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$$\mathcal{H} = \sum_i \left(\frac{p_i^2}{2m} + \frac{1}{2} k x_i^2 \right)$$

$$\mathcal{H}^* = \frac{1}{2} \sum_i \left(x_i^2 + p_i^2 \right) \qquad 2H = \sum_i (x_i^2 + p_i^2)$$

$V(E) =$ nº de estados c/ energía inferior.

$$V(E) = \int_{H<E} dx_1 \cdots dx_N \, dp_1 \cdots dp_N \qquad R = \sqrt{2E}$$

$$d = 2N$$

eq. superficie esférica

$$R^2 = x_1^2 + x_2^2 + x_3^2 + \cdots$$

$$V_d = \frac{\pi^{d/2} R^d}{(d/2)!} \qquad \therefore \quad V(E) = \frac{\pi^N (2E)^N}{N!}$$

$$\Omega(E) \, dE = V(E + dE) - V(E)$$

$$= \frac{dV}{dE} dE$$

$$\Omega(E) = \frac{(2\pi)^N}{N!} N E^{N-1} \simeq \frac{(2\pi)^N E^N}{N!}$$

$$S = \ln \Omega$$

$$\Omega = (2\pi)^N \frac{E^N}{N!}$$

$$S = N \ln 2\pi + N \ln E$$
$$- \ln N!$$

$$\ln N! \simeq N \ln N - N$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{N}{E}$$

$$\frac{\partial \ln E}{\partial E} = \frac{1}{E}$$

1 d

$$E = NT$$

3 d

$$E = 3NT$$

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Oscilladores quanticos

$$E = \hbar\omega \sum_{i=1}^{N} \left(n_i + \tfrac{1}{2}\right)$$

$n_{i,x}$

$n_{i,y}$

$n_{i,z}$

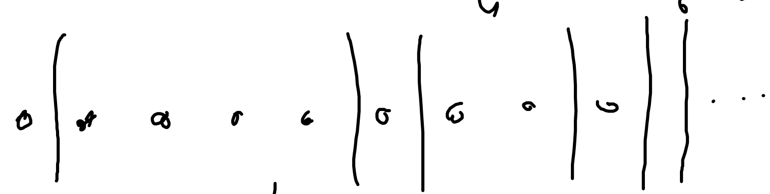
$$n_i = 0, 1, 2, \dots \qquad u_E = \hbar\omega$$

$$E = \frac{N}{2} + \sum_i n_i$$

$$\sum_i n_i = E - \frac{N}{2}$$

$$\Omega(E)$$

$n_4 = 2 \qquad n_6 = 0$



$n_1 = 1 \qquad n_2 = 4 \qquad n_3 = 1 \qquad n_5 = 1$

$$\Omega(E) = \frac{\left(N - 1 + E - \frac{N}{2}\right)!}{(N-1)!\,\left(E - N/2\right)!}$$

$$S = k_B \ln E = \left(N - 1 + E - \frac{N}{2}\right) \ln\left(N - 1 + E - \frac{N}{2}\right) - \left(N - 1 + E - \frac{N}{2}\right)$$
$$- (N-1)\ln(N-1) + N - 1$$
$$- \dots$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{V,N}$$

$$\frac{1}{T} = \ln\left(\frac{E + \frac{N}{2}}{E - \frac{N}{2}}\right)$$

$$e^{1/T} = \frac{E + N/2}{E - N/2}$$

$$E = \frac{N}{2} + \frac{N}{e^{1/T} - 1}$$

$$\frac{E}{\hbar \omega} = \frac{N}{2} + \frac{N}{e^{1/T / \hbar\omega / k_B} - 1}$$

$$E = N \hbar \omega \left(1/2 + \frac{1}{e^{\hbar\omega/k_B T} - 1} \right)$$

$$U_E = \hbar \omega$$

$$U_T = U \tau / k_B$$