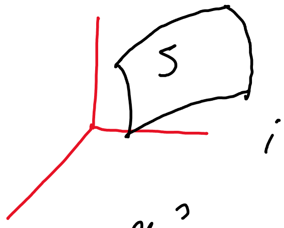
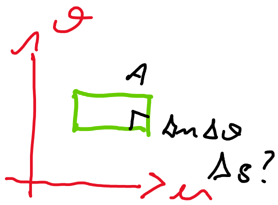
Área de Superfícies

terça-feira, 20 de outubro de 2020 16:00



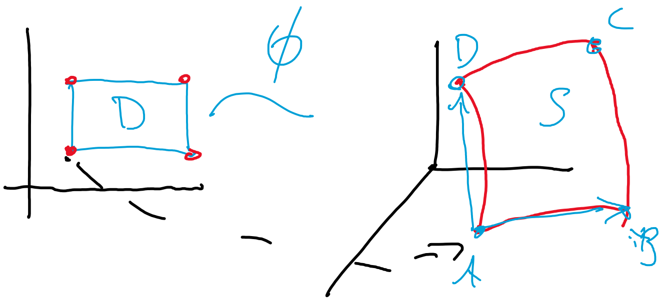


x?
y?

$a \leq u \leq b$
$c \leq v \leq d$

$$\phi(u,v) = (x(u,v), y(u,v), z(u,v))$$

$$\Delta s \Rightarrow \Delta u \, \Delta v$$



$$A = \phi(u,v), \quad B = \phi(u+\Delta u, v)$$

$$C = \phi(u+\Delta u, v+\Delta v), \quad D = \phi(u, v+\Delta v)$$

$$M = B - A, \quad N = D - A$$

$$M = \Delta u \cdot \phi_u, \quad N = \Delta v \cdot \phi_v$$

$$\Delta s = \Delta u \, \Delta v \, \|\phi_u \times \phi_v\|$$

$$\iint_D ds = \int_a^b \int_c^d \|\phi_u \times \phi_v\| \, du \, dv$$

* A integral de uma função escalar numa superfície, parametrizada por $\phi(u,v)$

$$\Rightarrow \iint_D ds = \boxed{\int_a^b \int_c^d \|\phi_u \times \phi_v\| \, du \, dv}$$

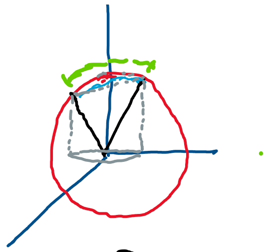
Ex $x^2+y^2+z^2=4$, $z=\sqrt{x^2+y^2}$

$z = \sqrt{x^2+y^2}$

$\Rightarrow z^2 = x^2+y^2$

$\rightarrow 2z^2 = 4$

$\rightarrow z = \sqrt{2}$, $z = 2\cos\varphi = \sqrt{2} \Rightarrow \cos\varphi = \frac{\sqrt{2}}{2}$



$\varphi \geq 0$; $\varphi = \pi/4$

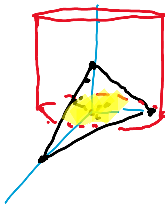
$\begin{cases} x = r\cos\theta \operatorname{sen}\varphi \\ y = r\operatorname{sen}\theta \operatorname{sen}\varphi \\ z = r\cos\varphi \end{cases}$; Coordenadas Esféricas

$\Rightarrow \boxed{\phi(\theta,\varphi) = (2\cos\theta\operatorname{sen}\varphi, \ldots) \quad 0 \leq \theta \leq 2\pi, \ 0 \leq \varphi \leq \frac{\pi}{4}}$

ou $z = \sqrt{4-x^2-y^2}$, $x^2+y^2 \leq 2$

Ex: $x + 2y + 3z = 1$; $x^2+y^2 = 3$

$\rightarrow \phi(x,y) = \left(x, y, \frac{1-x-2y}{3}\right)$



$N(-z_x, -z_y, 1)$

$z_x = -\frac{1}{3}$, $z_y = \frac{-2}{3}$; $\|\phi_x \times \phi_y\|$

$= N = \left|\left(1^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right)^{1/2}\right|$

$= \frac{1}{3}\sqrt{14}$; $N = \frac{\sqrt{14}}{3}$; $x^2+y^2 \leq 3$;

$\iint_D 1\,dS$; $\iint_R \left|\frac{\sqrt{14}}{3}\right| \cdot dx\,dy$

$= \frac{\sqrt{14}}{3} \cdot \iint_R 1 \cdot dx\,dy$; $A_R = \pi \cdot (\sqrt{3})^2 = 3\pi$

$\rightarrow \iint_D dS = \frac{\sqrt{14}}{3} \cdot 3\pi = \pi\sqrt{14}$ ///

Ex 3: $z = xy$, $x^2+y^2 = 1$

$\phi(x,y) = (x, y, xy)$;

$N = (-y, -x, 1) \Rightarrow \|N\| = \sqrt{x^2+y^2+1}$

$\Rightarrow \iint_D 1\,dS = \iint_R \sqrt{x^2+y^2+1}\,dx\,dy$

Usando coordenadas polares:

$\int_0^{2\pi}\int_0^1 r \cdot \sqrt{r^2+1}\,dr\,d\theta$; $u = r^2+1$; $r=0 \rightarrow u=1$

$du = r\,dr$, $r=1 \rightarrow u=2$

$$L_V = \int_0^{2\pi}\int_1^2 \sqrt{u}\, du\, d\theta = \int_0^{2\pi} d\theta \int_1^2 \sqrt{u}\, du$$

$$\to 2\pi \cdot \int_1^2 \sqrt{u}\, du = 2\pi \cdot \frac{2}{3} \cdot u^{3/2}\Big|_1^2$$

$$= \frac{4\pi}{3} \cdot (\sqrt{8} - 1) ///$$

16.2 / 47) b

$F(x) = KX$; $X = (x, y) . K$

$x^2 + y^2 = 1$; $c(t) = (\cos t, \operatorname{sen} t)$

$$\int_C F(X)\, ds = \int_0^{2\pi} \underbrace{F(c(t)) . c'(t)} dt$$

$c'(t) = (-\operatorname{sen} t, \cos t)$

$F(c(t)) = (K \cos t, K \operatorname{sen} t)$

$F . c' = K \cancel{\cos t . (-\operatorname{sen} t)} + K \operatorname{sen} \cancel{t \cos t}$

$\int_0^{2\pi} 0\, dt = 0$; sim, realiza trabalho nulo em $C: x^2+y^2=1$

16.6, 47) $y = 4x + z^2$, $X = 0$, $X = 1$, $z = 0$, $z = 1$

$\phi(X, Z) = (x, 4x + z^2, z)$

$N = (-4, 1, -2z)$; $\|N\| = \sqrt{16 + 1 + 4z^2}$

$$\iint_D ds = \int_0^1\int_0^1 \sqrt{16 + 4z^2}\, dx\, dz = \int_0^1 dx \int_0^1 \sqrt{17 + z^2}\, dz$$

$$= \int_0^1 \sqrt{17 + z^2}\, dz \; ; \; 17 \operatorname{tg}\theta = z$$

$17 \sec^2\theta\, d\theta = dz$

$$= \int_0^{\pi/4} \sec^2\theta \sqrt{17(1 + \operatorname{tg}^2\theta)}\, d\theta . 17$$

$$= 17\sqrt{17} \int_0^{\pi/4} \sec^2\theta . \sec\theta\, d\theta \Rightarrow \int_0^{\pi/4} \sec^3\theta\, d\theta = \sec^3\theta = \frac{\cos}{\cos^4\theta} \to (\cos^2\theta)^2$$

$\left\{ \frac{1}{2} \sec\theta . \operatorname{tg}\theta + \frac{1}{2} \ln|\sec\theta + \operatorname{tg}\theta| \right.$

$u = \operatorname{sen}\theta$

$du = \cos\theta\, d\theta$

$(1 - \operatorname{sen}^2\theta)^2 = (1 - u^2)^2$

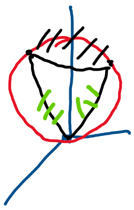
$$\int_0^{\sqrt{2}/2} \frac{du}{(1-u^2)^2} = \frac{Au + b}{(1-u^2)} + \frac{cu + d}{(1-u^2)^2}$$

$$\frac{(Au + b)(1 - u^2) + cu + d}{(1 - u^2)^2} ;$$

$$\boxed{\begin{array}{l} Au - Au^3 + b - bu^2 + Cu + d = 1 \\ u(A+C),\ u^2(-b),\ u^3(-A),\ b+d \\ -=0,\ B=0 \\ b+d=1;\ d=1,\ A+C=0 \to C=1 \end{array}}$$

61) $x^2+y^2+z^2=4z,\quad z=x^2+y^2$

$\cancel{x^2}+\cancel{y^2}+z^2=\cancel{4}\,3(x^2+y^2)$



$\to z^2=3z \to z=3$

$x^2+y^2+z^2=4 \iff x^2+y^2+(z-2)^2=4$

$z=3 \to x^2+y^2+(3-2)^2=4 \to x^2+y^2=3$

$\phi(x,y)=\left(x,y,2+\sqrt{4-x^2-y^2}\right)$

$N=\left(\frac{x}{\sqrt{4-x^2-y^2}},\frac{y}{\sqrt{4-x^2-y^2}},1\right)$

$\|N\|=\sqrt{\frac{x^2+y^2}{4-x^2-y^2}+1}$

$$\iint_D ds = \iint_R \sqrt{\frac{x^2+y^2}{4-x^2-y^2}+1}\,dx\,dy$$

Con coordenadas polares:

$$\int_0^{2\pi}\int_0^{\sqrt{3}} r\cdot\sqrt{\frac{r^2}{4-r^2}+1}\,dr\,d\theta = 2\int_0^{2\pi}\int_0^{\sqrt{3}} r\sqrt{\frac{1}{4-r^2}}\,dr\,d\theta$$

$4-r^2=u \to -\frac{du}{2}=r\,dr$

$$\to -2\int_0^{2\pi}\int_4^1 \frac{u+4}{\sqrt{u}}\,du = \dots = 4\pi$$