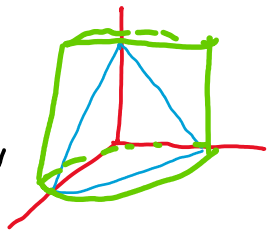
# Teste-2

terça-feira, 3 de novembro de 2020 15:43

1) $S: x^2+y^2=4;\ z=0,\ z\geq 0$

$x+y+z=2$

$f: A.$



$\phi(\theta, z) = (2\cos\theta, 2\operatorname{sen}\theta, z)$

$\frac{\pi}{2} \leq \theta \leq 2\pi, \quad 0 \leq z \leq 2-2(\cos\theta+\operatorname{sen}\theta)$

$z \geq 0 = 2(1-(\cos\theta+\operatorname{sen}\theta))$

$\Rightarrow \cos\theta + \operatorname{sen}\theta \geq 0, \quad [0, \pi/2]$

$\cos^2\theta + \operatorname{sen}^2\theta = 1 < \cos\theta + \operatorname{sen}^2\theta$

$< \cos\theta + \operatorname{sen}\theta$

$\Rightarrow \cos\theta > \cos^2\theta$

$\cos\theta + \operatorname{sen}\theta < 1$ ok

$\cos\theta + \operatorname{sen}\theta > 1$

II, III, IV

$\rightarrow$ Dentro de S.

$< 1$ $< 1$

$\phi_\theta = 2(-\operatorname{sen}\theta, \cos\theta, 0)$

$\phi_z = (0,0,1) \} N = \phi_\theta \times \phi_z$

$= \begin{vmatrix} 2\cos\theta & 0 & -2\operatorname{sen}\theta \\ 0 & 1 & 0 \end{vmatrix} \begin{matrix} \times \\ \times \\ \times \end{matrix} \begin{matrix} 2\cos\theta \\ 0 \end{matrix}$

$= (2\cos\theta, 2\operatorname{sen}\theta, 0) = N$

$\Rightarrow |N| = 2\sqrt{\cos^2\theta + \operatorname{sen}^2\theta} = 2.$

$\Rightarrow \int_{\pi/2}^{2\pi} \int_0^{2-2(\cos\theta+\operatorname{sen}\theta)} (A.2)\, dz\, d\theta =$

$6A\pi + 8A$

2) $n = 1, 2, \dots$ ; $n \in \mathbb{N}$.

$$\int_{\Gamma} y\,dx + (3y^3 - x)\,dy + z\,dz,$$

$\sigma(t) = (t, t^n, 0)$

$F = (y, 3y^3 - x, z)$

$F(\sigma(t)) = (t^n, 3t^{3n} - t, 0)$

$\sigma'(t) = (1, nt^{n-1}, 0)$

$\Rightarrow F \cdot \sigma' = t^n + 3nt^{4n-1} - nt^n$

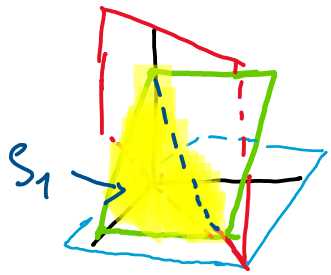
$$\Rightarrow \int_{\Gamma} F \cdot ds = \int_a^b F \cdot \sigma' dt$$

$$\Rightarrow \int_0^1 \left[ t^n + 3nt^{4n-1} - nt^n \right] dt$$

$$= \dots = \left[ \frac{7-n}{4n+4} \right]_0^1 \;/\!/\!/\; n \to \infty \longrightarrow -\frac{1}{4}$$

3) $\iint_S xy\,ds$ ; $z = 0$, $y = 0$

$x + z = 1$, $x = y$




$S_1$: $\phi(x, y) = (x, y, 1 - x)$,

$0 \le x \le 1$, $0 \le y \le x$

$\phi_x = (1, 0, -1)$ ; $\phi_y = (0, 1, 0)$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} = (1, 0, 1) = N$$

$\Rightarrow |N| = \sqrt{2}$

$S_2: \phi(x,z) = (x,0,z)$

$0 \le x \le 1, \quad 0 \le z \le 1-x$

$\phi_x = (1,0,0), \quad \phi_z = (0,0,1)$

$\phi_x \times \phi_z = (0,1,0) = N \Rightarrow \|N\| = 1$

$S_3: \phi(x,y) = (x,y,0)$

$0 \le x \le 1, \quad 0 \le y \le x \Rightarrow N = (0,0,1)$

$|N| = 1$

$S_4: \phi(x,z) = (x,x,z)$

$0 \le x \le 1, \quad 0 \le z \le 1-x$

$\phi_x = (1,1,0), \quad \phi_z = (0,0,1)$

$\phi_x \times \phi_z = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = (1,-1,0)$

$\Rightarrow |N| = \sqrt{2}$

I) $\int_0^1 \int_0^x xy\sqrt{2}\, dy\, dx = \sqrt{2}/8$

II) $\int_0^1 \int_0^{1-x} \sqrt{2} \cdot x \cdot 0\, dz\, dx = 0$

III) $\int_0^1 \int_0^x xy\, dy\, dx = 1/8$

IV) $\int_0^1 \int_0^{1-x} x^2\, dz\, dy = \sqrt{2}/12$

$\Rightarrow \iint_S xy\, ds = \iint_{S_1 \cup S_2 \cup S_3 \cup S_4} xy\, dS_i$

$= \sqrt{2}/8 + 0 + 1/8 + \sqrt{2}/12$

$= \boxed{\frac{3\sqrt{2}+3}{24}}$

) $T(x,y,z) = 3x^2 + 3z^2$

$S: x^2 + z^2 = 2$ ; $0 \le y \le 2$

$n = 1$

$= -\nabla T = -(6x, 0, 6z)$

$i(\theta, y) = (\sqrt{2}\cos\theta, y, \sqrt{2}\operatorname{sen}\theta)$

$\phi_\theta = (-\sqrt{2}\operatorname{sen}\theta, 0, \sqrt{2}\cos\theta)$

$\phi_y = (0, 1, 0)$ ; $\phi_\theta \times \phi_z$

$0 \quad \sqrt{2}\cos\theta \quad -\sqrt{2}\operatorname{sen}\theta \quad 0$

$1 \quad 0 \quad 0 \quad 1$

$= (-\sqrt{2}\cos\theta, 0, -\sqrt{2}\operatorname{sen}\theta)$

$\nabla N = (\sqrt{2}\cos\theta, 0, \sqrt{2}\operatorname{sen}\theta)$

$F(\phi) = -6\sqrt{2}(\cos\theta, 0, \operatorname{sen}\theta)$

$\nabla F \cdot N = -6\sqrt{2} \cdot \sqrt{2}(\cos\theta, 0, \operatorname{sen}\theta) \cdot (\cos\theta, 0, \operatorname{sen}\theta)$

$= -6 \cdot 2 \cdot 1$

$\nabla \int_0^{2\pi} \int_0^2 -12 \, dy \, d\theta = \boxed{-48\pi}$