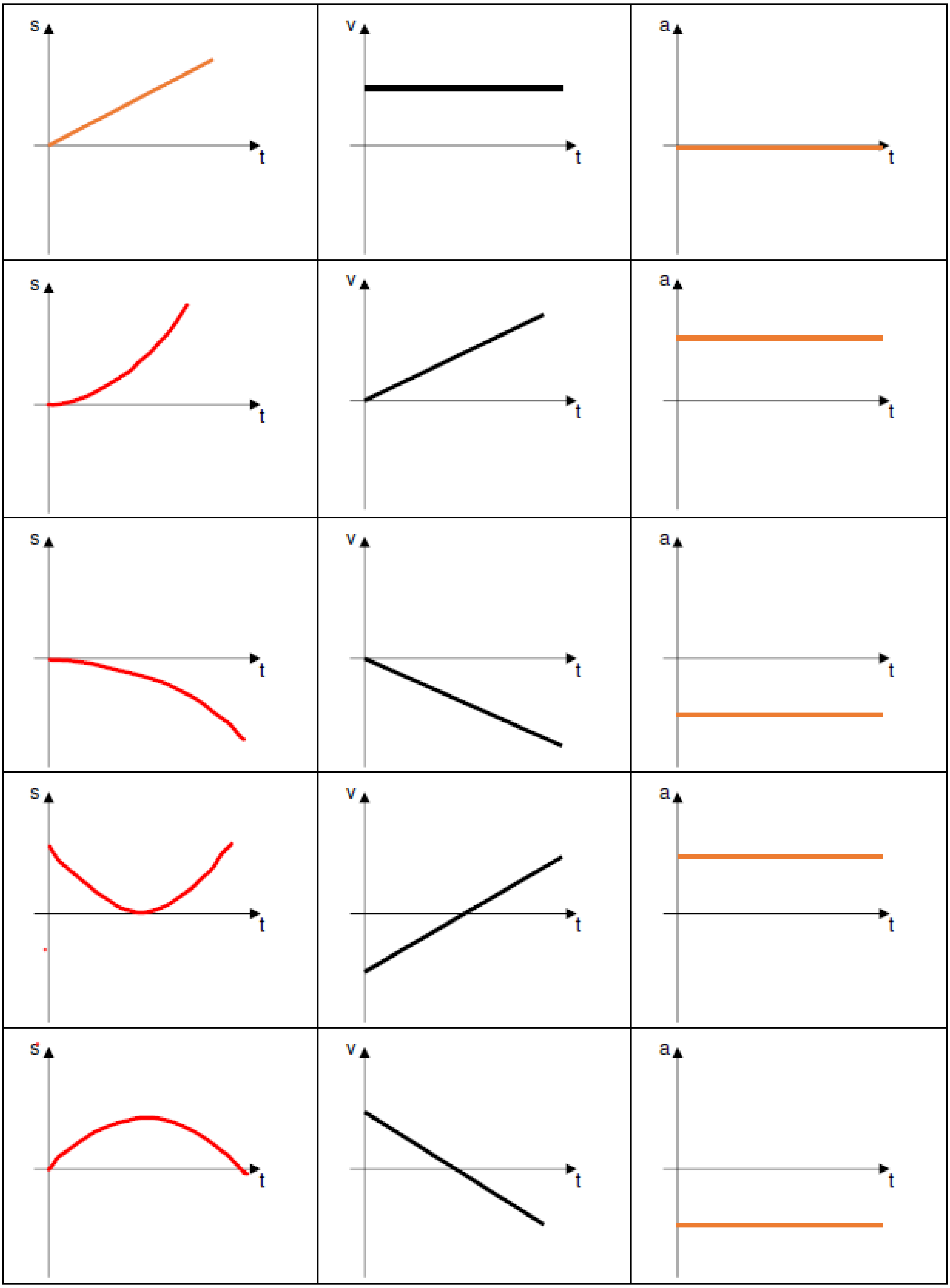
Solutions to Additional Questions

Topic 2: Kinematics: Basic Graph Sketching

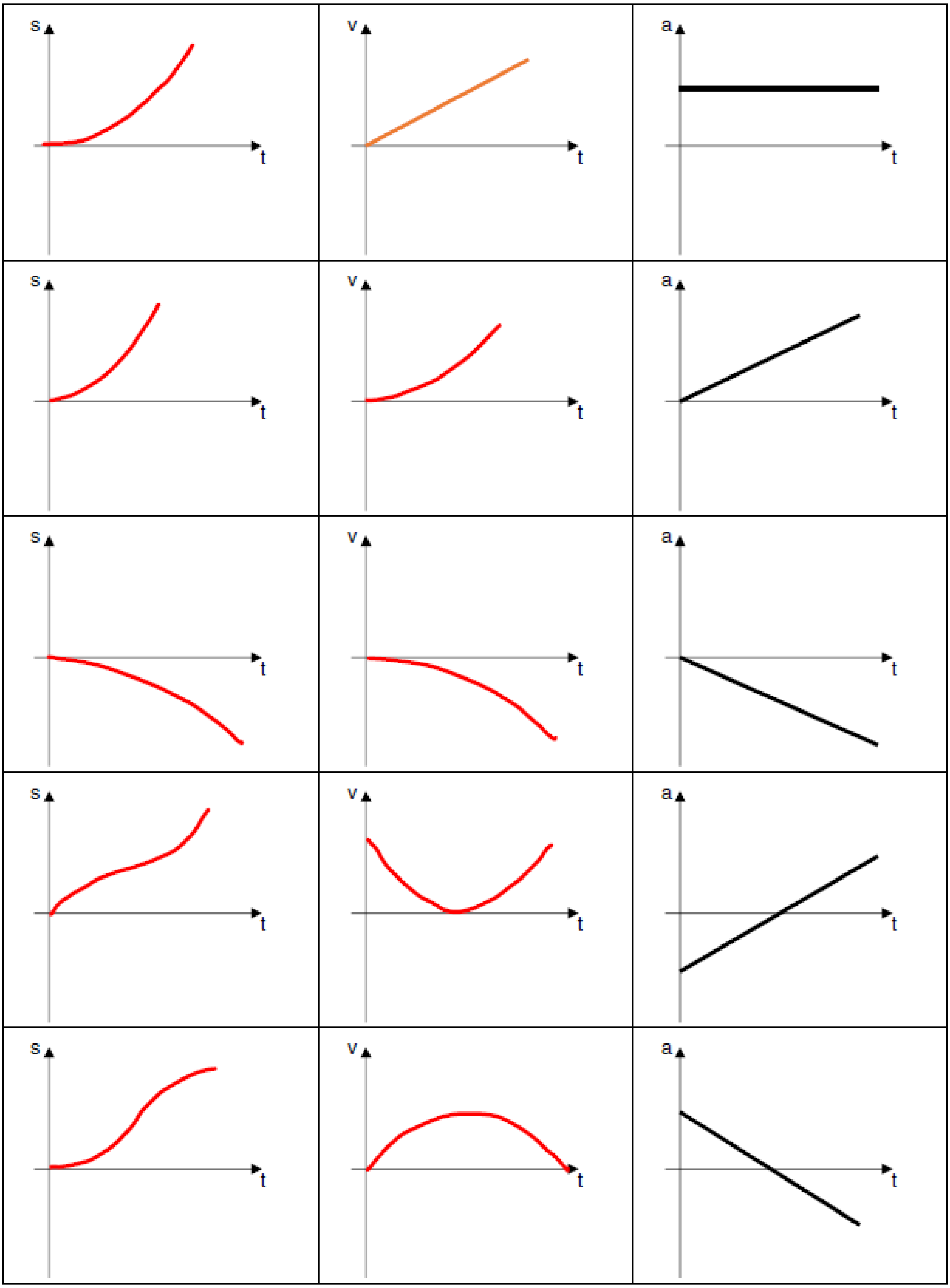
Starting from s-t graph



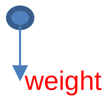
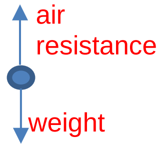
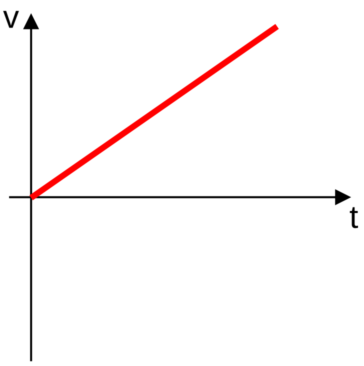
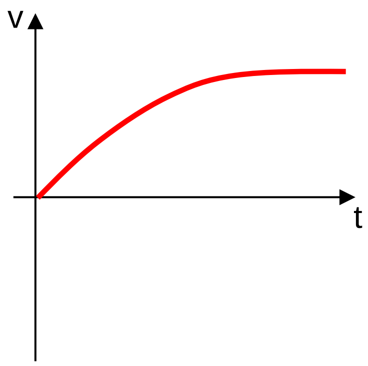
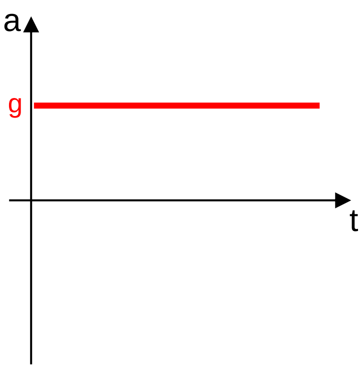
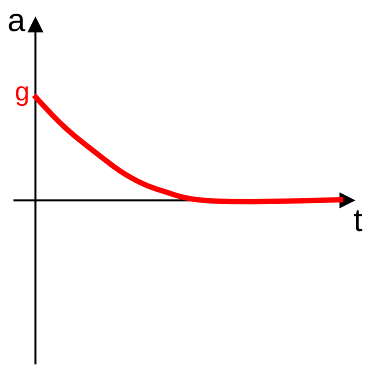
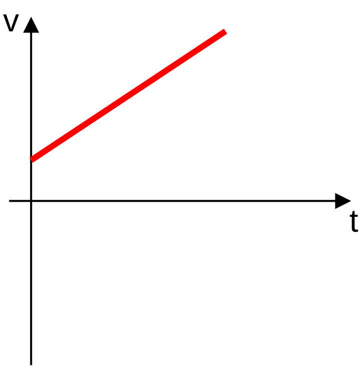
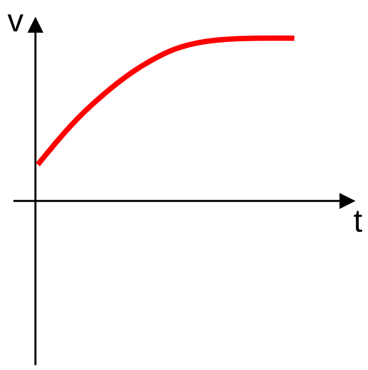
Starting from v-t graph



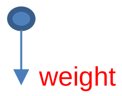
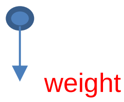
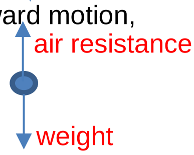
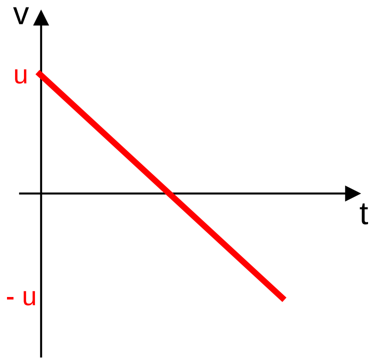
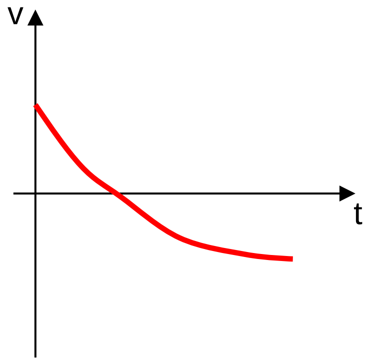
Starting from a-t graph



Case 1: Consider a ball released from rest from a height of h.

	Without air resistance	**With air resistance**
Draw the forces acting on the ball during the motion.		
What is the direction and magnitude of the acceleration of the ball? How does it change?	Constant acceleration of g downwards	Hint: As speed increases, air resistance increases. Downward acceleration decreases from g to zero
Draw the v-t graph. What is your positive direction? What can you say about the areas under the 2 graphs? What can you say about the time taken without and with air resistance?	 Take downwards as positive	 Take downwards as positive Equal areas since same height h Longer time taken with air resistance
Draw the a-t graph.		
What if the ball was thrown downwards instead of being dropped? Draw the v-t graph.		

Case 2: Consider a ball thrown upwards with velocity *u* and landed at the same level.

	Without air resistance	**With air resistance**
Draw the forces acting on the ball during the motion.	During upward motion,  During downward motion, 	During upward motion,  During downward motion, 
What is the direction and magnitude of the acceleration of the ball? How does it change?	During upward motion, Constant acceleration of g downwards During downward motion, Constant acceleration of g downwards	Hint: As speed increases, air resistance increases. During upward motion, Downward acceleration decreases to g During downward motion, Downward acceleration decreases from g to zero
Draw the v-t graph. What is your positive direction? What can you say about the areas under the 2 graphs? What can you say about the time taken for the upward motion and the downward motion?	 Take upwards as positive Larger area Equal time taken	 Take upwards as positive Smaller area due to lower max height reached Shorter time taken during upward motion

Paths of 3 soccer balls:

	Solution:
(a)	Same for all 3 paths. Hint: Using $v_y^2 = u_y^2 + 2a_y s_y$, since v_y is zero at the max height, and a_y and s_y are the same for all three paths, u_y must be the same.
(b)	Same for all 3 paths. Hint: using $s_y = u_y t + \frac{1}{2} a_y t^2$, since all three paths have the same vertical initial speed u_y (see part (a)) and height s_y, the same time is required to reach the peak position and also to come down to ground level.
(c)	c > b > a Hint: Using $s_x = u_x t$ and time taken is the same for all paths (refer to (b)), s_x is proportional to u_x.
(d)	c > b > a Hint: from (a) and (c), we can deduce that initial speed is highest for path c.