Lecture 5: The Equity Premium and Risk-Free Rate Puzzles

Raul Riva

FGV EPGE

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Intro

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- Lucas (1978) set the stage for modern comsumption-based asset pricing
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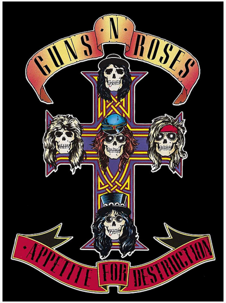
Then the 80s: the Guns n Roses release Appetite for Destruction

- The equity premium puzzle: the Lucas-like economy cannot fit equity data
- The risk-free rate puzzle: if you torture parameters to fit equity, risk-free rates explode
- The volatility puzzle: consumption growth is too smooth to explain asset return volatility
- Where is your Asset Pricing God now, huh?



Flight Plan

- Some empirical evidence about equity returns and consumption growth
- The Mehra-Prescott economy: setup and solution
- Taking the model to the data: the equity premium puzzle
- The risk-free rate puzzle
- Next class: the volatility puzzle and the early Campbell-Shiller papers
- After that: let's try to solve the equity premium puzzle!

The Equity Premium Puzzle

The Equity Premium Puzzle

- Mehra and Prescott (1985) is our reference
- In a traditional general equilibrium environment, how large can the equity premium be?
- This is very different from the statistical shenanigans of APT!
- Here: the SDF will heavily depend on consumption growth
- Back to "m high $\iff$ low consumption growth"

Some (Annual) Data for the US

Time	% growth rate of per capita real consumption		% real return on a relatively riskless security		% risk premium		% real return on S&P 500	
periods	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
1889–1978	1.83	3.57	0.80	5.67	6.18	16.67	6.98	16.54
	(Std. error = 0.38)		(Std. error = 0.60)		(Std. error = 1.76)		(Std. error = 1.74)	
1889–1898	2.30	4.90	5.80	3.23	1.78	11.57	7.58	10.02
1899–1908	2.55	5.31	2.62	2.59	5.08	16.86	7.71	17.21
1909–1918	0.44	3.07	-1.63	9.02	1.49	9.18	-0.14	12.81
1919–1928	3.00	3.97	4.30	6.61	14.64	15.94	18.94	16.18
1929–1938	-0.25	5.28	2.39	6.50	0.18	31.63	2.56	27.90
1939–1948	2.19	2.52	-5.82	4.05	8.89	14.23	3.07	14.67
1949–1958	1.48	1.00	-0.81	1.89	18.30	13.20	17.49	13.08
1959–1968	2.37	1.00	1.07	0.64	4.50	10.17	5.58	10.59
1969–1978	2.41	1.40	-0.72	2.06	0.75	11.64	0.03	13.11

SP500 Returns vs Consumption Growth (USA)

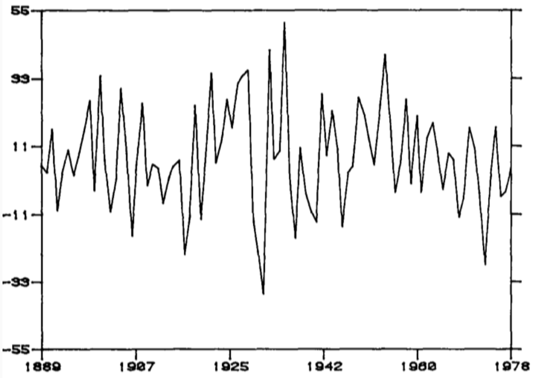


Fig. 1. Real annual return on S&P 500, 1889–1978 (percent).

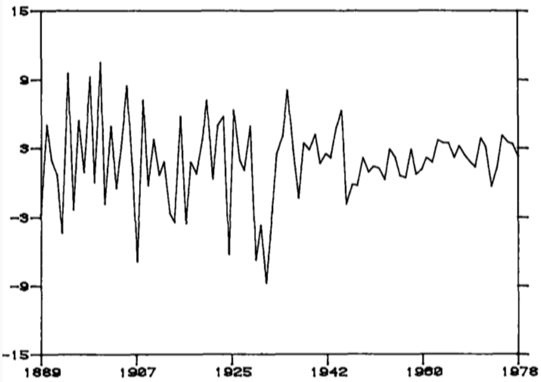


Fig. 2. Growth rate of real per capita consumption, 1889–1978 (percent).

Similar Evidence Across Developed Markets (Quarterly Data)

Country	Sample period	$\bar{r}_e$	$\sigma(r_e)$	$\rho(r_e)$	$\bar{r}_f$	$\sigma(r_f)$	$\rho(r_f)$	$\overline{\Delta c}$	$\sigma(\Delta c)$	$\rho(\Delta c)$
USA	1947Q2–2011Q2	6.85	15.98	0.10	0.72	1.78	0.36	1.74	1.64	0.04
Australia	1970Q1–2011Q2	3.84	20.75	0.03	2.00	2.18	0.55	1.82	1.77	−0.09
Canada	1970Q2–2011Q2	5.47	17.85	0.15	2.07	1.64	0.66	1.69	1.93	0.07
France	1973Q2–2011Q2	7.06	23.10	0.08	2.08	1.53	0.74	1.38	1.80	−0.13
Germany	1978Q4–2011Q2	7.54	23.85	0.04	2.38	1.09	0.40	1.74	4.19	−0.07
Italy	1971Q2–2011Q2	1.51	25.74	0.07	1.86	2.06	0.77	2.18	2.23	0.47
Japan	1970Q2–2011Q2	2.70	21.41	0.09	1.03	1.91	0.29	1.72	2.92	−0.10
Netherlands	1977Q2–2011Q2	8.57	19.76	0.09	2.29	1.42	0.45	1.05	2.21	−0.11
Sweden	1970Q1–2011Q2	8.93	25.16	0.12	1.68	2.23	0.40	1.23	1.81	−0.15
Switzerland	1982Q2–2011Q2	8.14	20.05	0.01	0.87	1.32	0.03	0.75	1.30	−0.22
UK	1970Q1–2011Q2	6.33	19.85	0.10	1.34	2.60	0.46	2.14	2.68	−0.03

Takeaways

- Average return on stocks is high compared to consumption growth across markets
- Consumption growth is *way* less volatile than stock returns
- Risk-free rates are low and smooth (for developed markets!)
- Stock returns have low autocorrelation
- Consumption growth even has negative autocorrelation sometimes. Italy seems an exception there

The Mehra-Prescott Economy

- They build upon Lucas (1978) (the Lucas tree model)
- There is a representative agent with preferences over consumption streams given by

$$U(c) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right]$$

where $\beta \in (0, 1)$ and $\gamma > 0$. If $\gamma = 1$, we have log-utility

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- There is one unit of an equity asset that pays perishable dividends y_t every period, evolving according to

$$y_{t+1} = x_{t+1} \cdot y_t, \quad y_0 > 0$$

- The growth rate process $x_t \in \{\lambda_1, \ldots, \lambda_n\}$ follows an ergodic Markov chain

$$\mathbb{P}\left(x_{t+1} = \lambda_j \mid x_t = \lambda_i\right) = \phi_{ij} \in (0,1)$$

Household Problem (Sequential)

- Choose $\{c_t, b_{t+1}, s_{t+1}\}_{t \geq 0} \implies$ consumption, bond holdings, and shares of the tree
- State variables: $(b_t, s_t, y_t, x_t) \implies$ current holdings of bonds, shares, dividend, and growth state
- Prices: q_t (bond), p_t (tree)

$$\max_{\{c_t, b_{t+1}, s_{t+1}\}_{t \geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \right]$$

$$c_t + q_t b_{t+1} + p_t s_{t+1} \leq b_t + (p_t + y_t) s_t, \qquad \forall t$$

Notice that marginal utility is given by $u'(c_t) = c_t^{-\gamma}$.

The First Order Conditions

If μ_t is the Lagrange multiplier on the budget constraint at time t, the FOCs are:

$$
\begin{aligned}
c_t^{-\gamma} &= \mu_t \\
q_t \mu_t &= \beta \mathbb{E}_t \left[\mu_{t+1} \right] \\
p_t \mu_t &= \beta \mathbb{E}_t \left[\mu_{t+1} (p_{t+1} + y_{t+1}) \right]
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Define the SDF:

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} = \beta \frac{\mu_{t+1}}{\mu_t}$$

$$\mathbb{E}_t \left[m_{t+1} R^f_{t+1}\right] = 1, \qquad R^f_{t+1} = \frac{1}{q_t}$$
$$\mathbb{E}_t \left[m_{t+1} R^e_{t+1}\right] = 1, \qquad R^e_{t+1} = \frac{p_{t+1} + y_{t+1}}{p_t}$$

Pricing the Tree Recursively

- These equations are no surprise if you got this far in the class
- If we iterate the equity pricing equation forward and impose the no-bubble condition, we get

$$p_t = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \beta^j \left(\frac{c_{t+j}}{c_t} \right)^{-\gamma} y_{t+j} \right]$$

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- Since the dividend is perishable, $c_t = y_t$ for all t. Then:

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- Since $q_t = \mathbb{E}_t[m_{t+1}]$, then

$$q_t = \beta \mathbb{E}_t \left[\left(\frac{y_{t+1}}{y_t} \right)^{-\gamma} \right] = \beta \mathbb{E}_t \left[x_{t+1}^{-\gamma} \right]$$

Solving the Mehra-Prescott Economy

- Guess a solution for the price-dividend ratio:

$$\frac{p_t}{y_t} = f(x_t)$$

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- Let's search for these unknowns using the notation $F \equiv [f_1, \dots, f_n]^\top$
- To ease computations, define the following matrices/vectors:
  - $\Phi \equiv [\phi_{ij}]$ (transition matrix)
  - $D \equiv \text{diag}(\lambda_1^{1-\gamma}, \dots, \lambda_n^{1-\gamma})$
  - $\mathbf{1} \equiv [1, \dots, 1]^\top$ (vector of ones)

Solving a Linear System

- Recall the recursive pricing equation:

$$p_t = \mathbb{E}_t\left[m_{t+1}(p_{t+1} + y_{t+1})\right]$$

- This equation can be written as

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- Using the notation from before:

$$F = \beta\Phi D F + \beta\Phi D \mathbf{1} \implies (I - \beta\Phi D)F = \beta\Phi D\mathbf{1}$$

- Mehra and Prescott (1985) shows an explicit condition on D and Φ such that $[I - \beta\Phi D]$ is invertible. In that case (show it at home):

$$F = \beta[I - \beta\Phi D]^{-1}\Phi D\mathbf{1}$$

Equilibrium Returns

- The price q_t is also a function of the state x_t:

$$q(x_t = \lambda_i) \equiv q_i = \beta \sum_{j=1}^{n} \phi_{ij} \lambda_j^{-\gamma}, \quad \forall i = 1, \ldots, n$$

- The risk-free rate in state $x_t = \lambda_i$ is given by

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- The (gross) return on equity from state i to state j is:

$$R_{i,j}^e = \frac{p_{t+1} + y_{t+1}}{p_t} = \lambda_j \left(\frac{f_j + 1}{f_i} \right)$$

Unconditional Averages

- Let $\pi = [\pi_1, \ldots, \pi_n]$ be the stationary distribution of the Markov chain for x_t
- This distribution is characterized as

$$\pi = \Phi^\top \pi, \quad \sum_{i=1}^{n} \pi_i = 1$$

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- The unconditional mean return on equity is

$$\bar{R}^e = \sum_{i=1}^{n} \pi_i \left(\sum_{j=1}^{n} \phi_{ij} R_{i,j}^e \right)$$

- The risk premium is then $\overline{RP} = \bar{R}^e - \bar{R}^f$

Questions?

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- They assumed two states for consumption growth: $n = 2$
- There are three parameters for technology: θ, δ, and ϕ:

$$\lambda_1 = 1 + \theta + \delta, \quad \lambda_2 = 1 + \theta - \delta$$
$$\Phi = \begin{bmatrix} \phi & 1-\phi \\ 1-\phi & \phi \end{bmatrix}$$

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- Parameters were calibrated to match the average, standard deviation, and first autocorrelation of US consumption growth

$$\theta = 0.018, \quad \delta = 0.036, \quad \phi = 0.43$$

Preference Parameters and Feasible Region

- Vast literature on γ, but typical values are less than 10.
- Theory implies $\beta \in (0, 1)$
- Given a pair $(\gamma, \beta) \in [1, 10] \times (0, 1)$, the model is determined
- Why not try many values to match the equity premium and the risk-free rate?
- Essentially, no pair is able to match the data!

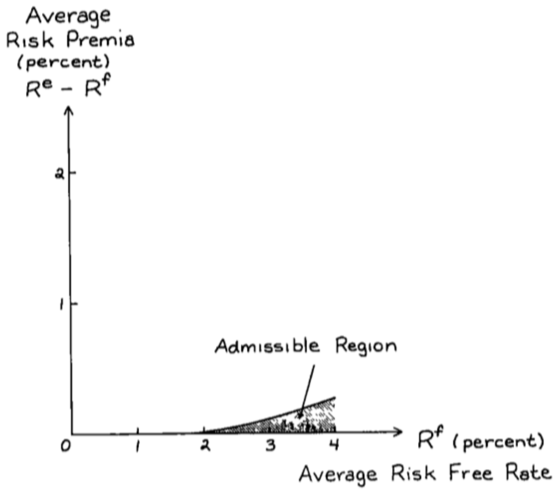

Fig. 4. Set of admissible average equity risk premia and real returns.

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Main Idea:

- What if we bump up the risk aversion parameter γ to match the equity premium?
- What happens to the risk-free rate then? Maybe the previous evidence on γ is all wrong!

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- We shall prove that, for realistic values of consumption growth volatility, $\uparrow \gamma \implies R_f \uparrow$
- Any intuition why? What would break this result?

A Quick Math Lemma

- We need to prove a lemma first. We want to show that, if X has enough moments, then

$$\log(\mathbb{E}\left[e^X\right]) \approx \mathbb{E}[X] + \frac{1}{2}\mathsf{Var}(X)$$

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- In case X is Gaussian, this is exact!
- Recall the moment generating function (MGF) of X:

$$M_X(t) \equiv \mathbb{E}\left[e^{tX}\right]$$

- Define also the cumulant generating function (CGF):

$$K_X(t) \equiv \log(M_X(t)) = \log\left(\mathbb{E}\left[e^{tX}\right]\right)$$

- Notice that we want to approximate $K_X(1)$

Mr. Taylor To The Rescue

- Let's do a second-order approximation around $t = 0$:

$$K_X(1) \approx K_X(0) + K'_X(0)(1-0) + \frac{1}{2}K''_X(0)(1-0)^2$$

- Recall that $M_X^{(n)}(0) = \mathbb{E}[X^n]$. Using this, we can compute the derivatives of $K_X(t)$:

$$K_X(0) = \log\left(\mathbb{E}\left[e^0\right]\right) = 0$$

$$K'_X(t) = \frac{M'_X(t)}{M_X(t)} \implies K'_X(0) = \mathbb{E}[X]$$

$$K''_X(t) = \frac{M''_X(t)M_X(t) - (M'_X(t))^2}{(M_X(t))^2} \implies K''_X(0) = \mathsf{Var}(X)$$

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- Then we have that

$$\log(\mathbb{E}\left[e^X\right]) = K_X(1) \approx \mathbb{E}[X] + \frac{1}{2}\mathrm{Var}(X)$$

Applying The Lemma

- Recall that $\mathbb{E}_t[m_{t+1}] = \frac{1}{R_t^f}$ and that

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} = \beta e^{-\gamma \Delta c_{t+1}}, \qquad \Delta c_{t+1} \equiv \log \left(\frac{c_{t+1}}{c_t} \right)$$

Applying The Lemma

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- Using the lemma (and using the unconditional version of the pricing equation):

$$\log \left(\mathbb{E}[m_{t+1}] \right) \approx \log(\beta) - \gamma \mathbb{E}[\Delta c_{t+1}] + \frac{1}{2}\gamma^2 \mathsf{Var}(\Delta c_{t+1})$$

- Let $r^f \equiv \log(R_f)$. Then:

$$r^f \approx -\log(\beta) + \gamma \mathbb{E}[\Delta c_{t+1}] - \frac{1}{2}\gamma^2 \mathsf{Var}(\Delta c_{t+1})$$

Two Economic Forces

$$r^f \approx -\log(\beta) + \gamma\mathbb{E}[\Delta c_{t+1}] - \frac{1}{2}\gamma^2\mathsf{Var}(\Delta c_{t+1})$$

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- In the US data, $\mathbb{E}[\Delta c] \approx 1.75\%$, with $\sigma^2(\Delta c) \approx 0.0165^2 = 0.00027225 \approx 0.027\%$
- The first term is two orders of magnitude larger than the second

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- The first term is two orders of magnitude larger than the second
- Bumping up γ to match the equity premium will make r^f explode!
- The average real risk-free rate in the US for 1947-2011 was 0.72% per year (see Campbell (2018))

Risk Aversion and Intertemporal Substitution

- Notice that γ controls the Arrow-Pratt measure of relative risk aversion;

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \qquad \text{RRA} \equiv -\frac{cu''(c)}{u'(c)} = \gamma$$

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- But it also controls the intertemporal substitution:

$$1 = \beta R_{t+1} \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \Rightarrow \frac{c_{t+1}}{c_t} = (\beta R_{t+1})^{1/\gamma}$$

$$\text{EIS} \equiv \frac{\partial \log(c_{t+1}/c_t)}{\partial \log R_{t+1}} = \frac{1}{\gamma}$$

- $\uparrow \gamma \implies$ more risk averse **and** less willing to shift consumption over time

Questions?

AI-Generated Summary

- Standard power utility with $\gamma < 10$ cannot generate 6% equity premium
- Increasing γ to match equity premium makes r^f explode
- CRRA confounds risk aversion and intertemporal substitution (EIS $= 1/\gamma$)
- Consumption too smooth: $\sigma(\Delta c) \ll \sigma(r^e)$

Readings and References i

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