Exercícios

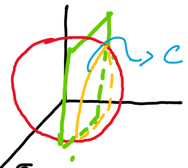
terça-feira, 13 de outubro de 2020 15:40

Bebel

2) $\int_C F\,ds$; $F=(x^2-y^2,\ z^2-x^2,\ y^2-z^2)$

C: $x^2+y^2+z^2=4$, $y=1$

$y=1$; $x^2+y^2+z^2$
$= x^2+1+z^2=4$
$\iff x^2+z^2=3$



$\sigma(\theta)=(\sqrt{3}\cos\theta,\ 1,\ \sqrt{3}\,sen\theta)$

$F(\sigma(\theta))=(3\cos^2\theta-1,\ 3\,sen^2\theta-3\cos^2\theta,\ 1-3\,sen^2\theta)$

$\int_C F\,ds$; $\int_\alpha^\beta F(\sigma(t))\cdot\sigma'(t)\,dt$

$\Rightarrow \sigma'(\theta)=(\sqrt{3}(-sen\theta),\ 0,\ \sqrt{3}\cos\theta)$

$F(\sigma(\theta))\cdot\sigma'(\theta)=(3\cos^2\theta-1,\ 3\,sen^2\theta-3\cos^2\theta,\ 1-3\,sen^2\theta)\cdot(-\sqrt{3}\,sen\theta,\ 0,\ \sqrt{3}\cos\theta)$

$= \sqrt{3}(\cos^2\theta\,sen\theta - sen\theta + \cos\theta - sen^3\theta)$

$\Rightarrow \int_C F\,ds = \sqrt{3}\Big[\int_0^{2\pi}\cos^2\theta\,sen\theta\,d\theta - \cancel{\int_0^{2\pi} sen\theta\,d\theta} \to -\int_0^{2\pi} sen\theta\,d\theta$
$+ \cancel{\int_0^{2\pi}\cos\theta\,d\theta} - \int_0^{2\pi} sen^3\theta\,d\theta\Big]$

$= \sqrt{3}.\Big[\int_0^{2\pi}\cos^2\theta\,\underline{sen\theta\,d\theta} + \int_0^{2\pi} sen^3\theta\,d\theta\Big]$

$u=\cos\theta$; $\theta=0 \to u=1$
$-du = sen\theta\,d\theta$; $\theta=2\pi \to u=1$

(I) $\int_1^1 -u^2\,du = 0$

$\Rightarrow \sqrt{3}\int_0^{2\pi} sen^3\theta\,d\theta =$; $\boxed{sen^2\theta = \frac{1-\cos 2\theta}{2}}$

$= \frac{\sqrt{3}}{2}\int_0^{2\pi}(1-\cos 2\theta)\,d\theta = \frac{\sqrt{3}}{2}\cdot 2\pi = \sqrt{3}\,\pi.$ //

4) $\oint_\sigma \frac{(x+y)dx-(x-y)dy}{x^2+y^2}$; $\sigma: x^2+y^2=a^2$

$\sigma(t)=(a\cos t,\ a\,sen t)$, $0\le t\le 2\pi$

$F=\frac{(a\cos t + a\,sen t,\ -a\cos t + a\,sen t)}{a^2}$

$\sigma'(t)=a(-sen t,\ \cos t)$

4. $\sigma'(t) = (-\cos \cancel{t}\,\text{sen} t - \text{sen}^2 t$
$\quad -\cos^2 t + \text{sen}\cancel{t}\cos t)$

$\quad = -(\cos^2 t + \text{sen}^2 t) = -1$

$\Rightarrow \int_0^{2\pi} -1\,dt = -2\pi$ ///

5) $\sigma: r^2 = a^2\cos 2\theta$

Com coordenadas polares:

$x = r.\cos\theta = a.\sqrt{\cos 2\theta}.\cos\theta$
$y = r.\text{sen}\theta = a.\sqrt{\cos 2\theta}.\text{sen}\theta$

$$F = \frac{(xy^2, -x^2y)}{x^2+y^2} = \frac{a^3(\cos\theta\,\text{sen}^2\theta\cos 2\theta^{3/2}, -\cos^2\theta\,\text{sen}\theta\cos(2\theta)^{3/2})}{a^2\cancel{\cos 2\theta}}$$

$= a.\sqrt{\cos 2\theta}.(P, Q).$

$$\sigma'(\theta) = a\left(\frac{1.2(-\text{sen}2\theta)}{2\sqrt{\cos 2\theta}}.(-\text{sen}\theta) - \text{sen}\theta\sqrt{\cos 2\theta},\ \frac{\text{sen}2\theta.\cos\theta}{\sqrt{\cos 2\theta}} + \cos\theta\sqrt{\cos 2\theta}\right)$$

$\frac{\sqrt{\cos 2\theta}}{\sqrt{\cos 2\theta}}$

$= \frac{a}{\sqrt{\cos 2\theta}}.(\sigma_1, \sigma_2) \Rightarrow F(\sigma(\theta)).(\sigma'(t))$

$\quad = a^2.(P.\sigma_1 + Q.\sigma_2)$ ///

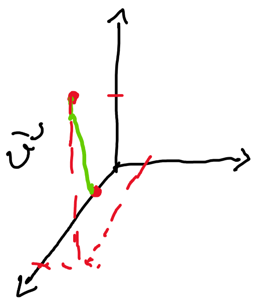
Livro 16.2

15) $\int_C z^2dx + x^2dy + y^2dz$;

C: reta de $(1,0,0) \to (4,1,2)$

$v = (a,b,c)$;

$P_0 = (1,0,0) + t.v$



$t = 1$; $1 + t.a = 4 \Rightarrow a = 3$
; $0 + t.b = 1 \Rightarrow b = 1$
; $0 + t.c = 2 \Rightarrow c = 2$

C: $P_0 + t.v = (1+3t, t, 2t)$; $0 \le t \le 1$

$C'(t) = (3,1,2)$

$F = (z^2, x^2, y^2) \Rightarrow F(C(t)) = (4t^2, 1+6t+9t^2, t^2)$

$$\Rightarrow F(c(t)) \cdot c'(t))$$
$$= 12t^2 + 1 + 6t + 9t^2 + 2t^2$$

$$\int_C F\,ds = \int_0^1 F(c(t)) \cdot c'(t)\,dt$$
$$= \int_0^1 (23t^2 + 6t + 1)\,dt = \frac{23}{3} + 3 + 1 = \frac{35}{3}. ///$$

47) F é constante; $c: x^2+y^2=1$

Prove que o trabalho realizado por F em C é nulo.

$F(K_1, K_2)$, $K_1, K_2 \in \mathbb{R}$ constantes.

$\int_C F\,ds = 0 \;=(\mathrm{I})$; $c(\theta) = (\cos\theta, \operatorname{sen}\theta)$
$c'(\theta) = (-\operatorname{sen}\theta, \cos\theta)$

$$F \cdot c'(\theta) = K_1 \cdot (-\operatorname{sen}\theta + K_2 \cos\theta$$

$$(\mathrm{I}) = \int_0^{2\pi} K_1(-\cancel{\operatorname{sen}}\theta)\,d\theta + \int_0^{2\pi} K_2 \cancel{\cos}\theta\,d\theta = 0$$

ou seja, em c; F realiza trabalho nulo.

(b) $F = (ax, by)$, a e b constantes ?

$$F_2(c(\theta)) = (a\cos\theta, b\operatorname{sen}\theta)$$

$$\Rightarrow F_2(c(\theta)) \cdot c'(\theta) = a\cos\theta \cdot (-\operatorname{sen}\theta) + b\operatorname{sen}\theta\cos\theta$$
$$= \operatorname{sen}\theta\cos\theta(b-a)$$
$$= \operatorname{sen}2\theta\left(\frac{b-a}{2}\right)$$

$$\Rightarrow \int_C F_2\,ds = \int_0^{2\pi} \operatorname{sen}(2\theta)\left(\frac{b-a}{2}\right)d\theta$$

$$= \left(\frac{b-a}{4}\right)\left(-\cos(2\theta)\Big|_0^{2\pi}\right) = \left(\frac{a-b}{4}\right)\cos u\Big|_0^{4\pi}$$

$$= 0 \cdot \left(\frac{a-b}{4}\right) = 0.$$

$F_2 = (ax, by)$ também realiza trabalho nulo em C. //