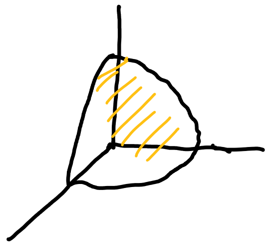
segunda-feira, 21 de setembro de 2020 17:58

SIMULADO

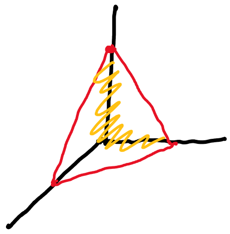
2) a) $\sqrt{x}+\sqrt{y}+\sqrt{z}=1$



$x=u^2,\ y=\vartheta^2,\ z=w^2$

$\sqrt{x}+\sqrt{y}+\sqrt{z}=1 \iff$

$u+\vartheta+w=1$ $\quad (u,\vartheta,w)\geq 0$



$$|S| = \left|\frac{\partial(x,y,z)}{\partial(u,\vartheta,w)}\right| =$$

$$= \begin{vmatrix} x_u & x_\vartheta & x_w \\ y_u & y_\vartheta & y_w \\ z_u & z_\vartheta & z_w \end{vmatrix} = \begin{vmatrix} 2u & 0 & 0 \\ 0 & 2v & 0 \\ 0 & 0 & 2w \end{vmatrix}$$

$$= |8u\vartheta w|$$

$\to$ $u+\vartheta+w=1$

$0\leq u\leq 1$

$0\leq \vartheta \leq 1-u$

$0\leq w\leq 1-u-\vartheta$

$$\int_0^1\int_0^{1-u}\int_0^{1-u-\vartheta} 8u\vartheta w \, du\, d\vartheta\, dw$$

$$= 4\int_0^1 \left[\int_0^{1-u} \cancel{u}\,u\,v\,(1-u-v)^2\,dv\right] du$$

$$\int_0^{1-u} v(1-u-v)^2\,dv \quad , \; p = 1-u-v$$
$$-dp = -du$$

$$-\int_{1-u}^{0} (1-u-p)\,p^2\,dp$$

$$\int_0^{1-u} (p^2 - p^2 u - p^3)\,dp$$

$$\frac{p^3}{3} - \frac{p^3 u}{3} - \frac{p^4}{4}\Bigg|_0^{1-u}$$

$$= (1-u)^3\left(\frac{1}{3} - \frac{u}{3} - \frac{(1-u)}{4}\right)$$
$$= \frac{(1-u)^4}{12} = i$$

$$4\int_0^1 u \cdot i\,du = \int_0^1 \frac{u(1-u)^4}{3}\,du$$

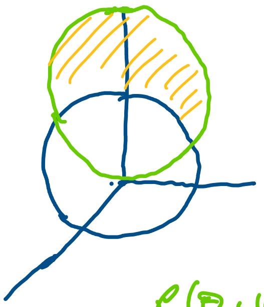
$$1-u = q \;;\; -\int_1^0 \frac{(1-q)}{}q^4\,dq$$

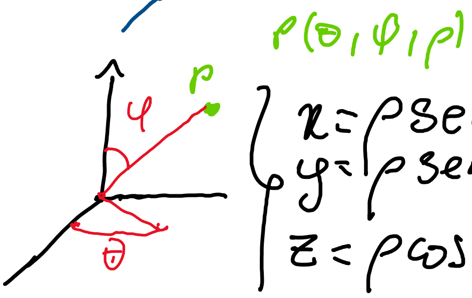
$$= \int_0^1 (-q^5 + q^4)\,dq = \left(-\frac{1}{6} + \frac{1}{5}\right)\frac{1}{3}$$
$$= \frac{1}{3}\cdot\left(\frac{1}{30}\right) = \frac{1}{90} \cdot ///$$

(b) $\underbrace{x^2+y^2+z^2}_{b_1} \geq 4 \;;\; \underbrace{x^2+y^2+z^2}_{b_2} \leq 4z$

b_2: $x^2 + y^2 + z^2 - 4z \leq 0$

$\Rightarrow x^2+y^2+(z-2)^2 \leq 4$





$\begin{cases} x = \rho \operatorname{sen}\phi \cos\theta \\ y = \rho \operatorname{sen}\phi \operatorname{sen}\theta \\ z = \rho\cos\phi \end{cases}$

$|J| = \rho^2 \operatorname{sen}\phi$

$b_1: x^2+y^2+z^2 = \rho^2 \geq 4; \ \rho \geq 2$

$b_2: x^2+y^2+z^2 \leq 4z$

$\hookrightarrow \rho^2 \leq 4\rho\cos\phi \to \rho \leq 4\cos\phi$

$2 \leq \rho \leq 4\cos\phi \to \cos\phi \geq \frac{1}{2}$

$\to \phi = \frac{\pi}{3}; \quad 0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \pi/3$

$2 \leq \rho \leq 4\cos\phi$

$$\int_0^{2\pi}\int_0^{\pi/3}\int_2^{4\cos\phi} 1.\ \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

$$2\pi . \int_0^{\pi/3} \operatorname{sen}\phi \left(\frac{1}{3}.(64\cos^3\phi - 8)\right) d\phi$$

$$= \frac{2\pi}{3}\int_0^{\pi/3} \operatorname{sen}\phi\,(64\cos^3\phi - 8)\, d\phi$$

$\cos\phi = u$

$-du = \operatorname{sen}\phi \, d\phi$

$$\hookrightarrow -\frac{2\pi}{3}\int_1^{1/2}(64u^3-8)\,du$$

$$= \frac{2\pi}{3}\int^1 64u^3 - \frac{2\pi}{3}\int^1 8\,du$$

$$3 \quad 1/2 \qquad\qquad 3 \quad 1/2$$

$$= \frac{2\pi}{3}\left(16u^4\left(\frac{1}{1/2} - 4\right)\right)$$

$$= \frac{2\pi}{3}\left(\underbrace{16 - 1}_{11} - 4\right) = \frac{22\pi}{3}. \quad ///$$

4) $f(x,y) = x\,g(x^2-y^2)$

Plano tangente em $P = (a, a, f(a,a))$

$: f_x(a,a)(x-a) + f_y(a,a)(y-a) + f(a,a)$

$\Rightarrow f_x = g(x^2-y^2) + x\,2x\,g'(x^2-y^2)$

$f_y = -2yx\,g'(x^2-y^2)$

$f_x(a,a) = g(0) + 2a^2 g'(0)$

$f_y(a,a) = -a^2 g'(0)$

$$z = a.\cancel{g(0)} + [\cancel{g(0)} + 2a^2 g'(0)](x - \cancel{a})$$

$$- 2a^2 g'(0)(y - \cancel{a})$$

$$= 2a^2 \cancel{x} g'(0) - 2a^2 g'(0) y$$

$$+ x\,g(0). \;; \text{em } (0,0) \;; z=0$$

ou seja, $(0,0,0)$ satisfaz!

4) a) $f(x,y) = 2y^2 - x$

$f(1,0) = 1$, $\qquad F_x = -1 \neq 0$

$f: 2y^2 - x$; $g = x^2 + y^2 = 1$

$\to \nabla f(-1, 4y) = \lambda \underset{\nabla g}{(2x, 2y)}$

$\to \lambda = -1/2x \to x = -1/4$

$\to 4y = \lambda 2y \to \lambda = 2$

$f(x,0) = -x$; $\to x = 0$

$f(0,y) = 2y^2$; $y = 1$

$f(0,0) = 0$; $\boxed{f(1,0) = 2}$ → Máximo

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(6) $A = \sqrt{s(s-x)(s-y)(s-z)}$,

$s = p/2$; $A =$ área

$x + y + z = p$

maximar $A \to$ max. $A^2 \Rightarrow \frac{A^2}{s}$

$\frac{A^2}{s} = (s-x)(s-y)(s-z)$

$\nabla \frac{A^2}{s} = (-(s-y)(s-z), -(s-x)(s-z), -(s-x)(s-y)) = \lambda(1,1,1)$

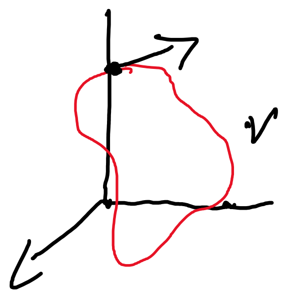
$\to \lambda = 0 \to x \vee y \vee z = s$

$s = p/2$ $\quad x + y + z = p \to x = y + z$

$(5-x)(5-y) = (5-y)(5-z) \to x = z$

$\to \boxed{z = y = x}$

A1 2019 |



$\to r(t) = (t, y(t), z(t))$

$r'(t) = (t', y'(t), z'(t))$

$2x + 3y + 4z + 2 = 0$

$\to (4z)^2 = -2 - 2x - 3y$

$z^2 = x^2 + y^2 \to 16z^2 = 16x^2 + 16x^2$

$= (2 + 2x + 3y)(2 + 2x + 3y)$

$= 4 + 8x + 12y + 12xy + 4x^4$
$+ 9y^2$

$\to 12x^2 + 7y^2 = 4 + 8x + 12y + 12xy$

Derivando implicitamente: (x)

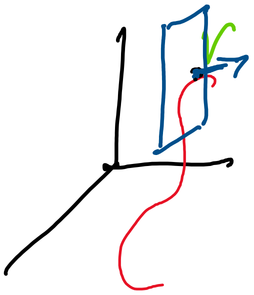
$24x + 14yy' = 8 + 12y' + 12y + 12xy'$

$\to y'(14y - 12x - 12) = 8 + 12y$

$\to y' = 8 + 12y \qquad \therefore P = (3, 4, -5)$

$$y = \frac{\ldots}{14y - 12x - 12}$$

$$y'(3,4) = \frac{8+48}{56-36-12} = \frac{56}{8} = 7$$



$; \ r(t) = (t, y(t), z(t))$

$t = x$

$r'(t)$ tangente à curva

$t = x \rightarrow x' = 1$
$y'(t=x) = 7$
z'?

$$z^2 = x^2 + y^2 - 6$$

$$z' = \frac{2x + 2yy'}{2z} \text{ em } (3,4,-5)$$

$$\rightarrow z' = \frac{6 - 8.7}{-10} = \boxed{5}$$

$$r'(t) = (1, 7, -5) \ ; \ n: x + 7y + 5z = d$$

Vamos encontrar d subs as coordenadas por P.

$$\rightarrow 3 + 7.4 + 5.(-5) = d = 6$$

$$\boxed{n: x + 7y + 5z = 6}$$

(3) $4x^4 + 4y^4 = 17x^2y^2$

$y(x) \ ; \ 16x^3 + 16y^3y' = 34xy^2 + 34x^2yy'$

$\rightarrow y'(16y^3 - 34x^2y) = 34xy^2 - 16x^3$

$$y' = \frac{34xy^2 - 16x^3}{16y^3 - 34x^2y} \ ; \text{ em } (2,4):$$

$$\rightarrow y' = \frac{34.2.16 - 16.8}{16.64 - 34.4.4} = 2$$

$$x' = \frac{1}{y'} \rightarrow x' = 1/2$$

$dy = 1{,}9 - 2 = -0{,}1$

$\rightarrow L(y) = 2 - 0{,}1 \cdot \frac{1}{2} \qquad = -0{,}05$

$\boxed{\cong 1{,}995}$ ///

$\rightarrow$ (6) $z = x+y$, $\boxed{xy = 1 , \; xy = 2 ,}$

$\boxed{y = x , \quad y = 2x}$, $x > 0$, $y > 0$.

$\left. \begin{array}{l} xy = u \\ \frac{y}{x} = v \end{array} \right\{ \quad \begin{array}{l} 1 \leq u \leq 2 \\ 1 \leq v \leq 2 \end{array}$

$z = x+y \left\{ \begin{array}{l} x = u/y \\ \frac{y}{u/y} = \frac{y^2}{u} = v \rightarrow y = \sqrt{uv} \end{array} \right.$

$x\sqrt{uv} = v \rightarrow x = \sqrt{\frac{v}{u}}$

$\rightarrow z = \sqrt{uv} + \sqrt{\frac{v}{u}}$ i

$\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \left| \frac{\partial(u,v)}{\partial(x,y)} \right|^{-1}$

$\rightarrow \begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix} = \left| \frac{y}{x} + \frac{y}{x} \right| = 2v$

$\int_1^2 \int_1^2 \left(\sqrt{uv} + \sqrt{\frac{v}{u}} \right) \cdot \frac{1}{2v} \, du \, dv$

Simulado:

1) a) $\int_0^1 \left[\int_{\sqrt{x}}^1 \frac{3y}{\sqrt{4-2y^4}} \, dy \right] dx$

$y=\sqrt{x}$

$\to x=y^2$

$0\le y\le 1$

$0\le x\le y^2$

$$\int_0^1\int_0^{y^2} \frac{3y}{\sqrt{4-2y^4}}\,dx\,dy=\int_0^1 \frac{3y\cdot y^2}{\sqrt{4-2y^4}}\,dx$$

$u=4-2y^4$

$-\frac{du}{8}=y^3\,dy$

$$=-\int_4^2 \frac{du}{\sqrt{u}}=$$

$$\int_2^4 \frac{du}{\sqrt{u}}=2\cdot\left(\sqrt{u}\,\Big|_2^4\right)=4-2\sqrt{2}$$

$\boxed{S_1:\ 3}$ $f(x,y)=\begin{cases}\dfrac{xy^3}{x^2+y^2}, & (x,y)\neq 0\\ 0, & (x,y)=(0,0)\end{cases}$

$$f_x=\frac{y^3(x^2+y^2)-xy^3\cdot 2x}{(x^2+y^2)^2}$$

$$f_y=\frac{3xy^2(x^2+y^2)-xy^3\cdot 2y}{(x^2+y^2)^2} \to \lim_{y\to 0} f_y=0$$

$(0,0)$

em $(0,0)$ é cont.?

$$f_x(0,0)=\lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h}$$

$$=\lim_{h\to 0}\frac{f(h,0)}{h}=\lim_{h\to 0}\frac{h\cdot 0^2}{h^2+0}=\frac{0}{h^2}=0$$

$$f_y=\lim_{h\to 0}\frac{0\cdot h^2}{0+h^2}=\frac{0}{h^2}=0$$

$$\lim_{(x,0)\to(0,0)} \frac{y^2(x^2+y^2)-xy^2\,2x}{(x^2+y^2)^2}$$

$$=\frac{y^2(x^2+y^2-2x^2)}{(x^2+y^2)^2}=\frac{y^2(y^2-x^2)}{(x^2+y^2)^2}$$

$= \frac{y^2}{x^2+y^2} \cdot \frac{y^2-x^2}{x^2+y^2}$

$; |f_y - 0| < \varepsilon.$

$\to \left|\frac{y^3}{x^2+y^2}\right| \left|\frac{y^2-x^2}{x^2+y^2}\right| < \varepsilon$

dado que $\sqrt{x^2+y^2} < \delta$

$\to \leq y \underbrace{\left|\frac{y^2-x^2}{x^2+y^2}\right|} \leq y = \sqrt{y^2}$

$\leq \sqrt{y^2+x^2} < \delta$

tome $\boxed{\varepsilon = \delta.}$

Provamos que $\boxed{\lim_{x\to 0} f_x = 0 = f_x(0)}$

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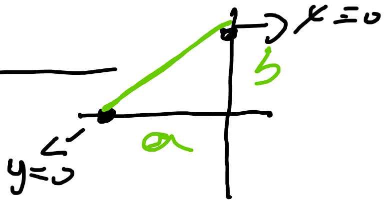
(5) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$;



$\nabla f = \left(\frac{2x_0}{a^2}, \frac{2y_0}{b^2}\right)$; $t: \frac{2x_0}{a^2}x + \frac{2y_0}{b^2}y = 1$

$x=0 \to \frac{y_0 y}{2b^2} = 1 \to y = \frac{1b^2}{2y_0}$

$y=0 \to x = \frac{a^2}{2x_0}$

$A = x'y' = \frac{a^2b^2}{4x_0y_0}$; $\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1$

$\to g =$

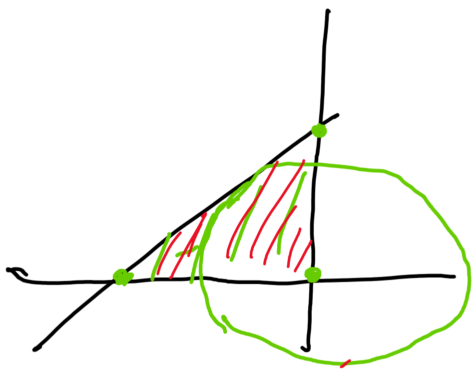
$\nabla A = \frac{a^2b}{4}\left(\frac{-1}{x_0^2 y_0}, \frac{-1}{x_0 y_0^2}\right) = \lambda\left(\frac{2x_0}{a^2}, \frac{2y_0}{b^2}\right)$

$\to \frac{x_0 y_0^2}{x_0^2 y_0^2} = \frac{\cancel{2}x_0}{a^2} \Big/ \frac{\cancel{2}y_0}{b^2} = \frac{x_0}{y_0}\frac{b^2}{a^2}$

$$= \frac{y_0}{x_0} = \frac{x_0 b^3}{y_0 a^2} \to y_0^2 = \frac{x_0^2 b^2}{a^2}$$

$$\to \frac{x_0^3}{a^3} + \frac{x_0^2 b^2}{a^2 b^2} = \frac{2x_0^2}{a^2} = 1$$

$$\to x_0^2 = \frac{a^2}{2} \to \boxed{\begin{array}{l} x_0 = \pm \frac{a}{\sqrt{2}} \\ \hookrightarrow y_0 = \pm \frac{b}{\sqrt{2}} \end{array}}$$



$\underbrace{f(x,y)}$ $\to$ $\underbrace{\text{elipse}}$