terça-feira, 10 de novembro de 2020 15:45

Operadores Diferenciais:

$$\nabla : \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_n}, \ldots, \frac{\partial}{\partial x_n}\right)$$

$$\nabla : \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

Divergente: Seja F um campo vetorial no $\mathbb{R}^3$

$$F(x, y, z) = (F_1, F_2, F_3)$$

$$\nabla \cdot F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Rotacional: $\nabla \times F$

$$\operatorname{Rot} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

(I) $\nabla \cdot (\nabla \times F) = 0$

(II) $\nabla \times (\nabla f) = 0$

F é conservativo $\Rightarrow \operatorname{rot} F = 0$

Ex 1: $F = (y \cos x, x \operatorname{sen} y, 0)$

Mostre que F não é um campo gradiente

- Se F é um campo gradiente

$\Rightarrow \operatorname{rot} F = 0, \forall (x, y, z)$.

$$\begin{vmatrix} i & j & k \end{vmatrix}$$

$$\text{rot } F = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y\cos x & x \,\text{sen}\, y & 0 \end{vmatrix} = (0, 0, x\cos y - \cos x)$$

$\neq (0,0,0)$, $p = (1,1,0)$

$\Rightarrow \neq 0.$

rot $F \neq 0 \Rightarrow F$ não é um campo gradiente.

Ex 2: $\exists F$ t.q. rot $F = (x \,\text{sen}\, y, \cos y, z - xy) = G$?

$\nabla \cdot (\text{rot } F) = 0 \rightarrow \frac{\partial}{\partial x}(x \,\text{sen}\, y) + \frac{\partial}{\partial y}(\cos y) + \frac{\partial}{\partial z}(z - xy)$

$= \cancel{\text{sen}\, y} - \cancel{\text{sen}\, y} + 1 = 1 \neq 0.$

$\Rightarrow \nexists F$, t.q rot $F = G$.

Ex 3: $K = x^2 + y^2 + z^2$

$F = (x, y, z) / K^{p/2}$

a) div F?

$$\frac{K^{p/2} - x(p/\cancel{2})K^{\frac{p-2}{2}} \cdot \cancel{2}x}{K^p} = \frac{K^{\frac{p-2}{2}}(K - x^2 p)}{K^p}$$

$+$

$$\frac{\partial F_2}{\partial y} = \frac{K^{\frac{p-2}{2}}(K - y^2 p)}{K^p};$$

$+$

$$\frac{\partial F_3}{\partial z} = \frac{K^{\frac{p-2}{2}}(K - z^2 p)}{K^p}$$

$$= \frac{K^{\frac{p-2}{2}}(K - P(K))}{K^p}$$

b) Para quais valores de P, F é o rotacional de algum campo.

$\nabla\cdot(\text{rot}\, G) = 0$

$\Rightarrow k - P.k = 0 \Rightarrow \boxed{P = 1}$

* Será que toda função contínua é o divergente de algum campo vetorial?

$f(x,y,z) = f.$

$\exists\, G,$ t.q. $\text{div}\, G = f.$

$\Rightarrow G = \left(\int_0^x f(t,y,z)\,dt,\ 0, 0\right)$

existe;

$\frac{\partial}{\partial x}\left[\int_0^x f(t,y,z)\,dt\right] = f(x,y,z)$

$\Rightarrow \text{div}\, G = f(x,y,z) + \frac{\partial 0}{\partial y} + \frac{\partial 0}{\partial z}$

$= f(x,y,z) + 0 + 0 = f.$

16.5, 25) $\boxed{\text{div}(fF) = f\,\text{div}\,F + F\cdot\nabla f}$

f é uma função escalar

e $F = (F_1, F_2, F_3)$

- $fF = (fF_1, fF_2, fF_3)$

$\text{div}(fF) = (\text{I})\ \frac{\partial}{\partial x}(fF_1)$

$$= f_x F_1 + f \cdot F_{1x}$$

(II) $\frac{\partial (f F_2)}{\partial y} = f_y F_2 + f \cdot F_{2y}$

(III) $\frac{\partial}{\partial z}(f F_3) = f_z F_3 + f \cdot F_{3z}$

$\bullet := \nabla f \cdot F$

$\bullet := f \, \nabla \cdot F$

$+ \; : = \boxed{\nabla f \cdot F + f \cdot \nabla \cdot f}$

<u>Campo</u> <u>Conservativo</u>: (Independentes do Caminho)

$\oint_C F \cdot dr = 0$, para qualquer C

$\Leftrightarrow F = \nabla f$

$\Rightarrow \int_C F \cdot dr = \int_a^b \nabla f \cdot dt = f(b) - f(a)$

<u>Ex</u> 5: Determinar a função potencial e usar p/ calcular a integral:

$F = (yz, xz, xy + 2z)$, f t.q. $\nabla f = F$?

$\int yz \, dx = f + h(y,z)$

$= xyz + h(y,z)$

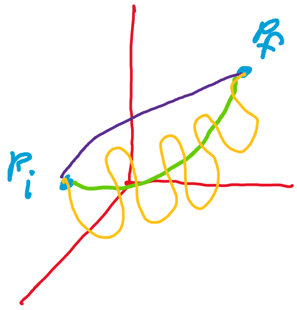
$\frac{\partial f}{\partial y} = xz + \frac{\partial h}{\partial y} \rightarrow = 0$ → h não depende de y.

$f = xyz + h(z)$

$$\frac{\partial f}{\partial z} = xy + \frac{\partial h}{\partial z} = 2z \rightarrow h = z^2$$

$$\rightarrow f = xyz + h(z) = xyz + z^2$$

$$C: \underset{a}{(1,0,-2)} \rightarrow \underset{b}{(4,6,3)}$$



$$\int_C F \cdot ds = \int_a^b F \cdot ds$$

$$= f(b) - f(a)$$

$$\cdot f(a) = 0 + 4 = 4$$

$$\cdot f(b) = 72 + 9 = 81$$

$$\int_C F \cdot dr = 81 - 4 = 77.$$

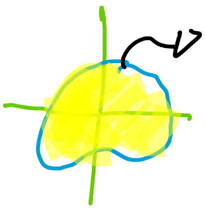
Obs: Se F é conservativo

$$\Rightarrow \frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$$

($\Leftarrow$) nem sempre é verdadeiro

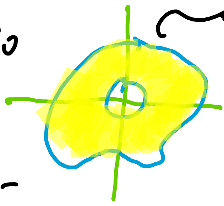
$$\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x} \not\Rightarrow F \text{ é conservativo.}$$

$$F = (-y, x)/(x^2+y^2)$$



Simplesmente conexa;

Basta calcular a integral de F no círculo unitário nas partes de cima e de baixo separadamente.



Não é simplesmente conexa