TUTORIAL 2: KINEMATICS SOLUTIONS

Level 1 Solutions

<table>
<tr><td>1</td><td>[C]
Acceleration is defined as the rate of CHANGE of velocity & By Newton's 2nd Law, Resultant force = ma. Hence the acceleration, a, is always in the same direction as the change in velocity and net force.

Option A is wrong as acceleration is not related to velocity at that instant. Should be change in velocity.
Option C is correct as this can occur at the highest point of the projectile motion.
Option B is wrong as at the highest point, weight is acting on the object. Hence since weight is the resultant force, acceleration of the object is downwards.
Option D is wrong. When acceleration is zero, the object can also move at constant velocity.</td><td></td></tr>
<tr><td>2</td><td>[D]
The gradient of the s-t graph of option D shows that the velocity increases (accelerating down the hill), becomes constant, then decreases.</td><td></td></tr>
<tr><td>10</td><td><table><tr><td></td><td>s</td><td>v</td><td>a</td></tr><tr><td>1</td><td>+</td><td>+</td><td>-</td></tr><tr><td>2</td><td>+</td><td>-</td><td>-</td></tr><tr><td>3</td><td>-</td><td>-</td><td>-</td></tr></table></td><td></td></tr>
<tr><td>11</td><td>[D]
Option A is wrong because acceleration should be $– 9.81 \text{ m s}^{-2}$
Option B is wrong because acceleration is not zero but $– 9.81 \text{ m s}^{-2}$
Option C is wrong because velocity should be $– 9.81 \text{ m s}^{-1}$ since ball is travelling downwards after reaching its maximum height at t = 2.0 s .
Using v = u + a t (taking upwards as positive),
$v = (19.6) + (-9.81)(5.0)$
$= - 29.45 \text{ m s}^{-1}$</td><td></td></tr>
<tr><td>12(a)</td><td>Using $v = u + at$,
Note: 1 km / 1 hour = 1000 m / 3600 s
i.e. 90 km hr^{-1} = 90000 m / 3600 s
(90 000 / 3 600) = (30 000 / 3 600) + 2 t
t = 8.3 s</td><td></td></tr>
<tr><td>(b)</td><td>Using $v = u + at$,
0 = (90 000 / 3 600) + (-5) t
Note: a = -5 as it is in the opposite direction of u
t = 5.00 s

Using $s = ut + ½ at^2$,
$s = 25 (5) + ½ (-5)(5.00)^2$
s = 62.5 m</td><td></td></tr>
<tr><td>(c)</td><td>Using $v^2 = u^2 + 2as$,
$(100\ 000 / 3600)^2 = 0 + 2 (a)(50)$
$a = 7.72 \text{ m s}^{-2}$</td><td></td></tr>
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16	[C] Using $v_y = u_y + a_y t$, (taking upwards as positive), $v_y = (40 \sin 45^o) + (-9.81)(5)$ $= -20.77$ m s^{-1} Hence $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(40 \cos 45°)^2 + (-20.77)^2}$ **Note:** v_x is constant $= 35.1$ m s^{-1}	
17	Let speed at take-off = u_x. Taking downwards as +ve, Using $s_y = u_y t + ½ g t^2$ $1.25 = 0 + ½ (9.81) t^2$ $\rightarrow t = 0.5$ s Using $s_x = u_x t$ $10 = u_x (0.5)$ $\rightarrow u_x = 19.8$ m s^{-1}	

Level 2 Solutions

3	[C]. Area under the acceleration-time graph is change in velocity. Positive area represents positive change in velocity (implying increasing velocity). Option B is wrong because between B and C, the car is still accelerating and its velocity will thus continue to increase until point C. Beyond C, the car starts to decelerate and slow down.	
4	[D] Question states taking downwards as positive. Points A, B, C are when the ball impacts on the ground. From C to D, the velocity is negative, indicating that the ball is moving upwards (away from ground). At D, the ball is at its maximum height.	
5	[D] Initially, the parachutist undergoes free fall for the first 2 s and his acceleration is constant at 9.81 m s^{-2}. At the instant the parachute opens, there is a sudden great upwards force acting on the parachutist / parachute. Hence, the acceleration becomes large and negative.	
6	[D] Let the entire distance fallen be s in time t. Initial velocity u = 0 Using $s = ut + ½ at^2$ $s = ½ gt^2$ --- (1) $0.25 s = ½ g (t - 1)^2$ --- (2) Solving simultaneous equations, t = 2 s	
7(a)	At E, the body <u>decelerates uniformly(constant value)</u> until it comes to <u>rest</u> and then experiences <u>uniform acceleration in the opposite direction</u> until its <u>speed is 5 m s^{-1}</u>. At F, it continues moving in this (opposite) direction at a <u>constant speed</u> of 5 m s^{-1} until it comes to an abrupt stop at 65 s.	[1] [1] [1] [1]

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<tr><td>(b)</td><td>

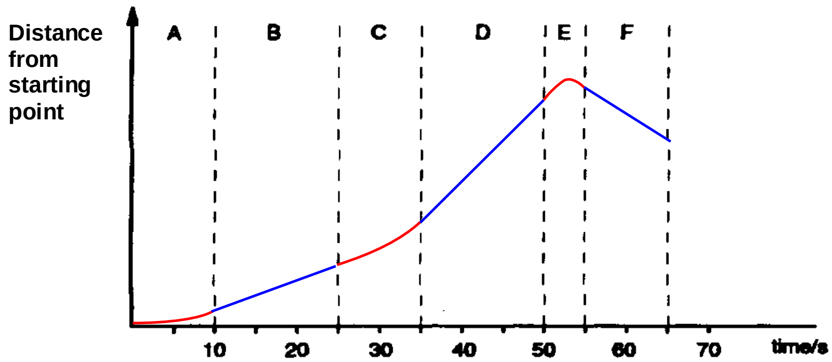

Fig 7.2

Explanation for section E:

- When *v* is positive and decreasing, gradient (of *s*-*t* graph) must be positive and decreasing;
- When *v* = 0, gradient (of *s*-*t* graph) = 0;
- When *v* is negative and increasing, gradient (of *s*-*t* graph) must be negative and increasing.

Explanation for section F:

- When v is negative and constant, gradient (of *s*-*t* graph) is negative and constant

</td><td>A & B, [1]

C & D, [1]

E & F, [1]</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>8</td><td>[C]
K.E = ½ Mv^2 = ½ $M(u^2 + 2\,as)$
Graph will be a linear line as it follows y = m x + c,
Where KE is the y-axis variable, s is the x-axis variable.
Option A is wrong as K.E cannot be negative.</td><td></td></tr>
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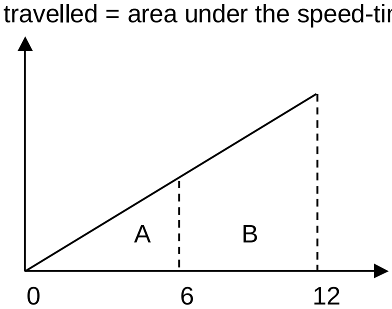
<tr><td>9</td><td>

Distance travelled = area under the speed-time graph.



$$\frac{\text{distance travelled between 6s and 12s}}{\text{distance travelled between 0s and 6s}} = \frac{\text{Area B}}{\text{Area A}}$$

By similar triangle, the larger triangle (A+B) has twice the base of the smaller triangle (A), the area of triangle (A+B) = 2^2 area of triangle (A).

Hence, Area A+B = 4 (Area A), thus, Area B is 3 times that of Area A.

$$\frac{\text{Area B}}{\text{Area A}} = 3.$$

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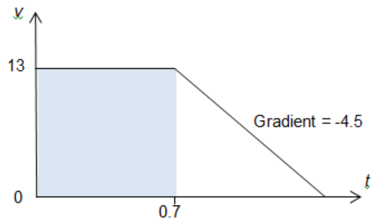
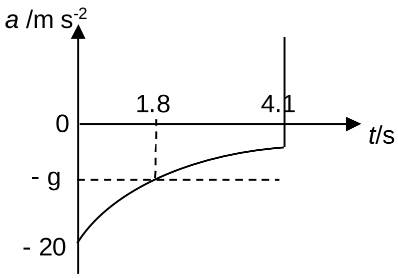
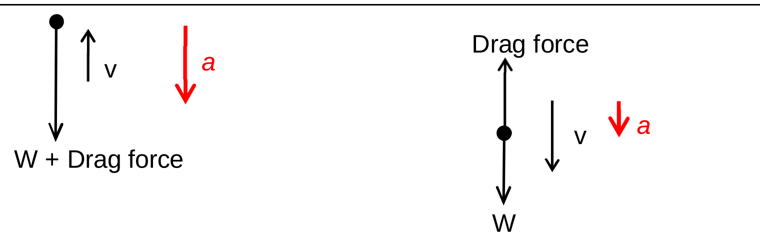
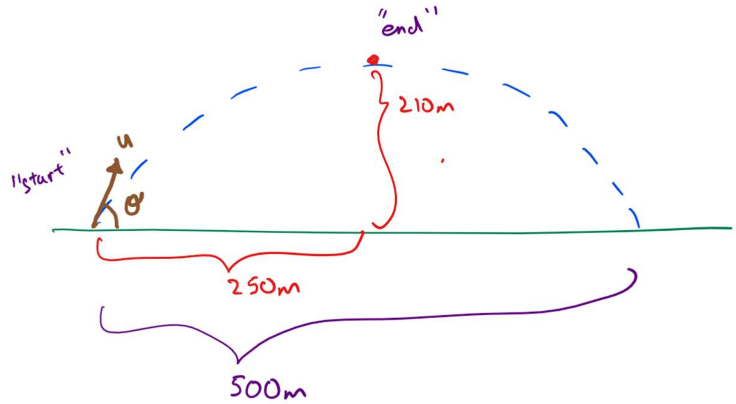
13	{For such questions, it is helpful to sketch a v-t graph}  Distance travelled during his reaction time = 0.7 x 13 = 9.1 m Braking distance: $v^2 = u^2 + 2as$ (**Note:** a is opposite direction of u) $0 = 13^2 + 2(-4.5)(s)$ s = 18.8 m Total distance covered = 18.8 m + 9.1 m = 27.9 m Therefore, the motorist stops 2.9 m after the stop line.	
14(i)	$v^2 = u^2 + 2\,a\,s$ (taking downwards as positive), $= 0 + 2\,(9.81)\,(2.50)$ v = 7.00 m s^{-1}	 [1] [1]
(ii)	$s = u\,t + ½\,a\,t^2$ (taking downwards as positive), $0.12 = 7\,t + ½\,(9.81)\,t^2$ $4.91\,t^2 + 7t - 0.12 = 0$ $t = \dfrac{-7 \pm \sqrt{7^2 - 4(4.91)(-0.12)}}{2(4.91)}$ = 0.0169 s	 [1] [1]
(iii)	Yes, it confirms this time. The shutter is open for 0.0169 s, which is close to the specification of $\frac{1}{60}$ s or 0.0167 s.	[1] [1]
15(a)(i)	1.8 s	[1]
(ii)	The <u>gradient</u> of the <u>tangent</u> to the velocity-time graph.	[1]
(iii)	The ball experiences the gravitational force (ie its own weight) and drag force due to air resistance, both acting downwards. Hence, the downward acceleration of the ball is more than 9.81 m s^{-2} and in this case, approximately 20 m s^{-2}.	[1] [1]
(iv)	At t = 1.8 s. At this instant, the ball is at its maximum height. Its velocity is momentarily zero and hence, drag force is also zero. The only force on the ball is the gravitational force (ie its own weight) and hence its acceleration is g.	[1] [1]

(v)	 for correct shape of graph; for correct labelling of g on vertical axis; for correct labelling of -20 m s^{-1}, 1.8 s and 4.1 s.	 [1] [1] [1]
(b)	 Ball moving upwards Ball moving downwards When the ball is moving upwards, weight and air resistance act in the same direction, (ie. ma = mg + AR) it will cause a large deceleration (i.e. the speed of the upward-traveling ball quickly decreases). However, when the ball is moving downwards, the air resistance acts opposite to its weight, resulting in a downward acceleration of smaller magnitude than the magnitude of its upward deceleration. (ie. ma = mg – AR) With the distance covered during the upward motion equaling to the distance covered during the downward motion, the time taken to reach maximum height must be shorter than the time taken to return to the starting point.	[1] [1]
18(a)	Using $v_y^2 = u_y^2 + 2a_y s_y$ (taking upwards as positive), $0 = u_y^2 + 2(-9.81)(1.8)$ $u_y = 5.94$ m s^{-1}	[1] [1]
(b)	From $\tan\theta = \frac{u_y}{u_x} = \frac{5.943}{6.6}$ $\Rightarrow \theta = 42^{o}$	[1] [1]
(c)	The time of the flight for it to rise before the collision is the same as the time it takes to drop after the collision. However, the horizontal component of velocity after the collision has reduced. As such, even with the same time interval, the ball will travel a shorter horizontal distance after the rebound and land closer to the wall.	[1] [1]
19(a)(i)	$v_y^2 = u_y^2 + 2a_y s_y$ (taking downwards as positive), $= 0 + 2(9.81)(32)$ $v_y = 25.1$ m s^{-1}	[1] [1]

(ii)	$\sin\theta = \frac{v_y}{v} = \frac{25}{34} = 0.7352$ $\theta = 47.5^0$	[1] [1]
(b)	If the stone causes a splash on hitting the sea, then some KE & GPE of the stone is transferred to the KE and GPE of the splashing water, sound and thermal energy (to a lesser extent). With less KE (of the stone) remaining, the stone will be slowed down by the viscous force of water in a shorter distance. (Since work done against viscous force of water = viscous force x displacement).	[1] [1]
20	 Consider the motion from the ball from the "ground level" to the "highest point" →: $s_x = u_x t$ $250 = u\cos\theta t$ ---(1) ↑: $v_y = u_y + at$ $0 = u\sin\theta - 9.81t$ --- (2) Making t the subject, to eliminate t: $t = \frac{u\sin\theta}{9.81}$ --- (3) Sub (3) in (1): $250 = u\cos\theta(\frac{u\sin\theta}{9.81})$ $(9.81)(250) = u^2\sin\theta\cos\theta$ ---(4) Hmmmmm. We need to find θ, so we need to eliminate u. So we need to set up another equation! ↑: $v_y^2 = u_y^2 + 2as$ $0 = (u\sin\theta)^2 - 2(9.81)(210)$ $u = \frac{\sqrt{2(9.81)(210)}}{\sin\theta}$ --- (5) Sub (5) in (4): $(9.81)(250) = (\frac{\sqrt{2(9.81)(210)}}{\sin\theta})^2 \sin\theta\cos\theta$ ---(4) $\frac{250}{2(210)} = \frac{\cos\theta}{\sin\theta}$ $\tan\theta = \frac{2(210)}{250}$ $\theta = 59.2^o$ Note: This is a rather hard A level question.	