Exercícios

terça-feira, 11 de agosto de 2020 15:28

9) $\lim\limits_{(x,y)\to(0,0)} \frac{x^4-4y^2}{x^2+2y^2}$; $y=x^2$

$\Rightarrow \lim\limits_{(x,x^2)\to(0,0)} \frac{x^4-4x^4}{x^2+2x^4} = \frac{x^2-4x^2}{1+2x^2} = 0$ ///

$\to x=y$; $\lim\limits_{(x,x)\to(0,0)} \frac{x^4-4x^2}{x^2+2x^2} = \frac{x^2(x^2-4)}{x^2(3)} = \frac{x^2-4}{3}$

$\to \lim\limits_{(x,x)\to(0,0)} \frac{x^2-4}{3} = -\frac{4}{3} \neq 0$. Logo; $\therefore$ limite não existe. ///

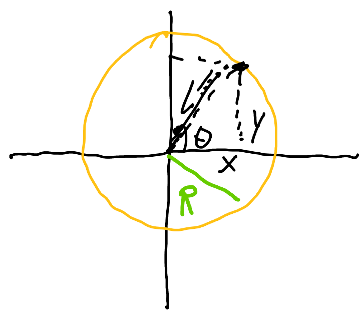
15) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2 y e^y}{x^4+4y^2}$; $x=0$

$\to \lim\limits_{(0,y)\to(0,0)} \frac{0ye^y}{4y^2} = 0$. $y=x^2$

$$\lim_{(x,x^2)\to(0,0)} \frac{x^2x^2e^{x^2}}{x^4+4x^4} = \lim_{(x,x^2)\to(0,0)} \frac{e^{x^2}}{5} = \frac{1}{5} \neq 0.$$

limite não existe.

41) $\lim\limits_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2} =$



Coord. polares.

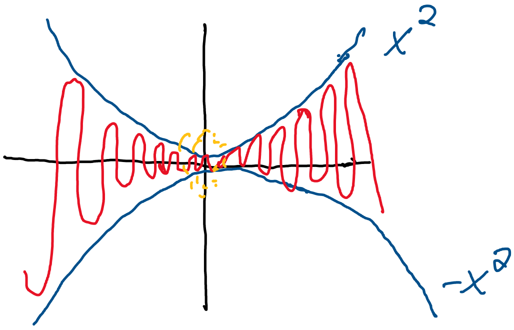
$x = r\cos\theta$
$y = r\,\mathrm{sen}\,\theta$ $\to x^2+y^2 \to r^2$

$(x,y)\to(0,0)$; $r\to 0$

$$\lim_{r\to0} \frac{e^{-r^2}-1}{r^2} \overset{H}{=} \lim_{r\to0}\frac{-\cancel{2r}e^{-r^2}}{\cancel{2r}} = \lim_{r\to0} -e^{-r^2} = -1.$$

$$\lim_{(x,y)\to 0} xy.\operatorname{sen}(1/xy)\ ;\ x=y$$

$$\lim_{(x,y)\to(0,0)} x^2\operatorname{sen}(1/x^2)$$



Afirmo: $\lim = 0$.

$$\varepsilon>0,\ \exists\,\delta>0;\ \sqrt{x^2+y^2}<\delta \to |xy\operatorname{sen}(1/xy)|<\varepsilon$$

$$xy \le x^2+y^2$$

$$(x-y)^2 = x^2-2xy+y^2 \ge 0 \Rightarrow x^2+y^2 \ge 2xy \ge xy \to \text{se}$$

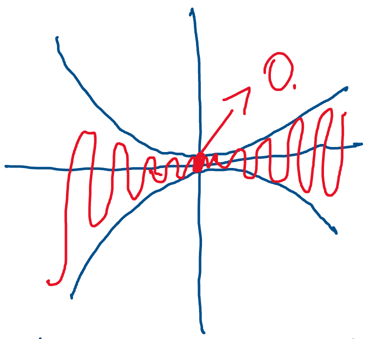
$$|xy\operatorname{sen}(1/xy)| = |xy||\operatorname{sen}(1/xy)| \le |x^2+y^2$$

$$\le |x^2+y^2| = |(\sqrt{x^2+y^2})^2| < \delta^2$$

$\therefore$ fazendo $\varepsilon = \delta^2$; $|xy\operatorname{sen}(1/xy$

$$\lim_{(x,y)\to(0,0)} xy\operatorname{sen}(1/xy).$$

$x=y$



$$\varepsilon>0,\ \exists\,\delta>0;\ \|X-X_0\|<\delta \Rightarrow |xy\operatorname{sen}(1/xy)| \le \varepsilon$$

$|xy| = |x||y|$.

(I) $|x| \ge |y| \to |x||y| \le |x||x| = x^2 \le |x^2+y^2|$

(II) $|x|<|y| \to |x||y| \le |y||y| = y^2 \le |x^2+y^2|$

$\Rightarrow |xy\, sen(1/xy)| = |xy|\,|sen(1/xy)| \leq |x^2+y^2|\,|se$

$\leq |x^2+y^2| = |(\sqrt{x^2+y^2})^2| = \|X - X_0\|^2 <$

$\therefore \varepsilon = \delta^2 \Rightarrow |xy\, sen(1/xy)| < \varepsilon.$

Teorema de Clairut: $f_{xy} = f_{yx}$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

59) $u = x^4y^3 - y^4$

$\frac{\partial u}{\partial x} = 4x^3y^3$; $\frac{\partial(4x^3y^3)}{\partial y} = 12x^3y^2 = f_{xy}$

$\frac{\partial u}{\partial y} = 3x^4y^2 - 4y^3$; $\frac{\partial(3x^4y^2 - 4y^3)}{\partial x} = 12x^3y^2$ ▩

Plano tangente em $z=f$ no ponto P_0.

$$z - z_0 = f_x(x-x_0) + \underbrace{f_y(y-y_0)}_{0}$$

1) $z = 3y^2 - 2x^2 + x$, $(2, -1, -3)$

$\Rightarrow \frac{\partial z}{\partial x} = -4x + 1 = -3 \;\Big|\; \frac{\partial z}{\partial y} = 6y = -6$

$z - (?) = -3(x-2) - 6(y-(-1)) = -3x - 6y$

$z = -3x - 6y - 6$

Linearização

: para valores pertos ponto P_0, onde passa tangente!

$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y$

31) $z = 5x^2 + y^2$; $P_0 = (1,2) \to (1,0$

$$\left.\begin{array}{l} z_x = 10x = 10 \\ z_y = 2y = 4 \end{array}\right\} L(x,y) = 9 + 10(x -$$

$= 10x + 4y - 9$

→ Substituindo: $10 \cdot (1,05) +$

$= 10,5 + 8,$

$z(1.05, 2.1) = 9,922$

$z - L = 0,0225$; a liv

aproxima bem z na vizin
$P_0 = (1,2)$.

diferencial: Queremos saber quanto z varia quando x e y variam em pequenas quantidades.

$$\boxed{dz = f_x\,dx + f_y\,dy}$$

14.4 | Ex. 33) comp. x e larg. y, erro 0,1 cm

$A(x,y) = x.y$

$dA = A_x\,dx + A_y\,dy = y\,dx + x\,dy$

$\Rightarrow$ em $(30,24)$; $30.\frac{1}{10} + 24.\frac{1}{10} = 5{,}4\ cm^2$