TOPIC 6: MOTION IN A CIRCLE

Contents

Learning Outcomes: Candidates should be able to:

a.	Express angular displacement in radians.
b.	Show an understanding of and use the concept of angular velocity to solve problems.
c.	Recall and use $v = r\,\omega$ to solve problems.
d.	Describe qualitatively motion in a curved path due to a perpendicular force and understand the centripetal acceleration in the case of uniform motion in a circle.
e.	Recall and use centripetal acceleration $a = r\,\omega^2$ and $a = \frac{v^2}{r}$ to solve problems.
f.	Recall and use centripetal force $F = m\,r\,\omega^2$ and $F = \frac{mv^2}{r}$ to solve problems.

Three broad areas:

- Kinematics of uniform circular motion
- Centripetal acceleration
- Centripetal force

Motion is a circle is an extension to the topic of kinematics and dynamics. Just like kinematics, we will begin with kinematics language for describing rotational motion. It is not possible to apply the kinematics equations found in Topic 2 to uniform circular motion because the direction of acceleration is constantly changing. Hence, we need a new set of equations. These new equations require the understanding of the following new terminologies:

angular displacement, angular velocity, period and frequency

We will also look at the dynamics involved that relate the forces on a body to the rotational motion using Newton's laws.

Key questions:

- Why must an object moving in a circle with uniform speed experience a force?
- How do you apply mathematical analysis to quantify motion in a circle (relating motion to forces and energy concepts)?

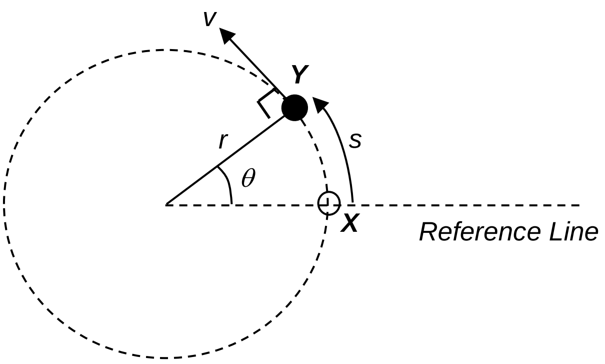
Why is the study of motion in a circle important?

Motion in a circle occurs at all scales, from planets revolving around the Sun to electrons orbiting around the nucleus. All these involve a body that rotates about a fixed axis. Grasping the concepts and mathematical analysis will allow you to study the motion of a satellite in the topic of Gravitational Field and the motion of charged particles moving in uniform magnetic fields in the topics of Electromagnetism and Nuclear Physics.

6.1 Angular Quantities

6.1.1 Angular Displacement and the Radian

To describe rotational motion, we make use of angular quantities, such as ***angular displacement*** and ***angular velocity***.



Consider an object moving along the circumference of a circular path from point **X** to point **Y**.

The angle ***θ*** is known as the **angular displacement** (the angle subtended by the arc) of the object and is defined by the relation:

$$\boxed{\theta = \frac{s}{r}} \quad \text{....... (eq. 6.1)}$$

where
s - arc length
r - radius of the circular path

When a body moves one complete revolution, the arc length *s* would be the circumference of the circle ($2\pi r$). Therefore,

$$\theta = \frac{2\pi r}{r} = 2\pi$$

Since in one complete revolution, the body moves through 360°, hence $2\pi \equiv 360^0$ and $\pi \equiv 180^0$.

Note:
- Angular displacement is a vector since it can be clockwise/anticlockwise
- The unit of angular displacement is the **radian**.
- This defines angles as a ratio of *s* to *r*, and thus, strictly speaking, has no unit. However, for clarity, angles are often written as *θ* rad.

One radian is the angle subtended by an arc equal in length to the radius of the circle.

when s = r, $\longrightarrow$ θ = 1 rad

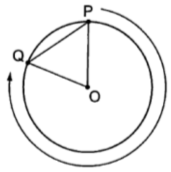
Example 1: Converting radian to degree

Complete the table by filling in the corresponding degree values.

	Radian	working	Degree
(a)	2π rad	Since π = 180°, 2π = 2(180°)	**360°**
(b)	$\frac{\pi}{3}$ rad		
(c)	3.1 rad		

Worked Example 2 (N11/H2/P1/13): Angular Displacement

A disc rotates clockwise about its centre O until point P has moved to point Q, such that OP equals the length of the line PQ.



What is the angular displacement of OQ relative to OP?

A $\frac{\pi}{3}$ rad **B** $\frac{2\pi}{3}$ rad **C** $\frac{4\pi}{3}$ rad **D** $\frac{5\pi}{3}$ rad

Ans: D

Acute angle POQ = 60° (since POQ is an equilateral triangle)

Angular displacement, θ = obtuse angle POQ = 360° – 60° = 300° = $\frac{2\pi}{360°} \times 300° = \frac{5\pi}{3}$ rad

Note: $\pi/3$ is not the answer as P does not move ANTI-clockwise from the starting position to Q. The question talks about the clockwise rotation.

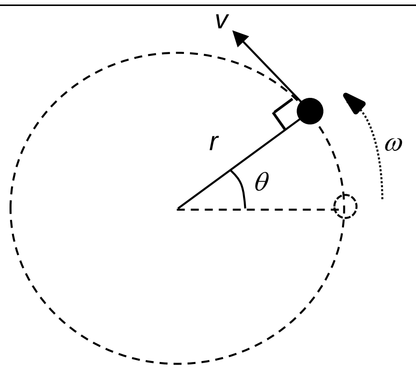
Complete tutorial Q1

6.1.2 Angular Velocity vs Linear Velocity

Angular velocity, ω (omega), of an object is the rate of change of angular displacement, θ, with respect to time, t.

Angular velocity is a vector (clockwise / anticlockwise).

It is measured in radians per second (rad s^{-1}).



Linear velocity, v, of an object is its instantaneous tangential velocity at any point in its circular path.

Linear velocity is a vector.

It is measured in m s^{-1}.

$$\boxed{\omega = \frac{\theta}{t}}$$ …… (eq. 6.2)

For one complete revolution,

$$T = \frac{displacement}{velocity}$$

$$T = \frac{2\pi r}{v}$$

from (6.3),

$$\boxed{T = \frac{2\pi}{\omega}}$$ …… (eq. 6.4)

from (eq. 6.1),

Differentiating $\theta = \frac{s}{r}$ with respect to t where r = constant

$$\frac{d\theta}{dt} = \frac{1}{r}\left(\frac{ds}{dt}\right)$$

from (6.2),

$$\boxed{\omega = \frac{1}{r}(v)}$$

$v = r\,\omega$ …… *(eq. 6.3)*

where r is the radius of the circular path.

The magnitude is called linear speed or tangential speed.

where 2π is the angular displacement and *T*, the period of the object. The magnitude is called angular speed.	If the object is moving at constant linear speed, we can also derive the above formula using: $v = \frac{s}{t} = \frac{\text{arc length}}{\text{time taken}}$ $= \frac{r\,\theta}{t}$ $= r\,\omega$

Comparing linear motion and circular motion:

Linear motion	Circular motion
displacement, s (m)	Angular displacement, θ (rad)
velocity, $v = \frac{ds}{dt}$ (m s^{-1})	Angular velocity, $\omega = \frac{d\theta}{dt}$ (rad s^{-1})

Worked Example 3 (N20/P1/11): Angular speed

The minute hand of a large clock is 3.0 m long. What is its mean angular speed?

A 1.4×10^{-4} rad s^{-1} **B** 1.7×10^{-3} rad s^{-1} **C** 5.2×10^{-3} rad s^{-1} **D** 1.0×10^{-1} rad s^{-1}

Ans: B

The minute hand takes 1 hr = 60 x 60 s = 3600 s to complete one revolution.

Thus $\omega = \frac{2\pi}{T} = \frac{2\pi}{3600} = 1.7 \times 10^{-3}$ rad s^{-1}

Example 4: Angular velocity vs linear velocity

A washing machine spins its tub at 1200 rpm (revolution per minute). If the diameter of the tub is 35 cm,

(a) calculate the period of the spin in seconds**,**

(b) calculate the angular velocity of the tub,

(c) calculate the linear velocity of the rim of the tub.

Solution:

(a)

(b)

(c)

6.1.3 Constant Angular Velocity

In many examples of circular motions, ω is constant for every point in the rotating object. Hence, $v \propto r$ *since* $v = r\omega$. This implies that the linear velocity, *v,* is greater at a point further away from the axis of rotation.

For example, the two balls as shown in Fig. 1 are connected by a thin rod at distance R and 2R from the centre and both are moving in a circle.

Both will be moving with the **<u>same</u>** angular velocity, ω but **<u>different</u>** linear velocities, with $v_2 = 2v_1$.

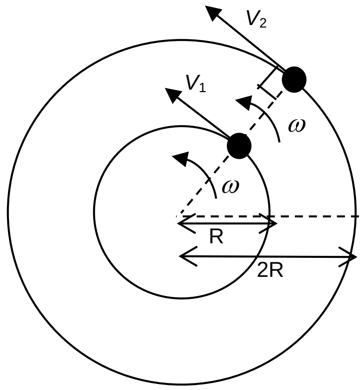


Fig. 1

Food for Thought:
State and explain whether a child sitting nearer to or further away from the axis of rotation in the merry-go-round (Fig. 2) will experience a more thrilling ride?

Ans:
The child sitting further from the axis of rotation where the linear velocity is the greatest.

Since v = rω and angular velocity (ω) is constant, v α r. The linear velocity will be increase with a larger radius.

Fig. 2

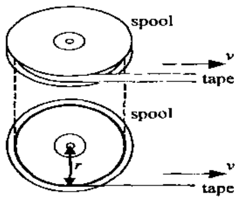
6.1.4 Constant Linear Velocity

In some cases of circular motions, linear velocity, *v,* might be kept constant.

For example, ribbon pulled from the spool at a constant *v* but at a decreasing distance *r* (radius) from the centre. Hence, from (eq. 6.3),

$$v = r\,\omega$$

$$\omega \propto \frac{1}{r}$$



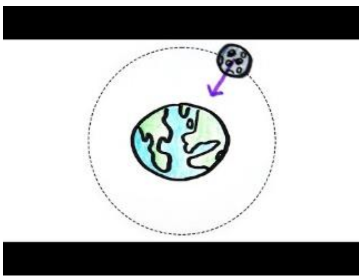
The angular velocity, *ω,* increases as the radius of the spool decreases.

6.2 Uniform Circular Motion

Uniform circular motion refers to the motion of an object along a circular path at **constant speed**.

However, it involves the **continuous change in the direction of velocity** over time i.e. an **acceleration**.

Hence by Newton's 2nd law, this must be due to a <u>net force acting on the object.</u> This net force is known as **centripetal force** (from Latin centrum, "center" and petere, "to seek").

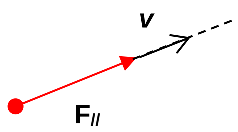
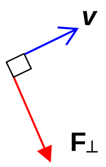
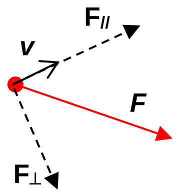


An introductory explanation

6.3 Dynamics of Motion in a Circle

6.3.1 Centripetal Force & Centripetal Acceleration

Let's look at the 3 cases below to understand how the angle between force and velocity affects the velocity.

Case 1: **Force parallel to v**	**Case 2:** **Force perpendicular to v**	**Case 3:** **Force at an angle (not equal to 0 or 90°) to v**
A force, $F_{//}$, acting in the direction of motion, changes only the magnitude of velocity	A force, $F_{\perp}$ acting perpendicularly to motion, changes only the direction of velocity.	A force, F, acting at an angle with the direction of motion, has a component along and perpendicular to the direction of velocity changes both the magnitude and direction of the velocity.
 **Fig. 3a**	 **Fig. 3b**	 **Fig. 3c**

In the case of uniform circular motion (no change in the magnitude of velocity), the force must have no component along the direction of velocity. This is only possible if the force is perpendicular to the direction of tangential velocity (Case 2: Fig. 3b).

Thus, an object needs a net force which is **always perpendicular** to the direction of velocity to maintain uniform circular motion. The force is always directed **towards the centre of the circular path.** (Fig. 3d)

Ball moving in circular path due to perpendicular force towards centre of circular motion.

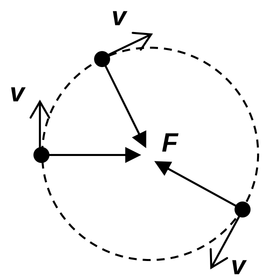


Fig. 3d

Complete Concept Worksheet: Q1 & Q2

The centripetal force (a net force) gives rise to an acceleration directed towards the centre of circular motion. This is known as the **centripetal acceleration, a_c** and is given by:

(refer to Annex A for derivation)

$$a_c = v\omega \quad \text{...... (eq. 6.5)}$$

Since $v = r\omega$,

$$a_c = r\omega^2 \quad \text{...... (eq. 6.6)}$$

$$a_c = \frac{v^2}{r} \quad \text{...... (eq. 6.7)}$$

Correspondingly, by Newton's 2nd Law, $F_{net} = ma$, the **centripetal force, F_c**, acting on an object is

$$F_c = mv\omega \quad \text{...... (eq. 6.8)}$$

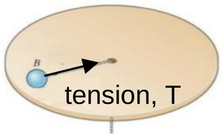
$$F_c = mr\omega^2 \quad \text{...... (eq. 6.9)}$$

$$F_c = m\frac{v^2}{r} \quad \text{...... (eq. 6.10)}$$

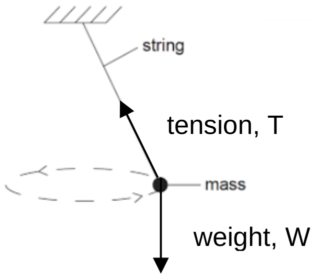
Concept Check:
Is centripetal force provided by a single force or a combination of forces?

Ans:
The centripetal force is essentially a resultant force that may be provided by a **single** force or a **combination** of several forces.

In the case of a ball attached to a cord that is pulled down through a hole travelling in a circular path on the table, the centripetal force is the tension, T in the cord which is a single force.



In the case of a ball attached to a string swinging in a horizontal circle, the centripetal force (towards the centre of the circle) is the resultant of tension, T in the string and the weight of the ball, W.



Error of "Double Counting"
In a question like: "Draw a diagram showing all the forces acting on an object in circular motion", you MUST NOT include an arrow indicating the centripetal force in a FBD. You must ONLY draw the individual forces involved.

Worked Example 5: Path of object just after the centripetal force is removed

Fig. 4 shows a ball attached to a string moving in a uniform horizontal circular motion. What will happen to the motion when the string breaks? Will the ball still move in circular motion? Will the ball fly out radially?

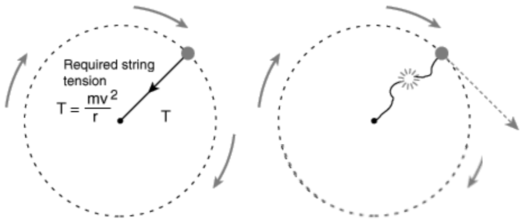


Fig 4

Marble moving tangentially after leaving a circular path.

Solution:
The centripetal force is **perpendicular** to the instantaneous **velocity** and directed towards the **centre** of the circular motion. Sudden removal of the force would allow the object to move along a straight path **tangential** to the initial circular path in accordance with Newton's first law. It will not fly out radially.

Common Error: Some students tend to believe that the ball will fly out radially, caused by *centrifugal* force. In reality, such a force does NOT exist.

FYI: What is Centrifugal Force? (NEVER use this term in A Level Physics)

Consider an object on the dashboard of a car turning to the left shown in Fig. 5.
By Newton's 1st Law, the object will continue in its straight line motion.
Since the car is turning to the left, the object appears to the driver (in the rotating frame of reference) to move outwards.
This apparent outward force is called centrifugal force. It is NOT a REAL force.

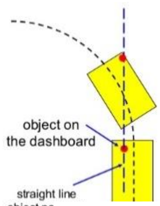


Fig. 5

Worked Example 6 (N09/1/6): Centripetal force

An object in a space capsule orbiting the Earth (moving in a circular path) seems to be floating. Which statement describes the forces acting on the object?

A There are no forces on the object
B The centrifugal force on the object is equal and opposite to its weight.
C The centripetal force on the object is equal and opposite to its weight.
D The weight of the object is the only force acting on it.

Ans: D

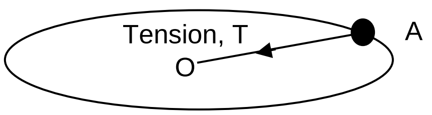
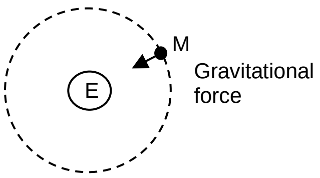
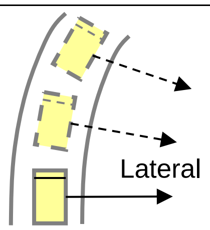
B is false as the centrifugal force is a fictitious force.
For C: Weight of object provides the centripetal force for the circular motion **& hence the portion "..opposite to the weight" is incorrect.** (Gravitational orbits will be studied further in Topic 7.)

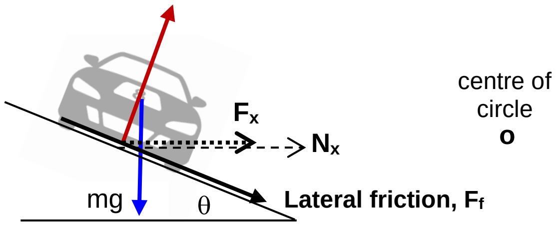
Food for Thought: Is there work done on an object by the centripetal force?

Ans:
Work done is defined as the product of the force and the displacement in the direction of the force (Topic 5). Since the displacement direction (in the direction of the linear velocity) and force direction are **perpendicular**, **zero** work is done by the centripetal force.

Complete tutorial Q2 to Q5

6.4 Identifying Centripetal Forces and Solving Problems

Example	Description
 (i) Swinging mass A on a horizontal frictionless surface with string around O	A mass is swung in a horizontal circle of centre O, with a string. **Tension** in the string provides the centripetal force, $T = mr\omega^2 = \frac{mv^2}{r}$
 (ii) Moon orbiting around Earth	When the moon, M, orbits around the Earth, E, the **gravitational force** between the Earth and the moon pulls the moon towards the earth. Gravitational force, F_G provides the centripetal force, $F_G = mr\omega^2 = \frac{mv^2}{r}$
 (iii) Car turning round an unbanked corner	When a car goes round an unbanked corner (flat horizontal road), the **lateral friction, F_f** between the tyres and road provides the centripetal force required to keep the car in the circular path. Lateral friction provides the centripetal force, $F_f = mr\omega^2 = \frac{mv^2}{r}$



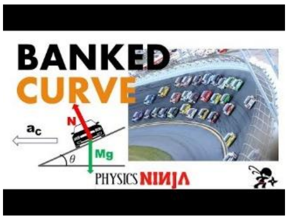
(iv) Car turning round a banked corner (with friction)

When a car goes round a banked corner (road tilted at an angle θ to the horizontal), the **horizontal component of the normal reaction, N_x**, and that of the **lateral friction, F_x**, provides the centripetal force required to keep the car in the circular path at a particular speed.

Horizontal component of normal reaction and lateral friction provides centripetal force,

$$N_x + F_x = mr\omega^2 = \frac{mv^2}{r}$$

Find out more about how banked tracks allow cars to turn even without a lateral friction (ideal banking).



Complete tutorial Q6 & Q7

Reminders:

- Centripetal force is simply a **term given to the resultant force** required to keep an object in circular motion.
- So, when you draw a free-body diagram, draw all the forces acting on the body. The centripetal force is the resultant of these forces; hence, it is wrong to include a force that is labelled as "centripetal force".

Example 7: Excluding "centripetal force" in FBD

Fig. 6 represents a cyclist making a left turn on a rough road surface at constant linear speed *v*. The total mass of the bicycle and rider is *m* and their combined centre of gravity is at G.

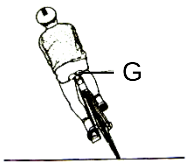
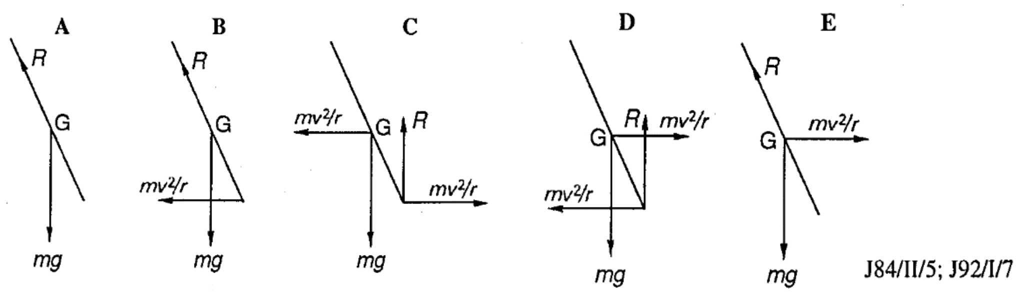


Fig. 6

If R is the resultant force of the normal reaction and frictional force, which vector diagram represents the directions of the forces acting on the bicycle and its rider?



J84/II/5; J92/I/7

Ans: ___. Since centripetal force (mv^2/r) is a resultant force, it should not be included in the diagram.

Concept check: What is the force which enable cyclists and vehicles to turn in the videos?

Ans: It is the horizontal component of normal reaction and the **<u>sideway / lateral friction</u>** between the tyre and the road, that provides the centripetal force.

Common Error: Simply citing the word 'friction' is not acceptable. Examiner will interpret 'friction' as that which opposes forward motion. In this case, turning is due to the sideway or lateral friction.

Worked Example 8: Drawing FBD of Conical Pendulum

Draw on Fig. 7, the forces acting on the object of mass, *m*, moving in a horizontal circular motion.

Solution:
There are only 2 forces, the tension *T* and the weight *mg*.

Since the object is moving in horizontal circular motion, net vertical force is zero since object has no vertical motion.

Resolving, we get $T \cos \theta = mv^2 / r$ (horizontal motion)
$T \sin \theta = mg$ (vertical motion)

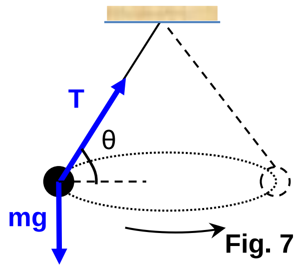


Fig. 7

Strategy for solving for circular motion problems:

Caution: NEVER include the centripetal force in FBD!

1. Sketch a FBD showing all the forces acting on the body.
2. Identify the centre of the circular path.
3. Resolve the forces along the direction towards the centre of the circular path.
4. Identify the force(s) that provides the centripetal force and write down the statement "________________ provides the centripetal force".
5. Apply Newton's 2nd law ($F_{net} = ma$) where the a is the centripetal acceleration

$$F_{net} = mr\omega^2 = m\frac{v^2}{r}$$

Complete tutorial Q8

6.4.1 The Banking Turn of Aircraft

When an aircraft makes a banking (incline) turn, the centripetal force is provided by the horizontal component of the "Lift" of the aircraft. This "Lift" is a force experienced by the wings of the aircraft due to the pressure differences above and below the wings. This "Lift" is always <u>perpendicular</u> to the wings.

Airbus A380 demonstrating high bank turn

Example 9: An Aircraft Performing a Horizontal Circular Turn

An aircraft has a mass of 30 800 kg. To move in a horizontal circle at a constant speed of 680 m s^{-1} (twice the speed of sound), the aircraft banks at 30° *to the vertical* as shown in Fig. 8.

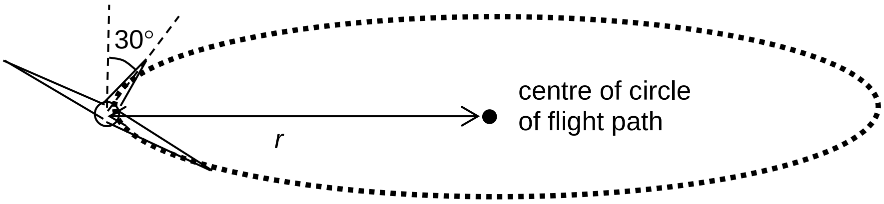
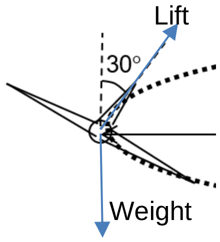


Fig. 8

(a) On Fig. 8, label the forces acting on the aircraft.
(b) Calculate the lift L of the aircraft.
(c) Determine the radius r of the circular path.
(d) Suggest and explain, whether it is possible for the plane to turn without banking at an angle.

Solution:

(a)



(b) For vertical equilibrium,

(c) Taking horizontal components, where the net force is the centripetal force.

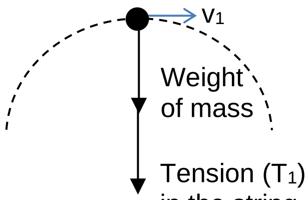
(d)

Complete Concept Worksheet: Q3

Complete tutorial Q9

6.4.2 Calculating Tension on Object Tied To A String In A Vertical Circular Motion

Top of the Loop

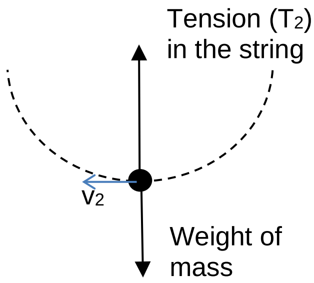


acc. is ↓ (F_{net} towards centre of the circular path),
∴ take ↓ to be +ve

$$F_c = ma_c$$

$$T_1 + mg = \frac{m v_1^2}{r}$$

Bottom of the Loop



acc. is ↑ (F_{net} towards centre of the circular path),
∴ take ↑ to be +ve

$$F_c = ma_c$$

$$T_2 - mg = \frac{m v_2^2}{r}$$

Tips:

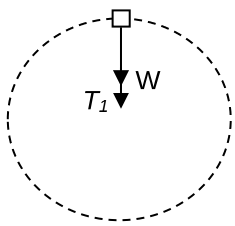
- You need to set a sign convention. Choose the +ve direction in the same direction as F_{net} or a_c.
- Use different symbols to represent the speed at different position e.g., v_1 and v_2 as they may be different.

Example 10: Vertical Circular Motion at Constant Speed

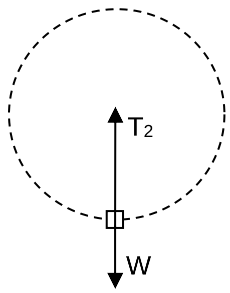
A mass of 2 kg, attached to a string, is whirled round a vertical circle of radius 4.0 m with a constant speed of 10 m s^{-1}. Compare the string's tension at the highest point and the lowest point of the circular path. Hence, deduce the points of maximum tension and minimum tension.

Solution:

At highest point, take ↓ to be +ve



At lowest point, take ↑ to be +ve



Maximum tension, T_2, occurs at the lowest point,
Minimum tension, T_1, occurs at the highest point.

6.4.3 Calculating Max / Min Speeds Required to Stay In Contact With The Surface

It is first necessary to define the terms "stay in contact" and "lose contact" in terms of the force diagrams. Whether or not a body remains in contact with the surface during the circular motion can be determined by looking at the contact force (aka normal reaction), N.

If the contact force N is zero, the object is no longer in contact with the surface.

The magnitude of the contact force differs at different points of the circular motion, just like tension in the previous case.

Worked Example 11: Car moving over a hump

A car of weight 2.0×10^4 N is approaching a hump **ABCD**. The stretch from **B** to **D** is shaped like the arc of a circle of radius 5.0 m and centred at **O** as shown in Fig. 9 below.

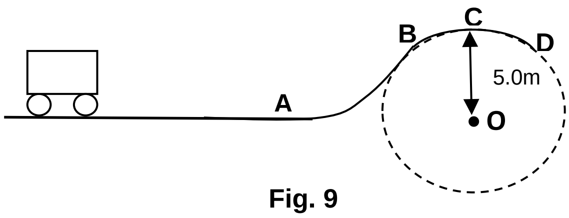
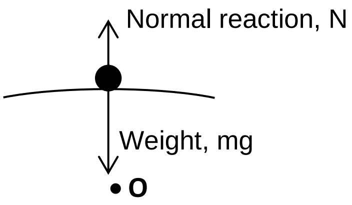


Fig. 9

Sketch the free body diagram of the car when it is over at point C. Hence, determine the minimum speed required for the car to just lose contact with the road at the highest point **C** of the ramp.

Solution:



At point C, the resultant (centripetal) force, F_c, is pointing towards O, acc. is ↓, ∴ take ↓ to be +ve

$$F_c = mg - N = \frac{mv^2}{r}$$

$$N = mg - \frac{mv^2}{r}$$

When the car is in contact with the ground, it will have a normal reaction (i.e., N > 0)

For the car to **<u>just</u>** lose contact,

$$N = 0$$

$$mg - \frac{mv^2}{r} = 0$$

$$\frac{mv^2}{r} = mg$$

$$v = 7.0 \text{ m s}^{-1}$$

Hence, the minimum speed where the car will lose contact with the road is 7.0 m s^{-1}.

Complete tutorial Q10

6.5 Non-uniform Circular Motion (vertical circular path)

Non-uniform circular motion is one where a body moving along a circular path with a speed that is varying.

In the example 10 vertical circular motion, the speed of the mass is given to be constant. However, if the total energy is conserved (TE = GPE + KE), the KE of the object must change as the GPE changes when the object moves to different height. Such situations may require one to consider the energy transformations along with the drawing of FBD and applying Newton's 2nd law to solve the problems.

Worked Example 12: Object tied to a string and swung in a vertical circle

With reference to Example 10, assuming the total energy is conserved in that question, and that the speed is no longer constant,

(a) show that, if the string is **just** taut (tension = 0) when the mass reaches the highest point, the speed is 6.3 m s^{-1},

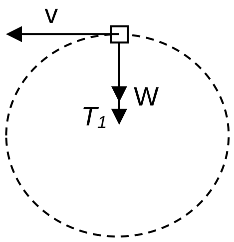
(b) find the speed v with which the mass should be at the lowest point for the mass to just complete the circular path with the string being taut at the highest point.

Solution:

(a) At highest point, take ↓ to be +ve

$$F_c = T_1 + W = \frac{mv^2}{r}$$

When the string is just taut, $T_1 = 0$



$$W = \frac{mv^2}{r}$$

$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{gr} = \sqrt{(9.81)(4)} = 6.3 \text{ m s}^{-1}$$

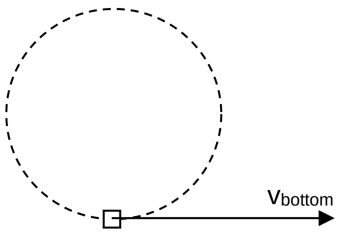
(b) At lowest point,

Using COE,

$$TE_{\text{highest point}} = TE_{\text{lowest point}}$$

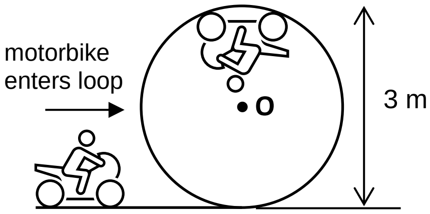
$$mg(2r) + \tfrac{1}{2}m(6.3)^2 = \tfrac{1}{2}m(v_{\text{bottom}})^2$$

$$v_{\text{bottom}} = 14 \text{ m s}^{-1}$$



Example 13: Motorcycle in a Loop Track

Scan the QR code to watch a motorbike entering a loop-the-loop in a vertical circle.

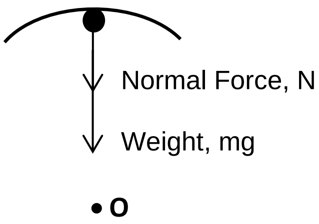


(a) Calculate the minimum speed of the motorbike at the top of the circular loop so that it remains in contact with the track.

(b) State and explain if the speed at the bottom of the loop is greater or lesser than this minimum speed.

Solution:

(a) The resultant (centripetal) force is pointing towards centre, acc. is ↓, ∴ take ↓ to be +ve



To just stay in contact with the track,

Extended Question:

1. Why is N downwards?

   N is contact force by track on the motorbike. It always pushes the motorbike away from its surface.

2. How will the FBD at the top of the loop be different if the motorbike is travelling on the "outside" of the loop instead?

   The N would be upwards and W downwards, thus $F_c = W - N$

Hence, the minimum speed required for the motorbike to stay in contact is 3.84 m s^{-1}.

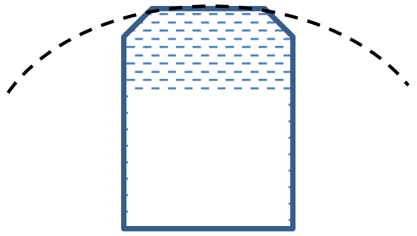
(b) Speed at the bottom would be greater. This is due to COE. The motorbike would gain KE as it loses GPE when it moves to lower height.

Concept Check:

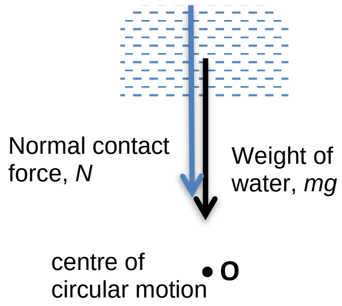
Scan the QR code to watch glasses of water swung in a vertical circle.

Why does the water stay in the glass?

Glass of water at the top of the circle



Free body diagram of water



Note: N and mg should be drawn along same line directed towards O. They are drawn far apart here for purpose of showing them clearly.

Do not do this in exams!

As the glass of water swings, the normal contact force *N* exerted on the water by the glass changes, and it is smallest when it reaches the top where the weight is in the same downward direction.

At the top, the normal contact force *N* and weight of water *mg* provides the centripetal force.

$$N + mg = \frac{mv^2}{r}$$

From the equation, since *r*, *m* and *g* are constants, minimum *v* must occur when *N* = 0.

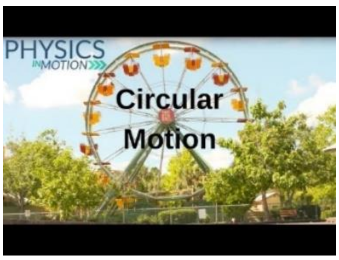
$$0 + mg = \frac{m{v_{min}}^2}{r}$$

$$v_{min} = \sqrt{rg} \quad \text{where } r = \text{radius of circular motion}$$

Thus, to make sure that the water stays in the glass, the velocity of the glass when it reaches the top must be above v_{min}. When the glass is swung at a speed higher than v_{min}, the glass will exert *N*.

Since *N* > 0, the water remains in contact with the glass and the water will not leave the glass.

But if the glass is swung at a gradually decreasing velocity, $\frac{mv^2}{r}$ (centripetal force needed) decreases and *N* will decrease (*mg* remains constant). When the velocity reaches a critical minimum value, *N* becomes zero and the water will start to lose contact with the glass.



Summarise your learning by watching this informative and interesting video on circular motion in a theme park!!

Worked Example 14 with Visible Thinking (N2010 P3 Q2)

A small ball of mass *m* is fixed to one end of a light rigid rod. The ball is made to move at constant speed around the circumference of a vertical circle with centre at C, as shown in fig. 10.

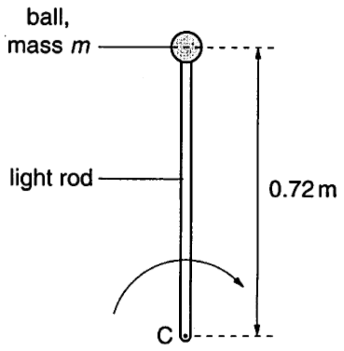


Fig. 10

When the rod is vertical with the ball above C, the tension *T* in the rod is given by

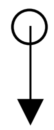
$$T = 2mg$$

where *g* is the acceleration of the free fall.

(a) (i) Explain why the centripetal force on the ball is greater than *2mg*. [1]

Thought process:

1. ***Trigger Keywords:*** *…. greater than 2 mg….*
   If the centripetal force is GREATER than 2mg, there must be another force other than tension that is providing for the centripetal force. This is both the weight and tension, and hence > 2mg!



When the ball is above C, both weight and tension acts downward on the ball, providing for the centripetal force.
Since the tension is given as 2mg, and weight is mg, centripetal force is > 2mg.

(ii) State, in terms of *mg*, the magnitude of the centripetal force. [1]

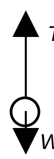
Centripetal force = T + mg = 3 mg

(iii) Determine the magnitude of the tension, in terms of *mg*, in the rod when the rod is vertical, with the ball below point C. [1]

Thought process:

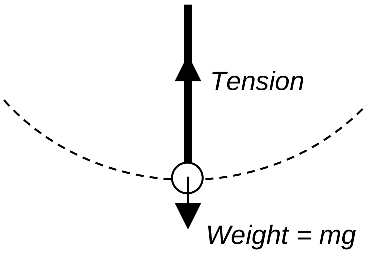
1. ***Trigger Keywords:*** *(in the first paragraph) … constant speed… vertical circle….*
   a. *The ball is moving at CONSTANT speed in a vertical circle (fixed radius).*
   b. *Hence, using $F_c = mv^2 / r$, given that all 3 variables are fixed, the magnitude of the centripetal force must be CONSTANT.*

2. ***Trigger Keywords:*** *…. when the rod is vertical, with the ball below point C….*
   When the ball is below point C, the tension on the ball must be acting UPWARD.



3. ***Key consideration:*** *Should tension be constant???*
   Taking upward as positive, *Centripetal force = 3mg*
   T – mg = 3mg
   T = 4mg (Tension is NOT a constant)

NOTE: *This is a unique vertical circular motion scenario. Normally for vertical circular motion, it is unlikely an object will move at constant speed. An object at the top will lose GPE and gain KE when it is circling down, vice versa.*
If KE changes, speed will change and generally centripetal force at the top and bottom will be different.



At bottom of vertical circle:

$T_B - mg = 3\ mg$
$T_B = 4\ mg$ [1]

(c) The distance from the centre of the ball to point C is 0.72 m.

Use your answer in (a)(ii) to determine, for the ball,

(i) the angular speed, [3]

$F_c = mr\omega^2$ **[1]**

From (aii), $3mg = mr\omega^2$ **[1]**

Solving, $\omega = 6.39\ rad\ s^{-1}$ **[1]**

(ii) the linear speed. [2]

Using $v = r\omega$, **[1]**

$v = (0.72)(6.39) = 4.6\ m\ s^{-1}$ **[1]**

(c) The ball has a constant angular speed.

(i) Explain why work has to be done for the ball to move from the position where it is vertically above point C to the position where it is vertically below C. [2]

Thought process:

1. ***Trigger Keywords:*** *… constant angular speed…*
   This means that KE should be constant through the motion.

2. *For the ball to circle from top to bottom, it must be losing GPE. If the GPE is not lost to the surrounding or used to do work against a resistive force, then the ball's KE should increase.*
   *However, since the ball has constant ω, then KE must be constant and the loss in GPE must be used to do work against a resistive force when it is circling **downward**.*

   NOTE*: If we are required to also explain the upward circular motion, then the ball must gain GPE yet maintain constant KE (as specified in this scenario). Hence, positive work must be done on the ball (i.e., transferring energy to the ball) for the ball to gain GPE without a change in KE.*

There is GPE loss but no KE gain for the system of rod-mass. **[1]**
This must mean that the ball must do positive work to the surrounding or against a resistive force (e.g., air resistance or internal friction at the hinge) to cause a decrease in GPE without it transforming into KE. **[1]**

(ii) Calculate the work done in (i) for a ball of mass 240 g. [2]

Work done = loss in GPE = mgh = (0.240 kg) (9.81) (2 x 0.72 m) **[1]**
= 3.4 J **[1]**

Complete Concept Worksheet: Q4

Worked Example 15 with Visible Thinking (SAJC Prelim 2017 P2 Q2. Modified from CIE N2007)

An elastic cord has an un-extended length of 13.0 cm. One end of the cord is attached to a fixed-point **C**. A small mass of weight 5.0 N is hung from the free end of the cord. The cord extends to a length of 14.8 cm, as shown in Fig. 11.1.

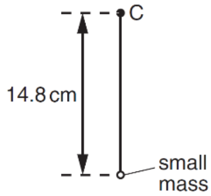
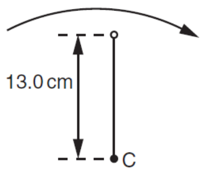
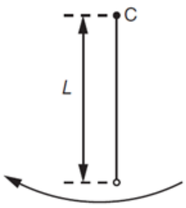


Fig. 11.1

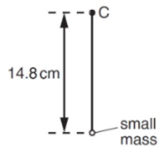
The cord and mass are now rotated at constant angular speed ω in a vertical plane about point **C**.

When the cord is vertical and above **C**, its length is the **<u>un-extended</u>** length of 13.0 cm, as shown in Fig. 11.2.	When the cord and mass rotate so that the cord is vertically below **C**, its length becomes an unknown length, L, as shown in Fig. 11.3.
 **Fig. 11.2**	 **Fig. 11.3**

The elastic cord obeys Hooke's law when stretched and behaves like a spring such that the tension (force) in the elastic cord can be represented by the equation $T = kx$, where k is the spring constant and x is the extension of the cord.

Calculate the length L of the cord. [4]

(From Fig. 11.1, at stationary equilibrium) $T_{equilibrium} = mg$

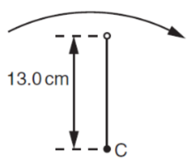


$$k\,(0.148 - 0.13) = 5$$

$$k = 278 \text{ N m}^{-1} \qquad [1]$$

(From Fig. 11.2, at top of motion)

Since cord is unextended, T = 0. Only the weight provides the centripetal force.



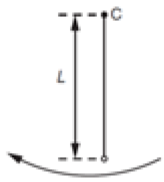
$$mg = m\, r_{top}\, \omega^2$$

eliminating m, $9.81 = (0.13)\,\omega^2$

$$\omega = 8.687 \text{ rad s}^{-1} \qquad [1]$$

(From Fig. 11.2, at bottom of motion)

Since the mass is moving at <u>constant</u> angular velocity, ω is also 8.687 rad s^{-1} at the bottom.



$$T_{bottom} - mg = mL\omega^2$$

$$ke - mg = mL\omega^2$$

substituting e = $L - 0.13$, $(277.8)\,(\mathbf{L - 0.13}) - (5.0) = (5/9.81)\,(L)\,(8.687^2)$ [1]

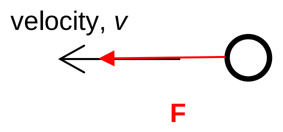
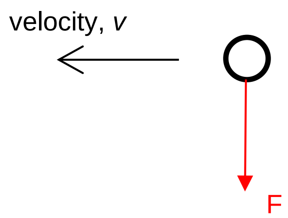
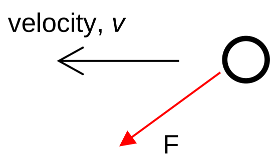
$$L = 0.172 \text{ m} \qquad [1]$$

Complete tutorial Q11 to Q15

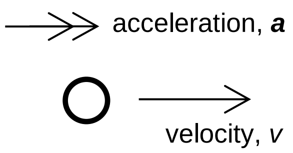
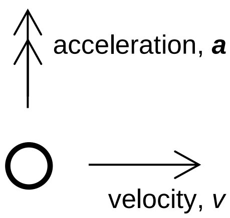
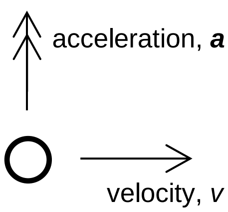
CONCEPT WORKSHEET

Topic 6: Circular Motion

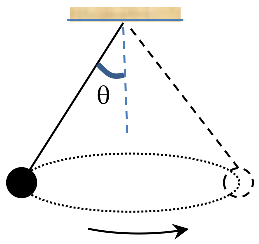
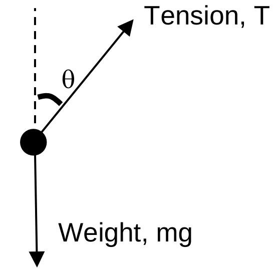
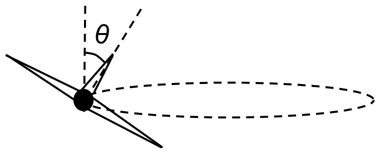
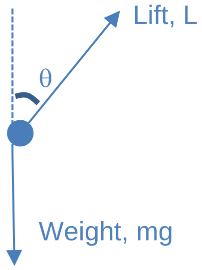
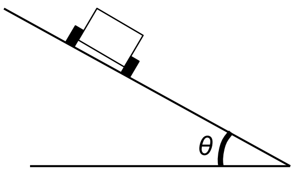
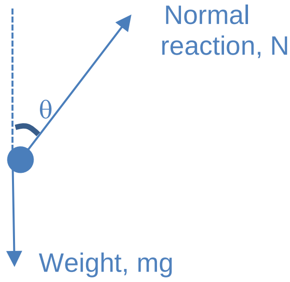
Question 1:

	Draw a force vector on the diagram such that it will **<u>only</u>** cause the object to increase in speed (no change in direction).
	Draw a force vector on the diagram such that it will **<u>only</u>** cause the object to change direction (no change in speed).
	Draw a force vector on the diagram such that it will cause the object to increase in speed **<u>and</u>** change direction.

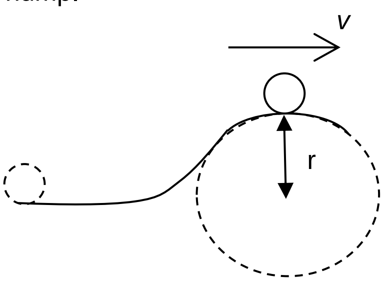
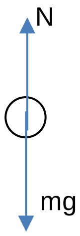
Question 2: Circle the type of motion based on the description.

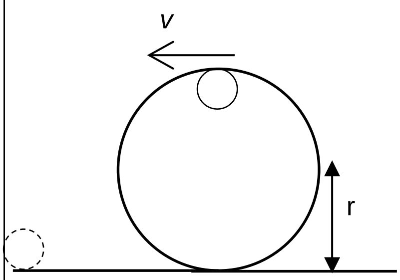
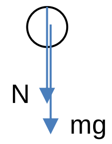
	Description	Type of Motion
	i. Acceleration, ***a***, is <u>parallel</u> to *v*. ii. The direction of ***a*** <u>remains parallel</u> to *v* throughout the motion.	Circular Linear Parabolic
	i. Acceleration, ***a***, is <u>perpendicular</u> to *v*. ii. The direction of ***a*** remains in the <u>same direction</u> (i.e., upwards) throughout the motion.	Circular Linear Parabolic
	i. Acceleration ***a*** is <u>perpendicular</u> to *v*. ii. The direction of ***a*** remains <u>perpendicular</u> to *v* throughout the motion.	Circular Linear Parabolic

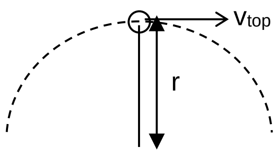
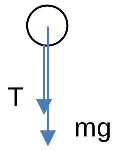
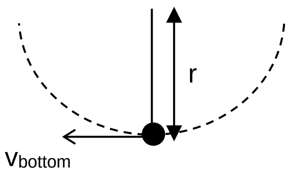
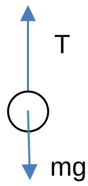
Question 3: Horizontal Circular Motion

Diagram	FBD	Force Equations that may be used to solve such problems
		Mass tied to a cord in uniform circular motion with velocity *v.*
		Airplane in uniform circular motion with velocity *v*
		Car on a smooth (frictionless) banked surface moving in uniform circular motion with velocity v.

Question 4: Vertical Circular Motion

Diagram	FBD	Force Equations that may be used to solve such problems
Object travelling over a circular hump. 		
		Derive the maximum velocity, v, such that the object can complete the circular motion.

<table>
<tr><th>Diagram</th><th>FBD</th><th>Force Equations that may be used to solve such problems</th></tr>
<tr><td>Object travelling in a loop-the-loop

</td><td>
</td><td>
Derive the minimum velocity, v, such that the object can complete the circular motion.</td></tr>
<tr><td>A mass tied to a cord under <u>non-uniform</u> vertical circular motion:

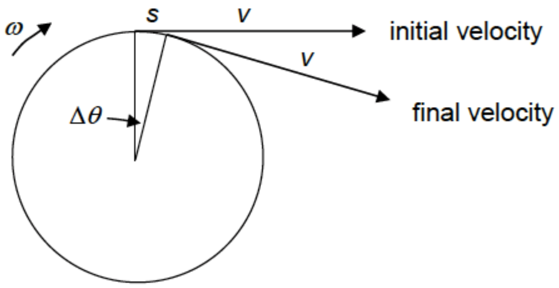
At the top of the circular motion:

</td><td>
</td><td>
Derive the minimum velocity, v, such that the object can complete the circular motion.</td></tr>
<tr><td>At the bottom of the circular motion:

</td><td>
</td><td>Force Equations</td></tr>
<tr><td rowspan="2">Using the COE, derive an expression to show the mathematical relationship between v_{bottom} and v_{top}</td><td colspan="2">By COE, $KE_{bottom} = KE_{top} + GPE_{top}$
$\frac{1}{2} mv_{bottom}^2 = \frac{1}{2} mv_{top}^2 + mg(2r)$
$\frac{1}{2} v_{bottom}^2 = \frac{1}{2} v_{top}^2 + 2gr$</td></tr>
<tr><td colspan="2">Conclusion:
$\mathbf{v_{bottom} > v_{top}}$</td></tr>
<tr><td rowspan="3">Show your mathematical reasoning that it is more likely for the string to break at the bottom than at the top of the non-uniform circular motion.</td><td>Bottom</td><td>Top</td></tr>
<tr><td>$T_{bottom} - mg = (m\frac{v^2}{r})_{bottom}$

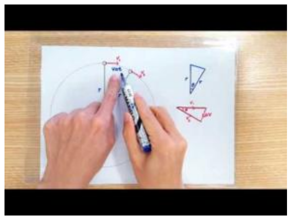
$T_{bottom} = (m\frac{v^2}{r})_{bottom} + mg$</td><td>$T_{top} + mg = (m\frac{v^2}{r})_{top}$

$T_{top} = (m\frac{v^2}{r})_{top} - mg$</td></tr>
<tr><td colspan="2">Conclusion:
Comparing the 2 equations and knowing $v_{bottom} > v_{top}$, it can be concluded that $T_{bottom} > T_{top}$.</td></tr>
</table>

Annex A – Derivation of Centripetal Acceleration (not in Syllabus)

Consider an object undergoing uniform circular motion with constant linear speed v.

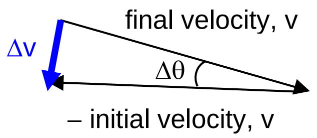
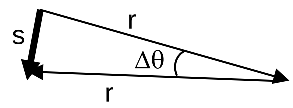




Deriving centripetal acceleration formula $a_c = \frac{v^2}{r}$

The object moves through an angular displacment of $\Delta\theta$ in a small time interval of Δt,

Average acceleration, $a = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\text{final velocity - iniital velocity}}{\text{time taken}}$

Vector diagram for the change in velocity	Diagram for the radius and arc length
	

By similar triangles,

$$\frac{\Delta v}{v} = \frac{s}{r}$$

from (eq. 6.1),

$$\Delta v = v\Delta\theta$$

Therefore,

$$a = \frac{\Delta v}{\Delta t} = \frac{v\Delta\theta}{\Delta t} = v\omega$$

Since $v = r\omega$,

$$a = r\omega^2 \text{ or } a = \frac{v^2}{r}$$

SUMMARY

1. Angular velocity is defined as the rate of change of angular displacement (about the centre of the circle).

$$\boxed{\omega = \frac{\theta}{t}} \qquad \boxed{\omega = \frac{2\pi}{T}}$$

Linear [or tangential] **velocity**, $\boxed{v = r\,\omega}$

2. A body moving in a circle at a <u>constant speed</u> has changing velocity (since its direction changes). Thus, it is *always* experiencing an acceleration**,** a net force and a change in momentum.

Centripetal acceleration, $\boxed{a = r\,\omega^2 = \frac{v^2}{r}}$ (in magnitude)

3. **Centripetal force** refers to the **<u>resultant</u>** of all the forces acting on a system in a circular motion.
   a. it is not a particular force
   b. "centripetal" simply means "centre-seeking"
   c. when asked to draw a diagram showing all the forces that act on a system, it is **wrong to include a force that is labelled as "centripetal force**"
   d. Centripetal force, $\boxed{F_{NET} = m\,r\,\omega^2 = \frac{mv^2}{r}}$

4. **Solving calculation questions involving centripetal forces**

1. Sketch a FBD showing all the forces acting on the body
2. Identify the centre of the circular path
3. Resolve the forces along the direction towards the centre of the circular path
4. Identify the force(s) that provides the centripetal force and write down the statement "_________________ provides the centripetal force"
5. Apply Newton's 2nd law ($F_{net} = ma$) where the a is the centripetal acceleration

$$F_{net} = mr\omega^2 = m\frac{v^2}{r}$$

Caution: Don't draw the centripetal force in free body diagrams!

TUTORIAL 6: MOTION IN A CIRCLE

Angular velocity, linear speed, centripetal acceleration and centripetal force

(L1)1. A disc in a CD player rotates at a constant rate of 300 revolution per minute (rpm).

(a) Calculate the angular velocity of the disc in rad s^{-1}. [1]

(b) Sketch a graph to show how the linear speeds $\boldsymbol{v}$ of a point on the disc varies with the radial distance $\boldsymbol{r}$ from the centre of rotation. [1]

(c) **P** and **Q** are two points on the disc, which are 30 mm and 50 mm respectively from the centre of rotation. Calculate the linear speeds for the two points. [2]

(L1)2. A pendulum hangs down from the ceiling of a train travelling along a circular arc bending to the right. Draw a FBD of the pendulum and indicate the direction of its acceleration. [3]

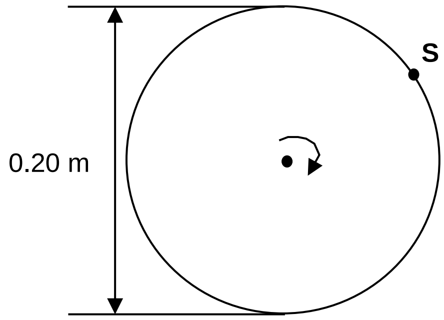
(L1)3. The diagram shows the top view of a grinding wheel of diameter 0.20 m spinning horizontally. **S** is a soyabean resting on the edge of the wheel.

(a) If the rate of revolution is 600 rpm, calculate the magnitude of the force holding **S** to the wheel if **S** has a mass of 1.0×10^{-4} kg. [1]

(b) If the maximum force that holds **S** to the wheel is 0.1 N.

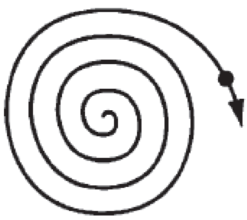
(i) Calculate the maximum angular velocity before **S** dislodges from the wheel. [2]

(ii) Find the linear velocity of **S** when it dislodges. [1]



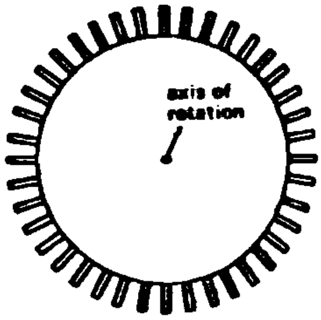
(L2)4. A proton of mass 1.67×10^{-27} kg is accelerated from rest in a particle accelerator (cyclotron).

It travels in a circular path of diameter gradually increasing over many revolutions to a final value of 1.2 m when the proton reaches a kinetic energy of 5.0×10^{-13} J.



What is the centripetal acceleration of the proton at that instant? [Modified N13/P1/Q12]

(L2)5. Each blade on a turbine wheel is attached separately on a small section of the rim of the wheel as shown. The blades are small, and each behaves like a point mass of 0.8 kg at 0.7 m from the axis of rotation.



The plane of the wheel is kept horizontal with the axis of rotation vertical when it spins at high speed. It is found that blades break off at angular velocities greater than 500 rad s^{-1}.

(a) Calculate (i) the linear speed and (ii) the corresponding centripetal acceleration of the blades when the angular velocity is 500 rad s^{-1}. [4]

(b) Use Newton's laws to explain why a blade might break off at high angular velocities. [4]

(c) Calculate the minimum radial force required to pull a blade off the wheel. [2]

[N89/P2/Q9 part]

(L2)6. (a) Explain how a vehicle travelling on a level circular road with uniform speed has an acceleration. [2]

(b) Circle the quantities that remain constant during the vehicle's circular motion. [2]

Period *Linear momentum* *Centripetal force* *Angular velocity*

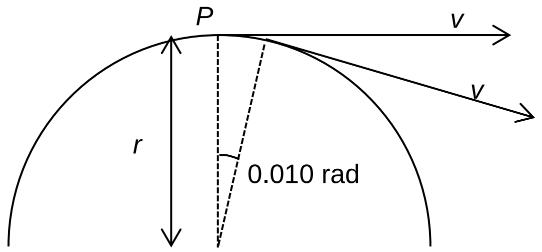
Kinetic energy *Linear velocity* *Centripetal acceleration*

(c) Suggest why a vehicle with a heavier mass may have to travel at a slower speed round a bend. [2]

(L2)7. The maximum safe speed of a car rounding an unbanked corner is 20 m s^{-1} when the road is dry. The maximum frictional force between the road surface and the wheels of the car is halved when the road is wet. What is the maximum safe speed for the car to round the corner when the road is wet? [N03/PI/Q9]

(L2)8. (a) An object travelling in a circle of radius *r* at the constant speed *v* is accelerating. By drawing a vector diagram to show the combination of vectors, explain how this is possible. [2]

(b) An object P moves at a constant speed *v* through an arc of a circle of radius *r*. The arc subtends an angle of 0.010 rad at the centre of the circle as shown in the figure below.



(i) Determine, in terms of *v*, the magnitude of the change in velocity. [1]

(ii) State the direction of the acceleration of P. [1]

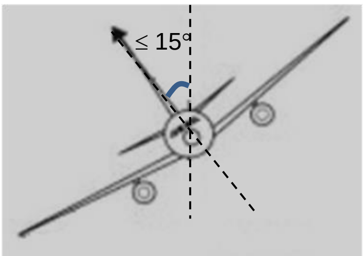
(iii) Deduce, in terms of *r* and *v*, the time taken for P to travel 0.010 rad. [2]

(iv) Hence, show that the magnitude of the acceleration P is $\frac{v^2}{r}$. [1]

[N06/P3/Q3]

Banking turn of aircraft

(L2)9. An SIA Airbus A380 has a mass of 560 tonnes. It is flown by a 70 kg pilot cruising at a constant speed of 900 km h^{-1}. For the passengers' comfort, the plane should not roll by more than 15° when it turns in a horizontal circle. (Note: 1 tonne = 1000 kg)



Determine, for an angle of 15°,

(a) the radius of its turning circle. [2]

(b) the ratio of the centripetal force to the weight of the aircraft. [1]

(c) the force exerted by the plane on the pilot. [1]

(d) the lift of the aircraft. [1]

Car moving over a hump

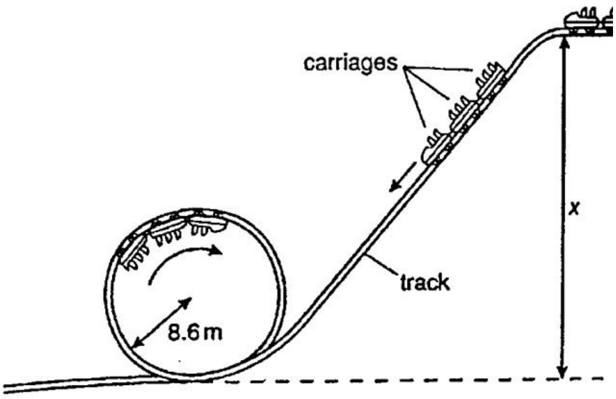
(L2)10. A car of mass 1500 kg moving at a constant speed of 20 m s^{-1} passes over a humpback bridge of radius of curvature r. The car remains in contact with the road when it passes over the top of the bridge.

(a) Express N, the total Normal force exerted by the road on the car, in terms of r. [2]

(b) Determine the minimum radius of curvature of the road required for the car to just remain in contact with the road. [2]

Vertical circular motion and non-uniform circular motion

(L2)11. A theme park ride is illustrated below:



(a) (i) Find the minimum speed of the carriages at the top of the circular track so that the carriages remain in contact with the track. [2]

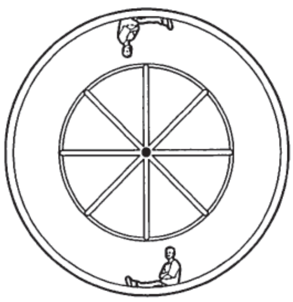
(ii) In practice, it is essential for designers to build in a considerable safety margin. Each carriage and its passengers have a total mass of 800kg, and at the top of the loop, the carriage is travelling at 17m s^{-1}. Calculate the force that the track is exerting on the carriage at the top of the loop and state its direction. [3]

(b) For the carriage in part **(a)** to have a speed of 17m s^{-1} at the top of the loop, it is necessary for it to have fallen from a height of at least x. Deduce a value for x, stating any assumptions you may have made in making this calculation. [4]

[N06/P3/Q3]

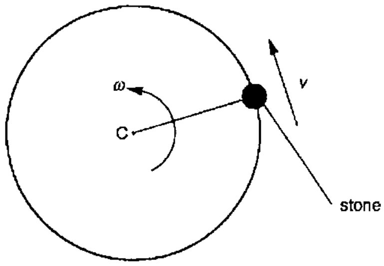
(L2)12. On a fairground ride, passengers are rotated in a vertical circle of radius 4.00 m. Passengers complete one revolution in 3.70 s.

A passenger of mass 77.0 kg is shown in the diagram when at the top of the circle and when at the bottom of the circle.



What force does the ride exert on the passenger in each position?

(L2)13. (a) A stone is tied to one end of a cord and then made to rotate in a horizontal circle about a point C with the cord horizontal as shown below.



The stone has speed v and angular velocity ω about C.

(i) Write down a relation between the speed v, the length r of the cord and the angular velocity ω. [1]

(ii) Explain how v can be made to vary when ω is constant. [1]

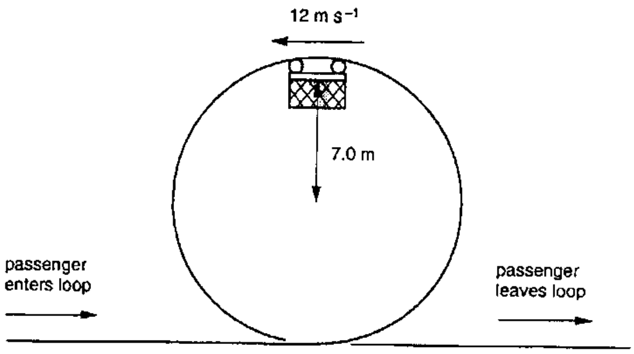
(iii) Explain why there needs to be a tension in the cord to maintain the horizontal circular motion. [2]

(iv) Write down an expression for the acceleration of the stone in terms of v and r. Hence, if the stone has mass m, show that the tension T in the cord is given by

$$T = mv\omega.$$

[2]

(b) On one ride in an amusement park, passengers 'loop-the-loop' in a vertical circle, as illustrated below.



The loop has a radius of 7.0 m and a passenger, mass 60 kg, is travelling at 12 m s^{-1} when at the highest point of the loop. Assume that frictional forces many be neglected.

(i) Calculate, for the passenger when at the highest point,

(1) the centripetal acceleration,

(2) the force the seat exerts on the passenger.

[4]

(ii) The passenger now moves round and descends to the bottom of the loop. Calculate

(1) the change in potential energy of the passenger in moving from the top of the loop to the bottom,

(2) the speed of the passenger on leaving on the loop.

[4]

(iii) Operators of this ride must ensure that the speed at which the passengers enter the loop is above a certain minimum value. Suggest a reason for this.

[2]

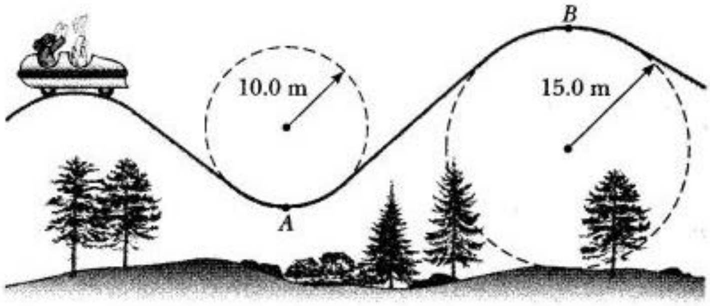
[J94/P3/Q1]

NUMERICAL ANSWERS:

1. (a) 31.4 rad s^{-1} (c) 0.943 m s^{-1}, 1.57 m s^{-1}
3. (a) 3.95 x 10^{-2} N (b)(i) 100 rad s^{-1}, 10 m s^{-1}
4. 9.98 ×10^{14} m s^{-2}
5. (a)(i) 350 m s^{-1} (ii) 1.75 x 10^{5} m s^{-2} (c) 1.40 x 10^{5} N
7. 14.1 m s^{-1}
8. (b) (i) 0.01 v, (iii) 0.01 $\frac{r}{v}$
9. (a) 2.38 x 10^{4} m (b) 0.268 (c) 711 N (d) 5.69 x 10^{6} N
10. (a) $mg - \frac{mv^2}{r}$ (b) 40.8 m
11. (a)(i) 9.19 m s^{-1} (ii) 19.0 x 10^{3} N (b) 31.9 m
12. 133 N down, 1640 N up
13. (b) (i) (1) 21 m s^{-2}, (2) 650 N (ii) (1) - 8240 J, (2) 20 m s^{-1}

ADDITIONAL QUESTIONS

1 A roller-coaster vehicle has a mass of 500 kg when fully loaded with passengers as shown in the diagram below.



(a) Name the forces which act on the vehicle at point **A** and draw a labelled diagram to show the directions of these forces. Assume that friction is negligible. Indicate the centre of the circle in your sketch and the direction of the centripetal acceleration. [3]

(b) If the vehicle has a speed of 20 m s^{-1} at point **A**, what is the force exerted by the track on the vehicle at this point? [2]

(c) What is the maximum speed the vehicle can have at point **B** and yet remain on the track? [2]

2 [2021 RI Prelims P2Q4]

(a) A pendulum with a bob of mass 10 g is suspended from a fixed point O by an inextensible string of length 30 cm. The bob is initially held at point A, at an angle of 25° to the vertical as shown in **Fig. 2.1**.

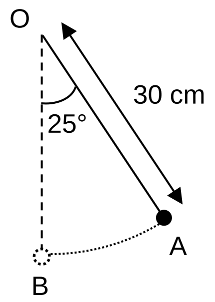


Fig. 2.1

(i) Show that the speed of the mass at point B is 0.74 m s^{-1}. [2]

(ii) Hence, determine the tension in the string at point B.

tension = …………………. N [2]

(iii) A rod is placed above point B such that part of the string remains vertical as the mass swings past B as shown in Fig. 2.2.

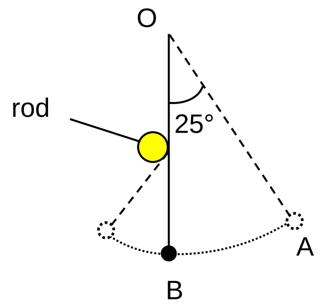


Fig. 2.2

Explain why the tension in the string just after the bob passes point B will be larger than the tension calculated in **(a)(ii)**.

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………[2]

(b) The bob is now set in uniform circular motion in a horizontal plane with the string making an angle *θ* to the vertical as shown in Fig. 2.3. The tension in the string is 0.20 N.

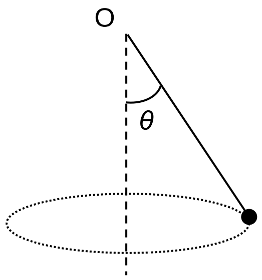


Fig. 2.3

(i) Calculate angle θ.

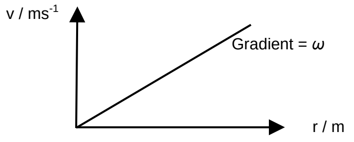
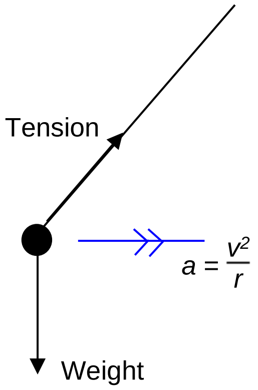
θ = …………………. ° [2]

(ii) Calculate the angular speed of the bob.

angular speed = …………………. rad s^{-1} [2]

TUTORIAL 6: MOTION IN A CIRCLE SOLUTIONS

<u>Level 1 Solutions</u>

1(a)	300 rpm = 5rps = 5 x 2π s^{-1} = 10π s^{-1} = 31.42 s^{-1} = 31.4 rad s^{-1}	[1]
(b)	Since $v = r\omega$, and ω is constant, v α r. Therefore, it is a linear graph. 	[1]
(c)	$v_1 = r_1\omega$ = (0.030)(31.42) = 0.943 m s^{-1} $v_2 = r_2\omega$ = (0.050)(31.42) = 1.57 m s^{-1}	[1] [1]
2	 {Tension [1], Weight [1] } Note: A common mistake is including the centripetal force. Students must remember that centripetal force is a resultant force that does not exist by itself but is the resultant of the forces acting on the object. In this case, the horizontal component of the tension.	
3(a)	$r = \frac{0.2}{2} = 0.1$ m ω = 600 rpm = 10 rps = 62.831 rad s^{-1} $F_c = mr\omega^2$ $= 10^{-4}$ (0.1) $(62.831)^2$ = 3.9478 x 10^{-2} N = 3.95 x 10^{-2} N (3.s.f)	[1]
(b)(i)	$F_{max} = mr\omega_{max}^2$ 0.1 $= 10^{-4}$ (0.1) ω^2 $\omega^2 = 10^4$ ω = 100 rad s^{-1}	[1] [1]
(ii)	$v = r\omega$ = 0.1 (100) = 10 m s^{-1} The moment the pea leaves the wheel, its linear velocity is the same as the linear velocity of the wheel.	[1]