Log linearização

Temos

$$\chi \frac{C_t}{1-N_t} = (1-\alpha) z_t K_t^{\alpha} N_t^{-\alpha} \tag{1}$$

No steady-state não estocástico temos $\log z_t = 0$ e

$$\log \chi + \log C - \log(1-N) = \log(1-\alpha) + \alpha \log K - \alpha \log N$$

No caso geral,

$$\log \chi + \log C_t - \log(1-N_t) = \log(1-\alpha) + \alpha \log K_t - \alpha \log N_t + \log z_t$$

Único termo não log-linear é

$$\log(1-N_t) = \log\left(1 - e^{\log N_t}\right) \approx \log(1-N) - \frac{N}{1-N}(\log N_t - \log N)$$

Portanto, em desvios para o estado estacionário (usando a notação de $\hat{X}_t = \log X_t - \log X$) e usando $\zeta := \frac{N}{1-N}$,

$$\hat{C}_t + (\zeta + \alpha) \hat{N}_t = \alpha \hat{K}_t + \hat{z}_t$$

ou

$$\hat{N}_t = \frac{1}{\alpha+\zeta} \hat{z}_t + \frac{\alpha}{\alpha+\zeta} \hat{K}_t - \frac{1}{\alpha+\zeta} \hat{C}_t \tag{2}$$

- Relação estática.
- $\hat{C}_t$ ainda endógeno.
- Resposta de N_t a choque em z_t ainda pode ser até negativa.

Agora, focamos em

$$1 = 1\beta\left[\left(1-\delta+\alpha K^{\alpha-1} N^{1-\alpha}\right)\right] \tag{3}$$

$$C_t^{-1} = \beta E_t\left[C_{t+1}^{-1}\left(1-\delta+\alpha z_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}\right)\right]$$

Temos

$$C_t^{-1} = e^{-\log C_t} \approx C^{-1} - C^{-1}\underbrace{(\log C_t - \log C)}_{\hat{C}_t},$$

$$C_{t+1}^{-1}(1-\delta) \approx C^{-1}(1-\delta)\left[1 - \hat{C}_{t+1}\right]$$

e

$$C_{t+1}^{-1} \alpha z_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} \approx C^{-1} \alpha K^{\alpha-1} N^{1-\alpha} \left[1 - \hat{C}_{t+1} - (1-\alpha)\hat{K}_{t+1} + (1-\alpha)\hat{N}_{t+1} + \hat{z}_{t+1}\right].$$

Logo, usando a condição no steady-state, temos

$$-\hat{C}_t = \underbrace{\beta\alpha \left(\frac{N}{K}\right)^{1-\alpha}}_{\alpha\frac{Y}{K}} E_t\left[\hat{z}_{t+1} - (1-\alpha)\hat{K}_{t+1} + (1-\alpha)\hat{N}_{t+1}\right] - \underbrace{\left(1 - \delta + \alpha K^{\alpha-1} N^{1-\alpha}\right)}_{\beta^{-1}} \beta E_t \hat{C}_{t+1}$$

e

$$E_t\hat{C}_{t+1} - \hat{C}_t = \beta\alpha\left(\frac{Y}{K}\right) E_t\left[\hat{z}_{t+1} - (1-\alpha)\hat{K}_{t+1} + (1-\alpha)\hat{N}_{t+1}\right] \tag{4}$$

Substituindo para N_t

$$\begin{aligned} E_t\hat{C}_{t+1} - \hat{C}_t &= \beta\alpha\left(\frac{Y}{K}\right) E_t\left[\hat{z}_{t+1} - (1-\alpha)\hat{K}_{t+1} + \frac{(1-\alpha)}{\alpha+\zeta}\hat{z}_{t+1} + \frac{\alpha(1-\alpha)}{\alpha+\zeta}\hat{K}_{t+1} - \frac{(1-\alpha)}{\alpha+\zeta}\hat{C}_{t+1}\right] \\ &= \beta\alpha\left(\frac{Y}{K}\right) E_t\left[\frac{\zeta+1}{\alpha+\zeta}\hat{z}_{t+1} - (1-\alpha)\frac{\zeta}{\alpha+\zeta}\hat{K}_{t+1} - \frac{(1-\alpha)}{\alpha+\zeta}\hat{C}_{t+1}\right] \end{aligned}$$

$$E_t\hat{C}_{t+1} - \hat{C}_t = \beta\alpha\left(\frac{Y}{K}\right) E_t\left[\frac{\zeta+1}{\alpha+\zeta}\hat{z}_{t+1} - (1-\alpha)\frac{\zeta}{\alpha+\zeta}\hat{K}_{t+1} - \frac{(1-\alpha)}{\alpha+\zeta}\hat{C}_{t+1}\right] \tag{5}$$

Note que, usando uma defasagem, o locus em que o consumo fica constante pode ser descrito $E_{t-1}\hat{C}_t - \hat{C}_{t-1} = 0$ como

$$0 = \beta\alpha\left(\frac{Y}{K}\right) E_{t-1}\left[\frac{\zeta+1}{\alpha+\zeta}\hat{z}_t - (1-\alpha)\frac{\zeta}{\alpha+\zeta}\hat{K}_t - \frac{(1-\alpha)}{\alpha+\zeta}\hat{C}_t\right].$$

Por fim, temos a dinâmica de acumulação

$$K_{t+1} = z_t K_t^{\alpha} N_t^{1-\alpha} + (1-\delta) K_t - C_t \tag{6}$$

com

$$z_t K_t^{\alpha} N_t^{1-\alpha} \approx K^{\alpha} N^{1-\alpha}\left[1 + \hat{z}_t + \alpha\hat{K}_t + (1-\alpha)\hat{N}_t\right]$$

Usando as equação em steady-state eliminamos os termos com o coeficiente 1, restando

$$K\hat{K}_{t+1} = Y\left[\hat{z}_t + \alpha\hat{K}_t + (1-\alpha)\hat{N}_t\right] + (1-\delta) K\hat{K}_t - C\hat{C}_t$$

e

$$\hat{K}_{t+1} = \frac{Y}{K}\left[\hat{z}_t + \alpha\hat{K}_t + (1-\alpha)\,\hat{N}_t\right] + (1-\delta)\,\hat{K}_t - \frac{C}{K}\hat{C}_t \tag{7}$$

Podemos usar a eq de Euler no steady-state, com

$$\beta^{-1} = (1-\delta) + \alpha\frac{Y}{K} \tag{8}$$

para fazer uma substituição, bem como a equação do eq no mercado de trabalho

$$\hat{K}_{t+1} = \frac{Y}{K}\left[\hat{z}_t + (1-\alpha)\left[\frac{1}{\alpha+\zeta}\hat{z}_t + \frac{\alpha}{\alpha+\zeta}\hat{K}_t - \frac{1}{\alpha+\zeta}\hat{C}_t\right]\right] + \beta^{-1}\hat{K}_t - \frac{C}{K}\hat{C}_t$$

rearrumando

$$\hat{K}_{t+1} = \frac{Y}{K}\frac{\zeta+1}{\alpha+\zeta}\hat{z}_t + \left(\beta^{-1} + \frac{\alpha\,(1-\alpha)}{\alpha+\zeta}\frac{Y}{K}\right)\hat{K}_t - \left(\frac{C}{K} + \frac{(1-\alpha)}{\alpha+\zeta}\frac{Y}{K}\right)\hat{C}_t. \tag{9}$$

Olharemos o locus

$$\hat{K}_{t+1} - \hat{K}_t = 0,$$

então note que $\left(\beta^{-1} + \frac{\alpha(1-\alpha)}{\alpha+\zeta}\frac{Y}{K}\right) - 1 > 0.$