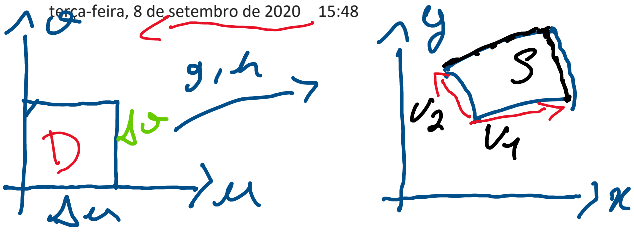
Mudança de Variáveis

terça-feira, 8 de setembro de 2020 15:48



$$\left.\begin{array}{l} x = g(u,v) \\ y = h(u,v) \end{array}\right\} v_1 = (a,b),\ v_2 = (c,d)$$

$$a = g(u+\Delta u, v) - g(u,v)$$

$$b = h(u+\Delta u, v) - h(u,v)$$

$$a = \boxed{\frac{g(u+\Delta u, v) - g(u,v)}{\Delta u}} \cdot \Delta u$$

$$a = \frac{\partial g}{\partial u} \cdot \Delta u,\quad b = \frac{\partial h}{\partial u} \cdot \Delta u$$

$$c = \frac{\partial g}{\partial v} \cdot \Delta v,\quad d = \frac{\partial h}{\partial v} \cdot \Delta v$$

$$v_1 = (g_u, h_u) \cdot \Delta u$$

$$v_2 = (g_v, h_v) \cdot \Delta v$$

$$A_S = \left\| \begin{matrix} \frac{\partial g}{\partial u} & \frac{\partial g}{\partial v} \\ \frac{\partial h}{\partial u} & \frac{\partial h}{\partial v} \end{matrix} \right\| \cdot \Delta u \Delta v$$

$$\iint_S f(x,y)\, |1| \cdot dx\, dy$$

$$= \iint_D f(u,v) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\, dv$$

$\star$ $x = g(u, v)$
$y = h(u, v)$

$\star$ $\iint_R f(x, y)\, dx\, dy = \iint_D f(u, v) \cdot |J| \cdot du\, dv$

Notas de Aula:

4) $D: \begin{cases} x^2 + y^2 = 4 \text{ (I)} \\ x^2 + y^2 = 2x \text{ (II)} \end{cases}$, $f(x, y) = x$

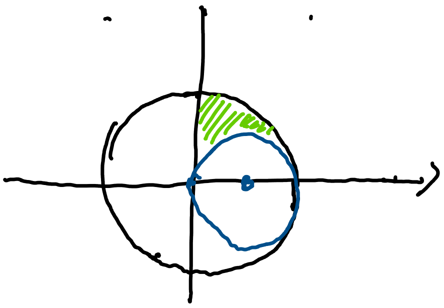
$\iint_D x \, dA$;

(I): Círculo de raio 2 centrado na origem

(II) Círculo de raio 1, centrado em (1,0).

$x^2 + y^2 = 2x \iff \underbrace{x^2 - 2x}_{} + y^2 = 0$

$\iff (x-1)^2 + y^2 = 1$



(I) $\left.\begin{array}{l} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{array}\right\} \begin{array}{l} x^2 + y^2 = r^2 = 4 \\ r \leq 2 \end{array}$

(II) $x^2 + y^2 = 2x \iff r^{\cancel{2}} = 2\cancel{r}\cos\theta$

$\Rightarrow r = 2\cos\theta \quad \Big| \; 2\cos\theta \leq r \leq 2$

$0 \leq \theta \leq \pi/2$

$$\left|\frac{\partial(x,y)}{\partial(r,\theta)}\right| = \left| \begin{vmatrix} \cos\theta & \text{sen}\,\theta \\ -r\,\text{sen}\,\theta & r\cos\theta \end{vmatrix} \right|$$

$= r.\left(\cos^2\theta - (-\text{sen}^2\theta)\right) = r.1 = r$

$$\iint_D x\,dx\,dy = \int_0^{\pi/2}\int_{2\cos\theta}^{2} r.r\cos\theta\,dr\,d\theta$$

$$= \int_0^{\pi/2} \frac{1}{3}(8 - 8\cos^3\theta).\cos\theta\,dr\,d\theta$$

$$= \frac{8}{3}\left(\int_0^{\pi/2}\cos\theta\,d\theta - \int_0^{\pi/2}\cos^4\theta\,d\theta\right)$$

$\cos^4\theta \Rightarrow (\text{h}\cos)^1$

$\cos 2\theta = \cos^2\theta - \text{sen}^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta)$

$\Rightarrow \cos 2\theta = 2\cos^2\theta - 1$

$$\Leftrightarrow \boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

$$\to \cos^4\theta = (\cos^2\theta)^2 = \left(\frac{1+\cos 2\theta}{2}\right)^2$$

...

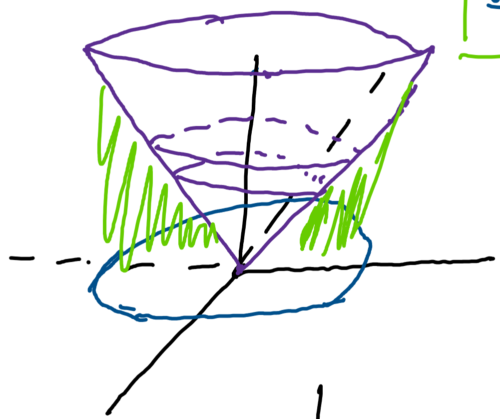
7) Volume : abaixo de $z = x^2+y^2$
acima de $x^2+y^2 \leq 9$

$f(x,y)=1$ | Por coordenadas Polares:

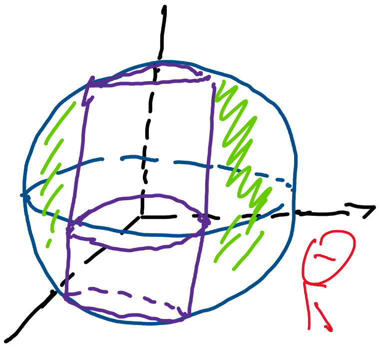
$\left.\begin{matrix} r^2 = z \\ r^2 \leq 3 \end{matrix}\right\} \begin{matrix} 0 \leq r \leq 3 \\ 0 \leq \theta \leq 2\pi \end{matrix}$

$$\to \int_0^{2\pi}\int_0^3 r\cdot r^2\, dr\, d\theta = 2\pi \cdot \frac{1}{4}\cdot(81)$$

$$= \boxed{\frac{81\pi}{2}}$$



8)



$x^2+y^2 \geq 4 \Rightarrow r \geq 2$

$z = \pm\sqrt{16-r^2} \Rightarrow r \leq 4$

$$2\cdot\int_0^{2\pi}\int_2^4 1\cdot r\cdot\sqrt{16-r^2}\, dr\, d\theta$$

$u = r^2 \to$ | $r = 2 \to u = 4$
$du = 2r\,dr$ | $r = 4 \to u = 16$

$2\pi \quad 16$

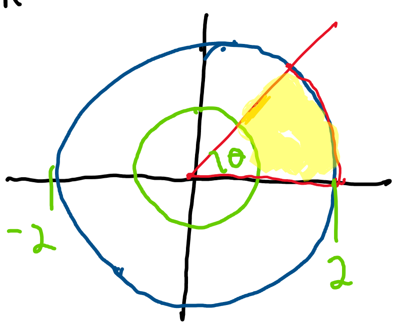
$$\to \int_0 \int_4 \sqrt{16-u}\, du\, d\theta$$

$$= -\int_0^{2\pi}\int_{12}^{0} \sqrt{v}\, dv\, d\theta = \int_0^{2\pi}\int_0^{12} \sqrt{v}\, dv\, d\theta$$

$$= \frac{2}{3} \cdot \int_0^{2\pi} 12^{3/2}\, d\theta = \boxed{\frac{4\pi}{3} \cdot 12^{3/2}}$$

Livro 13.4 #

13) $\iint_R \operatorname{arctg}(y/x)\, dA,\ R = \{(x,y) \mid 1 \le x^2+y^2 \le 4,\ 0 \le y \le x\}$



$0 \le \theta \le \pi/4$

$1 \le r \le 2$

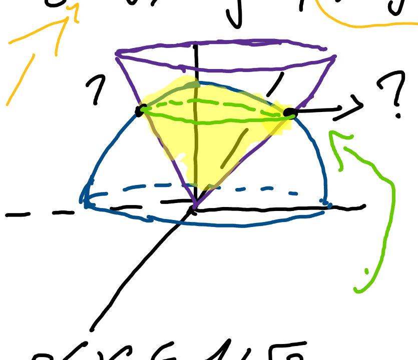
$f = \operatorname{arctg}(y/x)\ ;\ \frac{y}{x} = \frac{r\,\text{sen}\theta}{r\cos\theta} = tg\theta$

$f = \operatorname{arctg}(tg\theta) = \theta$

$$\to \int_0^{\pi/4}\int_1^2 r\cdot\theta\, dr\, d\theta = \int_0^{\pi/4}\theta\, d\theta \cdot \int_1^2 r\, dr$$

$$= \frac{1}{2}\cdot\left(\frac{\pi}{4}\right)^2 \cdot \frac{1}{2}\cdot(4-1) = \boxed{\frac{3\pi^2}{64}}$$

25) $z = \sqrt{x^2+y^2}$, $x^2+y^2+z^2=1$ $\to z_2$



$z = \sqrt{r^2} = r$

$z = \sqrt{1-r^2}$

$\to r = \sqrt{1-r^2}$

$2r^2 = 1 \Rightarrow r = \frac{1}{\sqrt{2}}$

$\to 0 \le r \le 1/\sqrt{2}$

$$0 \leq \theta \leq 2\pi$$

$$\int_0^{2\pi}\int_0^{1/\sqrt{2}} r \cdot \underbrace{\left(r - \sqrt{1-r^2}\right)} dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{1/\sqrt{2}} \left(r^2 - r\sqrt{1-r^2}\right) dr\, d\theta$$

$$= 2\pi \cdot \left(\int_0^{1/\sqrt{2}} r^2 dr - \int_0^{1/\sqrt{2}} r\sqrt{1-r^2}\, dr\right)$$

$$= 2\pi\left(\frac{1}{3} \cdot 2^{-3/2} - \underbrace{\int_0^{1/\sqrt{2}} r\sqrt{1-r^2}\, dr}_{*}\right)$$

$$*: \; u = 1-r^2 \;\Big|\; r = 0 \to u = 1$$
$$-\frac{du}{2} = r\,dr \;\Big|\; r = 1/\sqrt{2} \; ; u = 1/2$$

$$\to -\int_1^{1/2} \sqrt{u}\, du = \int_{1/2}^{1} \sqrt{u}\, du = \frac{3}{2}\left(1 - \left(\frac{1}{2}\right)^{3/2}\right)$$