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A Multi-Position Calibration Algorithm for Inertial Measurement Units

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As an inertial sensors assembly, the inertial measurement unit (IMU) must be calibrated before being used. This paper proposes a new IMU multi-position calibration algorithm that takes the Earth's rotation rate and gravity as inputs, and calculates calibration parameters based on the two facts that: 1) norm of the accelerometer measurement vector is equal to the magnitude of gravity; 2) the dot product of gyro measurement vector and accelerometer measurement vector is equal to minus dot product of the Earth's rotation rate and gravity. Two theorems about the rank of the involved matrix are given to prove the feasibility of estimation. The algorithm features that no high-precise instruments such as a turntable are in principle necessary. Simulations show feasibility of the proposed calibration algorithm.

I. Introduction

An inertial measurement unit (IMU) is generally composed of three gyroscopes and three accelerometers. As an inertial sensors assembly, the IMU must be calibrated before being used. Calibration is the process of comparing the outputs of the IMU with known reference information and determining parameters that force the outputs of IMU to agree with the reference information[1]. Bias, scale factor and nonorthogonality are the dominant elements among the calibration parameters and the main results calibrated by most calibration methods. The performance of an IMU is mainly determined by the accuracies of these three kinds of parameters. Take a navigation-grade IMU for example, the requirements of stability and calibration accuracy are: 0.01 deg/h for the biases and 50 ppm for the scale factors of gyroscopes; 10 μg for the biases and 20 ppm for the scale factors of accelerometers[2, 3].

Many methods have been designed to calibrate IMUs. Among them the multi-position static and rate tests are the most commonly used lab calibration methods[1, 4-8]. These fully controlled calibration methods require a precise 2-axies or 3-axies turntable to provide attitude or angular rate reference. During calibration, the IMU is mounted on a turntable, and by controlling the turntable, the input specific force and angular rate are exactly known. Comparing the output signals of IMU with the known inputs, we can estimate the calibration parameters. Such kind of lab calibration methods can get exact results if a precise and costly turntable is available to use, but a factory-based sensor calibration is an expensive and time-consuming process and is typically done for research specific high grade IMUs[9]. Instead, we try to design a calibration method that does not depend on any precise instruments and can be performed easily in the field. Such calibration of course is possible. In fact, the Earth's rotation and gravity act on an IMU anywhere on the Earth, and they can be considered as reference inputs in calibration. Some calibration procedures were ever designed to relax the requirements of a precise control of the IMU's orientation[2, 9-11]. These methods can calibrate gyroscopes and accelerometers by using the fact that the norms of the outputs of the accelerometer cluster and gyroscope cluster are equal to the magnitudes of referenced gravity and Earth's rotation rate, respectively. However, as the gyroscopes and accelerometers are

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calibrated independently, the misalignment between the gyroscope triad and accelerometer triad was seldom addressed. Some other modern calibrations utilize Kalman filters (KF) to estimate parameters of IMUs[12-15], where the calibration problem is converted to that of state estimation. A maneuver strategy must be carefully designed in order to make all the calibration parameters completely observable. As a Kalman filter tends to disconverge with a bad initial guess of the states, the modern calibrations must be provided first with good initial gyro and accelerometer parameters.

This paper proposes a multi-position calibration method that takes the Earth's rotation rate and gravity as inputs. Parameters of an IMU can be estimated that expresses the outputs of gyroscopes and accelerometers in one body frame. After calibration, the gyroscope triad and accelerometer triad both align with the same body frame. The proposed algorithm utilizes two facts to estimate the parameters of accelerometers and gyroscopes sequentially. One fact is the norm of output vector of accelerometer triad is equal to magnitude of gravity and the other is the dot product between angular rate and specific force measured by IMU is equal to minus dot product between the Earth's rotation rate and gravity. The calibration can be performed without requirement of precise control of the IMU's posture, thus no high-precise instruments such as a turntable are in principle necessary.

This paper is organized as follows. Section II presents the input-output model of IMUs. The matrixes and vectors encoding scale factors, nonorthogonalities and biases are regarded as parameters to be calibrated. The multi-position calibration algorithm is derived in Section III. Two Theorems about the rank of the involved matrix are given to prove the feasibility of estimation. Section IV reports simulation results of a navigation-grade IMU. Calibrated errors were compared with that only caused by sensors' noise. Conclusions are drawn in Section V.

II. Input-Output Model of IMUs

Assume that the three axes of the body frame are denoted by x^b, y^b and z^b, the three sensitive axes of gyroscope triad by x^g, y^g, z^g and the three sensitive axes of accelerometer triad by x^a, y^a, z^a, respectively. Ignoring the nonlinear parts, we express gyroscope outputs in the matrix form as

$$\begin{bmatrix} N_x^g \\ N_y^g \\ N_z^g \end{bmatrix} = \begin{bmatrix} S_x^g & 0 & 0 \\ 0 & S_y^g & 0 \\ 0 & 0 & S_z^g \end{bmatrix} \begin{bmatrix} x^g \cdot x^b & x^g \cdot y^b & x^g \cdot z^b \\ y^g \cdot x^b & y^g \cdot y^b & y^g \cdot z^b \\ z^g \cdot x^b & z^g \cdot y^b & z^g \cdot z^b \end{bmatrix} \begin{bmatrix} \omega_x^b \\ \omega_y^b \\ \omega_z^b \end{bmatrix} + \begin{bmatrix} b_x^g \\ b_y^g \\ b_z^g \end{bmatrix} + \begin{bmatrix} n_x^g \\ n_y^g \\ n_z^g \end{bmatrix} \tag{1}$$

where the superscripts g and b respectively stand for gyroscope triad and the body frame. $\omega_{ib}^b = \begin{bmatrix} \omega_x^b & \omega_y^b & \omega_z^b \end{bmatrix}^T$ denotes the input angular rate vector expressed in the body frame, $N^g = \begin{bmatrix} N_x^g & N_y^g & N_z^g \end{bmatrix}^T$ is the output vector of gyroscopes, S^g represents the scale factor, b^g is the gyroscope bias, and n^g denotes noise. The elements of the 3×3 matrix $\begin{bmatrix} x^g \cdot x^b & x^g \cdot y^b & x^g \cdot z^b \\ y^g \cdot x^b & y^g \cdot y^b & y^g \cdot z^b \\ z^g \cdot x^b & z^g \cdot y^b & z^g \cdot z^b \end{bmatrix}$ are dot products of unit gyroscope axes and unit body frame axes. The matrix is nonorthogonal because of the nonorthogonality of gyroscope axes.

Analogically, accelerometer outputs can be expressed as

$$\begin{bmatrix} N_x^a \\ N_y^a \\ N_z^a \end{bmatrix} = \begin{bmatrix} S_x^a & 0 & 0 \\ 0 & S_y^a & 0 \\ 0 & 0 & S_z^a \end{bmatrix} \begin{bmatrix} x^a \cdot x^b & x^a \cdot y^b & x^a \cdot z^b \\ y^a \cdot x^b & y^a \cdot y^b & y^a \cdot z^b \\ z^a \cdot x^b & z^a \cdot y^b & z^a \cdot z^b \end{bmatrix} \begin{bmatrix} f_x^b \\ f_y^b \\ f_z^b \end{bmatrix} + \begin{bmatrix} b_x^a \\ b_y^a \\ b_z^a \end{bmatrix} + \begin{bmatrix} n_x^a \\ n_y^a \\ n_z^a \end{bmatrix} \tag{2}$$

where the superscripts a and b respectively stand for accelerometer triad and the body frame. $f^b = \begin{bmatrix} f_x^b & f_y^b & f_z^b \end{bmatrix}^T$ denotes the input specific force expressed in the body frame, $N^a = \begin{bmatrix} N_x^a & N_y^a & N_z^a \end{bmatrix}^T$ denotes the output vector of accelerometers, S^a and b^a are the scale factor and the bias, respectively, $\begin{bmatrix} x^a \cdot x^b & x^a \cdot y^b & x^a \cdot z^b \\ y^a \cdot x^b & y^a \cdot y^b & y^a \cdot z^b \\ z^a \cdot x^b & z^a \cdot y^b & z^a \cdot z^b \end{bmatrix}$ is the nonorthogonal matrix transforming from body frame axes to accelerometer axes, and n^a is noise.

Define the three orthogonal axes of the body frame[1, 2, 11]: x^b coincides with the accelerometer sensitive axis x^a, y^b lies in the $x^a y^a$ plane, and z^b is defined to constitute a right-handed orthogonal frame with x^b and y^b. Then the transforming matrix from body axes to accelerometer axes becomes

$$\begin{bmatrix} x^a \cdot x^b & x^a \cdot y^b & x^a \cdot z^b \\ y^a \cdot x^b & y^a \cdot y^b & y^a \cdot z^b \\ z^a \cdot x^b & z^a \cdot y^b & z^a \cdot z^b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ y^a \cdot x^b & y^a \cdot y^b & 0 \\ z^a \cdot x^b & z^a \cdot y^b & z^a \cdot z^b \end{bmatrix} \tag{3}$$

From the input-output model (1) and (2), the angular rate and specific force measured by an IMU are

$$\omega_{ib}^b = \begin{bmatrix} x^g \cdot x^b & x^g \cdot y^b & x^g \cdot z^b \\ y^g \cdot x^b & y^g \cdot y^b & y^g \cdot z^b \\ z^g \cdot x^b & z^g \cdot y^b & z^g \cdot z^b \end{bmatrix}^{-1} \begin{bmatrix} S_x^g & 0 & 0 \\ 0 & S_y^g & 0 \\ 0 & 0 & S_z^g \end{bmatrix}^{-1} \begin{bmatrix} N_x^g - b_x^g - n_x^g \\ N_y^g - b_y^g - n_y^g \\ N_z^g - b_z^g - n_z^g \end{bmatrix} \triangleq K^g N^g - \omega_0 - \delta_\omega \tag{4}$$

$$f^b = \begin{bmatrix} 1 & 0 & 0 \\ y^a \cdot x^b & y^a \cdot y^b & 0 \\ z^a \cdot x^b & z^a \cdot y^b & z^a \cdot z^b \end{bmatrix}^{-1} \begin{bmatrix} S_x^a & 0 & 0 \\ 0 & S_y^a & 0 \\ 0 & 0 & S_z^a \end{bmatrix}^{-1} \begin{bmatrix} N_x^a - b_x^a - n_x^a \\ N_y^a - b_y^a - n_y^a \\ N_z^a - b_z^a - n_z^a \end{bmatrix} \triangleq K^a N^a - f_0 - \delta_f \tag{5}$$

where K^g and K^a combine scale factors and nonorthogonality as

$$K^g = \begin{bmatrix} S_x^g \left(x^g \cdot x^b\right) & S_x^g \left(x^g \cdot y^b\right) & S_x^g \left(x^g \cdot z^b\right) \\ S_y^g \left(y^g \cdot x^b\right) & S_y^g \left(y^g \cdot y^b\right) & S_y^g \left(y^g \cdot z^b\right) \\ S_z^g \left(z^g \cdot x^b\right) & S_z^g \left(z^g \cdot y^b\right) & S_z^g \left(z^g \cdot z^b\right) \end{bmatrix}^{-1} \tag{6}$$

$$K^a = \begin{bmatrix} S_x^a & 0 & 0 \\ S_y^a\left(y^a \cdot x^b\right) & S_y^a\left(y^a \cdot y^b\right) & 0 \\ S_z^a\left(z^a \cdot x^b\right) & S_z^a\left(z^a \cdot y^b\right) & S_z^a\left(z^a \cdot z^b\right) \end{bmatrix}^{-1} \tag{7}$$

$\omega_0 = \begin{bmatrix} \omega_{x0} & \omega_{y0} & \omega_{z0} \end{bmatrix}^T$ is a vector related to gyroscope biases, $f_0 = \begin{bmatrix} f_{x0} & f_{y0} & f_{z0} \end{bmatrix}^T$ is related to accelerometer biases, i.e.,

$$\omega_0 = K^g \begin{bmatrix} b_x^g \\ b_y^g \\ b_z^g \end{bmatrix}, f_0 = K^a \begin{bmatrix} b_x^a \\ b_y^a \\ b_z^a \end{bmatrix} \tag{8}$$

and δ_ω and δ_f are noise parts.

For typical IMUs, gyroscopes axes (or accelerometers axes) are almost orthogonal to each other, and gyroscopes axes are almost parallel to accelerometers axes, so K^g and K^a can be written approximately by

$$K^g \approx \begin{bmatrix} S_x^g & S_x^g\theta_{xy}^g & S_x^g\theta_{xz}^g \\ S_y^g\theta_{yx}^g & S_y^g & S_y^g\theta_{yz}^g \\ S_z^g\theta_{zx}^g & S_z^g\theta_{zy}^g & S_z^g \end{bmatrix}^{-1} \tag{9}$$

$$K^a \approx \begin{bmatrix} S_x^a & 0 & 0 \\ S_y^a\theta_{yx}^a & S_y^a & 0 \\ S_z^a\theta_{zx}^a & S_z^a\theta_{zy}^a & S_z^a \end{bmatrix}^{-1} \tag{10}$$

where θ_{ij}^g denotes the misalignment angle between the i-th body frame coordinate axis and the j-th gyroscope sensitive axis and θ_{ij}^a is the misalignment angle between the i-th body frame coordinate axis and the j-th accelerometer sensitive axis. So the diagonal elements of K^g and K^a can be approximately regarded as scale factors and the other elements as nonorthogonalities.

In this paper, we regard K^g, K^a, ω_0 and f_0 as the parameters to be calibrated, where K^g and K^a are 3×3 matrices defined by (6) and (7) encoding scale factors and nonorthogonalities (the superscripts *g* and *a* respectively stand for gyroscope and accelerometer) and ω_0 and f_0 defined by (8) refer to gyroscope biases and accelerometer biases, respectively. Using these calibration parameters, we can transform outputs of gyroscopes and accelerometers to the angular rate and specific force in the body frame by (4) and (5) respectively.

III. Calibration Algorithm

In this section, we develop formulations to compute the above calibration parameters based on the two facts for a static IMU that: 1) norm of the accelerometer measurement vector is equal to the magnitude of gravity; 2) the dot product of gyro measurement vector and accelerometer measurement vector is equal to minus dot product of the Earth's rotation rate and gravity. By the first fact, we are able to estimate K^a and f_0 in (7) and (8), and then the specific force is computed from (5). Finally, we can estimate K^g and ω_0 with the help of the second fact.

Take the local-level frame N as the navigation reference frame, with x directing north, y east, and z downward vertical. For a static IMU, we have $\omega_{eb}^n = 0$ and ω_{ib}^b and f^b satisfies

$$C_b^n \omega_{ib}^b = \omega_{ie}^n, C_b^n f^b = -g^n \tag{11}$$

where C_b^n denotes the body attitude matrix from the body frame to the navigation frame, $\omega_{ie}^n = \begin{bmatrix} \omega_{ie} \cos L & 0 & -\omega_{ie} \sin L \end{bmatrix}^T$ is the Earth's rotation rate in the navigation frame, L is the latitude and $g^n = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T$ is the gravity vector in the navigation frame.

Because $C_b^{nT} C_b^n = I_3$, from (11), we have

$$\begin{aligned} \left|\omega_{ie}^n\right|^2 &= \left(\omega_{ie}^n\right)^T \omega_{ie}^n = \left(C_b^n \omega_{ib}^b\right)^T \left(C_b^n \omega_{ib}^b\right) = \left(\omega_{ib}^b\right)^T \omega_{ib}^b = \left|\omega_{ib}^b\right|^2 \\ \left|g^n\right|^2 &= \left(g^n\right)^T g^n = \left(C_b^n f^b\right)^T \left(C_b^n f^b\right) = \left(f^b\right)^T f^b = \left|f^b\right|^2 \end{aligned} \tag{12}$$

This fact, i.e., the norm of the measured output of IMU being equal to the magnitude of input, has been used in Ref. 2 and Ref. 9-11 for calibration. Here only the second equation of (12) is used to estimate the accelerometer parameters, because as shown in Theorem 1 below only 9 elements of K^a or K^g can be calculated, not all the elements of K^g can be estimated. Note that we have another fact

$$\omega_{ib}^b \cdot f^b = -\left(C_n^b \omega_{ie}^n\right)^T \left(C_n^b g^n\right) = -\omega_{ie}^n \cdot g^n = \omega_{ie} g \sin L \tag{13}$$

which gives a relationship between measurement vectors of gyroscopes and accelerometers, and can be utilized to estimate gyroscopes parameters using the obtained accelerometer parameters.

A. Calibration of Accelerometers

Substituting (5) into the second equation of (12), we have

$$\begin{aligned} \left|g^n\right|^2 &= \left(K^a N^a - f_0 - \delta_f\right)^T \left(K^a N^a - f_0 - \delta_f\right) \\ &= \left(\begin{bmatrix} K^a & f_0 \end{bmatrix} \begin{bmatrix} N^a \\ -1 \end{bmatrix}\right)^T \left(\begin{bmatrix} K^a & f_0 \end{bmatrix} \begin{bmatrix} N^a \\ -1 \end{bmatrix}\right) + \Delta \\ &\triangleq \begin{bmatrix} \left(N^a\right)^T & -1 \end{bmatrix} \cdot K^A \cdot \begin{bmatrix} N^a \\ -1 \end{bmatrix} + \Delta \end{aligned} \tag{14}$$

where Δ is the noise term caused by δ_f and K^A is a symmetric matrix defined as

$$K^A = \begin{bmatrix} K^a & f_0 \end{bmatrix}^T \begin{bmatrix} K^a & f_0 \end{bmatrix} = \begin{bmatrix} \left(K^a\right)^T K^a & \left(K^a\right)^T f_0 \\ \left(f_0\right)^T K^a & \left(f_0\right)^T f_0 \end{bmatrix} \triangleq \begin{bmatrix} k_{11}^A & k_{12}^A & k_{13}^A & k_{14}^A \\ k_{12}^A & k_{22}^A & k_{23}^A & k_{24}^A \\ k_{13}^A & k_{23}^A & k_{33}^A & k_{34}^A \\ k_{14}^A & k_{24}^A & k_{34}^A & k_{44}^A \end{bmatrix} \tag{15}$$

If we know signs of the diagonal elements of K^a, we can get a unique solution to K^a and f_0 from the known K^A. The calculating process is provided in appendix A. The signs are readily available, for example, by preliminary tests at a couple of postures.

Substituting (15) into (14) and reorganizing the terms yield

$$\begin{aligned} \left|g^n\right|^2 &= \begin{bmatrix} \left(N_x^a\right)^2 & \left(N_y^a\right)^2 & \left(N_z^a\right)^2 & 2N_x^a N_y^a & 2N_x^a N_z^a & 2N_y^a N_z^a & -2N_x^a & -2N_y^a & -2N_z^a & 1 \end{bmatrix} \\ &\cdot \begin{bmatrix} k_{11}^A & k_{22}^A & k_{33}^A & k_{12}^A & k_{13}^A & k_{23}^A & k_{14}^A & k_{24}^A & k_{34}^A & k_{44}^A \end{bmatrix}^T + \Delta \\ &\triangleq N_v \cdot K_v + \Delta \end{aligned} \tag{16}$$

where N_v is defined as a vector constructed by elements about N^a and K_v is a vector constructed by 10 different elements of K^A. A least square algorithm can be used for (16) to estimate all elements of K^A, from which we can calculate K^a and f_0. However, it is not trivial to perform this process because of singularity in estimation.

Put an IMU in n (≥ 10) static but different postures, we can obtain n equations as

$$\left|g^n\right|^2 = N_{vi} \cdot K_v + \Delta_i, i = 1, 2, \cdots, n \tag{17}$$

In the matrix from, it gives

$$W = \begin{bmatrix} \left|g^n\right|^2 \\ \left|g^n\right|^2 \\ \vdots \\ \left|g^n\right|^2 \end{bmatrix} = \begin{bmatrix} N_{v1} \\ N_{v2} \\ \vdots \\ N_{vn} \end{bmatrix} \cdot K_v + \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \end{bmatrix} \triangleq N_m \cdot K_v + \nabla \tag{18}$$

If $N_m = \begin{bmatrix} N_{v1} & N_{v2} & \cdots & N_{vn} \end{bmatrix}^T$ is column nonsingular, K_v can be estimated by

$$K_v = \left(N_m{}^T N_m\right)^{-1} N_m{}^T \cdot W \tag{19}$$

Unfortunately, N_m is always column singular.

Theorem 1: Ignoring the noise, N_m is always column singular.

The proof is provided in appendix B.

However, noticing that K^a and f_0 justly have 9 independent elements, while K_v has 10 different elements, if $rank\left(N_m\right) = 9$, we can still uniquely determine K^a and f_0.

Theorem 2: If $k_{44}^A = (f_0)^T f_0 \neq |g^n|^2$, IMU outputs at enough positions can make $rank(N_m) = 9$.

The proof is provided in appendix C.

In the following, we briefly describe the calculate process of K^A, K^a and f_0.

Reorganizing (16) yields

$$|g^n|^2 - k_{44}^A = \begin{bmatrix} (N_x^a)^2 & (N_y^a)^2 & (N_z^a)^2 & 2N_x^a N_y^a & 2N_x^a N_z^a & 2N_y^a N_z^a & -2N_x^a & -2N_y^a & -2N_z^a \end{bmatrix} \cdot \begin{bmatrix} k_{11}^A & k_{22}^A & k_{33}^A & k_{12}^A & k_{13}^A & k_{23}^A & k_{14}^A & k_{24}^A & k_{34}^A \end{bmatrix}^T + \Delta \triangleq N_v' \cdot K_v' + \Delta \tag{20}$$

where N_v' consists of the first nine elements of N_v and K_v' has the first nine elements of K_v.

Denote

$$r = \frac{|g^n|^2}{|g^n|^2 - k_{44}^A} \tag{21}$$

We have

$$|g^n|^2 = N_v' \cdot K_v' \cdot r + \Delta \cdot r \tag{22}$$

By the least square algorithm, we can estimate $K_v' \cdot r$. Substituting elements of $K_v' \cdot r$ into (15), we can compute $\sqrt{|r|}K^a$ and $\sqrt{|r|}f_0$ (the calculate process is similar to the process described in appendix A and is omitted here for brevity). Then substituting $\sqrt{|r|}K^a$ and $\sqrt{|r|}f_0$ into (14), we can get K^a, f_0 and r, which finishes the calibration of accelerometers.

B. Calibration of Gyroscopes

As shown in Theorem 1 only 9 different elements of K^g can be calculated, we cannot calibrate gyroscopes based on (12) because there are 12 unknown elements in K^g. Instead, we utilize the relationship between measurement vectors of gyroscopes and accelerometers (13) to calibrate gyroscopes.

At a point with latitude L, the relationship of ω_{ib}^b and f^b always holds for a static IMU

$$(\omega_{ib}^b)^T f^b = -(C_n^b \omega_{ie}^n)^T (C_n^b g^n) = -(\omega_{ie}^n)^T g^n = \omega_{ie} g \sin L \tag{23}$$

which will help us calibrate gyroscopes.

Put the IMU in a static state, we can compute f^b from (5) using the calibrated accelerometer parameters K^a and f_0. Substituting (4) into (23), we have

$$\omega_{ie} g \sin L = \left(\omega_{ib}^b\right)^T f^b = \left(K^g N^g - \omega_0 - \delta_\omega\right)^T f^b = \left(\begin{bmatrix} K^g & \omega_0 \end{bmatrix} \begin{bmatrix} N^g \\ -1 \end{bmatrix}\right)^T f^b + \Delta'$$

$$= \begin{bmatrix} f_x^b N_x^g & f_x^b N_y^g & f_x^b N_z^g & f_y^b N_x^g & f_y^b N_y^g & f_y^b N_z^g & f_z^b N_x^g & f_z^b N_y^g & f_z^b N_z^g & -f_x^b & -f_y^b & -f_z^b \end{bmatrix}$$

$$\cdot \begin{bmatrix} k_{11}^g & k_{12}^g & k_{13}^g & k_{21}^g & k_{22}^g & k_{23}^g & k_{31}^g & k_{32}^g & k_{33}^g & \omega_{x0} & \omega_{y0} & \omega_{z0} \end{bmatrix}^T + \Delta' \tag{24}$$

where Δ' is the additional noise term caused by δ_ω and $k_{11}^g \sim k_{33}^g$ denote elements of matrix K^g. For an IMU capable of sensing the Earth's rotation rate, we can place it at more than twelve poses and estimate K^g and ω_0 using the least square algorithm.

In summary, the proposed algorithm requires the IMU to be put in n (≥ 12) different static postures and calibrates accelerometers and gyroscopes sequentially. No precise instruments such as a turntable are required for the proposed calibration algorithm mainly owed to no need of aligning the IMU to the local level frame.

IV. Simulations

Assume an IMU with fixed scale factors, nonrothogonalities and biases. The grade of IMU is classified by noises (1σ): 0.01 $°/h$ for gyroscopes and 50 micro-g for accelerometers. The measurements of the IMU are generated at 100HZ. The calibration is performed at a place of latitude 28 deg, longitude 112 deg and altitude 60 m. The IMU is sampled for 1 minute at each of 18 static postures (see Tab. 1) and the one-minute-duration outputs of gyroscopes and accelerometers at each position are averaged to depressing noises and then fed to the calibration algorithm.

Table 2 lists the calibration results of scale factors, nonorthogonalities and biases. As the misalignment angles set are small, we can approximately evaluate the calibrating accuracy of scale factors, nonrothogonalities and biases from errors of the diagonal elements of K^a, K^g, of non-diagonal elements of K^a, K^g and of f_0, ω_0, respectively. From Tab. 2 we know that for accelerometers, the biggest error of diagonal elements of K^a is 6.3×10^{-11}, implying 63 ppm for scale factor; the biggest error of the non-diagonal elements of K^a is 1.8×10^{-10}, which means misalignment angle error is about $\sin^{-1}(1.8\text{e-}4) = 1.8\text{e-}4\,\text{rad} \approx 38\,\text{arc-sec}$; and the biggest bias error is 1.84×10^{-4} m/s^2, i.e., 18.4 micro-g. Similarly, for gyroscopes, the biggest errors of scale factor, misalignment angle and bias are about 1348 ppm, 4.5 arc-min and 0.012 deg/h, respectively. The biggest calibrated errors are shown in Tab. 3.

To evaluate the proposed algorithm further, the calibrating simulation is performed 1000 times. Figures 1-6, show the biggest scale factor error, misalignment angle error, bias error of accelerometer triad and gyroscope triad for the 1000 runs respectively. The scale factor errors are both expressed in ppm, the misalignment angle errors are respectively in arc-sec and arc-min, and the bias errors are respectively in micro-g and deg/h. Table 4 gives the average errors of the 1000 runs. We will compare the errors with a perfect multi-position calibration method whose calibrated errors are only caused by sensors' noise. Generally speaking, using a perfect multi-position calibration method, for a 50 micro-g (1σ) accelerometer, the scale factor can be calibrated with accuracy 50μg/1g=5×10^{-5}=50 ppm, the misalignment angle with accuracy 5×10^{-5} rad=10.3 arc-sec and the bias with accuracy 50 micro-g. As the magnitude of the Earth's rotation rate is small, the calibration accuracy for gyroscopes is rather low. For a gyroscope with noise 0.01deg/h (1σ), as the Earth's rotation rate is about 15 deg/h, we can only achieve the scale factor with accuracy 0.01/15=667 ppm, the misalignment angle with accuracy 0.01/15rad=2.3 arc-min and the bias with accuracy 0.01 deg/h. As shown in Tab. 4: for accelerometers, the scale factor error and bias error are respectively 33.8 ppm and 27.2 micro-g, which are both smaller than errors calibrated by the perfect method. The misalignment angle error is 21.4 arc-sec, about twice of 10.3 arc-sec; for gyroscopes, the calibrated errors are much larger than the errors caused by gyroscopes' noises. The scale factor error is 5 times larger, the misalignment angle error and bias error are about 4 times and twice larger respectively. In summary, the proposed algorithm can achieve accelerometer parameters with accuracy comparable to conventional lab methods which are also multi-position methods when calibrating accelerometers, while much lower accuracy for the gyroscope triad because the proposed method uses a much smaller rotation rate as inputs to the gyroscopes.

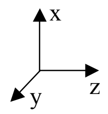
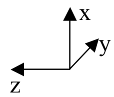
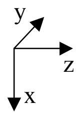
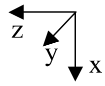
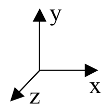
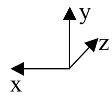
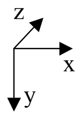
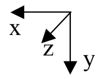
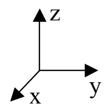
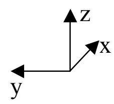
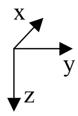
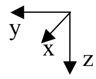
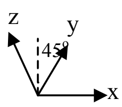
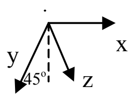
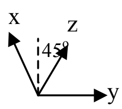
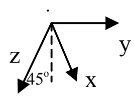
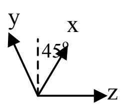
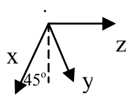
V. Conclusions

Calibration is a necessary step for any IMU since the accuracy of calibration parameters largely determines the navigating performance. This paper comes up with a calibration algorithm that relaxes the requirements of a precise control of the IMU's orientation. Based on the facts that norm of the accelerometer measurement vector is equal to the magnitude of gravity and the dot product of gyro measurement vector and accelerometer measurement vector is equal to minus dot product of the Earth's rotation rate and gravity, the proposed algorithm is able to calibrate a static IMU without the need of an accurate turntable.

When calibrating accelerometers, we first estimate elements of a symmetric matrix that encodes the scale factors, nonorthogonalities and biases, and then calculate the desired coefficients. Due to singularity, the estimate process is not as easy as it appears. Two theorems about the matrix rank are given to prove the feasibility of estimation. We do not utilize the norm equation of the Earth's rotation rate and gyroscopes' measurement vector to calibrate gyroscopes as we do for accelerometers, because from the equation not all parameters related to misalignment between the gyroscope triad and accelerometer triad can be determined. Instead, we use a dot product equation that makes calibration of gyroscopes feasible.

Simulation results show that the algorithm can calibrate accelerometer parameters with accuracy comparable to conventional methods. But for gyroscopes' parameters, the accuracy is unfortunately still unacceptable. The underlying reason is that the input Earth rate is as low as 15deg/h. Further study is under way to improve the accuracy of gyroscope calibration along the road of non accurate turntable. A low-precise rotating scheme will be considered to enlarge the input magnitude of rotation rate. A Kalman filter will be designed to refine the calibration parameters and the calibration results in this paper will be used as initial states.

Tab. 1: 18 IMU Positions

Posture #	1	2	3	4
Description	x-upward, y-south, z-east	x-upward, y-north, z-west	x-downward, y-north, z-east	x-downward, y-south, z-west
Illustration				
Posture #	5	6	7	8
Description	y-upward, x-east, z-south	y-upward, x-west, z-north	y-downward, x-east, z-north	y-downward, x-west, z-south
Illustration				
Posture #	9	10	11	12
Description	z-upward, x-south, y-east	z-upward, x-north, y-west	z-downward, x-north, y-east	z-downward, x- south, y- west
Illustration				
Posture #	13	14	15	16
Description	x-east, y-north-upward with 45deg pitch, z-south-upward with a 45deg pitch	x-east, y-south-downward with 45deg pitch, z-north-downward with a 45deg pitch	y-east, z-north-upward with 45deg pitch, x-south-upward with a 45deg pitch	y-east, z-south-downward with 45deg pitch, x-north-downward with a 45deg pitch
Illustration				
Posture #	17	18		
Description	z-east, x-north-upward with 45deg pitch, y-south-upward with a 45deg pitch	z-east, x-south-downward with 45deg pitch, y-north-downward with a 45deg pitch		
Illustration				

Tab. 2: Simulation Results

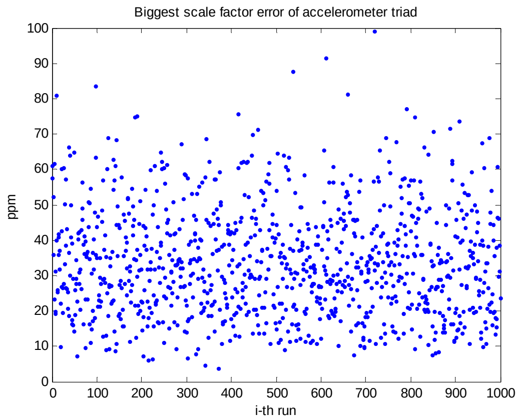
	True parameters	Parameters calibrated	Errors
K^a	$\begin{bmatrix} 1e\text{-}6 & 0 & 0 \\ 2e\text{-}9 & 1e\text{-}6 & 0 \\ 8e\text{-}10 & 1e\text{-}9 & 1e\text{-}6 \end{bmatrix}$	$\begin{bmatrix} 0.999975e\text{-}6 & 0 & 0 \\ 1.9476e\text{-}9 & 0.999937e\text{-}6 & 0 \\ 8.1580e\text{-}10 & 1.1851e\text{-}9 & 0.999995e\text{-}6 \end{bmatrix}$	$\begin{bmatrix} -2.5e\text{-}11 & 0 & 0 \\ -5.2e\text{-}11 & -6.3e\text{-}11 & 0 \\ 1.6e\text{-}11 & 1.8e\text{-}10 & -5.0e\text{-}12 \end{bmatrix}$
f_0	$[0.006 \quad -0.02 \quad -0.01]^T$	$[5.8206e\text{-}3 \quad -2.0032e\text{-}2 \quad -1.0008e\text{-}2]^T$	$[-1.8e\text{-}4 \quad -3.2e\text{-}5 \quad -8.0e\text{-}6]$
K^g	$\begin{bmatrix} 1e\text{-}8 & 7e\text{-}12 & 9e\text{-}12 \\ -2e\text{-}11 & 1e\text{-}8 & 3e\text{-}11 \\ 6e\text{-}12 & -1e\text{-}11 & 1e\text{-}8 \end{bmatrix}$	$\begin{bmatrix} 0.999863e\text{-}8 & 4.8981e\text{-}12 & -4.4157e\text{-}12 \\ -1.9450e\text{-}11 & 0.998841e\text{-}8 & 2.6014e\text{-}11 \\ 4.3303e\text{-}12 & -1.3957e\text{-}11 & 1.001348e\text{-}8 \end{bmatrix}$	$\begin{bmatrix} -1.4e\text{-}12 & -2.1e\text{-}12 & -1.3e\text{-}11 \\ 5.5e\text{-}13 & -1.2e\text{-}11 & -4.0e\text{-}12 \\ -1.7e\text{-}12 & -4.0e\text{-}12 & 1.3e\text{-}11 \end{bmatrix}$
ω_0	$[5e\text{-}5 \quad -2e\text{-}5 \quad 4e\text{-}5]^T$	$[4.9944e\text{-}5 \quad -1.9990e\text{-}5 \quad 4.0053e\text{-}5]^T$	$[-5.6e\text{-}8 \quad 1.0e\text{-}8 \quad 5.3e\text{-}8]$

Tab. 3: Biggest Calibrated Errors

	Scale factor error	Misalignment angle error	Bias error
Accelerometer triad	63 ppm	38 arc-sec	18.4 micro-g
Gyroscope triad	1348 ppm	4.5 arc-min	0.012 deg/h

Tab. 4: Average Errors of 1000 Runs

	Scale factor error	Misalignment angle error	Bias error
Accelerometer triad	33.8 ppm	21.4 arc-sec	27.2 micro-g
Gyroscope triad	3377 ppm	9.2 arc-min	0.023 deg/h



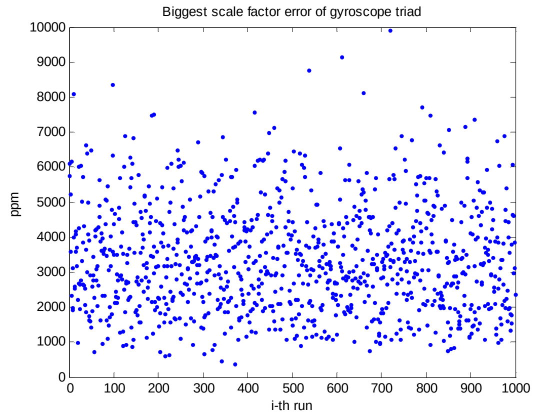
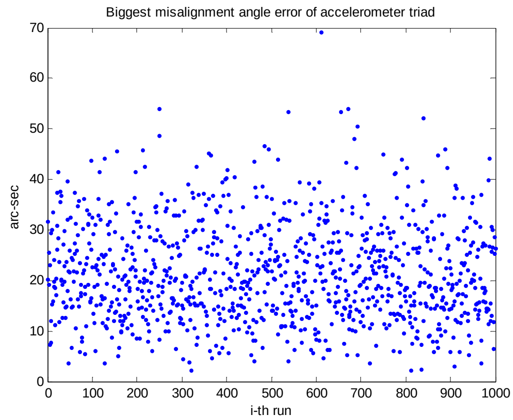


Fig. 1: Biggest Scale Factor Error of Accelerometer Triad

Fig. 2: Biggest Scale Factor Error of Gyroscope Triad



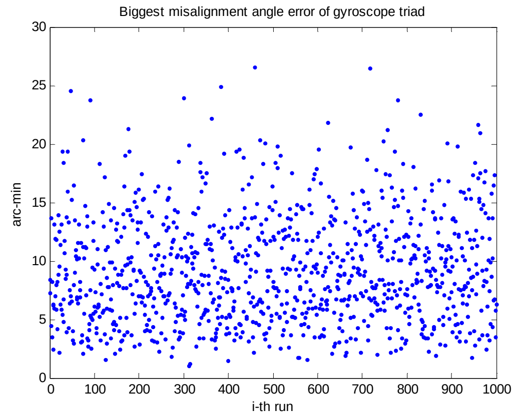
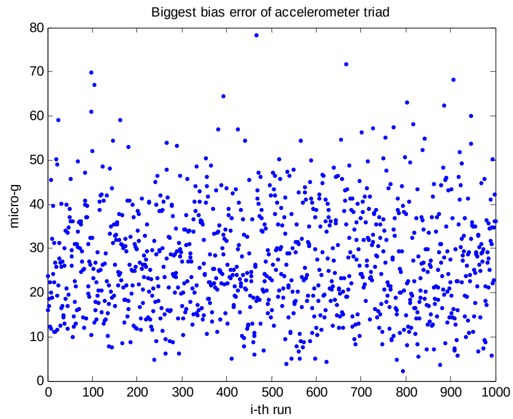


Fig. 3: Biggest Misalignment Angle Error of Accelerometer Triad

Fig. 4: Biggest Misalignment Angle Error of Gyroscope Triad



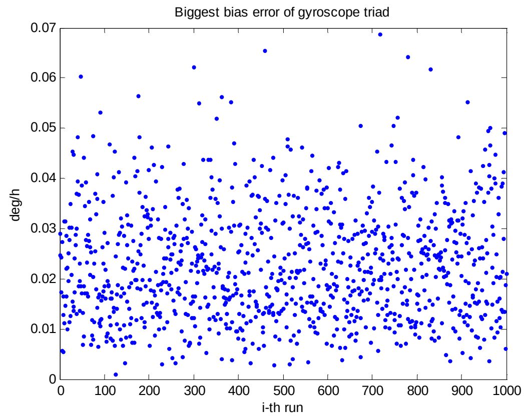


Fig. 5: Biggest Bias Error of Accelerometer Triad

Fig. 6: Biggest Bias Error of Gyroscope Triad

Appendix

A. Calculating K^a and f_0 from K^A

Denoting $K^a = \begin{bmatrix} k_{11}^a & 0 & 0 \\ k_{21}^a & k_{22}^a & 0 \\ k_{31}^a & k_{32}^a & k_{33}^a \end{bmatrix}$, (15) can be written as

$$K^A = \begin{bmatrix} k_{11}^A & k_{12}^A & k_{13}^A & k_{14}^A \\ k_{12}^A & k_{22}^A & k_{23}^A & k_{24}^A \\ k_{13}^A & k_{23}^A & k_{33}^A & k_{34}^A \\ k_{14}^A & k_{24}^A & k_{34}^A & k_{44}^A \end{bmatrix} = \begin{bmatrix} k_{11}^a & 0 & 0 & f_{x0} \\ k_{21}^a & k_{22}^a & 0 & f_{y0} \\ k_{31}^a & k_{32}^a & k_{33}^a & f_{z0} \end{bmatrix}^T \begin{bmatrix} k_{11}^a & 0 & 0 & f_{x0} \\ k_{21}^a & k_{22}^a & 0 & f_{y0} \\ k_{31}^a & k_{32}^a & k_{33}^a & f_{z0} \end{bmatrix}$$

$$= \begin{bmatrix} k_{11}^{a\,2} + k_{21}^{a\,2} + k_{31}^{a\,2} & k_{21}^a k_{22}^a + k_{31}^a k_{32}^a & k_{31}^a k_{33}^a & k_{11}^a f_{x0} + k_{21}^a f_{y0} + k_{31}^a f_{z0} \\ k_{21}^a k_{22}^a + k_{31}^a k_{32}^a & k_{22}^{a\,2} + k_{32}^{a\,2} & k_{32}^a k_{33}^a & k_{22}^a f_{y0} + k_{32}^a f_{z0} \\ k_{31}^a k_{33}^a & k_{32}^a k_{33}^a & k_{33}^{a\,2} & k_{33}^a f_{z0} \\ k_{11}^a f_{x0} + k_{21}^a f_{y0} + k_{31}^a f_{z0} & k_{22}^a f_{y0} + k_{32}^a f_{z0} & k_{33}^a f_{z0} & f_{x0}^{\,2} + f_{y0}^{\,2} + f_{z0}^{\,2} \end{bmatrix} \tag{A-1}$$

If we know signs of k_{11}^a, k_{22}^a and k_{33}^a, and assume $k_{11}^a = \lambda_1 \left|k_{11}^a\right|$, $k_{22}^a = \lambda_2 \left|k_{22}^a\right|$ and $k_{33}^a = \lambda_3 \left|k_{33}^a\right|$, where $\lambda_1, \lambda_2, \lambda_3$ =+1 or –1, then from (A-1) we can obtain unique K^a and f_0 from K^A

$$\begin{aligned} & k_{33}^a = \lambda_3 \sqrt{k_{33}^A}, k_{32}^a = k_{23}^A / k_{33}^a, k_{31}^a = k_{13}^A / k_{33}^a \\ & k_{22}^a = \lambda_2 \sqrt{k_{22}^A - k_{32}^{a\,2}}, k_{21}^a = \left(k_{12}^A - k_{31}^a k_{32}^a\right) / k_{22}^a \\ & k_{11}^a = \lambda_1 \sqrt{k_{11}^A - k_{21}^{a\,2} - k_{31}^{a\,2}} \\ & f_{z0} = k_{34}^A / k_{33}^a,\ f_{y0} = \left(k_{24}^A - k_{32}^a f_{z0}\right) / k_{22}^a \\ & f_{x0} = \left(k_{14}^A - k_{21}^a f_{y0} - k_{31}^a f_{z0}\right) / k_{11}^a \end{aligned} \tag{A-2}$$

B. Proof of Theorem 1

From (16), we have

$$\begin{aligned} f\left(N_v\right) = & k_{11}^A \left(N_x^a\right)^2 + k_{22}^A \left(N_y^a\right)^2 + k_{33}^A \left(N_z^a\right)^2 + 2k_{12}^A N_x^a N_y^a + 2k_{13}^A N_x^a N_z^a \\ & + 2k_{23}^A N_y^a N_z^a - 2k_{14}^A N_x^a - 2k_{24}^A N_y^a - 2k_{34}^A N_z^a + \left(k_{44}^A - \left|g^n\right|^2\right) \cdot 1 = 0 \end{aligned} \tag{A-3}$$

Because $k_{11}^A \sim k_{44}^A$ are 10 constants, and not all of them are 0, the elements of $N_v = \left[\left(N_x^a\right)^2 \quad \left(N_y^a\right)^2 \quad \left(N_z^a\right)^2 \quad 2N_x^a N_y^a \quad 2N_x^a N_z^a \quad 2N_y^a N_z^a \quad -2N_x^a \quad -2N_y^a \quad -2N_z^a \quad 1\right]$ are linearly dependent. Equation (A-3) holds for each row of $N_m = \left[N_{v1} \quad N_{v2} \quad \cdots \quad N_{vn}\right]^T$, thus the columns of N_m are linearly dependent, that is to say, the matrix N_m is column singular and $rank\left(N_m\right) < 10$.

The proof ends here.

C. Proof of Theorem 2

If $rank\left(N_m\right) < 9$, there exist some constants $d_1 \sim d_9$, not all zero, such that

$$\begin{aligned} g\left(N_v\right) = d_1\left(N_x^a\right)^2 + d_2\left(N_y^a\right)^2 + d_3\left(N_z^a\right)^2 + 2d_4N_x^aN_y^a + 2d_5N_x^aN_z^a \\ + 2d_6N_y^aN_z^a - 2d_7N_x^a - 2d_8N_y^a - 2d_9N_z^a = 0 \end{aligned} \quad \text{(A-4)}$$

Written in the matrix norm, it gives

$$g\left(N_v\right) = \begin{bmatrix} N_x^a & N_y^a & N_z^a \end{bmatrix} \begin{bmatrix} d_1 & d_4 & d_5 \\ d_4 & d_2 & d_6 \\ d_5 & d_6 & d_3 \end{bmatrix} \begin{bmatrix} N_x^a \\ N_y^a \\ N_z^a \end{bmatrix} - \begin{bmatrix} 2d_7 & 2d_8 & 2d_9 \end{bmatrix} \begin{bmatrix} N_x^a \\ N_y^a \\ N_z^a \end{bmatrix} = 0 \quad \text{(A-5)}$$

The output vector of accelerometers N^a satisfies

$$N^a = \left(K^a\right)^{-1}\left(f^b + f_0 + \delta_f\right) \quad \text{(A-6)}$$

Substituting (A-6) into (A-5), and ignoring noise, we have

$$\left(f^b + f_0\right)^T \left(\left(K^a\right)^{-1}\right)^T \begin{bmatrix} d_1 & d_4 & d_5 \\ d_4 & d_2 & d_6 \\ d_5 & d_6 & d_3 \end{bmatrix} \left(K^a\right)^{-1}\left(f^b + f_0\right) - \begin{bmatrix} 2d_7 & 2d_8 & 2d_9 \end{bmatrix} \left(K^a\right)^{-1}\left(f^b + f_0\right) = 0 \quad \text{(A-7)}$$

Equation (A-7) means that the vector vertex of f^b lies on a conicoid which pass the vertex of vector $-f_0$. On the other hand, from (12) we know that the vector vertex of f^b lies on a spherical surface with radius $\left|g^n\right|$. If $k_{44}^A = f_0^T f_0 \neq \left|g^n\right|^2$, the spherical surface doesn't pass the vertex of vector $-f_0$. This is contrary. So $rank\left(N_m\right) \geq 9$. With Theorem 1, we know $rank\left(N_m\right) = 9$.

The proof ends here.

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