The M(j) process

February 3, 2021

1 Definitions

$$\{a_1, \ldots, a_n\} \sim U[0,1] \tag{1}$$

$$M(j) = \max(\{a_1, \ldots, a_j\}), 1 \le j \le n \tag{2}$$

$$\{k_1, \ldots, k_n\} \in [0,1] \tag{3}$$

2 n=3

From simulation of 10 million draws

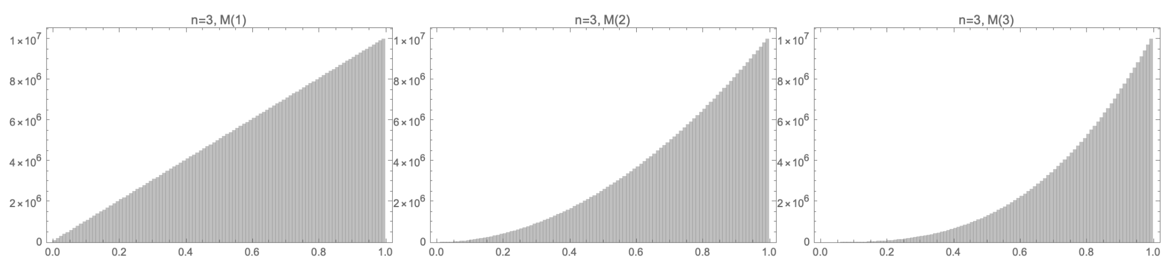

Figure 1: Unconditional CDF for M(j), n=3

Defining the stopping regions for rounds 1, 2 and 3 as:

$$R_1 : a_1 > k_1 = 0.689897948556636 \tag{4}$$

$$R_2 : a_2 > k_2 = 0.5 \tag{5}$$

$$R_3 : a_3 > k_3 = 0 \tag{6}$$

And assuming that if we do not stop at rounds $\{1, \ldots, n-1\}$ we will know the maximum only after round n (delay knowledge of losses), $M(j)$ will only be stopped with wins and immediate losses on regions R_j.

For R_1:

$$CDF[M(1)]_{WIN} = \begin{cases} \frac{\left(z^3 - k_1^3\right)}{3} & z > k_1 \\ 0 & otherwise \end{cases} \tag{7}$$

$$CDF[M(1)]_{LOSS} = \begin{cases} \frac{\left(3 \cdot z - z^3 - 3 \cdot k_1 + k_1^3\right)}{3} & z > k_1 \\ 0 & otherwise \end{cases} \tag{8}$$

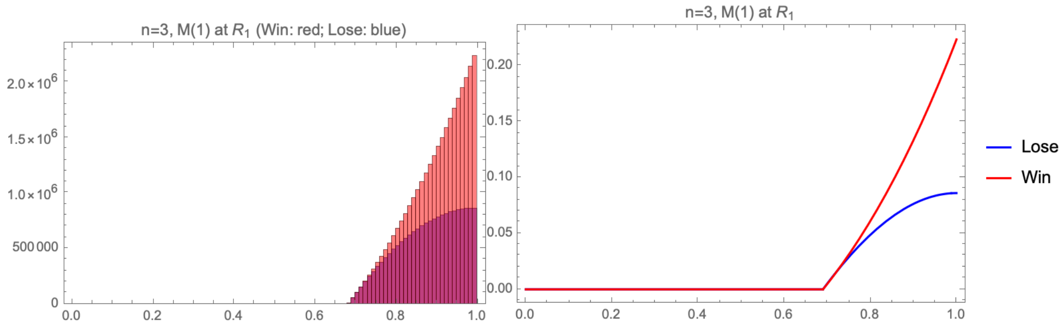

Figure 2: Wins and (immediate) losses at R_1

Which will truncate further realizations of $M(j)$.

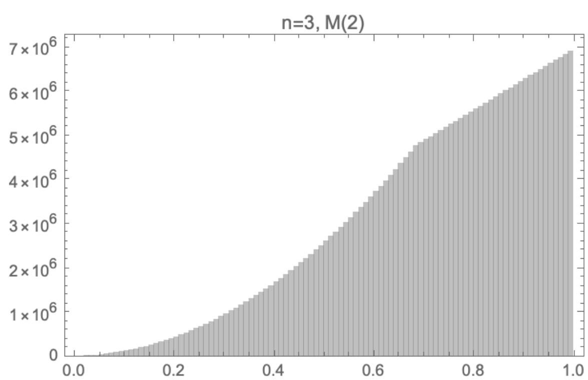

Figure 3: New M(2)

For R_2:

$$CDF\left[M\left(2\right)\right]_{WIN} = \begin{cases} \frac{\left(3\cdot z^2\cdot k_1 - 2\cdot k_2^3 - k_1^3\right)}{6} & z > k_1 \\ \frac{\left(z^3 - k_1^3\right)}{3} & k_1 > z > k_2 \\ 0 & otherwise \end{cases} \tag{9}$$

$$CDF\left[M\left(2\right)\right]_{LOSS} = \begin{cases} \frac{\left(6\cdot z\cdot k_1 - 3\cdot z^2\cdot k_1 + 2\cdot k_2^3 + k_1^3 - 6\cdot k_1\cdot k_2\right)}{6} & z > k_1 \\ \frac{\left(3\cdot z^2 - z^3 - 3\cdot z^2\cdot k_2 + k_2^3\right)}{3} & k_1 > z > k_2 \\ 0 & otherwise \end{cases} \tag{10}$$

Cross term on the loss alert!

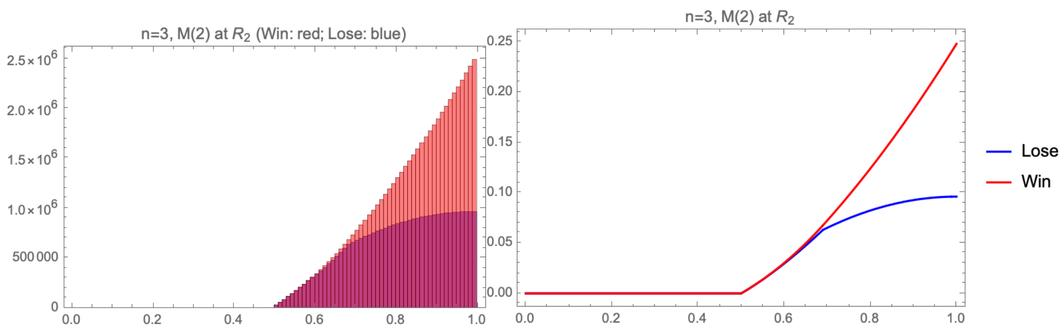

Figure 4: Wins and (immediate) losses at R_2

On to round 3 and the reckoning with the hidden losses:

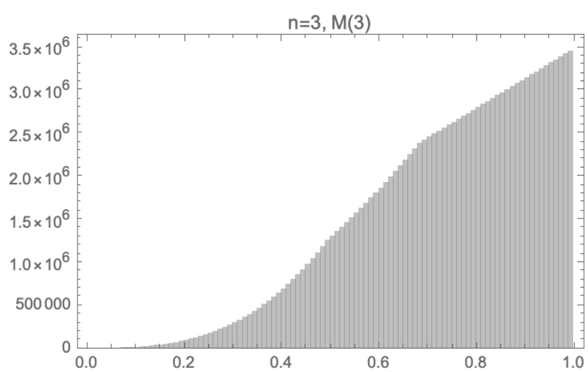

Figure 5: New M(3)

For R_3:

$$CDF\left[M\left(3\right)\right]_{WIN} = \begin{cases} \frac{\left(6\cdot z\cdot k_1\cdot k_2-3\cdot k_1^2\cdot k_2-k_2^3\right)}{6} & z > k_1 \\ \frac{\left(3\cdot z^2\cdot k_2-k_2^3\right)}{6} & k_1 > z > k_2 \\ \frac{z^3}{3} & otherwise \end{cases} \tag{11}$$

$$CDF\left[M\left(3\right)\right]_{LOSS} = \begin{cases} \frac{\left(3\cdot k_1^2\cdot k_2+k_2^3\right)}{6} & z > k_1 \\ \frac{\left(3\cdot z^2\cdot k_2+k_2^3\right)}{6} & k_1 > z > k_2 \\ \frac{2\cdot z^3}{3} & otherwise \end{cases} \tag{12}$$

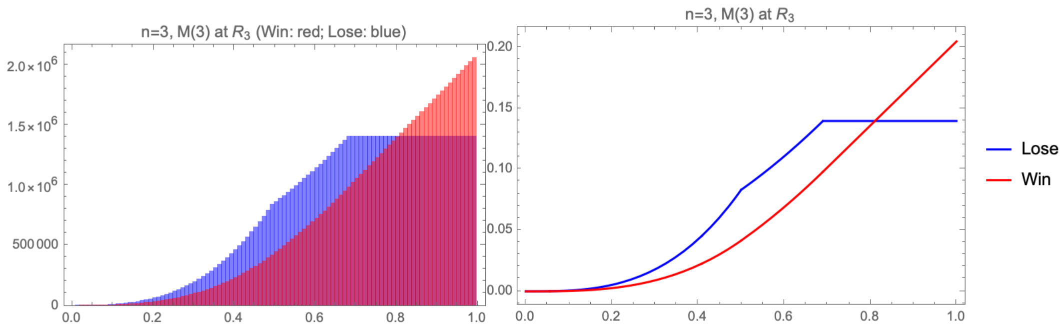

Figure 6: Wins and (previously unrevealed) losses at R_3