Sintaxe Abstrata

$$
\begin{array}{lcl}
e & ::= & n \mid b \mid x \mid e_1 \; op \; e_2 \\
 & \mid & \langle e_1, e_2 \rangle \mid \texttt{fst } e \mid \texttt{snd } e \\
 & \mid & \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \\
 & \mid & \texttt{fn } x \Rightarrow e \mid e_1 \; e_2 \\
 & \mid & \texttt{let } x = e_1 \texttt{ in } e_2 \\
 & \mid & \texttt{let rec } \; = \texttt{fn } x \Rightarrow e_1 \texttt{ in } e_2 \\
(*) & \mid & e_1 \mid\!> e_2 \\
(*) & \mid & \texttt{nil} \mid e_1 \texttt{::} e_2 \\
(*) & \mid & \texttt{match } e_1 \texttt{ with nil} \Rightarrow e_2 \mid x \texttt{::} xs \Rightarrow \; e_3 \\
(*) & \mid & \texttt{nothing} \mid \texttt{just } e \\
(*) & \mid & \texttt{match } e_1 \texttt{ with nothing } \Rightarrow e_2 \mid \texttt{just } x \Rightarrow \; e_3 \\
(*) & \mid & \texttt{left } e \mid \texttt{right } e \\
(*) & \mid & \texttt{match } e_1 \texttt{ with left } x \Rightarrow e_2 \mid \texttt{right } y \; \Rightarrow \; e_3
\end{array}
$$

$$op \in \{+, -, *, <, \leq, >, \geq, =, and, or\}$$

$$v ::= n \mid b \mid \texttt{fn } x \Rightarrow e \mid \langle v_1, v_2 \rangle \mid \texttt{nil} \mid \texttt{just } v \mid \texttt{nothing} \mid \texttt{left } v \mid \texttt{right } v$$

Sistema de Tipos

$$T ::= \mathsf{int} \mid \mathsf{bool} \mid T_1 \to T_2 \mid T_1 * T_2 \mid \mathsf{either}\ T_1\ T_2 \mid T\ \mathsf{list} \mid \mathsf{maybe}\ T \mid X$$

$$\sigma ::= \forall X.\sigma \mid T \mid$$

$$\frac{}{\Gamma \vdash n : int \mid \{\ \}} \qquad \frac{}{\Gamma \vdash b : bool \mid \{\ \}} \qquad \frac{\Gamma(x) = \sigma}{\Gamma \vdash x : \mathsf{instance\_of}(\sigma) \mid \{\ \}}$$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2}{\Gamma \vdash e_1 + e_2 : int \mid C_1 \cup C_2 \cup \{T_1 = int, T_2 = int\}}$$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad \Gamma \vdash e_3 : T_3 \mid C_3}{\Gamma \vdash \texttt{if } (e_1, e_2, e_3) : T_2 \mid C_1 \cup C_2 \cup C_3 \cup \{T_1 = bool, T_2 = T_3\}}$$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2}{\Gamma \vdash \langle e_1, e_2 \rangle : T_1 * T_2 \mid C_1 \cup C_2}$$

$$\frac{\Gamma \vdash e : T \mid C \qquad X, Y \; new}{\Gamma \vdash \texttt{fst } e : X \mid C \cup \{T = X * Y\}} \qquad \frac{\Gamma \vdash e : T \mid C \qquad X, Y \; new}{\Gamma \vdash \texttt{snd } e : Y \mid C \cup \{T = X * Y\}}$$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X \; new}{\Gamma \vdash e_1 \; e_2 : X \mid C_1 \cup C_2 \cup \{T_1 = T_2 \to X\}}$$

$$\frac{\Gamma, x : X \vdash e : T \mid C \qquad X \; new}{\Gamma \vdash \texttt{fn } \; x \Rightarrow e : X \to T \mid C}$$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma, x : \mathsf{Gen}(\Gamma, T_1, C_1) \vdash e_2 : T_2 \mid C_2}{\texttt{let } x = e_1 \texttt{ in } e_2 : T_2 \mid C_1 \cup C_2}$$

$$\frac{\Gamma, f : X, x : Y \vdash e_1 : T_1 \mid C_1 \qquad \Gamma, f : X \vdash e_2 : T_2 \mid C_2 \qquad X, Y \ \textit{new}}{\Gamma \vdash \texttt{let rec } f = \texttt{fn } x \Rightarrow e_1 \texttt{ in } e_2 : T_2 \mid C_1 \cup C_2 \cup \{X = Y \to T_1\}}$$

Complemento

$e_1 \mid> e_2$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad X \ \textit{new}}{\Gamma \vdash e_1 \mid> e_2 : X \mid C_1 \cup C_2 \cup \{T_2 = T_1 \to X\}}$$

nil

$$\frac{X \ \textit{new}}{\Gamma \vdash \texttt{nil} : X \ \mathsf{list} \mid \{\}}$$

$e_1 :: e_2$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2}{\Gamma \vdash e_1 :: e_2 : T_2 \ \mathsf{list} \mid C_1 \cup C_2 \cup \{T_1 = T_2\}}$$

match e_1 *with nil* $\Rightarrow e_2 | x :: xs \Rightarrow e_3$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad \Gamma, x : T, xs : \mathsf{list}\ T \vdash e_3 : T_3 \mid C_3 \qquad X \ \textit{new}}{\Gamma \vdash \texttt{match } e_1 \texttt{ with nil } \Rightarrow e_2 \ \texttt{|} \ x :: xs \Rightarrow e_3 : T_3 \mid C_1 \cup C_2 \cup C_3 \cup \{T_1 = \mathsf{list}\ X, T_2 = T_3\}}$$

nothing

$$\frac{X \ \textit{new}}{\Gamma \vdash \texttt{nothing} : \mathsf{maybe}\ X \mid \{\}}$$

just e

$$\frac{\Gamma \vdash e : T \mid C}{\Gamma \vdash \texttt{just } e : \mathsf{maybe}\ T \mid C}$$

match e_1 *with nothing* $\Rightarrow e_2 |$ *just* $x \Rightarrow e_3$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma \vdash e_2 : T_2 \mid C_2 \qquad \Gamma \vdash e_3 : T_3 \mid C_3 \qquad X \ \textit{new}}{\Gamma \vdash \texttt{match } e_1 \texttt{ with nothing } \Rightarrow e_2 \ \texttt{| just } x \Rightarrow e_3 : T_3 \mid C_1 \cup C_2 \cup C_3 \cup \{T_1 = \mathsf{maybe}\ X, T_2 = T_3\}}$$

left e_1

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad X \ \textit{new}}{\Gamma \vdash \texttt{left } e_1 : \mathsf{either}\ T_1 \ \ X \mid C_1}$$

right e_1

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad X \; new}{\Gamma \vdash \texttt{right} \; e_1 : \textsf{either} \; X \;\; T_1 \mid C_1}$$

match e_1 *with left* $x \Rightarrow e_2 | right \; y \Rightarrow e_3$

$$\frac{\Gamma \vdash e_1 : T_1 \mid C_1 \qquad \Gamma, x : T_1 \vdash e_2 : T_2 \mid C_2 \qquad \Gamma, y : T_2 \vdash e_3 : T_3 \mid C_3 \qquad X \; Y \; new}{\Gamma \vdash \texttt{match} \; e_1 \; \texttt{with left} \; x \Rightarrow e_2 \; \texttt{|} \; \texttt{right} \; y \Rightarrow \; e_3 : T_3 \mid C_1 \cup C_2 \cup C_3 \cup \{T_1 = \textsf{either} \; X \; Y, T_2 = T_3\}}$$

Semântica Operacional

$$\rho \vdash n \Downarrow n \qquad \text{(BS-NUM)} \qquad\qquad \rho \vdash b \Downarrow b \qquad \text{(BS-BOOL)}$$

$$\frac{\rho \vdash e_1 \Downarrow n_1 \qquad \rho \vdash e_2 \Downarrow n_2 \qquad n = n_1 + n_2}{\rho \vdash e_1 \texttt{+} \; e_2 \Downarrow n} \qquad \text{(BS-SUM)}$$

$$\frac{\rho(x) = v}{\rho \vdash x \Downarrow v} \qquad \text{(BS-ID)}$$

$$\frac{\rho \vdash e_1 \Downarrow \texttt{true} \qquad \rho \vdash e_2 \Downarrow v}{\rho \vdash \texttt{if} \; (e_1, e_2, e_3) \Downarrow v} \; \text{(BS-IFTR)} \qquad \frac{\rho \vdash e_1 \Downarrow \texttt{false} \qquad \rho \vdash e_3 \Downarrow v}{\rho \vdash \texttt{if} \; (e_1, e_2, e_3) \Downarrow v} \; \text{(BS-IFFLS)}$$

$$\rho \vdash (\texttt{fn} \; x \Rightarrow e) \Downarrow \langle x, e, \rho \rangle \qquad \text{(BS-FN)}$$

$$\frac{\rho \vdash e_1 \Downarrow \langle x, e, \rho' \rangle \qquad \rho \vdash e_2 \Downarrow v' \qquad \rho', x \mapsto v' \vdash e \Downarrow v}{\rho \vdash e_1 \; e_2 \Downarrow v} \qquad \text{(BS-APP)}$$

$$\frac{\rho \vdash e_1 \Downarrow v' \qquad \rho, x \mapsto v' \vdash e_2 \Downarrow v}{\rho \vdash \texttt{let} \; x = e_1 \; \texttt{in} \; e_2 \Downarrow v} \qquad \text{(BS-LET)}$$

$$\frac{\rho \vdash e_1 \Downarrow \langle f, x, e, \rho' \rangle \qquad \rho \vdash e_2 \Downarrow v' \qquad \rho', \; x \mapsto v', \; f \mapsto \langle f, x, e, \rho' \rangle \vdash e \Downarrow v}{\rho \vdash e_1 \; e_2 \Downarrow v} \; \text{(BS-APPREC)}$$

$$\frac{\rho, f \mapsto \langle f, x, e_1, \rho \rangle \vdash e_2 \Downarrow v}{\rho \vdash \texttt{let rec} \; f = \texttt{fn} \; x \Rightarrow e_1 \; \texttt{in} \; e_2 \Downarrow v} \qquad \text{(BS-LETREC)}$$

Complemento

$e_1 \; |\!\!> e_2$

$$\frac{\rho \vdash e_1 \Downarrow v \qquad \rho \vdash e_2 \Downarrow \langle x, e, \rho' \rangle \qquad \rho', x \mapsto v \vdash e \Downarrow v'}{\rho \vdash e_1 \; |\!\!> e_2 \Downarrow v'} \qquad \text{(BS-PIPE)}$$

$$\frac{\rho \vdash e_1 \Downarrow v \qquad \rho \vdash e_2 \Downarrow \langle f, x, e, \rho' \rangle \qquad \rho', x \mapsto v, f \mapsto \langle f, x, e, \rho' \rangle \vdash e \Downarrow v'}{\rho \vdash e_1 \; |\!\!> e_2 \Downarrow v'} \; \text{(BS-PIPEREC)}$$

nil

$$\rho \vdash \texttt{nil} \Downarrow \texttt{nil} \qquad \text{(BS-NIL)}$$

$e_1 :: e_2$

$$\frac{\rho \vdash e_1 \Downarrow v_1 \qquad \rho \vdash e_2 \Downarrow v_2}{\rho \vdash e_1 :: e_2 \Downarrow v_1 :: v_2} \tag{BS-Cons}$$

$match\ e_1\ with\ nil \Rightarrow e_2 | x :: xs \Rightarrow\ e_3$

$$\frac{\rho \vdash e_1 \Downarrow \texttt{nil} \qquad \rho \vdash e_2 \Downarrow v'}{\rho \vdash \texttt{match}\ e_1\ \texttt{with}\ \texttt{nil} \Rightarrow e_2\ \texttt{|}\ x :: xs \Rightarrow\ e_3 \Downarrow v'} \tag{BS-MatchNil}$$

$nothing$

$$\rho \vdash \texttt{nothing} \Downarrow \texttt{nothing} \tag{BS-Nothing}$$

$just\ e$

$$\frac{\rho \vdash e \Downarrow v}{\rho \vdash \texttt{just}\ e \Downarrow \texttt{just}\ v} \tag{BS-Just}$$

$match\ e_1\ with\ nothing \Rightarrow e_2 | just\ x \Rightarrow e_3$

$$\frac{\rho \vdash e_1 \Downarrow \texttt{nothing} \qquad \rho \vdash e_2 \Downarrow v'}{\rho \vdash \texttt{match}\ e_1\ \texttt{with}\ \texttt{nothing}\ \Rightarrow e_2\ \texttt{|}\ \texttt{just}\ x \Rightarrow\ e_3 \Downarrow v'} \tag{BS-MatchNothing}$$

$left\ e_1$

$$\frac{\rho \vdash e_1 \Downarrow v}{\rho \vdash \texttt{left}\ e_1 \Downarrow \texttt{left}\ v} \tag{BS-Left}$$

$right\ e_1$

$$\frac{\rho \vdash e_1 \Downarrow v}{\rho \vdash \texttt{right}\ e_1 \Downarrow \texttt{right}\ v} \tag{BS-Right}$$

$match\ e_1\ with\ left\ x \Rightarrow e_2 | right\ y \Rightarrow e_3$

$$\frac{\rho \vdash e_1 \Downarrow \texttt{left}\ v \qquad \rho, x \mapsto v \vdash e_2 \Downarrow v'}{\rho \vdash \texttt{match}\ e_1\ \texttt{with}\ \texttt{left}\ x \Rightarrow e_2\ \texttt{|}\ \texttt{right}\ y \Rightarrow\ e_3 \Downarrow v'} \tag{BS-MatchLeft}$$

$$\frac{\rho \vdash e_1 \Downarrow \texttt{right}\ v \qquad \rho, y \mapsto v \vdash e_3 \Downarrow v'}{\rho \vdash \texttt{match}\ e_1\ \texttt{with}\ \texttt{left}\ x \Rightarrow e_2\ \texttt{|}\ \texttt{right}\ y \Rightarrow\ e_3 \Downarrow v'} \tag{BS-MatchRight}$$