Problema 1 — Resolución usando eliminación de Gauss

$$\min\{c^\top x : Ax = b,\ x \geq 0\}$$

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \quad c = \begin{pmatrix} 1 \\ 3 \\ -1 \\ 3 \end{pmatrix}$$

Base inicial

$$B = \{x_1, x_2\}, \quad A_B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Cálculo de A_B^{-1} por Gauss

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftarrow R_2 - R_1} \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \leftarrow -\frac{1}{2} R_2} \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

$$A_B^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Solución básica

$$x_B = A_B^{-1} b = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$x = (3,\ 1,\ 0,\ 0)$$

Entrada de x_3

$$A_3 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad d = A_B^{-1} A_3 = \begin{pmatrix} \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}$$

$$\theta = \text{mín} \left\{ \frac{3}{\frac{3}{2}} \right\} = 2$$

$$x = (0,\, 2,\, 2,\, 0)$$

Solución óptima

$$\boxed{x^* = (0,\, 2,\, 2,\, 0), \quad z^* = 4}$$

Problema 2 — Método simplex revisado con Gauss

$$\text{mín}\, 3x_1 + 2x_2 - x_3$$

$$\begin{aligned} x_1 - 2x_2 + x_3 - x_4 &= 0 \\ x_1 + x_2 + 2x_3 - x_4 &= 1 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Datos

$$A = \begin{pmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & 2 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad c = \begin{pmatrix} 3 \\ 2 \\ -1 \\ 0 \end{pmatrix}$$

Base inicial

$$B = \{x_1, x_2\}, \quad A_B = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$

Cálculo de A_B^{-1} por Gauss

$$\left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \leftarrow R_2 - R_1} \left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 3 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \leftarrow \frac{1}{3} R_2} \left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \end{array}\right]$$

$$\xrightarrow{R_1 \leftarrow R_1 + 2R_2} \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \end{array}\right]$$

$$A_B^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$$

Solución básica

$$x_B = A_B^{-1} b = \frac{1}{3} \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$$

$$x = \left(\frac{2}{3},\ \frac{1}{3},\ 0,\ 0 \right)$$

Entrada de x_3

$$A_3 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad d = A_B^{-1} A_3 = \begin{pmatrix} \frac{5}{3} \\ \frac{1}{3} \end{pmatrix}$$

$$\theta = \text{mín} \left\{ \frac{\frac{2}{3}}{\frac{5}{3}}, \frac{\frac{1}{3}}{\frac{1}{3}} \right\} = \frac{2}{5}$$

$$x = \left(0,\ \frac{1}{5},\ \frac{2}{5},\ 0\right)$$

Solución óptima

$$\boxed{x^* = \left(0,\ \frac{1}{5},\ \frac{2}{5},\ 0\right), \quad z^* = -\frac{1}{5}}$$