terça-feira, 25 de agosto de 2020 15:52

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$$

1) b) $\displaystyle \lim_{(x,y)\to(0,2)} \frac{\operatorname{sen}(xy)}{x}$

$$\lim_{(x,y)\to(0,2)} y \cdot \frac{\operatorname{sen}(xy)}{xy}$$

$$\to \lim_{(x,y)\to(0,2)} y \cdot \left[\lim_{(x,y)\to(0,2)} \frac{\operatorname{sen}(xy)}{xy}\right] = 1$$

$x = f(t)$; $y = g(t)$ z

$$\lim_{t\to t_0} \frac{\operatorname{sen}(x(t)\cdot y(t))}{x(t)\cdot y(t)} \to \frac{0}{0}$$

$$\lim_{t\to t_0} \frac{(\cancel{x'(t)y(t) + y'(t)x(t)})\cos xy}{(\cancel{x'(t)y(t) + y'(t)x(t)})}$$

$$= \lim_{t\to t_0} \frac{\cos xy}{1} = \frac{\cos 0}{1} = 1.$$

$$\Rightarrow \lim_{(x,y)} \frac{\operatorname{sen} xy}{x} = \lim_{(x,y)\to(0,2)} y \cdot 1 = 2. ///$$

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1) a) $\displaystyle \lim_{(x,y)\to(0,0)} xy \operatorname{sen}\left(\frac{1}{xy}\right)$

lim; $\varepsilon \to 0$

$$\left| xy \operatorname{sen}\left(\frac{1}{xy}\right) \right| < \varepsilon \ ; \ \delta > \sqrt{x^2+y^2}$$

$$\to \left| xy \operatorname{sen}\left(\frac{1}{xy}\right) \right| = |xy| \left|\operatorname{sen}\left(\frac{1}{xy}\right)\right|$$

$\leq |xy|$

$|x| \geq |y| \to |xy| \leq x^2$

$|x| < |y| \to |xy| < y^2$

$\to |xy| \leq |x^2 + y^2| < \delta^2$

$\to |xy| \leq \max\{x^2, y^2\}$

$\to \leq x^2 + y^2$

$\delta > \sqrt{x^2+y^2} \Rightarrow \delta^2 > x^2 + y^2$

$; \delta = \sqrt{\varepsilon} ; \left| xy \operatorname{sen}\left(\frac{1}{xy}\right) \right| < \varepsilon$

limite $\to 0$

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$\frac{\operatorname{sen}(1/x^2)}{1/x^2}$; $\frac{1}{x^2} \to 0$

$1/x^2 \to \infty$, $\to 0$, ∞

$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$ $\neq$

— // — // —

$$\lim_{(x,y)\to(0,2)} y \cdot \frac{\operatorname{sen}(xy)}{xy} = 2$$

$$\to \left| y \cdot \frac{\operatorname{sen}(xy)}{xy} - 2 \right| < \varepsilon$$

$$\rightarrow \leq |y| \left|\frac{\operatorname{sen}(xy)}{xy}\right| + 2$$

$$\leq |y| + 2 = \sqrt{y^2} + 2$$

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2) $f(x,y) = 6x^2 - xy^3$; $P_0 = (2,1,22)$

$$z - z_0 = f_x(x - x_0) + f_y(y - y_0)$$

$f_x = 12x - y^3$; em P_0

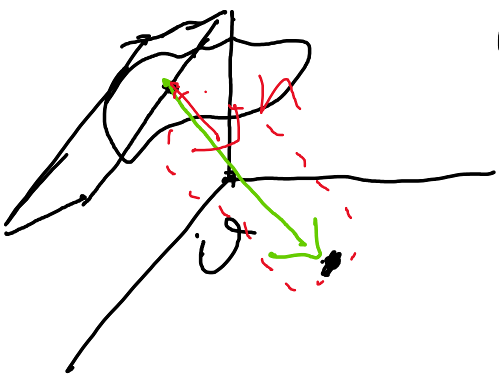
$f_y = -3xy^2$

$$\rightarrow f_x = 24 - 1 = 23, \quad f_y = -6$$

$$z - 22 = 23(x-2) - 6(y-1)$$

$$\rightarrow \boxed{23x - 6y - z = 38} \rightarrow n = (23, -6, -1)$$

b)



$$r = P_0 + t.n$$

$$= (2,1,22) + t(23,-6,-1)$$

$$22 - t = 0$$

$$\rightarrow t = 22$$

$\vartheta = (2, 1, 22) + 22(23, -6, -1)$

$= (508, -131, 0) = \vartheta$

$|\vartheta| = \sqrt{508^2 + (-131)^2 + 0} = \underbrace{\sqrt{275225}}$

3) $f(x,y) = \begin{cases} \dfrac{xy^3}{x^2+y^2} ; & x,y \neq 0 \\ \boxed{0, \quad x,y = 0} \end{cases}$ é diferenciável $\mathbb{R}^2$

$\lim_{(x,y)\to(0,0)} \dfrac{xy^3}{x^2+y^2} = 0$

$\left|\dfrac{xy^3}{x^2+y^2}\right| \leq \left|\dfrac{xy^3}{y^2}\right| = |xy| \leq |x^2+y^2| < \delta^2$

$< \delta^2 ; \left|\dfrac{xy^3}{x^2+y^2}\right| < \varepsilon ; \delta = \sqrt{\varepsilon}$

Teo: f_x e f_y existem e são contínuas no ponto P; então f é diferenciável em P.

$f_x = \dfrac{y^3(x^2+y^2) - 2x^2y^3}{(x^2+y^2)^2}$

$f_y = 3y^2x(x^2+y^2) - 2xy^4$

$$(x^2+y^2)^2$$

$$f_x = \lim_{(h,0)\to} \frac{f(h,0)-f(0,0)}{h} <-$$

$$= \frac{1}{h}\left(\frac{h.y^3}{h^2+y^2}\right) = \frac{1}{h}.\frac{0}{h^2} = 0$$

$$= 0$$

$$f_y = \lim_{(0,h)\to(0,0)} \frac{f(0,h)-f(0,0)}{h}$$

$$= \lim_{(0,h)\to(0,0)} \frac{1}{h}.\left(\frac{0.h^3}{0^2+h^2}\right) = \frac{0}{h^3} = 0$$

$$\lim_{(x,y)\to 0} \frac{y^3(x^2+y^2)-2x^2y^3}{(x^2+y^2)^2}$$

$$1.1 = 1$$

$$= \left|\frac{y^3(y^2-x^2)}{(x^2+y^2)^2}\right| = |y| \left|\frac{y^2}{(x^2+y^2)}\right| . \left|\frac{(y^2-x^2)}{x^2+y^2}\right|$$

$$\leq 1$$

$$\leq |y| = \sqrt{y^2} \leq \sqrt{y^2+x^2} < \delta$$

$$\varepsilon = \delta.$$

$$\lim \frac{3y^2x(x^2+y^2)-2xy^4}{\ldots}$$

$(x,y)\to(0,0) \quad (x^2+y^2)^2$

$$= \frac{y^2x\,(3x^2+3y^2-2y^2)}{(x^2+y^2)^2} = x.\frac{y^2(3x^2+y^2)}{(x^2+y^2)^2}$$

$$= x.\underbrace{\left|\frac{y^2}{x^2+y^2}\right|}_{\leq 1}.3.\underbrace{\left|\frac{(x^2+y^2/3)}{x^2+y^2}\right|}_{\leq 1}$$

$$\leq |3x| \leq 3\sqrt{x^2+y^2} < 3\delta$$

$$\to \boxed{\delta = \varepsilon/3.}$$

I) f é contínua em (0,0)

(II) f_x é contínua em (0,0)

(III) f_y é contínua em (0,0)

Logo, f é diferenciável no $\mathbb{R}^2$

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4) $r=50$; $h=50$; $dr=0,2$; $dh=0,2$

$v=\pi r^2 h$; $dv=$?

$dv = v_r\,dr + v_h\,.dh$

$v_r = 2\pi r h$, $v_h = \pi r^2$ — substituindo em (50,50):

$v_r = \pi.5000$

$v_h = \pi.2500$

$$V_r = 0{,}2 \cdot \pi \,(5000 + 2500) = \frac{\pi}{5} \cdot 7500$$
$$= \boxed{1500\pi \ u.v}$$

5) $f(x,y)$; $x = r\cos\theta$, $y = r\,sen\theta$

$$(f_x)^2 + (f_y)^2 = (f_r)^2 + \frac{1}{r^2}(f_\theta)^2$$

$$\frac{\partial f}{\partial r} = \underbrace{\frac{\partial f}{\partial x}}_{f_x} \cdot \frac{\partial x}{\partial r} + \underbrace{\frac{\partial f}{\partial y}}_{f_y} \cdot \frac{\partial y}{\partial r} = \boxed{f_x \cdot \cos\theta + f_y \cdot sen\theta}$$

$$f_\theta = f_x \cdot (-r\,sen\theta) + f_y \cdot (r\cos\theta)$$

$$f_\theta^2 = (f_x)^2 \cancel{r^2} sen^2\theta - f_x f_y \cancel{r^2} sen(2\theta) + (f_y)^2 \cancel{r^2} \cos^2\theta \quad \div r^2$$

$$(f_r)^2 = f_x^2 \cos^2\theta + f_x f_y\, sen(2\theta) + f_y^2\, sen^2\theta$$

$$f_r^2 + \frac{1}{r^2} f_\theta^2 = f_x^2(\cos^2\theta + sen^2\theta) + f_y^2(sen^2\theta + \cos^2\theta)$$
$$= \boxed{f_x^2 + f_y^2}$$

$x(\theta)$, $y(\theta)$; $f(x,y)$

$$\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

f_x ↙ f_y → $r\cos\theta$

$x = (r\cos\theta)' \Rightarrow -r\,\text{sen}\,\theta$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} \leq 0 \quad f_{xx} = 0$$

$D > 0$, $f_{xx} > 0$; mínimo local

$D > 0$, $f_{xx} < 0$; máximo local

$D < 0$; Sela.

$D = 0$. nada podemos afirmar

$$D = \begin{vmatrix} 0 & f_{xy} \\ f_{xy} & f_{xy} \end{vmatrix} = -(f_{xy})^2 \boxed{\leq 0}$$

$$f(x,y) = (x^2 - 2xy + y^2) = (x-y)^2$$

$f_x = 2x - 2y$

$f_y = -2x + 2y$; $\rightarrow 2x = 2y \Rightarrow x = y$

$P_1 = (0,0)$

$f_{xx} = 2$

$f_{yy} = 2$

$f_{xy} = -2$

$\rightarrow D = \begin{vmatrix} 2 & -2 \end{vmatrix} = 4 - (-2)^2 = 0$

$\begin{vmatrix} -2 & 2 \end{vmatrix}$

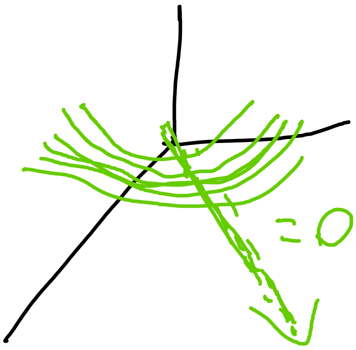
$f(x,y) \geq 0$

$f(0,0) = 0^2 - 2\cdot 0 \cdot 0 + 0 = 0$

$\forall x,y \in \mathbb{R}^2;\ f(x,y) \geq f(0,0)$

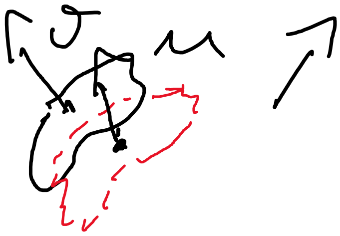
$\to (0,0,0)$; mínimo local.

$\to$ absoluto



f: $x^2 + 2y^2 + 3z^2 = 21;\ x + 4y + 6z = 0$

$\to \nabla f = (2x, 4y, 6z) = \lambda (1, 4, 6)$



$\to 2x = \lambda \to 4y = 4 \cdot 2x = 8x$

$4y = 4\lambda \rightarrow \boxed{y = 2x}$

$6z = 6\lambda$

$6z = 6 \cdot 2x$

$\rightarrow z = 2x = y$

$\rightarrow x + 8x + 12x = 0$

$\Rightarrow x = y = z = 0$;

$f = y^2 - x^2$; $\frac{1}{4}x^2 + y^2 = 1$

$\nabla f = (-2x, 2y) = \lambda\left(\frac{x}{2}, 2y\right)$

$\rightarrow 2y = \lambda 2y \neq 0$

$\lambda = 1$

$\rightarrow \boxed{-2x = \frac{x}{2}} \rightarrow x = 0$

$\rightarrow \boxed{x = 0 \; ; \; y = \pm 1}$

$-2x = \frac{\lambda}{2}x$

$\rightarrow x = \pm 2$

$\frac{1}{4} \cdot 0 + y^2 = 1$

$y^2 = 1$

$y = \pm 1$

$\frac{1}{4}x^2 + y^2 = 1$

$\frac{1}{4}x^2 = 1$

$\rightarrow x^2 = 4$

$x = \pm 2$

$P_1 = (-2, 0)$ | $P_3 = (0, -1)$

$P_2 = (2, 0)$ | $P_4 = (0, 1)$

$f(P_1) = -4$ | $f(P_3) = 1$

$f(P_2) = -4$ | $f(P_4) = 1$

máximos mínimos