Problema de Planejador

$$\max_{\{C_t,N_t,I_t\}} E\sum\beta^t\left[u\left(C_t,1-N_t\right)\right]$$

s.a.

$$C_t+I_t=z_tK_t^{\alpha}N_t^{1-\alpha}$$

$$K_{t+1}=\left(1-\delta\right)K_t+I_t$$

$$\ln z_t=\rho\ln z_{t-1}+\sigma\epsilon_t,\quad \epsilon_t\overset{iid}{\sim}N\left(0,1\right).$$

Versão recursiva (K_t,z_t)

$$V\left(K_t,z_t\right)=\max_{C_t,N_t}u\left(C_t,1-N_t\right)+\beta E_t\left[V\left(\left(1-\delta\right)K_t+z_tK_t^{\alpha}N_t^{1-\alpha}-C_t,z_{t+1}\right)\right]$$

CPOs:

C_t

$$u_{c,t}=\beta E_t\left[V_k\left(K_{t+1},z_{t+1}\right)\right]$$

N_t

$$-u_{l,t}+\beta E_t\left[V_K\left(K_{t+1},z_{t+1}\right)\right]\left(1-\alpha\right)z_tK_t^{\alpha}N_t^{-\alpha}=0$$

$$u_{l,t}=u_{c,t}MPL_t$$

Envelope:

$$\begin{aligned}V_k\left(K_t,z_t\right)&=\beta E_t\left[V_k\left(K_{t+1},z_{t+1}\right)\right]\left(1-\delta+\alpha z_tK_t^{\alpha-1}N_t^{1-\alpha}\right)\\&=u_{c,t}\left(1-\delta+\alpha z_tK_t^{\alpha-1}N_t^{1-\alpha}\right)\end{aligned}$$

EE

$$u_{c,t}=\beta E_t\left[u_{c,t+1}\left(1-\delta+\alpha z_{t+1}K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha}\right)\right]$$