terça-feira, 27 de outubro de 2020 16:03

16.7) 23)

$F = (xy, yz, zx)$;

$S: z = 4 - x^2 y^2$; $0 \leq x \leq 1$, $0 \leq y \leq 1$

$\iint_S F ds$;

$\phi(x,y) = (x, y, 4 - x^2 - y^2)$

$$\iint_S F\, ds = \boxed{\int_a^b \int_c^d F(\phi(u,v)) \cdot \phi_u \times \phi_v \, du\, dv}$$

$\phi(u,v)$; $a \leq u \leq b$, $c \leq v \leq d$

Caso especial: Parâmetros são as próprias variáveis

$N = \phi_x \times \phi_y = (-z_x, -z_y, 1)$

$$\left.\begin{aligned} z_x &= -2x \\ z_y &= -2y \end{aligned}\right\} N = (2x, 2y, 1)$$

$F(\phi(x,y)) \cdot (2x, 2y, 1)$

$F = (xy, yz, zx)$; $F \cdot N = 2x^2y + 2y^2z + zx$

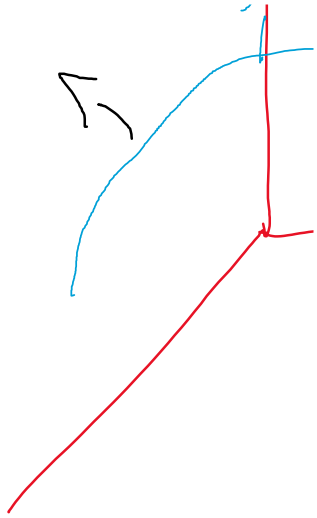
$= 2x^2y + 2y^2(4 - x^2 - y^2) + x(4 - x^2 - y^2)$

$= 2x^2y + 8y^2 - 2x^2y^2 - 2y^4 + 4x - x^3 - xy^2$

$$\Rightarrow \int_0^1 \int_0^1 (2x^2y + 8y^2 - 2x^2y^2 - 2y^4 + 4x - x^3 - x$$

N aponta p/ fora da superfície,

⇒ N aponta p/ dentro



47) $u = 2y^2 + 2z^2$, $K = 6,5$

$u(1,y,z)$; $= K\nabla u(x,y,z)$

$\nabla u = (0, 4y, 4z)$; $\therefore f = K.\nabla u$

$= 6,5.(0, 4y, 4z)$

S: $y^2 + z^2 = 6$, $0 \le x \le 4$

$\phi(\theta, x)$; $x = x$, $y = \sqrt{6}\cos\theta$, $z = \sqrt{6}\,\text{sen}\,\theta$

$\phi(\theta, x) = (x, \sqrt{6}\cos\theta, \sqrt{6}\,\text{sen}\,\theta)$

$\phi_\theta = (0, -\sqrt{6}\,\text{sen}\,\theta, \sqrt{6}\cos\theta)$

$\phi_x = (1, 0, 0)$

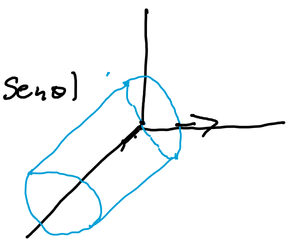
$$\Rightarrow \phi_\theta \times \phi_x = \begin{array}{cccc} -\sqrt{6}\,\text{sen}\,\theta & \sqrt{6}\cos\theta & 0 & -\sqrt{6}\,\text{sen}\,\theta \\ 0 & 0 & 1 & 0 \end{array}$$

$$= (0, \sqrt{6}\cos\theta, \sqrt{6}\,\text{sen}\,\theta)$$

$N = -\phi_x \times \phi_\theta$

$= -(0, \sqrt{6}\cos\theta, \sqrt{6}\,\text{sen}\,\theta)$

$\uparrow\sqrt{6}$



$F(\phi) = (0, 4\sqrt{6}\cos\theta, 4\sqrt{6}\,\text{sen}\,\theta)$

$F.N = 0 - 4\sqrt{6}.\sqrt{6}(\cos^2\theta + \text{sen}^2\theta) = -24$

$$\int_0^{2\pi}\int_0^4 -24\,dx\,d\theta = \boxed{-192\pi} \rightarrow -192\pi.(-6,5) = 1248\pi$$

Teste 2019:

1) $F(x,y) = (x, y, 0)$;

$C(t) = (\cos^3 t, \operatorname{sen}^3 t, t)$; $0 \leq t \leq 2\pi$

$\int_C F\,ds$; F é um campo vetorial,

$\Rightarrow \int_C F\,ds = \int_0^{2\pi} F(c(t)) \cdot c'(t)\,dt$

$F(c(t)) = (\cos^3 t, \operatorname{sen}^3 t, 0)$

$c'(t) = (3\cos^2 t \cdot (-\operatorname{sen} t), 3\operatorname{sen}^2 t \cdot \cos t, 1)$

$\Rightarrow F \cdot c'(t) = 3\cos^5 t(-\operatorname{sen} t) +$
$3\operatorname{sen}^5 t \cos t + 0$

$\Rightarrow \int_C F\,ds = 3\int_0^{2\pi} \cos^5 t(-\operatorname{sen} t)\,dt$ (I)
$+ 3\int_0^{2\pi} \operatorname{sen}^5 t \cos t\,dt$ (II)

(I): $\cos t = u$; $-du = \operatorname{sen} t\,dt$ $\Big\}$ $t=0 \to u=1$; $t=2\pi \to u=1$

$\int_0^{2\pi} \cos^5 t(-\operatorname{sen} t)\,dt = \int_1^1 u^5\,du = 0$

(II) $\operatorname{sen} t\ u$; $t=0 \to u=0$; $t=2\pi \to u=0$

$\Rightarrow \int_0^0 u^5\,du = 0$; logo; $\int_C F\,ds = 0$.

2) $C_1: x^2+y^2=x$, $x^2+y^2+z^2=1$;

S_1: A parte da esfera que está dentro do cilindro;

S_2: A parte da esfera fora

$\frac{A(S_2)}{A(S_1)} = ?$ $\iint_S ds$; a área de S = *

$$\phi(u,v) = (x(u,v), \dots)$$

$$* = \iint_D \|\phi_u \times \phi_v\| \, du\, dv$$

$D: \{(u,v) \mid a \le u \le b, c \le v \le d\}$

$x^2+y^2 = x \Leftrightarrow x^2 - x + y^2 = 0$

$\Leftrightarrow (x-\frac{1}{2})^2 + y^2 = \frac{1}{4}$; cilindro centrado em $(\frac{1}{2}, 0)$

Por coordenadas polares:

$x^2+y^2 = r^{\cancel{2}} = \cancel{r}\cos\theta \Leftrightarrow r = \cos\theta$

$r \le \cos\theta \; ; \; -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\phi(x,y) = (x, y, \sqrt{1-x^2-y^2})$

$N = \left(\frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{-x^2-y^2}}, 1\right)$

$\Rightarrow \|N\| = \sqrt{\frac{x^2+y^2}{1-x^2-y^2} + 1} = \sqrt{\frac{1}{1-x^2-y^2}}$

$A(S_1) = \iint_D \frac{1}{\sqrt{1-x^2-y^2}} dS$;

$= 2\int_0^{\pi/2}\int_0^{\cos\theta} \frac{r}{\sqrt{1-r^2}} dr\, d\theta$; $\quad u = 1-r^2$, $-\frac{du}{2} = r\,dr$ $\quad$ $r=0 \rightsquigarrow u=1$; $r=\cos\theta \rightsquigarrow u = 1-\cos^2\theta$

$= \cancel{2}\cdot(-\frac{1}{\cancel{2}}) \int_0^{\pi/2}\int_1^{1-\cos^2\theta} \frac{1}{\sqrt{u}} du\, d\theta$

$= \frac{1}{1/2}(-1)\int_0^{\pi/2} (\sqrt{1-\cos^2\theta} - 1) d\theta$

$= -2\int_0^{\pi/2} (\operatorname{sen}\theta - 1) d\theta = 2\cdot\frac{\pi}{2} - 2\int_0^{\pi/2} \operatorname{sen}\theta \, d\theta$

$= \pi - 2(1-0) = \pi - 2$

$S_1 = \pi - 2$; $\Rightarrow A(S_1) = (\pi-2)\cdot 2 \rightarrow$ Parte de cima e parte de baixo

$A(S_2) = A_{esfera} - A(S_1) = 4\pi - (2\pi - 4)$

$= 2\pi + 4$

$$\frac{A(S_2)}{A(S_1)} = \boxed{\frac{2\pi+4}{2\pi-4}}$$

3) $\iint_S (x^2 z + y^2 z) \, dS$; $z = 4 + x + y$

$$x^2+y^2=4$$

$$\iint_S f\,dS = \iint_D f(\phi(u,v)) \cdot \|\phi_u \times \phi_v\|\,du\,dv$$

$\Rightarrow \phi(u,v) = (x, y, 4+x+y)$

$\Rightarrow N = (-1,-1,1) \Rightarrow \|N\| = \sqrt{3}$

$$\iint_S f\,dS = \iint_D f(\phi) \cdot \sqrt{3}\,du\,dv$$

$F = x^2 z + y^2 z = z(x^2+y^2)$

$0 \leq \theta \leq \pi$
$0 \leq r \leq 2$; $2\int_0^{\pi}\int_0^2 (4 + r\cos\theta + r\,sen\theta)\, r^2 \cdot r\,dr\,d\theta$

$$= \int_0^{2\pi}\int_0^2 r^3(4 + r\cos\theta + r\,sen\theta)\,dr\,d\theta$$

$(I) = \int_0^{2\pi}\int_0^2 4r^3\,dr\,d\theta = \int_0^{2\pi} 16\,d\theta = 32\pi$

$(II) = \int_0^{2\pi}\int_0^2 r^4\cos\theta\,dr\,d\theta = \frac{1}{5}\int_0^{2\pi} 32 \cdot \cos\theta\,d\theta$

$= \frac{64}{5} \cdot 0 = 0$

$(III) = \int_0^{\pi}\int_0^2 r^4\,sen\theta\,d\theta = \frac{1}{5} \cdot 32 \int_0^{\pi} sen\theta\,d\theta = \frac{32}{5}\left(\cos\theta\Big|_0^{2\pi}\right) = 0$

$= A(S) =$

4) 4) $x^2+y^2+z^2=1$, $z \geq 0$, $x^2+y^2=1$

$F = (2x, 2y, 2z)$; $\iint_S F \cdot dS$

$\phi(x,y) = (x, y, \sqrt{1-x^2-y^2})$;

$N = \left(\frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, 1\right)$

$$F \cdot N = \frac{2x^2+2y^2}{\sqrt{1-x^2-y^2}} + 2\sqrt{1-x^2-y^2} = \frac{2}{\sqrt{1-x^2-y^2}}$$

Com coordenadas polares:

$\Rightarrow F \cdot N = \frac{2}{\sqrt{1-r^2}}$

$$\int_0^{2\pi}\int_0^1 r \cdot \frac{1}{\sqrt{1-r^2}}$$

.

31) $F=(x^2, y^2, z^2)$

$0\leq z\leq \sqrt{1-y^2}$; $0\leq x\leq 2$;

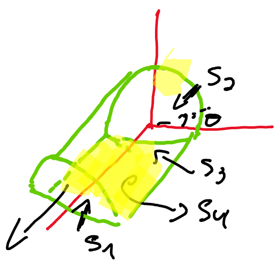
S_1: $x=2$;

$y=r\cos\theta$

$z=r\,\text{sen}\,\theta$

$0\leq r\leq 1$

$0\leq\theta\leq\pi$



$\phi(r,\theta)=(2, r\cos\theta, r\,\text{sen}\,\theta)$

$\phi_r=(0, \cos\theta, \text{sen}\,\theta)$

$\phi_\theta=(0, -r\,\text{sen}\,\theta, r\cos\theta)$

$\begin{matrix}\cos\theta & \text{sen}\,\theta & 0 & \cos\theta \\ -r\,\text{sen}\,\theta & r\cos\theta & 0 & -r\,\text{sen}\,\theta\end{matrix}$

$=(1,0,0)$

$\int_0^\pi\int_0^1 0^2\cdot 1\,dr\,d\theta = 0$

S_2: $\phi(r,\theta)=(0, r\cos\theta, r\,\text{sen}\,\theta)$

$N=(1,0,0)$

$\int_0^\pi\int_0^1 1.0^2\,dr\,d\theta=0$

S_3: $\phi(x,\theta)=(x, \cos\theta, \text{sen}\,\theta)$

$\phi_x=(1,0,0)$

$\phi_\theta=(0, -\text{sen}\,\theta, \cos\theta)$

$\phi_x\times\phi_\theta=\begin{matrix}0 & 0 & 1 & 0\\ -\text{sen}\,\theta & \cos\theta & 0 & -\text{sen}\,\theta\end{matrix}=(0, -\cos\theta, -\text{sen}\,\theta)$

$N(0)=(0,-1,0)$

$\Rightarrow N=(0, \cos\theta, \text{sen}\,\theta)$

$F=0.0+\cos^3\theta+\text{sen}^3\theta$

$\Rightarrow \int_0^\pi\int_0^2(\cos^3\theta+\text{sen}^3\theta)\,d\theta = \ldots$

S_4: $x = x$, $-1 \leq y \leq 1$
$y = y$, $0 \leq x \leq 2$
$z = 0$

$\phi(x,y) = (x, y, 0)$

$\left.\begin{array}{l} \phi_x = (1, 0, 0) \\ \phi_y = (0, 1, 0) \end{array}\right\}$ $\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}$

$= (0, 0, 1)$; $N = (0, 0, -1)$

$F(\phi(x,y)) \cdot N = x^2 \cdot 0 + y^2 \cdot 0 - z \cdot 1 = 0$

$\iint_S F\,ds = \iint_{S_3} F\,ds = \int_0^{\pi}\int_0^2 (\cos^3\theta + \mathrm{sen}^3\theta)\,dx\,d\theta$