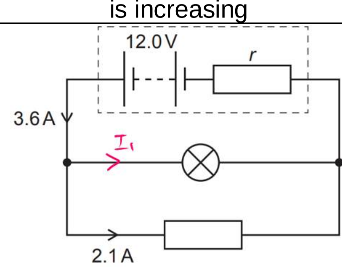
TUTORIAL 13: CURRENT OF ELECTRICITY SOLUTIONS

Level 1 Solutions

1	Q = I t = 0.1 × 20 = 2 C	[1]
5	**Ans: C** Potential difference = $\frac{\text{Energy transferred}}{\text{Charge}}$ Dividing the equation by time Potential difference ($\frac{\text{Charge}}{\text{time}}$) = Power Hence Power = VI	[1]
8(a)	P = VI 24 = 12 × I I = 2.0 A	[1]
	V= R I R = $\frac{12}{2}$ = 6.0 Ω	[1]
(b)	p.d across the variable resistor = 18 – 12 = 6.0 V	[1]
	Power dissipation = I V = 6.0 × 2.0 = 12 W	[1]
11	Resistance, R = $\frac{\rho L}{A}$ $= \frac{(1.1\times10^{\ 6})(500\times10^{\ 3})}{\pi\left(\frac{1.0\times10^{\ 3}}{2}\right)^2}$ = 0.70 Ω	[1]
13(a)(i)	*I* = *P* / *V* = 72 / 12 = 6.0 A	[1]
(ii)	*W* = *P t* = 72 × (20 × 60) = 8.6 x 10^4 J	[1]
(b)	The lamp is operating at 12 V constantly.	[1]
14(a)	Energy = *P t* = 60 x (3 x 60 x 60) = 6.5 x 10^5 J	[1]
(b)	At rated power, *P* = *V I* *I* = 60 / 240 = 0.25 A	[1]
(c)	*Q* = *I t* = 0.25 x (3 x 60 x 60) = 2700 C	[1]
15(a)	*E* = *I r* + *I R* *I* = $\frac{E}{r+R}$	[1]

	$= \dfrac{12.0}{(65.0+0.5)}$ $= 0.183$ A	
(b)	$E = Ir + IR$, where IR is the terminal p.d. of battery. $V = IR = 0.183 \times 65$ $= 11.9$ V	[1]

Solutions to Additional Questions

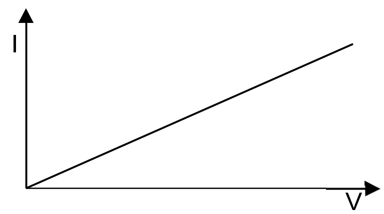
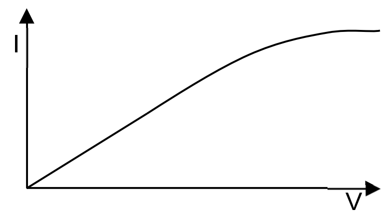
1(a)(i)	Since resistance is the ratio of potential difference to current, and from the graph, From graph, this ratio is increasing and thus resistance increases. Accept: Resistance at any point is the reciprocal of the gradient of the line joining the point to the origin, and since reciprocal of this gradient is increasing, resistance is increasing	[M1] [A1]
(ii)	 Current across filament lamp, $I_1 = 3.6 - 2.1 = 1.5$ A Reading off Fig. 4.2, p.d across filament lamp V = 4.4 V p.d across internal r = 12.0 – 4.4 = 7.6 V Internal resistance, r = 7.6 / 3.6 = 2.11 Ω	[1] [1] [1]
(iii)	I = nAvq and current in series connection is constant $(nAvq)_{filament} = (nAvq)_{copper}$ $nAv_{filament}e = (2.5n)(360A)(v_{copper})(e)$ ratio = (2.5)(360) = 900	[1]
(b)	Potential difference across each lamp in series is 6 V, while potential difference across each lamp in parallel is 12 V. Since power = V^2 / R, each lamp in parallel receives more power than each lamp in series. Hence, temperature increases more for each lamp in parallel. Since resistance of filament lamp increases with temperature, resistance of each lamp in parallel will be greater.	[1] [1] [1]
(c)	When the switch is closed, the emf of the battery will set up an electric field almost immediately across the circuit. The electric field will cause all the free electrons to start moving. Free electrons are distributed throughout the wire, including those near and at the filament lamp. Hence, those electrons near and at the filament can power the lamp immediately even though the drift velocity is less than 1 mm s^{-1}.	[1] [1]

2	Entire length of the wire = 30 turns × circumference of each turns = 1.4137 m Cross-sectional area $= \pi r^2$ $= \pi(0.25 \times 10^{-3})^2$ $= 1.9634 \times 10^{-7}$ m^2 $R = \rho L / A = (1.3 \times 10^{-8})(1.4137) / (1.9634 \times 10^{-7})$ $= 9.4 \times 10^{-2}\ \Omega$	
3	Resistance at X = V_X / I_X, Resistance at Y = V_Y / I_Y Change in resistance = final – initial = $V_Y / I_Y - V_X / I_X$ Note: Resistance is NOT gradient.	
4	**Ans: A** When resistance of R decreases, current in the circuit increases. Hence, potential drop across r increases since $V_r = Ir$ Hence, terminal PD decreases (V_R = EMF – Ir) and power wasted in r increases ($P_r = I^2r$) Using maximum power theorem, P delivered to the load increases as R gets <u>closer</u> to r. Hence, power delievered to the load R will also increase.	
5(a)(i)	$P = \frac{V^2}{R}$ $R = \frac{V^2}{P} = \frac{(6.0)^2}{3.0} = 12\ \Omega$	[1]
(ii)	$R = \frac{\rho L}{A}$ $\rho = \frac{RA}{L}$ $= \frac{(12)\left(\pi \times \left(\frac{78 \times 10^{-6}}{2}\right)^2\right)}{0.020}$ $= 2.8670 \times 10^{-6} = 2.87 \times 10^{-6}\ \Omega$ m	[1] [1]
(iii)	The <u>table of constants states the resistivity of tungsten at room temperature</u> whereas the resistivity value in (a)(i) is the resistivity at a much <u>higher temperature when the light bulb is in use</u>. When <u>temperature increases, the lattice ions</u> in tungsten gain thermal energy and <u>vibrate with larger amplitudes</u>. This <u>increases the collisions</u> between the free electrons and the lattice ions which hinder the movement of the electrons. Hence, <u>resistivity increases with increasing temperature</u>.	[1] [1]
(b)(i)	p.d. across Y, $V_Y = \left(\frac{\frac{R}{2}}{\frac{R}{2} + \frac{R}{4}}\right)(6.0) = 4.0$ V current through Y, $I_Y = \frac{V_Y}{R} = \frac{4.0}{12}$ A $Q = I_Y t$ $= \left(\frac{4.0}{12}\right)(2 \times 60)$ $= 40$ C	

(ii)	Consider $I = Anvq = Anev$. Since both filaments are identical, *Ane* is constant. Hence the current I through the filament is directly proportional to the mean drift velocity *v* of the electrons in the filament i.e. $I \propto v$. The current through Y is twice the current through Z as the potential difference across Y is twice the potential difference across Z (OR the total current flowing through 2 bulbs and 4 bulbs in parallel are equal). Therefore, the mean drift velocity of the electrons in Y is twice that of the electrons in Z.	
6(i)	(Using maximum power theorem, r = R when P across R is maximum.) Hence, at max power, $P = V^2/R$ $1.13 = (1.50^2)/R$ $R = 1.99\ \Omega$ Hence, $r = 1.99\ \Omega$	 [1] [1]
(ii)	Larger p.d. across *R* means smaller p.d. across *r*, since e.m.f. is a constant. Smaller power dissipation across *r* at larger values of *V* since power = current × potential difference, and current is the same for *R* and *r*	[1] [1] [1]
7(i)	Since current = 0.30 A, p.d. across each lamp = 2.5 V (read off from graph) Hence, p.d. across each connecting wire = (8.7 – 7.5) / 2 = 0.60 V Therefore, resistance of each connecting wire = 0.60 / 0.30 = 2.0 Ω	[1] [1]
(ii)	straight line through origin with gradient of 0.5	[1] [1]
(iii)	power loss $= I^2R$ $= (0.30^2)(2.0)$ $= 0.18$ W	[1] [1]
(iv) 1.	$R = \rho L/A$ $2.0 = (1.7 \times 10^{-8})(L) / (0.40 \times 10^{-6})$ $L = 47$ m	[1] [1]
2.	$I = nAvq$ $v = 0.30 / (8.5 \times 10^{28} \times 0.40 \times 10^{-6} \times 1.6 \times 10^{-19})$ $= 5.5 \times 10^{-5}$ m s^{-1}	 [1] [1]

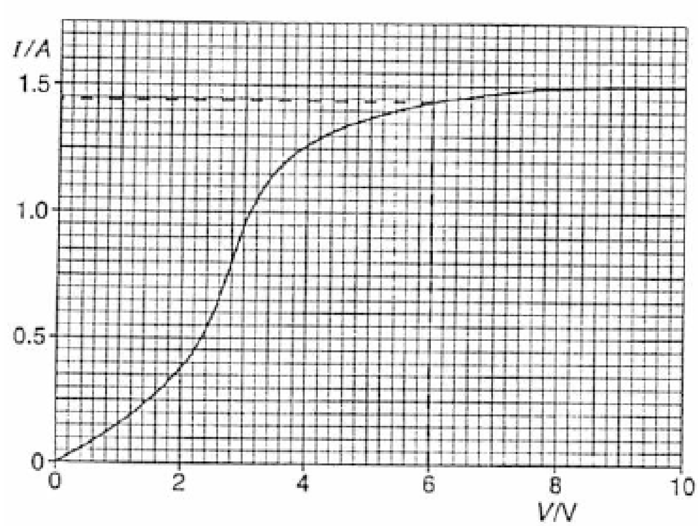
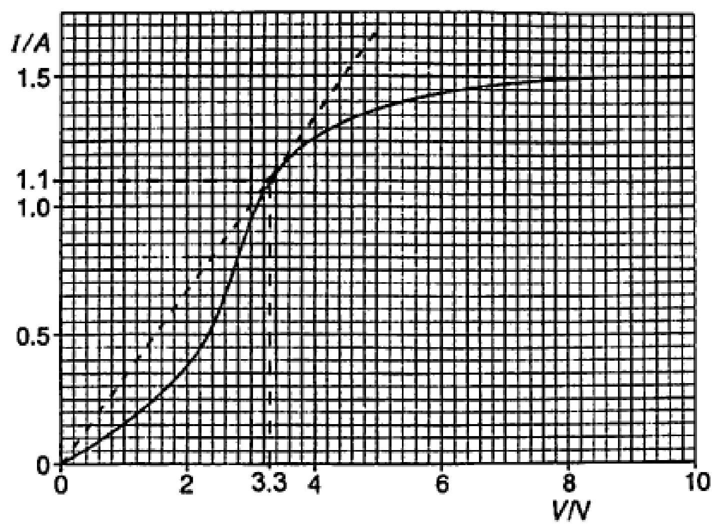
Level 2 Solutions

2	Equivalent current = Total charge per rotation / time per rotation = 4Q / (1/f) = 4Qf	[1] [1]
3(a)	Current due to electrons per second = $4.4 \times 10^{15} \times 1.6 \times 10^{-19}$ = 7.04×10^{-4} A Current due to protons per second = $1.5 \times 10^{15} \times 1.6 \times 10^{-19}$ = 2.4×10^{-4} A Total current per second = $7.04 \times 10^{-4} + 2.4 \times 10^{-4}$ = 9.44×10^{-4} A	 [1] [1]
(b)	Current flow from <u>positive electrode to negative</u> electrode.	[1]
4	**Ans: C** $v = \frac{I}{nAe}$ with *I*, e, and A being constants. $v \propto \frac{1}{n}$ $v = \frac{8.6 \times 10^{28}}{4.3 \times 10^{21}} \times 0.58 \times 10^{-3}$ = 11,600 m s^{-1}	[1]
6	**Ans: D** Total energy dissipated in the external circuit and in internal resistance should be equal to the energy supplied by battery, which is e.m.f. x total charge = E Q.	 [1]
7	**Ans: B** At linear portion, I proportional to V, → R constant. At point where graph is no longer linear, V increases as I decreases → R increases.	[1]
9(a)(i)	0 to 0.12 V: I = 0. This indicates the resistance is infinitely/very high. 0.12 to 0.80 V: I increases non-linearly/exponentially as p.d. increases. Resistance (ratio of V to I) decreases as p.d. increases. 0.80 V to 1.08 V: I increases linearly as p.d. increases. Resistance (ratio of V to I) continue to decreases as p.d. increases. [*Examiner's comments: A significant number of candidates incorrectly considered the straight line part of the graph to indicate constant resistance and that Ohm's Law is obeyed. Many also referred incorrectly to the inverse of the gradient as the resistance of the diode.*]	[1] [1] [1] [1]
(ii)	From the graph, the current is 4.4 mA when the voltage is 0.80 V. Resistance of the diode R = $\frac{V}{I}$ = $\frac{0.8}{4.4 \times 10^{-3}}$ = 181 Ω	

10(a) (i)	Total charge, $Q = N e = 3 \times 10^{20} \times 1.6 \times 10^{-19} = 48$ C	[1]
(ii)	$I = Q / t$ $= 48 / 60$ $= 0.8$ A	 [1]
(iii)	$V = R I$ $R = 9.0 / 0.8$ $= 11.3\ \Omega$	[1]
(b)	(i)  (ii) 	[1] each

12(a)	R = *p* L / A = (1.72 x 10^{-8} x 8.5 x 10^{-2}) / [π (0.068 x 10^{-3})^2 /4] = 0.403 Ω	[1]
(b)	$\frac{R_{new}}{R_{old}} = \frac{\frac{l_{new}}{A_{new}}}{\frac{l_{old}}{A_{old}}} = \frac{\frac{1.02 l_{old}}{(0.99 D_{old})^2}}{\frac{l_{old}}{D_{old}^2}}$ $R_{new} = R_{old}(\frac{1.02}{0.99^2})$ = 0.419 Ω	[1]
16	**Ans: C** Power supplied = power dissipated in internal resistance + power delivered to lamp Total power = $I^2 r + I^2 R$ Power delivered to the lamp = $I^2 R$ $\Rightarrow$ fraction of the total power delivered to the lamp = $\frac{I^2 R}{I^2 (R+r)} = \frac{R}{(R+r)}$	[1]
17	From $E = IR + Ir$ Where I is the common current through circuit, and $V = IR$ So $E = V + \frac{Vr}{R}$ $\Rightarrow V = \frac{ER}{R+r}$ With variable resistance of R = 6.0 Ω, and voltmeter reading of V = 4.0 V $4 = \frac{6 \times E}{6+r}$ -----(1) With variable resistance of R = 10.0 Ω, and voltmeter reading of V = 4.4 V $4.4 = \frac{10 \times E}{10+r}$ -------(2) Solving: r = 1.76 Ω E = 5.17 V	[1 for both eq] [1] [1]
18(a)	From $E = Ir + IR$, Using R = 1.0, I = 3.0, E = 3.0 r + 3.0 ----------------(1) Using R = 2.0, I = 2.0, E = 2.0 r + 4.0 ----------------(2) Solving the equation gives, ir = 1.0 Ω and E = 6.0 V.	[1 for both eq] [1 for each]
(b)	V = E – Ir and $I = \frac{E}{r+R}$ $\Rightarrow V = E - \frac{Er}{r+R} = \frac{Er + ER - Er}{r+R} = \frac{ER}{r+R}$	[1] [1]

19.	To determine e.m.f and internal resistance, we will think of using E = V + Ir . However terminal p.d and external resistance are not given. Given current = 0.025 A , time = 80 s , using $Q = I\,t$, Total charge = 0.025 × 80 = 2 C Given the battery produces 18 J of electrical energy and the external resistors M & N receive a total of 11 + 4 = 15 J , we can deduce that 3 J of electrical energy is converted to other forms of energy when current passes through the internal resistor. From the above information, we can link charge Q and Energy using $V = \frac{W}{Q}$ e.m.f of the battery = 18 J ÷ 2 C = 9 V Hence, potential difference across the internal resistance = 3 J ÷ 2 C = 1.5 V Therefore, internal resistance, $r = \frac{V}{I} = \frac{1.5}{0.025} = 60\ \Omega$.	[1] [1] [1]
20(a) (i)	When R = 4 Ω, P_R = 9 W $P = I^2 R$ $9 = I^2 \times 4$ $\Rightarrow I = 1.5$ A (shown)	
(ii)	When R = 4 Ω, P_T = 13.5 W and I = 1.5 A, $P = E \times I$ $13.5 = E \times 1.5$ E = 9.00 V	[1] [1]
(b)(i)	Power dissipated in the internal resistance, r in the battery.	[1]
(ii)	$P_T - P_R = I^2 r$ $13.5 - 9 = 1.5^2 \times r$ r = 2.0 Ω	
(ci)	From the Fig, R = 2.0 Ω	[1]
(ii)	Efficiency = $\frac{P_R}{P_T} = \frac{10}{20} \times 100\% = 50\%$	[1]
(iii)	At 4 Ω, efficiency = 66%; at 10 Ω, efficiency = 73.5% The efficiency increases as the P_T decrease more than the decrease of P_R.	 [1]

21 (a)(i)	$R = V / I \quad = 6.0 / 1.45 \quad = 4.1\ \Omega$ (as illustrated below) 	[1]
(ii)	The min value occurs at a point where the ratio if V to I is the lowest. See line drawn below. i.e. steepest line drawn from origin to a point on the curve.  Hence $R = V / I$ $= 3.3 / 1.1$ $= 3.0\ \Omega$	[1] [1]
(b)(i)	P.d. across C = p.d. across 5.0 Ω resistor = $I R$ $= 0.85 \times 5.0$ $= 4.25$ V	[1] [1]
(ii)	Total current from supply = current thru 5.0 Ω + current thru C = 0.85 + 1.3 (from graph when p.d. across C is 4.25 V) = 2.15 A	[1] [1]
(iii)	E.m.f. = $V + I R$ $= 4.25 + (2.15)(0.80)$ $= 5.97$ V	[1] [1]
(iv)	Energy = $I V t$ $= (1.3)(4.25)(20 \times 60)$ $= 6.6 \times 10^3$ J	[1] [1] [1]

22(i)	e.m.f = 12 V as voltmeter has infinite resistance when switch is open. When switch closed, I_S= 10.8/200 = 0.054 A, I_T= 10.8/300 = 0.036 A, Total I = 0.09 A V = E – Ir; 10.8 = 12 – 0.09 r r = 13.3 Ω	[1] [2] [1] [1]
(ii)	Energy = $I^2r\,t$ = $(0.09)^2(13.3)(5\text{x}60)$ = 32.4 J	[1] [1]
(iii)	Higher temperature causes resistance of T to decrease Larger current flows through T, total current from e.m.f. source increases. Terminal potential = E – Ir will decrease when I increases as E and r remain unchanged.	[1] [1] [1]

End of tutorial solutions -