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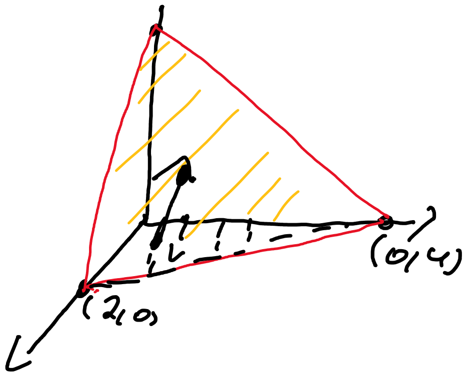
15.7) 19)

$S: 2x + y + z = 4$



$0 \leq x \leq 2 \; ; \; y \leq 4-2x$

$0 \leq y \leq 4-2x$

$2x + y + z = 4 \iff z = 4 - 2x - y$

$0 \leq z \; 4 - 2x - y$

$$\int_0^2 \left[\int_0^{4-2x} \int_0^{4-2x-y} |1| \, dz \, dy \, dx \right]$$

$$\int_0^2 \left[\int_0^{4-2x} (4-2x-y) \, dy \, dx \right]$$

$$\Rightarrow \int_0^2 \left(4y - 2xy - \frac{y^2}{2} \Big|_0^{4-2x} \right) dx$$

$$= \int_0^2 \left(16 - \cancel{8x} - 2\cancel{x}(\cancel{4} - 2x) - \frac{(4-2x)^2}{2} \right) dx$$

$$= \int_0^2 (8 - 12x - 2x^2) \, dx$$

$$= \left(8x - 6x^2 - \frac{2}{3} x^3 \Big|_0^2 \right) =$$

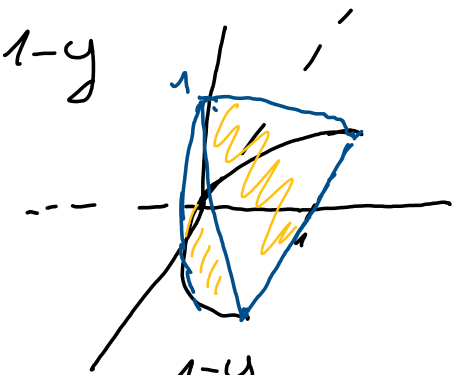
$16 - 24 - 16 = -8 - 16 = -40$

$\overline{3} \quad - \quad \overline{3} \quad \overline{3}$

21) $y = x^2, \quad z = 0, \quad y + z = 1$

$\left.\begin{array}{l} 0 \le y \le 1 \\ -1 \le x \le 1 \end{array}\right\} \; x^2 \le y \le 1$

$z = 1 - y$



$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} 1 \cdot dz\,dy\,dx$$

$$= \int_{-1}^{1}\int_{x^2}^{1} (1-y)\,dy\,dx$$

$$= \int_{-1}^{1} \left(y - \frac{y^2}{2}\Big|_{x^2}^{1}\right) dx$$

$$= \int_{-1}^{1} \left(1 - x^2 - \frac{1}{2} - (-x^4)\right) dx$$

$$= \int_{-1}^{1} \left(\frac{1}{2} - x^2 - \frac{x^4}{2}\right) dx$$

$$= \left(\frac{x}{2} - \frac{x^3}{3} - \frac{x^4}{2}\Big|_{-1}^{1}\right)$$

$$= \left(\frac{1}{2} \overline{- \frac{1}{3} - \frac{1}{2}}\right) - \left(-\frac{1}{2} + \frac{1}{3} - \frac{1}{2}\right)$$

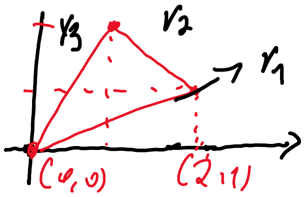
$$= 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

15.10) 15) $\iint_{\Omega} (x - 3y)\,dv$

$R_1 : (0,0)\,,\ (1,2)\,,\ (2,1)$

$x = 2u + v\,, \quad y = u + 2v$

$$|J| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3$$



$r_1 : \ y = \frac{x}{2}\,, \quad r_2 : \ y - 2 = -(x-1)$

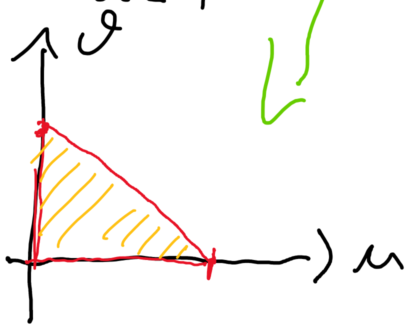
$\rightarrow y = -x + 3$

$r_3 : \ y = 2x$

• $y = \frac{x}{2} \rightarrow u + 2v = \frac{1}{2}(2u + v)$

$\rightarrow v = 0$

$im(r_1) : \ v = 0\,, \quad 0 \le u \le 1$



$im(r_2) : ? \ ; \ y = -x + 3$

$\rightarrow u + 2v = -(2u + v) + 3$

$\rightarrow u + 2v = -2u - v + 3$

$\rightarrow u + v = 1$

$\rightarrow v = 1 - u$

$r_3 : \ y = 2x \ ; \quad u + 2v = 4u + 2v$

$\rightarrow u = 4u \ ; \ u = 0$

$0 \le u \le 1$

$0 \le v \le 1 - u$

$$\int_0^1 \int_0^{1-u} (2u + v - 3u - 6v)\, dv\, du$$

$$= \int_0^1 \left[2uv - v^2 - 3uv - 3v^2 \right]_0^{1-u} \Big) \, du$$

$$\int_0^1 \left(2u(1-u) - \frac{1-2u+u^2}{2} - \cancel{3u} + 3\cancel{u^2} - 3(1-2\ldots\right.$$

$$= \int_0^1 \left(2u - 2u^2 + u - \frac{7}{2} - u\right) du$$

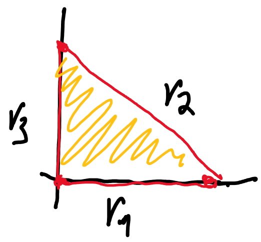
$$\int_0^1 \left(3u - 2u^2 - \frac{7}{2}\right) du$$

$$= \frac{3}{2} - 1 - \frac{7}{2} = \boxed{-1}$$

2) f é derivável em $[0,1]$

R: $(0,0)$, $(1,0)$, $(0,1)$

$$\iint_R f(x+y)\,dA = \int u f(u)\,du$$



; $x+y=u$, $x=v$

r_1: $y=0$, $0 \le x \le 1$

$0 \le u \le 1$, $u=v$
$0 \le v \le 1$

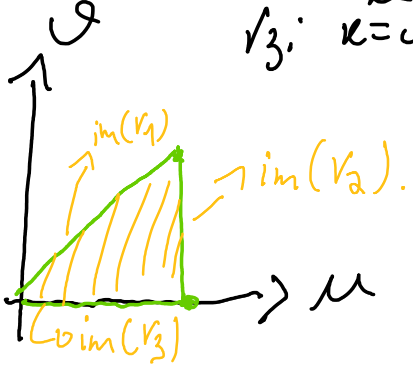
$$J = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$\to |-1| = 1$

r_2: $y = 1-x$

$\to x+y = 1 = u$
$x = x$

r_3: $x=0$, $v=0$
$0 \le u \le 1$



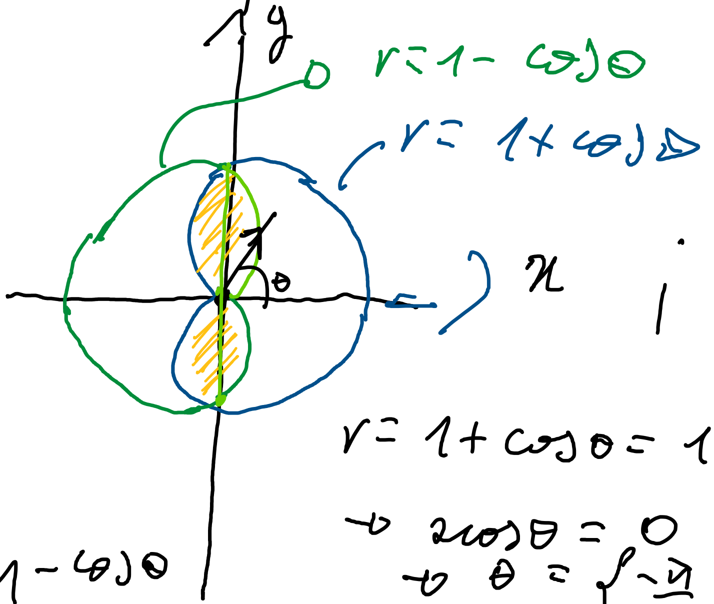
$0 \le u \le 1$
$0 \le v \le u$

$$\Rightarrow \iint f(x+y)\,dA = \int \int^{u} f(u)\,|J|\,dA$$

$$\to \iint_R f(x+y)\,dA = \int_0 \int_0 f(u)\cdot|J|\,dA$$

$$= \int_0^1 u f(u)\,du$$

$r = 1+\cos\theta \; , \; r = 1-\cos\theta$



$; \; |J| = r$

$r = 1+\cos\theta = 1-\cos\theta$

$\to 2\cos\theta = 0$

$\to \theta = \left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

$$2\cdot\int_{-\frac{\pi}{2}}^{\pi/2} \int_0^{1-\cos\theta} r\cdot|1|\,dr\,d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\pi/2} (1 - 2\cos\theta + \cos^2\theta)\,d\theta$$

$$= \pi - 2\left(\operatorname{sen}\theta \Big|_{-\pi/2}^{\pi/2}\right) + \int_{-\frac{\pi}{2}}^{\pi/2} \cos^2\theta\,d\theta$$

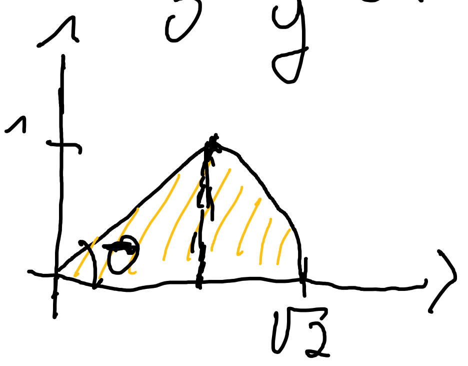
$$= \pi - 2.2 + \underbrace{\int_{-\frac{\pi}{2}}^{\pi/2} \left(\frac{1+\cos 2\theta}{2}\right) d\theta}_{(I)}$$

$$I: \frac{1}{2}\pi + \frac{1}{4}\int_{-\pi}^{\pi} \cos u\,du = \frac{\pi}{2} + 0.$$

$$\iint_R f = \pi - 4 + \frac{\pi}{2} = \boxed{\frac{5}{2}\pi - 4}$$

$$31) \int^1 \int^{\sqrt{2-y^2}} (x \ldots)\,dx\,dy$$ $\to$ círculo de raio $\sqrt{2}$

$\int_0 \int_y (x+y)\,dx\,dy$



$y = x \;;\; r\,\mathrm{sen}\theta = r\cos\theta$

$\to \mathrm{sen}\theta = \cos\theta$

$\to \mathrm{tg}\,\theta = 1$

$\theta = \pi/4$

$0 \leq \theta \leq \pi/4$

$x = \sqrt{2-y^2} \;;\; x^2 = 2 - y^2$

$\to x^2 + y^2 = 2$

$0 \leq r \leq \sqrt{2}$

$$\int_0^{\pi/4} \int_0^{\sqrt{2}} r.(r\cos\theta + r\,\mathrm{sen}\theta)\,dr\,d\theta$$

$$= \frac{1}{3} \int_0^{\pi/4} (\cos\theta + \mathrm{sen}\theta)\,d\theta \,.\, 2\sqrt{2}$$

$$\Rightarrow \int_0^{\pi/4} \cos\theta\,d\theta = \mathrm{sen}\theta \Big|_0^{\pi/4} = \frac{\sqrt{2}}{2} \;;$$

$$\int_0^{\pi/4} \mathrm{sen}\theta\,d\theta = -\cos\theta \Big|_0^{\pi/4} - \frac{\sqrt{2}}{2} + 1$$

$$\to \iint_R f\,dA = \frac{2\sqrt{2}}{3}$$