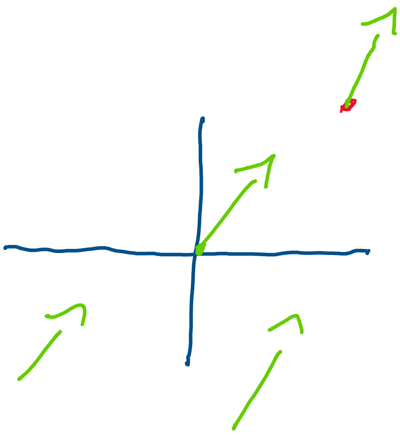
terça-feira, 6 de outubro de 2020 15:56

Campo Vetorial: Uma função que associa a cada ponto do domínio um vetor

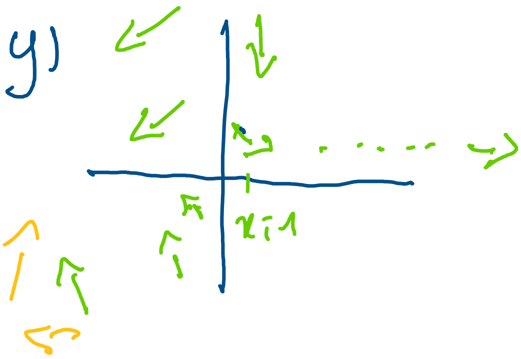
$|\vec{F}| = \frac{mMG}{d^2}$; $\vec{v}$ unitário

$\vec{F} = \frac{mMG}{|r^3|} \cdot \vec{r}$.

$\rightarrow F(x,y) = (1,2)$



$F_2(x,y) = (x, -y)$



$f(x,y)$ \ $\nabla f = F(x,y) = (F_1, F_2)$

F é um campo conservativo

$f = (F_1, F_2)$; $\int F_1\, dx = f + h(y)$

$\int F_2\, dy = f + g(x)$

$f(x,y,z) \mid \nabla f = F(x,y,z)$
$\Rightarrow F$ é conservativo

$F(x,y,z) = (yz, xz, xy)$. ?

$\int yz\,dx = f + h(y,z)$
$\hookrightarrow = xyz + c$; $f = xyz + h(y,z)$

$f_y = xz = xz + \frac{\partial h}{\partial y}(y,z) = 0$
h não depende de y
$f = xyz + h(z)$.
$\frac{\partial f}{\partial z} = xy = xy + \frac{\partial h}{\partial z}(z) = 0$

h não depende de nenhuma variável; ou seja:
$f = xyz + c$; F é conservativo
Ex2: $F(x,y,z) = (yz\,e^{xz}, e^{xz}, xy\,e^{xy})$
$f(x,y,z)$: $\nabla f = F$
$F = (F_1, F_2, F_3) \Rightarrow \int F_1\,dx = f + h(y,z)$
$\Rightarrow \int yz\,e^{xz}\,dx$; $u = xz$
$\frac{du}{z} = dx$
$\Rightarrow \int y\,e^{u}\,du = y\,e^{u} = y\,e^{xz}$

$\Rightarrow f = y\,e^{xz} + h(y,z)$
$\frac{\nabla f}{\partial y} = e^{xz} + h_y = e^{xy}$
$\hookrightarrow$ h ñ depende de y

$f = y\,e^{xz} + h(z)$

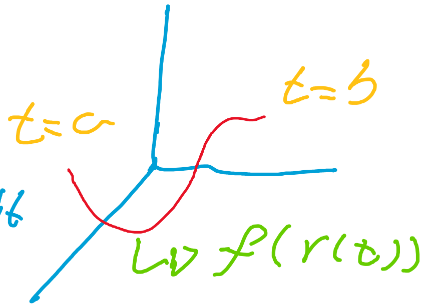
$\Rightarrow \frac{\partial f}{\partial z} = xye^{xz} + h_z = xye^{xz}$

f existe ; $f = ye^{xz} + C$.

$\therefore F$ é conservativo.

$L = \int_a^b \sqrt{\left(\frac{dx}{dx}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\Rightarrow ds = \underbrace{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}}_{\rho} dt$



$\rho = |r'(t)|$

$r(t) = (r_1, r_2)$

$r'(t) = (r_1', r_2') = (x'(t), y'(t))$

$|r'(t)| = \sqrt{x'^2 + y'^2}$

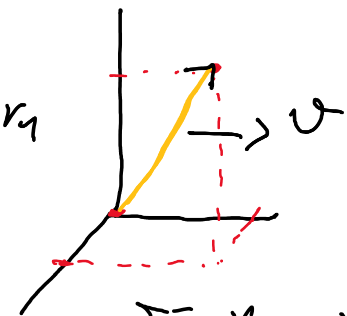
$\int_C f(x,y)\, ds \Rightarrow \boxed{\int_C f(x(t), y(t)) \cdot |r'(t)|\, dt}$

Ex | $\int_C xe^{yz}\, ds$; $C: (0,0,0) \Rightarrow (1,2,3)$

$r(t) = (1-t)r_0 + tr_1$

$r(t) = r_0 + t \cdot v$

$\Rightarrow 0 \leq t \leq 1$



, $v = r_1 - r_0$

$\Rightarrow \vec{v} = (1,2,3) - (0,0,0) = (1,2,3)$

$r(t) = (0,0,0) + t \cdot (1,2,3) = (t, 2t, 3t)$

$ds = |r'(t)|$. ; $r'(t) = (1,2,3)$

$$|r'(t)| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$\int_C f\, ds = \int_1^2 f(r(t)) \cdot \sqrt{14}\, dt$$

$$= \int_1^2 t\, e^{2t \cdot 3t}\, dt = \int_1^2 t\, e^{6t^2} dt$$

$$\rightarrow u = 6t^2 \; ; \; \int_0^6 \frac{1}{12} \cdot \sqrt{14}\, e^u\, du$$

$$\rightarrow \frac{du}{12} = t\, dt$$

$$= \frac{\sqrt{14}}{12} \cdot (e^6 - 1) \text{ ///}$$

$$\int_C f\, dx \; , \; \int_C f\, dz$$

$$x = x(t)$$

$$\frac{dx}{dt} = x'(t) \rightarrow dx = x'(t)\, dt$$

$$\rightarrow \int_C f\, dx = \int_a^b f\, x'(t)\, dt$$

$$\rightarrow ds = \rightarrow \sqrt{x'^2 + y'^2}\, dt$$

Ex 4: $\int_C (x^2 y^3 - \sqrt{x})\, dy$; $y = \sqrt{x}$

$(1,1) \rightarrow (4,2)$

$t = x$

$y = \sqrt{t}$

$x = t$.

$r(t) = (t, \sqrt{t})$

$t = y$; $y = t$, $x = t^2$

$\rightarrow r(t) = (t^2, t)$; $\frac{d(y)}{dt} = 1$

$dy = y'(t)\, dt$

$$\int f\, \underline{ds}$$

$$\sqrt{x'^2 + y'^2}$$

$\Rightarrow dy = dt$

$\int F \, \underline{dx}$

$x'(t)\, dt$

$\int_1^2 (x^2 y^3 - \sqrt{x})\, dt = \int_1^2 (t^4 t^3 - t)\, dt$

$= \frac{1}{8} t^8 - \frac{1}{2} t^2 \Big|_1^2 = 32 - 2 - \left(\frac{1}{8} - \frac{1}{2}\right)$

$= 30 + \frac{3}{8} = \frac{243}{8}$ ///

$* \int_C f\, d\underline{s} = \int_C f(r(t)) \cdot \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$* \int_C f\, d\underline{x} = \int_C f(r(t)) \cdot x'(t)\, dt$

$\int_C F \cdot ds \; ; \; F = (F_1, F_2, F_3)$

$\Rightarrow W = \vec{F} \cdot \vec{D} \; ; \; T$ unitário

$\int_C F \cdot T\, ds \; ; \; \int_C F \cdot \left(\frac{r'(t)}{|r'(t)|}\right) \cdot ds$

$ds = |r'(t)|\, dt$

$\Rightarrow \int_C F \cdot T\, ds = \int_C F \cdot \left(\frac{r'(t)}{\cancel{|r'(t)|}}\right) \cdot \cancel{|r'(t)|}\, dt$

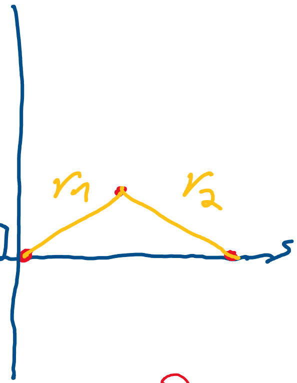
$\int_C F \cdot T\, ds = \int_a^b F \cdot r'(t)\, dt$

<u>Ex5</u>| $\int_C (x + 2y)\, dx + x^2 dy$

$F = (F_1, F_2) \; ; \; \begin{matrix} F_1 dx = F_1 \cdot x'(t)\, dt \\ F_2 dy = F_2 \cdot y'(t)\, dt \end{matrix}$

$$\to (f_1, f_2)\cdot(x'(t), y'(t)) = f_1\,dx + f_2\,dy$$

$C_1: (0,0) \to (2,1)$
$C_2: (2,1) \to (3,0)$



$$r_1 = (0,0) + t([(2,1)-(0,0)]$$
$$= (0,0) + t(2,1) = (2t, t)$$

$$\left.\begin{array}{l} r_2 = (2,1) + t[(3,0)-(2,1)] \\ = (2,1) + t(\ \ ,-1) = (2+t, 1-t) \end{array}\right\} 0 \leq t \leq 1$$

$r_1'(t) = (2,1)$
$r_2'(t) = (1,-1)$

$$// \int_0^1 (2t+2t, 4t^2)\cdot(2,1)\,dt \quad (I)$$
$$+ \int_0^1 (4-t, 4+4t+t^2)\cdot(1,-1)\,dt \quad ($$

$$(I): \int_0^1 (8t + 4t^2)\,dt = 4t^2 + \frac{4}{3}t^3 \Big|_0^1 = 4 + \frac{4}{3} = \frac{16}{3}$$

$$(II): \int_0^1 (\cancel{4} - t - (\cancel{4} - 4t + t^2))\,dt = \int_0^1 (3t - t^2)\,dt$$
$$= \frac{3t^2}{2} - \frac{t^3}{3}\Big|_0^1 = \frac{3}{2} - \frac{1}{3}$$

$$\therefore \int_c F\cdot ds = \frac{16}{3} + \frac{3}{2} - \frac{1}{3} = \frac{32+9-2}{6} = \frac{13}{3}. ///$$

<u>Ex</u> 8| $F = (x, y+2)$

$$r(t) = (t - \text{sen}\,t, 1 - \cos t)$$
$$0 \leq t \leq 2\pi$$

$$F(r(t)) = (t - \text{sen}\,t, 3 - \cos t)$$
$$r'(t) = (1 - \cos t, \text{sen}\,t)$$

$$F \cdot r'(t) = t - \text{sen}\,t - t\cos t + \cancel{\text{sen}\,t\cos t}$$
$$+ 3\,\text{sen}\,t - \cancel{\text{sen}\,t\cos t}$$
$$= 2\,\text{sen}\,t + t - t\cos t$$

$$\int_0^{2\pi} (2\,\text{sen}\,t + t - t\cos t)\,dt$$

$$\int_0^{2\pi} 2\,\text{sen}\,t\,dt = -2\cos t\,\Big|_0^{2\pi} = -2(1-1) = 0$$

$$\int_0^{2\pi} t\,dt = \frac{t^2}{2}\Big|_0^{2\pi} = 2\pi^2$$

$$\int_0^{2\pi} t\cos t\,dt \;; \quad \cos t = dv$$
$$v = \text{sen}\,t$$
$$t = u \;; \; dt = 1$$

$$\Rightarrow t \cdot \text{sen}\,t\,\Big|_0^{2\pi} - \int_0^{2\pi} 1 \cdot \cancel{\cos t}\,dt =$$

$$t \cdot \text{sen}\,t\,\Big|_0^{2\pi} = 0.$$

$$\Rightarrow \int_0^{2\pi} (2\,\text{sen}\,t + t - \cos t \cdot t)\,dt = 2\pi^2.$$