Dia 18

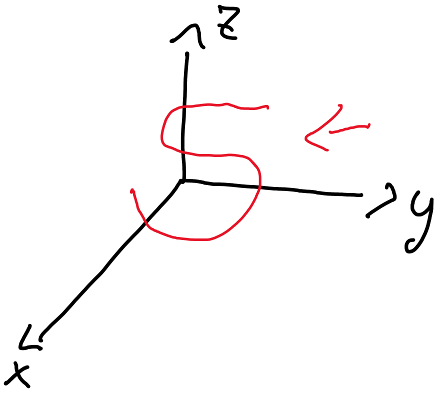
terça-feira, 18 de agosto de 2020 15:54

$f(x)\ ;\ x = g(t)$

$\to \dfrac{df}{dt}\ ;\ \dfrac{df}{dt} = \dfrac{df}{dt}\dfrac{dx}{dt}$

$\to f(x,y) \Rightarrow x(t),\ y(t)$

$$\boxed{\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}}$$

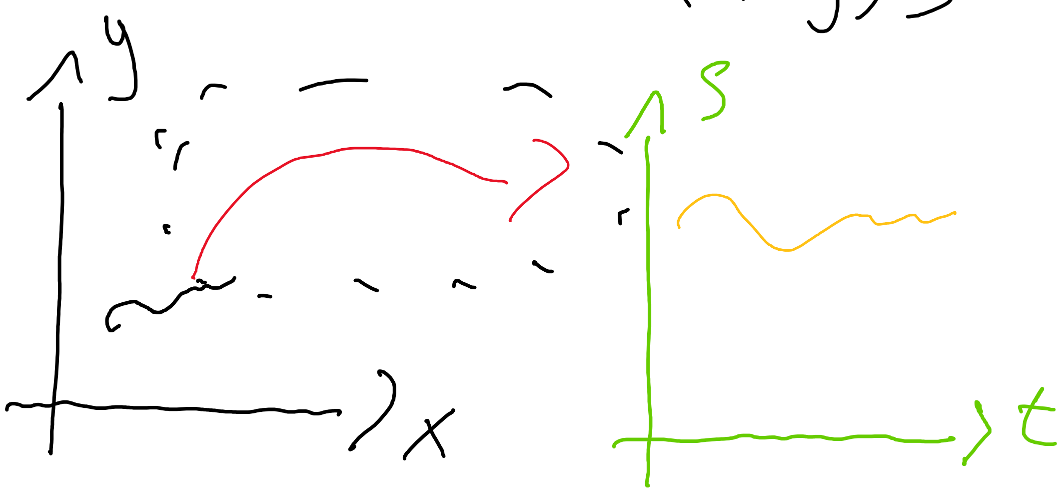


$x(t_1, t_2, \dots, t_m)$
$y(t_1, \dots, t_m)$

$f(x,y) = 2xy + y^2$
$x(t,s) = 2s + t^2$
$y(t,s) = st$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$= 2y \cdot 2t + (2x + 2y)\,s$



$$g(x,y)\,;\ x(t),\ y(t)$$

$$f(x,y)\,;\ x = r\cos\theta$$
$$y = r\,\mathrm{sen}\,\theta$$

$$f(x(r,\theta),\, y(r,\theta))$$

$f: \to (r,\theta)$, indiretamente.

* Matrizes:

$f: \mathbb{R}^n \to \mathbb{R}$

$f(x_1, \dots, x_m)\,;\ x_i(t_1, \dots, t_n)$

$$\begin{bmatrix} \frac{\partial f}{\partial t_1} \\ \vdots \\ \partial f \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \cdots & \frac{\partial f}{\partial x_m} \end{bmatrix} \begin{bmatrix} \frac{\partial x_1}{t_1} & \cdots & \frac{\partial x_1}{t_n} \\ \vdots & & \end{bmatrix}$$

$$\left. \partial t_n \right] \qquad \left. \frac{\partial x_m}{\partial t_1} \cdots \frac{\partial f_n}{\partial t_n} \right]$$

Derivação Implícita:

$$xy^2 - 3e^t y + 4 = 0$$

$$\frac{dy}{dx};\ y^2 + 2xy \cdot y' - 3e^t y - 3e^t$$

$$F(x, y(x)) = 0$$

$$\frac{\partial F}{\partial x} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial x} = 0$$

$$\rightarrow \boxed{\frac{dy}{dx} = -\frac{F_x}{F_y}}$$

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35) $T(x, y);\ x = \sqrt{1+t}$ $\left.\begin{matrix} \\ \end{matrix}\right\} x(t)$

$y = 2 + \frac{1}{3}t$ $\left.\begin{matrix} \\ \end{matrix}\right\} y(t)$

$$\frac{dT}{dt} = \frac{\partial T}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt}$$

$$\left.\begin{matrix} T_x = 4 \\ T_y = 3 \end{matrix}\right\} (2, 3);\quad \frac{dx}{dt} = \frac{1}{2\sqrt{1+t}}$$

$$\frac{dy}{dt} = \frac{1}{3}$$

$$dx(3)$$

$$\frac{\quad}{dt} = \frac{1}{4} \; ; \; \frac{dY}{dt}(3) = \frac{1}{3}$$

$$\frac{dT}{dt} = \frac{1}{4} \cdot 4 + \frac{1}{3} \cdot 3 = 2C^{\circ}/s.$$

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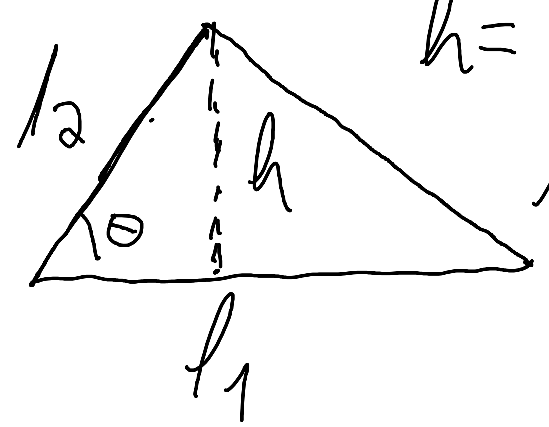
43) T ; l_1, l_2

$$\frac{dl_1}{dt} = 3 \; ; \; \frac{dl_2}{dt} = -2$$

$$A(T) = C$$

$$A(l_1, l_2, \theta) = C$$



$$h = l_2 \operatorname{sen} \theta$$

$$A = \frac{l_1 \cdot l_2 \operatorname{sen} \theta}{2}$$

$$\left.\begin{matrix} l_1 = 20 \\ l_2 = 30 \\ \theta = \pi/6 \end{matrix}\right\} \quad \frac{20 \cdot 30}{2} \cdot \frac{1}{2} = 150$$

$$A = 150\, m^2$$

$$\rightarrow 150 = \frac{l_1 l_2 \operatorname{sen}\theta}{2}$$

$$\Rightarrow \frac{300}{l_1 l_2} = \operatorname{sen}\theta$$

$$\frac{d\theta}{dt} ; \quad \frac{dA}{dt} = \frac{\partial A}{\partial l_1} \frac{\partial l_1}{\partial t} + \frac{\partial A}{\partial l_2} \frac{\partial l_2}{\partial t}$$

$$+ \frac{\partial A}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} = 0$$

$$\rightarrow \frac{\partial A}{\partial l_1} = \frac{l_2 \operatorname{sen}\theta}{2} = \frac{30}{4}$$

$$\frac{\partial A}{\partial l_2} = \frac{l_1 \operatorname{sen}\theta}{2} = 5$$

$$\frac{\partial A}{\partial \theta} = \frac{l_1 l_2 \cos\theta}{2} = 150\sqrt{3}$$

$$\rightarrow 90 - 5 + 150\sqrt{3} \cdot \theta =$$

$$\overline{4} \qquad \overline{2} \qquad t$$

$$\Leftrightarrow \frac{\partial \theta}{\partial t} = \frac{10 - \frac{90}{4}}{150\sqrt{3}} \;.\;///$$

Derivadas direcionais

Seja $u = (a, b)$ um vetor unitário; a derivada direcionada de uma função $f(x,y)$ na direção de u $D_u f$ é dada por:

$$\lim_{h \to 0} \frac{f(x + ha, y + hb) - f(x, y)}{h}$$

Teo: $D_u f = \nabla f \cdot u$

$$= \langle \nabla f, u \rangle$$

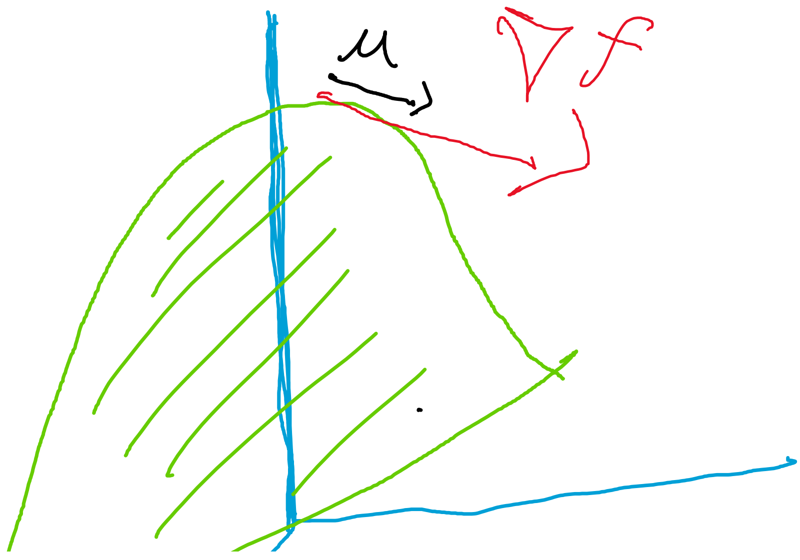
→ Seja θ o ângulo entre ∇f e u; $\cos\theta = \frac{u \cdot v}{|u||v|}$

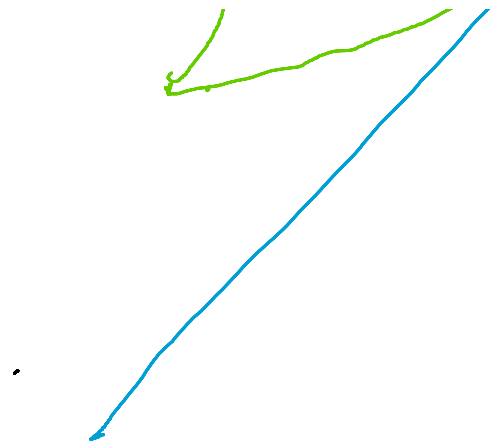
$\Rightarrow u \cdot v = \cos\theta \cdot |u||v|$

maximizar $\langle \nabla f, u \rangle$ implica em maximizar $\cos\theta$; ou seja, $\cos\theta = 1$

$\rightarrow \theta = 0.$

Com isso, concluímos que a direção que f cresce mais rapidamente é a própria direção do vetor gradiente.





Planos tangentes à sup. de nível.

→ Seja $F(x,y,z) = K$, constante; o gradiente de F será perpendicular à superfície de nível.

Seja π um plano tangente

a r em $P_0 = (x_0, y_0, t_0)$ e

$r'(t) = (x - x_0, y - y_0, z - z_0)$,

um vetor de π;

$\Rightarrow \nabla F \perp r'(t)$

logo; $\nabla F \cdot r'(t) = 0$

Portanto; podemos escrever

$$\pi: \boxed{F_x(x - x_0) + F_y(y - y_0) + F_z(x - z_0) = 0}$$

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35) $f(x,y)$; $A = (1,3)$, $B = (3,3)$.

$C=(1,7),\ D=(6,15)$

$D_{\vec{AB}} = 3$ | $D_{\vec{AD}} = ?$

$D_{\vec{AC}} = 26$ | $(1,0)$

$\vec{AB} = B-A = \overbrace{(2,0)}^{(1,0)}$ | $\vec{AC} = C-A = (0,4)$, $\underbrace{}_{(0,1)}$

$\vec{AD} = D-A = (5,12)$

$\to 5\frac{\vec{AB}}{|\vec{AB}|} + 12\frac{\vec{BC}}{|\vec{BC}|} = (5,12) = \vec{AD}$

$\nabla f \cdot \left(5\frac{\vec{AB}}{|\vec{AB}|} + 12\frac{\vec{BC}}{|\vec{BC}|}\right) \cdot \frac{1}{|\vec{AD}|}$

$$\frac{5\nabla f_{\vec{AB}} + 12\nabla f_{\vec{BC}}}{\sqrt{25+144}} = \frac{5.3 + 12.26}{\sqrt{169}}$$

$$= \frac{15+312}{13} = \frac{327}{13}.\ //$$

w; tem u, v

$w = \alpha u + \beta v.$

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49) $f(x,y) = xy$; $P=(3,2)$

$f(x,y) = 6$; $r(P)$: $ax + by = c$

$\rightarrow xy = 6 \; ; \; F_x = y \, , \; F_y = x$

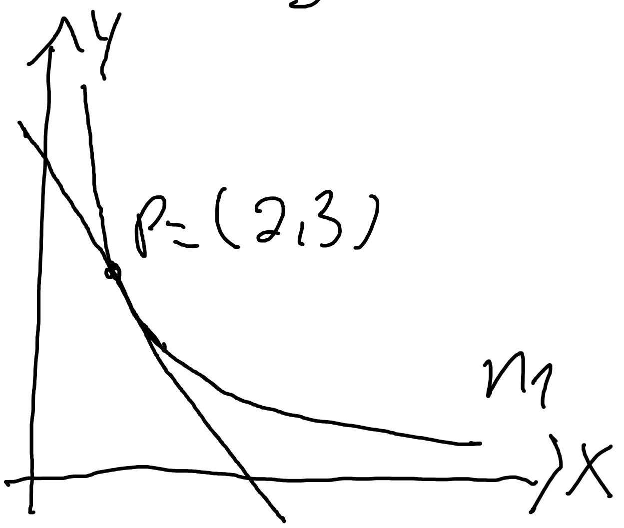
$\nabla F = (y, x) \; ; \; \nabla f(3,2) = (2,3)$

$\rightarrow (y, x)(x - x_0, y - y_0) = 0$

$\rightarrow (2,3)(x-3, y-2)$

$= 2x + 3y = 12$

$r: 2x + 3y = 12 \iff \boxed{y = 4 - \frac{2}{3}x} \, , \; P = (3$



$r: y = -\frac{2}{3}x + 4$

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55) $x^2 - y^2 - z^2 = 1 \; ;$

$z = x + y$

$F = x^2 - y^2 - z^2$

$\nabla F = (2x, -2y, -2z)$

$z = x + y \Rightarrow x + y - z = 0$

$\rightarrow v_{n_2} = (1, 1, -1)$

$P = (x_0, y_0, z_0)$

$\rightarrow (2x_0, -2y_0, -2z_0) = k(1, 1, -1)$

$(x_0, -y_0, -z_0) = (1, 1, -1)$

$\rightarrow x_0 = x_0 ; \; y_0 = x_0, \; z_0 = -x_0$

$\rightarrow x_0^2 - x_0^2 - (-x_0)^2 = 1$

$= -x_0^2 \Rightarrow x_0^2 = -1;$

Absurdo;

logo, não existem vetores paralelos.