$$\mathrm{Var}\, T(X) \geq \frac{[\psi(\theta)-\psi(\varphi)]^2}{\mathrm{Var}\left[\frac{f_\varphi(X)}{f_\theta(X)}\right]}$$

razão de verossimilhança

Problemas 8.2

$\boxed{1}$ Se Y for VA contínua não negativa,

$$EY = \int_0^\infty y\, f_\theta(y)\, dy = \int_0^\infty P(Y > \epsilon)\, d\epsilon$$

$$Y = |T_n - \theta|^2$$

Exercícios 8.2

$$EY = \int_0^\infty y \cdot f_\theta(y)\,dy = \int_0^\infty P(Y > \varepsilon)\,d\varepsilon$$

$$Y = |T_n - \theta|^2$$

$$E|T_n - \theta|^2 \longrightarrow 0 \quad n \uparrow 0 \qquad E|T_n - \theta|^2 = \int_0^\infty P(|T_n - \theta|^2 > \varepsilon)\,d\varepsilon$$

$$= \int_0^\infty P(|T_n - \theta| > \sqrt{\varepsilon})\,d\varepsilon$$

$$T_n \xrightarrow{P} \theta$$

$$\epsilon = \varepsilon^2$$

$$d\epsilon = 2 \cdot \varepsilon\, d\varepsilon$$

$$E|T_n - \theta|^2 = 2\int_0^\infty \varepsilon \cdot P(|T_n - \theta| > \varepsilon)\,d\varepsilon \leq 2 \cdot A \int_0^A P(|T_n - \theta| > \varepsilon)\,d\varepsilon$$

no caso de A_n resolver da mesma forma

(2) $X_1 \ldots X_n \sim U(0,\theta)$

$X_{(1)} \le X_{(2)} \le \ldots \le X_{(n)} \le x$

$\downarrow$ mínimo

$X_{(n)} = \max\{X_1 \ldots X_n\}$

$X_{(n)} \xrightarrow{P} \theta \longrightarrow P\left(\left|X_{(n)} - \theta\right| > \varepsilon\right) \to 0$

$\forall \varepsilon > 0$

$E_\theta X_{(n)} =$

$Var_\theta X_{(n)} =$

$EMC\left(X_{(n)}, \theta\right) = Var_\theta X_{(n)} + \left[E_\theta X_{(n)} - \theta\right]^2 \longrightarrow 0$

à medida que n aumenta

$(n) \le \quad \le X_{(n)} \le x$

$) \to 0$

0

dade

n aumenta

$$F_{X_{(n)}}(x) = P(X_{(n)} \le x) \quad \text{p/ } x \in [0,\theta]$$

$$= P(X_1 \le x, \dots, X_n \le x)$$

$$= [P(X \le x)]^n = \left[\frac{x}{\theta}\right]^n \quad \text{p/ } x \in [0,\theta]$$

$$F_{X_{(n)}}(x) = \begin{cases} 1 & \text{p/ } x \ge \theta \\ \left(\frac{x}{\theta}\right)^n & \text{p/ } x \in [0,\theta] \\ 0, & \text{p/ } x \le 0 \end{cases}$$

$$\mathrm{Var}\,\underbrace{T(X)} \geq \frac{[\psi(\theta)-\psi(\varphi)]^2}{\mathrm{Var}\left[\dfrac{f_\varphi(\underline{X})}{f_\theta(\underline{X})}\right]}$$

razão de verossimilhança

12)

E X

$$f_{X_{(n)}}(x) = \begin{cases} \dfrac{d}{dx}\,\dfrac{x^n}{\theta^n} = \dfrac{n\,x^{n-1}}{\theta^n} & p/\ x \in [0,\theta] \\ 0 & p/\ x \notin [0,\theta] \end{cases}$$

$$E\,X_{(n)} = \int_0^\theta x\,\frac{n\,x^{n-1}}{\theta^n}\,dx = \frac{n}{n+1}\cdot\theta$$

$$\text{ou} = \int_0^\theta \left(1-\frac{x^n}{\theta^n}\right)dx = \theta - \frac{\theta^{n+1}}{\theta^n(n+1)} = \frac{n}{n+1}\cdot\theta.$$

(12) $X_1 \ldots X_n \sim U(0,\theta)$

$X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)} \leq x$

minimum

$$E X_{(n)}^2 = \int_0^\theta \frac{n}{\theta^n} x^{n+1}\,dx = \frac{n}{\theta^n}\frac{\theta^{n+2}}{n+2} = \frac{n}{n+2}\theta^2 \to \theta$$

$$\operatorname{Var} X_{(n)} = \frac{n}{n+2}\theta^2 - \frac{n^2}{(n+1)^2}\theta^2 \to \theta^2 - \theta^2 = 0$$

6

(12) $X_1 \dots X_n \sim U(0,\theta)$ $\qquad X_{(1)} \le X_{(2)} \le \dots \le X_{(n)} \le x$

$X_{(1)} \downarrow$ mínimo

$$E X_{(n)}^2 = \int_0^\theta \frac{n}{\theta^n} x^{n+1} \, dx = \frac{n}{\theta^n} \frac{\theta^{n+2}}{n+2} = \frac{n}{n+2}\theta^2 \to \theta$$

$$\text{Var } X_{(n)} = \frac{n}{n+2}\theta^2 - \frac{n^2}{(n+1)^2}\theta^2 \to \theta^2 - \theta^2 = 0$$

$$= \frac{n(n+1)^2 - n^2(n+2)}{(n+2)(n+1)^2}\theta = \frac{n}{(n+2)(n+1)^2}\theta$$

$$\text{viés} = E X_{(n)} - \theta = \frac{n}{n+1}\theta - \theta = -\frac{1}{n+1}\theta$$

$\leq \quad \leq X_{(n)} \leq x$

$\frac{n}{n+2}\theta^2 \to \theta$

$-\theta^2 = 0$

$)^2\,\theta$

$\frac{1}{n+1}\theta$

(6)

$$E\left[\frac{(n+2)X_{(n)}}{(n+1)}\right] =$$

$$\frac{n+2}{n+1} E X_{(n)} = \frac{n+2}{n+1}\,\frac{n}{n+1}\,\theta$$

$$V(\hat{\theta}) = \left[\frac{n+2}{n+1}\,\frac{n}{n+1} - 1\right]\theta$$

$$= \frac{\cancel{n^2} + \cancel{2n} - \cancel{n^2} - \cancel{2n} - 1}{(n+1)^2}\,\theta$$

$$= \frac{1}{(n+1)^2}\,\theta$$

Exercícios 18.2

2) $X_1 \ldots X_n \sim U[0, \theta]$

$X_{(n)} \xrightarrow{P} \theta$

$T(X_1, \ldots, X_n) = X_{(n)}$

$$\operatorname{Var} X_{(n)} = \frac{n}{(n+2)(n+1)^2}\,\theta^2 \xrightarrow[n\uparrow\infty]{} 0$$

$$E X_{(n)} = \frac{n}{n+1}\,\theta \xrightarrow[n\uparrow\infty]{} \theta$$

$E|X_{(n)} - \theta|^2 =$

$$EQM(X_{(n)}, \theta) = \operatorname{Var} X_{(n)} + (E X_{(n)} - \theta)^2$$

$$P(|X_{(n)} - \theta| > \varepsilon) \leq \frac{EQM(X_{(n)}, \theta)}{\varepsilon^2} \xrightarrow[n\uparrow\infty]{} 0$$

a) $\boxed{X_{(n)} \xrightarrow{P} \theta}$

Lo

b) $EX = \frac{\theta}{2}$

$$E\bar{X} = \frac{\theta}{2} \Rightarrow E\,2\bar{X} = \theta$$

$n,\theta)$

$\longrightarrow 0$
$n \to \infty$

$= \theta$

Proponha um estimador para θ com base na relação:

c). $\operatorname{Var} X = \dfrac{\theta^2}{12} = \sigma^2$

- $\hat{\sigma}^2 = S^2$ variância amostral
- $12 \cdot S^2$ é estimador de θ^2

Logo, $2\bar{X} \xrightarrow{P} \theta$

$\operatorname{Var} \bar{X} = \dfrac{\operatorname{Var} X}{n} = \dfrac{\theta^2}{12n} \to 0$

6) Seja

$T(\underset{\sim}{X}) = c\, X_{(n)}, \quad c > 0$

uma classe de estimadores

por ex.

Se $c = \dfrac{n+1}{n} \Rightarrow T(\underset{\sim}{X}) = \dfrac{n+1}{n} X_{(n)}$

$$\begin{cases} E\left(\dfrac{n+1}{n} X_{(n)}\right) = \theta \\ \operatorname{Var}\left(\dfrac{n+1}{n} X_{(n)}\right) = \left(\dfrac{n+1}{n}\right)^2 \operatorname{Var}(X_{(n)}) > \operatorname{Var}(X_{(n)}) \end{cases}$$

Exercícios 8.2

$$EQM(c X_{(n)}, \theta) = c^2 \, Var X_{(n)} + \left(c \, E(X_{(n)}) - \theta\right)^2$$

$$= c^2 \frac{n}{(n+2)(n+1)^2} \theta^2 + \left(c \frac{n}{n+1} - 1\right)^2 \cdot \theta^2$$

$$\frac{EQM(c)}{\theta^2} = c^2 \cdot \frac{n}{(n+2)(n+1)^2} + c^2 \frac{n^2}{(n+1)^2} - 2c \frac{n}{n+1} + 1$$

$$\boxed{\frac{d\, EQM(c)}{dc} = 0} \Rightarrow \boxed{c = \frac{n+2}{n+1}}$$

$E|X_{(n)} -$

$EQM(X$

$P(|$

(7) $X_1 \ldots X_n$ a.a.s. $f_\theta(x) = \begin{cases} \exp\{-(x-\theta)\}, & \boxed{x > \theta} \\ 0 & ; x < \theta \end{cases}$

si $Y \sim EXP(1)$

$EY = 1$ | $EX = 1 + \theta$

$Var\,Y = 1$ | $Var\,X = 1$

$f_\theta(x_1 \ldots x_n) = \prod_{i=1}^{n} \exp\{-(x_i - \theta)\} \cdot I_{[0,\infty)}(x_i)$

$T(\underset{\sim}{X}) = X_{(1)} + b$

$\prod_{i=1}^{n} I_{[0,+\infty)}(x_i) = \begin{cases} 1, & \text{si } \theta \le x_{(1)} \le x_{(2)} \le \ldots \le x_{(n)} \\ 0, & \text{si c.c.} \end{cases}$

$x_{(1)} = \min\{x_1 \ldots x_n\}$

$f(x) = \lambda e^{-\lambda x}$, $E X = \frac{1}{\lambda}$

$\boxed{x \geq \theta}$

$x < \theta$

$= 1 + \theta$

$X = 1$

$(x_i - \theta)\} \cdot I_{[\theta,\infty)}(x_i)$

$\leq X_{(2)} \leq \dots \leq X_{(n)}$

$\{X_1 \dots X_n\}$

$$F_X(x) = \begin{cases} 1 - e^{-(x-\theta)}, & x \geq \theta \\ 0 & x < \theta \end{cases}$$

$$P(X_{(1)} \leq x) = 1 - P(X_{(1)} \geq x)$$

$$= 1 - P[X_1 \geq x, X_2 \geq x, \dots X_n \geq x]$$

$$= 1 - e^{-n(x-\theta)}$$

$$E\left[X_{(1)} - \frac{1}{n}\right] = \theta$$

Exercícios 18 2

17

$T_n(\underset{\sim}{X}) = X_{(1)} - \frac{1}{n}$ é estimador ñ viciado

$T_b(\underset{\sim}{X}) = X_{(1)} + b$

$$EQM(X_{(1)}, \theta) = var(X_{(1)}) + (EX_{(1)} - \theta)^2$$

$$EQM(X_{(1)}+b, \theta) = var(X_{(1)}) + \underbrace{\{E(X_{(1)}+b) - \theta\}^2}_{\text{qdo vício é zero}}$$

T

Exercícios: 8.2

14) $T(X_1 \ldots X_n) = \prod_{i=1}^{n} (X_i)^{1/n} \longrightarrow \frac{\theta}{e}$

$\ln T = \sum_{i=1}^{n} \frac{\ln X_i}{n}$

$E \ln X_i = \ln \theta - 1$

$E\left[\sum \frac{\ln X_i}{n}\right] = \ln \theta - 1$

$X_i \sim U[0, \theta]$

$P(\ln X_i \leq x) = P(X_i \leq e^x) = \frac{e^x}{\theta}$, p/ $x \in (-\infty, \ln \theta]$

$\sum \frac{\ln X_i}{n}$ P

$\exp\{\sum \frac{\ln X_i}{n}$

(resul

$T(X$

$\exp(\lambda) \quad f_\lambda(x) = \lambda e^{-\lambda x}$

$$\frac{\sum \ln X_i}{n} \xrightarrow{P} \ln\theta - 1$$

$-\theta)\}, \quad \boxed{x \geq}$

$$\exp\left\{\frac{\sum \ln X_i}{n}\right\} \xrightarrow{P} \exp\{\ln\theta - 1\}$$

(resultado de Probabilidad)

$E X = 1 + \theta$

$\operatorname{Var} X = 1$

1

$- 1$

$$T(\underset{\sim}{X}) = X_{(1)} + b$$

$$f_\theta(x_1 \ldots x_n) = \prod_{i=1}^{n} \exp\{-(x_i - \theta)\}$$

$$\prod_{i=1}^{n} I_{[0,+\infty)}(x_i) = \begin{cases} 1, & \text{si } \theta \leq X_{(1)} \leq X_{(n)} \leq \\ 0, & \text{si c.c} \end{cases}$$

$) = \dfrac{e^x}{\theta}$

$x \in (-\infty, \ln\theta]$

$X_{(1)} = \min\{X_1 \ldots X_n\}$

$$\frac{\sum \ln X_i}{n} \xrightarrow{P} \ln\theta - 1$$

$$\exp\left\{\frac{\sum \ln X_i}{n}\right\} \xrightarrow{P} \exp\{\ln\theta - 1\}$$

(resultado de Probabilidad)

exp(λ) $f_{(x)} = \lambda e^{-\lambda x}$

$-\theta)\}$, $\boxed{x > \theta}$

$x < \theta$

$E X = 1 + \theta$

$\operatorname{Var} X = 1$

1

-1

$\boxed{5}$ $E\, X_{(n)} = \frac{n}{n+1}\theta \to \theta$

$\exp\{-(x_i - \theta)\} \cdot I$

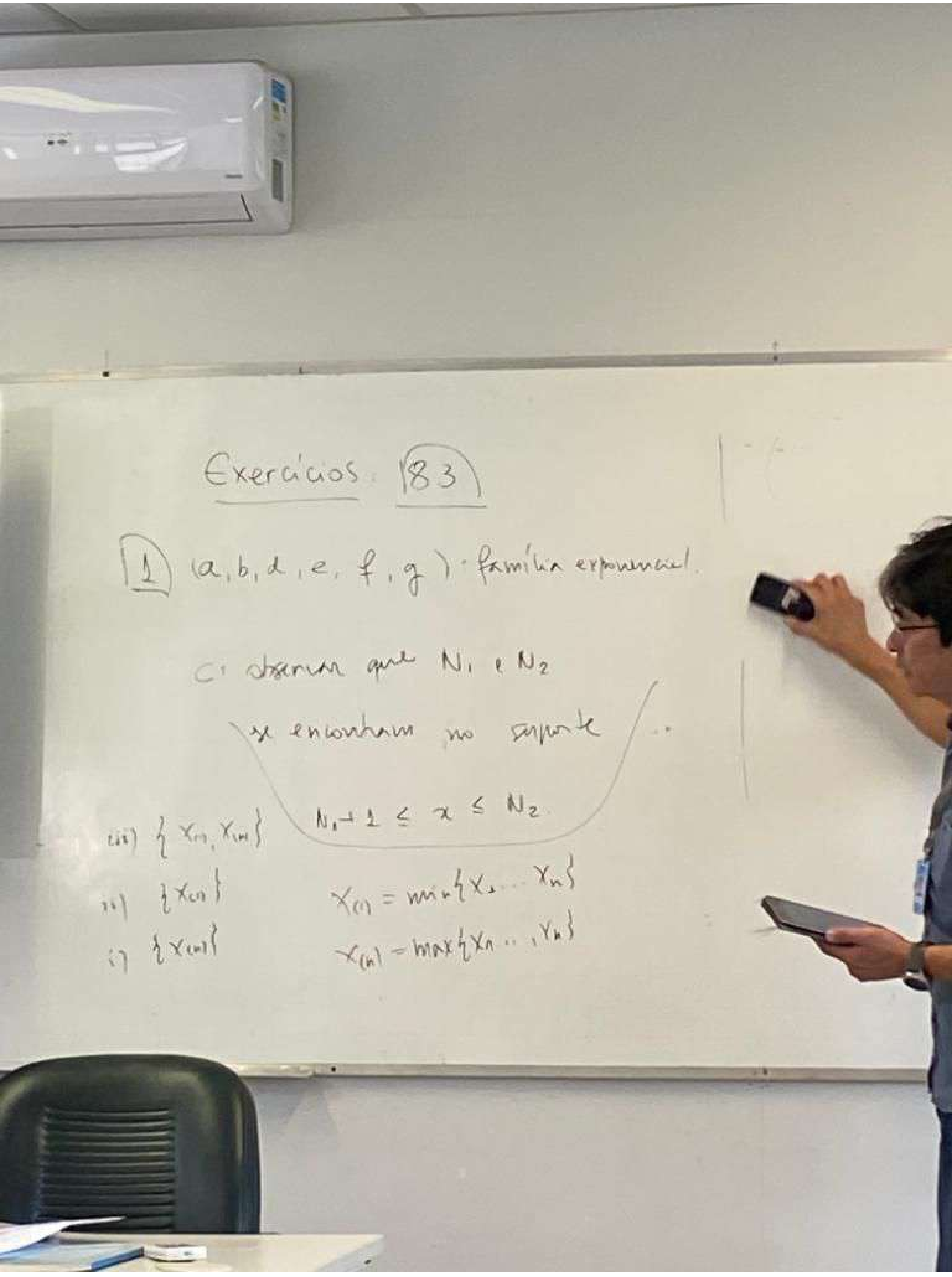
$) = \frac{e^x}{\theta}$,

$e^{-\lambda x} = \lim_{n\to\infty}\left(1 - \frac{\lambda}{n}\right)^n$

$X_{(1)} \leq X_{(2)} \leq$

$x \in (-\infty, \ln\theta]$

$\{X_1 \ldots X_n\}$


Exercícios 183
1 (a, b, d, e, f, g): família exponencial
c: observar que N1 e N2
se encontram no suporte
N1+1 ≤ x ≤ N2
iii) {X(1), X(n)}
ii) {X(1)}
i) {X(n)}
X(1) = min{X1 ... Xn}
X(n) = max{X1, ..., Xn}

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

(2) $X \sim N(\alpha\sigma, \sigma^2)$ α ct.

$$T(X) = \left(\sum_{i=1}^{n} X_i ,\ \sum_{i=1}^{n} X_i^2\right)$$

$\forall\, 0 \le \pi \le 1$

$$E\left(\pi\left(\tfrac{1}{\alpha}\bar{X}\right) + (1-\pi)\left[\tfrac{1}{r}S\right]\right) = \sigma$$

$$\bar{X} = \sum_{i=1}^{n} \frac{X_i}{n} \to E\bar{X} = \alpha\sigma$$

$$S^2 = \sum_{i=1}^{n} \frac{(X_i-\bar{X})^2}{n-1} \to ES^2 = \sigma^2$$

$$S = \sqrt{S^2} \to ES = r\sigma$$

$\exists\, r > 0$ t.q.

$$E_\theta[g(T)] = 0$$

$$\Rightarrow P_\theta(g(T) = 0) = 1 \ ?$$

$$h_1(T) = \frac{1}{\alpha}\bar{X} \to$$

$$h_2(T) = rS$$

$$g(T) = h_1(T) - h_2(T)$$

$$E(g(T)) = 0$$

$$P(g(T) = 0) \neq 1$$

$(n-1)\frac{S^2}{\sigma^2}$

rcícios: 183

5) $f_\theta(x_1, \dots, x_n) = g_\theta(T) \cdot h(x)$

$= (g_\theta \circ g(x))\, h(x)$

$= m_\theta(x)\, h(x)$

$N(\alpha\sigma, \sigma^2)$

$T(X) = \left(\sum_{i=1}^{n} x_i \, , \sum_{i=1}^{n} x_i^2 \right)$

d, e, f, g) · família exponencial.

servar que N_1 e N_2

encontram no suporte

$N_1 + 1 \le x \le N_2$

$X_{(1)} = \min\{X_1, \dots, X_n\}$

$X_{(n)} = \max\{X_1, \dots, X_n\}$

$E\left(\pi\left(\frac{1}{\alpha}\bar{X}\right) + (1-\pi)[\beta S] \right) =$

$\bar{X} = \sum_{i=1}^{n} \frac{x_i}{n} \rightarrow E\bar{X} =$

$S^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} \rightarrow ES$

$S = \sqrt{S^2} \rightarrow$

17o). $(X_1 \ldots X_n)$ for amostra -

$T(\underset{\sim}{X}) = (X_1 \ldots X_{n-1})$ é suficiente?

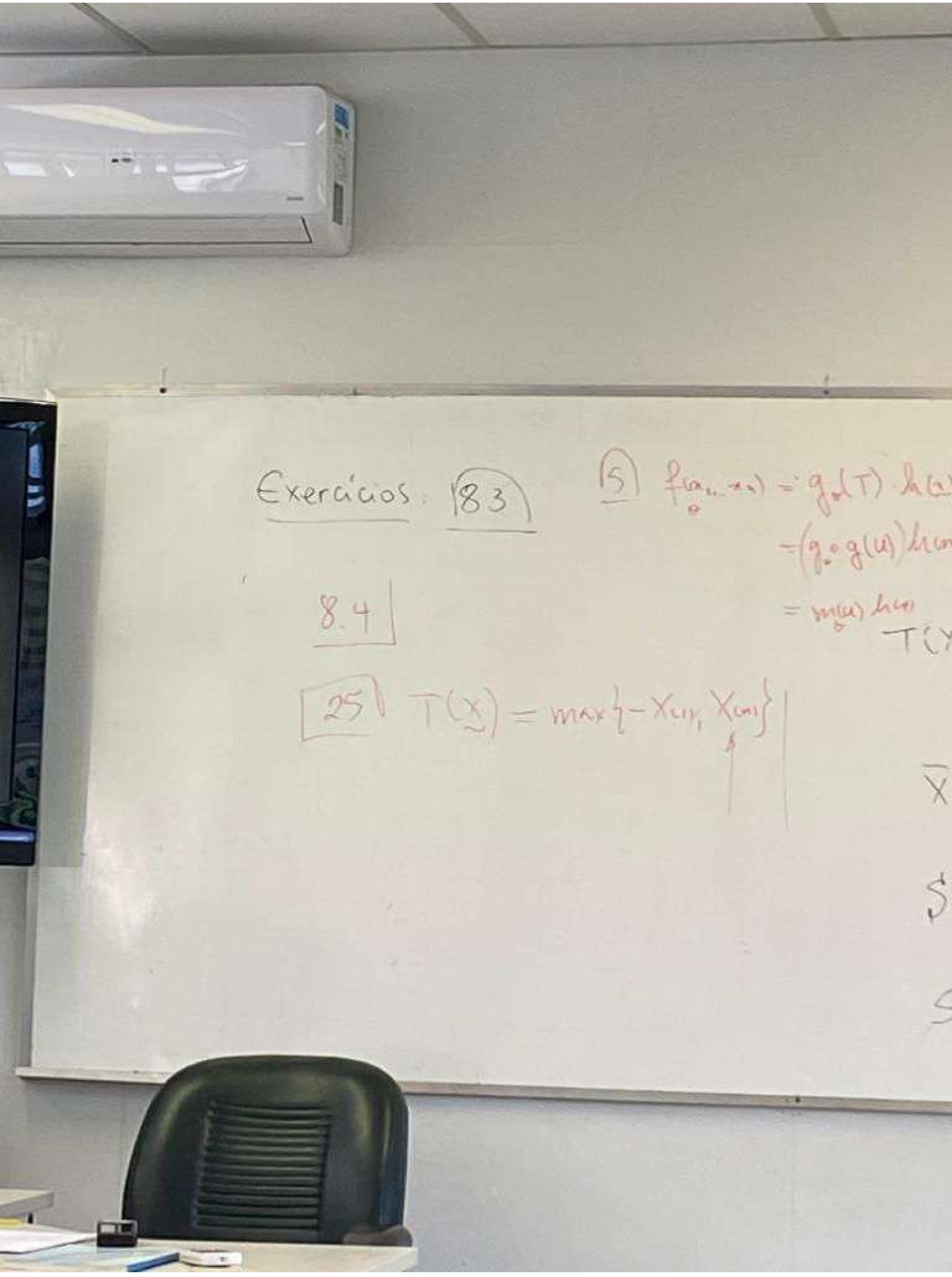
Lembrete.

Se houver dependência do tipo $p(x_n | x_{n-1}) = \pi$ c/ π conhecido.

$X_1 \ldots X_n \mid x_1 \ldots x_{n-1}$

$$p(x_1 \ldots x_n \mid x_1 \ldots x_{n-1}) = \frac{p(x_1 \ldots x_{n-1}, x_n)}{p(x_1 \ldots x_{n-1})} = \frac{p(x_n \mid x_1 \ldots x_{n-1})\, \cancel{p(x_1 \ldots x_{n-1})}}{\cancel{p(x_1 \ldots x_{n-1})}}$$

$$p(x_n \mid x_{n-1}) = \pi$$


Exercícios 8.3
8.4
25
T(X) = max{-X(1), X(n)}