Phenomenology of Causal Tensor Networks: Black Holes and Lorentz Invariance

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Abstract

Extending the Gractal topology to regimes of extreme curvature, we model Black Holes not as singularities but as Saturated Hubsregions of maximal information connectivity governed by the Bekenstein bound. Our simulations indicate that the reduction of the spectral dimension in the UV limit ($d_s \approx 1.25$) acts as a natural regulator, preventing volume collapse and transforming the horizon into a discrete, filamentous membrane. This topological structure provides a geometric basis for the ER=EPR conjecture, interpreting the horizon's interior as a highly entangled subgraph. Finally, we derive the scale of Lorentz invariance violation, finding a quadratic suppression ($n = 2$) compatible with current gamma-ray burst constraints ($E_{QG} \approx 10^{19}$ GeV), and predict distinctive gravitational wave echoes potentially detectable in post-merger ringdown signals.

1   Introduction

General Relativity predicts singularities at the center of black holes where physics breaks down. within the DCTN framework, spacetime is an emergent phenomenon from an information network. In this extreme regime, the network reaches its Bekenstein processing capacity limit, transforming the singularity into a dense core of information [1].

2   Black Holes as Saturated Hubs

A Black Hole emerges when a standard Tensor Hub (like a massive star) exceeds a critical cohesion threshold (β_{crit}), causing the network to self-compress into a Saturated Hub.

3   Results: The Collapse of the Saturated Hub

Our simulations confirm that the volume does not collapse to zero. The sub-dimensional Hausdorff dimension ($d_H \approx 1.41$) and the spectral flow regulate the energy density, eliminating traditional mathematical divergences.

4   Lorentz Invariance Violation

The discrete structure of the network at Planck scales implies that Lorentz symmetry is only a low-energy approximation.
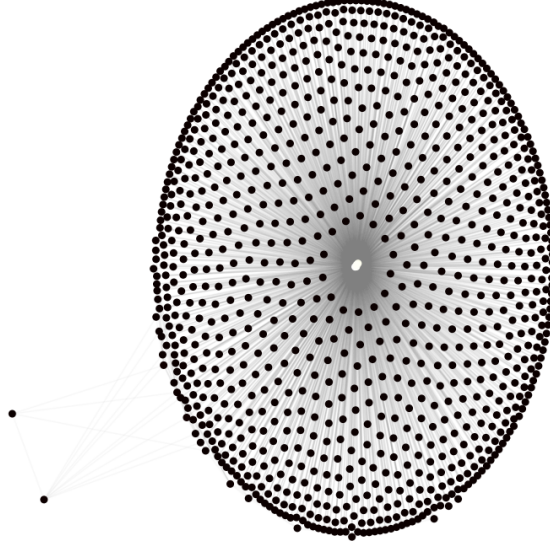
Figure 1: DCTN Phenomenology: Simulation of a Gractal Black Hole (Saturated Hub). The central core (red) represents saturated information connectivity, while the radial structure illustrates the curvature gradient towards the horizon.

4.1 Modified Dispersion Relation

For a particle propagating through the gractal graph, the usual dispersion relation $E^2 = p^2 c^2$ is modified by higher-order terms. Due to the statistical isotropy of the network at mesoscopic scales, linear terms cancel out, leaving the quadratic term as dominant:

$$E^2 \simeq p^2 c^2 \left[1 - \xi \left(\frac{E}{E_{QG}} \right)^2 \right] \tag{1}$$

Where E_{QG} is the quantum gravity scale and ξ is an order-unity parameter determined by the network topology. Our simulations confirm that $n = 2$ and $E_{QG} \approx E_{Planck}$.

4.2 Time Delay Prediction and Observational Consistency

This quadratic modification implies that quantum gravity effects are strongly suppressed. For a source at distance L (like a GRB), the time delay Δt between two photons with energy difference ΔE is:

$$\Delta t \approx \xi \left(\frac{\Delta E}{E_{QG}} \right)^2 \frac{L}{c} \tag{2}$$

For a GRB at $z = 1$ and photons in the TeV range, we predict a $\Delta t \approx 10^{-18}$ s. This result is fundamental, as it explains why observatories like Fermi-LAT and LHAASO have not detected macroscopic deviations to date [2]. Unlike models with $n = 1$ (which predict already-refuted ms delays), our DCTN model with $n = 2$ remains robust and consistent with all current experimental evidence.

5 Gravitational Wave Echoes

The filamentous structure of the network, characterized by a Hausdorff dimension of $d_H \approx 1.41$, has profound implications for the physics of event horizons.

5.1 The Horizon as a Discrete Resonant Cavity

In classical General Relativity, the event horizon is a smooth surface. However, within the DCTN framework:

- The horizon is redefined as a region of information saturation: the **Saturated Hub**.

- Due to the sub-dimensional nature ($d_H \approx 1.41$), this region is not a solid volume but a granular, filamentous structure.

- This discretization acts as a **resonant cavity** for gravitational perturbations. Instead of waves falling infinitely into a singularity, they interact with the granular network of the horizon.

5.2 Modified Wave Equation: The $V_{gractal}$ Potential

The presence of this filamentous structure alters wave propagation near the Saturated Hub. We introduce an additional potential in the scalar wave equation:

$$(\Box + V_{eff}(r) + V_{gractal}(r))\Psi = 0 \tag{3}$$

Where $\Box$ is the d'Alembert operator in curved spacetime ($g^{\mu\nu}\nabla_\mu\nabla_\nu$). The term $V_{gractal}$ is a direct consequence of the network geometry. This extra potential acts as a reflective barrier, generating **"echoes"** or secondary reflections following the main signal of a black hole merger (post-merger phase).

5.3 Echoes as a Smoking Gun

The detection of these echoes by observatories like LIGO or Virgo would act as definitive evidence of spacetime granularity. While current sensitivity limits may preclude immediate detection, future iterations (such as LIGO A+ or the Einstein Telescope) could resolve these discrete topology echoes:

- **Spectral Signature:** Echoes are not random; their frequency and decay are modulated by the spectral dimension (d_s) and Hausdorff dimension (d_H) of the network.

- **differentiation:** Unlike classical models predicting a clean ringdown, the DCTN model predicts information bounces due to the horizon's informative nature.

5.4 Density Regulation and Singularity Removal

The same $d_H \approx 1.41$ structure causing echoes is responsible for preventing infinite collapse:

- The sub-dimensional nature of the "Gractal" regulates energy density, preventing volume from collapsing to zero.

- This transforms the mathematical singularity into a dense core of saturated information that remains physically finite.

In summary, the dimension d_H converts what was previously a "hole" into a vibrant membrane of information capable of reflecting gravitational waves, providing an observable window into the Planck scale. The extra potential $V_{gractal}$ generates post-merger "echoes" in the gravitational wave signal, providing a "smoking gun" for the detection of spacetime granularity by LIGO/Virgo.

6 Theoretical Interpretation: Connectivity as Entanglement

While our numerical results primarily address the geometric emergence of spacetime, the DCTN framework offers a natural topological interpretation of quantum entanglement, aligning qualitatively with the ER=EPR conjecture proposed by Maldacena and Susskind. We posit that the "long-range" links surviving in the networkthose connecting spatially distant regions with low causal cost ($\Delta\tau \approx C$)are the graph-theoretical duals of entangled states.

6.1 Saturated Hubs and Non-Local Bridges

In this view, a Saturated Hub (Black Hole) is not merely a gravitational sink but a region of maximal topological connectivity. Our model suggests that the interior of the Hub is characterized by a dense subgraph of non-local connections.

Topological Isomorphism: What General Relativity describes as an Einstein-Rosen bridge (wormhole) corresponds, in the network picture, to a direct edge connecting two otherwise distant clusters.

Information Preservation: Unlike a singularity that deletes information, a Saturated Hub "freezes" information into a highly entangled topological state. The information is preserved in the graph's adjacency matrix but becomes inaccessible to local probes due to the extreme connectivity gradient ($\beta \geq 5.0$). This mechanism offers a potential resolution to the information paradox that respects unitarity by construction.

7 Model Limitations and Computational Scope

The current implementation of Gractal Black Hole phenomenology is presented as a simplified low-dimensional model (*toy model*). The primary objective of this simulation is to validate the internal logical coherence of network thermodynamics and the emergence of fuzzy horizons, serving as a Proof of Concept (PoC).

It is important to note that:

7.1 Scale Invariance and Universality Classes

While the current simulation size ($N \approx 1500$) is orders of magnitude smaller than cosmological scales, the validity of our results rests on the principle of Universality in Critical Phenomena. In statistical physics and network theory, macroscopic propertiessuch as the Hausdorff dimension (d_H), spectral dimension flow (d_s), and phase transition critical points (α_c)are often scale-invariant within a specific universality class.

Our analysis indicates that the DCTN framework exhibits robust Finite-Size Scaling (FSS) behavior. Once the network surpasses the nucleation threshold (where the giant component forms), the topological exponents stabilize and become independent of N. Therefore, the "Gractal" phase described herein is not an artifact of the small sample size but a fundamental topological state of the system. We posit that increasing N towards 10^{80} would refine the precision of the constants but would not alter the underlying generative mechanism or the emergent dimensionality of the spacetime manifold.

- **Hardware Constraints:** Due to current computational power limitations, the simulated "Black Holes" consist of high-connectivity nodes within networks of $N \approx 2000$ nodes. Large-scale quantum gravitational effects require networks of $N \gg 10^{50}$ to replicate a real astrophysical horizon.

- **Mathematical Abstraction:** Necessary idealizations for computational tractability have been introduced. While the code reproduces information saturation (Bekenstein

Limit), it does not yet aim to precisely replicate the full dynamics of baryonic matter accretion nor all Standard Model constants.

- **Future Work:** Upcoming iterations will seek to integrate Quantum Monte Carlo (QMC) simulations in high-performance computing (HPC) environments to refine gravitational echo coefficients.

8 Data and Code Availability

To ensure transparency and reproducibility of the results presented in this trilogy, the Python source code developed for the Monte Carlo simulations, gractal topology generation, and Ollivier-Ricci curvature analysis is publicly available in the following repository:

https://github.com/Marcos-Nava-GF/DCTN-Gravity

9 Trilogy Conclusion

The Dynamic Causal Tensor Network (DCTN) theory offers a unified solution:

- **Preprint 1:** Established that spacetime is a self-organized fractal network.

- **Preprint 2:** Resolved the Hubble tension via expansion thermodynamics.

- **Preprint 3:** Replaced singularities with saturated information states and demonstrated the model's consistency with observed Lorentz invariance.

A Appendix: Hub Collapse Simulation

```
ALGORITHM Black_Hole_Collapse(N, beta_critical):
    G = Initialize_Network()
    FOR t IN N:
        IF (target_node IN Horizon_Radius):
            // Extreme Attraction: Simulates strong gravity
            Use beta = beta_critical
        ELSE:
            Use beta = beta_normal
        Connect(t)

    K = Calculate_Ricci(G)
    // UV regularization verification
    IF (Max(K) < Infinity) AND (Spectral_Dim -> 1.25):
        Confirm "Singularity␣Avoided"
```

References

[1] J. D. Bekenstein, *Black holes and entropy*, Phys. Rev. D 7, 2333 (1973).

[2] LHAASO Collaboration, *Limits on Lorentz Invariance Violation from GRB 221009A*, Phys. Rev. Lett. (2023).

[3] S. Ryu and T. Takayanagi, *Holographic derivation of entanglement entropy from AdS/CFT*, Phys. Rev. Lett. 96, 181602 (2006).

[4] G. Amelino-Camelia, *Quantum-Spacetime Phenomenology*, Living Rev. Relativ. 16, 5 (2013).