Fundamentos de Programação

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Summary

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- Recursive functions
  - How it works.
  - The program stack.
  - The rules for termination
- Examples
  - Operations on a list
  - Towers of Hanoi
  - A sorting algorithm (kind of quicksort)

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A crazy idea

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- What does this function do?

  `sumsq([1, 2, 3])    #-> 14`

  - Does it work on an empty list?
  - Canyou write it with a generator expression? (Homework!)

```python
def sumsq(lst):
    s = 0
    for x in lst:
        s += x**2
    return s
```

- Check this weird version!
  - It squares first element;
  - Calls `sumsq` on the rest;
  - And adds.

```python
def sumsq2(lst):
    s = 0
    if len(lst) > 0:
        sq0 = lst[0]**2
        s = sq0 + sumsq(lst[1:])
    return s
```

- It works, but must call the original `sumsq`. Not very useful.
- But if `sumsq2` works, why not **call itself**?

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Recursive functions

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- This is what would result.

```python
def sumsqR(lst):
    s = 0
    if len(lst) > 0:
        sq0 = lst[0]**2
        s = sq0 + sumsqR(lst[1:])
    return s
```

- This is a **recursive function**: a function that calls itself.
- Notice that there is no loop instruction, but code gets executed several times, anyway.
- How does it work?

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How recursion works

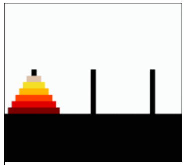
- What happens when we call `sumsqR([1, 2, 3])` ?
  - Watch the execution in PythonTutor.

```
sumsqR([1, 2, 3])
|   sumsqR([2, 3])
|   |   sumsqR([3])
|   |   |   sumsqR([])
|   |   |   └>0
|   |   └>9   ( = 3**2 + 0 )
|   └>13   ( = 2**2 + 9 )
└>14   ( = 1**2 + 13 )
```

- Notice that at one point, there are 4 frames in memory.
  - 4 variables named `lst`, 4 named `s`, 3 named sq0, but all distinct!
- The frames are stored in the **program stack**.

Example: Towers of Hanoi

- The Towers of Hanoi (Édouard Lucas, 1883)
- Move tower from A to C, using B temporarily.
  - Move only one disk at a time;
  - No disk may be put on top of a smaller disk.



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- Now solve it in 4 lines of code!

Example: quicksort

- The quicksort algorithm (C.A.R. Hoare) goes like this:
  - Pick one of the values in the list (generally the first) and store in T.
  - Put values smaller than T into a list L1, the others into a list L2.
  - Sort L1 and L2 (using same algorithm, by the way)
  - Result is L1 + [T] + L2.
- Of course, there's a few more details (the base case).

```
def qsorted(lst):
    if len(lst)<=1:     # no need to sort
        return lst[:]   # just return a copy
    T = lst[0]
    L1 = [x for x in lst[1:] if x<T]
    L2 = [x for x in lst[1:] if x>=T]
    return qsorted(L1) + [T] + qsorted(L2)
```

- This is simple to understand and quite efficient!

The actual quicksort modifies the list in-place, and is slightly harder.

The rules of recursion

To guarantee that a recursive function **terminates**, it must obey some rules!

1. There must be **base cases**, which can be solved without recursive calls.
   - In `sumsqR`, the base case is `len(lst)==0`. In that case, return 0.
2. In the other cases, the *context* passed to recursive calls **must differ** from the context received.
   - In `sumsqR`, the argument `lst[1:] != lst`.
3. The context in successive recursive calls must **converge** towards the base cases.
   - In `sumsqR`, the `lst` is shortened each time, until it's empty.

The ***context*** is the set of arguments (and global values) that have an impact on the base case / recursive case selection.

Recursion vs repetition

- Any problem that can be solved by repetition may be solved by recursion, and vice-versa.
- For certain complex problems, recursive solutions are usually more concise and easier to understand.
- Recursive implementations imply some time and memory cost because of functions calls and stack usage.
- If the problem has a simple iterative solution, that is usually the most efficient, too.

Writing recursive functions

- To develop a recursive function to solve some problem, there are some tricks that help.
  1. Start by defining the **arguments** you need, what they **mean**, and the **result** you **expect**, as *rigorously* as possible.
  2. Now, **assume** the function will work. Describe how the solution to a problem can be obtained from the solutions of **smaller** versions of the problem. This will be the recursive part of the algorithm.
  3. Finally, determine what are the **base cases**: which conditions have a trivial solution? This will be the non-recursive part of the algorithm. (Hint: usually, base cases are conditions outside the domain of the recursive call.)
- While in step 2, you may realize that you need extra arguments. Just add them and go back to step 1.