terça-feira, 1 de setembro de 2020 15:52

4) $f(x,y,z) = x^2 + y^2 + z^2 + kzy$

a) $P = (0,0,0)$ é ponto crítico.

$f_x = 2x$ | $f_x(P_0) = 0$ ✓
$f_y = 2y + kz$ | $f_y(P_0) = 0$ ✓
$f_z = 2z + ky$ | $f_z(P_0) = 0$ ✓

b) Pra que valores de k, f tem mínimo local em P

$f_{xx} = 2$ | $f_{xy} = 0$ | $f_{yz} = k$
$f_{yy} = 2$ | $f_{xz} = 0$ |
$f_{zz} = 2$ | |

$$D = \begin{array}{c|ccc|} & x & y & z \\ x & 2 & 0 & 0 \\ y & 0 & 2 & k \\ z & 0 & k & 2 \end{array} = 8 - 2k$$

; $8 - 2k^2 > 0 \iff k^2 > 4$

$\to -2 < k < 2$

$k = 2$; $x^2 + (y^2 + z^2 + 2yz)$
$= \underbrace{x^2}_{\geq 0} + \underbrace{(y+z)^2}_{\geq 0}$

$K=2$; $f \geq 0$; $f(0,0,0)=0$

→ $K=2$; f também tem mínimo local em P.

$K=-2$; $f = x^2 + y^2 + (z^2 - 2yz)$

$$= x^2 + (y-z)^2 \geq 0$$

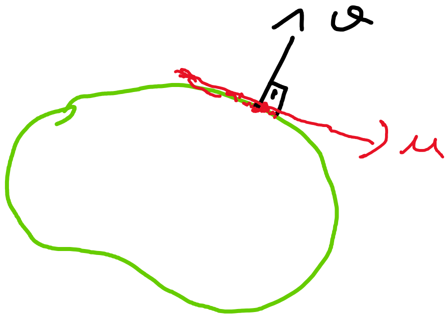
∴ f tem mínimo local em

$P=(0,0,0)$; $-2 \leq K \leq 2$. //

3) $f(x,y) = y\cos(\pi x) - x\cos(\pi y) + 10$.

$P = (2,1,13)$, u?

$u \perp v$.



$\nabla F(P)$

$f_x = -y\pi sen(\pi x) - \cos(\pi y)$

$f_y = \cos(\pi x) + \pi x sen(\pi y)$

em P: $f_x = -1.\pi sen(2\pi) - \cos(\pi)$
$= 0 - (-1) = 1$

$f_y(P) = 1 + 0 = 1$

$\nabla f(P) = (1,1)$;

$v=(a,b)$; $w=(-b,a)$ → $w \perp v$

$u = (-1,1) \perp \nabla F$; $u = \frac{1}{\sqrt{2}}(-1,1)$ ///

$$f(x,y) = \begin{cases} \dfrac{x^2 y^4}{x^4 + 6y^8}, & (x,y) \neq (0,0) \\ 0, & (x,y) = 0. \end{cases}$$

a) $f_x(0,0)$ e $f_y(0,0)$ existem.

$$f_x(0,0) = \lim_{h \to 0} \frac{f(0+h,0) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \left(\frac{h^2 \cdot 0^4}{h^4} \right) \cdot \frac{1}{h} = \frac{0}{h^5} = 0.$$

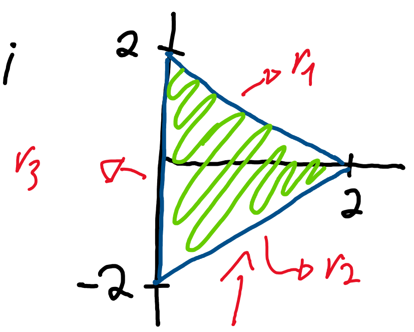
$$f_y = \lim_{h \to 0} \frac{f(0,0+h) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \left(\frac{0^2 \cdot h^4}{h^8} \right) \cdot \frac{1}{h} = \frac{0}{h^9} = 0.$$

$$\boxed{\begin{gathered} f_x(0,0) = 0 \\ f_y(0,0) = 0. \end{gathered}}$$

14.7 (29) $f(x,y) = x^2 + y^2 - 2x.$

D:



$f_x = 2x - 2 \Rightarrow x = 1$

$f_y = 2y \rightarrow y = 0$

$f_{xx} = 2$, $f_{yy} = 2$, $f_{xy} = 0$; $D = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$

$D > 0$, $f_{xx} > 0$; $f(1,0)$ mínimo local

$r_1: y = 2 - x$

$\rightarrow f(x, 2-x) = x^2 + (2-x)^2 - 2x$

$\rightarrow \cancel{x^2} + 4 - 4x + \cancel{x^2} - 2x = 2x^2 - 6x + 4$

$\rightarrow 4x - 6 = 0 \rightarrow \boxed{x = 3/2}$

$r_2: y = x - 2$

$f(x, x-2) = x^2 + (x-2)^2 - 2x$

$\hookrightarrow \boxed{x = 3/2}$

$r_3: x = 0$

$f(0, y) = y^2$, $\boxed{-2 \leq y \leq 2}$

$f(3/2, 2 - 3/2) = \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - 2 \cdot \frac{3}{2}$

$= \frac{9+1}{4} - 3 = \boxed{-\frac{1}{2}}$

$f(0,-2) = f(0,2) = 0 + 4^2 + 0$

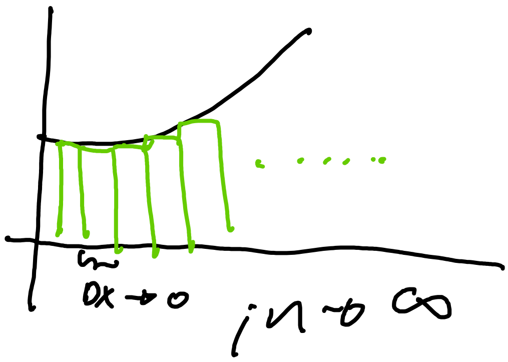
$$\boxed{=4}$$

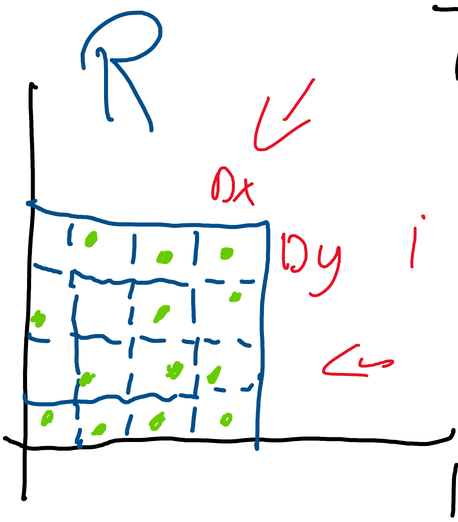
$$f(1,0) = 1 - 2.1 = \boxed{-1}$$

$f(1,0)$ mínimo absoluto em D

$f(0,\pm 2)$ máximo absoluto em D.

$\iint_R f \, dA$;

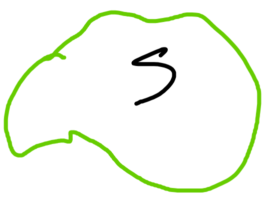




; $(\Delta x, \Delta y) \to 0$

f é integrável em R; se a soma dos valores da função nos pontos amostrais converge e não depende do ponto amostral.

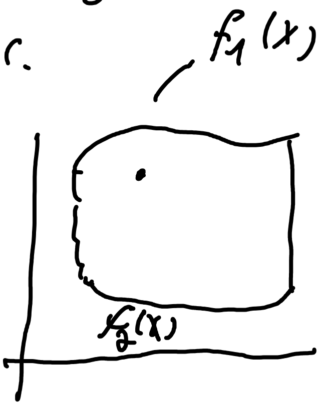
Teo: Se f é contínua em R então é integrável; ...



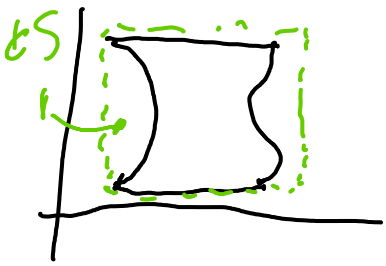
;

Def: S é simples se existe um retângulo R tal que para $(x,y) \in S$, $(x,y) \in R$.

$$\tilde{f}: \tilde{R} \to \mathbb{R} \; ; \begin{cases} f, & \text{se } (x,y) \in S \\ 0, & \text{c.c.} \end{cases}$$



Tipo 1: $f_1 \le y \le f_2$

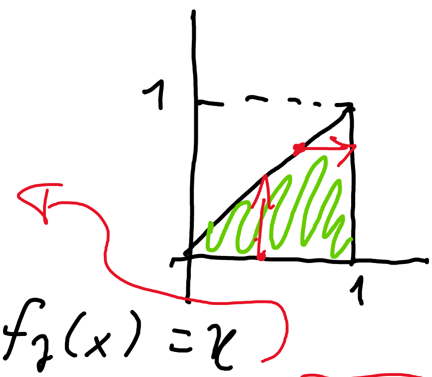


Tipo 2: $f_1(y) \le x \le f_2(y)$

$$\int_a^b \int_{f_1(x)}^{f_2(x)} f \, dy \, dx$$

$f = e^{x^2}$;

$0 \le x \le 1$
$0 \le y \le x$



$\to f_1(x) = 0$, $f_2(x) = x$

$$\int_0^1 \int_0^x e^{x^2} dy \, dx$$

$0 \le y \le 1$
$y \le x \le 1$

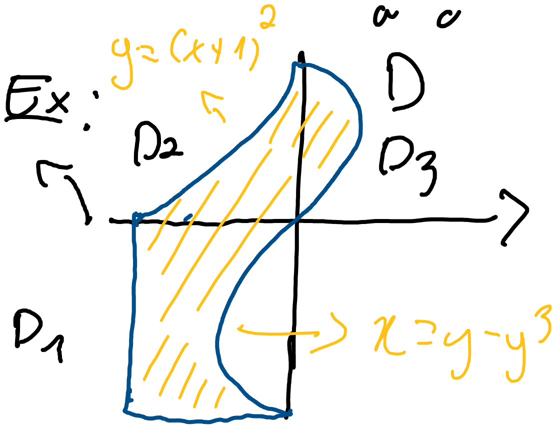
$$\int_0 \int_y e^{x^2}\,dx\,dy \quad \leftarrow$$

$$\int_0^1 \left[\int_0^x e^{x^2}\,dy\right] dx = \int_0^1 e^{x^2}\cdot x\,dx$$

$\iint$; Pelo teorema de Fubini:

$$\int_a^b \int_c^d f\,dy\,dx = \int_c^d \int_a^b f\,dx\,dy$$

Ex:



$$D = D_1 \cup D_2 \cup D_3$$

$$\iint_D f = \iint_{D_1} f + \iint_{D_2} f + \iint_{D_3} f$$

Aditividade de integrais

$f = x^2$, $\iint_{D_1} x^2 =$ $\quad -1 \leq y \leq 0$

$$\boxed{\int_{-1}^0 \int_{-1}^{y-y^3} x^2\,dx\,dy} \qquad -1 \leq x \leq y-y^3$$

D_2: $y = (x+1)^2 \Rightarrow x = \pm\sqrt{y} - 1$

$x = \sqrt{y} - 1$

$-1 \leq y \leq 1$

$$\sqrt{y-1} \le x \le 0$$

$$\boxed{\int_0^1 \int_{\sqrt{y-1}}^0 x^2 \, dx \, dy}$$

D_3: $0 \le y \le 1$
$0 \le x \le y - y^3$

$$\Rightarrow \boxed{\iint_{D_3} f = \int_0^1 \int_0^{y-y^3} x^2 \, dx \, dy}$$

$$\iint_D f = \iint_{D_1} f + \iint_{D_2} f + \iint_{D_3} f$$

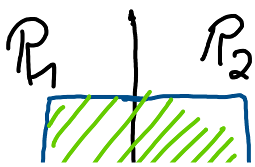
$D = D_1 \cup D_2 \cup D_3$

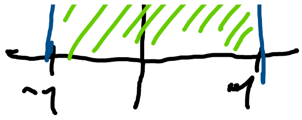
$$= \int_{-1}^0 \int_{-1}^{y-y^3} x^2 \, dx \, dy + \int_0^1 \int_{\sqrt{y-1}}^0 x^2 \, dx \, dy + \int_0^1 \int_0^{y-y^3} x^2 \, dx \, dy$$

* Integrais iteradas
* Divisão em regiões do tipo I ou II.

15.2|37)

$$\iint_R \frac{xy}{1+x^4} \, dA, \quad R = \{(x,y) \mid -1 \le x \le 1, \ 0 \le y \le 1\}$$





Afirmações: $\iint_R \frac{xy}{1+x^4} = 0$

$$\iint_{R_2} f + \iint_{R_1} f \Rightarrow \int_{-1}^{0}\int_{0}^{1} \frac{xy}{1+x^4}\,dy\,dx \,;\, R_1$$

$$R_2: \int_0^1\int_0^1 \frac{xy}{1+x^4}\,dx\,dy \,;\, u=-x$$

$$du = -dx$$
$$-du = dx$$

$$\Rightarrow \iint_{R_2} f = \int_0^{-1}\int_0^1 -\frac{(-u\,x)}{1+u^4}\,dy\,dx \quad \Big|\; \begin{array}{l} x=0;\ u=0 \\ x=1;\ u=-1 \end{array}$$

$$= \boxed{-\int_{-1}^{0} \frac{u\,x}{1+u^4}\,dy\,dx} = -\iint_{R_1} f$$

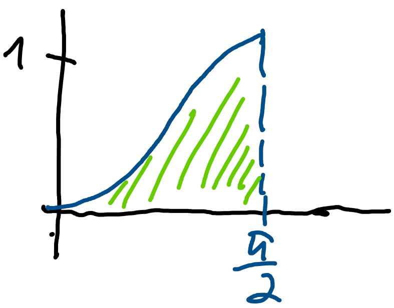
$$\Rightarrow \iint_{R_1} f + \iint_{R_2} f = = \iint_{R_1} f - \left(\iint_{R_1} f\right)$$

$$= 0.$$

15.3 | 53) $\int_0^1 \int_{\arcsin y}^{\pi/2} \left[\cos x\sqrt{1+\cos^2 x}\right] dx\,dy$

$x = \operatorname{arcsen} y$

$\Rightarrow y = \operatorname{sen} x$



$\Rightarrow \quad 0 \le x \le \pi/2$
$\quad\;\; 0 \le y \le \operatorname{sen} x$

$$\Rightarrow \iint f = \int^{\pi/2}\int^{\operatorname{sen}x} \cos x\sqrt{1+\cos^2 x}\,dy\,dx$$

$\to \cos^2 x = u$

$du = -\operatorname{sen}x \cos x \, dx$

$$\hookrightarrow \int_0^{\pi/2} \underbrace{\operatorname{sen}x \cos x}\sqrt{1+\cos^2 x}\, dx$$

$$= \int_1^0 -\sqrt{1+u}\, du = \int_0^1 \sqrt{1+u}\, du$$

$v = 1+u$

$dv = du$; $\displaystyle\int_1^2 \sqrt{v}\, dv = \frac{2}{3}\cdot(4-1)$

$= 2.$

37) $$\left[\int_0^1 \left[\int_0^{1-x} (1-x-y)\, dy\right] dx\right]$$

$z = 1-x-y$

$\Leftrightarrow 1 = x+y+z$